Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.4.2-a+b-cos-^m-c+d-cos-^n-A+B-cos+C-cos^2-

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1541]. This is test number [94].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1541)	% 0. (0)
Mathematica	% 99.55 (1534)	% 0.45 (7)
Maple	% 99.42 (1532)	% 0.58 (9)
Maxima	% 29.27 (451)	% 70.73 (1090)
Fricas	% 45.94 (708)	% 54.06 (833)
Sympy	% 7.79 (120)	% 92.21 (1421)
Giac	% 34.07 (525)	% 65.93 (1016)

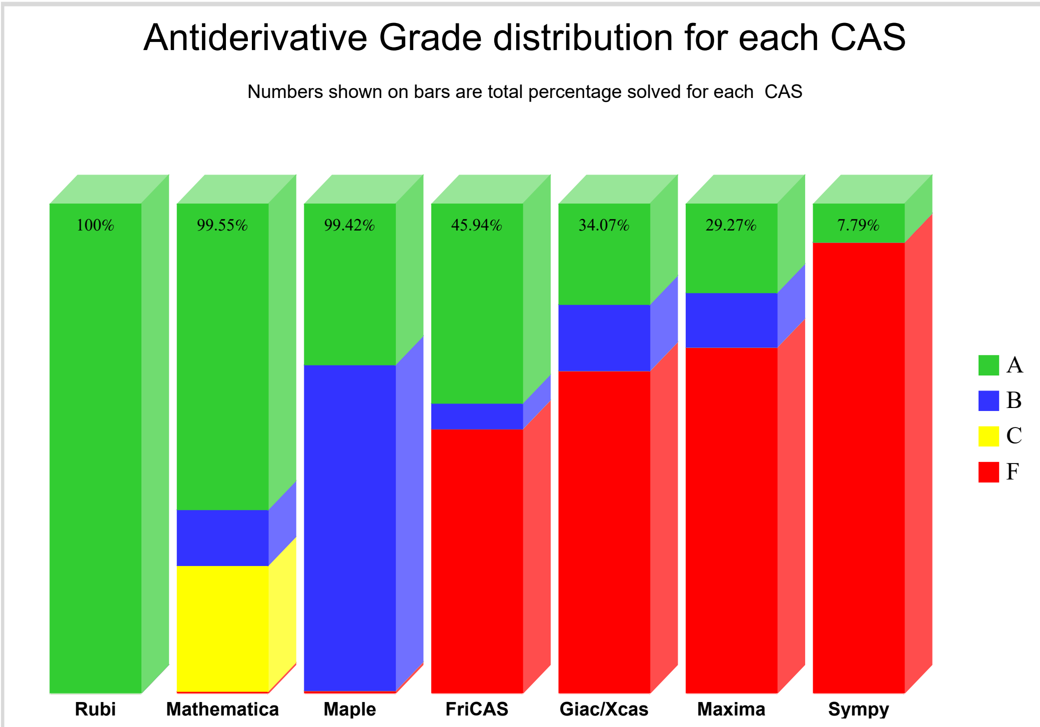
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

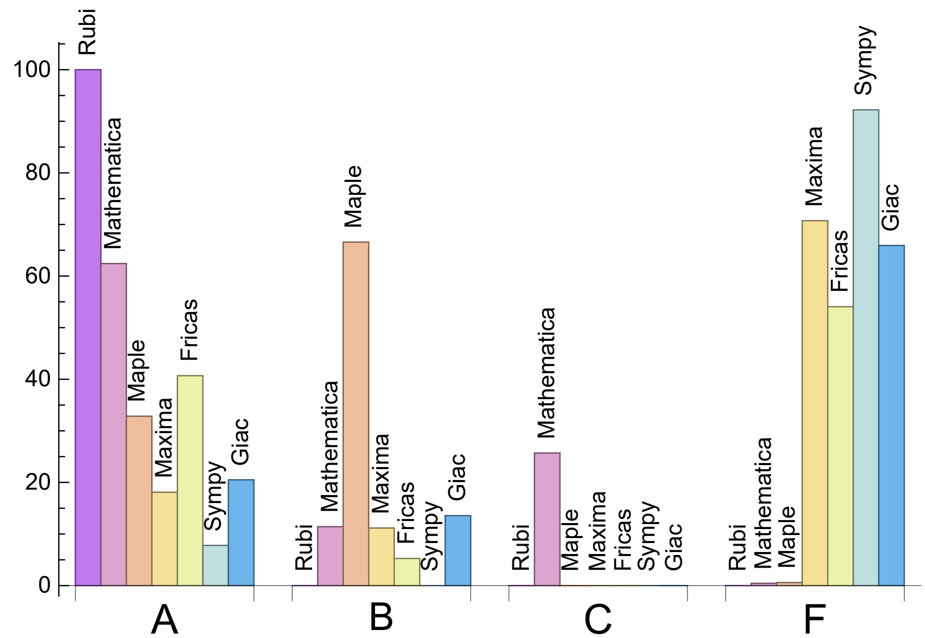
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	62.43	11.42	25.7	0.45
Maple	32.84	66.58	0.	0.58
Maxima	18.11	11.16	0.	70.73
Fricas	40.69	5.26	0.	54.06
Sympy	7.79	0.	0.	92.21
Giac	20.51	13.56	0.	65.93

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.79	258.93	1.	221.	1.
Mathematica	4.3	700.43	2.47	257.	1.08
Maple	0.88	1101.22	3.46	569.	2.79
Maxima	1.7	928.21	5.15	323.	2.19
Fricas	5.87	666.1	3.68	464.	2.66
Sympy	11.5	606.22	3.7	422.	2.65
Giac	1.68	455.15	2.51	309.	2.24

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {46, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 163, 164, 165, 170, 201, 202, 203, 204, 208, 209, 210, 276, 415, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 461, 462, 463, 464, 468, 469, 470, 471, 472, 474, 475, 506, 507, 508, 509, 514, 515, 516, 567, 568, 574, 577, 582, 584, 591, 592, 653, 654, 662, 668, 727, 730, 732, 733, 734, 737, 739, 740, 741, 742, 744, 745, 746, 747, 748, 750, 751, 754, 755, 760, 761, 762, 763, 764, 765, 768, 769, 834, 851, 852, 897, 899, 901, 903, 904, 905, 910, 911, 912, 913, 914, 916, 917, 918, 919, 920, 921, 922, 925, 927, 928, 932, 933, 934, 936, 937, 982, 990, 991, 999, 1007, 1020, 1027, 1028, 1036, 1037, 1044, 1046, 1051, 1052, 1058, 1059, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1141, 1142, 1145, 1146, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1158, 1159, 1175, 1202, 1203, 1232, 1233, 1234, 1235, 1239, 1240, 1241, 1242, 1337, 1338, 1339, 1341, 1343, 1344, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1391, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1426, 1427, 1428, 1429, 1430, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1482, 1486, 1487, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1510, 1511, 1512, 1513, 1515, 1517, 1518, 1519, 1523, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications.

Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

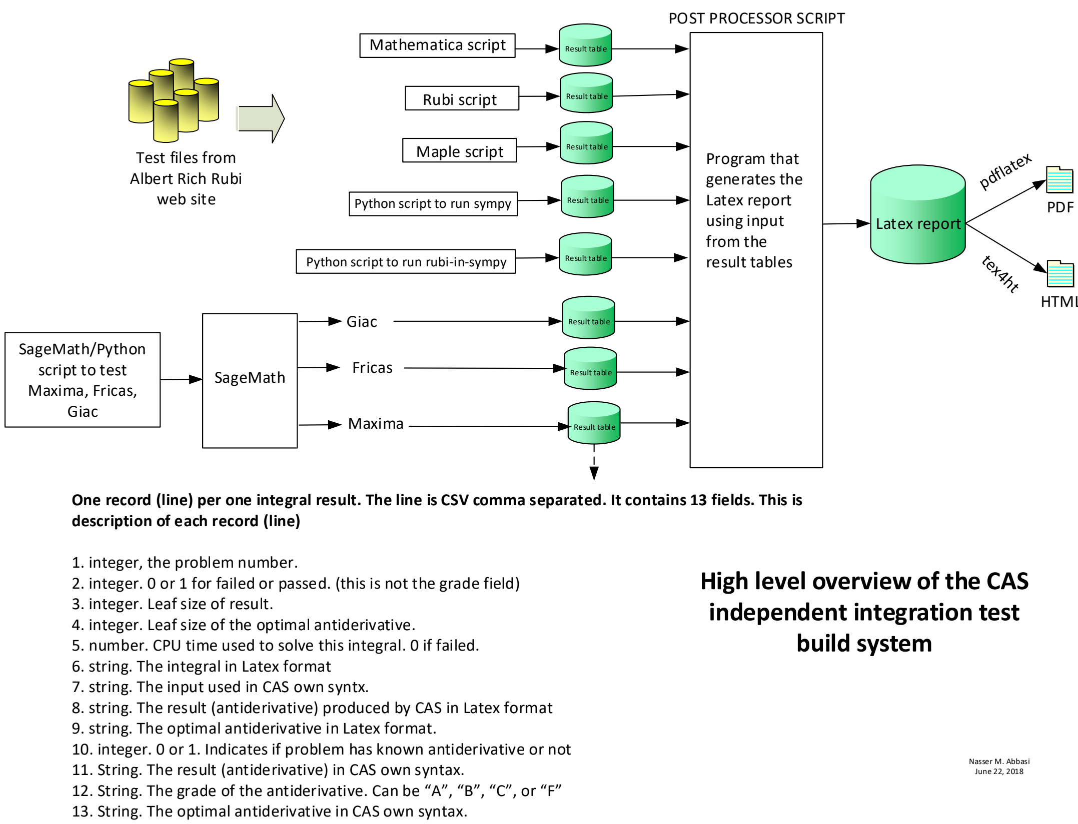
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507,

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1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 49, 59, 60, 61, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 249, 251, 252, 254, 263, 265, 272, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 318, 319, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 341, 359, 362, 363, 364, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 505, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 537, 538, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 570, 571, 572, 573, 575, 577, 578, 579, 580, 581, 588, 589, 591, 592, 593, 594, 595, 596, 597, 598, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 630, 631, 632, 638, 639, 640, 646, 647, 648, 649, 650, 651, 656, 657, 658, 659, 663, 664, 665, 666, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 730, 750, 752, 766, 767, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 791, 792, 793, 794, 795, 796, 797, 798, 799, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 820, 821, 822, 827, 828, 829, 835, 836, 837, 838, 841, 842, 843, 844, 847, 848, 849, 850, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 900, 901, 923, 924, 925, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 952, 953, 954, 955, 956, 957, 958, 959, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 985, 986, 987, 988, 989, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1002, 1004, 1005, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1021, 1022, 1023, 1029, 1030, 1031, 1038, 1039, 1040, 1041, 1042, 1047, 1048, 1049, 1053, 1054, 1055, 1056, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1144, 1156, 1157, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1236, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1342, 1345, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1392, 1393, 1394, 1398, 1402, 1403, 1404, 1405, 1409, 1410, 1412, 1413, 1414, 1415, 1420, 1425, 1427, 1432, 1436, 1437, 1438, 1442, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1483, 1484, 1485, 1488, 1489, 1490, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1504, 1506, 1508, 1509, 1516, 1517, 1520, 1528, 1529, 1532, 1535, 1539, 1540 }

B grade: { 14, 15, 22, 24, 33, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 71, 72, 240, 241, 248, 250, 253, 255, 256, 257, 258, 259, 260, 261, 262, 264, 266, 267, 268, 269, 270, 271, 275, 276, 314, 315, 316, 317, 324, 325, 326, 335, 336, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 365, 366, 369, 370, 536, 545, 555, 569, 576, 583, 584, 585, 586, 587, 599, 609, 610, 751, 768, 769, 790, 800, 881, 899, 951, 960, 983, 984, 1003, 1158, 1159, 1391, 1395, 1396, 1397, 1399, 1400, 1401, 1406, 1407, 1408, 1411, 1417, 1418, 1419, 1421, 1422, 1423, 1424, 1426, 1428, 1429, 1430, 1431, 1433, 1434, 1435, 1440, 1441, 1443, 1444, 1446, 1447, 1448, 1449, 1450, 1482, 1486, 1487, 1491, 1492, 1502, 1503, 1505, 1510, 1511, 1512, 1513, 1514, 1515, 1518, 1519, 1521, 1522, 1523, 1525, 1526, 1527, 1530, 1531, 1533, 1534, 1536, 1537, 1541 }

C grade: { 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 566, 567, 568, 574, 582, 590, 626, 627, 628, 629, 633, 634, 635, 636, 637, 641, 642, 643, 644, 645, 653, 654, 655, 661, 662, 668, 727, 728, 729, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 818, 819, 823, 824, 825, 826, 830, 831, 832, 833, 834, 839, 840, 845, 846, 851, 852, 897, 898, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 981, 982, 990, 998, 1001, 1006, 1017, 1018, 1019, 1020, 1024, 1025, 1026, 1027, 1028, 1032, 1033, 1034, 1035, 1036, 1037, 1044, 1045, 1046, 1051, 1052, 1058, 1059, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1231, 1232, 1233, 1234, 1235, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1337, 1338, 1339, 1341, 1343, 1344, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1416, 1439, 1445, 1507, 1524, 1538 }

F grade: { 652, 660, 667, 1043, 1050, 1057, 1340}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 44, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 92, 93, 94, 102, 103, 104, 105, 111, 112, 113, 119, 120, 126, 134, 135, 136, 142, 143, 144, 151, 152, 153, 154, 155, 158, 164, 165, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 199, 200, 209, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 286, 287, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 337, 338, 349, 350, 351, 356, 357, 358, 359, 360, 364, 365, 366, 367, 368, 369, 373, 374, 375, 376, 382, 383, 384, 391, 392, 393, 401, 402, 428, 439, 447, 448, 449, 456, 457, 458, 459, 463, 479, 480, 481, 482, 483, 489, 490, 491, 492, 499, 500, 501, 502, 505, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 565, 566, 604, 605, 606, 614, 619, 620, 652, 659, 660, 667, 671, 672, 708, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 797, 798, 817, 837, 838, 844, 845, 879, 880, 923, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 974, 975, 976, 1009, 1043, 1049, 1050, 1057, 1062, 1063, 1098, 1164, 1165, 1166, 1171, 1172, 1173, 1174, 1181, 1182, 1183, 1186, 1187, 1188, 1189, 1190, 1192, 1194, 1195, 1196, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1225, 1227, 1228, 1229, 1230, 1236, 1237, 1238, 1243, 1244, 1245, 1255, 1256, 1257, 1262, 1263, 1264, 1265, 1278, 1279, 1280, 1287, 1288, 1289, 1292, 1293, 1294, 1295, 1296, 1301, 1302, 1309, 1310, 1311, 1312, 1318, 1319, 1320, 1327, 1328, 1329, 1342, 1343, 1344, 1345, 1363, 1365, 1366, 1392, 1393, 1394, 1438, 1483, 1484, 1485 }

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465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 484, 485, 486, 487, 488, 493, 494, 495, 496, 497, 498, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 562, 563, 564, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 607, 608, 609, 610, 611, 612, 613, 615, 616, 617, 618, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 661, 662, 663, 664, 665, 666, 668, 669, 670, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 794, 795, 796, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 839, 840, 841, 842, 843, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 972, 973, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1044, 1045, 1046, 1047, 1048, 1051, 1052, 1053, 1054, 1055, 1056, 1058, 1059, 1060, 1061, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1160, 1161, 1162, 1163, 1167, 1168, 1169, 1170, 1175, 1176, 1177, 1178, 1179, 1180, 1184, 1185, 1191, 1193, 1197, 1198, 1207, 1215, 1224, 1226, 1231, 1232, 1233, 1234, 1235, 1239, 1240, 1241, 1242, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1258, 1259, 1260, 1261, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1281, 1282, 1283, 1284, 1285, 1286, 1290, 1291, 1297, 1298, 1299, 1300, 1303, 1304, 1305, 1306, 1307, 1308, 1313, 1314, 1315, 1316, 1317, 1321, 1322, 1323, 1324, 1325, 1326, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1364, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541 }

C grade: { }

F grade: { 85, 766, 767, 768, 769, 1156, 1157, 1158, 1159 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 41, 49, 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 92, 93, 94, 95, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 264, 265, 266, 269, 270, 271, 272, 273, 274, 277, 278, 279, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 331, 332, 333, 334, 358, 359, 360, 365, 366, 367, 368, 373, 374, 375, 376, 377, 382, 383, 384, 385, 391, 392, 393, 394, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 770, 771, 772, 773, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976

B grade: { 37, 39, 40, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 79, 80, 81, 87, 88, 90, 96, 97, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 221, 231, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 267, 268, 275, 276, 306, 316, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 369, 370, 371, 372, 378, 379, 380, 386, 387, 395, 477, 478, 479, 480, 481, 482, 483, 486, 487, 488, 489, 490, 491, 492, 496, 497, 498, 499, 500, 501, 502, 774, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1228, 1229, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1318, 1319, 1320, 1321, 1322, 1324, 1327, 1328, 1329, 1330, 1331 }

C grade: { }

F grade: { 82, 89, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 381, 388, 389, 390, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 484, 485, 493, 494, 495, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1217, 1226, 1227, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1317, 1323, 1325, 1326, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 125, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 408, 409, 410, 411, 412, 413, 416, 417, 418, 419, 420, 424, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 505, 506, 507, 508, 509, 514, 515, 516, 519, 520, 521, 522, 523, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 577, 581, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 610, 611, 614, 619, 620, 621, 770, 771, 772, 773, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 801, 804, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 984, 986, 987, 1009, 1010, 1011, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1346, 1347, 1348, 1349, 1350, 1353, 1354, 1355, 1356 }

B grade: { 5, 106, 107, 108, 115, 116, 122, 123, 124, 221, 222, 231, 267, 282, 296, 405, 406, 407, 414, 415, 421, 422, 423, 526, 572, 574, 575, 576, 578, 579, 580, 582, 583, 585, 586, 587, 588, 589, 590, 605, 606, 607, 608, 609, 612, 613, 615, 616, 617, 618, 622, 774, 799, 800, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 982, 983, 988, 989, 990, 991, 994, 995, 996, 997, 1001, 1002, 1003, 1004, 1005, 1012, 1013 }

C grade: { }

F grade: { 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 283, 284, 285, 286, 287, 288, 289, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 503, 504, 510, 511, 512, 513, 517, 518, 584, 591, 592, 597, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 985, 992, 993, 998, 999, 1000, 1006, 1007, 1008, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1343, 1344, 1345, 1351, 1352, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541 }

2.1.6 Sympy

A grade: { 1, 2, 3, 9, 10, 11, 18, 19, 20, 28, 29, 30, 39, 40, 41, 42, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 216, 217, 218, 219, 226, 227, 228, 235, 236, 244, 245, 253, 254, 255, 261, 262, 263, 264, 269, 270, 271, 272, 290, 291, 292, 293, 294, 301, 302, 303, 309, 310, 311, 318, 319, 320, 328, 329, 330, 339, 340, 341, 342, 347, 348, 349, 350, 355, 356, 357, 358, 359, 364, 365, 366, 367, 368, 522, 523, 524, 531, 532, 533, 540, 541, 549, 550, 559, 560, 561, 619, 770, 771, 778, 779, 786, 938, 939, 940, 947, 948, 955, 956, 964, 965, 974, 975, 976, 1009 }

B grade: { }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 52, 53, 54, 55, 61, 62, 63, 64, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 256, 257, 258, 259, 260, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 295, 296, 297, 298, 299, 300, 304, 305, 306, 307, 308, 312, 313, 314, 315, 316, 317, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 346, 351, 352, 353, 354, 360, 361, 362, 363, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 525, 526, 527, 528, 529, 530, 534, 535, 536, 537, 538, 539, 542, 543, 544, 545, 546, 547, 548, 551, 552, 553, 554, 555, 556, 557, 558, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726,

727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747,
748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768,
769, 772, 773, 774, 775, 776, 777, 780, 781, 782, 783, 784, 785, 787, 788, 789, 790, 791, 792, 793, 794,
795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815,
816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836,
837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857,
858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878,
879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899,
900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920,
921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 941, 942, 943, 944,
945, 946, 949, 950, 951, 952, 953, 954, 957, 958, 959, 960, 961, 962, 963, 966, 967, 968, 969, 970, 971,
972, 973, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995,
996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1011, 1012, 1013,
1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030,
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1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115,
1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132,
1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149,
1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166,
1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183,
1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200,
1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217,
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1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268,
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1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336,
1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353,
1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370,
1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387,
1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404,
1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421,
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1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506,
1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523,
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1541 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 86, 95, 102, 103, 104, 105, 106, 111, 112, 113, 114, 119, 120, 121, 122, 123, 216, 217, 218, 219, 226, 227, 228, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 290, 291, 292, 293, 294, 301, 302, 303, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 394, 401, 402, 403, 404, 410, 411, 412, 413, 418, 419, 420, 421, 422, 522, 523, 524, 531, 532, 533, 535, 536, 540, 541, 543, 549, 550, 553, 554, 559, 560, 561, 564, 565, 566, 567, 568, 570, 571, 573, 574, 576, 577, 579, 594, 595, 596, 597, 600, 601, 602, 603, 605, 606, 608, 609, 610, 612, 614, 616, 618, 620, 621, 622, 770, 771, 778, 779, 786, 789, 795, 796, 797, 798, 799, 801, 803, 804, 805, 938, 939, 940, 947, 948, 950, 955, 956, 964, 965, 969, 974, 975, 976, 979, 980, 981, 982, 986, 987, 989, 990, 992, 993, 1010, 1011 }

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1536, 1537, 1538, 1539, 1540, 1541 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	86	117	153	242	279	147
normalized size	1	1.	0.66	0.89	1.17	1.85	2.13	1.12
time (sec)	N/A	0.176	0.284	0.464	1.029	1.402	2.618	1.139

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	77	96	122	189	226	116
normalized size	1	1.	0.71	0.89	1.13	1.75	2.09	1.07
time (sec)	N/A	0.101	0.223	0.024	1.018	1.397	1.328	1.187

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	59	68	90	140	121	86
normalized size	1	1.	0.73	0.84	1.11	1.73	1.49	1.06
time (sec)	N/A	0.064	0.131	0.021	1.126	1.422	0.633	1.184

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	77	85	167	0	134
normalized size	1	1.	0.9	1.33	1.47	2.88	0.	2.31
time (sec)	N/A	0.109	0.081	0.063	1.117	1.47	0.	1.194

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	57	80	232	0	158
normalized size	1	1.	1.29	1.36	1.9	5.52	0.	3.76
time (sec)	N/A	0.102	0.024	0.085	1.113	1.434	0.	1.193

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	85	128	267	0	142
normalized size	1	1.	1.16	1.47	2.21	4.6	0.	2.45
time (sec)	N/A	0.125	0.03	0.089	1.262	1.36	0.	1.198

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	56	108	144	285	0	211
normalized size	1	1.	0.65	1.26	1.67	3.31	0.	2.45
time (sec)	N/A	0.167	0.255	0.087	1.06	1.495	0.	1.278

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	149	205	335	0	254
normalized size	1	1.	0.64	1.27	1.75	2.86	0.	2.17
time (sec)	N/A	0.19	0.388	0.098	1.054	1.481	0.	1.233

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	123	211	275	315	592	213
normalized size	1	1.	0.63	1.09	1.42	1.62	3.05	1.1
time (sec)	N/A	0.472	0.482	0.056	1.131	1.442	6.313	1.166

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	97	160	211	258	350	174
normalized size	1	1.	0.6	0.98	1.29	1.58	2.15	1.07
time (sec)	N/A	0.29	0.375	0.025	1.106	1.393	3.095	1.174

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	142	178	207	309	139
normalized size	1	1.	0.59	1.15	1.45	1.68	2.51	1.13
time (sec)	N/A	0.139	0.247	0.024	1.126	1.437	1.56	1.156

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	109	128	144	236	0	242
normalized size	1	1.	1.14	1.33	1.5	2.46	0.	2.52
time (sec)	N/A	0.298	0.22	0.055	1.012	1.453	0.	1.192

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	109	107	136	296	0	193
normalized size	1	1.	0.97	0.96	1.21	2.64	0.	1.72
time (sec)	N/A	0.391	0.394	0.066	1.053	1.486	0.	1.215

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	293	114	192	320	0	205
normalized size	1	1.	2.62	1.02	1.71	2.86	0.	1.83
time (sec)	N/A	0.359	2.138	0.071	1.085	1.504	0.	1.231

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	748	134	186	331	0	252
normalized size	1	1.	6.8	1.22	1.69	3.01	0.	2.29
time (sec)	N/A	0.354	6.385	0.07	1.087	1.566	0.	1.226

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	262	166	316	356	0	286
normalized size	1	1.	1.78	1.13	2.15	2.42	0.	1.95
time (sec)	N/A	0.449	1.14	0.076	1.195	1.463	0.	1.283

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	292	210	294	409	0	332
normalized size	1	1.	1.64	1.18	1.65	2.3	0.	1.87
time (sec)	N/A	0.474	1.398	0.095	1.099	1.527	0.	1.205

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	145	286	383	382	750	250
normalized size	1	1.	0.61	1.21	1.62	1.61	3.16	1.05
time (sec)	N/A	0.607	0.674	0.053	1.063	1.513	10.449	1.238

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	123	245	323	321	646	213
normalized size	1	1.	0.65	1.3	1.72	1.71	3.44	1.13
time (sec)	N/A	0.334	0.412	0.027	1.05	1.437	6.472	1.239

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	97	197	257	266	422	177
normalized size	1	1.	0.66	1.33	1.74	1.8	2.85	1.2
time (sec)	N/A	0.198	0.364	0.025	1.023	1.427	3.251	1.224

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	124	175	220	284	0	288
normalized size	1	1.	0.84	1.19	1.5	1.93	0.	1.96
time (sec)	N/A	0.439	0.324	0.06	1.035	1.493	0.	1.319

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	298	146	185	350	0	284
normalized size	1	1.	2.06	1.01	1.28	2.41	0.	1.96
time (sec)	N/A	0.452	1.922	0.07	1.045	1.56	0.	1.257

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	214	151	236	365	0	311
normalized size	1	1.	1.34	0.94	1.48	2.28	0.	1.94
time (sec)	N/A	0.48	1.985	0.075	1.043	1.717	0.	1.289

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	832	152	239	379	0	296
normalized size	1	1.	5.33	0.97	1.53	2.43	0.	1.9
time (sec)	N/A	0.5	6.341	0.077	1.045	1.499	0.	1.219

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	334	180	347	386	0	300
normalized size	1	1.	1.98	1.07	2.05	2.28	0.	1.78
time (sec)	N/A	0.496	1.422	0.078	1.012	1.468	0.	1.264

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	294	212	394	419	0	332
normalized size	1	1.	1.52	1.09	2.03	2.16	0.	1.71
time (sec)	N/A	0.572	1.415	0.08	1.08	1.505	0.	1.271

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	358	257	516	474	0	378
normalized size	1	1.	1.59	1.14	2.29	2.11	0.	1.68
time (sec)	N/A	0.618	1.943	0.114	1.008	1.503	0.	1.267

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	167	393	531	448	1149	285
normalized size	1	1.	0.6	1.41	1.9	1.61	4.12	1.02
time (sec)	N/A	0.793	0.929	0.057	1.069	1.535	20.364	1.244

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	145	322	431	377	799	250
normalized size	1	1.	0.66	1.47	1.97	1.72	3.65	1.14
time (sec)	N/A	0.412	0.574	0.032	1.027	1.469	11.455	1.23

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	119	284	369	319	707	213
normalized size	1	1.	0.66	1.59	2.06	1.78	3.95	1.19
time (sec)	N/A	0.232	0.376	0.026	1.034	1.423	6.307	1.225

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	147	221	300	351	0	335
normalized size	1	1.	0.83	1.25	1.69	1.98	0.	1.89
time (sec)	N/A	0.539	0.468	0.066	1.028	1.596	0.	1.323

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	338	191	262	408	0	329
normalized size	1	1.	1.87	1.06	1.45	2.25	0.	1.82
time (sec)	N/A	0.603	2.13	0.08	1.065	1.483	0.	1.271

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	756	190	285	432	0	335
normalized size	1	1.	4.06	1.02	1.53	2.32	0.	1.8
time (sec)	N/A	0.608	6.226	0.086	1.02	1.571	0.	1.296

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	386	189	285	425	0	335
normalized size	1	1.	1.95	0.95	1.44	2.15	0.	1.69
time (sec)	N/A	0.686	6.209	0.089	1.074	1.601	0.	1.283

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	350	197	400	437	0	342
normalized size	1	1.	1.75	0.98	2.	2.18	0.	1.71
time (sec)	N/A	0.669	2.116	0.086	1.076	1.622	0.	1.406

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	389	226	425	452	0	347
normalized size	1	1.	1.88	1.09	2.05	2.18	0.	1.68
time (sec)	N/A	0.665	1.792	0.089	1.028	1.583	0.	1.265

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	358	258	616	471	0	378
normalized size	1	1.	1.54	1.11	2.66	2.03	0.	1.63
time (sec)	N/A	0.761	2.123	0.092	1.066	1.521	0.	1.283

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	390	303	624	531	0	424
normalized size	1	1.	1.48	1.15	2.37	2.02	0.	1.61
time (sec)	N/A	0.802	3.103	0.115	1.055	1.478	0.	1.288

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	283	352	474	290	1795	243
normalized size	1	1.	1.81	2.26	3.04	1.86	11.51	1.56
time (sec)	N/A	0.191	0.53	0.057	1.531	1.465	12.583	1.179

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	225	280	363	243	1163	205
normalized size	1	1.	1.81	2.26	2.93	1.96	9.38	1.65
time (sec)	N/A	0.173	0.593	0.032	1.579	1.413	7.031	1.217

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	159	144	248	192	665	130
normalized size	1	1.	1.62	1.47	2.53	1.96	6.79	1.33
time (sec)	N/A	0.094	0.332	0.03	1.529	1.397	3.787	1.181

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	108	88	158	132	202	100
normalized size	1	1.	2.25	1.83	3.29	2.75	4.21	2.08
time (sec)	N/A	0.097	0.243	0.028	1.546	1.334	2.044	1.17

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	114	98	169	242	0	108
normalized size	1	1.	2.38	2.04	3.52	5.04	0.	2.25
time (sec)	N/A	0.104	0.317	0.05	1.568	1.371	0.	1.23

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	229	121	194	288	0	136
normalized size	1	1.	3.75	1.98	3.18	4.72	0.	2.23
time (sec)	N/A	0.139	1.658	0.059	1.003	1.416	0.	1.281

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	284	209	323	377	0	176
normalized size	1	1.	2.7	1.99	3.08	3.59	0.	1.68
time (sec)	N/A	0.176	2.62	0.06	1.024	1.504	0.	1.255

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	765	294	439	429	0	250
normalized size	1	1.	5.75	2.21	3.3	3.23	0.	1.88
time (sec)	N/A	0.183	6.465	0.062	1.009	1.494	0.	1.25

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	399	392	560	432	2161	297
normalized size	1	1.	2.09	2.05	2.93	2.26	11.31	1.55
time (sec)	N/A	0.338	0.779	0.034	1.559	1.418	34.681	1.181

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	341	322	439	369	1426	258
normalized size	1	1.	2.09	1.98	2.69	2.26	8.75	1.58
time (sec)	N/A	0.327	0.652	0.033	1.567	1.465	19.718	1.316

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	273	184	319	332	845	185
normalized size	1	1.	1.94	1.3	2.26	2.35	5.99	1.31
time (sec)	N/A	0.263	0.629	0.031	1.516	1.396	11.88	1.284

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	195	130	223	261	335	154
normalized size	1	1.	2.17	1.44	2.48	2.9	3.72	1.71
time (sec)	N/A	0.241	0.519	0.03	1.521	1.329	6.214	1.257

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	141	97	161	228	104	113
normalized size	1	1.	2.14	1.47	2.44	3.45	1.58	1.71
time (sec)	N/A	0.125	0.324	0.023	1.531	1.368	3.234	1.295

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	166	119	197	338	0	151
normalized size	1	1.	2.16	1.55	2.56	4.39	0.	1.96
time (sec)	N/A	0.227	0.554	0.051	1.034	1.391	0.	1.235

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	288	164	258	429	0	192
normalized size	1	1.	3.16	1.8	2.84	4.71	0.	2.11
time (sec)	N/A	0.296	1.394	0.059	1.028	1.415	0.	1.3

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	484	249	389	554	0	231
normalized size	1	1.	3.32	1.71	2.66	3.79	0.	1.58
time (sec)	N/A	0.316	3.121	0.069	1.046	1.47	0.	1.365

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	594	338	512	591	0	304
normalized size	1	1.	3.45	1.97	2.98	3.44	0.	1.77
time (sec)	N/A	0.335	5.064	0.065	1.078	1.701	0.	1.3

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	463	362	493	525	1586	308
normalized size	1	1.	2.14	1.68	2.28	2.43	7.34	1.43
time (sec)	N/A	0.485	0.853	0.036	1.593	1.741	55.932	1.233

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	393	224	373	478	967	235
normalized size	1	1.	2.08	1.19	1.97	2.53	5.12	1.24
time (sec)	N/A	0.46	0.66	0.034	1.559	1.685	29.494	1.31

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	283	170	277	386	422	204
normalized size	1	1.	2.08	1.25	2.04	2.84	3.1	1.5
time (sec)	N/A	0.463	0.905	0.031	1.49	1.644	16.142	1.299

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	227	117	189	351	128	140
normalized size	1	1.	1.99	1.03	1.66	3.08	1.12	1.23
time (sec)	N/A	0.256	0.524	0.027	1.511	1.614	9.563	1.253

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	129	88	181	221	136	120
normalized size	1	1.	1.32	0.9	1.85	2.26	1.39	1.22
time (sec)	N/A	0.121	0.276	0.021	1.025	1.319	5.947	1.248

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	203	139	225	481	0	177
normalized size	1	1.	1.77	1.21	1.96	4.18	0.	1.54
time (sec)	N/A	0.315	1.046	0.053	1.062	1.423	0.	1.286

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	596	204	315	576	0	240
normalized size	1	1.	4.62	1.58	2.44	4.47	0.	1.86
time (sec)	N/A	0.443	6.307	0.061	1.034	1.464	0.	1.333

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	597	289	446	740	0	279
normalized size	1	1.	3.11	1.51	2.32	3.85	0.	1.45
time (sec)	N/A	0.513	4.6	0.071	1.036	1.371	0.	1.348

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	798	378	568	786	0	352
normalized size	1	1.	3.55	1.68	2.52	3.49	0.	1.56
time (sec)	N/A	0.554	6.454	0.072	1.063	1.539	0.	1.258

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	513	264	429	632	1086	279
normalized size	1	1.	2.3	1.18	1.92	2.83	4.87	1.25
time (sec)	N/A	0.612	1.041	0.031	1.6	1.513	71.126	1.249

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	371	210	332	516	462	248
normalized size	1	1.	2.13	1.21	1.91	2.97	2.66	1.43
time (sec)	N/A	0.577	0.729	0.031	1.66	1.417	40.214	1.162

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	315	177	271	477	192	208
normalized size	1	1.	2.07	1.16	1.78	3.14	1.26	1.37
time (sec)	N/A	0.445	0.709	0.027	1.566	1.382	24.123	1.215

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	179	90	236	309	182	158
normalized size	1	1.	1.3	0.65	1.71	2.24	1.32	1.14
time (sec)	N/A	0.349	0.392	0.026	1.028	1.338	16.402	1.162

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	159	88	236	308	178	158
normalized size	1	1.	1.17	0.65	1.74	2.26	1.31	1.16
time (sec)	N/A	0.171	0.355	0.023	1.027	1.275	11.398	1.153

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	245	199	308	624	0	246
normalized size	1	1.	1.69	1.37	2.12	4.3	0.	1.7
time (sec)	N/A	0.466	1.716	0.058	1.117	1.48	0.	1.212

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	680	244	370	722	0	286
normalized size	1	1.	4.22	1.52	2.3	4.48	0.	1.78
time (sec)	N/A	0.616	6.412	0.066	1.149	1.448	0.	1.263

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	784	329	502	923	0	325
normalized size	1	1.	3.5	1.47	2.24	4.12	0.	1.45
time (sec)	N/A	0.665	6.479	0.074	1.126	1.456	0.	1.267

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	361	418	622	965	0	398
normalized size	1	1.	1.4	1.63	2.42	3.75	0.	1.55
time (sec)	N/A	0.706	4.286	0.077	1.167	1.591	0.	1.254

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	114	135	216	308	0	0
normalized size	1	1.	0.51	0.61	0.97	1.38	0.	0.
time (sec)	N/A	0.506	0.937	0.272	2.125	1.312	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	92	116	177	257	0	0
normalized size	1	1.	0.51	0.64	0.98	1.43	0.	0.
time (sec)	N/A	0.42	0.484	0.049	2.232	1.46	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	74	97	139	207	0	0
normalized size	1	1.	0.54	0.71	1.01	1.51	0.	0.
time (sec)	N/A	0.305	0.279	0.044	2.068	1.416	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	58	78	97	159	0	0
normalized size	1	1.	0.61	0.82	1.02	1.67	0.	0.
time (sec)	N/A	0.128	0.112	0.042	2.001	1.405	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	82	248	50	375	0	236
normalized size	1	1.	0.85	2.58	0.52	3.91	0.	2.46
time (sec)	N/A	0.259	0.142	0.205	1.951	1.45	0.	4.003

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	436	987	413	0	374
normalized size	1	1.	0.97	4.64	10.5	4.39	0.	3.98
time (sec)	N/A	0.296	0.226	0.102	2.132	1.501	0.	2.535

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	103	943	3568	450	0	463
normalized size	1	1.	0.94	8.57	32.44	4.09	0.	4.21
time (sec)	N/A	0.311	0.38	0.105	21.622	1.589	0.	2.653

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	115	1311	4169	495	0	861
normalized size	1	1.	0.75	8.57	27.25	3.24	0.	5.63
time (sec)	N/A	0.388	1.058	0.11	3.203	1.681	0.	2.723

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	145	1631	0	551	0	1083
normalized size	1	1.	0.74	8.32	0.	2.81	0.	5.53
time (sec)	N/A	0.471	1.836	0.116	0.	1.837	0.	2.774

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	115	137	230	332	0	0
normalized size	1	1.	0.51	0.61	1.02	1.48	0.	0.
time (sec)	N/A	0.638	0.912	0.044	2.044	1.407	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	93	118	186	275	0	0
normalized size	1	1.	0.53	0.68	1.07	1.58	0.	0.
time (sec)	N/A	0.364	0.541	0.043	1.935	1.35	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	75	0	146	223	0	0
normalized size	1	1.	0.57	0.	1.11	1.69	0.	0.
time (sec)	N/A	0.175	0.244	180.	1.906	1.365	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	95	307	73	429	0	306
normalized size	1	1.	0.71	2.31	0.55	3.23	0.	2.3
time (sec)	N/A	0.41	0.349	0.076	1.727	1.54	0.	4.062

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	106	474	1828	463	0	423
normalized size	1	1.	0.78	3.49	13.44	3.4	0.	3.11
time (sec)	N/A	0.441	0.341	0.089	1.953	1.73	0.	3.969

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	118	1018	2734	491	0	512
normalized size	1	1.	0.8	6.93	18.6	3.34	0.	3.48
time (sec)	N/A	0.467	0.575	0.094	2.219	1.936	0.	3.023

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	124	1311	0	516	0	861
normalized size	1	1.	0.8	8.46	0.	3.33	0.	5.55
time (sec)	N/A	0.517	0.883	0.094	0.	1.968	0.	3.052

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	152	1630	9430	566	0	1081
normalized size	1	1.	0.76	8.15	47.15	2.83	0.	5.4
time (sec)	N/A	0.607	1.451	0.099	7.622	2.255	0.	3.107

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	174	1951	0	635	0	1304
normalized size	1	1.	0.71	7.96	0.	2.59	0.	5.32
time (sec)	N/A	0.684	2.223	0.118	0.	2.262	0.	3.143

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	138	156	301	427	0	0
normalized size	1	1.	0.51	0.57	1.1	1.56	0.	0.
time (sec)	N/A	0.861	1.275	0.102	2.193	1.676	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	117	137	255	339	0	0
normalized size	1	1.	0.55	0.65	1.21	1.61	0.	0.
time (sec)	N/A	0.41	0.865	0.043	2.08	1.638	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	95	118	209	290	0	0
normalized size	1	1.	0.56	0.7	1.24	1.72	0.	0.
time (sec)	N/A	0.208	0.466	0.043	2.179	1.637	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	115	346	105	500	0	348
normalized size	1	1.	0.68	2.04	0.62	2.94	0.	2.05
time (sec)	N/A	0.567	0.526	0.084	1.933	1.729	0.	3.828

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	127	533	11036	529	0	478
normalized size	1	1.	0.73	3.08	63.79	3.06	0.	2.76
time (sec)	N/A	0.589	0.593	0.087	3.49	1.768	0.	2.865

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	137	1052	4952	547	0	544
normalized size	1	1.	0.74	5.72	26.91	2.97	0.	2.96
time (sec)	N/A	0.633	0.763	0.093	22.417	1.971	0.	3.094

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	142	1337	0	560	0	909
normalized size	1	1.	0.74	6.96	0.	2.92	0.	4.73
time (sec)	N/A	0.659	1.122	0.098	0.	2.079	0.	3.294

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	153	1630	0	594	0	1083
normalized size	1	1.	0.76	8.15	0.	2.97	0.	5.42
time (sec)	N/A	0.717	1.646	0.103	0.	2.195	0.	3.475

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	176	1951	0	655	0	1304
normalized size	1	1.	0.72	7.96	0.	2.67	0.	5.32
time (sec)	N/A	0.799	2.046	0.11	0.	2.242	0.	3.588

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	198	2271	0	711	0	1526
normalized size	1	1.	0.68	7.83	0.	2.45	0.	5.26
time (sec)	N/A	0.907	2.718	0.13	0.	2.3	0.	3.814

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	121	340	0	532	0	306
normalized size	1	1.	0.51	1.44	0.	2.25	0.	1.3
time (sec)	N/A	0.817	0.613	0.079	0.	1.72	0.	1.9

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	89	253	0	485	0	213
normalized size	1	1.	0.46	1.31	0.	2.51	0.	1.1
time (sec)	N/A	0.56	0.321	0.061	0.	1.702	0.	1.766

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	87	247	0	436	0	223
normalized size	1	1.	0.57	1.62	0.	2.87	0.	1.47
time (sec)	N/A	0.331	0.232	0.061	0.	1.691	0.	1.744

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	63	173	0	392	0	116
normalized size	1	1.	0.58	1.59	0.	3.6	0.	1.06
time (sec)	N/A	0.141	0.104	0.054	0.	1.621	0.	1.792

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	295	0	613	0	258
normalized size	1	1.	0.72	2.57	0.	5.33	0.	2.24
time (sec)	N/A	0.292	0.254	0.099	0.	1.813	0.	2.85

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	89	554	0	676	0	392
normalized size	1	1.	0.79	4.9	0.	5.98	0.	3.47
time (sec)	N/A	0.315	0.331	0.106	0.	1.812	0.	2.967

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	113	1192	0	741	0	536
normalized size	1	1.	0.71	7.5	0.	4.66	0.	3.37
time (sec)	N/A	0.49	0.726	0.117	0.	2.144	0.	3.092

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	131	1645	0	790	0	933
normalized size	1	1.	0.66	8.22	0.	3.95	0.	4.66
time (sec)	N/A	0.649	1.326	0.125	0.	2.19	0.	3.179

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	174	2049	0	852	0	1154
normalized size	1	1.	0.72	8.43	0.	3.51	0.	4.75
time (sec)	N/A	0.843	1.97	0.129	0.	2.512	0.	3.194

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	157	442	0	637	0	343
normalized size	1	1.	0.61	1.71	0.	2.46	0.	1.32
time (sec)	N/A	0.792	0.999	0.071	0.	1.785	0.	2.038

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	136	362	0	575	0	271
normalized size	1	1.	0.64	1.69	0.	2.69	0.	1.27
time (sec)	N/A	0.595	0.641	0.066	0.	1.743	0.	1.995

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	94	292	0	533	0	225
normalized size	1	1.	0.56	1.73	0.	3.15	0.	1.33
time (sec)	N/A	0.341	0.606	0.065	0.	1.683	0.	1.968

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	77	254	0	485	0	174
normalized size	1	1.	0.68	2.23	0.	4.25	0.	1.53
time (sec)	N/A	0.144	0.447	0.061	0.	1.742	0.	1.789

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	129	373	0	745	0	288
normalized size	1	1.	1.03	2.98	0.	5.96	0.	2.3
time (sec)	N/A	0.32	0.74	0.106	0.	1.891	0.	3.205

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	167	746	0	830	0	468
normalized size	1	1.	1.06	4.72	0.	5.25	0.	2.96
time (sec)	N/A	0.491	2.211	0.115	0.	1.881	0.	3.352

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	211	1540	0	927	0	612
normalized size	1	1.	0.97	7.1	0.	4.27	0.	2.82
time (sec)	N/A	0.709	3.042	0.124	0.	2.298	0.	3.537

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	205	2028	0	980	0	1010
normalized size	1	1.	0.77	7.62	0.	3.68	0.	3.8
time (sec)	N/A	0.881	3.866	0.133	0.	2.34	0.	3.657

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	129	432	0	725	0	346
normalized size	1	1.	0.5	1.67	0.	2.8	0.	1.34
time (sec)	N/A	0.804	1.303	0.071	0.	1.752	0.	2.351

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	112	362	0	671	0	274
normalized size	1	1.	0.53	1.71	0.	3.17	0.	1.29
time (sec)	N/A	0.606	0.864	0.067	0.	1.713	0.	2.306

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	95	327	0	610	0	240
normalized size	1	1.	0.58	1.98	0.	3.7	0.	1.45
time (sec)	N/A	0.367	0.7	0.071	0.	1.916	0.	2.181

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	292	0	589	0	180
normalized size	1	1.	0.72	2.35	0.	4.75	0.	1.45
time (sec)	N/A	0.16	0.455	0.066	0.	1.858	0.	1.931

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	124	445	0	899	0	336
normalized size	1	1.	0.77	2.75	0.	5.55	0.	2.07
time (sec)	N/A	0.481	1.686	0.111	0.	1.997	0.	3.362

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	185	815	0	1002	0	518
normalized size	1	1.	0.93	4.1	0.	5.04	0.	2.6
time (sec)	N/A	0.698	4.446	0.127	0.	2.009	0.	3.881

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	408	1610	0	1106	0	662
normalized size	1	1.	1.56	6.15	0.	4.22	0.	2.53
time (sec)	N/A	0.913	6.17	0.141	0.	2.468	0.	4.467

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	964	434	0	0	0	0
normalized size	1	1.	4.92	2.21	0.	0.	0.	0.
time (sec)	N/A	0.236	6.335	0.128	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	918	406	0	0	0	0
normalized size	1	1.	5.56	2.46	0.	0.	0.	0.
time (sec)	N/A	0.214	6.268	0.078	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	872	378	0	0	0	0
normalized size	1	1.	6.51	2.82	0.	0.	0.	0.
time (sec)	N/A	0.186	6.245	0.075	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	824	345	0	0	0	0
normalized size	1	1.	8.16	3.42	0.	0.	0.	0.
time (sec)	N/A	0.164	6.266	0.072	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	813	458	0	0	0	0
normalized size	1	1.	8.56	4.82	0.	0.	0.	0.
time (sec)	N/A	0.168	6.331	0.09	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	817	437	0	0	0	0
normalized size	1	1.	8.6	4.6	0.	0.	0.	0.
time (sec)	N/A	0.172	6.332	0.147	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	851	729	0	0	0	0
normalized size	1	1.	6.45	5.52	0.	0.	0.	0.
time (sec)	N/A	0.199	6.413	0.202	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	895	838	0	0	0	0
normalized size	1	1.	5.42	5.08	0.	0.	0.	0.
time (sec)	N/A	0.218	6.473	0.307	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	982	436	0	0	0	0
normalized size	1	1.	4.27	1.9	0.	0.	0.	0.
time (sec)	N/A	0.478	6.287	0.083	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	936	408	0	0	0	0
normalized size	1	1.	4.75	2.07	0.	0.	0.	0.
time (sec)	N/A	0.433	6.267	0.084	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	890	380	0	0	0	0
normalized size	1	1.	5.43	2.32	0.	0.	0.	0.
time (sec)	N/A	0.419	6.321	0.086	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	658	440	0	0	0	0
normalized size	1	1.	4.11	2.75	0.	0.	0.	0.
time (sec)	N/A	0.414	6.372	0.091	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	865	651	0	0	0	0
normalized size	1	1.	5.54	4.17	0.	0.	0.	0.
time (sec)	N/A	0.425	6.44	0.175	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	656	756	0	0	0	0
normalized size	1	1.	4.21	4.85	0.	0.	0.	0.
time (sec)	N/A	0.436	6.479	0.22	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	913	918	0	0	0	0
normalized size	1	1.	4.63	4.66	0.	0.	0.	0.
time (sec)	N/A	0.474	6.566	0.273	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	955	1168	0	0	0	0
normalized size	1	1.	4.15	5.08	0.	0.	0.	0.
time (sec)	N/A	0.507	6.646	0.34	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	1028	464	0	0	0	0
normalized size	1	1.	3.68	1.66	0.	0.	0.	0.
time (sec)	N/A	0.656	6.333	0.099	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	982	436	0	0	0	0
normalized size	1	1.	3.99	1.77	0.	0.	0.	0.
time (sec)	N/A	0.604	6.305	0.098	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	936	408	0	0	0	0
normalized size	1	1.	4.39	1.92	0.	0.	0.	0.
time (sec)	N/A	0.58	6.378	0.095	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	926	569	0	0	0	0
normalized size	1	1.	4.27	2.62	0.	0.	0.	0.
time (sec)	N/A	0.587	6.481	0.106	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	909	704	0	0	0	0
normalized size	1	1.	4.31	3.34	0.	0.	0.	0.
time (sec)	N/A	0.59	6.537	0.115	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	905	939	0	0	0	0
normalized size	1	1.	4.25	4.41	0.	0.	0.	0.
time (sec)	N/A	0.585	6.576	0.273	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	920	1012	0	0	0	0
normalized size	1	1.	4.32	4.75	0.	0.	0.	0.
time (sec)	N/A	0.613	6.68	0.374	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	955	1246	0	0	0	0
normalized size	1	1.	3.88	5.07	0.	0.	0.	0.
time (sec)	N/A	0.636	6.723	0.394	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	997	1408	0	0	0	0
normalized size	1	1.	3.57	5.05	0.	0.	0.	0.
time (sec)	N/A	0.67	6.803	0.463	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	1219	295	0	0	0	0
normalized size	1	1.	6.35	1.54	0.	0.	0.	0.
time (sec)	N/A	0.216	6.624	0.112	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	1170	276	0	0	0	0
normalized size	1	1.	7.36	1.74	0.	0.	0.	0.
time (sec)	N/A	0.188	6.566	0.109	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	1126	262	0	0	0	0
normalized size	1	1.	9.23	2.15	0.	0.	0.	0.
time (sec)	N/A	0.172	6.495	0.11	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	1095	247	0	0	0	0
normalized size	1	1.	13.19	2.98	0.	0.	0.	0.
time (sec)	N/A	0.154	6.473	0.118	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	1128	316	0	0	0	0
normalized size	1	1.	9.98	2.8	0.	0.	0.	0.
time (sec)	N/A	0.173	6.611	0.196	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	1163	486	0	0	0	0
normalized size	1	1.	7.75	3.24	0.	0.	0.	0.
time (sec)	N/A	0.193	6.897	0.387	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	1207	803	0	0	0	0
normalized size	1	1.	6.29	4.18	0.	0.	0.	0.
time (sec)	N/A	0.212	7.095	0.365	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	1248	451	0	0	0	0
normalized size	1	1.	6.37	2.3	0.	0.	0.	0.
time (sec)	N/A	0.351	6.762	0.133	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	1209	437	0	0	0	0
normalized size	1	1.	7.51	2.71	0.	0.	0.	0.
time (sec)	N/A	0.333	6.671	0.138	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	814	348	0	0	0	0
normalized size	1	1.	6.46	2.76	0.	0.	0.	0.
time (sec)	N/A	0.287	6.537	0.137	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	1176	419	0	0	0	0
normalized size	1	1.	9.41	3.35	0.	0.	0.	0.
time (sec)	N/A	0.299	6.57	0.133	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	834	452	0	0	0	0
normalized size	1	1.	5.38	2.92	0.	0.	0.	0.
time (sec)	N/A	0.323	6.633	0.152	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	1245	738	0	0	0	0
normalized size	1	1.	6.59	3.9	0.	0.	0.	0.
time (sec)	N/A	0.352	7.186	0.357	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	1333	479	0	0	0	0
normalized size	1	1.	5.33	1.92	0.	0.	0.	0.
time (sec)	N/A	0.541	7.021	0.144	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	1296	465	0	0	0	0
normalized size	1	1.	6.2	2.22	0.	0.	0.	0.
time (sec)	N/A	0.495	6.871	0.142	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	1271	451	0	0	0	0
normalized size	1	1.	7.14	2.53	0.	0.	0.	0.
time (sec)	N/A	0.48	6.764	0.148	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	1259	451	0	0	0	0
normalized size	1	1.	6.99	2.51	0.	0.	0.	0.
time (sec)	N/A	0.469	6.675	0.148	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	1265	451	0	0	0	0
normalized size	1	1.	6.88	2.45	0.	0.	0.	0.
time (sec)	N/A	0.482	6.671	0.15	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	1301	685	0	0	0	0
normalized size	1	1.	5.94	3.13	0.	0.	0.	0.
time (sec)	N/A	0.524	6.885	0.17	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	1331	876	0	0	0	0
normalized size	1	1.	5.5	3.62	0.	0.	0.	0.
time (sec)	N/A	0.524	7.427	0.19	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	129	434	9933	416	0	0
normalized size	1	1.	0.6	2.03	46.42	1.94	0.	0.
time (sec)	N/A	0.475	0.818	0.228	3.473	2.851	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	112	362	3663	360	0	0
normalized size	1	1.	0.66	2.14	21.67	2.13	0.	0.
time (sec)	N/A	0.387	0.49	0.187	2.872	2.366	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	98	202	1629	316	0	0
normalized size	1	1.	0.79	1.63	13.14	2.55	0.	0.
time (sec)	N/A	0.314	0.273	0.131	2.309	2.246	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	100	166	1202	324	0	0
normalized size	1	1.	0.85	1.42	10.27	2.77	0.	0.
time (sec)	N/A	0.32	0.244	0.135	2.086	2.098	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	90	127	458	338	0	0
normalized size	1	1.	0.78	1.09	3.95	2.91	0.	0.
time (sec)	N/A	0.309	0.225	0.124	1.946	2.158	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	77	454	211	0	0
normalized size	1	1.	0.59	0.63	3.69	1.72	0.	0.
time (sec)	N/A	0.316	0.276	0.107	1.698	1.768	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	101	99	641	261	0	0
normalized size	1	1.	0.6	0.59	3.82	1.55	0.	0.
time (sec)	N/A	0.408	0.44	0.112	1.699	1.376	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	124	121	765	308	0	0
normalized size	1	1.	0.58	0.57	3.59	1.45	0.	0.
time (sec)	N/A	0.463	0.642	0.127	1.73	1.493	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	147	507	6035	505	0	0
normalized size	1	1.	0.55	1.91	22.77	1.91	0.	0.
time (sec)	N/A	0.695	1.516	0.144	3.676	2.342	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	128	435	10855	435	0	0
normalized size	1	1.	0.59	2.	49.79	2.	0.	0.
time (sec)	N/A	0.626	0.862	0.115	3.892	2.396	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	113	363	3707	386	0	0
normalized size	1	1.	0.66	2.12	21.68	2.26	0.	0.
time (sec)	N/A	0.518	0.561	0.169	2.898	1.941	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	119	325	2805	401	0	0
normalized size	1	1.	0.68	1.86	16.03	2.29	0.	0.
time (sec)	N/A	0.529	0.547	0.175	2.506	1.954	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	116	150	1256	383	0	0
normalized size	1	1.	0.72	0.93	7.8	2.38	0.	0.
time (sec)	N/A	0.517	0.518	0.138	2.095	1.561	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	121	232	1642	390	0	0
normalized size	1	1.	0.74	1.42	10.07	2.39	0.	0.
time (sec)	N/A	0.481	0.697	0.15	2.205	1.612	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	102	100	525	271	0	0
normalized size	1	1.	0.59	0.58	3.05	1.58	0.	0.
time (sec)	N/A	0.524	0.536	0.104	1.896	1.496	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	123	122	711	324	0	0
normalized size	1	1.	0.56	0.56	3.25	1.48	0.	0.
time (sec)	N/A	0.61	0.718	0.117	1.781	1.461	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	146	144	837	381	0	0
normalized size	1	1.	0.55	0.54	3.15	1.43	0.	0.
time (sec)	N/A	0.7	0.801	0.135	1.986	1.53	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	170	581	0	582	0	0
normalized size	1	1.	0.54	1.86	0.	1.87	0.	0.
time (sec)	N/A	0.913	2.465	0.162	0.	2.502	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	148	509	6151	525	0	0
normalized size	1	1.	0.56	1.92	23.21	1.98	0.	0.
time (sec)	N/A	0.793	1.59	0.127	4.013	2.431	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	131	437	11552	464	0	0
normalized size	1	1.	0.6	2.	52.99	2.13	0.	0.
time (sec)	N/A	0.713	0.946	0.119	4.469	2.447	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	142	399	3966	471	0	0
normalized size	1	1.	0.64	1.8	17.86	2.12	0.	0.
time (sec)	N/A	0.721	0.905	0.12	2.92	1.999	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	141	354	3379	463	0	0
normalized size	1	1.	0.65	1.62	15.5	2.12	0.	0.
time (sec)	N/A	0.723	0.769	0.109	2.621	1.981	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	141	245	1520	451	0	0
normalized size	1	1.	0.67	1.17	7.24	2.15	0.	0.
time (sec)	N/A	0.725	0.893	0.161	2.208	1.702	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	151	327	2214	462	0	0
normalized size	1	1.	0.72	1.56	10.54	2.2	0.	0.
time (sec)	N/A	0.668	1.331	0.103	2.227	1.712	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	127	124	595	339	0	0
normalized size	1	1.	0.58	0.57	2.72	1.55	0.	0.
time (sec)	N/A	0.719	0.882	0.114	1.696	1.532	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	149	146	782	387	0	0
normalized size	1	1.	0.56	0.55	2.94	1.45	0.	0.
time (sec)	N/A	0.803	1.008	0.127	1.722	1.614	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	171	168	906	475	0	0
normalized size	1	1.	0.55	0.54	2.89	1.52	0.	0.
time (sec)	N/A	0.91	0.911	0.148	3.608	1.466	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	349	429	0	555	0	0
normalized size	1	1.	1.54	1.9	0.	2.46	0.	0.
time (sec)	N/A	0.749	2.101	0.126	0.	29.499	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	344	253	0	508	0	0
normalized size	1	1.	1.88	1.38	0.	2.78	0.	0.
time (sec)	N/A	0.564	1.58	0.138	0.	30.855	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	104	178	0	447	0	0
normalized size	1	1.	0.78	1.34	0.	3.36	0.	0.
time (sec)	N/A	0.387	0.23	0.117	0.	12.655	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	235	271	0	513	0	0
normalized size	1	1.	1.74	2.01	0.	3.8	0.	0.
time (sec)	N/A	0.392	3.435	0.132	0.	12.691	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	556	264	0	436	0	0
normalized size	1	1.	4.09	1.94	0.	3.21	0.	0.
time (sec)	N/A	0.348	6.749	0.136	0.	1.986	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	1765	418	0	482	0	0
normalized size	1	1.	9.75	2.31	0.	2.66	0.	0.
time (sec)	N/A	0.511	7.952	0.113	0.	1.996	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	2490	554	0	532	0	0
normalized size	1	1.	11.12	2.47	0.	2.38	0.	0.
time (sec)	N/A	0.689	10.241	0.123	0.	2.035	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	370	477	0	678	0	0
normalized size	1	1.	1.51	1.95	0.	2.77	0.	0.
time (sec)	N/A	0.787	2.403	0.16	0.	88.82	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	238	394	0	601	0	0
normalized size	1	1.	1.27	2.1	0.	3.2	0.	0.
time (sec)	N/A	0.57	2.028	0.148	0.	50.028	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	227	365	0	579	0	0
normalized size	1	1.	1.57	2.52	0.	3.99	0.	0.
time (sec)	N/A	0.411	1.889	0.127	0.	49.173	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	434	341	0	522	0	0
normalized size	1	1.	2.86	2.24	0.	3.43	0.	0.
time (sec)	N/A	0.378	3.832	0.135	0.	2.351	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	1192	328	0	576	0	0
normalized size	1	1.	5.93	1.63	0.	2.87	0.	0.
time (sec)	N/A	0.549	6.834	0.167	0.	2.324	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	2422	472	0	620	0	0
normalized size	1	1.	9.77	1.9	0.	2.5	0.	0.
time (sec)	N/A	0.739	8.084	0.116	0.	2.425	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	256	583	0	772	0	0
normalized size	1	1.	1.08	2.46	0.	3.26	0.	0.
time (sec)	N/A	0.793	2.174	0.171	0.	107.747	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	244	553	0	733	0	0
normalized size	1	1.	1.27	2.88	0.	3.82	0.	0.
time (sec)	N/A	0.58	1.911	0.195	0.	97.303	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	200	449	0	581	0	0
normalized size	1	1.	1.3	2.92	0.	3.77	0.	0.
time (sec)	N/A	0.399	1.465	0.13	0.	2.332	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	211	479	0	648	0	0
normalized size	1	1.	1.06	2.41	0.	3.26	0.	0.
time (sec)	N/A	0.579	2.599	0.147	0.	2.088	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	239	472	0	713	0	0
normalized size	1	1.	0.97	1.92	0.	2.9	0.	0.
time (sec)	N/A	0.77	3.438	0.111	0.	2.077	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	70	93	173	173	104
normalized size	1	1.	0.97	0.76	1.01	1.88	1.88	1.13
time (sec)	N/A	0.091	0.14	0.015	1.051	1.663	2.375	1.171

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	77	136	150	84
normalized size	1	1.	0.96	0.79	1.01	1.79	1.97	1.11
time (sec)	N/A	0.085	0.096	0.013	1.269	1.685	1.379	1.186

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	62	105	95	63
normalized size	1	1.	1.06	0.91	1.15	1.94	1.76	1.17
time (sec)	N/A	0.063	0.062	0.012	1.219	1.561	0.612	1.272

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	40	47	72	63	43
normalized size	1	1.	0.92	1.05	1.24	1.89	1.66	1.13
time (sec)	N/A	0.023	0.061	0.014	1.269	1.571	0.352	1.242

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	27	38	0	53
normalized size	1	1.	1.73	1.4	1.8	2.53	0.	3.53
time (sec)	N/A	0.03	0.008	0.026	1.291	1.596	0.	1.343

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	30	50	95	0	58
normalized size	1	1.	1.	1.88	3.12	5.94	0.	3.62
time (sec)	N/A	0.048	0.006	0.035	1.327	1.675	0.	1.262

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	51	162	0	85
normalized size	1	1.	1.	1.33	2.12	6.75	0.	3.54
time (sec)	N/A	0.062	0.008	0.033	1.323	1.657	0.	1.406

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	78	198	0	142
normalized size	1	1.	1.	1.09	1.66	4.21	0.	3.02
time (sec)	N/A	0.076	0.016	0.034	1.37	1.639	0.	1.328

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	95	236	0	165
normalized size	1	1.	0.95	1.14	1.51	3.75	0.	2.62
time (sec)	N/A	0.08	0.151	0.039	1.338	1.673	0.	1.328

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	128	266	0	221
normalized size	1	1.	0.89	1.08	1.51	3.13	0.	2.6
time (sec)	N/A	0.091	0.21	0.036	1.337	1.679	0.	1.378

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	102	128	167	239	338	151
normalized size	1	1.	0.82	1.02	1.34	1.91	2.7	1.21
time (sec)	N/A	0.219	0.335	0.023	1.173	1.685	2.655	1.327

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	76	107	136	193	255	120
normalized size	1	1.	0.78	1.1	1.4	1.99	2.63	1.24
time (sec)	N/A	0.182	0.266	0.023	1.127	1.635	1.504	1.296

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	85	107	146	170	92
normalized size	1	1.	0.76	1.	1.26	1.72	2.	1.08
time (sec)	N/A	0.077	0.174	0.019	1.105	1.59	0.699	1.337

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	74	99	0	126
normalized size	1	1.	0.94	1.21	1.57	2.11	0.	2.68
time (sec)	N/A	0.063	0.098	0.042	1.3	1.618	0.	1.254

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	46	56	78	139	0	107
normalized size	1	1.	1.44	1.75	2.44	4.34	0.	3.34
time (sec)	N/A	0.145	0.025	0.056	1.208	1.715	0.	1.361

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	65	99	220	0	113
normalized size	1	1.	1.34	2.03	3.09	6.88	0.	3.53
time (sec)	N/A	0.156	0.019	0.053	1.566	1.756	0.	1.451

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	128	239	0	167
normalized size	1	1.	1.34	1.54	2.29	4.27	0.	2.98
time (sec)	N/A	0.187	0.025	0.055	1.128	1.629	0.	1.296

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	56	128	171	288	0	208
normalized size	1	1.	0.65	1.49	1.99	3.35	0.	2.42
time (sec)	N/A	0.21	0.312	0.059	1.298	1.687	0.	1.647

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	171	220	339	0	254
normalized size	1	1.	0.73	1.61	2.08	3.2	0.	2.4
time (sec)	N/A	0.216	0.359	0.063	1.185	1.734	0.	1.439

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	104	186	240	271	462	185
normalized size	1	1.	0.65	1.16	1.5	1.69	2.89	1.16
time (sec)	N/A	0.339	0.325	0.024	1.082	1.659	3.357	1.647

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	86	154	194	213	340	149
normalized size	1	1.	0.67	1.19	1.5	1.65	2.64	1.16
time (sec)	N/A	0.141	0.331	0.023	1.14	1.652	1.518	1.846

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	61	116	149	165	0	192
normalized size	1	1.	0.65	1.23	1.59	1.76	0.	2.04
time (sec)	N/A	0.125	0.189	0.049	1.057	1.642	0.	1.762

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	96	108	136	194	0	196
normalized size	1	1.	1.17	1.32	1.66	2.37	0.	2.39
time (sec)	N/A	0.273	0.178	0.065	1.077	1.718	0.	1.472

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	143	107	142	278	0	209
normalized size	1	1.	1.93	1.45	1.92	3.76	0.	2.82
time (sec)	N/A	0.291	0.317	0.062	1.086	1.723	0.	1.562

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	277	113	192	297	0	208
normalized size	1	1.	3.15	1.28	2.18	3.38	0.	2.36
time (sec)	N/A	0.3	1.437	0.063	1.14	1.749	0.	1.697

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	451	141	235	315	0	240
normalized size	1	1.	3.99	1.25	2.08	2.79	0.	2.12
time (sec)	N/A	0.36	5.727	0.066	1.276	1.693	0.	1.584

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	262	187	311	362	0	286
normalized size	1	1.	1.82	1.3	2.16	2.51	0.	1.99
time (sec)	N/A	0.386	1.128	0.07	1.317	1.707	0.	1.414

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	280	235	375	421	0	332
normalized size	1	1.	1.66	1.39	2.22	2.49	0.	1.96
time (sec)	N/A	0.394	1.305	0.074	1.277	1.725	0.	1.69

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	130	266	354	332	699	224
normalized size	1	1.	0.65	1.32	1.76	1.65	3.48	1.11
time (sec)	N/A	0.487	0.415	0.027	1.482	1.736	6.186	1.483

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	108	223	288	278	532	184
normalized size	1	1.	0.7	1.45	1.87	1.81	3.45	1.19
time (sec)	N/A	0.194	0.432	0.023	1.206	1.623	3.373	1.557

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	86	176	225	216	0	238
normalized size	1	1.	0.74	1.52	1.94	1.86	0.	2.05
time (sec)	N/A	0.166	0.302	0.055	1.061	1.682	0.	1.497

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	153	200	254	0	243
normalized size	1	1.	1.02	1.38	1.8	2.29	0.	2.19
time (sec)	N/A	0.381	0.253	0.078	1.359	1.793	0.	1.639

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	272	145	189	323	0	259
normalized size	1	1.	2.47	1.32	1.72	2.94	0.	2.35
time (sec)	N/A	0.4	1.711	0.067	1.21	1.808	0.	1.416

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	208	144	223	342	0	259
normalized size	1	1.	1.82	1.26	1.96	3.	0.	2.27
time (sec)	N/A	0.425	1.837	0.071	1.076	1.99	0.	1.37

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	786	158	286	356	0	255
normalized size	1	1.	6.29	1.26	2.29	2.85	0.	2.04
time (sec)	N/A	0.424	6.359	0.074	1.369	2.09	0.	1.546

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	273	188	363	366	0	286
normalized size	1	1.	1.77	1.22	2.36	2.38	0.	1.86
time (sec)	N/A	0.506	1.216	0.076	1.276	1.754	0.	1.65

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	294	234	455	431	0	332
normalized size	1	1.	1.59	1.26	2.46	2.33	0.	1.79
time (sec)	N/A	0.533	1.424	0.08	1.334	1.727	0.	1.604

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	249	281	419	242	1166	204
normalized size	1	1.	2.04	2.3	3.43	1.98	9.56	1.67
time (sec)	N/A	0.263	0.544	0.033	1.609	1.65	10.551	1.579

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	197	211	304	204	668	167
normalized size	1	1.	1.99	2.13	3.07	2.06	6.75	1.69
time (sec)	N/A	0.171	0.439	0.03	1.91	1.594	6.48	1.616

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	126	108	193	147	265	105
normalized size	1	1.	2.33	2.	3.57	2.72	4.91	1.94
time (sec)	N/A	0.089	0.235	0.027	1.872	1.587	3.594	1.61

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	72	56	99	105	0	58
normalized size	1	1.	2.12	1.65	2.91	3.09	0.	1.71
time (sec)	N/A	0.121	0.117	0.04	1.865	1.605	0.	1.822

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	109	78	134	197	0	96
normalized size	1	1.	2.48	1.77	3.05	4.48	0.	2.18
time (sec)	N/A	0.166	0.247	0.053	1.217	1.674	0.	1.58

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	201	163	265	320	0	149
normalized size	1	1.	2.91	2.36	3.84	4.64	0.	2.16
time (sec)	N/A	0.239	1.075	0.054	1.237	1.681	0.	1.421

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	289	252	381	385	0	212
normalized size	1	1.	2.7	2.36	3.56	3.6	0.	1.98
time (sec)	N/A	0.249	3.112	0.064	1.311	1.64	0.	1.512

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	490	340	497	419	0	246
normalized size	1	1.	3.74	2.6	3.79	3.2	0.	1.88
time (sec)	N/A	0.265	4.396	0.066	1.209	1.762	0.	1.34

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	369	322	502	387	1430	259
normalized size	1	1.	2.17	1.89	2.95	2.28	8.41	1.52
time (sec)	N/A	0.397	0.608	0.033	2.012	1.709	28.494	1.439

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	315	252	382	343	848	221
normalized size	1	1.	2.14	1.71	2.6	2.33	5.77	1.5
time (sec)	N/A	0.363	0.861	0.031	2.03	1.687	17.415	1.391

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	137	149	258	294	415	161
normalized size	1	1.	1.38	1.51	2.61	2.97	4.19	1.63
time (sec)	N/A	0.315	0.68	0.031	1.995	1.67	10.386	1.392

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	153	97	162	228	107	116
normalized size	1	1.	2.19	1.39	2.31	3.26	1.53	1.66
time (sec)	N/A	0.111	0.337	0.023	1.863	1.619	5.522	1.42

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	126	144	0	81
normalized size	1	1.	1.17	0.92	1.94	2.22	0.	1.25
time (sec)	N/A	0.124	0.173	0.043	1.3	1.548	0.	1.414

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	170	119	196	338	0	153
normalized size	1	1.	2.15	1.51	2.48	4.28	0.	1.94
time (sec)	N/A	0.266	0.493	0.057	1.361	1.703	0.	1.492

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	264	205	329	502	0	209
normalized size	1	1.	2.47	1.92	3.07	4.69	0.	1.95
time (sec)	N/A	0.38	1.53	0.06	1.364	1.722	0.	1.748

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	496	294	454	564	0	267
normalized size	1	1.	3.26	1.93	2.99	3.71	0.	1.76
time (sec)	N/A	0.401	3.12	0.067	1.351	1.761	0.	1.458

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	435	292	435	497	971	270
normalized size	1	1.	2.25	1.51	2.25	2.58	5.03	1.4
time (sec)	N/A	0.535	0.833	0.032	1.5	1.704	40.098	1.36

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	361	189	312	429	502	209
normalized size	1	1.	2.46	1.29	2.12	2.92	3.41	1.42
time (sec)	N/A	0.515	0.861	0.031	1.969	1.671	25.222	1.522

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	241	137	216	351	151	162
normalized size	1	1.	2.08	1.18	1.86	3.03	1.3	1.4
time (sec)	N/A	0.338	0.539	0.026	1.926	1.646	15.375	1.466

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	155	227	119	101
normalized size	1	1.	1.32	0.63	1.52	2.23	1.17	0.99
time (sec)	N/A	0.134	0.328	0.022	1.141	1.523	9.585	1.485

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	155	227	0	101
normalized size	1	1.	0.94	0.63	1.52	2.23	0.	0.99
time (sec)	N/A	0.154	0.258	0.043	1.369	1.579	0.	1.613

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	197	159	252	481	0	200
normalized size	1	1.	1.68	1.36	2.15	4.11	0.	1.71
time (sec)	N/A	0.402	0.904	0.063	1.045	1.742	0.	1.69

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	482	245	386	670	0	257
normalized size	1	1.	3.32	1.69	2.66	4.62	0.	1.77
time (sec)	N/A	0.546	2.976	0.065	1.041	1.702	0.	1.769

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	610	334	509	756	0	315
normalized size	1	1.	3.11	1.7	2.6	3.86	0.	1.61
time (sec)	N/A	0.566	4.728	0.07	1.059	1.705	0.	1.442

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	64	83	119	170	0	0
normalized size	1	1.	0.63	0.82	1.18	1.68	0.	0.
time (sec)	N/A	0.132	0.24	0.06	1.861	1.57	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	81	104	166	236	0	0
normalized size	1	1.	0.59	0.75	1.2	1.71	0.	0.
time (sec)	N/A	0.188	0.393	0.065	1.868	1.575	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	105	123	232	302	0	0
normalized size	1	1.	0.6	0.7	1.33	1.73	0.	0.
time (sec)	N/A	0.223	0.691	0.063	1.947	1.616	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	78	194	0	400	0	153
normalized size	1	1.	0.66	1.64	0.	3.39	0.	1.3
time (sec)	N/A	0.151	0.167	0.102	0.	1.672	0.	1.837

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	104	256	0	493	0	177
normalized size	1	1.	0.88	2.17	0.	4.18	0.	1.5
time (sec)	N/A	0.152	0.438	0.112	0.	1.945	0.	1.889

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	87	292	0	586	0	181
normalized size	1	1.	0.69	2.32	0.	4.65	0.	1.44
time (sec)	N/A	0.172	0.585	0.105	0.	1.946	0.	2.134

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	0	0	0
normalized size	1	1.	0.69	2.61	0.	0.	0.	0.
time (sec)	N/A	0.107	0.496	0.128	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.088	0.2	0.122	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	229	0	0	0	0
normalized size	1	1.	0.87	3.75	0.	0.	0.	0.
time (sec)	N/A	0.079	0.107	0.297	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	152	0	0	0	0
normalized size	1	1.	1.	4.34	0.	0.	0.	0.
time (sec)	N/A	0.066	0.063	0.115	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	148	0	0	0	0
normalized size	1	1.	0.89	2.6	0.	0.	0.	0.
time (sec)	N/A	0.075	0.137	0.124	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	0	0	0
normalized size	1	1.	0.78	4.78	0.	0.	0.	0.
time (sec)	N/A	0.084	0.399	0.277	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	0	0	0
normalized size	1	1.	0.86	4.52	0.	0.	0.	0.
time (sec)	N/A	0.099	0.296	0.352	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	102	115	155	244	321	149
normalized size	1	1.	0.77	0.87	1.17	1.85	2.43	1.13
time (sec)	N/A	0.114	0.276	0.015	1.01	1.886	4.592	1.204

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	87	89	120	194	209	120
normalized size	1	1.	0.77	0.79	1.06	1.72	1.85	1.06
time (sec)	N/A	0.107	0.177	0.013	1.107	1.963	2.395	1.219

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	70	84	104	163	197	95
normalized size	1	1.	0.8	0.95	1.18	1.85	2.24	1.08
time (sec)	N/A	0.095	0.154	0.013	0.967	1.963	1.292	1.175

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	53	57	74	113	107	72
normalized size	1	1.	0.77	0.83	1.07	1.64	1.55	1.04
time (sec)	N/A	0.047	0.094	0.014	0.989	1.876	0.605	1.2

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	43	51	82	66	47
normalized size	1	1.	1.34	1.05	1.24	2.	1.61	1.15
time (sec)	N/A	0.025	0.044	0.003	1.001	1.848	0.277	1.189

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	41	49	120	0	95
normalized size	1	1.	1.41	1.52	1.81	4.44	0.	3.52
time (sec)	N/A	0.052	0.02	0.032	0.974	1.891	0.	1.217

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	41	62	193	0	95
normalized size	1	1.	1.	1.52	2.3	7.15	0.	3.52
time (sec)	N/A	0.054	0.019	0.035	0.972	1.948	0.	1.263

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	70	111	220	0	153
normalized size	1	1.	1.16	1.37	2.18	4.31	0.	3.
time (sec)	N/A	0.079	0.02	0.038	1.005	2.028	0.	1.272

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	51	83	107	250	0	219
normalized size	1	1.	0.65	1.06	1.37	3.21	0.	2.81
time (sec)	N/A	0.103	0.205	0.038	0.977	1.966	0.	1.331

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	130	188	306	0	311
normalized size	1	1.	0.73	1.34	1.94	3.15	0.	3.21
time (sec)	N/A	0.104	0.257	0.04	1.003	2.007	0.	1.324

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	144	171	329	0	332
normalized size	1	1.	0.66	1.18	1.4	2.7	0.	2.72
time (sec)	N/A	0.118	0.561	0.042	0.989	2.016	0.	1.262

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	93	173	224	282	428	174
normalized size	1	1.	0.65	1.21	1.57	1.97	2.99	1.22
time (sec)	N/A	0.229	0.442	0.026	1.011	1.876	2.959	1.268

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	96	141	178	221	320	138
normalized size	1	1.	0.81	1.19	1.51	1.87	2.71	1.17
time (sec)	N/A	0.138	0.395	0.024	0.988	1.895	1.424	1.237

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	65	102	132	162	189	103
normalized size	1	1.	0.71	1.12	1.45	1.78	2.08	1.13
time (sec)	N/A	0.083	0.228	0.022	0.989	2.119	0.703	1.192

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	100	111	184	0	177
normalized size	1	1.	0.94	1.59	1.76	2.92	0.	2.81
time (sec)	N/A	0.136	0.134	0.051	0.979	2.346	0.	1.251

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	71	88	124	257	0	178
normalized size	1	1.	1.54	1.91	2.7	5.59	0.	3.87
time (sec)	N/A	0.136	0.029	0.063	0.997	2.287	0.	1.248

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	92	117	176	292	0	190
normalized size	1	1.	1.48	1.89	2.84	4.71	0.	3.06
time (sec)	N/A	0.158	0.034	0.066	1.013	2.033	0.	1.309

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	160	219	312	0	277
normalized size	1	1.	0.66	1.76	2.41	3.43	0.	3.04
time (sec)	N/A	0.211	0.367	0.068	1.014	1.987	0.	1.261

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	84	223	294	375	0	343
normalized size	1	1.	0.67	1.78	2.35	3.	0.	2.74
time (sec)	N/A	0.254	0.592	0.072	1.022	2.04	0.	1.281

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	171	304	400	373	821	265
normalized size	1	1.	0.8	1.43	1.88	1.75	3.85	1.24
time (sec)	N/A	0.503	0.712	0.029	1.007	1.977	6.166	1.242

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	132	247	319	305	570	216
normalized size	1	1.	0.73	1.36	1.76	1.69	3.15	1.19
time (sec)	N/A	0.336	0.613	0.026	1.018	2.057	3.385	1.22

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	94	203	257	239	420	174
normalized size	1	1.	0.68	1.47	1.86	1.73	3.04	1.26
time (sec)	N/A	0.175	0.387	0.024	1.008	1.947	1.649	1.207

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	121	181	207	269	0	317
normalized size	1	1.	1.01	1.51	1.72	2.24	0.	2.64
time (sec)	N/A	0.343	0.311	0.059	0.992	2.053	0.	1.309

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	174	160	204	331	0	267
normalized size	1	1.	1.44	1.32	1.69	2.74	0.	2.21
time (sec)	N/A	0.384	0.579	0.072	1.031	2.04	0.	1.196

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	259	166	259	358	0	275
normalized size	1	1.	2.11	1.35	2.11	2.91	0.	2.24
time (sec)	N/A	0.392	1.444	0.076	1.012	2.054	0.	1.288

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	315	193	302	379	0	338
normalized size	1	1.	2.35	1.44	2.25	2.83	0.	2.52
time (sec)	N/A	0.385	5.163	0.078	1.02	2.161	0.	1.277

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	404	246	427	397	0	392
normalized size	1	1.	2.52	1.54	2.67	2.48	0.	2.45
time (sec)	N/A	0.475	3.948	0.082	1.047	2.035	0.	1.266

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	502	315	486	463	0	460
normalized size	1	1.	2.56	1.61	2.48	2.36	0.	2.35
time (sec)	N/A	0.513	5.31	0.088	1.018	2.091	0.	1.234

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	204	427	574	454	1149	309
normalized size	1	1.	0.77	1.61	2.17	1.71	4.34	1.17
time (sec)	N/A	0.678	1.066	0.029	1.013	2.145	11.124	1.249

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	171	364	478	378	932	265
normalized size	1	1.	0.83	1.76	2.31	1.83	4.5	1.28
time (sec)	N/A	0.396	0.63	0.03	1.023	1.96	6.314	1.188

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	129	295	381	315	658	220
normalized size	1	1.	0.78	1.78	2.3	1.9	3.96	1.33
time (sec)	N/A	0.226	0.482	0.027	1.003	1.924	3.46	1.148

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	147	251	315	336	0	386
normalized size	1	1.	0.91	1.55	1.94	2.07	0.	2.38
time (sec)	N/A	0.478	0.48	0.067	1.02	1.94	0.	1.265

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	227	221	284	398	0	379
normalized size	1	1.	1.46	1.42	1.82	2.55	0.	2.43
time (sec)	N/A	0.51	0.958	0.082	1.	2.125	0.	1.269

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	256	219	320	410	0	378
normalized size	1	1.	1.46	1.25	1.83	2.34	0.	2.16
time (sec)	N/A	0.534	1.477	0.086	1.022	2.151	0.	1.25

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	354	226	370	425	0	389
normalized size	1	1.	2.09	1.34	2.19	2.51	0.	2.3
time (sec)	N/A	0.572	4.259	0.088	1.016	2.076	0.	1.287

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	793	262	494	447	0	406
normalized size	1	1.	4.33	1.43	2.7	2.44	0.	2.22
time (sec)	N/A	0.554	6.183	0.098	1.03	2.167	0.	1.268

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	931	316	602	477	0	460
normalized size	1	1.	4.39	1.49	2.84	2.25	0.	2.17
time (sec)	N/A	0.637	6.195	0.095	1.069	2.048	0.	1.23

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	265	385	755	540	0	529
normalized size	1	1.	1.09	1.58	3.09	2.21	0.	2.17
time (sec)	N/A	0.703	1.889	0.098	1.081	2.099	0.	1.278

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	237	577	782	536	1640	352
normalized size	1	1.	0.78	1.9	2.57	1.76	5.39	1.16
time (sec)	N/A	0.863	1.424	0.032	1.079	2.063	19.167	1.26

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	204	490	652	454	1258	309
normalized size	1	1.	0.84	2.02	2.68	1.87	5.18	1.27
time (sec)	N/A	0.454	0.94	0.033	1.04	2.159	11.494	1.25

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	163	416	540	378	1005	265
normalized size	1	1.	0.82	2.08	2.7	1.89	5.02	1.32
time (sec)	N/A	0.28	0.535	0.028	1.049	1.926	6.619	1.291

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	182	320	439	408	0	455
normalized size	1	1.	0.93	1.64	2.25	2.09	0.	2.33
time (sec)	N/A	0.603	0.744	0.075	1.037	2.123	0.	1.256

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	246	289	392	466	0	448
normalized size	1	1.	1.26	1.47	2.	2.38	0.	2.29
time (sec)	N/A	0.679	1.874	0.09	1.037	2.217	0.	1.311

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	299	280	400	486	0	468
normalized size	1	1.	1.45	1.36	1.94	2.36	0.	2.27
time (sec)	N/A	0.686	3.668	0.095	1.11	2.18	0.	1.247

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	354	279	432	487	0	468
normalized size	1	1.	1.62	1.27	1.97	2.22	0.	2.14
time (sec)	N/A	0.714	5.694	0.099	1.025	2.291	0.	1.286

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	838	294	562	493	0	458
normalized size	1	1.	3.86	1.35	2.59	2.27	0.	2.11
time (sec)	N/A	0.743	6.193	0.098	1.063	2.5	0.	1.295

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	971	331	670	521	0	475
normalized size	1	1.	4.32	1.47	2.98	2.32	0.	2.11
time (sec)	N/A	0.691	6.207	0.101	1.039	2.165	0.	1.246

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	265	385	871	539	0	529
normalized size	1	1.	1.05	1.52	3.44	2.13	0.	2.09
time (sec)	N/A	0.833	2.057	0.106	1.061	2.1	0.	1.335

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	298	454	987	617	0	598
normalized size	1	1.	1.04	1.58	3.44	2.15	0.	2.08
time (sec)	N/A	0.868	3.441	0.114	1.055	2.097	0.	1.287

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	393	526	709	344	2688	336
normalized size	1	1.	2.26	3.02	4.07	1.98	15.45	1.93
time (sec)	N/A	0.234	0.762	0.039	1.543	1.916	19.304	1.171

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	307	420	540	288	1739	279
normalized size	1	1.	2.21	3.02	3.88	2.07	12.51	2.01
time (sec)	N/A	0.212	0.796	0.036	1.486	1.949	11.746	1.192

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	213	248	369	227	993	186
normalized size	1	1.	1.94	2.25	3.35	2.06	9.03	1.69
time (sec)	N/A	0.123	0.466	0.033	1.501	1.977	6.834	1.167

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	136	125	223	153	330	119
normalized size	1	1.	2.52	2.31	4.13	2.83	6.11	2.2
time (sec)	N/A	0.112	0.28	0.027	1.474	1.905	3.582	1.191

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	163	115	197	247	0	124
normalized size	1	1.	3.2	2.25	3.86	4.84	0.	2.43
time (sec)	N/A	0.119	0.541	0.055	1.484	1.979	0.	1.192

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	256	180	294	325	0	163
normalized size	1	1.	3.61	2.54	4.14	4.58	0.	2.3
time (sec)	N/A	0.173	1.302	0.062	1.049	1.88	0.	1.245

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	256	311	481	428	0	235
normalized size	1	1.	2.19	2.66	4.11	3.66	0.	2.01
time (sec)	N/A	0.202	1.44	0.066	1.016	1.97	0.	1.221

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	351	442	655	489	0	328
normalized size	1	1.	2.37	2.99	4.43	3.3	0.	2.22
time (sec)	N/A	0.217	3.875	0.07	1.032	1.995	0.	1.241

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	481	482	657	437	2134	359
normalized size	1	1.	2.6	2.61	3.55	2.36	11.54	1.94
time (sec)	N/A	0.379	0.915	0.036	1.472	2.02	30.978	1.154

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	385	309	475	383	1261	267
normalized size	1	1.	2.41	1.93	2.97	2.39	7.88	1.67
time (sec)	N/A	0.317	0.845	0.033	1.533	1.917	20.011	1.16

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	275	187	317	308	536	204
normalized size	1	1.	2.67	1.82	3.08	2.99	5.2	1.98
time (sec)	N/A	0.259	0.672	0.033	1.543	1.94	11.234	1.173

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	175	135	221	242	148	157
normalized size	1	1.	2.43	1.88	3.07	3.36	2.06	2.18
time (sec)	N/A	0.12	0.407	0.026	1.462	1.89	5.718	1.151

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	221	157	257	351	0	194
normalized size	1	1.	2.66	1.89	3.1	4.23	0.	2.34
time (sec)	N/A	0.215	0.85	0.057	0.997	2.047	0.	1.187

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	321	243	387	514	0	251
normalized size	1	1.	2.94	2.23	3.55	4.72	0.	2.3
time (sec)	N/A	0.345	1.889	0.065	1.029	1.98	0.	1.206

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	578	373	582	630	0	317
normalized size	1	1.	3.5	2.26	3.53	3.82	0.	1.92
time (sec)	N/A	0.361	6.177	0.075	1.012	1.981	0.	1.245

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	763	506	765	689	0	409
normalized size	1	1.	3.93	2.61	3.94	3.55	0.	2.11
time (sec)	N/A	0.388	6.21	0.078	1.044	2.022	0.	1.218

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	663	542	738	613	2373	432
normalized size	1	1.	2.8	2.29	3.11	2.59	10.01	1.82
time (sec)	N/A	0.556	1.953	0.037	1.51	1.981	78.099	1.291

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	565	369	555	556	1445	340
normalized size	1	1.	2.73	1.78	2.68	2.69	6.98	1.64
time (sec)	N/A	0.501	0.953	0.037	1.569	2.038	40.07	1.203

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	423	247	398	454	665	274
normalized size	1	1.	2.78	1.62	2.62	2.99	4.38	1.8
time (sec)	N/A	0.476	1.001	0.033	1.551	1.915	24.158	1.226

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	289	175	277	375	192	207
normalized size	1	1.	2.35	1.42	2.25	3.05	1.56	1.68
time (sec)	N/A	0.281	0.739	0.029	1.538	1.887	14.419	1.18

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	164	113	242	251	180	155
normalized size	1	1.	1.5	1.04	2.22	2.3	1.65	1.42
time (sec)	N/A	0.132	0.402	0.024	1.073	1.777	9.514	1.207

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	276	197	313	505	0	243
normalized size	1	1.	2.23	1.59	2.52	4.07	0.	1.96
time (sec)	N/A	0.353	1.636	0.059	1.029	1.98	0.	1.258

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	839	303	473	691	0	323
normalized size	1	1.	5.59	2.02	3.15	4.61	0.	2.15
time (sec)	N/A	0.517	6.347	0.072	1.05	2.345	0.	1.269

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	206	433	666	849	0	389
normalized size	1	1.	0.98	2.06	3.17	4.04	0.	1.85
time (sec)	N/A	0.557	1.453	0.08	1.1	2.324	0.	1.334

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	270	566	851	910	0	481
normalized size	1	1.	1.1	2.3	3.46	3.7	0.	1.96
time (sec)	N/A	0.576	0.989	0.085	1.093	2.111	0.	1.24

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	299	429	640	730	1624	408
normalized size	1	1.	1.22	1.75	2.61	2.98	6.63	1.67
time (sec)	N/A	0.691	2.797	0.034	1.571	2.112	99.888	1.257

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	571	307	481	606	746	344
normalized size	1	1.	2.93	1.57	2.47	3.11	3.83	1.76
time (sec)	N/A	0.648	1.11	0.034	1.576	1.999	58.61	1.248

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	405	255	386	512	279	297
normalized size	1	1.	2.47	1.55	2.35	3.12	1.7	1.81
time (sec)	N/A	0.478	0.902	0.032	1.573	2.002	35.86	1.223

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	239	108	350	343	267	231
normalized size	1	1.	1.61	0.73	2.36	2.32	1.8	1.56
time (sec)	N/A	0.348	0.513	0.027	1.022	1.98	24.066	1.164

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	208	106	350	343	264	231
normalized size	1	1.	1.41	0.72	2.36	2.32	1.78	1.56
time (sec)	N/A	0.176	0.458	0.025	1.039	1.78	16.438	1.178

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	334	277	423	660	0	335
normalized size	1	1.	2.13	1.76	2.69	4.2	0.	2.13
time (sec)	N/A	0.492	2.549	0.067	1.413	1.991	0.	1.332

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	1190	363	555	873	0	392
normalized size	1	1.	6.43	1.96	3.	4.72	0.	2.12
time (sec)	N/A	0.717	6.402	0.077	1.33	2.079	0.	1.29

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	271	493	751	1062	0	458
normalized size	1	1.	1.09	1.99	3.03	4.28	0.	1.85
time (sec)	N/A	0.761	1.477	0.086	1.423	2.165	0.	1.258

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	304	626	932	1135	0	549
normalized size	1	1.	1.06	2.18	3.25	3.95	0.	1.91
time (sec)	N/A	0.798	1.655	0.092	1.408	2.184	0.	1.217

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	145	152	320	363	0	0
normalized size	1	1.	0.61	0.64	1.34	1.52	0.	0.
time (sec)	N/A	0.547	1.253	0.071	2.806	1.914	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	114	130	262	298	0	0
normalized size	1	1.	0.59	0.67	1.36	1.54	0.	0.
time (sec)	N/A	0.467	0.694	0.074	2.07	1.91	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	86	108	205	238	0	0
normalized size	1	1.	0.59	0.73	1.39	1.62	0.	0.
time (sec)	N/A	0.347	0.407	0.069	2.265	1.809	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	67	86	143	180	0	0
normalized size	1	1.	0.64	0.83	1.38	1.73	0.	0.
time (sec)	N/A	0.148	0.172	0.07	2.134	1.859	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	84	272	77	383	0	263
normalized size	1	1.	0.84	2.72	0.77	3.83	0.	2.63
time (sec)	N/A	0.275	0.186	0.197	1.915	2.05	0.	4.36

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	694	987	435	0	396
normalized size	1	1.	0.97	7.08	10.07	4.44	0.	4.04
time (sec)	N/A	0.306	0.281	0.201	1.987	2.252	0.	3.119

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	111	1376	4525	477	0	653
normalized size	1	1.	0.95	11.76	38.68	4.08	0.	5.58
time (sec)	N/A	0.348	0.545	0.215	20.817	3.045	0.	3.057

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	1897	7733	532	0	1162
normalized size	1	1.	0.85	11.64	47.44	3.26	0.	7.13
time (sec)	N/A	0.425	1.151	0.234	22.175	3.118	0.	3.118

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	168	2370	0	601	0	1494
normalized size	1	1.	0.8	11.34	0.	2.88	0.	7.15
time (sec)	N/A	0.509	1.472	0.262	0.	4.768	0.	3.093

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	145	154	340	392	0	0
normalized size	1	1.	0.6	0.63	1.4	1.61	0.	0.
time (sec)	N/A	0.7	1.28	0.067	2.236	2.189	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	113	132	277	320	0	0
normalized size	1	1.	0.6	0.71	1.48	1.71	0.	0.
time (sec)	N/A	0.415	0.756	0.066	2.214	1.9	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	87	110	215	257	0	0
normalized size	1	1.	0.6	0.76	1.49	1.78	0.	0.
time (sec)	N/A	0.197	0.349	0.223	2.636	1.944	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	105	335	126	456	0	343
normalized size	1	1.	0.74	2.36	0.89	3.21	0.	2.42
time (sec)	N/A	0.427	0.421	0.188	1.829	2.023	0.	4.808

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	118	781	1828	502	0	460
normalized size	1	1.	0.82	5.42	12.69	3.49	0.	3.19
time (sec)	N/A	0.5	0.508	0.211	2.077	2.219	0.	3.121

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	127	1453	4508	521	0	702
normalized size	1	1.	0.8	9.14	28.35	3.28	0.	4.42
time (sec)	N/A	0.504	0.751	0.229	2.836	3.13	0.	4.074

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	139	1897	0	558	0	1162
normalized size	1	1.	0.84	11.5	0.	3.38	0.	7.04
time (sec)	N/A	0.559	1.25	0.237	0.	3.011	0.	3.471

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	174	2370	0	622	0	1494
normalized size	1	1.	0.81	11.02	0.	2.89	0.	6.95
time (sec)	N/A	0.642	2.05	0.27	0.	4.171	0.	3.631

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	208	2843	0	703	0	1827
normalized size	1	1.	0.79	10.81	0.	2.67	0.	6.95
time (sec)	N/A	0.763	3.14	0.289	0.	4.221	0.	3.839

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	180	176	448	498	0	0
normalized size	1	1.	0.61	0.6	1.52	1.69	0.	0.
time (sec)	N/A	0.959	1.764	0.077	3.261	2.008	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	147	154	381	413	0	0
normalized size	1	1.	0.64	0.67	1.66	1.8	0.	0.
time (sec)	N/A	0.494	1.247	0.08	2.142	1.934	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	114	132	311	331	0	0
normalized size	1	1.	0.62	0.72	1.69	1.8	0.	0.
time (sec)	N/A	0.245	0.658	0.065	2.305	1.757	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	127	377	188	540	0	397
normalized size	1	1.	0.7	2.07	1.03	2.97	0.	2.18
time (sec)	N/A	0.662	0.735	0.201	1.871	2.165	0.	3.934

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	145	846	11036	579	0	539
normalized size	1	1.	0.79	4.6	59.98	3.15	0.	2.93
time (sec)	N/A	0.676	0.784	0.24	3.189	2.248	0.	3.209

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	153	1512	0	591	0	761
normalized size	1	1.	0.77	7.6	0.	2.97	0.	3.82
time (sec)	N/A	0.701	1.079	0.244	0.	3.089	0.	3.502

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	156	1925	0	606	0	1210
normalized size	1	1.	0.75	9.3	0.	2.93	0.	5.85
time (sec)	N/A	0.727	1.515	0.257	0.	3.113	0.	3.849

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	176	2369	0	649	0	1494
normalized size	1	1.	0.82	11.02	0.	3.02	0.	6.95
time (sec)	N/A	0.79	1.956	0.29	0.	4.096	0.	4.125

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	210	2843	0	722	0	1827
normalized size	1	1.	0.8	10.89	0.	2.77	0.	7.
time (sec)	N/A	0.902	2.813	0.296	0.	4.193	0.	4.376

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	242	3316	0	810	0	2159
normalized size	1	1.	0.78	10.66	0.	2.6	0.	6.94
time (sec)	N/A	0.966	3.946	0.411	0.	4.255	0.	4.814

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	144	392	0	582	0	363
normalized size	1	1.	0.57	1.54	0.	2.29	0.	1.43
time (sec)	N/A	0.838	0.786	0.15	0.	1.991	0.	2.008

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	118	324	0	524	0	271
normalized size	1	1.	0.57	1.56	0.	2.52	0.	1.3
time (sec)	N/A	0.612	0.638	0.143	0.	2.019	0.	1.871

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	98	306	0	466	0	255
normalized size	1	1.	0.6	1.87	0.	2.84	0.	1.55
time (sec)	N/A	0.38	0.367	0.133	0.	2.006	0.	2.036

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	79	233	0	410	0	154
normalized size	1	1.	0.67	1.97	0.	3.47	0.	1.31
time (sec)	N/A	0.158	0.144	0.128	0.	1.941	0.	1.974

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	86	337	0	624	0	277
normalized size	1	1.	0.73	2.86	0.	5.29	0.	2.35
time (sec)	N/A	0.314	0.33	0.242	0.	2.38	0.	3.354

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	96	893	0	710	0	440
normalized size	1	1.	0.8	7.44	0.	5.92	0.	3.67
time (sec)	N/A	0.351	0.51	0.249	0.	2.868	0.	3.309

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	118	1750	0	779	0	745
normalized size	1	1.	0.7	10.36	0.	4.61	0.	4.41
time (sec)	N/A	0.54	0.486	0.317	0.	3.393	0.	3.364

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	147	2374	0	838	0	1253
normalized size	1	1.	0.69	11.15	0.	3.93	0.	5.88
time (sec)	N/A	0.737	1.367	0.311	0.	3.562	0.	3.679

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	200	2997	0	910	0	1585
normalized size	1	1.	0.77	11.57	0.	3.51	0.	6.12
time (sec)	N/A	0.934	2.799	0.335	0.	4.907	0.	3.951

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	180	577	0	709	0	412
normalized size	1	1.	0.65	2.08	0.	2.56	0.	1.49
time (sec)	N/A	0.873	1.503	0.16	0.	2.01	0.	2.839

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	153	533	0	641	0	308
normalized size	1	1.	0.67	2.33	0.	2.8	0.	1.34
time (sec)	N/A	0.67	0.722	0.159	0.	1.989	0.	2.595

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	131	407	0	575	0	262
normalized size	1	1.	0.72	2.25	0.	3.18	0.	1.45
time (sec)	N/A	0.396	0.479	0.151	0.	1.978	0.	2.288

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	83	334	0	514	0	194
normalized size	1	1.	0.69	2.78	0.	4.28	0.	1.62
time (sec)	N/A	0.162	0.502	0.143	0.	1.935	0.	2.398

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	135	453	0	767	0	305
normalized size	1	1.	1.03	3.46	0.	5.85	0.	2.33
time (sec)	N/A	0.348	0.941	0.292	0.	2.274	0.	5.066

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	196	1222	0	900	0	518
normalized size	1	1.	1.13	7.06	0.	5.2	0.	2.99
time (sec)	N/A	0.567	1.596	0.287	0.	2.719	0.	4.269

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	186	2261	0	999	0	819
normalized size	1	1.	0.8	9.75	0.	4.31	0.	3.53
time (sec)	N/A	0.78	1.544	0.316	0.	4.17	0.	4.671

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	223	2993	0	1069	0	1328
normalized size	1	1.	0.79	10.54	0.	3.76	0.	4.68
time (sec)	N/A	0.992	2.634	0.351	0.	4.262	0.	4.932

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	152	617	0	809	0	414
normalized size	1	1.	0.55	2.23	0.	2.92	0.	1.49
time (sec)	N/A	0.898	1.582	0.171	0.	2.044	0.	3.279

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	126	512	0	743	0	311
normalized size	1	1.	0.56	2.26	0.	3.27	0.	1.37
time (sec)	N/A	0.687	1.164	0.157	0.	2.093	0.	3.234

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	107	442	0	674	0	285
normalized size	1	1.	0.6	2.47	0.	3.77	0.	1.59
time (sec)	N/A	0.407	0.799	0.158	0.	2.039	0.	2.851

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	96	407	0	635	0	200
normalized size	1	1.	0.72	3.06	0.	4.77	0.	1.5
time (sec)	N/A	0.177	0.567	0.217	0.	2.029	0.	2.635

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	200	560	0	948	0	360
normalized size	1	1.	1.17	3.27	0.	5.54	0.	2.11
time (sec)	N/A	0.521	1.752	0.273	0.	2.378	0.	4.12

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	189	1327	0	1112	0	575
normalized size	1	1.	0.87	6.12	0.	5.12	0.	2.65
time (sec)	N/A	0.802	3.77	0.317	0.	2.835	0.	5.598

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	248	2366	0	1218	0	876
normalized size	1	1.	0.89	8.45	0.	4.35	0.	3.13
time (sec)	N/A	1.018	5.267	0.343	0.	4.94	0.	6.924

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	342	0	0	0	0
normalized size	1	1.	0.7	2.78	0.	0.	0.	0.
time (sec)	N/A	0.117	0.546	0.161	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	72	308	0	0	0	0
normalized size	1	1.	0.77	3.31	0.	0.	0.	0.
time (sec)	N/A	0.101	0.237	0.21	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	274	0	0	0	0
normalized size	1	1.	0.88	4.22	0.	0.	0.	0.
time (sec)	N/A	0.087	0.126	0.156	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	194	0	0	0	0
normalized size	1	1.	0.89	3.18	0.	0.	0.	0.
time (sec)	N/A	0.088	0.186	0.184	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	69	500	0	0	0	0
normalized size	1	1.	0.79	5.75	0.	0.	0.	0.
time (sec)	N/A	0.102	0.522	0.376	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	112	799	0	0	0	0
normalized size	1	1.	0.91	6.5	0.	0.	0.	0.
time (sec)	N/A	0.12	0.453	0.509	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	1344	543	0	0	0	0
normalized size	1	1.	6.37	2.57	0.	0.	0.	0.
time (sec)	N/A	0.287	6.432	0.2	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	1292	512	0	0	0	0
normalized size	1	1.	7.3	2.89	0.	0.	0.	0.
time (sec)	N/A	0.267	6.358	0.206	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	1240	481	0	0	0	0
normalized size	1	1.	8.61	3.34	0.	0.	0.	0.
time (sec)	N/A	0.228	6.314	0.183	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	1186	447	0	0	0	0
normalized size	1	1.	11.08	4.18	0.	0.	0.	0.
time (sec)	N/A	0.197	6.362	0.191	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	1173	380	0	0	0	0
normalized size	1	1.	11.61	3.76	0.	0.	0.	0.
time (sec)	N/A	0.201	6.442	0.207	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	1180	515	0	0	0	0
normalized size	1	1.	11.8	5.15	0.	0.	0.	0.
time (sec)	N/A	0.219	6.469	0.431	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	1228	739	0	0	0	0
normalized size	1	1.	8.83	5.32	0.	0.	0.	0.
time (sec)	N/A	0.254	6.541	0.582	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	1284	849	0	0	0	0
normalized size	1	1.	7.25	4.8	0.	0.	0.	0.
time (sec)	N/A	0.271	6.616	0.817	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1374	545	0	0	0	0
normalized size	1	1.	5.47	2.17	0.	0.	0.	0.
time (sec)	N/A	0.537	6.41	0.189	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	1322	514	0	0	0	0
normalized size	1	1.	6.15	2.39	0.	0.	0.	0.
time (sec)	N/A	0.504	6.357	0.188	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	1270	483	0	0	0	0
normalized size	1	1.	7.09	2.7	0.	0.	0.	0.
time (sec)	N/A	0.477	6.454	0.212	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	1039	595	0	0	0	0
normalized size	1	1.	6.04	3.46	0.	0.	0.	0.
time (sec)	N/A	0.479	6.55	0.233	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	1025	800	0	0	0	0
normalized size	1	1.	5.96	4.65	0.	0.	0.	0.
time (sec)	N/A	0.472	6.624	0.486	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	1041	906	0	0	0	0
normalized size	1	1.	5.98	5.21	0.	0.	0.	0.
time (sec)	N/A	0.487	6.724	0.597	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	1310	932	0	0	0	0
normalized size	1	1.	6.09	4.33	0.	0.	0.	0.
time (sec)	N/A	0.524	6.826	0.807	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1364	1181	0	0	0	0
normalized size	1	1.	5.43	4.71	0.	0.	0.	0.
time (sec)	N/A	0.552	6.937	1.128	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	1426	576	0	0	0	0
normalized size	1	1.	4.71	1.9	0.	0.	0.	0.
time (sec)	N/A	0.735	6.47	0.275	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	1374	545	0	0	0	0
normalized size	1	1.	5.15	2.04	0.	0.	0.	0.
time (sec)	N/A	0.685	6.418	0.232	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1322	514	0	0	0	0
normalized size	1	1.	5.72	2.23	0.	0.	0.	0.
time (sec)	N/A	0.66	6.522	0.319	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	1313	727	0	0	0	0
normalized size	1	1.	5.73	3.17	0.	0.	0.	0.
time (sec)	N/A	0.672	6.665	0.339	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	1297	950	0	0	0	0
normalized size	1	1.	5.71	4.19	0.	0.	0.	0.
time (sec)	N/A	0.661	6.764	0.688	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	1298	1328	0	0	0	0
normalized size	1	1.	5.64	5.77	0.	0.	0.	0.
time (sec)	N/A	0.678	6.808	0.893	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1317	1097	0	0	0	0
normalized size	1	1.	5.7	4.75	0.	0.	0.	0.
time (sec)	N/A	0.669	6.899	0.933	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	1364	1262	0	0	0	0
normalized size	1	1.	5.11	4.73	0.	0.	0.	0.
time (sec)	N/A	0.702	6.958	1.089	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	1418	1424	0	0	0	0
normalized size	1	1.	4.68	4.7	0.	0.	0.	0.
time (sec)	N/A	0.749	7.118	1.316	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	1752	341	0	0	0	0
normalized size	1	1.	8.34	1.62	0.	0.	0.	0.
time (sec)	N/A	0.253	6.855	0.313	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	1697	319	0	0	0	0
normalized size	1	1.	9.75	1.83	0.	0.	0.	0.
time (sec)	N/A	0.234	6.764	0.261	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	1644	300	0	0	0	0
normalized size	1	1.	12.27	2.24	0.	0.	0.	0.
time (sec)	N/A	0.203	6.609	0.272	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	1607	281	0	0	0	0
normalized size	1	1.	17.86	3.12	0.	0.	0.	0.
time (sec)	N/A	0.183	6.596	0.415	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	1642	353	0	0	0	0
normalized size	1	1.	13.14	2.82	0.	0.	0.	0.
time (sec)	N/A	0.207	6.776	0.485	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	1686	494	0	0	0	0
normalized size	1	1.	10.22	2.99	0.	0.	0.	0.
time (sec)	N/A	0.229	7.128	0.686	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	1745	812	0	0	0	0
normalized size	1	1.	8.31	3.87	0.	0.	0.	0.
time (sec)	N/A	0.247	7.477	1.003	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	1789	491	0	0	0	0
normalized size	1	1.	8.36	2.29	0.	0.	0.	0.
time (sec)	N/A	0.403	7.061	0.272	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	1741	472	0	0	0	0
normalized size	1	1.	9.67	2.62	0.	0.	0.	0.
time (sec)	N/A	0.375	6.895	0.302	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	1347	507	0	0	0	0
normalized size	1	1.	9.69	3.65	0.	0.	0.	0.
time (sec)	N/A	0.343	6.729	0.288	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	1342	507	0	0	0	0
normalized size	1	1.	10.09	3.81	0.	0.	0.	0.
time (sec)	N/A	0.347	6.707	0.289	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	1380	563	0	0	0	0
normalized size	1	1.	7.89	3.22	0.	0.	0.	0.
time (sec)	N/A	0.376	6.859	0.749	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	1782	751	0	0	0	0
normalized size	1	1.	8.45	3.56	0.	0.	0.	0.
time (sec)	N/A	0.402	7.436	0.905	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	1888	666	0	0	0	0
normalized size	1	1.	6.92	2.44	0.	0.	0.	0.
time (sec)	N/A	0.602	7.368	0.315	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	1841	638	0	0	0	0
normalized size	1	1.	7.94	2.75	0.	0.	0.	0.
time (sec)	N/A	0.575	7.127	0.314	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	1809	624	0	0	0	0
normalized size	1	1.	9.28	3.2	0.	0.	0.	0.
time (sec)	N/A	0.53	7.082	0.298	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	1799	624	0	0	0	0
normalized size	1	1.	9.42	3.27	0.	0.	0.	0.
time (sec)	N/A	0.524	6.898	0.376	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	1802	624	0	0	0	0
normalized size	1	1.	9.34	3.23	0.	0.	0.	0.
time (sec)	N/A	0.53	6.908	0.39	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	1841	793	0	0	0	0
normalized size	1	1.	7.77	3.35	0.	0.	0.	0.
time (sec)	N/A	0.58	7.213	0.367	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	1883	1040	0	0	0	0
normalized size	1	1.	6.97	3.85	0.	0.	0.	0.
time (sec)	N/A	0.615	7.936	1.267	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	144	622	0	467	0	0
normalized size	1	1.	0.63	2.74	0.	2.06	0.	0.
time (sec)	N/A	0.528	0.896	0.146	0.	7.384	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	124	514	5090	398	0	0
normalized size	1	1.	0.69	2.87	28.44	2.22	0.	0.
time (sec)	N/A	0.431	0.5	0.11	3.334	4.566	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	103	328	2695	340	0	0
normalized size	1	1.	0.79	2.5	20.57	2.6	0.	0.
time (sec)	N/A	0.355	0.359	0.151	2.75	4.576	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	210	1397	346	0	0
normalized size	1	1.	0.86	1.74	11.55	2.86	0.	0.
time (sec)	N/A	0.357	0.304	0.136	2.374	2.697	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	105	147	587	348	0	0
normalized size	1	1.	0.88	1.22	4.89	2.9	0.	0.
time (sec)	N/A	0.343	0.399	0.127	2.02	2.196	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	85	97	707	231	0	0
normalized size	1	1.	0.65	0.75	5.44	1.78	0.	0.
time (sec)	N/A	0.369	0.371	0.109	1.782	1.869	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	121	130	959	292	0	0
normalized size	1	1.	0.68	0.73	5.39	1.64	0.	0.
time (sec)	N/A	0.439	0.595	0.133	1.811	2.035	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	155	163	1145	348	0	0
normalized size	1	1.	0.69	0.72	5.07	1.54	0.	0.
time (sec)	N/A	0.518	0.854	0.129	1.816	1.994	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	170	731	0	574	0	0
normalized size	1	1.	0.6	2.58	0.	2.03	0.	0.
time (sec)	N/A	0.752	1.575	0.136	0.	6.845	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	145	623	0	493	0	0
normalized size	1	1.	0.62	2.67	0.	2.12	0.	0.
time (sec)	N/A	0.645	0.95	0.112	0.	6.819	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	125	515	5162	428	0	0
normalized size	1	1.	0.69	2.85	28.52	2.36	0.	0.
time (sec)	N/A	0.566	0.732	0.117	3.387	4.492	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	127	443	3887	431	0	0
normalized size	1	1.	0.7	2.45	21.48	2.38	0.	0.
time (sec)	N/A	0.58	0.604	0.111	2.872	4.507	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	128	302	2599	423	0	0
normalized size	1	1.	0.75	1.77	15.2	2.47	0.	0.
time (sec)	N/A	0.577	0.74	0.102	2.513	2.577	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	134	263	1808	419	0	0
normalized size	1	1.	0.78	1.53	10.51	2.44	0.	0.
time (sec)	N/A	0.53	0.85	0.106	2.179	2.231	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	122	131	815	305	0	0
normalized size	1	1.	0.66	0.71	4.43	1.66	0.	0.
time (sec)	N/A	0.579	0.703	0.114	1.757	2.045	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	157	164	1064	369	0	0
normalized size	1	1.	0.68	0.71	4.59	1.59	0.	0.
time (sec)	N/A	0.679	0.968	0.125	1.791	2.411	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	187	197	1251	440	0	0
normalized size	1	1.	0.66	0.69	4.4	1.55	0.	0.
time (sec)	N/A	0.763	1.041	0.102	1.817	2.386	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	205	841	0	679	0	0
normalized size	1	1.	0.62	2.53	0.	2.04	0.	0.
time (sec)	N/A	0.99	2.165	0.146	0.	6.932	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	171	733	0	593	0	0
normalized size	1	1.	0.61	2.61	0.	2.11	0.	0.
time (sec)	N/A	0.881	1.702	0.128	0.	5.841	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	146	625	0	520	0	0
normalized size	1	1.	0.63	2.68	0.	2.23	0.	0.
time (sec)	N/A	0.785	1.067	0.124	0.	5.817	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	156	553	5457	517	0	0
normalized size	1	1.	0.68	2.39	23.62	2.24	0.	0.
time (sec)	N/A	0.793	1.065	0.12	3.447	3.928	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	156	514	4690	508	0	0
normalized size	1	1.	0.67	2.21	20.13	2.18	0.	0.
time (sec)	N/A	0.799	1.074	0.117	3.129	3.977	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	156	479	3401	501	0	0
normalized size	1	1.	0.7	2.15	15.25	2.25	0.	0.
time (sec)	N/A	0.809	1.33	0.105	2.797	2.377	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	172	369	2415	502	0	0
normalized size	1	1.	0.77	1.66	10.88	2.26	0.	0.
time (sec)	N/A	0.724	1.551	0.107	2.231	2.089	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	158	166	921	379	0	0
normalized size	1	1.	0.68	0.71	3.94	1.62	0.	0.
time (sec)	N/A	0.809	1.216	0.123	1.734	1.825	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	190	199	1170	462	0	0
normalized size	1	1.	0.67	0.7	4.12	1.63	0.	0.
time (sec)	N/A	0.895	0.934	0.104	1.797	1.859	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	224	232	1355	547	0	0
normalized size	1	1.	0.67	0.69	4.06	1.64	0.	0.
time (sec)	N/A	0.989	1.216	0.11	1.806	1.944	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	449	613	0	0	0	0
normalized size	1	1.	1.86	2.54	0.	0.	0.	0.
time (sec)	N/A	0.845	2.924	0.125	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	431	421	0	0	0	0
normalized size	1	1.	2.21	2.16	0.	0.	0.	0.
time (sec)	N/A	0.625	2.153	0.157	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	112	240	0	477	0	0
normalized size	1	1.	0.79	1.7	0.	3.38	0.	0.
time (sec)	N/A	0.44	0.282	0.118	0.	86.897	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	266	303	0	524	0	0
normalized size	1	1.	1.93	2.2	0.	3.8	0.	0.
time (sec)	N/A	0.425	5.102	0.135	0.	67.565	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	701	379	0	458	0	0
normalized size	1	1.	4.9	2.65	0.	3.2	0.	0.
time (sec)	N/A	0.377	6.789	0.161	0.	2.081	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	1950	601	0	512	0	0
normalized size	1	1.	10.21	3.15	0.	2.68	0.	0.
time (sec)	N/A	0.559	8.	0.119	0.	2.094	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	2716	805	0	571	0	0
normalized size	1	1.	11.46	3.4	0.	2.41	0.	0.
time (sec)	N/A	0.76	10.35	0.137	0.	2.121	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	540	571	0	0	0	0
normalized size	1	1.	2.54	2.68	0.	0.	0.	0.
time (sec)	N/A	0.753	2.927	0.104	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	462	685	0	0	0	0
normalized size	1	1.	1.78	2.63	0.	0.	0.	0.
time (sec)	N/A	0.869	3.211	0.109	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	413	542	0	0	0	0
normalized size	1	1.	2.04	2.68	0.	0.	0.	0.
time (sec)	N/A	0.627	2.404	0.153	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	366	460	0	0	0	0
normalized size	1	1.	2.46	3.09	0.	0.	0.	0.
time (sec)	N/A	0.434	3.083	0.142	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	455	438	0	552	0	0
normalized size	1	1.	2.83	2.72	0.	3.43	0.	0.
time (sec)	N/A	0.418	4.65	0.149	0.	2.684	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	1207	471	0	618	0	0
normalized size	1	1.	5.67	2.21	0.	2.9	0.	0.
time (sec)	N/A	0.598	6.876	0.115	0.	2.432	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	2437	683	0	684	0	0
normalized size	1	1.	9.27	2.6	0.	2.6	0.	0.
time (sec)	N/A	0.821	8.116	0.127	0.	2.147	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	434	881	0	0	0	0
normalized size	1	1.	1.71	3.47	0.	0.	0.	0.
time (sec)	N/A	0.872	4.129	0.11	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	385	747	0	0	0	0
normalized size	1	1.	1.92	3.72	0.	0.	0.	0.
time (sec)	N/A	0.63	3.163	0.1	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	209	643	0	626	0	0
normalized size	1	1.	1.28	3.94	0.	3.84	0.	0.
time (sec)	N/A	0.441	1.889	0.144	0.	2.312	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	225	675	0	711	0	0
normalized size	1	1.	1.07	3.2	0.	3.37	0.	0.
time (sec)	N/A	0.64	2.446	0.111	0.	2.452	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	262	683	0	784	0	0
normalized size	1	1.	1.	2.62	0.	3.	0.	0.
time (sec)	N/A	0.85	4.08	0.123	0.	2.155	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	89	117	153	242	279	147
normalized size	1	1.	0.68	0.89	1.17	1.85	2.13	1.12
time (sec)	N/A	0.195	0.304	0.017	1.006	1.403	3.11	1.543

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	84	96	122	189	226	116
normalized size	1	1.	0.78	0.89	1.13	1.75	2.09	1.07
time (sec)	N/A	0.108	0.211	0.016	1.004	1.409	1.749	1.576

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	64	68	90	140	121	86
normalized size	1	1.	0.67	0.71	0.94	1.46	1.26	0.9
time (sec)	N/A	0.074	0.116	0.016	0.99	1.37	0.665	1.324

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	77	85	167	0	171
normalized size	1	1.	1.26	1.33	1.47	2.88	0.	2.95
time (sec)	N/A	0.116	0.131	0.037	0.979	1.442	0.	1.521

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	57	80	232	0	158
normalized size	1	1.	1.29	1.36	1.9	5.52	0.	3.76
time (sec)	N/A	0.106	0.019	0.042	0.991	1.436	0.	1.495

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	85	128	267	0	178
normalized size	1	1.	1.16	1.47	2.21	4.6	0.	3.07
time (sec)	N/A	0.133	0.023	0.045	1.005	1.425	0.	1.343

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	59	108	144	285	0	248
normalized size	1	1.	0.69	1.26	1.67	3.31	0.	2.88
time (sec)	N/A	0.173	0.324	0.048	1.019	1.512	0.	1.58

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	80	149	205	335	0	410
normalized size	1	1.	0.68	1.27	1.75	2.86	0.	3.5
time (sec)	N/A	0.191	0.446	0.047	0.993	1.417	0.	1.276

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	96	192	236	389	0	451
normalized size	1	1.	0.69	1.37	1.69	2.78	0.	3.22
time (sec)	N/A	0.204	0.786	0.05	1.003	1.448	0.	1.504

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	160	209	273	383	592	247
normalized size	1	1.	0.75	0.98	1.28	1.79	2.77	1.15
time (sec)	N/A	0.489	0.705	0.02	1.016	1.495	6.527	1.413

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	126	158	208	300	350	192
normalized size	1	1.	0.71	0.89	1.17	1.69	1.97	1.08
time (sec)	N/A	0.296	0.427	0.017	1.005	1.576	2.984	1.5

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	106	140	176	248	309	157
normalized size	1	1.	0.66	0.87	1.09	1.54	1.92	0.98
time (sec)	N/A	0.215	0.368	0.019	0.98	1.57	1.606	1.28

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	145	137	142	250	0	346
normalized size	1	1.	1.41	1.33	1.38	2.43	0.	3.36
time (sec)	N/A	0.279	0.249	0.044	0.987	1.557	0.	1.376

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	132	120	134	309	0	236
normalized size	1	1.	1.21	1.1	1.23	2.83	0.	2.17
time (sec)	N/A	0.315	0.713	0.051	1.017	1.461	0.	1.675

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	249	133	189	347	0	255
normalized size	1	1.	2.42	1.29	1.83	3.37	0.	2.48
time (sec)	N/A	0.313	1.283	0.053	1.009	1.6	0.	1.255

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	145	184	347	0	354
normalized size	1	1.	0.68	1.29	1.64	3.1	0.	3.16
time (sec)	N/A	0.315	0.44	0.052	1.015	1.457	0.	1.4

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	107	229	313	424	0	575
normalized size	1	1.	0.69	1.49	2.03	2.75	0.	3.73
time (sec)	N/A	0.422	0.618	0.059	1.018	1.664	0.	1.449

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	115	257	292	455	0	718
normalized size	1	1.	0.61	1.37	1.56	2.43	0.	3.84
time (sec)	N/A	0.447	1.094	0.056	1.	1.477	0.	1.283

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	252	249	328	450	668	292
normalized size	1	1.	0.95	0.94	1.24	1.7	2.53	1.11
time (sec)	N/A	0.539	0.687	0.022	1.022	1.584	7.449	1.205

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	160	201	262	367	440	235
normalized size	1	1.	0.71	0.89	1.16	1.63	1.96	1.04
time (sec)	N/A	0.341	0.682	0.018	1.01	1.438	3.888	1.195

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	180	252	225	354	0	679
normalized size	1	1.	1.08	1.51	1.35	2.12	0.	4.07
time (sec)	N/A	0.542	0.561	0.049	0.999	1.603	0.	1.337

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	185	183	190	394	0	413
normalized size	1	1.	1.11	1.1	1.14	2.36	0.	2.47
time (sec)	N/A	0.504	0.866	0.055	1.022	1.595	0.	1.709

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	285	196	242	419	0	520
normalized size	1	1.	1.7	1.17	1.44	2.49	0.	3.1
time (sec)	N/A	0.58	1.512	0.06	1.011	1.582	0.	1.307

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	377	195	244	440	0	435
normalized size	1	1.	2.31	1.2	1.5	2.7	0.	2.67
time (sec)	N/A	0.536	4.293	0.062	1.034	1.526	0.	1.329

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	127	267	352	485	0	710
normalized size	1	1.	0.7	1.47	1.93	2.66	0.	3.9
time (sec)	N/A	0.6	0.943	0.061	1.021	1.558	0.	1.373

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	150	338	400	552	0	886
normalized size	1	1.	0.66	1.49	1.76	2.43	0.	3.9
time (sec)	N/A	0.714	2.248	0.066	1.02	1.6	0.	1.527

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	184	430	521	636	0	1258
normalized size	1	1.	0.67	1.58	1.91	2.33	0.	4.61
time (sec)	N/A	0.786	1.665	0.064	1.052	1.554	0.	1.643

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	351	332	444	595	850	392
normalized size	1	1.	1.02	0.96	1.29	1.72	2.46	1.14
time (sec)	N/A	0.863	0.843	0.021	0.999	1.623	13.185	1.263

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	301	294	382	504	748	333
normalized size	1	1.	1.	0.98	1.27	1.67	2.49	1.11
time (sec)	N/A	0.528	0.841	0.02	1.013	1.637	8.081	1.682

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	226	364	313	473	0	1017
normalized size	1	1.	1.	1.6	1.38	2.08	0.	4.48
time (sec)	N/A	0.793	1.038	0.054	1.006	1.629	0.	1.788

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	274	296	275	504	0	753
normalized size	1	1.	1.2	1.29	1.2	2.2	0.	3.29
time (sec)	N/A	0.802	1.341	0.061	1.079	1.699	0.	1.712

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	323	259	298	516	0	535
normalized size	1	1.	1.47	1.18	1.36	2.36	0.	2.44
time (sec)	N/A	0.903	3.1	0.065	1.063	1.655	0.	1.385

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	412	258	298	509	0	536
normalized size	1	1.	1.64	1.03	1.19	2.03	0.	2.14
time (sec)	N/A	0.962	6.017	0.069	1.025	1.674	0.	1.717

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	612	316	413	575	0	797
normalized size	1	1.	2.49	1.28	1.68	2.34	0.	3.24
time (sec)	N/A	0.935	6.334	0.071	1.042	1.532	0.	1.539

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	169	377	439	610	0	1050
normalized size	1	1.	0.68	1.51	1.76	2.44	0.	4.2
time (sec)	N/A	0.902	1.081	0.07	1.025	1.622	0.	1.321

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	204	511	629	716	0	1485
normalized size	1	1.	0.66	1.66	2.05	2.33	0.	4.84
time (sec)	N/A	1.125	4.906	0.073	1.038	1.727	0.	1.553

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	233	591	637	783	0	1728
normalized size	1	1.	0.66	1.66	1.79	2.21	0.	4.87
time (sec)	N/A	1.237	2.201	0.071	1.047	1.662	0.	1.466

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	139	151	197	308	321	198
normalized size	1	1.	0.76	0.83	1.08	1.68	1.75	1.08
time (sec)	N/A	0.323	0.586	0.026	1.008	1.494	2.752	1.426

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	89	87	113	188	190	123
normalized size	1	1.	0.69	0.67	0.88	1.46	1.47	0.95
time (sec)	N/A	0.201	0.22	0.021	0.998	1.407	1.33	1.531

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	75	75	99	149	131	100
normalized size	1	1.	0.82	0.82	1.08	1.62	1.42	1.09
time (sec)	N/A	0.112	0.168	0.018	0.995	1.387	0.963	1.384

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	194	1060	0	1312	0	775
normalized size	1	1.	0.83	4.55	0.	5.63	0.	3.33
time (sec)	N/A	0.786	0.637	0.062	0.	1.774	0.	1.339

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	152	551	0	1050	0	440
normalized size	1	1.	0.86	3.11	0.	5.93	0.	2.49
time (sec)	N/A	0.474	0.436	0.033	0.	1.689	0.	1.279

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	126	117	296	0	832	0	269
normalized size	1	0.98	0.91	2.31	0.	6.5	0.	2.1
time (sec)	N/A	0.258	0.404	0.033	0.	1.671	0.	1.226

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	149	0	640	0	184
normalized size	1	1.	0.95	1.73	0.	7.44	0.	2.14
time (sec)	N/A	0.126	0.21	0.028	0.	1.566	0.	1.24

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	234	158	0	786	0	193
normalized size	1	1.	2.66	1.8	0.	8.93	0.	2.19
time (sec)	N/A	0.131	0.553	0.057	0.	5.247	0.	1.365

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	306	183	0	952	0	221
normalized size	1	1.	3.22	1.93	0.	10.02	0.	2.33
time (sec)	N/A	0.225	2.183	0.064	0.	5.591	0.	1.447

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	399	362	0	1239	0	327
normalized size	1	1.	2.91	2.64	0.	9.04	0.	2.39
time (sec)	N/A	0.474	2.037	0.068	0.	16.64	0.	1.768

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	413	554	0	1472	0	502
normalized size	1	1.	2.24	3.01	0.	8.	0.	2.73
time (sec)	N/A	0.741	2.881	0.072	0.	16.469	0.	1.42

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	215	828	0	2171	0	593
normalized size	1	1.	0.65	2.49	0.	6.54	0.	1.79
time (sec)	N/A	1.118	1.119	0.043	0.	2.086	0.	1.469

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	178	569	0	1804	0	420
normalized size	1	1.	0.68	2.17	0.	6.89	0.	1.6
time (sec)	N/A	0.69	0.98	0.039	0.	1.916	0.	1.724

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	359	0	1388	0	478
normalized size	1	1.	0.94	2.49	0.	9.64	0.	3.32
time (sec)	N/A	0.346	1.035	0.037	0.	1.735	0.	1.438

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	123	320	0	1172	0	271
normalized size	1	1.	0.98	2.54	0.	9.3	0.	2.15
time (sec)	N/A	0.196	0.679	0.029	0.	1.63	0.	1.501

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	306	342	0	1497	0	305
normalized size	1	1.	2.28	2.55	0.	11.17	0.	2.28
time (sec)	N/A	0.332	1.826	0.068	0.	14.024	0.	1.268

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	219	394	0	1887	0	516
normalized size	1	1.	1.22	2.19	0.	10.48	0.	2.87
time (sec)	N/A	0.582	1.743	0.076	0.	20.901	0.	1.306

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	712	638	0	2538	0	477
normalized size	1	1.	2.69	2.41	0.	9.58	0.	1.8
time (sec)	N/A	1.064	6.321	0.089	0.	79.708	0.	1.572

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	593	830	0	2917	0	652
normalized size	1	1.	1.77	2.48	0.	8.71	0.	1.95
time (sec)	N/A	1.47	6.281	0.089	0.	70.918	0.	1.693

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	256	1428	0	3391	0	1544
normalized size	1	1.	0.69	3.84	0.	9.12	0.	4.15
time (sec)	N/A	1.489	2.399	0.046	0.	2.997	0.	1.721

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	214	1094	0	2638	0	660
normalized size	1	1.	0.82	4.18	0.	10.07	0.	2.52
time (sec)	N/A	0.841	1.704	0.04	0.	2.535	0.	1.725

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	194	1093	0	2279	0	647
normalized size	1	1.	0.96	5.38	0.	11.23	0.	3.19
time (sec)	N/A	0.464	1.286	0.04	0.	2.277	0.	1.335

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	170	810	0	1562	0	498
normalized size	1	1.	0.96	4.58	0.	8.82	0.	2.81
time (sec)	N/A	0.263	0.786	0.033	0.	2.043	0.	1.388

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	409	1115	0	2851	0	680
normalized size	1	1.	1.94	5.28	0.	13.51	0.	3.22
time (sec)	N/A	0.649	3.58	0.073	0.	46.969	0.	1.686

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	649	1129	0	3401	0	698
normalized size	1	1.	2.36	4.11	0.	12.37	0.	2.54
time (sec)	N/A	1.189	6.333	0.085	0.	82.808	0.	1.634

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	856	1497	0	0	0	1608
normalized size	1	1.	2.26	3.96	0.	0.	0.	4.25
time (sec)	N/A	1.876	6.384	0.099	0.	0.	0.	1.831

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	514	1452	2919	0	5520	0	1389
normalized size	1	1.	2.82	5.68	0.	10.74	0.	2.7
time (sec)	N/A	2.21	6.688	0.05	0.	5.455	0.	1.954

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	849	2199	0	4302	0	1142
normalized size	1	1.	2.3	5.96	0.	11.66	0.	3.09
time (sec)	N/A	1.579	4.082	0.049	0.	3.892	0.	1.849

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	723	2314	0	3852	0	1141
normalized size	1	1.	2.38	7.61	0.	12.67	0.	3.75
time (sec)	N/A	1.084	6.235	0.043	0.	3.153	0.	1.802

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	224	1727	0	2457	0	930
normalized size	1	1.	0.86	6.62	0.	9.41	0.	3.56
time (sec)	N/A	0.567	1.152	0.039	0.	2.423	0.	1.712

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	224	1726	0	2456	0	930
normalized size	1	1.	0.89	6.85	0.	9.75	0.	3.69
time (sec)	N/A	0.463	1.081	0.036	0.	2.391	0.	1.392

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	498	2337	0	4764	0	1172
normalized size	1	1.	1.65	7.76	0.	15.83	0.	3.89
time (sec)	N/A	1.302	5.248	0.082	0.	159.947	0.	1.498

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	515	2234	0	0	0	1176
normalized size	1	1.	1.37	5.94	0.	0.	0.	3.13
time (sec)	N/A	2.127	3.109	0.097	0.	0.	0.	1.702

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	740	2988	0	0	0	1445
normalized size	1	1.	1.42	5.72	0.	0.	0.	2.77
time (sec)	N/A	2.659	5.916	0.119	0.	0.	0.	1.935

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	168	653	0	842	0	500
normalized size	1	1.	0.87	3.38	0.	4.36	0.	2.59
time (sec)	N/A	0.615	0.967	0.035	0.	1.883	0.	1.485

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	125	350	0	703	0	309
normalized size	1	1.	0.83	2.33	0.	4.69	0.	2.06
time (sec)	N/A	0.39	0.423	0.031	0.	1.779	0.	1.651

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	98	269	0	586	0	251
normalized size	1	1.	0.9	2.47	0.	5.38	0.	2.3
time (sec)	N/A	0.2	0.304	0.024	0.	1.467	0.	1.359

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	69	145	0	475	0	165
normalized size	1	1.	0.95	1.99	0.	6.51	0.	2.26
time (sec)	N/A	0.105	0.135	0.022	0.	1.433	0.	1.563

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	115	153	0	0	0	176
normalized size	1	1.	1.51	2.01	0.	0.	0.	2.32
time (sec)	N/A	0.117	0.136	0.043	0.	0.	0.	1.345

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	112	177	0	748	0	203
normalized size	1	1.	1.37	2.16	0.	9.12	0.	2.48
time (sec)	N/A	0.206	0.271	0.049	0.	1.674	0.	1.291

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	236	309	0	907	0	296
normalized size	1	1.	2.02	2.64	0.	7.75	0.	2.53
time (sec)	N/A	0.374	1.1	0.052	0.	1.884	0.	1.399

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	256	407	0	1045	0	359
normalized size	1	1.	1.65	2.63	0.	6.74	0.	2.32
time (sec)	N/A	0.57	2.51	0.057	0.	2.046	0.	1.243

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	271	708	0	1634	0	568
normalized size	1	1.	1.14	2.99	0.	6.89	0.	2.4
time (sec)	N/A	0.898	3.75	0.038	0.	2.029	0.	1.257

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	217	403	0	1374	0	378
normalized size	1	1.	1.15	2.13	0.	7.27	0.	2.
time (sec)	N/A	0.616	2.25	0.035	0.	1.719	0.	1.245

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	131	321	0	1191	0	321
normalized size	1	1.	0.85	2.08	0.	7.73	0.	2.08
time (sec)	N/A	0.397	0.292	0.034	0.	1.627	0.	1.677

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	132	198	0	1002	0	279
normalized size	1	1.	1.18	1.77	0.	8.95	0.	2.49
time (sec)	N/A	0.258	0.999	0.032	0.	1.606	0.	1.545

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	116	0	829	0	189
normalized size	1	1.	0.94	1.36	0.	9.75	0.	2.22
time (sec)	N/A	0.118	0.233	0.024	0.	1.555	0.	1.39

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	123	137	0	1056	0	223
normalized size	1	1.	1.31	1.46	0.	11.23	0.	2.37
time (sec)	N/A	0.17	0.19	0.047	0.	1.946	0.	1.613

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	143	231	0	1403	0	317
normalized size	1	1.	1.21	1.96	0.	11.89	0.	2.69
time (sec)	N/A	0.399	0.623	0.058	0.	2.176	0.	1.594

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	271	364	0	1688	0	363
normalized size	1	1.	1.69	2.28	0.	10.55	0.	2.27
time (sec)	N/A	0.672	3.423	0.065	0.	3.349	0.	1.814

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	475	458	0	1886	0	427
normalized size	1	1.	2.44	2.35	0.	9.67	0.	2.19
time (sec)	N/A	0.916	6.211	0.069	0.	2.788	0.	1.506

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	979	786	0	2471	0	587
normalized size	1	1.	3.	2.41	0.	7.58	0.	1.8
time (sec)	N/A	1.047	7.203	0.039	0.	2.169	0.	1.71

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	374	704	0	2171	0	814
normalized size	1	1.	1.4	2.63	0.	8.1	0.	3.04
time (sec)	N/A	0.751	5.001	0.04	0.	2.065	0.	1.419

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	159	576	0	1879	0	450
normalized size	1	1.	0.87	3.16	0.	10.32	0.	2.47
time (sec)	N/A	0.458	0.978	0.037	0.	1.849	0.	1.738

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	291	475	0	1628	0	392
normalized size	1	1.	1.95	3.19	0.	10.93	0.	2.63
time (sec)	N/A	0.292	1.592	0.033	0.	1.66	0.	1.685

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	94	160	0	1015	0	239
normalized size	1	1.	0.8	1.37	0.	8.68	0.	2.04
time (sec)	N/A	0.13	0.28	0.021	0.	1.248	0.	2.083

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	180	496	0	2036	0	420
normalized size	1	1.	1.16	3.2	0.	13.14	0.	2.71
time (sec)	N/A	0.412	0.949	0.053	0.	3.271	0.	1.62

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	200	609	0	2483	0	482
normalized size	1	1.	0.98	2.99	0.	12.17	0.	2.36
time (sec)	N/A	0.735	2.711	0.067	0.	4.45	0.	1.691

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	414	747	0	2919	0	860
normalized size	1	1.	1.53	2.76	0.	10.77	0.	3.17
time (sec)	N/A	1.048	6.207	0.076	0.	6.817	0.	1.457

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	563	843	0	3258	0	636
normalized size	1	1.	1.68	2.52	0.	9.73	0.	1.9
time (sec)	N/A	1.398	6.247	0.08	0.	7.263	0.	1.548

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	28	22	0	38	32	53
normalized size	1	1.	1.75	1.38	0.	2.38	2.	3.31
time (sec)	N/A	0.045	0.01	0.025	0.	1.315	0.801	1.358

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	61	0	478	0	115
normalized size	1	1.	0.98	1.13	0.	8.85	0.	2.13
time (sec)	N/A	0.1	0.069	0.033	0.	1.496	0.	1.68

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	91	177	0	783	0	193
normalized size	1	1.	0.98	1.9	0.	8.42	0.	2.08
time (sec)	N/A	0.117	0.244	0.037	0.	1.479	0.	1.491

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	116	547	0	1287	0	344
normalized size	1	1.	0.83	3.91	0.	9.19	0.	2.46
time (sec)	N/A	0.233	0.624	0.038	0.	1.688	0.	1.77

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	269	1527	0	0	0	0
normalized size	1	1.	0.74	4.2	0.	0.	0.	0.
time (sec)	N/A	0.806	1.297	0.457	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	216	1131	0	0	0	0
normalized size	1	1.	0.74	3.89	0.	0.	0.	0.
time (sec)	N/A	0.48	0.841	0.36	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	181	821	0	0	0	0
normalized size	1	1.	0.83	3.77	0.	0.	0.	0.
time (sec)	N/A	0.326	0.93	0.355	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	371	601	0	0	0	0
normalized size	1	1.	1.61	2.6	0.	0.	0.	0.
time (sec)	N/A	0.647	2.359	0.383	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	374	833	0	0	0	0
normalized size	1	1.	1.82	4.06	0.	0.	0.	0.
time (sec)	N/A	0.62	2.254	0.537	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	406	1264	0	0	0	0
normalized size	1	1.	1.47	4.56	0.	0.	0.	0.
time (sec)	N/A	0.99	3.39	0.908	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	601	2309	0	0	0	0
normalized size	1	1.	1.65	6.33	0.	0.	0.	0.
time (sec)	N/A	1.36	6.5	1.423	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	331	1791	0	0	0	0
normalized size	1	1.	0.75	4.04	0.	0.	0.	0.
time (sec)	N/A	1.063	1.75	0.428	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	269	1527	0	0	0	0
normalized size	1	1.	0.76	4.29	0.	0.	0.	0.
time (sec)	N/A	0.653	1.319	0.378	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	224	1131	0	0	0	0
normalized size	1	1.	0.79	3.97	0.	0.	0.	0.
time (sec)	N/A	0.469	0.837	0.403	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	421	962	0	0	0	0
normalized size	1	1.	1.5	3.42	0.	0.	0.	0.
time (sec)	N/A	0.923	3.348	0.415	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	406	1221	0	0	0	0
normalized size	1	1.	1.5	4.52	0.	0.	0.	0.
time (sec)	N/A	0.932	3.516	0.44	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	411	1526	0	0	0	0
normalized size	1	1.	1.49	5.53	0.	0.	0.	0.
time (sec)	N/A	0.951	4.759	0.46	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	607	2424	0	0	0	0
normalized size	1	1.	1.66	6.64	0.	0.	0.	0.
time (sec)	N/A	1.441	6.572	1.127	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	696	3534	0	0	0	0
normalized size	1	1.	1.6	8.11	0.	0.	0.	0.
time (sec)	N/A	1.806	6.739	1.719	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	395	2223	0	0	0	0
normalized size	1	1.	0.76	4.25	0.	0.	0.	0.
time (sec)	N/A	1.28	2.58	0.424	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	328	1791	0	0	0	0
normalized size	1	1.	0.75	4.12	0.	0.	0.	0.
time (sec)	N/A	0.879	1.647	0.434	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	274	1527	0	0	0	0
normalized size	1	1.	0.78	4.36	0.	0.	0.	0.
time (sec)	N/A	0.649	1.319	0.492	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	468	1209	0	0	0	0
normalized size	1	1.	1.37	3.54	0.	0.	0.	0.
time (sec)	N/A	1.223	3.085	0.456	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	462	1714	0	0	0	0
normalized size	1	1.	1.41	5.24	0.	0.	0.	0.
time (sec)	N/A	1.274	3.55	0.466	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	445	1897	0	0	0	0
normalized size	1	1.	1.35	5.77	0.	0.	0.	0.
time (sec)	N/A	1.273	4.149	0.517	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	477	2673	0	0	0	0
normalized size	1	1.	1.31	7.36	0.	0.	0.	0.
time (sec)	N/A	1.416	5.967	1.378	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	704	3651	0	0	0	0
normalized size	1	1.	1.61	8.35	0.	0.	0.	0.
time (sec)	N/A	1.875	6.827	1.708	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	212	824	0	0	0	0
normalized size	1	1.	0.86	3.35	0.	0.	0.	0.
time (sec)	N/A	0.458	1.164	0.437	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	178	662	0	0	0	0
normalized size	1	1.	0.9	3.36	0.	0.	0.	0.
time (sec)	N/A	0.338	0.854	0.395	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	272	1527	0	0	0	0
normalized size	1	1.	0.72	4.04	0.	0.	0.	0.
time (sec)	N/A	0.927	1.397	0.422	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	217	1131	0	0	0	0
normalized size	1	1.	0.71	3.71	0.	0.	0.	0.
time (sec)	N/A	0.589	0.882	0.393	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	190	892	0	0	0	0
normalized size	1	1.	0.82	3.83	0.	0.	0.	0.
time (sec)	N/A	0.348	0.929	0.41	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	148	532	0	0	0	0
normalized size	1	1.	0.85	3.06	0.	0.	0.	0.
time (sec)	N/A	0.206	0.684	0.378	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	0	249	0	0	0	0
normalized size	1	1.	0.	1.36	0.	0.	0.	0.
time (sec)	N/A	0.411	8.426	0.434	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	559	638	0	0	0	0
normalized size	1	1.	2.61	2.98	0.	0.	0.	0.
time (sec)	N/A	0.63	11.77	0.672	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	603	814	0	0	0	0
normalized size	1	1.	2.17	2.93	0.	0.	0.	0.
time (sec)	N/A	0.884	6.732	0.661	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	604	1562	0	0	0	0
normalized size	1	1.	1.63	4.22	0.	0.	0.	0.
time (sec)	N/A	1.319	6.746	1.063	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	358	1788	0	0	0	0
normalized size	1	1.	0.76	3.78	0.	0.	0.	0.
time (sec)	N/A	1.13	1.613	1.828	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	289	1289	0	0	0	0
normalized size	1	1.	0.77	3.44	0.	0.	0.	0.
time (sec)	N/A	0.735	1.606	1.456	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	209	885	0	0	0	0
normalized size	1	1.	0.82	3.46	0.	0.	0.	0.
time (sec)	N/A	0.424	1.393	1.092	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	166	490	0	0	0	0
normalized size	1	1.	0.82	2.43	0.	0.	0.	0.
time (sec)	N/A	0.247	0.669	0.949	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	0	539	0	0	0	0
normalized size	1	1.	0.	2.08	0.	0.	0.	0.
time (sec)	N/A	0.702	29.745	0.821	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	511	904	0	0	0	0
normalized size	1	1.	1.73	3.05	0.	0.	0.	0.
time (sec)	N/A	0.968	6.183	1.043	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	727	1561	0	0	0	0
normalized size	1	1.	1.96	4.22	0.	0.	0.	0.
time (sec)	N/A	1.341	6.801	1.335	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	521	521	350	1735	0	0	0	0
normalized size	1	1.	0.67	3.33	0.	0.	0.	0.
time (sec)	N/A	1.336	3.564	2.425	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	306	1323	0	0	0	0
normalized size	1	1.	0.78	3.38	0.	0.	0.	0.
time (sec)	N/A	0.83	2.717	1.934	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	227	926	0	0	0	0
normalized size	1	1.	0.72	2.95	0.	0.	0.	0.
time (sec)	N/A	0.51	2.133	1.764	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	205	856	0	0	0	0
normalized size	1	1.	0.69	2.87	0.	0.	0.	0.
time (sec)	N/A	0.39	1.764	1.582	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	375	375	0	875	0	0	0	0
normalized size	1	1.	0.	2.33	0.	0.	0.	0.
time (sec)	N/A	1.081	36.646	1.57	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	786	1331	0	0	0	0
normalized size	1	1.	1.89	3.2	0.	0.	0.	0.
time (sec)	N/A	1.405	7.032	2.185	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	314	1305	0	0	0	0
normalized size	1	1.	0.81	3.35	0.	0.	0.	0.
time (sec)	N/A	0.605	2.568	2.747	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	450	0	0	0	0
normalized size	1	1.	0.85	2.87	0.	0.	0.	0.
time (sec)	N/A	0.233	0.498	0.718	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	83	218	0	0	0	0
normalized size	1	1.	0.72	1.88	0.	0.	0.	0.
time (sec)	N/A	0.157	0.166	0.475	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	134	371	0	0	0	0
normalized size	1	1.	0.81	2.25	0.	0.	0.	0.
time (sec)	N/A	0.239	0.355	0.533	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	158	792	0	0	0	0
normalized size	1	1.	0.65	3.26	0.	0.	0.	0.
time (sec)	N/A	0.341	0.98	1.384	0.	0.	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	134	481	0	0	0	0
normalized size	1	1.	0.68	2.45	0.	0.	0.	0.
time (sec)	N/A	0.237	1.648	0.355	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	119	443	0	0	0	0
normalized size	1	1.	0.72	2.68	0.	0.	0.	0.
time (sec)	N/A	0.207	0.912	0.431	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	98	401	0	0	0	0
normalized size	1	1.	0.73	2.99	0.	0.	0.	0.
time (sec)	N/A	0.185	0.678	0.506	0.	0.	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	363	0	0	0	0
normalized size	1	1.	0.78	3.59	0.	0.	0.	0.
time (sec)	N/A	0.173	0.416	0.387	0.	0.	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	78	294	0	0	0	0
normalized size	1	1.	0.82	3.09	0.	0.	0.	0.
time (sec)	N/A	0.173	0.468	0.388	0.	0.	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	76	614	0	0	0	0
normalized size	1	1.	0.8	6.46	0.	0.	0.	0.
time (sec)	N/A	0.186	0.759	0.891	0.	0.	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	122	732	0	0	0	0
normalized size	1	1.	0.92	5.55	0.	0.	0.	0.
time (sec)	N/A	0.195	0.733	1.153	0.	0.	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	160	841	0	0	0	0
normalized size	1	1.	0.97	5.1	0.	0.	0.	0.
time (sec)	N/A	0.215	0.774	1.403	0.	0.	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	187	649	0	0	0	0
normalized size	1	1.	0.74	2.56	0.	0.	0.	0.
time (sec)	N/A	0.48	1.504	0.502	0.	0.	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	148	587	0	0	0	0
normalized size	1	1.	0.72	2.86	0.	0.	0.	0.
time (sec)	N/A	0.424	1.186	0.401	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	126	532	0	0	0	0
normalized size	1	1.	0.74	3.11	0.	0.	0.	0.
time (sec)	N/A	0.409	0.985	0.4	0.	0.	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	119	694	0	0	0	0
normalized size	1	1.	0.72	4.18	0.	0.	0.	0.
time (sec)	N/A	0.406	1.03	0.457	0.	0.	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	108	871	0	0	0	0
normalized size	1	1.	0.7	5.66	0.	0.	0.	0.
time (sec)	N/A	0.4	1.417	0.493	0.	0.	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	158	913	0	0	0	0
normalized size	1	1.	0.93	5.4	0.	0.	0.	0.
time (sec)	N/A	0.408	0.833	1.161	0.	0.	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	198	930	0	0	0	0
normalized size	1	1.	0.98	4.58	0.	0.	0.	0.
time (sec)	N/A	0.449	1.145	1.512	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	215	793	0	0	0	0
normalized size	1	1.	0.73	2.69	0.	0.	0.	0.
time (sec)	N/A	0.76	1.569	0.395	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	181	718	0	0	0	0
normalized size	1	1.	0.74	2.93	0.	0.	0.	0.
time (sec)	N/A	0.702	1.525	0.516	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	172	943	0	0	0	0
normalized size	1	1.	0.7	3.86	0.	0.	0.	0.
time (sec)	N/A	0.738	1.77	0.624	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	150	1267	0	0	0	0
normalized size	1	1.	0.69	5.81	0.	0.	0.	0.
time (sec)	N/A	0.691	1.694	1.385	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	196	1333	0	0	0	0
normalized size	1	1.	0.86	5.82	0.	0.	0.	0.
time (sec)	N/A	0.679	1.361	1.411	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	241	1113	0	0	0	0
normalized size	1	1.	0.99	4.58	0.	0.	0.	0.
time (sec)	N/A	0.725	1.515	1.839	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	250	1270	0	0	0	0
normalized size	1	1.	0.85	4.33	0.	0.	0.	0.
time (sec)	N/A	0.788	5.287	2.273	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	281	1017	0	0	0	0
normalized size	1	1.	0.74	2.66	0.	0.	0.	0.
time (sec)	N/A	1.154	3.016	0.531	0.	0.	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	243	924	0	0	0	0
normalized size	1	1.	0.74	2.81	0.	0.	0.	0.
time (sec)	N/A	1.086	2.198	0.465	0.	0.	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	216	1209	0	0	0	0
normalized size	1	1.	0.68	3.78	0.	0.	0.	0.
time (sec)	N/A	1.179	3.877	0.626	0.	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	206	1715	0	0	0	0
normalized size	1	1.	0.69	5.72	0.	0.	0.	0.
time (sec)	N/A	1.149	2.739	1.575	0.	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	233	1622	0	0	0	0
normalized size	1	1.	0.73	5.05	0.	0.	0.	0.
time (sec)	N/A	1.225	1.576	1.729	0.	0.	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	276	1531	0	0	0	0
normalized size	1	1.	0.87	4.84	0.	0.	0.	0.
time (sec)	N/A	1.143	2.536	1.888	0.	0.	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	268	1451	0	0	0	0
normalized size	1	1.	0.82	4.46	0.	0.	0.	0.
time (sec)	N/A	1.156	5.164	2.263	0.	0.	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	284	1521	0	0	0	0
normalized size	1	1.	0.75	4.03	0.	0.	0.	0.
time (sec)	N/A	1.233	4.451	2.655	0.	0.	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	364	1554	0	0	0	0
normalized size	1	1.	1.22	5.2	0.	0.	0.	0.
time (sec)	N/A	1.502	2.583	0.716	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	293	1244	0	0	0	0
normalized size	1	1.	1.23	5.21	0.	0.	0.	0.
time (sec)	N/A	1.12	2.261	0.577	0.	0.	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	248	948	0	0	0	0
normalized size	1	1.	1.37	5.24	0.	0.	0.	0.
time (sec)	N/A	0.789	2.284	0.558	0.	0.	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	200	686	0	0	0	0
normalized size	1	1.	1.54	5.28	0.	0.	0.	0.
time (sec)	N/A	0.534	1.728	0.454	0.	0.	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	131	259	0	0	0	0
normalized size	1	1.	1.54	3.05	0.	0.	0.	0.
time (sec)	N/A	0.272	0.893	0.442	0.	0.	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	209	407	0	0	0	0
normalized size	1	1.	1.87	3.63	0.	0.	0.	0.
time (sec)	N/A	0.501	3.747	1.113	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	223	463	0	0	0	0
normalized size	1	1.	1.59	3.31	0.	0.	0.	0.
time (sec)	N/A	0.73	5.725	1.135	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	299	786	0	0	0	0
normalized size	1	1.	1.45	3.82	0.	0.	0.	0.
time (sec)	N/A	1.073	4.195	1.759	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	342	982	0	0	0	0
normalized size	1	1.	1.27	3.64	0.	0.	0.	0.
time (sec)	N/A	1.494	3.923	2.205	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	431	1320	0	0	0	0
normalized size	1	1.	1.25	3.84	0.	0.	0.	0.
time (sec)	N/A	1.936	4.112	2.906	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	358	1337	0	0	0	0
normalized size	1	1.	0.97	3.61	0.	0.	0.	0.
time (sec)	N/A	1.376	4.177	1.872	0.	0.	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	303	1102	0	0	0	0
normalized size	1	1.	1.04	3.77	0.	0.	0.	0.
time (sec)	N/A	0.969	2.876	1.661	0.	0.	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	284	834	0	0	0	0
normalized size	1	1.	1.31	3.84	0.	0.	0.	0.
time (sec)	N/A	0.662	3.241	1.713	0.	0.	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	273	804	0	0	0	0
normalized size	1	1.	1.28	3.76	0.	0.	0.	0.
time (sec)	N/A	0.705	2.416	1.197	0.	0.	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	310	899	0	0	0	0
normalized size	1	1.	1.15	3.33	0.	0.	0.	0.
time (sec)	N/A	1.066	4.047	1.803	0.	0.	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	336	1019	0	0	0	0
normalized size	1	1.	1.	3.03	0.	0.	0.	0.
time (sec)	N/A	1.453	5.231	2.176	0.	0.	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	496	1353	0	0	0	0
normalized size	1	1.	1.16	3.17	0.	0.	0.	0.
time (sec)	N/A	1.807	7.018	2.956	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	432	2240	0	0	0	0
normalized size	1	1.	1.	5.17	0.	0.	0.	0.
time (sec)	N/A	1.601	4.404	2.78	0.	0.	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	372	1966	0	0	0	0
normalized size	1	1.	1.08	5.7	0.	0.	0.	0.
time (sec)	N/A	1.141	4.518	2.484	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	368	1934	0	0	0	0
normalized size	1	1.	1.06	5.56	0.	0.	0.	0.
time (sec)	N/A	1.13	3.621	2.326	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	370	1846	0	0	0	0
normalized size	1	1.	1.07	5.35	0.	0.	0.	0.
time (sec)	N/A	1.073	4.903	2.148	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	429	2023	0	0	0	0
normalized size	1	1.	1.03	4.85	0.	0.	0.	0.
time (sec)	N/A	1.547	5.371	2.79	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	547	2140	0	0	0	0
normalized size	1	1.	1.11	4.33	0.	0.	0.	0.
time (sec)	N/A	2.096	7.233	3.81	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	553	553	1220	2526	0	0	0	0
normalized size	1	1.	2.21	4.57	0.	0.	0.	0.
time (sec)	N/A	1.532	6.306	0.291	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	455	455	1169	2165	0	0	0	0
normalized size	1	1.	2.57	4.76	0.	0.	0.	0.
time (sec)	N/A	1.083	8.742	0.191	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	1166	1585	0	0	0	0
normalized size	1	1.	2.66	3.61	0.	0.	0.	0.
time (sec)	N/A	1.061	20.104	0.212	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	394	394	318	1753	0	0	0	0
normalized size	1	1.	0.81	4.45	0.	0.	0.	0.
time (sec)	N/A	0.769	8.745	0.247	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	1288	2434	0	0	0	0
normalized size	1	1.	3.73	7.06	0.	0.	0.	0.
time (sec)	N/A	0.84	6.337	0.203	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	415	415	1373	2767	0	0	0	0
normalized size	1	1.	3.31	6.67	0.	0.	0.	0.
time (sec)	N/A	1.189	6.41	0.26	0.	0.	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	1270	3800	0	0	0	0
normalized size	1	1.	1.99	5.96	0.	0.	0.	0.
time (sec)	N/A	1.995	6.377	0.354	0.	0.	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	553	553	1221	2716	0	0	0	0
normalized size	1	1.	2.21	4.91	0.	0.	0.	0.
time (sec)	N/A	1.581	6.415	0.329	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	1209	2610	0	0	0	0
normalized size	1	1.	2.38	5.13	0.	0.	0.	0.
time (sec)	N/A	1.553	6.428	0.245	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	500	500	1219	2126	0	0	0	0
normalized size	1	1.	2.44	4.25	0.	0.	0.	0.
time (sec)	N/A	1.525	6.398	0.172	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	465	465	1296	2819	0	0	0	0
normalized size	1	1.	2.79	6.06	0.	0.	0.	0.
time (sec)	N/A	1.176	6.495	0.19	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	1371	2971	0	0	0	0
normalized size	1	1.	3.28	7.11	0.	0.	0.	0.
time (sec)	N/A	1.232	6.486	0.248	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	1485	4110	0	0	0	0
normalized size	1	1.	2.96	8.19	0.	0.	0.	0.
time (sec)	N/A	1.736	6.62	0.395	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	746	746	1341	4724	0	0	0	0
normalized size	1	1.	1.8	6.33	0.	0.	0.	0.
time (sec)	N/A	2.758	6.482	0.546	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	635	635	1275	3991	0	0	0	0
normalized size	1	1.	2.01	6.29	0.	0.	0.	0.
time (sec)	N/A	2.058	6.573	0.555	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	609	609	1262	3513	0	0	0	0
normalized size	1	1.	2.07	5.77	0.	0.	0.	0.
time (sec)	N/A	2.114	6.589	0.35	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	567	1256	3195	0	0	0	0
normalized size	1	1.	2.22	5.63	0.	0.	0.	0.
time (sec)	N/A	1.981	6.545	0.194	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	606	606	1309	3489	0	0	0	0
normalized size	1	1.	2.16	5.76	0.	0.	0.	0.
time (sec)	N/A	2.031	6.576	0.22	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	540	540	1378	3373	0	0	0	0
normalized size	1	1.	2.55	6.25	0.	0.	0.	0.
time (sec)	N/A	1.562	6.635	0.264	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	504	504	1485	4330	0	0	0	0
normalized size	1	1.	2.95	8.59	0.	0.	0.	0.
time (sec)	N/A	1.758	6.694	0.376	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	587	587	1591	4694	0	0	0	0
normalized size	1	1.	2.71	8.	0.	0.	0.	0.
time (sec)	N/A	2.545	6.849	0.575	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	554	554	1216	2336	0	0	0	0
normalized size	1	1.	2.19	4.22	0.	0.	0.	0.
time (sec)	N/A	1.523	13.543	0.233	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	455	455	1169	1635	0	0	0	0
normalized size	1	1.	2.57	3.59	0.	0.	0.	0.
time (sec)	N/A	1.055	12.006	0.161	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	342	939	0	0	0	0
normalized size	1	1.	0.87	2.39	0.	0.	0.	0.
time (sec)	N/A	0.728	15.108	0.123	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	2642	992	0	0	0	0
normalized size	1	1.	7.7	2.89	0.	0.	0.	0.
time (sec)	N/A	0.445	16.942	0.191	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	349	1185	0	0	0	0
normalized size	1	1.	1.23	4.19	0.	0.	0.	0.
time (sec)	N/A	0.508	11.936	0.172	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	1298	2235	0	0	0	0
normalized size	1	1.	3.67	6.31	0.	0.	0.	0.
time (sec)	N/A	0.81	6.414	0.18	0.	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	1376	2767	0	0	0	0
normalized size	1	1.	3.21	6.45	0.	0.	0.	0.
time (sec)	N/A	1.198	6.464	0.24	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	604	604	1276	3546	0	0	0	0
normalized size	1	1.	2.11	5.87	0.	0.	0.	0.
time (sec)	N/A	1.714	6.507	0.249	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	1234	2499	0	0	0	0
normalized size	1	1.	2.45	4.97	0.	0.	0.	0.
time (sec)	N/A	1.238	6.368	0.183	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	1225	2046	0	0	0	0
normalized size	1	1.	2.91	4.86	0.	0.	0.	0.
time (sec)	N/A	0.802	6.386	0.219	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	1269	2278	0	0	0	0
normalized size	1	1.	4.12	7.4	0.	0.	0.	0.
time (sec)	N/A	0.625	6.467	0.29	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	1327	2667	0	0	0	0
normalized size	1	1.	3.39	6.8	0.	0.	0.	0.
time (sec)	N/A	0.979	6.612	0.185	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	1418	4066	0	0	0	0
normalized size	1	1.	2.87	8.23	0.	0.	0.	0.
time (sec)	N/A	1.446	6.724	0.243	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	650	650	1366	6463	0	0	0	0
normalized size	1	1.	2.1	9.94	0.	0.	0.	0.
time (sec)	N/A	2.472	6.64	0.378	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	563	563	1388	6425	0	0	0	0
normalized size	1	1.	2.47	11.41	0.	0.	0.	0.
time (sec)	N/A	1.613	6.539	0.326	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	1364	4582	0	0	0	0
normalized size	1	1.	3.27	10.99	0.	0.	0.	0.
time (sec)	N/A	1.03	6.501	0.598	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	1421	6176	0	0	0	0
normalized size	1	1.	3.16	13.76	0.	0.	0.	0.
time (sec)	N/A	1.158	6.729	0.841	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	549	549	1471	7087	0	0	0	0
normalized size	1	1.	2.68	12.91	0.	0.	0.	0.
time (sec)	N/A	1.672	6.815	0.347	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	250	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.858	2.378	1.61	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.371	1.001	0.85	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	10459	0	0	0	0	0
normalized size	1	1.	29.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	28.829	0.612	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	514	514	17999	0	0	0	0	0
normalized size	1	1.	35.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.881	34.231	0.523	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	136	205	255	120
normalized size	1	1.	0.87	1.02	1.3	1.95	2.43	1.14
time (sec)	N/A	0.208	0.221	0.016	1.129	1.691	4.784	1.576

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	104	75	85	107	149	170	92
normalized size	1	1.24	0.89	1.01	1.27	1.77	2.02	1.1
time (sec)	N/A	0.081	0.153	0.018	1.021	1.689	2.459	1.506

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	74	104	0	163
normalized size	1	1.	0.98	1.1	1.42	2.	0.	3.13
time (sec)	N/A	0.065	0.081	0.035	1.027	1.676	0.	1.324

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	56	78	142	0	107
normalized size	1	1.	1.31	1.6	2.23	4.06	0.	3.06
time (sec)	N/A	0.159	0.025	0.046	1.054	1.466	0.	1.902

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	65	99	225	0	113
normalized size	1	1.	1.23	1.86	2.83	6.43	0.	3.23
time (sec)	N/A	0.171	0.013	0.039	1.012	1.396	0.	1.519

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	128	247	0	204
normalized size	1	1.	1.23	1.41	2.1	4.05	0.	3.34
time (sec)	N/A	0.196	0.023	0.042	1.028	1.389	0.	1.522

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	67	128	171	298	0	284
normalized size	1	1.	0.72	1.38	1.84	3.2	0.	3.05
time (sec)	N/A	0.237	0.262	0.047	1.118	1.521	0.	1.507

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	85	171	220	352	0	410
normalized size	1	1.	0.75	1.5	1.93	3.09	0.	3.6
time (sec)	N/A	0.227	0.576	0.047	1.077	1.578	0.	1.507

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	146	184	238	350	462	211
normalized size	1	1.	0.77	0.97	1.26	1.85	2.44	1.12
time (sec)	N/A	0.357	0.456	0.019	1.009	1.556	8.602	1.373

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	118	152	192	274	340	167
normalized size	1	1.	0.69	0.89	1.13	1.61	2.	0.98
time (sec)	N/A	0.193	0.433	0.016	1.038	1.503	3.946	1.525

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	146	201	0	343
normalized size	1	1.	0.84	1.07	1.36	1.88	0.	3.21
time (sec)	N/A	0.159	0.216	0.039	1.035	1.45	0.	1.565

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	120	120	134	213	0	240
normalized size	1	1.	1.4	1.4	1.56	2.48	0.	2.79
time (sec)	N/A	0.245	0.219	0.05	1.052	1.471	0.	1.613

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	109	104	139	294	0	205
normalized size	1	1.	1.82	1.73	2.32	4.9	0.	3.42
time (sec)	N/A	0.243	0.473	0.046	1.105	1.6	0.	1.693

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	67	133	189	335	0	257
normalized size	1	1.	0.84	1.66	2.36	4.19	0.	3.21
time (sec)	N/A	0.28	0.257	0.051	1.143	1.501	0.	1.472

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	92	174	232	371	0	397
normalized size	1	1.	0.79	1.5	2.	3.2	0.	3.42
time (sec)	N/A	0.36	0.447	0.052	1.003	1.383	0.	1.79

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	120	241	308	443	0	645
normalized size	1	1.	0.77	1.54	1.97	2.84	0.	4.13
time (sec)	N/A	0.377	0.719	0.054	1.045	1.458	0.	1.758

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	176	227	293	423	552	254
normalized size	1	1.	0.72	0.93	1.21	1.74	2.27	1.05
time (sec)	N/A	0.293	0.662	0.016	1.027	1.523	10.077	1.621

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	140	180	231	321	0	724
normalized size	1	1.	0.82	1.05	1.35	1.88	0.	4.23
time (sec)	N/A	0.262	0.401	0.045	1.05	1.482	0.	1.578

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	159	207	205	317	0	424
normalized size	1	1.	1.16	1.51	1.5	2.31	0.	3.09
time (sec)	N/A	0.479	0.38	0.057	1.075	1.561	0.	1.78

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	217	168	194	369	0	316
normalized size	1	1.	1.66	1.28	1.48	2.82	0.	2.41
time (sec)	N/A	0.465	0.633	0.053	1.013	1.456	0.	1.431

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	277	172	228	401	0	323
normalized size	1	1.	2.23	1.39	1.84	3.23	0.	2.6
time (sec)	N/A	0.413	2.166	0.056	1.018	1.519	0.	1.339

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	108	223	292	458	0	454
normalized size	1	1.	0.74	1.54	2.01	3.16	0.	3.13
time (sec)	N/A	0.429	0.563	0.06	1.036	1.546	0.	1.544

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	140	290	369	510	0	791
normalized size	1	1.	0.74	1.54	1.96	2.71	0.	4.21
time (sec)	N/A	0.549	0.8	0.059	1.072	1.603	0.	1.955

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	181	382	460	612	0	975
normalized size	1	1.	0.77	1.62	1.95	2.59	0.	4.13
time (sec)	N/A	0.562	3.411	0.059	1.064	1.58	0.	1.865

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	152	641	0	1164	0	486
normalized size	1	1.	0.85	3.6	0.	6.54	0.	2.73
time (sec)	N/A	0.57	0.464	0.037	0.	1.65	0.	1.807

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	367	0	918	0	306
normalized size	1	1.	0.9	2.74	0.	6.85	0.	2.28
time (sec)	N/A	0.355	0.318	0.034	0.	1.562	0.	1.597

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	172	0	689	0	192
normalized size	1	1.	0.96	1.93	0.	7.74	0.	2.16
time (sec)	N/A	0.131	0.213	0.033	0.	1.553	0.	1.664

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	524	0	135
normalized size	1	1.	1.01	1.69	0.	7.82	0.	2.01
time (sec)	N/A	0.144	0.12	0.049	0.	1.458	0.	1.678

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	689	0	173
normalized size	1	1.	1.47	1.78	0.	9.07	0.	2.28
time (sec)	N/A	0.208	0.16	0.058	0.	5.189	0.	1.611

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	129	228	0	1049	0	236
normalized size	1	1.	1.3	2.3	0.	10.6	0.	2.38
time (sec)	N/A	0.273	0.527	0.061	0.	2.394	0.	1.72

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	300	410	0	1334	0	363
normalized size	1	1.	2.1	2.87	0.	9.33	0.	2.54
time (sec)	N/A	0.585	1.647	0.07	0.	27.519	0.	1.718

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	184	643	0	2120	0	456
normalized size	1	1.	0.7	2.44	0.	8.06	0.	1.73
time (sec)	N/A	0.749	1.022	0.043	0.	2.042	0.	1.404

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	147	445	0	1701	0	502
normalized size	1	1.	0.95	2.87	0.	10.97	0.	3.24
time (sec)	N/A	0.468	0.789	0.041	0.	1.971	0.	1.49

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	119	320	0	1210	0	269
normalized size	1	1.	0.98	2.62	0.	9.92	0.	2.2
time (sec)	N/A	0.174	0.544	0.036	0.	1.791	0.	1.337

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	234	0	846	0	212
normalized size	1	1.	0.97	2.34	0.	8.46	0.	2.12
time (sec)	N/A	0.154	0.322	0.055	0.	1.674	0.	1.387

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	191	342	0	1534	0	304
normalized size	1	1.	1.44	2.57	0.	11.53	0.	2.29
time (sec)	N/A	0.386	0.588	0.074	0.	29.914	0.	1.35

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	240	502	0	2419	0	545
normalized size	1	1.	1.27	2.66	0.	12.8	0.	2.88
time (sec)	N/A	0.788	1.792	0.075	0.	76.54	0.	1.467

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	734	1504	0	4001	0	1821
normalized size	1	1.	1.84	3.78	0.	10.05	0.	4.58
time (sec)	N/A	1.765	3.487	0.05	0.	2.928	0.	1.623

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	232	1301	0	3380	0	733
normalized size	1	1.	0.83	4.65	0.	12.07	0.	2.62
time (sec)	N/A	1.284	2.177	0.045	0.	2.577	0.	1.444

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	204	1023	0	2462	0	614
normalized size	1	1.	0.97	4.85	0.	11.67	0.	2.91
time (sec)	N/A	0.615	1.381	0.042	0.	2.072	0.	1.324

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	172	886	0	1616	0	528
normalized size	1	1.	0.96	4.92	0.	8.98	0.	2.93
time (sec)	N/A	0.228	0.821	0.036	0.	1.877	0.	1.429

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	157	886	0	1616	0	527
normalized size	1	1.	0.96	5.4	0.	9.85	0.	3.21
time (sec)	N/A	0.253	0.653	0.059	0.	2.001	0.	1.354

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	269	1045	0	3032	0	649
normalized size	1	1.	1.26	4.88	0.	14.17	0.	3.03
time (sec)	N/A	0.802	1.274	0.084	0.	140.4	0.	1.422

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	352	1358	0	0	0	775
normalized size	1	1.	1.18	4.54	0.	0.	0.	2.59
time (sec)	N/A	1.803	5.857	0.091	0.	0.	0.	1.449

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	232	1305	0	0	0	0
normalized size	1	1.	0.77	4.31	0.	0.	0.	0.
time (sec)	N/A	0.605	0.981	0.746	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	179	993	0	0	0	0
normalized size	1	1.	0.77	4.3	0.	0.	0.	0.
time (sec)	N/A	0.346	0.857	0.74	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	146	600	0	0	0	0
normalized size	1	1.	0.85	3.51	0.	0.	0.	0.
time (sec)	N/A	0.303	0.583	0.715	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	107	247	0	0	0	0
normalized size	1	1.	0.6	1.39	0.	0.	0.	0.
time (sec)	N/A	0.461	2.385	0.791	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	372	746	0	0	0	0
normalized size	1	1.	1.75	3.5	0.	0.	0.	0.
time (sec)	N/A	0.706	10.643	1.405	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	420	1290	0	0	0	0
normalized size	1	1.	1.44	4.42	0.	0.	0.	0.
time (sec)	N/A	1.047	4.289	1.334	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	291	1635	0	0	0	0
normalized size	1	1.	0.77	4.33	0.	0.	0.	0.
time (sec)	N/A	0.789	1.566	0.79	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	233	1305	0	0	0	0
normalized size	1	1.	0.78	4.39	0.	0.	0.	0.
time (sec)	N/A	0.455	1.093	0.85	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	203	993	0	0	0	0
normalized size	1	1.	0.9	4.41	0.	0.	0.	0.
time (sec)	N/A	0.449	0.769	0.703	0.	0.	0.	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	406	738	0	0	0	0
normalized size	1	1.	1.72	3.13	0.	0.	0.	0.
time (sec)	N/A	0.833	2.383	0.67	0.	0.	0.	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	398	1167	0	0	0	0
normalized size	1	1.	1.72	5.03	0.	0.	0.	0.
time (sec)	N/A	0.805	2.453	0.705	0.	0.	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	422	1403	0	0	0	0
normalized size	1	1.	1.43	4.76	0.	0.	0.	0.
time (sec)	N/A	1.182	4.861	1.904	0.	0.	0.	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	634	2327	0	0	0	0
normalized size	1	1.	1.69	6.21	0.	0.	0.	0.
time (sec)	N/A	1.551	6.557	2.221	0.	0.	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	357	1983	0	0	0	0
normalized size	1	1.	0.77	4.29	0.	0.	0.	0.
time (sec)	N/A	0.98	2.031	1.177	0.	0.	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	291	1635	0	0	0	0
normalized size	1	1.	0.78	4.4	0.	0.	0.	0.
time (sec)	N/A	0.629	1.532	0.863	0.	0.	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	254	1305	0	0	0	0
normalized size	1	1.	0.88	4.53	0.	0.	0.	0.
time (sec)	N/A	0.578	1.039	0.827	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	453	1067	0	0	0	0
normalized size	1	1.	1.55	3.65	0.	0.	0.	0.
time (sec)	N/A	1.117	2.693	0.731	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	442	1563	0	0	0	0
normalized size	1	1.	1.49	5.28	0.	0.	0.	0.
time (sec)	N/A	1.119	3.741	0.813	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	451	1742	0	0	0	0
normalized size	1	1.	1.43	5.53	0.	0.	0.	0.
time (sec)	N/A	1.147	5.591	2.168	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	486	2438	0	0	0	0
normalized size	1	1.	1.29	6.48	0.	0.	0.	0.
time (sec)	N/A	1.565	5.962	2.523	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	465	465	729	3548	0	0	0	0
normalized size	1	1.	1.57	7.63	0.	0.	0.	0.
time (sec)	N/A	1.997	6.71	3.414	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	180	993	0	0	0	0
normalized size	1	1.	0.73	4.04	0.	0.	0.	0.
time (sec)	N/A	0.487	0.89	0.779	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	154	671	0	0	0	0
normalized size	1	1.	0.84	3.67	0.	0.	0.	0.
time (sec)	N/A	0.222	0.774	0.747	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	93	249	0	0	0	0
normalized size	1	1.	0.72	1.92	0.	0.	0.	0.
time (sec)	N/A	0.237	3.184	0.65	0.	0.	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0
normalized size	1	1.	0.69	1.64	0.	0.	0.	0.
time (sec)	N/A	0.401	0.186	0.682	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	320	639	0	0	0	0
normalized size	1	1.	1.48	2.96	0.	0.	0.	0.
time (sec)	N/A	0.72	6.425	1.052	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	420	1182	0	0	0	0
normalized size	1	1.	1.4	3.95	0.	0.	0.	0.
time (sec)	N/A	1.126	5.71	1.417	0.	0.	0.	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	304	1308	0	0	0	0
normalized size	1	1.	0.79	3.38	0.	0.	0.	0.
time (sec)	N/A	0.875	1.738	2.812	0.	0.	0.	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	189	954	0	0	0	0
normalized size	1	1.	0.72	3.64	0.	0.	0.	0.
time (sec)	N/A	0.567	1.387	2.411	0.	0.	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	170	515	0	0	0	0
normalized size	1	1.	0.83	2.52	0.	0.	0.	0.
time (sec)	N/A	0.282	0.799	1.668	0.	0.	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	151	428	0	0	0	0
normalized size	1	1.	0.82	2.31	0.	0.	0.	0.
time (sec)	N/A	0.336	0.547	1.431	0.	0.	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	460	429	0	0	0	0
normalized size	1	1.	2.42	2.26	0.	0.	0.	0.
time (sec)	N/A	0.643	3.823	1.636	0.	0.	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	482	908	0	0	0	0
normalized size	1	1.	1.59	3.	0.	0.	0.	0.
time (sec)	N/A	1.164	5.523	1.92	0.	0.	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	334	1389	0	0	0	0
normalized size	1	1.	0.81	3.36	0.	0.	0.	0.
time (sec)	N/A	0.951	2.834	3.297	0.	0.	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	274	950	0	0	0	0
normalized size	1	1.	0.83	2.87	0.	0.	0.	0.
time (sec)	N/A	0.639	2.218	2.745	0.	0.	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	224	860	0	0	0	0
normalized size	1	1.	0.73	2.8	0.	0.	0.	0.
time (sec)	N/A	0.415	1.909	2.59	0.	0.	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	193	750	0	0	0	0
normalized size	1	1.	0.7	2.73	0.	0.	0.	0.
time (sec)	N/A	0.483	1.542	2.373	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	743	854	0	0	0	0
normalized size	1	1.	2.13	2.45	0.	0.	0.	0.
time (sec)	N/A	1.25	6.775	2.623	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	437	437	750	1341	0	0	0	0
normalized size	1	1.	1.72	3.07	0.	0.	0.	0.
time (sec)	N/A	1.637	7.172	3.721	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	125	451	0	0	0	0
normalized size	1	1.	0.74	2.65	0.	0.	0.	0.
time (sec)	N/A	0.267	1.347	0.59	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	0	0	0
normalized size	1	1.	0.74	2.95	0.	0.	0.	0.
time (sec)	N/A	0.24	0.913	0.752	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	0	0	0
normalized size	1	1.	0.8	3.44	0.	0.	0.	0.
time (sec)	N/A	0.219	0.422	0.622	0.	0.	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	0	0	0
normalized size	1	1.	0.89	4.35	0.	0.	0.	0.
time (sec)	N/A	0.205	0.232	0.75	0.	0.	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	244	0	0	0	0
normalized size	1	1.	0.9	3.44	0.	0.	0.	0.
time (sec)	N/A	0.211	0.347	0.575	0.	0.	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	428	0	0	0	0
normalized size	1	1.	1.04	4.16	0.	0.	0.	0.
time (sec)	N/A	0.227	0.466	1.444	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	134	663	0	0	0	0
normalized size	1	1.	0.96	4.74	0.	0.	0.	0.
time (sec)	N/A	0.248	0.798	1.744	0.	0.	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	196	666	0	0	0	0
normalized size	1	1.	0.74	2.52	0.	0.	0.	0.
time (sec)	N/A	0.462	1.679	0.647	0.	0.	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	167	610	0	0	0	0
normalized size	1	1.	0.75	2.74	0.	0.	0.	0.
time (sec)	N/A	0.423	1.387	0.641	0.	0.	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.394	1.033	0.722	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	0	0	0
normalized size	1	1.	0.76	3.48	0.	0.	0.	0.
time (sec)	N/A	0.356	0.57	0.742	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	404	0	0	0	0
normalized size	1	1.	0.84	3.34	0.	0.	0.	0.
time (sec)	N/A	0.335	0.624	0.66	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	105	677	0	0	0	0
normalized size	1	1.	0.83	5.37	0.	0.	0.	0.
time (sec)	N/A	0.36	1.097	1.567	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	175	750	0	0	0	0
normalized size	1	1.	1.02	4.36	0.	0.	0.	0.
time (sec)	N/A	0.391	1.054	2.03	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	191	859	0	0	0	0
normalized size	1	1.	0.89	4.01	0.	0.	0.	0.
time (sec)	N/A	0.438	4.198	2.485	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	235	825	0	0	0	0
normalized size	1	1.	0.77	2.7	0.	0.	0.	0.
time (sec)	N/A	0.638	1.911	0.684	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	197	745	0	0	0	0
normalized size	1	1.	0.77	2.92	0.	0.	0.	0.
time (sec)	N/A	0.581	1.154	0.661	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	158	664	0	0	0	0
normalized size	1	1.	0.77	3.24	0.	0.	0.	0.
time (sec)	N/A	0.554	1.259	0.707	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	150	867	0	0	0	0
normalized size	1	1.	0.74	4.29	0.	0.	0.	0.
time (sec)	N/A	0.558	1.115	0.718	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	165	1212	0	0	0	0
normalized size	1	1.	0.86	6.31	0.	0.	0.	0.
time (sec)	N/A	0.532	1.019	1.844	0.	0.	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	176	997	0	0	0	0
normalized size	1	1.	0.86	4.89	0.	0.	0.	0.
time (sec)	N/A	0.547	2.026	2.143	0.	0.	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	221	944	0	0	0	0
normalized size	1	1.	0.87	3.7	0.	0.	0.	0.
time (sec)	N/A	0.595	3.429	2.678	0.	0.	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	266	1193	0	0	0	0
normalized size	1	1.	0.87	3.91	0.	0.	0.	0.
time (sec)	N/A	0.626	4.698	3.329	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	309	1375	0	0	0	0
normalized size	1	1.	1.26	5.59	0.	0.	0.	0.
time (sec)	N/A	1.204	2.6	0.807	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	264	1074	0	0	0	0
normalized size	1	1.	1.45	5.9	0.	0.	0.	0.
time (sec)	N/A	0.899	2.273	0.71	0.	0.	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	209	786	0	0	0	0
normalized size	1	1.	1.53	5.74	0.	0.	0.	0.
time (sec)	N/A	0.605	1.353	0.819	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	131	295	0	0	0	0
normalized size	1	1.	1.47	3.31	0.	0.	0.	0.
time (sec)	N/A	0.298	0.856	0.706	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0
normalized size	1	1.	0.95	3.56	0.	0.	0.	0.
time (sec)	N/A	0.242	0.197	0.67	0.	0.	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	210	327	0	0	0	0
normalized size	1	1.	2.44	3.8	0.	0.	0.	0.
time (sec)	N/A	0.409	2.391	1.307	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	262	468	0	0	0	0
normalized size	1	1.	1.75	3.12	0.	0.	0.	0.
time (sec)	N/A	0.872	2.167	1.961	0.	0.	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	313	787	0	0	0	0
normalized size	1	1.	1.44	3.63	0.	0.	0.	0.
time (sec)	N/A	1.236	3.863	2.697	0.	0.	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	373	1348	0	0	0	0
normalized size	1	1.	0.96	3.47	0.	0.	0.	0.
time (sec)	N/A	1.382	4.567	3.045	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	322	1066	0	0	0	0
normalized size	1	1.	1.06	3.52	0.	0.	0.	0.
time (sec)	N/A	1.025	3.019	2.637	0.	0.	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	284	849	0	0	0	0
normalized size	1	1.	1.27	3.79	0.	0.	0.	0.
time (sec)	N/A	0.705	2.532	2.114	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	262	808	0	0	0	0
normalized size	1	1.	1.32	4.08	0.	0.	0.	0.
time (sec)	N/A	0.636	2.31	1.829	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	276	721	0	0	0	0
normalized size	1	1.	1.38	3.6	0.	0.	0.	0.
time (sec)	N/A	0.714	2.55	1.646	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	320	883	0	0	0	0
normalized size	1	1.	1.25	3.45	0.	0.	0.	0.
time (sec)	N/A	1.013	3.935	2.086	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	367	1031	0	0	0	0
normalized size	1	1.	1.06	2.99	0.	0.	0.	0.
time (sec)	N/A	1.385	6.371	3.378	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	466	2195	0	0	0	0
normalized size	1	1.	1.01	4.76	0.	0.	0.	0.
time (sec)	N/A	1.545	4.673	3.932	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	394	1977	0	0	0	0
normalized size	1	1.	1.07	5.39	0.	0.	0.	0.
time (sec)	N/A	1.1	4.673	3.355	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	364	1937	0	0	0	0
normalized size	1	1.	1.06	5.63	0.	0.	0.	0.
time (sec)	N/A	1.097	3.356	3.077	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	369	1850	0	0	0	0
normalized size	1	1.	1.09	5.49	0.	0.	0.	0.
time (sec)	N/A	1.026	4.234	3.082	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	387	1744	0	0	0	0
normalized size	1	1.	1.12	5.06	0.	0.	0.	0.
time (sec)	N/A	1.172	4.734	3.027	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	462	2002	0	0	0	0
normalized size	1	1.	1.1	4.77	0.	0.	0.	0.
time (sec)	N/A	1.575	5.167	3.873	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	1224	2949	0	0	0	0
normalized size	1	1.	2.19	5.27	0.	0.	0.	0.
time (sec)	N/A	1.659	6.329	0.227	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	1175	2052	0	0	0	0
normalized size	1	1.	2.48	4.34	0.	0.	0.	0.
time (sec)	N/A	1.174	21.045	0.142	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	3054	1693	0	0	0	0
normalized size	1	1.	7.93	4.4	0.	0.	0.	0.
time (sec)	N/A	0.842	18.209	0.215	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	275	1687	0	0	0	0
normalized size	1	1.	0.78	4.81	0.	0.	0.	0.
time (sec)	N/A	0.641	13.14	0.158	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	407	1727	0	0	0	0
normalized size	1	1.	1.43	6.08	0.	0.	0.	0.
time (sec)	N/A	0.627	14.201	0.116	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	1315	2481	0	0	0	0
normalized size	1	1.	3.76	7.09	0.	0.	0.	0.
time (sec)	N/A	0.952	6.35	0.156	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1408	3427	0	0	0	0
normalized size	1	1.	3.25	7.91	0.	0.	0.	0.
time (sec)	N/A	1.291	6.424	0.239	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	670	670	1284	4048	0	0	0	0
normalized size	1	1.	1.92	6.04	0.	0.	0.	0.
time (sec)	N/A	2.273	6.427	0.347	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	566	566	1227	3139	0	0	0	0
normalized size	1	1.	2.17	5.55	0.	0.	0.	0.
time (sec)	N/A	1.858	6.315	0.223	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	1198	2430	0	0	0	0
normalized size	1	1.	2.54	5.15	0.	0.	0.	0.
time (sec)	N/A	1.317	6.334	0.249	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	1196	2185	0	0	0	0
normalized size	1	1.	2.66	4.87	0.	0.	0.	0.
time (sec)	N/A	1.305	6.356	0.138	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	1236	2318	0	0	0	0
normalized size	1	1.	2.96	5.55	0.	0.	0.	0.
time (sec)	N/A	0.994	6.351	0.135	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	1314	2666	0	0	0	0
normalized size	1	1.	3.72	7.55	0.	0.	0.	0.
time (sec)	N/A	1.051	6.427	0.176	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1407	3413	0	0	0	0
normalized size	1	1.	3.25	7.88	0.	0.	0.	0.
time (sec)	N/A	1.417	6.52	0.261	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	779	779	1353	5164	0	0	0	0
normalized size	1	1.	1.74	6.63	0.	0.	0.	0.
time (sec)	N/A	3.29	6.522	0.647	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	664	664	1287	4238	0	0	0	0
normalized size	1	1.	1.94	6.38	0.	0.	0.	0.
time (sec)	N/A	2.361	6.405	0.357	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	563	563	1251	3512	0	0	0	0
normalized size	1	1.	2.22	6.24	0.	0.	0.	0.
time (sec)	N/A	1.824	6.518	0.36	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	547	547	1241	3270	0	0	0	0
normalized size	1	1.	2.27	5.98	0.	0.	0.	0.
time (sec)	N/A	1.785	6.506	0.178	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	1269	3204	0	0	0	0
normalized size	1	1.	2.37	5.98	0.	0.	0.	0.
time (sec)	N/A	1.76	6.504	0.155	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	493	493	1319	3274	0	0	0	0
normalized size	1	1.	2.68	6.64	0.	0.	0.	0.
time (sec)	N/A	1.362	6.539	0.18	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	434	434	1409	3628	0	0	0	0
normalized size	1	1.	3.25	8.36	0.	0.	0.	0.
time (sec)	N/A	1.452	6.598	0.26	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	1517	4392	0	0	0	0
normalized size	1	1.	2.91	8.41	0.	0.	0.	0.
time (sec)	N/A	1.955	6.718	0.37	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	622	1640	5373	0	0	0	0
normalized size	1	1.	2.64	8.64	0.	0.	0.	0.
time (sec)	N/A	2.753	6.841	0.592	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	571	571	1229	2949	0	0	0	0
normalized size	1	1.	2.15	5.16	0.	0.	0.	0.
time (sec)	N/A	1.727	6.409	0.238	0.	0.	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	479	1175	1871	0	0	0	0
normalized size	1	1.	2.45	3.91	0.	0.	0.	0.
time (sec)	N/A	1.196	12.603	0.148	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	427	4017	1002	0	0	0	0
normalized size	1	1.09	10.27	2.56	0.	0.	0.	0.
time (sec)	N/A	1.213	18.059	0.183	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	146	197	0	0	0	0
normalized size	1	1.	0.64	0.86	0.	0.	0.	0.
time (sec)	N/A	0.393	1.464	0.123	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	299	935	0	0	0	0
normalized size	1	1.	1.3	4.07	0.	0.	0.	0.
time (sec)	N/A	0.438	13.232	0.149	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	416	1536	0	0	0	0
normalized size	1	1.	1.43	5.3	0.	0.	0.	0.
time (sec)	N/A	0.629	16.448	0.12	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	1319	2480	0	0	0	0
normalized size	1	1.	3.63	6.83	0.	0.	0.	0.
time (sec)	N/A	0.965	6.399	0.163	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	620	620	1297	4001	0	0	0	0
normalized size	1	1.	2.09	6.45	0.	0.	0.	0.
time (sec)	N/A	1.891	6.563	0.224	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	500	500	1234	2885	0	0	0	0
normalized size	1	1.	2.47	5.77	0.	0.	0.	0.
time (sec)	N/A	1.412	6.379	0.142	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	1012	2013	0	0	0	0
normalized size	1	1.	2.43	4.84	0.	0.	0.	0.
time (sec)	N/A	0.726	18.208	0.154	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	1223	1633	0	0	0	0
normalized size	1	1.	4.31	5.75	0.	0.	0.	0.
time (sec)	N/A	0.64	6.359	0.162	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	1281	2282	0	0	0	0
normalized size	1	1.	4.2	7.48	0.	0.	0.	0.
time (sec)	N/A	0.745	6.489	0.162	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	1357	3334	0	0	0	0
normalized size	1	1.	3.45	8.48	0.	0.	0.	0.
time (sec)	N/A	1.08	6.68	0.15	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	674	674	1396	8611	0	0	0	0
normalized size	1	1.	2.07	12.78	0.	0.	0.	0.
time (sec)	N/A	2.124	6.679	0.282	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	545	545	1342	5749	0	0	0	0
normalized size	1	1.	2.46	10.55	0.	0.	0.	0.
time (sec)	N/A	1.496	6.513	0.233	0.	0.	0.	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	1335	4237	0	0	0	0
normalized size	1	1.	3.41	10.84	0.	0.	0.	0.
time (sec)	N/A	0.989	6.446	0.181	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	1384	5200	0	0	0	0
normalized size	1	1.	3.23	12.12	0.	0.	0.	0.
time (sec)	N/A	1.107	6.551	0.512	0.	0.	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	456	456	1431	6498	0	0	0	0
normalized size	1	1.	3.14	14.25	0.	0.	0.	0.
time (sec)	N/A	1.295	6.716	0.751	0.	0.	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	117	173	224	305	428	174
normalized size	1	1.	0.75	1.11	1.44	1.96	2.74	1.12
time (sec)	N/A	0.229	0.507	0.02	0.966	1.775	3.024	1.174

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	118	141	178	239	320	138
normalized size	1	1.	0.92	1.1	1.39	1.87	2.5	1.08
time (sec)	N/A	0.139	0.323	0.018	0.988	1.705	1.552	1.218

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	113	85	102	132	173	189	103
normalized size	1	1.41	1.06	1.27	1.65	2.16	2.36	1.29
time (sec)	N/A	0.099	0.181	0.018	0.991	1.658	0.758	1.149

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	100	111	192	0	215
normalized size	1	1.	0.99	1.45	1.61	2.78	0.	3.12
time (sec)	N/A	0.14	0.116	0.042	1.224	1.749	0.	1.208

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	71	88	124	265	0	178
normalized size	1	1.	1.37	1.69	2.38	5.1	0.	3.42
time (sec)	N/A	0.138	0.02	0.049	0.999	1.75	0.	1.194

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	92	117	176	305	0	227
normalized size	1	1.	1.33	1.7	2.55	4.42	0.	3.29
time (sec)	N/A	0.168	0.028	0.053	1.011	1.841	0.	1.243

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	73	160	219	331	0	352
normalized size	1	1.	0.72	1.58	2.17	3.28	0.	3.49
time (sec)	N/A	0.22	0.419	0.058	0.977	1.726	0.	1.231

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	100	223	294	400	0	578
normalized size	1	1.	0.73	1.63	2.15	2.92	0.	4.22
time (sec)	N/A	0.239	0.654	0.058	1.022	1.814	0.	1.267

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	123	287	359	467	0	639
normalized size	1	1.	0.75	1.74	2.18	2.83	0.	3.87
time (sec)	N/A	0.255	1.289	0.062	1.033	1.775	0.	1.202

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	169	244	315	414	570	248
normalized size	1	1.	0.75	1.09	1.41	1.85	2.54	1.11
time (sec)	N/A	0.325	0.814	0.02	1.003	1.76	3.504	1.212

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	137	200	252	320	420	197
normalized size	1	1.	0.72	1.05	1.32	1.68	2.2	1.03
time (sec)	N/A	0.226	0.592	0.019	0.982	1.68	1.73	1.181

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	158	204	203	313	0	467
normalized size	1	1.	1.18	1.52	1.51	2.34	0.	3.49
time (sec)	N/A	0.332	0.5	0.053	0.989	1.838	0.	1.286

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	155	171	200	366	0	309
normalized size	1	1.	1.23	1.36	1.59	2.9	0.	2.45
time (sec)	N/A	0.323	1.026	0.063	1.022	1.814	0.	1.244

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	277	184	255	406	0	323
normalized size	1	1.	2.35	1.56	2.16	3.44	0.	2.74
time (sec)	N/A	0.36	1.487	0.068	1.022	1.805	0.	1.286

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	104	225	298	444	0	491
normalized size	1	1.	0.74	1.6	2.11	3.15	0.	3.48
time (sec)	N/A	0.366	0.615	0.066	1.001	1.815	0.	1.239

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	137	321	423	510	0	851
normalized size	1	1.	0.74	1.74	2.3	2.77	0.	4.62
time (sec)	N/A	0.465	1.077	0.069	0.985	1.917	0.	1.281

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	167	404	482	598	0	1034
normalized size	1	1.	0.72	1.74	2.08	2.58	0.	4.46
time (sec)	N/A	0.507	2.476	0.069	1.033	1.878	0.	1.208

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	368	370	486	610	966	382
normalized size	1	1.	1.13	1.13	1.49	1.87	2.95	1.17
time (sec)	N/A	0.607	1.108	0.023	1.025	1.925	6.986	1.189

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	288	301	389	498	685	306
normalized size	1	1.	1.04	1.09	1.4	1.8	2.47	1.1
time (sec)	N/A	0.419	0.904	0.022	0.979	1.738	3.881	1.208

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	218	362	323	463	0	976
normalized size	1	1.	1.05	1.75	1.56	2.24	0.	4.71
time (sec)	N/A	0.564	0.934	0.061	0.995	1.964	0.	1.225

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	266	278	292	486	0	564
normalized size	1	1.	1.39	1.45	1.52	2.53	0.	2.94
time (sec)	N/A	0.587	1.234	0.064	1.	1.846	0.	1.258

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	318	267	328	500	0	726
normalized size	1	1.	1.56	1.31	1.61	2.45	0.	3.56
time (sec)	N/A	0.646	3.2	0.076	1.	1.929	0.	1.302

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	429	294	378	543	0	591
normalized size	1	1.	2.19	1.5	1.93	2.77	0.	3.02
time (sec)	N/A	0.642	5.743	0.075	1.027	2.062	0.	1.252

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	165	389	502	624	0	1025
normalized size	1	1.	0.74	1.74	2.25	2.8	0.	4.6
time (sec)	N/A	0.671	1.253	0.085	1.004	2.196	0.	1.293

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	204	504	610	711	0	1335
normalized size	1	1.	0.73	1.81	2.19	2.56	0.	4.8
time (sec)	N/A	0.918	4.607	0.079	1.011	2.303	0.	1.272

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	252	644	763	824	0	1850
normalized size	1	1.	0.75	1.92	2.27	2.45	0.	5.51
time (sec)	N/A	0.913	2.911	0.076	1.066	2.016	0.	1.362

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	528	505	672	852	1334	527
normalized size	1	1.	1.19	1.13	1.51	1.91	3.	1.18
time (sec)	N/A	1.042	1.426	0.024	1.043	2.089	13.122	1.213

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	432	431	560	698	1066	440
normalized size	1	1.	1.15	1.15	1.49	1.86	2.84	1.17
time (sec)	N/A	0.683	1.351	0.022	0.995	1.927	7.49	1.194

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	382	543	459	640	0	1477
normalized size	1	1.	1.32	1.87	1.58	2.21	0.	5.09
time (sec)	N/A	0.906	1.222	0.063	1.014	1.962	0.	1.335

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	383	434	412	636	0	1083
normalized size	1	1.	1.4	1.59	1.51	2.33	0.	3.97
time (sec)	N/A	0.906	2.946	0.072	1.001	2.031	0.	1.315

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	367	374	420	628	0	730
normalized size	1	1.	1.34	1.36	1.53	2.29	0.	2.66
time (sec)	N/A	0.974	4.733	0.079	1.015	2.072	0.	1.298

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	351	377	452	644	0	743
normalized size	1	1.	1.16	1.24	1.49	2.13	0.	2.45
time (sec)	N/A	1.077	2.252	0.081	1.046	2.08	0.	1.383

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	462	457	582	725	0	1134
normalized size	1	1.	1.58	1.56	1.99	2.47	0.	3.87
time (sec)	N/A	1.065	2.456	0.082	1.01	2.074	0.	1.436

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	230	572	690	824	0	1539
normalized size	1	1.	0.73	1.82	2.2	2.62	0.	4.9
time (sec)	N/A	1.048	1.802	0.086	1.038	2.01	0.	1.429

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	366	745	891	941	0	2238
normalized size	1	1.	0.96	1.96	2.34	2.47	0.	5.87
time (sec)	N/A	1.323	6.307	0.087	1.057	2.149	0.	1.367

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	341	905	1007	1085	0	2549
normalized size	1	1.	0.75	1.99	2.22	2.39	0.	5.61
time (sec)	N/A	1.397	3.509	0.1	1.048	2.234	0.	1.34

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	287	276	355	500	619	306
normalized size	1	1.	1.12	1.08	1.39	1.95	2.42	1.2
time (sec)	N/A	0.552	1.108	0.021	1.011	1.871	3.848	1.185

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	134	168	219	311	357	194
normalized size	1	1.	0.76	0.95	1.24	1.77	2.03	1.1
time (sec)	N/A	0.35	0.578	0.022	0.994	1.669	1.716	1.149

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	102	131	169	224	241	144
normalized size	1	1.	0.85	1.09	1.41	1.87	2.01	1.2
time (sec)	N/A	0.214	0.387	0.018	1.034	1.649	0.864	1.138

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	238	1580	0	1674	0	1081
normalized size	1	1.	0.85	5.66	0.	6.	0.	3.87
time (sec)	N/A	0.935	0.874	0.042	0.	2.234	0.	1.227

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	179	814	0	1288	0	572
normalized size	1	1.	0.87	3.95	0.	6.25	0.	2.78
time (sec)	N/A	0.567	0.651	0.036	0.	2.001	0.	1.302

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	142	133	434	0	988	0	323
normalized size	1	0.99	0.92	3.01	0.	6.86	0.	2.24
time (sec)	N/A	0.334	0.404	0.034	0.	1.948	0.	1.24

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	92	216	0	710	0	198
normalized size	1	1.	0.95	2.23	0.	7.32	0.	2.04
time (sec)	N/A	0.147	0.225	0.03	0.	1.869	0.	1.183

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	256	202	0	807	0	200
normalized size	1	1.	2.72	2.15	0.	8.59	0.	2.13
time (sec)	N/A	0.137	0.608	0.067	0.	19.906	0.	1.234

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	339	272	0	1071	0	243
normalized size	1	1.	3.17	2.54	0.	10.01	0.	2.27
time (sec)	N/A	0.262	2.622	0.073	0.	34.678	0.	1.236

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	314	499	0	1450	0	387
normalized size	1	1.	2.04	3.24	0.	9.42	0.	2.51
time (sec)	N/A	0.55	2.069	0.079	0.	76.968	0.	1.274

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	466	825	0	1796	0	652
normalized size	1	1.	2.18	3.86	0.	8.39	0.	3.05
time (sec)	N/A	0.882	2.736	0.091	0.	160.163	0.	1.23

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	406	1335	0	0	0	1185
normalized size	1	1.	1.42	4.68	0.	0.	0.	4.16
time (sec)	N/A	1.282	1.426	0.098	0.	0.	0.	1.308

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	256	1229	0	2986	0	760
normalized size	1	1.	0.64	3.09	0.	7.5	0.	1.91
time (sec)	N/A	1.597	1.745	0.047	0.	2.888	0.	1.245

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	208	845	0	2381	0	508
normalized size	1	1.	0.69	2.79	0.	7.86	0.	1.68
time (sec)	N/A	1.117	1.375	0.043	0.	2.546	0.	1.248

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	159	561	0	1782	0	554
normalized size	1	1.	0.95	3.34	0.	10.61	0.	3.3
time (sec)	N/A	0.461	0.96	0.042	0.	2.311	0.	1.21

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	131	436	0	1280	0	297
normalized size	1	1.	0.94	3.14	0.	9.21	0.	2.14
time (sec)	N/A	0.226	0.658	0.034	0.	1.721	0.	1.205

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	319	458	0	1605	0	329
normalized size	1	1.	2.17	3.12	0.	10.92	0.	2.24
time (sec)	N/A	0.351	2.826	0.077	0.	61.17	0.	1.262

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	331	618	0	2498	0	597
normalized size	1	1.	1.57	2.93	0.	11.84	0.	2.83
time (sec)	N/A	0.768	1.687	0.088	0.	175.453	0.	1.276

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	389	914	0	0	0	571
normalized size	1	1.	1.27	2.98	0.	0.	0.	1.86
time (sec)	N/A	1.402	5.238	0.101	0.	0.	0.	1.273

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	519	1242	0	0	0	836
normalized size	1	1.	1.28	3.07	0.	0.	0.	2.06
time (sec)	N/A	2.004	3.048	0.109	0.	0.	0.	1.272

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	456	456	883	2133	0	4652	0	2284
normalized size	1	1.	1.94	4.68	0.	10.2	0.	5.01
time (sec)	N/A	4.637	4.411	0.053	0.	3.995	0.	1.357

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	573	1693	0	3621	0	899
normalized size	1	1.	1.82	5.39	0.	11.53	0.	2.86
time (sec)	N/A	2.822	2.799	0.049	0.	3.197	0.	1.372

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	225	1485	0	2651	0	814
normalized size	1	1.	0.97	6.37	0.	11.38	0.	3.49
time (sec)	N/A	0.771	1.668	0.044	0.	2.568	0.	1.269

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	192	1290	0	1837	0	675
normalized size	1	1.	0.95	6.39	0.	9.09	0.	3.34
time (sec)	N/A	0.359	1.044	0.035	0.	2.175	0.	1.266

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	473	1507	0	0	0	842
normalized size	1	1.	1.99	6.33	0.	0.	0.	3.54
time (sec)	N/A	0.978	4.316	0.087	0.	0.	0.	1.307

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	444	1750	0	0	0	944
normalized size	1	1.	1.31	5.16	0.	0.	0.	2.78
time (sec)	N/A	3.419	2.408	0.104	0.	0.	0.	1.34

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	606	2202	0	0	0	2354
normalized size	1	1.	1.31	4.77	0.	0.	0.	5.1
time (sec)	N/A	5.159	3.917	0.121	0.	0.	0.	1.389

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	658	4367	0	7788	0	1939
normalized size	1	1.	1.01	6.73	0.	12.	0.	2.99
time (sec)	N/A	12.199	6.902	0.059	0.	5.502	0.	1.355

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	532	3571	0	6165	0	1654
normalized size	1	1.	1.15	7.75	0.	13.37	0.	3.59
time (sec)	N/A	9.761	6.836	0.054	0.	4.346	0.	1.35

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	863	3098	0	4518	0	1490
normalized size	1	1.	2.47	8.88	0.	12.95	0.	4.27
time (sec)	N/A	2.394	4.589	0.048	0.	2.834	0.	1.331

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	307	2667	0	3094	0	1304
normalized size	1	1.	0.98	8.49	0.	9.85	0.	4.15
time (sec)	N/A	0.919	1.705	0.042	0.	2.389	0.	1.277

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	301	2667	0	3094	0	1304
normalized size	1	1.	1.01	8.92	0.	10.35	0.	4.36
time (sec)	N/A	0.775	1.541	0.039	0.	2.155	0.	1.344

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	587	3121	0	0	0	1521
normalized size	1	1.	1.7	9.05	0.	0.	0.	4.41
time (sec)	N/A	2.55	6.72	0.098	0.	0.	0.	1.399

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	480	709	3628	0	0	0	1694
normalized size	1	1.	1.48	7.56	0.	0.	0.	3.53
time (sec)	N/A	10.538	4.459	0.115	0.	0.	0.	1.364

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	657	657	686	4436	0	0	0	2001
normalized size	1	1.	1.04	6.75	0.	0.	0.	3.05
time (sec)	N/A	12.859	6.512	0.135	0.	0.	0.	1.472

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	34	32	0	55	58	65
normalized size	1	1.	1.48	1.39	0.	2.39	2.52	2.83
time (sec)	N/A	0.023	0.011	0.024	0.	1.565	0.908	1.248

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	68	108	0	524	0	128
normalized size	1	1.	1.11	1.77	0.	8.59	0.	2.1
time (sec)	N/A	0.116	0.119	0.036	0.	1.572	0.	1.192

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	107	299	0	933	0	230
normalized size	1	1.	0.97	2.72	0.	8.48	0.	2.09
time (sec)	N/A	0.181	0.326	0.038	0.	1.678	0.	1.246

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	171	964	0	1735	0	514
normalized size	1	1.	0.98	5.51	0.	9.91	0.	2.94
time (sec)	N/A	0.406	0.716	0.041	0.	1.946	0.	1.271

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	246	1817	0	2859	0	960
normalized size	1	1.	0.99	7.3	0.	11.48	0.	3.86
time (sec)	N/A	0.746	1.005	0.049	0.	2.198	0.	1.445

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	321	2143	0	0	0	0
normalized size	1	1.	0.77	5.15	0.	0.	0.	0.
time (sec)	N/A	0.931	1.737	1.063	0.	0.	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	249	1635	0	0	0	0
normalized size	1	1.	0.78	5.09	0.	0.	0.	0.
time (sec)	N/A	0.574	1.264	0.961	0.	0.	0.	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	189	1187	0	0	0	0
normalized size	1	1.	0.8	5.01	0.	0.	0.	0.
time (sec)	N/A	0.37	0.852	0.908	0.	0.	0.	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	393	740	0	0	0	0
normalized size	1	1.	1.64	3.08	0.	0.	0.	0.
time (sec)	N/A	0.693	3.522	0.953	0.	0.	0.	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	385	1035	0	0	0	0
normalized size	1	1.	1.77	4.77	0.	0.	0.	0.
time (sec)	N/A	0.666	2.584	0.993	0.	0.	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	428	1395	0	0	0	0
normalized size	1	1.	1.43	4.67	0.	0.	0.	0.
time (sec)	N/A	1.057	5.419	1.908	0.	0.	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	661	2319	0	0	0	0
normalized size	1	1.	1.66	5.81	0.	0.	0.	0.
time (sec)	N/A	1.53	6.643	2.849	0.	0.	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	407	2603	0	0	0	0
normalized size	1	1.	0.79	5.03	0.	0.	0.	0.
time (sec)	N/A	1.287	2.768	1.13	0.	0.	0.	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	321	2143	0	0	0	0
normalized size	1	1.	0.79	5.25	0.	0.	0.	0.
time (sec)	N/A	0.82	1.711	1.161	0.	0.	0.	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	257	1635	0	0	0	0
normalized size	1	1.	0.82	5.19	0.	0.	0.	0.
time (sec)	N/A	0.519	1.266	0.983	0.	0.	0.	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	455	1330	0	0	0	0
normalized size	1	1.	1.49	4.35	0.	0.	0.	0.
time (sec)	N/A	1.014	3.32	0.961	0.	0.	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	434	1635	0	0	0	0
normalized size	1	1.	1.52	5.72	0.	0.	0.	0.
time (sec)	N/A	1.034	4.031	1.118	0.	0.	0.	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	438	1743	0	0	0	0
normalized size	1	1.	1.43	5.68	0.	0.	0.	0.
time (sec)	N/A	1.076	6.25	2.104	0.	0.	0.	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	667	2441	0	0	0	0
normalized size	1	1.	1.67	6.12	0.	0.	0.	0.
time (sec)	N/A	1.55	6.858	3.121	0.	0.	0.	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	503	503	783	3551	0	0	0	0
normalized size	1	1.	1.56	7.06	0.	0.	0.	0.
time (sec)	N/A	2.043	6.964	4.403	0.	0.	0.	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	629	629	501	3165	0	0	0	0
normalized size	1	1.	0.8	5.03	0.	0.	0.	0.
time (sec)	N/A	1.515	3.852	1.168	0.	0.	0.	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	510	405	2603	0	0	0	0
normalized size	1	1.	0.79	5.1	0.	0.	0.	0.
time (sec)	N/A	1.096	2.599	1.078	0.	0.	0.	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	327	2143	0	0	0	0
normalized size	1	1.	0.81	5.33	0.	0.	0.	0.
time (sec)	N/A	0.756	1.836	1.022	0.	0.	0.	0.

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	526	1713	0	0	0	0
normalized size	1	1.	1.37	4.47	0.	0.	0.	0.
time (sec)	N/A	1.378	4.037	1.082	0.	0.	0.	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	502	2274	0	0	0	0
normalized size	1	1.	1.41	6.37	0.	0.	0.	0.
time (sec)	N/A	1.401	4.159	1.233	0.	0.	0.	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	492	2375	0	0	0	0
normalized size	1	1.	1.32	6.38	0.	0.	0.	0.
time (sec)	N/A	1.448	5.012	2.752	0.	0.	0.	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	519	2791	0	0	0	0
normalized size	1	1.	1.28	6.86	0.	0.	0.	0.
time (sec)	N/A	1.571	6.169	3.318	0.	0.	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	792	3673	0	0	0	0
normalized size	1	1.	1.58	7.32	0.	0.	0.	0.
time (sec)	N/A	2.11	7.063	4.656	0.	0.	0.	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	624	624	930	5171	0	0	0	0
normalized size	1	1.	1.49	8.29	0.	0.	0.	0.
time (sec)	N/A	2.71	7.234	6.298	0.	0.	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	259	1302	0	0	0	0
normalized size	1	1.	0.91	4.57	0.	0.	0.	0.
time (sec)	N/A	0.656	1.221	1.026	0.	0.	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	178	990	0	0	0	0
normalized size	1	1.	0.81	4.48	0.	0.	0.	0.
time (sec)	N/A	0.494	1.008	1.032	0.	0.	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	252	1635	0	0	0	0
normalized size	1	1.	0.73	4.75	0.	0.	0.	0.
time (sec)	N/A	0.715	1.299	1.027	0.	0.	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	186	1258	0	0	0	0
normalized size	1	1.	0.72	4.88	0.	0.	0.	0.
time (sec)	N/A	0.424	1.07	1.251	0.	0.	0.	0.

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	160	740	0	0	0	0
normalized size	1	1.	0.85	3.94	0.	0.	0.	0.
time (sec)	N/A	0.235	0.754	0.894	0.	0.	0.	0.

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	275	0	0	0	0
normalized size	1	1.	0.	1.46	0.	0.	0.	0.
time (sec)	N/A	0.449	16.894	0.918	0.	0.	0.	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	600	738	0	0	0	0
normalized size	1	1.	2.73	3.35	0.	0.	0.	0.
time (sec)	N/A	0.675	13.763	1.425	0.	0.	0.	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	424	1282	0	0	0	0
normalized size	1	1.	1.4	4.23	0.	0.	0.	0.
time (sec)	N/A	1.02	6.502	2.076	0.	0.	0.	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	665	2205	0	0	0	0
normalized size	1	1.	1.64	5.44	0.	0.	0.	0.
time (sec)	N/A	1.486	6.78	2.885	0.	0.	0.	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	328	1331	0	0	0	0
normalized size	1	1.	0.77	3.12	0.	0.	0.	0.
time (sec)	N/A	0.922	2.428	3.402	0.	0.	0.	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	236	1036	0	0	0	0
normalized size	1	1.	0.84	3.7	0.	0.	0.	0.
time (sec)	N/A	0.516	1.721	2.792	0.	0.	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	182	522	0	0	0	0
normalized size	1	1.	0.83	2.38	0.	0.	0.	0.
time (sec)	N/A	0.289	0.948	2.236	0.	0.	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	0	543	0	0	0	0
normalized size	1	1.	0.	2.	0.	0.	0.	0.
time (sec)	N/A	0.787	31.894	1.847	0.	0.	0.	0.

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	751	915	0	0	0	0
normalized size	1	1.	2.4	2.92	0.	0.	0.	0.
time (sec)	N/A	1.074	6.946	2.73	0.	0.	0.	0.

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	723	1577	0	0	0	0
normalized size	1	1.	1.74	3.79	0.	0.	0.	0.
time (sec)	N/A	1.586	7.105	3.339	0.	0.	0.	0.

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	622	622	422	1780	0	0	0	0
normalized size	1	1.	0.68	2.86	0.	0.	0.	0.
time (sec)	N/A	1.675	5.353	5.57	0.	0.	0.	0.

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	377	1480	0	0	0	0
normalized size	1	1.	0.83	3.27	0.	0.	0.	0.
time (sec)	N/A	1.025	3.725	5.096	0.	0.	0.	0.

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	323	963	0	0	0	0
normalized size	1	1.	0.9	2.68	0.	0.	0.	0.
time (sec)	N/A	0.63	3.043	4.58	0.	0.	0.	0.

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	278	867	0	0	0	0
normalized size	1	1.	0.83	2.6	0.	0.	0.	0.
time (sec)	N/A	0.486	2.399	3.935	0.	0.	0.	0.

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	401	0	879	0	0	0	0
normalized size	1	1.	0.	2.19	0.	0.	0.	0.
time (sec)	N/A	1.229	48.033	3.95	0.	0.	0.	0.

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	461	461	915	1348	0	0	0	0
normalized size	1	1.	1.98	2.92	0.	0.	0.	0.
time (sec)	N/A	1.613	7.286	5.882	0.	0.	0.	0.

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	572	572	922	2019	0	0	0	0
normalized size	1	1.	1.61	3.53	0.	0.	0.	0.
time (sec)	N/A	2.312	7.764	7.628	0.	0.	0.	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	433	1316	0	0	0	0
normalized size	1	1.	0.96	2.93	0.	0.	0.	0.
time (sec)	N/A	0.736	3.629	6.512	0.	0.	0.	0.

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	144	598	0	0	0	0
normalized size	1	1.	0.86	3.58	0.	0.	0.	0.
time (sec)	N/A	0.342	0.633	1.104	0.	0.	0.	0.

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	90	246	0	0	0	0
normalized size	1	1.	0.73	1.98	0.	0.	0.	0.
time (sec)	N/A	0.165	0.233	0.871	0.	0.	0.	0.

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	150	422	0	0	0	0
normalized size	1	1.	0.83	2.34	0.	0.	0.	0.
time (sec)	N/A	0.289	0.594	2.326	0.	0.	0.	0.

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	193	744	0	0	0	0
normalized size	1	1.	0.71	2.75	0.	0.	0.	0.
time (sec)	N/A	0.451	1.545	3.672	0.	0.	0.	0.

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	143	565	0	0	0	0
normalized size	1	1.	0.75	2.97	0.	0.	0.	0.
time (sec)	N/A	0.255	0.994	0.929	0.	0.	0.	0.

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	117	515	0	0	0	0
normalized size	1	1.	0.76	3.34	0.	0.	0.	0.
time (sec)	N/A	0.238	0.858	1.096	0.	0.	0.	0.

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	94	465	0	0	0	0
normalized size	1	1.	0.81	4.01	0.	0.	0.	0.
time (sec)	N/A	0.217	0.593	0.844	0.	0.	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	90	388	0	0	0	0
normalized size	1	1.	0.84	3.63	0.	0.	0.	0.
time (sec)	N/A	0.224	0.486	1.392	0.	0.	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	115	666	0	0	0	0
normalized size	1	1.	1.04	6.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.6	1.873	0.	0.	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	136	742	0	0	0	0
normalized size	1	1.	0.89	4.88	0.	0.	0.	0.
time (sec)	N/A	0.255	1.382	3.295	0.	0.	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	173	851	0	0	0	0
normalized size	1	1.	0.91	4.48	0.	0.	0.	0.
time (sec)	N/A	0.282	4.181	3.511	0.	0.	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	239	863	0	0	0	0
normalized size	1	1.	0.78	2.83	0.	0.	0.	0.
time (sec)	N/A	0.608	1.609	0.947	0.	0.	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	195	784	0	0	0	0
normalized size	1	1.	0.78	3.12	0.	0.	0.	0.
time (sec)	N/A	0.537	1.267	1.02	0.	0.	0.	0.

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	160	706	0	0	0	0
normalized size	1	1.	0.79	3.48	0.	0.	0.	0.
time (sec)	N/A	0.506	1.303	0.999	0.	0.	0.	0.

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	144	932	0	0	0	0
normalized size	1	1.	0.76	4.93	0.	0.	0.	0.
time (sec)	N/A	0.519	1.247	1.089	0.	0.	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	157	1303	0	0	0	0
normalized size	1	1.	0.87	7.24	0.	0.	0.	0.
time (sec)	N/A	0.501	1.239	3.009	0.	0.	0.	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	202	1000	0	0	0	0
normalized size	1	1.	1.01	5.	0.	0.	0.	0.
time (sec)	N/A	0.539	1.59	2.845	0.	0.	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	217	947	0	0	0	0
normalized size	1	1.	0.88	3.82	0.	0.	0.	0.
time (sec)	N/A	0.56	4.56	3.714	0.	0.	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	266	1196	0	0	0	0
normalized size	1	1.	0.88	3.96	0.	0.	0.	0.
time (sec)	N/A	0.638	5.246	4.865	0.	0.	0.	0.

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	285	1082	0	0	0	0
normalized size	1	1.	0.79	3.	0.	0.	0.	0.
time (sec)	N/A	0.92	1.88	0.889	0.	0.	0.	0.

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	230	975	0	0	0	0
normalized size	1	1.	0.78	3.29	0.	0.	0.	0.
time (sec)	N/A	0.839	2.01	1.05	0.	0.	0.	0.

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	212	1278	0	0	0	0
normalized size	1	1.	0.76	4.58	0.	0.	0.	0.
time (sec)	N/A	0.831	1.855	1.321	0.	0.	0.	0.

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	186	1837	0	0	0	0
normalized size	1	1.	0.69	6.78	0.	0.	0.	0.
time (sec)	N/A	0.858	2.324	3.149	0.	0.	0.	0.

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	248	1419	0	0	0	0
normalized size	1	1.	0.91	5.2	0.	0.	0.	0.
time (sec)	N/A	0.824	2.134	3.658	0.	0.	0.	0.

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	251	1205	0	0	0	0
normalized size	1	1.	0.85	4.1	0.	0.	0.	0.
time (sec)	N/A	0.851	4.867	4.113	0.	0.	0.	0.

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	414	1292	0	0	0	0
normalized size	1	1.	1.16	3.62	0.	0.	0.	0.
time (sec)	N/A	0.924	6.901	4.806	0.	0.	0.	0.

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	477	381	1407	0	0	0	0
normalized size	1	1.	0.8	2.95	0.	0.	0.	0.
time (sec)	N/A	1.316	3.519	1.152	0.	0.	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	319	1273	0	0	0	0
normalized size	1	1.	0.79	3.15	0.	0.	0.	0.
time (sec)	N/A	1.259	2.456	1.142	0.	0.	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	275	1652	0	0	0	0
normalized size	1	1.	0.73	4.36	0.	0.	0.	0.
time (sec)	N/A	1.271	4.316	1.459	0.	0.	0.	0.

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	257	2507	0	0	0	0
normalized size	1	1.	0.69	6.72	0.	0.	0.	0.
time (sec)	N/A	1.258	2.596	3.875	0.	0.	0.	0.

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	316	1884	0	0	0	0
normalized size	1	1.	0.82	4.88	0.	0.	0.	0.
time (sec)	N/A	1.295	2.369	4.677	0.	0.	0.	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	271	1624	0	0	0	0
normalized size	1	1.	0.71	4.24	0.	0.	0.	0.
time (sec)	N/A	1.272	5.13	4.952	0.	0.	0.	0.

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	463	1550	0	0	0	0
normalized size	1	1.	1.15	3.87	0.	0.	0.	0.
time (sec)	N/A	1.306	7.217	5.803	0.	0.	0.	0.

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	475	381	1550	0	0	0	0
normalized size	1	1.	0.8	3.26	0.	0.	0.	0.
time (sec)	N/A	1.396	6.163	6.718	0.	0.	0.	0.

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	339	1097	0	0	0	0
normalized size	1	1.	1.19	3.85	0.	0.	0.	0.
time (sec)	N/A	1.286	2.603	2.984	0.	0.	0.	0.

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	276	803	0	0	0	0
normalized size	1	1.	1.31	3.82	0.	0.	0.	0.
time (sec)	N/A	0.879	2.312	2.404	0.	0.	0.	0.

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	216	945	0	0	0	0
normalized size	1	1.	1.47	6.43	0.	0.	0.	0.
time (sec)	N/A	0.61	1.318	1.157	0.	0.	0.	0.

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	177	323	0	0	0	0
normalized size	1	1.	1.82	3.33	0.	0.	0.	0.
time (sec)	N/A	0.314	1.573	1.196	0.	0.	0.	0.

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	216	411	0	0	0	0
normalized size	1	1.	1.83	3.48	0.	0.	0.	0.
time (sec)	N/A	0.528	1.232	1.899	0.	0.	0.	0.

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	266	474	0	0	0	0
normalized size	1	1.	1.68	3.	0.	0.	0.	0.
time (sec)	N/A	0.837	2.463	2.854	0.	0.	0.	0.

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	336	802	0	0	0	0
normalized size	1	1.	1.44	3.43	0.	0.	0.	0.
time (sec)	N/A	1.236	4.924	3.932	0.	0.	0.	0.

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	420	1003	0	0	0	0
normalized size	1	1.	1.32	3.15	0.	0.	0.	0.
time (sec)	N/A	1.744	4.794	5.277	0.	0.	0.	0.

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	408	1382	0	0	0	0
normalized size	1	1.	0.92	3.11	0.	0.	0.	0.
time (sec)	N/A	1.592	5.497	4.115	0.	0.	0.	0.

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	343	1129	0	0	0	0
normalized size	1	1.	1.	3.29	0.	0.	0.	0.
time (sec)	N/A	1.126	3.626	3.989	0.	0.	0.	0.

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	304	862	0	0	0	0
normalized size	1	1.	1.21	3.43	0.	0.	0.	0.
time (sec)	N/A	0.72	3.896	3.278	0.	0.	0.	0.

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	299	815	0	0	0	0
normalized size	1	1.	1.23	3.35	0.	0.	0.	0.
time (sec)	N/A	0.708	3.354	2.797	0.	0.	0.	0.

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	355	903	0	0	0	0
normalized size	1	1.	1.16	2.95	0.	0.	0.	0.
time (sec)	N/A	1.089	4.337	3.482	0.	0.	0.	0.

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	428	1038	0	0	0	0
normalized size	1	1.	1.09	2.65	0.	0.	0.	0.
time (sec)	N/A	1.562	7.081	5.498	0.	0.	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	654	654	555	2520	0	0	0	0
normalized size	1	1.	0.85	3.85	0.	0.	0.	0.
time (sec)	N/A	2.639	7.441	6.424	0.	0.	0.	0.

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	524	2267	0	0	0	0
normalized size	1	1.	0.98	4.23	0.	0.	0.	0.
time (sec)	N/A	1.901	5.851	6.158	0.	0.	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	441	2000	0	0	0	0
normalized size	1	1.	1.04	4.73	0.	0.	0.	0.
time (sec)	N/A	1.363	6.308	5.379	0.	0.	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	429	1950	0	0	0	0
normalized size	1	1.	1.03	4.67	0.	0.	0.	0.
time (sec)	N/A	1.333	4.413	4.97	0.	0.	0.	0.

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	443	1857	0	0	0	0
normalized size	1	1.	1.07	4.5	0.	0.	0.	0.
time (sec)	N/A	1.275	6.128	4.942	0.	0.	0.	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	502	502	510	2027	0	0	0	0
normalized size	1	1.	1.02	4.04	0.	0.	0.	0.
time (sec)	N/A	1.858	7.037	5.839	0.	0.	0.	0.

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	609	609	676	2165	0	0	0	0
normalized size	1	1.	1.11	3.56	0.	0.	0.	0.
time (sec)	N/A	2.494	7.704	10.188	0.	0.	0.	0.

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	586	586	1242	3765	0	0	0	0
normalized size	1	1.	2.12	6.42	0.	0.	0.	0.
time (sec)	N/A	1.795	6.415	0.286	0.	0.	0.	0.

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	483	483	1183	2999	0	0	0	0
normalized size	1	1.	2.45	6.21	0.	0.	0.	0.
time (sec)	N/A	1.153	6.345	0.19	0.	0.	0.	0.

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	1176	2507	0	0	0	0
normalized size	1	1.	2.62	5.58	0.	0.	0.	0.
time (sec)	N/A	1.135	20.81	0.337	0.	0.	0.	0.

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	1240	2585	0	0	0	0
normalized size	1	1.	3.05	6.35	0.	0.	0.	0.
time (sec)	N/A	0.843	6.389	0.385	0.	0.	0.	0.

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	1340	3333	0	0	0	0
normalized size	1	1.	3.72	9.26	0.	0.	0.	0.
time (sec)	N/A	0.932	6.501	0.2	0.	0.	0.	0.

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	1464	4337	0	0	0	0
normalized size	1	1.	3.28	9.7	0.	0.	0.	0.
time (sec)	N/A	1.332	6.657	0.316	0.	0.	0.	0.

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	704	704	1317	5493	0	0	0	0
normalized size	1	1.	1.87	7.8	0.	0.	0.	0.
time (sec)	N/A	2.485	6.558	0.479	0.	0.	0.	0.

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	587	587	1250	4145	0	0	0	0
normalized size	1	1.	2.13	7.06	0.	0.	0.	0.
time (sec)	N/A	1.787	6.58	0.461	0.	0.	0.	0.

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	535	535	1232	3598	0	0	0	0
normalized size	1	1.	2.3	6.73	0.	0.	0.	0.
time (sec)	N/A	1.793	6.555	0.328	0.	0.	0.	0.

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	528	528	1260	3352	0	0	0	0
normalized size	1	1.	2.39	6.35	0.	0.	0.	0.
time (sec)	N/A	1.667	6.548	0.2	0.	0.	0.	0.

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	490	490	1353	3922	0	0	0	0
normalized size	1	1.	2.76	8.	0.	0.	0.	0.
time (sec)	N/A	1.291	6.653	0.215	0.	0.	0.	0.

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	1463	4526	0	0	0	0
normalized size	1	1.	3.25	10.06	0.	0.	0.	0.
time (sec)	N/A	1.432	6.731	0.332	0.	0.	0.	0.

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	550	550	1614	5956	0	0	0	0
normalized size	1	1.	2.93	10.83	0.	0.	0.	0.
time (sec)	N/A	2.016	6.958	0.566	0.	0.	0.	0.

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	834	834	1410	7062	0	0	0	0
normalized size	1	1.	1.69	8.47	0.	0.	0.	0.
time (sec)	N/A	3.792	6.718	0.873	0.	0.	0.	0.

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	700	700	1326	5873	0	0	0	0
normalized size	1	1.	1.89	8.39	0.	0.	0.	0.
time (sec)	N/A	2.534	6.815	0.902	0.	0.	0.	0.

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	647	647	1302	5130	0	0	0	0
normalized size	1	1.	2.01	7.93	0.	0.	0.	0.
time (sec)	N/A	2.369	6.82	0.513	0.	0.	0.	0.

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	622	1316	4889	0	0	0	0
normalized size	1	1.	2.12	7.86	0.	0.	0.	0.
time (sec)	N/A	2.207	6.789	0.247	0.	0.	0.	0.

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	643	643	1370	4986	0	0	0	0
normalized size	1	1.	2.13	7.75	0.	0.	0.	0.
time (sec)	N/A	2.362	6.857	0.268	0.	0.	0.	0.

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	580	580	1472	5143	0	0	0	0
normalized size	1	1.	2.54	8.87	0.	0.	0.	0.
time (sec)	N/A	1.798	6.961	0.348	0.	0.	0.	0.

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	552	552	1616	6176	0	0	0	0
normalized size	1	1.	2.93	11.19	0.	0.	0.	0.
time (sec)	N/A	2.049	7.101	0.509	0.	0.	0.	0.

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	593	593	1241	3575	0	0	0	0
normalized size	1	1.	2.09	6.03	0.	0.	0.	0.
time (sec)	N/A	1.861	6.524	0.309	0.	0.	0.	0.

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	1182	2248	0	0	0	0
normalized size	1	1.	2.44	4.64	0.	0.	0.	0.
time (sec)	N/A	1.123	12.719	0.173	0.	0.	0.	0.

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	1117	1325	0	0	0	0
normalized size	1	1.	2.79	3.3	0.	0.	0.	0.
time (sec)	N/A	0.792	18.98	0.137	0.	0.	0.	0.

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	1169	1321	0	0	0	0
normalized size	1	1.	3.37	3.81	0.	0.	0.	0.
time (sec)	N/A	0.535	19.013	0.192	0.	0.	0.	0.

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	1244	1823	0	0	0	0
normalized size	1	1.	4.25	6.22	0.	0.	0.	0.
time (sec)	N/A	0.571	6.476	0.24	0.	0.	0.	0.

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	1351	3134	0	0	0	0
normalized size	1	1.	3.63	8.42	0.	0.	0.	0.
time (sec)	N/A	0.933	6.579	0.214	0.	0.	0.	0.

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	466	466	1468	4337	0	0	0	0
normalized size	1	1.	3.15	9.31	0.	0.	0.	0.
time (sec)	N/A	1.415	6.708	0.315	0.	0.	0.	0.

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	1175	2052	0	0	0	0
normalized size	1	1.	2.48	4.34	0.	0.	0.	0.
time (sec)	N/A	1.476	6.204	0.158	0.	0.	0.	0.

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	160	919	0	0	0	0
normalized size	1	1.	0.62	3.59	0.	0.	0.	0.
time (sec)	N/A	0.495	4.753	0.153	0.	0.	0.	0.

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	660	660	1322	5209	0	0	0	0
normalized size	1	1.	2.	7.89	0.	0.	0.	0.
time (sec)	N/A	2.047	6.724	0.259	0.	0.	0.	0.

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	535	535	1256	3691	0	0	0	0
normalized size	1	1.	2.35	6.9	0.	0.	0.	0.
time (sec)	N/A	1.456	6.493	0.21	0.	0.	0.	0.

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	1245	2856	0	0	0	0
normalized size	1	1.	2.86	6.55	0.	0.	0.	0.
time (sec)	N/A	0.9	6.523	0.276	0.	0.	0.	0.

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	1306	3086	0	0	0	0
normalized size	1	1.	4.06	9.58	0.	0.	0.	0.
time (sec)	N/A	0.683	6.673	0.403	0.	0.	0.	0.

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	424	424	1402	4192	0	0	0	0
normalized size	1	1.	3.31	9.89	0.	0.	0.	0.
time (sec)	N/A	1.11	6.87	0.423	0.	0.	0.	0.

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	545	545	1511	5884	0	0	0	0
normalized size	1	1.	2.77	10.8	0.	0.	0.	0.
time (sec)	N/A	1.738	7.09	0.299	0.	0.	0.	0.

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	723	723	1448	10402	0	0	0	0
normalized size	1	1.	2.	14.39	0.	0.	0.	0.
time (sec)	N/A	2.63	6.817	0.516	0.	0.	0.	0.

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	589	589	1441	8239	0	0	0	0
normalized size	1	1.	2.45	13.99	0.	0.	0.	0.
time (sec)	N/A	1.618	6.777	0.386	0.	0.	0.	0.

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	1440	7003	0	0	0	0
normalized size	1	1.	3.15	15.32	0.	0.	0.	0.
time (sec)	N/A	1.147	6.7	1.071	0.	0.	0.	0.

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	1516	8926	0	0	0	0
normalized size	1	1.	3.06	18.03	0.	0.	0.	0.
time (sec)	N/A	1.32	6.926	1.44	0.	0.	0.	0.

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	620	620	1601	10927	0	0	0	0
normalized size	1	1.	2.58	17.62	0.	0.	0.	0.
time (sec)	N/A	2.301	7.146	0.454	0.	0.	0.	0.

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	268	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.94	3.266	1.783	0.	0.	0.	0.

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	205	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	1.855	1.175	0.	0.	0.	0.

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	15557	0	0	0	0	0
normalized size	1	1.	41.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.421	30.184	0.754	0.	0.	0.	0.

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	564	564	25789	0	0	0	0	0
normalized size	1	1.	45.73	0.	0.	0.	0.	0.
time (sec)	N/A	1.029	35.464	0.608	0.	0.	0.	0.

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	392	838	0	0	0	0
normalized size	1	1.	1.91	4.09	0.	0.	0.	0.
time (sec)	N/A	0.285	3.973	3.601	0.	0.	0.	0.

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	277	729	0	0	0	0
normalized size	1	1.	1.61	4.24	0.	0.	0.	0.
time (sec)	N/A	0.263	1.737	3.153	0.	0.	0.	0.

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	173	437	0	0	0	0
normalized size	1	1.	1.28	3.24	0.	0.	0.	0.
time (sec)	N/A	0.236	1.14	2.268	0.	0.	0.	0.

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	169	458	0	0	0	0
normalized size	1	1.	1.25	3.39	0.	0.	0.	0.
time (sec)	N/A	0.238	1.139	1.201	0.	0.	0.	0.

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	169	345	0	0	0	0
normalized size	1	1.	1.2	2.45	0.	0.	0.	0.
time (sec)	N/A	0.233	1.473	0.865	0.	0.	0.	0.

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	188	378	0	0	0	0
normalized size	1	1.	1.08	2.17	0.	0.	0.	0.
time (sec)	N/A	0.257	1.987	1.145	0.	0.	0.	0.

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	204	406	0	0	0	0
normalized size	1	1.	1.	1.98	0.	0.	0.	0.
time (sec)	N/A	0.28	2.6	0.949	0.	0.	0.	0.

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	655	1168	0	0	0	0
normalized size	1	1.	2.43	4.33	0.	0.	0.	0.
time (sec)	N/A	0.603	6.778	4.419	0.	0.	0.	0.

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	399	918	0	0	0	0
normalized size	1	1.	1.68	3.87	0.	0.	0.	0.
time (sec)	N/A	0.563	4.204	3.711	0.	0.	0.	0.

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	301	756	0	0	0	0
normalized size	1	1.	1.54	3.86	0.	0.	0.	0.
time (sec)	N/A	0.542	2.653	3.109	0.	0.	0.	0.

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	191	651	0	0	0	0
normalized size	1	1.	0.97	3.32	0.	0.	0.	0.
time (sec)	N/A	0.543	1.677	2.576	0.	0.	0.	0.

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	281	440	0	0	0	0
normalized size	1	1.	1.4	2.2	0.	0.	0.	0.
time (sec)	N/A	0.505	2.307	1.26	0.	0.	0.	0.

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	189	380	0	0	0	0
normalized size	1	1.	0.93	1.86	0.	0.	0.	0.
time (sec)	N/A	0.509	1.834	0.911	0.	0.	0.	0.

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	206	408	0	0	0	0
normalized size	1	1.	0.87	1.72	0.	0.	0.	0.
time (sec)	N/A	0.531	2.579	0.939	0.	0.	0.	0.

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	228	436	0	0	0	0
normalized size	1	1.	0.84	1.61	0.	0.	0.	0.
time (sec)	N/A	0.588	2.905	0.916	0.	0.	0.	0.

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	697	1408	0	0	0	0
normalized size	1	1.	2.18	4.41	0.	0.	0.	0.
time (sec)	N/A	0.769	6.925	5.112	0.	0.	0.	0.

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	655	1246	0	0	0	0
normalized size	1	1.	2.29	4.36	0.	0.	0.	0.
time (sec)	N/A	0.728	6.819	4.206	0.	0.	0.	0.

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	302	1012	0	0	0	0
normalized size	1	1.	1.19	4.	0.	0.	0.	0.
time (sec)	N/A	0.698	4.385	3.586	0.	0.	0.	0.

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	279	939	0	0	0	0
normalized size	1	1.	1.1	3.71	0.	0.	0.	0.
time (sec)	N/A	0.682	3.031	3.352	0.	0.	0.	0.

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	221	704	0	0	0	0
normalized size	1	1.	0.88	2.8	0.	0.	0.	0.
time (sec)	N/A	0.689	2.186	1.125	0.	0.	0.	0.

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	218	569	0	0	0	0
normalized size	1	1.	0.85	2.21	0.	0.	0.	0.
time (sec)	N/A	0.68	1.777	1.214	0.	0.	0.	0.

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	206	408	0	0	0	0
normalized size	1	1.	0.81	1.61	0.	0.	0.	0.
time (sec)	N/A	0.67	2.582	1.328	0.	0.	0.	0.

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	228	436	0	0	0	0
normalized size	1	1.	0.8	1.52	0.	0.	0.	0.
time (sec)	N/A	0.707	2.725	1.101	0.	0.	0.	0.

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	250	464	0	0	0	0
normalized size	1	1.	0.78	1.45	0.	0.	0.	0.
time (sec)	N/A	0.759	3.154	1.006	0.	0.	0.	0.

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	685	803	0	0	0	0
normalized size	1	1.	2.95	3.46	0.	0.	0.	0.
time (sec)	N/A	0.326	7.325	3.63	0.	0.	0.	0.

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	651	486	0	0	0	0
normalized size	1	1.	3.43	2.56	0.	0.	0.	0.
time (sec)	N/A	0.295	7.109	2.984	0.	0.	0.	0.

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	396	316	0	0	0	0
normalized size	1	1.	2.59	2.07	0.	0.	0.	0.
time (sec)	N/A	0.279	2.295	2.277	0.	0.	0.	0.

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	421	247	0	0	0	0
normalized size	1	1.	3.42	2.01	0.	0.	0.	0.
time (sec)	N/A	0.244	3.342	0.91	0.	0.	0.	0.

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	439	262	0	0	0	0
normalized size	1	1.	2.71	1.62	0.	0.	0.	0.
time (sec)	N/A	0.27	4.011	1.002	0.	0.	0.	0.

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	458	276	0	0	0	0
normalized size	1	1.	2.3	1.39	0.	0.	0.	0.
time (sec)	N/A	0.291	2.886	1.223	0.	0.	0.	0.

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	542	295	0	0	0	0
normalized size	1	1.	2.34	1.27	0.	0.	0.	0.
time (sec)	N/A	0.314	3.835	1.041	0.	0.	0.	0.

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	444	738	0	0	0	0
normalized size	1	1.	1.94	3.22	0.	0.	0.	0.
time (sec)	N/A	0.453	7.413	3.254	0.	0.	0.	0.

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	275	452	0	0	0	0
normalized size	1	1.	1.41	2.32	0.	0.	0.	0.
time (sec)	N/A	0.414	1.377	1.206	0.	0.	0.	0.

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	450	419	0	0	0	0
normalized size	1	1.	2.73	2.54	0.	0.	0.	0.
time (sec)	N/A	0.394	4.77	1.344	0.	0.	0.	0.

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	267	348	0	0	0	0
normalized size	1	1.	1.61	2.1	0.	0.	0.	0.
time (sec)	N/A	0.389	1.4	1.096	0.	0.	0.	0.

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	762	437	0	0	0	0
normalized size	1	1.	3.79	2.17	0.	0.	0.	0.
time (sec)	N/A	0.432	6.766	1.247	0.	0.	0.	0.

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	813	451	0	0	0	0
normalized size	1	1.	3.44	1.91	0.	0.	0.	0.
time (sec)	N/A	0.461	6.866	1.15	0.	0.	0.	0.

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	822	876	0	0	0	0
normalized size	1	1.	2.91	3.11	0.	0.	0.	0.
time (sec)	N/A	0.619	7.816	1.612	0.	0.	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	359	685	0	0	0	0
normalized size	1	1.	1.39	2.64	0.	0.	0.	0.
time (sec)	N/A	0.616	5.151	1.28	0.	0.	0.	0.

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	792	451	0	0	0	0
normalized size	1	1.	3.54	2.01	0.	0.	0.	0.
time (sec)	N/A	0.572	6.89	1.224	0.	0.	0.	0.

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	787	451	0	0	0	0
normalized size	1	1.	3.58	2.05	0.	0.	0.	0.
time (sec)	N/A	0.565	6.971	1.468	0.	0.	0.	0.

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	813	451	0	0	0	0
normalized size	1	1.	3.73	2.07	0.	0.	0.	0.
time (sec)	N/A	0.575	6.957	1.124	0.	0.	0.	0.

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	573	465	0	0	0	0
normalized size	1	1.	2.3	1.87	0.	0.	0.	0.
time (sec)	N/A	0.597	4.15	1.295	0.	0.	0.	0.

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	623	479	0	0	0	0
normalized size	1	1.	2.15	1.65	0.	0.	0.	0.
time (sec)	N/A	0.657	5.391	1.222	0.	0.	0.	0.

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	124	129	890	311	0	0
normalized size	1	1.	0.58	0.61	4.18	1.46	0.	0.
time (sec)	N/A	0.582	0.708	0.22	1.773	1.492	0.	0.

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	101	107	765	263	0	0
normalized size	1	1.	0.6	0.64	4.55	1.57	0.	0.
time (sec)	N/A	0.506	0.523	0.189	1.831	1.468	0.	0.

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	85	640	213	0	0
normalized size	1	1.	0.59	0.69	5.2	1.73	0.	0.
time (sec)	N/A	0.437	0.293	0.175	1.718	1.382	0.	0.

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	90	271	1836	333	0	0
normalized size	1	1.	0.66	1.99	13.5	2.45	0.	0.
time (sec)	N/A	0.416	0.223	0.203	2.323	1.601	0.	0.

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	100	185	1202	285	0	0
normalized size	1	1.	0.73	1.35	8.77	2.08	0.	0.
time (sec)	N/A	0.431	0.267	0.193	2.224	1.573	0.	0.

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	118	204	1629	338	0	0
normalized size	1	1.	0.82	1.42	11.31	2.35	0.	0.
time (sec)	N/A	0.428	0.34	0.218	2.433	1.857	0.	0.

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	274	3663	385	0	0
normalized size	1	1.	0.71	1.45	19.38	2.04	0.	0.
time (sec)	N/A	0.505	0.51	0.228	2.94	1.892	0.	0.

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	151	344	10395	440	0	0
normalized size	1	1.	0.65	1.47	44.42	1.88	0.	0.
time (sec)	N/A	0.594	0.54	0.193	3.811	2.299	0.	0.

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	146	152	961	383	0	0
normalized size	1	1.	0.55	0.57	3.61	1.44	0.	0.
time (sec)	N/A	0.836	0.839	0.192	1.834	1.5	0.	0.

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	123	130	836	327	0	0
normalized size	1	1.	0.56	0.59	3.82	1.49	0.	0.
time (sec)	N/A	0.748	0.744	0.173	1.858	1.489	0.	0.

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	102	108	711	274	0	0
normalized size	1	1.	0.59	0.63	4.13	1.59	0.	0.
time (sec)	N/A	0.655	0.551	0.168	1.766	1.444	0.	0.

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	121	371	2295	392	0	0
normalized size	1	1.	0.66	2.03	12.54	2.14	0.	0.
time (sec)	N/A	0.619	0.716	0.204	2.421	1.75	0.	0.

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	116	290	1881	379	0	0
normalized size	1	1.	0.64	1.6	10.39	2.09	0.	0.
time (sec)	N/A	0.64	0.529	0.203	2.305	1.601	0.	0.

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	119	327	0	362	0	0
normalized size	1	1.	0.61	1.68	0.	1.86	0.	0.
time (sec)	N/A	0.653	0.626	0.206	0.	1.961	0.	0.

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	133	269	3707	408	0	0
normalized size	1	1.	0.7	1.41	19.41	2.14	0.	0.
time (sec)	N/A	0.643	0.667	0.203	3.074	1.925	0.	0.

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	150	345	10799	456	0	0
normalized size	1	1.	0.63	1.45	45.37	1.92	0.	0.
time (sec)	N/A	0.745	0.647	0.178	4.037	2.501	0.	0.

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	169	417	6035	527	0	0
normalized size	1	1.	0.59	1.46	21.18	1.85	0.	0.
time (sec)	N/A	0.824	0.996	0.197	3.718	2.456	0.	0.

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	171	176	1030	478	0	0
normalized size	1	1.	0.55	0.56	3.29	1.53	0.	0.
time (sec)	N/A	1.056	0.927	0.207	1.832	1.548	0.	0.

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	149	154	906	390	0	0
normalized size	1	1.	0.56	0.58	3.41	1.47	0.	0.
time (sec)	N/A	0.958	1.103	0.184	1.766	1.539	0.	0.

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	127	132	782	342	0	0
normalized size	1	1.	0.58	0.6	3.57	1.56	0.	0.
time (sec)	N/A	0.862	0.968	0.181	1.784	1.675	0.	0.

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	151	473	3163	463	0	0
normalized size	1	1.	0.66	2.06	13.75	2.01	0.	0.
time (sec)	N/A	0.8	1.421	0.228	2.579	1.85	0.	0.

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	141	391	2259	452	0	0
normalized size	1	1.	0.61	1.7	9.82	1.97	0.	0.
time (sec)	N/A	0.855	1.007	0.211	2.425	1.722	0.	0.

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	141	494	0	459	0	0
normalized size	1	1.	0.59	2.08	0.	1.93	0.	0.
time (sec)	N/A	0.852	0.859	0.214	0.	2.151	0.	0.

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	142	361	0	432	0	0
normalized size	1	1.	0.59	1.49	0.	1.79	0.	0.
time (sec)	N/A	0.86	1.003	0.168	0.	2.239	0.	0.

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	153	341	11549	486	0	0
normalized size	1	1.	0.64	1.43	48.53	2.04	0.	0.
time (sec)	N/A	0.852	0.764	0.181	4.324	2.468	0.	0.

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	170	419	6151	547	0	0
normalized size	1	1.	0.6	1.47	21.58	1.92	0.	0.
time (sec)	N/A	0.949	1.3	0.18	4.017	2.673	0.	0.

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	192	491	0	603	0	0
normalized size	1	1.	0.58	1.48	0.	1.82	0.	0.
time (sec)	N/A	1.046	1.257	0.203	0.	2.625	0.	0.

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	271	775	0	529	0	0
normalized size	1	1.	0.94	2.68	0.	1.83	0.	0.
time (sec)	N/A	1.023	9.086	0.194	0.	1.837	0.	0.

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	2480	639	0	483	0	0
normalized size	1	1.	10.16	2.62	0.	1.98	0.	0.
time (sec)	N/A	0.829	10.271	0.229	0.	1.704	0.	0.

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	1757	503	0	433	0	0
normalized size	1	1.	8.74	2.5	0.	2.15	0.	0.
time (sec)	N/A	0.641	7.988	0.203	0.	1.901	0.	0.

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	576	366	0	382	0	0
normalized size	1	1.	3.69	2.35	0.	2.45	0.	0.
time (sec)	N/A	0.463	6.798	0.187	0.	1.951	0.	0.

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	251	353	0	454	0	0
normalized size	1	1.	1.43	2.02	0.	2.59	0.	0.
time (sec)	N/A	0.504	3.524	0.181	0.	11.835	0.	0.

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	124	185	0	447	0	0
normalized size	1	1.	0.72	1.07	0.	2.58	0.	0.
time (sec)	N/A	0.506	0.323	0.191	0.	10.836	0.	0.

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	496	270	0	529	0	0
normalized size	1	1.	2.22	1.21	0.	2.37	0.	0.
time (sec)	N/A	0.687	1.21	0.192	0.	21.44	0.	0.

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	439	340	0	578	0	0
normalized size	1	1.	1.65	1.28	0.	2.17	0.	0.
time (sec)	N/A	0.88	1.547	0.166	0.	26.994	0.	0.

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	3121	719	0	630	0	0
normalized size	1	1.	9.91	2.28	0.	2.	0.	0.
time (sec)	N/A	1.085	10.547	0.244	0.	2.239	0.	0.

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	2280	583	0	567	0	0
normalized size	1	1.	8.51	2.18	0.	2.12	0.	0.
time (sec)	N/A	0.882	8.058	0.205	0.	2.361	0.	0.

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	1055	445	0	521	0	0
normalized size	1	1.	4.77	2.01	0.	2.36	0.	0.
time (sec)	N/A	0.719	6.809	0.188	0.	2.409	0.	0.

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	460	311	0	427	0	0
normalized size	1	1.	2.67	1.81	0.	2.48	0.	0.
time (sec)	N/A	0.535	4.67	0.185	0.	2.064	0.	0.

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	245	283	0	579	0	0
normalized size	1	1.	1.32	1.53	0.	3.13	0.	0.
time (sec)	N/A	0.557	1.651	0.175	0.	42.267	0.	0.

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	251	321	0	624	0	0
normalized size	1	1.	1.1	1.41	0.	2.74	0.	0.
time (sec)	N/A	0.722	1.684	0.18	0.	42.106	0.	0.

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	385	404	0	701	0	0
normalized size	1	1.	1.35	1.42	0.	2.46	0.	0.
time (sec)	N/A	0.931	6.198	0.199	0.	75.416	0.	0.

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	261	717	0	717	0	0
normalized size	1	1.	0.83	2.28	0.	2.28	0.	0.
time (sec)	N/A	1.086	8.06	0.223	0.	2.053	0.	0.

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	243	573	0	657	0	0
normalized size	1	1.	0.91	2.15	0.	2.47	0.	0.
time (sec)	N/A	0.917	3.438	0.206	0.	1.856	0.	0.

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	213	457	0	552	0	0
normalized size	1	1.	0.97	2.09	0.	2.52	0.	0.
time (sec)	N/A	0.71	2.169	0.187	0.	1.921	0.	0.

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	216	376	0	554	0	0
normalized size	1	1.	1.24	2.16	0.	3.18	0.	0.
time (sec)	N/A	0.523	1.738	0.177	0.	2.524	0.	0.

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	262	475	0	756	0	0
normalized size	1	1.	1.13	2.05	0.	3.26	0.	0.
time (sec)	N/A	0.719	2.36	0.178	0.	77.736	0.	0.

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	274	509	0	795	0	0
normalized size	1	1.	0.99	1.84	0.	2.87	0.	0.
time (sec)	N/A	0.935	2.745	0.188	0.	78.324	0.	0.

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	968	642	0	886	0	0
normalized size	1	1.	2.9	1.92	0.	2.65	0.	0.
time (sec)	N/A	1.154	7.2	0.211	0.	132.385	0.	0.

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	0	0	0
normalized size	1	1.	0.64	3.32	0.	0.	0.	0.
time (sec)	N/A	0.138	0.318	2.656	0.	0.	0.	0.

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	85	397	0	0	0	0
normalized size	1	1.	0.69	3.23	0.	0.	0.	0.
time (sec)	N/A	0.122	0.225	2.335	0.	0.	0.	0.

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	148	0	0	0	0
normalized size	1	1.	0.73	1.53	0.	0.	0.	0.
time (sec)	N/A	0.105	0.108	1.177	0.	0.	0.	0.

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	0	0	0
normalized size	1	1.	0.69	2.03	0.	0.	0.	0.
time (sec)	N/A	0.093	0.072	0.797	0.	0.	0.	0.

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	229	0	0	0	0
normalized size	1	1.	0.75	2.27	0.	0.	0.	0.
time (sec)	N/A	0.106	0.132	0.983	0.	0.	0.	0.

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	88	383	0	0	0	0
normalized size	1	1.	0.69	3.02	0.	0.	0.	0.
time (sec)	N/A	0.119	0.311	1.984	0.	0.	0.	0.

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	99	403	0	0	0	0
normalized size	1	1.	0.66	2.67	0.	0.	0.	0.
time (sec)	N/A	0.13	0.534	1.079	0.	0.	0.	0.

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	112	799	0	0	0	0
normalized size	1	1.	0.69	4.9	0.	0.	0.	0.
time (sec)	N/A	0.153	1.199	3.406	0.	0.	0.	0.

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	89	500	0	0	0	0
normalized size	1	1.	0.7	3.94	0.	0.	0.	0.
time (sec)	N/A	0.135	0.307	2.295	0.	0.	0.	0.

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	194	0	0	0	0
normalized size	1	1.	0.74	1.92	0.	0.	0.	0.
time (sec)	N/A	0.121	0.177	1.03	0.	0.	0.	0.

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	274	0	0	0	0
normalized size	1	1.	0.76	2.61	0.	0.	0.	0.
time (sec)	N/A	0.12	0.178	1.222	0.	0.	0.	0.

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	94	308	0	0	0	0
normalized size	1	1.	0.71	2.32	0.	0.	0.	0.
time (sec)	N/A	0.136	0.288	1.104	0.	0.	0.	0.

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	108	342	0	0	0	0
normalized size	1	1.	0.66	2.1	0.	0.	0.	0.
time (sec)	N/A	0.157	0.652	1.376	0.	0.	0.	0.

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	172	849	0	0	0	0
normalized size	1	1.	0.79	3.91	0.	0.	0.	0.
time (sec)	N/A	0.352	2.216	4.046	0.	0.	0.	0.

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	147	739	0	0	0	0
normalized size	1	1.	0.82	4.13	0.	0.	0.	0.
time (sec)	N/A	0.316	0.98	3.262	0.	0.	0.	0.

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	99	515	0	0	0	0
normalized size	1	1.	0.71	3.68	0.	0.	0.	0.
time (sec)	N/A	0.313	0.69	2.748	0.	0.	0.	0.

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	101	380	0	0	0	0
normalized size	1	1.	0.72	2.7	0.	0.	0.	0.
time (sec)	N/A	0.262	0.46	1.242	0.	0.	0.	0.

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	105	447	0	0	0	0
normalized size	1	1.	0.71	3.04	0.	0.	0.	0.
time (sec)	N/A	0.27	0.626	1.217	0.	0.	0.	0.

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	125	481	0	0	0	0
normalized size	1	1.	0.68	2.61	0.	0.	0.	0.
time (sec)	N/A	0.298	0.964	0.941	0.	0.	0.	0.

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	149	512	0	0	0	0
normalized size	1	1.	0.69	2.36	0.	0.	0.	0.
time (sec)	N/A	0.345	0.861	1.378	0.	0.	0.	0.

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	209	1181	0	0	0	0
normalized size	1	1.	0.72	4.06	0.	0.	0.	0.
time (sec)	N/A	0.652	2.299	5.187	0.	0.	0.	0.

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	177	932	0	0	0	0
normalized size	1	1.	0.69	3.65	0.	0.	0.	0.
time (sec)	N/A	0.631	2.569	5.027	0.	0.	0.	0.

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	135	906	0	0	0	0
normalized size	1	1.	0.63	4.23	0.	0.	0.	0.
time (sec)	N/A	0.609	1.328	3.902	0.	0.	0.	0.

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	118	800	0	0	0	0
normalized size	1	1.	0.56	3.77	0.	0.	0.	0.
time (sec)	N/A	0.589	1.437	3.35	0.	0.	0.	0.

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	121	595	0	0	0	0
normalized size	1	1.	0.57	2.81	0.	0.	0.	0.
time (sec)	N/A	0.589	0.701	1.412	0.	0.	0.	0.

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	133	483	0	0	0	0
normalized size	1	1.	0.61	2.21	0.	0.	0.	0.
time (sec)	N/A	0.579	0.991	1.319	0.	0.	0.	0.

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	151	514	0	0	0	0
normalized size	1	1.	0.59	2.02	0.	0.	0.	0.
time (sec)	N/A	0.611	0.937	1.345	0.	0.	0.	0.

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	174	545	0	0	0	0
normalized size	1	1.	0.6	1.87	0.	0.	0.	0.
time (sec)	N/A	0.65	1.392	1.187	0.	0.	0.	0.

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	242	1424	0	0	0	0
normalized size	1	1.	0.71	4.15	0.	0.	0.	0.
time (sec)	N/A	0.843	3.145	5.989	0.	0.	0.	0.

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	209	1262	0	0	0	0
normalized size	1	1.	0.68	4.11	0.	0.	0.	0.
time (sec)	N/A	0.816	2.344	6.28	0.	0.	0.	0.

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	176	1097	0	0	0	0
normalized size	1	1.	0.65	4.05	0.	0.	0.	0.
time (sec)	N/A	0.776	3.426	4.606	0.	0.	0.	0.

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	157	1328	0	0	0	0
normalized size	1	1.	0.58	4.92	0.	0.	0.	0.
time (sec)	N/A	0.791	2.109	4.743	0.	0.	0.	0.

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	149	950	0	0	0	0
normalized size	1	1.	0.56	3.56	0.	0.	0.	0.
time (sec)	N/A	0.762	1.042	4.498	0.	0.	0.	0.

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	149	727	0	0	0	0
normalized size	1	1.	0.55	2.7	0.	0.	0.	0.
time (sec)	N/A	0.762	1.004	1.378	0.	0.	0.	0.

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	153	514	0	0	0	0
normalized size	1	1.	0.56	1.9	0.	0.	0.	0.
time (sec)	N/A	0.784	1.541	1.49	0.	0.	0.	0.

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	174	545	0	0	0	0
normalized size	1	1.	0.57	1.78	0.	0.	0.	0.
time (sec)	N/A	0.827	1.567	1.523	0.	0.	0.	0.

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	197	576	0	0	0	0
normalized size	1	1.	0.57	1.68	0.	0.	0.	0.
time (sec)	N/A	0.845	2.149	1.352	0.	0.	0.	0.

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	200	812	0	0	0	0
normalized size	1	1.	0.8	3.25	0.	0.	0.	0.
time (sec)	N/A	0.377	3.912	4.773	0.	0.	0.	0.

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	162	494	0	0	0	0
normalized size	1	1.	0.79	2.41	0.	0.	0.	0.
time (sec)	N/A	0.348	2.501	3.162	0.	0.	0.	0.

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	132	353	0	0	0	0
normalized size	1	1.	0.8	2.14	0.	0.	0.	0.
time (sec)	N/A	0.324	1.183	2.537	0.	0.	0.	0.

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	126	281	0	0	0	0
normalized size	1	1.	0.97	2.16	0.	0.	0.	0.
time (sec)	N/A	0.294	0.743	0.986	0.	0.	0.	0.

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	163	300	0	0	0	0
normalized size	1	1.	0.94	1.72	0.	0.	0.	0.
time (sec)	N/A	0.318	0.781	1.113	0.	0.	0.	0.

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	178	319	0	0	0	0
normalized size	1	1.	0.83	1.49	0.	0.	0.	0.
time (sec)	N/A	0.34	1.123	1.235	0.	0.	0.	0.

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	198	341	0	0	0	0
normalized size	1	1.	0.79	1.36	0.	0.	0.	0.
time (sec)	N/A	0.365	1.801	1.234	0.	0.	0.	0.

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	212	751	0	0	0	0
normalized size	1	1.	0.84	2.99	0.	0.	0.	0.
time (sec)	N/A	0.514	4.242	4.203	0.	0.	0.	0.

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	172	563	0	0	0	0
normalized size	1	1.	0.8	2.62	0.	0.	0.	0.
time (sec)	N/A	0.482	3.191	3.256	0.	0.	0.	0.

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	164	507	0	0	0	0
normalized size	1	1.	0.95	2.93	0.	0.	0.	0.
time (sec)	N/A	0.436	2.009	1.456	0.	0.	0.	0.

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	162	507	0	0	0	0
normalized size	1	1.	0.91	2.83	0.	0.	0.	0.
time (sec)	N/A	0.444	2.108	1.577	0.	0.	0.	0.

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	183	472	0	0	0	0
normalized size	1	1.	0.83	2.15	0.	0.	0.	0.
time (sec)	N/A	0.497	2.744	1.332	0.	0.	0.	0.

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	200	491	0	0	0	0
normalized size	1	1.	0.79	1.93	0.	0.	0.	0.
time (sec)	N/A	0.521	3.642	1.254	0.	0.	0.	0.

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	249	1040	0	0	0	0
normalized size	1	1.	0.8	3.35	0.	0.	0.	0.
time (sec)	N/A	0.711	5.848	5.271	0.	0.	0.	0.

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	215	793	0	0	0	0
normalized size	1	1.	0.78	2.86	0.	0.	0.	0.
time (sec)	N/A	0.688	1.946	1.602	0.	0.	0.	0.

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	188	624	0	0	0	0
normalized size	1	1.	0.81	2.68	0.	0.	0.	0.
time (sec)	N/A	0.659	1.472	1.252	0.	0.	0.	0.

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	188	624	0	0	0	0
normalized size	1	1.	0.81	2.7	0.	0.	0.	0.
time (sec)	N/A	0.673	1.508	1.228	0.	0.	0.	0.

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	190	624	0	0	0	0
normalized size	1	1.	0.81	2.66	0.	0.	0.	0.
time (sec)	N/A	0.65	2.168	1.413	0.	0.	0.	0.

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	206	638	0	0	0	0
normalized size	1	1.	0.76	2.35	0.	0.	0.	0.
time (sec)	N/A	0.685	2.903	1.493	0.	0.	0.	0.

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	229	666	0	0	0	0
normalized size	1	1.	0.73	2.13	0.	0.	0.	0.
time (sec)	N/A	0.721	3.123	1.556	0.	0.	0.	0.

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	155	171	1331	351	0	0
normalized size	1	1.	0.69	0.76	5.89	1.55	0.	0.
time (sec)	N/A	0.646	0.955	0.22	1.935	1.548	0.	0.

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	121	138	1145	294	0	0
normalized size	1	1.	0.68	0.78	6.43	1.65	0.	0.
time (sec)	N/A	0.592	0.71	0.193	1.903	1.599	0.	0.

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	85	105	957	234	0	0
normalized size	1	1.	0.65	0.81	7.36	1.8	0.	0.
time (sec)	N/A	0.5	0.354	0.191	1.844	1.532	0.	0.

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	105	286	2088	344	0	0
normalized size	1	1.	0.75	2.04	14.91	2.46	0.	0.
time (sec)	N/A	0.473	0.387	0.202	2.388	1.653	0.	0.

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	104	305	2288	304	0	0
normalized size	1	1.	0.74	2.16	16.23	2.16	0.	0.
time (sec)	N/A	0.486	0.304	0.202	2.61	2.083	0.	0.

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	123	270	2695	365	0	0
normalized size	1	1.	0.81	1.79	17.85	2.42	0.	0.
time (sec)	N/A	0.486	0.427	0.209	2.78	3.254	0.	0.

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	144	374	5090	458	0	0
normalized size	1	1.	0.72	1.88	25.58	2.3	0.	0.
time (sec)	N/A	0.563	0.795	0.2	3.465	1.64	0.	0.

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	164	480	0	490	0	0
normalized size	1	1.	0.66	1.94	0.	1.98	0.	0.
time (sec)	N/A	0.653	1.006	0.196	0.	4.938	0.	0.

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	187	205	1438	443	0	0
normalized size	1	1.	0.66	0.72	5.06	1.56	0.	0.
time (sec)	N/A	0.907	1.147	0.221	1.949	1.551	0.	0.

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	157	172	1250	371	0	0
normalized size	1	1.	0.68	0.74	5.39	1.6	0.	0.
time (sec)	N/A	0.81	1.055	0.198	1.905	1.63	0.	0.

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	122	139	1064	308	0	0
normalized size	1	1.	0.66	0.76	5.78	1.67	0.	0.
time (sec)	N/A	0.714	0.785	0.182	1.867	1.6	0.	0.

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	134	404	2585	420	0	0
normalized size	1	1.	0.7	2.1	13.46	2.19	0.	0.
time (sec)	N/A	0.661	0.882	0.205	2.51	1.852	0.	0.

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	128	490	3680	419	0	0
normalized size	1	1.	0.67	2.57	19.27	2.19	0.	0.
time (sec)	N/A	0.718	0.715	0.223	2.835	2.16	0.	0.

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	127	462	0	392	0	0
normalized size	1	1.	0.63	2.3	0.	1.95	0.	0.
time (sec)	N/A	0.717	0.57	0.173	0.	3.474	0.	0.

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	145	369	5162	450	0	0
normalized size	1	1.	0.72	1.84	25.68	2.24	0.	0.
time (sec)	N/A	0.727	0.89	0.185	3.578	3.628	0.	0.

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	167	481	0	514	0	0
normalized size	1	1.	0.66	1.9	0.	2.03	0.	0.
time (sec)	N/A	0.807	0.931	0.204	0.	4.936	0.	0.

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	190	589	0	595	0	0
normalized size	1	1.	0.63	1.94	0.	1.96	0.	0.
time (sec)	N/A	0.908	1.954	0.222	0.	4.921	0.	0.

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	224	240	1542	549	0	0
normalized size	1	1.	0.67	0.72	4.62	1.64	0.	0.
time (sec)	N/A	1.176	1.244	0.238	1.973	1.712	0.	0.

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	190	207	1357	464	0	0
normalized size	1	1.	0.67	0.73	4.78	1.63	0.	0.
time (sec)	N/A	1.058	0.982	0.22	1.979	1.511	0.	0.

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	158	174	1169	382	0	0
normalized size	1	1.	0.68	0.74	5.	1.63	0.	0.
time (sec)	N/A	0.953	1.352	0.203	2.018	1.511	0.	0.

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	172	522	3488	504	0	0
normalized size	1	1.	0.71	2.16	14.41	2.08	0.	0.
time (sec)	N/A	0.878	1.668	0.181	2.894	1.855	0.	0.

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	156	673	4362	502	0	0
normalized size	1	1.	0.64	2.77	17.95	2.07	0.	0.
time (sec)	N/A	0.952	1.409	0.183	3.106	2.309	0.	0.

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	156	711	0	504	0	0
normalized size	1	1.	0.62	2.81	0.	1.99	0.	0.
time (sec)	N/A	0.949	1.137	0.184	0.	3.924	0.	0.

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	156	513	0	478	0	0
normalized size	1	1.	0.62	2.04	0.	1.9	0.	0.
time (sec)	N/A	0.954	0.995	0.185	0.	3.261	0.	0.

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	166	477	0	541	0	0
normalized size	1	1.	0.66	1.89	0.	2.14	0.	0.
time (sec)	N/A	0.941	1.466	0.203	0.	4.951	0.	0.

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	193	591	0	614	0	0
normalized size	1	1.	0.64	1.96	0.	2.04	0.	0.
time (sec)	N/A	1.038	1.33	0.209	0.	5.191	0.	0.

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	227	699	0	701	0	0
normalized size	1	1.	0.64	1.98	0.	1.99	0.	0.
time (sec)	N/A	1.159	1.775	0.22	0.	5.373	0.	0.

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	7123	1131	0	579	0	0
normalized size	1	1.	23.35	3.71	0.	1.9	0.	0.
time (sec)	N/A	1.132	34.588	0.21	0.	1.901	0.	0.

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	2646	927	0	522	0	0
normalized size	1	1.	10.3	3.61	0.	2.03	0.	0.
time (sec)	N/A	0.91	10.021	0.192	0.	1.977	0.	0.

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	1882	723	0	463	0	0
normalized size	1	1.	8.92	3.43	0.	2.19	0.	0.
time (sec)	N/A	0.717	8.017	0.227	0.	1.888	0.	0.

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	0	518	0	404	0	0
normalized size	1	1.	0.	3.18	0.	2.48	0.	0.
time (sec)	N/A	0.507	0.	0.219	0.	1.941	0.	0.

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	277	439	0	464	0	0
normalized size	1	1.	1.56	2.47	0.	2.61	0.	0.
time (sec)	N/A	0.557	3.862	0.191	0.	68.497	0.	0.

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	132	247	0	477	0	0
normalized size	1	1.	0.73	1.36	0.	2.64	0.	0.
time (sec)	N/A	0.558	0.389	0.201	0.	85.904	0.	0.

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	16885	363	0	0	0	0
normalized size	1	1.	71.85	1.54	0.	0.	0.	0.
time (sec)	N/A	0.778	27.462	0.218	0.	0.	0.	0.

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	16934	470	0	0	0	0
normalized size	1	1.	60.26	1.67	0.	0.	0.	0.
time (sec)	N/A	1.041	27.551	0.187	0.	0.	0.	0.

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	143	317	0	0	0	0
normalized size	1	1.	0.74	1.65	0.	0.	0.	0.
time (sec)	N/A	0.699	0.44	0.219	0.	0.	0.	0.

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	3136	1047	0	702	0	0
normalized size	1	1.	9.42	3.14	0.	2.11	0.	0.
time (sec)	N/A	1.226	10.132	0.2	0.	2.749	0.	0.

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	2295	843	0	632	0	0
normalized size	1	1.	8.11	2.98	0.	2.23	0.	0.
time (sec)	N/A	0.981	8.111	0.235	0.	2.522	0.	0.

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	1070	637	0	563	0	0
normalized size	1	1.	4.59	2.73	0.	2.42	0.	0.
time (sec)	N/A	0.773	6.816	0.209	0.	2.439	0.	0.

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	481	434	0	456	0	0
normalized size	1	1.	2.66	2.4	0.	2.52	0.	0.
time (sec)	N/A	0.569	5.687	0.204	0.	2.116	0.	0.

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	16028	365	0	601	0	0
normalized size	1	1.	84.8	1.93	0.	3.18	0.	0.
time (sec)	N/A	0.599	27.788	0.183	0.	166.405	0.	0.

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	16853	450	0	0	0	0
normalized size	1	1.	69.64	1.86	0.	0.	0.	0.
time (sec)	N/A	0.807	27.85	0.204	0.	0.	0.	0.

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	17684	567	0	0	0	0
normalized size	1	1.	58.95	1.89	0.	0.	0.	0.
time (sec)	N/A	1.051	28.124	0.173	0.	0.	0.	0.

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	7162	1045	0	801	0	0
normalized size	1	1.	21.51	3.14	0.	2.41	0.	0.
time (sec)	N/A	1.241	27.831	0.195	0.	2.32	0.	0.

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	7114	833	0	729	0	0
normalized size	1	1.	25.32	2.96	0.	2.59	0.	0.
time (sec)	N/A	1.018	25.42	0.247	0.	1.843	0.	0.

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	7100	649	0	616	0	0
normalized size	1	1.	30.74	2.81	0.	2.67	0.	0.
time (sec)	N/A	0.805	25.07	0.204	0.	1.695	0.	0.

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	7093	524	0	599	0	0
normalized size	1	1.	38.76	2.86	0.	3.27	0.	0.
time (sec)	N/A	0.573	24.805	0.202	0.	1.787	0.	0.

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	241	241	16100	624	0	0	0	0
normalized size	1	1.	66.8	2.59	0.	0.	0.	0.
time (sec)	N/A	0.779	28.002	0.198	0.	0.	0.	0.

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	16926	758	0	0	0	0
normalized size	1	1.	57.57	2.58	0.	0.	0.	0.
time (sec)	N/A	1.036	28.143	0.211	0.	0.	0.	0.

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	17757	924	0	0	0	0
normalized size	1	1.	50.45	2.62	0.	0.	0.	0.
time (sec)	N/A	1.274	28.488	0.191	0.	0.	0.	0.

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	155	841	0	0	0	0
normalized size	1	1.	0.76	4.1	0.	0.	0.	0.
time (sec)	N/A	0.296	2.528	4.181	0.	0.	0.	0.

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	122	732	0	0	0	0
normalized size	1	1.	0.71	4.26	0.	0.	0.	0.
time (sec)	N/A	0.267	1.633	3.631	0.	0.	0.	0.

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	96	614	0	0	0	0
normalized size	1	1.	0.71	4.55	0.	0.	0.	0.
time (sec)	N/A	0.242	0.443	2.783	0.	0.	0.	0.

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	98	294	0	0	0	0
normalized size	1	1.	0.73	2.18	0.	0.	0.	0.
time (sec)	N/A	0.235	0.401	1.164	0.	0.	0.	0.

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	101	363	0	0	0	0
normalized size	1	1.	0.72	2.57	0.	0.	0.	0.
time (sec)	N/A	0.232	0.481	1.2	0.	0.	0.	0.

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	120	401	0	0	0	0
normalized size	1	1.	0.69	2.3	0.	0.	0.	0.
time (sec)	N/A	0.256	0.834	1.123	0.	0.	0.	0.

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	141	443	0	0	0	0
normalized size	1	1.	0.69	2.16	0.	0.	0.	0.
time (sec)	N/A	0.294	1.234	1.315	0.	0.	0.	0.

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	286	1179	0	0	0	0
normalized size	1	1.	0.98	4.04	0.	0.	0.	0.
time (sec)	N/A	0.63	6.414	5.346	0.	0.	0.	0.

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	218	930	0	0	0	0
normalized size	1	1.	0.9	3.83	0.	0.	0.	0.
time (sec)	N/A	0.568	1.349	4.542	0.	0.	0.	0.

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	147	913	0	0	0	0
normalized size	1	1.	0.7	4.37	0.	0.	0.	0.
time (sec)	N/A	0.528	2.15	3.69	0.	0.	0.	0.

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	133	871	0	0	0	0
normalized size	1	1.	0.69	4.49	0.	0.	0.	0.
time (sec)	N/A	0.52	1.574	1.561	0.	0.	0.	0.

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	139	694	0	0	0	0
normalized size	1	1.	0.67	3.37	0.	0.	0.	0.
time (sec)	N/A	0.528	1.103	1.385	0.	0.	0.	0.

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	148	532	0	0	0	0
normalized size	1	1.	0.7	2.52	0.	0.	0.	0.
time (sec)	N/A	0.513	1.076	1.145	0.	0.	0.	0.

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	170	587	0	0	0	0
normalized size	1	1.	0.69	2.4	0.	0.	0.	0.
time (sec)	N/A	0.554	1.575	1.229	0.	0.	0.	0.

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	209	649	0	0	0	0
normalized size	1	1.	0.71	2.21	0.	0.	0.	0.
time (sec)	N/A	0.633	2.443	1.162	0.	0.	0.	0.

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	324	1270	0	0	0	0
normalized size	1	1.	0.97	3.81	0.	0.	0.	0.
time (sec)	N/A	0.953	6.546	5.788	0.	0.	0.	0.

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	261	1113	0	0	0	0
normalized size	1	1.	0.92	3.93	0.	0.	0.	0.
time (sec)	N/A	0.862	2.249	4.525	0.	0.	0.	0.

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	216	1333	0	0	0	0
normalized size	1	1.	0.8	4.96	0.	0.	0.	0.
time (sec)	N/A	0.817	2.094	4.251	0.	0.	0.	0.

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	179	1267	0	0	0	0
normalized size	1	1.	0.69	4.91	0.	0.	0.	0.
time (sec)	N/A	0.827	2.612	4.072	0.	0.	0.	0.

Problem 1379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	193	943	0	0	0	0
normalized size	1	1.	0.68	3.32	0.	0.	0.	0.
time (sec)	N/A	0.895	1.96	1.411	0.	0.	0.	0.

Problem 1380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	203	718	0	0	0	0
normalized size	1	1.	0.71	2.52	0.	0.	0.	0.
time (sec)	N/A	0.843	1.672	1.281	0.	0.	0.	0.

Problem 1381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	236	793	0	0	0	0
normalized size	1	1.	0.7	2.37	0.	0.	0.	0.
time (sec)	N/A	0.927	2.402	1.281	0.	0.	0.	0.

Problem 1382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	276	873	0	0	0	0
normalized size	1	1.	0.72	2.26	0.	0.	0.	0.
time (sec)	N/A	1.016	2.624	1.377	0.	0.	0.	0.

Problem 1383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	425	1521	0	0	0	0
normalized size	1	1.	1.02	3.65	0.	0.	0.	0.
time (sec)	N/A	1.375	6.853	6.893	0.	0.	0.	0.

Problem 1384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	356	1451	0	0	0	0
normalized size	1	1.	0.98	3.98	0.	0.	0.	0.
time (sec)	N/A	1.277	6.741	6.142	0.	0.	0.	0.

Problem 1385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	296	1531	0	0	0	0
normalized size	1	1.	0.83	4.3	0.	0.	0.	0.
time (sec)	N/A	1.301	3.271	5.316	0.	0.	0.	0.

Problem 1386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	233	1622	0	0	0	0
normalized size	1	1.	0.65	4.49	0.	0.	0.	0.
time (sec)	N/A	1.267	3.171	5.208	0.	0.	0.	0.

Problem 1387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	243	1715	0	0	0	0
normalized size	1	1.	0.71	5.04	0.	0.	0.	0.
time (sec)	N/A	1.261	1.996	4.536	0.	0.	0.	0.

Problem 1388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	252	1209	0	0	0	0
normalized size	1	1.	0.7	3.36	0.	0.	0.	0.
time (sec)	N/A	1.344	1.805	1.743	0.	0.	0.	0.

Problem 1389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	265	924	0	0	0	0
normalized size	1	1.	0.72	2.5	0.	0.	0.	0.
time (sec)	N/A	1.246	1.75	1.287	0.	0.	0.	0.

Problem 1390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	303	1017	0	0	0	0
normalized size	1	1.	0.72	2.41	0.	0.	0.	0.
time (sec)	N/A	1.324	2.74	1.27	0.	0.	0.	0.

Problem 1391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	648	786	0	0	0	0
normalized size	1	1.	2.44	2.95	0.	0.	0.	0.
time (sec)	N/A	1.202	6.874	4.78	0.	0.	0.	0.

Problem 1392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	220	463	0	0	0	0
normalized size	1	1.	1.1	2.32	0.	0.	0.	0.
time (sec)	N/A	0.844	2.908	3.33	0.	0.	0.	0.

Problem 1393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	128	407	0	0	0	0
normalized size	1	1.	0.74	2.37	0.	0.	0.	0.
time (sec)	N/A	0.616	1.236	2.424	0.	0.	0.	0.

Problem 1394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	242	259	0	0	0	0
normalized size	1	1.	1.67	1.79	0.	0.	0.	0.
time (sec)	N/A	0.369	6.457	1.416	0.	0.	0.	0.

Problem 1395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	539	686	0	0	0	0
normalized size	1	1.	2.84	3.61	0.	0.	0.	0.
time (sec)	N/A	0.633	6.742	1.37	0.	0.	0.	0.

Problem 1396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	241	241	603	948	0	0	0	0
normalized size	1	1.	2.5	3.93	0.	0.	0.	0.
time (sec)	N/A	0.902	6.829	1.318	0.	0.	0.	0.

Problem 1397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	663	1244	0	0	0	0
normalized size	1	1.	2.22	4.16	0.	0.	0.	0.
time (sec)	N/A	1.259	6.967	1.519	0.	0.	0.	0.

Problem 1398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	724	1019	0	0	0	0
normalized size	1	1.	1.83	2.57	0.	0.	0.	0.
time (sec)	N/A	1.563	7.083	5.693	0.	0.	0.	0.

Problem 1399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	682	899	0	0	0	0
normalized size	1	1.	2.07	2.72	0.	0.	0.	0.
time (sec)	N/A	1.148	7.006	4.196	0.	0.	0.	0.

Problem 1400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	657	804	0	0	0	0
normalized size	1	1.	2.4	2.93	0.	0.	0.	0.
time (sec)	N/A	0.805	6.881	2.994	0.	0.	0.	0.

Problem 1401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	663	834	0	0	0	0
normalized size	1	1.	2.39	3.01	0.	0.	0.	0.
time (sec)	N/A	0.787	6.858	3.837	0.	0.	0.	0.

Problem 1402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	698	1102	0	0	0	0
normalized size	1	1.	1.98	3.13	0.	0.	0.	0.
time (sec)	N/A	1.114	6.997	4.283	0.	0.	0.	0.

Problem 1403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	430	768	1337	0	0	0	0
normalized size	1	1.	1.79	3.11	0.	0.	0.	0.
time (sec)	N/A	1.548	7.149	4.954	0.	0.	0.	0.

Problem 1404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	554	554	886	2140	0	0	0	0
normalized size	1	1.	1.6	3.86	0.	0.	0.	0.
time (sec)	N/A	2.238	7.257	9.041	0.	0.	0.	0.

Problem 1405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	846	2023	0	0	0	0
normalized size	1	1.	1.77	4.24	0.	0.	0.	0.
time (sec)	N/A	1.718	7.174	6.506	0.	0.	0.	0.

Problem 1406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	816	1846	0	0	0	0
normalized size	1	1.	2.01	4.56	0.	0.	0.	0.
time (sec)	N/A	1.255	6.989	5.434	0.	0.	0.	0.

Problem 1407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	821	1934	0	0	0	0
normalized size	1	1.	2.01	4.74	0.	0.	0.	0.
time (sec)	N/A	1.318	7.008	6.118	0.	0.	0.	0.

Problem 1408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	824	1966	0	0	0	0
normalized size	1	1.	2.03	4.85	0.	0.	0.	0.
time (sec)	N/A	1.34	7.067	6.187	0.	0.	0.	0.

Problem 1409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	493	493	863	2240	0	0	0	0
normalized size	1	1.	1.75	4.54	0.	0.	0.	0.
time (sec)	N/A	1.872	7.223	6.854	0.	0.	0.	0.

Problem 1410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	579	579	931	2466	0	0	0	0
normalized size	1	1.	1.61	4.26	0.	0.	0.	0.
time (sec)	N/A	2.427	7.422	7.436	0.	0.	0.	0.

Problem 1411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	3619	4134	0	0	0	0
normalized size	1	1.	6.65	7.6	0.	0.	0.	0.
time (sec)	N/A	1.953	25.429	0.503	0.	0.	0.	0.

Problem 1412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	455	478	2775	0	0	0	0
normalized size	1	1.	1.05	6.1	0.	0.	0.	0.
time (sec)	N/A	1.42	19.22	0.339	0.	0.	0.	0.

Problem 1413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	429	2442	0	0	0	0
normalized size	1	1.	1.11	6.34	0.	0.	0.	0.
time (sec)	N/A	1.049	17.806	0.24	0.	0.	0.	0.

Problem 1414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	400	1489	0	0	0	0
normalized size	1	1.	0.88	3.28	0.	0.	0.	0.
time (sec)	N/A	0.937	12.39	0.25	0.	0.	0.	0.

Problem 1415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	499	499	703	1596	0	0	0	0
normalized size	1	1.	1.41	3.2	0.	0.	0.	0.
time (sec)	N/A	1.204	18.195	0.202	0.	0.	0.	0.

Problem 1416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	1391	1818	0	0	0	0
normalized size	1	1.	2.7	3.53	0.	0.	0.	0.
time (sec)	N/A	1.245	18.779	0.198	0.	0.	0.	0.

Problem 1417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	613	613	1317	2528	0	0	0	0
normalized size	1	1.	2.15	4.12	0.	0.	0.	0.
time (sec)	N/A	1.725	19.497	0.262	0.	0.	0.	0.

Problem 1418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	698	698	4845	3615	0	0	0	0
normalized size	1	1.	6.94	5.18	0.	0.	0.	0.
time (sec)	N/A	2.174	26.228	0.36	0.	0.	0.	0.

Problem 1419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	542	542	3622	4118	0	0	0	0
normalized size	1	1.	6.68	7.6	0.	0.	0.	0.
time (sec)	N/A	1.951	25.753	0.449	0.	0.	0.	0.

Problem 1420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	458	458	482	2979	0	0	0	0
normalized size	1	1.	1.05	6.5	0.	0.	0.	0.
time (sec)	N/A	1.423	19.595	0.304	0.	0.	0.	0.

Problem 1421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	525	525	6049	2827	0	0	0	0
normalized size	1	1.	11.52	5.38	0.	0.	0.	0.
time (sec)	N/A	1.364	24.676	0.263	0.	0.	0.	0.

Problem 1422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	3392	2134	0	0	0	0
normalized size	1	1.	6.06	3.81	0.	0.	0.	0.
time (sec)	N/A	1.696	23.638	0.22	0.	0.	0.	0.

Problem 1423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	1178	2618	0	0	0	0
normalized size	1	1.	2.07	4.6	0.	0.	0.	0.
time (sec)	N/A	1.729	18.373	0.254	0.	0.	0.	0.

Problem 1424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	613	613	1285	2718	0	0	0	0
normalized size	1	1.	2.1	4.43	0.	0.	0.	0.
time (sec)	N/A	1.906	18.576	0.257	0.	0.	0.	0.

Problem 1425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	698	698	536	3803	0	0	0	0
normalized size	1	1.	0.77	5.45	0.	0.	0.	0.
time (sec)	N/A	2.364	11.139	0.569	0.	0.	0.	0.

Problem 1426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	627	627	3885	4702	0	0	0	0
normalized size	1	1.	6.2	7.5	0.	0.	0.	0.
time (sec)	N/A	2.615	26.146	0.62	0.	0.	0.	0.

Problem 1427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	621	4338	0	0	0	0
normalized size	1	1.	1.14	7.97	0.	0.	0.	0.
time (sec)	N/A	2.007	21.739	0.465	0.	0.	0.	0.

Problem 1428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	600	600	3979	3381	0	0	0	0
normalized size	1	1.	6.63	5.64	0.	0.	0.	0.
time (sec)	N/A	1.824	25.357	0.325	0.	0.	0.	0.

Problem 1429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	666	666	6720	3497	0	0	0	0
normalized size	1	1.	10.09	5.25	0.	0.	0.	0.
time (sec)	N/A	2.339	25.548	0.325	0.	0.	0.	0.

Problem 1430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	627	627	4880	3203	0	0	0	0
normalized size	1	1.	7.78	5.11	0.	0.	0.	0.
time (sec)	N/A	2.28	25.544	0.299	0.	0.	0.	0.

Problem 1431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	669	669	1405	3521	0	0	0	0
normalized size	1	1.	2.1	5.26	0.	0.	0.	0.
time (sec)	N/A	2.419	19.428	0.345	0.	0.	0.	0.

Problem 1432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	695	695	603	3993	0	0	0	0
normalized size	1	1.	0.87	5.75	0.	0.	0.	0.
time (sec)	N/A	2.323	20.295	0.392	0.	0.	0.	0.

Problem 1433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	806	806	2064	4726	0	0	0	0
normalized size	1	1.	2.56	5.86	0.	0.	0.	0.
time (sec)	N/A	3.096	22.169	0.525	0.	0.	0.	0.

Problem 1434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	3164	2775	0	0	0	0
normalized size	1	1.	6.75	5.92	0.	0.	0.	0.
time (sec)	N/A	1.433	23.258	0.307	0.	0.	0.	0.

Problem 1435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	394	394	2920	2243	0	0	0	0
normalized size	1	1.	7.41	5.69	0.	0.	0.	0.
time (sec)	N/A	1.01	22.144	0.247	0.	0.	0.	0.

Problem 1436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	303	1102	0	0	0	0
normalized size	1	1.	0.94	3.41	0.	0.	0.	0.
time (sec)	N/A	0.679	13.289	0.243	0.	0.	0.	0.

Problem 1437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	624	1000	0	0	0	0
normalized size	1	1.	1.55	2.48	0.	0.	0.	0.
time (sec)	N/A	0.62	15.964	0.222	0.	0.	0.	0.

Problem 1438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	453	453	340	818	0	0	0	0
normalized size	1	1.	0.75	1.81	0.	0.	0.	0.
time (sec)	N/A	0.915	11.576	0.222	0.	0.	0.	0.

Problem 1439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	1399	1636	0	0	0	0
normalized size	1	1.	2.72	3.18	0.	0.	0.	0.
time (sec)	N/A	1.239	14.849	0.199	0.	0.	0.	0.

Problem 1440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	3767	4077	0	0	0	0
normalized size	1	1.	7.05	7.63	0.	0.	0.	0.
time (sec)	N/A	1.7	25.613	0.317	0.	0.	0.	0.

Problem 1441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	432	432	3204	2676	0	0	0	0
normalized size	1	1.	7.42	6.19	0.	0.	0.	0.
time (sec)	N/A	1.192	22.935	0.219	0.	0.	0.	0.

Problem 1442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	348	348	456	2287	0	0	0	0
normalized size	1	1.	1.31	6.57	0.	0.	0.	0.
time (sec)	N/A	0.816	18.012	0.248	0.	0.	0.	0.

Problem 1443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	481	481	1027	2049	0	0	0	0
normalized size	1	1.	2.14	4.26	0.	0.	0.	0.
time (sec)	N/A	0.997	17.938	0.234	0.	0.	0.	0.

Problem 1444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	563	563	1163	2502	0	0	0	0
normalized size	1	1.	2.07	4.44	0.	0.	0.	0.
time (sec)	N/A	1.498	19.251	0.2	0.	0.	0.	0.

Problem 1445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	664	664	2415	3555	0	0	0	0
normalized size	1	1.	3.64	5.35	0.	0.	0.	0.
time (sec)	N/A	1.984	16.971	0.271	0.	0.	0.	0.

Problem 1446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	589	589	3973	7095	0	0	0	0
normalized size	1	1.	6.75	12.05	0.	0.	0.	0.
time (sec)	N/A	1.883	26.028	0.363	0.	0.	0.	0.

Problem 1447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	489	489	3741	6184	0	0	0	0
normalized size	1	1.	7.65	12.65	0.	0.	0.	0.
time (sec)	N/A	1.347	25.803	0.436	0.	0.	0.	0.

Problem 1448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	456	456	3279	4579	0	0	0	0
normalized size	1	1.	7.19	10.04	0.	0.	0.	0.
time (sec)	N/A	1.241	23.008	0.293	0.	0.	0.	0.

Problem 1449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	618	618	1588	6427	0	0	0	0
normalized size	1	1.	2.57	10.4	0.	0.	0.	0.
time (sec)	N/A	1.784	19.669	0.502	0.	0.	0.	0.

Problem 1450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	710	710	1609	6471	0	0	0	0
normalized size	1	1.	2.27	9.11	0.	0.	0.	0.
time (sec)	N/A	2.428	20.706	0.312	0.	0.	0.	0.

Problem 1451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	191	851	0	0	0	0
normalized size	1	1.	0.83	3.7	0.	0.	0.	0.
time (sec)	N/A	0.376	3.179	4.913	0.	0.	0.	0.

Problem 1452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	149	742	0	0	0	0
normalized size	1	1.	0.78	3.86	0.	0.	0.	0.
time (sec)	N/A	0.346	1.223	4.082	0.	0.	0.	0.

Problem 1453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	112	666	0	0	0	0
normalized size	1	1.	0.74	4.41	0.	0.	0.	0.
time (sec)	N/A	0.32	1.061	2.967	0.	0.	0.	0.

Problem 1454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	109	388	0	0	0	0
normalized size	1	1.	0.74	2.64	0.	0.	0.	0.
time (sec)	N/A	0.302	0.714	1.436	0.	0.	0.	0.

Problem 1455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	116	465	0	0	0	0
normalized size	1	1.	0.74	2.98	0.	0.	0.	0.
time (sec)	N/A	0.296	0.909	1.324	0.	0.	0.	0.

Problem 1456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	139	515	0	0	0	0
normalized size	1	1.	0.72	2.65	0.	0.	0.	0.
time (sec)	N/A	0.313	1.112	1.309	0.	0.	0.	0.

Problem 1457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	165	565	0	0	0	0
normalized size	1	1.	0.72	2.46	0.	0.	0.	0.
time (sec)	N/A	0.359	1.521	1.242	0.	0.	0.	0.

Problem 1458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	357	1196	0	0	0	0
normalized size	1	1.	1.04	3.5	0.	0.	0.	0.
time (sec)	N/A	0.763	6.652	6.62	0.	0.	0.	0.

Problem 1459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	221	947	0	0	0	0
normalized size	1	1.	0.77	3.29	0.	0.	0.	0.
time (sec)	N/A	0.698	4.363	5.326	0.	0.	0.	0.

Problem 1460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	193	1000	0	0	0	0
normalized size	1	1.	0.8	4.17	0.	0.	0.	0.
time (sec)	N/A	0.652	2.275	4.415	0.	0.	0.	0.

Problem 1461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	158	1303	0	0	0	0
normalized size	1	1.	0.72	5.92	0.	0.	0.	0.
time (sec)	N/A	0.62	1.182	3.646	0.	0.	0.	0.

Problem 1462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	165	932	0	0	0	0
normalized size	1	1.	0.72	4.07	0.	0.	0.	0.
time (sec)	N/A	0.649	1.28	1.51	0.	0.	0.	0.

Problem 1463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	183	706	0	0	0	0
normalized size	1	1.	0.75	2.91	0.	0.	0.	0.
time (sec)	N/A	0.635	0.982	1.256	0.	0.	0.	0.

Problem 1464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	218	784	0	0	0	0
normalized size	1	1.	0.75	2.69	0.	0.	0.	0.
time (sec)	N/A	0.686	1.401	1.284	0.	0.	0.	0.

Problem 1465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	259	863	0	0	0	0
normalized size	1	1.	0.75	2.5	0.	0.	0.	0.
time (sec)	N/A	0.749	2.072	1.498	0.	0.	0.	0.

Problem 1466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	416	1292	0	0	0	0
normalized size	1	1.	1.05	3.25	0.	0.	0.	0.
time (sec)	N/A	1.09	6.911	7.183	0.	0.	0.	0.

Problem 1467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	255	1205	0	0	0	0
normalized size	1	1.	0.76	3.61	0.	0.	0.	0.
time (sec)	N/A	0.986	4.259	5.757	0.	0.	0.	0.

Problem 1468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	276	1419	0	0	0	0
normalized size	1	1.	0.88	4.53	0.	0.	0.	0.
time (sec)	N/A	0.963	2.227	5.409	0.	0.	0.	0.

Problem 1469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	224	1837	0	0	0	0
normalized size	1	1.	0.72	5.91	0.	0.	0.	0.
time (sec)	N/A	1.002	2.063	4.546	0.	0.	0.	0.

Problem 1470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	233	1278	0	0	0	0
normalized size	1	1.	0.73	4.01	0.	0.	0.	0.
time (sec)	N/A	0.994	1.924	1.794	0.	0.	0.	0.

Problem 1471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	253	975	0	0	0	0
normalized size	1	1.	0.75	2.9	0.	0.	0.	0.
time (sec)	N/A	1.007	1.735	1.29	0.	0.	0.	0.

Problem 1472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	304	1082	0	0	0	0
normalized size	1	1.	0.76	2.7	0.	0.	0.	0.
time (sec)	N/A	1.061	3.477	1.485	0.	0.	0.	0.

Problem 1473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	355	1188	0	0	0	0
normalized size	1	1.	0.77	2.57	0.	0.	0.	0.
time (sec)	N/A	1.141	3.877	1.567	0.	0.	0.	0.

Problem 1474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	563	1550	0	0	0	0
normalized size	1	1.	1.09	3.01	0.	0.	0.	0.
time (sec)	N/A	1.59	7.372	9.52	0.	0.	0.	0.

Problem 1475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	459	1550	0	0	0	0
normalized size	1	1.	1.04	3.51	0.	0.	0.	0.
time (sec)	N/A	1.492	7.265	7.894	0.	0.	0.	0.

Problem 1476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	294	1624	0	0	0	0
normalized size	1	1.	0.7	3.84	0.	0.	0.	0.
time (sec)	N/A	1.469	4.439	7.289	0.	0.	0.	0.

Problem 1477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	307	1884	0	0	0	0
normalized size	1	1.	0.72	4.42	0.	0.	0.	0.
time (sec)	N/A	1.472	4.991	6.056	0.	0.	0.	0.

Problem 1478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	316	2507	0	0	0	0
normalized size	1	1.	0.77	6.07	0.	0.	0.	0.
time (sec)	N/A	1.451	3.103	5.94	0.	0.	0.	0.

Problem 1479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	327	1652	0	0	0	0
normalized size	1	1.	0.78	3.94	0.	0.	0.	0.
time (sec)	N/A	1.444	2.793	2.28	0.	0.	0.	0.

Problem 1480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	338	1273	0	0	0	0
normalized size	1	1.	0.76	2.87	0.	0.	0.	0.
time (sec)	N/A	1.46	3.886	1.618	0.	0.	0.	0.

Problem 1481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	400	1407	0	0	0	0
normalized size	1	1.	0.77	2.72	0.	0.	0.	0.
time (sec)	N/A	1.527	4.432	1.67	0.	0.	0.	0.

Problem 1482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	698	802	0	0	0	0
normalized size	1	1.	2.37	2.73	0.	0.	0.	0.
time (sec)	N/A	1.422	7.059	5.628	0.	0.	0.	0.

Problem 1483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	273	474	0	0	0	0
normalized size	1	1.	1.25	2.17	0.	0.	0.	0.
time (sec)	N/A	0.981	4.09	4.031	0.	0.	0.	0.

Problem 1484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	140	411	0	0	0	0
normalized size	1	1.	0.79	2.31	0.	0.	0.	0.
time (sec)	N/A	0.667	1.497	2.814	0.	0.	0.	0.

Problem 1485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	282	323	0	0	0	0
normalized size	1	1.	1.8	2.06	0.	0.	0.	0.
time (sec)	N/A	0.435	2.055	1.48	0.	0.	0.	0.

Problem 1486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	560	945	0	0	0	0
normalized size	1	1.	2.71	4.57	0.	0.	0.	0.
time (sec)	N/A	0.754	6.78	1.674	0.	0.	0.	0.

Problem 1487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	632	803	0	0	0	0
normalized size	1	1.	2.34	2.97	0.	0.	0.	0.
time (sec)	N/A	1.059	7.014	3.438	0.	0.	0.	0.

Problem 1488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	538	1097	0	0	0	0
normalized size	1	1.	1.56	3.18	0.	0.	0.	0.
time (sec)	N/A	1.478	6.592	4.052	0.	0.	0.	0.

Problem 1489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	452	452	791	1038	0	0	0	0
normalized size	1	1.	1.75	2.3	0.	0.	0.	0.
time (sec)	N/A	1.736	7.277	7.284	0.	0.	0.	0.

Problem 1490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	366	366	723	903	0	0	0	0
normalized size	1	1.	1.98	2.47	0.	0.	0.	0.
time (sec)	N/A	1.274	7.064	4.791	0.	0.	0.	0.

Problem 1491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	688	815	0	0	0	0
normalized size	1	1.	2.27	2.69	0.	0.	0.	0.
time (sec)	N/A	0.862	6.914	3.699	0.	0.	0.	0.

Problem 1492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	695	862	0	0	0	0
normalized size	1	1.	2.23	2.77	0.	0.	0.	0.
time (sec)	N/A	0.881	7.004	4.384	0.	0.	0.	0.

Problem 1493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	748	1129	0	0	0	0
normalized size	1	1.	1.86	2.8	0.	0.	0.	0.
time (sec)	N/A	1.325	7.109	5.191	0.	0.	0.	0.

Problem 1494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	837	1382	0	0	0	0
normalized size	1	1.	1.66	2.74	0.	0.	0.	0.
time (sec)	N/A	1.806	7.358	5.506	0.	0.	0.	0.

Problem 1495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	669	669	1019	2165	0	0	0	0
normalized size	1	1.	1.52	3.24	0.	0.	0.	0.
time (sec)	N/A	2.816	7.662	12.504	0.	0.	0.	0.

Problem 1496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	562	950	2027	0	0	0	0
normalized size	1	1.	1.69	3.61	0.	0.	0.	0.
time (sec)	N/A	2.09	7.489	8.282	0.	0.	0.	0.

Problem 1497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	473	473	909	1857	0	0	0	0
normalized size	1	1.	1.92	3.93	0.	0.	0.	0.
time (sec)	N/A	1.508	7.294	6.544	0.	0.	0.	0.

Problem 1498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	478	478	908	1950	0	0	0	0
normalized size	1	1.	1.9	4.08	0.	0.	0.	0.
time (sec)	N/A	1.582	7.239	6.814	0.	0.	0.	0.

Problem 1499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	483	483	921	2000	0	0	0	0
normalized size	1	1.	1.91	4.14	0.	0.	0.	0.
time (sec)	N/A	1.683	7.349	7.813	0.	0.	0.	0.

Problem 1500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	596	596	978	2267	0	0	0	0
normalized size	1	1.	1.64	3.8	0.	0.	0.	0.
time (sec)	N/A	2.13	7.647	9.055	0.	0.	0.	0.

Problem 1501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	714	714	1065	2520	0	0	0	0
normalized size	1	1.	1.49	3.53	0.	0.	0.	0.
time (sec)	N/A	3.048	7.926	9.429	0.	0.	0.	0.

Problem 1502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	592	592	4669	5979	0	0	0	0
normalized size	1	1.	7.89	10.1	0.	0.	0.	0.
time (sec)	N/A	2.247	27.759	0.65	0.	0.	0.	0.

Problem 1503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	487	487	3574	4345	0	0	0	0
normalized size	1	1.	7.34	8.92	0.	0.	0.	0.
time (sec)	N/A	1.589	25.795	0.418	0.	0.	0.	0.

Problem 1504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	400	400	466	3343	0	0	0	0
normalized size	1	1.	1.16	8.36	0.	0.	0.	0.
time (sec)	N/A	1.101	19.985	0.299	0.	0.	0.	0.

Problem 1505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	467	467	5188	2323	0	0	0	0
normalized size	1	1.	11.11	4.97	0.	0.	0.	0.
time (sec)	N/A	1.006	24.454	0.225	0.	0.	0.	0.

Problem 1506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	904	2150	0	0	0	0
normalized size	1	1.	1.78	4.22	0.	0.	0.	0.
time (sec)	N/A	1.321	18.324	0.224	0.	0.	0.	0.

Problem 1507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	543	543	1816	2618	0	0	0	0
normalized size	1	1.	3.34	4.82	0.	0.	0.	0.
time (sec)	N/A	1.353	19.452	0.244	0.	0.	0.	0.

Problem 1508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	646	646	513	3767	0	0	0	0
normalized size	1	1.	0.79	5.83	0.	0.	0.	0.
time (sec)	N/A	1.979	11.597	0.346	0.	0.	0.	0.

Problem 1509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	766	766	854	5307	0	0	0	0
normalized size	1	1.	1.11	6.93	0.	0.	0.	0.
time (sec)	N/A	2.657	15.383	0.503	0.	0.	0.	0.

Problem 1510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	4669	5965	0	0	0	0
normalized size	1	1.	7.91	10.11	0.	0.	0.	0.
time (sec)	N/A	2.219	27.853	0.645	0.	0.	0.	0.

Problem 1511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	490	490	3611	4534	0	0	0	0
normalized size	1	1.	7.37	9.25	0.	0.	0.	0.
time (sec)	N/A	1.568	26.004	0.416	0.	0.	0.	0.

Problem 1512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	550	550	6852	3930	0	0	0	0
normalized size	1	1.	12.46	7.15	0.	0.	0.	0.
time (sec)	N/A	1.43	26.192	0.331	0.	0.	0.	0.

Problem 1513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	588	588	7588	3360	0	0	0	0
normalized size	1	1.	12.9	5.71	0.	0.	0.	0.
time (sec)	N/A	1.867	25.882	0.283	0.	0.	0.	0.

Problem 1514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	595	595	1469	3606	0	0	0	0
normalized size	1	1.	2.47	6.06	0.	0.	0.	0.
time (sec)	N/A	1.889	19.345	0.309	0.	0.	0.	0.

Problem 1515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	647	647	4966	4147	0	0	0	0
normalized size	1	1.	7.68	6.41	0.	0.	0.	0.
time (sec)	N/A	1.981	24.246	0.344	0.	0.	0.	0.

Problem 1516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	603	5495	0	0	0	0
normalized size	1	1.	0.79	7.19	0.	0.	0.	0.
time (sec)	N/A	2.769	15.331	0.508	0.	0.	0.	0.

Problem 1517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	705	705	959	7237	0	0	0	0
normalized size	1	1.	1.36	10.27	0.	0.	0.	0.
time (sec)	N/A	3.372	21.626	0.911	0.	0.	0.	0.

Problem 1518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	592	592	4718	6184	0	0	0	0
normalized size	1	1.	7.97	10.45	0.	0.	0.	0.
time (sec)	N/A	2.284	27.951	0.679	0.	0.	0.	0.

Problem 1519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	640	640	7891	5151	0	0	0	0
normalized size	1	1.	12.33	8.05	0.	0.	0.	0.
time (sec)	N/A	1.994	28.367	0.444	0.	0.	0.	0.

Problem 1520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	703	703	1372	4994	0	0	0	0
normalized size	1	1.	1.95	7.1	0.	0.	0.	0.
time (sec)	N/A	2.556	20.475	0.426	0.	0.	0.	0.

Problem 1521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	682	1640	4897	0	0	0	0
normalized size	1	1.	2.4	7.18	0.	0.	0.	0.
time (sec)	N/A	2.428	20.209	0.398	0.	0.	0.	0.

Problem 1522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	707	707	1940	5138	0	0	0	0
normalized size	1	1.	2.74	7.27	0.	0.	0.	0.
time (sec)	N/A	2.59	20.628	0.434	0.	0.	0.	0.

Problem 1523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	760	760	5555	5875	0	0	0	0
normalized size	1	1.	7.31	7.73	0.	0.	0.	0.
time (sec)	N/A	2.803	25.661	0.568	0.	0.	0.	0.

Problem 1524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	894	894	667	7064	0	0	0	0
normalized size	1	1.	0.75	7.9	0.	0.	0.	0.
time (sec)	N/A	4.058	20.616	0.753	0.	0.	0.	0.

Problem 1525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	506	506	3704	4345	0	0	0	0
normalized size	1	1.	7.32	8.59	0.	0.	0.	0.
time (sec)	N/A	1.608	25.914	0.44	0.	0.	0.	0.

Problem 1526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	412	412	3208	3142	0	0	0	0
normalized size	1	1.	7.79	7.63	0.	0.	0.	0.
time (sec)	N/A	1.094	24.876	0.304	0.	0.	0.	0.

Problem 1527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	2616	1740	0	0	0	0
normalized size	1	1.	7.86	5.23	0.	0.	0.	0.
time (sec)	N/A	0.712	21.936	0.22	0.	0.	0.	0.

Problem 1528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	407	407	625	1182	0	0	0	0
normalized size	1	1.	1.54	2.9	0.	0.	0.	0.
time (sec)	N/A	0.681	17.791	0.242	0.	0.	0.	0.

Problem 1529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	461	461	777	1188	0	0	0	0
normalized size	1	1.	1.69	2.58	0.	0.	0.	0.
time (sec)	N/A	0.928	17.66	0.272	0.	0.	0.	0.

Problem 1530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	545	545	1376	2251	0	0	0	0
normalized size	1	1.	2.52	4.13	0.	0.	0.	0.
time (sec)	N/A	1.298	21.234	0.242	0.	0.	0.	0.

Problem 1531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	653	653	15695	3583	0	0	0	0
normalized size	1	1.	24.04	5.49	0.	0.	0.	0.
time (sec)	N/A	2.086	27.549	0.329	0.	0.	0.	0.

Problem 1532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	795	1369	0	0	0	0
normalized size	1	1.	1.79	3.08	0.	0.	0.	0.
time (sec)	N/A	1.254	6.137	0.254	0.	0.	0.	0.

Problem 1533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	585	585	4900	5893	0	0	0	0
normalized size	1	1.	8.38	10.07	0.	0.	0.	0.
time (sec)	N/A	1.955	28.333	0.451	0.	0.	0.	0.

Problem 1534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	464	464	3736	4201	0	0	0	0
normalized size	1	1.	8.05	9.05	0.	0.	0.	0.
time (sec)	N/A	1.283	26.081	0.292	0.	0.	0.	0.

Problem 1535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	482	3093	0	0	0	0
normalized size	1	1.	1.33	8.54	0.	0.	0.	0.
time (sec)	N/A	0.842	20.368	0.29	0.	0.	0.	0.

Problem 1536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	496	1141	2859	0	0	0	0
normalized size	1	1.	2.3	5.76	0.	0.	0.	0.
time (sec)	N/A	1.069	18.852	0.293	0.	0.	0.	0.

Problem 1537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	595	595	1683	3698	0	0	0	0
normalized size	1	1.	2.83	6.22	0.	0.	0.	0.
time (sec)	N/A	1.686	21.069	0.246	0.	0.	0.	0.

Problem 1538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	720	720	3665	5218	0	0	0	0
normalized size	1	1.	5.09	7.25	0.	0.	0.	0.
time (sec)	N/A	2.345	25.784	0.332	0.	0.	0.	0.

Problem 1539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	660	660	867	10935	0	0	0	0
normalized size	1	1.	1.31	16.57	0.	0.	0.	0.
time (sec)	N/A	2.734	22.097	0.505	0.	0.	0.	0.

Problem 1540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	535	535	790	8934	0	0	0	0
normalized size	1	1.	1.48	16.7	0.	0.	0.	0.
time (sec)	N/A	1.646	21.012	0.498	0.	0.	0.	0.

Problem 1541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	3853	7005	0	0	0	0
normalized size	1	1.	7.78	14.15	0.	0.	0.	0.
time (sec)	N/A	1.376	26.224	0.356	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [636] had the largest ratio of [0.3143]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.	31	0.194
2	A	3	3	1.	29	0.103
3	A	2	2	1.	23	0.087
4	A	4	4	1.	29	0.138
5	A	4	4	1.	31	0.129
6	A	4	4	1.	31	0.129
7	A	6	6	1.	31	0.194
8	A	7	7	1.	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	9	8	1.	33	0.242
10	A	5	5	1.	31	0.161
11	A	3	3	1.	25	0.12
12	A	6	6	1.	31	0.194
13	A	6	6	1.	33	0.182
14	A	6	6	1.	33	0.182
15	A	6	6	1.	33	0.182
16	A	8	8	1.	33	0.242
17	A	9	9	1.	33	0.273
18	A	10	8	1.	33	0.242
19	A	11	9	1.	31	0.29
20	A	9	7	1.	25	0.28
21	A	7	6	1.	31	0.194
22	A	7	6	1.	33	0.182
23	A	7	7	1.	33	0.212
24	A	7	6	1.	33	0.182
25	A	7	6	1.	33	0.182
26	A	9	8	1.	33	0.242
27	A	10	9	1.	33	0.273
28	A	11	8	1.	33	0.242
29	A	14	9	1.	31	0.29
30	A	12	7	1.	25	0.28
31	A	8	6	1.	31	0.194
32	A	8	6	1.	33	0.182
33	A	8	7	1.	33	0.212
34	A	8	7	1.	33	0.212
35	A	8	6	1.	33	0.182
36	A	8	6	1.	33	0.182
37	A	10	8	1.	33	0.242
38	A	11	9	1.	33	0.273
39	A	7	5	1.	33	0.152
40	A	6	5	1.	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	2	2	1.	31	0.065
42	A	3	3	1.	25	0.12
43	A	3	3	1.	31	0.097
44	A	5	5	1.	33	0.152
45	A	6	6	1.	33	0.182
46	A	6	5	1.	33	0.152
47	A	8	6	1.	33	0.182
48	A	7	6	1.	33	0.182
49	A	3	3	1.	33	0.091
50	A	6	6	1.	31	0.194
51	A	3	3	1.	25	0.12
52	A	4	4	1.	31	0.129
53	A	6	6	1.	33	0.182
54	A	7	7	1.	33	0.212
55	A	7	6	1.	33	0.182
56	A	8	6	1.	33	0.182
57	A	4	3	1.	33	0.091
58	A	7	7	1.	33	0.212
59	A	5	5	1.	31	0.161
60	A	3	3	1.	25	0.12
61	A	5	4	1.	31	0.129
62	A	7	6	1.	33	0.182
63	A	8	7	1.	33	0.212
64	A	8	6	1.	33	0.182
65	A	5	3	1.	33	0.091
66	A	8	7	1.	33	0.212
67	A	6	6	1.	33	0.182
68	A	5	5	1.	31	0.161
69	A	4	4	1.	25	0.16
70	A	6	4	1.	31	0.129
71	A	8	6	1.	33	0.182
72	A	9	7	1.	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
73	A	9	6	1.	33	0.182
74	A	6	6	1.	35	0.171
75	A	5	5	1.	35	0.143
76	A	5	5	1.	33	0.152
77	A	3	3	1.	27	0.111
78	A	4	4	1.	33	0.121
79	A	4	4	1.	35	0.114
80	A	4	4	1.	35	0.114
81	A	5	5	1.	35	0.143
82	A	6	5	1.	35	0.143
83	A	6	6	1.	35	0.171
84	A	6	6	1.	33	0.182
85	A	4	4	1.	27	0.148
86	A	5	5	1.	33	0.152
87	A	5	5	1.	35	0.143
88	A	5	5	1.	35	0.143
89	A	5	5	1.	35	0.143
90	A	6	6	1.	35	0.171
91	A	7	6	1.	35	0.171
92	A	7	6	1.	35	0.171
93	A	7	6	1.	33	0.182
94	A	5	4	1.	27	0.148
95	A	6	5	1.	33	0.152
96	A	6	5	1.	35	0.143
97	A	6	6	1.	35	0.171
98	A	6	5	1.	35	0.143
99	A	6	5	1.	35	0.143
100	A	7	6	1.	35	0.171
101	A	8	6	1.	35	0.171
102	A	8	7	1.	35	0.2
103	A	7	7	1.	35	0.2
104	A	6	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	4	4	1.	27	0.148
106	A	6	5	1.	33	0.152
107	A	6	5	1.	35	0.143
108	A	7	6	1.	35	0.171
109	A	8	6	1.	35	0.171
110	A	9	6	1.	35	0.171
111	A	8	7	1.	35	0.2
112	A	7	7	1.	35	0.2
113	A	6	6	1.	33	0.182
114	A	4	4	1.	27	0.148
115	A	6	5	1.	33	0.152
116	A	7	6	1.	35	0.171
117	A	8	6	1.	35	0.171
118	A	9	6	1.	35	0.171
119	A	8	8	1.	35	0.229
120	A	7	7	1.	35	0.2
121	A	6	6	1.	33	0.182
122	A	4	4	1.	27	0.148
123	A	7	6	1.	33	0.182
124	A	8	7	1.	35	0.2
125	A	9	7	1.	35	0.2
126	A	8	6	1.	33	0.182
127	A	7	6	1.	33	0.182
128	A	6	6	1.	33	0.182
129	A	5	5	1.	33	0.152
130	A	5	5	1.	33	0.152
131	A	5	5	1.	33	0.152
132	A	6	6	1.	33	0.182
133	A	7	6	1.	33	0.182
134	A	9	8	1.	35	0.229
135	A	8	8	1.	35	0.229
136	A	7	7	1.	35	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
137	A	7	7	1.	35	0.2
138	A	7	7	1.	35	0.2
139	A	7	7	1.	35	0.2
140	A	8	8	1.	35	0.229
141	A	9	8	1.	35	0.229
142	A	10	8	1.	35	0.229
143	A	9	8	1.	35	0.229
144	A	8	7	1.	35	0.2
145	A	8	7	1.	35	0.2
146	A	8	8	1.	35	0.229
147	A	8	7	1.	35	0.2
148	A	8	7	1.	35	0.2
149	A	9	8	1.	35	0.229
150	A	10	8	1.	35	0.229
151	A	7	5	1.	35	0.143
152	A	6	5	1.	35	0.143
153	A	5	5	1.	35	0.143
154	A	4	4	1.	35	0.114
155	A	5	5	1.	35	0.143
156	A	6	5	1.	35	0.143
157	A	7	5	1.	35	0.143
158	A	7	6	1.	35	0.171
159	A	6	6	1.	35	0.171
160	A	5	5	1.	35	0.143
161	A	5	5	1.	35	0.143
162	A	6	6	1.	35	0.171
163	A	7	6	1.	35	0.171
164	A	8	6	1.	35	0.171
165	A	7	6	1.	35	0.171
166	A	6	5	1.	35	0.143
167	A	6	6	1.	35	0.171
168	A	6	5	1.	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
169	A	7	6	1.	35	0.171
170	A	8	6	1.	35	0.171
171	A	6	5	1.	37	0.135
172	A	5	5	1.	37	0.135
173	A	4	4	1.	37	0.108
174	A	4	4	1.	37	0.108
175	A	4	4	1.	37	0.108
176	A	3	3	1.	37	0.081
177	A	4	4	1.	37	0.108
178	A	5	4	1.	37	0.108
179	A	7	6	1.	37	0.162
180	A	6	6	1.	37	0.162
181	A	5	5	1.	37	0.135
182	A	5	5	1.	37	0.135
183	A	5	5	1.	37	0.135
184	A	5	5	1.	37	0.135
185	A	4	4	1.	37	0.108
186	A	5	5	1.	37	0.135
187	A	6	5	1.	37	0.135
188	A	8	6	1.	37	0.162
189	A	7	6	1.	37	0.162
190	A	6	5	1.	37	0.135
191	A	6	5	1.	37	0.135
192	A	6	6	1.	37	0.162
193	A	6	5	1.	37	0.135
194	A	6	5	1.	37	0.135
195	A	5	4	1.	37	0.108
196	A	6	5	1.	37	0.135
197	A	7	5	1.	37	0.135
198	A	8	7	1.	37	0.189
199	A	7	7	1.	37	0.189
200	A	6	6	1.	37	0.162

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
201	A	6	6	1.	37	0.162
202	A	5	5	1.	37	0.135
203	A	6	5	1.	37	0.135
204	A	7	5	1.	37	0.135
205	A	8	7	1.	37	0.189
206	A	7	7	1.	37	0.189
207	A	6	6	1.	37	0.162
208	A	5	5	1.	37	0.135
209	A	6	5	1.	37	0.135
210	A	7	5	1.	37	0.135
211	A	8	8	1.	37	0.216
212	A	7	7	1.	37	0.189
213	A	5	5	1.	37	0.135
214	A	6	6	1.	37	0.162
215	A	7	6	1.	37	0.162
216	A	7	5	1.	28	0.179
217	A	7	5	1.	28	0.179
218	A	6	5	1.	26	0.192
219	A	4	3	1.	19	0.158
220	A	3	2	1.	26	0.077
221	A	3	3	1.	28	0.107
222	A	5	5	1.	28	0.179
223	A	6	6	1.	28	0.214
224	A	6	5	1.	28	0.179
225	A	7	5	1.	28	0.179
226	A	9	7	1.	38	0.184
227	A	8	7	1.	36	0.194
228	A	2	2	1.	30	0.067
229	A	2	2	1.	36	0.056
230	A	5	5	1.	38	0.132
231	A	5	5	1.	38	0.132
232	A	7	7	1.	38	0.184

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	8	8	1.	38	0.21
234	A	8	7	1.	38	0.184
235	A	9	8	1.	38	0.21
236	A	3	3	1.	32	0.094
237	A	3	3	1.	38	0.079
238	A	6	6	1.	40	0.15
239	A	6	6	1.	40	0.15
240	A	6	6	1.	40	0.15
241	A	8	8	1.	40	0.2
242	A	9	9	1.	40	0.225
243	A	9	8	1.	40	0.2
244	A	10	8	1.	38	0.21
245	A	9	7	1.	32	0.219
246	A	9	7	1.	38	0.184
247	A	7	6	1.	40	0.15
248	A	7	7	1.	40	0.175
249	A	7	6	1.	40	0.15
250	A	7	6	1.	40	0.15
251	A	9	8	1.	40	0.2
252	A	10	9	1.	40	0.225
253	A	7	6	1.	40	0.15
254	A	3	3	1.	38	0.079
255	A	4	4	1.	32	0.125
256	A	3	3	1.	38	0.079
257	A	4	4	1.	40	0.1
258	A	6	6	1.	40	0.15
259	A	7	7	1.	40	0.175
260	A	7	6	1.	40	0.15
261	A	8	6	1.	40	0.15
262	A	4	3	1.	40	0.075
263	A	7	7	1.	38	0.184
264	A	3	3	1.	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
265	A	3	3	1.	38	0.079
266	A	5	4	1.	40	0.1
267	A	7	6	1.	40	0.15
268	A	8	7	1.	40	0.175
269	A	5	3	1.	40	0.075
270	A	8	7	1.	40	0.175
271	A	6	6	1.	38	0.158
272	A	3	3	1.	32	0.094
273	A	4	4	1.	38	0.105
274	A	6	4	1.	40	0.1
275	A	8	6	1.	40	0.15
276	A	9	7	1.	40	0.175
277	A	3	3	1.	34	0.088
278	A	4	4	1.	34	0.118
279	A	5	4	1.	34	0.118
280	A	4	4	1.	34	0.118
281	A	4	4	1.	34	0.118
282	A	4	4	1.	34	0.118
283	A	7	5	1.	30	0.167
284	A	6	5	1.	30	0.167
285	A	5	5	1.	30	0.167
286	A	4	4	1.	30	0.133
287	A	5	5	1.	30	0.167
288	A	6	5	1.	30	0.167
289	A	7	5	1.	30	0.167
290	A	7	5	1.	29	0.172
291	A	7	5	1.	29	0.172
292	A	6	5	1.	29	0.172
293	A	2	2	1.	27	0.074
294	A	4	3	1.	20	0.15
295	A	3	3	1.	27	0.111
296	A	3	3	1.	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	5	5	1.	29	0.172
298	A	6	6	1.	29	0.207
299	A	6	5	1.	29	0.172
300	A	7	5	1.	29	0.172
301	A	7	6	1.	39	0.154
302	A	3	3	1.	37	0.081
303	A	2	2	1.	31	0.065
304	A	4	4	1.	37	0.108
305	A	4	4	1.	39	0.103
306	A	4	4	1.	39	0.103
307	A	6	6	1.	39	0.154
308	A	7	7	1.	39	0.18
309	A	9	8	1.	41	0.195
310	A	5	5	1.	39	0.128
311	A	3	3	1.	33	0.091
312	A	6	6	1.	39	0.154
313	A	6	6	1.	41	0.146
314	A	6	6	1.	41	0.146
315	A	6	6	1.	41	0.146
316	A	8	8	1.	41	0.195
317	A	9	9	1.	41	0.22
318	A	10	8	1.	41	0.195
319	A	11	9	1.	39	0.231
320	A	9	7	1.	33	0.212
321	A	7	6	1.	39	0.154
322	A	7	6	1.	41	0.146
323	A	7	7	1.	41	0.171
324	A	7	6	1.	41	0.146
325	A	7	6	1.	41	0.146
326	A	9	8	1.	41	0.195
327	A	10	9	1.	41	0.22
328	A	11	8	1.	41	0.195

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	14	9	1.	39	0.231
330	A	12	7	1.	33	0.212
331	A	8	6	1.	39	0.154
332	A	8	6	1.	41	0.146
333	A	8	7	1.	41	0.171
334	A	8	7	1.	41	0.171
335	A	8	6	1.	41	0.146
336	A	8	6	1.	41	0.146
337	A	10	8	1.	41	0.195
338	A	11	9	1.	41	0.22
339	A	7	5	1.	41	0.122
340	A	6	5	1.	41	0.122
341	A	2	2	1.	39	0.051
342	A	3	3	1.	33	0.091
343	A	3	3	1.	39	0.077
344	A	5	5	1.	41	0.122
345	A	6	6	1.	41	0.146
346	A	6	5	1.	41	0.122
347	A	7	6	1.	41	0.146
348	A	3	3	1.	41	0.073
349	A	6	6	1.	39	0.154
350	A	3	3	1.	33	0.091
351	A	4	4	1.	39	0.103
352	A	6	6	1.	41	0.146
353	A	7	7	1.	41	0.171
354	A	7	6	1.	41	0.146
355	A	8	6	1.	41	0.146
356	A	4	3	1.	41	0.073
357	A	7	7	1.	41	0.171
358	A	5	5	1.	39	0.128
359	A	3	3	1.	33	0.091
360	A	5	4	1.	39	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
361	A	7	6	1.	41	0.146
362	A	8	7	1.	41	0.171
363	A	8	6	1.	41	0.146
364	A	5	3	1.	41	0.073
365	A	8	7	1.	41	0.171
366	A	6	6	1.	41	0.146
367	A	5	5	1.	39	0.128
368	A	4	4	1.	33	0.121
369	A	6	4	1.	39	0.103
370	A	8	6	1.	41	0.146
371	A	9	7	1.	41	0.171
372	A	9	6	1.	41	0.146
373	A	6	6	1.	43	0.14
374	A	5	5	1.	43	0.116
375	A	5	5	1.	41	0.122
376	A	3	3	1.	35	0.086
377	A	4	4	1.	41	0.098
378	A	4	4	1.	43	0.093
379	A	4	4	1.	43	0.093
380	A	5	5	1.	43	0.116
381	A	6	5	1.	43	0.116
382	A	6	6	1.	43	0.14
383	A	6	6	1.	41	0.146
384	A	4	4	1.	35	0.114
385	A	5	5	1.	41	0.122
386	A	5	5	1.	43	0.116
387	A	5	5	1.	43	0.116
388	A	5	5	1.	43	0.116
389	A	6	6	1.	43	0.14
390	A	7	6	1.	43	0.14
391	A	7	6	1.	43	0.14
392	A	7	6	1.	41	0.146

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	5	4	1.	35	0.114
394	A	6	5	1.	41	0.122
395	A	6	5	1.	43	0.116
396	A	6	6	1.	43	0.14
397	A	6	5	1.	43	0.116
398	A	6	5	1.	43	0.116
399	A	7	6	1.	43	0.14
400	A	8	6	1.	43	0.14
401	A	8	7	1.	43	0.163
402	A	7	7	1.	43	0.163
403	A	6	6	1.	41	0.146
404	A	4	4	1.	35	0.114
405	A	6	5	1.	41	0.122
406	A	6	5	1.	43	0.116
407	A	7	6	1.	43	0.14
408	A	8	6	1.	43	0.14
409	A	9	6	1.	43	0.14
410	A	8	7	1.	43	0.163
411	A	7	7	1.	43	0.163
412	A	6	6	1.	41	0.146
413	A	4	4	1.	35	0.114
414	A	6	5	1.	41	0.122
415	A	7	6	1.	43	0.14
416	A	8	6	1.	43	0.14
417	A	9	6	1.	43	0.14
418	A	8	8	1.	43	0.186
419	A	7	7	1.	43	0.163
420	A	6	6	1.	41	0.146
421	A	4	4	1.	35	0.114
422	A	7	6	1.	41	0.146
423	A	8	7	1.	43	0.163
424	A	9	7	1.	43	0.163

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
425	A	6	5	1.	31	0.161
426	A	5	5	1.	31	0.161
427	A	4	4	1.	31	0.129
428	A	4	4	1.	31	0.129
429	A	5	5	1.	31	0.161
430	A	6	5	1.	31	0.161
431	A	8	6	1.	41	0.146
432	A	7	6	1.	41	0.146
433	A	6	6	1.	41	0.146
434	A	5	5	1.	41	0.122
435	A	5	5	1.	41	0.122
436	A	5	5	1.	41	0.122
437	A	6	6	1.	41	0.146
438	A	7	6	1.	41	0.146
439	A	9	8	1.	43	0.186
440	A	8	8	1.	43	0.186
441	A	7	7	1.	43	0.163
442	A	7	7	1.	43	0.163
443	A	7	7	1.	43	0.163
444	A	7	7	1.	43	0.163
445	A	8	8	1.	43	0.186
446	A	9	8	1.	43	0.186
447	A	10	8	1.	43	0.186
448	A	9	8	1.	43	0.186
449	A	8	7	1.	43	0.163
450	A	8	7	1.	43	0.163
451	A	8	8	1.	43	0.186
452	A	8	7	1.	43	0.163
453	A	8	7	1.	43	0.163
454	A	9	8	1.	43	0.186
455	A	10	8	1.	43	0.186
456	A	7	5	1.	43	0.116

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
457	A	6	5	1.	43	0.116
458	A	5	5	1.	43	0.116
459	A	4	4	1.	43	0.093
460	A	5	5	1.	43	0.116
461	A	6	5	1.	43	0.116
462	A	7	5	1.	43	0.116
463	A	7	6	1.	43	0.14
464	A	6	6	1.	43	0.14
465	A	5	5	1.	43	0.116
466	A	5	5	1.	43	0.116
467	A	6	6	1.	43	0.14
468	A	7	6	1.	43	0.14
469	A	8	6	1.	43	0.14
470	A	7	6	1.	43	0.14
471	A	6	5	1.	43	0.116
472	A	6	6	1.	43	0.14
473	A	6	5	1.	43	0.116
474	A	7	6	1.	43	0.14
475	A	8	6	1.	43	0.14
476	A	6	5	1.	45	0.111
477	A	5	5	1.	45	0.111
478	A	4	4	1.	45	0.089
479	A	4	4	1.	45	0.089
480	A	4	4	1.	45	0.089
481	A	3	3	1.	45	0.067
482	A	4	4	1.	45	0.089
483	A	5	4	1.	45	0.089
484	A	7	6	1.	45	0.133
485	A	6	6	1.	45	0.133
486	A	5	5	1.	45	0.111
487	A	5	5	1.	45	0.111
488	A	5	5	1.	45	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
489	A	5	5	1.	45	0.111
490	A	4	4	1.	45	0.089
491	A	5	5	1.	45	0.111
492	A	6	5	1.	45	0.111
493	A	8	6	1.	45	0.133
494	A	7	6	1.	45	0.133
495	A	6	5	1.	45	0.111
496	A	6	5	1.	45	0.111
497	A	6	6	1.	45	0.133
498	A	6	5	1.	45	0.111
499	A	6	5	1.	45	0.111
500	A	5	4	1.	45	0.089
501	A	6	5	1.	45	0.111
502	A	7	5	1.	45	0.111
503	A	8	7	1.	45	0.156
504	A	7	7	1.	45	0.156
505	A	6	6	1.	45	0.133
506	A	6	6	1.	45	0.133
507	A	5	5	1.	45	0.111
508	A	6	5	1.	45	0.111
509	A	7	5	1.	45	0.111
510	A	7	7	1.	54	0.13
511	A	8	7	1.	45	0.156
512	A	7	7	1.	45	0.156
513	A	6	6	1.	45	0.133
514	A	5	5	1.	45	0.111
515	A	6	5	1.	45	0.111
516	A	7	5	1.	45	0.111
517	A	8	8	1.	45	0.178
518	A	7	7	1.	45	0.156
519	A	5	5	1.	45	0.111
520	A	6	6	1.	45	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
521	A	7	6	1.	45	0.133
522	A	7	6	1.	31	0.194
523	A	3	3	1.	29	0.103
524	A	2	2	1.	23	0.087
525	A	4	4	1.	29	0.138
526	A	4	4	1.	31	0.129
527	A	4	4	1.	31	0.129
528	A	6	6	1.	31	0.194
529	A	7	7	1.	31	0.226
530	A	7	6	1.	31	0.194
531	A	8	7	1.	33	0.212
532	A	4	4	1.	31	0.129
533	A	3	3	1.	25	0.12
534	A	5	5	1.	31	0.161
535	A	5	5	1.	33	0.152
536	A	5	5	1.	33	0.152
537	A	5	5	1.	33	0.152
538	A	7	7	1.	33	0.212
539	A	8	8	1.	33	0.242
540	A	5	5	1.	31	0.161
541	A	4	3	1.	25	0.12
542	A	6	6	1.	31	0.194
543	A	6	6	1.	33	0.182
544	A	6	6	1.	33	0.182
545	A	6	6	1.	33	0.182
546	A	6	6	1.	33	0.182
547	A	8	8	1.	33	0.242
548	A	9	9	1.	33	0.273
549	A	6	5	1.	31	0.161
550	A	5	3	1.	25	0.12
551	A	7	6	1.	31	0.194
552	A	7	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
553	A	7	7	1.	33	0.212
554	A	7	6	1.	33	0.182
555	A	7	6	1.	33	0.182
556	A	7	6	1.	33	0.182
557	A	9	8	1.	33	0.242
558	A	10	9	1.	33	0.273
559	A	5	3	1.	30	0.1
560	A	4	3	1.	30	0.1
561	A	3	3	1.	28	0.107
562	A	7	6	1.	33	0.182
563	A	6	6	1.	33	0.182
564	A	5	5	0.98	31	0.161
565	A	4	4	1.	25	0.16
566	A	4	4	1.	31	0.129
567	A	5	5	1.	33	0.152
568	A	6	6	1.	33	0.182
569	A	7	6	1.	33	0.182
570	A	7	6	1.	33	0.182
571	A	6	6	1.	33	0.182
572	A	5	5	1.	31	0.161
573	A	4	4	1.	25	0.16
574	A	5	5	1.	31	0.161
575	A	6	6	1.	33	0.182
576	A	7	6	1.	33	0.182
577	A	8	6	1.	33	0.182
578	A	7	7	1.	33	0.212
579	A	6	6	1.	33	0.182
580	A	5	5	1.	31	0.161
581	A	5	5	1.	25	0.2
582	A	6	6	1.	31	0.194
583	A	7	6	1.	33	0.182
584	A	8	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	8	7	1.	33	0.212
586	A	7	7	1.	33	0.212
587	A	6	6	1.	33	0.182
588	A	6	6	1.	31	0.194
589	A	6	5	1.	25	0.2
590	A	7	6	1.	31	0.194
591	A	8	6	1.	33	0.182
592	A	9	6	1.	33	0.182
593	A	7	6	1.	33	0.182
594	A	6	6	1.	33	0.182
595	A	5	5	1.	31	0.161
596	A	4	4	1.	25	0.16
597	A	4	4	1.	31	0.129
598	A	5	5	1.	33	0.152
599	A	6	6	1.	33	0.182
600	A	7	6	1.	33	0.182
601	A	8	7	1.	33	0.212
602	A	7	7	1.	33	0.212
603	A	6	6	1.	33	0.182
604	A	5	5	1.	31	0.161
605	A	5	5	1.	25	0.2
606	A	6	6	1.	31	0.194
607	A	6	5	1.	33	0.152
608	A	7	6	1.	33	0.182
609	A	8	6	1.	33	0.182
610	A	8	6	1.	33	0.182
611	A	7	6	1.	33	0.182
612	A	6	6	1.	33	0.182
613	A	5	5	1.	31	0.161
614	A	6	5	1.	25	0.2
615	A	6	5	1.	31	0.161
616	A	7	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
617	A	8	6	1.	33	0.182
618	A	9	6	1.	33	0.182
619	A	3	2	1.	30	0.067
620	A	4	4	1.	30	0.133
621	A	5	5	1.	30	0.167
622	A	6	5	1.	30	0.167
623	A	9	9	1.	35	0.257
624	A	8	8	1.	33	0.242
625	A	7	7	1.	27	0.259
626	A	9	9	1.	33	0.273
627	A	9	9	1.	35	0.257
628	A	10	10	1.	35	0.286
629	A	11	10	1.	35	0.286
630	A	10	9	1.	35	0.257
631	A	9	8	1.	33	0.242
632	A	8	7	1.	27	0.259
633	A	10	10	1.	33	0.303
634	A	10	10	1.	35	0.286
635	A	10	10	1.	35	0.286
636	A	11	11	1.	35	0.314
637	A	12	11	1.	35	0.314
638	A	11	9	1.	35	0.257
639	A	10	8	1.	33	0.242
640	A	9	7	1.	27	0.259
641	A	11	10	1.	33	0.303
642	A	11	10	1.	35	0.286
643	A	11	11	1.	35	0.314
644	A	11	10	1.	35	0.286
645	A	12	11	1.	35	0.314
646	A	9	7	1.	32	0.219
647	A	8	7	1.	32	0.219
648	A	9	8	1.	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
649	A	8	8	1.	35	0.229
650	A	7	7	1.	33	0.212
651	A	6	6	1.	27	0.222
652	A	8	8	1.	33	0.242
653	A	9	9	1.	35	0.257
654	A	10	10	1.	35	0.286
655	A	11	10	1.	35	0.286
656	A	9	8	1.	35	0.229
657	A	8	8	1.	35	0.229
658	A	7	7	1.	33	0.212
659	A	6	6	1.	27	0.222
660	A	9	9	1.	33	0.273
661	A	10	10	1.	35	0.286
662	A	11	10	1.	35	0.286
663	A	9	9	1.	35	0.257
664	A	8	8	1.	35	0.229
665	A	7	7	1.	33	0.212
666	A	7	7	1.	27	0.259
667	A	10	10	1.	33	0.303
668	A	11	10	1.	35	0.286
669	A	8	7	1.	27	0.259
670	A	7	7	1.	32	0.219
671	A	6	6	1.	32	0.188
672	A	7	7	1.	32	0.219
673	A	8	7	1.	32	0.219
674	A	8	6	1.	33	0.182
675	A	7	6	1.	33	0.182
676	A	6	6	1.	33	0.182
677	A	5	5	1.	33	0.152
678	A	5	5	1.	33	0.152
679	A	5	5	1.	33	0.152
680	A	6	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
681	A	7	6	1.	33	0.182
682	A	8	7	1.	35	0.2
683	A	7	7	1.	35	0.2
684	A	6	6	1.	35	0.171
685	A	6	6	1.	35	0.171
686	A	6	6	1.	35	0.171
687	A	6	6	1.	35	0.171
688	A	7	7	1.	35	0.2
689	A	8	8	1.	35	0.229
690	A	7	7	1.	35	0.2
691	A	7	7	1.	35	0.2
692	A	7	7	1.	35	0.2
693	A	7	7	1.	35	0.2
694	A	7	7	1.	35	0.2
695	A	8	8	1.	35	0.229
696	A	9	8	1.	35	0.229
697	A	8	7	1.	35	0.2
698	A	8	7	1.	35	0.2
699	A	8	8	1.	35	0.229
700	A	8	7	1.	35	0.2
701	A	8	7	1.	35	0.2
702	A	8	7	1.	35	0.2
703	A	9	8	1.	35	0.229
704	A	9	7	1.	35	0.2
705	A	8	7	1.	35	0.2
706	A	7	7	1.	35	0.2
707	A	6	6	1.	35	0.171
708	A	5	5	1.	35	0.143
709	A	6	6	1.	35	0.171
710	A	7	7	1.	35	0.2
711	A	8	7	1.	35	0.2
712	A	9	7	1.	35	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
713	A	10	7	1.	35	0.2
714	A	8	7	1.	35	0.2
715	A	7	7	1.	35	0.2
716	A	6	6	1.	35	0.171
717	A	6	6	1.	35	0.171
718	A	7	7	1.	35	0.2
719	A	8	7	1.	35	0.2
720	A	9	7	1.	35	0.2
721	A	8	8	1.	35	0.229
722	A	7	7	1.	35	0.2
723	A	7	7	1.	35	0.2
724	A	7	7	1.	35	0.2
725	A	8	7	1.	35	0.2
726	A	9	7	1.	35	0.2
727	A	8	8	1.	37	0.216
728	A	7	7	1.	37	0.189
729	A	7	7	1.	37	0.189
730	A	6	6	1.	37	0.162
731	A	5	5	1.	37	0.135
732	A	6	5	1.	37	0.135
733	A	9	8	1.	37	0.216
734	A	8	8	1.	37	0.216
735	A	8	8	1.	37	0.216
736	A	8	8	1.	37	0.216
737	A	7	7	1.	37	0.189
738	A	6	6	1.	37	0.162
739	A	7	6	1.	37	0.162
740	A	10	8	1.	37	0.216
741	A	9	8	1.	37	0.216
742	A	9	8	1.	37	0.216
743	A	9	9	1.	37	0.243
744	A	9	8	1.	37	0.216

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
745	A	8	7	1.	37	0.189
746	A	7	6	1.	37	0.162
747	A	8	6	1.	37	0.162
748	A	8	8	1.	37	0.216
749	A	7	7	1.	37	0.189
750	A	6	6	1.	37	0.162
751	A	6	6	1.	37	0.162
752	A	4	4	1.	37	0.108
753	A	5	5	1.	37	0.135
754	A	6	5	1.	37	0.135
755	A	8	8	1.	37	0.216
756	A	7	7	1.	37	0.189
757	A	6	6	1.	37	0.162
758	A	4	4	1.	37	0.108
759	A	5	5	1.	37	0.135
760	A	6	5	1.	37	0.135
761	A	8	8	1.	37	0.216
762	A	7	7	1.	37	0.189
763	A	5	5	1.	37	0.135
764	A	5	5	1.	37	0.135
765	A	6	5	1.	37	0.135
766	A	6	5	1.	33	0.152
767	A	5	4	1.	31	0.129
768	A	8	5	1.	33	0.152
769	A	9	6	1.	33	0.182
770	A	8	7	1.	36	0.194
771	A	2	2	1.24	30	0.067
772	A	2	2	1.	36	0.056
773	A	5	5	1.	38	0.132
774	A	5	5	1.	38	0.132
775	A	7	7	1.	38	0.184
776	A	8	8	1.	38	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
777	A	8	7	1.	38	0.184
778	A	8	7	1.	38	0.184
779	A	3	3	1.	32	0.094
780	A	3	3	1.	38	0.079
781	A	5	5	1.	40	0.125
782	A	5	5	1.	40	0.125
783	A	5	5	1.	40	0.125
784	A	7	7	1.	40	0.175
785	A	8	8	1.	40	0.2
786	A	4	3	1.	32	0.094
787	A	4	3	1.	38	0.079
788	A	6	6	1.	40	0.15
789	A	6	6	1.	40	0.15
790	A	6	6	1.	40	0.15
791	A	6	6	1.	40	0.15
792	A	8	8	1.	40	0.2
793	A	9	9	1.	40	0.225
794	A	7	7	1.	40	0.175
795	A	6	6	1.	38	0.158
796	A	5	5	1.	32	0.156
797	A	4	4	1.	38	0.105
798	A	5	5	1.	40	0.125
799	A	7	7	1.	40	0.175
800	A	7	7	1.	40	0.175
801	A	7	7	1.	40	0.175
802	A	6	6	1.	38	0.158
803	A	4	4	1.	32	0.125
804	A	5	5	1.	38	0.132
805	A	6	6	1.	40	0.15
806	A	7	7	1.	40	0.175
807	A	8	8	1.	40	0.2
808	A	7	7	1.	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
809	A	6	6	1.	38	0.158
810	A	5	5	1.	32	0.156
811	A	6	5	1.	38	0.132
812	A	7	7	1.	40	0.175
813	A	8	7	1.	40	0.175
814	A	9	9	1.	40	0.225
815	A	7	7	1.	34	0.206
816	A	7	7	1.	40	0.175
817	A	9	9	1.	42	0.214
818	A	10	10	1.	42	0.238
819	A	11	11	1.	42	0.262
820	A	10	9	1.	40	0.225
821	A	8	7	1.	34	0.206
822	A	8	7	1.	40	0.175
823	A	10	10	1.	42	0.238
824	A	10	10	1.	42	0.238
825	A	11	11	1.	42	0.262
826	A	12	11	1.	42	0.262
827	A	11	9	1.	40	0.225
828	A	9	7	1.	34	0.206
829	A	9	7	1.	40	0.175
830	A	11	11	1.	42	0.262
831	A	11	11	1.	42	0.262
832	A	11	11	1.	42	0.262
833	A	12	12	1.	42	0.286
834	A	13	12	1.	42	0.286
835	A	8	8	1.	40	0.2
836	A	6	6	1.	34	0.176
837	A	6	6	1.	40	0.15
838	A	6	6	1.	42	0.143
839	A	10	10	1.	42	0.238
840	A	11	11	1.	42	0.262

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
841	A	9	9	1.	42	0.214
842	A	8	8	1.	40	0.2
843	A	6	6	1.	34	0.176
844	A	7	7	1.	40	0.175
845	A	8	8	1.	42	0.19
846	A	11	11	1.	42	0.262
847	A	9	9	1.	42	0.214
848	A	8	8	1.	40	0.2
849	A	7	7	1.	34	0.206
850	A	8	7	1.	40	0.175
851	A	11	11	1.	42	0.262
852	A	12	12	1.	42	0.286
853	A	9	7	1.	40	0.175
854	A	8	7	1.	40	0.175
855	A	7	7	1.	40	0.175
856	A	6	6	1.	40	0.15
857	A	6	6	1.	40	0.15
858	A	7	7	1.	40	0.175
859	A	8	7	1.	40	0.175
860	A	9	7	1.	42	0.167
861	A	8	7	1.	42	0.167
862	A	7	7	1.	42	0.167
863	A	6	6	1.	42	0.143
864	A	6	6	1.	42	0.143
865	A	6	6	1.	42	0.143
866	A	7	7	1.	42	0.167
867	A	8	7	1.	42	0.167
868	A	9	8	1.	42	0.19
869	A	8	8	1.	42	0.19
870	A	7	7	1.	42	0.167
871	A	7	7	1.	42	0.167
872	A	7	7	1.	42	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
873	A	7	7	1.	42	0.167
874	A	8	8	1.	42	0.19
875	A	9	8	1.	42	0.19
876	A	9	8	1.	42	0.19
877	A	8	8	1.	42	0.19
878	A	7	7	1.	42	0.167
879	A	6	6	1.	42	0.143
880	A	4	4	1.	42	0.095
881	A	6	6	1.	42	0.143
882	A	8	8	1.	42	0.19
883	A	9	8	1.	42	0.19
884	A	9	8	1.	42	0.19
885	A	8	8	1.	42	0.19
886	A	7	7	1.	42	0.167
887	A	7	7	1.	42	0.167
888	A	7	7	1.	42	0.167
889	A	8	8	1.	42	0.19
890	A	9	8	1.	42	0.19
891	A	9	9	1.	42	0.214
892	A	8	8	1.	42	0.19
893	A	8	8	1.	42	0.19
894	A	8	8	1.	42	0.19
895	A	8	8	1.	42	0.19
896	A	9	8	1.	42	0.19
897	A	9	9	1.	44	0.204
898	A	8	8	1.	44	0.182
899	A	7	7	1.	44	0.159
900	A	6	6	1.	44	0.136
901	A	5	5	1.	44	0.114
902	A	6	6	1.	44	0.136
903	A	7	6	1.	44	0.136
904	A	10	9	1.	44	0.204

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
905	A	9	9	1.	44	0.204
906	A	8	8	1.	44	0.182
907	A	8	8	1.	44	0.182
908	A	7	7	1.	44	0.159
909	A	6	6	1.	44	0.136
910	A	7	6	1.	44	0.136
911	A	11	9	1.	44	0.204
912	A	10	9	1.	44	0.204
913	A	9	9	1.	44	0.204
914	A	9	9	1.	44	0.204
915	A	9	9	1.	44	0.204
916	A	8	8	1.	44	0.182
917	A	7	7	1.	44	0.159
918	A	8	7	1.	44	0.159
919	A	9	7	1.	44	0.159
920	A	9	9	1.	44	0.204
921	A	8	8	1.	44	0.182
922	A	8	8	1.09	44	0.182
923	A	4	4	1.	44	0.091
924	A	4	4	1.	44	0.091
925	A	5	5	1.	44	0.114
926	A	6	6	1.	44	0.136
927	A	9	9	1.	44	0.204
928	A	8	8	1.	44	0.182
929	A	7	7	1.	44	0.159
930	A	5	5	1.	44	0.114
931	A	5	5	1.	44	0.114
932	A	6	6	1.	44	0.136
933	A	9	9	1.	44	0.204
934	A	8	8	1.	44	0.182
935	A	6	6	1.	44	0.136
936	A	6	6	1.	44	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
937	A	6	6	1.	44	0.136
938	A	7	6	1.	39	0.154
939	A	3	3	1.	37	0.081
940	A	2	2	1.41	31	0.065
941	A	4	4	1.	37	0.108
942	A	4	4	1.	39	0.103
943	A	4	4	1.	39	0.103
944	A	6	6	1.	39	0.154
945	A	7	7	1.	39	0.18
946	A	7	6	1.	39	0.154
947	A	4	4	1.	39	0.103
948	A	3	3	1.	33	0.091
949	A	5	5	1.	39	0.128
950	A	5	5	1.	41	0.122
951	A	5	5	1.	41	0.122
952	A	5	5	1.	41	0.122
953	A	7	7	1.	41	0.171
954	A	8	8	1.	41	0.195
955	A	5	4	1.	39	0.103
956	A	4	3	1.	33	0.091
957	A	6	5	1.	39	0.128
958	A	6	6	1.	41	0.146
959	A	6	5	1.	41	0.122
960	A	6	5	1.	41	0.122
961	A	6	5	1.	41	0.122
962	A	8	7	1.	41	0.171
963	A	9	8	1.	41	0.195
964	A	6	4	1.	39	0.103
965	A	5	3	1.	33	0.091
966	A	7	5	1.	39	0.128
967	A	7	6	1.	41	0.146
968	A	7	6	1.	41	0.146

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
969	A	7	5	1.	41	0.122
970	A	7	5	1.	41	0.122
971	A	7	5	1.	41	0.122
972	A	9	7	1.	41	0.171
973	A	10	8	1.	41	0.195
974	A	5	3	1.	48	0.062
975	A	4	3	1.	48	0.062
976	A	3	3	1.	46	0.065
977	A	7	5	1.	41	0.122
978	A	6	5	1.	41	0.122
979	A	5	5	0.99	39	0.128
980	A	4	4	1.	33	0.121
981	A	4	4	1.	39	0.103
982	A	5	5	1.	41	0.122
983	A	6	5	1.	41	0.122
984	A	7	5	1.	41	0.122
985	A	8	5	1.	41	0.122
986	A	7	6	1.	41	0.146
987	A	6	6	1.	41	0.146
988	A	5	5	1.	39	0.128
989	A	4	4	1.	33	0.121
990	A	5	5	1.	39	0.128
991	A	6	5	1.	41	0.122
992	A	7	5	1.	41	0.122
993	A	8	5	1.	41	0.122
994	A	7	6	1.	41	0.146
995	A	6	6	1.	41	0.146
996	A	5	5	1.	39	0.128
997	A	5	5	1.	33	0.152
998	A	6	5	1.	39	0.128
999	A	7	5	1.	41	0.122
1000	A	8	5	1.	41	0.122

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1001	A	8	6	1.	41	0.146
1002	A	7	6	1.	41	0.146
1003	A	6	6	1.	41	0.146
1004	A	6	6	1.	39	0.154
1005	A	6	5	1.	33	0.152
1006	A	7	5	1.	39	0.128
1007	A	8	5	1.	41	0.122
1008	A	9	5	1.	41	0.122
1009	A	3	2	1.	48	0.042
1010	A	4	4	1.	48	0.083
1011	A	5	5	1.	48	0.104
1012	A	6	5	1.	48	0.104
1013	A	7	5	1.	48	0.104
1014	A	9	8	1.	43	0.186
1015	A	8	8	1.	41	0.195
1016	A	7	7	1.	35	0.2
1017	A	9	9	1.	41	0.22
1018	A	9	9	1.	43	0.209
1019	A	10	10	1.	43	0.233
1020	A	11	10	1.	43	0.233
1021	A	10	8	1.	43	0.186
1022	A	9	8	1.	41	0.195
1023	A	8	7	1.	35	0.2
1024	A	10	9	1.	41	0.22
1025	A	10	10	1.	43	0.233
1026	A	10	9	1.	43	0.209
1027	A	11	10	1.	43	0.233
1028	A	12	10	1.	43	0.233
1029	A	11	8	1.	43	0.186
1030	A	10	8	1.	41	0.195
1031	A	9	7	1.	35	0.2
1032	A	11	9	1.	41	0.22

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1033	A	11	10	1.	43	0.233
1034	A	11	10	1.	43	0.233
1035	A	11	9	1.	43	0.209
1036	A	12	10	1.	43	0.233
1037	A	13	10	1.	43	0.233
1038	A	9	7	1.	50	0.14
1039	A	8	7	1.	50	0.14
1040	A	8	7	1.	43	0.163
1041	A	7	7	1.	41	0.171
1042	A	6	6	1.	35	0.171
1043	A	8	8	1.	41	0.195
1044	A	9	9	1.	43	0.209
1045	A	10	9	1.	43	0.209
1046	A	11	9	1.	43	0.209
1047	A	8	8	1.	43	0.186
1048	A	7	7	1.	41	0.171
1049	A	6	6	1.	35	0.171
1050	A	9	9	1.	41	0.22
1051	A	10	9	1.	43	0.209
1052	A	11	9	1.	43	0.209
1053	A	9	8	1.	43	0.186
1054	A	8	8	1.	43	0.186
1055	A	7	7	1.	41	0.171
1056	A	7	7	1.	35	0.2
1057	A	10	9	1.	41	0.22
1058	A	11	9	1.	43	0.209
1059	A	12	9	1.	43	0.209
1060	A	8	7	1.	35	0.2
1061	A	7	7	1.	50	0.14
1062	A	6	6	1.	50	0.12
1063	A	7	7	1.	50	0.14
1064	A	8	7	1.	50	0.14

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1065	A	7	6	1.	41	0.146
1066	A	6	6	1.	41	0.146
1067	A	5	5	1.	41	0.122
1068	A	5	5	1.	41	0.122
1069	A	5	5	1.	41	0.122
1070	A	6	6	1.	41	0.146
1071	A	7	6	1.	41	0.146
1072	A	8	7	1.	43	0.163
1073	A	7	7	1.	43	0.163
1074	A	6	6	1.	43	0.14
1075	A	6	6	1.	43	0.14
1076	A	6	6	1.	43	0.14
1077	A	6	6	1.	43	0.14
1078	A	7	7	1.	43	0.163
1079	A	8	7	1.	43	0.163
1080	A	8	7	1.	43	0.163
1081	A	7	6	1.	43	0.14
1082	A	7	7	1.	43	0.163
1083	A	7	6	1.	43	0.14
1084	A	7	6	1.	43	0.14
1085	A	7	6	1.	43	0.14
1086	A	8	7	1.	43	0.163
1087	A	9	7	1.	43	0.163
1088	A	8	6	1.	43	0.14
1089	A	8	7	1.	43	0.163
1090	A	8	7	1.	43	0.163
1091	A	8	6	1.	43	0.14
1092	A	8	6	1.	43	0.14
1093	A	8	6	1.	43	0.14
1094	A	9	7	1.	43	0.163
1095	A	8	6	1.	43	0.14
1096	A	7	6	1.	43	0.14

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1097	A	6	6	1.	43	0.14
1098	A	5	5	1.	43	0.116
1099	A	6	6	1.	43	0.14
1100	A	7	6	1.	43	0.14
1101	A	8	6	1.	43	0.14
1102	A	9	6	1.	43	0.14
1103	A	8	7	1.	43	0.163
1104	A	7	7	1.	43	0.163
1105	A	6	6	1.	43	0.14
1106	A	6	6	1.	43	0.14
1107	A	7	6	1.	43	0.14
1108	A	8	6	1.	43	0.14
1109	A	9	7	1.	43	0.163
1110	A	8	7	1.	43	0.163
1111	A	7	6	1.	43	0.14
1112	A	7	7	1.	43	0.163
1113	A	7	6	1.	43	0.14
1114	A	8	6	1.	43	0.14
1115	A	9	6	1.	43	0.14
1116	A	8	7	1.	45	0.156
1117	A	7	7	1.	45	0.156
1118	A	7	7	1.	45	0.156
1119	A	6	6	1.	45	0.133
1120	A	5	5	1.	45	0.111
1121	A	6	5	1.	45	0.111
1122	A	9	7	1.	45	0.156
1123	A	8	7	1.	45	0.156
1124	A	8	8	1.	45	0.178
1125	A	8	7	1.	45	0.156
1126	A	7	6	1.	45	0.133
1127	A	6	5	1.	45	0.111
1128	A	7	5	1.	45	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1129	A	10	7	1.	45	0.156
1130	A	9	7	1.	45	0.156
1131	A	9	8	1.	45	0.178
1132	A	9	8	1.	45	0.178
1133	A	9	7	1.	45	0.156
1134	A	8	6	1.	45	0.133
1135	A	7	5	1.	45	0.111
1136	A	8	7	1.	45	0.156
1137	A	7	7	1.	45	0.156
1138	A	6	6	1.	45	0.133
1139	A	5	5	1.	45	0.111
1140	A	4	4	1.	45	0.089
1141	A	5	4	1.	45	0.089
1142	A	6	4	1.	45	0.089
1143	A	8	8	1.	54	0.148
1144	A	4	4	1.	46	0.087
1145	A	8	8	1.	45	0.178
1146	A	7	7	1.	45	0.156
1147	A	6	6	1.	45	0.133
1148	A	4	4	1.	45	0.089
1149	A	5	4	1.	45	0.089
1150	A	6	4	1.	45	0.089
1151	A	8	7	1.	45	0.156
1152	A	7	7	1.	45	0.156
1153	A	5	5	1.	45	0.111
1154	A	5	4	1.	45	0.089
1155	A	6	4	1.	45	0.089
1156	A	6	5	1.	41	0.122
1157	A	5	4	1.	39	0.103
1158	A	8	5	1.	41	0.122
1159	A	9	6	1.	41	0.146
1160	A	8	7	1.	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1161	A	7	7	1.	33	0.212
1162	A	6	6	1.	33	0.182
1163	A	6	6	1.	33	0.182
1164	A	6	6	1.	33	0.182
1165	A	7	7	1.	33	0.212
1166	A	8	7	1.	33	0.212
1167	A	10	9	1.	35	0.257
1168	A	9	9	1.	35	0.257
1169	A	8	8	1.	35	0.229
1170	A	8	8	1.	35	0.229
1171	A	8	8	1.	35	0.229
1172	A	8	8	1.	35	0.229
1173	A	9	9	1.	35	0.257
1174	A	10	9	1.	35	0.257
1175	A	11	9	1.	35	0.257
1176	A	10	9	1.	35	0.257
1177	A	9	8	1.	35	0.229
1178	A	9	8	1.	35	0.229
1179	A	9	9	1.	35	0.257
1180	A	9	8	1.	35	0.229
1181	A	9	8	1.	35	0.229
1182	A	10	9	1.	35	0.257
1183	A	11	9	1.	35	0.257
1184	A	8	6	1.	35	0.171
1185	A	7	6	1.	35	0.171
1186	A	6	6	1.	35	0.171
1187	A	5	5	1.	35	0.143
1188	A	6	6	1.	35	0.171
1189	A	7	6	1.	35	0.171
1190	A	8	6	1.	35	0.171
1191	A	8	7	1.	35	0.2
1192	A	7	7	1.	35	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1193	A	6	6	1.	35	0.171
1194	A	6	6	1.	35	0.171
1195	A	7	7	1.	35	0.2
1196	A	8	7	1.	35	0.2
1197	A	9	7	1.	35	0.2
1198	A	8	7	1.	35	0.2
1199	A	7	6	1.	35	0.171
1200	A	7	7	1.	35	0.2
1201	A	7	6	1.	35	0.171
1202	A	8	7	1.	35	0.2
1203	A	9	7	1.	35	0.2
1204	A	6	5	1.	37	0.135
1205	A	5	5	1.	37	0.135
1206	A	4	4	1.	37	0.108
1207	A	5	5	1.	37	0.135
1208	A	5	5	1.	37	0.135
1209	A	5	5	1.	37	0.135
1210	A	6	6	1.	37	0.162
1211	A	7	6	1.	37	0.162
1212	A	7	6	1.	37	0.162
1213	A	6	6	1.	37	0.162
1214	A	5	5	1.	37	0.135
1215	A	6	6	1.	37	0.162
1216	A	6	6	1.	37	0.162
1217	A	6	6	1.	37	0.162
1218	A	6	6	1.	37	0.162
1219	A	7	7	1.	37	0.189
1220	A	8	7	1.	37	0.189
1221	A	8	6	1.	37	0.162
1222	A	7	6	1.	37	0.162
1223	A	6	5	1.	37	0.135
1224	A	7	6	1.	37	0.162

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1225	A	7	6	1.	37	0.162
1226	A	7	7	1.	37	0.189
1227	A	7	6	1.	37	0.162
1228	A	7	6	1.	37	0.162
1229	A	8	7	1.	37	0.189
1230	A	9	7	1.	37	0.189
1231	A	9	6	1.	37	0.162
1232	A	8	6	1.	37	0.162
1233	A	7	6	1.	37	0.162
1234	A	6	6	1.	37	0.162
1235	A	7	7	1.	37	0.189
1236	A	7	7	1.	37	0.189
1237	A	8	8	1.	37	0.216
1238	A	9	8	1.	37	0.216
1239	A	9	6	1.	37	0.162
1240	A	8	6	1.	37	0.162
1241	A	7	6	1.	37	0.162
1242	A	6	6	1.	37	0.162
1243	A	7	7	1.	37	0.189
1244	A	8	8	1.	37	0.216
1245	A	9	8	1.	37	0.216
1246	A	9	7	1.	37	0.189
1247	A	8	7	1.	37	0.189
1248	A	7	7	1.	37	0.189
1249	A	6	6	1.	37	0.162
1250	A	8	8	1.	37	0.216
1251	A	9	9	1.	37	0.243
1252	A	10	9	1.	37	0.243
1253	A	8	6	1.	30	0.2
1254	A	7	6	1.	30	0.2
1255	A	6	6	1.	30	0.2
1256	A	5	5	1.	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1257	A	6	6	1.	30	0.2
1258	A	7	6	1.	30	0.2
1259	A	8	6	1.	30	0.2
1260	A	7	6	1.	31	0.194
1261	A	6	6	1.	31	0.194
1262	A	5	5	1.	31	0.161
1263	A	5	5	1.	31	0.161
1264	A	6	6	1.	31	0.194
1265	A	7	6	1.	31	0.194
1266	A	8	7	1.	41	0.171
1267	A	7	7	1.	41	0.171
1268	A	6	6	1.	41	0.146
1269	A	6	6	1.	41	0.146
1270	A	6	6	1.	41	0.146
1271	A	7	7	1.	41	0.171
1272	A	8	7	1.	41	0.171
1273	A	10	9	1.	43	0.209
1274	A	9	9	1.	43	0.209
1275	A	8	8	1.	43	0.186
1276	A	8	8	1.	43	0.186
1277	A	8	8	1.	43	0.186
1278	A	8	8	1.	43	0.186
1279	A	9	9	1.	43	0.209
1280	A	10	9	1.	43	0.209
1281	A	11	9	1.	43	0.209
1282	A	10	9	1.	43	0.209
1283	A	9	8	1.	43	0.186
1284	A	9	8	1.	43	0.186
1285	A	9	9	1.	43	0.209
1286	A	9	8	1.	43	0.186
1287	A	9	8	1.	43	0.186
1288	A	10	9	1.	43	0.209

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1289	A	11	9	1.	43	0.209
1290	A	8	6	1.	43	0.14
1291	A	7	6	1.	43	0.14
1292	A	6	6	1.	43	0.14
1293	A	5	5	1.	43	0.116
1294	A	6	6	1.	43	0.14
1295	A	7	6	1.	43	0.14
1296	A	8	6	1.	43	0.14
1297	A	8	7	1.	43	0.163
1298	A	7	7	1.	43	0.163
1299	A	6	6	1.	43	0.14
1300	A	6	6	1.	43	0.14
1301	A	7	7	1.	43	0.163
1302	A	8	7	1.	43	0.163
1303	A	9	7	1.	43	0.163
1304	A	8	7	1.	43	0.163
1305	A	7	6	1.	43	0.14
1306	A	7	7	1.	43	0.163
1307	A	7	6	1.	43	0.14
1308	A	8	7	1.	43	0.163
1309	A	9	7	1.	43	0.163
1310	A	6	5	1.	45	0.111
1311	A	5	5	1.	45	0.111
1312	A	4	4	1.	45	0.089
1313	A	5	5	1.	45	0.111
1314	A	5	5	1.	45	0.111
1315	A	5	5	1.	45	0.111
1316	A	6	6	1.	45	0.133
1317	A	7	6	1.	45	0.133
1318	A	7	6	1.	45	0.133
1319	A	6	6	1.	45	0.133
1320	A	5	5	1.	45	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1321	A	6	6	1.	45	0.133
1322	A	6	6	1.	45	0.133
1323	A	6	6	1.	45	0.133
1324	A	6	6	1.	45	0.133
1325	A	7	7	1.	45	0.156
1326	A	8	7	1.	45	0.156
1327	A	8	6	1.	45	0.133
1328	A	7	6	1.	45	0.133
1329	A	6	5	1.	45	0.111
1330	A	7	6	1.	45	0.133
1331	A	7	6	1.	45	0.133
1332	A	7	7	1.	45	0.156
1333	A	7	6	1.	45	0.133
1334	A	7	6	1.	45	0.133
1335	A	8	7	1.	45	0.156
1336	A	9	7	1.	45	0.156
1337	A	9	6	1.	45	0.133
1338	A	8	6	1.	45	0.133
1339	A	7	6	1.	45	0.133
1340	A	6	6	1.	45	0.133
1341	A	7	7	1.	45	0.156
1342	A	7	7	1.	45	0.156
1343	A	8	8	1.	45	0.178
1344	A	9	8	1.	45	0.178
1345	A	7	7	1.	54	0.13
1346	A	9	6	1.	45	0.133
1347	A	8	6	1.	45	0.133
1348	A	7	6	1.	45	0.133
1349	A	6	6	1.	45	0.133
1350	A	7	7	1.	45	0.156
1351	A	8	8	1.	45	0.178
1352	A	9	8	1.	45	0.178

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1353	A	9	7	1.	45	0.156
1354	A	8	7	1.	45	0.156
1355	A	7	7	1.	45	0.156
1356	A	6	6	1.	45	0.133
1357	A	8	8	1.	45	0.178
1358	A	9	9	1.	45	0.2
1359	A	10	9	1.	45	0.2
1360	A	8	7	1.	33	0.212
1361	A	7	7	1.	33	0.212
1362	A	6	6	1.	33	0.182
1363	A	6	6	1.	33	0.182
1364	A	6	6	1.	33	0.182
1365	A	7	7	1.	33	0.212
1366	A	8	7	1.	33	0.212
1367	A	9	8	1.	35	0.229
1368	A	8	8	1.	35	0.229
1369	A	7	7	1.	35	0.2
1370	A	7	7	1.	35	0.2
1371	A	7	7	1.	35	0.2
1372	A	7	7	1.	35	0.2
1373	A	8	8	1.	35	0.229
1374	A	9	8	1.	35	0.229
1375	A	9	9	1.	35	0.257
1376	A	8	8	1.	35	0.229
1377	A	8	8	1.	35	0.229
1378	A	8	8	1.	35	0.229
1379	A	8	8	1.	35	0.229
1380	A	8	8	1.	35	0.229
1381	A	9	9	1.	35	0.257
1382	A	10	9	1.	35	0.257
1383	A	10	9	1.	35	0.257
1384	A	9	8	1.	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1385	A	9	8	1.	35	0.229
1386	A	9	8	1.	35	0.229
1387	A	9	9	1.	35	0.257
1388	A	9	8	1.	35	0.229
1389	A	9	8	1.	35	0.229
1390	A	10	9	1.	35	0.257
1391	A	9	8	1.	35	0.229
1392	A	8	8	1.	35	0.229
1393	A	7	7	1.	35	0.2
1394	A	6	6	1.	35	0.171
1395	A	7	7	1.	35	0.2
1396	A	8	8	1.	35	0.229
1397	A	9	8	1.	35	0.229
1398	A	9	8	1.	35	0.229
1399	A	8	8	1.	35	0.229
1400	A	7	7	1.	35	0.2
1401	A	7	7	1.	35	0.2
1402	A	8	8	1.	35	0.229
1403	A	9	8	1.	35	0.229
1404	A	10	8	1.	35	0.229
1405	A	9	8	1.	35	0.229
1406	A	8	8	1.	35	0.229
1407	A	8	8	1.	35	0.229
1408	A	8	8	1.	35	0.229
1409	A	9	9	1.	35	0.257
1410	A	10	9	1.	35	0.257
1411	A	8	6	1.	37	0.162
1412	A	7	6	1.	37	0.162
1413	A	6	6	1.	37	0.162
1414	A	7	7	1.	37	0.189
1415	A	8	8	1.	37	0.216
1416	A	8	8	1.	37	0.216

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1417	A	9	9	1.	37	0.243
1418	A	10	9	1.	37	0.243
1419	A	8	7	1.	37	0.189
1420	A	7	7	1.	37	0.189
1421	A	8	8	1.	37	0.216
1422	A	9	9	1.	37	0.243
1423	A	9	9	1.	37	0.243
1424	A	9	9	1.	37	0.243
1425	A	10	9	1.	37	0.243
1426	A	9	7	1.	37	0.189
1427	A	8	7	1.	37	0.189
1428	A	9	8	1.	37	0.216
1429	A	10	9	1.	37	0.243
1430	A	10	10	1.	37	0.27
1431	A	10	9	1.	37	0.243
1432	A	10	9	1.	37	0.243
1433	A	11	9	1.	37	0.243
1434	A	7	6	1.	37	0.162
1435	A	6	6	1.	37	0.162
1436	A	5	5	1.	37	0.135
1437	A	7	7	1.	37	0.189
1438	A	7	7	1.	37	0.189
1439	A	8	8	1.	37	0.216
1440	A	7	6	1.	37	0.162
1441	A	6	6	1.	37	0.162
1442	A	5	5	1.	37	0.135
1443	A	7	7	1.	37	0.189
1444	A	8	8	1.	37	0.216
1445	A	9	9	1.	37	0.243
1446	A	7	6	1.	37	0.162
1447	A	6	6	1.	37	0.162
1448	A	6	6	1.	37	0.162

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1449	A	8	8	1.	37	0.216
1450	A	9	9	1.	37	0.243
1451	A	8	7	1.	41	0.171
1452	A	7	7	1.	41	0.171
1453	A	6	6	1.	41	0.146
1454	A	6	6	1.	41	0.146
1455	A	6	6	1.	41	0.146
1456	A	7	7	1.	41	0.171
1457	A	8	7	1.	41	0.171
1458	A	9	8	1.	43	0.186
1459	A	8	8	1.	43	0.186
1460	A	7	7	1.	43	0.163
1461	A	7	7	1.	43	0.163
1462	A	7	7	1.	43	0.163
1463	A	7	7	1.	43	0.163
1464	A	8	8	1.	43	0.186
1465	A	9	8	1.	43	0.186
1466	A	9	8	1.	43	0.186
1467	A	8	7	1.	43	0.163
1468	A	8	7	1.	43	0.163
1469	A	8	7	1.	43	0.163
1470	A	8	8	1.	43	0.186
1471	A	8	7	1.	43	0.163
1472	A	9	8	1.	43	0.186
1473	A	10	8	1.	43	0.186
1474	A	10	8	1.	43	0.186
1475	A	9	7	1.	43	0.163
1476	A	9	7	1.	43	0.163
1477	A	9	7	1.	43	0.163
1478	A	9	8	1.	43	0.186
1479	A	9	8	1.	43	0.186
1480	A	9	7	1.	43	0.163

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1481	A	10	8	1.	43	0.186
1482	A	9	7	1.	43	0.163
1483	A	8	7	1.	43	0.163
1484	A	7	7	1.	43	0.163
1485	A	6	6	1.	43	0.14
1486	A	7	7	1.	43	0.163
1487	A	8	7	1.	43	0.163
1488	A	9	7	1.	43	0.163
1489	A	9	7	1.	43	0.163
1490	A	8	7	1.	43	0.163
1491	A	7	7	1.	43	0.163
1492	A	7	7	1.	43	0.163
1493	A	8	8	1.	43	0.186
1494	A	9	8	1.	43	0.186
1495	A	10	7	1.	43	0.163
1496	A	9	7	1.	43	0.163
1497	A	8	7	1.	43	0.163
1498	A	8	8	1.	43	0.186
1499	A	8	7	1.	43	0.163
1500	A	9	8	1.	43	0.186
1501	A	10	8	1.	43	0.186
1502	A	8	6	1.	45	0.133
1503	A	7	6	1.	45	0.133
1504	A	6	6	1.	45	0.133
1505	A	7	7	1.	45	0.156
1506	A	8	8	1.	45	0.178
1507	A	8	8	1.	45	0.178
1508	A	9	8	1.	45	0.178
1509	A	10	8	1.	45	0.178
1510	A	8	6	1.	45	0.133
1511	A	7	6	1.	45	0.133
1512	A	8	7	1.	45	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1513	A	9	8	1.	45	0.178
1514	A	9	9	1.	45	0.2
1515	A	9	8	1.	45	0.178
1516	A	10	8	1.	45	0.178
1517	A	9	6	1.	45	0.133
1518	A	8	6	1.	45	0.133
1519	A	9	7	1.	45	0.156
1520	A	10	8	1.	45	0.178
1521	A	10	9	1.	45	0.2
1522	A	10	9	1.	45	0.2
1523	A	10	8	1.	45	0.178
1524	A	11	8	1.	45	0.178
1525	A	7	5	1.	45	0.111
1526	A	6	5	1.	45	0.111
1527	A	5	5	1.	45	0.111
1528	A	6	6	1.	45	0.133
1529	A	7	7	1.	45	0.156
1530	A	8	8	1.	45	0.178
1531	A	9	8	1.	45	0.178
1532	A	8	8	1.	54	0.148
1533	A	7	5	1.	45	0.111
1534	A	6	5	1.	45	0.111
1535	A	5	5	1.	45	0.111
1536	A	7	7	1.	45	0.156
1537	A	8	8	1.	45	0.178
1538	A	9	9	1.	45	0.2
1539	A	7	5	1.	45	0.111
1540	A	6	5	1.	45	0.111
1541	A	6	6	1.	45	0.133

Chapter 3

Listing of integrals

3.1 $\int \cos^2(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=131

$$-\frac{a(5A+4C)\sin^3(c+dx)}{15d}+\frac{a(5A+4C)\sin(c+dx)}{5d}+\frac{a(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3C)+\frac{aC\sin(}{}$$

```
[Out] (a*(4*A + 3*C)*x)/8 + (a*(5*A + 4*C)*Sin[c + d*x])/(5*d) + (a*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (a*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A + 4*C)*Sin[c + d*x]^3)/(
15*d)
```

Rubi [A] time = 0.17577, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3034, 3023, 2748, 2635, 8, 2633}

$$-\frac{a(5A+4C)\sin^3(c+dx)}{15d}+\frac{a(5A+4C)\sin(c+dx)}{5d}+\frac{a(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3C)+\frac{aC\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(4*A + 3*C)*x)/8 + (a*(5*A + 4*C)*Sin[c + d*x])/(5*d) + (a*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (a*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A + 4*C)*Sin[c + d*x]^3)/(
```

```
15*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx = \frac{aC\cos^4(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^2(c+dx)\left(5aA+\right.$$
$$=\frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aC\cos^4(c+dx)\sin(c+dx}{5d}$$
$$=\frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aC\cos^4(c+dx)\sin(c+dx}{5d}$$
$$=\frac{a(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aC\cos^3(c+dx)\sin}{4d}$$
$$=\frac{1}{8}a(4A+3C)x+\frac{a(5A+4C)\sin(c+dx)}{5d}+\frac{a(4A+3C)}{}$$

Mathematica [A] time = 0.283941, size = 86, normalized size = 0.66

$$\frac{a\left(-160(A+2C)\sin^3(c+dx)+480(A+C)\sin(c+dx)+15(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(480*(A + C)*Sin[c + d*x] - 160*(A + 2*C)*Sin[c + d*x]^3 + 96*C*Sin[c +
d*x]^5 + 15*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4
*(c + d*x)])))/(480*d)
```

Maple [A] time = 0.464, size = 117, normalized size = 0.9

$$\frac{1}{d}\left(\frac{aC\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+aC\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\,\cos(dx+c)}{2}\right)+\frac{3\,a}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*A*(2+cos(d*x+c)^2)*
sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.02914, size = 153, normalized size = 1.17

$$-\frac{160\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa-120\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa-32\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 120*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
 c))*C*a - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a)/
d
```

Fricas [A] time = 1.40241, size = 242, normalized size = 1.85

$$\frac{15\,(4\,A+3\,C)adx+\left(24\,Ca\cos\left(dx+c\right)^4+30\,Ca\cos\left(dx+c\right)^3+8\,(5\,A+4\,C)a\cos\left(dx+c\right)^2+15\,(4\,A+3\,C)a\cos\left(dx+\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(4*A + 3*C)*a*d*x + (24*C*a*cos(d*x + c)^4 + 30*C*a*cos(d*x + c)^
3 + 8*(5*A + 4*C)*a*cos(d*x + c)^2 + 15*(4*A + 3*C)*a*cos(d*x + c) + 16*(5*
A + 4*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 2.61774, size = 279, normalized size = 2.13

$$\begin{cases}\frac{Aax\sin^2(c+dx)}{2}+\frac{Aax\cos^2(c+dx)}{2}+\frac{2Aa\sin^3(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^2(c+dx)}{d}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Cax\sin^4(c+dx)}{8}+\frac{3Cax\sin^2(c+dx}{4}\\ x\left(A+C\cos^2(c)\right)(a\cos(c)+a)\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + 2*A*a*sin(c
+ d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + A*a*sin(c + d*x)*cos
(c + d*x)/(2*d) + 3*C*a*x*sin(c + d*x)**4/8 + 3*C*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 8*C*a*sin(c + d*x)**5/(15*d) + 4
*C*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*C*a*sin(c + d*x)**3*cos(c +
d*x)/(8*d) + C*a*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*a*sin(c + d*x)*cos(c
+ d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)*cos(c)**2,
True))
```

Giac [A] time = 1.13894, size = 147, normalized size = 1.12

$$\frac{1}{8}(4\,Aa+3\,Ca)x+\frac{Ca\sin(5\,dx+5\,c)}{80\,d}+\frac{Ca\sin(4\,dx+4\,c)}{32\,d}+\frac{(4\,Aa+5\,Ca)\sin(3\,dx+3\,c)}{48\,d}+\frac{(Aa+Ca)\sin(2\,dx+2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(4*A*a + 3*C*a)*x + 1/80*C*a*sin(5*d*x + 5*c)/d + 1/32*C*a*sin(4*d*x +
4*c)/d + 1/48*(4*A*a + 5*C*a)*sin(3*d*x + 3*c)/d + 1/4*(A*a + C*a)*sin(2*d*
x + 2*c)/d + 1/8*(6*A*a + 5*C*a)*sin(d*x + c)/d
```

3.2 $\int \cos(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=108

$$\frac{a(3A+2C)\sin(c+dx)}{3d}+\frac{a(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3C)+\frac{aC\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{aC\sin(}{}$$

```
[Out] (a*(4*A + 3*C)*x)/8 + (a*(3*A + 2*C)*Sin[c + d*x])/(3*d) + (a*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) +
 (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.101298, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3034, 3023, 2734}

$$\frac{a(3A+2C)\sin(c+dx)}{3d}+\frac{a(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3C)+\frac{aC\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{aC\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(4*A + 3*C)*x)/8 + (a*(3*A + 2*C)*Sin[c + d*x])/(3*d) + (a*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) +
 (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\int \cos(c+dx)\left(4aA+a(4\right.$$

$$= \frac{aC\cos^2(c+dx)\sin(c+dx)}{3d} + \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}$$

$$= \frac{1}{8}a(4A+3C)x + \frac{a(3A+2C)\sin(c+dx)}{3d} + \frac{a(4A+3C)\,\mathrm{co}}{}$$

Mathematica [A] time = 0.222567, size = 77, normalized size = 0.71

$$\frac{a(24(4A+3C)\sin(c+dx)+24(A+C)\sin(2(c+dx))+48Ac+48Adx+8C\sin(3(c+dx))+3C\sin(4(c+dx))+36c}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 24*(4*A + 3*C)*Sin[c + d*x] + 2
4*(A + C)*Sin[2*(c + d*x)] + 8*C*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/
(96*d)
```

Maple [A] time = 0.024, size = 96, normalized size = 0.9

$$\frac{1}{d}\left(aC\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{aC\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+aA\left(\frac{\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+a*A*sin(d*x+c))
```

Maxima [A] time = 1.01826, size = 122, normalized size = 1.13

$$\frac{24\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa - 32\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ca + 3\,(12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\,\sin(2\,c}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*C*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C
*a + 96*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.39691, size = 189, normalized size = 1.75

$$\frac{3\,(4\,A + 3\,C)adx + \left(6\,Ca\cos(dx + c)^3 + 8\,Ca\cos(dx + c)^2 + 3\,(4\,A + 3\,C)a\cos(dx + c) + 8\,(3\,A + 2\,C)a\right)\sin(dx + c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A + 3*C)*a*d*x + (6*C*a*cos(d*x + c)^3 + 8*C*a*cos(d*x + c)^2 +
3*(4*A + 3*C)*a*cos(d*x + c) + 8*(3*A + 2*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.32846, size = 226, normalized size = 2.09

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{Aa\sin(c+dx)}{d} + \frac{3Cax\sin^4(c+dx)}{8} + \frac{3Cax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Cax\cos^4(c+d}{8} \\ x\left(A + C\cos^2(c)\right)(a\cos(c) + a)\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + A*a*sin(c + d*x)/d + 3*C*a*x*sin(c + d*x)**4/8 +
3*C*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 3*C
*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*C*a*sin(c + d*x)**3/(3*d) + 5*C*a
*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a*sin(c + d*x)*cos(c + d*x)**2/d, N
e(d, 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)*cos(c), True))
```

Giac [A] time = 1.18732, size = 116, normalized size = 1.07

$$\frac{1}{8}(4\,Aa+3\,Ca)x+\frac{Ca\sin(4\,dx+4\,c)}{32\,d}+\frac{Ca\sin(3\,dx+3\,c)}{12\,d}+\frac{(Aa+Ca)\sin(2\,dx+2\,c)}{4\,d}+\frac{(4\,Aa+3\,Ca)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*C*a)*x + 1/32*C*a*sin(4*d*x + 4*c)/d + 1/12*C*a*sin(3*d*x +
3*c)/d + 1/4*(A*a + C*a)*sin(2*d*x + 2*c)/d + 1/4*(4*A*a + 3*C*a)*sin(d*x +
 c)/d
```

3.3 $\int (a + a\cos(c + dx)) \left(A + C\cos^2(c + dx)\right) dx$

Optimal. Leaf size=81

$$\frac{a(3A+C)\sin(c+dx)}{3d} + \frac{1}{2}ax(2A+C) + \frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3ad} - \frac{aC\sin(c+dx)\cos(c+dx)}{6d}$$

```
[Out] (a*(2*A + C)*x)/2 + (a*(3*A + C)*Sin[c + d*x])/(3*d) - (a*C*Cos[c + d*x]*Si
n[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.0635726, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3024, 2734}

$$\frac{a(3A+C)\sin(c+dx)}{3d} + \frac{1}{2}ax(2A+C) + \frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3ad} - \frac{aC\sin(c+dx)\cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a*(2*A + C)*x)/2 + (a*(3*A + C)*Sin[c + d*x])/(3*d) - (a*C*Cos[c + d*x]*Si
n[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*a*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))\left(A + C\cos^2(c+dx)\right)dx = \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3ad} + \frac{\int (a+a\cos(c+dx))(a(3A+2C)}{3a}$$
$$= \frac{1}{2}a(2A+C)x + \frac{a(3A+C)\sin(c+dx)}{3d} - \frac{aC\cos(c+dx)\sin(c+dx)}{6d} +$$

Mathematica [A] time = 0.13119, size = 59, normalized size = 0.73

$$\frac{a(3(4A+3C)\sin(c+dx) + 12Adx + 3C\sin(2(c+dx)) + C\sin(3(c+dx)) + 6cC + 6Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(6*c*C + 12*A*d*x + 6*C*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*C*Sin[2*(c
+ d*x)] + C*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.021, size = 68, normalized size = 0.8

$$\frac{1}{d}\left(\frac{aC\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aA\sin(dx+c) + aA(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*A*sin(d*x+c)+a*A*(d*x+c))
```

Maxima [A] time = 1.12617, size = 90, normalized size = 1.11

$$\frac{12(dx+c)Aa - 4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Ca + 3(2dx+2c+\sin(2dx+2c))Ca + 12Aa\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a + 3*(2*d*x
 + 2*c + sin(2*d*x + 2*c))*C*a + 12*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.42238, size = 140, normalized size = 1.73

$$\frac{3\,(2\,A+C)adx+\left(2\,Ca\cos\left(dx+c\right)^{2}+3\,Ca\cos\left(dx+c\right)+2\,(3\,A+2\,C)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A + C)*a*d*x + (2*C*a*cos(d*x + c)^2 + 3*C*a*cos(d*x + c) + 2*(3*
A + 2*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.632881, size = 121, normalized size = 1.49

$$\begin{cases} Aax+\frac{Aa\sin\left(c+dx\right)}{d}+\frac{Cax\sin^{2}\left(c+dx\right)}{2}+\frac{Cax\cos^{2}\left(c+dx\right)}{2}+\frac{2Ca\sin^{3}\left(c+dx\right)}{3d}+\frac{Ca\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}+\frac{Ca\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\left(A+C\cos^{2}\left(c\right)\right)\left(a\cos\left(c\right)+a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x + A*a*sin(c + d*x)/d + C*a*x*sin(c + d*x)**2/2 + C*a*x*cos
(c + d*x)**2/2 + 2*C*a*sin(c + d*x)**3/(3*d) + C*a*sin(c + d*x)*cos(c + d*x
)**2/d + C*a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + C*cos(c)**
2)*(a*cos(c) + a), True))
```

Giac [A] time = 1.18375, size = 86, normalized size = 1.06

$$\frac{1}{2}\,(2\,Aa+Ca)x+\frac{Ca\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{Ca\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(4\,Aa+3\,Ca)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + C*a)*x + 1/12*C*a*sin(3*d*x + 3*c)/d + 1/4*C*a*sin(2*d*x + 2*c
)/d + 1/4*(4*A*a + 3*C*a)*sin(d*x + c)/d
```

3.4 $\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}ax(2A+C)+\frac{aC\sin(c+dx)}{d}+\frac{aC\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (a*(2*A + C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d +
(a*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.108718, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3034, 3023, 2735, 3770}

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}ax(2A+C)+\frac{aC\sin(c+dx)}{d}+\frac{aC\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a*(2*A + C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d +
(a*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{aC\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\int\left(2aA+a(2A+C)\cos(c\right.$$
$$=\frac{aC\sin(c+dx)}{d}+\frac{aC\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\int(2aA+$$
$$=\frac{1}{2}a(2A+C)x+\frac{aC\sin(c+dx)}{d}+\frac{aC\cos(c+dx)\sin(c+d}{2d}$$
$$=\frac{1}{2}a(2A+C)x+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0807273, size = 52, normalized size = 0.9

$$\frac{a\left(4A\tanh^{-1}(\sin(c+dx))+4Adx+4C\sin(c+dx)+C\sin(2(c+dx))+2cC+2Cdx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a*(2*c*C + 4*A*d*x + 2*C*d*x + 4*A*ArcTanh[Sin[c + d*x]] + 4*C*Sin[c + d*x
] + C*Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.063, size = 77, normalized size = 1.3

$$aAx+\frac{Aac}{d}+\frac{aC\cos(dx+c)\sin(dx+c)}{2d}+\frac{aCx}{2}+\frac{aCc}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] a*A*x+1/d*A*a*c+1/2*a*C*cos(d*x+c)*sin(d*x+c)/d+1/2*a*C*x+1/2/d*a*C*c+1/d*a
*A*ln(sec(d*x+c)+tan(d*x+c))+a*C*sin(d*x+c)/d
```

Maxima [A] time = 1.11724, size = 85, normalized size = 1.47

$$\frac{4(dx+c)Aa + (2dx+2c+\sin(2dx+2c))Ca + 4Aa\log(\sec(dx+c)+\tan(dx+c)) + 4Ca\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*a + 4*A*a*log(sec
(d*x + c) + tan(d*x + c)) + 4*C*a*sin(d*x + c))/d
```

Fricas [A] time = 1.4704, size = 167, normalized size = 2.88

$$\frac{(2A+C)adx + Aa\log(\sin(dx+c)+1) - Aa\log(-\sin(dx+c)+1) + (Ca\cos(dx+c)+2Ca)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*((2*A + C)*a*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c) +
1) + (C*a*cos(d*x + c) + 2*C*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec(c+dx)\,dx + \int A\cos(c+dx)\sec(c+dx)\,dx + \int C\cos^2(c+dx)\sec(c+dx)\,dx + \int C\cos^3(c+dx)\sec(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(C*cos(c + d*x)**2*sec(c + d*x), x) + Integral(C*cos(c + d*x)**3*s
ec(c + d*x), x))
```

<u>**Giac**</u> **[A]** time = 1.19362, size = 134, normalized size = 2.31

$$\frac{2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(2\,Aa+Ca)(dx+c)+\frac{2\left(Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,Ca\tan(}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (2*A*a + C*a)*(d*x + c) + 2*(C*a*tan(1/2*d*x + 1/2*c)^3 + 3
*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.5 $\int (a+a\cos(c+dx)) \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx) \, dx$

Optimal. Leaf size=42

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}+aCx$$

```
[Out] a*C*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d + (a*A*Tan[c +
 d*x])/d
```

Rubi [A] time = 0.101608, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3032, 3023, 2735, 3770}

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}+aCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] a*C*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d + (a*A*Tan[c +
 d*x])/d
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{aA\tan(c+dx)}{d}+\int \left(aA+aC\cos(c+dx)+aC\cos^2(c+\right.$$
$$= \frac{aC\sin(c+dx)}{d}+\frac{aA\tan(c+dx)}{d}+\int (aA+aC\cos(c+a$$
$$= aCx+\frac{aC\sin(c+dx)}{d}+\frac{aA\tan(c+dx)}{d}+(aA)\int \sec(c-$$
$$= aCx+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}+\frac{aA\,\text{ta}}{}$$

Mathematica [A] time = 0.0240613, size = 54, normalized size = 1.29

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c)\cos(dx)}{d}+\frac{aC\cos(c)\sin(dx)}{d}+aCx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] a*C*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Cos[d*x]*Sin[c])/d + (a*C*Cos[
c]*Sin[d*x])/d + (a*A*Tan[c + d*x])/d
```

Maple [A] time = 0.085, size = 57, normalized size = 1.4

$$aCx + \frac{aA\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{aA\tan\left(dx+c\right)}{d} + \frac{aC\sin\left(dx+c\right)}{d} + \frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] a*C*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+a*A*tan(d*x+c)/d+a*C*sin(d*x+c)/d+1
/d*a*C*c
```

Maxima [A] time = 1.11274, size = 80, normalized size = 1.9

$$\frac{2\left(dx+c\right)Ca + Aa(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 2\,Ca\sin\left(dx+c\right) + 2\,Aa\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + A*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*C*a*sin(d*x + c) + 2*A*a*tan(d*x + c))/d
```

Fricas [B] time = 1.43401, size = 232, normalized size = 5.52

$$\frac{2\,Cadx\cos\left(dx+c\right) + Aa\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right) - Aa\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(Ca\cos\left(dx+c\right)\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*C*a*d*x*cos(d*x + c) + A*a*cos(d*x + c)*log(sin(d*x + c) + 1) - A*a*
cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a*cos(d*x + c) + A*a)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1934, size = 158, normalized size = 3.76

$$\frac{(dx+c)Ca + Aa\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - Aa\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + Aa\tan\cdots\right.}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - \cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((d*x + c)*C*a + A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - C*a*tan(1/2*d*x + 1
/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^4 - 1))/d
```

3.6 $\int (a + a\cos(c + dx)) \left(A + C\cos^2(c + dx)\right)\sec^3(c + dx)\,dx$

Optimal. Leaf size=58

$$\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)}{d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+aCx$$

```
[Out] a*C*x + (a*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*A*Tan[c + d*x])/d +
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.124825, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3032, 3021, 2735, 3770}

$$\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)}{d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+aCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] a*C*x + (a*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*A*Tan[c + d*x])/d +
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2aA+a(A+2C)\cos\right.$$
$$= \frac{aA\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a(A$$
$$= aCx+\frac{aA\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}($$
$$= aCx+\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0301834, size = 67, normalized size = 1.16

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+aCx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] a*C*x + (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*ArcTanh[Sin[c + d*x]])/d +
 (a*A*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.089, size = 85, normalized size = 1.5

$$\frac{aA\tan(dx+c)}{d}+aCx+\frac{Cac}{d}+\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aC\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] a*A*tan(d*x+c)/d+a*C*x+1/d*a*C*c+1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*
ln(sec(d*x+c)+tan(d*x+c))+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.26223, size = 128, normalized size = 2.21

$$\frac{4(dx+c)Ca-Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+2Ca(\log(\sin(dx+c)+1)-\log(\sin(dx}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a - A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 4*A*a*tan(d*x + c))/d
```

Fricas [A] time = 1.36035, size = 267, normalized size = 4.6

$$\frac{4\,Cadx\cos(dx+c)^2+(A+2\,C)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(A+2\,C)a\cos(dx+c)^2\log(-\sin(dx+c)+1)+}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*C*a*d*x*cos(d*x + c)^2 + (A + 2*C)*a*cos(d*x + c)^2*log(sin(d*x + c)
 + 1) - (A + 2*C)*a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*a*cos(d*
```

```
x + c) + A*a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19806, size = 142, normalized size = 2.45

$$\frac{2\,(dx+c)Ca+(Aa+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + (A*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
A*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*
c)^3 - 3*A*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.7 $\int (a + a\cos(c + dx)) \left(A + C\cos^2(c + dx)\right) \sec^4(c + dx)\, dx$

Optimal. Leaf size=86

$$\frac{a(2A+3C)\tan(c+dx)}{3d}+\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*C)*Tan[c + d*x])/(3
*d) + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 0.167052, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3032, 3021, 2748, 3767, 8, 3770}

$$\frac{a(2A+3C)\tan(c+dx)}{3d}+\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (a*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*C)*Tan[c + d*x])/(3
*d) + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \left(3aA+a(2A+3C)\,\mathrm{c}\right.$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+dx}{3d}$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+dx}{3d}$$
$$= \frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\sec(c+dx)\tan(c+}{2d}$$
$$= \frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2A+3C)\tan(c+dx}{3d}$$

Mathematica [A] time = 0.254944, size = 56, normalized size = 0.65

$$\frac{a\left(3(A+2C)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2A\tan^2(c+dx)+3A\sec(c+dx)+6(A+C)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (a*(3*(A + 2*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + C) + 3*A*Sec[c
 + d*x] + 2*A*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.087, size = 108, normalized size = 1.3

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{2aA\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*C
*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*
x+c)/d+1/d*a*C*tan(d*x+c)
```

Maxima [A] time = 1.0598, size = 144, normalized size = 1.67

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa-3Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6Ca(\log(\sin}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*a*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*C*a*tan(d*x + c))/d
```

Fricas [A] time = 1.49501, size = 285, normalized size = 3.31

$$\frac{3\,(A+2\,C)a\cos{(dx+c)}^{3}\log{(\sin{(dx+c)}+1)}-3\,(A+2\,C)a\cos{(dx+c)}^{3}\log{(-\sin{(dx+c)}+1)}+2\,\big(2\,(2\,A+3\,C)a}{12\,d\cos{(dx+c)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(A + 2*C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + 2*C)*a*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*C)*a*cos(d*x + c)^2 + 3
*A*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27824, size = 211, normalized size = 2.45

$$\frac{3\,(Aa+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aa+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ca\tan\left(\frac{1}{2}\,dx\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(A*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + 2*C*a)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a
```

```
*tan(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 6*C*a*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.8 $\int (a + a\cos(c + dx)) \left(A + C\cos^2(c + dx)\right) \sec^5(c + dx)\,dx$

Optimal. Leaf size=117

$$\frac{a(2A+3C)\tan(c+dx)}{3d}+\frac{a(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[Out] (a*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(2*A + 3*C)*Tan[c + d*x])/
(3*d) + (a*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.18974, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3032, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a(2A+3C)\tan(c+dx)}{3d}+\frac{a(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] (a*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(2*A + 3*C)*Tan[c + d*x])/
(3*d) + (a*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))\left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int \left(4aA + a(3A+4C)\,\mathrm{c}\right.$$
$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{aA\sec^3(c+dx)\tan(c+d}{4d}$$
$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{aA\sec^3(c+dx)\tan(c+d}{4d}$$
$$= \frac{a(3A+4C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{aA\sec^2(c+dx)\,\mathrm{ta}}{3d}$$
$$= \frac{a(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a(2A+3C)\tan(c+d}{3d}$$

Mathematica [A] time = 0.388413, size = 75, normalized size = 0.64

$$\frac{a\left(3(3A+4C)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(3(3A+4C)\sec(c+dx) + 8A\tan^2(c+dx) + 6A\sec^3(c+dx) + 24(A\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(A + C) + 3*(3*A
 + 4*C)*Sec[c + d*x] + 6*A*Sec[c + d*x]^3 + 8*A*Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.098, size = 149, normalized size = 1.3

$$\frac{2\,aA\tan(dx+c)}{3\,d} + \frac{aA(\sec(dx+c))^2\tan(dx+c)}{3\,d} + \frac{aC\tan(dx+c)}{d} + \frac{aA(\sec(dx+c))^3\tan(dx+c)}{4\,d} + \frac{3\,aA\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/d*a*C*tan(d*x+c)+1
/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A*
ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(
d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.05411, size = 205, normalized size = 1.75

$$\frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa-3\,Aa\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*C*a*tan(d*x +
 c))/d
```

Fricas [A] time = 1.48064, size = 335, normalized size = 2.86

$$\frac{3\,(3\,A+4\,C)a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+4\,C)a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(2\,A+3\,C)\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A + 4*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*C)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(2*A + 3*C)*a*cos(d*x + c)^3
 + 3*(3*A + 4*C)*a*cos(d*x + c)^2 + 8*A*a*cos(d*x + c) + 6*A*a)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23292, size = 254, normalized size = 2.17

$$\frac{3\,(3\,Aa+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ca\tan(\cdots}{\cdots}}{24\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a + 4*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*C
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 +
12*C*a*tan(1/2*d*x + 1/2*c)^7 - 49*A*a*tan(1/2*d*x + 1/2*c)^5 - 60*C*a*tan(
1/2*d*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 84*C*a*tan(1/2*d*x + 1
/2*c)^3 - 39*A*a*tan(1/2*d*x + 1/2*c) - 36*C*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.9 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=194

$$-\frac{2a^2(5A+4C)\sin^3(c+dx)}{15d}+\frac{2a^2(5A+4C)\sin(c+dx)}{5d}+\frac{a^2(10A+9C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^2(14A+11C)\sin(}{16}$$

```
[Out] (a^2*(14*A + 11*C)*x)/16 + (2*a^2*(5*A + 4*C)*Sin[c + d*x])/(5*d) + (a^2*(1
4*A + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(10*A + 9*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(6*d) + (C*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(
15*d) - (2*a^2*(5*A + 4*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.472005, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3046, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{2a^2(5A+4C)\sin^3(c+dx)}{15d}+\frac{2a^2(5A+4C)\sin(c+dx)}{5d}+\frac{a^2(10A+9C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^2(14A+11C)\sin(}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a^2*(14*A + 11*C)*x)/16 + (2*a^2*(5*A + 4*C)*Sin[c + d*x])/(5*d) + (a^2*(1
4*A + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(10*A + 9*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(6*d) + (C*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(
15*d) - (2*a^2*(5*A + 4*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{\int \cos^2(c}{}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{C\cos^3(c}{}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{C\cos^3(c}{}$$

$$= \frac{a^2(10A+9C)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{C\cos^3(c+dx)(a}{}$$

$$= \frac{a^2(10A+9C)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{C\cos^3(c+dx)(a}{}$$

$$= \frac{a^2(14A+11C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^2(10A+9C)\,\mathrm{co}}{}$$

$$= \frac{1}{16}a^2(14A+11C)x + \frac{2a^2(5A+4C)\sin(c+dx)}{5d} + \frac{a^2(14A}{}$$

Mathematica [A] time = 0.482144, size = 123, normalized size = 0.63

$$\frac{a^2(240(6A+5C)\sin(c+dx)+15(32A+31C)\sin(2(c+dx))+160A\sin(3(c+dx))+30A\sin(4(c+dx))+840Adx+2}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(420*c*C + 840*A*d*x + 660*C*d*x + 240*(6*A + 5*C)*Sin[c + d*x] + 15*(
32*A + 31*C)*Sin[2*(c + d*x)] + 160*A*Sin[3*(c + d*x)] + 200*C*Sin[3*(c + d
*x)] + 30*A*Sin[4*(c + d*x)] + 75*C*Sin[4*(c + d*x)] + 24*C*Sin[5*(c + d*x)
] + 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.056, size = 211, normalized size = 1.1

$$\frac{1}{d}\left(Aa^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2C\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2
*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c)+2/3*A*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2/5*a^2*C*(8/3+cos(d*x+c)^4+
4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.13132, size = 275, normalized size = 1.42

$$-\frac{640\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-30\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c))Aa^2-240\,(2\,dx+2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/960*(640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 30*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 240*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^2 - 128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c)
)*C*a^2 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48
*sin(2*d*x + 2*c))*C*a^2 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*C*a^2)/d
```

Fricas [A] time = 1.44155, size = 315, normalized size = 1.62

$$\frac{15\,(14\,A+11\,C)a^2dx+\left(40\,Ca^2\cos(dx+c)^5+96\,Ca^2\cos(dx+c)^4+10\,(6\,A+11\,C)a^2\cos(dx+c)^3+32\,(5\,A+4\,C)a\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(14*A + 11*C)*a^2*d*x + (40*C*a^2*cos(d*x + c)^5 + 96*C*a^2*cos(d
*x + c)^4 + 10*(6*A + 11*C)*a^2*cos(d*x + c)^3 + 32*(5*A + 4*C)*a^2*cos(d*x
 + c)^2 + 15*(14*A + 11*C)*a^2*cos(d*x + c) + 64*(5*A + 4*C)*a^2)*sin(d*x +
 c))/d
```

Sympy [A] time = 6.31262, size = 592, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/8 + A
*a**2*x*cos(c + d*x)**2/2 + 3*A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4
*A*a**2*sin(c + d*x)**3/(3*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 2*A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 5*C*a**2*x*sin(c + d*x)**6/16 + 15*C*a**2*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 3*C*a**2*x*sin(c + d*x)**4/8 + 15*C*a**2*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 3*C*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*C*a*
*2*x*cos(c + d*x)**6/16 + 3*C*a**2*x*cos(c + d*x)**4/8 + 5*C*a**2*sin(c + d
*x)**5*cos(c + d*x)/(16*d) + 16*C*a**2*sin(c + d*x)**5/(15*d) + 5*C*a**2*si
n(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*C*a**2*sin(c + d*x)**3*cos(c + d*x)
**2/(3*d) + 3*C*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 11*C*a**2*sin(c +
 d*x)*cos(c + d*x)**5/(16*d) + 2*C*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*
C*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*
(a*cos(c) + a)**2*cos(c)**2, True))
```

Giac [A] time = 1.16631, size = 213, normalized size = 1.1

$$\frac{Ca^2 \sin(6\,dx + 6\,c)}{192\,d} + \frac{Ca^2 \sin(5\,dx + 5\,c)}{40\,d} + \frac{1}{16}\left(14\,Aa^2 + 11\,Ca^2\right)x + \frac{\left(2\,Aa^2 + 5\,Ca^2\right)\sin(4\,dx + 4\,c)}{64\,d} + \frac{\left(4\,Aa^2 + 5\,Ca^2\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/192*C*a^2*sin(6*d*x + 6*c)/d + 1/40*C*a^2*sin(5*d*x + 5*c)/d + 1/16*(14*A
*a^2 + 11*C*a^2)*x + 1/64*(2*A*a^2 + 5*C*a^2)*sin(4*d*x + 4*c)/d + 1/24*(4*
A*a^2 + 5*C*a^2)*sin(3*d*x + 3*c)/d + 1/64*(32*A*a^2 + 31*C*a^2)*sin(2*d*x
+ 2*c)/d + 1/4*(6*A*a^2 + 5*C*a^2)*sin(d*x + c)/d
```

3.10 $\int \cos(c+dx)(a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=163

$$\frac{a^2(4A+3C)\sin(c+dx)}{3d}+\frac{a^2(4A+3C)\sin(c+dx)\cos(c+dx)}{12d}+\frac{1}{4}a^2x(4A+3C)+\frac{(10A+3C)\sin(c+dx)(a\cos(c+d}{30d}$$

```
[Out] (a^2*(4*A + 3*C)*x)/4 + (a^2*(4*A + 3*C)*Sin[c + d*x])/(3*d) + (a^2*(4*A +
3*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d) + ((10*A + 3*C)*(a + a*Cos[c + d*x])
^2*Sin[c + d*x])/(30*d) + (C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*Sin[c +
d*x])/(5*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(10*a*d)
```

Rubi [A] time = 0.290271, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3046, 2968, 3023, 2751, 2644}

$$\frac{a^2(4A+3C)\sin(c+dx)}{3d}+\frac{a^2(4A+3C)\sin(c+dx)\cos(c+dx)}{12d}+\frac{1}{4}a^2x(4A+3C)+\frac{(10A+3C)\sin(c+dx)(a\cos(c+d}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(4*A + 3*C)*x)/4 + (a^2*(4*A + 3*C)*Sin[c + d*x])/(3*d) + (a^2*(4*A +
3*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d) + ((10*A + 3*C)*(a + a*Cos[c + d*x])
^2*Sin[c + d*x])/(30*d) + (C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*Sin[c +
d*x])/(5*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(10*a*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{5d} + \frac{\int \cos(c}{}$$

$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{5d} + \frac{\int (a+a}{}$$

$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{5d} + \frac{C(a+a}{}$$

$$= \frac{(10A+3C)(a+a\cos(c+dx))^2\sin(c+dx)}{30d} + \frac{C\cos^2(c+}{}$$

$$= \frac{1}{4}a^2(4A+3C)x + \frac{a^2(4A+3C)\sin(c+dx)}{3d} + \frac{a^2(4A+3C}{}$$

Mathematica [A] time = 0.374725, size = 97, normalized size = 0.6

$$\frac{a^2(30(14A+11C)\sin(c+dx)+120(A+C)\sin(2(c+dx))+20A\sin(3(c+dx))+240Adx+45C\sin(3(c+dx))+15C\sin}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(120*c*C + 240*A*d*x + 180*C*d*x + 30*(14*A + 11*C)*Sin[c + d*x] + 120
*(A + C)*Sin[2*(c + d*x)] + 20*A*Sin[3*(c + d*x)] + 45*C*Sin[3*(c + d*x)] +
 15*C*Sin[4*(c + d*x)] + 3*C*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.025, size = 160, normalized size = 1.

$$\frac{1}{d}\left(\frac{Aa^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{a^2C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+2Aa^2(1/2\cos(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+1/5*a^2*C*(8/3+cos(d*x+c)^4+4/3*
cos(d*x+c)^2)*sin(d*x+c)+2*A*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
2*a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^2*
sin(d*x+c)+1/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.10604, size = 211, normalized size = 1.29

$$-\frac{80\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-120(2dx+2c+\sin(2dx+2c))Aa^2-16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/240*(80*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 120*(2*d*x + 2*c + sin
(2*d*x + 2*c))*A*a^2 - 16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*
```

```
x + c))*C*a^2 + 80*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 - 240*A*a^2*sin(d*x + c)
)/d
```

Fricas [A] time = 1.39308, size = 258, normalized size = 1.58

$$\frac{15\,(4\,A+3\,C)a^2dx+\left(12\,Ca^2\cos\left(dx+c\right)^4+30\,Ca^2\cos\left(dx+c\right)^3+4\,(5\,A+9\,C)a^2\cos\left(dx+c\right)^2+15\,(4\,A+3\,C)a^2\cos\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*(4*A + 3*C)*a^2*d*x + (12*C*a^2*cos(d*x + c)^4 + 30*C*a^2*cos(d*x
+ c)^3 + 4*(5*A + 9*C)*a^2*cos(d*x + c)^2 + 15*(4*A + 3*C)*a^2*cos(d*x + c)
 + 4*(25*A + 18*C)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 3.09497, size = 350, normalized size = 2.15

$$\begin{cases} Aa^2x\sin^2\left(c+dx\right)+Aa^2x\cos^2\left(c+dx\right)+\frac{2Aa^2\sin^3\left(c+dx\right)}{3d}+\frac{Aa^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{Aa^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{Aa^2\sin\left(c+dx\right)}{d} \\ x\left(A+C\cos^2\left(c\right)\right)\left(a\cos\left(c\right)+a\right)^2\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2 + A*a**2*x*cos(c + d*x)**2 + 2*A*a**2*s
in(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c
 + d*x)*cos(c + d*x)/d + A*a**2*sin(c + d*x)/d + 3*C*a**2*x*sin(c + d*x)**4
/4 + 3*C*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*C*a**2*x*cos(c + d*x)
**4/4 + 8*C*a**2*sin(c + d*x)**5/(15*d) + 4*C*a**2*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + 3*C*a**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 2*C*a**2*sin(
c + d*x)**3/(3*d) + C*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*a**2*sin(c
+ d*x)*cos(c + d*x)**3/(4*d) + C*a**2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d,
 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)**2*cos(c), True))
```

Giac [A] time = 1.17389, size = 174, normalized size = 1.07

$$\frac{Ca^2 \sin(5\,dx + 5\,c)}{80\,d} + \frac{Ca^2 \sin(4\,dx + 4\,c)}{16\,d} + \frac{1}{4}\left(4\,Aa^2 + 3\,Ca^2\right)x + \frac{\left(4\,Aa^2 + 9\,Ca^2\right)\sin(3\,dx + 3\,c)}{48\,d} + \frac{\left(Aa^2 + Ca^2\right)\sin(2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/80*C*a^2*sin(5*d*x + 5*c)/d + 1/16*C*a^2*sin(4*d*x + 4*c)/d + 1/4*(4*A*a^
2 + 3*C*a^2)*x + 1/48*(4*A*a^2 + 9*C*a^2)*sin(3*d*x + 3*c)/d + 1/2*(A*a^2 +
 C*a^2)*sin(2*d*x + 2*c)/d + 1/8*(14*A*a^2 + 11*C*a^2)*sin(d*x + c)/d
```

3.11 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=123

$$\frac{a^2(12A+7C)\sin(c+dx)}{6d}+\frac{a^2(12A+7C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(12A+7C)+\frac{C\sin(c+dx)(a\cos(c+dx)+}{4ad}$$

```
[Out] (a^2*(12*A + 7*C)*x)/8 + (a^2*(12*A + 7*C)*Sin[c + d*x])/(6*d) + (a^2*(12*A
 + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) - (C*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(12*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.139384, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3024, 2751, 2644}

$$\frac{a^2(12A+7C)\sin(c+dx)}{6d}+\frac{a^2(12A+7C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(12A+7C)+\frac{C\sin(c+dx)(a\cos(c+dx)+}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(12*A + 7*C)*x)/8 + (a^2*(12*A + 7*C)*Sin[c + d*x])/(6*d) + (a^2*(12*A
 + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) - (C*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(12*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right) dx = \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{\int (a+a\cos(c+dx))^2(a(4A+3C)}{4a}$$
$$= -\frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{C(a+a\cos(c+dx))^3\sin(c+dx}{4ad}$$
$$= \frac{1}{8}a^2(12A+7C)x + \frac{a^2(12A+7C)\sin(c+dx)}{6d} + \frac{a^2(12A+7C)\cos(c+}{24d}$$

Mathematica [A] time = 0.24651, size = 73, normalized size = 0.59

$$\frac{a^2(48(4A+3C)\sin(c+dx)+24(A+2C)\sin(2(c+dx))+144Adx+16C\sin(3(c+dx))+3C\sin(4(c+dx))+84Cdx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(144*A*d*x + 84*C*d*x + 48*(4*A + 3*C)*Sin[c + d*x] + 24*(A + 2*C)*Sin
[2*(c + d*x)] + 16*C*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.024, size = 142, normalized size = 1.2

$$\frac{1}{d}\left(a^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Aa^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3
*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a^2*sin(d*x+c)+
```

```
A*a^2*(d*x+c))
```

Maxima [A] time = 1.12641, size = 178, normalized size = 1.45

$$\frac{24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^2+96\,(dx+c)Aa^2-64\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ca^2+3\,(12\,dx+12\,c+\sin(4}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 96*(d*x + c)*A*a^2 - 64*(
sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c
) + 8*sin(2*d*x + 2*c))*C*a^2 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 +
 192*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.43682, size = 207, normalized size = 1.68

$$\frac{3\,(12\,A+7\,C)a^2dx+\left(6\,Ca^2\cos\left(dx+c\right)^3+16\,Ca^2\cos\left(dx+c\right)^2+3\,(4\,A+7\,C)a^2\cos\left(dx+c\right)+16\,(3\,A+2\,C)a^2\right)\sin(}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(12*A + 7*C)*a^2*d*x + (6*C*a^2*cos(d*x + c)^3 + 16*C*a^2*cos(d*x +
 c)^2 + 3*(4*A + 7*C)*a^2*cos(d*x + c) + 16*(3*A + 2*C)*a^2)*sin(d*x + c))/
d
```

Sympy [A] time = 1.55956, size = 309, normalized size = 2.51

$$\begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2}+\frac{Aa^2x\cos^2(c+dx)}{2}+Aa^2x+\frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Aa^2\sin(c+dx)}{d}+\frac{3Ca^2x\sin^4(c+dx)}{8}+\frac{3Ca^2x\sin^2(c+dx)\cos^2(c+dx}{4} \\ x\left(A+C\cos^2(c)\right)(a\cos(c)+a)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*x + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*sin(c + d*x)/d + 3*C
*a**2*x*sin(c + d*x)**4/8 + 3*C*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
C*a**2*x*sin(c + d*x)**2/2 + 3*C*a**2*x*cos(c + d*x)**4/8 + C*a**2*x*cos(c
+ d*x)**2/2 + 3*C*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*C*a**2*sin(c
+ d*x)**3/(3*d) + 5*C*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*C*a**2*si
n(c + d*x)*cos(c + d*x)**2/d + C*a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d
, 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.15564, size = 139, normalized size = 1.13

$$\frac{Ca^2\sin(4\,dx+4\,c)}{32\,d}+\frac{Ca^2\sin(3\,dx+3\,c)}{6\,d}+\frac{1}{8}\left(12\,Aa^2+7\,Ca^2\right)x+\frac{\left(Aa^2+2\,Ca^2\right)\sin(2\,dx+2\,c)}{4\,d}+\frac{\left(4\,Aa^2+3\,Ca^2\right)\sin}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/32*C*a^2*sin(4*d*x + 4*c)/d + 1/6*C*a^2*sin(3*d*x + 3*c)/d + 1/8*(12*A*a^
2 + 7*C*a^2)*x + 1/4*(A*a^2 + 2*C*a^2)*sin(2*d*x + 2*c)/d + 1/2*(4*A*a^2 +
3*C*a^2)*sin(d*x + c)/d
```

3.12 $\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{a^2(A+C)\sin(c+dx)}{d}+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2A+C)+\frac{C\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}+\frac{C\sin(c+dx}{}$$

```
[Out] a^2*(2*A + C)*x + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Sin[c + d*
x])/d + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (C*(a^2 + a^2*Cos[c
 + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.298016, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3046, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^2(A+C)\sin(c+dx)}{d}+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2A+C)+\frac{C\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}+\frac{C\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] a^2*(2*A + C)*x + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Sin[c + d*
x])/d + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (C*(a^2 + a^2*Cos[c
 + d*x])*Sin[c + d*x])/(3*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{\int (a+a\cos(c+dx))^2}{}$$
$$= \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{C\left(a^2+a^2\cos(c+dx\right.}{3d}$$
$$= \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{C\left(a^2+a^2\cos(c+dx\right.}{3d}$$
$$= \frac{a^2(A+C)\sin(c+dx)}{d}+\frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d}$$
$$= a^2(2A+C)x+\frac{a^2(A+C)\sin(c+dx)}{d}+\frac{C(a+a\cos(c+}{3a}$$
$$= a^2(2A+C)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(A+C)\sin}{d}$$

Mathematica [A] time = 0.220093, size = 109, normalized size = 1.14

$$\frac{a^2\left(3(4A+7C)\sin(c+dx)-12A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a^2*(24*A*d*x + 12*C*d*x - 12*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(4*A + 7*C)*Sin[c + d*x]
 + 6*C*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.055, size = 128, normalized size = 1.3

$$\frac{Aa^2\sin(dx+c)}{d}+\frac{C\sin(dx+c)(\cos(dx+c))^2a^2}{3d}+\frac{5a^2C\sin(dx+c)}{3d}+2Aa^2x+2\frac{Aa^2c}{d}+\frac{a^2C\cos(dx+c)\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^2*sin(d*x+c)+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^2+5/3/d*a^2*C*sin(d*
x+c)+2*A*a^2*x+2/d*A*a^2*c+1/d*a^2*C*cos(d*x+c)*sin(d*x+c)+a^2*C*x+1/d*a^2*
```

```
C*c+1/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01176, size = 144, normalized size = 1.5

$$\frac{12\,(dx+c)Aa^2 - 2\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)Ca^2 + 3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ca^2 + 6\,Aa^2\log(\sec(dx+c) + \text{ta}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/6*(12*(d*x + c)*A*a^2 - 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 3*(2*
d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 6*A*a^2*log(sec(d*x + c) + tan(d*x +
c)) + 6*A*a^2*sin(d*x + c) + 6*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.45344, size = 236, normalized size = 2.46

$$\frac{6\,(2\,A+C)a^2dx + 3\,Aa^2\log(\sin(dx+c)+1) - 3\,Aa^2\log(-\sin(dx+c)+1) + 2\left(Ca^2\cos(dx+c)^2 + 3\,Ca^2\cos(dx+c)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] 1/6*(6*(2*A + C)*a^2*d*x + 3*A*a^2*log(sin(d*x + c) + 1) - 3*A*a^2*log(-sin
(d*x + c) + 1) + 2*(C*a^2*cos(d*x + c)^2 + 3*C*a^2*cos(d*x + c) + (3*A + 5*
C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

[Out] Timed out

Giac [A] time = 1.19238, size = 242, normalized size = 2.52

$$\frac{3\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(2\,Aa^2+Ca^2\right)(dx+c)+\frac{2\left(3\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+\cdots\right.}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*A*a^2 + C*a^2)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x
+ 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*tan(1/2*d*x + 1/2*c)^
3 + 8*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^2*tan(1/2*d*x + 1/2*c) + 9*C*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.13 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{a^2(2A-3C)\sin(c+dx)}{2d}-\frac{(2A-C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+\frac{2a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x(2A+3C)+$$

```
[Out] (a^2*(2*A + 3*C)*x)/2 + (2*a^2*A*ArcTanh[Sin[c + d*x]])/d - (a^2*(2*A - 3*C
)*Sin[c + d*x])/(2*d) - ((2*A - C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(
2*d) + (A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.39064, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2976, 2968, 3023, 2735, 3770}

$$-\frac{a^2(2A-3C)\sin(c+dx)}{2d}-\frac{(2A-C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+\frac{2a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x(2A+3C)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] (a^2*(2*A + 3*C)*x)/2 + (2*a^2*A*ArcTanh[Sin[c + d*x]])/d - (a^2*(2*A - 3*C
)*Sin[c + d*x])/(2*d) - ((2*A - C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(
2*d) + (A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\tan(c+dx)}{d} + \frac{\int (a+a\cos(c+dx))^2 \ldots}{\ldots}$$

$$= -\frac{(2A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{A(a+a\cos \ldots}{\ldots}$$

$$= -\frac{(2A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{A(a+a\cos \ldots}{\ldots}$$

$$= -\frac{a^2(2A-3C)\sin(c+dx)}{2d} - \frac{(2A-C)\left(a^2+a^2\cos(c+dx \ldots\right.}{2d}$$

$$= \frac{1}{2}a^2(2A+3C)x - \frac{a^2(2A-3C)\sin(c+dx)}{2d} - \frac{(2A-C)\left(a^2 \ldots\right.}{\ldots}$$

$$= \frac{1}{2}a^2(2A+3C)x + \frac{2a^2A\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^2(2A-3 \ldots}{\ldots}$$

Mathematica [A] time = 0.394473, size = 109, normalized size = 0.97

$$\frac{a^2\left(4A\tan(c+dx) - 8A\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 8A\log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right) + 4Ac + 4 \ldots\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*(4*A*c + 6*c*C + 4*A*d*x + 6*C*d*x - 8*A*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 8*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*C*Sin[c + d*
x] + C*Sin[2*(c + d*x)] + 4*A*Tan[c + d*x]))/(4*d)
```

Maple [A] time = 0.066, size = 107, normalized size = 1.

$$Aa^2x + \frac{Aa^2c}{d} + \frac{a^2C\cos(dx+c)\sin(dx+c)}{2d} + \frac{3a^2Cx}{2} + \frac{3a^2Cc}{2d} + 2\frac{Aa^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\frac{a^2C\sin(d \ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] A*a^2*x+1/d*A*a^2*c+1/2/d*a^2*C*cos(d*x+c)*sin(d*x+c)+3/2*a^2*C*x+3/2/d*a^2
*C*c+2/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*C*sin(d*x+c)+1/d*A*a^2*tan
```

```
(d*x+c)
```

Maxima [A] time = 1.05289, size = 136, normalized size = 1.21

$$\frac{4\,(dx+c)Aa^2+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Ca^2+4\,(dx+c)Ca^2+4\,Aa^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 4*(d*x +
c)*C*a^2 + 4*A*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*C*a^
2*sin(d*x + c) + 4*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.4864, size = 296, normalized size = 2.64

$$\frac{(2\,A+3\,C)a^2dx\cos{(dx+c)}+2\,Aa^2\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-2\,Aa^2\cos{(dx+c)}\log{(-\sin{(dx+c)}+1)}+\left(C\right.}{2\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] 1/2*((2*A + 3*C)*a^2*d*x*cos(d*x + c) + 2*A*a^2*cos(d*x + c)*log(sin(d*x +
c) + 1) - 2*A*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (C*a^2*cos(d*x + c)
^2 + 4*C*a^2*cos(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

[Out] Timed out

Giac [A] time = 1.21522, size = 193, normalized size = 1.72

$$\frac{4\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\left(2\,Aa^2+3\,Ca^2\right)(dx+c)+\frac{2\left(\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] 1/2*(4*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (2*A*a^2 + 3*C*a^2)*(d*x + c) + 2*(3*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*
C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.14 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{a^2(3A-2C)\sin(c+dx)}{2d}+\frac{a^2(3A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}+2a^2Cx+\frac{A\tan}{}$$

```
[Out] 2*a^2*C*x + (a^2*(3*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (a^2*(3*A - 2*C
)*Sin[c + d*x])/(2*d) + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d + (A*(a
 + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.358719, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2975, 2968, 3023, 2735, 3770}

$$-\frac{a^2(3A-2C)\sin(c+dx)}{2d}+\frac{a^2(3A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}+2a^2Cx+\frac{A\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] 2*a^2*C*x + (a^2*(3*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (a^2*(3*A - 2*C
)*Sin[c + d*x])/(2*d) + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d + (A*(a
 + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{\int (a+a}{}$$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d} + \frac{A(a+a\cos(c+d}{}$$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d} + \frac{A(a+a\cos(c+d}{}$$
$$= -\frac{a^2(3A-2C)\sin(c+dx)}{2d} + \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan}{d}$$
$$= 2a^2Cx - \frac{a^2(3A-2C)\sin(c+dx)}{2d} + \frac{A\left(a^2+a^2\cos(c+d\right.}{d}$$
$$= 2a^2Cx + \frac{a^2(3A+2C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^2(3A-2C}{}$$

Mathematica [B] time = 2.1381, size = 293, normalized size = 2.62

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2(3A+2C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{2(3A+2C)\log\left(\sin\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(8*C*x - (2*(3*A + 2*C)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(3*A + 2*C)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + (4*C*Cos[d*x]*Sin[c])/d + (4*C*Cos[c]*Sin[d*x])/d
 + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (8*A*Sin[(d*x)/2])/(d*(C
os[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - A/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^2) + (8*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/16
```

Maple [A] time = 0.071, size = 114, normalized size = 1.

$$\frac{3\,Aa^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + \frac{a^2C\sin(dx+c)}{d} + 2\,\frac{Aa^2\tan(dx+c)}{d} + 2\,a^2Cx + 2\,\frac{Ca^2c}{d} + \frac{Aa^2\sec(dx+c)\,\mathrm{ta}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 3/2/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*sin(d*x+c)+2/d*A*a^2*tan(d*
x+c)+2*a^2*C*x+2/d*a^2*C*c+1/2/d*A*a^2*sec(d*x+c)*tan(d*x+c)+1/d*a^2*C*ln(s
ec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.08465, size = 192, normalized size = 1.71

$$\frac{8\,(dx+c)Ca^2 - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Aa^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] 1/4*(8*(d*x + c)*C*a^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*C*a^2*sin(d*x + c) + 8*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.50356, size = 320, normalized size = 2.86

$$\frac{8\,Ca^2dx\cos\left(dx+c\right)^2 + (3\,A+2\,C)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - (3\,A+2\,C)a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] 1/4*(8*C*a^2*d*x*cos(d*x + c)^2 + (3*A + 2*C)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (3*A + 2*C)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
C*a^2*cos(d*x + c)^2 + 4*A*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23067, size = 205, normalized size = 1.83

$$\frac{4\,(dx+c)Ca^2 + \frac{4\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + \left(3\,Aa^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(3\,Aa^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] 1/2*(4*(d*x + c)*C*a^2 + 4*C*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)
^2 + 1) + (3*A*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (3*A*a^2
 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*tan(1/2*d*x + 1
/2*c)^3 - 5*A*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.15 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{a^2(A+C)\tan(c+dx)}{d}+\frac{a^2(A+2C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}+a^2Cx+$$

```
[Out] a^2*C*x + (a^2*(A + 2*C)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Tan[c + d*
x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (A*(
a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.353821, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2975, 2968, 3021, 2735, 3770}

$$\frac{a^2(A+C)\tan(c+dx)}{d}+\frac{a^2(A+2C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}+a^2Cx+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] a^2*C*x + (a^2*(A + 2*C)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Tan[c + d*
x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (A*(
a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{\int (a+a\,$$

$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{3d} + \frac{A(a+a}{}$$

$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{3d} + \frac{A(a+a}{}$$

$$= \frac{a^2(A+C)\tan(c+dx)}{d} + \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+}{3d}$$

$$= a^2Cx + \frac{a^2(A+C)\tan(c+dx)}{d} + \frac{A\left(a^2+a^2\cos(c+dx)\right)s}{3d}$$

$$= a^2Cx + \frac{a^2(A+2C)\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2(A+C)\tan(}{d}$$

Mathematica [B] time = 6.38516, size = 748, normalized size = 6.8

$$\frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(5A\sin\left(\frac{dx}{2}\right)+3C\sin\left(\frac{dx}{2}\right)\right)}{12d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(5A\sin\left(\frac{dx}{2}\right)+3C\sin\right.}{12d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (C*x*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4)/4 + ((-A - 2*C)*(a + a*Co
s[c + d*x])^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/
2]^4)/(4*d) + ((A + 2*C)*(a + a*Cos[c + d*x])^2*Log[Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4)/(4*d) + (A*(a + a*Cos[c + d*x])^2*S
ec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d
*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2
]^4*(7*A*Cos[c/2] - 5*A*Sin[c/2]))/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (
d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/
2]^4*(5*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(12*d*(Cos[c/2] - Sin[c/2])*(Co
s[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (A*(a + a*Cos[c + d*x])^2*Sec[c/2
 + (d*x)/2]^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-
7*A*Cos[c/2] - 5*A*Sin[c/2]))/(48*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*
(5*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2
 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.07, size = 134, normalized size = 1.2

$$\frac{5\,Aa^2\tan(dx+c)}{3\,d}+a^2Cx+\frac{Ca^2c}{d}+\frac{Aa^2\sec(dx+c)\tan(dx+c)}{d}+\frac{Aa^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\,\frac{a^2C\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 5/3/d*A*a^2*tan(d*x+c)+a^2*C*x+1/d*a^2*C*c+1/d*A*a^2*sec(d*x+c)*tan(d*x+c)+
1/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/3
/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+1/d*a^2*C*tan(d*x+c)
```

Maxima [A] time = 1.08735, size = 186, normalized size = 1.69

$$\frac{2\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2+6\,(dx+c)Ca^2-3\,Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 6*(d*x + c)*C*a^2 - 3*A*a^
2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 6*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*
A*a^2*tan(d*x + c) + 6*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.56567, size = 331, normalized size = 3.01

$$\frac{6\,Ca^2dx\cos(dx+c)^3+3\,(A+2\,C)a^2\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(A+2\,C)a^2\cos(dx+c)^3\log(-\sin(dx+c}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(6*C*a^2*d*x*cos(d*x + c)^3 + 3*(A + 2*C)*a^2*cos(d*x + c)^3*log(sin(d*
x + c) + 1) - 3*(A + 2*C)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*((5
*A + 3*C)*a^2*cos(d*x + c)^2 + 3*A*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22598, size = 252, normalized size = 2.29

$$\frac{3\,(dx+c)Ca^2 + 3\left(Aa^2 + 2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(Aa^2 + 2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(3\,Aa^2\tan\left(\frac{1}{2}\right.\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] 1/3*(3*(d*x + c)*C*a^2 + 3*(A*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 3*(A*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*
tan(1/2*d*x + 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*A*a^2*tan(1/2*d
*x + 1/2*c)^3 - 6*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 9*A*a^2*tan(1/2*d*x + 1/2*
c) + 3*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.16 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(2A+3C)\tan(c+dx)}{3d}+\frac{a^2(7A+12C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(11A+12C)\tan(c+dx)\sec(c+dx)}{24d}+\frac{A\tan(c+$$

```
[Out] (a^2*(7*A + 12*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*(2*A + 3*C)*Tan[c +
 d*x])/(3*d) + (a^2*(11*A + 12*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + (A*(a
^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (A*(a + a*Cos[c
 + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.449169, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3044, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{2a^2(2A+3C)\tan(c+dx)}{3d}+\frac{a^2(7A+12C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(11A+12C)\tan(c+dx)\sec(c+dx)}{24d}+\frac{A\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] (a^2*(7*A + 12*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*(2*A + 3*C)*Tan[c +
 d*x])/(3*d) + (a^2*(11*A + 12*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + (A*(a
^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (A*(a + a*Cos[c
 + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{\int(a+}{}$$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{A(a}{}$$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{A(a}{}$$
$$= \frac{a^2(11A+12C)\sec(c+dx)\tan(c+dx)}{24d} + \frac{A\left(a^2+a^2\cos\right.}{}$$
$$= \frac{a^2(11A+12C)\sec(c+dx)\tan(c+dx)}{24d} + \frac{A\left(a^2+a^2\cos\right.}{}$$
$$= \frac{a^2(7A+12C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(11A+12C)\sec}{2}$$
$$= \frac{a^2(7A+12C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2a^2(2A+3C)\tan(}{3d}$$

Mathematica [A] time = 1.13972, size = 262, normalized size = 1.78

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(24(7A+12C)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(7*A + 12*
C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-48*(2*A + 3*C)*Sin[c] + 3*(15*A + 4
*C)*Sin[d*x] + 45*A*Sin[2*c + d*x] + 12*C*Sin[2*c + d*x] + 128*A*Sin[c + 2*
d*x] + 144*C*Sin[c + 2*d*x] - 48*C*Sin[3*c + 2*d*x] + 21*A*Sin[2*c + 3*d*x]
 + 12*C*Sin[2*c + 3*d*x] + 21*A*Sin[4*c + 3*d*x] + 12*C*Sin[4*c + 3*d*x] +
32*A*Sin[3*c + 4*d*x] + 48*C*Sin[3*c + 4*d*x])))/(768*d)
```

Maple [A] time = 0.076, size = 166, normalized size = 1.1

$$\frac{7\,Aa^{2}\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}+\frac{7\,Aa^{2}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8\,d}+\frac{3\,a^{2}C\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2\,d}+\frac{4\,Aa^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 7/8/d*A*a^2*sec(d*x+c)*tan(d*x+c)+7/8/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+3/2
/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*A*a^2*tan(d*x+c)+2/3/d*A*a^2*tan(d
*x+c)*sec(d*x+c)^2+2/d*a^2*C*tan(d*x+c)+1/4/d*A*a^2*tan(d*x+c)*sec(d*x+c)^3
+1/2/d*a^2*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.19544, size = 316, normalized size = 2.15

$$\frac{32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{2}-3\,Aa^{2}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 3*A*a^2*(2*(3*sin(d*x +
c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*C*a^2*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 24*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*C
*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.46305, size = 356, normalized size = 2.42

$$\frac{3\left(7\,A+12\,C\right)a^{2}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(7\,A+12\,C\right)a^{2}\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(2\,A+\right.\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/48*(3*(7*A + 12*C)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*A + 12
*C)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(2*A + 3*C)*a^2*cos(d
*x + c)^3 + 3*(7*A + 4*C)*a^2*cos(d*x + c)^2 + 16*A*a^2*cos(d*x + c) + 6*A*
a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28254, size = 286, normalized size = 1.95

$$\frac{3\left(7\,Aa^2+12\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Aa^2+12\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 12*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*A*a^
2 + 12*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*a^2*tan(1/2*d*x
+ 1/2*c)^7 + 36*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*A*a^2*tan(1/2*d*x + 1/2*c
)^5 - 132*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 +
156*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*A*a^2*tan(1/2*d*x + 1/2*c) - 60*C*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.17 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{a^2(18A+25C)\tan(c+dx)}{15d}+\frac{a^2(3A+4C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2(9A+10C)\tan(c+dx)\sec^2(c+dx)}{30d}+\frac{a^2(3A+4C}{}$$

```
[Out] (a^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^2*(18*A + 25*C)*Tan[c +
d*x])/(15*d) + (a^2*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*(9*
A + 10*C)*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (A*(a^2 + a^2*Cos[c + d*x])
*Sec[c + d*x]^3*Tan[c + d*x])/(10*d) + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*
x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.4736, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3044, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^2(18A+25C)\tan(c+dx)}{15d}+\frac{a^2(3A+4C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2(9A+10C)\tan(c+dx)\sec^2(c+dx)}{30d}+\frac{a^2(3A+4C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (a^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^2*(18*A + 25*C)*Tan[c +
d*x])/(15*d) + (a^2*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*(9*
A + 10*C)*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (A*(a^2 + a^2*Cos[c + d*x])
*Sec[c + d*x]^3*Tan[c + d*x])/(10*d) + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*
x]^4*Tan[c + d*x])/(5*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{\int(a+a\,$$

$$=\frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{10d}+\frac{A(a+}{}$$

$$=\frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{10d}+\frac{A(a+}{}$$

$$=\frac{a^2(9A+10C)\sec^2(c+dx)\tan(c+dx)}{30d}+\frac{A\left(a^2+a^2\cos(c\right.}{}$$

$$=\frac{a^2(9A+10C)\sec^2(c+dx)\tan(c+dx)}{30d}+\frac{A\left(a^2+a^2\cos(c\right.}{}$$

$$=\frac{a^2(3A+4C)\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2(9A+10C)\sec^2}{3}$$

$$=\frac{a^2(3A+4C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2(18A+25C)\tan(c}{15d}$$

Mathematica [A] time = 1.39847, size = 292, normalized size = 1.64

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(3A+4C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^5*(240*(3*A + 4*
C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(40*(15*A + 16*C)*Sin[d*x] - 120*(A +
 3*C)*Sin[2*c + d*x] + 210*A*Sin[c + 2*d*x] + 120*C*Sin[c + 2*d*x] + 210*A*
Sin[3*c + 2*d*x] + 120*C*Sin[3*c + 2*d*x] + 360*A*Sin[2*c + 3*d*x] + 440*C*
Sin[2*c + 3*d*x] - 60*C*Sin[4*c + 3*d*x] + 45*A*Sin[3*c + 4*d*x] + 60*C*Sin
[3*c + 4*d*x] + 45*A*Sin[5*c + 4*d*x] + 60*C*Sin[5*c + 4*d*x] + 72*A*Sin[4*
c + 5*d*x] + 100*C*Sin[4*c + 5*d*x])))/(3840*d)
```

Maple [A] time = 0.095, size = 210, normalized size = 1.2

$$\frac{6\,Aa^2\tan(dx+c)}{5d}+\frac{3\,Aa^2\tan(dx+c)(\sec(dx+c))^2}{5d}+\frac{5\,a^2C\tan(dx+c)}{3d}+\frac{Aa^2\tan(dx+c)(\sec(dx+c))^3}{2d}+\frac{3\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 6/5/d*A*a^2*tan(d*x+c)+3/5/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+5/3/d*a^2*C*tan(
d*x+c)+1/2/d*A*a^2*tan(d*x+c)*sec(d*x+c)^3+3/4/d*A*a^2*sec(d*x+c)*tan(d*x+c
)+3/4/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/d
*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/5/d*A*a^2*tan(d*x+c)*sec(d*x+c)^4+1/3/d*
a^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.09912, size = 294, normalized size = 1.65

$$\frac{8\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^2+40\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2+40\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 + 4
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 40*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*C*a^2 - 15*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 60*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
```

```
) + log(sin(d*x + c) - 1)) + 120*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.52733, size = 409, normalized size = 2.3

$$\frac{15\,(3\,A+4\,C)a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(3\,A+4\,C)a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,(18\,A+\right.}{120\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(3*A + 4*C)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*A +
4*C)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(4*(18*A + 25*C)*a^2*cos
(d*x + c)^4 + 15*(3*A + 4*C)*a^2*cos(d*x + c)^3 + 4*(9*A + 5*C)*a^2*cos(d*x
 + c)^2 + 30*A*a^2*cos(d*x + c) + 12*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20525, size = 332, normalized size = 1.87

$$15\left(3\,Aa^2+4\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(3\,Aa^2+4\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] 1/60*(15*(3*A*a^2 + 4*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*a
^2 + 4*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a^2*tan(1/2*d*x
+ 1/2*c)^9 + 60*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 210*A*a^2*tan(1/2*d*x + 1/2*
c)^7 - 280*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 432*A*a^2*tan(1/2*d*x + 1/2*c)^5
+ 560*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 270*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 520
*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 195*A*a^2*tan(1/2*d*x + 1/2*c) + 180*C*a^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.18 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=237

$$-\frac{a^3(133A+108C)\sin^3(c+dx)}{105d}+\frac{a^3(133A+108C)\sin(c+dx)}{35d}+\frac{a^3(154A+129C)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{(A+C)}{}$$

```
[Out] (a^3*(26*A + 21*C)*x)/16 + (a^3*(133*A + 108*C)*Sin[c + d*x])/(35*d) + (a^3
*(26*A + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(154*A + 129*C)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^
3*Sin[c + d*x])/(7*d) + (C*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c
+ d*x])/(14*a*d) + ((A + C)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Sin[c +
 d*x])/(5*d) - (a^3*(133*A + 108*C)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.606788, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3046, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^3(133A+108C)\sin^3(c+dx)}{105d}+\frac{a^3(133A+108C)\sin(c+dx)}{35d}+\frac{a^3(154A+129C)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{(A+C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(26*A + 21*C)*x)/16 + (a^3*(133*A + 108*C)*Sin[c + d*x])/(35*d) + (a^3
*(26*A + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(154*A + 129*C)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^
3*Sin[c + d*x])/(7*d) + (C*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c
+ d*x])/(14*a*d) + ((A + C)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Sin[c +
 d*x])/(5*d) - (a^3*(133*A + 108*C)*Sin[c + d*x]^3)/(105*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{\int \cos^2(c$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{C\cos^3(c$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{C\cos^3(c$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{C\cos^3(c$$

$$= \frac{a^3(154A+129C)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{C\cos^3(c+d$$

$$= \frac{a^3(154A+129C)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{C\cos^3(c+d$$

$$= \frac{a^3(26A+21C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^3(154A+129C$$

$$= \frac{1}{16}a^3(26A+21C)x + \frac{a^3(133A+108C)\sin(c+dx)}{35d} + \frac{a^3(\cdots$$

Mathematica [A] time = 0.673984, size = 145, normalized size = 0.61

$$\frac{a^3(105(184A+155C)\sin(c+dx)+105(64A+61C)\sin(2(c+dx))+2380A\sin(3(c+dx))+630A\sin(4(c+dx))+84A\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(5460*c*C + 10920*A*d*x + 8820*C*d*x + 105*(184*A + 155*C)*Sin[c + d*x
] + 105*(64*A + 61*C)*Sin[2*(c + d*x)] + 2380*A*Sin[3*(c + d*x)] + 2835*C*S
```

```
in[3*(c + d*x)] + 630*A*Sin[4*(c + d*x)] + 1155*C*Sin[4*(c + d*x)] + 84*A*S
in[5*(c + d*x)] + 399*C*Sin[5*(c + d*x)] + 105*C*Sin[6*(c + d*x)] + 15*C*Si
n[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.053, size = 286, normalized size = 1.2

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+\frac{a^3C\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6\,(\cos(dx+c}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/7*a^3*C*(16
/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*a^3*C*(1/6*(cos(d
*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+A*a^3
*(2+cos(d*x+c)^2)*sin(d*x+c)+3/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*
sin(d*x+c)+A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*C*(1/4*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.0631, size = 383, normalized size = 1.62

$$\frac{448\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-6720\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3+630\,(12\,dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/6720*(448*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3
- 6720*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 630*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 1680*(2*d*x + 2*c + sin(2*d*x + 2
*c))*A*a^3 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3
- 35*sin(d*x + c))*C*a^3 + 1344*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*C*a^3 - 105*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d
*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^3 + 210*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*C*a^3)/d
```

Fricas [A] time = 1.51285, size = 382, normalized size = 1.61

$$\frac{105\,(26\,A+21\,C)a^3dx+\left(240\,Ca^3\cos\left(dx+c\right)^6+840\,Ca^3\cos\left(dx+c\right)^5+48\,(7\,A+27\,C)a^3\cos\left(dx+c\right)^4+210\,(6\,A+7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/1680*(105*(26*A + 21*C)*a^3*d*x + (240*C*a^3*cos(d*x + c)^6 + 840*C*a^3*c
os(d*x + c)^5 + 48*(7*A + 27*C)*a^3*cos(d*x + c)^4 + 210*(6*A + 7*C)*a^3*co
s(d*x + c)^3 + 16*(133*A + 108*C)*a^3*cos(d*x + c)^2 + 105*(26*A + 21*C)*a^
3*cos(d*x + c) + 32*(133*A + 108*C)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 10.449, size = 750, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((9*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 9*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 8*A*a**3*sin(c + d*x)**5/(15*d) + 4*A*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 2*A*a**3*sin(c + d*x)**3/d + A*a**3*sin(c + d*x)*cos(c + d*x)**4/d +
 15*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*A*a**3*sin(c + d*x)*cos(c
 + d*x)**2/d + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 15*C*a**3*x*sin(c +
 d*x)**6/16 + 45*C*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*C*a**3*x*s
in(c + d*x)**4/8 + 45*C*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*C*a**
3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 15*C*a**3*x*cos(c + d*x)**6/16 + 3*
C*a**3*x*cos(c + d*x)**4/8 + 16*C*a**3*sin(c + d*x)**7/(35*d) + 8*C*a**3*si
n(c + d*x)**5*cos(c + d*x)**2/(5*d) + 15*C*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + 8*C*a**3*sin(c + d*x)**5/(5*d) + 2*C*a**3*sin(c + d*x)**3*cos(c
+ d*x)**4/d + 5*C*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 4*C*a**3*sin
(c + d*x)**3*cos(c + d*x)**2/d + 3*C*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + C*a**3*sin(c + d*x)*cos(c + d*x)**6/d + 33*C*a**3*sin(c + d*x)*cos(c +
d*x)**5/(16*d) + 3*C*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*a**3*sin(c +
```

```
 d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)
**3*cos(c)**2, True))
```

Giac [A] time = 1.23773, size = 250, normalized size = 1.05

$$\frac{Ca^3\sin(7\,dx+7\,c)}{448\,d}+\frac{Ca^3\sin(6\,dx+6\,c)}{64\,d}+\frac{1}{16}\left(26\,Aa^3+21\,Ca^3\right)x+\frac{\left(4\,Aa^3+19\,Ca^3\right)\sin(5\,dx+5\,c)}{320\,d}+\frac{\left(6\,Aa^3+11\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/448*C*a^3*sin(7*d*x + 7*c)/d + 1/64*C*a^3*sin(6*d*x + 6*c)/d + 1/16*(26*A
*a^3 + 21*C*a^3)*x + 1/320*(4*A*a^3 + 19*C*a^3)*sin(5*d*x + 5*c)/d + 1/64*(
6*A*a^3 + 11*C*a^3)*sin(4*d*x + 4*c)/d + 1/192*(68*A*a^3 + 81*C*a^3)*sin(3*
d*x + 3*c)/d + 1/64*(64*A*a^3 + 61*C*a^3)*sin(2*d*x + 2*c)/d + 1/64*(184*A*
a^3 + 155*C*a^3)*sin(d*x + c)/d
```

3.19 $\int \cos(c+dx)(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=188

$$-\frac{a^3(30A+23C)\sin^3(c+dx)}{120d}+\frac{a^3(30A+23C)\sin(c+dx)}{10d}+\frac{3a^3(30A+23C)\sin(c+dx)\cos(c+dx)}{80d}+\frac{1}{16}a^3x(30A+2$$

```
[Out] (a^3*(30*A + 23*C)*x)/16 + (a^3*(30*A + 23*C)*Sin[c + d*x])/(10*d) + (3*a^3
*(30*A + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + ((30*A + 7*C)*(a + a*Cos
[c + d*x])^3*Sin[c + d*x])/(120*d) + (C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])
^3*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(10*a*d) -
 (a^3*(30*A + 23*C)*Sin[c + d*x]^3)/(120*d)
```

Rubi [A] time = 0.333739, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3046, 2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(30A+23C)\sin^3(c+dx)}{120d}+\frac{a^3(30A+23C)\sin(c+dx)}{10d}+\frac{3a^3(30A+23C)\sin(c+dx)\cos(c+dx)}{80d}+\frac{1}{16}a^3x(30A+2$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a^3*(30*A + 23*C)*x)/16 + (a^3*(30*A + 23*C)*Sin[c + d*x])/(10*d) + (3*a^3
*(30*A + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + ((30*A + 7*C)*(a + a*Cos
[c + d*x])^3*Sin[c + d*x])/(120*d) + (C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])
^3*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(10*a*d) -
 (a^3*(30*A + 23*C)*Sin[c + d*x]^3)/(120*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right) dx &= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{6d} + \frac{\int \cos(c+\\
&= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{6d} + \frac{\int (a+a\,c\\
&= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{6d} + \frac{C(a+a\,c\\
&= \frac{(30A+7C)(a+a\cos(c+dx))^3\sin(c+dx)}{120d} + \frac{C\cos^2(c+d\\
&= \frac{(30A+7C)(a+a\cos(c+dx))^3\sin(c+dx)}{120d} + \frac{C\cos^2(c+d\\
&= \frac{1}{40}a^3(30A+23C)x + \frac{(30A+7C)(a+a\cos(c+dx))^3\sin(\\
&= \frac{1}{40}a^3(30A+23C)x + \frac{3a^3(30A+23C)\sin(c+dx)}{40d} + \frac{3a^3(3\\
&= \frac{1}{16}a^3(30A+23C)x + \frac{a^3(30A+23C)\sin(c+dx)}{10d} + \frac{3a^3(30
\end{aligned}$$

Mathematica [A] time = 0.412154, size = 123, normalized size = 0.65

$$\frac{a^3(120(26A+21C)\sin(c+dx)+15(64A+63C)\sin(2(c+dx))+240A\sin(3(c+dx))+30A\sin(4(c+dx))+1800Adx}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(900*c*C + 1800*A*d*x + 1380*C*d*x + 120*(26*A + 21*C)*Sin[c + d*x] +
15*(64*A + 63*C)*Sin[2*(c + d*x)] + 240*A*Sin[3*(c + d*x)] + 380*C*Sin[3*(c
 + d*x)] + 30*A*Sin[4*(c + d*x)] + 135*C*Sin[4*(c + d*x)] + 36*C*Sin[5*(c +
 d*x)] + 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.027, size = 245, normalized size = 1.3

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3C\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3
*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c)+A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*
cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
3*a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^3*
sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.0501, size = 323, normalized size = 1.72

$$-\frac{960\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-30\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^3-720\left(2dx+2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/960*(960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 30*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c)
)*C*a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48
*sin(2*d*x + 2*c))*C*a^3 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 90
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 - 960*A*a^3*
sin(d*x + c))/d
```

Fricas [A] time = 1.43713, size = 321, normalized size = 1.71

$$\frac{15(30A+23C)a^3dx+\left(40Ca^3\cos(dx+c)^5+144Ca^3\cos(dx+c)^4+10(6A+23C)a^3\cos(dx+c)^3+16(15A+17\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(30*A + 23*C)*a^3*d*x + (40*C*a^3*cos(d*x + c)^5 + 144*C*a^3*cos(
d*x + c)^4 + 10*(6*A + 23*C)*a^3*cos(d*x + c)^3 + 16*(15*A + 17*C)*a^3*cos(
d*x + c)^2 + 15*(30*A + 23*C)*a^3*cos(d*x + c) + 16*(45*A + 34*C)*a^3)*sin(
d*x + c))/d
```

Sympy [A] time = 6.47182, size = 646, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 +
 3*A*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*A*a**3*sin(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**3*sin(c + d*x)*cos(c +
d*x)/(2*d) + A*a**3*sin(c + d*x)/d + 5*C*a**3*x*sin(c + d*x)**6/16 + 15*C*a
**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*C*a**3*x*sin(c + d*x)**4/8 + 1
5*C*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*C*a**3*x*sin(c + d*x)**2*
cos(c + d*x)**2/4 + 5*C*a**3*x*cos(c + d*x)**6/16 + 9*C*a**3*x*cos(c + d*x)
**4/8 + 5*C*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8*C*a**3*sin(c + d*x
)**5/(5*d) + 5*C*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 4*C*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/d + 9*C*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*C*a**3*sin(c + d*x)**3/(3*d) + 11*C*a**3*sin(c + d*x)*cos(c + d*x)**5/
(16*d) + 3*C*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*C*a**3*sin(c + d*x)*c
os(c + d*x)**3/(8*d) + C*a**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x
*(A + C*cos(c)**2)*(a*cos(c) + a)**3*cos(c), True))
```

Giac [A] time = 1.23903, size = 213, normalized size = 1.13

$$\frac{Ca^3\sin(6\,dx+6\,c)}{192\,d}+\frac{3\,Ca^3\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{16}\left(30\,Aa^3+23\,Ca^3\right)x+\frac{\left(2\,Aa^3+9\,Ca^3\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{\left(12\,Aa^3+19\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/192*C*a^3*sin(6*d*x + 6*c)/d + 3/80*C*a^3*sin(5*d*x + 5*c)/d + 1/16*(30*A
*a^3 + 23*C*a^3)*x + 1/64*(2*A*a^3 + 9*C*a^3)*sin(4*d*x + 4*c)/d + 1/48*(12
*A*a^3 + 19*C*a^3)*sin(3*d*x + 3*c)/d + 1/64*(64*A*a^3 + 63*C*a^3)*sin(2*d*
x + 2*c)/d + 1/8*(26*A*a^3 + 21*C*a^3)*sin(d*x + c)/d
```

3.20 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=148

$$-\frac{a^3(20A+13C)\sin^3(c+dx)}{60d}+\frac{a^3(20A+13C)\sin(c+dx)}{5d}+\frac{3a^3(20A+13C)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(20A+13$$

```
[Out] (a^3*(20*A + 13*C)*x)/8 + (a^3*(20*A + 13*C)*Sin[c + d*x])/(5*d) + (3*a^3*(
20*A + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) - (C*(a + a*Cos[c + d*x])^3*
Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a*d) - (a
^3*(20*A + 13*C)*Sin[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.198492, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3024, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(20A+13C)\sin^3(c+dx)}{60d}+\frac{a^3(20A+13C)\sin(c+dx)}{5d}+\frac{3a^3(20A+13C)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(20A+13$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] (a^3*(20*A + 13*C)*x)/8 + (a^3*(20*A + 13*C)*Sin[c + d*x])/(5*d) + (3*a^3*(
20*A + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) - (C*(a + a*Cos[c + d*x])^3*
Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a*d) - (a
^3*(20*A + 13*C)*Sin[c + d*x]^3)/(60*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx = \frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{5ad}+\frac{\int (a+a\cos(c+dx))^3(a(5A+4C)}{5a}$$
$$= -\frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{20d}+\frac{C(a+a\cos(c+dx))^4\sin(c+dx}{5ad}$$
$$= -\frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{20d}+\frac{C(a+a\cos(c+dx))^4\sin(c+dx}{5ad}$$
$$= \frac{1}{20}a^3(20A+13C)x-\frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{20d}+\frac{C(a+a\cos(}{}$$
$$= \frac{1}{20}a^3(20A+13C)x+\frac{3a^3(20A+13C)\sin(c+dx)}{20d}+\frac{3a^3(20A+13C)}{}$$
$$= \frac{1}{8}a^3(20A+13C)x+\frac{a^3(20A+13C)\sin(c+dx)}{5d}+\frac{3a^3(20A+13C)\cos}{40}$$

Mathematica [A] time = 0.364388, size = 97, normalized size = 0.66

$$\frac{a^3(60(30A+23C)\sin(c+dx)+120(3A+4C)\sin(2(c+dx))+40A\sin(3(c+dx))+1200Adx+170C\sin(3(c+dx))+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(1200*A*d*x + 780*C*d*x + 60*(30*A + 23*C)*Sin[c + d*x] + 120*(3*A + 4
*C)*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 170*C*Sin[3*(c + d*x)] + 45*
C*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.025, size = 197, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^3C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3a^3C\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*C*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*a^3*(2+cos(d*
x+c)^2)*sin(d*x+c)+a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/2*cos(d*x+c
```

```
)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
+3*A*a^3*sin(d*x+c)+A*a^3*(d*x+c))
```

Maxima [A] time = 1.02303, size = 257, normalized size = 1.74

$$-\frac{160\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}-360\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-480\left(dx+c\right)Aa^{3}-32\left(3\,\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 360*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a^3 - 480*(d*x + c)*A*a^3 - 32*(3*sin(d*x + c)^5 - 10*sin
(d*x + c)^3 + 15*sin(d*x + c))*C*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d*x + c)
)*C*a^3 - 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3
- 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 - 1440*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.42726, size = 266, normalized size = 1.8

$$\frac{15\left(20\,A+13\,C\right)a^{3}dx+\left(24\,Ca^{3}\cos\left(dx+c\right)^{4}+90\,Ca^{3}\cos\left(dx+c\right)^{3}+8\left(5\,A+19\,C\right)a^{3}\cos\left(dx+c\right)^{2}+15\left(12\,A+13\,C\right)\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(20*A + 13*C)*a^3*d*x + (24*C*a^3*cos(d*x + c)^4 + 90*C*a^3*cos(d
*x + c)^3 + 8*(5*A + 19*C)*a^3*cos(d*x + c)^2 + 15*(12*A + 13*C)*a^3*cos(d*
x + c) + 8*(55*A + 38*C)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 3.25116, size = 422, normalized size = 2.85

$$\begin{cases}\frac{3Aa^{3}x\sin^{2}\left(c+dx\right)}{2}+\frac{3Aa^{3}x\cos^{2}\left(c+dx\right)}{2}+Aa^{3}x+\frac{2Aa^{3}\sin^{3}\left(c+dx\right)}{3d}+\frac{Aa^{3}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}+\frac{3Aa^{3}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{3Aa^{3}\sin\left(c+dx\right)}{d} \\ x\left(A+C\cos^{2}\left(c\right)\right)\left(a\cos\left(c\right)+a\right)^{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**2/2 + A*
a**3*x + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*A*a**3*sin(c + d*x)/d +
 9*C*a**3*x*sin(c + d*x)**4/8 + 9*C*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + C*a**3*x*sin(c + d*x)**2/2 + 9*C*a**3*x*cos(c + d*x)**4/8 + C*a**3*x*co
s(c + d*x)**2/2 + 8*C*a**3*sin(c + d*x)**5/(15*d) + 4*C*a**3*sin(c + d*x)**
3*cos(c + d*x)**2/(3*d) + 9*C*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*C
*a**3*sin(c + d*x)**3/d + C*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*C*a**3
*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*C*a**3*sin(c + d*x)*cos(c + d*x)**2
/d + C*a**3*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + C*cos(c)**2
)*(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.22378, size = 177, normalized size = 1.2

$$\frac{Ca^3\sin(5\,dx+5\,c)}{80\,d}+\frac{3\,Ca^3\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(20\,Aa^3+13\,Ca^3\right)x+\frac{\left(4\,Aa^3+17\,Ca^3\right)\sin(3\,dx+3\,c)}{48\,d}+\frac{\left(3\,Aa^3+4\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/80*C*a^3*sin(5*d*x + 5*c)/d + 3/32*C*a^3*sin(4*d*x + 4*c)/d + 1/8*(20*A*a
^3 + 13*C*a^3)*x + 1/48*(4*A*a^3 + 17*C*a^3)*sin(3*d*x + 3*c)/d + 1/4*(3*A*
a^3 + 4*C*a^3)*sin(2*d*x + 2*c)/d + 1/8*(30*A*a^3 + 23*C*a^3)*sin(d*x + c)/
d
```

3.21 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=147

$$\frac{5a^3(4A+3C)\sin(c+dx)}{8d}+\frac{(4A+5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{8d}+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{8}a^3x(28A+15$$

```
[Out] (a^3*(28*A + 15*C)*x)/8 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(4*A + 3
*C)*Sin[c + d*x])/(8*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (
C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(4*a*d) + ((4*A + 5*C)*(a^3 + a^
3*Cos[c + d*x])*Sin[c + d*x])/(8*d)
```

Rubi [A] time = 0.438851, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3046, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^3(4A+3C)\sin(c+dx)}{8d}+\frac{(4A+5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{8d}+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{8}a^3x(28A+15$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a^3*(28*A + 15*C)*x)/8 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(4*A + 3
*C)*Sin[c + d*x])/(8*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (
C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(4*a*d) + ((4*A + 5*C)*(a^3 + a^
3*Cos[c + d*x])*Sin[c + d*x])/(8*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{\int (a+a\cos(c+dx))^3}{}$$

$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{C\left(a^2+a^2\cos(c+dx\right.}{4ad}$$

$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{C\left(a^2+a^2\cos(c+dx\right.}{4ad}$$

$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{C\left(a^2+a^2\cos(c+dx\right.}{4ad}$$

$$= \frac{5a^3(4A+3C)\sin(c+dx)}{8d} + \frac{C(a+a\cos(c+dx))^3\sin(c+}{4d}$$

$$= \frac{1}{8}a^3(28A+15C)x + \frac{5a^3(4A+3C)\sin(c+dx)}{8d} + \frac{C(a+a}{}$$

$$= \frac{1}{8}a^3(28A+15C)x + \frac{a^3A\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3(4A-}{}$$

Mathematica [A] time = 0.324389, size = 124, normalized size = 0.84

$$\frac{a^3\left(8(12A+13C)\sin(c+dx) + 8(A+4C)\sin(2(c+dx)) - 32A\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 32A\log\left(\sin\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a^3*(112*A*d*x + 60*C*d*x - 32*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 32*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*(12*A + 13*C)*Sin[c + d
*x] + 8*(A + 4*C)*Sin[2*(c + d*x)] + 8*C*Sin[3*(c + d*x)] + C*Sin[4*(c + d*
x)]))/(32*d)
```

Maple [A] time = 0.06, size = 175, normalized size = 1.2

$$\frac{Aa^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{7Aa^3x}{2} + \frac{7Aa^3c}{2d} + \frac{a^3C\sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{15a^3C\cos(dx+c)\sin(dx+c)}{8d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/2/d*A*a^3*cos(d*x+c)*sin(d*x+c)+7/2*A*a^3*x+7/2/d*A*a^3*c+1/4/d*a^3*C*sin
(d*x+c)*cos(d*x+c)^3+15/8/d*a^3*C*cos(d*x+c)*sin(d*x+c)+15/8*a^3*C*x+15/8/d
*a^3*C*c+3*a^3*A*sin(d*x+c)/d+1/d*C*sin(d*x+c)*cos(d*x+c)^2*a^3+3*a^3*C*sin
(d*x+c)/d+1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.03475, size = 220, normalized size = 1.5

$$8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3+96\,(dx+c)Aa^3-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^3+(12\,dx+12\,c+\sin(4\,dx\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 96*(d*x + c)*A*a^3 - 32*(s
in(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + (12*d*x + 12*c + sin(4*d*x + 4*c) +
 8*sin(2*d*x + 2*c))*C*a^3 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 32
*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 96*A*a^3*sin(d*x + c) + 32*C*a^3*
sin(d*x + c))/d
```

Fricas [A] time = 1.49299, size = 284, normalized size = 1.93

$$\frac{(28\,A+15\,C)a^3dx+4\,Aa^3\log(\sin(dx+c)+1)-4\,Aa^3\log(-\sin(dx+c)+1)+\left(2\,Ca^3\cos(dx+c)^3+8\,Ca^3\cos(dx+\cdots\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] 1/8*((28*A + 15*C)*a^3*d*x + 4*A*a^3*log(sin(d*x + c) + 1) - 4*A*a^3*log(-s
in(d*x + c) + 1) + (2*C*a^3*cos(d*x + c)^3 + 8*C*a^3*cos(d*x + c)^2 + (4*A
+ 15*C)*a^3*cos(d*x + c) + 24*(A + C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31912, size = 288, normalized size = 1.96

$$8\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(28\,Aa^3+15\,Ca^3\right)(dx+c)+\frac{2\left(20\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] 1/8*(8*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (28*A*a^3 + 15*C*a^3)*(d*x + c) + 2*(20*A*a^3*tan(1/2*d
*x + 1/2*c)^7 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 68*A*a^3*tan(1/2*d*x + 1/
2*c)^5 + 55*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 76*A*a^3*tan(1/2*d*x + 1/2*c)^3
+ 73*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 28*A*a^3*tan(1/2*d*x + 1/2*c) + 49*C*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.22 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=145

$$-\frac{(6A-5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}-\frac{(3A-C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3ad}+\frac{3a^3A\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a^3*(6*A + 5*C)*x)/2 + (3*a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*C*Sin[c
+ d*x])/(2*d) - ((3*A - C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(3*a*d)
 - ((6*A - 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d) + (A*(a + a*Co
s[c + d*x])^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.452336, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2976, 2968, 3023, 2735, 3770}

$$-\frac{(6A-5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}-\frac{(3A-C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3ad}+\frac{3a^3A\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] (a^3*(6*A + 5*C)*x)/2 + (3*a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*C*Sin[c
+ d*x])/(2*d) - ((3*A - C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(3*a*d)
 - ((6*A - 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d) + (A*(a + a*Co
s[c + d*x])^3*Tan[c + d*x])/d
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\tan(c+dx)}{d} + \frac{\int (a+a\cos(c+dx))^3}{}$$

$$= -\frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3ad} + \frac{A(a+a\,\mathrm{co}}{}$$

$$= -\frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3ad} - \frac{(6A-5C)}{}$$

$$= -\frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3ad} - \frac{(6A-5C)}{}$$

$$= \frac{5a^3C\sin(c+dx)}{2d} - \frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{3ad}$$

$$= \frac{1}{2}a^3(6A+5C)x + \frac{5a^3C\sin(c+dx)}{2d} - \frac{(3A-C)\left(a^2+a^2\,\mathrm{co}\right.}{}$$

$$= \frac{1}{2}a^3(6A+5C)x + \frac{3a^3A\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3C\sin(c}{2d}$$

Mathematica [B] time = 1.9216, size = 298, normalized size = 2.06

$$\frac{1}{96}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(4A+15C)\sin(c)\cos(dx)}{d} + \frac{3(4A+15C)\cos(c)\sin(dx)}{d} + \frac{}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(6*(6*A + 5*C)*x - (36*A*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (36*A*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]])/d + (3*(4*A + 15*C)*Cos[d*x]*Sin[c])/d + (9*C*Cos[2*d*x]*Sin[
2*c])/d + (C*Cos[3*d*x]*Sin[3*c])/d + (3*(4*A + 15*C)*Cos[c]*Sin[d*x])/d +
(9*C*Cos[2*c]*Sin[2*d*x])/d + (C*Cos[3*c]*Sin[3*d*x])/d + (12*A*Sin[(d*x)/2
])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (12*A*
Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))))/96
```

Maple [A] time = 0.07, size = 146, normalized size = 1.

$$\frac{Aa^3\sin(dx+c)}{d} + \frac{C(\cos(dx+c))^2\sin(dx+c)\,a^3}{3d} + \frac{11\,a^3C\sin(dx+c)}{3d} + 3\,Aa^3x + 3\,\frac{Aa^3c}{d} + \frac{3\,a^3C\cos(dx+c)\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] a^3*A*sin(d*x+c)/d+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^3+11/3*a^3*C*sin(d*x+c
)/d+3*A*a^3*x+3/d*A*a^3*c+3/2/d*a^3*C*cos(d*x+c)*sin(d*x+c)+5/2*a^3*C*x+5/2
/d*a^3*C*c+3/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^3*tan(d*x+c)
```

Maxima [A] time = 1.04451, size = 185, normalized size = 1.28

$$\frac{36\,(dx+c)Aa^3 - 4\left(\sin\,(dx+c)^3 - 3\,\sin\,(dx+c)\right)Ca^3 + 9\,(2\,dx + 2\,c + \sin\,(2\,dx + 2\,c))Ca^3 + 12\,(dx+c)Ca^3 + 18\,Aa^3(}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] 1/12*(36*(d*x + c)*A*a^3 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 9*(2
*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 12*(d*x + c)*C*a^3 + 18*A*a^3*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^3*sin(d*x + c) + 36*C*a^
3*sin(d*x + c) + 12*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.56, size = 350, normalized size = 2.41

$$\frac{3\,(6\,A + 5\,C)a^3dx\cos\,(dx+c) + 9\,Aa^3\cos\,(dx+c)\log\,(\sin\,(dx+c)+1) - 9\,Aa^3\cos\,(dx+c)\log\,(-\sin\,(dx+c)+1) + (}{6\,d\cos\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(6*A + 5*C)*a^3*d*x*cos(d*x + c) + 9*A*a^3*cos(d*x + c)*log(sin(d*x
+ c) + 1) - 9*A*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*C*a^3*cos(d*x
+ c)^3 + 9*C*a^3*cos(d*x + c)^2 + 2*(3*A + 11*C)*a^3*cos(d*x + c) + 6*A*a^3
)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25744, size = 284, normalized size = 1.96

$$\frac{18\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+3\left(6\,Aa^3+5\,Ca^3\right)(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] 1/6*(18*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*A*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 - 1) + 3*(6*A*a^3 + 5*C*a^3)*(d*x + c) + 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5
 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*C
*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 33*C*a^3*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.23 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=160

$$-\frac{5a^3(A-C)\sin(c+dx)}{2d}+\frac{a^3(7A+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(4A-C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{3A\tan(c}{}$$

```
[Out] (a^3*(2*A + 7*C)*x)/2 + (a^3*(7*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*
a^3*(A - C)*Sin[c + d*x])/(2*d) - ((4*A - C)*(a^3 + a^3*Cos[c + d*x])*Sin[c
 + d*x])/(2*d) + (3*A*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/(2*a*d) + (A
*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.480221, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3044, 2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^3(A-C)\sin(c+dx)}{2d}+\frac{a^3(7A+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(4A-C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{3A\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```



```
[Out] (a^3*(2*A + 7*C)*x)/2 + (a^3*(7*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*
a^3*(A - C)*Sin[c + d*x])/(2*d) - ((4*A - C)*(a^3 + a^3*Cos[c + d*x])*Sin[c
 + d*x])/(2*d) + (3*A*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/(2*a*d) + (A
*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{\int (a+a}{}$$

$$= \frac{3A\left(a^2+a^2\cos(c+dx)\right)^2\tan(c+dx)}{2ad} + \frac{A(a+a\cos(c+}{}$$

$$= -\frac{(4A-C)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{3A\left(a^2+\right.}{}$$

$$= -\frac{(4A-C)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{3A\left(a^2+\right.}{}$$

$$= -\frac{5a^3(A-C)\sin(c+dx)}{2d} - \frac{(4A-C)\left(a^3+a^3\cos(c+dx\right.}{2d}$$

$$= \frac{1}{2}a^3(2A+7C)x - \frac{5a^3(A-C)\sin(c+dx)}{2d} - \frac{(4A-C)\left(a^3\right.}{}$$

$$= \frac{1}{2}a^3(2A+7C)x + \frac{a^3(7A+2C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a}{}$$

Mathematica [A] time = 1.98469, size = 214, normalized size = 1.34

$$a^3\left(12A\tan(c+dx) + \frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{A}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 14A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (a^3*(4*A*c + 14*c*C + 4*A*d*x + 14*C*d*x - 14*A*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 4*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 14*A*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2 + 12*C*Sin[c + d*x] + C*Sin[2*(c + d*x)] + 12*A*Tan[c +
```

```
d*x]))/(4*d)
```

Maple [A] time = 0.075, size = 151, normalized size = 0.9

$$Aa^3x+\frac{Aa^3c}{d}+\frac{a^3C\cos(dx+c)\sin(dx+c)}{2d}+\frac{7a^3Cx}{2}+\frac{7a^3Cc}{2d}+\frac{7Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3\frac{a^3C\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] A*a^3*x+1/d*A*a^3*c+1/2/d*a^3*C*cos(d*x+c)*sin(d*x+c)+7/2*a^3*C*x+7/2/d*a^3
*C*c+7/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*sin(d*x+c)/d+3/d*A*a^3*t
an(d*x+c)+1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x
+c))
```

Maxima [A] time = 1.0431, size = 236, normalized size = 1.48

$$\frac{4(dx+c)Aa^3+(2dx+2c+\sin(2dx+2c))Ca^3+12(dx+c)Ca^3-Aa^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 12*(d*x +
 c)*C*a^3 - A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 2*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*
C*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.71711, size = 365, normalized size = 2.28

$$\frac{2(2A+7C)a^3dx\cos(dx+c)^2+(7A+2C)a^3\cos(dx+c)^2\log(\sin(dx+c)+1)-(7A+2C)a^3\cos(dx+c)^2\log(-\sin}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] 1/4*(2*(2*A + 7*C)*a^3*d*x*cos(d*x + c)^2 + (7*A + 2*C)*a^3*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (7*A + 2*C)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) +
1) + 2*(C*a^3*cos(d*x + c)^3 + 6*C*a^3*cos(d*x + c)^2 + 6*A*a^3*cos(d*x + c
) + A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28877, size = 311, normalized size = 1.94

$$\frac{\left(2\,Aa^{3}+7\,Ca^{3}\right)(dx+c)+\left(7\,Aa^{3}+2\,Ca^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(7\,Aa^{3}+2\,Ca^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] 1/2*((2*A*a^3 + 7*C*a^3)*(d*x + c) + (7*A*a^3 + 2*C*a^3)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) - (7*A*a^3 + 2*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 3*A*a
^3*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^3*tan(1/
2*d*x + 1/2*c)^3 + 9*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^3*tan(1/2*d*x + 1
/2*c) - 7*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.24 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{a^3(5A+6C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3C)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{3d}-\frac{5a^3A\sin(c+dx)}{2d}+\frac{A\tan(c+dx)\text{se}}{}$$

```
[Out] 3*a^3*C*x + (a^3*(5*A + 6*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*A*Sin[c
+ d*x])/(2*d) + ((5*A + 3*C)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/(3*d) +
 (A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + (A*(a +
 a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.499718, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2975, 2968, 3023, 2735, 3770}

$$\frac{a^3(5A+6C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3C)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{3d}-\frac{5a^3A\sin(c+dx)}{2d}+\frac{A\tan(c+dx)\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] 3*a^3*C*x + (a^3*(5*A + 6*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*A*Sin[c
+ d*x])/(2*d) + ((5*A + 3*C)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/(3*d) +
 (A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + (A*(a +
 a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{\int(a+a}{}$$

$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx)\tan(c+dx)}{2ad} + \frac{A(a+}{}$$

$$= \frac{(5A+3C)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{3d} + \frac{A\left(a^2+a^2\right.}{}$$

$$= \frac{(5A+3C)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{3d} + \frac{A\left(a^2+a^2\right.}{}$$

$$= -\frac{5a^3A\sin(c+dx)}{2d} + \frac{(5A+3C)\left(a^3+a^3\cos(c+dx)\right)\tan}{3d}$$

$$= 3a^3Cx - \frac{5a^3A\sin(c+dx)}{2d} + \frac{(5A+3C)\left(a^3+a^3\cos(c+d\right.}{3d}$$

$$= 3a^3Cx + \frac{a^3(5A+6C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^3A\sin(c+}{2d}$$

Mathematica [B] time = 6.34054, size = 832, normalized size = 5.33

$$\frac{3}{8}Cx(\cos(c+dx)a+a)^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{(-5A-6C)(\cos(c+dx)a+a)^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (3*C*x*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/8 + ((-5*A - 6*C)*(a +
a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d
*x)/2]^6)/(16*d) + ((5*A + 6*C)*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(16*d) + (C*Cos[d*x]*(a + a*
Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[c])/(8*d) + (C*Cos[c]*(a + a*Cos[c
 + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[d*x])/(8*d) + (A*(a + a*Cos[c + d*x])^3
*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
(d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)
/2]^6*(5*A*Cos[c/2] - 4*A*Sin[c/2]))/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x
)/2]^6*(11*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(24*d*(Cos[c/2] - Sin[c/2])*
(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (A*(a + a*Cos[c + d*x])^3*Sec[
c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)
```

```
/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6
*(-5*A*Cos[c/2] - 4*A*Sin[c/2]))/(48*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]
^6*(11*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(24*d*(Cos[c/2] + Sin[c/2])*(Cos
[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.077, size = 152, normalized size = 1.

$$\frac{5\,Aa^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{a^3C\sin\left(dx+c\right)}{d}+\frac{11\,Aa^3\tan\left(dx+c\right)}{3d}+3\,a^3Cx+3\,\frac{Ca^3c}{d}+\frac{3\,Aa^3\sec\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*C*sin(d*x+c)/d+11/3/d*A*a^3*tan(d
*x+c)+3*a^3*C*x+3/d*a^3*C*c+3/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+3/d*a^3*C*ln(
sec(d*x+c)+tan(d*x+c))+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/d*a^3*C*tan(d*
x+c)
```

Maxima [A] time = 1.04456, size = 239, normalized size = 1.53

$$4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+36\left(dx+c\right)Ca^3-9\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 36*(d*x + c)*C*a^3 - 9*A*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
18*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*C*a^3*sin(d*x
 + c) + 36*A*a^3*tan(d*x + c) + 12*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.49857, size = 379, normalized size = 2.43

$$\frac{36\,Ca^3dx\cos\left(dx+c\right)^3+3\,(5\,A+6\,C)a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A+6\,C)a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx\right.\right.}{12\,d\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] 1/12*(36*C*a^3*d*x*cos(d*x + c)^3 + 3*(5*A + 6*C)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(5*A + 6*C)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(6*C*a^3*cos(d*x + c)^3 + 2*(11*A + 3*C)*a^3*cos(d*x + c)^2 + 9*A*a^3*co
s(d*x + c) + 2*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2188, size = 296, normalized size = 1.9

$$\frac{18\,(dx+c)Ca^3+\frac{12\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+3\left(5\,Aa^3+6\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(5\,Aa^3+6\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] 1/6*(18*(d*x + c)*C*a^3 + 12*C*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*
c)^2 + 1) + 3*(5*A*a^3 + 6*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(5
```

```
*A*a^3 + 6*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A*a^3*tan(1/2*
d*x + 1/2*c)^5 + 6*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^3*tan(1/2*d*x + 1/
2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d*x + 1/2*c) +
6*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.25 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{5a^3(3A+4C)\tan(c+dx)}{8d}+\frac{a^3(15A+28C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(5A+4C)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+\right.}{8d}$$

```
[Out] a^3*C*x + (a^3*(15*A + 28*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(3*A + 4
*C)*Tan[c + d*x])/(8*d) + ((5*A + 4*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x
]*Tan[c + d*x])/(8*d) + (A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c
+ d*x])/(4*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d)
```

Rubi [A] time = 0.496396, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2975, 2968, 3021, 2735, 3770}

$$\frac{5a^3(3A+4C)\tan(c+dx)}{8d}+\frac{a^3(15A+28C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(5A+4C)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] a^3*C*x + (a^3*(15*A + 28*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(3*A + 4
*C)*Tan[c + d*x])/(8*d) + ((5*A + 4*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x
]*Tan[c + d*x])/(8*d) + (A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c
+ d*x])/(4*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3 \left(A + C\cos^2(c + dx)\right)\sec^5(c + dx)\,dx = \frac{A(a + a\cos(c + dx))^3\sec^3(c + dx)\tan(c + dx)}{4d} + \frac{\int (a + a}{}$$

$$= \frac{A\left(a^2 + a^2\cos(c + dx)\right)^2\sec^2(c + dx)\tan(c + dx)}{4ad} + \frac{A(a +}{}$$

$$= \frac{(5A + 4C)\left(a^3 + a^3\cos(c + dx)\right)\sec(c + dx)\tan(c + dx)}{8d} +$$

$$= \frac{(5A + 4C)\left(a^3 + a^3\cos(c + dx)\right)\sec(c + dx)\tan(c + dx)}{8d} +$$

$$= \frac{5a^3(3A + 4C)\tan(c + dx)}{8d} + \frac{(5A + 4C)\left(a^3 + a^3\cos(c + d\right.}{8a}$$

$$= a^3Cx + \frac{5a^3(3A + 4C)\tan(c + dx)}{8d} + \frac{(5A + 4C)\left(a^3 + a^3 c\right.}{}$$

$$= a^3Cx + \frac{a^3(15A + 28C)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{5a^3(3A + 4}{}$$

Mathematica [A] time = 1.42177, size = 334, normalized size = 1.98

$$\frac{a^3(\cos(c + dx) + 1)^3\sec^6\left(\frac{1}{2}(c + dx)\right)\sec^4(c + dx)\left(\sec(c)(23A\sin(2c + dx) + 88A\sin(c + 2dx) - 8A\sin(3c + 2dx) + 15A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(-8*(15*A + 28*
C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(24*C*d*x*Cos[c] + 16*C*d*x*Cos[c + 2
*d*x] + 16*C*d*x*Cos[3*c + 2*d*x] + 4*C*d*x*Cos[3*c + 4*d*x] + 4*C*d*x*Cos[
5*c + 4*d*x] - 72*A*Sin[c] - 72*C*Sin[c] + 23*A*Sin[d*x] + 4*C*Sin[d*x] + 2
3*A*Sin[2*c + d*x] + 4*C*Sin[2*c + d*x] + 88*A*Sin[c + 2*d*x] + 72*C*Sin[c
+ 2*d*x] - 8*A*Sin[3*c + 2*d*x] - 24*C*Sin[3*c + 2*d*x] + 15*A*Sin[2*c + 3*
d*x] + 4*C*Sin[2*c + 3*d*x] + 15*A*Sin[4*c + 3*d*x] + 4*C*Sin[4*c + 3*d*x]
+ 24*A*Sin[3*c + 4*d*x] + 24*C*Sin[3*c + 4*d*x])))/(512*d)
```

Maple [A] time = 0.078, size = 180, normalized size = 1.1

$$3\,\frac{Aa^3\tan(dx + c)}{d} + a^3Cx + \frac{Ca^3c}{d} + \frac{15\,Aa^3\sec(dx + c)\tan(dx + c)}{8\,d} + \frac{15\,Aa^3\ln(\sec(dx + c) + \tan(dx + c))}{8\,d} + \frac{7\,a^3C\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 3/d*A*a^3*tan(d*x+c)+a^3*C*x+1/d*a^3*C*c+15/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)
+15/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c
))+1/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/d*a^3*C*tan(d*x+c)+1/4/d*A*a^3*tan(d
*x+c)*sec(d*x+c)^3+1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.01213, size = 347, normalized size = 2.05

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+16(dx+c)Ca^3-Aa^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\ \mathrm{lo}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(d*x + c)*C*a^3 - A*a
^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^
2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a^3*(2*s
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) - 4*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) + 24*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 16*A*a^3*tan(d*x + c) + 48*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.46781, size = 386, normalized size = 2.28

$$\frac{16\,Ca^3dx\cos(dx+c)^4+(15\,A+28\,C)a^3\cos(dx+c)^4\log(\sin(dx+c)+1)-(15\,A+28\,C)a^3\cos(dx+c)^4\log(-\sin(\cdots}{16\,d\cos(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/16*(16*C*a^3*d*x*cos(d*x + c)^4 + (15*A + 28*C)*a^3*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - (15*A + 28*C)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) +
```

```
 2*(24*(A + C)*a^3*cos(d*x + c)^3 + (15*A + 4*C)*a^3*cos(d*x + c)^2 + 8*A*a
^3*cos(d*x + c) + 2*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2644, size = 300, normalized size = 1.78

$$8\,(dx+c)Ca^3+\left(15\,Aa^3+28\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(15\,Aa^3+28\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] 1/8*(8*(d*x + c)*C*a^3 + (15*A*a^3 + 28*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - (15*A*a^3 + 28*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^7 + 20*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*A*a^3*ta
n(1/2*d*x + 1/2*c)^5 - 68*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 73*A*a^3*tan(1/2*d
*x + 1/2*c)^3 + 76*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*A*a^3*tan(1/2*d*x + 1/
2*c) - 28*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.26 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=194

$$\frac{a^3(38A+55C)\tan(c+dx)}{15d}+\frac{a^3(13A+20C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(109A+140C)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(11A$$

```
[Out] (a^3*(13*A + 20*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 55*C)*Tan[c
+ d*x])/(15*d) + (a^3*(109*A + 140*C)*Sec[c + d*x]*Tan[c + d*x])/(120*d) +
((11*A + 10*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(30*d)
 + (3*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*a*d) +
(A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.57228, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3044, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^3(38A+55C)\tan(c+dx)}{15d}+\frac{a^3(13A+20C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(109A+140C)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(11A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] (a^3*(13*A + 20*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 55*C)*Tan[c
+ d*x])/(15*d) + (a^3*(109*A + 140*C)*Sec[c + d*x]*Tan[c + d*x])/(120*d) +
((11*A + 10*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(30*d)
 + (3*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*a*d) +
(A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{\int (a+}{}$$

$$= \frac{3A\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c+dx)\tan(c+dx)}{20ad} + \frac{A(}{}$$

$$= \frac{(11A+10C)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+d}{30d}$$

$$= \frac{(11A+10C)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+d}{30d}$$

$$= \frac{a^3(109A+140C)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(11A+10C}{}$$

$$= \frac{a^3(109A+140C)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(11A+10C}{}$$

$$= \frac{a^3(13A+20C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(109A+140C)}{}$$

$$= \frac{a^3(13A+20C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(38A+55C)\,\mathrm{ta}}{15d}$$

Mathematica [A] time = 1.41525, size = 294, normalized size = 1.52

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(13A+20C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(13*A + 2
0*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(29*A + 34*C)*Sin[d*x] - 240*(3
*A + 7*C)*Sin[2*c + d*x] + 750*A*Sin[c + 2*d*x] + 360*C*Sin[c + 2*d*x] + 75
0*A*Sin[3*c + 2*d*x] + 360*C*Sin[3*c + 2*d*x] + 1520*A*Sin[2*c + 3*d*x] + 1
840*C*Sin[2*c + 3*d*x] - 360*C*Sin[4*c + 3*d*x] + 195*A*Sin[3*c + 4*d*x] +
180*C*Sin[3*c + 4*d*x] + 195*A*Sin[5*c + 4*d*x] + 180*C*Sin[5*c + 4*d*x] +
304*A*Sin[4*c + 5*d*x] + 440*C*Sin[4*c + 5*d*x])))/(15360*d)
```

Maple [A] time = 0.08, size = 212, normalized size = 1.1

$$\frac{13\,Aa^3\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{13\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{5\,a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{38\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 13/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+5
/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+38/15/d*A*a^3*tan(d*x+c)+19/15/d*A*a^3
*tan(d*x+c)*sec(d*x+c)^2+11/3/d*a^3*C*tan(d*x+c)+3/4/d*A*a^3*tan(d*x+c)*sec
(d*x+c)^3+3/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+
c)^4+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.07989, size = 394, normalized size = 2.03

$$\frac{16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^3+240\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3+80\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 80*(tan(d*x + c)^3 + 3*tan(d*
x + c))*C*a^3 - 45*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 60*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
```

```
 1) + log(sin(d*x + c) - 1)) - 180*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 -
1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*C*a^3*(log(sin(d*
x + c) + 1) - log(sin(d*x + c) - 1)) + 720*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.50459, size = 419, normalized size = 2.16

$$\frac{15\,(13\,A+20\,C)a^3\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(13\,A+20\,C)a^3\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(13*A + 20*C)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(13*A
 + 20*C)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38*A + 55*C)*a^3
*cos(d*x + c)^4 + 15*(13*A + 12*C)*a^3*cos(d*x + c)^3 + 8*(19*A + 5*C)*a^3*
cos(d*x + c)^2 + 90*A*a^3*cos(d*x + c) + 24*A*a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27092, size = 332, normalized size = 1.71

$$15\left(13\,Aa^3+20\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(13\,Aa^3+20\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 20*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(13
*A*a^3 + 20*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 300*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1400*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 2560*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 2120*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c)
+ 660*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.27 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=225

$$\frac{a^3(34A+45C)\tan(c+dx)}{15d}+\frac{a^3(23A+30C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(73A+90C)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{a^3(23}{}$$

```
[Out] (a^3*(23*A + 30*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(34*A + 45*C)*Tan[c
 + d*x])/(15*d) + (a^3*(23*A + 30*C)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a
^3*(73*A + 90*C)*Sec[c + d*x]^2*Tan[c + d*x])/(120*d) + ((31*A + 30*C)*(a^3
 + a^3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (A*(a^2 + a^2*C
os[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*a*d) + (A*(a + a*Cos[c + d*
x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.617659, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3044, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^3(34A+45C)\tan(c+dx)}{15d}+\frac{a^3(23A+30C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(73A+90C)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{a^3(23}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] (a^3*(23*A + 30*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(34*A + 45*C)*Tan[c
 + d*x])/(15*d) + (a^3*(23*A + 30*C)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a
^3*(73*A + 90*C)*Sec[c + d*x]^2*Tan[c + d*x])/(120*d) + ((31*A + 30*C)*(a^3
 + a^3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (A*(a^2 + a^2*C
os[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*a*d) + (A*(a + a*Cos[c + d*
x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
```

```
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{\int (a+}{}$$

$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)^2\sec^4(c+dx)\tan(c+dx)}{10ad} + \frac{A(a}{}$$

$$= \frac{(31A+30C)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c+}{120d}$$

$$= \frac{(31A+30C)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c+}{120d}$$

$$= \frac{a^3(73A+90C)\sec^2(c+dx)\tan(c+dx)}{120d} + \frac{(31A+30C)}{}$$

$$= \frac{a^3(73A+90C)\sec^2(c+dx)\tan(c+dx)}{120d} + \frac{(31A+30C)}{}$$

$$= \frac{a^3(23A+30C)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a^3(73A+90C}{}$$

$$= \frac{a^3(23A+30C)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^3(34A+45C)\,\mathrm{ta}}{15d}$$

Mathematica [A] time = 1.94335, size = 358, normalized size = 1.59

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(480(23A+30C)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(480*(23*A + 3
0*C)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(34*A + 45*C)*Sin[c] + 30*(75
*A + 38*C)*Sin[d*x] + 2250*A*Sin[2*c + d*x] + 1140*C*Sin[2*c + d*x] + 7680*
A*Sin[c + 2*d*x] + 8160*C*Sin[c + 2*d*x] - 480*A*Sin[3*c + 2*d*x] - 2640*C*
Sin[3*c + 2*d*x] + 1955*A*Sin[2*c + 3*d*x] + 1590*C*Sin[2*c + 3*d*x] + 1955
*A*Sin[4*c + 3*d*x] + 1590*C*Sin[4*c + 3*d*x] + 3264*A*Sin[3*c + 4*d*x] + 4
080*C*Sin[3*c + 4*d*x] - 240*C*Sin[5*c + 4*d*x] + 345*A*Sin[4*c + 5*d*x] +
450*C*Sin[4*c + 5*d*x] + 345*A*Sin[6*c + 5*d*x] + 450*C*Sin[6*c + 5*d*x] +
544*A*Sin[5*c + 6*d*x] + 720*C*Sin[5*c + 6*d*x])))/(61440*d)
```

Maple [A] time = 0.114, size = 257, normalized size = 1.1

$$\frac{34\,Aa^3\tan(dx+c)}{15\,d}+\frac{17\,Aa^3\tan(dx+c)(\sec(dx+c))^2}{15\,d}+3\,\frac{a^3C\tan(dx+c)}{d}+\frac{23\,Aa^3\tan(dx+c)(\sec(dx+c))^3}{24\,d}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 34/15/d*A*a^3*tan(d*x+c)+17/15/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/d*a^3*C*ta
n(d*x+c)+23/24/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+23/16/d*A*a^3*sec(d*x+c)*tan
(d*x+c)+23/16/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+15/8/d*a^3*C*sec(d*x+c)*tan
(d*x+c)+15/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/5/d*A*a^3*tan(d*x+c)*sec(d
*x+c)^4+1/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+1/6/d*A*a^3*tan(d*x+c)*sec(d*x+c)
^5+1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 1.00758, size = 516, normalized size = 2.29

$$\frac{96\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^3+160\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3+480\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
maxima")
```

```
[Out] 1/480*(96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
160*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 480*(tan(d*x + c)^3 + 3*tan(d
*x + c))*C*a^3 - 5*A*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin
(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 90*A*a^3*(2*(3*sin(d*x
+ c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*C*a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) - 360*C*a^3*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*C*a^3*tan(
d*x + c))/d
```

Fricas [A] time = 1.50272, size = 474, normalized size = 2.11

$$\frac{15\,(23\,A+30\,C)a^3\cos(dx+c)^6\log(\sin(dx+c)+1)-15\,(23\,A+30\,C)a^3\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\,(16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
fricas")
```

```
[Out] 1/480*(15*(23*A + 30*C)*a^3*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(23*A
 + 30*C)*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(34*A + 45*C)*a^
3*cos(d*x + c)^5 + 15*(23*A + 30*C)*a^3*cos(d*x + c)^4 + 16*(17*A + 15*C)*a
^3*cos(d*x + c)^3 + 10*(23*A + 6*C)*a^3*cos(d*x + c)^2 + 144*A*a^3*cos(d*x
+ c) + 40*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26656, size = 378, normalized size = 1.68

$$15\left(23\,Aa^3+30\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(23\,Aa^3+30\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(345\,Aa^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
giac")
```

```
[Out] 1/240*(15*(23*A*a^3 + 30*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(23
*A*a^3 + 30*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(345*A*a^3*tan(1/
2*d*x + 1/2*c)^11 + 450*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 1955*A*a^3*tan(1/2*
d*x + 1/2*c)^9 - 2550*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 + 5940*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 5814*A*a^3*tan(1/2*d*x +
1/2*c)^5 - 7500*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*tan(1/2*d*x + 1/2
*c)^3 + 5130*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1575*A*a^3*tan(1/2*d*x + 1/2*c)
 - 1470*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.28 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=279

$$-\frac{4a^4(63A+52C)\sin^3(c+dx)}{105d}+\frac{4a^4(63A+52C)\sin(c+dx)}{35d}+\frac{a^4(2408A+2007C)\sin(c+dx)\cos^3(c+dx)}{2240d}+\frac{(56A}{}$$

```
[Out] (a^4*(392*A + 323*C)*x)/128 + (4*a^4*(63*A + 52*C)*Sin[c + d*x])/(35*d) + (
a^4*(392*A + 323*C)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a^4*(2408*A + 200
7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2240*d) + (a*C*Cos[c + d*x]^3*(a + a*Cos
[c + d*x])^3*Sin[c + d*x])/(14*d) + (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^
4*Sin[c + d*x])/(8*d) + ((56*A + 61*C)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*
x])^2*Sin[c + d*x])/(336*d) + (7*(8*A + 7*C)*Cos[c + d*x]^3*(a^4 + a^4*Cos[
c + d*x])*Sin[c + d*x])/(120*d) - (4*a^4*(63*A + 52*C)*Sin[c + d*x]^3)/(105
*d)
```

Rubi [A] time = 0.792916, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3046, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{4a^4(63A+52C)\sin^3(c+dx)}{105d}+\frac{4a^4(63A+52C)\sin(c+dx)}{35d}+\frac{a^4(2408A+2007C)\sin(c+dx)\cos^3(c+dx)}{2240d}+\frac{(56A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a^4*(392*A + 323*C)*x)/128 + (4*a^4*(63*A + 52*C)*Sin[c + d*x])/(35*d) + (
a^4*(392*A + 323*C)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a^4*(2408*A + 200
7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2240*d) + (a*C*Cos[c + d*x]^3*(a + a*Cos
[c + d*x])^3*Sin[c + d*x])/(14*d) + (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^
4*Sin[c + d*x])/(8*d) + ((56*A + 61*C)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*
x])^2*Sin[c + d*x])/(336*d) + (7*(8*A + 7*C)*Cos[c + d*x]^3*(a^4 + a^4*Cos[
c + d*x])*Sin[c + d*x])/(120*d) - (4*a^4*(63*A + 52*C)*Sin[c + d*x]^3)/(105
*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
```

```
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^4\sin(c+dx)}{8d}+\frac{\int\cos^2}{}$$

$$=\frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{14d}+\frac{C\cos}{}$$

$$=\frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{14d}+\frac{C\cos}{}$$

$$=\frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{14d}+\frac{C\cos}{}$$

$$=\frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{14d}+\frac{C\cos}{}$$

$$=\frac{a^4(2408A+2007C)\cos^3(c+dx)\sin(c+dx)}{2240d}+\frac{aC\cos^3}{}$$

$$=\frac{a^4(2408A+2007C)\cos^3(c+dx)\sin(c+dx)}{2240d}+\frac{aC\cos^3}{}$$

$$=\frac{a^4(392A+323C)\cos(c+dx)\sin(c+dx)}{128d}+\frac{a^4(2408A+}{}$$

$$=\frac{1}{128}a^4(392A+323C)x+\frac{4a^4(63A+52C)\sin(c+dx)}{35d}+$$

Mathematica [A] time = 0.928706, size = 167, normalized size = 0.6

$$\frac{a^4(6720(88A+75C)\sin(c+dx)+1680(127A+120C)\sin(2(c+dx))+80640A\sin(3(c+dx))+25200A\sin(4(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^4*(164640*c*C + 329280*A*d*x + 271320*C*d*x + 6720*(88*A + 75*C)*Sin[c +
 d*x] + 1680*(127*A + 120*C)*Sin[2*(c + d*x)] + 80640*A*Sin[3*(c + d*x)] +
91840*C*Sin[3*(c + d*x)] + 25200*A*Sin[4*(c + d*x)] + 39480*C*Sin[4*(c + d*
x)] + 5376*A*Sin[5*(c + d*x)] + 14784*C*Sin[5*(c + d*x)] + 560*A*Sin[6*(c +
 d*x)] + 4480*C*Sin[6*(c + d*x)] + 960*C*Sin[7*(c + d*x)] + 105*C*Sin[8*(c
+ d*x)]))/(107520*d)
```

Maple [A] time = 0.057, size = 393, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+a^4*C*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)
^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c)+4/5*A*a^4*(8/3+cos(d*x
+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/7*a^4*C*(16/5+cos(d*x+c)^6+6/5*cos(d*x
+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c
))*sin(d*x+c)+3/8*d*x+3/8*c)+6*a^4*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15
/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4/3*A*a^4*(2+cos(d*x+c)^2)*sin(d
*x+c)+4/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/2*c
os(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c)
)*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.06925, size = 531, normalized size = 1.9

$$28672\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{4}-560\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/107520*(28672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*
a^4 - 560*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
```

```
in(2*d*x + 2*c))*A*a^4 - 143360*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 2
0160*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 26880*
(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 12288*(5*sin(d*x + c)^7 - 21*sin(d
*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*C*a^4 + 28672*(3*sin(d*x +
 c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^4 - 35*(128*sin(2*d*x + 2*
c)^3 - 840*d*x - 840*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*si
n(2*d*x + 2*c))*C*a^4 - 3360*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(
4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^4 + 3360*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4)/d
```

Fricas [A] time = 1.53497, size = 448, normalized size = 1.61

$$\frac{105\,(392\,A+323\,C)a^4dx+\left(1680\,Ca^4\cos\left(dx+c\right)^7+7680\,Ca^4\cos\left(dx+c\right)^6+280\,(8\,A+55\,C)a^4\cos\left(dx+c\right)^5+1536\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/13440*(105*(392*A + 323*C)*a^4*d*x + (1680*C*a^4*cos(d*x + c)^7 + 7680*C*
a^4*cos(d*x + c)^6 + 280*(8*A + 55*C)*a^4*cos(d*x + c)^5 + 1536*(7*A + 13*C
)*a^4*cos(d*x + c)^4 + 70*(328*A + 323*C)*a^4*cos(d*x + c)^3 + 512*(63*A +
52*C)*a^4*cos(d*x + c)^2 + 105*(392*A + 323*C)*a^4*cos(d*x + c) + 1024*(63*
A + 52*C)*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 20.364, size = 1149, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((5*A*a**4*x*sin(c + d*x)**6/16 + 15*A*a**4*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 9*A*a**4*x*sin(c + d*x)**4/4 + 15*A*a**4*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 9*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a**4
*x*sin(c + d*x)**2/2 + 5*A*a**4*x*cos(c + d*x)**6/16 + 9*A*a**4*x*cos(c + d
*x)**4/4 + A*a**4*x*cos(c + d*x)**2/2 + 5*A*a**4*sin(c + d*x)**5*cos(c + d*
```

```
x)/(16*d) + 32*A*a**4*sin(c + d*x)**5/(15*d) + 5*A*a**4*sin(c + d*x)**3*cos
(c + d*x)**3/(6*d) + 16*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*
a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) +
11*A*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*A*a**4*sin(c + d*x)*cos(c
 + d*x)**4/d + 15*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*A*a**4*sin(
c + d*x)*cos(c + d*x)**2/d + A*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 35*C*
a**4*x*sin(c + d*x)**8/128 + 35*C*a**4*x*sin(c + d*x)**6*cos(c + d*x)**2/32
 + 15*C*a**4*x*sin(c + d*x)**6/8 + 105*C*a**4*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 45*C*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 3*C*a**4*x*sin(c
+ d*x)**4/8 + 35*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 45*C*a**4*x*
sin(c + d*x)**2*cos(c + d*x)**4/8 + 3*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 35*C*a**4*x*cos(c + d*x)**8/128 + 15*C*a**4*x*cos(c + d*x)**6/8 + 3
*C*a**4*x*cos(c + d*x)**4/8 + 35*C*a**4*sin(c + d*x)**7*cos(c + d*x)/(128*d
) + 64*C*a**4*sin(c + d*x)**7/(35*d) + 385*C*a**4*sin(c + d*x)**5*cos(c + d
*x)**3/(384*d) + 32*C*a**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 15*C*a**
4*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 32*C*a**4*sin(c + d*x)**5/(15*d) + 5
11*C*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 8*C*a**4*sin(c + d*x)**
3*cos(c + d*x)**4/d + 5*C*a**4*sin(c + d*x)**3*cos(c + d*x)**3/d + 16*C*a**
4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*C*a**4*sin(c + d*x)**3*cos(c +
d*x)/(8*d) + 93*C*a**4*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 4*C*a**4*sin(
c + d*x)*cos(c + d*x)**6/d + 33*C*a**4*sin(c + d*x)*cos(c + d*x)**5/(8*d) +
 4*C*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*a**4*sin(c + d*x)*cos(c + d*
x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)**4*cos(c)**2, T
rue))
```

Giac [A] time = 1.24415, size = 285, normalized size = 1.02

$$\frac{Ca^4 \sin(8\,dx + 8\,c)}{1024\,d} + \frac{Ca^4 \sin(7\,dx + 7\,c)}{112\,d} + \frac{1}{128}\left(392\,Aa^4 + 323\,Ca^4\right)x + \frac{\left(Aa^4 + 8\,Ca^4\right)\sin(6\,dx + 6\,c)}{192\,d} + \frac{\left(4\,Aa^4 + 11\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/1024*C*a^4*sin(8*d*x + 8*c)/d + 1/112*C*a^4*sin(7*d*x + 7*c)/d + 1/128*(3
92*A*a^4 + 323*C*a^4)*x + 1/192*(A*a^4 + 8*C*a^4)*sin(6*d*x + 6*c)/d + 1/80
*(4*A*a^4 + 11*C*a^4)*sin(5*d*x + 5*c)/d + 1/128*(30*A*a^4 + 47*C*a^4)*sin(
4*d*x + 4*c)/d + 1/48*(36*A*a^4 + 41*C*a^4)*sin(3*d*x + 3*c)/d + 1/64*(127*
A*a^4 + 120*C*a^4)*sin(2*d*x + 2*c)/d + 1/16*(88*A*a^4 + 75*C*a^4)*sin(d*x
+ c)/d
```

3.29 $\int \cos(c+dx)(a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=219

$$-\frac{8a^4(14A+11C)\sin^3(c+dx)}{105d}+\frac{16a^4(14A+11C)\sin(c+dx)}{35d}+\frac{a^4(14A+11C)\sin(c+dx)\cos^3(c+dx)}{70d}+\frac{27a^4(14A\ -}{}$$

```
[Out] (a^4*(14*A + 11*C)*x)/4 + (16*a^4*(14*A + 11*C)*Sin[c + d*x])/(35*d) + (27*
a^4*(14*A + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(140*d) + (a^4*(14*A + 11*C)*C
os[c + d*x]^3*Sin[c + d*x])/(70*d) + ((21*A + 4*C)*(a + a*Cos[c + d*x])^4*S
in[c + d*x])/(105*d) + (C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*Sin[c + d*x
])/(7*d) + (2*C*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(21*a*d) - (8*a^4*(14*
A + 11*C)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.412092, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3046, 2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{8a^4(14A+11C)\sin^3(c+dx)}{105d}+\frac{16a^4(14A+11C)\sin(c+dx)}{35d}+\frac{a^4(14A+11C)\sin(c+dx)\cos^3(c+dx)}{70d}+\frac{27a^4(14A\ -}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^4*(14*A + 11*C)*x)/4 + (16*a^4*(14*A + 11*C)*Sin[c + d*x])/(35*d) + (27*
a^4*(14*A + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(140*d) + (a^4*(14*A + 11*C)*C
os[c + d*x]^3*Sin[c + d*x])/(70*d) + ((21*A + 4*C)*(a + a*Cos[c + d*x])^4*S
in[c + d*x])/(105*d) + (C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*Sin[c + d*x
])/(7*d) + (2*C*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(21*a*d) - (8*a^4*(14*
A + 11*C)*Sin[c + d*x]^3)/(105*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^4\sin(c+dx)}{7d} + \frac{\int \cos(c\ldots}{\ldots}$$

$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^4\sin(c+dx)}{7d} + \frac{\int (a+a\ldots}{\ldots}$$

$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^4\sin(c+dx)}{7d} + \frac{2C(a+\ldots}{\ldots}$$

$$= \frac{(21A+4C)(a+a\cos(c+dx))^4\sin(c+dx)}{105d} + \frac{C\cos^2(c+\ldots}{\ldots}$$

$$= \frac{(21A+4C)(a+a\cos(c+dx))^4\sin(c+dx)}{105d} + \frac{C\cos^2(c+\ldots}{\ldots}$$

$$= \frac{2}{35}a^4(14A+11C)x + \frac{(21A+4C)(a+a\cos(c+dx))^4\sin\ldots}{105d}$$

$$= \frac{2}{35}a^4(14A+11C)x + \frac{8a^4(14A+11C)\sin(c+dx)}{35d} + \frac{6a^4(\ldots}{\ldots}$$

$$= \frac{8}{35}a^4(14A+11C)x + \frac{16a^4(14A+11C)\sin(c+dx)}{35d} + \frac{27a\ldots}{\ldots}$$

$$= \frac{1}{4}a^4(14A+11C)x + \frac{16a^4(14A+11C)\sin(c+dx)}{35d} + \frac{27a^4\ldots}{\ldots}$$

Mathematica [A] time = 0.574277, size = 145, normalized size = 0.66

$$\frac{a^4(105(392A+323C)\sin(c+dx)+420(32A+31C)\sin(2(c+dx))+4060A\sin(3(c+dx))+840A\sin(4(c+dx))+84\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^4*(11760*c*C + 23520*A*d*x + 18480*C*d*x + 105*(392*A + 323*C)*Sin[c + d
*x] + 420*(32*A + 31*C)*Sin[2*(c + d*x)] + 4060*A*Sin[3*(c + d*x)] + 5495*C
*Sin[3*(c + d*x)] + 840*A*Sin[4*(c + d*x)] + 2100*C*Sin[4*(c + d*x)] + 84*A
*Sin[5*(c + d*x)] + 651*C*Sin[5*(c + d*x)] + 140*C*Sin[6*(c + d*x)] + 15*C*
Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.032, size = 322, normalized size = 1.5

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+\frac{a^4C\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/7*a^4*C*(16
/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^4*C*(1/6*(cos(d
*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+2*A*a
^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2
)*sin(d*x+c)+4*A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^4*C*(1/4
*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^4*sin(d*x+c)+1
/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.02724, size = 431, normalized size = 1.97

$$\frac{112\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-3360\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4+210(12dx+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/1680*(112*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4
- 3360*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 210*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 1680*(2*d*x + 2*c + sin(2*d*x + 2
*c))*A*a^4 - 48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 -
 35*sin(d*x + c))*C*a^4 + 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
```

```
n(d*x + c))*C*a^4 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x
+ 4*c) - 48*sin(2*d*x + 2*c))*C*a^4 - 560*(sin(d*x + c)^3 - 3*sin(d*x + c))
*C*a^4 + 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4
+ 1680*A*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.46895, size = 377, normalized size = 1.72

$$\frac{105\,(14\,A+11\,C)a^4dx+\left(60\,Ca^4\cos\left(dx+c\right)^6+280\,Ca^4\cos\left(dx+c\right)^5+12\,(7\,A+48\,C)a^4\cos\left(dx+c\right)^4+70\,(6\,A+11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/420*(105*(14*A + 11*C)*a^4*d*x + (60*C*a^4*cos(d*x + c)^6 + 280*C*a^4*cos
(d*x + c)^5 + 12*(7*A + 48*C)*a^4*cos(d*x + c)^4 + 70*(6*A + 11*C)*a^4*cos(
d*x + c)^3 + 4*(238*A + 227*C)*a^4*cos(d*x + c)^2 + 105*(14*A + 11*C)*a^4*c
os(d*x + c) + 4*(581*A + 454*C)*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 11.4555, size = 799, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/2 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2 + 2*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/2 + 2*A
*a**4*x*cos(c + d*x)**2 + 8*A*a**4*sin(c + d*x)**5/(15*d) + 4*A*a**4*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 4*A*a**4*sin(c + d*x)**3/d + A*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5
*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 6*A*a**4*sin(c + d*x)*cos(c +
d*x)**2/d + 2*A*a**4*sin(c + d*x)*cos(c + d*x)/d + A*a**4*sin(c + d*x)/d +
5*C*a**4*x*sin(c + d*x)**6/4 + 15*C*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/
4 + 3*C*a**4*x*sin(c + d*x)**4/2 + 15*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)
**4/4 + 3*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*C*a**4*x*cos(c + d*x
)**6/4 + 3*C*a**4*x*cos(c + d*x)**4/2 + 16*C*a**4*sin(c + d*x)**7/(35*d) +
```

```
8*C*a**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 5*C*a**4*sin(c + d*x)**5*c
os(c + d*x)/(4*d) + 16*C*a**4*sin(c + d*x)**5/(5*d) + 2*C*a**4*sin(c + d*x)
**3*cos(c + d*x)**4/d + 10*C*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 8
*C*a**4*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*C*a**4*sin(c + d*x)**3*cos(c
+ d*x)/(2*d) + 2*C*a**4*sin(c + d*x)**3/(3*d) + C*a**4*sin(c + d*x)*cos(c +
 d*x)**6/d + 11*C*a**4*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 6*C*a**4*sin(c
+ d*x)*cos(c + d*x)**4/d + 5*C*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + C*
a**4*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*(a*cos
(c) + a)**4*cos(c), True))
```

Giac [A] time = 1.23046, size = 250, normalized size = 1.14

$$\frac{Ca^4\sin(7\,dx+7\,c)}{448\,d}+\frac{Ca^4\sin(6\,dx+6\,c)}{48\,d}+\frac{1}{4}\left(14\,Aa^4+11\,Ca^4\right)x+\frac{\left(4\,Aa^4+31\,Ca^4\right)\sin(5\,dx+5\,c)}{320\,d}+\frac{\left(2\,Aa^4+5\,Ca^4\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/448*C*a^4*sin(7*d*x + 7*c)/d + 1/48*C*a^4*sin(6*d*x + 6*c)/d + 1/4*(14*A*
a^4 + 11*C*a^4)*x + 1/320*(4*A*a^4 + 31*C*a^4)*sin(5*d*x + 5*c)/d + 1/16*(2
*A*a^4 + 5*C*a^4)*sin(4*d*x + 4*c)/d + 1/192*(116*A*a^4 + 157*C*a^4)*sin(3*
d*x + 3*c)/d + 1/16*(32*A*a^4 + 31*C*a^4)*sin(2*d*x + 2*c)/d + 1/64*(392*A*
a^4 + 323*C*a^4)*sin(d*x + c)/d
```

3.30 $\int (a + a\cos(c + dx))^4 \left(A + C\cos^2(c + dx)\right) dx$

Optimal. Leaf size=179

$$-\frac{2a^4(10A + 7C)\sin^3(c + dx)}{15d} + \frac{4a^4(10A + 7C)\sin(c + dx)}{5d} + \frac{a^4(10A + 7C)\sin(c + dx)\cos^3(c + dx)}{40d} + \frac{27a^4(10A + 7C}{}$$

```
[Out] (7*a^4*(10*A + 7*C)*x)/16 + (4*a^4*(10*A + 7*C)*Sin[c + d*x])/(5*d) + (27*a
^4*(10*A + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(10*A + 7*C)*Cos[c
 + d*x]^3*Sin[c + d*x])/(40*d) - (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(3
0*d) + (C*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) - (2*a^4*(10*A + 7*C
)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.232356, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3024, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{2a^4(10A + 7C)\sin^3(c + dx)}{15d} + \frac{4a^4(10A + 7C)\sin(c + dx)}{5d} + \frac{a^4(10A + 7C)\sin(c + dx)\cos^3(c + dx)}{40d} + \frac{27a^4(10A + 7C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (7*a^4*(10*A + 7*C)*x)/16 + (4*a^4*(10*A + 7*C)*Sin[c + d*x])/(5*d) + (27*a
^4*(10*A + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(10*A + 7*C)*Cos[c
 + d*x]^3*Sin[c + d*x])/(40*d) - (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(3
0*d) + (C*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) - (2*a^4*(10*A + 7*C
)*Sin[c + d*x]^3)/(15*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right) dx = \frac{C(a+a\cos(c+dx))^5\sin(c+dx)}{6ad} + \frac{\int (a+a\cos(c+dx))^4(a(6A+5C}{6a}$$
$$= -\frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{C(a+a\cos(c+dx))^5\sin(c+d}{6ad}$$
$$= -\frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{C(a+a\cos(c+dx))^5\sin(c+d}{6ad}$$
$$= \frac{1}{10}a^4(10A+7C)x - \frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{C(a+a\cos}{}$$
$$= \frac{1}{10}a^4(10A+7C)x + \frac{2a^4(10A+7C)\sin(c+dx)}{5d} + \frac{3a^4(10A+7C)\cos}{1}$$
$$= \frac{2}{5}a^4(10A+7C)x + \frac{4a^4(10A+7C)\sin(c+dx)}{5d} + \frac{27a^4(10A+7C)\cos}{8}$$
$$= \frac{7}{16}a^4(10A+7C)x + \frac{4a^4(10A+7C)\sin(c+dx)}{5d} + \frac{27a^4(10A+7C)\cos}{}$$

Mathematica [A] time = 0.375509, size = 119, normalized size = 0.66

$$\frac{a^4(480(14A+11C)\sin(c+dx)+15(112A+127C)\sin(2(c+dx))+320A\sin(3(c+dx))+30A\sin(4(c+dx))+4200A}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^4*(4200*A*d*x + 2940*C*d*x + 480*(14*A + 11*C)*Sin[c + d*x] + 15*(112*A
+ 127*C)*Sin[2*(c + d*x)] + 320*A*Sin[3*(c + d*x)] + 720*C*Sin[3*(c + d*x)]
 + 30*A*Sin[4*(c + d*x)] + 225*C*Sin[4*(c + d*x)] + 48*C*Sin[5*(c + d*x)] +
 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.026, size = 284, normalized size = 1.6

$$\frac{1}{d}\left(a^4C\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4C\sin(dx+c)}{5}\left(\frac{8}{3}+(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a^4*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A
*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*a^4*C*(
1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c)+4/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/2*c
os(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*
x+1/2*c)+4*A*a^4*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [A] time = 1.03415, size = 369, normalized size = 2.06

$$-\frac{1280\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^4-30\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^4-1440\left(2\,dx+2\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/960*(1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 30*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 1440*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a^4 - 960*(d*x + c)*A*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x
+ c)^3 + 15*sin(d*x + c))*C*a^4 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c -
 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^4 + 1280*(sin(d*x + c)^3 - 3
*sin(d*x + c))*C*a^4 - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
+ 2*c))*C*a^4 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 - 3840*A*a^4*sin
(d*x + c))/d
```

Fricas [A] time = 1.42339, size = 319, normalized size = 1.78

$$\frac{105\left(10\,A+7\,C\right)a^4dx+\left(40\,Ca^4\cos\left(dx+c\right)^5+192\,Ca^4\cos\left(dx+c\right)^4+10\left(6\,A+41\,C\right)a^4\cos\left(dx+c\right)^3+64\left(5\,A+9\,C\right)a\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(105*(10*A + 7*C)*a^4*d*x + (40*C*a^4*cos(d*x + c)^5 + 192*C*a^4*cos(
d*x + c)^4 + 10*(6*A + 41*C)*a^4*cos(d*x + c)^3 + 64*(5*A + 9*C)*a^4*cos(d*
x + c)^2 + 15*(54*A + 49*C)*a^4*cos(d*x + c) + 64*(25*A + 18*C)*a^4)*sin(d*
x + c))/d
```

Sympy [A] time = 6.30688, size = 707, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/8 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/8 + 3
*A*a**4*x*cos(c + d*x)**2 + A*a**4*x + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) + 5*A*a**4*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + 4*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**4*sin(c + d*
x)*cos(c + d*x)/d + 4*A*a**4*sin(c + d*x)/d + 5*C*a**4*x*sin(c + d*x)**6/16
 + 15*C*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*C*a**4*x*sin(c + d*x)
**4/4 + 15*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*C*a**4*x*sin(c +
 d*x)**2*cos(c + d*x)**2/2 + C*a**4*x*sin(c + d*x)**2/2 + 5*C*a**4*x*cos(c
+ d*x)**6/16 + 9*C*a**4*x*cos(c + d*x)**4/4 + C*a**4*x*cos(c + d*x)**2/2 +
5*C*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 32*C*a**4*sin(c + d*x)**5/(1
5*d) + 5*C*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 16*C*a**4*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) + 9*C*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d)
+ 8*C*a**4*sin(c + d*x)**3/(3*d) + 11*C*a**4*sin(c + d*x)*cos(c + d*x)**5/(
16*d) + 4*C*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*C*a**4*sin(c + d*x)*co
s(c + d*x)**3/(4*d) + 4*C*a**4*sin(c + d*x)*cos(c + d*x)**2/d + C*a**4*sin(
c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a*cos(c) + a)
**4, True))
```

Giac [A] time = 1.22546, size = 213, normalized size = 1.19

$$\frac{Ca^4\sin(6\,dx+6\,c)}{192\,d}+\frac{Ca^4\sin(5\,dx+5\,c)}{20\,d}+\frac{7}{16}\left(10\,Aa^4+7\,Ca^4\right)x+\frac{\left(2\,Aa^4+15\,Ca^4\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{\left(4\,Aa^4+9\,Ca\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/192*C*a^4*sin(6*d*x + 6*c)/d + 1/20*C*a^4*sin(5*d*x + 5*c)/d + 7/16*(10*A
*a^4 + 7*C*a^4)*x + 1/64*(2*A*a^4 + 15*C*a^4)*sin(4*d*x + 4*c)/d + 1/12*(4*
A*a^4 + 9*C*a^4)*sin(3*d*x + 3*c)/d + 1/64*(112*A*a^4 + 127*C*a^4)*sin(2*d*
x + 2*c)/d + 1/2*(14*A*a^4 + 11*C*a^4)*sin(d*x + c)/d
```

3.31 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=177

$$\frac{a^4(10A+7C)\sin(c+dx)}{2d}+\frac{(5A+7C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{15d}+\frac{(8A+7C)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{6d}$$

```
[Out] (a^4*(12*A + 7*C)*x)/2 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (a^4*(10*A + 7*C
)*Sin[c + d*x])/(2*d) + (a*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(5*d) + (
C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + ((5*A + 7*C)*(a^2 + a^2*Cos[
c + d*x])^2*Sin[c + d*x])/(15*d) + ((8*A + 7*C)*(a^4 + a^4*Cos[c + d*x])*Si
n[c + d*x])/(6*d)
```

Rubi [A] time = 0.538812, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3046, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^4(10A+7C)\sin(c+dx)}{2d}+\frac{(5A+7C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{15d}+\frac{(8A+7C)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] (a^4*(12*A + 7*C)*x)/2 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (a^4*(10*A + 7*C
)*Sin[c + d*x])/(2*d) + (a*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(5*d) + (
C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + ((5*A + 7*C)*(a^2 + a^2*Cos[
c + d*x])^2*Sin[c + d*x])/(15*d) + ((8*A + 7*C)*(a^4 + a^4*Cos[c + d*x])*Si
n[c + d*x])/(6*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{\int (a+a\cos(c+dx))^4(5}{}$$

$$= \frac{aC(a+a\cos(c+dx))^3\sin(c+dx)}{5d} + \frac{C(a+a\cos(c+dx))^4}{5d}$$

$$= \frac{aC(a+a\cos(c+dx))^3\sin(c+dx)}{5d} + \frac{C(a+a\cos(c+dx))^4}{5d}$$

$$= \frac{aC(a+a\cos(c+dx))^3\sin(c+dx)}{5d} + \frac{C(a+a\cos(c+dx))^4}{5d}$$

$$= \frac{aC(a+a\cos(c+dx))^3\sin(c+dx)}{5d} + \frac{C(a+a\cos(c+dx))^4}{5d}$$

$$= \frac{a^4(10A+7C)\sin(c+dx)}{2d} + \frac{aC(a+a\cos(c+dx))^3\sin(c+}{5d}$$

$$= \frac{1}{2}a^4(12A+7C)x + \frac{a^4(10A+7C)\sin(c+dx)}{2d} + \frac{aC(a+a\,\mathrm{c}}{}$$

$$= \frac{1}{2}a^4(12A+7C)x + \frac{a^4A\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^4(10A+7}{}$$

Mathematica [A] time = 0.468441, size = 147, normalized size = 0.83

$$\frac{a^4\left(30(54A+49C)\sin(c+dx)+240(A+2C)\sin(2(c+dx))+20A\sin(3(c+dx))-240A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a^4*(1440*A*d*x + 840*C*d*x - 240*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 240*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 30*(54*A + 49*C)*Sin[
c + d*x] + 240*(A + 2*C)*Sin[2*(c + d*x)] + 20*A*Sin[3*(c + d*x)] + 145*C*S
in[3*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 3*C*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.066, size = 221, normalized size = 1.3

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^4}{3d} + \frac{20Aa^4\sin(dx+c)}{3d} + \frac{83a^4C\sin(dx+c)}{15d} + \frac{a^4C\sin(dx+c)(\cos(dx+c))^4}{5d} + \frac{34a^4C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+83/15/d*a^4*C*s
in(d*x+c)+1/5/d*a^4*C*sin(d*x+c)*cos(d*x+c)^4+34/15/d*a^4*C*sin(d*x+c)*cos(
d*x+c)^2+2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+6*A*a^4*x+6/d*A*a^4*c+1/d*a^4*C*si
n(d*x+c)*cos(d*x+c)^3+7/2/d*a^4*C*cos(d*x+c)*sin(d*x+c)+7/2*a^4*C*x+7/2/d*a
^4*C*c+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02808, size = 300, normalized size = 1.69

$$-\frac{40\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}-120\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{4}-480\left(dx+c\right)Aa^{4}-8\left(3\,\sin\left(dx+c\right)^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] -1/120*(40*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 120*(2*d*x + 2*c + sin
(2*d*x + 2*c))*A*a^4 - 480*(d*x + c)*A*a^4 - 8*(3*sin(d*x + c)^5 - 10*sin(d
*x + c)^3 + 15*sin(d*x + c))*C*a^4 + 240*(sin(d*x + c)^3 - 3*sin(d*x + c))*
C*a^4 - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4 -
120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 - 120*A*a^4*log(sec(d*x + c) + t
an(d*x + c)) - 720*A*a^4*sin(d*x + c) - 120*C*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.5962, size = 351, normalized size = 1.98

$$\frac{15\left(12\,A+7\,C\right)a^{4}dx+15\,Aa^{4}\log\left(\sin\left(dx+c\right)+1\right)-15\,Aa^{4}\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Ca^{4}\cos\left(dx+c\right)^{4}+30\,Ca^{4}\cos\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(12*A + 7*C)*a^4*d*x + 15*A*a^4*log(sin(d*x + c) + 1) - 15*A*a^4*l
og(-sin(d*x + c) + 1) + (6*C*a^4*cos(d*x + c)^4 + 30*C*a^4*cos(d*x + c)^3 +
```

```
 2*(5*A + 34*C)*a^4*cos(d*x + c)^2 + 15*(4*A + 7*C)*a^4*cos(d*x + c) + 2*(1
00*A + 83*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32295, size = 335, normalized size = 1.89

$$\frac{30\,Aa^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 30\,Aa^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + 15\left(12\,Aa^4 + 7\,Ca^4\right)(dx + c) + \frac{2\left(150\,Aa^4 \tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] 1/30*(30*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 30*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 15*(12*A*a^4 + 7*C*a^4)*(d*x + c) + 2*(150*A*a^4*tan
(1/2*d*x + 1/2*c)^9 + 105*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 680*A*a^4*tan(1/2*
d*x + 1/2*c)^7 + 490*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 1180*A*a^4*tan(1/2*d*x
+ 1/2*c)^5 + 896*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 920*A*a^4*tan(1/2*d*x + 1/2
*c)^3 + 790*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 270*A*a^4*tan(1/2*d*x + 1/2*c) +
 375*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.32 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{5a^4(4A+7C)\sin(c+dx)}{8d}-\frac{(12A-7C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d}-\frac{(12A-35C)\sin(c+dx)\left(a^4\cos(c+dx)+\right.}{24d}$$

```
[Out] (a^4*(52*A + 35*C)*x)/8 + (4*a^4*A*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(4*A +
 7*C)*Sin[c + d*x])/(8*d) - (a*(4*A - C)*(a + a*Cos[c + d*x])^3*Sin[c + d*x
])/(4*d) - ((12*A - 7*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) -
((12*A - 35*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d) + (A*(a + a*Co
s[c + d*x])^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.602695, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^4(4A+7C)\sin(c+dx)}{8d}-\frac{(12A-7C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d}-\frac{(12A-35C)\sin(c+dx)\left(a^4\cos(c+dx)+\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] (a^4*(52*A + 35*C)*x)/8 + (4*a^4*A*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(4*A +
 7*C)*Sin[c + d*x])/(8*d) - (a*(4*A - C)*(a + a*Cos[c + d*x])^3*Sin[c + d*x
])/(4*d) - ((12*A - 7*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) -
((12*A - 35*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d) + (A*(a + a*Co
s[c + d*x])^4*Tan[c + d*x])/d
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\tan(c+dx)}{d} + \frac{\int (a+a\cos(c+dx)}{}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{A(a+a\,\mathrm{co}}{}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx)}{4d} - \frac{(12A-7C}{}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx)}{4d} - \frac{(12A-7C}{}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx)}{4d} - \frac{(12A-7C}{}$$

$$= \frac{5a^4(4A+7C)\sin(c+dx)}{8d} - \frac{a(4A-C)(a+a\cos(c+dx)}{4d}$$

$$= \frac{1}{8}a^4(52A+35C)x + \frac{5a^4(4A+7C)\sin(c+dx)}{8d} - \frac{a(4A-}{}$$

$$= \frac{1}{8}a^4(52A+35C)x + \frac{4a^4A\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^4(4A}{}$$

Mathematica [A] time = 2.12994, size = 338, normalized size = 1.87

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{96(4A+7C)\sin(c)\cos(dx)}{d} + \frac{24(A+7C)\sin(2c)\cos(2dx)}{d} + \frac{96(4A+7C)\cos(c)\sin(dx)}{d} + \frac{24(A+7C)\cos(}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(12*(52*A + 35*C)*x - (384*A*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (384*A*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/d + (96*(4*A + 7*C)*Cos[d*x]*Sin[c])/d + (24*(A + 7*C)*C
os[2*d*x]*Sin[2*c])/d + (32*C*Cos[3*d*x]*Sin[3*c])/d + (3*C*Cos[4*d*x]*Sin[
4*c])/d + (96*(4*A + 7*C)*Cos[c]*Sin[d*x])/d + (24*(A + 7*C)*Cos[2*c]*Sin[2
*d*x])/d + (32*C*Cos[3*c]*Sin[3*d*x])/d + (3*C*Cos[4*c]*Sin[4*d*x])/d + (96
*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])) + (96*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]))))/1536
```

Maple [A] time = 0.08, size = 191, normalized size = 1.1

$$\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13\,Aa^4x}{2}+\frac{13\,Aa^4c}{2d}+\frac{a^4C\sin(dx+c)(\cos(dx+c))^3}{4d}+\frac{27\,a^4C\cos(dx+c)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*A*a^4*x+13/2/d*A*a^4*c+1/4/d*a^4*C*s
in(d*x+c)*cos(d*x+c)^3+27/8/d*a^4*C*cos(d*x+c)*sin(d*x+c)+35/8*a^4*C*x+35/8
/d*a^4*C*c+4/d*A*a^4*sin(d*x+c)+4/3/d*a^4*C*sin(d*x+c)*cos(d*x+c)^2+20/3/d*
a^4*C*sin(d*x+c)+4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^4*tan(d*x+c)
```

Maxima [A] time = 1.06523, size = 262, normalized size = 1.45

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+576\,(dx+c)Aa^4-128\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^4+3\,(12\,dx+12\,c+\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 576*(d*x + c)*A*a^4 - 128
*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*C*a^4 + 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^
4 + 96*(d*x + c)*C*a^4 + 192*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 384*A*a^4*sin(d*x + c) + 384*C*a^4*sin(d*x + c) + 96*A*a^4*tan(d*
x + c))/d
```

Fricas [A] time = 1.483, size = 408, normalized size = 2.25

$$\frac{3\,(52\,A+35\,C)a^4dx\cos(dx+c)+48\,Aa^4\cos(dx+c)\log(\sin(dx+c)+1)-48\,Aa^4\cos(dx+c)\log(-\sin(dx+c)+1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] 1/24*(3*(52*A + 35*C)*a^4*d*x*cos(d*x + c) + 48*A*a^4*cos(d*x + c)*log(sin(
d*x + c) + 1) - 48*A*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*C*a^4*cos
(d*x + c)^4 + 32*C*a^4*cos(d*x + c)^3 + 3*(4*A + 27*C)*a^4*cos(d*x + c)^2 +
 32*(3*A + 5*C)*a^4*cos(d*x + c) + 24*A*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27138, size = 329, normalized size = 1.82

$$96\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-96\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{48\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+3\left(52\,Aa^4+35\,Ca^4\right)(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] 1/24*(96*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 96*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 - 1) + 3*(52*A*a^4 + 35*C*a^4)*(d*x + c) + 2*(84*A*a^4*tan(1/2*d*x + 1/2*
c)^7 + 105*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 276*A*a^4*tan(1/2*d*x + 1/2*c)^5
+ 385*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 300*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 511
*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 108*A*a^4*tan(1/2*d*x + 1/2*c) + 279*C*a^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.33 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=186

$$-\frac{5a^4(A-2C)\sin(c+dx)}{2d}+\frac{a^4(13A+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(15A-2C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{6d}-\frac{(9A-$$

```
[Out] 2*a^4*(2*A + 3*C)*x + (a^4*(13*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a
^4*(A - 2*C)*Sin[c + d*x])/(2*d) - ((15*A - 2*C)*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(6*d) - ((9*A - 4*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/
(3*d) + (2*a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d + (A*(a + a*Cos[c + d
*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.607624, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3044, 2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(A-2C)\sin(c+dx)}{2d}+\frac{a^4(13A+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(15A-2C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{6d}-\frac{(9A-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```



```
[Out] 2*a^4*(2*A + 3*C)*x + (a^4*(13*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a
^4*(A - 2*C)*Sin[c + d*x])/(2*d) - ((15*A - 2*C)*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(6*d) - ((9*A - 4*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/
(3*d) + (2*a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d + (A*(a + a*Cos[c + d
*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2d}+\frac{\int(a+a\,\mathrm{c}}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^3\tan(c+dx)}{d}+\frac{A(a+a\cos(c+dx}{}$$

$$= -\frac{(15A-2C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{6d}+\frac{2aA(a+}{}$$

$$= -\frac{(15A-2C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{6d}-\frac{(9A-4}{}$$

$$= -\frac{(15A-2C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{6d}-\frac{(9A-4}{}$$

$$= -\frac{5a^4(A-2C)\sin(c+dx)}{2d}-\frac{(15A-2C)\left(a^2+a^2\cos(c+a\right.}{6d}$$

$$= 2a^4(2A+3C)x-\frac{5a^4(A-2C)\sin(c+dx)}{2d}-\frac{(15A-2C)\left(\right.}{}$$

$$= 2a^4(2A+3C)x+\frac{a^4(13A+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a}{}$$

Mathematica [B] time = 6.22573, size = 756, normalized size = 4.06

$$\frac{1}{8}x(2A+3C)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4+\frac{(4A+27C)\sin(c)\cos(dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4}{64d}+\frac{(4A+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] ((2*A + 3*C)*x*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/8 + ((-13*A - 2
*C)*(a + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec
[c/2 + (d*x)/2]^8)/(32*d) + ((13*A + 2*C)*(a + a*Cos[c + d*x])^4*Log[Cos[c/
2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8)/(32*d) + ((4*A + 2
7*C)*Cos[d*x]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[c])/(64*d) +
(C*Cos[2*d*x]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[2*c])/(16*d)
+ (C*Cos[3*d*x]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[3*c])/(192*
d) + ((4*A + 27*C)*Cos[c]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[d
*x])/(64*d) + (C*Cos[2*c]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[2
*d*x])/(16*d) + (C*Cos[3*c]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin
[3*d*x])/(192*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(64*d*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (A*(a + a*Cos[c + d*x])^4*Sec[
c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(4*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/
2] - Sin[c/2 + (d*x)/2])) - (A*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)
/(64*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (A*(a + a*Cos[c + d*x
])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(4*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2
 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.086, size = 190, normalized size = 1.

$$\frac{Aa^4\sin(dx+c)}{d}+\frac{a^4C\sin(dx+c)(\cos(dx+c))^2}{3d}+\frac{20\,a^4C\sin(dx+c)}{3d}+4\,Aa^4x+4\,\frac{Aa^4c}{d}+2\,\frac{a^4C\cos(dx+c)\sin(a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+1/3/d*a^4*C*sin(d*x+c)*cos(d*x+c)^2+20/3/d*a^4*C*sin(d
*x+c)+4*A*a^4*x+4/d*A*a^4*c+2/d*a^4*C*cos(d*x+c)*sin(d*x+c)+6*a^4*C*x+6/d*a
^4*C*c+13/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^4*tan(d*x+c)+1/2/d*A*
a^4*sec(d*x+c)*tan(d*x+c)+1/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01986, size = 285, normalized size = 1.53

$$\frac{48(dx+c)Aa^4-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^4+12(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^4+48(dx+c)Ca^4-3\,Aa^4\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] 1/12*(48*(d*x + c)*A*a^4 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 12*(
2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 48*(d*x + c)*C*a^4 - 3*A*a^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 36*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*C*a^4*(
log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^4*sin(d*x + c) + 72
*C*a^4*sin(d*x + c) + 48*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.57148, size = 432, normalized size = 2.32

$$\frac{24\,(2\,A+3\,C)a^4dx\cos\left(dx+c\right)^2+3\,(13\,A+2\,C)a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,(13\,A+2\,C)a^4\cos\left(dx+c\right)^2\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] 1/12*(24*(2*A + 3*C)*a^4*d*x*cos(d*x + c)^2 + 3*(13*A + 2*C)*a^4*cos(d*x +
c)^2*log(sin(d*x + c) + 1) - 3*(13*A + 2*C)*a^4*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*C*a^4*cos(d*x + c)^4 + 12*C*a^4*cos(d*x + c)^3 + 2*(3*A +
 20*C)*a^4*cos(d*x + c)^2 + 24*A*a^4*cos(d*x + c) + 3*A*a^4)*sin(d*x + c))/
(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29635, size = 335, normalized size = 1.8

$$12\left(2\,Aa^{4}+3\,Ca^{4}\right)(dx+c)+3\left(13\,Aa^{4}+2\,Ca^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(13\,Aa^{4}+2\,Ca^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] 1/6*(12*(2*A*a^4 + 3*C*a^4)*(d*x + c) + 3*(13*A*a^4 + 2*C*a^4)*log(abs(tan(
1/2*d*x + 1/2*c) + 1)) - 3*(13*A*a^4 + 2*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - 6*(7*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 4*(3*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 15*C*
a^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 38*C*a^4*tan(
1/2*d*x + 1/2*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) + 27*C*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.34 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=198

$$-\frac{5a^4(2A-C)\sin(c+dx)}{2d}+\frac{2a^4(3A+2C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(22A+3C)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{6d}+\frac{(8A+$$

```
[Out] (a^4*(2*A + 13*C)*x)/2 + (2*a^4*(3*A + 2*C)*ArcTanh[Sin[c + d*x]])/d - (5*a
^4*(2*A - C)*Sin[c + d*x])/(2*d) - ((22*A + 3*C)*(a^4 + a^4*Cos[c + d*x])*S
in[c + d*x])/(6*d) + ((8*A + 3*C)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/
(3*d) + (2*a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (A
*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.686172, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3044, 2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(2A-C)\sin(c+dx)}{2d}+\frac{2a^4(3A+2C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(22A+3C)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{6d}+\frac{(8A+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] (a^4*(2*A + 13*C)*x)/2 + (2*a^4*(3*A + 2*C)*ArcTanh[Sin[c + d*x]])/d - (5*a
^4*(2*A - C)*Sin[c + d*x])/(2*d) - ((22*A + 3*C)*(a^4 + a^4*Cos[c + d*x])*S
in[c + d*x])/(6*d) + ((8*A + 3*C)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/
(3*d) + (2*a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (A
*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{\int(a+a}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{A(a+}{}$$

$$= \frac{(8A+3C)\left(a^2+a^2\cos(c+dx)\right)^2\tan(c+dx)}{3d} + \frac{2aA(a+}{}$$

$$= -\frac{(22A+3C)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{6d} + \frac{(8A+3C}{}$$

$$= -\frac{(22A+3C)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{6d} + \frac{(8A+3C}{}$$

$$= -\frac{5a^4(2A-C)\sin(c+dx)}{2d} - \frac{(22A+3C)\left(a^4+a^4\cos(c+\right.}{6d}$$

$$= \frac{1}{2}a^4(2A+13C)x - \frac{5a^4(2A-C)\sin(c+dx)}{2d} - \frac{(22A+3C)}{}$$

$$= \frac{1}{2}a^4(2A+13C)x + \frac{2a^4(3A+2C)\tanh^{-1}(\sin(c+dx))}{d} - \frac{5}{}$$

Mathematica [A] time = 6.2087, size = 386, normalized size = 1.95

$$a^4\left(\frac{(2A+13C)(c+dx)}{2d} + \frac{20A\sin\left(\frac{1}{2}(c+dx)\right)+3C\sin\left(\frac{1}{2}(c+dx)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{20A\sin\left(\frac{1}{2}(c+dx)\right)+3C\sin\left(\frac{1}{2}(c+dx)\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{2(3A}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(((2*A + 13*C)*(c + d*x))/(2*d) - (2*(3*A + 2*C)*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]])/d + (2*(3*A + 2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]])/d + (13*A)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (A*Sin[(c
 + d*x)/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (A*Sin[(c + d*x
)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (13*A)/(12*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2) + (20*A*Sin[(c + d*x)/2] + 3*C*Sin[(c + d*
x)/2])/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (20*A*Sin[(c + d*x)/2]
 + 3*C*Sin[(c + d*x)/2])/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (4*C
*Sin[c + d*x])/d + (C*Sin[2*(c + d*x)])/(4*d))
```

Maple [A] time = 0.089, size = 189, normalized size = 1.

$$Aa^4x + \frac{Aa^4c}{d} + \frac{a^4C\cos(dx+c)\sin(dx+c)}{2d} + \frac{13\,a^4Cx}{2} + \frac{13\,a^4Cc}{2d} + 6\,\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d} + 4\,\frac{a^4C\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] A*a^4*x+1/d*A*a^4*c+1/2/d*a^4*C*cos(d*x+c)*sin(d*x+c)+13/2*a^4*C*x+13/2/d*a
^4*C*c+6/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^4*C*sin(d*x+c)+20/3/d*A*a^
4*tan(d*x+c)+2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+4/d*a^4*C*ln(sec(d*x+c)+tan(d*
x+c))+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/d*a^4*C*tan(d*x+c)
```

Maxima [A] time = 1.07418, size = 285, normalized size = 1.44

$$\frac{4\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Aa^4 + 12\,(dx+c)Aa^4 + 3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ca^4 + 72\,(dx+c)Ca^4 - 12\,Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 12*(d*x + c)*A*a^4 + 3*(2
*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 72*(d*x + c)*C*a^4 - 12*A*a^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*C*a^4*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*C*a^4*sin(d*x + c) + 7
```

```
2*A*a^4*tan(d*x + c) + 12*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.60054, size = 425, normalized size = 2.15

$$3\,(2\,A+13\,C)a^4dx\cos\left(dx+c\right)^3+6\,(3\,A+2\,C)a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-6\,(3\,A+2\,C)a^4\cos\left(dx+c\right)^3\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(2*A + 13*C)*a^4*d*x*cos(d*x + c)^3 + 6*(3*A + 2*C)*a^4*cos(d*x + c)
^3*log(sin(d*x + c) + 1) - 6*(3*A + 2*C)*a^4*cos(d*x + c)^3*log(-sin(d*x +
c) + 1) + (3*C*a^4*cos(d*x + c)^4 + 24*C*a^4*cos(d*x + c)^3 + 2*(20*A + 3*C
)*a^4*cos(d*x + c)^2 + 12*A*a^4*cos(d*x + c) + 2*A*a^4)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28259, size = 335, normalized size = 1.69

$$3\left(2\,Aa^4+13\,Ca^4\right)(dx+c)+12\left(3\,Aa^4+2\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\left(3\,Aa^4+2\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] 1/6*(3*(2*A*a^4 + 13*C*a^4)*(d*x + c) + 12*(3*A*a^4 + 2*C*a^4)*log(abs(tan(
1/2*d*x + 1/2*c) + 1)) - 12*(3*A*a^4 + 2*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) + 6*(7*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*C*a^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 4*(15*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 3*C*
a^4*tan(1/2*d*x + 1/2*c)^5 - 38*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 6*C*a^4*tan(
1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.35 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=200

$$-\frac{5a^4(7A+4C)\sin(c+dx)}{8d}+\frac{a^4(35A+52C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(35A+36C)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{12d}+\frac{(7}{}$$

```
[Out] 4*a^4*C*x + (a^4*(35*A + 52*C)*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^4*(7*A +
 4*C)*Sin[c + d*x])/(8*d) + ((35*A + 36*C)*(a^4 + a^4*Cos[c + d*x])*Tan[c +
 d*x])/(12*d) + ((7*A + 4*C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(8*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3
*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.668955, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2975, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(7A+4C)\sin(c+dx)}{8d}+\frac{a^4(35A+52C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(35A+36C)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{12d}+\frac{(7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] 4*a^4*C*x + (a^4*(35*A + 52*C)*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^4*(7*A +
 4*C)*Sin[c + d*x])/(8*d) + ((35*A + 36*C)*(a^4 + a^4*Cos[c + d*x])*Tan[c +
 d*x])/(12*d) + ((7*A + 4*C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(8*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3
*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{\int (a+a\ldots}{}$$

$$= \frac{aA(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{A(a+\ldots}{}$$

$$= \frac{(7A+4C)\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx)\tan(c+dx)}{8d} \ldots$$

$$= \frac{(35A+36C)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{12d} + \frac{(7A+4\ldots}{}$$

$$= \frac{(35A+36C)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{12d} + \frac{(7A+4\ldots}{}$$

$$= -\frac{5a^4(7A+4C)\sin(c+dx)}{8d} + \frac{(35A+36C)\left(a^4+a^4\cos(c\ldots\right.}{12d}$$

$$= 4a^4Cx - \frac{5a^4(7A+4C)\sin(c+dx)}{8d} + \frac{(35A+36C)\left(a^4+a\ldots\right.}{}$$

$$= 4a^4Cx + \frac{a^4(35A+52C)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{5a^4(7A+\ldots}{}$$

Mathematica [A] time = 2.11598, size = 350, normalized size = 1.75

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\sec(c)(105A\sin(2c+dx)+544A\sin(c+2dx)-96A\sin(3c+2dx)+81A\sin(2c\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(35*A + 52*C)*Cos[c + d*x
]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + Sec[c]*(288*C*d*x*Cos[c] + 192*C*d*x*Cos[c + 2*d*x] + 192*
C*d*x*Cos[3*c + 2*d*x] + 48*C*d*x*Cos[3*c + 4*d*x] + 48*C*d*x*Cos[5*c + 4*d
*x] - 480*A*Sin[c] - 288*C*Sin[c] + 105*A*Sin[d*x] + 24*C*Sin[d*x] + 105*A*
Sin[2*c + d*x] + 24*C*Sin[2*c + d*x] + 544*A*Sin[c + 2*d*x] + 288*C*Sin[c +
 2*d*x] - 96*A*Sin[3*c + 2*d*x] - 96*C*Sin[3*c + 2*d*x] + 81*A*Sin[2*c + 3*
d*x] + 30*C*Sin[2*c + 3*d*x] + 81*A*Sin[4*c + 3*d*x] + 30*C*Sin[4*c + 3*d*x
] + 160*A*Sin[3*c + 4*d*x] + 96*C*Sin[3*c + 4*d*x] + 6*C*Sin[4*c + 5*d*x] +
 6*C*Sin[6*c + 5*d*x])))/(3072*d)
```

Maple [A] time = 0.086, size = 197, normalized size = 1.

$$\frac{35\,A a^4 \ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{a^4 C \sin\left(dx+c\right)}{d}+\frac{20\,A a^4 \tan\left(dx+c\right)}{3d}+4\,a^4 C x+4\,\frac{C a^4 c}{d}+\frac{27\,A a^4 \sec\left(dx\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 35/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*C*sin(d*x+c)+20/3/d*A*a^4*ta
n(d*x+c)+4*a^4*C*x+4/d*a^4*C*c+27/8/d*A*a^4*sec(d*x+c)*tan(d*x+c)+13/2/d*a^
4*C*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+4/d*a^4*C
*tan(d*x+c)+1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+1/2/d*a^4*C*sec(d*x+c)*tan(
d*x+c)
```

Maxima [A] time = 1.07609, size = 400, normalized size = 2.

$$64\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)A a^4+192\left(dx+c\right)C a^4-3\,A a^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 192*(d*x + c)*C*a^4 - 3*
A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*A*a^4*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 12*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) + 144*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 48*C*a^4*sin(d*x + c) + 192*A*a^4*tan(d*x + c) + 192*C*a^4*tan(d*x + c))
/d
```

Fricas [A] time = 1.62247, size = 437, normalized size = 2.18

$$192\,C a^4 d x \cos\left(dx+c\right)^4+3\left(35\,A+52\,C\right)a^4 \cos\left(dx+c\right)^4 \log\left(\sin\left(dx+c\right)+1\right)-3\left(35\,A+52\,C\right)a^4 \cos\left(dx+c\right)^4 \log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/48*(192*C*a^4*d*x*cos(d*x + c)^4 + 3*(35*A + 52*C)*a^4*cos(d*x + c)^4*log
(sin(d*x + c) + 1) - 3*(35*A + 52*C)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
 1) + 2*(24*C*a^4*cos(d*x + c)^4 + 32*(5*A + 3*C)*a^4*cos(d*x + c)^3 + 3*(2
7*A + 4*C)*a^4*cos(d*x + c)^2 + 32*A*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40601, size = 342, normalized size = 1.71

$$96\,(dx+c)Ca^4 + \frac{48\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 3\left(35\,Aa^4+52\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\left(35\,Aa^4+52\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] 1/24*(96*(d*x + c)*C*a^4 + 48*C*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 + 1) + 3*(35*A*a^4 + 52*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(35*A*a^4 + 52*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(105*A*a^4*ta
n(1/2*d*x + 1/2*c)^7 + 84*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*A*a^4*tan(1/2*
d*x + 1/2*c)^5 - 276*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 300*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*A*a^4*tan(1/2*d*x + 1/2*
```

```
c) - 108*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.36 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=207

$$\frac{a^4(7A+10C)\tan(c+dx)}{2d}+\frac{a^4(7A+12C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(7A+5C)\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+\right.}{15d}$$

```
[Out] a^4*C*x + (a^4*(7*A + 12*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*(7*A + 10*C
)*Tan[c + d*x])/(2*d) + ((7*A + 8*C)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]*
Tan[c + d*x])/(6*d) + ((7*A + 5*C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^
2*Tan[c + d*x])/(15*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c +
 d*x])/(5*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.665237, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3044, 2975, 2968, 3021, 2735, 3770}

$$\frac{a^4(7A+10C)\tan(c+dx)}{2d}+\frac{a^4(7A+12C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(7A+5C)\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] a^4*C*x + (a^4*(7*A + 12*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*(7*A + 10*C
)*Tan[c + d*x])/(2*d) + ((7*A + 8*C)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]*
Tan[c + d*x])/(6*d) + ((7*A + 5*C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^
2*Tan[c + d*x])/(15*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c +
 d*x])/(5*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{\int (a+a}{}$$

$$= \frac{aA(a+a\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{5d} + \frac{A(a+}{}$$

$$= \frac{(7A+5C)\left(a^2+a^2\cos(c+dx)\right)^2\sec^2(c+dx)\tan(c+dx)}{15d}$$

$$= \frac{(7A+8C)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{6d} +$$

$$= \frac{(7A+8C)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{6d} +$$

$$= \frac{a^4(7A+10C)\tan(c+dx)}{2d} + \frac{(7A+8C)\left(a^4+a^4\cos(c+d\right.}{6d}$$

$$= a^4Cx + \frac{a^4(7A+10C)\tan(c+dx)}{2d} + \frac{(7A+8C)\left(a^4+a^4\cos\right.}{}$$

$$= a^4Cx + \frac{a^4(7A+12C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^4(7A+10C}{2}$$

Mathematica [A] time = 1.79188, size = 389, normalized size = 1.88

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\sec(c)(-480A\sin(2c+dx)+330A\sin(c+2dx)+330A\sin(3c+2dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^5*(-240*(7*A + 12
*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(150*C*d*x*Cos[d*x] + 150*C*d*x*Cos[
2*c + d*x] + 75*C*d*x*Cos[2*c + 3*d*x] + 75*C*d*x*Cos[4*c + 3*d*x] + 15*C*d
*x*Cos[4*c + 5*d*x] + 15*C*d*x*Cos[6*c + 5*d*x] + 1180*A*Sin[d*x] + 1220*C*
Sin[d*x] - 480*A*Sin[2*c + d*x] - 780*C*Sin[2*c + d*x] + 330*A*Sin[c + 2*d*
x] + 120*C*Sin[c + 2*d*x] + 330*A*Sin[3*c + 2*d*x] + 120*C*Sin[3*c + 2*d*x]
 + 800*A*Sin[2*c + 3*d*x] + 820*C*Sin[2*c + 3*d*x] - 30*A*Sin[4*c + 3*d*x]
- 180*C*Sin[4*c + 3*d*x] + 105*A*Sin[3*c + 4*d*x] + 60*C*Sin[3*c + 4*d*x] +
 105*A*Sin[5*c + 4*d*x] + 60*C*Sin[5*c + 4*d*x] + 166*A*Sin[4*c + 5*d*x] +
200*C*Sin[4*c + 5*d*x])))/(7680*d)
```

Maple [A] time = 0.089, size = 226, normalized size = 1.1

$$\frac{83\,Aa^4\tan(dx+c)}{15\,d}+a^4Cx+\frac{Ca^4c}{d}+\frac{7\,Aa^4\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{7\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+6\,\frac{a^4C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 83/15/d*A*a^4*tan(d*x+c)+a^4*C*x+1/d*a^4*C*c+7/2/d*A*a^4*sec(d*x+c)*tan(d*x
+c)+7/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c
))+34/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+20/3/d*a^4*C*tan(d*x+c)+1/d*A*a^4*
tan(d*x+c)*sec(d*x+c)^3+2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+1/5/d*A*a^4*tan(d*x
+c)*sec(d*x+c)^4+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.02836, size = 425, normalized size = 2.05

$$\frac{4\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^4+120\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+20\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 + 12
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 20*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*C*a^4 + 60*(d*x + c)*C*a^4 - 15*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*
x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) +
 3*log(sin(d*x + c) - 1)) - 60*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
 log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*C*a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120
*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 60*A*a^4*tan(d*x +
 c) + 360*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.58336, size = 452, normalized size = 2.18

$$\frac{60\,Ca^4dx\cos(dx+c)^5+15\,(7\,A+12\,C)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(7\,A+12\,C)a^4\cos(dx+c)^5\log(-\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] 1/60*(60*C*a^4*d*x*cos(d*x + c)^5 + 15*(7*A + 12*C)*a^4*cos(d*x + c)^5*log(
sin(d*x + c) + 1) - 15*(7*A + 12*C)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) +
1) + 2*(2*(83*A + 100*C)*a^4*cos(d*x + c)^4 + 15*(7*A + 4*C)*a^4*cos(d*x +
c)^3 + 2*(34*A + 5*C)*a^4*cos(d*x + c)^2 + 30*A*a^4*cos(d*x + c) + 6*A*a^4)
*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26489, size = 347, normalized size = 1.68

$$\frac{30\,(dx+c)Ca^4 + 15\left(7\,Aa^4 + 12\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(7\,Aa^4 + 12\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] 1/30*(30*(d*x + c)*C*a^4 + 15*(7*A*a^4 + 12*C*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 15*(7*A*a^4 + 12*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(105*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 150*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 490
*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 680*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 896*A*a^
4*tan(1/2*d*x + 1/2*c)^5 + 1180*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*A*a^4*ta
n(1/2*d*x + 1/2*c)^3 - 920*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 375*A*a^4*tan(1/2
```

```
*d*x + 1/2*c) + 270*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^5)/d
```

3.37 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=232

$$\frac{4a^4(18A+25C)\tan(c+dx)}{15d}+\frac{7a^4(7A+10C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(417A+550C)\tan(c+dx)\sec(c+dx)}{240d}+\frac{(37A$$

```
[Out] (7*a^4*(7*A + 10*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(18*A + 25*C)*Ta
n[c + d*x])/(15*d) + (a^4*(417*A + 550*C)*Sec[c + d*x]*Tan[c + d*x])/(240*d
) + ((43*A + 50*C)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6
0*d) + ((37*A + 30*C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x
])/(120*d) + (2*a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(15
*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.761468, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3044, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{4a^4(18A+25C)\tan(c+dx)}{15d}+\frac{7a^4(7A+10C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(417A+550C)\tan(c+dx)\sec(c+dx)}{240d}+\frac{(37A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```


```
[Out] (7*a^4*(7*A + 10*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(18*A + 25*C)*Ta
n[c + d*x])/(15*d) + (a^4*(417*A + 550*C)*Sec[c + d*x]*Tan[c + d*x])/(240*d
) + ((43*A + 50*C)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6
0*d) + ((37*A + 30*C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x
])/(120*d) + (2*a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(15
*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
```

```
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{\int(a+a}{}$$

$$=\frac{2aA(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{15d}+\frac{A(a+}{}$$

$$=\frac{(37A+30C)\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c+dx)\tan(c+d}{120d}$$

$$=\frac{(43A+50C)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx}{60d}$$

$$=\frac{(43A+50C)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx}{60d}$$

$$=\frac{a^4(417A+550C)\sec(c+dx)\tan(c+dx)}{240d}+\frac{(43A+50C)}{}$$

$$=\frac{a^4(417A+550C)\sec(c+dx)\tan(c+dx)}{240d}+\frac{(43A+50C)}{}$$

$$=\frac{7a^4(7A+10C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(417A+550C)\mathrm{s}}{2}$$

$$=\frac{7a^4(7A+10C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^4(18A+25C)\mathrm{ta}}{15d}$$

Mathematica [A] time = 2.12287, size = 358, normalized size = 1.54

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(3360(7A+10C)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360*(7*A + 1
0*C)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-640*(18*A + 25*C)*Sin[c] + 30*(12
5*A + 62*C)*Sin[d*x] + 3750*A*Sin[2*c + d*x] + 1860*C*Sin[2*c + d*x] + 1536
0*A*Sin[c + 2*d*x] + 17280*C*Sin[c + 2*d*x] - 1920*A*Sin[3*c + 2*d*x] - 672
0*C*Sin[3*c + 2*d*x] + 3845*A*Sin[2*c + 3*d*x] + 2670*C*Sin[2*c + 3*d*x] +
3845*A*Sin[4*c + 3*d*x] + 2670*C*Sin[4*c + 3*d*x] + 6912*A*Sin[3*c + 4*d*x]
 + 8640*C*Sin[3*c + 4*d*x] - 960*C*Sin[5*c + 4*d*x] + 735*A*Sin[4*c + 5*d*x
] + 810*C*Sin[4*c + 5*d*x] + 735*A*Sin[6*c + 5*d*x] + 810*C*Sin[6*c + 5*d*x
] + 1152*A*Sin[5*c + 6*d*x] + 1600*C*Sin[5*c + 6*d*x])))/(122880*d)
```

Maple [A] time = 0.092, size = 258, normalized size = 1.1

$$\frac{49\,Aa^4\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{49\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{16\,d}+\frac{35\,a^4C\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 49/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)+49/16/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
+35/8/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+24/5/d*A*a^4*tan(d*x+c)+12/5/d*A*a^
4*tan(d*x+c)*sec(d*x+c)^2+20/3/d*a^4*C*tan(d*x+c)+41/24/d*A*a^4*tan(d*x+c)*
sec(d*x+c)^3+27/8/d*a^4*C*sec(d*x+c)*tan(d*x+c)+4/5/d*A*a^4*tan(d*x+c)*sec(
d*x+c)^4+4/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+1/6/d*A*a^4*tan(d*x+c)*sec(d*x
+c)^5+1/4/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [B] time = 1.06607, size = 616, normalized size = 2.66

$$128\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^4+640\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+640\left(\tan(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 640*(tan(d*x + c)^3 + 3*tan(
d*x + c))*C*a^4 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*si
n(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15
*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*A*a^4*(2*(3*sin(d*
x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(
sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*C*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 720*C*a^4*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 240*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 1920
*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.52145, size = 471, normalized size = 2.03

$$\frac{105\,(7\,A+10\,C)a^4\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-105\,(7\,A+10\,C)a^4\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+2\left(64\,(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
fricas")
```

```
[Out] 1/480*(105*(7*A + 10*C)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(7*A
 + 10*C)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(64*(18*A + 25*C)*a^
4*cos(d*x + c)^5 + 15*(49*A + 54*C)*a^4*cos(d*x + c)^4 + 64*(9*A + 5*C)*a^4
*cos(d*x + c)^3 + 10*(41*A + 6*C)*a^4*cos(d*x + c)^2 + 192*A*a^4*cos(d*x +
c) + 40*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28318, size = 378, normalized size = 1.63

$$105\left(7\,Aa^4+10\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(7\,Aa^4+10\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,Aa^4\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
giac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 10*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(7
*A*a^4 + 10*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(735*A*a^4*tan(1/
2*d*x + 1/2*c)^11 + 1050*C*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*A*a^4*tan(1/2
*d*x + 1/2*c)^9 - 5950*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*
x + 1/2*c)^7 + 13860*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*A*a^4*tan(1/2*d*x
 + 1/2*c)^5 - 16860*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 10690*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*A*a^4*tan(1/2*d*x + 1
/2*c) - 2790*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.38 $\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx$

Optimal. Leaf size=263

$$\frac{a^4(454A+581C)\tan(c+dx)}{105d}+\frac{a^4(11A+14C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(247A+308C)\tan(c+dx)\sec^2(c+dx)}{210d}+\frac{a^4(}{}$$

```
[Out] (a^4*(11*A + 14*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^4*(454*A + 581*C)*Tan[
c + d*x])/(105*d) + (a^4*(11*A + 14*C)*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (
a^4*(247*A + 308*C)*Sec[c + d*x]^2*Tan[c + d*x])/(210*d) + ((109*A + 126*C)
*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(210*d) + ((8*A + 7*
C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(35*d) + (2*a*A*
(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(21*d) + (A*(a + a*Cos[
c + d*x])^4*Sec[c + d*x]^6*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.802422, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3044, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^4(454A+581C)\tan(c+dx)}{105d}+\frac{a^4(11A+14C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(247A+308C)\tan(c+dx)\sec^2(c+dx)}{210d}+\frac{a^4(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```


```
[Out] (a^4*(11*A + 14*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^4*(454*A + 581*C)*Tan[
c + d*x])/(105*d) + (a^4*(11*A + 14*C)*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (
a^4*(247*A + 308*C)*Sec[c + d*x]^2*Tan[c + d*x])/(210*d) + ((109*A + 126*C)
*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(210*d) + ((8*A + 7*
C)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(35*d) + (2*a*A*
(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(21*d) + (A*(a + a*Cos[
c + d*x])^4*Sec[c + d*x]^6*Tan[c + d*x])/(7*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
```

```
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^6(c+dx)\tan(c+dx)}{7d} + \frac{\int (a+\ldots}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{21d} + \frac{A(a\ldots}{}$$

$$= \frac{(8A+7C)\left(a^2+a^2\cos(c+dx)\right)^2\sec^4(c+dx)\tan(c+d\ldots}{35d}$$

$$= \frac{(109A+126C)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(c\ldots}{210d}$$

$$= \frac{(109A+126C)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(c\ldots}{210d}$$

$$= \frac{a^4(247A+308C)\sec^2(c+dx)\tan(c+dx)}{210d} + \frac{(109A+1\ldots}{}$$

$$= \frac{a^4(247A+308C)\sec^2(c+dx)\tan(c+dx)}{210d} + \frac{(109A+1\ldots}{}$$

$$= \frac{a^4(11A+14C)\sec(c+dx)\tan(c+dx)}{4d} + \frac{a^4(247A+30\ldots}{}$$

$$= \frac{a^4(11A+14C)\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a^4(454A+581C)\ldots}{105d}$$

Mathematica [A] time = 3.10264, size = 390, normalized size = 1.48

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(6720(11A+14C)\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\ldots\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^7*(6720*(11*A +
14*C)*Cos[c + d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(560*(83*A + 91*C)*Sin[d*x] - 140*
(122*A + 217*C)*Sin[2*c + d*x] + 16415*A*Sin[c + 2*d*x] + 10710*C*Sin[c + 2
*d*x] + 16415*A*Sin[3*c + 2*d*x] + 10710*C*Sin[3*c + 2*d*x] + 37296*A*Sin[2
*c + 3*d*x] + 41244*C*Sin[2*c + 3*d*x] - 840*A*Sin[4*c + 3*d*x] - 7560*C*Si
n[4*c + 3*d*x] + 7700*A*Sin[3*c + 4*d*x] + 7560*C*Sin[3*c + 4*d*x] + 7700*A
*Sin[5*c + 4*d*x] + 7560*C*Sin[5*c + 4*d*x] + 12712*A*Sin[4*c + 5*d*x] + 15
848*C*Sin[4*c + 5*d*x] - 420*C*Sin[6*c + 5*d*x] + 1155*A*Sin[5*c + 6*d*x] +
```

```
1470*C*Sin[5*c + 6*d*x] + 1155*A*Sin[7*c + 6*d*x] + 1470*C*Sin[7*c + 6*d*x
] + 1816*A*Sin[6*c + 7*d*x] + 2324*C*Sin[6*c + 7*d*x])))/(430080*d)
```

Maple [A] time = 0.115, size = 303, normalized size = 1.2

$$\frac{454\,Aa^4\tan(dx+c)}{105\,d}+\frac{227\,Aa^4\tan(dx+c)(\sec(dx+c))^2}{105\,d}+\frac{83\,a^4C\tan(dx+c)}{15\,d}+\frac{11\,Aa^4\tan(dx+c)(\sec(dx+c))^3}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x)
```

```
[Out] 454/105/d*A*a^4*tan(d*x+c)+227/105/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+83/15/d*
a^4*C*tan(d*x+c)+11/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+11/4/d*A*a^4*sec(d*x+
c)*tan(d*x+c)+11/4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*a^4*C*sec(d*x+c)
*tan(d*x+c)+7/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+48/35/d*A*a^4*tan(d*x+c)*
sec(d*x+c)^4+34/15/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+2/3/d*A*a^4*tan(d*x+c)*s
ec(d*x+c)^5+1/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+1/7/d*A*a^4*tan(d*x+c)*sec(d*
x+c)^6+1/5/d*a^4*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 1.05522, size = 624, normalized size = 2.37

$$\frac{24\left(5\,\tan(dx+c)^7+21\,\tan(dx+c)^5+35\,\tan(dx+c)^3+35\,\tan(dx+c)\right)Aa^4+336\left(3\,\tan(dx+c)^5+10\,\tan(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="
maxima")
```

```
[Out] 1/840*(24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*ta
n(d*x + c))*A*a^4 + 336*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
+ c))*A*a^4 + 280*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 56*(3*tan(d*x +
 c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^4 + 1680*(tan(d*x + c)^3 +
 3*tan(d*x + c))*C*a^4 - 35*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3
 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 -
 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 210*A*a^4*(2*(
3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1)
- 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 210*C*a^4*(2*(3*sin(
```

```
d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*C*a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 840
*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.47776, size = 531, normalized size = 2.02

$$\frac{105\,(11\,A + 14\,C)a^4\cos{(dx + c)}^7\log{(\sin{(dx + c)} + 1)} - 105\,(11\,A + 14\,C)a^4\cos{(dx + c)}^7\log{(-\sin{(dx + c)} + 1)} + 2\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="
fricas")
```

```
[Out] 1/840*(105*(11*A + 14*C)*a^4*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(11
*A + 14*C)*a^4*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 2*(4*(454*A + 581*C)
*a^4*cos(d*x + c)^6 + 105*(11*A + 14*C)*a^4*cos(d*x + c)^5 + 4*(227*A + 238
*C)*a^4*cos(d*x + c)^4 + 70*(11*A + 6*C)*a^4*cos(d*x + c)^3 + 12*(48*A + 7*
C)*a^4*cos(d*x + c)^2 + 280*A*a^4*cos(d*x + c) + 60*A*a^4)*sin(d*x + c))/(d
*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2876, size = 424, normalized size = 1.61

$$\frac{105\left(11\,Aa^4 + 14\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 105\left(11\,Aa^4 + 14\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(1155\,Aa^4\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="
giac")
```

```
[Out] 1/420*(105*(11*A*a^4 + 14*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(
11*A*a^4 + 14*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(1155*A*a^4*tan
(1/2*d*x + 1/2*c)^13 + 1470*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 7700*A*a^4*tan(
1/2*d*x + 1/2*c)^11 - 9800*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 21791*A*a^4*tan(
1/2*d*x + 1/2*c)^9 + 27734*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 33792*A*a^4*tan(1
/2*d*x + 1/2*c)^7 - 43008*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 31521*A*a^4*tan(1/
2*d*x + 1/2*c)^5 + 39914*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 14700*A*a^4*tan(1/2
*d*x + 1/2*c)^3 - 21560*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 5565*A*a^4*tan(1/2*d
*x + 1/2*c) + 5250*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^7)/d
```

3.39 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{(3A+4C)\sin^3(c+dx)}{3ad}-\frac{(3A+4C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(4A+5C)\sin(c+dx)\cos^3(c+}{4ad}$$

```
[Out] (3*(4*A + 5*C)*x)/(8*a) - ((3*A + 4*C)*Sin[c + d*x])/(a*d) + (3*(4*A + 5*C)
*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((4*A + 5*C)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a*d) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
)) + ((3*A + 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.191381, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2748, 2633, 2635, 8}

$$\frac{(3A+4C)\sin^3(c+dx)}{3ad}-\frac{(3A+4C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(4A+5C)\sin(c+dx)\cos^3(c+}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(4*A + 5*C)*x)/(8*a) - ((3*A + 4*C)*Sin[c + d*x])/(a*d) + (3*(4*A + 5*C)
*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((4*A + 5*C)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a*d) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
)) + ((3*A + 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx &= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^3(c+dx)(-a(3A+4C)+a(4A+5}{a^2}\\ &= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A+4C)\int \cos^3(c+dx)\,dx}{a}+\frac{(4A+5}{}\\ &= \frac{(4A+5C)\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{(3(}{}\\ &= -\frac{(3A+4C)\sin(c+dx)}{ad}+\frac{3(4A+5C)\cos(c+dx)\sin(c+dx)}{8ad}+\frac{(4A+5C)\,c}{}\\ &= \frac{3(4A+5C)x}{8a}-\frac{(3A+4C)\sin(c+dx)}{ad}+\frac{3(4A+5C)\cos(c+dx)\sin(c+dx)}{8ad}\end{aligned}$$

Mathematica [A] time = 0.530179, size = 283, normalized size = 1.81

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(72dx(4A+5C)\cos\left(c+\frac{dx}{2}\right)-96A\sin\left(c+\frac{dx}{2}\right)-72A\sin\left(c+\frac{3dx}{2}\right)-72A\sin\left(2c+\frac{3dx}{2}\right)+24A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(72*(4*A + 5*C)*d*x*Cos[(d*x)/2] + 72*(4*A + 5*C
)*d*x*Cos[c + (d*x)/2] - 480*A*Sin[(d*x)/2] - 552*C*Sin[(d*x)/2] - 96*A*Sin
[c + (d*x)/2] - 168*C*Sin[c + (d*x)/2] - 72*A*Sin[c + (3*d*x)/2] - 120*C*Si
n[c + (3*d*x)/2] - 72*A*Sin[2*c + (3*d*x)/2] - 120*C*Sin[2*c + (3*d*x)/2] +
 24*A*Sin[2*c + (5*d*x)/2] + 40*C*Sin[2*c + (5*d*x)/2] + 24*A*Sin[3*c + (5*
d*x)/2] + 40*C*Sin[3*c + (5*d*x)/2] - 5*C*Sin[3*c + (7*d*x)/2] - 5*C*Sin[4*
c + (7*d*x)/2] + 3*C*Sin[4*c + (9*d*x)/2] + 3*C*Sin[5*c + (9*d*x)/2]))/(192
*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.057, size = 352, normalized size = 2.3

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{(\tan(1/2\,dx+c/2))^7 A}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^4}-\frac{25\,C}{4\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(tan(1/2*d*x+1
/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*A-25/4/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan
(1/2*d*x+1/2*c)^7*C-7/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*A
-115/12/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*C-5/a/d/(tan(1/
2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*A-109/12/a/d/(tan(1/2*d*x+1/2*c)^2
+1)^4*tan(1/2*d*x+1/2*c)^3*C-1/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*A*tan(1/2*d*x
+1/2*c)-7/4/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*C*tan(1/2*d*x+1/2*c)+3/a/d*arcta
n(tan(1/2*d*x+1/2*c))*A+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.53064, size = 474, normalized size = 3.04

$$-\frac{C\left(\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+12\,A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+\cdots}{(\cos(dx+c)\cdots}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(\cdots}{(\cos(dx\cdots}}\cdots\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(C*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) + 12*A*((sin(d*x +
c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) -
 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c
) + 1))))/d
```

Fricas [A] time = 1.46508, size = 290, normalized size = 1.86

$$\frac{9\,(4\,A+5\,C)dx\cos{(dx+c)}+9\,(4\,A+5\,C)dx+\left(6\,C\cos{(dx+c)}^4-2\,C\cos{(dx+c)}^3+(12\,A+13\,C)\cos{(dx+c)}^2-(12}{24\,(ad\cos{(dx+c)}+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(9*(4*A + 5*C)*d*x*cos(d*x + c) + 9*(4*A + 5*C)*d*x + (6*C*cos(d*x + c
)^4 - 2*C*cos(d*x + c)^3 + (12*A + 13*C)*cos(d*x + c)^2 - (12*A + 19*C)*cos
(d*x + c) - 48*A - 64*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 12.5831, size = 1795, normalized size = 11.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((36*A*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*
d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) + 144*A*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8
```

```
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) + 216*A*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d
*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*
d*tan(c/2 + d*x/2)**2 + 24*a*d) + 144*A*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan
(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 36*A*d*x/(24*a*d*tan(c/2 + d*x/2)
**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan
(c/2 + d*x/2)**2 + 24*a*d) - 24*A*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x
/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*
tan(c/2 + d*x/2)**2 + 24*a*d) - 168*A*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 +
 d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*
a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 312*A*tan(c/2 + d*x/2)**5/(24*a*d*tan(c
/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 +
 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 216*A*tan(c/2 + d*x/2)**3/(24*a*d*t
an(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)*
*4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 48*A*tan(c/2 + d*x/2)/(24*a*d*t
an(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)*
*4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*C*d*x*tan(c/2 + d*x/2)**8/(2
4*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*C*d*x*tan(c/2 + d*x/
2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*ta
n(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 270*C*d*x*tan(c/
2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 14
4*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*C*d*
x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)
**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) +
45*C*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d
*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 24*C*tan(c/2
+ d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*
a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 246*C*tan(
c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 +
144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 374*C*
tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**
6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 31
4*C*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/
2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
- 66*C*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/
2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d),
 Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**3/(a*cos(c) + a), True))
```

<u>Giac</u> [A] time = 1.17939, size = 243, normalized size = 1.56

$$\frac{\frac{9\,(dx+c)(4\,A+5\,C)}{a} - \frac{24\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a} - \frac{2\left(36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+75\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+84\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+115\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(9*(d*x + c)*(4*A + 5*C)/a - 24*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*
x + 1/2*c))/a - 2*(36*A*tan(1/2*d*x + 1/2*c)^7 + 75*C*tan(1/2*d*x + 1/2*c)^
7 + 84*A*tan(1/2*d*x + 1/2*c)^5 + 115*C*tan(1/2*d*x + 1/2*c)^5 + 60*A*tan(1
/2*d*x + 1/2*c)^3 + 109*C*tan(1/2*d*x + 1/2*c)^3 + 12*A*tan(1/2*d*x + 1/2*c
) + 21*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.40 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{(3A+4C)\sin^3(c+dx)}{3ad}+\frac{(3A+4C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A+3C)\sin(c+dx)\cos(c+$$

```
[Out] -((2*A + 3*C)*x)/(2*a) + ((3*A + 4*C)*Sin[c + d*x])/(a*d) - ((2*A + 3*C)*Co
s[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(d
*(a + a*Cos[c + d*x])) - ((3*A + 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.173477, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2748, 2635, 8, 2633}

$$-\frac{(3A+4C)\sin^3(c+dx)}{3ad}+\frac{(3A+4C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A+3C)\sin(c+dx)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((2*A + 3*C)*x)/(2*a) + ((3*A + 4*C)*Sin[c + d*x])/(a*d) - ((2*A + 3*C)*Co
s[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(d
*(a + a*Cos[c + d*x])) - ((3*A + 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx &= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^2(c+dx)(-a(2A+3C)+a(3A+4}{a^2}\\
&= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A+3C)\int \cos^2(c+dx)\,dx}{a}+\frac{(3A+4C}{}\\
&= -\frac{(2A+3C)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A}{}\\
&= -\frac{(2A+3C)x}{2a}+\frac{(3A+4C)\sin(c+dx)}{ad}-\frac{(2A+3C)\cos(c+dx)\sin(c+dx)}{2ad}-
\end{aligned}$$

Mathematica [A] time = 0.592904, size = 225, normalized size = 1.81

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-12dx(2A+3C)\cos\left(c+\frac{dx}{2}\right)+12A\sin\left(c+\frac{dx}{2}\right)+12A\sin\left(c+\frac{3dx}{2}\right)+12A\sin\left(2c+\frac{3dx}{2}\right)-12a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-12*(2*A + 3*C)*d*x*Cos[(d*x)/2] - 12*(2*A + 3*
C)*d*x*Cos[c + (d*x)/2] + 60*A*Sin[(d*x)/2] + 69*C*Sin[(d*x)/2] + 12*A*Sin[
c + (d*x)/2] + 21*C*Sin[c + (d*x)/2] + 12*A*Sin[c + (3*d*x)/2] + 18*C*Sin[c
 + (3*d*x)/2] + 12*A*Sin[2*c + (3*d*x)/2] + 18*C*Sin[2*c + (3*d*x)/2] - 2*C
*Sin[2*c + (5*d*x)/2] - 2*C*Sin[3*c + (5*d*x)/2] + C*Sin[3*c + (7*d*x)/2] +
 C*Sin[4*c + (7*d*x)/2]))/(24*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.032, size = 280, normalized size = 2.3

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{(\tan(1/2\,dx+c/2))^5 A}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^3}+5\,\frac{(\tan(1/2\,dx+c/2))^5 C}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^3}+4\,\frac{(\mathrm{ta}}{ad\left((\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d/(tan(1/2*d*x+1/
2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*A+5/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2
*d*x+1/2*c)^5*C+4/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*A+16/
3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*C+2/a/d/(tan(1/2*d*x+
1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*c)+3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1
/2*d*x+1/2*c)-2/a/d*arctan(tan(1/2*d*x+1/2*c))*A-3/a/d*arctan(tan(1/2*d*x+1
/2*c))*C
```

Maxima [B] time = 1.57856, size = 363, normalized size = 2.93

$$\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(d}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
```

```
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*A*(2*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.41263, size = 243, normalized size = 1.96

$$-\frac{3\,(2\,A+3\,C)dx\cos\left(dx+c\right)+3\,(2\,A+3\,C)dx-\left(2\,C\cos\left(dx+c\right)^3-C\cos\left(dx+c\right)^2+(6\,A+7\,C)\cos\left(dx+c\right)+12\,A-}{6\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(2*A + 3*C)*d*x*cos(d*x + c) + 3*(2*A + 3*C)*d*x - (2*C*cos(d*x + c
)^3 - C*cos(d*x + c)^2 + (6*A + 7*C)*cos(d*x + c) + 12*A + 16*C)*sin(d*x +
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 7.0312, size = 1163, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-6*A*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d
*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 18*A*d*x*tan(c
/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18
*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 18*A*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan
(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2
+ 6*a*d) - 6*A*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
+ 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*A*tan(c/2 + d*x/2)**7/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 30*A*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(
c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 42*A*tan(c/2 + d*x/
2)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(
c/2 + d*x/2)**2 + 6*a*d) + 18*A*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6
```

```
 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*C*d
*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)
**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*C*d*x*tan(c/2 + d*x/2)**4/(6
*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*
x/2)**2 + 6*a*d) - 27*C*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*C*d*
x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2
+ d*x/2)**2 + 6*a*d) + 6*C*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*C*ta
n(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*C*tan(c/2 + d*x/2)**3/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 24*C*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*(A + C*co
s(c)**2)*cos(c)**2/(a*cos(c) + a), True))
```

Giac [A] time = 1.2175, size = 205, normalized size = 1.65

$$-\frac{\frac{3(dx+c)(2A+3C)}{a} - \frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} - \frac{2\left(6A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+16C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3 a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(2*A + 3*C)/a - 6*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x
 + 1/2*c))/a - 2*(6*A*tan(1/2*d*x + 1/2*c)^5 + 15*C*tan(1/2*d*x + 1/2*c)^5
+ 12*A*tan(1/2*d*x + 1/2*c)^3 + 16*C*tan(1/2*d*x + 1/2*c)^3 + 6*A*tan(1/2*d
*x + 1/2*c) + 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))
/d
```

3.41 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=98

$$-\frac{(A+2C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(2A+3C)\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x(2A+3C)}{2a}$$

```
[Out] ((2*A + 3*C)*x)/(2*a) - ((A + 2*C)*Sin[c + d*x])/(a*d) + ((2*A + 3*C)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
 + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0935063, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3042, 2734}

$$-\frac{(A+2C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(2A+3C)\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x(2A+3C)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((2*A + 3*C)*x)/(2*a) - ((A + 2*C)*Sin[c + d*x])/(a*d) + ((2*A + 3*C)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
 + a*Cos[c + d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos(c+dx)(-a(A+2C)+a(2A+3C\cdots}{a^2}$$
$$=\frac{(2A+3C)x}{2a}-\frac{(A+2C)\sin(c+dx)}{ad}+\frac{(2A+3C)\cos(c+dx)\sin(c+dx)}{2ad}-\cdots$$

Mathematica [A] time = 0.332249, size = 159, normalized size = 1.62

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4dx(2A+3C)\cos\left(c+\frac{dx}{2}\right)+4dx(2A+3C)\cos\left(\frac{dx}{2}\right)-16A\sin\left(\frac{dx}{2}\right)-4C\sin\left(c+\frac{dx}{2}\right)-3C\sin\cdots\right.}{8ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(2*A + 3*C)*d*x*Cos[(d*x)/2] + 4*(2*A + 3*C)*
d*x*Cos[c + (d*x)/2] - 16*A*Sin[(d*x)/2] - 20*C*Sin[(d*x)/2] - 4*C*Sin[c +
(d*x)/2] - 3*C*Sin[c + (3*d*x)/2] - 3*C*Sin[2*c + (3*d*x)/2] + C*Sin[2*c +
(5*d*x)/2] + C*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.03, size = 144, normalized size = 1.5

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{C\left(\tan\left(1/2\,dx+c/2\right)\right)^3}{ad\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2+1\right)^2}-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(tan(1/2*d*x+1
/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3-1/a/d/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(
1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))*A+3/a/d*arctan(tan(1/2*d*x+
```

```
1/2*c))*C
```

Maxima [A] time = 1.52889, size = 248, normalized size = 2.53

$$-\frac{C\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(C*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) - A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.39718, size = 192, normalized size = 1.96

$$\frac{(2A+3C)dx\cos(dx+c)+(2A+3C)dx+\left(C\cos(dx+c)^2-C\cos(dx+c)-2A-4C\right)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((2*A + 3*C)*d*x*cos(d*x + c) + (2*A + 3*C)*d*x + (C*cos(d*x + c)^2 - C
*cos(d*x + c) - 2*A - 4*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 3.78659, size = 665, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*A*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*t
an(c/2 + d*x/2)**2 + 2*a*d) + 4*A*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 +
d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*A*d*x/(2*a*d*tan(c/2 + d
*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*A*tan(c/2 + d*x/2)**5/(2*
a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*A*tan(c/2
+ d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d)
 - 2*A*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)
**2 + 2*a*d) + 3*C*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a
*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*C*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*C*d*x/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*C*tan(c/2 + d*x/2)**5
/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*C*tan
(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2
*a*d) - 4*C*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d
*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)/(a*cos(c) + a), T
rue))
```

Giac [A] time = 1.18069, size = 130, normalized size = 1.33

$$\frac{\frac{(dx+c)(2A+3C)}{a} - \frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} - \frac{2\left(3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((d*x + c)*(2*A + 3*C)/a - 2*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x +
1/2*c))/a - 2*(3*C*tan(1/2*d*x + 1/2*c)^3 + C*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.42 $\int \frac{A+C\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{(A+C)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{C\sin(c+dx)}{ad}-\frac{Cx}{a}$$

```
[Out] -((C*x)/a) + (C*Sin[c + d*x])/(a*d) + ((A + C)*Sin[c + d*x])/(a*d*(1 + Cos[
c + d*x]))
```

Rubi [A] time = 0.0966013, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3024, 2735, 2648}

$$\frac{(A+C)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{C\sin(c+dx)}{ad}-\frac{Cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((C*x)/a) + (C*Sin[c + d*x])/(a*d) + ((A + C)*Sin[c + d*x])/(a*d*(1 + Cos[
c + d*x]))
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{C\sin(c+dx)}{ad}+\frac{\int \frac{aA-aC\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\ &= -\frac{Cx}{a}+\frac{C\sin(c+dx)}{ad}+(A+C)\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= -\frac{Cx}{a}+\frac{C\sin(c+dx)}{ad}+\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.243367, size = 108, normalized size = 2.25

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(4A\sin\left(\frac{dx}{2}\right)+C\sin\left(c+\frac{dx}{2}\right)+C\sin\left(c+\frac{3dx}{2}\right)+C\sin\left(2c+\frac{3dx}{2}\right)-2Cdx\cos\left(c+\frac{dx}{2}\right)+5C\sin\right.}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*C*d*x*Cos[(d*x)/2] - 2*C*d*x*Cos[c + (d*x)/2
] + 4*A*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2] + C*Sin[c + (d*x)/2] + C*Sin[c + (3
*d*x)/2] + C*Sin[2*c + (3*d*x)/2]))/(4*a*d)
```

Maple [A] time = 0.028, size = 88, normalized size = 1.8

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{C\tan(1/2\,dx+c/2)}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)}-2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,C}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d*C*tan(1/2*d*x+1
/2*c)/(tan(1/2*d*x+1/2*c)^2+1)-2/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.54586, size = 158, normalized size = 3.29

$$-\frac{C\left(\frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\,\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.3344, size = 132, normalized size = 2.75

$$-\frac{Cdx\cos(dx+c)+Cdx-(C\cos(dx+c)+A+2\,C)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(C*d*x*cos(d*x + c) + C*d*x - (C*cos(d*x + c) + A + 2*C)*sin(d*x + c))/(a*
d*cos(d*x + c) + a*d)
```

Sympy [A] time = 2.04413, size = 202, normalized size = 4.21

$$\begin{cases}\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{Cdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{Cdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{3C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\left(A+C\cos^2(c)\right)}{a\cos(c)+a} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) + A*tan(c/
2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) - C*d*x*tan(c/2 + d*x/2)**2/(a*d
```

```
*tan(c/2 + d*x/2)**2 + a*d) - C*d*x/(a*d*tan(c/2 + d*x/2)**2 + a*d) + C*tan
(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) + 3*C*tan(c/2 + d*x/2)/(a*
d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*(A + C*cos(c)**2)/(a*cos(c) + a
), True))
```

Giac [A] time = 1.1696, size = 100, normalized size = 2.08

$$-\frac{\frac{(dx+c)C}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*C/a - (A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*C
*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.43 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=48

$$-\frac{(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{a}$$

```
[Out] (C*x)/a + (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A + C)*Sin[c + d*x])/(d*(a +
a*Cos[c + d*x]))
```

Rubi [A] time = 0.103687, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3042, 2735, 3770}

$$-\frac{(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (C*x)/a + (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A + C)*Sin[c + d*x])/(d*(a +
a*Cos[c + d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int(aA+aC\cos(c+dx))\sec(c+dx)\,dx}{a^2}\\ &= \frac{Cx}{a}-\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{A\int\sec(c+dx)\,dx}{a}\\ &= \frac{Cx}{a}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.317462, size = 114, normalized size = 2.38

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2]*(C*d*x - A*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - (A + C)*Sec[
c/2]*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.05, size = 98, normalized size = 2.

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,d\right.\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*A*ln(tan
(1/2*d*x+1/2*c)+1)-1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*
c))*C
```

Maxima [B] time = 1.56837, size = 169, normalized size = 3.52

$$\frac{C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] (C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.37114, size = 242, normalized size = 5.04

$$\frac{2\,Cdx\cos\left(dx+c\right)+2\,Cdx+\left(A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(A\cos\left(dx+c\right)+A\right)\log\left(-\sin\left(dx+c\right)+1\right)-2}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*C*d*x*cos(d*x + c) + 2*C*d*x + (A*cos(d*x + c) + A)*log(sin(d*x + c)
 + 1) - (A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*(A + C)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx+\int \frac{C\cos^{2}\left(c+dx\right)\sec\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(C*cos(c + d*x)**
2*sec(c + d*x)/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.23023, size = 108, normalized size = 2.25

$$\frac{\frac{(dx+c)C}{a} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((d*x + c)*C/a + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a
)/d
```

3.44 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{(2A+C)\tan(c+dx)}{ad}-\frac{(A+C)\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{A\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] -((A*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A + C)*Tan[c + d*x])/(a*d) - ((A +
 C)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.139345, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2748, 3767, 8, 3770}

$$\frac{(2A+C)\tan(c+dx)}{ad}-\frac{(A+C)\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{A\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((A*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A + C)*Tan[c + d*x])/(a*d) - ((A +
 C)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A+C)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int (a(2A+C)-aA\cos(c+dx))\sec^2(c+dx)\,dx}{a^2}\\ &= -\frac{(A+C)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{A\int \sec(c+dx)\,dx}{a}+\frac{(2A+C)\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A+C)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A+C)\,\mathrm{Subst}(\int 1\,dx,x}{ad}\\ &= -\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(2A+C)\tan(c+dx)}{ad}-\frac{(A+C)\tan(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 1.65843, size = 229, normalized size = 3.75

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left((A+C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+A\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)-\right.}\right.\right.}{ad(\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((A + C)*Sec[c/2]
*Sin[(d*x)/2] + A*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
```

```
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/
2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])))))/(a*d*(1 + Cos[c + d*x])*(2*A + C + C*Cos[2*(
c + d*x)]))
```

Maple [A] time = 0.059, size = 121, normalized size = 2.

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-1/a/d*A/(tan(1/2*d*x+
1/2*c)-1)+1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1
/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.00284, size = 194, normalized size = 3.18

$$-\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-\frac{C\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - C*sin(d*x
 + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.41619, size = 288, normalized size = 4.72

$$-\frac{\left(A\cos(dx+c)^2+A\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left(A\cos(dx+c)^2+A\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*((A*cos(d*x + c)^2 + A*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*cos(d*
x + c)^2 + A*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*((2*A + C)*cos(d*x +
c) + A)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{C\cos^2(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(C*cos(c + d*x
)**2*sec(c + d*x)**2/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.28109, size = 136, normalized size = 2.23

$$-\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -(A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
 1))/a - (A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*A*tan(1/2*
d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.45 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{(2A+C)\tan(c+dx)}{ad}+\frac{(3A+2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A+2C)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A+C)\tan(c+dx)\,\mathrm{se}}{d(a\cos(c+dx)-}$$

```
[Out] ((3*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((2*A + C)*Tan[c + d*x])/(a*d
) + ((3*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.175834, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2748, 3768, 3770, 3767, 8}

$$-\frac{(2A+C)\tan(c+dx)}{ad}+\frac{(3A+2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A+2C)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A+C)\tan(c+dx)\,\mathrm{se}}{d(a\cos(c+dx)-}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```


```
[Out] ((3*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((2*A + C)*Tan[c + d*x])/(a*d
) + ((3*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int(a(3A+2C)-a(2A+C)\cos(c+dx)}{a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A+C)\int\sec^2(c+dx)\,dx}{a}+\frac{(3A+2C}{}$$

$$= \frac{(3A+2C)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A+C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{(3A}{}$$

$$= \frac{(3A+2C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(2A+C)\tan(c+dx)}{ad}+\frac{(3A+2C)\sec(c}{2}$$

Mathematica [B] time = 2.61969, size = 284, normalized size = 2.7

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-2(3A+2C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\cdots}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\cdots\right.\right.}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-4*(A + C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-2*
(3*A + 2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*A*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + 4*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + A
/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (4*A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))
)/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.06, size = 209, normalized size = 2.

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3A}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{3A}{2da}\ln\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)+1/2/a/d*A/(tan(1/2*d
*x+1/2*c)-1)^2+3/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*A*ln(tan(1/2*d*x+1/
2*c)-1)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2
+3/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)+3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)+1/a/d*
ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [B] time = 1.02371, size = 323, normalized size = 3.08

$$-\frac{A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-2C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+\cdots}{\cos(dx+c\cdots}\right.}{a}\cdots\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2*(A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) - 2*C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.50407, size = 377, normalized size = 3.59

$$\frac{\big((3A+2C)\cos(dx+c)^3 + (3A+2C)\cos(dx+c)^2\big)\log(\sin(dx+c)+1) - \big((3A+2C)\cos(dx+c)^3 + (3A+2C)\,c}{4\big(ad\cos(dx+c)^3 + ad\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*(((3*A + 2*C)*cos(d*x + c)^3 + (3*A + 2*C)*cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - ((3*A + 2*C)*cos(d*x + c)^3 + (3*A + 2*C)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(2*(2*A + C)*cos(d*x + c)^2 + A*cos(d*x + c) - A)*sin
(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25451, size = 176, normalized size = 1.68

$$\frac{\frac{(3A+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(3A+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-A\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((3*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*A + 2*C)*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x +
 1/2*c))/a + 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - A*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.46 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=133

$$\frac{(4A+3C)\tan^3(c+dx)}{3ad}+\frac{(4A+3C)\tan(c+dx)}{ad}-\frac{(3A+2C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(3A+2C)\tan(c+dx)\sec(c+d}{2ad}$$

```
[Out] -((3*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A + 3*C)*Tan[c + d*x])/(
a*d) - ((3*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d
*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A + 3*C)*Tan[c + d*x]^3)
/(3*a*d)
```

Rubi [A] time = 0.183244, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2748, 3767, 3768, 3770}

$$\frac{(4A+3C)\tan^3(c+dx)}{3ad}+\frac{(4A+3C)\tan(c+dx)}{ad}-\frac{(3A+2C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(3A+2C)\tan(c+dx)\sec(c+d}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```


```
[Out] -((3*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A + 3*C)*Tan[c + d*x])/(
a*d) - ((3*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d
*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A + 3*C)*Tan[c + d*x]^3)
/(3*a*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int(a(4A+3C)-a(3A+2C)\cos(c+dx}{a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3A+2C)\int\sec^3(c+dx)\,dx}{a} + \frac{(4A+3}{}$$

$$= -\frac{(3A+2C)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3}{}$$

$$= -\frac{(3A+2C)\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(4A+3C)\tan(c+dx)}{ad} - \frac{(3A+2C)\sec(}{}$$

Mathematica [B] time = 6.46505, size = 765, normalized size = 5.75

$$\frac{2\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(5A\sin\left(\frac{dx}{2}\right)+3C\sin\left(\frac{dx}{2}\right)\right)}{3d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(a\cos(c+dx)+a)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{2\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(5A\sin\left(\frac{dx}{2}\right)+3\right.}{3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)(a\cos(c+dx)+a)\left(\sin\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((3*A + 2*C)*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2]])/(d*(a + a*Cos[c + d*x])) + ((-3*A - 2*C)*Cos[c/2 + (d*x)/2]^2*Log[Cos[
c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])) + (2*Cos[c/2
 + (d*x)/2]*Sec[c/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(d*(a + a*Cos[c + d
*x])) + (A*Cos[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(3*d*(a + a*Cos[c + d*x])*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c/2
+ (d*x)/2]^2*(-(A*Cos[c/2]) + 2*A*Sin[c/2]))/(3*d*(a + a*Cos[c + d*x])*(Cos
[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (2*Cos[c/2
 + (d*x)/2]^2*(5*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(3*d*(a + a*Cos[c + d*
x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (A*C
os[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(3*d*(a + a*Cos[c + d*x])*(Cos[c/2] + Sin
[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2
*(A*Cos[c/2] + 2*A*Sin[c/2]))/(3*d*(a + a*Cos[c + d*x])*(Cos[c/2] + Sin[c/2
])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (2*Cos[c/2 + (d*x)/2]^2*(
5*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(3*d*(a + a*Cos[c + d*x])*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.062, size = 294, normalized size = 2.2

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3\,A}{2\,da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-1/3/a/d*A/(tan(1/2*d*
x+1/2*c)-1)^3-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2+3/2/a/d*A*ln(tan(1/2*d*x+1/2
*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-5/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)-1/
a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/3/a/d*A/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d*A/(ta
n(1/2*d*x+1/2*c)+1)^2-3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(tan(1/2*d
*x+1/2*c)+1)*C-5/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1/a/d/(tan(1/2*d*x+1/2*c)+1
)*C
```

Maxima [B] time = 1.00923, size = 439, normalized size = 3.3

$$\frac{A\left(\frac{2\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\,\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-6\,C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 6*C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin
(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c)
+ 1))))/d
```

Fricas [A] time = 1.49427, size = 429, normalized size = 3.23

$$-\frac{3\left((3\,A+2\,C)\cos(dx+c)^4+(3\,A+2\,C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((3\,A+2\,C)\cos(dx+c)^4+(3\,A+2\,C\right.}{12\left(ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((3*A + 2*C)*cos(d*x + c)^4 + (3*A + 2*C)*cos(d*x + c)^3)*log(sin(
d*x + c) + 1) - 3*((3*A + 2*C)*cos(d*x + c)^4 + (3*A + 2*C)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) - 2*(4*(4*A + 3*C)*cos(d*x + c)^3 + (7*A + 6*C)*cos
(d*x + c)^2 - A*cos(d*x + c) + 2*A)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d
*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2504, size = 250, normalized size = 1.88

$$-\frac{\frac{3\,(3\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{3\,(3\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}+\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,C\,t\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(3*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*(3*A + 2*C)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*
d*x + 1/2*c))/a + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)
^5 - 16*A*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/
2*d*x + 1/2*c) + 6*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*
a))/d
```

3.47 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=191

$$\frac{8(A+2C)\sin^3(c+dx)}{3a^2d}-\frac{8(A+2C)\sin(c+dx)}{a^2d}-\frac{2(A+2C)\sin(c+dx)\cos^4(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{(28A+55C)\sin(c+dx)\cos^3(c}{12a^2d}$$

```
[Out] ((28*A + 55*C)*x)/(8*a^2) - (8*(A + 2*C)*Sin[c + d*x])/(a^2*d) + ((28*A + 5
5*C)*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + ((28*A + 55*C)*Cos[c + d*x]^3*S
in[c + d*x])/(12*a^2*d) - (2*(A + 2*C)*Cos[c + d*x]^4*Sin[c + d*x])/(a^2*d*
(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^5*Sin[c + d*x])/(3*d*(a + a*Cos
[c + d*x])^2) + (8*(A + 2*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.3378, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2977, 2748, 2633, 2635, 8}

$$\frac{8(A+2C)\sin^3(c+dx)}{3a^2d}-\frac{8(A+2C)\sin(c+dx)}{a^2d}-\frac{2(A+2C)\sin(c+dx)\cos^4(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{(28A+55C)\sin(c+dx)\cos^3(c}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((28*A + 55*C)*x)/(8*a^2) - (8*(A + 2*C)*Sin[c + d*x])/(a^2*d) + ((28*A + 5
5*C)*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + ((28*A + 55*C)*Cos[c + d*x]^3*S
in[c + d*x])/(12*a^2*d) - (2*(A + 2*C)*Cos[c + d*x]^4*Sin[c + d*x])/(a^2*d*
(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^5*Sin[c + d*x])/(3*d*(a + a*Cos
[c + d*x])^2) + (8*(A + 2*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^4(c+dx)(-a(2A+5C)+a(4A+7C)\cos(c+dx))}{a+a\cos(c+dx)}\,d}{3a^2}$$

$$= -\frac{2(A+2C)\cos^4(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \int$$

$$= -\frac{2(A+2C)\cos^4(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - (8$$

$$= \frac{(28A+55C)\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \frac{2(A+2C)\cos^4(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} -$$

$$= -\frac{8(A+2C)\sin(c+dx)}{a^2d} + \frac{(28A+55C)\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{(28A+55}{}$$

$$= \frac{(28A+55C)x}{8a^2} - \frac{8(A+2C)\sin(c+dx)}{a^2d} + \frac{(28A+55C)\cos(c+dx)\sin(c+d}{8a^2d}$$

Mathematica [B] time = 0.779352, size = 399, normalized size = 2.09

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(72dx(28A+55C)\cos\left(c+\frac{dx}{2}\right)+1176A\sin\left(c+\frac{dx}{2}\right)-1912A\sin\left(c+\frac{3dx}{2}\right)-504A\sin\left(2c+\frac{3dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(72*(28*A + 55*C)*d*x*Cos[(d*x)/2] + 72*(28*A +
55*C)*d*x*Cos[c + (d*x)/2] + 672*A*d*x*Cos[c + (3*d*x)/2] + 1320*C*d*x*Cos[
c + (3*d*x)/2] + 672*A*d*x*Cos[2*c + (3*d*x)/2] + 1320*C*d*x*Cos[2*c + (3*d
*x)/2] - 3048*A*Sin[(d*x)/2] - 5184*C*Sin[(d*x)/2] + 1176*A*Sin[c + (d*x)/2
] + 1344*C*Sin[c + (d*x)/2] - 1912*A*Sin[c + (3*d*x)/2] - 3488*C*Sin[c + (3
*d*x)/2] - 504*A*Sin[2*c + (3*d*x)/2] - 1312*C*Sin[2*c + (3*d*x)/2] - 120*A
*Sin[2*c + (5*d*x)/2] - 285*C*Sin[2*c + (5*d*x)/2] - 120*A*Sin[3*c + (5*d*x
)/2] - 285*C*Sin[3*c + (5*d*x)/2] + 24*A*Sin[3*c + (7*d*x)/2] + 57*C*Sin[3*
c + (7*d*x)/2] + 24*A*Sin[4*c + (7*d*x)/2] + 57*C*Sin[4*c + (7*d*x)/2] - 7*
C*Sin[4*c + (9*d*x)/2] - 7*C*Sin[5*c + (9*d*x)/2] + 3*C*Sin[5*c + (11*d*x)/
2] + 3*C*Sin[6*c + (11*d*x)/2]))/(384*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.034, size = 392, normalized size = 2.1

$$\frac{A}{6\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{C}{6\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{7\,A}{2\,da^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{11\,C}{2\,da^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\,\frac{(\tan(1/2\,dx+c\ldots}{da^{2}\left((\tan(1/2\,dx+c\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-11/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(tan(1/2*d*x+
1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*A-65/4/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^4*
tan(1/2*d*x+1/2*c)^7*C-13/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*
c)^5*A-395/12/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*C-11/d/
a^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*A-341/12/d/a^2/(tan(1/2
*d*x+1/2*c)^2+1)^4*C*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^
4*A*tan(1/2*d*x+1/2*c)-31/4/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^4*C*tan(1/2*d*x+
1/2*c)+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A+55/4/d/a^2*arctan(tan(1/2*d*x+1
/2*c))*C
```

Maxima [B] time = 1.55908, size = 560, normalized size = 2.93

$$-\frac{C\left(\frac{\frac{93\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{341\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{395\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{195\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{2}+\frac{4\,a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{2\left(\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}}-\frac{165\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}\right)+2\,A\left(\frac{6\left(\frac{3\sin\ldots}{\cos(d\ldots}\right.}{a^{2}+\frac{2\,a^{2}\sin\ldots}{(\cos(d\ldots}}\right.\ldots}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/12*(C*((93*sin(d*x + c)/(cos(d*x + c) + 1) + 341*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 195*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 2*(33*sin(d*x + c)/(
cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 165*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a^2) + 2*A*(6*(3*sin(d*x + c)/(cos(d*x + c)
 + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d
```

```
*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*
arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.41821, size = 432, normalized size = 2.26

$$\frac{3\,(28\,A+55\,C)dx\cos\left(dx+c\right)^{2}+6\,(28\,A+55\,C)dx\cos\left(dx+c\right)+3\,(28\,A+55\,C)dx+\left(6\,C\cos\left(dx+c\right)^{5}-4\,C\cos\left(dx+\right.\right.}{24\left(a^{2}d\cos\left(dx+c\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/24*(3*(28*A + 55*C)*d*x*cos(d*x + c)^2 + 6*(28*A + 55*C)*d*x*cos(d*x + c)
 + 3*(28*A + 55*C)*d*x + (6*C*cos(d*x + c)^5 - 4*C*cos(d*x + c)^4 + (12*A +
 19*C)*cos(d*x + c)^3 - 6*(4*A + 9*C)*cos(d*x + c)^2 - (172*A + 347*C)*cos(
d*x + c) - 128*A - 256*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

Sympy [A] time = 34.6805, size = 2161, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((84*A*d*x*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96
*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*ta
n(c/2 + d*x/2)**2 + 24*a**2*d) + 336*A*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*t
an(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 504*A*d*x*tan(c/2 +
 d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 +
 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d
) + 336*A*d*x*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*
d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2
+ d*x/2)**2 + 24*a**2*d) + 84*A*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**
2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/
2 + d*x/2)**2 + 24*a**2*d) + 4*A*tan(c/2 + d*x/2)**11/(24*a**2*d*tan(c/2 +
```

```
d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4
+ 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 68*A*tan(c/2 + d*x/2)**9/(24
*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*ta
n(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 432*A*tan(
c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)
**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a
**2*d) - 800*A*tan(c/2 + d*x/2)**5/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2
*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2
 + d*x/2)**2 + 24*a**2*d) - 596*A*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 +
d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4
+ 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 156*A*tan(c/2 + d*x/2)/(24*a
**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(
c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 165*C*d*x*ta
n(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/
2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24
*a**2*d) + 660*C*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 + d*x/2)**8 + 9
6*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*t
an(c/2 + d*x/2)**2 + 24*a**2*d) + 990*C*d*x*tan(c/2 + d*x/2)**4/(24*a**2*d*
tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 +
d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 660*C*d*x*tan(c/2
+ d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6
+ 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*
d) + 165*C*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)*
*6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a*
*2*d) + 4*C*tan(c/2 + d*x/2)**11/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d
*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 +
 d*x/2)**2 + 24*a**2*d) - 116*C*tan(c/2 + d*x/2)**9/(24*a**2*d*tan(c/2 + d*
x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 +
96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 894*C*tan(c/2 + d*x/2)**7/(24*
a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 1566*C*tan(
c/2 + d*x/2)**5/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)
**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a
**2*d) - 1206*C*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**
2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/
2 + d*x/2)**2 + 24*a**2*d) - 318*C*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*
x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 +
96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d), Ne(d, 0)), (x*(A + C*cos(c)**2)
*cos(c)**4/(a*cos(c) + a)**2, True))
```

<u>Giac</u> [A] time = 1.18072, size = 297, normalized size = 1.55

$$\frac{\frac{3\,(dx+c)(28\,A+55\,C)}{a^2}+\frac{4\left(Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-33\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6}-\frac{2\left(60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+195\,C\tan\left(\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/24*(3*(d*x + c)*(28*A + 55*C)/a^2 + 4*(A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a
^4*tan(1/2*d*x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) - 33*C*a^4*tan(1/
2*d*x + 1/2*c))/a^6 - 2*(60*A*tan(1/2*d*x + 1/2*c)^7 + 195*C*tan(1/2*d*x +
1/2*c)^7 + 156*A*tan(1/2*d*x + 1/2*c)^5 + 395*C*tan(1/2*d*x + 1/2*c)^5 + 13
2*A*tan(1/2*d*x + 1/2*c)^3 + 341*C*tan(1/2*d*x + 1/2*c)^3 + 36*A*tan(1/2*d*
x + 1/2*c) + 93*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^2
))/d
```

3.48 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(5A+12C)\sin^3(c+dx)}{3a^2d}+\frac{(5A+12C)\sin(c+dx)}{a^2d}-\frac{2(2A+5C)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(2A+5C)\sin(c+dx)\cos}{a^2d}$$

```
[Out] -(((2*A + 5*C)*x)/a^2) + ((5*A + 12*C)*Sin[c + d*x])/(a^2*d) - ((2*A + 5*C)
*Cos[c + d*x]*Sin[c + d*x])/(a^2*d) - (2*(2*A + 5*C)*Cos[c + d*x]^3*Sin[c +
 d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])
/(3*d*(a + a*Cos[c + d*x])^2) - ((5*A + 12*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.327101, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2977, 2748, 2635, 8, 2633}

$$-\frac{(5A+12C)\sin^3(c+dx)}{3a^2d}+\frac{(5A+12C)\sin(c+dx)}{a^2d}-\frac{2(2A+5C)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(2A+5C)\sin(c+dx)\cos}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```



```
[Out] -(((2*A + 5*C)*x)/a^2) + ((5*A + 12*C)*Sin[c + d*x])/(a^2*d) - ((2*A + 5*C)
*Cos[c + d*x]*Sin[c + d*x])/(a^2*d) - (2*(2*A + 5*C)*Cos[c + d*x]^3*Sin[c +
 d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])
/(3*d*(a + a*Cos[c + d*x])^2) - ((5*A + 12*C)*Sin[c + d*x]^3)/(3*a^2*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(-a(A+4C)+3a(A+2C)\cos(c+dx))}{a+a\cos(c+dx)}\,d}{3a^2}$$

$$= -\frac{2(2A+5C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} +$$

$$= -\frac{2(2A+5C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} -$$

$$= -\frac{(2A+5C)\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{2(2A+5C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}$$

$$= -\frac{(2A+5C)x}{a^2} + \frac{(5A+12C)\sin(c+dx)}{a^2d} - \frac{(2A+5C)\cos(c+dx)\sin(c+dx}{a^2d}$$

Mathematica [B] time = 0.651771, size = 341, normalized size = 2.09

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-72dx(2A+5C)\cos\left(c+\frac{dx}{2}\right)-120A\sin\left(c+\frac{dx}{2}\right)+164A\sin\left(c+\frac{3dx}{2}\right)+36A\sin\left(2c+\frac{3dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-72*(2*A + 5*C)*d*x*Cos[(d*x)/2] - 72*(2*A + 5*
C)*d*x*Cos[c + (d*x)/2] - 48*A*d*x*Cos[c + (3*d*x)/2] - 120*C*d*x*Cos[c + (
3*d*x)/2] - 48*A*d*x*Cos[2*c + (3*d*x)/2] - 120*C*d*x*Cos[2*c + (3*d*x)/2]
+ 264*A*Sin[(d*x)/2] + 516*C*Sin[(d*x)/2] - 120*A*Sin[c + (d*x)/2] - 156*C*
Sin[c + (d*x)/2] + 164*A*Sin[c + (3*d*x)/2] + 342*C*Sin[c + (3*d*x)/2] + 36
*A*Sin[2*c + (3*d*x)/2] + 118*C*Sin[2*c + (3*d*x)/2] + 12*A*Sin[2*c + (5*d*
x)/2] + 30*C*Sin[2*c + (5*d*x)/2] + 12*A*Sin[3*c + (5*d*x)/2] + 30*C*Sin[3*
c + (5*d*x)/2] - 3*C*Sin[3*c + (7*d*x)/2] - 3*C*Sin[4*c + (7*d*x)/2] + C*Si
n[4*c + (9*d*x)/2] + C*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^
2)
```

Maple [B] time = 0.033, size = 322, normalized size = 2.

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{5\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{9\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\,\frac{(\tan(1/2\,dx+}{da^2\left((\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+9/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2/(tan(1/2*d*x+
1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*A+10/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*ta
n(1/2*d*x+1/2*c)^5*C+4/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^
3*A+40/3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(t
an(1/2*d*x+1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*c)+6/d/a^2/(tan(1/2*d*x+1/2*c)^2
+1)^3*C*tan(1/2*d*x+1/2*c)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A-10/d/a^2*ar
ctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.56672, size = 439, normalized size = 2.69

$$\frac{C\left(\frac{4\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)+A\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\mathrm{ar}\ldots}{\ldots}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] 1/6*(C*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) + A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)
^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^2 + 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(
d*x + c) + 1))))/d
```

Fricas [A] time = 1.46484, size = 369, normalized size = 2.26

$$-\frac{3\,(2\,A+5\,C)dx\cos(dx+c)^2+6\,(2\,A+5\,C)dx\cos(dx+c)+3\,(2\,A+5\,C)dx-\left(C\cos(dx+c)^4-C\cos(dx+c)^3+3\,(\ldots\right.}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] -1/3*(3*(2*A + 5*C)*d*x*cos(d*x + c)^2 + 6*(2*A + 5*C)*d*x*cos(d*x + c) + 3
*(2*A + 5*C)*d*x - (C*cos(d*x + c)^4 - C*cos(d*x + c)^3 + 3*(A + 2*C)*cos(d
*x + c)^2 + (14*A + 33*C)*cos(d*x + c) + 10*A + 24*C)*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 19.7183, size = 1426, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*A*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
6*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 36*A*d*x*tan(c
/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*A*d*x/(6*a**2*d*tan(c/2
+ d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) - A*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**
2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 12*A*
tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x
/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 54*A*tan(c/2 + d*x/2)*
*5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*
d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 68*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(
c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) + 27*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
0*C*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*C*d*x*tan(c
/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*C*d*x*tan(c/2 + d*x/2)**
2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
- C*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
```

```
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*C*tan(c/2 + d*x
/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*C*tan(c/2 + d*x/2)**5/(6*a**2*d
*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 +
d*x/2)**2 + 6*a**2*d) + 160*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2
)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 63*C*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan
(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x
*(A + C*cos(c)**2)*cos(c)**3/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.31605, size = 258, normalized size = 1.58

$$-\frac{\frac{6\,(dx+c)(2\,A+5\,C)}{a^2}-\frac{4\left(3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/6*(6*(d*x + c)*(2*A + 5*C)/a^2 - 4*(3*A*tan(1/2*d*x + 1/2*c)^5 + 15*C*ta
n(1/2*d*x + 1/2*c)^5 + 6*A*tan(1/2*d*x + 1/2*c)^3 + 20*C*tan(1/2*d*x + 1/2*
c)^3 + 3*A*tan(1/2*d*x + 1/2*c) + 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x +
 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) - 27*C*a^4*tan(1/2*d*x + 1/2*c))/
a^6)/d
```

3.49 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$-\frac{4(A+4C)\sin(c+dx)}{3a^2d}-\frac{2(A+4C)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(2A+7C)\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{x(2A+7C)}{2a^2}-\frac{(A$$

```
[Out] ((2*A + 7*C)*x)/(2*a^2) - (4*(A + 4*C)*Sin[c + d*x])/(3*a^2*d) + ((2*A + 7*
C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - (2*(A + 4*C)*Cos[c + d*x]^2*Sin[c
 + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x
])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.262512, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 2977, 2734}

$$-\frac{4(A+4C)\sin(c+dx)}{3a^2d}-\frac{2(A+4C)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(2A+7C)\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{x(2A+7C)}{2a^2}-\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] ((2*A + 7*C)*x)/(2*a^2) - (4*(A + 4*C)*Sin[c + d*x])/(3*a^2*d) + ((2*A + 7*
C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - (2*(A + 4*C)*Cos[c + d*x]^2*Sin[c
 + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x
])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(-3aC+a(2A+5C)\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{2(A+4C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \int$$

$$= \frac{(2A+7C)x}{2a^2} - \frac{4(A+4C)\sin(c+dx)}{3a^2d} + \frac{(2A+7C)\cos(c+dx)\sin(c+dx)}{2a^2d} -$$

Mathematica [A] time = 0.628729, size = 273, normalized size = 1.94

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(2A+7C)\cos\left(c+\frac{dx}{2}\right)+96A\sin\left(c+\frac{dx}{2}\right)-80A\sin\left(c+\frac{3dx}{2}\right)+24Adx\cos\left(c+\frac{3dx}{2}\right)+24\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(2*A + 7*C)*d*x*Cos[(d*x)/2] + 36*(2*A + 7*C
)*d*x*Cos[c + (d*x)/2] + 24*A*d*x*Cos[c + (3*d*x)/2] + 84*C*d*x*Cos[c + (3*
d*x)/2] + 24*A*d*x*Cos[2*c + (3*d*x)/2] + 84*C*d*x*Cos[2*c + (3*d*x)/2] - 1
44*A*Sin[(d*x)/2] - 381*C*Sin[(d*x)/2] + 96*A*Sin[c + (d*x)/2] + 147*C*Sin[
```

```
c + (d*x)/2] - 80*A*Sin[c + (3*d*x)/2] - 239*C*Sin[c + (3*d*x)/2] - 63*C*Si
n[2*c + (3*d*x)/2] - 15*C*Sin[2*c + (5*d*x)/2] - 15*C*Sin[3*c + (5*d*x)/2]
+ 3*C*Sin[3*c + (7*d*x)/2] + 3*C*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + Cos[
c + d*x])^2)
```

Maple [A] time = 0.031, size = 184, normalized size = 1.3

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\,\frac{C\,(\tan(1/2\,dx+}{da^2\left((\tan(1/2\,dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(tan(1/2*d*x+1
/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^2*C*ta
n(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A+7/d/a^2*arctan(tan(1/
2*d*x+1/2*c))*C
```

Maxima [A] time = 1.51558, size = 319, normalized size = 2.26

$$-\frac{C\left(\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)+A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(}{\cos(}\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/6*(C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) + A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.39598, size = 332, normalized size = 2.35

$$\frac{3\,(2\,A+7\,C)dx\cos\left(dx+c\right)^{2}+6\,(2\,A+7\,C)dx\cos\left(dx+c\right)+3\,(2\,A+7\,C)dx+\left(3\,C\cos\left(dx+c\right)^{3}-6\,C\cos\left(dx+c\right)^{2}-(\right.}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(2*A + 7*C)*d*x*cos(d*x + c)^2 + 6*(2*A + 7*C)*d*x*cos(d*x + c) + 3*
(2*A + 7*C)*d*x + (3*C*cos(d*x + c)^3 - 6*C*cos(d*x + c)^2 - (10*A + 43*C)*
cos(d*x + c) - 8*A - 32*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [A] time = 11.8798, size = 845, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((6*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 12*A*d*x*tan(c/2 + d*x/2)**2/(6*a**
2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 6*A*d
*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d
) + A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2
 + d*x/2)**2 + 6*a**2*d) - 7*A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/
2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 17*A*tan(c/2 + d*x/2)**
3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
 - 9*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) + 21*C*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d
*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*C*d*x*tan(c/2 + d
*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*
a**2*d) + 21*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/
2)**2 + 6*a**2*d) + C*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 1
2*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*C*tan(c/2 + d*x/2)**5/(6*a**2
*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 71*C*t
```

```
an(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/
2)**2 + 6*a**2*d) - 39*C*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 1
2*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*c
os(c)**2/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.28356, size = 185, normalized size = 1.31

$$\frac{\frac{3\,(dx+c)(2\,A+7\,C)}{a^2} - \frac{6\left(5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2} + \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-21\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/6*(3*(d*x + c)*(2*A + 7*C)/a^2 - 6*(5*C*tan(1/2*d*x + 1/2*c)^3 + 3*C*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x
 + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) -
 21*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.50 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{(A+4C)\sin(c+dx)}{3a^2d}+\frac{2C\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{2Cx}{a^2}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (-2*C*x)/a^2 + ((A + 4*C)*Sin[c + d*x])/(3*a^2*d) + (2*C*Sin[c + d*x])/(a^2
*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*
Cos[c + d*x])^2)
```

Rubi [A] time = 0.241035, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 2968, 3023, 12, 2735, 2648}

$$\frac{(A+4C)\sin(c+dx)}{3a^2d}+\frac{2C\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{2Cx}{a^2}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-2*C*x)/a^2 + ((A + 4*C)*Sin[c + d*x])/(3*a^2*d) + (2*C*Sin[c + d*x])/(a^2
*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*
Cos[c + d*x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(a(A-2C)+a(A+4C)\cos(c+dx))}{a+a\cos(c+dx)}dx}{3a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{a(A-2C)\cos(c+dx)+a(A+4C)\cos^2(c+dx)}{a+a\cos(c+dx)}dx}{3a^2}$$

$$= \frac{(A+4C)\sin(c+dx)}{3a^2d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int -\frac{6a^2C\cos(c+dx)}{a+a\cos(c+dx)}dx}{3a^3}$$

$$= \frac{(A+4C)\sin(c+dx)}{3a^2d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(2C)\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}}{a}$$

$$= -\frac{2Cx}{a^2} + \frac{(A+4C)\sin(c+dx)}{3a^2d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2C)\int \frac{}{a+}}{}$$

$$= -\frac{2Cx}{a^2} + \frac{(A+4C)\sin(c+dx)}{3a^2d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{2C\,\mathrm{si}}{d\left(a^2+a^2\right.}$$

Mathematica [B] time = 0.519144, size = 195, normalized size = 2.17

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-12A\sin\left(c+\frac{dx}{2}\right)+8A\sin\left(c+\frac{3dx}{2}\right)+12A\sin\left(\frac{dx}{2}\right)-30C\sin\left(c+\frac{dx}{2}\right)+41C\sin\left(c+\frac{3dx}{2}\right)+9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-36*C*d*x*Cos[(d*x)/2] - 36*C*d*x*Cos[c + (d*
x)/2] - 12*C*d*x*Cos[c + (3*d*x)/2] - 12*C*d*x*Cos[2*c + (3*d*x)/2] + 12*A*
Sin[(d*x)/2] + 66*C*Sin[(d*x)/2] - 12*A*Sin[c + (d*x)/2] - 30*C*Sin[c + (d*
x)/2] + 8*A*Sin[c + (3*d*x)/2] + 41*C*Sin[c + (3*d*x)/2] + 9*C*Sin[2*c + (3
*d*x)/2] + 3*C*Sin[2*c + (5*d*x)/2] + 3*C*Sin[3*c + (5*d*x)/2]))/(48*a^2*d)
```

Maple [A] time = 0.03, size = 130, normalized size = 1.4

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\,\frac{C\tan(1/2\,dx+}{da^2\left((\tan(1/2\,dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*C*tan(1/2*d*x
+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.52127, size = 223, normalized size = 2.48

$$\frac{C\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)+\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(C*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) +
 A*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/a^2)/d
```

Fricas [A] time = 1.32886, size = 261, normalized size = 2.9

$$-\frac{6\,Cdx\cos(dx+c)^2+12\,Cdx\cos(dx+c)+6\,Cdx-\left(3\,C\cos(dx+c)^2+2\,(A+7\,C)\cos(dx+c)+A+10\,C\right)\sin(dx}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*C*d*x*cos(d*x + c)^2 + 12*C*d*x*cos(d*x + c) + 6*C*d*x - (3*C*cos(d
*x + c)^2 + 2*(A + 7*C)*cos(d*x + c) + A + 10*C)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 6.21401, size = 335, normalized size = 3.72

$$\begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{2A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{3A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{12Cdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{12Cdx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{C\tan^5}{6a^2d\tan^2(} \\ \frac{x\left(A+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
 + 2*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 3*A*
tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*C*d*x*tan(c
/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*C*d*x/(6*a**2
*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - C*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/
2 + d*x/2)**2 + 6*a**2*d) + 14*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*
x/2)**2 + 6*a**2*d) + 27*C*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 +
 6*a**2*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)/(a*cos(c) + a)**2, True)
)
```

Giac [A] time = 1.25654, size = 154, normalized size = 1.71

$$-\frac{\frac{12(dx+c)C}{a^2} - \frac{12C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^2} + \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-15Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(d*x + c)*C/a^2 - 12*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3
 - 3*A*a^4*tan(1/2*d*x + 1/2*c) - 15*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.51 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{(A-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Cx}{a^2}+\frac{(A+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (C*x)/a^2 + ((A - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A + C
)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.125207, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3020, 2735, 2648}

$$\frac{(A-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Cx}{a^2}+\frac{(A+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (C*x)/a^2 + ((A - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A + C
)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3020

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> Simp[(b*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[a*A*(m + 1) - a*C*m + b*C*(2*m + 1)*Sin[e + f*x], x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-a(A-2C)-3aC\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= \frac{Cx}{a^2} + \frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A-5C)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= \frac{Cx}{a^2} + \frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A-5C)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [B] time = 0.323995, size = 141, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(2A\sin\left(c+\frac{3dx}{2}\right)+6A\sin\left(\frac{dx}{2}\right)+12C\sin\left(c+\frac{dx}{2}\right)-10C\sin\left(c+\frac{3dx}{2}\right)+9Cdx\cos\left(c+\frac{dx}{2}\right)+3C\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*C*d*x*Cos[(d*x)/2] + 9*C*d*x*Cos[c + (d*x)/
2] + 3*C*d*x*Cos[c + (3*d*x)/2] + 3*C*d*x*Cos[2*c + (3*d*x)/2] + 6*A*Sin[(d
*x)/2] - 18*C*Sin[(d*x)/2] + 12*C*Sin[c + (d*x)/2] + 2*A*Sin[c + (3*d*x)/2]
 - 10*C*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.023, size = 97, normalized size = 1.5

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan(\tan(1/2\,dx+}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2
```

```
*d*x+1/2*c))*C
```

Maxima [A] time = 1.53094, size = 161, normalized size = 2.44

$$-\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - A*(3*sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.3679, size = 228, normalized size = 3.45

$$\frac{3\,Cdx\cos{(dx+c)}^2+6\,Cdx\cos{(dx+c)}+3\,Cdx+((A-5\,C)\cos{(dx+c)}+2\,A-4\,C)\sin{(dx+c)}}{3\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*C*d*x*cos(d*x + c)^2 + 6*C*d*x*cos(d*x + c) + 3*C*d*x + ((A - 5*C)*c
os(d*x + c) + 2*A - 4*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [A] time = 3.23375, size = 104, normalized size = 1.58

$$\begin{cases} \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}+\frac{Cx}{a^2}+\frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x\left(A+C\cos^2{(c)}\right)}{(a\cos{(c)}+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d)
 + C*x/a**2 + C*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*C*tan(c/2 + d*x/2)/(2*a*
*2*d), Ne(d, 0)), (x*(A + C*cos(c)**2)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.29483, size = 113, normalized size = 1.71

$$\frac{\frac{6\,(dx+c)C}{a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*C/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) - 9*C*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.52 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2(2A-C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - (2*(2*A - C)*Sin[c + d*x])/(3*a^2*d*(1
+ Cos[c + d*x])) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.226968, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3042, 2978, 12, 3770}

$$-\frac{2(2A-C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - (2*(2*A - C)*Sin[c + d*x])/(3*a^2*d*(1
+ Cos[c + d*x])) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3aA-a(A-2C)\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= -\frac{2(2A-C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int 3a^2A\sec(c+dx)\,dx}{3a^4}\\
&= -\frac{2(2A-C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{A\int \sec(c+dx)\,dx}{a^2}\\
&= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2(2A-C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}
\end{aligned}$$

Mathematica [B] time = 0.5535, size = 166, normalized size = 2.16

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A+C)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A+C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+4(2A-C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^2\left(\frac{1}{2}(c+\right.\right.}{3a^2d(\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*A*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A + C)*Sec[c/2]*
```

```
Sin[(d*x)/2] + 4*(2*A - C)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A +
C)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.051, size = 119, normalized size = 1.6

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2*A*ln(tan(1/2*
d*x+1/2*c)-1)+1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.03448, size = 197, normalized size = 2.56

$$-\frac{A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{C\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - C*(3*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.39146, size = 338, normalized size = 4.39

$$\frac{3\left(A\cos(dx+c)^2+2\,A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-3\left(A\cos(dx+c)^2+2\,A\cos(dx+c)+A\right)\log(-\sin\ldots}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 3*
(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*(2*(2*
A - C)*cos(d*x + c) + 5*A - C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{C\cos^2(c+dx)\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + Integ
ral(C*cos(c + d*x)**2*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),
x))/a**2
```

Giac [A] time = 1.23462, size = 151, normalized size = 1.96

$$\frac{\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} - \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*A*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2
*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.53 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{(10A+C)\tan(c+dx)}{3a^2d}-\frac{2A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2A\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A+C)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (-2*A*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((10*A + C)*Tan[c + d*x])/(3*a^2*d)
- (2*A*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Tan[c + d*x])/(3
*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.296305, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2748, 3767, 8, 3770}

$$\frac{(10A+C)\tan(c+dx)}{3a^2d}-\frac{2A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2A\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A+C)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-2*A*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((10*A + C)*Tan[c + d*x])/(3*a^2*d)
- (2*A*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Tan[c + d*x])/(3
*d*(a + a*Cos[c + d*x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(4A+C)-a(2A-C)\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{2A\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(a^2(10A+C)-6a^2A\,c\right.}{3}$$
$$= -\frac{2A\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(2A)\int\sec(c+dx)\,dx}{a^2} + $$
$$= -\frac{2A\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{2A\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$
$$= -\frac{2A\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{(10A+C)\tan(c+dx)}{3a^2d} - \frac{2A\tan(c+dx)}{a^2d(1+\cos(c+dx))}$$

Mathematica [B] time = 1.39357, size = 288, normalized size = 3.16

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left((A+C)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A+C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(7A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((A + C)*Sec[c/2]
*Sin[(d*x)/2] + 2*(7*A + C)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*A*
Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2
] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]))) + (A + C)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c
+ d*x])^2*(2*A + C + C*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.059, size = 164, normalized size = 1.8

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{5A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{A}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2*A/(tan(1/2*d*x
+1/2*c)-1)+2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*A/(tan(1/2*d*x+1/2*c)
+1)-2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.02808, size = 258, normalized size = 2.84

$$\frac{A\left(\frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12 \sin(dx+c)}{\left(a^2-\frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)+\frac{C\left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)}\right.}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) + C*(3*sin(d*x + c)/(cos(
d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.41459, size = 429, normalized size = 4.71

$$-\frac{3\left(A\cos(dx+c)^3+2\,A\cos(dx+c)^2+A\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left(A\cos(dx+c)^3+2\,A\cos(dx+c)^2+\right.}{3\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] -1/3*(3*(A*cos(d*x + c)^3 + 2*A*cos(d*x + c)^2 + A*cos(d*x + c))*log(sin(d*
x + c) + 1) - 3*(A*cos(d*x + c)^3 + 2*A*cos(d*x + c)^2 + A*cos(d*x + c))*lo
g(-sin(d*x + c) + 1) - ((10*A + C)*cos(d*x + c)^2 + 2*(7*A + C)*cos(d*x + c
) + 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
```

```
*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29955, size = 192, normalized size = 2.11

$$-\frac{\frac{12\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{12\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Aa^4\tan\left(\frac{1}{2}\,d}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/6*(12*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^2 + 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 1
5*A*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.54 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$-\frac{4(4A+C)\tan(c+dx)}{3a^2d}+\frac{(7A+2C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A+2C)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{2(4A+C)\tan(c+dx)}{3a^2d(\cos(c+d$$

```
[Out] ((7*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(4*A + C)*Tan[c + d*x])/
(3*a^2*d) + ((7*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - (2*(4*A + C
)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Sec[c
+ d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.315835, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{4(4A+C)\tan(c+dx)}{3a^2d}+\frac{(7A+2C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A+2C)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{2(4A+C)\tan(c+dx)}{3a^2d(\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```



```
[Out] ((7*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(4*A + C)*Tan[c + d*x])/
(3*a^2*d) + ((7*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - (2*(4*A + C
)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Sec[c
+ d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(5A+2C)-3aA\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{2(4A+C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int (\ldots}{\ldots}$$

$$= -\frac{2(4A+C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(4(\ldots}{\ldots}$$

$$= \frac{(7A+2C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{2(4A+C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A\ldots}{\ldots}$$

$$= \frac{(7A+2C)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{4(4A+C)\tan(c+dx)}{3a^2d} + \frac{(7A+2C)\sec(c\ldots}{2a\ldots}$$

Mathematica [B] time = 3.12071, size = 484, normalized size = 3.32

$$-\frac{96(7A+2C)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(96*(7*A + 2*C)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]^2*(-2*(7*A + 10*C)*Sin[(d*x)/2] + (97*A + 22*C)*Sin[(3*
d*x)/2] - 126*A*Sin[c - (d*x)/2] - 36*C*Sin[c - (d*x)/2] + 42*A*Sin[c + (d*
x)/2] + 36*C*Sin[c + (d*x)/2] - 98*A*Sin[2*c + (d*x)/2] - 20*C*Sin[2*c + (d
*x)/2] - 3*A*Sin[c + (3*d*x)/2] - 18*C*Sin[c + (3*d*x)/2] + 37*A*Sin[2*c +
(3*d*x)/2] + 22*C*Sin[2*c + (3*d*x)/2] - 63*A*Sin[3*c + (3*d*x)/2] - 18*C*S
in[3*c + (3*d*x)/2] + 75*A*Sin[c + (5*d*x)/2] + 18*C*Sin[c + (5*d*x)/2] + 1
5*A*Sin[2*c + (5*d*x)/2] - 6*C*Sin[2*c + (5*d*x)/2] + 39*A*Sin[3*c + (5*d*x
)/2] + 18*C*Sin[3*c + (5*d*x)/2] - 21*A*Sin[4*c + (5*d*x)/2] - 6*C*Sin[4*c
+ (5*d*x)/2] + 32*A*Sin[2*c + (7*d*x)/2] + 8*C*Sin[2*c + (7*d*x)/2] + 12*A*
Sin[3*c + (7*d*x)/2] + 20*A*Sin[4*c + (7*d*x)/2] + 8*C*Sin[4*c + (7*d*x)/2]
))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.069, size = 249, normalized size = 1.7

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{3\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{7\,A}{2\,da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)-7/2/d/a^2*A*ln(tan(1/
2*d*x+1/2*c)-1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^2*A/(tan(1/2*d*x
+1/2*c)-1)^2+5/2/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)+7/2/d/a^2*A*ln(tan(1/2*d*x+
1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^2*A/(tan(1/2*d*x+1/2*c
)+1)^2+5/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.04645, size = 389, normalized size = 2.66

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)+C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) + C*((9*sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a^2))/d
```

Fricas [A] time = 1.46977, size = 554, normalized size = 3.79

$$\frac{3\left((7A+2C)\cos(dx+c)^4+2(7A+2C)\cos(dx+c)^3+(7A+2C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((7A+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/12*(3*((7*A + 2*C)*cos(d*x + c)^4 + 2*(7*A + 2*C)*cos(d*x + c)^3 + (7*A +
 2*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*A + 2*C)*cos(d*x + c)^4
 + 2*(7*A + 2*C)*cos(d*x + c)^3 + (7*A + 2*C)*cos(d*x + c)^2)*log(-sin(d*x
+ c) + 1) - 2*(8*(4*A + C)*cos(d*x + c)^3 + (43*A + 10*C)*cos(d*x + c)^2 +
6*A*cos(d*x + c) - 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d
*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3645, size = 231, normalized size = 1.58

$$\frac{\frac{3\,(7\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\,(7\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{6\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\cdots}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/6*(3*(7*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A + 2*C)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 3*A
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/
2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/
2*c) + 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.55 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=172

$$\frac{(12A+5C)\tan^3(c+dx)}{3a^2d}+\frac{(12A+5C)\tan(c+dx)}{a^2d}-\frac{(5A+2C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(5A+2C)\tan(c+dx)\sec(c+$$

```
[Out] -(((5*A + 2*C)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + ((12*A + 5*C)*Tan[c + d*x]
)/(a^2*d) - ((5*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(a^2*d) - (2*(5*A + 2*C
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Sec[
c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((12*A + 5*C)*Tan[c
 + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.334533, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2748, 3767, 3768, 3770}

$$\frac{(12A+5C)\tan^3(c+dx)}{3a^2d}+\frac{(12A+5C)\tan(c+dx)}{a^2d}-\frac{(5A+2C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(5A+2C)\tan(c+dx)\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] -(((5*A + 2*C)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + ((12*A + 5*C)*Tan[c + d*x]
)/(a^2*d) - ((5*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(a^2*d) - (2*(5*A + 2*C
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Sec[
c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((12*A + 5*C)*Tan[c
 + d*x]^3)/(3*a^2*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(3a(2A+C)-a(4A+C)\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)} d}{3a^2}$$

$$= -\frac{2(5A+2C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{2(5A+2C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(5A+2C)\sec(c+dx)\tan(c+dx)}{a^2d} - \frac{2(5A+2C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}$$

$$= -\frac{(5A+2C)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{(12A+5C)\tan(c+dx)}{a^2d} - \frac{(5A+2C)\,\mathrm{se}}{}$$

Mathematica [B] time = 5.06395, size = 594, normalized size = 3.45

$$\frac{192(5A+2C)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (192*(5*A + 2*C)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]^3*(-3*(A + 8*C)*Sin[(d*x)/2] + (155*A + 66*C)*Sin[(3*d*
x)/2] - 153*A*Sin[c - (d*x)/2] - 60*C*Sin[c - (d*x)/2] + 21*A*Sin[c + (d*x)
/2] + 24*C*Sin[c + (d*x)/2] - 135*A*Sin[2*c + (d*x)/2] - 60*C*Sin[2*c + (d*
x)/2] + 25*A*Sin[c + (3*d*x)/2] - 4*C*Sin[c + (3*d*x)/2] + 45*A*Sin[2*c + (
3*d*x)/2] + 36*C*Sin[2*c + (3*d*x)/2] - 85*A*Sin[3*c + (3*d*x)/2] - 34*C*Si
n[3*c + (3*d*x)/2] + 99*A*Sin[c + (5*d*x)/2] + 42*C*Sin[c + (5*d*x)/2] + 21
*A*Sin[2*c + (5*d*x)/2] + 33*A*Sin[3*c + (5*d*x)/2] + 24*C*Sin[3*c + (5*d*x
)/2] - 45*A*Sin[4*c + (5*d*x)/2] - 18*C*Sin[4*c + (5*d*x)/2] + 57*A*Sin[2*c
 + (7*d*x)/2] + 24*C*Sin[2*c + (7*d*x)/2] + 18*A*Sin[3*c + (7*d*x)/2] + 3*C
*Sin[3*c + (7*d*x)/2] + 24*A*Sin[4*c + (7*d*x)/2] + 15*C*Sin[4*c + (7*d*x)/
2] - 15*A*Sin[5*c + (7*d*x)/2] - 6*C*Sin[5*c + (7*d*x)/2] + 24*A*Sin[3*c +
(9*d*x)/2] + 10*C*Sin[3*c + (9*d*x)/2] + 11*A*Sin[4*c + (9*d*x)/2] + 3*C*Si
n[4*c + (9*d*x)/2] + 13*A*Sin[5*c + (9*d*x)/2] + 7*C*Sin[5*c + (9*d*x)/2]))
/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.065, size = 338, normalized size = 2.

$$\frac{A}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9\,A}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\,C}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\,\frac{A}{d a^2\,(\tan(1/2\,dx+c/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2*A/(tan(1/2*d*x
+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C+5/d/a^2*A*ln(tan(1/2*d*x+1/2*c)
-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)^3
-3/2/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)^2-5/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)-2/
d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-5/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2/(
tan(1/2*d*x+1/2*c)+1)*C-1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^3+3/2/d/a^2*A/(t
an(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.07824, size = 512, normalized size = 2.98

$$\frac{A\left(\frac{4\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)+C\left(\frac{15\,\sin(dx+c}{\cos(dx+c)+} \cdots\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) + C*
((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.70117, size = 591, normalized size = 3.44

$$-\frac{3\left((5A+2C)\cos(dx+c)^5+2(5A+2C)\cos(dx+c)^4+(5A+2C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((5A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] -1/6*(3*((5*A + 2*C)*cos(d*x + c)^5 + 2*(5*A + 2*C)*cos(d*x + c)^4 + (5*A +
 2*C)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((5*A + 2*C)*cos(d*x + c)^5
 + 2*(5*A + 2*C)*cos(d*x + c)^4 + (5*A + 2*C)*cos(d*x + c)^3)*log(-sin(d*x
+ c) + 1) - 2*(2*(12*A + 5*C)*cos(d*x + c)^4 + (33*A + 14*C)*cos(d*x + c)^3
 + 3*(2*A + C)*cos(d*x + c)^2 - A*cos(d*x + c) + A)*sin(d*x + c))/(a^2*d*co
s(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29958, size = 304, normalized size = 1.77

$$-\frac{\frac{6(5A+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6(5A+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{4\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-20A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6C\,\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right.}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/6*(6*(5*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(5*A + 2*C)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(15*A*tan(1/2*d*x + 1/2*c)^5 + 3
*C*tan(1/2*d*x + 1/2*c)^5 - 20*A*tan(1/2*d*x + 1/2*c)^3 - 6*C*tan(1/2*d*x +
 1/2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*
d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*
d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) + 15*C*a^4*tan(1/2*d*x + 1/2
*c))/a^6)/d
```

3.56 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$-\frac{4(9A+34C)\sin^3(c+dx)}{15a^3d}+\frac{4(9A+34C)\sin(c+dx)}{5a^3d}-\frac{(6A+23C)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A+23C)\sin(c+d}{2a^3d}$$

```
[Out] -((6*A + 23*C)*x)/(2*a^3) + (4*(9*A + 34*C)*Sin[c + d*x])/(5*a^3*d) - ((6*A
 + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]^5*Sin
[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((3*A + 13*C)*Cos[c + d*x]^4*Sin[
c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((6*A + 23*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) - (4*(9*A + 34*C)*Sin[c + d*x]^3
)/(15*a^3*d)
```

Rubi [A] time = 0.484895, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2977, 2748, 2635, 8, 2633}

$$-\frac{4(9A+34C)\sin^3(c+dx)}{15a^3d}+\frac{4(9A+34C)\sin(c+dx)}{5a^3d}-\frac{(6A+23C)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A+23C)\sin(c+d}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((6*A + 23*C)*x)/(2*a^3) + (4*(9*A + 34*C)*Sin[c + d*x])/(5*a^3*d) - ((6*A
 + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]^5*Sin
[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((3*A + 13*C)*Cos[c + d*x]^4*Sin[
c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((6*A + 23*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) - (4*(9*A + 34*C)*Sin[c + d*x]^3
)/(15*a^3*d)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^4(c+dx)(-5aC+a(3A+8C)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A+13C)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A+13C)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A+13C)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= -\frac{(6A+23C)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} -$$

$$= -\frac{(6A+23C)x}{2a^3} + \frac{4(9A+34C)\sin(c+dx)}{5a^3d} - \frac{(6A+23C)\cos(c+dx)\sin(c+$$

Mathematica [B] time = 0.852709, size = 463, normalized size = 2.14

$$-\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(6A+23C)\cos\left(c+\frac{dx}{2}\right)+4500A\sin\left(c+\frac{dx}{2}\right)-4860A\sin\left(c+\frac{3dx}{2}\right)+900A\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c/2]*(600*(6*A + 23*C)*d*x*Cos[(d*x)/2] + 600*(6*A +
 23*C)*d*x*Cos[c + (d*x)/2] + 1800*A*d*x*Cos[c + (3*d*x)/2] + 6900*C*d*x*Co
s[c + (3*d*x)/2] + 1800*A*d*x*Cos[2*c + (3*d*x)/2] + 6900*C*d*x*Cos[2*c + (
3*d*x)/2] + 360*A*d*x*Cos[2*c + (5*d*x)/2] + 1380*C*d*x*Cos[2*c + (5*d*x)/2
] + 360*A*d*x*Cos[3*c + (5*d*x)/2] + 1380*C*d*x*Cos[3*c + (5*d*x)/2] - 7020
*A*Sin[(d*x)/2] - 20410*C*Sin[(d*x)/2] + 4500*A*Sin[c + (d*x)/2] + 11110*C*
Sin[c + (d*x)/2] - 4860*A*Sin[c + (3*d*x)/2] - 15380*C*Sin[c + (3*d*x)/2] +
 900*A*Sin[2*c + (3*d*x)/2] + 380*C*Sin[2*c + (3*d*x)/2] - 1452*A*Sin[2*c +
 (5*d*x)/2] - 4777*C*Sin[2*c + (5*d*x)/2] - 300*A*Sin[3*c + (5*d*x)/2] - 16
25*C*Sin[3*c + (5*d*x)/2] - 60*A*Sin[3*c + (7*d*x)/2] - 230*C*Sin[3*c + (7*
d*x)/2] - 60*A*Sin[4*c + (7*d*x)/2] - 230*C*Sin[4*c + (7*d*x)/2] + 20*C*Sin
[4*c + (9*d*x)/2] + 20*C*Sin[5*c + (9*d*x)/2] - 5*C*Sin[5*c + (11*d*x)/2] -
 5*C*Sin[6*c + (11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.036, size = 362, normalized size = 1.7

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{5\,C}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A-5/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+49/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3/(tan(1/2*d*x+1/2*c)
^2+1)^3*A*tan(1/2*d*x+1/2*c)^5+17/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/
2*d*x+1/2*c)^5+4/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*A+76
/3/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)^3+2/d/a^3/(tan(1/2
*d*x+1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*c)+11/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3
*C*tan(1/2*d*x+1/2*c)-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A-23/d/a^3*arctan(
tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.59266, size = 493, normalized size = 2.28

$$\frac{C\left(\frac{20\left(\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3+\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1380\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+3\,A\left(\frac{40\,\mathrm{s}}{\left(a^3+\frac{a^3\sin(dx+}{(\cos(dx+c)}\right.}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(C*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 3*A*
(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x
+ c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d
```

```
*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 1.74121, size = 525, normalized size = 2.43

$$-\frac{15\,(6\,A+23\,C)dx\cos{(dx+c)}^{3}+45\,(6\,A+23\,C)dx\cos{(dx+c)}^{2}+45\,(6\,A+23\,C)dx\cos{(dx+c)}+15\,(6\,A+23\,C)d}{30\,\left(a^{3}d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] -1/30*(15*(6*A + 23*C)*d*x*cos(d*x + c)^3 + 45*(6*A + 23*C)*d*x*cos(d*x + c
)^2 + 45*(6*A + 23*C)*d*x*cos(d*x + c) + 15*(6*A + 23*C)*d*x - (10*C*cos(d*
x + c)^5 - 15*C*cos(d*x + c)^4 + 5*(6*A + 19*C)*cos(d*x + c)^3 + (234*A + 8
69*C)*cos(d*x + c)^2 + 9*(38*A + 143*C)*cos(d*x + c) + 144*A + 544*C)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [A] time = 55.9321, size = 1586, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-180*A*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 +
180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d
) - 540*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3
*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 540*
A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c
/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*A*d*x/(6
0*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*A*tan(c/2 + d*x/2)**11/(60*a**3*d*tan(
c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x
/2)**2 + 60*a**3*d) - 21*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)*
*6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
```

```
**3*d) + 174*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**
3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 798
*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2
+ d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 975*A*tan(c/2 +
 d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4
+ 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*A*tan(c/2 + d*x/2)/(60*
a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*ta
n(c/2 + d*x/2)**2 + 60*a**3*d) - 690*C*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*t
an(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 +
d*x/2)**2 + 60*a**3*d) - 2070*C*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2
+ d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)*
*2 + 60*a**3*d) - 2070*C*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2
)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) - 690*C*d*x/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 +
d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*C*tan(c/2 + d*x
/2)**11/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 1
80*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 41*C*tan(c/2 + d*x/2)**9/(60*a
**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) + 594*C*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/
2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) + 3078*C*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)*
*6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
**3*d) + 3675*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a*
*3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 13
95*C*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 +
 d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*(A
+ C*cos(c)**2)*cos(c)**4/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.23347, size = 308, normalized size = 1.43

$$-\frac{\frac{30\,(dx+c)(6\,A+23\,C)}{a^3}-\frac{20\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+51\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+76\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+33\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/60*(30*(d*x + c)*(6*A + 23*C)/a^3 - 20*(6*A*tan(1/2*d*x + 1/2*c)^5 + 51*
C*tan(1/2*d*x + 1/2*c)^5 + 12*A*tan(1/2*d*x + 1/2*c)^3 + 76*C*tan(1/2*d*x +
```

```
 1/2*c)^3 + 6*A*tan(1/2*d*x + 1/2*c) + 33*C*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*t
an(1/2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 50*C*a^12*tan(1/
2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) + 735*C*a^12*tan(1/2*d*x
 + 1/2*c))/a^15)/d
```

3.57 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=189

$$-\frac{2(11A+76C)\sin(c+dx)}{15a^3d}-\frac{(11A+76C)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+13C)\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{x(2A+13C}{2a^3}$$

```
[Out] ((2*A + 13*C)*x)/(2*a^3) - (2*(11*A + 76*C)*Sin[c + d*x])/(15*a^3*d) + ((2*
A + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((A + 11*C)*Cos[c + d*x]^3*Sin[c
 + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((11*A + 76*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.46004, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 2977, 2734}

$$-\frac{2(11A+76C)\sin(c+dx)}{15a^3d}-\frac{(11A+76C)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+13C)\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{x(2A+13C}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((2*A + 13*C)*x)/(2*a^3) - (2*(11*A + 76*C)*Sin[c + d*x])/(15*a^3*d) + ((2*
A + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((A + 11*C)*Cos[c + d*x]^3*Sin[c
 + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((11*A + 76*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(a(A-4C)+a(2A+7C)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(A+11C)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + $$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(A+11C)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - $$

$$= \frac{(2A+13C)x}{2a^3} - \frac{2(11A+76C)\sin(c+dx)}{15a^3d} + \frac{(2A+13C)\cos(c+dx)\sin(c+}{2a^3d}$$

Mathematica [B] time = 0.659566, size = 393, normalized size = 2.08

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(2A+13C)\cos\left(c+\frac{dx}{2}\right)+2160A\sin\left(c+\frac{dx}{2}\right)-1840A\sin\left(c+\frac{3dx}{2}\right)+720A\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(2*A + 13*C)*d*x*Cos[(d*x)/2] + 600*(2*A +
13*C)*d*x*Cos[c + (d*x)/2] + 600*A*d*x*Cos[c + (3*d*x)/2] + 3900*C*d*x*Cos[
c + (3*d*x)/2] + 600*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*C*d*x*Cos[2*c + (3*d
*x)/2] + 120*A*d*x*Cos[2*c + (5*d*x)/2] + 780*C*d*x*Cos[2*c + (5*d*x)/2] +
120*A*d*x*Cos[3*c + (5*d*x)/2] + 780*C*d*x*Cos[3*c + (5*d*x)/2] - 2960*A*Si
n[(d*x)/2] - 12760*C*Sin[(d*x)/2] + 2160*A*Sin[c + (d*x)/2] + 7560*C*Sin[c
+ (d*x)/2] - 1840*A*Sin[c + (3*d*x)/2] - 9230*C*Sin[c + (3*d*x)/2] + 720*A*
Sin[2*c + (3*d*x)/2] + 930*C*Sin[2*c + (3*d*x)/2] - 512*A*Sin[2*c + (5*d*x)
/2] - 2782*C*Sin[2*c + (5*d*x)/2] - 750*C*Sin[3*c + (5*d*x)/2] - 105*C*Sin[
3*c + (7*d*x)/2] - 105*C*Sin[4*c + (7*d*x)/2] + 15*C*Sin[4*c + (9*d*x)/2] +
 15*C*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.034, size = 224, normalized size = 1.2

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{2\,C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A+2/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*C*tan(1/2*d*x+1/2*c)-7/d/a^3/(tan(1/2*d*x+1/2*c)
^2+1)^2*C*tan(1/2*d*x+1/2*c)^3-5/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2
*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A+13/d/a^3*arctan(tan(1/2*d*
x+1/2*c))*C
```

Maxima [A] time = 1.55897, size = 373, normalized size = 1.97

$$-\frac{C\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3}{(c}}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] -1/60*(C*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + A*((105*sin(d*x
+ c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^3))/d
```

Fricas [A] time = 1.68491, size = 478, normalized size = 2.53

$$\frac{15\,(2\,A+13\,C)dx\cos{(dx+c)}^{3}+45\,(2\,A+13\,C)dx\cos{(dx+c)}^{2}+45\,(2\,A+13\,C)dx\cos{(dx+c)}+15\,(2\,A+13\,C)dx}{30\left(a^{3}d\cos{(dx+c)}^{3}+3\,a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(2*A + 13*C)*d*x*cos(d*x + c)^3 + 45*(2*A + 13*C)*d*x*cos(d*x + c)
^2 + 45*(2*A + 13*C)*d*x*cos(d*x + c) + 15*(2*A + 13*C)*d*x + (15*C*cos(d*x
 + c)^4 - 45*C*cos(d*x + c)^3 - (64*A + 479*C)*cos(d*x + c)^2 - 3*(34*A + 2
39*C)*cos(d*x + c) - 44*A - 304*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 29.4935, size = 967, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 120*A*d*x*tan(c/2 + d*x/2)**2/(
```

```
60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 60*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
 + 60*a**3*d) - 3*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 14*A*tan(c/2 + d*x/2)**7/(60*a*
*3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 68
*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2
+ d*x/2)**2 + 60*a**3*d) - 190*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d
*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*A*tan(c/2 + d*
x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
**3*d) + 390*C*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120
*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*C*d*x*tan(c/2 + d*x/2)**2/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
+ 390*C*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
 + 60*a**3*d) - 3*C*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*C*tan(c/2 + d*x/2)**7/(60*a*
*3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 38
8*C*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2
 + d*x/2)**2 + 60*a**3*d) - 1310*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 +
 d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*C*tan(c/2 +
d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**3/(a*cos(c) + a)**3, True
))
```

Giac [A] time = 1.30967, size = 235, normalized size = 1.24

$$\frac{\frac{30\,(dx+c)(2\,A+13\,C)}{a^3}-\frac{60\left(7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,Ca^{12}\tan}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(30*(d*x + c)*(2*A + 13*C)/a^3 - 60*(7*C*tan(1/2*d*x + 1/2*c)^3 + 5*C*
tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(
1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*
x + 1/2*c)^3 - 40*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x +
1/2*c) + 465*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.58 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=136

$$\frac{(2A+27C)\sin(c+dx)}{15a^3d}+\frac{3C\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3Cx}{a^3}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(A-9C)\sin(c+dx)\text{co}}{15ad(a\cos(c+dx)}$$

```
[Out] (-3*C*x)/a^3 + ((2*A + 27*C)*Sin[c + d*x])/(15*a^3*d) - ((A + C)*Cos[c + d*
x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((A - 9*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (3*C*Sin[c + d*x])/(d*(a^3
 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.462583, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{(2A+27C)\sin(c+dx)}{15a^3d}+\frac{3C\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3Cx}{a^3}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(A-9C)\sin(c+dx)\text{co}}{15ad(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (-3*C*x)/a^3 + ((2*A + 27*C)*Sin[c + d*x])/(15*a^3*d) - ((A + C)*Cos[c + d*
x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((A - 9*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (3*C*Sin[c + d*x])/(d*(a^3
 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(a(2A-3C)+a(A+6C)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-9C)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-9C)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= \frac{(2A+27C)\sin(c+dx)}{15a^3d} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-9C)\cos}{15ad(a+}$$

$$= \frac{(2A+27C)\sin(c+dx)}{15a^3d} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-9C)\cos}{15ad(a+}$$

$$= -\frac{3Cx}{a^3} + \frac{(2A+27C)\sin(c+dx)}{15a^3d} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-}{}$$

$$= -\frac{3Cx}{a^3} + \frac{(2A+27C)\sin(c+dx)}{15a^3d} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-}{}$$

Mathematica [B] time = 0.905447, size = 283, normalized size = 2.08

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(120A\sin\left(c+\frac{dx}{2}\right)-80A\sin\left(c+\frac{3dx}{2}\right)+60A\sin\left(2c+\frac{3dx}{2}\right)-28A\sin\left(2c+\frac{5dx}{2}\right)-160A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^5*(900*C*d*x*Cos[(d*x)/2] + 900*C*d*x*Cos[c + (
d*x)/2] + 450*C*d*x*Cos[c + (3*d*x)/2] + 450*C*d*x*Cos[2*c + (3*d*x)/2] + 9
0*C*d*x*Cos[2*c + (5*d*x)/2] + 90*C*d*x*Cos[3*c + (5*d*x)/2] - 160*A*Sin[(d
*x)/2] - 1755*C*Sin[(d*x)/2] + 120*A*Sin[c + (d*x)/2] + 1125*C*Sin[c + (d*x
)/2] - 80*A*Sin[c + (3*d*x)/2] - 1215*C*Sin[c + (3*d*x)/2] + 60*A*Sin[2*c +
 (3*d*x)/2] + 225*C*Sin[2*c + (3*d*x)/2] - 28*A*Sin[2*c + (5*d*x)/2] - 363*
C*Sin[2*c + (5*d*x)/2] - 75*C*Sin[3*c + (5*d*x)/2] - 15*C*Sin[3*c + (7*d*x)
/2] - 15*C*Sin[4*c + (7*d*x)/2]))/(960*a^3*d)
```

Maple [A] time = 0.031, size = 170, normalized size = 1.3

$$\frac{A}{20\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\frac{C}{20\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{A}{6\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{C}{2\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{A}{4\,d a^{3}}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*A-1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(
1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*C*tan(1/2*d*x+1/2*c)
/(tan(1/2*d*x+1/2*c)^2+1)-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.49045, size = 277, normalized size = 2.04

$$\frac{3\,C\left(\frac{40\,\sin(dx+c)}{\left(a^{3}+\frac{a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{3}}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}\right)+\frac{A\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,\mathrm{s}\ldots}{(\cos\ldots}\right.}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(3*C*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + A*(15*sin(d*x + c)/(cos(d*x +
 c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.64396, size = 386, normalized size = 2.84

$$-\frac{45\,Cdx\cos(dx+c)^{3}+135\,Cdx\cos(dx+c)^{2}+135\,Cdx\cos(dx+c)+45\,Cdx-\left(15\,C\cos(dx+c)^{3}+(7\,A+117\,C)\cos\ldots\right.}{15\left(a^{3}d\cos(dx+c)^{3}+3\,a^{3}d\cos(dx+c)^{2}+3\,a^{3}d\cos(dx+c\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] -1/15*(45*C*d*x*cos(d*x + c)^3 + 135*C*d*x*cos(d*x + c)^2 + 135*C*d*x*cos(d
*x + c) + 45*C*d*x - (15*C*cos(d*x + c)^3 + (7*A + 117*C)*cos(d*x + c)^2 +
3*(2*A + 57*C)*cos(d*x + c) + 2*A + 72*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 16.1421, size = 422, normalized size = 3.1

$$\begin{cases} \frac{3A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{7A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{5A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{15A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \frac{180Cdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} - \\ \frac{x\left(A+C\cos^2(c)\right)\cos^2(c)}{(a\cos(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((3*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3
*d) - 7*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 5*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 15*A
*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*C*d*x*t
an(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*C*d*x/
(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*C*tan(c/2 + d*x/2)**7/(60*a
**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 27*C*tan(c/2 + d*x/2)**5/(60*a**3*
d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 225*C*tan(c/2 + d*x/2)**3/(60*a**3*d*t
an(c/2 + d*x/2)**2 + 60*a**3*d) + 375*C*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2
 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**2/(a*cos
(c) + a)**3, True))
```

Giac [A] time = 1.29864, size = 204, normalized size = 1.5

$$-\frac{\frac{180\,(dx+c)C}{a^3} - \frac{120\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-30\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Aa^{12}\mathrm{ta}}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/60*(180*(d*x + c)*C/a^3 - 120*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x +
 1/2*c)^5 - 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*C*a^12*tan(1/2*d*x + 1/2*
c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) + 255*C*a^12*tan(1/2*d*x + 1/2*c))/a^
15)/d
```

3.59 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=114

$$\frac{(6A-29C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Cx}{a^3}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(3A-7C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] (C*x)/a^3 - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((3*A - 7*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((6*A -
29*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.255591, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 2968, 3019, 2735, 2648}

$$\frac{(6A-29C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Cx}{a^3}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(3A-7C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (C*x)/a^3 - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((3*A - 7*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((6*A -
29*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx &= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{\cos(c+dx)(a(3A-2C)+5aC\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}\\
&= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{a(3A-2C)\cos(c+dx)+5aC\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}\\
&= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(3A-7C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{\int \frac{-2a^2(3A-7C)-\cdots}{a+a\,c\cdots}}{1\cdots}\\
&= \frac{Cx}{a^3}-\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(3A-7C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(6A-29\cdots}{\cdots}\\
&= \frac{Cx}{a^3}-\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(3A-7C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(6A-2\cdots}{15d\left(a^3+\cdots\right.}
\end{aligned}
$$

Mathematica [A] time = 0.523532, size = 227, normalized size = 1.99

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-30A\sin\left(c+\frac{dx}{2}\right)+30A\sin\left(c+\frac{3dx}{2}\right)+6A\sin\left(2c+\frac{5dx}{2}\right)+30A\sin\left(\frac{dx}{2}\right)+270C\sin\left(c+\frac{dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*C*d*x*Cos[(d*x)/2] + 150*C*d*x*Cos[c + (d
*x)/2] + 75*C*d*x*Cos[c + (3*d*x)/2] + 75*C*d*x*Cos[2*c + (3*d*x)/2] + 15*C
*d*x*Cos[2*c + (5*d*x)/2] + 15*C*d*x*Cos[3*c + (5*d*x)/2] + 30*A*Sin[(d*x)/
2] - 370*C*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2] + 270*C*Sin[c + (d*x)/2] +
30*A*Sin[c + (3*d*x)/2] - 230*C*Sin[c + (3*d*x)/2] + 90*C*Sin[2*c + (3*d*x)
/2] + 6*A*Sin[2*c + (5*d*x)/2] - 64*C*Sin[2*c + (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.027, size = 117, normalized size = 1.

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7\,C}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1
/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.51142, size = 189, normalized size = 1.66

$$-\frac{C\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(C*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.61388, size = 351, normalized size = 3.08

$$\frac{15\,Cdx\cos\left(dx+c\right)^{3}+45\,Cdx\cos\left(dx+c\right)^{2}+45\,Cdx\cos\left(dx+c\right)+15\,Cdx+\left(\left(3\,A-32\,C\right)\cos\left(dx+c\right)^{2}+3\left(3\,A-17\,C\right.\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*C*d*x*cos(d*x + c)^3 + 45*C*d*x*cos(d*x + c)^2 + 45*C*d*x*cos(d*x
+ c) + 15*C*d*x + ((3*A - 32*C)*cos(d*x + c)^2 + 3*(3*A - 17*C)*cos(d*x + c
) + 3*A - 22*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 9.56269, size = 128, normalized size = 1.12

$$\begin{cases} -\frac{A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d}+\frac{Cx}{a^{3}}-\frac{C\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d}+\frac{C\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{3}d}-\frac{7C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d} & \text{for } d\neq 0 \\ \frac{x\left(A+C\cos^{2}\left(c\right)\right)\cos\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)/(4*a**3*
d) + C*x/a**3 - C*tan(c/2 + d*x/2)**5/(20*a**3*d) + C*tan(c/2 + d*x/2)**3/(
3*a**3*d) - 7*C*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + C*cos(c)**2
)*cos(c)/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.25333, size = 140, normalized size = 1.23

$$\frac{\frac{60\,(dx+c)C}{a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*C/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(
1/2*d*x + 1/2*c)^5 - 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^12*tan(1/2*d
*x + 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.60 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{(2A+7C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2(A-4C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (2*(A - 4*C)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 7*C)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.120647, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3020, 2750, 2648}

$$\frac{(2A+7C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2(A-4C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (2*(A - 4*C)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 7*C)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 3020

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> Simp[(b*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[a*A*(m + 1) - a*C*m + b*C*(2*m + 1)*Sin[e + f*x], x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{-a(2A-3C)-5aC\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(A-4C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2A+7C)\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}$$
$$= \frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(A-4C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2A+7C)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] time = 0.27611, size = 129, normalized size = 1.32

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(10A\sin\left(c+\frac{3dx}{2}\right)+2A\sin\left(2c+\frac{5dx}{2}\right)+20(A+2C)\sin\left(\frac{dx}{2}\right)-30C\sin\left(c+\frac{dx}{2}\right)+20C\sin\left(c-\right.\right.}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(20*(A + 2*C)*Sin[(d*x)/2] - 30*C*Sin[c + (d*x)/
2] + 10*A*Sin[c + (3*d*x)/2] + 20*C*Sin[c + (3*d*x)/2] - 15*C*Sin[2*c + (3*
d*x)/2] + 2*A*Sin[2*c + (5*d*x)/2] + 7*C*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(
1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.021, size = 88, normalized size = 0.9

$$\frac{1}{4\,da^3}\left(\frac{A}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{2\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*A*tan(1/2*d*x+1/2*c)^5+1/5*C*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/
2*d*x+1/2*c)^3*A-2/3*C*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+C*tan(1/2*
d*x+1/2*c))
```

Maxima [A] time = 1.02457, size = 181, normalized size = 1.85

$$\frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + C*(15*sin(d*x + c)
/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.31873, size = 221, normalized size = 2.26

$$\frac{\left((2\,A+7\,C)\cos\left(dx+c\right)^2+6\,(A+C)\cos\left(dx+c\right)+7\,A+2\,C\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*A + 7*C)*cos(d*x + c)^2 + 6*(A + C)*cos(d*x + c) + 7*A + 2*C)*sin(
d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [A] time = 5.94721, size = 136, normalized size = 1.39

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\left(A+C\cos^2(c)\right)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) + C*tan(c/2 + d*x/2)**5/(20*a**3*d) -
C*tan(c/2 + d*x/2)**3/(6*a**3*d) + C*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0))
, (x*(A + C*cos(c)**2)/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.24804, size = 120, normalized size = 1.22

$$\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 + 3*C*tan(1/2*d*x + 1/2*c)^5 + 10*A*tan(1/
2*d*x + 1/2*c)^3 - 10*C*tan(1/2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c)
+ 15*C*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

3.61 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=115

$$-\frac{(22A-3C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-3C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*A - 3*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 ((22*A - 3*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.314572, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3042, 2978, 12, 3770}

$$-\frac{(22A-3C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-3C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```



```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*A - 3*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 ((22*A - 3*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5aA-a(2A-3C)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(15a^2A-a^2(7A-3C)\cos(c}{a+a\cos(c+}}{15a^4}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(22A-3C)\sin(c+d}{15d\left(a^3+a^3\cos(c+d\right.}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(22A-3C)\sin(c+d}{15d\left(a^3+a^3\cos(c+d\right.}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

Mathematica [A] time = 1.04608, size = 203, normalized size = 1.77

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(15(5A-C)\sin\left(c+\frac{dx}{2}\right)-95A\sin\left(c+\frac{3dx}{2}\right)+15A\sin\left(2c+\frac{3dx}{2}\right)-22A\sin\left(2c+\frac{5dx}{2}\right)-5(29.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-240*A*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-5*(29*A
 - 3*C)*Sin[(d*x)/2] + 15*(5*A - C)*Sin[c + (d*x)/2] - 95*A*Sin[c + (3*d*x)
/2] + 15*C*Sin[c + (3*d*x)/2] + 15*A*Sin[2*c + (3*d*x)/2] - 22*A*Sin[2*c +
(5*d*x)/2] + 3*C*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.053, size = 139, normalized size = 1.2

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{4\,da^3}\tan\left(\frac{dx}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-7/4/d/a^3*A*tan(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1
/2*d*x+1/2*c)-1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*A*ln(tan(1/2*d*x+1
/2*c)+1)
```

Maple [A] is followed by

Maxima [A] time = 1.06165, size = 225, normalized size = 1.96

$$-\frac{A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{3\,C\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
```

```
1)/a^3) - 3*C*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.42269, size = 481, normalized size = 4.18

$$\frac{15\left(A\cos(dx+c)^3+3\,A\cos(dx+c)^2+3\,A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-15\left(A\cos(dx+c)^3+3\,A\cos(dx\right.}{30\left(a^3d\cos(dx+c)^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*log
(sin(d*x + c) + 1) - 15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*
x + c) + A)*log(-sin(d*x + c) + 1) - 2*((22*A - 3*C)*cos(d*x + c)^2 + 3*(17
*A - 3*C)*cos(d*x + c) + 32*A - 3*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{C\cos^2(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x) + Integral(C*cos(c + d*x)**2*sec(c + d*x)/(cos(c + d*x)**3 + 3
*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.28601, size = 177, normalized size = 1.54

$$\frac{\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,Aa^{12}\tan\left(\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*
d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x
+ 1/2*c) - 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.62 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$\frac{2(36A+C)\tan(c+dx)}{15a^3d}-\frac{3A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3A\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-C)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A+C)\,\text{ta}}{5d(a\cos(c-}$$

```
[Out] (-3*A*ArcTanh[Sin[c + d*x]])/(a^3*d) + (2*(36*A + C)*Tan[c + d*x])/(15*a^3*
d) - ((A + C)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*A - C)*Tan[c
 + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (3*A*Tan[c + d*x])/(d*(a^3 + a^3
*Cos[c + d*x]))
```

Rubi [A] time = 0.442715, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2748, 3767, 8, 3770}

$$\frac{2(36A+C)\tan(c+dx)}{15a^3d}-\frac{3A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3A\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-C)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A+C)\,\text{ta}}{5d(a\cos(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-3*A*ArcTanh[Sin[c + d*x]])/(a^3*d) + (2*(36*A + C)*Tan[c + d*x])/(15*a^3*
d) - ((A + C)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*A - C)*Tan[c
 + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (3*A*Tan[c + d*x])/(d*(a^3 + a^3
*Cos[c + d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(6A+C)-a(3A-2C)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(a^2(27A+2C)-2a^2(9A-C}{a+a\cos}}{15}$$

$$= -\frac{(A+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{3A\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{(A+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{3A\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{3A\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-C)\tan(c+dx}{15ad(a+a\cos(c+dx}$$

$$= -\frac{3A\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2(36A+C)\tan(c+dx)}{15a^3d} - \frac{(A+C)\tan(c+dx}{5d(a+a\cos(c+dx)}$$

Mathematica [B] time = 6.30674, size = 596, normalized size = 4.62

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos(c+dx)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-600A\sin\left(c-\frac{dx}{2}\right)+375A\sin\left(c+\frac{dx}{2}\right)-480A\sin\left(2c+\frac{dx}{2}\right)-60A\sin\left(c+\frac{3dx}{2}\right)+402A\sin\left(2c+\frac{3dx}{2}\right)-225A\sin\left(3c+\frac{3dx}{2}\right)+315A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((48*A*Cos[c/2 + (d*x)/2]^6*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2
 + (d*x)/2]]*(C + A*Sec[c + d*x]^2))/(d*(1 + Cos[c + d*x])^3*(2*A + C + C*C
os[2*c + 2*d*x])) - (48*A*Cos[c/2 + (d*x)/2]^6*Cos[c + d*x]^2*Log[Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2]]*(C + A*Sec[c + d*x]^2))/(d*(1 + Cos[c + d*x
])^3*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c/2 + (d*x)/2]*Cos[c + d*x]*Sec
[c/2]*Sec[c]*(C + A*Sec[c + d*x]^2)*(-255*A*Sin[(d*x)/2] - 20*C*Sin[(d*x)/2
] + 567*A*Sin[(3*d*x)/2] + 22*C*Sin[(3*d*x)/2] - 600*A*Sin[c - (d*x)/2] - 1
0*C*Sin[c - (d*x)/2] + 375*A*Sin[c + (d*x)/2] + 10*C*Sin[c + (d*x)/2] - 480
*A*Sin[2*c + (d*x)/2] - 20*C*Sin[2*c + (d*x)/2] - 60*A*Sin[c + (3*d*x)/2] +
 402*A*Sin[2*c + (3*d*x)/2] + 22*C*Sin[2*c + (3*d*x)/2] - 225*A*Sin[3*c + (
3*d*x)/2] + 315*A*Sin[c + (5*d*x)/2] + 10*C*Sin[c + (5*d*x)/2] + 30*A*Sin[2
*c + (5*d*x)/2] + 240*A*Sin[3*c + (5*d*x)/2] + 10*C*Sin[3*c + (5*d*x)/2] -
45*A*Sin[4*c + (5*d*x)/2] + 72*A*Sin[2*c + (7*d*x)/2] + 2*C*Sin[2*c + (7*d*
x)/2] + 15*A*Sin[3*c + (7*d*x)/2] + 57*A*Sin[4*c + (7*d*x)/2] + 2*C*Sin[4*c
```

```
 + (7*d*x)/2]))/(60*d*(1 + Cos[c + d*x])^3*(2*A + C + C*Cos[2*c + 2*d*x])))
/a^3
```

Maple [A] time = 0.061, size = 204, normalized size = 1.6

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)-1/d/a^3*A/(tan(1/2*d*x+1/2*c
)-1)+3/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-3/
d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.03376, size = 315, normalized size = 2.44

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)+\frac{C\left(\frac{15\,\sin(d}{\cos(dx+c}\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) + C*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.46351, size = 576, normalized size = 4.47

$$-\frac{45\left(A\cos\left(dx+c\right)^{4}+3\,A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-45\left(A\cos\left(dx+c\right.\right.}{30\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] -1/30*(45*(A*cos(d*x + c)^4 + 3*A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + A*c
os(d*x + c))*log(sin(d*x + c) + 1) - 45*(A*cos(d*x + c)^4 + 3*A*cos(d*x + c
)^3 + 3*A*cos(d*x + c)^2 + A*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(3
6*A + C)*cos(d*x + c)^3 + 3*(57*A + 2*C)*cos(d*x + c)^2 + (117*A + 7*C)*cos
(d*x + c) + 15*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33288, size = 240, normalized size = 1.86

$$-\frac{\frac{180\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{180\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+30\,Aa^{12}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/60*(180*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 180*A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^3 + 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/
2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 10*C*a^12*tan(1/2*d*x + 1/2*c)^
3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) + 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.63 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{2(76A+11C)\tan(c+dx)}{15a^3d}+\frac{(13A+2C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A+2C)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{(76A+11C)\,\mathrm{t}}{15d\left(a^3\,\mathrm{c}\right.}$$

```
[Out] ((13*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(76*A + 11*C)*Tan[c + d
*x])/(15*a^3*d) + ((13*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A
+ C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*A + C)*
Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((76*A + 11*C)
*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.513009, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(76A+11C)\tan(c+dx)}{15a^3d}+\frac{(13A+2C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A+2C)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{(76A+11C)\,\mathrm{t}}{15d\left(a^3\,\mathrm{c}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] ((13*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(76*A + 11*C)*Tan[c + d
*x])/(15*a^3*d) + ((13*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A
+ C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*A + C)*
Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((76*A + 11*C)
*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(7A+2C)-a(4A-C)\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A+C)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A+C)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - (7$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A+C)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - (7$$

$$= \frac{(13A+2C)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - (1$$

$$= \frac{(13A+2C)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{2(76A+11C)\tan(c+dx)}{15a^3d} + \frac{(13A+2C}{}$$

Mathematica [B] time = 4.60043, size = 597, normalized size = 3.11

$$-\frac{1920(13A+2C)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(1920*(13*A + 2*C)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/
2]*Sec[c]*Sec[c + d*x]^2*(-5*(247*A + 98*C)*Sin[(d*x)/2] + 5*(761*A + 106*C
)*Sin[(3*d*x)/2] - 4329*A*Sin[c - (d*x)/2] - 654*C*Sin[c - (d*x)/2] + 1989*
A*Sin[c + (d*x)/2] + 654*C*Sin[c + (d*x)/2] - 3575*A*Sin[2*c + (d*x)/2] - 4
90*C*Sin[2*c + (d*x)/2] - 475*A*Sin[c + (3*d*x)/2] - 350*C*Sin[c + (3*d*x)/
2] + 2005*A*Sin[2*c + (3*d*x)/2] + 530*C*Sin[2*c + (3*d*x)/2] - 2275*A*Sin[
3*c + (3*d*x)/2] - 350*C*Sin[3*c + (3*d*x)/2] + 2673*A*Sin[c + (5*d*x)/2] +
 378*C*Sin[c + (5*d*x)/2] + 105*A*Sin[2*c + (5*d*x)/2] - 150*C*Sin[2*c + (5
*d*x)/2] + 1593*A*Sin[3*c + (5*d*x)/2] + 378*C*Sin[3*c + (5*d*x)/2] - 975*A
*Sin[4*c + (5*d*x)/2] - 150*C*Sin[4*c + (5*d*x)/2] + 1325*A*Sin[2*c + (7*d*
x)/2] + 190*C*Sin[2*c + (7*d*x)/2] + 255*A*Sin[3*c + (7*d*x)/2] - 30*C*Sin[
3*c + (7*d*x)/2] + 875*A*Sin[4*c + (7*d*x)/2] + 190*C*Sin[4*c + (7*d*x)/2]
- 195*A*Sin[5*c + (7*d*x)/2] - 30*C*Sin[5*c + (7*d*x)/2] + 304*A*Sin[3*c +
```

```
(9*d*x)/2] + 44*C*Sin[3*c + (9*d*x)/2] + 90*A*Sin[4*c + (9*d*x)/2] + 214*A*
Sin[5*c + (9*d*x)/2] + 44*C*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c +
d*x])^3)
```

Maple [A] time = 0.071, size = 289, normalized size = 1.5

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*ta
n(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)-13/2/d/a^3*A*ln(tan(1/2*d*x
+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^3*A/(tan(1/2*d*x+1/2*
c)-1)^2+7/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)+13/2/d/a^3*A*ln(tan(1/2*d*x+1/2*
c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)
^2+7/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.03626, size = 446, normalized size = 2.32

$$-\frac{A\left(\frac{60\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)+C\left(\frac{105\,\mathrm{s}}{\cos(}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) + C*((105*sin(d*x + c)/(cos(d*x + c) + 1)
```

```
 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 1.37142, size = 740, normalized size = 3.85

$$\frac{15\left((13\,A+2\,C)\cos\left(dx+c\right)^{5}+3\,(13\,A+2\,C)\cos\left(dx+c\right)^{4}+3\,(13\,A+2\,C)\cos\left(dx+c\right)^{3}+(13\,A+2\,C)\cos\left(dx+c\right)^{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/60*(15*((13*A + 2*C)*cos(d*x + c)^5 + 3*(13*A + 2*C)*cos(d*x + c)^4 + 3*(
13*A + 2*C)*cos(d*x + c)^3 + (13*A + 2*C)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - 15*((13*A + 2*C)*cos(d*x + c)^5 + 3*(13*A + 2*C)*cos(d*x + c)^4 + 3*
(13*A + 2*C)*cos(d*x + c)^3 + (13*A + 2*C)*cos(d*x + c)^2)*log(-sin(d*x + c
) + 1) - 2*(4*(76*A + 11*C)*cos(d*x + c)^4 + 3*(239*A + 34*C)*cos(d*x + c)^
3 + (479*A + 64*C)*cos(d*x + c)^2 + 45*A*cos(d*x + c) - 15*A)*sin(d*x + c))
/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34848, size = 279, normalized size = 1.45

$$\frac{\frac{30\,(13\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{30\,(13\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(30*(13*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13*A + 2
*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*A*tan(1/2*d*x + 1/2*c)^3
 - 5*A*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^
12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^12*tan
(1/2*d*x + 1/2*c)^3 + 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2
*d*x + 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.64 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=225

$$\frac{4(34A+9C)\tan^3(c+dx)}{15a^3d}+\frac{4(34A+9C)\tan(c+dx)}{5a^3d}-\frac{(23A+6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(23A+6C)\tan(c+dx)\,\mathrm{se}}{2a^3d}$$

```
[Out] -((23*A + 6*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (4*(34*A + 9*C)*Tan[c + d
*x])/(5*a^3*d) - ((23*A + 6*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A +
 C)*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((13*A + 3*
C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((23*A +
6*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) + (4*(34*A
 + 9*C)*Tan[c + d*x]^3)/(15*a^3*d)
```

Rubi [A] time = 0.554484, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2748, 3767, 3768, 3770}

$$\frac{4(34A+9C)\tan^3(c+dx)}{15a^3d}+\frac{4(34A+9C)\tan(c+dx)}{5a^3d}-\frac{(23A+6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(23A+6C)\tan(c+dx)\,\mathrm{se}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -((23*A + 6*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (4*(34*A + 9*C)*Tan[c + d
*x])/(5*a^3*d) - ((23*A + 6*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A +
 C)*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((13*A + 3*
C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((23*A +
6*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) + (4*(34*A
 + 9*C)*Tan[c + d*x]^3)/(15*a^3*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c + dx)\right)\sec^4(c + dx)}{(a + a\cos(c + dx))^3}\,dx = -\frac{(A + C)\sec^2(c + dx)\tan(c + dx)}{5d(a + a\cos(c + dx))^3} + \frac{\int \frac{(a(8A+3C)-5aA\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A + C)\sec^2(c + dx)\tan(c + dx)}{5d(a + a\cos(c + dx))^3} - \frac{(13A + 3C)\sec^2(c + dx)\tan(c + dx)}{15ad(a + a\cos(c + dx))^2} -$$

$$= -\frac{(A + C)\sec^2(c + dx)\tan(c + dx)}{5d(a + a\cos(c + dx))^3} - \frac{(13A + 3C)\sec^2(c + dx)\tan(c + dx)}{15ad(a + a\cos(c + dx))^2} -$$

$$= -\frac{(A + C)\sec^2(c + dx)\tan(c + dx)}{5d(a + a\cos(c + dx))^3} - \frac{(13A + 3C)\sec^2(c + dx)\tan(c + dx)}{15ad(a + a\cos(c + dx))^2} -$$

$$= -\frac{(23A + 6C)\sec(c + dx)\tan(c + dx)}{2a^3d} - \frac{(A + C)\sec^2(c + dx)\tan(c + dx)}{5d(a + a\cos(c + dx))^3} -$$

$$= -\frac{(23A + 6C)\tanh^{-1}(\sin(c + dx))}{2a^3d} + \frac{4(34A + 9C)\tan(c + dx)}{5a^3d} - \frac{(23A + 6C}{}$$

Mathematica [B] time = 6.45378, size = 798, normalized size = 3.55

$$\frac{4(23A + 6C)\log\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)\cos^6\left(\frac{c}{2} + \frac{dx}{2}\right)}{d(\cos(c + dx)a + a)^3} - \frac{4(23A + 6C)\log\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)\cos^6\left(\frac{c}{2}\right.}{d(\cos(c + dx)a + a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (4*(23*A + 6*C)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*
x)/2]])/(d*(a + a*Cos[c + d*x])^3) - (4*(23*A + 6*C)*Cos[c/2 + (d*x)/2]^6*L
og[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^3) + (
Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^3*(-2484*A*Sin[(d*x)/2] - 1
764*C*Sin[(d*x)/2] + 12622*A*Sin[(3*d*x)/2] + 3372*C*Sin[(3*d*x)/2] - 13340
*A*Sin[c - (d*x)/2] - 3480*C*Sin[c - (d*x)/2] + 4140*A*Sin[c + (d*x)/2] + 2
100*C*Sin[c + (d*x)/2] - 11684*A*Sin[2*c + (d*x)/2] - 3144*C*Sin[2*c + (d*x
)/2] - 450*A*Sin[c + (3*d*x)/2] - 960*C*Sin[c + (3*d*x)/2] + 5022*A*Sin[2*c
 + (3*d*x)/2] + 2232*C*Sin[2*c + (3*d*x)/2] - 8050*A*Sin[3*c + (3*d*x)/2] -
 2100*C*Sin[3*c + (3*d*x)/2] + 9230*A*Sin[c + (5*d*x)/2] + 2460*C*Sin[c + (
5*d*x)/2] + 630*A*Sin[2*c + (5*d*x)/2] - 390*C*Sin[2*c + (5*d*x)/2] + 4230*
A*Sin[3*c + (5*d*x)/2] + 1710*C*Sin[3*c + (5*d*x)/2] - 4370*A*Sin[4*c + (5*
d*x)/2] - 1140*C*Sin[4*c + (5*d*x)/2] + 5347*A*Sin[2*c + (7*d*x)/2] + 1422*
C*Sin[2*c + (7*d*x)/2] + 875*A*Sin[3*c + (7*d*x)/2] - 60*C*Sin[3*c + (7*d*x
```

```
)/2] + 2747*A*Sin[4*c + (7*d*x)/2] + 1032*C*Sin[4*c + (7*d*x)/2] - 1725*A*S
in[5*c + (7*d*x)/2] - 450*C*Sin[5*c + (7*d*x)/2] + 2375*A*Sin[3*c + (9*d*x)
/2] + 630*C*Sin[3*c + (9*d*x)/2] + 655*A*Sin[4*c + (9*d*x)/2] + 60*C*Sin[4*
c + (9*d*x)/2] + 1375*A*Sin[5*c + (9*d*x)/2] + 480*C*Sin[5*c + (9*d*x)/2] -
 345*A*Sin[6*c + (9*d*x)/2] - 90*C*Sin[6*c + (9*d*x)/2] + 544*A*Sin[4*c + (
11*d*x)/2] + 144*C*Sin[4*c + (11*d*x)/2] + 200*A*Sin[5*c + (11*d*x)/2] + 30
*C*Sin[5*c + (11*d*x)/2] + 344*A*Sin[6*c + (11*d*x)/2] + 114*C*Sin[6*c + (1
1*d*x)/2]))/(960*d*(a + a*Cos[c + d*x])^3)
```

Maple [A] time = 0.072, size = 378, normalized size = 1.7

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{5\,A}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{49\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+5/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+49/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)-17/2/d/a^3*A/(tan(1/2*d*x+1
/2*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*C+23/2/d/a^3*A*ln(tan(1/2*d*x+1/2*c
)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^
3-2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^2-23/2/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)-
3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-17/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-1/d/
a^3/(tan(1/2*d*x+1/2*c)+1)*C-1/3/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)^3+2/d/a^3*A
/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [A] time = 1.06271, size = 568, normalized size = 2.52

$$\frac{A\left(\frac{20\left(\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{76\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3-\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{50\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{690\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{690\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(A*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) - 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 - 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) + 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 690*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 690*l
og(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3) + 3*C*(40*sin(d*x + c)/((a^3 -
 a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x
 + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 1.53899, size = 786, normalized size = 3.49

$$-\frac{15\left((23\,A+6\,C)\cos\left(dx+c\right)^{6}+3\,(23\,A+6\,C)\cos\left(dx+c\right)^{5}+3\,(23\,A+6\,C)\cos\left(dx+c\right)^{4}+(23\,A+6\,C)\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] -1/60*(15*((23*A + 6*C)*cos(d*x + c)^6 + 3*(23*A + 6*C)*cos(d*x + c)^5 + 3*
(23*A + 6*C)*cos(d*x + c)^4 + (23*A + 6*C)*cos(d*x + c)^3)*log(sin(d*x + c)
 + 1) - 15*((23*A + 6*C)*cos(d*x + c)^6 + 3*(23*A + 6*C)*cos(d*x + c)^5 + 3
*(23*A + 6*C)*cos(d*x + c)^4 + (23*A + 6*C)*cos(d*x + c)^3)*log(-sin(d*x +
c) + 1) - 2*(16*(34*A + 9*C)*cos(d*x + c)^5 + 9*(143*A + 38*C)*cos(d*x + c)
^4 + (869*A + 234*C)*cos(d*x + c)^3 + 5*(19*A + 6*C)*cos(d*x + c)^2 - 15*A*
cos(d*x + c) + 10*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6 + 3*a^3*d*cos(d*x
+ c)^5 + 3*a^3*d*cos(d*x + c)^4 + a^3*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25775, size = 352, normalized size = 1.56

$$-\frac{\frac{30\,(23\,A+6\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(23\,A+6\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{20\left(51\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-76\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/60*(30*(23*A + 6*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(23*A +
6*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 20*(51*A*tan(1/2*d*x + 1/2*c)
^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 - 76*A*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/
2*d*x + 1/2*c)^3 + 33*A*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x + 1/2*c))/((
tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C
*a^12*tan(1/2*d*x + 1/2*c)^5 + 50*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*C*a^12
*tan(1/2*d*x + 1/2*c)^3 + 735*A*a^12*tan(1/2*d*x + 1/2*c) + 255*C*a^12*tan(
1/2*d*x + 1/2*c))/a^15)/d
```

3.65 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=223

$$-\frac{32(5A+54C)\sin(c+dx)}{105a^4d}-\frac{(10A+129C)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{16(5A+54C)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}+\frac{(2A}{}$$

```
[Out] ((2*A + 21*C)*x)/(2*a^4) - (32*(5*A + 54*C)*Sin[c + d*x])/(105*a^4*d) + ((2
*A + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((10*A + 129*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (16*(5*A + 54*C)*Cos
[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c +
 d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*C*Cos[c + d*x]^4*Si
n[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.612062, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 2977, 2734}

$$-\frac{32(5A+54C)\sin(c+dx)}{105a^4d}-\frac{(10A+129C)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{16(5A+54C)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}+\frac{(2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((2*A + 21*C)*x)/(2*a^4) - (32*(5*A + 54*C)*Sin[c + d*x])/(105*a^4*d) + ((2
*A + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((10*A + 129*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (16*(5*A + 54*C)*Cos
[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c +
 d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*C*Cos[c + d*x]^4*Si
n[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^4(c+dx)(a(2A-5C)+a(2A+9C)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2C\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+d}{}}{}$$

$$= -\frac{(10A+129C)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{(10A+129C)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= \frac{(2A+21C)x}{2a^4} - \frac{32(5A+54C)\sin(c+dx)}{105a^4d} + \frac{(2A+21C)\cos(c+dx)\sin(c+}{2a^4d}$$

Mathematica [B] time = 1.04144, size = 513, normalized size = 2.3

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(14700dx(2A+21C)\cos\left(c+\frac{dx}{2}\right)+66080A\sin\left(c+\frac{dx}{2}\right)-57120A\sin\left(c+\frac{3dx}{2}\right)+30240A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(14700*(2*A + 21*C)*d*x*Cos[(d*x)/2] + 14700*(2*
A + 21*C)*d*x*Cos[c + (d*x)/2] + 17640*A*d*x*Cos[c + (3*d*x)/2] + 185220*C*
d*x*Cos[c + (3*d*x)/2] + 17640*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*C*d*x*Co
s[2*c + (3*d*x)/2] + 5880*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*C*d*x*Cos[2*c
+ (5*d*x)/2] + 5880*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*C*d*x*Cos[3*c + (5*d
*x)/2] + 840*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*C*d*x*Cos[3*c + (7*d*x)/2] +
 840*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*C*d*x*Cos[4*c + (7*d*x)/2] - 79520*A
*Sin[(d*x)/2] - 539490*C*Sin[(d*x)/2] + 66080*A*Sin[c + (d*x)/2] + 386190*C
*Sin[c + (d*x)/2] - 57120*A*Sin[c + (3*d*x)/2] - 422478*C*Sin[c + (3*d*x)/2
] + 30240*A*Sin[2*c + (3*d*x)/2] + 132930*C*Sin[2*c + (3*d*x)/2] - 22400*A*
Sin[2*c + (5*d*x)/2] - 181461*C*Sin[2*c + (5*d*x)/2] + 6720*A*Sin[3*c + (5*
d*x)/2] + 3675*C*Sin[3*c + (5*d*x)/2] - 4160*A*Sin[3*c + (7*d*x)/2] - 36003
*C*Sin[3*c + (7*d*x)/2] - 9555*C*Sin[4*c + (7*d*x)/2] - 945*C*Sin[4*c + (9*
d*x)/2] - 945*C*Sin[5*c + (9*d*x)/2] + 105*C*Sin[5*c + (11*d*x)/2] + 105*C*
Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.031, size = 264, normalized size = 1.2

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{9\,C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{11\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C-1/8/d/a
^4*A*tan(1/2*d*x+1/2*c)^5-9/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*tan
(1/2*d*x+1/2*c)^3*A+13/8/d/a^4*C*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2*
d*x+1/2*c)-111/8/d/a^4*C*tan(1/2*d*x+1/2*c)-9/d/a^4/(tan(1/2*d*x+1/2*c)^2+1
)^2*C*tan(1/2*d*x+1/2*c)^3-7/d/a^4/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x
+1/2*c)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*A+21/d/a^4*arctan(tan(1/2*d*x+1/
```

```
2*c))*C
```

Maxima [A] time = 1.60033, size = 429, normalized size = 1.92

$$-\frac{3\,C\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)+5\,A\left(\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}\cdots\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(3*C*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) + 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) -
 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c
)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 1.51275, size = 632, normalized size = 2.83

$$\frac{105\,(2\,A+21\,C)dx\cos{(dx+c)}^4+420\,(2\,A+21\,C)dx\cos{(dx+c)}^3+630\,(2\,A+21\,C)dx\cos{(dx+c)}^2+420\,(2\,A+21\,C\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/210*(105*(2*A + 21*C)*d*x*cos(d*x + c)^4 + 420*(2*A + 21*C)*d*x*cos(d*x +
 c)^3 + 630*(2*A + 21*C)*d*x*cos(d*x + c)^2 + 420*(2*A + 21*C)*d*x*cos(d*x
+ c) + 105*(2*A + 21*C)*d*x + (105*C*cos(d*x + c)^5 - 420*C*cos(d*x + c)^4
- 4*(130*A + 1509*C)*cos(d*x + c)^3 - 4*(310*A + 3411*C)*cos(d*x + c)^2 - (
1070*A + 11619*C)*cos(d*x + c) - 320*A - 3456*C)*sin(d*x + c))/(a^4*d*cos(d
```

```
*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d
*x + c) + a^4*d)
```

Sympy [A] time = 71.1258, size = 1086, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((840*A*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 +
1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 1680*A*d*x*tan(c/2 + d*x/2)
**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840
*a**4*d) + 840*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2
+ d*x/2)**2 + 840*a**4*d) + 15*A*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 +
 d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 75*A*tan(c/2 +
 d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**
2 + 840*a**4*d) + 190*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4
 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 910*A*tan(c/2 + d*x/2)**
5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a
**4*d) - 2765*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*
a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1575*A*tan(c/2 + d*x/2)/(840*a**
4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 8
820*C*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d
*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 17640*C*d*x*tan(c/2 + d*x/2)**2/(840*a
**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) +
 8820*C*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)*
*2 + 840*a**4*d) + 15*C*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 + d*x/2)**
4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 159*C*tan(c/2 + d*x/2)*
*9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*
a**4*d) + 1002*C*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680
*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 9114*C*tan(c/2 + d*x/2)**5/(840
*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d)
 - 29505*C*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*
d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 17535*C*tan(c/2 + d*x/2)/(840*a**4*d*
tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d,
0)), (x*(A + C*cos(c)**2)*cos(c)**4/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.24915, size = 279, normalized size = 1.25

$$\frac{\frac{420\,(dx+c)(2\,A+21\,C)}{a^4} - \frac{840\left(9\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-189}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(420*(d*x + c)*(2*A + 21*C)/a^4 - 840*(9*C*tan(1/2*d*x + 1/2*c)^3 + 7
*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*
tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(
1/2*d*x + 1/2*c)^5 - 189*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2
*d*x + 1/2*c)^3 + 1365*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*
d*x + 1/2*c) - 11655*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.66 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=174

$$\frac{2(3A+122C)\sin(c+dx)}{105a^4d}+\frac{(3A-88C)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4C\sin(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{4Cx}{a^4}-\frac{(A+C)\sin(c+dx)}{7d(a\cos(c+d}$$

```
[Out] (-4*C*x)/a^4 + (2*(3*A + 122*C)*Sin[c + d*x])/(105*a^4*d) + ((3*A - 88*C)*C
os[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*C*Sin[c +
 d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(
7*d*(a + a*Cos[c + d*x])^4) + (2*(A - 6*C)*Cos[c + d*x]^3*Sin[c + d*x])/(35
*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.577124, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{2(3A+122C)\sin(c+dx)}{105a^4d}+\frac{(3A-88C)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4C\sin(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{4Cx}{a^4}-\frac{(A+C)\sin(c+dx)}{7d(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (-4*C*x)/a^4 + (2*(3*A + 122*C)*Sin[c + d*x])/(105*a^4*d) + ((3*A - 88*C)*C
os[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*C*Sin[c +
 d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(
7*d*(a + a*Cos[c + d*x])^4) + (2*(A - 6*C)*Cos[c + d*x]^3*Sin[c + d*x])/(35
*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(a(3A-4C)+a(A+8C)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{2(A-6C)\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} +$$

$$= \frac{(3A-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} +$$

$$= \frac{(3A-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} +$$

$$= \frac{2(3A+122C)\sin(c+dx)}{105a^4d} + \frac{(3A-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C}{7a}$$

$$= \frac{2(3A+122C)\sin(c+dx)}{105a^4d} + \frac{(3A-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C}{7a}$$

$$= -\frac{4Cx}{a^4} + \frac{2(3A+122C)\sin(c+dx)}{105a^4d} + \frac{(3A-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}$$

$$= -\frac{4Cx}{a^4} + \frac{2(3A+122C)\sin(c+dx)}{105a^4d} + \frac{(3A-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}$$

Mathematica [B] time = 0.729315, size = 371, normalized size = 2.13

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(2520A\sin\left(c+\frac{dx}{2}\right)-1764A\sin\left(c+\frac{3dx}{2}\right)+1260A\sin\left(2c+\frac{3dx}{2}\right)-588A\sin\left(2c+\frac{5dx}{2}\right)+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*C*d*x*Cos[(d*x)/2] + 29400*C*d*x*Cos[c
 + (d*x)/2] + 17640*C*d*x*Cos[c + (3*d*x)/2] + 17640*C*d*x*Cos[2*c + (3*d*x
)/2] + 5880*C*d*x*Cos[2*c + (5*d*x)/2] + 5880*C*d*x*Cos[3*c + (5*d*x)/2] +
840*C*d*x*Cos[3*c + (7*d*x)/2] + 840*C*d*x*Cos[4*c + (7*d*x)/2] - 2520*A*Si
n[(d*x)/2] - 60830*C*Sin[(d*x)/2] + 2520*A*Sin[c + (d*x)/2] + 46130*C*Sin[c
```

```
 + (d*x)/2] - 1764*A*Sin[c + (3*d*x)/2] - 46116*C*Sin[c + (3*d*x)/2] + 1260
*A*Sin[2*c + (3*d*x)/2] + 18060*C*Sin[2*c + (3*d*x)/2] - 588*A*Sin[2*c + (5
*d*x)/2] - 19292*C*Sin[2*c + (5*d*x)/2] + 420*A*Sin[3*c + (5*d*x)/2] + 2100
*C*Sin[3*c + (5*d*x)/2] - 144*A*Sin[3*c + (7*d*x)/2] - 3791*C*Sin[3*c + (7*
d*x)/2] - 735*C*Sin[4*c + (7*d*x)/2] - 105*C*Sin[4*c + (9*d*x)/2] - 105*C*S
in[5*c + (9*d*x)/2]))/(26880*a^4*d)
```

Maple [A] time = 0.031, size = 210, normalized size = 1.2

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C+3/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*tan
(1/2*d*x+1/2*c)^3*A-23/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*A*tan(1/2*
d*x+1/2*c)+49/8/d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*C*tan(1/2*d*x+1/2*c)/(ta
n(1/2*d*x+1/2*c)^2+1)-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.65982, size = 332, normalized size = 1.91

$$\frac{C\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)+\frac{3\,A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\cdots\right.}{\cdots}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(C*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x +
```

```
c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.41748, size = 516, normalized size = 2.97

$$-\frac{420\,Cdx\cos\left(dx+c\right)^{4}+1680\,Cdx\cos\left(dx+c\right)^{3}+2520\,Cdx\cos\left(dx+c\right)^{2}+1680\,Cdx\cos\left(dx+c\right)+420\,Cdx-\left(105\,C\right.}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] -1/105*(420*C*d*x*cos(d*x + c)^4 + 1680*C*d*x*cos(d*x + c)^3 + 2520*C*d*x*c
os(d*x + c)^2 + 1680*C*d*x*cos(d*x + c) + 420*C*d*x - (105*C*cos(d*x + c)^4
 + 4*(9*A + 296*C)*cos(d*x + c)^3 + (39*A + 2636*C)*cos(d*x + c)^2 + 4*(6*A
 + 559*C)*cos(d*x + c) + 6*A + 664*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 +
 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a
^4*d)
```

Sympy [A] time = 40.2136, size = 462, normalized size = 2.66

$$\begin{cases} -\frac{15A\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^{4}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^{4}d}+\frac{48A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^{4}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^{4}d}-\frac{42A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^{4}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^{4}d}+\frac{105A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^{4}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^{4}d}-\frac{3360Cdx\tan^{2}\left(\right.}{840a^{4}d\tan^{2}\left(\frac{c}{2}+\frac{d}{2}\right.} \\ \frac{x\left(A+C\cos^{2}\left(c\right)\right)\cos^{3}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-15*A*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*
a**4*d) + 48*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a*
*4*d) - 42*A*tan(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4
*d) + 105*A*tan(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d)
- 3360*C*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4
*d) - 3360*C*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*C*tan(c
/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*C*tan(c/
2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*C*tan(c/2
```

```
+ d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*C*tan(c/2
+ d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*C*tan(c/2
+ d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*(A +
C*cos(c)**2)*cos(c)**3/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.16189, size = 248, normalized size = 1.43

$$-\frac{\frac{3360\,(dx+c)C}{a^4} - \frac{1680\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-147\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Aa}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/840*(3360*(d*x + c)*C/a^4 - 1680*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*
d*x + 1/2*c)^7 - 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*C*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 805*C*a^24*tan(1/2*d*x + 1
/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c) - 5145*C*a^24*tan(1/2*d*x + 1/2*c
))/a^28)/d
```

3.67 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=152

$$\frac{(16A-215C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A-55C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Cx}{a^4}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{2(2A-5C)\sin(c+dx)\cos^2(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] (C*x)/a^4 - ((8*A - 55*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) +
((16*A - 215*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + (2*(2*A - 5*C)*Cos
[c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.444877, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2977, 2968, 3019, 2735, 2648}

$$\frac{(16A-215C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A-55C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Cx}{a^4}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{2(2A-5C)\sin(c+dx)\cos^2(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (C*x)/a^4 - ((8*A - 55*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) +
((16*A - 215*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + (2*(2*A - 5*C)*Cos
[c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}dx = -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(a(4A-3C)+7aC\cos(c+dx))}{(a+a\cos(c+dx))^3}dx}{7a^2}$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{2(2A-5C)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} +$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{2(2A-5C)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} +$$

$$= -\frac{(8A-55C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{2(2A-5C}{35ad}$$

$$= \frac{Cx}{a^4} - \frac{(8A-55C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{2(2A}{}$$

$$= \frac{Cx}{a^4} - \frac{(8A-55C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{2(2A}{}$$

Mathematica [B] time = 0.709024, size = 315, normalized size = 2.07

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-350A\sin\left(c+\frac{dx}{2}\right)+336A\sin\left(c+\frac{3dx}{2}\right)-210A\sin\left(2c+\frac{3dx}{2}\right)+182A\sin\left(2c+\frac{5dx}{2}\right)+26A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*C*d*x*Cos[(d*x)/2] + 3675*C*d*x*Cos[c +
(d*x)/2] + 2205*C*d*x*Cos[c + (3*d*x)/2] + 2205*C*d*x*Cos[2*c + (3*d*x)/2]
+ 735*C*d*x*Cos[2*c + (5*d*x)/2] + 735*C*d*x*Cos[3*c + (5*d*x)/2] + 105*C*d
*x*Cos[3*c + (7*d*x)/2] + 105*C*d*x*Cos[4*c + (7*d*x)/2] + 560*A*Sin[(d*x)/
2] - 9940*C*Sin[(d*x)/2] - 350*A*Sin[c + (d*x)/2] + 8260*C*Sin[c + (d*x)/2]
 + 336*A*Sin[c + (3*d*x)/2] - 7140*C*Sin[c + (3*d*x)/2] - 210*A*Sin[2*c + (
3*d*x)/2] + 3780*C*Sin[2*c + (3*d*x)/2] + 182*A*Sin[2*c + (5*d*x)/2] - 2800
*C*Sin[2*c + (5*d*x)/2] + 840*C*Sin[3*c + (5*d*x)/2] + 26*A*Sin[3*c + (7*d*
x)/2] - 520*C*Sin[3*c + (7*d*x)/2]))/(13440*a^4*d)
```

Maple [A] time = 0.027, size = 177, normalized size = 1.2

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C-1/40/d/
a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5-1/24/d/a^4*tan(
1/2*d*x+1/2*c)^3*A+11/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*A*tan(1/2*d
*x+1/2*c)-15/8/d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c)
)*C
```

Maxima [A] time = 1.56561, size = 271, normalized size = 1.78

$$-\frac{5\,C\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{A\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(d}{(\cos(dx+}\right.}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.38198, size = 477, normalized size = 3.14

$$\frac{105\,Cdx\cos(dx+c)^4+420\,Cdx\cos(dx+c)^3+630\,Cdx\cos(dx+c)^2+420\,Cdx\cos(dx+c)+105\,Cdx+\left(13\,(A-20\,C\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/105*(105*C*d*x*cos(d*x + c)^4 + 420*C*d*x*cos(d*x + c)^3 + 630*C*d*x*cos(
d*x + c)^2 + 420*C*d*x*cos(d*x + c) + 105*C*d*x + (13*(A - 20*C)*cos(d*x +
c)^3 + 4*(13*A - 155*C)*cos(d*x + c)^2 + (32*A - 535*C)*cos(d*x + c) + 8*A
- 160*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a
^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 24.1228, size = 192, normalized size = 1.26

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{Cx}{a^4} + \frac{C\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{11C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} - \frac{15C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} \\ \frac{x\left(A+C\cos^2(c)\right)\cos^2(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a*
*4*d) - A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d) +
 C*x/a**4 + C*tan(c/2 + d*x/2)**7/(56*a**4*d) - C*tan(c/2 + d*x/2)**5/(8*a*
*4*d) + 11*C*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*C*tan(c/2 + d*x/2)/(8*a**
4*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**2/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.21489, size = 208, normalized size = 1.37

$$\frac{\frac{840(dx+c)C}{a^4} + \frac{15Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 15Ca^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 21Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 105Ca^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 35Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 385Ca^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 105Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1575Ca^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{28}}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(840*(d*x + c)*C/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*
tan(1/2*d*x + 1/2*c)^7 - 21*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*C*a^24*tan(
1/2*d*x + 1/2*c)^5 - 35*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 385*C*a^24*tan(1/2*
d*x + 1/2*c)^3 + 105*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*C*a^24*tan(1/2*d*x
+ 1/2*c))/a^28)/d
```

3.68 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{4(2A+9C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}+\frac{(23A-54C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{2(3A-4C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] ((23*A - 54*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*(2*A + 9
*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^2*
Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*(3*A - 4*C)*Sin[c + d*x])/(
35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.349296, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 2968, 3019, 2750, 2648}

$$\frac{4(2A+9C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}+\frac{(23A-54C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{2(3A-4C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((23*A - 54*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*(2*A + 9
*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^2*
Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*(3*A - 4*C)*Sin[c + d*x])/(
35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(a(5A-2C)-a(A-6C)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$
$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{a(5A-2C)\cos(c+dx)-a(A-6C)\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$
$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(3A-4C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{-6a^2(3A-4C)\cdots}{(a+\cdots}}{\cdots}$$
$$= \frac{(23A-54C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(3A-4C)\sin\cdots}{35ad(a+a\cos\cdots}$$
$$= \frac{(23A-54C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(3A-4C)\sin\cdots}{35ad(a+a\cos\cdots}$$

Mathematica [A] time = 0.391696, size = 179, normalized size = 1.3

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-70(2A+9C)\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+56A\sin\left(2c+\frac{5dx}{2}\right)+8A\sin\left(3c+\frac{7dx}{2}\right)+70(\cdots\right)}{420a^4d(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(2*A + 9*C)*Sin[(d*x)/2] - 70*(2*A + 9*C)*Si
n[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 441*C*Sin[c + (3*d*x)/2] - 315*
C*Sin[2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] + 147*C*Sin[2*c + (5*d*x
)/2] - 105*C*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] + 36*C*Sin[3*c
 + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.026, size = 90, normalized size = 0.7

$$\frac{1}{8\,da^4}\left(\frac{-A-C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A+3\,C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A-3\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A-C)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*C)*tan(1/2*d*x+1/2*c)^
5+1/3*(A-3*C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.0281, size = 236, normalized size = 1.71

$$\frac{\frac{A\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,C\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*C*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.33755, size = 309, normalized size = 2.24

$$\frac{\left(4\,(2\,A+9\,C)\cos\left(dx+c\right)^3+(32\,A+39\,C)\cos\left(dx+c\right)^2+4\,(13\,A+6\,C)\cos\left(dx+c\right)+13\,A+6\,C\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(4*(2*A + 9*C)*cos(d*x + c)^3 + (32*A + 39*C)*cos(d*x + c)^2 + 4*(13*
A + 6*C)*cos(d*x + c) + 13*A + 6*C)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [A] time = 16.4021, size = 182, normalized size = 1.32

$$\begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} - \frac{C\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} \\ \frac{x\left(A+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a
**4*d) + A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
- C*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*C*tan(c/2 + d*x/2)**5/(40*a**4*d) -
 C*tan(c/2 + d*x/2)**3/(8*a**4*d) + C*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)
), (x*(A + C*cos(c)**2)*cos(c)/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.16239, size = 158, normalized size = 1.14

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-63\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 + 15*C*tan(1/2*d*x + 1/2*c)^7 + 21*A*ta
n(1/2*d*x + 1/2*c)^5 - 63*C*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2
*c)^3 + 105*C*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*C*t
an(1/2*d*x + 1/2*c))/(a^4*d)
```

3.69 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=136

$$\frac{(6A+13C)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(6A+13C)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A-11C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A+C)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] ((A + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A - 11*C)*Sin[c +
 d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((6*A + 13*C)*Sin[c + d*x])/(105*d
*(a^2 + a^2*Cos[c + d*x])^2) + ((6*A + 13*C)*Sin[c + d*x])/(105*d*(a^4 + a^
4*Cos[c + d*x]))
```

Rubi [A] time = 0.170927, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3020, 2750, 2650, 2648}

$$\frac{(6A+13C)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(6A+13C)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A-11C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A+C)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((A + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A - 11*C)*Sin[c +
 d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((6*A + 13*C)*Sin[c + d*x])/(105*d
*(a^2 + a^2*Cos[c + d*x])^2) + ((6*A + 13*C)*Sin[c + d*x])/(105*d*(a^4 + a^
4*Cos[c + d*x]))
```

Rule 3020

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> Simp[(b*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[a*A*(m + 1) - a*C*m + b*C*(2*m + 1)*Sin[e + f*x], x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{-a(3A-4C)-7aC\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}\\
&= \frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-11C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(6A+13C)\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}\\
&= \frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-11C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(6A+13C)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{(6A+1}{}\\
&= \frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-11C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(6A+13C)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{(6A+}{105d\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.354827, size = 159, normalized size = 1.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(126A\sin\left(c+\frac{3dx}{2}\right)+42A\sin\left(2c+\frac{5dx}{2}\right)+6A\sin\left(3c+\frac{7dx}{2}\right)+70(3A+4C)\sin\left(\frac{dx}{2}\right)-175C\sin\right.}{420a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(3*A + 4*C)*Sin[(d*x)/2] - 175*C*Sin[c + (d*
x)/2] + 126*A*Sin[c + (3*d*x)/2] + 168*C*Sin[c + (3*d*x)/2] - 105*C*Sin[2*c
 + (3*d*x)/2] + 42*A*Sin[2*c + (5*d*x)/2] + 91*C*Sin[2*c + (5*d*x)/2] + 6*A
*Sin[3*c + (7*d*x)/2] + 13*C*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c +
 d*x])^4)
```

Maple [A] time = 0.023, size = 88, normalized size = 0.7

$$\frac{1}{8\,da^4}\left(\frac{A+C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A-C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,A-C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A+C)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-C)*tan(1/2*d*x+1/2*c)^5+
1/3*(3*A-C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.02696, size = 236, normalized size = 1.74

$$\frac{\frac{C\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(C*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.27544, size = 308, normalized size = 2.26

$$\frac{\left((6\,A+13\,C)\cos\left(dx+c\right)^3+4\,(6\,A+13\,C)\cos\left(dx+c\right)^2+(39\,A+32\,C)\cos\left(dx+c\right)+36\,A+8\,C\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*((6*A + 13*C)*cos(d*x + c)^3 + 4*(6*A + 13*C)*cos(d*x + c)^2 + (39*A
+ 32*C)*cos(d*x + c) + 36*A + 8*C)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)
```

Sympy [A] time = 11.3982, size = 178, normalized size = 1.31

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{C\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{f} \\ \frac{x\left(A+C\cos^2(c)\right)}{(a\cos(c)+a)^4} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40*
a**4*d) + A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
+ C*tan(c/2 + d*x/2)**7/(56*a**4*d) - C*tan(c/2 + d*x/2)**5/(40*a**4*d) - C
*tan(c/2 + d*x/2)**3/(24*a**4*d) + C*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + C*cos(c)**2)/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.15321, size = 158, normalized size = 1.16

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 21\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 + 15*C*tan(1/2*d*x + 1/2*c)^7 + 63*A*tan
(1/2*d*x + 1/2*c)^5 - 21*C*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2
*c)^3 - 35*C*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*C*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

3.70 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{8(20A-C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-8C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2(5A-2C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A+C}{7d(a\cos}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 8*C)*Sin[c + d*x])/(105*a^4*d*
(1 + Cos[c + d*x])^2) - (8*(20*A - C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c +
 d*x])) - ((A + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*(5*A - 2
*C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.465526, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3042, 2978, 12, 3770}

$$-\frac{8(20A-C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-8C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2(5A-2C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A+C}{7d(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 8*C)*Sin[c + d*x])/(105*a^4*d*
(1 + Cos[c + d*x])^2) - (8*(20*A - C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c +
 d*x])) - ((A + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*(5*A - 2
*C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{\int \frac{(7aA-a(3A-4C)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{2(5A-2C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{\int \frac{\left(35a^2A-4a^2(5A-2C)\cos(c+\right.}{(a+a\cos(c+dx)}}{35a^4}$$

$$= -\frac{(55A-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{2(5A-2C)\sin(c+dx}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{(55A-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{2(5A-2C)\sin(c+dx}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{(55A-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{2(5A-2C)\sin(c+dx}{35ad(a+a\cos(c+dx))}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(55A-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{(A+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-$$

Mathematica [A] time = 1.71607, size = 245, normalized size = 1.69

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(70(31A-2C)\sin\left(c+\frac{dx}{2}\right)-2625A\sin\left(c+\frac{3dx}{2}\right)+735A\sin\left(2c+\frac{3dx}{2}\right)-1015A\sin\left(2c+\frac{5dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-6720*A*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-70*(49
*A - 2*C)*Sin[(d*x)/2] + 70*(31*A - 2*C)*Sin[c + (d*x)/2] - 2625*A*Sin[c +
(3*d*x)/2] + 168*C*Sin[c + (3*d*x)/2] + 735*A*Sin[2*c + (3*d*x)/2] - 1015*A
*Sin[2*c + (5*d*x)/2] + 56*C*Sin[2*c + (5*d*x)/2] + 105*A*Sin[3*c + (5*d*x)
/2] - 160*A*Sin[3*c + (7*d*x)/2] + 8*C*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1
 + Cos[c + d*x])^4)
```

Maple [A] time = 0.058, size = 199, normalized size = 1.4

$$-\frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{A}{da^4}\ln\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/8/d/a^4*A*tan(1/2*d*x+1/2*c)^5-1/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-1/56/d/
a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C+1/d/a^4*A*ln(t
an(1/2*d*x+1/2*c)+1)-1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-11/24/d/a^4*tan(1/2
*d*x+1/2*c)^3*A+1/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2*d*x+
1/2*c)+1/8/d/a^4*C*tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.11692, size = 308, normalized size = 2.12

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-\frac{C\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx}{(\cos(dx+c)}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - C*(105*sin(d*x + c
)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

Fricas [A] time = 1.48021, size = 624, normalized size = 4.3

$$\frac{105\left(A\cos\left(dx+c\right)^4+4\,A\cos\left(dx+c\right)^3+6\,A\cos\left(dx+c\right)^2+4\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-105\left(A\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*(A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*
A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 105*(A*cos(d*x + c)^4 + 4*A*cos
(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + A)*log(-sin(d*x + c)
+ 1) - 2*(8*(20*A - C)*cos(d*x + c)^3 + (535*A - 32*C)*cos(d*x + c)^2 + 4*(
155*A - 13*C)*cos(d*x + c) + 260*A - 13*C)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21191, size = 246, normalized size = 1.7

$$\frac{\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,Ca^{24}}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan
(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 21*C*a^24*tan(1/2
*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 35*C*a^24*tan(1/2*d*x
 + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 105*C*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

3.71 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=161

$$\frac{2(332A+3C)\tan(c+dx)}{105a^4d}-\frac{(88A-3C)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{4A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{4A\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{2(6A-C)\tan(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] (-4*A*ArcTanh[Sin[c + d*x]])/(a^4*d) + (2*(332*A + 3*C)*Tan[c + d*x])/(105*
a^4*d) - ((88*A - 3*C)*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (4*
A*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Tan[c + d*x])/(7*d*(a
 + a*Cos[c + d*x])^4) - (2*(6*A - C)*Tan[c + d*x])/(35*a*d*(a + a*Cos[c + d
*x])^3)
```

Rubi [A] time = 0.616374, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2748, 3767, 8, 3770}

$$\frac{2(332A+3C)\tan(c+dx)}{105a^4d}-\frac{(88A-3C)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{4A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{4A\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{2(6A-C)\tan(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-4*A*ArcTanh[Sin[c + d*x]])/(a^4*d) + (2*(332*A + 3*C)*Tan[c + d*x])/(105*
a^4*d) - ((88*A - 3*C)*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (4*
A*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A + C)*Tan[c + d*x])/(7*d*(a
 + a*Cos[c + d*x])^4) - (2*(6*A - C)*Tan[c + d*x])/(35*a*d*(a + a*Cos[c + d
*x])^3)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(8A+C)-a(4A-3C)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(6A-C)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(a^2(52A+3C)-6a^2(6A-C)\right.}{(a+a\cos(c}}{35a}$$

$$= -\frac{(88A-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(6A-C)\tan(c+dx}{35ad(a+a\cos(c+dx)}$$

$$= -\frac{(88A-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(6A-C)\tan(c+dx}{35ad(a+a\cos(c+dx)}$$

$$= -\frac{(88A-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(6A-C)\tan(c+dx}{35ad(a+a\cos(c+dx)}$$

$$= -\frac{4A\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(88A-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\cos(c+dx))}$$

$$= -\frac{4A\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{2(332A+3C)\tan(c+dx)}{105a^4d} - \frac{(88A-3C)\tan(c}{105a^4d(1+\cos(c+}$$

Mathematica [B] time = 6.41192, size = 680, normalized size = 4.22

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos(c+dx)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-20524A\sin\left(c-\frac{dx}{2}\right)+14644A\sin\left(c+\frac{dx}{2}\right)-16660A\sin\left(2c+\frac{dx}{2}\right)-4690A\sin\left(c+\frac{3dx}{2}\right)+14378A\sin\left(2c+\frac{3dx}{2}\right)-9100A\sin\left(3c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((128*A*Cos[c/2 + (d*x)/2]^8*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/
2 + (d*x)/2]]*(C + A*Sec[c + d*x]^2))/(d*(1 + Cos[c + d*x])^4*(2*A + C + C*
Cos[2*c + 2*d*x])) - (128*A*Cos[c/2 + (d*x)/2]^8*Cos[c + d*x]^2*Log[Cos[c/2
 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(C + A*Sec[c + d*x]^2))/(d*(1 + Cos[c + d
*x])^4*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c/2 + (d*x)/2]*Cos[c + d*x]*S
ec[c/2]*Sec[c]*(C + A*Sec[c + d*x]^2)*(-10780*A*Sin[(d*x)/2] - 210*C*Sin[(d
*x)/2] + 18788*A*Sin[(3*d*x)/2] + 252*C*Sin[(3*d*x)/2] - 20524*A*Sin[c - (d
*x)/2] - 126*C*Sin[c - (d*x)/2] + 14644*A*Sin[c + (d*x)/2] + 126*C*Sin[c +
(d*x)/2] - 16660*A*Sin[2*c + (d*x)/2] - 210*C*Sin[2*c + (d*x)/2] - 4690*A*S
in[c + (3*d*x)/2] + 14378*A*Sin[2*c + (3*d*x)/2] + 252*C*Sin[2*c + (3*d*x)/
2] - 9100*A*Sin[3*c + (3*d*x)/2] + 11668*A*Sin[c + (5*d*x)/2] + 132*C*Sin[c
```

```
 + (5*d*x)/2] - 630*A*Sin[2*c + (5*d*x)/2] + 9358*A*Sin[3*c + (5*d*x)/2] +
132*C*Sin[3*c + (5*d*x)/2] - 2940*A*Sin[4*c + (5*d*x)/2] + 4228*A*Sin[2*c +
 (7*d*x)/2] + 42*C*Sin[2*c + (7*d*x)/2] + 315*A*Sin[3*c + (7*d*x)/2] + 3493
*A*Sin[4*c + (7*d*x)/2] + 42*C*Sin[4*c + (7*d*x)/2] - 420*A*Sin[5*c + (7*d*
x)/2] + 664*A*Sin[3*c + (9*d*x)/2] + 6*C*Sin[3*c + (9*d*x)/2] + 105*A*Sin[4
*c + (9*d*x)/2] + 559*A*Sin[5*c + (9*d*x)/2] + 6*C*Sin[5*c + (9*d*x)/2]))/(
840*d*(1 + Cos[c + d*x])^4*(2*A + C + C*Cos[2*c + 2*d*x])))/a^4
```

Maple [A] time = 0.066, size = 244, normalized size = 1.5

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{23\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C+7/40/d/
a^4*A*tan(1/2*d*x+1/2*c)^5+3/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4*ta
n(1/2*d*x+1/2*c)^3*A+1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/2*
d*x+1/2*c)+1/8/d/a^4*C*tan(1/2*d*x+1/2*c)-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)+
4/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-4/d/a^4
*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.1494, size = 370, normalized size = 2.3

$$\frac{A\left(\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx+c}{\cos(dx+c)\cdot}\right.}{a^4}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
```

```
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) + 3
*C*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.44844, size = 722, normalized size = 4.48

$$-\frac{210\left(A\cos\left(dx+c\right)^{5}+4\,A\cos\left(dx+c\right)^{4}+6\,A\cos\left(dx+c\right)^{3}+4\,A\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] -1/105*(210*(A*cos(d*x + c)^5 + 4*A*cos(d*x + c)^4 + 6*A*cos(d*x + c)^3 + 4
*A*cos(d*x + c)^2 + A*cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(A*cos(d*x
+ c)^5 + 4*A*cos(d*x + c)^4 + 6*A*cos(d*x + c)^3 + 4*A*cos(d*x + c)^2 + A*c
os(d*x + c))*log(-sin(d*x + c) + 1) - (2*(332*A + 3*C)*cos(d*x + c)^4 + 4*(
559*A + 6*C)*cos(d*x + c)^3 + (2636*A + 39*C)*cos(d*x + c)^2 + 4*(296*A + 9
*C)*cos(d*x + c) + 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos
(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26265, size = 286, normalized size = 1.78

$$-\frac{\frac{3360\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{3360\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+147}{}}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/840*(3360*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*A*log(abs(tan(
1/2*d*x + 1/2*c) - 1))/a^4 + 1680*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*
x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 63*C*a^24*tan(1/2*d*x +
1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 105*C*a^24*tan(1/2*d*x + 1/2
*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*c) + 105*C*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

3.72 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=224

$$-\frac{32(54A+5C)\tan(c+dx)}{105a^4d}+\frac{(21A+2C)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A+2C)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{16(54A+5C)\tan(c+dx)\sec(c+dx)}{105a^4d(\cos(c+dx)+1)}$$

```
[Out] ((21*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (32*(54*A + 5*C)*Tan[c + d
*x])/(105*a^4*d) + ((21*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((1
29*A + 10*C)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) -
(16*(54*A + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x]))
- ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*A*S
ec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.664869, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{32(54A+5C)\tan(c+dx)}{105a^4d}+\frac{(21A+2C)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A+2C)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{16(54A+5C)\tan(c+dx)\sec(c+dx)}{105a^4d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] ((21*A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (32*(54*A + 5*C)*Tan[c + d
*x])/(105*a^4*d) + ((21*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((1
29*A + 10*C)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) -
(16*(54*A + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x]))
- ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*A*S
ec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(9A+2C)-a(5A-2C)\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2A\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(a^2(73A+1}{}}{}$$

$$= -\frac{(129A+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{(129A+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{(129A+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= \frac{(21A+2C)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(129A+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2}$$

$$= \frac{(21A+2C)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{32(54A+5C)\tan(c+dx)}{105a^4d} + \frac{(21A+2C)}{}$$

Mathematica [B] time = 6.47914, size = 784, normalized size = 3.5

$$-\frac{8(21A+2C)\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^4} + \frac{8(21A+2C)\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}\right.\right.}{d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-8*(21*A + 2*C)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (8*(21*A + 2*C)*Cos[c/2 + (d*x)/2]^8*
Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) +
(Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(73206*A*Sin[(d*x)/2] +
14140*C*Sin[(d*x)/2] - 166668*A*Sin[(3*d*x)/2] - 15160*C*Sin[(3*d*x)/2] + 1
83162*A*Sin[c - (d*x)/2] + 17220*C*Sin[c - (d*x)/2] - 100842*A*Sin[c + (d*x
)/2] - 17220*C*Sin[c + (d*x)/2] + 155526*A*Sin[2*c + (d*x)/2] + 14140*C*Sin
[2*c + (d*x)/2] + 37380*A*Sin[c + (3*d*x)/2] + 9800*C*Sin[c + (3*d*x)/2] -
101148*A*Sin[2*c + (3*d*x)/2] - 15160*C*Sin[2*c + (3*d*x)/2] + 102900*A*Sin
[3*c + (3*d*x)/2] + 9800*C*Sin[3*c + (3*d*x)/2] - 119364*A*Sin[c + (5*d*x)/
2] - 10920*C*Sin[c + (5*d*x)/2] + 8820*A*Sin[2*c + (5*d*x)/2] + 4760*C*Sin[
```

```
2*c + (5*d*x)/2] - 78204*A*Sin[3*c + (5*d*x)/2] - 10920*C*Sin[3*c + (5*d*x)
/2] + 49980*A*Sin[4*c + (5*d*x)/2] + 4760*C*Sin[4*c + (5*d*x)/2] - 64053*A*
Sin[2*c + (7*d*x)/2] - 5890*C*Sin[2*c + (7*d*x)/2] - 3885*A*Sin[3*c + (7*d*
x)/2] + 1470*C*Sin[3*c + (7*d*x)/2] - 44733*A*Sin[4*c + (7*d*x)/2] - 5890*C
*Sin[4*c + (7*d*x)/2] + 15435*A*Sin[5*c + (7*d*x)/2] + 1470*C*Sin[5*c + (7*
d*x)/2] - 21987*A*Sin[3*c + (9*d*x)/2] - 2030*C*Sin[3*c + (9*d*x)/2] - 3675
*A*Sin[4*c + (9*d*x)/2] + 210*C*Sin[4*c + (9*d*x)/2] - 16107*A*Sin[5*c + (9
*d*x)/2] - 2030*C*Sin[5*c + (9*d*x)/2] + 2205*A*Sin[6*c + (9*d*x)/2] + 210*
C*Sin[6*c + (9*d*x)/2] - 3456*A*Sin[4*c + (11*d*x)/2] - 320*C*Sin[4*c + (11
*d*x)/2] - 840*A*Sin[5*c + (11*d*x)/2] - 2616*A*Sin[6*c + (11*d*x)/2] - 320
*C*Sin[6*c + (11*d*x)/2]))/(6720*d*(a + a*Cos[c + d*x])^4)
```

Maple [A] time = 0.074, size = 329, normalized size = 1.5

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{13\,A}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C-9/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4*tan
(1/2*d*x+1/2*c)^3*A-11/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*A*tan(1/
2*d*x+1/2*c)-15/8/d/a^4*C*tan(1/2*d*x+1/2*c)-21/2/d/a^4*A*ln(tan(1/2*d*x+1/
2*c)-1)-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-
1)^2+9/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)+21/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+
1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)^2+
9/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.12618, size = 502, normalized size = 2.24

$$-\frac{3\,A\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\,\log\left(\frac{\sin(d}{\cos(dx}\right.}{a^4}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) + 5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [A] time = 1.45579, size = 923, normalized size = 4.12

$$\frac{105\left((21\,A + 2\,C)\cos\left(dx + c\right)^{6} + 4\,(21\,A + 2\,C)\cos\left(dx + c\right)^{5} + 6\,(21\,A + 2\,C)\cos\left(dx + c\right)^{4} + 4\,(21\,A + 2\,C)\cos\left(dx + c\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/420*(105*((21*A + 2*C)*cos(d*x + c)^6 + 4*(21*A + 2*C)*cos(d*x + c)^5 + 6
*(21*A + 2*C)*cos(d*x + c)^4 + 4*(21*A + 2*C)*cos(d*x + c)^3 + (21*A + 2*C)
*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((21*A + 2*C)*cos(d*x + c)^6 +
 4*(21*A + 2*C)*cos(d*x + c)^5 + 6*(21*A + 2*C)*cos(d*x + c)^4 + 4*(21*A +
2*C)*cos(d*x + c)^3 + (21*A + 2*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) -
 2*(64*(54*A + 5*C)*cos(d*x + c)^5 + (11619*A + 1070*C)*cos(d*x + c)^4 + 4*
(3411*A + 310*C)*cos(d*x + c)^3 + 4*(1509*A + 130*C)*cos(d*x + c)^2 + 420*A
*cos(d*x + c) - 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*
x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26721, size = 325, normalized size = 1.45

$$\frac{\frac{420\,(21\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(21\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(420*(21*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(21*A
+ 2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*A*tan(1/2*d*x + 1/2*
c)^3 - 7*A*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15
*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 189*A*a
^24*tan(1/2*d*x + 1/2*c)^5 + 105*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^2
4*tan(1/2*d*x + 1/2*c)^3 + 385*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 11655*A*a^24
*tan(1/2*d*x + 1/2*c) + 1575*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.73 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=257

$$\frac{4(454A+83C)\tan^3(c+dx)}{105a^4d}+\frac{4(454A+83C)\tan(c+dx)}{35a^4d}-\frac{2(11A+2C)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2(11A+2C)\tan(c+}{a^4d}$$

```
[Out] (-2*(11*A + 2*C)*ArcTanh[Sin[c + d*x]])/(a^4*d) + (4*(454*A + 83*C)*Tan[c +
 d*x])/(35*a^4*d) - (2*(11*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(a^4*d) - ((
178*A + 31*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2)
 - (4*(11*A + 2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^4*d*(1 + Cos[c + d*x])
) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2
*(8*A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (
4*(454*A + 83*C)*Tan[c + d*x]^3)/(105*a^4*d)
```

Rubi [A] time = 0.706449, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2748, 3767, 3768, 3770}

$$\frac{4(454A+83C)\tan^3(c+dx)}{105a^4d}+\frac{4(454A+83C)\tan(c+dx)}{35a^4d}-\frac{2(11A+2C)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2(11A+2C)\tan(c+}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (-2*(11*A + 2*C)*ArcTanh[Sin[c + d*x]])/(a^4*d) + (4*(454*A + 83*C)*Tan[c +
 d*x])/(35*a^4*d) - (2*(11*A + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(a^4*d) - ((
178*A + 31*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2)
 - (4*(11*A + 2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^4*d*(1 + Cos[c + d*x])
) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2
*(8*A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (
4*(454*A + 83*C)*Tan[c + d*x]^3)/(105*a^4*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
```

```
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(10A+3C)-a(6A-C)\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2(8A+C)\sec^2(c+dx)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \ldots$$

$$= -\frac{(178A+31C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{(178A+31C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{(178A+31C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{2(11A+2C)\sec(c+dx)\tan(c+dx)}{a^4d} - \frac{(178A+31C)\sec^2(c+dx)\tan(c+\ldots}{105a^4d(1+\cos(c+dx))^2}$$

$$= -\frac{2(11A+2C)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{4(454A+83C)\tan(c+dx)}{35a^4d} - \frac{2(11A+\ldots}{}$$

Mathematica [A] time = 4.28583, size = 361, normalized size = 1.4

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(4(412A+139C)\tan\left(\frac{c}{2}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+6(31A+17C)\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+15(A+C)\tan\left(\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(15*(A + C)*Sec[c/2]*Sin[(d*x)/2] + 6*(31*A + 17*C)*Cos
[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 4*(412*A + 139*C)*Cos[(c + d*x)/2]^
4*Sec[c/2]*Sin[(d*x)/2] + 8*(2512*A + 559*C)*Cos[(c + d*x)/2]^6*Sec[c/2]*Si
n[(d*x)/2] + 15*(A + C)*Cos[(c + d*x)/2]*Tan[c/2] + 6*(31*A + 17*C)*Cos[(c
+ d*x)/2]^3*Tan[c/2] + 4*(412*A + 139*C)*Cos[(c + d*x)/2]^5*Tan[c/2] + 280*
Cos[(c + d*x)/2]^7*(6*(11*A + 2*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]*(32*A +
 3*C - 6*A*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x] - 6*A*Sec[c + d*x]*Tan
[c] + A*Sec[c + d*x]^2*Tan[c])))/(105*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.077, size = 418, normalized size = 1.6

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{11\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{59\,A}{24\,da^4}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*C+11/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+59/24/d/a^4*t
an(1/2*d*x+1/2*c)^3*A+23/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3+209/8/d/a^4*A*tan(
1/2*d*x+1/2*c)+49/8/d/a^4*C*tan(1/2*d*x+1/2*c)-13/d/a^4*A/(tan(1/2*d*x+1/2*
c)-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*C+22/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)+
4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^3-5/2
/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^2-22/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-4/d/a
^4*ln(tan(1/2*d*x+1/2*c)+1)*C-13/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4/(ta
n(1/2*d*x+1/2*c)+1)*C-1/3/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)^3+5/2/d/a^4*A/(tan
(1/2*d*x+1/2*c)+1)^2
```

Maxima [A] time = 1.16716, size = 622, normalized size = 2.42

$$A\left(\frac{560\left(\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{62\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{39\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4-\frac{3\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{21945\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{2065\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{231\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{18480\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(A*(560*(27*sin(d*x + c)/(cos(d*x + c) + 1) - 62*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 39*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4 - 3*a^4*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (21945*sin(d*x + c)/(cos(d*x
+ c) + 1) + 2065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 231*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 18480*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 18480*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^4) + C*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c)
```

```
+ 1) + 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c)
+ 1) - 1)/a^4))/d
```

Fricas [A] time = 1.59124, size = 965, normalized size = 3.75

$$-\frac{105\left((11\,A+2\,C)\cos\left(dx+c\right)^{7}+4\,(11\,A+2\,C)\cos\left(dx+c\right)^{6}+6\,(11\,A+2\,C)\cos\left(dx+c\right)^{5}+4\,(11\,A+2\,C)\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] -1/105*(105*((11*A + 2*C)*cos(d*x + c)^7 + 4*(11*A + 2*C)*cos(d*x + c)^6 +
6*(11*A + 2*C)*cos(d*x + c)^5 + 4*(11*A + 2*C)*cos(d*x + c)^4 + (11*A + 2*C
)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 105*((11*A + 2*C)*cos(d*x + c)^7
+ 4*(11*A + 2*C)*cos(d*x + c)^6 + 6*(11*A + 2*C)*cos(d*x + c)^5 + 4*(11*A +
 2*C)*cos(d*x + c)^4 + (11*A + 2*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1)
- (8*(454*A + 83*C)*cos(d*x + c)^6 + 2*(6109*A + 1118*C)*cos(d*x + c)^5 + 4
*(3592*A + 659*C)*cos(d*x + c)^4 + 8*(799*A + 148*C)*cos(d*x + c)^3 + 35*(1
4*A + 3*C)*cos(d*x + c)^2 - 70*A*cos(d*x + c) + 35*A)*sin(d*x + c))/(a^4*d*
cos(d*x + c)^7 + 4*a^4*d*cos(d*x + c)^6 + 6*a^4*d*cos(d*x + c)^5 + 4*a^4*d*
cos(d*x + c)^4 + a^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25371, size = 398, normalized size = 1.55

$$-\frac{\frac{1680\,(11\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{1680\,(11\,A+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{560\left(39\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-62\,A\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,\cdots\right.\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/840*(1680*(11*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 1680*(11
*A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 560*(39*A*tan(1/2*d*x +
1/2*c)^5 + 3*C*tan(1/2*d*x + 1/2*c)^5 - 62*A*tan(1/2*d*x + 1/2*c)^3 - 6*C*t
an(1/2*d*x + 1/2*c)^3 + 27*A*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7
 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 231*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 1
47*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 2065*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 805
*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 21945*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*C
*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.74 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=223

$$\frac{2a(99A+80C)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4(99A+80C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+80C)\sin(c+dx)\sqrt{}}{3465d}$$

```
[Out] (4*a*(99*A + 80*C)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(9
9*A + 80*C)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a*C*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) - (8*(
99*A + 80*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3465*d) + (2*C*Cos[c +
 d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(11*d) + (4*(99*A + 80*C)*(a
 + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1155*a*d)
```

Rubi [A] time = 0.50594, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3046, 2981, 2770, 2759, 2751, 2646}

$$\frac{2a(99A+80C)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4(99A+80C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+80C)\sin(c+dx)\sqrt{}}{3465d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (4*a*(99*A + 80*C)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(9
9*A + 80*C)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a*C*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) - (8*(
99*A + 80*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3465*d) + (2*C*Cos[c +
 d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(11*d) + (4*(99*A + 80*C)*(a
 + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1155*a*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{11d} + \frac{2\int\cos^3(}{}$$
$$= \frac{2aC\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^4(c+dx)\sqrt{a+a\cos}}{11d}$$
$$= \frac{2a(99A+80C)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\cos^4(c+d}{99d\sqrt{a+a\cos}}$$
$$= \frac{2a(99A+80C)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\cos^4(c+d}{99d\sqrt{a+a\cos}}$$
$$= \frac{2a(99A+80C)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\cos^4(c+d}{99d\sqrt{a+a\cos}}$$
$$= \frac{4a(99A+80C)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a(99A+80C)\cos^3(c+dx)}{693d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.936908, size = 114, normalized size = 0.51

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(9306A+9095C)\cos(c+dx)+16(297A+415C)\cos(2(c+dx))+1980A\cos(3(c}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(30096*A + 26420*C + 2*(9306*A + 9095*C)*Cos[c
+ d*x] + 16*(297*A + 415*C)*Cos[2*(c + d*x)] + 1980*A*Cos[3*(c + d*x)] + 31
75*C*Cos[3*(c + d*x)] + 700*C*Cos[4*(c + d*x)] + 315*C*Cos[5*(c + d*x)])*Ta
n[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 0.272, size = 135, normalized size = 0.6

$$\frac{2a\sqrt{2}}{3465d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\Bigg(-10080C(\sin(1/2dx+c/2))^{10}+30800C(\sin(1/2dx+c/2))^{8}+(-3960A-39600C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-10080*C*sin(1/2*d*x+1/2*c)
^10+30800*C*sin(1/2*d*x+1/2*c)^8+(-3960*A-39600*C)*sin(1/2*d*x+1/2*c)^6+(83
16*A+27720*C)*sin(1/2*d*x+1/2*c)^4+(-6930*A-11550*C)*sin(1/2*d*x+1/2*c)^2+3
465*A+3465*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.12468, size = 216, normalized size = 0.97

$$\frac{396\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+5\left(63\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/55440*(396*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*
c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) + 5*(63*sqrt(2)*sin(11/2*d*x + 11/2*c) + 77*sqrt(2)*sin(9/2*d*x + 9
/2*c) + 495*sqrt(2)*sin(7/2*d*x + 7/2*c) + 693*sqrt(2)*sin(5/2*d*x + 5/2*c)
 + 2310*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6930*sqrt(2)*sin(1/2*d*x + 1/2*c))*C
*sqrt(a))/d
```

Fricas [A] time = 1.31158, size = 308, normalized size = 1.38

$$\frac{2\left(315\,C\cos\left(dx+c\right)^{5}+350\,C\cos\left(dx+c\right)^{4}+5\left(99\,A+80\,C\right)\cos\left(dx+c\right)^{3}+6\left(99\,A+80\,C\right)\cos\left(dx+c\right)^{2}+8\left(99\,A+\right.\right.}{3465\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2/3465*(315*C*cos(d*x + c)^5 + 350*C*cos(d*x + c)^4 + 5*(99*A + 80*C)*cos(d
*x + c)^3 + 6*(99*A + 80*C)*cos(d*x + c)^2 + 8*(99*A + 80*C)*cos(d*x + c) +
 1584*A + 1280*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)\sqrt{a\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^3, x
)
```

3.75 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=180

$$\frac{2(21A+16C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a(21A+16C)\sin(c}{45d\sqrt{a\cos(c+dx)}}$$

```
[Out] (2*a*(21*A + 16*C)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*C*C
os[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) - (4*(21*A + 16
*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*C*Cos[c + d*x]^3*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(21*A + 16*C)*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.419835, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3046, 2981, 2759, 2751, 2646}

$$\frac{2(21A+16C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a(21A+16C)\sin(c}{45d\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*a*(21*A + 16*C)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*C*C
os[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) - (4*(21*A + 16
*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*C*Cos[c + d*x]^3*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(21*A + 16*C)*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d} + \frac{2\int \cos}{}$$

$$= \frac{2aC\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sqrt{a+a\cos}}{9d}$$

$$= \frac{2aC\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sqrt{a+a\cos}}{9d}$$

$$= \frac{2aC\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} - \frac{4(21A+16C)\sqrt{a+a\cos}}{315d}$$

$$= \frac{2a(21A+16C)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\cos^3(c+dx)\sin(c+d}{63d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.484212, size = 92, normalized size = 0.51

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(16(42A+47C)\cos(c+dx)+4(63A+83C)\cos(2(c+dx))+1596A+80C\cos(3(}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(1596*A + 1321*C + 16*(42*A + 47*C)*Cos[c + d*x
] + 4*(63*A + 83*C)*Cos[2*(c + d*x)] + 80*C*Cos[3*(c + d*x)] + 35*C*Cos[4*(
c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.049, size = 116, normalized size = 0.6

$$\frac{2a\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560C(\sin(1/2dx+c/2))^8-1440C(\sin(1/2dx+c/2))^6+(252A+1512C)\left(\sin\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*C*sin(1/2*d*x+1/2*c)^8-1
440*C*sin(1/2*d*x+1/2*c)^6+(252*A+1512*C)*sin(1/2*d*x+1/2*c)^4+(-420*A-840*
```

```
C)*sin(1/2*d*x+1/2*c)^2+315*A+315*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)
/d
```

Maxima [A] time = 2.23202, size = 177, normalized size = 0.98

$$\frac{84\left(3\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(35\sqrt{2}\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\sqrt{2}\sin\left(\frac{7}{2}\right.\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2520*(84*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
 + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*sin(9/2*d*x + 9
/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c) + 252*sqrt(2)*sin(5/2*d*x + 5/2*c)
+ 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 1890*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*s
qrt(a))/d
```

Fricas [A] time = 1.46039, size = 257, normalized size = 1.43

$$\frac{2\left(35\,C\cos\left(dx+c\right)^{4}+40\,C\cos\left(dx+c\right)^{3}+3\left(21\,A+16\,C\right)\cos\left(dx+c\right)^{2}+4\left(21\,A+16\,C\right)\cos\left(dx+c\right)+168\,A+128\,C\right)}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2/315*(35*C*cos(d*x + c)^4 + 40*C*cos(d*x + c)^3 + 3*(21*A + 16*C)*cos(d*x
+ c)^2 + 4*(21*A + 16*C)*cos(d*x + c) + 168*A + 128*C)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^2, x
)
```

3.76 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=137

$$\frac{2(35A+18C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(35A+27C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)\cos^2(c+dx)\sqrt{a\cos(c+dx)}}{7d}$$

```
[Out] (2*a*(35*A + 27*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(35*
A + 18*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Cos[c + d*x
]^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (2*C*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.305165, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3046, 2968, 3023, 2751, 2646}

$$\frac{2(35A+18C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(35A+27C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)\cos^2(c+dx)\sqrt{a\cos(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(35*A + 27*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(35*
A + 18*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Cos[c + d*x
]^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (2*C*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*a*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d} + \frac{2\int \cos(\cdots}{\cdots}$$

$$= \frac{2C\cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d} + \frac{2\int \sqrt{a\cdots}}{\cdots}$$

$$= \frac{2C\cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d} + \frac{2C(a+\cdots}{\cdots}$$

$$= \frac{2(35A+18C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C\cos^2(c\cdots}{\cdots}$$

$$= \frac{2a(35A+27C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(35A+18C)\sqrt{a+a\cos(c\cdots}}{105d}$$

Mathematica [A] time = 0.279413, size = 74, normalized size = 0.54

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((140A+141C)\cos(c+dx)+280A+36C\cos(2(c+dx))+15C\cos(3(c+dx))+22}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(280*A + 228*C + (140*A + 141*C)*Cos[c + d*x] +
 36*C*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.044, size = 97, normalized size = 0.7

$$\frac{2a\sqrt{2}}{105d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-120C(\sin(1/2dx+c/2))^6+252C(\sin(1/2dx+c/2))^4+(-70A-210C)\left(\sin\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-120*C*sin(1/2*d*x+1/2*c)^6+
252*C*sin(1/2*d*x+1/2*c)^4+(-70*A-210*C)*sin(1/2*d*x+1/2*c)^2+105*A+105*C)*
2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.06793, size = 139, normalized size = 1.01

$$\frac{140\left(\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+3\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}\right.\right.}{420d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/420*(140*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*
A*sqrt(a) + 3*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2
```

```
*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*C
*sqrt(a))/d
```

Fricas [A] time = 1.41552, size = 207, normalized size = 1.51

$$\frac{2\left(15\,C\cos\left(dx+c\right)^{3}+18\,C\cos\left(dx+c\right)^{2}+\left(35\,A+24\,C\right)\cos\left(dx+c\right)+70\,A+48\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/105*(15*C*cos(d*x + c)^3 + 18*C*cos(d*x + c)^2 + (35*A + 24*C)*cos(d*x +
c) + 70*A + 48*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.77 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=95

$$\frac{2a(15A+7C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}-\frac{4C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}$$

```
[Out] (2*a*(15*A + 7*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (4*C*Sqrt
[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^(3/2)
*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.127987, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3024, 2751, 2646}

$$\frac{2a(15A+7C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}-\frac{4C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*a*(15*A + 7*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (4*C*Sqrt
[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^(3/2)
*Sin[c + d*x])/(5*a*d)
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{2\int \sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(5A\right.}{5a}$$

$$= -\frac{4C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2C(a+a\cos(c+dx))^{3/2}\sin(c-}{5ad}$$

$$= \frac{2a(15A+7C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{4C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2C(}{}$$

Mathematica [A] time = 0.112477, size = 58, normalized size = 0.61

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(30A+8C\cos(c+dx)+3C\cos(2(c+dx))+19C)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(30*A + 19*C + 8*C*Cos[c + d*x] + 3*C*Cos[2*(c
+ d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.042, size = 78, normalized size = 0.8

$$\frac{2a\sqrt{2}}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12C(\cos(1/2dx+c/2))^4-4C(\cos(1/2dx+c/2))^2+15A+7C\right)\frac{1}{\sqrt{a\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*C*cos(1/2*d*x+1/2*c)^4-4*C
*cos(1/2*d*x+1/2*c)^2+15*A+7*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.0009, size = 97, normalized size = 1.02

$$\frac{60\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\left(3\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)C\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(60*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + (3*sqrt(2)*sin(5/2*d*x +
5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*
C*sqrt(a))/d
```

Fricas [A] time = 1.40498, size = 159, normalized size = 1.67

$$\frac{2\left(3\,C\cos\left(dx+c\right)^{2}+4\,C\cos\left(dx+c\right)+15\,A+8\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*C*cos(d*x + c)^2 + 4*C*cos(d*x + c) + 15*A + 8*C)*sqrt(a*cos(d*x +
c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.78 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2aC\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*C*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.259046, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3046, 2981, 2773, 206}

$$\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2aC\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*C*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2\int\sqrt{a+a\cos(c+d}}{}$$

$$=\frac{2aC\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

$$=\frac{2aC\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

$$=\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2aC\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+$$

Mathematica [A] time = 0.141806, size = 82, normalized size = 0.85

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*A*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]] + C*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(3*d)
```

Maple [B] time = 0.205, size = 248, normalized size = 2.6

$$\frac{1}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4C\sqrt{2}\sqrt{a(\sin(1/2\,dx+c/2))^2}\sqrt{a}(\sin(1/2\,dx+c/2))^2+3A\ln\left(-4\frac{\sqrt{a}\sqrt{2}\sqrt{a(\mathrm{si}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+3*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a+6*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*
x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.95112, size = 50, normalized size = 0.52

$$\frac{\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)C\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/3*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*sqrt(
a)/d
```

Fricas [A] time = 1.45046, size = 375, normalized size = 3.91

$$\frac{3\left(A\cos\left(dx+c\right)+A\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}-4\,\sqrt{a\cos\left(dx+c\right)+a}\sqrt{a}\left(\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)+4\left(C\cos\left(dx+c\right)+2\,C\right)}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/6*(3*(A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(C*cos(d*x + c) + 2*C)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.00292, size = 236, normalized size = 2.46

$$\frac{\frac{3\,Aa^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(\sqrt{2}Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,\sqrt{2}Ca^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/3*(3*A*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))
/abs(a) + 2*(sqrt(2)*C*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)*C*a^2)*tan(1/
2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

3.79 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=94

$$-\frac{a(A-2C)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}+\frac{\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a
*(A - 2*C)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a + a*Cos[c
 + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.296169, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3044, 2981, 2773, 206}

$$-\frac{a(A-2C)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}+\frac{\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a
*(A - 2*C)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a + a*Cos[c
 + d*x]]*Tan[c + d*x])/d
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d} + \frac{\int \sqrt{a+a\cos(c+dx)}\,(}{}$$

$$= -\frac{a(A-2C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+dx)}\tan(c+dx}{d}$$

$$= -\frac{a(A-2C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+dx)}\tan(c+dx}{d}$$

$$= \frac{\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a(A-2C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{A}{}$$

Mathematica [A] time = 0.226289, size = 91, normalized size = 0.97

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(A+2C\cos(c+dx))+\sqrt{2}A\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\,\mathrm{s}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*A*ArcTan
h[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 2*C*Cos[c + d*x])*Sin[(c
+ d*x)/2]))/(2*d)
```

Maple [B] time = 0.102, size = 436, normalized size = 4.6

$$\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\left(-2\,A\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}-a\sqrt{2}\cos\left(1/2\,dx+c/2\right)+2\,a}{-2\,\cos\left(1/2\,dx+c/2\right)+\sqrt{2}}\right)a-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-2*A*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a-2*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
))*a-8*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)
^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+A*ln(-4/(-2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2
)*cos(1/2*d*x+1/2*c)+2*a))*a+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
a+4*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*
x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(
1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 2.13161, size = 987, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*sqrt(2)*C*sqrt(a)*sin(1/2*d*x + 1/2*c) - (4*sqrt(2)*cos(5/2*d*x + 5/
2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4
*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
qrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c
) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
4*sqrt(2)*sin(3/2*d*x + 3/2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.50052, size = 413, normalized size = 4.39

$$\frac{\left(A\cos\left(dx+c\right)^2 + A\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(2\,C\cos\left(dx\right.\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/4*((A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*si
n(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*C*cos(d*x + c)
+ A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.53461, size = 374, normalized size = 3.98

$$\frac{4\sqrt{2}Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} + A\sqrt{a}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right) - A\sqrt{a}\log\left(\left|\left(\sqrt{a}\tan \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*C*a*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
+ A*sqrt(a)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - A*sqrt(a)*log(abs((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) +
 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*A*a^(3/2) - A*a^(5/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.80 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{a}(3A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] (Sqrt[a]*(3*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a*A*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.311477, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3044, 2980, 2773, 206}

$$\frac{\sqrt{a}(3A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a]*(3*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a*A*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{\int \sqrt{a+a}}{}$$

$$= \frac{aA\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+dx)}\sec(c+dx)}{2d}$$

$$= \frac{aA\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+dx)}\sec(c+dx)}{2d}$$

$$= \frac{\sqrt{a}(3A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{aA\tan(c+d}{4d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.380317, size = 103, normalized size = 0.94

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(3A+8C)\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(3*A +
 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + A*(Sin[(c + d*x)/2
] + 3*Sin[(3*(c + d*x))/2])))/(8*d)
```

Maple [B] time = 0.105, size = 943, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(3*A*ln(-4/(-2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+8*C*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(3*A*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+8*C*ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+10*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+3*A*ln(-4/(-2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a))*a+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(1/2)/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 21.6225, size = 3568, normalized size = 32.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/16*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*c
os(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*
cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(
2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
```

```
t(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*si
n(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*
sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*
x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*c
os(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*A*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1)*d)
```

Fricas [A] time = 1.58865, size = 450, normalized size = 4.09

$$\frac{\left((3\,A+8\,C)\cos\left(dx+c\right)^{3}+(3\,A+8\,C)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/16*(((3*A + 8*C)*cos(d*x + c)^3 + (3*A + 8*C)*cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 + 4*(3*A*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos
(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.65309, size = 463, normalized size = 4.21

$$\left(3\,A\sqrt{a}+8\,C\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-\left(3\,A\sqrt{a}+8\,C\sqrt{a}\right)\log\left(\left|\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/8*((3*A*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(a) +
8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(3/2) + 19*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(5/2) - 17*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(7/
2) + A*a^(9/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)^2)/d
```

3.81 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=153

$$\frac{a(5A+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(5A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{aA}{1}$$

```
[Out] (Sqrt[a]*(5*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*d) + (a*(5*A + 8*C)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*A*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a
+ a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.388036, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2980, 2772, 2773, 206}

$$\frac{a(5A+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(5A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{aA}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a]*(5*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*d) + (a*(5*A + 8*C)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*A*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a
+ a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{\int \sqrt{a+}}{}$$
$$= \frac{aA\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+dx)}\sec^2(}{3d}$$
$$= \frac{a(5A+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} +$$
$$= \frac{a(5A+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} +$$
$$= \frac{\sqrt{a}(5A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(5A+8C)\tan(}{8d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 1.058, size = 115, normalized size = 0.75

$$\frac{\sqrt{a(\cos(c+dx)+1)}\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)(3(5A+8C)\cos(2(c+dx))+20A\cos(c+dx)+31A+24C)+3\sqrt{2}(\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(5*A + 8*C)*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]]*Sec[(c + d*x)/2] + (31*A + 24*C + 20*A*Cos[c + d*x] + 3*(5*A + 8*
C)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Tan[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.11, size = 1311, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(5*A*ln(-4/(-2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+5*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
```

```
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a))+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+8*C*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(10*A*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin
(1/2*d*x+1/2*c)^4-2*(80*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+96
*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+45*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+45*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a+72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+72*C*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+15*A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+66*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+24*C*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*
d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 3.20296, size = 4169, normalized size = 27.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/96*((120*(sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*co
s(13/2*d*x + 13/2*c) - 8*(15*sin(11/2*d*x + 11/2*c) + 50*sin(9/2*d*x + 9/2*
c) + 42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x + 3/2
*c))*cos(6*d*x + 6*c) + 360*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(11/2*
d*x + 11/2*c) + 1200*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(9/2*d*x + 9/
2*c) - 24*(42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x
 + 3/2*c))*cos(4*d*x + 4*c) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*co
s(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)
^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2
*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2
 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*si
n(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x +
4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) + 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^
2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*s
in(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2
)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 120*(cos(6*d*x
```

```
+ 6*c) + 3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(13/2*d*x + 13/2*c
) + 8*(15*cos(11/2*d*x + 11/2*c) + 50*cos(9/2*d*x + 9/2*c) + 42*cos(7/2*d*x
 + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*
c) - 120*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(11/2*d*x + 11/2*
c) - 400*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(9/2*d*x + 9/2*c)
 + 24*(42*cos(7/2*d*x + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3
/2*c))*sin(4*d*x + 4*c) - 336*(3*cos(2*d*x + 2*c) + 1)*sin(7/2*d*x + 7/2*c)
 - 24*(3*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c) + 1008*cos(7/2*d*x + 7/
2*c)*sin(2*d*x + 2*c) + 72*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 120*cos(
3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 120*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + 120*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 40*sin(3/2*d*x + 3/2*c))
*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*s
qrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*
x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) + 24*(4*sqrt(2)*cos(5/
2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d
*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.68059, size = 495, normalized size = 3.24

$$\frac{3\left((5A+8C)\cos(dx+c)^4+(5A+8C)\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c)^4+d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/96*(3*((5*A + 8*C)*cos(d*x + c)^4 + (5*A + 8*C)*cos(d*x + c)^3)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*(3*(5*A + 8*C)*cos(d*x + c)^2 + 10*A*cos(d*x + c) + 8*A)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.72307, size = 861, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/48*(3*(5*A*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(
a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(63*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(3/2) + 72*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(3/2)
 - 369*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
8*A*a^(5/2) - 888*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^8*C*a^(5/2) + 1638*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*A*a^(7/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(7/2) - 1074*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(9/2) - 1776*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(9/2) + 17
1*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a
^(11/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*C*a^(11/2) - 13*A*a^(13/2) - 24*C*a^(13/2))/((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.82 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=196

$$\frac{a(35A+48C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(35A+48C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(35A+48C)\tan(c+dx)\sec(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan}{}$$

```
[Out] (Sqrt[a]*(35*A + 48*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a*(35*A + 48*C)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]
]) + (a*(35*A + 48*C)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d
*x]]) + (a*A*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) +
 (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.471043, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2980, 2772, 2773, 206}

$$\frac{a(35A+48C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(35A+48C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(35A+48C)\tan(c+dx)\sec(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] (Sqrt[a]*(35*A + 48*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a*(35*A + 48*C)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]
]) + (a*(35*A + 48*C)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d
*x]]) + (a*A*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) +
 (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{\int \sqrt{a+\cdots}}{\cdots}$$
$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+dx)}\sec^3\cdots}{4d}$$
$$= \frac{a(35A+48C)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec^2(c+dx)\cdots}{24d\sqrt{a+a\,\mathrm{co}\cdots}}$$
$$= \frac{a(35A+48C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a(35A+48C)\sec(c+dx)\,\mathrm{t}\cdots}{96d\sqrt{a+a\cos(c+\cdots}}$$
$$= \frac{a(35A+48C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a(35A+48C)\sec(c+dx)\,\mathrm{t}\cdots}{96d\sqrt{a+a\cos(c+\cdots}}$$
$$= \frac{\sqrt{a}(35A+48C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a(35A+48C)\cdots}{64d\sqrt{a+a\,\mathrm{c}\cdots}}$$

Mathematica [A] time = 1.83642, size = 145, normalized size = 0.74

$$\frac{\sqrt{a(\cos(c+dx)+1)}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)((539A+432C)\cos(c+dx)+4(35A+48C)\cos(2(c+dx))+105A\cdots\right.}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(35*A + 48*C)*ArcTanh[Sqrt[2]*Sin[(c
 + d*x)/2]]*Sec[(c + d*x)/2] + ((332*A + 192*C + (539*A + 432*C)*Cos[c + d*
x] + 4*(35*A + 48*C)*Cos[2*(c + d*x)] + 105*A*Cos[3*(c + d*x)] + 144*C*Cos[
3*(c + d*x)])*Sec[c + d*x]^4*Tan[(c + d*x)/2])/2))/(384*d)
```

Maple [B] time = 0.116, size = 1631, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/24*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(35*A*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+35*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))+48*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+48*C*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(35*A*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+70*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+70*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+96*C*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a+96*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)
*sin(1/2*d*x+1/2*c)^6+8*(385*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+528*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+315*A*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+315*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a))*a+432*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+432*C*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(511*A*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+624*C*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)+210*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a+210*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+288*C*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+288*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+558*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+105*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+105*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+480*C*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+144*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1
44*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/2*d*x+1/2*c)/(a*co
s(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.83699, size = 551, normalized size = 2.81

$$\frac{3\left((35\,A+48\,C)\cos\left(dx+c\right)^{5}+(35\,A+48\,C)\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{768\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/768*(3*((35*A + 48*C)*cos(d*x + c)^5 + (35*A + 48*C)*cos(d*x + c)^4)*sqrt
(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)
*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)) + 4*(3*(35*A + 48*C)*cos(d*x + c)^3 + 2*(35*A + 48*C)*cos(d*x + c)^
2 + 56*A*cos(d*x + c) + 48*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos
(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.77409, size = 1083, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/384*(3*(35*A*sqrt(a) + 48*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(35*A*s
qrt(a) + 48*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(279*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(3/2) + 24
0*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*
a^(3/2) + 285*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^12*A*a^(5/2) - 1968*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^12*C*a^(5/2) - 4605*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(7/2) - 2640*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(7/2) + 37281*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(9/2) + 4
1616*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*
C*a^(9/2) - 35643*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^6*A*a^(11/2) - 42288*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^6*C*a^(11/2) + 9175*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(13/2) + 12528*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(13/2) - 1311*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(15/
2) - 1392*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*C*a^(15/2) + 43*A*a^(17/2) + 48*C*a^(17/2))/((sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.83 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=225

$$\frac{2a^2(33A+28C)\sin(c+dx)\cos^3(c+dx)}{231d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(143A+112C)\sin(c+dx)}{165d\sqrt{a\cos(c+dx)+a}}+\frac{2(143A+112C)\sin(c+dx)(a\cos(c+dx)}{385d}$$

```
[Out] (2*a^2*(143*A + 112*C)*Sin[c + d*x])/(165*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*(33*A + 28*C)*Cos[c + d*x]^3*Sin[c + d*x])/(231*d*Sqrt[a + a*Cos[c + d*
x]]) - (4*a*(143*A + 112*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1155*d)
 + (2*a*C*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(33*d) + (2
*(143*A + 112*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(385*d) + (2*C*Co
s[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.638437, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3046, 2976, 2981, 2759, 2751, 2646}

$$\frac{2a^2(33A+28C)\sin(c+dx)\cos^3(c+dx)}{231d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(143A+112C)\sin(c+dx)}{165d\sqrt{a\cos(c+dx)+a}}+\frac{2(143A+112C)\sin(c+dx)(a\cos(c+dx)}{385d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*a^2*(143*A + 112*C)*Sin[c + d*x])/(165*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*(33*A + 28*C)*Cos[c + d*x]^3*Sin[c + d*x])/(231*d*Sqrt[a + a*Cos[c + d*
x]]) - (4*a*(143*A + 112*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1155*d)
 + (2*a*C*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(33*d) + (2
*(143*A + 112*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(385*d) + (2*C*Co
s[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d} + \frac{2\int}{}$$

$$= \frac{2aC\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{33d} + \frac{2C}{}$$

$$= \frac{2a^2(33A+28C)\cos^3(c+dx)\sin(c+dx)}{231d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\cos^3(}{}$$

$$= \frac{2a^2(33A+28C)\cos^3(c+dx)\sin(c+dx)}{231d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\cos^3(}{}$$

$$= \frac{2a^2(33A+28C)\cos^3(c+dx)\sin(c+dx)}{231d\sqrt{a+a\cos(c+dx)}} - \frac{4a(143A+}{}$$

$$= \frac{2a^2(143A+112C)\sin(c+dx)}{165d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(33A+28C)\cos^3}{231d\sqrt{a+a}}$$

Mathematica [A] time = 0.912158, size = 115, normalized size = 0.51

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(5566A+5789C)\cos(c+dx)+8(429A+581C)\cos(2(c+dx))+660A\cos(3(}{9240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(21736*A + 18494*C + 2*(5566*A + 5789*C)*Cos[
c + d*x] + 8*(429*A + 581*C)*Cos[2*(c + d*x)] + 660*A*Cos[3*(c + d*x)] + 16
45*C*Cos[3*(c + d*x)] + 490*C*Cos[4*(c + d*x)] + 105*C*Cos[5*(c + d*x)])*Ta
n[(c + d*x)/2])/(9240*d)
```

Maple [A] time = 0.044, size = 137, normalized size = 0.6

$$\frac{4a^2\sqrt{2}}{1155d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1680C(\sin(1/2dx+c/2))^{10}+6160C(\sin(1/2dx+c/2))^{8}+(-660A-9240C)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 4/1155*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-1680*C*sin(1/2*d*x+1/2*c
)^10+6160*C*sin(1/2*d*x+1/2*c)^8+(-660*A-9240*C)*sin(1/2*d*x+1/2*c)^6+(1848
*A+7392*C)*sin(1/2*d*x+1/2*c)^4+(-1925*A-3465*C)*sin(1/2*d*x+1/2*c)^2+1155*
A+1155*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.04439, size = 230, normalized size = 1.02

$$44\left(15\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+175\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+735\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+7\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] 1/18480*(44*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x +
 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x +
1/2*c))*A*sqrt(a) + 7*(15*sqrt(2)*a*sin(11/2*d*x + 11/2*c) + 55*sqrt(2)*a*s
in(9/2*d*x + 9/2*c) + 165*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 429*sqrt(2)*a*si
n(5/2*d*x + 5/2*c) + 990*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 3630*sqrt(2)*a*si
n(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.40677, size = 332, normalized size = 1.48

$$\frac{2\left(105\,Ca\cos\left(dx+c\right)^{5}+245\,Ca\cos\left(dx+c\right)^{4}+5\left(33\,A+56\,C\right)a\cos\left(dx+c\right)^{3}+3\left(143\,A+112\,C\right)a\cos\left(dx+c\right)^{2}+4\left(14\right.\right.}{1155\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/1155*(105*C*a*cos(d*x + c)^5 + 245*C*a*cos(d*x + c)^4 + 5*(33*A + 56*C)*a
*cos(d*x + c)^3 + 3*(143*A + 112*C)*a*cos(d*x + c)^2 + 4*(143*A + 112*C)*a*
cos(d*x + c) + 8*(143*A + 112*C)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(
```

```
d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
 x)
```

3.84 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=174

$$\frac{8a^2(63A+47C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(63A+22C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{315d}+\frac{2a(63A+47C)\sin(c+dx)\sqrt{a\cos(c+a}}{315d}$$

```
[Out] (8*a^2*(63*A + 47*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(63*A + 47*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*(63*A + 2
2*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*d) + (2*C*Cos[c + d*x]^2
*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d) + (2*C*(a + a*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(21*a*d)
```

Rubi [A] time = 0.364194, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3046, 2968, 3023, 2751, 2647, 2646}

$$\frac{8a^2(63A+47C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(63A+22C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{315d}+\frac{2a(63A+47C)\sin(c+dx)\sqrt{a\cos(c+a}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (8*a^2*(63*A + 47*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(63*A + 47*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*(63*A + 2
2*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*d) + (2*C*Cos[c + d*x]^2
*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d) + (2*C*(a + a*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(21*a*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d} + \frac{2\int c\ldots}{\ldots}$$

$$= \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d} + \frac{2\int(a\ldots}{\ldots}$$

$$= \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d} + \frac{2C(a\ldots}{\ldots}$$

$$= \frac{2(63A+22C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{315d} + \frac{2C\cos\ldots}{\ldots}$$

$$= \frac{2a(63A+47C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{2(63A+\ldots}{\ldots}$$

$$= \frac{8a^2(63A+47C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a(63A+47C)\sqrt{a+a\cos\ldots}}{315d\ldots}$$

Mathematica [A] time = 0.540507, size = 93, normalized size = 0.53

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(756A+799C)\cos(c+dx)+4(63A+137C)\cos(2(c+dx))+3276A+170C\cos\ldots}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(3276*A + 2689*C + 2*(756*A + 799*C)*Cos[c +
d*x] + 4*(63*A + 137*C)*Cos[2*(c + d*x)] + 170*C*Cos[3*(c + d*x)] + 35*C*Co
s[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.043, size = 118, normalized size = 0.7

$$\frac{4a^2\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280C(\sin(1/2dx+c/2))^8-900C(\sin(1/2dx+c/2))^6+(126A+1134C)\left(\sin\left(\frac{dx}{2}\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*C*sin(1/2*d*x+1/2*c)^8
-900*C*sin(1/2*d*x+1/2*c)^6+(126*A+1134*C)*sin(1/2*d*x+1/2*c)^4+(-315*A-735
*C)*sin(1/2*d*x+1/2*c)^2+315*A+315*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2
)/d
```

Maxima [A] time = 1.93453, size = 186, normalized size = 1.07

$$\frac{252\left(\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+20\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(35\sqrt{2}a\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+135\sqrt{\cdot}\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/2520*(252*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*a*sin(9/2*
d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d
*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*
d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.34964, size = 275, normalized size = 1.58

$$\frac{2\left(35\,Ca\cos\left(dx+c\right)^{4}+85\,Ca\cos\left(dx+c\right)^{3}+3\,(21\,A+34\,C)a\cos\left(dx+c\right)^{2}+(189\,A+136\,C)a\cos\left(dx+c\right)+2\,(189\,A\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*C*a*cos(d*x + c)^4 + 85*C*a*cos(d*x + c)^3 + 3*(21*A + 34*C)*a*co
s(d*x + c)^2 + (189*A + 136*C)*a*cos(d*x + c) + 2*(189*A + 136*C)*a)*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + A\right)(a\cos{(dx+c)} + a)^{\frac{3}{2}}\cos{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x
)
```

3.85 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=132

$$\frac{8a^2(35A+19C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a(35A+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}-$$

```
[Out] (8*a^2*(35*A + 19*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(35*A + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (4*C*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a + a*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.174563, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3024, 2751, 2647, 2646}

$$\frac{8a^2(35A+19C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a(35A+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (8*a^2*(35*A + 19*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(35*A + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (4*C*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a + a*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(7*a*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}+\frac{2\int (a+a\cos(c+dx))^{3/2}\left(\frac{1}{2}a(7\right.}{7}$$
$$= -\frac{4C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2C(a+a\cos(c+dx))^{5/2}\sin(}{7ad}$$
$$= \frac{2a(35A+19C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d}-\frac{4C(a+a\cos(c+dx}{35d}$$
$$= \frac{8a^2(35A+19C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(35A+19C)\sqrt{a+a\cos(c+dx)}\sin}{105d}$$

Mathematica [A] time = 0.243845, size = 75, normalized size = 0.57

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((140A+253C)\cos(c+dx)+700A+78C\cos(2(c+dx))+15C\cos(3(c+dx))+}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(700*A + 494*C + (140*A + 253*C)*Cos[c + d*x]
 + 78*C*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(210*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

Maxima [A] time = 1.90594, size = 146, normalized size = 1.11

$$\frac{140\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(15\,\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+175\,\sqrt{2}\cdots}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/420*(140*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*
c))*A*sqrt(a) + (15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d
*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*
x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.3654, size = 223, normalized size = 1.69

$$\frac{2\left(15\,Ca\cos\left(dx+c\right)^{3}+39\,Ca\cos\left(dx+c\right)^{2}+(35\,A+52\,C)a\cos\left(dx+c\right)+(175\,A+104\,C)a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(\cdots\right.}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*C*a*cos(d*x + c)^3 + 39*C*a*cos(d*x + c)^2 + (35*A + 52*C)*a*cos(
d*x + c) + (175*A + 104*C)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(
d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.86 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=133

$$\frac{2a^2(5A+4C)\sin(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aC\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2C\sin(c+dx)(a\,\mathrm{co}}{5d}$$

```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(5*A + 4*C)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*C*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + (2*C*(a + a*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.409798, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3046, 2976, 2981, 2773, 206}

$$\frac{2a^2(5A+4C)\sin(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aC\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2C\sin(c+dx)(a\,\mathrm{co}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(5*A + 4*C)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*C*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + (2*C*(a + a*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(5*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2\int(a+a\cos(c+ \ldots}{\ldots}$$

$$= \frac{2aC\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2C(a+a\cos(c+ \ldots}{5\ldots}$$

$$= \frac{2a^2(5A+4C)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\sqrt{a+a\cos(c+dx)}\sin(\ldots}{5d}$$

$$= \frac{2a^2(5A+4C)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2aC\sqrt{a+a\cos(c+dx)}\sin(\ldots}{5d}$$

$$= \frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(5A+4C)\sin(c+ \ldots}{5d\sqrt{a+a\cos(c+dx\ldots}}$$

Mathematica [A] time = 0.349019, size = 95, normalized size = 0.71

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(10A+6C\cos(c+dx)+C\cos(2(c+dx))+13C)+5\sqrt{2}A\tanh\ldots\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(5*Sqrt[2]*A*ArcTanh[Sqrt[2]
*Sin[(c + d*x)/2]] + (10*A + 13*C + 6*C*Cos[c + d*x] + C*Cos[2*(c + d*x)])*
Sin[(c + d*x)/2]))/(5*d)
```

Maple [B] time = 0.076, size = 307, normalized size = 2.3

$$\frac{1}{5\,d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(8\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^4-20\,C\sqrt{2}\sqrt{a\,(\sin(1/2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/5*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*C*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+10*A*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a+5*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+20*C*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1
/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.72651, size = 73, normalized size = 0.55

$$\frac{\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)C\sqrt{a}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="maxima")
```

```
[Out] 1/10*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 2
0*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*sqrt(a)/d
```

Fricas [A] time = 1.53968, size = 429, normalized size = 3.23

$$\frac{5\,(Aa\cos\left(dx+c\right)+Aa)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(Ca\cos\left(dx+c\right)^2+3\right.}{10\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="fricas")
```

```
[Out] 1/10*(5*(A*a*cos(d*x + c) + A*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x +
c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(C*a*cos(d*x + c)^2 + 3*C*
a*cos(d*x + c) + (5*A + 6*C)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*c
os(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.06203, size = 306, normalized size = 2.3

$$\frac{\frac{5\,Aa^{\frac{5}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(5\,\sqrt{2}Aa^{4}+10\,\sqrt{2}Ca^{4}+\left(10\,\sqrt{2}Aa^{4}+10\,\sqrt{2}Ca^{4}+\left(5\,\sqrt{2}Aa^{4}+4\,\sqrt{2}Ca^{4}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{5}{2}}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="giac")
```

```
[Out] 1/5*(5*A*a^(5/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))
/abs(a) + 2*(5*sqrt(2)*A*a^4 + 10*sqrt(2)*C*a^4 + (10*sqrt(2)*A*a^4 + 10*sq
rt(2)*C*a^4 + (5*sqrt(2)*A*a^4 + 4*sqrt(2)*C*a^4)*tan(1/2*d*x + 1/2*c)^2)*t
an(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^
(5/2))/d
```

3.87 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=136

$$-\frac{a^2(3A-8C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{A\tan(c+dx}{}$$

```
[Out] (3*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d -
(a^2*(3*A - 8*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (a*(3*A - 2
*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (A*(a + a*Cos[c + d*x])^
(3/2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.440984, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2976, 2981, 2773, 206}

$$-\frac{a^2(3A-8C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{A\tan(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] (3*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d -
(a^2*(3*A - 8*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (a*(3*A - 2
*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (A*(a + a*Cos[c + d*x])^
(3/2)*Tan[c + d*x])/d
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a + a\cos(c+dx))^{3/2}\left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \frac{\int (a + a\cos(c + dx}{}$$

$$= -\frac{a(3A-2C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{A(a + a\cos}{}$$

$$= -\frac{a^2(3A-8C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a(3A-2C)\sqrt{a+a\cos(c+}}{3d}$$

$$= -\frac{a^2(3A-8C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a(3A-2C)\sqrt{a+a\cos(c+}}{3d}$$

$$= \frac{3a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(3A-8C)\sin(c+dx}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.341132, size = 106, normalized size = 0.78

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(3A+10C\cos(c+dx)+C\cos(2(c+dx))+C)+9\sqrt{}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,
x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(9*Sqrt[2]*A*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(3*A + C + 10*C*Cos[c + d*
x] + C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Maple [B] time = 0.089, size = 474, normalized size = 3.5

$$\frac{1}{3\,d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(16\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^4+\left(-18\,A\ln\left(4\,\frac{a\sqrt{2}\cos}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+(-18*A*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-18*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a-56*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1
/2*d*x+1/2*c)^2+6*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+9*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2*a))*a+24*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*
c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 1.95261, size = 1828, normalized size = 13.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="maxima")
```

```
[Out] 1/12*(4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))
*C*sqrt(a) - 3*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(
2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2
```

```
*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x
+ 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/
2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt
(2)*a)*sin(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/2*d*x + 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/
2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.73008, size = 463, normalized size = 3.4

$$\frac{9\left(Aa\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(2\,Ca\,\mathrm{c}\right.}{12\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="fricas")
```

```
[Out] 1/12*(9*(A*a*cos(d*x + c)^2 + A*a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c)
- 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*C*a*cos(
d*x + c)^2 + 10*C*a*cos(d*x + c) + 3*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.96866, size = 423, normalized size = 3.11

$$\frac{3\sqrt{2}Aa^{\frac{7}{2}}\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{a|a|}+\frac{8\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\cdots}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\cdots}\right.\right.}\cdots\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="giac")
```

```
[Out] 1/12*(3*sqrt(2)*A*a^(7/2)*(3*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a)) + 16*(2*sqrt(2)*C*a^3*tan(1
/2*d*x + 1/2*c)^2 + 3*sqrt(2)*C*a^3)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x +
1/2*c)^2 + a)^(3/2))/d
```

3.88 $\int (a+a\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=147

$$-\frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{A\tan(c+dx)}{}$$

```
[Out] (a^(3/2)*(7*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) - (a^2*(5*A - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) +
 (3*a*A*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (A*(a + a*Cos[c + d*
x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.466666, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2975, 2981, 2773, 206}

$$-\frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{A\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] (a^(3/2)*(7*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) - (a^2*(5*A - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) +
 (3*a*A*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (A*(a + a*Cos[c + d*
x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d} + \frac{\int (a+}{}$$

$$= \frac{3aA\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{4d} + \frac{A(a+a\cos(c+d}{}$$

$$= -\frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{3aA\sqrt{a+a\cos(c+dx)}\tan}{4d}$$

$$= -\frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{3aA\sqrt{a+a\cos(c+dx)}\tan}{4d}$$

$$= \frac{a^{3/2}(7A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^2(5A-8C)\sin}{4d\sqrt{a+a\cos(}}$$

Mathematica [A] time = 0.575402, size = 118, normalized size = 0.8

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(7A\cos(c+dx)+2A+4C\cos(2(c+dx))+4C)+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,
x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(7*A
 + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*A + 4*C + 7
*A*Cos[c + d*x] + 4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.094, size = 1018, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(16*C*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
```

```
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+8*C*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(7*A*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))*a+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2
+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+7*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+18*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+8*C*ln(-4/(-2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2
/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 2.21853, size = 2734, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="maxima")
```

```
[Out] -1/16*(12*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*c)
^2*sin(3/2*d*x + 3/2*c) + 12*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48
*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)*sin
(2*d*x + 2*c) + 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos(3/2*
d*x + 3/2*c)*sin(2*d*x + 2*c) - 24*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
- 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 12
*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 48*(a
```

```
*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 4*(12*a*co
s(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 20*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(
5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 7*(sqrt(2)*
a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x
 + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin
(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x +
 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*s
qrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2
)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d
*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2) - 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^
2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(
2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*c
os(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x +
4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x +
2*c) + a)*sin(13/2*d*x + 13/2*c) - 12*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x +
 2*c) + a)*sin(11/2*d*x + 11/2*c) - 48*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
+ 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(12*a*sin(2*d*x + 2*c)*sin(3/2*d*x + 3
/2*c) + 20*a*cos(7/2*d*x + 7/2*c) + 21*a*cos(5/2*d*x + 5/2*c) + 9*a*cos(3/2
*d*x + 3/2*c))*sin(4*d*x + 4*c) - 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*x
 + 7/2*c) - 84*(2*a*cos(2*d*x + 2*c) + a)*sin(5/2*d*x + 5/2*c) - 24*a*sin(3
/2*d*x + 3/2*c) - 4*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(
4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*
c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c)
 + a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 56*(a*
cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*si
```

```
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*x
 + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/((sqrt(2)*cos(4*d*x
+ 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sq
rt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*d)
```

Fricas [A] time = 1.93602, size = 491, normalized size = 3.34

$$\frac{\left((7\,A+8\,C)a\cos\left(dx+c\right)^{3}+(7\,A+8\,C)a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(d}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="fricas")
```

```
[Out] 1/16*((((7*A + 8*C)*a*cos(d*x + c)^3 + (7*A + 8*C)*a*cos(d*x + c)^2)*sqrt(a)
*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)) + 4*(8*C*a*cos(d*x + c)^2 + 7*A*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.02283, size = 512, normalized size = 3.48

$$\frac{16\sqrt{2}Ca^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\left(7Aa^{\frac{3}{2}}+8Ca^{\frac{3}{2}}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)-\left(7Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="giac")
```

```
[Out] 1/8*(16*sqrt(2)*C*a^2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a) + (7*A*a^(3/2) + 8*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (7*A*a^(3/2) +
8*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(5/2) - 95*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(7/2) + 53*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(9/
2) - 5*A*a^(11/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2*a + a^2)^2)/d
```

3.89 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=155

$$\frac{a^2(19A+24C)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{A\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

```
[Out] (a^(3/2)*(11*A + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^2*(19*A + 24*C)*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (A*(
a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.516846, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2975, 2980, 2773, 206}

$$\frac{a^2(19A+24C)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{A\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (a^(3/2)*(11*A + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^2*(19*A + 24*C)*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (A*(
a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{\int(a}{}$$

$$= \frac{aA\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{4d} + \frac{A(a+}{}$$

$$= \frac{a^2(19A+24C)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}{4d}$$

$$= \frac{a^2(19A+24C)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}{4d}$$

$$= \frac{a^{3/2}(11A+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(19A+24}{24d\sqrt{a+}}$$

Mathematica [A] time = 0.883064, size = 124, normalized size = 0.8

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(11A+8C)\cos(2(c+dx))+44A\cos(c+dx)+49\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,
x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(1
1*A + 24*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (49*A + 24*C
 + 44*A*Cos[c + d*x] + 3*(11*A + 8*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/
(48*d)
```

Maple [B] time = 0.094, size = 1311, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(11*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+11*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+24*C*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(22*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+16*C*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a
+33*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+72*C*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a))*a+72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a)*sin(1/2*d*x+1/2*c)^4-2*(176*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+99*A*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+99*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a+216*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+216*C*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+33*A*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+33*A*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a))*a+126*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+72*C*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
```

```
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.9678, size = 516, normalized size = 3.33

$$\frac{3\left((11\,A+24\,C)a\cos\left(dx+c\right)^{4}+(11\,A+24\,C)a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{96\left(d\cos\left(dx+c\right)^{4}+d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="fricas")
```

```
[Out] 1/96*(3*((11*A + 24*C)*a*cos(d*x + c)^4 + (11*A + 24*C)*a*cos(d*x + c)^3)*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) +
 a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*(3*(11*A + 8*C)*a*cos(d*x + c)^2 + 22*A*a*cos(d*x + c) + 8*A
*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x +
c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.05204, size = 861, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="giac")
```

```
[Out] 1/48*(3*(11*A*a^(3/2) + 24*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(11*A*a^
(3/2) + 24*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(33*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(5/2) + 72*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(
5/2) - 303*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^8*A*a^(7/2) - 888*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^8*C*a^(7/2) + 2394*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(9/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(9/2) - 1806*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(11/2) - 1776*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(11/2
) + 309*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*A*a^(13/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*C*a^(13/2) - 19*A*a^(15/2) - 24*C*a^(15/2))/((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.90 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{a^2(75A+112C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(13A+16C)\tan(c+dx)\sec(c+dx)}{32d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (a^(3/2)*(75*A + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^2*(75*A + 112*C)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c +
d*x]]) + (a^2*(13*A + 16*C)*Sec[c + d*x]*Tan[c + d*x])/(32*d*Sqrt[a + a*Cos
[c + d*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(8
*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.607056, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2975, 2980, 2772, 2773, 206}

$$\frac{a^2(75A+112C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(13A+16C)\tan(c+dx)\sec(c+dx)}{32d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] (a^(3/2)*(75*A + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^2*(75*A + 112*C)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c +
d*x]]) + (a^2*(13*A + 16*C)*Sec[c + d*x]*Tan[c + d*x])/(32*d*Sqrt[a + a*Cos
[c + d*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(8
*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{\int (a$$

$$= \frac{aA\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{8d} + \frac{A(a}{}$$

$$= \frac{a^2(13A+16C)\sec(c+dx)\tan(c+dx)}{32d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos}}{}$$

$$= \frac{a^2(75A+112C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(13A+16C)\sec(c+}{32d\sqrt{a+a\cos(}}$$

$$= \frac{a^2(75A+112C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(13A+16C)\sec(c+}{32d\sqrt{a+a\cos(}}$$

$$= \frac{a^{3/2}(75A+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(75A+1}{64d\sqrt{a-}}$$

Mathematica [A] time = 1.45082, size = 152, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(7(55A+48C)\cos(c+dx)+4(25A+16C)\cos(2(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,
x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(2*Sqrt[2]*(7
5*A + 112*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (164*A + 64
*C + 7*(55*A + 48*C)*Cos[c + d*x] + 4*(25*A + 16*C)*Cos[2*(c + d*x)] + 75*A
*Cos[3*(c + d*x)] + 112*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(256*d)
```

Maple [B] time = 0.099, size = 1630, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/8*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*a*(75*A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))+112*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+112*C*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-16*(75*
A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+112*C*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+150*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a+150*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+224*C*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+224*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(275*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+368*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+225*A
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+225*A*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a+336*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))
*a+336*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c
)^4+(-1460*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-600*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-600*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a-1600*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-896*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-896*C*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+362*A*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
```

```
a+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+288*C*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+112*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))*a+112*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/2*d*x+1/2*c)/
(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 7.62221, size = 9430, normalized size = 47.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="maxima")
```

```
[Out] -1/256*((140*a*cos(8*d*x + 8*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*cos(6*d*x +
 6*c)^2*sin(3/2*d*x + 3/2*c) + 5040*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*
c) + 2240*a*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 140*a*sin(8*d*x + 8*c
)^2*sin(3/2*d*x + 3/2*c) + 2240*a*sin(6*d*x + 6*c)^2*sin(3/2*d*x + 3/2*c) +
 5040*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*sin(2*d*x + 2*c)^2
*sin(3/2*d*x + 3/2*c) + 4064*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 336*
a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 240*a*cos(3/2*d*x + 3/2*c)*sin(2*
d*x + 2*c) + 1360*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 36*(a*sin(8*d*x
 + 8*c) + 4*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + 4*a*sin(2*d*x + 2*c
))*cos(21/2*d*x + 21/2*c) + 140*(a*sin(8*d*x + 8*c) + 4*a*sin(6*d*x + 6*c)
+ 6*a*sin(4*d*x + 4*c) + 4*a*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c) + 456
*(a*sin(8*d*x + 8*c) + 4*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + 4*a*si
n(2*d*x + 2*c))*cos(17/2*d*x + 17/2*c) + 4*(280*a*cos(6*d*x + 6*c)*sin(3/2*
d*x + 3/2*c) + 420*a*cos(4*d*x + 4*c)*sin(3/2*d*x + 3/2*c) + 280*a*cos(2*d*
x + 2*c)*sin(3/2*d*x + 3/2*c) - 290*a*sin(15/2*d*x + 15/2*c) - 596*a*sin(13
/2*d*x + 13/2*c) - 780*a*sin(11/2*d*x + 11/2*c) - 750*a*sin(9/2*d*x + 9/2*c
) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) + 85*a*sin(3/2*d
*x + 3/2*c))*cos(8*d*x + 8*c) + 2320*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x
+ 4*c) + 2*a*sin(2*d*x + 2*c))*cos(15/2*d*x + 15/2*c) + 4768*(2*a*sin(6*d*x
 + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*
c) + 16*(420*a*cos(4*d*x + 4*c)*sin(3/2*d*x + 3/2*c) + 280*a*cos(2*d*x + 2*
c)*sin(3/2*d*x + 3/2*c) - 780*a*sin(11/2*d*x + 11/2*c) - 750*a*sin(9/2*d*x
+ 9/2*c) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) + 85*a*si
n(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) + 6240*(3*a*sin(4*d*x + 4*c) + 2*a*sin
```

```
(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 6000*(3*a*sin(4*d*x + 4*c) + 2*a*si
n(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 24*(280*a*cos(2*d*x + 2*c)*sin(3/2*d
*x + 3/2*c) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) + 85*a
*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 75*(sqrt(2)*a*cos(8*d*x + 8*c)^2
+ 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a*cos(4*d*x + 4*c)^2 + 16*sq
rt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x + 8*c)^2 + 16*sqrt(2)*a*si
n(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)^2 + 48*sqrt(2)*a*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 8*sqrt(2)*a*co
s(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x +
4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(8*d*x + 8*c) + 8*(6*sq
rt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(6*
d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c)
 + 4*(2*sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(
2)*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sqrt(2)*a)*log(2*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(sqrt(2)*a*cos(8*d*x
+ 8*c)^2 + 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a*cos(4*d*x + 4*c)^
2 + 16*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x + 8*c)^2 + 16*sqr
t(2)*a*sin(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)^2 + 48*sqrt(2)*a*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 8*sqr
t(2)*a*cos(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos
(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(8*d*x + 8*c)
+ 8*(6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*
a)*cos(6*d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d
*x + 4*c) + 4*(2*sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*sqrt(2)*a*sin(4*d*
x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sqrt(2)*a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(sqrt(2)*a*c
os(8*d*x + 8*c)^2 + 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a*cos(4*d*
x + 4*c)^2 + 16*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x + 8*c)^2
 + 16*sqrt(2)*a*sin(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)^2 + 48*s
qrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*x + 2*c)^
2 + 8*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt
(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(8*d*
x + 8*c) + 8*(6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) +
 sqrt(2)*a)*cos(6*d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
)*cos(4*d*x + 4*c) + 4*(2*sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*
x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*sqrt(2)*a
*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sqrt(2
```

```
)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(sq
rt(2)*a*cos(8*d*x + 8*c)^2 + 16*sqrt(2)*a*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*a
*cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(8*d*x
 + 8*c)^2 + 16*sqrt(2)*a*sin(6*d*x + 6*c)^2 + 36*sqrt(2)*a*sin(4*d*x + 4*c)
^2 + 48*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*a*sin(2*d*
x + 2*c)^2 + 8*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(4*sqrt(2)*a*cos(6*d*x + 6*c)
 + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)
*cos(8*d*x + 8*c) + 8*(6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*cos(6*d*x + 6*c) + 12*(4*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a)*cos(4*d*x + 4*c) + 4*(2*sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a
*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*
sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
 + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 + 36*(a*cos(8*d*x + 8*c) + 4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4
*a*cos(2*d*x + 2*c) + a)*sin(21/2*d*x + 21/2*c) - 140*(a*cos(8*d*x + 8*c) +
 4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*si
n(19/2*d*x + 19/2*c) - 456*(a*cos(8*d*x + 8*c) + 4*a*cos(6*d*x + 6*c) + 6*a
*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(17/2*d*x + 17/2*c) + 4*(2
80*a*sin(6*d*x + 6*c)*sin(3/2*d*x + 3/2*c) + 420*a*sin(4*d*x + 4*c)*sin(3/2
*d*x + 3/2*c) + 280*a*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 290*a*cos(15/
2*d*x + 15/2*c) + 596*a*cos(13/2*d*x + 13/2*c) + 780*a*cos(11/2*d*x + 11/2*
c) + 750*a*cos(9/2*d*x + 9/2*c) + 254*a*cos(7/2*d*x + 7/2*c) + 21*a*cos(5/2
*d*x + 5/2*c) - 15*a*cos(3/2*d*x + 3/2*c))*sin(8*d*x + 8*c) - 1160*(4*a*cos
(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(15/2*d
*x + 15/2*c) - 2384*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(
2*d*x + 2*c) + a)*sin(13/2*d*x + 13/2*c) + 16*(420*a*sin(4*d*x + 4*c)*sin(3
/2*d*x + 3/2*c) + 280*a*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 780*a*cos(1
1/2*d*x + 11/2*c) + 750*a*cos(9/2*d*x + 9/2*c) + 254*a*cos(7/2*d*x + 7/2*c)
 + 21*a*cos(5/2*d*x + 5/2*c) - 15*a*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c)
 - 3120*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(11/2*d*x + 11/
2*c) - 3000*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(9/2*d*x +
 9/2*c) + 24*(280*a*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 254*a*cos(7/2*d
*x + 7/2*c) + 21*a*cos(5/2*d*x + 5/2*c) - 15*a*cos(3/2*d*x + 3/2*c))*sin(4*
d*x + 4*c) - 1016*(4*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*x + 7/2*c) - 84*(4*a
*cos(2*d*x + 2*c) + a)*sin(5/2*d*x + 5/2*c) + 200*a*sin(3/2*d*x + 3/2*c) -
36*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^
2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^
2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a
*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*
```

```
cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(
2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*
x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x +
 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) +
 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*sin(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 600*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x
 + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x
 + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x +
6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) +
8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(
4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*
sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x +
 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 a)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)
/(sqrt(2)*cos(8*d*x + 8*c)^2 + 16*sqrt(2)*cos(6*d*x + 6*c)^2 + 36*sqrt(2)*c
os(4*d*x + 4*c)^2 + 16*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(8*d*x + 8*c
)^2 + 16*sqrt(2)*sin(6*d*x + 6*c)^2 + 36*sqrt(2)*sin(4*d*x + 4*c)^2 + 48*sq
rt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sqrt(2)*sin(2*d*x + 2*c)^2 + 2
*(4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*cos(8*d*x + 8*c) + 8*(6*sqrt(2)*cos(4*d*x + 4*c) + 4
*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 12*(4*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(2*sqrt(2)*sin(6*d*x + 6*c) + 3
*sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
16*(3*sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + 8*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) + 16*(12*a*cos(4*d*x + 4*c)^2*si
n(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 12*a*si
n(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x
 + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 168*a*cos(5/2*d*x
 + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 2
4*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 4*(a*sin(4*d*x + 4*c) + 2*a*sin
(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 12*(a*sin(4*d*x + 4*c) + 2*a*sin(2*
d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 48*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x
 + 2*c))*cos(9/2*d*x + 9/2*c) + 4*(12*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) - 20*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*
x + 3/2*c))*cos(4*d*x + 4*c) - 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*
a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos
(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*
c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqr
t(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
```

```
 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(
2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(sqrt(2)*a*cos(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2
+ 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2
*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*c
os(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*
d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c)
+ sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(13/2*d*x + 13/2*c)
- 12*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(11/2*d*x + 11/2*c)
 - 48*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(9/2*d*x + 9/2*c)
+ 4*(12*a*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 20*a*cos(7/2*d*x + 7/2*c)
 + 21*a*cos(5/2*d*x + 5/2*c) + 9*a*cos(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c) -
 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*x + 7/2*c) - 84*(2*a*cos(2*d*x + 2
*c) + a)*sin(5/2*d*x + 5/2*c) - 24*a*sin(3/2*d*x + 3/2*c) - 4*(a*cos(4*d*x
+ 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) +
a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(5/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 56*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*
x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4
*a*cos(2*d*x + 2*c) + a)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*C*sqrt(a)/(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2
*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)))/d
```

Fricas [A] time = 2.25532, size = 566, normalized size = 2.83

$$\frac{\left((75\,A+112\,C)a\cos\left(dx+c\right)^{5}+(75\,A+112\,C)a\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{256\left(d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="fricas")
```

```
[Out] 1/256*(((75*A + 112*C)*a*cos(d*x + c)^5 + (75*A + 112*C)*a*cos(d*x + c)^4)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*((75*A + 112*C)*a*cos(d*x + c)^3 + 2*(25*A + 16*C)*a*cos(d*
x + c)^2 + 40*A*a*cos(d*x + c) + 16*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.10683, size = 1081, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="giac")
```

```
[Out] 1/128*((75*A*a^(3/2) + 112*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (75*A*a^(3
```

```
/2) + 112*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(75*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(5/2) + 112*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(
5/2) - 2087*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^12*A*a^(7/2) - 2864*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^12*C*a^(7/2) + 11975*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(9/2) + 23344*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(9/2) - 42483*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(11/2) -
69360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8
*C*a^(11/2) + 33889*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^6*A*a^(13/2) + 51536*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(13/2) - 8693*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(15/2) - 14736*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(15/2) + 1101*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(1
7/2) + 1808*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*C*a^(17/2) - 49*A*a^(19/2) - 80*C*a^(19/2))/((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.91 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=245

$$\frac{a^2(133A+176C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(133A+176C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(67A+80C)\tan(c+dx)\sec^2(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (a^(3/2)*(133*A + 176*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^2*(133*A + 176*C)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^2*(133*A + 176*C)*Sec[c + d*x]*Tan[c + d*x])/(192*d*Sqrt[a
+ a*Cos[c + d*x]]) + (a^2*(67*A + 80*C)*Sec[c + d*x]^2*Tan[c + d*x])/(240*d
*Sqrt[a + a*Cos[c + d*x]]) + (3*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3
*Tan[c + d*x])/(40*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*Tan[c
+ d*x])/(5*d)
```

Rubi [A] time = 0.68439, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2975, 2980, 2772, 2773, 206}

$$\frac{a^2(133A+176C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(133A+176C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(67A+80C)\tan(c+dx)\sec^2(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (a^(3/2)*(133*A + 176*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^2*(133*A + 176*C)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^2*(133*A + 176*C)*Sec[c + d*x]*Tan[c + d*x])/(192*d*Sqrt[a
+ a*Cos[c + d*x]]) + (a^2*(67*A + 80*C)*Sec[c + d*x]^2*Tan[c + d*x])/(240*d
*Sqrt[a + a*Cos[c + d*x]]) + (3*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3
*Tan[c + d*x])/(40*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*Tan[c
+ d*x])/(5*d)
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{\int (a+}{}$$

$$= \frac{3aA\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{40d} + \frac{A(a+}{}$$

$$= \frac{a^2(67A+80C)\sec^2(c+dx)\tan(c+dx)}{240d\sqrt{a+a\cos(c+dx)}} + \frac{3aA\sqrt{a+a\,\mathrm{c}}}{}$$

$$= \frac{a^2(133A+176C)\sec(c+dx)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(67A+80}{240}$$

$$= \frac{a^2(133A+176C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(133A+176C)\sec(c}{192d\sqrt{a+a\cos}}$$

$$= \frac{a^2(133A+176C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(133A+176C)\sec(c}{192d\sqrt{a+a\cos}}$$

$$= \frac{a^{3/2}(133A+176C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^2(133A+}{128d\sqrt{a}}$$

Mathematica [A] time = 2.22283, size = 174, normalized size = 0.71

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(1273A+880C)\cos(c+dx)+4(3059A+3280C)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,
x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]*(
133*A + 176*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (13313*A
```

```
+ 10480*C + 12*(1273*A + 880*C)*Cos[c + d*x] + 4*(3059*A + 3280*C)*Cos[2*(c
 + d*x)] + 2660*A*Cos[3*(c + d*x)] + 3520*C*Cos[3*(c + d*x)] + 1995*A*Cos[4
*(c + d*x)] + 2640*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

Maple [B] time = 0.118, size = 1951, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/120*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(13
3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+133*A*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))+176*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+
176*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^10+
240*(266*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+352*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+665*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a+665*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+880*C*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+880*C*ln(-4/(-2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^8-80*(1862*A*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+2464*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+1995*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1995*A*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2640*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a))*a+2640*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*s
in(1/2*d*x+1/2*c)^6+8*(17024*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+21760*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9975*A*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+9975*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
```

```
+1/2*c)+2*a))*a+13200*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+13200
*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-10
*(6004*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+6848*C*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1995*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a+1995*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2640*C*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2640*C*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+11370*A*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+1995*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a+1995*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+10080*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2640*C*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+2640*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))^5/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^5/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.26181, size = 635, normalized size = 2.59

$$15\left((133\,A+176\,C)a\cos\left(dx+c\right)^{6}+(133\,A+176\,C)a\cos\left(dx+c\right)^{5}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(d}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorit
hm="fricas")
```

```
[Out] 1/7680*(15*((133*A + 176*C)*a*cos(d*x + c)^6 + (133*A + 176*C)*a*cos(d*x +
c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x
 + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)) + 4*(15*(133*A + 176*C)*a*cos(d*x + c)^4 + 10*(133*A + 17
6*C)*a*cos(d*x + c)^3 + 8*(133*A + 80*C)*a*cos(d*x + c)^2 + 912*A*a*cos(d*x
 + c) + 384*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^6 +
 d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.14264, size = 1304, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorit
hm="giac")
```

```
[Out] 1/3840*(15*(133*A*a^(3/2) + 176*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(1
33*A*a^(3/2) + 176*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(1995*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*a^(5
/2) + 2640*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^18*C*a^(5/2) - 38505*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
```

```
 + 1/2*c)^2 + a))^16*A*a^(7/2) - 55920*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*a^(7/2) + 561660*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(9/2) + 582720*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(9/2)
- 2684100*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^12*A*a^(11/2) - 3395520*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^12*C*a^(11/2) + 7371738*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(13/2) + 9329760*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(13/2) - 64074
70*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*
a^(15/2) - 8110880*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^8*C*a^(15/2) + 2176620*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(17/2) + 2882880*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(17/2) - 399860*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(19/2) - 4
98880*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4
*C*a^(19/2) + 34035*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*A*a^(21/2) + 42960*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(21/2) - 1201*A*a^(23/2) - 1520*C*a^(23/2)
)/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 -
6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
 a^2)^5)/d
```

3.92 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=273

$$\frac{2a^3(2717A+2224C)\sin(c+dx)\cos^3(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(143A+136C)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{1287d}+\frac{2a^3(\cdots}{\cdots}$$

```
[Out] (2*a^3*(10439*A + 8368*C)*Sin[c + d*x])/(6435*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a^3*(2717*A + 2224*C)*Cos[c + d*x]^3*Sin[c + d*x])/(9009*d*Sqrt[a + a*C
os[c + d*x]]) - (4*a^2*(10439*A + 8368*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(45045*d) + (2*a^2*(143*A + 136*C)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(1287*d) + (2*a*(10439*A + 8368*C)*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(15015*d) + (10*a*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(143*d) + (2*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(13*d)
```

Rubi [A] time = 0.861483, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3046, 2976, 2981, 2759, 2751, 2646}

$$\frac{2a^3(2717A+2224C)\sin(c+dx)\cos^3(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(143A+136C)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{1287d}+\frac{2a^3(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a^3*(10439*A + 8368*C)*Sin[c + d*x])/(6435*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a^3*(2717*A + 2224*C)*Cos[c + d*x]^3*Sin[c + d*x])/(9009*d*Sqrt[a + a*C
os[c + d*x]]) - (4*a^2*(10439*A + 8368*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(45045*d) + (2*a^2*(143*A + 136*C)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(1287*d) + (2*a*(10439*A + 8368*C)*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(15015*d) + (10*a*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(143*d) + (2*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(13*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
```

```
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{13d} + \frac{2\int}{}$$

$$= \frac{10aC\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{143d} +$$

$$= \frac{2a^2(143A+136C)\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(}{1287d}$$

$$= \frac{2a^3(2717A+2224C)\cos^3(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(1}{}$$

$$= \frac{2a^3(2717A+2224C)\cos^3(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(1}{}$$

$$= \frac{2a^3(2717A+2224C)\cos^3(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}} - \frac{4a^2(1}{}$$

$$= \frac{2a^3(10439A+8368C)\sin(c+dx)}{6435d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(2717A+222}{9009d}$$

Mathematica [A] time = 1.2751, size = 138, normalized size = 0.51

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(8(222794A+226573C)\cos(c+dx)+(581152A+746519C)\cos(2(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3233516*A + 2798182*C + 8*(222794*A + 2265
73*C)*Cos[c + d*x] + (581152*A + 746519*C)*Cos[2*(c + d*x)] + 148720*A*Cos[
3*(c + d*x)] + 287060*C*Cos[3*(c + d*x)] + 20020*A*Cos[4*(c + d*x)] + 94010
*C*Cos[4*(c + d*x)] + 23940*C*Cos[5*(c + d*x)] + 3465*C*Cos[6*(c + d*x)])*T
```

```
an[(c + d*x)/2])/(720720*d)
```

Maple [A] time = 0.102, size = 156, normalized size = 0.6

$$\frac{8\,a^3\sqrt{2}}{45045\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(55440\,C\,(\sin(1/2\,dx+c/2))^{12}-262080\,C\,(\sin(1/2\,dx+c/2))^{10}+(20020\,A+52052\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 8/45045*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(55440*C*sin(1/2*d*x+1/2*
c)^12-262080*C*sin(1/2*d*x+1/2*c)^10+(20020*A+520520*C)*sin(1/2*d*x+1/2*c)^
8+(-77220*A-566280*C)*sin(1/2*d*x+1/2*c)^6+(117117*A+369369*C)*sin(1/2*d*x+
1/2*c)^4+(-90090*A-150150*C)*sin(1/2*d*x+1/2*c)^2+45045*A+45045*C)*2^(1/2)/
(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.19278, size = 301, normalized size = 1.1

$$\frac{572\left(35\sqrt{2}a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+225\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+756\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+2100\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+81\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] 1/1441440*(572*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7
/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*s
in(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (3
465*sqrt(2)*a^2*sin(13/2*d*x + 13/2*c) + 20475*sqrt(2)*a^2*sin(11/2*d*x + 1
1/2*c) + 70070*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 193050*sqrt(2)*a^2*sin(7/
2*d*x + 7/2*c) + 459459*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 1066065*sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c) + 3783780*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt
(a))/d
```

Fricas [A] time = 1.67566, size = 427, normalized size = 1.56

$$\frac{2\left(3465\,Ca^2\cos\left(dx+c\right)^6+11970\,Ca^2\cos\left(dx+c\right)^5+35\left(143\,A+523\,C\right)a^2\cos\left(dx+c\right)^4+10\left(1859\,A+2092\,C\right)a^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/45045*(3465*C*a^2*cos(d*x + c)^6 + 11970*C*a^2*cos(d*x + c)^5 + 35*(143*A
 + 523*C)*a^2*cos(d*x + c)^4 + 10*(1859*A + 2092*C)*a^2*cos(d*x + c)^3 + 3*
(10439*A + 8368*C)*a^2*cos(d*x + c)^2 + 4*(10439*A + 8368*C)*a^2*cos(d*x +
c) + 8*(10439*A + 8368*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos
(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
 x)
```

3.93 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=211

$$\frac{16a^2(33A+25C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{693d}+\frac{64a^3(33A+25C)\sin(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2(99A+26C)\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{693d}$$

```
[Out] (64*a^3*(33*A + 25*C)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(33*A + 25*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(693*d) + (2*a*(33
*A + 25*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(231*d) + (2*(99*A + 26
*C)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*d) + (2*C*Cos[c + d*x]^2*
(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*d) + (10*C*(a + a*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(99*a*d)
```

Rubi [A] time = 0.410216, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3046, 2968, 3023, 2751, 2647, 2646}

$$\frac{16a^2(33A+25C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{693d}+\frac{64a^3(33A+25C)\sin(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2(99A+26C)\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{693d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (64*a^3*(33*A + 25*C)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(33*A + 25*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(693*d) + (2*a*(33
*A + 25*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(231*d) + (2*(99*A + 26
*C)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*d) + (2*C*Cos[c + d*x]^2*
(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*d) + (10*C*(a + a*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(99*a*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{11d} + \frac{2\int \mathrm{c}}{}$$

$$= \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{11d} + \frac{2\int (a}{}$$

$$= \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{11d} + \frac{10C(}{}$$

$$= \frac{2(99A+26C)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{693d} + \frac{2C\cos}{}$$

$$= \frac{2a(33A+25C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{231d} + \frac{2(99A}{}$$

$$= \frac{16a^2(33A+25C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{693d} + \frac{2a(33}{}$$

$$= \frac{64a^3(33A+25C)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(33A+25C)\sqrt{a+a}}{693}$$

Mathematica [A] time = 0.864536, size = 117, normalized size = 0.55

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(6666A+6989C)\cos(c+dx)+16(198A+325C)\cos(2(c+dx))+396A\cos(3}{5544d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(27456*A + 22928*C + 2*(6666*A + 6989*C)*Co
s[c + d*x] + 16*(198*A + 325*C)*Cos[2*(c + d*x)] + 396*A*Cos[3*(c + d*x)] +
 1735*C*Cos[3*(c + d*x)] + 448*C*Cos[4*(c + d*x)] + 63*C*Cos[5*(c + d*x)])*
Tan[(c + d*x)/2])/(5544*d)
```

Maple [A] time = 0.043, size = 137, normalized size = 0.7

$$\frac{8a^3\sqrt{2}}{693d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-504C(\sin(1/2dx+c/2))^{10}+2156C(\sin(1/2dx+c/2))^{8}+(-198A-3762C)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 8/693*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-504*C*sin(1/2*d*x+1/2*c)^
10+2156*C*sin(1/2*d*x+1/2*c)^8+(-198*A-3762*C)*sin(1/2*d*x+1/2*c)^6+(693*A+
3465*C)*sin(1/2*d*x+1/2*c)^4+(-924*A-1848*C)*sin(1/2*d*x+1/2*c)^2+693*A+693
*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.07976, size = 255, normalized size = 1.21

$$132\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/11088*(132*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d
*x + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2
*d*x + 1/2*c))*A*sqrt(a) + (63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqr
t(2)*a^2*sin(9/2*d*x + 9/2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 346
5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
+ 31878*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.63788, size = 339, normalized size = 1.61

$$\frac{2\left(63Ca^2\cos(dx+c)^5+224Ca^2\cos(dx+c)^4+(99A+355C)a^2\cos(dx+c)^3+6(66A+71C)a^2\cos(dx+c)^2+(75\right.}{693(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/693*(63*C*a^2*cos(d*x + c)^5 + 224*C*a^2*cos(d*x + c)^4 + (99*A + 355*C)*
a^2*cos(d*x + c)^3 + 6*(66*A + 71*C)*a^2*cos(d*x + c)^2 + (759*A + 568*C)*a
^2*cos(d*x + c) + 2*(759*A + 568*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x
)
```

3.94 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=169

$$\frac{16a^2(21A+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(21A+13C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a(21A+13C)\sin(c+dx)(a\cos(c+dx)+a)}{105d}$$

```
[Out] (64*a^3*(21*A + 13*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(21*A + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(21
*A + 13*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) - (4*C*(a + a*C
os[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*C*(a + a*Cos[c + d*x])^(7/2)*S
in[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.208198, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3024, 2751, 2647, 2646}

$$\frac{16a^2(21A+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(21A+13C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a(21A+13C)\sin(c+dx)(a\cos(c+dx)+a)}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] (64*a^3*(21*A + 13*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(21*A + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(21
*A + 13*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) - (4*C*(a + a*C
os[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*C*(a + a*Cos[c + d*x])^(7/2)*S
in[c + d*x])/(9*a*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int (a+a\cos(c+dx))^{5/2}\left(\frac{1}{2}a(9\right.}{9}$$

$$= -\frac{4C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2C(a+a\cos(c+dx))^{7/2}\sin(}{9ad}$$

$$= \frac{2a(21A+13C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} - \frac{4C(a+a\cos(c+a}{63}$$

$$= \frac{16a^2(21A+13C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{2a(21A+13C)(a}{}$$

$$= \frac{64a^3(21A+13C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(21A+13C)\sqrt{a+a\cos(c+dx)}}{315d}$$

Mathematica [A] time = 0.465865, size = 95, normalized size = 0.56

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(4(588A+779C)\cos(c+dx)+4(63A+254C)\cos(2(c+dx))+7476A+260C\,c}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(7476*A + 5653*C + 4*(588*A + 779*C)*Cos[c
+ d*x] + 4*(63*A + 254*C)*Cos[2*(c + d*x)] + 260*C*Cos[3*(c + d*x)] + 35*C*
Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.043, size = 118, normalized size = 0.7

$$\frac{8a^3\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\,C\,(\sin(1/2\,dx+c/2))^8-540\,C\,(\sin(1/2\,dx+c/2))^6+(63\,A+819\,C)\left(\sin\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*C*sin(1/2*d*x+1/2*c)^8
-540*C*sin(1/2*d*x+1/2*c)^6+(63*A+819*C)*sin(1/2*d*x+1/2*c)^4+(-210*A-630*C
)*sin(1/2*d*x+1/2*c)^2+315*A+315*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/
d
```

Maxima [A] time = 2.17856, size = 209, normalized size = 1.24

$$\frac{84\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(35\sqrt{2}a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+\right.}{252}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2520*(84*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*a
^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2
)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*s
qrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.63681, size = 290, normalized size = 1.72

$$\frac{2\left(35\,Ca^2\cos(dx+c)^4+130\,Ca^2\cos(dx+c)^3+3\,(21\,A+73\,C)a^2\cos(dx+c)^2+2\,(147\,A+146\,C)a^2\cos(dx+c)+\right(}{315\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*C*a^2*cos(d*x + c)^4 + 130*C*a^2*cos(d*x + c)^3 + 3*(21*A + 73*C)
*a^2*cos(d*x + c)^2 + 2*(147*A + 146*C)*a^2*cos(d*x + c) + (903*A + 584*C)*
a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos\left(dx+c\right)^2+A\right)\left(a \cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2), x)
```

3.95 $\int (a+a\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec(c+dx)\, dx$

Optimal. Leaf size=170

$$\frac{2a^3(49A+32C)\sin(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(7A+8C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aC\sin}{}$$

```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(49*A + 32*C)*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(7*A + 8*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*C*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*C*(a + a*Cos[c + d*x])^(5/2)*S
in[c + d*x])/(7*d)
```

Rubi [A] time = 0.566954, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3046, 2976, 2981, 2773, 206}

$$\frac{2a^3(49A+32C)\sin(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(7A+8C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aC\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(49*A + 32*C)*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(7*A + 8*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*C*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*C*(a + a*Cos[c + d*x])^(5/2)*S
in[c + d*x])/(7*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\,dx = \frac{2C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2\int (a + a\cos(c +}{}$$

$$= \frac{2aC(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2C(a+a\cos(c}{}$$

$$= \frac{2a^2(7A+8C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2aC(a+}{}$$

$$= \frac{2a^3(49A+32C)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(7A+8C)\sqrt{a+a\cos}}{21d}$$

$$= \frac{2a^3(49A+32C)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(7A+8C)\sqrt{a+a\cos}}{21d}$$

$$= \frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(49A+32C)\sin(c}{21d\sqrt{a+a\cos(c+d}}$$

Mathematica [A] time = 0.525522, size = 115, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((28A+101C)\cos(c+dx)+224A+24C\cos(2(c+dx))+3C}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(84*Sqrt[2]*A*ArcTanh[Sqrt
[2]*Sin[(c + d*x)/2]] + 2*(224*A + 208*C + (28*A + 101*C)*Cos[c + d*x] + 24
*C*Cos[2*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(84*d)
```

Maple [B] time = 0.084, size = 346, normalized size = 2.

$$\frac{1}{21\,d}a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-48\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^6+168\,C\sqrt{2}\sqrt{a\,(\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/21*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-48*C*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-28*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(A+8*C)*sin(1/2*d*x+1/2*c)^2+126*A*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+21*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+21*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+16
8*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*c
os(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.93268, size = 105, normalized size = 0.62

$$\frac{\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)C\sqrt{a}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="maxima")
```

```
[Out] 1/84*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2
*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c))*C*sqrt(a)/d
```

Fricas [A] time = 1.72862, size = 500, normalized size = 2.94

$$\frac{21\left(Aa^2\cos\left(dx+c\right)+Aa^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3\,Ca^2\cos\left(dx+c\right.\right.}{42\left(d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="fricas")
```

```
[Out] 1/42*(21*(A*a^2*cos(d*x + c) + A*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d
```

```
*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*C*a^2*cos(d*x + c)
^3 + 12*C*a^2*cos(d*x + c)^2 + (7*A + 23*C)*a^2*cos(d*x + c) + 2*(28*A + 23
*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.82816, size = 348, normalized size = 2.05

$$\frac{\frac{21\,Aa^{\frac{7}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(63\,\sqrt{2}Aa^{6}+84\,\sqrt{2}Ca^{6}+\left(175\,\sqrt{2}Aa^{6}+140\,\sqrt{2}Ca^{6}+\left(161\,\sqrt{2}Aa^{6}+112\,\sqrt{2}Ca^{6}+\left(49\,\sqrt{2}A\right.\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\right.\right.}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="giac")
```

```
[Out] 1/21*(21*A*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a
))/abs(a) + 2*(63*sqrt(2)*A*a^6 + 84*sqrt(2)*C*a^6 + (175*sqrt(2)*A*a^6 + 1
40*sqrt(2)*C*a^6 + (161*sqrt(2)*A*a^6 + 112*sqrt(2)*C*a^6 + (49*sqrt(2)*A*a
^6 + 32*sqrt(2)*C*a^6)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(
1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/
2))/d
```

3.96 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=173

$$\frac{a^3(15A+64C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(15A-16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{5a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a(5A-2\cdots}{}$$

```
[Out] (5*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(a^3*(15*A + 64*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (a^2*(15
*A - 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (a*(5*A - 2*C)*(
a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (A*(a + a*Cos[c + d*x])^(5/
2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.589302, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2976, 2981, 2773, 206}

$$\frac{a^3(15A+64C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(15A-16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{5a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a(5A-2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] (5*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(a^3*(15*A + 64*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (a^2*(15
*A - 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (a*(5*A - 2*C)*(
a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (A*(a + a*Cos[c + d*x])^(5/
2)*Tan[c + d*x])/d
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\tan(c+dx)}{d} + \frac{\int (a+a\cos(c+dx}{}$$

$$= -\frac{a(5A-2C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{A(a+a}{}$$

$$= -\frac{a^2(15A-16C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} - \frac{a(5A-}{}$$

$$= \frac{a^3(15A+64C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(15A-16C)\sqrt{a+a\cos(}}{15d}$$

$$= \frac{a^3(15A+64C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(15A-16C)\sqrt{a+a\cos(}}{15d}$$

$$= \frac{5a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3(15A+64C)\sin(c+}{15d\sqrt{a+a\cos(c+dx}}$$

Mathematica [A] time = 0.59266, size = 127, normalized size = 0.73

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((60A+181C)\cos(c+dx)+30A+28C\cos(2(c+d}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(150*Sqrt[2]*
A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(30*A + 28*C + (60*A +
 181*C)*Cos[c + d*x] + 28*C*Cos[2*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sin[(c
 + d*x)/2]))/(60*d)
```

Maple [B] time = 0.087, size = 533, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*C*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+368*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+(-120*A*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-150*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a))*a-150*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-64
0*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+90
*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a))*a+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+240*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*
d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 3.49041, size = 11036, normalized size = 63.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="maxima")
```

```
[Out] 1/1260*(42*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a) - 5*(1449*sqrt(
2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d
*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
+ 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 60*sqrt(2)*a^2*s
in(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^
2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*cos(5/2*d*x
+ 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*
c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 69*sqrt
(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 198*sqrt(2)*a^2*sin(1/2*d*x
 + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 198*sqrt(2)*a^2*sin(1/2*
```

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d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 12
*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c)^2
 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c))*sin(2*
d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*
sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sq
rt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(5/2*d
*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1/2*d*
x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(13
/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c)
 + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)
 + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(
5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*c
os(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x +
2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x +
2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*
sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c
))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x
 + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x
 + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a
^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x +
2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*
d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin
(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)
*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x
+ 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(2*
d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 18
9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) - 21*(60*sqrt(2)*a^2*sin(1/2*d*x
+ 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) - 315*(a^2*cos(
1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 +
 a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c
)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*
sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(
1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x +
 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2
*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*
```

```
x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*
c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/2*d*x + 1/2*c)^2 +
 a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d
*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2
*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(
5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)
^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a
^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2
*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos
(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2
*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d
*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x
 + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1
/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*
sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*
sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 +
 (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)
^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2
*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2
+ a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*
sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*
cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d
*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2
*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(
5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)
^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x +
1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*
x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)
^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)
```

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)*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x +
1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)
 + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin
(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 35*(sqrt(2)*a^2*co
s(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*co
s(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d
*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x
+ 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x
 + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x +
 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x +
 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(13/2*
d*x + 13/2*c) + 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*
x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5
/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)
*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c))*sin(5/2*d*x + 5/2*c))*sin(11/2*d*x + 11/2*c) + 7*(9*sqrt(2)*a^2*cos(1/2
*d*x + 1/2*c)^2 + 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 - (5*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*
x + 2*c) - 9*sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d
*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 - (5
*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 - 5*(sqrt(
2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d
*x + 2*c)^2 - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + 5*
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d
*x + 2*c)*cos(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/
2*d*x + 5/2*c) + 4*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*si
n(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)
- 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(9/2*d*x + 9/2*c) - 390*(sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*
a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*c
os(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(
2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5
/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2
*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin
```

(7/2*d*x + 7/2*c) - 21*(69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 69*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*cos(2*d*x + 2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*sin(2*d*x + 2*c)^2 + 12*sqrt(2)*a^2 + 138*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 - 50*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 24*sqrt(2)*a^2)*cos(2*d*x + 2*c) - 10*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c) + 105*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) - 252*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c) - 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 63*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2

```
)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(5/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1260*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2
+ sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)
*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x +
 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) +
 sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2
+ sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*c
os(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*s
in(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) +
 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2
*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(5/2*d*x +
 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2
*c)^2 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*
sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + cos(1/2*d
*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) +
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + cos(1/2*d*x + 1/2*c)^2 + 2*(cos(2*
d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)
 + 2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(5/2*
d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2))/d
```

Fricas [A] time = 1.76775, size = 529, normalized size = 3.06

$$\frac{75\left(Aa^2\cos\left(dx+c\right)^2+Aa^2\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(6\,C\right.}{60\left(d\cos\left(dx+c\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="fricas")
```

```
[Out] 1/60*(75*(A*a^2*cos(d*x + c)^2 + A*a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
```

```
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(6*C*a
^2*cos(d*x + c)^3 + 28*C*a^2*cos(d*x + c)^2 + 2*(15*A + 43*C)*a^2*cos(d*x +
 c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 +
d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.86502, size = 478, normalized size = 2.76

$$75\,Aa^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-75\,Aa^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="giac")
```

```
[Out] 1/30*(75*A*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 75*A*a^(5/2)*log(abs((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2
) - 3))) + 60*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*A*a^(7/2) - A*a^(9/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2) + 4*(15*sqrt(2)*A*a^5 + 60*s
qrt(2)*C*a^5 + (30*sqrt(2)*A*a^5 + 80*sqrt(2)*C*a^5 + (15*sqrt(2)*A*a^5 + 3
2*sqrt(2)*C*a^5)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*
x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.97 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=184

$$-\frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(21A-8C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{12d}+\frac{a^{5/2}(19A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+$$

```
[Out] (a^(5/2)*(19*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) - (a^3*(27*A - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]
]) - (a^2*(21*A - 8*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (5*a
*A*(a + a*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(4*d) + (A*(a + a*Cos[c + d*x])
^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.633111, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2975, 2976, 2981, 2773, 206}

$$-\frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(21A-8C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{12d}+\frac{a^{5/2}(19A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] (a^(5/2)*(19*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) - (a^3*(27*A - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]
]) - (a^2*(21*A - 8*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (5*a
*A*(a + a*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(4*d) + (A*(a + a*Cos[c + d*x])
^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{2d}+\frac{\int(a+}{}$$

$$= \frac{5aA(a+a\cos(c+dx))^{3/2}\tan(c+dx)}{4d}+\frac{A(a+a\cos(c+}{}$$

$$= -\frac{a^2(21A-8C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{12d}+\frac{5aA(a+}{}$$

$$= -\frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}-\frac{a^2(21A-8C)\sqrt{a+a\cos(}}{12d}$$

$$= -\frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}-\frac{a^2(21A-8C)\sqrt{a+a\cos(}}{12d}$$

$$= \frac{a^{5/2}(19A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}-\frac{a^3(27A-56C)}{12d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.762644, size = 137, normalized size = 0.74

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((33A+6C)\cos(c+dx)+6A+32C\cos(2(c+dx)\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(6*Sqrt[2]*
(19*A + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 4*(6*A + 32
*C + (33*A + 6*C)*Cos[c + d*x] + 32*C*Cos[2*(c + d*x)] + 2*C*Cos[3*(c + d*x
)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.093, size = 1052, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-128*C*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+(228*A*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+228*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+704*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+96
*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+96*C*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(33*A*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)+152*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+57*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+57*A*ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a))*a+24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d
*x+1/2*c)^2+78*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+57*A*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+57*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+144*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+24*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1
/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 22.4174, size = 4952, normalized size = 26.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="maxima")
```

```
[Out] -1/16*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sqrt(2)*
a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d*x + 7
/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - (3*
sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*
sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
```

```
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)
^2 - 3*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(
15/2*d*x + 15/2*c) - 5*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*
d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 11*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*s
qrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 45*(sqrt(2)*a^2*sin(4
*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) - (11*sq
rt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 38*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(4*sqrt(2)*a^2*co
s(2*d*x + 2*c) + 27*sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + (20*sqrt(2)*a^2*cos
(2*d*x + 2*c) + 87*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) - 2*
(11*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
+ 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/2*d*x + 15/2*c) + 5*(sqr
t(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*s
in(13/2*d*x + 13/2*c) - 11*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*co
s(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/2*d*x + 11/2*c) - 45*(sqrt(2)*a^2*cos(
4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/2*d*x +
9/2*c) - (12*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 20*sqrt(2)
*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 75*sqrt(2)*a^2*cos(7/2*d*x + 7
```

```
/2*c) - 77*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) - 45*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 4*(17*sqrt(2)*a^2*sin(3/2*d*
x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*
d*x + 2*c))*sin(4*d*x + 4*c) - 6*(2*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 2*sqrt
(2)*a^2*sin(2*d*x + 2*c)^2 + 27*sqrt(2)*a^2*cos(2*d*x + 2*c) + 13*sqrt(2)*a
^2)*sin(7/2*d*x + 7/2*c) - 2*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 10*sqrt(2
)*a^2*sin(2*d*x + 2*c)^2 + 87*sqrt(2)*a^2*cos(2*d*x + 2*c) + 41*sqrt(2)*a^2
)*sin(5/2*d*x + 5/2*c) + 2*(45*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 11*sqrt(2
)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt(a)/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.97101, size = 547, normalized size = 2.97

$$\frac{3\left((19A+8C)a^2\cos(dx+c)^3+(19A+8C)a^2\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{48\left(d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="fricas")
```

```
[Out] 1/48*(3*((19*A + 8*C)*a^2*cos(d*x + c)^3 + (19*A + 8*C)*a^2*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 4*(8*C*a^2*cos(d*x + c)^3 + 64*C*a^2*cos(d*x + c)^2 + 33*A*a
^2*cos(d*x + c) + 6*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*
x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.09359, size = 544, normalized size = 2.96

$$3\left(19\,Aa^{\frac{5}{2}} + 8\,Ca^{\frac{5}{2}}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \sqrt{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\right)^2 - a\left(2\,\sqrt{2} + 3\right)\right|\right) - 3\left(19\,Aa^{\frac{5}{2}} + 8\,Ca^{\frac{5}{2}}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="giac")
```

```
[Out] 1/24*(3*(19*A*a^(5/2) + 8*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(19*A*a^(
5/2) + 8*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 16*(7*sqrt(2)*C*a^4*tan(1/2*
d*x + 1/2*c)^2 + 9*sqrt(2)*C*a^4)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2
*c)^2 + a)^(3/2) + 12*sqrt(2)*(19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(7/2) - 171*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(9/2) + 89*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(11/2) - 9*A*a^(13/2)
)/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 -
6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
 a^2)^2)/d
```

3.98 $\int (a+a\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=192

$$-\frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(31A+24C)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{5a^{5/2}(5A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}$$

```
[Out] (5*a^(5/2)*(5*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) - (a^3*(49*A - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*(31*A + 24*C)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(24*d) + (5
*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (A*(a +
 a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.658965, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2975, 2981, 2773, 206}

$$-\frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(31A+24C)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{5a^{5/2}(5A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] (5*a^(5/2)*(5*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) - (a^3*(49*A - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*(31*A + 24*C)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(24*d) + (5
*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (A*(a +
 a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{\int (a+$$

$$= \frac{5aA(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{12d}+\frac{A(a$$

$$= \frac{a^2(31A+24C)\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{24d}+\frac{5aA(a$$

$$= -\frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(31A+24C)\sqrt{a+a\cos}}{24d}$$

$$= -\frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(31A+24C)\sqrt{a+a\cos}}{24d}$$

$$= \frac{5a^{5/2}(5A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}-\frac{a^3(49A-24C)}{24d\sqrt{a+a\cos}}$$

Mathematica [A] time = 1.12183, size = 142, normalized size = 0.74

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((68A+72C)\cos(c+dx)+3(25A+8C)\cos(2(c+}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(15*Sqrt[2]
*(5*A + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (91*A + 24*
C + (68*A + 72*C)*Cos[c + d*x] + 3*(25*A + 8*C)*Cos[2*(c + d*x)] + 24*C*Cos
[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.098, size = 1337, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*(32*C*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+25*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a))*a+25*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4
0*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+40*C*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+12*(50*A*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+112*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+75*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+120*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a+120*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1
/2*d*x+1/2*c)^4+(-450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-450*A
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-736*A*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)-720*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a-720*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-768*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+75*A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a+234*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+120*C*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+120*C*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+144*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.07896, size = 560, normalized size = 2.92

$$\frac{15\left((5\,A+8\,C)a^2\cos\left(dx+c\right)^4+(5\,A+8\,C)a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\,\mathrm{si}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="fricas")
```

```
[Out] 1/96*(15*((5*A + 8*C)*a^2*cos(d*x + c)^4 + (5*A + 8*C)*a^2*cos(d*x + c)^3)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(48*C*a^2*cos(d*x + c)^3 + 3*(25*A + 8*C)*a^2*cos(d*x + c)^
2 + 34*A*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))
/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.29396, size = 909, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="giac")
```

```
[Out] 1/48*(96*sqrt(2)*C*a^3*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a) + 15*(5*A*a^(5/2) + 8*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(5*A*a^(
5/2) + 8*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(75*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(7/2) + 72*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(7/
2) - 1125*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*A*a^(9/2) - 888*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*C*a^(9/2) + 6174*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(11/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(11/2) - 4314*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(13/2) - 1776*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(13/
2) + 807*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2*A*a^(15/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*C*a^(15/2) - 49*A*a^(17/2) - 24*C*a^(17/2))/((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.99 $\int (a+a\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{a^3(299A+432C)\tan(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+16C)\tan(c+dx)\sec(c+dx)\sqrt{a}}{32d}$$

```
[Out] (a^(5/2)*(163*A + 304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(64*d) + (a^3*(299*A + 432*C)*Tan[c + d*x])/(192*d*Sqrt[a + a*Cos[c
 + d*x]]) + (a^2*(17*A + 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c
+ d*x])/(32*d) + (5*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d
*x])/(24*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d)
```

Rubi [A] time = 0.71687, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2975, 2980, 2773, 206}

$$\frac{a^3(299A+432C)\tan(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+16C)\tan(c+dx)\sec(c+dx)\sqrt{a}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```



```
[Out] (a^(5/2)*(163*A + 304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(64*d) + (a^3*(299*A + 432*C)*Tan[c + d*x])/(192*d*Sqrt[a + a*Cos[c
 + d*x]]) + (a^2*(17*A + 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c
+ d*x])/(32*d) + (5*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d
*x])/(24*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{\int (a+$$

$$= \frac{5aA(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{24d} + \frac{A(}{}$$

$$= \frac{a^2(17A+16C)\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx}{32d}$$

$$= \frac{a^3(299A+432C)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(17A+16C)\sqrt{a+a\,c}}{}$$

$$= \frac{a^3(299A+432C)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(17A+16C)\sqrt{a+a\,c}}{}$$

$$= \frac{a^{5/2}(163A+304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(299A+}{192d\sqrt{a}}$$

Mathematica [A] time = 1.64602, size = 153, normalized size = 0.76

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((2203A+1584C)\cos(c+dx)+4(163A+48C)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*
(163*A + 304*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (844*A +
 192*C + (2203*A + 1584*C)*Cos[c + d*x] + 4*(163*A + 48*C)*Cos[2*(c + d*x)]
 + 489*A*Cos[3*(c + d*x)] + 528*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768
*d)
```

Maple [B] time = 0.103, size = 1630, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(163*A
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+163*A*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))+304*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+304
*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(
163*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+176*C*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+326*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a))*a+326*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+608*C*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+608*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(1793*A*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+1680*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+1467*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1467*A*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2736*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a))*a+2736*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2
*d*x+1/2*c)^4+(-9212*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-3912*
A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-3912*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a))*a-7104*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)-7296*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-7296*C*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+2094*A*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+489*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+489*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+124
8*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+912*C*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+912*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
```

```
*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.19496, size = 594, normalized size = 2.97

$$\frac{3\left(\left(163\,A+304\,C\right)a^{2}\cos\left(dx+c\right)^{5}+\left(163\,A+304\,C\right)a^{2}\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="fricas")
```

```
[Out] 1/768*(3*((163*A + 304*C)*a^2*cos(d*x + c)^5 + (163*A + 304*C)*a^2*cos(d*x
+ c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(3*(163*A + 176*C)*a^2*cos(d*x + c)^3 + 2*(163*A +
48*C)*a^2*cos(d*x + c)^2 + 184*A*a^2*cos(d*x + c) + 48*A*a^2)*sqrt(a*cos(d*
x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.4747, size = 1083, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="giac")
```

```
[Out] 1/384*(3*(163*A*a^(5/2) + 304*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*
A*a^(5/2) + 304*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(489*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(7/2)
+ 912*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^1
4*C*a^(7/2) - 10269*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^12*A*a^(9/2) - 19152*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(9/2) + 69885*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(11/2) + 137424*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(11/2) - 25
9233*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*
A*a^(13/2) - 374544*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^8*C*a^(13/2) + 209979*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(15/2) + 266928*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(15/2) - 55511*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(17/2) - 758
88*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*
a^(17/2) + 6687*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*A*a^(19/2) + 9456*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*C*a^(19/2) - 299*A*a^(21/2) - 432*C*a^(21/2))/((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4
)/d
```

3.100 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=245

$$\frac{a^3(283A+400C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(283A+400C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(79A+80C)\tan(c+dx)\sec^2(c+dx)\sqrt{}}{240d}$$

```
[Out] (a^(5/2)*(283*A + 400*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^3*(283*A + 400*C)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^3*(787*A + 1040*C)*Sec[c + d*x]*Tan[c + d*x])/(960*d*Sqrt[a
 + a*Cos[c + d*x]]) + (a^2*(79*A + 80*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^2*Tan[c + d*x])/(240*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^
3*Tan[c + d*x])/(8*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4*Tan[c
+ d*x])/(5*d)
```

Rubi [A] time = 0.798867, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2975, 2980, 2772, 2773, 206}

$$\frac{a^3(283A+400C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(283A+400C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(79A+80C)\tan(c+dx)\sec^2(c+dx)\sqrt{}}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] (a^(5/2)*(283*A + 400*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^3*(283*A + 400*C)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^3*(787*A + 1040*C)*Sec[c + d*x]*Tan[c + d*x])/(960*d*Sqrt[a
 + a*Cos[c + d*x]]) + (a^2*(79*A + 80*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^2*Tan[c + d*x])/(240*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^
3*Tan[c + d*x])/(8*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4*Tan[c
+ d*x])/(5*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{\int(a+}{}$$

$$= \frac{aA(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{A(a}{}$$

$$= \frac{a^2(79A+80C)\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+d}{240d}$$

$$= \frac{a^3(787A+1040C)\sec(c+dx)\tan(c+dx)}{960d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(79A+8}{}$$

$$= \frac{a^3(283A+400C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(787A+1040C)\sec(}{960d\sqrt{a+a\,co}}$$

$$= \frac{a^3(283A+400C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(787A+1040C)\sec(}{960d\sqrt{a+a\,co}}$$

$$= \frac{a^{5/2}(283A+400C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^3(283A+}{128d\sqrt{a}}$$

Mathematica [A] time = 2.04562, size = 176, normalized size = 0.72

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(2343A+1360C)\cos(c+dx)+4(6509A+6640}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]
*(283*A + 400*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (24863*
```

```
A + 20560*C + 12*(2343*A + 1360*C)*Cos[c + d*x] + 4*(6509*A + 6640*C)*Cos[2
*(c + d*x)] + 5660*A*Cos[3*(c + d*x)] + 5440*C*Cos[3*(c + d*x)] + 4245*A*Co
s[4*(c + d*x)] + 6000*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

Maple [B] time = 0.11, size = 1951, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/120*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(28
3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+283*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))+400*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)+400*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^10+
240*(566*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+800*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1415*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+1415*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2000*C
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2000*C*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^8-80*(3962*A*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+5344*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4245*A*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+6000*C*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a+6000*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
a)*sin(1/2*d*x+1/2*c)^6+8*(36224*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+44800*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+21225*A*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+21225*A*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
```

```
2*c)^2)^(1/2)+2*a))*a+30000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a+30000*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c
)^4-10*(12556*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+13376*C*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*a+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a
+6000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6000*C*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+4245*A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+22230*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6000*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+18720*C*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+6000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^5/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^5/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.24222, size = 655, normalized size = 2.67

$$15\left(\left(283\,A+400\,C\right)a^{2}\cos\left(dx+c\right)^{6}+\left(283\,A+400\,C\right)a^{2}\cos\left(dx+c\right)^{5}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\mathrm{co}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorit
hm="fricas")
```

```
[Out] 1/7680*(15*((283*A + 400*C)*a^2*cos(d*x + c)^6 + (283*A + 400*C)*a^2*cos(d*
x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(15*(283*A + 400*C)*a^2*cos(d*x + c)^4 + 10*(283*
A + 272*C)*a^2*cos(d*x + c)^3 + 8*(283*A + 80*C)*a^2*cos(d*x + c)^2 + 1392*
A*a^2*cos(d*x + c) + 384*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*c
os(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.58755, size = 1304, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorit
hm="giac")
```

```
[Out] 1/3840*(15*(283*A*a^(5/2) + 400*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(2
83*A*a^(5/2) + 400*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(4245*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*a^(7
/2) + 6000*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^18*C*a^(7/2) - 114615*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
```

```
x + 1/2*c)^2 + a))^16*A*a^(9/2) - 162000*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*a^(9/2) + 1298820*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(11/2) + 1801920*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(
11/2) - 6176700*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^12*A*a^(13/2) - 9764160*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(13/2) + 16394598*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(15/2) + 24060960*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(15/2)
 - 14042770*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*A*a^(17/2) - 19910240*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^8*C*a^(17/2) + 4791060*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(19/2) + 7135680*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(19/2) - 860300
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^
(21/2) - 1268800*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*C*a^(21/2) + 75885*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*A*a^(23/2) + 111600*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(23/2) - 2671*A*a^(25/2) - 392
0*C*a^(25/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*a + a^2)^5)/d
```

3.101 $\int (a+a\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=290

$$\frac{a^3(1015A+1304C)\tan(c+dx)}{512d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(1015A+1304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{512d}+\frac{a^2(23A+24C)\tan(c+dx)\sec^3(c+}{96d}$$

```
[Out] (a^(5/2)*(1015*A + 1304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(512*d) + (a^3*(1015*A + 1304*C)*Tan[c + d*x])/(512*d*Sqrt[a + a*
Cos[c + d*x]]) + (a^3*(1015*A + 1304*C)*Sec[c + d*x]*Tan[c + d*x])/(768*d*S
qrt[a + a*Cos[c + d*x]]) + (a^3*(109*A + 136*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(23*A + 24*C)*Sqrt[a + a*Cos[c +
d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(96*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)
*Sec[c + d*x]^4*Tan[c + d*x])/(12*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c
+ d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.906508, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2975, 2980, 2772, 2773, 206}

$$\frac{a^3(1015A+1304C)\tan(c+dx)}{512d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(1015A+1304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{512d}+\frac{a^2(23A+24C)\tan(c+dx)\sec^3(c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```



```
[Out] (a^(5/2)*(1015*A + 1304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(512*d) + (a^3*(1015*A + 1304*C)*Tan[c + d*x])/(512*d*Sqrt[a + a*
Cos[c + d*x]]) + (a^3*(1015*A + 1304*C)*Sec[c + d*x]*Tan[c + d*x])/(768*d*S
qrt[a + a*Cos[c + d*x]]) + (a^3*(109*A + 136*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(23*A + 24*C)*Sqrt[a + a*Cos[c +
d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(96*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)
*Sec[c + d*x]^4*Tan[c + d*x])/(12*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c
+ d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
```

```
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{\int (a}{}$$

$$=\frac{aA(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(c+dx)}{12d}+\frac{A(}{}$$

$$=\frac{a^2(23A+24C)\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+}{96d}$$

$$=\frac{a^3(109A+136C)\sec^2(c+dx)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(23A+}{}$$

$$=\frac{a^3(1015A+1304C)\sec(c+dx)\tan(c+dx)}{768d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(109A}{}$$

$$=\frac{a^3(1015A+1304C)\tan(c+dx)}{512d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(1015A+1304C)}{768d\sqrt{a+}}$$

$$=\frac{a^3(1015A+1304C)\tan(c+dx)}{512d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(1015A+1304C)}{768d\sqrt{a+}}$$

$$=\frac{a^{5/2}(1015A+1304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{512d}+\frac{a^3(1015}{512}$$

Mathematica [A] time = 2.7184, size = 198, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(14(4591A+4056C)\cos(c+dx)+16(1711A+149\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,
x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^6*(24*Sqrt[2]
*(1015*A + 1304*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^6 + (2741
2*A + 18720*C + 14*(4591*A + 4056*C)*Cos[c + d*x] + 16*(1711*A + 1496*C)*Co
s[2*(c + d*x)] + 21721*A*Cos[3*(c + d*x)] + 25448*C*Cos[3*(c + d*x)] + 4060
*A*Cos[4*(c + d*x)] + 5216*C*Cos[4*(c + d*x)] + 3045*A*Cos[5*(c + d*x)] + 3
912*C*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]))/(24576*d)
```

Maple [B] time = 0.13, size = 2271, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/48*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*a*(1015
*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+1015*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))+1304*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))+1304*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^1
2-192*(1015*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1304*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3045*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+3045*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+39
12*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3912*C*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^10+16*(34510*A*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+44336*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+45675*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4
5675*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+58680*C*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+58680*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
```

```
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^8-96*(6699*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+8504*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+507
5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+5075*A*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a))*a+6520*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+6520*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x
+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x
+1/2*c)^6+12*(32596*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+39712*
C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+15225*A*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+15225*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a))*a+19560*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+19560*C*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(31897*
A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+35176*C*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9135*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+9135*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+11736*C*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+11736*C*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+18486*A*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)+3045*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
a+3045*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+16752*C*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3912*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a+3912*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))^6/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^6/sin(1/2*d*x+1
/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.29953, size = 711, normalized size = 2.45

$$\frac{3\left((1015\,A+1304\,C)a^{2}\cos\left(dx+c\right)^{7}+(1015\,A+1304\,C)a^{2}\cos\left(dx+c\right)^{6}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{}}{\cos(dx+c)^{3}+\cos(dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorit
hm="fricas")
```

```
[Out] 1/6144*(3*((1015*A + 1304*C)*a^2*cos(d*x + c)^7 + (1015*A + 1304*C)*a^2*cos
(d*x + c)^6)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*
cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(3*(1015*A + 1304*C)*a^2*cos(d*x + c)^5 + 2*(1
015*A + 1304*C)*a^2*cos(d*x + c)^4 + 8*(203*A + 184*C)*a^2*cos(d*x + c)^3 +
 48*(29*A + 8*C)*a^2*cos(d*x + c)^2 + 896*A*a^2*cos(d*x + c) + 256*A*a^2)*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.81402, size = 1526, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorit
hm="giac")
```

```
[Out] 1/3072*(3*(1015*A*a^(5/2) + 1304*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(1
015*A*a^(5/2) + 1304*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3045*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^22*A*a^
(7/2) + 3912*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^22*C*a^(7/2) - 100485*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^20*A*a^(9/2) - 129096*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^20*C*a^(9/2) + 1303699*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*a^(11/2) + 1693560
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*a
^(11/2) - 9936699*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^16*A*a^(13/2) - 11951544*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^16*C*a^(13/2) + 38257266*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(15/2) + 48800976*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(15
/2) - 83779026*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^12*A*a^(17/2) - 106200016*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(17/2) + 74917446*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(19/2) + 94661616*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(19/2
) - 30850806*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^8*A*a^(21/2) - 39751536*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^8*C*a^(21/2) + 7187801*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(23/2) + 9070440*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(23/2) - 92981
7*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a
^(25/2) - 1176936*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*C*a^(25/2) + 64887*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*A*a^(27/2) + 82200*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(27/2) - 1887*A*a^(29/2) - 239
2*C*a^(29/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*a + a^2)^6)/d
```

3.102 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=236

$$\frac{2(21A+19C)\sin(c+dx)\cos^2(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(21A+29C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315ad}+\frac{4(147A+143C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}-$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (4*(147*A + 143*C)*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) + (2*(21*A + 19*C)*Cos[c + d*x]^2*Sin[c + d*x])/(105*d*Sq
rt[a + a*Cos[c + d*x]]) - (2*C*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c
+ d*x]]) - (2*(21*A + 29*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d
)
```

Rubi [A] time = 0.816877, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3046, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(21A+19C)\sin(c+dx)\cos^2(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(21A+29C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315ad}+\frac{4(147A+143C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (4*(147*A + 143*C)*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) + (2*(21*A + 19*C)*Cos[c + d*x]^2*Sin[c + d*x])/(105*d*Sq
rt[a + a*Cos[c + d*x]]) - (2*C*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c
+ d*x]]) - (2*(21*A + 29*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d
)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
```

```
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^3(c+dx)\left(\frac{1}{2}a(9A+8C)-\frac{1}{2}aC\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{9a}$$

$$= -\frac{2C\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{4\int \frac{\cos^2(c+dx)\left(-\right.}{}}{}$$

$$= \frac{2(21A+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\,\mathrm{c}}{9a}$$

$$= \frac{2(21A+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\,\mathrm{c}}{9a}$$

$$= \frac{2(21A+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\,\mathrm{c}}{9a}$$

$$= \frac{4(147A+143C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(21A+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\,\mathrm{c}}{63}$$

$$= \frac{4(147A+143C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(21A+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\,\mathrm{c}}{63}$$

$$= -\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(147A+143C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(21}{}$$

Mathematica [A] time = 0.613118, size = 121, normalized size = 0.51

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)(-2(84A+131C)\cos(c+dx)+4(63A+92C)\cos(2(c+dx))+2436A-10C\cos(3(c\right.\right.}{1260d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] (Cos[(c + d*x)/2]*(-2520*(A + C)*ArcTanh[Sin[(c + d*x)/2]] + 2*(2436*A + 23
89*C - 2*(84*A + 131*C)*Cos[c + d*x] + 4*(63*A + 92*C)*Cos[2*(c + d*x)] - 1
0*C*Cos[3*(c + d*x)] + 35*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(1260*d*Sq
rt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.079, size = 340, normalized size = 1.4

$$\frac{1}{315\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(1120\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^{2}}\sqrt{a}\,(\sin(1/2\,dx+c/2))^{8}-2160\,C\sqrt{2}\sqrt{a\,(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/315*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1120*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-2160*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+504*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+4*C)*sin(1/2*d*x+1/2*c)^4-420*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*C)*sin(1/2*d*x+1/2*c)^2-315*2^(1
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A
-315*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+a))*a*C+630*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+630*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos
(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.72025, size = 532, normalized size = 2.25

$$\frac{4\left(35\,C\cos\left(dx+c\right)^{4}-5\,C\cos\left(dx+c\right)^{3}+3\left(21\,A+19\,C\right)\cos\left(dx+c\right)^{2}-\left(21\,A+29\,C\right)\cos\left(dx+c\right)+273\,A+257\,C\right)\sqrt{a}}{630\left(ad\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/630*(4*(35*C*cos(d*x + c)^4 - 5*C*cos(d*x + c)^3 + 3*(21*A + 19*C)*cos(d*
x + c)^2 - (21*A + 29*C)*cos(d*x + c) + 273*A + 257*C)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c) + 315*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(
cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*co
s(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.90032, size = 306, normalized size = 1.3

$$\frac{\frac{315\left(\sqrt{2}A+\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\left(315\,\sqrt{2}Aa^{4}+315\,\sqrt{2}Ca^{4}+\left(1050\,\sqrt{2}Aa^{4}+840\,\sqrt{2}Ca^{4}+\left(1512\,\sqrt{2}Aa^{4}+1638\,\sqrt{2}Ca^{4}+\left(\right.\right.\right.\right.}{}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/315*(315*(sqrt(2)*A + sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) +
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(315*sqrt(2)*A*a^4 + 315*s
qrt(2)*C*a^4 + (1050*sqrt(2)*A*a^4 + 840*sqrt(2)*C*a^4 + (1512*sqrt(2)*A*a^
4 + 1638*sqrt(2)*C*a^4 + (1134*sqrt(2)*A*a^4 + 936*sqrt(2)*C*a^4 + (357*sqr
t(2)*A*a^4 + 383*sqrt(2)*C*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c
)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(
a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2))/d
```

3.103 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=193

$$\frac{2(35A+31C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{4(35A+37C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\,\mathrm{s}}{7}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*(35*A + 37*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) - (2*C*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*C*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*(35*A + 31*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.559901, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3046, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(35A+31C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{4(35A+37C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\,\mathrm{s}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*(35*A + 37*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) - (2*C*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*C*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*(35*A + 31*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(\frac{1}{2}a(7A+6C)-\frac{1}{2}aC\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{7a}$$

$$= -\frac{2C\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{4\int \frac{\cos(c+dx)\left(-a\right.}{}}{}$$

$$= -\frac{2C\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{4\int \frac{-a^2C\cos(c+}{}}{}$$

$$= -\frac{2C\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2(35A+31C)}{}$$

$$= -\frac{4(35A+37C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)}{7d\sqrt{a+a\cos}}$$

$$= -\frac{4(35A+37C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)}{7d\sqrt{a+a\cos}}$$

$$= \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(35A+37C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2C\cos^2(}{35d\sqrt{a}}$$

Mathematica [A] time = 0.320768, size = 89, normalized size = 0.46

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(105(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin^3\left(\frac{1}{2}(c+dx)\right)(70A+24C\cos(c+dx)+15C\cos(2(c+dx\right.}{105d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(105*(A + C)*ArcTanh[Sin[(c + d*x)/2]] - 2*(70*A + 101*
C + 24*C*Cos[c + d*x] + 15*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^3))/(105*d*
```

```
Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.061, size = 253, normalized size = 1.3

$$\frac{1}{105\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-240\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^{2}}\sqrt{a}\,(\sin(1/2\,dx+c/2))^{6}+336\,C\sqrt{2}\sqrt{a\,(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/105*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+336*C*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-140*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*C)*sin(1/2*d*x+1/2*c)^2+105*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+105*2
^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*
a*C)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.70162, size = 485, normalized size = 2.51

$$\frac{4\left(15\,C\cos(dx+c)^{3}-3\,C\cos(dx+c)^{2}+(35\,A+31\,C)\cos(dx+c)-35\,A-43\,C\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)+\cdots}{210\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/210*(4*(15*C*cos(d*x + c)^3 - 3*C*cos(d*x + c)^2 + (35*A + 31*C)*cos(d*x
+ c) - 35*A - 43*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 105*sqrt(2)*((A
 + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76573, size = 213, normalized size = 1.1

$$-\frac{\frac{105\sqrt{2}(A+C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{4\left(\left(\sqrt{2}\left(35Aa^3+46Ca^3\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+14\sqrt{2}\left(5Aa^3+4Ca^3\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+35\sqrt{2}\cdots\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{7}{2}}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/105*(105*sqrt(2)*(A + C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 4*((sqrt(2)*(35*A*a^3 + 46*C*a^3)*t
an(1/2*d*x + 1/2*c)^2 + 14*sqrt(2)*(5*A*a^3 + 4*C*a^3))*tan(1/2*d*x + 1/2*c
)^2 + 35*sqrt(2)*(A*a^3 + 2*C*a^3))*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x +
 1/2*c)^2 + a)^(7/2))/d
```

3.104 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{2(15A+14C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(15*A + 14*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*
Cos[c + d*x]]) + (2*C*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c +
d*x]]) - (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.331004, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3046, 2968, 3023, 2751, 2649, 206}

$$\frac{2(15A+14C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(15*A + 14*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*
Cos[c + d*x]]) + (2*C*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c +
d*x]]) - (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos(c+dx)\left(\frac{1}{2}a(5A+4C)-\frac{1}{2}aC\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}a(5A+4C)\cos(c+dx)-\frac{1}{2}aC\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad} + \frac{4\int \frac{-\frac{a^2C}{4}+ \cdots}{\cdots}}{\cdots}$$

$$= \frac{2(15A+14C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2C\sqrt{a+a\cos(c \cdots}}{15 \cdots}$$

$$= \frac{2(15A+14C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2C\sqrt{a+a\cos(c \cdots}}{15 \cdots}$$

$$= -\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(15A+14C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos \cdots}{5d \cdots}$$

Mathematica [A] time = 0.232137, size = 87, normalized size = 0.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)(30A-2C\cos(c+dx)+3C\cos(2(c+dx))+29C)-30(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+ \cdots\right.\right.\right.}{15d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-30*(A + C)*ArcTanh[Sin[(c + d*x)/2]] + 2*(30*A + 29*C -
 2*C*Cos[c + d*x] + 3*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d*Sqrt[a*(
1 + Cos[c + d*x])])
```

Maple [A] time = 0.061, size = 247, normalized size = 1.6

$$\frac{\sqrt{2}}{15\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,C\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^4-20\,C\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+ \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*C*a^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+30*A*a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-15*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+a))*a+30*C*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-15*C*ln(4/cos(1/2*
d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.6909, size = 436, normalized size = 2.87

$$\frac{4\left(3\,C\cos\left(dx+c\right)^{2}-C\cos\left(dx+c\right)+15\,A+13\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)+\frac{15\,\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\log\left(-\frac{\cos(\cdots}{\cdots}\right)}{\cdots}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/30*(4*(3*C*cos(d*x + c)^2 - C*cos(d*x + c) + 15*A + 13*C)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c) + 15*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*lo
g(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*
```

```
d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74363, size = 223, normalized size = 1.47

$$\frac{\frac{15\left(\sqrt{2}A+\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(15\sqrt{2}Aa^2+15\sqrt{2}Ca^2+\left(30\sqrt{2}Aa^2+20\sqrt{2}Ca^2+\left(15\sqrt{2}Aa^2+17\sqrt{2}Ca^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/15*(15*(sqrt(2)*A + sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(15*sqrt(2)*A*a^2 + 15*sqrt(
2)*C*a^2 + (30*sqrt(2)*A*a^2 + 20*sqrt(2)*C*a^2 + (15*sqrt(2)*A*a^2 + 17*sq
rt(2)*C*a^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.105 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}-\frac{4C\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*C*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.140764, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3024, 2751, 2649, 206}

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}-\frac{4C\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*C*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{1}{2}a(3A+C)-aC\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= -\frac{4C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + (A+C)\int \frac{1}{\sqrt{a+a\cos(c+\ldots}}$$

$$= -\frac{4C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(2(A+C))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\ldots\right.}{d}$$

$$= \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2C\sqrt{a+a\cos(c+dx)}\sin\ldots}{3ad}$$

Mathematica [A] time = 0.104136, size = 63, normalized size = 0.58

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(3(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4C\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(3*(A + C)*ArcTanh[Sin[(c + d*x)/2]] - 4*C*Sin[(c + d*x
)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.054, size = 173, normalized size = 1.6

$$\frac{\sqrt{2}}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-4C\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}+3A\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,d\right.\right.}}{\cos\left(1/2\,dx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+3*A*ln(4/cos(1/2*d*x+1
/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+3*C*ln(4/cos(1/2*d*x+1/
2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2
*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.62127, size = 392, normalized size = 3.6

$$\frac{4\left(C\cos\left(dx+c\right)-C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)+\frac{3\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(\ldots}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*(C*cos(d*x + c) - C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 3*sqrt(
2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.79155, size = 116, normalized size = 1.06

$$-\frac{\frac{4\sqrt{2}Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{3\sqrt{2}(A+C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(4*sqrt(2)*C*a*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(
3/2) + 3*sqrt(2)*(A + C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a))/d
```

3.106 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d) + (2*C*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.291672, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3046, 2985, 2649, 206, 2773}

$$-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```



```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d) + (2*C*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{aA}{2}-\frac{1}{2}aC\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{A\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{a} + (-A-C)\int$$

$$= \frac{2C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2A)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{(2(A+C}{}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2C\,\mathrm{s}}{d\sqrt{a+}}$$

Mathematica [A] time = 0.253816, size = 83, normalized size = 0.72

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-((A + C)*ArcTanh[Sin[(c + d*x)/2]]) + Sqrt[2]*A*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*C*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Cos[c
+ d*x])])
```

Maple [B] time = 0.099, size = 295, normalized size = 2.6

$$-\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}+a}{\cos\left(1/2\,dx+c/2\right)}\right)aA+\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right.}}{\cos\left(1/2\,dx+c/2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+2^(1/2)*ln(4/cos(1
/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-A*ln(-4/(-2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))*a-2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.81318, size = 613, normalized size = 5.33

$$\frac{(A\cos(dx+c)+A)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos(dx+c)+a}C\sin}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*C*sin(d*
x + c) + sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2
+ 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c)
- 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x
)
```

Giac [A] time = 2.8496, size = 258, normalized size = 2.24

$$\frac{\frac{\sqrt{2}(A+C)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}+\frac{4\sqrt{2}C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{2A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/2*(sqrt(2)*(A + C)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2)/sqrt(a) + 4*sqrt(2)*C*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a) + 2*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/sqrt(a) - 2*A*log(ab
s((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a
*(2*sqrt(2) - 3)))/sqrt(a))/d
```

3.107 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d))
 + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.315482, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3044, 2985, 2649, 206, 2773}

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d))
 + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{aA}{2}+\frac{1}{2}a(A+2C)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{A\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{2a} + (A+C)\int -$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{(2(A+C))\,\mathrm{S}}{}$$

$$= -\frac{A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{A\,\mathrm{ta}}{d\sqrt{a+}}$$

Mathematica [A] time = 0.331381, size = 89, normalized size = 0.79

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+d$$
$$d\sqrt{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A + C)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*A*ArcTanh[
Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sec[c + d*x]*Sin[(c + d*x)/2]))/(d*Sqrt[a*(
1 + Cos[c + d*x])])
```

Maple [B] time = 0.106, size = 554, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-2*A*2^(1/2)*ln(4/c
os(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-2*C*2^(1/2)*l
n(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+A*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+2*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(
a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C+2*A*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)-A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-A*ln
(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x
+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.81176, size = 676, normalized size = 5.98

$$\frac{\left(A\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos\left(dx\right.}}{4\left(ad\cos\left(dx+c\right)^2+ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/4*((A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*si
n(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c
) + a)*A*sin(d*x + c) + 2*sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos
(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.96715, size = 392, normalized size = 3.47

$$-\frac{\frac{\sqrt{2}(A+C)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{\sqrt{a}}+\frac{A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{a}}-\frac{A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\right.\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/2*(sqrt(2)*(A + C)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2)/sqrt(a) + A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/sqrt(a) - A*log(
abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
 a*(2*sqrt(2) - 3)))/sqrt(a) - 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.108 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{(7A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{A\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((7*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d) - (A*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c
+ d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.489791, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2984, 2985, 2649, 206, 2773}

$$\frac{(7A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{A\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((7*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d) - (A*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c
+ d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{aA}{2}+\frac{1}{2}a(3A+4C)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a}$$

$$= -\frac{A\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}a^2(7A+8C)-\frac{1}{4}a^2A\cos(\right.}{\sqrt{a+a\cos(c+}}}{2a^2}$$

$$= -\frac{A\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + (-A-C)\int \frac{1}{\sqrt{a+a\,\mathrm{co}}}$$

$$= -\frac{A\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(2(A+C))\,\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{(7A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - $$

Mathematica [A] time = 0.725951, size = 113, normalized size = 0.71

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-8(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(7A+8C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\left(5\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.}{4d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] (Cos[(c + d*x)/2]*(-8*(A + C)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(7*A + 8*
C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + A*Sec[c + d*x]^2*(5*Sin[(c + d*x)/2]
 - Sin[(3*(c + d*x))/2])))/(4*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.117, size = 1192, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(8*A*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+8*C*2^(1/
2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-7*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))-8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-8*C*ln(-
4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(A*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*
(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+8*2^(1/2)*ln(4/cos(1/2*d*x+
1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-7*A*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a))*a-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a-8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-8*C*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4/cos(
1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+8*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-7*A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a-2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-8*
C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-8*C*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1
/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.14422, size = 741, normalized size = 4.66

$$\frac{\left((7\,A+8\,C)\cos\left(dx+c\right)^3+(7\,A+8\,C)\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/16*(((7*A + 8*C)*cos(d*x + c)^3 + (7*A + 8*C)*cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 - 4*(A*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 8*sqrt(
2)*((A + C)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*log(-(cos(d*x + c)
^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x +
c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3
 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.09186, size = 536, normalized size = 3.37

$$\frac{\frac{4\sqrt{2}(A+C)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}+\frac{(7A\sqrt{a}+8C\sqrt{a})\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a(2\sqrt{2}+3)\right|\right)}{a}-\frac{(7A}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*(A + C)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2)/sqrt(a) + (7*A*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt
(2) + 3)))/a - (7*A*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a - 4*s
qrt(2)*(17*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*A*sqrt(a) - 57*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^4*A*a^(3/2) + 19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2) - 3*A*a^(7/2))/((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.109 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{(7A+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{(9A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)\sec}{3d\sqrt{a\cos(c+d}}$$

```
[Out] -((9*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((7*A + 8*C)*Tan[c + d*x])/(8*d*Sqrt[a
 + a*Cos[c + d*x]]) - (A*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c
+ d*x]]) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.649236, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2984, 2985, 2649, 206, 2773}

$$\frac{(7A+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{(9A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)\sec}{3d\sqrt{a\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((9*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((7*A + 8*C)*Tan[c + d*x])/(8*d*Sqrt[a
 + a*Cos[c + d*x]]) - (A*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c
+ d*x]]) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{aA}{2}+\frac{1}{2}a(5A+6C)\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= -\frac{A\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{3}{4}a^2(7A+8C)-\frac{3}{4}a^2 \cdots\right.}{\sqrt{a+\cdots}}}{\cdots}$$

$$= \frac{(7A+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^2(c+dx)\tan(c+\cdots}{3d\sqrt{a+a\cos(c+d\cdots}}$$

$$= \frac{(7A+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^2(c+dx)\tan(c+\cdots}{3d\sqrt{a+a\cos(c+d\cdots}}$$

$$= \frac{(7A+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^2(c+dx)\tan(c+\cdots}{3d\sqrt{a+a\cos(c+d\cdots}}$$

$$= -\frac{(9A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \cdots$$

Mathematica [A] time = 1.32645, size = 131, normalized size = 0.66

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(48(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\sqrt{2}(9A+8C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec\cdots\right.}{24d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] (Cos[(c + d*x)/2]*(48*(A + C)*ArcTanh[Sin[(c + d*x)/2]] - 3*Sqrt[2]*(9*A +
8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (37*A + 24*C - 4*A*Cos[c + d*x] +
3*(7*A + 8*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]))/(24*d*Sqr
t[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.125, size = 1645, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(16*A*2^(1/2)*
ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+16*C*2^
(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-9
*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-9*A*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))-8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-8*C*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(14*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+48*2^(1/2)*ln(4/cos(1/2*d*x
+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+16*C*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-27*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a))*a-27*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-24*C*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-24*C*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-2*(80*A*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+144*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+144*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a))*a*C-81*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-81*
A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-72*C*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a))*a-72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a)*sin(1/2*d*x+1/2*c)^2+48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C+54*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)-27*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-27*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+48*C*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
```

```
*a-24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/a^(3/2)/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a
*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.19046, size = 790, normalized size = 3.95

$$\frac{3\left((9A+8C)\cos(dx+c)^4+(9A+8C)\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/96*(3*((9*A + 8*C)*cos(d*x + c)^4 + (9*A + 8*C)*cos(d*x + c)^3)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*(3*(7*A + 8*C)*cos(d*x + c)^2 - 2*A*cos(d*x + c) + 8*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c) + 48*sqrt(2)*((A + C)*a*cos(d*x + c)^4 + (A + C)*
a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.17918, size = 933, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/48*(24*sqrt(2)*(A + C)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) + 3*(9*A*sqrt(a) + 8*C*sqrt(a))*log(abs((
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2
*sqrt(2) + 3)))/a - 3*(9*A*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/
a - 4*sqrt(2)*(165*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^10*A*sqrt(a) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^10*C*sqrt(a) - 1323*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(3/2) - 888*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(3/2) + 3906*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(5/2) + 30
24*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*
a^(5/2) - 2118*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^4*A*a^(7/2) - 1776*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*C*a^(7/2) + 393*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(9/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(9/2) - 31*A*a^(11/2) - 24*
C*a^(11/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*a + a^2)^3)/d
```

3.110 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=243

$$-\frac{(21A+16C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{(107A+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(43A+48}{96}$$

```
[Out] ((107*A + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/
(64*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((21*A + 16*C)*Tan[c + d*x])/(64*d
*Sqrt[a + a*Cos[c + d*x]]) + ((43*A + 48*C)*Sec[c + d*x]*Tan[c + d*x])/(96*
d*Sqrt[a + a*Cos[c + d*x]]) - (A*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a
+ a*Cos[c + d*x]]) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c
+ d*x]])
```

Rubi [A] time = 0.843198, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3044, 2984, 2985, 2649, 206, 2773}

$$-\frac{(21A+16C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{(107A+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(43A+48}{96}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((107*A + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/
(64*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((21*A + 16*C)*Tan[c + d*x])/(64*d
*Sqrt[a + a*Cos[c + d*x]]) + ((43*A + 48*C)*Sec[c + d*x]*Tan[c + d*x])/(96*
d*Sqrt[a + a*Cos[c + d*x]]) - (A*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a
+ a*Cos[c + d*x]]) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c
+ d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec^3(c+dx)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{aA}{2}+\frac{1}{2}a(7A+8C)\cos(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{A\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^3(c+dx)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}a^2(43A+48C)-\right.}{\sqrt{}}}{}$$

$$= \frac{(43A+48C)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^3(}{4d\sqrt{}}$$

$$= -\frac{(21A+16C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{(43A+48C)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec^2(}{24d\sqrt{}}$$

$$= -\frac{(21A+16C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{(43A+48C)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec^2(}{24d\sqrt{}}$$

$$= -\frac{(21A+16C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{(43A+48C)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} - \frac{A\sec^2(}{24d\sqrt{}}$$

$$= \frac{(107A+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 1.96958, size = 174, normalized size = 0.72

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)((221A+144C)\cos(c+dx)-4(43A+48C)\cos(2(c+dx))+63A\cos(3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(768*(A + C)*ArcTanh[Sin[(c + d*x)/2]]*Co
s[c + d*x]^4 - 6*Sqrt[2]*(107*A + 112*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*
Cos[c + d*x]^4 + (-364*A - 192*C + (221*A + 144*C)*Cos[c + d*x] - 4*(43*A +
```

```
 48*C)*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)] + 48*C*Cos[3*(c + d*x)])*Si
n[(c + d*x)/2]))/(384*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.129, size = 2049, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/24*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-48*a*(128*A*2^(1/2
)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+128*C
*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)
)-107*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-107*A*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))-112*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a))-112*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)
^8+48*(21*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+256*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+16*C*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+256*2^(1/2)*ln(4/cos(1/2*d*x+1
/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-214*A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-214*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a-224*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-224*C*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6-8*(103*A*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1152*2^(1/2)*ln(4/cos(1/2*d*x+1
/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+48*C*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1152*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-963*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a))*a-963*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-100
8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-1008*C*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
```

```
2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+4*(25*A*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+768*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+768*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+a))*a*C-642*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a-642*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-672*C*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-672*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2-384*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-384*2^(1/2)*ln(4/cos(1/2*d*x+1
/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C+321*A*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+126*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+321*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x
+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+336*C*ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+336*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/a^(
3/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(
1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.51214, size = 852, normalized size = 3.51

$$\frac{3\left((107\,A+112\,C)\cos\left(dx+c\right)^{5}+(107\,A+112\,C)\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/768*(3*((107*A + 112*C)*cos(d*x + c)^5 + (107*A + 112*C)*cos(d*x + c)^4)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) - 4*(3*(21*A + 16*C)*cos(d*x + c)^3 - 2*(43*A + 48*C)*cos(d*x +
 c)^2 + 8*A*cos(d*x + c) - 48*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 38
4*sqrt(2)*((A + C)*a*cos(d*x + c)^5 + (A + C)*a*cos(d*x + c)^4)*log(-(cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos
(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x
 + c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.19392, size = 1154, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/384*(192*sqrt(2)*(A + C)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) + 3*(107*A*sqrt(a) + 112*C*sqrt(a))*log(
abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
 a*(2*sqrt(2) + 3)))/a - 3*(107*A*sqrt(a) + 112*C*sqrt(a))*log(abs((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2
) - 3)))/a - 4*sqrt(2)*(1599*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^14*A*sqrt(a) + 816*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(a) - 18219*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*a^(3/2) - 12528*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(3/2)
+ 91467*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^10*A*a^(5/2) + 64752*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^10*C*a^(5/2) - 177735*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(7/2) - 124848*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(7/2) + 100413*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(9/2) + 700
32*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*
a^(9/2) - 26881*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*A*a^(11/2) - 19152*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*C*a^(11/2) + 3321*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(13/2) + 2640*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(13/2) - 205*A*a^(15
/2) - 144*C*a^(15/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.111 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{(245A+397C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}+\frac{(11A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^4(c+}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((11*A + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) - ((455*A + 799*C)*Sin[c + d*x])/(105*a*d*Sqrt[
a + a*Cos[c + d*x]]) - ((35*A + 67*C)*Cos[c + d*x]^2*Sin[c + d*x])/(70*a*d*
Sqrt[a + a*Cos[c + d*x]]) + ((7*A + 11*C)*Cos[c + d*x]^3*Sin[c + d*x])/(14*
a*d*Sqrt[a + a*Cos[c + d*x]]) + ((245*A + 397*C)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(210*a^2*d)
```

Rubi [A] time = 0.791716, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3042, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{(245A+397C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}+\frac{(11A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^4(c+}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((11*A + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) - ((455*A + 799*C)*Sin[c + d*x])/(105*a*d*Sqrt[
a + a*Cos[c + d*x]]) - ((35*A + 67*C)*Cos[c + d*x]^2*Sin[c + d*x])/(70*a*d*
Sqrt[a + a*Cos[c + d*x]]) + ((7*A + 11*C)*Cos[c + d*x]^3*Sin[c + d*x])/(14*
a*d*Sqrt[a + a*Cos[c + d*x]]) + ((245*A + 397*C)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(210*a^2*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
```

```
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(-2a(A+2C)+\frac{1}{2}a(7A+11C)\cos(c+d\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A+11C)\cos^3(c+dx)\sin(c+dx)}{14ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(35A+67C)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(35A+67C)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(35A+67C)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(455A+799C)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} - \frac{(35A+}{7}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(455A+799C)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} - \frac{(35A+}{7}$$

$$= \frac{(11A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} -$$

Mathematica [A] time = 0.998765, size = 157, normalized size = 0.61

$$\frac{\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\left(6(140A+277C)\cos(c+dx)-4(35A+64C)\cos(2(c+dx))+1190A+18C\cos(3(c\right.}{105d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (-105*(11*A + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (Cos[(c
+ d*x)/2]^3*(1190*A + 2161*C + 6*(140*A + 277*C)*Cos[c + d*x] - 4*(35*A + 6
4*C)*Cos[2*(c + d*x)] + 18*C*Cos[3*(c + d*x)] - 15*C*Cos[4*(c + d*x)])*Sin[
(c + d*x)/2])/2)/(105*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]
^2))
```

Maple [A] time = 0.071, size = 442, normalized size = 1.7

$$\frac{1}{420\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(960\,C\sqrt{2}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin\,(1/2\,dx+c/2))^8-1632\,C\sqrt{2}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/420*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(960*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-1632*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+112*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)*(5*A+16*C)*sin(1/2*d*x+1/2*c)^4-35*2^(1/2)*(33*A*ln(4/cos(1/2*
d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-8*A*a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+57*C*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+a))*a-16*C*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))*sin(1
/2*d*x+1/2*c)^2+1155*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+a))*a*A+1995*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-945*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-1785*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/
2*d*x+1/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.7848, size = 637, normalized size = 2.46

$$\frac{105\sqrt{2}\left((11A+19C)\cos(dx+c)^2+2(11A+19C)\cos(dx+c)+11A+19C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}}{\cos(dx+c)^2+2\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/840*(105*sqrt(2)*((11*A + 19*C)*cos(d*x + c)^2 + 2*(11*A + 19*C)*cos(d*x
+ c) + 11*A + 19*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1)) + 4*(60*C*cos(d*x + c)^4 - 36*C*cos(d*x + c)^3 + 28
*(5*A + 7*C)*cos(d*x + c)^2 - 12*(35*A + 67*C)*cos(d*x + c) - 665*A - 1201*
C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.03828, size = 343, normalized size = 1.32

$$-\frac{\frac{105\left(11\sqrt{2}A+19\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\left(\frac{105\left(\sqrt{2}Aa^5+\sqrt{2}Ca^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3}+\frac{4\left(455\sqrt{2}Aa^5+877\sqrt{2}Ca^5\right)}{a^3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\cdots}{\cdots}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] -1/420*(105*(11*sqrt(2)*A + 19*sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/
2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((((105*(sqrt(2)*A*a^
5 + sqrt(2)*C*a^5)*tan(1/2*d*x + 1/2*c)^2/a^3 + 4*(455*sqrt(2)*A*a^5 + 877*
sqrt(2)*C*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 14*(305*sqrt(2)*A*a^5 + 517*sq
rt(2)*C*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 140*(25*sqrt(2)*A*a^5 + 47*sqrt(
2)*C*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 105*(9*sqrt(2)*A*a^5 + 17*sqrt(2)*C
*a^5)/a^3)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

3.112 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{(5A+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{10a^2d}-\frac{(7A+15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -((7*A + 15*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) + ((15*A + 31*C)*Sin[c + d*x])/(5*a*d*Sqrt[a +
a*Cos[c + d*x]]) + ((5*A + 9*C)*Cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[a
 + a*Cos[c + d*x]]) - ((5*A + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(10*a^2*d)
```

Rubi [A] time = 0.594538, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3042, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{(5A+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{10a^2d}-\frac{(7A+15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] -((7*A + 15*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) + ((15*A + 31*C)*Sin[c + d*x])/(5*a*d*Sqrt[a +
a*Cos[c + d*x]]) + ((5*A + 9*C)*Cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[a
 + a*Cos[c + d*x]]) - ((5*A + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(10*a^2*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
```

```
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(-a(A+3C)+\frac{1}{2}a(5A+9C)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+9C)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+9C)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+9C)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(15A+31C)\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{(5A+9C)}{10ad}$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(15A+31C)\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{(5A+9C)}{10ad}$$

$$= -\frac{(7A+15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.640961, size = 136, normalized size = 0.64

$$\frac{5(7A+15C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)((20A+39C)\cos(c+dx)+}{5d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (5*(7*A + 15*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - Cos[(c + d*x
)/2]^3*(25*A + 47*C + (20*A + 39*C)*Cos[c + d*x] - 2*C*Cos[2*(c + d*x)] + C
*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/(5*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1
+ Sin[(c + d*x)/2]^2))
```

Maple [A] time = 0.066, size = 362, normalized size = 1.7

$$\frac{1}{20\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(32\,C\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{a}\left(\cos\left(1/2\,dx+c/2\right)\right)^6-64\,C\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{a}\,(\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/20/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(32*C*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-64*C*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-35*A*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1
/2*c)^2*a-75*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*
d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+40*A*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+112*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+5*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)+5*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)
/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.74336, size = 575, normalized size = 2.69

$$\frac{5\sqrt{2}\Big((7A+15C)\cos(dx+c)^2+2(7A+15C)\cos(dx+c)+7A+15C\Big)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx}{\cos(dx+c)^2+2\cos(dx+c)}\right.}{40\left(a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/40*(5*sqrt(2)*((7*A + 15*C)*cos(d*x + c)^2 + 2*(7*A + 15*C)*cos(d*x + c)
+ 7*A + 15*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c
) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 4*(4*C*cos(d*x + c)^3 - 4*C*cos(d*x + c)^2 + 4*(5*A + 9
*C)*cos(d*x + c) + 25*A + 49*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2
*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.99479, size = 271, normalized size = 1.27

$$\frac{\frac{5\sqrt{2}(7A+15C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\frac{5\sqrt{2}(Aa^3+Ca^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2}+\frac{\sqrt{2}(55Aa^3+127Ca^3)}{a^2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{5\sqrt{2}(19Aa^3+35}{a^2}\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] 1/20*(5*sqrt(2)*(7*A + 15*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + (((5*sqrt(2)*(A*a^3 + C*a^3)*tan(1
/2*d*x + 1/2*c)^2/a^2 + sqrt(2)*(55*A*a^3 + 127*C*a^3)/a^2)*tan(1/2*d*x + 1
/2*c)^2 + 5*sqrt(2)*(19*A*a^3 + 35*C*a^3)/a^2)*tan(1/2*d*x + 1/2*c)^2 + 5*s
qrt(2)*(9*A*a^3 + 17*C*a^3)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*
c)^2 + a)^(5/2))/d
```

3.113 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(3A+7C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{(3A+11C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-$$

```
[Out] ((3*A + 11*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(
a + a*Cos[c + d*x])^(3/2)) - ((3*A + 13*C)*Sin[c + d*x])/(3*a*d*Sqrt[a + a*
Cos[c + d*x]]) + ((3*A + 7*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^2
*d)
```

Rubi [A] time = 0.341496, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2968, 3023, 2751, 2649, 206}

$$\frac{(3A+7C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{(3A+11C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((3*A + 11*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(
a + a*Cos[c + d*x])^(3/2)) - ((3*A + 13*C)*Sin[c + d*x])/(3*a*d*Sqrt[a + a*
Cos[c + d*x]]) + ((3*A + 7*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^2
*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(-2aC+\frac{1}{2}a(3A+7C)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{-2aC\cos(c+dx)+\frac{1}{2}a(3A+7C)\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+7C)\sqrt{a+a\cos(c+dx)}\sin(c+d}{6a^2d}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A+13C)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{(3A+7C)\sqrt{}}{}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A+13C)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{(3A+7C)\sqrt{}}{}$$

$$= \frac{(3A+11C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(}{3}$$

Mathematica [A] time = 0.605505, size = 94, normalized size = 0.56

$$\frac{3(3A+11C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \tan\left(\frac{1}{2}(c+dx)\right)(3A+12C\cos(c+dx)-2C\cos(2(c+dx))+1}{6ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (3*(3*A + 11*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - (3*A + 17*C +
12*C*Cos[c + d*x] - 2*C*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(6*a*d*Sqrt[a*(
1 + Cos[c + d*x])])
```

Maple [A] time = 0.065, size = 292, normalized size = 1.7

$$\frac{1}{12\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(16\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\cos(1/2\,dx+c/2))^4 + 9\,A\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2)}}{\cos(1/2\,dx+c/2)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+9*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+33*C*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2
)*cos(1/2*d*x+1/2*c)^2*a-40*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*cos(1/2*d*x+1/2*c)^2-3*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-3
*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5/
2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.68336, size = 533, normalized size = 3.15

$$\frac{3\sqrt{2}\left((3A+11C)\cos(dx+c)^2+2(3A+11C)\cos(dx+c)+3A+11C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*((3*A + 11*C)*cos(d*x + c)^2 + 2*(3*A + 11*C)*cos(d*x + c)
+ 3*A + 11*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c
) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 4*(4*C*cos(d*x + c)^2 - 12*C*cos(d*x + c) - 3*A - 19*C)
```

```
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.9682, size = 225, normalized size = 1.33

$$-\frac{\frac{3\left(3\sqrt{2}A+11\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\frac{3\left(\sqrt{2}Aa+\sqrt{2}Ca\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}+\frac{2\left(3\sqrt{2}Aa+23\sqrt{2}Ca\right)}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3\left(\sqrt{2}Aa+9\right.}{a}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/12*(3*(3*sqrt(2)*A + 11*sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c)
 + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((3*(sqrt(2)*A*a + sqrt(2
)*C*a)*tan(1/2*d*x + 1/2*c)^2/a + 2*(3*sqrt(2)*A*a + 23*sqrt(2)*C*a)/a)*tan
(1/2*d*x + 1/2*c)^2 + 3*(sqrt(2)*A*a + 9*sqrt(2)*C*a)/a)*tan(1/2*d*x + 1/2*
c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

3.114 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{(A-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2C\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((A - 7*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) + ((A + C)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2)) + (2*C*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.143886, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3020, 2751, 2649, 206}

$$\frac{(A-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2C\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A - 7*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) + ((A + C)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2)) + (2*C*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3020

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> Simp[(b*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[a*A*(m + 1) - a*C*m + b*C*(2*m + 1)*Sin[e + f*x], x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(A-3C)-2aC\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{(A-7C)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= \frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-7C)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\,\text{si}}{\sqrt{a+a}}\right.}{2ad}$$

$$= \frac{(A-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.446512, size = 77, normalized size = 0.68

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(A+4C\cos(c+dx)+5C)+(A-7C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A - 7*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A + 5*C + 4*C*Cos[
c + d*x])*Tan[(c + d*x)/2])/(2*a*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.061, size = 254, normalized size = 2.2

$$\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(A\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}+2\,a}{\cos\left(1/2\,dx+c/2\right)}\right)\sqrt{2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a-7\,C\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2 \cdots\right.\right.}}{\cos\left(1/2\,d \cdots\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1
/2*c)^2*a-7*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d
*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+8*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(
1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.74155, size = 485, normalized size = 4.25

$$-\frac{\sqrt{2}\left((A-7\,C)\cos\left(dx+c\right)^{2}+2\,(A-7\,C)\cos\left(dx+c\right)+A-7\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2\,a\cos(\cdots}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.\cdots}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((A - 7*C)*cos(d*x + c)^2 + 2*(A - 7*C)*cos(d*x + c) + A - 7*
C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)) - 4*(4*C*cos(d*x + c) + A + 5*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.78933, size = 174, normalized size = 1.53

$$\frac{\frac{\left(\frac{\sqrt{2}\left(Aa^2+Ca^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{\sqrt{2}\left(Aa^2+9\,Ca^2\right)}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}-\frac{\sqrt{2}(A-7\,C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*(A*a^2 + C*a^2)*tan(1/2*d*x + 1/2*c)^2/a^3 + sqrt(2)*(A*a^2 +
 9*C*a^2)/a^3)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - sq
rt(2)*(A - 7*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a)))/a^(3/2))/d
```

3.115 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{(5A-3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*(a + a*Cos[c +
d*x])^(3/2))
```

Rubi [A] time = 0.319639, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2985, 2649, 206, 2773}

$$-\frac{(5A-3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*(a + a*Cos[c +
d*x])^(3/2))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{\left(2aA-\frac{1}{2}a(A-3C)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{A\int\sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{a^2}-\frac{(5A-3\ldots}{\ldots}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{(2A)\operatorname{Subst}\left(\int\frac{1}{a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad}+\frac{(5A\ldots}{\ldots}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d}-\frac{(5A-3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C\ldots}{2d(a+a\ldots}$$

Mathematica [A] time = 0.740238, size = 129, normalized size = 1.03

$$\frac{(A+C)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(5A-3C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sqrt{2}A\cos^5\left(\frac{1}{2}(c+dx)\right.}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((5*A - 3*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 4*Sqrt[2]*A*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (A + C)*Cos[(c + d*x)/2
]^3*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2
]^2))
```

Maple [B] time = 0.106, size = 373, normalized size = 3.

$$-\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5A\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}+2a}{\cos(1/2\,dx+c/2)}\right)\sqrt{2}(\cos(1/2\,dx+c/2))^2a-3C\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin}}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*co
s(1/2*d*x+1/2*c)^2*a-3*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(-4*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*co
s(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(4/(2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a+A*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.89077, size = 745, normalized size = 5.96

$$-\frac{\sqrt{2}\left((5A-3C)\cos(dx+c)^2+2(5A-3C)\cos(dx+c)+5A-3C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/8*(sqrt(2)*((5*A - 3*C)*cos(d*x + c)^2 + 2*(5*A - 3*C)*cos(d*x + c) + 5*
A - 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(co
s(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*
sqrt(a*cos(d*x + c) + a)*(A + C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2)
, x)
```

Giac [B] time = 3.20511, size = 288, normalized size = 2.3

$$\frac{\frac{\sqrt{2}\left(5\,A\sqrt{a}-3\,C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a^{2}}+\frac{8\,A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)}{a^{\frac{3}{2}}}-\frac{8\,A\log\left(\left|\left(\sqrt{\right.\right.\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/8*(sqrt(2)*(5*A*sqrt(a) - 3*C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 8*A*log(abs((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))
/a^(3/2) - 8*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) - 2*sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a)*(sqrt(2)*A*a + sqrt(2)*C*a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.116 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{(9A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A+C)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (-3*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
 + ((9*A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Tan[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)) + ((3*A + C)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.491487, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2984, 2985, 2649, 206, 2773}

$$\frac{(9A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A+C)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-3*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
 + ((9*A + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Tan[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)) + ((3*A + C)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(3A+C)-\frac{1}{2}a(3A-C)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+C)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-3a^2A+\frac{1}{2}a^2(3A+C)\cos\right.}{\sqrt{a+a\cos(c}}}{2a^3}$$

$$= -\frac{(A+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+C)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A)\int\sqrt{a+a\cos(c-}}{2a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+C)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{(3A)\,\text{Subst}\left(\int\frac{1}{a-x^2}\,d\right.}{ad}$$

$$= -\frac{3A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+}{2d(a+}$$

Mathematica [A] time = 2.21139, size = 167, normalized size = 1.06

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(2(9A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{12\sqrt{2}A\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\right.}{s}\right.}{d(a(\cos(c+dx)+1))^{3/2}(2A+C\cos(2(c+dx))+C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(2*(9*A + C)*ArcT
anh[Sin[(c + d*x)/2]] + (12*Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos
[(c + d*x)/2]^2 - 2*(3*A + C + 2*A*Sec[c + d*x])*Sin[(c + d*x)/2])/(-1 + Si
n[(c + d*x)/2]^2)))/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(2*A + C + C*Cos[2*(c +
 d*x)]))
```

Maple [B] time = 0.115, size = 746, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+2*C*2^
(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c
))*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
cos(1/2*d*x+1/2*c)^4*a-12*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*co
s(1/2*d*x+1/2*c)^4*a-9*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-C*ln(2*(2*a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2
*c)^2*a+6*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)^2+6*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a
+6*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+2*
C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-A*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.88059, size = 830, normalized size = 5.25

$$\sqrt{2}\Big((9\,A+C)\cos\left(dx+c\right)^3+2\,(9\,A+C)\cos\left(dx+c\right)^2+(9\,A+C)\cos\left(dx+c\right)\Big)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\,\mathrm{si}}{\cos(dx+c)^2+2\,\cos(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((9*A + C)*cos(d*x + c)^3 + 2*(9*A + C)*cos(d*x + c)^2 + (9*A
+ C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2
+ 2*cos(d*x + c) + 1)) + 6*(A*cos(d*x + c)^3 + 2*A*cos(d*x + c)^2 + A*cos(d
*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^
3 + cos(d*x + c)^2)) + 4*((3*A + C)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*
cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.35156, size = 468, normalized size = 2.96

$$-\frac{\frac{\sqrt{2}\left(9\,A\sqrt{a}+C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a^{2}}+\frac{12\,A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{a^{\frac{3}{2}}}-\frac{12\,A\log\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] -1/8*(sqrt(2)*(9*A*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 12*A*log(abs((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))
/a^(3/2) - 12*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) - 16*sqrt(2)*(3*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a
^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2*a + a^2)*a) - 2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a + sqrt(
2)*C*a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.117 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(19A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A+5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A+2C)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A+C)\tan(}{2ad\sqrt{a\cos}}$$

```
[Out] ((19*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*
a^(3/2)*d) - ((13*A + 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A + 2*C)*Tan[c + d*x])/(4*a
*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a
+ a*Cos[c + d*x])^(3/2)) + ((2*A + C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.709254, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2984, 2985, 2649, 206, 2773}

$$\frac{(19A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A+5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A+2C)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A+C)\tan(}{2ad\sqrt{a\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((19*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*
a^(3/2)*d) - ((13*A + 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A + 2*C)*Tan[c + d*x])/(4*a
*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a
+ a*Cos[c + d*x])^(3/2)) + ((2*A + C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqr
t[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
```

```
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2a(2A+C)-\frac{1}{2}a(5A+C)\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,d}{2a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A+C)\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int}{}$$

$$= -\frac{(7A+2C)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A+C)\sec}{2ad\sqrt{a}}$$

$$= -\frac{(7A+2C)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A+C)\sec}{2ad\sqrt{a}}$$

$$= -\frac{(7A+2C)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A+C)\sec}{2ad\sqrt{a}}$$

$$= \frac{(19A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A+5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 3.04186, size = 211, normalized size = 0.97

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)((7A+2C)\cos(2(c+dx))+6A\cos(c+dx)+3A\right.}{4ad\sqrt{a(\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2
),x]
```

```
[Out] -((A + C*Cos[c + d*x]^2)*Sec[(c + d*x)/2]*Sec[c + d*x]^2*((13*A + 5*C)*ArcT
anh[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2 - ((19*A
+ 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*
x))/2])^2)/Sqrt[2] + (3*A + 2*C + 6*A*Cos[c + d*x] + (7*A + 2*C)*Cos[2*(c +
 d*x)])*Sin[(c + d*x)/2]))/(4*a*d*Sqrt[a*(1 + Cos[c + d*x])]*(2*A + C + C*C
os[2*(c + d*x)]))
```

Maple [B] time = 0.124, size = 1540, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a+40*C
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
))*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))
*cos(1/2*d*x+1/2*c)^6*a-32*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*co
s(1/2*d*x+1/2*c)^6*a-32*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(
1/2*d*x+1/2*c)^6*a-104*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-40*C*2^(1/2)*ln(2*(2*a
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+
1/2*c)^4*a+28*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+
1/2*c)^4+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c
)^4*a+76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^
4*a+8*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4
+32*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+32
*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+26*A
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^2*a+10*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-22*A*a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-19*A*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-19*A*ln(-4*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/
(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-8*C*a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-8*C*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-8*C*ln(-4*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2
```

```
*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^
(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/
2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.29836, size = 927, normalized size = 4.27

$$\frac{2\sqrt{2}\left((13A+5C)\cos(dx+c)^4+2(13A+5C)\cos(dx+c)^3+(13A+5C)\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos}}{\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*((13*A + 5*C)*cos(d*x + c)^4 + 2*(13*A + 5*C)*cos(d*x + c)^
3 + (13*A + 5*C)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A + 8*C)*cos(d*x + c)^4 + 2*(19
*A + 8*C)*cos(d*x + c)^3 + (19*A + 8*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(
d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d
*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((7
*A + 2*C)*cos(d*x + c)^2 + 3*A*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d
*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.53705, size = 612, normalized size = 2.82

$$\frac{\sqrt{2}\left(13\,A\sqrt{a}+5\,C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{a^2}+\frac{\left(19\,A\sqrt{a}+8\,C\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] 1/8*(sqrt(2)*(13*A*sqrt(a) + 5*C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + (19*A*sqrt(a) + 8*C*sqrt(a)
)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a*(2*sqrt(2) + 3)))/a^2 - (19*A*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqr
t(2) - 3)))/a^2 - 2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a + sqrt(
2)*C*a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(29*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 133*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 55*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2)
 - 7*A*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)^2*a))/d
```

3.118 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{(47A+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3(7A+4C)\tan(c+dx)}{8ad\sqrt{a\cos(c+dx)+a}}+\frac{(5A+3C)}{6ad}$$

```
[Out] -((47*A + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(
8*a^(3/2)*d) + ((17*A + 9*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + (3*(7*A + 4*C)*Tan[c + d*x])/
(8*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((13*A + 6*C)*Sec[c + d*x]*Tan[c + d*x])
/(12*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/
(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((5*A + 3*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.88116, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2984, 2985, 2649, 206, 2773}

$$-\frac{(47A+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3(7A+4C)\tan(c+dx)}{8ad\sqrt{a\cos(c+dx)+a}}+\frac{(5A+3C)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] -((47*A + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(
8*a^(3/2)*d) + ((17*A + 9*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + (3*(7*A + 4*C)*Tan[c + d*x])/
(8*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((13*A + 6*C)*Sec[c + d*x]*Tan[c + d*x])
/(12*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/
(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((5*A + 3*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(5A+3C)-\frac{1}{2}a(7A+3C)\cos(c+dx)\right)\sec^4(c+dx}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+3C)\sec^2(c+dx)\tan(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(13A+6C)\sec(c+dx)\tan(c+dx)}{12ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= \frac{3(7A+4C)\tan(c+dx)}{8ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A+6C)\sec(c+dx)\tan(c+dx)}{12ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\mathrm{se}}{2d(a}$$

$$= \frac{3(7A+4C)\tan(c+dx)}{8ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A+6C)\sec(c+dx)\tan(c+dx)}{12ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\mathrm{se}}{2d(a}$$

$$= \frac{3(7A+4C)\tan(c+dx)}{8ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A+6C)\sec(c+dx)\tan(c+dx)}{12ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\mathrm{se}}{2d(a}$$

$$= -\frac{(47A+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+d}}\right.}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 3.86616, size = 205, normalized size = 0.77

$$\frac{-48(17A+9C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\sqrt{2}(47A+24C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (-48*(17*A + 9*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 12*Sqrt[2]
*(47*A + 24*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - Cos[(
```

```
c + d*x)/2]^3*(106*A + 48*C + 3*(55*A + 36*C)*Cos[c + d*x] + (74*A + 48*C)*
Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)] + 36*C*Cos[3*(c + d*x)])*Sec[c + d
*x]^3*Sin[(c + d*x)/2])/(48*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d
*x)/2]^2))
```

Maple [B] time = 0.133, size = 2028, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-576*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-1128*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-1128*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)
-2^(1/2)))*cos(1/2*d*x+1/2*c)^8*a-576*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-
2^(1/2)))*cos(1/2*d*x+1/2*c)^8*a+72*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a))*cos(1/2*d*x+1/2*c)^2*a+72*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/
2)))*cos(1/2*d*x+1/2*c)^2*a-432*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2
)))*cos(1/2*d*x+1/2*c)^4*a-432*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*cos(1/2*d*x+1/2*c)^4*a+864*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
cos(1/2*d*x+1/2*c)^6*a+864*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*c
os(1/2*d*x+1/2*c)^6*a+1692*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*co
s(1/2*d*x+1/2*c)^6*a+1692*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*co
s(1/2*d*x+1/2*c)^6*a-846*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(
1/2*d*x+1/2*c)^4*a-846*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1
/2*d*x+1/2*c)^4*a+141*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)
```

```
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/
2*d*x+1/2*c)^2*a+141*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*
d*x+1/2*c)^2*a-12*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-12*A*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+864*C*2^(1/2)*ln(2*(2*a^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8
*a+1632*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1
/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a+504*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-2448*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-129
6*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))
*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-608*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+1224*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+648*C*2^(
1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c)
)*cos(1/2*d*x+1/2*c)^4*a+288*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)*cos(1/2*d*x+1/2*c)^6-336*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)*cos(1/2*d*x+1/2*c)^4-204*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-108*C*ln(2*(2*a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2
*d*x+1/2*c)^2*a+218*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/
2*d*x+1/2*c)^2+120*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2
*d*x+1/2*c)^2)/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.33969, size = 980, normalized size = 3.68

$$\frac{12\sqrt{2}\left((17A+9C)\cos(dx+c)^5+2(17A+9C)\cos(dx+c)^4+(17A+9C)\cos(dx+c)^3\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos\ldots}}{\cos(\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/96*(12*sqrt(2)*((17*A + 9*C)*cos(d*x + c)^5 + 2*(17*A + 9*C)*cos(d*x + c)
^4 + (17*A + 9*C)*cos(d*x + c)^3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*((47*A + 24*C)*cos(d*x + c)^5 + 2
*(47*A + 24*C)*cos(d*x + c)^4 + (47*A + 24*C)*cos(d*x + c)^3)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) +
 4*(9*(7*A + 4*C)*cos(d*x + c)^3 + (37*A + 24*C)*cos(d*x + c)^2 - 6*A*cos(d
*x + c) + 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^5
 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.65745, size = 1010, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] -1/48*(6*sqrt(2)*(17*A*sqrt(a) + 9*C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 3*(47*A*sqrt(a) + 24*C*
sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^2 - 3*(47*A*sqrt(a) + 24*C*sqrt(a))*log
(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ a*(2*sqrt(2) - 3)))/a^2 - 12*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*
A*a + sqrt(2)*C*a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(339*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(a) + 72*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(
a) - 3165*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*A*a^(3/2) - 888*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*C*a^(3/2) + 9198*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(5/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(5/2) - 4938*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(7/2) - 1776*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(7/2) +
 975*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*
A*a^(9/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*C*a^(9/2) - 73*A*a^(11/2) - 24*C*a^(11/2))/(((sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3*a))/d
```

3.119 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=259

$$\frac{(45A+157C)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(195A+787C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}+\frac{(465A+1729C)\sin(c+dx)}{120a^2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -((75*A + 283*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A + 21*C)*Cos[c + d*x]^3*Sin[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((465*A + 1729*C)*Sin[c + d*x])/(120
*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((45*A + 157*C)*Cos[c + d*x]^2*Sin[c + d
*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((195*A + 787*C)*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(240*a^3*d)
```

Rubi [A] time = 0.803956, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3042, 2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{(45A+157C)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(195A+787C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}+\frac{(465A+1729C)\sin(c+dx)}{120a^2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] -((75*A + 283*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^4*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A + 21*C)*Cos[c + d*x]^3*Sin[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((465*A + 1729*C)*Sin[c + d*x])/(120
*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((45*A + 157*C)*Cos[c + d*x]^2*Sin[c + d
*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((195*A + 787*C)*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(240*a^3*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)\left(-4aC+\frac{1}{2}a(5A+13C)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A+21C)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A+21C)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A+21C)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A+21C)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A+21C)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A+21C)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(75A+283C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}$$

<u>Mathematica</u> [A] time = 1.30275, size = 129, normalized size = 0.5

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(5(255A+887C)\cos(c+dx)+16(15A+52C)\cos(2(c+dx))+975A-40C\cos(3(c+dx))+12C\cos}{240ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (-30*(75*A + 283*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (975*A +
 3491*C + 5*(255*A + 887*C)*Cos[c + d*x] + 16*(15*A + 52*C)*Cos[2*(c + d*x)
] - 40*C*Cos[3*(c + d*x)] + 12*C*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(240*a
*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.071, size = 432, normalized size = 1.7

$$\frac{1}{480\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(768\,C\sqrt{2}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^2}\sqrt{a}\,(\cos\,(1/2\,dx+c/2))^8-2176\,C\sqrt{2}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/480/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(768*C*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^8-2176*C*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-1125*A*2^(1/2)*ln
(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1
/2*d*x+1/2*c)^4*a-4245*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+960*A*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+5248*C*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+315*A*a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+555*C*a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-30*A*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)-30*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.75169, size = 725, normalized size = 2.8

$$\frac{15\sqrt{2}\left((75\,A+283\,C)\cos(dx+c)^3+3\,(75\,A+283\,C)\cos(dx+c)^2+3\,(75\,A+283\,C)\cos(dx+c)+75\,A+283\,C\right)\sqrt{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/960*(15*sqrt(2)*((75*A + 283*C)*cos(d*x + c)^3 + 3*(75*A + 283*C)*cos(d*x
 + c)^2 + 3*(75*A + 283*C)*cos(d*x + c) + 75*A + 283*C)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(96*C*cos(d
*x + c)^4 - 160*C*cos(d*x + c)^3 + 32*(15*A + 49*C)*cos(d*x + c)^2 + 5*(255
*A + 911*C)*cos(d*x + c) + 735*A + 2671*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c
) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.35054, size = 346, normalized size = 1.34

$$\frac{\frac{15\left(75\sqrt{2}A+283\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}-\frac{\left(\left(\left(15\left(\frac{2\left(\sqrt{2}Aa^{2}+\sqrt{2}Ca^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}}-\frac{13\sqrt{2}Aa^{2}+29\sqrt{2}Ca^{2}}{a^{2}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\cdots}{\cdots}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] 1/480*(15*(75*sqrt(2)*A + 283*sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2
*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2) - (((15*(2*(sqrt(2)*A*a^
```

```
2 + sqrt(2)*C*a^2)*tan(1/2*d*x + 1/2*c)^2/a^2 - (13*sqrt(2)*A*a^2 + 29*sqrt
(2)*C*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - (1725*sqrt(2)*A*a^2 + 6733*sqrt(2)
*C*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 5*(549*sqrt(2)*A*a^2 + 1973*sqrt(2)*C
*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 15*(83*sqrt(2)*A*a^2 + 291*sqrt(2)*C*a^
2)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.120 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{5(3A+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{(21A+197C)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A+163C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-$$

```
[Out] ((19*A + 163*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(4*
d*(a + a*Cos[c + d*x])^(5/2)) - ((A + 17*C)*Cos[c + d*x]^2*Sin[c + d*x])/(1
6*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((21*A + 197*C)*Sin[c + d*x])/(24*a^2*d
*Sqrt[a + a*Cos[c + d*x]]) + (5*(3*A + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c
 + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.605957, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3042, 2977, 2968, 3023, 2751, 2649, 206}

$$\frac{5(3A+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{(21A+197C)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A+163C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] ((19*A + 163*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(4*
d*(a + a*Cos[c + d*x])^(5/2)) - ((A + 17*C)*Cos[c + d*x]^2*Sin[c + d*x])/(1
6*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((21*A + 197*C)*Sin[c + d*x])/(24*a^2*d
*Sqrt[a + a*Cos[c + d*x]]) + (5*(3*A + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c
 + d*x])/(48*a^3*d)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
```

```
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(a(A-3C)+\frac{1}{2}a(3A+11C)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A+17C)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A+17C)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A+17C)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A+17C)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A+17C)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{(19A+163C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.863963, size = 112, normalized size = 0.53

$$\frac{6(19A+163C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \tan\left(\frac{1}{2}(c+dx)\right)((39A+479C)\cos(c+dx)+27A+80C\,c}{48ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (6*(19*A + 163*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 - (27*A + 37
9*C + (39*A + 479*C)*Cos[c + d*x] + 80*C*Cos[2*(c + d*x)] - 8*C*Cos[3*(c +
d*x)])*Tan[(c + d*x)/2])/(48*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.067, size = 362, normalized size = 1.7

$$\frac{1}{96\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(128\,C\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{a}\left(\cos\left(1/2\,dx+c/2\right)\right)^6+57\,A\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+\right.\right.}}{\cos\left(1/2\,dx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+57*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+489*C*
2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2
*c))*cos(1/2*d*x+1/2*c)^4*a-512*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)*cos(1/2*d*x+1/2*c)^4-39*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*cos(1/2*d*x+1/2*c)^2-87*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*cos(1/2*d*x+1/2*c)^2+6*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+6
*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(
7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.71271, size = 671, normalized size = 3.17

$$\frac{3\sqrt{2}\left((19A+163C)\cos(dx+c)^3+3(19A+163C)\cos(dx+c)^2+3(19A+163C)\cos(dx+c)+19A+163C\right)\sqrt{a}\,\mathrm{l}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/192*(3*sqrt(2)*((19*A + 163*C)*cos(d*x + c)^3 + 3*(19*A + 163*C)*cos(d*x
+ c)^2 + 3*(19*A + 163*C)*cos(d*x + c) + 19*A + 163*C)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*C*cos(d*
x + c)^3 - 160*C*cos(d*x + c)^2 - (39*A + 503*C)*cos(d*x + c) - 27*A - 299*
C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.30582, size = 274, normalized size = 1.29

$$\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}(Aa^5+Ca^5)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6}-\frac{\sqrt{2}(7Aa^5+23Ca^5)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}(15Aa^5+167Ca^5)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{3\sqrt{2}(11Aa^5+155Ca^5)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}-\frac{3\sqrt{2}(1}{}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] 1/96*(((3*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(1/2*d*x + 1/2*c)^2/a^6 - sqrt(2)*(
7*A*a^5 + 23*C*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(15*A*a^5 + 167
*C*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*(11*A*a^5 + 155*C*a^5)/a^6)
*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 3*sqrt(2)*(19*
A + 163*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a)))/a^(5/2))/d
```

3.121 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=165

$$\frac{(A+9C)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{5(A-15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(3A-13C)\sin}{16ad(a\cos(c+d}$$

```
[Out] (5*(A - 15*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((3*A - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*Co
s[c + d*x])^(3/2)) + ((A + 9*C)*Sin[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c + d
*x]])
```

Rubi [A] time = 0.367333, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2968, 3019, 2751, 2649, 206}

$$\frac{(A+9C)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{5(A-15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A+C)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(3A-13C)\sin}{16ad(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (5*(A - 15*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((3*A - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*Co
s[c + d*x])^(3/2)) + ((A + 9*C)*Sin[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c + d
*x]])
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A-C)+\frac{1}{2}a(A+9C)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{2a(A-C)\cos(c+dx)+\frac{1}{2}a(A+9C)\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(3A-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{4}a^2(3A-}{}}{}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(3A-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(A+9C)}{4a^2d\sqrt{a+}}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(3A-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(A+9C)}{4a^2d\sqrt{a+}}$$

$$= \frac{5(A-15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{}{1}$$

Mathematica [A] time = 0.699797, size = 95, normalized size = 0.58

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(5(A+17C)\cos(c+dx)+A+16C\cos(2(c+dx))+65C)+10(A-15C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\right.}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (10*(A - 15*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A + 65*C + 5
*(A + 17*C)*Cos[c + d*x] + 16*C*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(16*a*d
*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.071, size = 327, normalized size = 2.

$$\frac{1}{32d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5A\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2))^2}+2a}{\cos(1/2dx+c/2)}\right)(\cos(1/2dx+c/2))^4a-75C\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a-75*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+64*C*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+5*A*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+21*C*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)-2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.91618, size = 610, normalized size = 3.7

$$-\frac{5\sqrt{2}\left((A-15C)\cos(dx+c)^3+3(A-15C)\cos(dx+c)^2+3(A-15C)\cos(dx+c)+A-15C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+\cdots}{\cdots}\right)\cdots}{64\left(a^3d\cos(dx+c)^3+3a\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(5*sqrt(2)*((A - 15*C)*cos(d*x + c)^3 + 3*(A - 15*C)*cos(d*x + c)^2 +
 3*(A - 15*C)*cos(d*x + c) + A - 15*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*s
qrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3
```

```
*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*C*cos(d*x + c)^2 + 5*(A
+ 17*C)*cos(d*x + c) + A + 49*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.1808, size = 240, normalized size = 1.45

$$-\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Aa^{6}+\sqrt{2}Ca^{6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{8}}-\frac{\sqrt{2}Aa^{6}+17\sqrt{2}Ca^{6}}{a^{8}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{3\sqrt{2}Aa^{6}+83\sqrt{2}Ca^{6}}{a^{8}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}+\frac{5\left(\sqrt{2}A-15\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{\cdots}\right|\right)}{a^{\frac{5}{2}}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/32*(((2*(sqrt(2)*A*a^6 + sqrt(2)*C*a^6)*tan(1/2*d*x + 1/2*c)^2/a^8 - (sq
rt(2)*A*a^6 + 17*sqrt(2)*C*a^6)/a^8)*tan(1/2*d*x + 1/2*c)^2 - (3*sqrt(2)*A*
a^6 + 83*sqrt(2)*C*a^6)/a^8)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a) + 5*(sqrt(2)*A - 15*sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2
*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.122 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=124

$$\frac{(3A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A-13C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A+C)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((3*A + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((3*A - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rubi [A] time = 0.159818, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3020, 2750, 2649, 206}

$$\frac{(3A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A-13C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A+C)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((3*A - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rule 3020

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> Simp[(b*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[a*A*(m + 1) - a*C*m + b*C*(2*m + 1)*Sin[e + f*x], x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3A-5C)-4aC\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3A+19C)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$

$$= \frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(3A+19C)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{16a^2d}$$

$$= \frac{(3A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-13C)\sin(c+}{16ad(a+a\cos(c+d}$$

Mathematica [A] time = 0.454865, size = 89, normalized size = 0.72

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((3A-13C)\cos(c+dx)+7A-9C)+2(3A+19C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*A + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (7*A - 9*C +
 (3*A - 13*C)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x])
```

```
)^(3/2))
```

Maple [B] time = 0.066, size = 292, normalized size = 2.4

$$\frac{1}{32d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3A\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2))^2}+2a}{\cos(1/2dx+c/2)}\right)(\cos(1/2dx+c/2))^4a+19C\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a+19*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+3*A*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-13*C*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.85836, size = 589, normalized size = 4.75

$$\frac{\sqrt{2}\left((3A+19C)\cos(dx+c)^3+3(3A+19C)\cos(dx+c)^2+3(3A+19C)\cos(dx+c)+3A+19C\right)\sqrt{a}\log\left(-\frac{a\cos(dx}{}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 19*C)*cos(d*x + c)^3 + 3*(3*A + 19*C)*cos(d*x + c)^2
+ 3*(3*A + 19*C)*cos(d*x + c) + 3*A + 19*C)*sqrt(a)*log(-(a*cos(d*x + c)^2
- 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A - 13*C)*cos(d*x +
 c) + 7*A - 9*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.93082, size = 180, normalized size = 1.45

$$\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Aa^5+Ca^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}\left(5\,Aa^5-11\,Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}(3\,A+19\,C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(1/2
*d*x + 1/2*c)^2/a^8 + sqrt(2)*(5*A*a^5 - 11*C*a^5)/a^8)*tan(1/2*d*x + 1/2*c
) - sqrt(2)*(3*A + 19*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.123 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{(43A-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-5C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A+C)\sin(c+dx}{4d(a\cos(c+dx)+a}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((11*A - 5*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2))
```

Rubi [A] time = 0.480825, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 2978, 2985, 2649, 206, 2773}

$$-\frac{(43A-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-5C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A+C)\sin(c+dx}{4d(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((11*A - 5*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4aA-\frac{1}{2}a(3A-5C)\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-5C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(8a^2A-\frac{1}{4}a^2(11A-5C)\cos\right.}{\sqrt{a+a\cos(}}}{8a^4}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-5C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\cos(c+}}{a^3}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-5C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\,\text{Subst}\left(\int\frac{1}{a-x^2}d\right.}{a^2}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C}{4d(a+a}$$

Mathematica [A] time = 1.68602, size = 124, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((5C-11A)\cos(c+dx)-15A+C)-2(43A-5C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64\sqrt{2}A}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (-2*(43*A - 5*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqrt[2]*
A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-15*A + C + (-11*
A + 5*C)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^(3/
2))
```

Maple [B] time = 0.111, size = 445, normalized size = 2.8

$$-\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(43\,A\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^4\,a-5\,C\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A*2^(
1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c)
)*cos(1/2*d*x+1/2*c)^4*a-5*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-32*A*ln(-4*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/
(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-32*A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+11*A*a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-5*C*a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.99696, size = 899, normalized size = 5.55

$$-\frac{\sqrt{2}\left((43\,A-5\,C)\cos\left(dx+c\right)^{3}+3\,(43\,A-5\,C)\cos\left(dx+c\right)^{2}+3\,(43\,A-5\,C)\cos\left(dx+c\right)+43\,A-5\,C\right)\sqrt{a}\log\left(-\frac{a\cos}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((43*A - 5*C)*cos(d*x + c)^3 + 3*(43*A - 5*C)*cos(d*x + c)^2
 + 3*(43*A - 5*C)*cos(d*x + c) + 43*A - 5*C)*sqrt(a)*log(-(a*cos(d*x + c)^2
```

```
 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x +
c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*(A*cos(d*x + c)^3 + 3
*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 5*C)*cos
(d*x + c) + 15*A - C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.3623, size = 336, normalized size = 2.07

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5+Ca^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{\sqrt{2}\left(13Aa^5-3Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\sqrt{2}\left(43A\sqrt{a}-5C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\cdots\right.\right.\right.}{a^3\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(
1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(13*A*a^5 - 3*C*a^5)/a^8)*tan(1/2*d*x + 1/
2*c) - sqrt(2)*(43*A*sqrt(a) - 5*C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 64*A*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) +
3)))/a^(5/2) + 64*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/d
```

3.124 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(35A+3C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(115A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(15A-C)\tan(c+}{16ad(a\cos(c+dx)+}$$

```
[Out] (-5*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
 + ((115*A + 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
+ d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Tan[c + d*x])/(4*d*(a + a*Cos[
c + d*x])^(5/2)) - ((15*A - C)*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2)) + ((35*A + 3*C)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.698175, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3042, 2978, 2984, 2985, 2649, 206, 2773}

$$\frac{(35A+3C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(115A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(15A-C)\tan(c+}{16ad(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-5*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
 + ((115*A + 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
+ d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Tan[c + d*x])/(4*d*(a + a*Cos[
c + d*x])^(5/2)) - ((15*A - C)*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2)) + ((35*A + 3*C)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{\int \frac{\left(a(5A+C)-\frac{1}{2}a(5A-3C)\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(15A-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{\left(\frac{1}{2}a^2(35A+3C)-\frac{3}{4}a^2 \cdots\right.}{\sqrt{a \cdots}}}{\cdots}$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(15A-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{(35A+3C)\tan(c \cdots}{16a^2d\sqrt{a+a\cos(c \cdots}}$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(15A-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{(35A+3C)\tan(c \cdots}{16a^2d\sqrt{a+a\cos(c \cdots}}$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(15A-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{(35A+3C)\tan(c \cdots}{16a^2d\sqrt{a+a\cos(c \cdots}}$$

$$= -\frac{5A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d}+\frac{(115A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A \cdots}{4d(\cdots}$$

Mathematica [A] time = 4.44601, size = 185, normalized size = 0.93

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left((230A+6C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\sec(c+ \cdots\right.}{4d(a(\cos(c+dx)+1 \cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((230*A + 6*C)*Ar
cTanh[Sin[(c + d*x)/2]] - 160*Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] +
 ((67*A + 3*C + 2*(55*A + 7*C)*Cos[c + d*x] + (35*A + 3*C)*Cos[2*(c + d*x)]
)*Sec[(c + d*x)/2]^3*Sec[c + d*x]*Tan[(c + d*x)/2])/2))/(4*d*(a*(1 + Cos[c
+ d*x]))^(5/2)*(2*A + C + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.127, size = 815, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a+6*C*
ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(
1/2)*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
))*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
)*cos(1/2*d*x+1/2*c)^6*a-115*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-3*C*2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)^4*a+70*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2
*d*x+1/2*c)^4+80*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+
1/2*c)^4*a+80*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/
2*c)^4*a+6*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2
*c)^4-15*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)^2+C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2
*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*C*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*
x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.00874, size = 1002, normalized size = 5.04

$$\frac{\sqrt{2}\left((115\,A + 3\,C)\cos\left(dx + c\right)^{4} + 3\,(115\,A + 3\,C)\cos\left(dx + c\right)^{3} + 3\,(115\,A + 3\,C)\cos\left(dx + c\right)^{2} + (115\,A + 3\,C)\cos\left(dx + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((115*A + 3*C)*cos(d*x + c)^4 + 3*(115*A + 3*C)*cos(d*x + c)^
3 + 3*(115*A + 3*C)*cos(d*x + c)^2 + (115*A + 3*C)*cos(d*x + c))*sqrt(a)*lo
g(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x +
 c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(
A*cos(d*x + c)^4 + 3*A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + A*cos(d*x + c)
)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c
) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)) + 4*((35*A + 3*C)*cos(d*x + c)^2 + (55*A + 7*C)*cos(d*x + c)
+ 16*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.8807, size = 518, normalized size = 2.6

$$2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5+Ca^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}\left(21\,Aa^5+5\,Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}\left(115\,A\sqrt{a}+3\,C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\right.\right.\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] 1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(1
/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(21*A*a^5 + 5*C*a^5)/a^8)*tan(1/2*d*x + 1/2
*c) - sqrt(2)*(115*A*sqrt(a) + 3*C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 160*A*log(abs((sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) +
 3)))/a^(5/2) + 160*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2) + 128*sqrt(2)*(3*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a
) - A*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)*a^2))/d
```

3.125 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{(63A+11C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A+43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A+7\ldots}{16a\ldots}$$

```
[Out] ((39*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*
a^(5/2)*d) - ((219*A + 43*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((63*A + 11*C)*Tan[c + d*x])
/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((31*A + 7*C)*Sec[c + d*x]*Tan[c + d
*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.913386, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3042, 2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{(63A+11C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A+43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A+7\ldots}{16a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] ((39*A + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*
a^(5/2)*d) - ((219*A + 43*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((63*A + 11*C)*Tan[c + d*x])
/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((31*A + 7*C)*Sec[c + d*x]*Tan[c + d
*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(2a(3A+C)-\frac{1}{2}a(7A-C)\cos(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,d}{4a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A+3C)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A+3C)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(63A+11C)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A+3C)}{16ad(\cdots}$$

$$= -\frac{(63A+11C)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A+3C)}{16ad(\cdots}$$

$$= -\frac{(63A+11C)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A+3C)}{16ad(\cdots}$$

$$= \frac{(39A+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A+43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx}}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 6.16988, size = 408, normalized size = 1.56

$$\frac{4\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(-\frac{A+C}{16\left(1-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{A+C}{16\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^2}-\frac{27A+11C}{16\left(1-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{27A+11C}{16\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (4*Cos[c/2 + (d*x)/2]^5*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(-((219*A + 4
3*C)*ArcTanh[Sin[c/2 + (d*x)/2]])/8 - 6*Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[c/2 +
 (d*x)/2]] + 4*Sqrt[2]*(6*A + C)*ArcTanh[Sqrt[2]*Sin[c/2 + (d*x)/2]] - (A +
 C)/(16*(1 - Sin[c/2 + (d*x)/2])^2) - (27*A + 11*C)/(16*(1 - Sin[c/2 + (d*x
)/2])) + (A + C)/(16*(1 + Sin[c/2 + (d*x)/2])^2) + (27*A + 11*C)/(16*(1 + S
in[c/2 + (d*x)/2])) + (2*A*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)
^2 - (12*A*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2) + (3*A*(Sqrt[2]
*ArcTanh[Sqrt[2]*Sin[c/2 + (d*x)/2]] + (2*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/
2 + (d*x)/2]^2)))/2))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(2*A + C + C*Cos[2*c
+ 2*d*x]))
```

Maple [B] time = 0.141, size = 1610, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/8*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-128*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-
2^(1/2)))*cos(1/2*d*x+1/2*c)^8*a-128*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2
^(1/2)))*cos(1/2*d*x+1/2*c)^8*a-32*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(
1/2)))*cos(1/2*d*x+1/2*c)^4*a-32*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
```

```
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a))*cos(1/2*d*x+1/2*c)^4*a+128*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*cos(1/2*d*x+1/2*c)^6*a+128*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))
*cos(1/2*d*x+1/2*c)^6*a+624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*c
os(1/2*d*x+1/2*c)^6*a+624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*co
s(1/2*d*x+1/2*c)^6*a-156*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(
1/2*d*x+1/2*c)^4*a-156*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1
/2*d*x+1/2*c)^4*a+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*A*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+172*C*2^(1/2)*ln(2*(2*a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^
8*a+876*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1
/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a+252*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-876*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-172*
C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2
^(1/2)*cos(1/2*d*x+1/2*c)^6*a-188*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)*cos(1/2*d*x+1/2*c)^4+219*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+43*C*2^(1/2)
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*co
s(1/2*d*x+1/2*c)^4*a+44*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*co
s(1/2*d*x+1/2*c)^6-36*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(
1/2*d*x+1/2*c)^4+19*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/
2*d*x+1/2*c)^2+3*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d
*x+1/2*c)^2)/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.46756, size = 1106, normalized size = 4.22

$$\frac{\sqrt{2}\Big((219\,A + 43\,C)\cos\left(dx + c\right)^{5} + 3\,(219\,A + 43\,C)\cos\left(dx + c\right)^{4} + 3\,(219\,A + 43\,C)\cos\left(dx + c\right)^{3} + (219\,A + 43\,C)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((219*A + 43*C)*cos(d*x + c)^5 + 3*(219*A + 43*C)*cos(d*x + c
)^4 + 3*(219*A + 43*C)*cos(d*x + c)^3 + (219*A + 43*C)*cos(d*x + c)^2)*sqrt
(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin
(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
+ 4*((39*A + 8*C)*cos(d*x + c)^5 + 3*(39*A + 8*C)*cos(d*x + c)^4 + 3*(39*A
+ 8*C)*cos(d*x + c)^3 + (39*A + 8*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((63*A
 + 11*C)*cos(d*x + c)^3 + 5*(19*A + 3*C)*cos(d*x + c)^2 + 20*A*cos(d*x + c)
 - 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.46668, size = 662, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(
1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(29*A*a^5 + 13*C*a^5)/a^8)*tan(1/2*d*x + 1
/2*c) - sqrt(2)*(219*A*sqrt(a) + 43*C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 8*(39*A*sqrt(a) + 8*C*
sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^3 + 8*(39*A*sqrt(a) + 8*C*sqrt(a))*log(
abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
 a*(2*sqrt(2) - 3)))/a^3 + 32*sqrt(2)*(41*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 209*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 91*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2) - 11*A*
a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*a + a^2)^2*a^2))/d
```

3.126 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=196

$$\frac{10a(11A+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(11A+9C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{2a(9A+7C}{}$$

```
[Out] (2*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(11*A + 9*C)*Ell
ipticF[(c + d*x)/2, 2])/(231*d) + (10*a*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(231*d) + (2*a*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*
d) + (2*a*(11*A + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*a*C*Cos
[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*a*C*Cos[c + d*x]^(9/2)*Sin[c + d*x
])/(11*d)
```

Rubi [A] time = 0.236391, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3034, 3023, 2748, 2635, 2639, 2641}

$$\frac{10a(11A+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(11A+9C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{2a(9A+7C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(11*A + 9*C)*Ell
ipticF[(c + d*x)/2, 2])/(231*d) + (10*a*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(231*d) + (2*a*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*
d) + (2*a*(11*A + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*a*C*Cos
[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*a*C*Cos[c + d*x]^(9/2)*Sin[c + d*x
])/(11*d)
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
```

```
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{2aC\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{2}{11}\int \cos^{\frac{5}{2}}(c+dx)\left(\frac{11aA}{2}\right.$$

$$= \frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2aC\cos^{\frac{9}{2}}(c+dx)\sin(c+}{11d}$$

$$= \frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2aC\cos^{\frac{9}{2}}(c+dx)\sin(c+}{11d}$$

$$= \frac{2a(9A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2a(11A+9C)\cos}{}$$

$$= \frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(11A+9C)\sqrt{\cos(c+}}{231d}$$

$$= \frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(11A+9C)F\left(\frac{1}{2}(c+d\right.}{231d}$$

Mathematica [C] time = 6.33458, size = 964, normalized size = 4.92

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(9A+7C)\cot(c)}{15d} + \frac{(506A+435C)\cos(dx)\sin(c)}{1848d} + \frac{(18A+19C)\cos(2dx)\sin(2c)}{180d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((9*A + 7*C
)*Cot[c])/(15*d) + ((506*A + 435*C)*Cos[d*x]*Sin[c])/(1848*d) + ((18*A + 19
*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + ((44*A + 57*C)*Cos[3*d*x]*Sin[3*c])/(123
2*d) + (C*Cos[4*d*x]*Sin[4*c])/(72*d) + (C*Cos[5*d*x]*Sin[5*c])/(176*d) + (
(506*A + 435*C)*Cos[c]*Sin[d*x])/(1848*d) + ((18*A + 19*C)*Cos[2*c]*Sin[2*d
*x])/(180*d) + ((44*A + 57*C)*Cos[3*c]*Sin[3*d*x])/(1232*d) + (C*Cos[4*c]*S
in[4*d*x])/(72*d) + (C*Cos[5*c]*Sin[5*d*x])/(176*d)) - (5*A*(1 + Cos[c + d*
x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
```

```
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (15*C*(1 + Cos
[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(77*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1
 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (7*C*(1 + Cos[c
+ d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d))
```

Maple [A] time = 0.128, size = 434, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(20160*C*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-62720*C*sin(1/2*d*x+1/2*c)^10*cos(
1/2*d*x+1/2*c)+(7920*A+81520*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1
7424*A-57712*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(14784*A+24332*C)*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4026*A-4638*C)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+825*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+675*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^5+Ca\cos\left(dx+c\right)^4+Aa\cos\left(dx+c\right)^3+Aa\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^5 + C*a*cos(d*x + c)^4 + A*a*cos(d*x + c)^3 + A*
a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x
)
```

3.127 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=165

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(9A+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a(7A+5C)\mathrm{si}}{}$$

```
[Out] (2*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*a*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (
2*a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*C*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```

Rubi [A] time = 0.214212, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3034, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(9A+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a(7A+5C)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*a*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (
2*a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*C*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{9aA}{2}+\right.$$

$$= \frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+}{9d}$$

$$= \frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+}{9d}$$

$$= \frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a(9A+7C)\cos}{}$$

$$= \frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

<u>Mathematica</u> [C] time = 6.26839, size = 918, normalized size = 5.56

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(9A+7C)\cot(c)}{15d}+\frac{(28A+23C)\cos(dx)\sin(c)}{84d}+\frac{(18A+19C)\cos(2dx)\sin(2c)}{180d}+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((9*A + 7*C
)*Cot[c])/(15*d) + ((28*A + 23*C)*Cos[d*x]*Sin[c])/(84*d) + ((18*A + 19*C)*
Cos[2*d*x]*Sin[2*c])/(180*d) + (C*Cos[3*d*x]*Sin[3*c])/(28*d) + (C*Cos[4*d*
x]*Sin[4*c])/(72*d) + ((28*A + 23*C)*Cos[c]*Sin[d*x])/(84*d) + ((18*A + 19*
C)*Cos[2*c]*Sin[2*d*x])/(180*d) + (C*Cos[3*c]*Sin[3*d*x])/(28*d) + (C*Cos[4
*c]*Sin[4*d*x])/(72*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(3*d*Sqrt[1 + Cot[c]^2]) - (5*C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec
```

```
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 +
 (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(10*d) - (7*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]
^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(30*d))
```

Maple [B] time = 0.078, size = 406, normalized size = 2.5

$$-\frac{2a}{315d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-1120\,C\,(\sin(1/2\,dx+c/2))^{10}\cos(1/2\,dx+c/2)+2960\,C\,(\sin(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*C*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2960*C*sin(1/2*d*x+1/2*c)^8*cos(1/2
*d*x+1/2*c)+(-504*A-3152*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(924*A+
1792*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-336*A-408*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos (dx+c)^2 + A\right)(a \cos (dx+c) + a) \cos (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca \cos (dx+c)^4 + Ca \cos (dx+c)^3 + Aa \cos (dx+c)^2 + Aa \cos (dx+c)\right)\sqrt{\cos (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^4 + C*a*cos(d*x + c)^3 + A*a*cos(d*x + c)^2 + A*
a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2+A\right)(a\cos{(dx+c)}+a)\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x
)
```

3.128 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=134

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aC\sin(c+dx}{7d}$$

```
[Out] (2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(21*d) + (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*C*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.185885, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3034, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aC\sin(c+dx}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(21*d) + (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*C*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sqrt{\cos(c+dx)}\left(\frac{7aA}{2}\right.$$
$$= \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+}{7d}$$
$$= \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+}{7d}$$
$$= \frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5C)\sqrt{\cos(c+dx}}{21d}$$
$$= \frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\right.}{21d}$$

Mathematica [C] time = 6.24549, size = 872, normalized size = 6.51

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(5A+3C)\cot(c)}{5d}+\frac{(28A+23C)\cos(dx)\sin(c)}{84d}+\frac{C\cos(2dx)\sin(2c)}{10d}+\frac{C\cos(3dx)}{28d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((5*A + 3*C
)*Cot[c])/(5*d) + ((28*A + 23*C)*Cos[d*x]*Sin[c])/(84*d) + (C*Cos[2*d*x]*Si
n[2*c])/(10*d) + (C*Cos[3*d*x]*Sin[3*c])/(28*d) + ((28*A + 23*C)*Cos[c]*Sin
[d*x])/(84*d) + (C*Cos[2*c]*Sin[2*d*x])/(10*d) + (C*Cos[3*c]*Sin[3*d*x])/(2
8*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Co
t[c]^2]) - (5*C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
```

```
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[
1 + Cot[c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*
d) - (3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 0.075, size = 378, normalized size = 2.8

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,C\left(\sin(1/2\,dx+c/2)\right)^8\cos(1/2\,dx+c/2)-528\,C\left(\sin(1/2\,dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*C*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-528*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(140*A+448*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A-122*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.129 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aC\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+}}{3d}$$

```
[Out] (2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + C)*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.163871, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3034, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aC\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + C)*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int\frac{\frac{5aA}{2}+\frac{1}{2}a(5A+3C)\cos(c+dx)}{\sqrt{\cos(c+dx)}}$$

$$=\frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{4}{15}\int$$

$$=\frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{3}(a($$

$$=\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aC\sqrt{}}{}$$

Mathematica [C] time = 6.26599, size = 824, normalized size = 8.16

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(5A+3C)\cot(c)}{5d}+\frac{C\cos(dx)\sin(c)}{3d}+\frac{C\cos(2dx)\sin(2c)}{10d}+\frac{C\cos(c)\sin(dx)}{3d}+\frac{C\,\mathrm{c}}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((5*A + 3*C
)*Cot[c])/(5*d) + (C*Cos[d*x]*Sin[c])/(3*d) + (C*Cos[2*d*x]*Sin[2*c])/(10*d
) + (C*Cos[c]*Sin[d*x])/(3*d) + (C*Cos[2*c]*Sin[2*d*x])/(10*d)) - (A*(1 + C
os[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (C*(1 +
Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (A*(1
 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (3*C*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 0.072, size = 345, normalized size = 3.4

$$-\frac{2a}{15d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,C\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+44\,C\,(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+44*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*C*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x
)
```

3.130 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (-2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a*C*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.168062, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3032, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a*C*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{aA}{2}-\frac{1}{2}a(A-C)\cos(c+dx)+\frac{1}{2}aC\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{4}{3}\int \frac{\frac{1}{4}a(3A+C)-}{\sqrt{c}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}-(a(A-C))\int \sqrt{\cos}$$

$$= -\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(}{d\sqrt{\cos(c}}$$

Mathematica [C] time = 6.33077, size = 813, normalized size = 8.56

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(-2A+C+C\cos(2c))\csc(c)\sec(c)}{2d}+\frac{A\sec(c+dx)\sin(dx)\sec(c)}{d}+\frac{C\cos(dx)\sin}{3d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((-2*A + C
+ C*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (C*Cos[d*x]*Sin[c])/(3*d) + (C*Cos[c]*
Sin[d*x])/(3*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d) - (A*(1 + Cos[c + d*x
])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*
x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (C*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.09, size = 458, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*a*(4*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(3*A+C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x
)
```

3.131 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*A*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.171574, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3032, 3021, 2748, 2641, 2639}

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*A*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{1}{2}a(A+3C)\cos(c+dx)+\frac{3}{2}aC\cos^2(c \cdots}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{4}{3}\int \frac{\frac{1}{4}a(A+3C)-\frac{3}{4}a(A-C) \cdots}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (a(A-C))\int \sqrt{\cos(c+dx)}\,dx \cdots$$

$$= -\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin \cdots}{3d\cos^{\frac{3}{2}} \cdots}$$

Mathematica [C] time = 6.33194, size = 817, normalized size = 8.6

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{3d}+\frac{\sec(c)(A\sin(c)+3A\sin(dx))\sec(c+dx)}{3d}-\frac{(-2A+C\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((-2*A + C
+ C*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*
d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 3*A*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c
 + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Co
s[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (A*(1 + C
os[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt
[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (C*(1 + Cos[c + d*x])
*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.147, size = 437, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2
*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x
)
```

3.132 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(}{5d\cos^{\frac{5}{2}}(}$$

```
[Out] (-2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + 3*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*C)*Sin[c + d
*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.198598, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3032, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(}{5d\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + 3*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*C)*Sin[c + d
*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5aA}{2}+\frac{1}{2}a(3A+5C)\cos(c+dx)+\frac{5}{2}aC\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4}{15}\int \frac{\frac{3}{4}a(3A+5C)+\frac{5}{4}a(A+3}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a(A+3C))\int \frac{1}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA}{5d\,\mathrm{co}}$$

Mathematica [C] time = 6.41253, size = 851, normalized size = 6.45

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{5d} + \frac{\sec(c)(3A\sin(c)+5A\sin(dx))\sec^2(c+dx)}{15d} + \frac{\sec(c)}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*C)
*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*A*Sin[c] + 5*A*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(
5*A*Sin[c] + 9*A*Sin[d*x] + 15*C*Sin[d*x]))/(15*d)) - (A*(1 + Cos[c + d*x])
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*
x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
```

```
]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (3*A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) + (C*(1 + Cos[c + d*x])*Csc[
c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.202, size = 729, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/10*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4
-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+1/2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
```

```
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x
)
```

3.133 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+$$

```
[Out] (-2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)
) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(5*A + 7*C)*Sin[c
+ d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqr
t[Cos[c + d*x]])
```

Rubi [A] time = 0.217699, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3032, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)
) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(5*A + 7*C)*Sin[c
+ d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqr
t[Cos[c + d*x]])
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
```

```
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{7aA}{2}+\frac{1}{2}a(5A+7C)\cos(c+dx)+\frac{7}{2}aC\cos^2(c}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{35}\int \frac{\frac{5}{4}a(5A+7C)+\frac{7}{4}a(3A+}{\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(a(3A+5C))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(}{}$$

$$= -\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a}{7d}$$

Mathematica [C] time = 6.47296, size = 895, normalized size = 5.42

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{7d} + \frac{\sec(c)(5A\sin(c)+7A\sin(dx))\sec^3(c+dx)}{35d} + \frac{\sec(c}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*C)
*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(7*d) + (Sec[c]*
Sec[c + d*x]^3*(5*A*Sin[c] + 7*A*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]^2
*(21*A*Sin[c] + 25*A*Sin[d*x] + 35*C*Sin[d*x]))/(105*d) + (Sec[c]*Sec[c + d
*x]*(25*A*Sin[c] + 35*C*Sin[c] + 63*A*Sin[d*x] + 105*C*Sin[d*x]))/(105*d))
- (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c
```

```
]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Co
t[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sq
rt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
+ (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin
[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
+ (C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.307, size = 838, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/10*A/(8*
sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)+1/2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/
2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
```

```
2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)+1/2*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+
5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x
)
```

3.134 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=230

$$\frac{8a^2(33A+25C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(99A+89C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+\frac{4a^2(9$$

```
[Out] (4*a^2*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^2*(33*A + 25*C)
*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^2*(33*A + 25*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(231*d) + (4*a^2*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(45*d) + (2*a^2*(99*A + 89*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d)
+ (2*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d) + (8*
C*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```

Rubi [A] time = 0.478117, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3046, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{8a^2(33A+25C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(99A+89C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+\frac{4a^2(9$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] (4*a^2*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^2*(33*A + 25*C)
*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^2*(33*A + 25*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(231*d) + (4*a^2*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(45*d) + (2*a^2*(99*A + 89*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d)
+ (2*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d) + (8*
C*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2\int c}{}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{8C\,c}{}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{8C\,c}{}$$

$$= \frac{2a^2(99A+89C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2C\cos^{\frac{5}{2}}(c}{}$$

$$= \frac{2a^2(99A+89C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2C\cos^{\frac{5}{2}}(c}{}$$

$$= \frac{8a^2(33A+25C)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} + \frac{4a^2(9A+7}{}$$

$$= \frac{4a^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{8a^2(33A+25C)F\left(\frac{1}{2}(c\right.}{231d}$$

Mathematica [C] time = 6.28686, size = 982, normalized size = 4.27

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(-\frac{(9A+7C)\cot(c)}{15d} + \frac{(1122A+941C)\cos(dx)\sin(c)}{3696d} + \frac{(18A+19C)\cos(2dx)\sin(2c}{180d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((9*A + 7*
C)*Cot[c])/(15*d) + ((1122*A + 941*C)*Cos[d*x]*Sin[c])/(3696*d) + ((18*A +
19*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + ((44*A + 101*C)*Cos[3*d*x]*Sin[3*c])/(
2464*d) + (C*Cos[4*d*x]*Sin[4*c])/(72*d) + (C*Cos[5*d*x]*Sin[5*c])/(352*d)
+ ((1122*A + 941*C)*Cos[c]*Sin[d*x])/(3696*d) + ((18*A + 19*C)*Cos[2*c]*Sin
[2*d*x])/(180*d) + ((44*A + 101*C)*Cos[3*c]*Sin[3*d*x])/(2464*d) + (C*Cos[4
*c]*Sin[4*d*x])/(72*d) + (C*Cos[5*c]*Sin[5*d*x])/(352*d)) - (2*A*(a + a*Cos
[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (50*C*
(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*d*Sqrt[1 + Cot[c]^
2]) - (3*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqr
t[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
 (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[
c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) -
 (7*C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d)
```

Maple [A] time = 0.083, size = 436, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-37520*C*sin(1/2*d*x+1/2*c)^10*co
s(1/2*d*x+1/2*c)+(3960*A+57040*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-11484*A-46192*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12474*A+22022*C)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-3861*A-4563*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+990*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+750*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^{2}\cos\left(dx+c\right)^{5}+2\,Ca^{2}\cos\left(dx+c\right)^{4}+(A+C)a^{2}\cos\left(dx+c\right)^{3}+2\,Aa^{2}\cos\left(dx+c\right)^{2}+Aa^{2}\cos\left(dx+c\right)\right)\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^5 + 2*C*a^2*cos(d*x + c)^4 + (A + C)*a^2*cos(d
*x + c)^3 + 2*A*a^2*cos(d*x + c)^2 + A*a^2*cos(d*x + c))*sqrt(cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
 x)
```

3.135 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=197

$$\frac{4a^2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(21A+19C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(7A$$

```
[Out] (16*a^2*(3*A + 2*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A + 5*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(21*d) + (2*a^2*(21*A + 19*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(105*d) + (2*C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d) + (8*C*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d)
```

Rubi [A] time = 0.433429, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3046, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(21A+19C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(7A$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (16*a^2*(3*A + 2*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A + 5*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(21*d) + (2*a^2*(21*A + 19*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(105*d) + (2*C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d) + (8*C*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{2\int \cdots}{\cdots}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{8C\,\cdots}{\cdots}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{8C\,\cdots}{\cdots}$$

$$= \frac{2a^2(21A+19C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2C\cos^{\frac{3}{2}}(c\cdots}{\cdots}$$

$$= \frac{2a^2(21A+19C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2C\cos^{\frac{3}{2}}(c\cdots}{\cdots}$$

$$= \frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(7A+5C)\sqrt{\cos(\cdots}}{21\cdots}$$

$$= \frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(7A+5C)F\left(\frac{1}{2}(c\cdots\right.}{21d}$$

Mathematica [C] time = 6.26685, size = 936, normalized size = 4.75

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(-\frac{4(3A+2C)\cot(c)}{15d} + \frac{(28A+23C)\cos(dx)\sin(c)}{84d} + \frac{(18A+37C)\cos(2dx)\sin(2c)}{360d} \cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((-4*(3*A +
2*C)*Cot[c])/(15*d) + ((28*A + 23*C)*Cos[d*x]*Sin[c])/(84*d) + ((18*A + 37*
C)*Cos[2*d*x]*Sin[2*c])/(360*d) + (C*Cos[3*d*x]*Sin[3*c])/(28*d) + (C*Cos[4
*d*x]*Sin[4*c])/(144*d) + ((28*A + 23*C)*Cos[c]*Sin[d*x])/(84*d) + ((18*A +
 37*C)*Cos[2*c]*Sin[2*d*x])/(360*d) + (C*Cos[3*c]*Sin[3*d*x])/(28*d) + (C*C
os[4*c]*Sin[4*d*x])/(144*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (5*C*(a + a*Cos[c + d*x])^2*Csc[c]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (
d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (2*A*(a + a*Cos[c + d*x])^2
*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) - (4*C*(a + a*Cos[c + d*x])^2*Csc[c
]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

Maple [A] time = 0.084, size = 408, normalized size = 2.1

$$-\frac{4\,a^2}{315\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Big(-560\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}\cos\left(1/2\,dx+c/2\right)+1840\,C\left(\sin\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1840*C*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-252*A-2368*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(672*A
```

```
+1568*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-273*A-387*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2+A\right)\left(a\cos{(dx+c)}+a\right)^2\sqrt{\cos{(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos{(dx+c)}^4+2\,Ca^2\cos{(dx+c)}^3+(A+C)a^2\cos{(dx+c)}^2+2\,Aa^2\cos{(dx+c)}+Aa^2\right)\sqrt{\cos{(dx+c)}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
 x)
```

3.136 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=164

$$\frac{8a^2(7A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+33C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{8C\sin(c}{}$$

```
[Out] (4*a^2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(7*A + 3*C)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 33*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(7*d) + (8*C*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(35*d)
```

Rubi [A] time = 0.419466, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3046, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^2(7A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+33C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{8C\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```



```
[Out] (4*a^2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(7*A + 3*C)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 33*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(7*d) + (8*C*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(35*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2\int \frac{(a+a\cos(c+dx))^2(}{}}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{8C\sqrt{\cos(c+dx)}\,(}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{8C\sqrt{\cos(c+dx)}\,(}{}$$

$$= \frac{2a^2(35A+33C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\cos(c+dx)}(a+a\,c}{7}$$

$$= \frac{2a^2(35A+33C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\cos(c+dx)}(a+a\,c}{7}$$

$$= \frac{4a^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2(7A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a}{}$$

Mathematica [C] time = 6.32073, size = 890, normalized size = 5.43

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(-\frac{(5A+3C)\cot(c)}{5d} + \frac{(28A+51C)\cos(dx)\sin(c)}{168d} + \frac{C\cos(2dx)\sin(2c)}{10d} + \frac{C\cos(3dx)}{56d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((5*A + 3*
C)*Cot[c])/(5*d) + ((28*A + 51*C)*Cos[d*x]*Sin[c])/(168*d) + (C*Cos[2*d*x]*
Sin[2*c])/(10*d) + (C*Cos[3*d*x]*Sin[3*c])/(56*d) + ((28*A + 51*C)*Cos[c]*S
in[d*x])/(168*d) + (C*Cos[2*c]*Sin[2*d*x])/(10*d) + (C*Cos[3*c]*Sin[3*d*x])
/(56*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
```

```
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sq
rt[1 + Cot[c]^2]) - (2*C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(7*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*
x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(2*d) - (3*C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^
4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
+ Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[
1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(
Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(10*d)
```

Maple [A] time = 0.086, size = 380, normalized size = 2.3

$$-\frac{4\,a^2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^8\cos\left(1/2\,dx+c/2\right)-348\,C\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-348*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(70*A+378*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-35*A-117*C
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+30*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
```

```
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca^{2}\cos\left(dx+c\right)^{4}+2\,Ca^{2}\cos\left(dx+c\right)^{3}+\left(A+C\right)a^{2}\cos\left(dx+c\right)^{2}+2\,Aa^{2}\cos\left(dx+c\right)+Aa^{2}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
 x)
```

3.137 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=160

$$\frac{4a^2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(15A-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c\right.}{5d}$$

```
[Out] (16*a^2*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + C)*EllipticF[(c
+ d*x)/2, 2])/(3*d) - (2*a^2*(15*A - 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) -
 (2*(5*A - C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*
d)
```

Rubi [A] time = 0.414087, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3044, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(15A-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (16*a^2*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + C)*EllipticF[(c
+ d*x)/2, 2])/(3*d) - (2*a^2*(15*A - 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) -
 (2*(5*A - C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*
d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))^2\left(2aA-\frac{1}{2}a(5A-C)\right.}{\sqrt{\cos(c+dx)}}}{a}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-C)\sqrt{\cos(c+dx)}\left(a^2+\right.}{5d}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-C)\sqrt{\cos(c+dx)}\left(a^2+\right.}{5d}$$

$$= -\frac{2a^2(15A-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\cos(c+dx))^2}{d\sqrt{\cos(c+dx}}$$

$$= -\frac{2a^2(15A-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\cos(c+dx))^2}{d\sqrt{\cos(c+dx}}$$

$$= \frac{16a^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a^2(15A-}{}$$

Mathematica [C] time = 6.37171, size = 658, normalized size = 4.11

$$-\frac{A\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\mathrm{co}\right.}}{d\sqrt{\cot^2(c)+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-5*A + 8
*C + 5*A*Cos[2*c] + 8*C*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + (C*Cos[d*x]*Sin[c
```

```
])/(3*d) + (C*Cos[2*d*x]*Sin[2*c])/(20*d) + (C*Cos[c]*Sin[d*x])/(3*d) + (A*
Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d) + (C*Cos[2*c]*Sin[2*d*x])/(20*d)) - (A*
(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2])
- (C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[
c]^2]) - (2*C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [B] time = 0.091, size = 440, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^2*(-12*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A+13*C)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-12*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3+(A+C)a^2\cos\left(dx+c\right)^2+2\,Aa^2\cos\left(dx+c\right)+Aa^2}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
 x)
```

3.138 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{8a^2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a^2(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8A\sin(c+dx)\left(a\right.}{3d\sqrt{\cos}}$$

```
[Out] (-4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*(A + C)*EllipticF[(c
+ d*x)/2, 2])/(3*d) - (2*a^2*(5*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (
8*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.424525, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3044, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a^2(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8A\sin(c+dx)\left(a\right.}{3d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*(A + C)*EllipticF[(c
+ d*x)/2, 2])/(3*d) - (2*a^2*(5*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (
8*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^2\left(2aA-\frac{3}{2}a(A-C)\cos\right.}{\cos^{\frac{3}{2}}(c+dx)}}{3a}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2a^2(5A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+a\cos(c+dx))^2\sin}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2a^2(5A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+a\cos(c+dx))^2\sin}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a^2(5}{}$$

Mathematica [C] time = 6.43974, size = 865, normalized size = 5.54

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{6d} + \frac{\sec(c)(A\sin(c)+6A\sin(dx))\sec(c+dx)}{6d} - \frac{(-2A+}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-2*A + C
 + C*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (C*Cos[d*x]*Sin[c])/(6*d) + (C*Cos[c]
```

```
*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c
 + d*x]*(A*Sin[c] + 6*A*Sin[d*x]))/(6*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*C*(a + a*Cos[c + d*
x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTa
n[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(a + a*Cos
[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt
[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (C*(a + a*Cos[c + d*x
])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d)
```

Maple [B] time = 0.175, size = 651, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+4*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+4*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
```

```
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-4*C*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+7*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3+(A+C)a^2\cos\left(dx+c\right)^2+2\,Aa^2\cos\left(dx+c\right)+Aa^2}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
 x)
```

3.139 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{4a^2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(17A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{16a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8A\sin(c+dx)\left(a^2\cos(c+dx)\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-16*a^2*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 3*C)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a^2*(17*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c
+ d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.435676, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3044, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(17A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{16a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8A\sin(c+dx)\left(a^2\cos(c+dx)\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-16*a^2*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 3*C)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a^2*(17*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c
+ d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int\frac{(a+a\cos(c+dx))^2\left(2aA-\frac{1}{2}a(A-5C)\right.}{\cos^{\frac{5}{2}}(c+dx)}}{5a}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(17A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2a^2(17A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= -\frac{16a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(17A}{15d}$$

Mathematica [C] time = 6.47852, size = 656, normalized size = 4.21

$$\frac{2A\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\mathrm{ta}\right.}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-16*A -
5*C + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*
x])/(10*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 10*A*Sin[d*x]))/(30*d) +
(Sec[c]*Sec[c + d*x]*(10*A*Sin[c] + 24*A*Sin[d*x] + 15*C*Sin[d*x]))/(30*d))
 - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot
[c]^2]) - (C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1
+ Cot[c]^2]) + (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(5*d)
```

Maple [B] time = 0.22, size = 756, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+48*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-96*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^6+60*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*C*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-20*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-48*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+116*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2+60*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-37*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^{2}\cos\left(dx+c\right)^{4}+2\,Ca^{2}\cos\left(dx+c\right)^{3}+(A+C)a^{2}\cos\left(dx+c\right)^{2}+2\,Aa^{2}\cos\left(dx+c\right)+Aa^{2}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
 x)
```

3.140 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{8a^2(3A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(3A+5C)\sin(c+}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(3*A + 7*C)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 35*C)*Sin[c + d*x])/(105*
d*Cos[c + d*x]^(3/2)) + (4*a^2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
 + (8*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.47447, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3044, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{8a^2(3A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(3A+5C)\sin(c+}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(3*A + 7*C)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 35*C)*Sin[c + d*x])/(105*
d*Cos[c + d*x]^(3/2)) + (4*a^2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
 + (8*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^2\left(2aA+\frac{1}{2}a(A+7C)\right.}{\cos^{\frac{7}{2}}(c+dx)}}{7a}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(33A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} +$$

$$= \frac{2a^2(33A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} +$$

$$= \frac{8a^2(3A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^2(33A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(}{}$$

$$= -\frac{4a^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2(3A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} +$$

Mathematica [C] time = 6.56553, size = 913, normalized size = 4.63

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{14d}+\frac{\sec(c)(5A\sin(c)+14A\sin(dx))\sec^3(c+dx)}{70d}+\frac{\sec(c)(\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((3*A + 5*C
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c
]*Sec[c + d*x]^3*(5*A*Sin[c] + 14*A*Sin[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x
]^2*(42*A*Sin[c] + 60*A*Sin[d*x] + 35*C*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c
+ d*x]*(60*A*Sin[c] + 35*C*Sin[c] + 126*A*Sin[d*x] + 210*C*Sin[d*x]))/(210*
d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1
+ Cot[c]^2]) - (2*C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*Sqrt[1 + Cot[c]^2]) + (3*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/
2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sq
rt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
)/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]))/(10*d) + (C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((
HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[
c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
))/(2*d)
```

Maple [B] time = 0.273, size = 918, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/10*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/4*C)*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*
c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+1/2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1)+1/4*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ca^2\cos(dx+c)^4 + 2\,Ca^2\cos(dx+c)^3 + (A+C)a^2\cos(dx+c)^2 + 2\,Aa^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c) + a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2),
 x)
```

3.141 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{4a^2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(19A+21C)\sin(c+}{105d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-16*a^2*(2*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 7*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(19*A + 21*C)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(5/2)) + (4*a^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*Cos[c + d*
x]^(3/2)) + (16*a^2*(2*A + 3*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (
2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (8*A*(a
^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.506819, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3044, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(19A+21C)\sin(c+}{105d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```


```
[Out] (-16*a^2*(2*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 7*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(19*A + 21*C)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(5/2)) + (4*a^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*Cos[c + d*
x]^(3/2)) + (16*a^2*(2*A + 3*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (
2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (8*A*(a
^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^2\left(2aA+\frac{3}{2}a(A+3C)\,\mathrm{co}\right.}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(19A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(19A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(19A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2(2A+}{15d\sqrt{}}$$

$$= -\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2}{}$$

Mathematica [C] time = 6.64595, size = 955, normalized size = 4.15

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^5(c+dx)}{18d}+\frac{\sec(c)(7A\sin(c)+18A\sin(dx))\sec^4(c+dx)}{126d}+\sec(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((4*(2*A + 3
*C)*Csc[c]*Sec[c])/(15*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(18*d) + (Se
c[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 18*A*Sin[d*x]))/(126*d) + (Sec[c]*Sec[c +
 d*x]^3*(90*A*Sin[c] + 112*A*Sin[d*x] + 63*C*Sin[d*x]))/(630*d) + (Sec[c]*S
ec[c + d*x]*(25*A*Sin[c] + 35*C*Sin[c] + 56*A*Sin[d*x] + 84*C*Sin[d*x]))/(1
05*d) + (Sec[c]*Sec[c + d*x]^2*(112*A*Sin[c] + 63*C*Sin[c] + 150*A*Sin[d*x]
 + 210*C*Sin[d*x]))/(630*d)) - (5*A*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]
^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (C*(a + a*Cos[c + d*x])^2*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (4*A*(a + a*Cos[c + d*x])^2
*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d) + (2*C*(a + a*Cos[c + d*x])^2*Csc[
c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [B] time = 0.34, size = 1168, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/5*(1/4
*A+1/4*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+1/4*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)+1/4*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*
d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))))+1/2*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3+(A+C)a^2\cos\left(dx+c\right)^2+2\,Aa^2\cos\left(dx+c\right)+Aa^2}{\cos\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(11/2)
, x)
```

3.142 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=279

$$\frac{4a^3(121A+95C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(221A+175C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{40a^3(143A+118C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{9009d}$$

```
[Out] (4*a^3*(221*A + 175*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (4*a^3*(121*A +
 95*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A + 95*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(221*A + 175*C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(585*d) + (40*a^3*(143*A + 118*C)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(9009*d) + (2*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*
x])/(13*d) + (12*C*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(143*a*d) + (2*(143*A + 145*C)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*
x])*Sin[c + d*x])/(1287*d)
```

Rubi [A] time = 0.656285, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3046, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(121A+95C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(221A+175C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{40a^3(143A+118C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{9009d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (4*a^3*(221*A + 175*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (4*a^3*(121*A +
 95*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A + 95*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(221*A + 175*C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(585*d) + (40*a^3*(143*A + 118*C)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(9009*d) + (2*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*
x])/(13*d) + (12*C*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(143*a*d) + (2*(143*A + 145*C)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*
x])*Sin[c + d*x])/(1287*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx=\frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d}+\frac{2\int c}{}$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d}+\frac{12C}{}$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d}+\frac{12C}{}$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d}+\frac{12C}{}$$

$$=\frac{40a^3(143A+118C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9009d}+\frac{2C\cos^{\frac{5}{2}}}{}$$

$$=\frac{40a^3(143A+118C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9009d}+\frac{2C\cos^{\frac{5}{2}}}{}$$

$$=\frac{4a^3(121A+95C)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d}+\frac{4a^3(221A}{}$$

$$=\frac{4a^3(221A+175C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{4a^3(121A+95C)}{231}$$

Mathematica [C] time = 6.33281, size = 1028, normalized size = 3.68

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2), x]

[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((221*A + 175*C)*Cot[c])/(390*d) + ((2134*A + 1811*C)*Cos[d*x]*Sin[c])/(7392*d) + ((7592*A + 7825*C)*Cos[2*d*x]*Sin[2*c])/(74880*d) + ((132*A + 215*C)*Cos[3*d*x]*Sin[3*c])/(4928*d) + ((13*A + 59*C)*Cos[4*d*x]*Sin[4*c])/(3744*d) + (3*C*Cos[5*d*x]*Sin[5*c])/(704*d) + (C*Cos[6*d*x]*Sin[6*c])/(1664*d) + ((2134*A + 1811*C)*Cos[c]*Sin[d*x])/(7392*d) + ((7592*A + 7825*C)*Cos[2*c]*Sin[2*d*x])/(74880*d) + ((132*A + 215*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + ((13*A + 59*C)*Cos[4*c]*Sin[4*d*x])/(3744*d) + (3*C*Cos[5*c]*Sin[5*d*x])/(704*d) + (C*Cos[6*c]*Sin[6*d*x])/(1664*d)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (95*C*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(462*d*Sqrt[1 + Cot[c]^2]) - (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d) - (35*C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(156*d)

Maple [A] time = 0.099, size = 464, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+1058400*C*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^12+(-80080*A-2122400*C)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*
x+1/2*c)+(314600*A+2331040*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-487
916*A-1535860*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(386386*A+633710*C
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-105534*A-121230*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+23595*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-51051*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+18525*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-40425*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^6+3\,Ca^3\cos\left(dx+c\right)^5+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^4+\left(3\,A+C\right)a^3\cos\left(dx+c\right)^3+3\,Aa^3\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^6 + 3*C*a^3*cos(d*x + c)^5 + (A + 3*C)*a^3*cos
(d*x + c)^4 + (3*A + C)*a^3*cos(d*x + c)^3 + 3*A*a^3*cos(d*x + c)^2 + A*a^3
*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2),
 x)
```

3.143 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=246

$$\frac{4a^3(143A+105C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^3(44A+35C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{385d}+\frac{2(3}{}$$

```
[Out] (4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(143*A + 105*C
)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(143*A + 105*C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(231*d) + (8*a^3*(44*A + 35*C)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(385*d) + (2*C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(11*d) + (4*C*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(33*a*d) + (2*(33*A + 35*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])
*Sin[c + d*x])/(231*d)
```

Rubi [A] time = 0.603976, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3046, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(143A+105C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^3(44A+35C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{385d}+\frac{2(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(143*A + 105*C
)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(143*A + 105*C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(231*d) + (8*a^3*(44*A + 35*C)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(385*d) + (2*C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(11*d) + (4*C*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(33*a*d) + (2*(33*A + 35*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])
*Sin[c + d*x])/(231*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
```

```
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx=\frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d}+\frac{2\int}{}$$

$$=\frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d}+\frac{4C\,c}{}$$

$$=\frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d}+\frac{4C\,c}{}$$

$$=\frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d}+\frac{4C\,c}{}$$

$$=\frac{8a^3(44A+35C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{385d}+\frac{2C\cos^{\frac{3}{2}}(c}{}$$

$$=\frac{8a^3(44A+35C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{385d}+\frac{2C\cos^{\frac{3}{2}}(c}{}$$

$$=\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(143A+105C)\sqrt{c}}{23}$$

$$=\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(143A+105C)F\Big(}{231d}$$

Mathematica [C] time = 6.30452, size = 982, normalized size = 3.99

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(7A+5C)\cot(c)}{10d}+\frac{(2354A+1953C)\cos(dx)\sin(c)}{7392d}+\frac{(18A+25C)\cos(2dx)\sin(2}{240d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((7*A + 5*
C)*Cot[c])/(10*d) + ((2354*A + 1953*C)*Cos[d*x]*Sin[c])/(7392*d) + ((18*A +
 25*C)*Cos[2*d*x]*Sin[2*c])/(240*d) + ((44*A + 189*C)*Cos[3*d*x]*Sin[3*c])/
(4928*d) + (C*Cos[4*d*x]*Sin[4*c])/(96*d) + (C*Cos[5*d*x]*Sin[5*c])/(704*d)
 + ((2354*A + 1953*C)*Cos[c]*Sin[d*x])/(7392*d) + ((18*A + 25*C)*Cos[2*c]*S
in[2*d*x])/(240*d) + ((44*A + 189*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + (C*Cos
[4*c]*Sin[4*d*x])/(96*d) + (C*Cos[5*c]*Sin[5*d*x])/(704*d)) - (13*A*(a + a*
Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (5
*C*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c
]^2]) - (7*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*S
qrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
 + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Si
n[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
 - (C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [A] time = 0.098, size = 436, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/1155*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(3360*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-14560*C*sin(1/2*d*x+1/2*c)^10*cos
(1/2*d*x+1/2*c)+(1320*A+25760*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-
4752*A-24080*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(6622*A+13090*C)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2288*A-2940*C)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+715*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+525*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1155*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{5}+3\,Ca^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+C\right)a^{3}\cos\left(dx+c\right)^{2}+3\,Aa^{3}\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
 x)
```

3.144 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{4a^3(21A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(27A+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(21A+16C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(63\ldots}{}$$

```
[Out] (4*a^3*(27*A + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21*A + 11*
C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(21*A + 16*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*
Sin[c + d*x])/(9*d) + (4*C*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Si
n[c + d*x])/(21*a*d) + (2*(63*A + 73*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c
 + d*x])*Sin[c + d*x])/(315*d)
```

Rubi [A] time = 0.579679, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3046, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(21A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(27A+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(21A+16C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(63\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (4*a^3*(27*A + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21*A + 11*
C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(21*A + 16*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*
Sin[c + d*x])/(9*d) + (4*C*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Si
n[c + d*x])/(21*a*d) + (2*(63*A + 73*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c
 + d*x])*Sin[c + d*x])/(315*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{2\int \frac{(a+a\cos(c+dx))^3(}{}}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{4C\sqrt{\cos(c+dx)}\,(}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{4C\sqrt{\cos(c+dx)}\,(}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{4C\sqrt{\cos(c+dx)}\,(}{}$$

$$= \frac{8a^3(21A+16C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\cos(c+dx)}(a+a\,c}{9}$$

$$= \frac{8a^3(21A+16C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\cos(c+dx)}(a+a\,c}{9}$$

$$= \frac{4a^3(27A+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(21A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [C] time = 6.37843, size = 936, normalized size = 4.39

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(27A+17C)\cot(c)}{30d} + \frac{(84A+97C)\cos(dx)\sin(c)}{336d} + \frac{(18A+73C)\cos(2dx)\sin(2c)}{720d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((27*A + 1
7*C)*Cot[c])/(30*d) + ((84*A + 97*C)*Cos[d*x]*Sin[c])/(336*d) + ((18*A + 73
*C)*Cos[2*d*x]*Sin[2*c])/(720*d) + (3*C*Cos[3*d*x]*Sin[3*c])/(112*d) + (C*C
os[4*d*x]*Sin[4*c])/(288*d) + ((84*A + 97*C)*Cos[c]*Sin[d*x])/(336*d) + ((1
8*A + 73*C)*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*C*Cos[3*c]*Sin[3*d*x])/(112*d
) + (C*Cos[4*c]*Sin[4*d*x])/(288*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (11*C*(a + a*Cos[c + d*x])^3*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (9*A*(a + a*Cos[c
+ d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
- Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (17*C*(a + a*Cos[c + d*
x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

Maple [A] time = 0.095, size = 408, normalized size = 1.9

$$-\frac{4a^3}{315d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Big(-560\,C\,(\sin(1/2\,dx+c/2))^{10}\cos(1/2\,dx+c/2)+2200\,C\,(\sin(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2200*C*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-252*A-3412*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(882*A
```

```
+2702*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-378*A-738*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+165*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^4+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^3+\left(3\,A+C\right)a^3\cos\left(dx+c\right)^2+3\,Aa^3\cos\left(\right.}{\sqrt{\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
 x)
```

3.145 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=217

$$\frac{4a^3(35A+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(35A-41C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}-\frac{2(35A}{}$$

```
[Out] (4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A + 13*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) - (4*a^3*(35*A - 41*C)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]]) - (2*(7*A - C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(7*a*d) - (2*(35*A - 11*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos
[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.587426, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3044, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(35A+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(35A-41C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}-\frac{2(35A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```



```
[Out] (4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A + 13*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) - (4*a^3*(35*A - 41*C)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]]) - (2*(7*A - C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(7*a*d) - (2*(35*A - 11*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos
[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
```

```
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int\frac{(a+a\cos(c+dx))^3\left(3aA-\frac{1}{2}a(7A-C)\right.}{\sqrt{\cos(c+dx)}}}{a}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(7A-C)\sqrt{\cos(c+dx)}\left(a^2+a\right.}{7ad}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(7A-C)\sqrt{\cos(c+dx)}\left(a^2+a\right.}{7ad}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(7A-C)\sqrt{\cos(c+dx)}\left(a^2+a\right.}{7ad}$$

$$= -\frac{4a^3(35A-41C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2A(a+a\cos(c+dx))^3}{d\sqrt{\cos(c+d}}$$

$$= -\frac{4a^3(35A-41C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2A(a+a\cos(c+dx))^3}{d\sqrt{\cos(c+d}}$$

$$= \frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(35A+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} -$$

Mathematica [C] time = 6.48084, size = 926, normalized size = 4.27

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(-\frac{(15\cos(2c)A+5A+14C+14C\cos(2c))\csc(c)\sec(c)}{40d} + \frac{A\sec(c+dx)\sin(dx)\sec(}{4d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((5*A + 14
*C + 15*A*Cos[2*c] + 14*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + ((28*A + 107*C)
*Cos[d*x]*Sin[c])/(336*d) + (3*C*Cos[2*d*x]*Sin[2*c])/(40*d) + (C*Cos[3*d*x
]*Sin[3*c])/(112*d) + ((28*A + 107*C)*Cos[c]*Sin[d*x])/(336*d) + (A*Sec[c]*
Sec[c + d*x]*Sin[d*x])/(4*d) + (3*C*Cos[2*c]*Sin[2*d*x])/(40*d) + (C*Cos[3*
c]*Sin[3*d*x])/(112*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) - (13*C*(a + a*Cos[c + d*x])^3*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^3*Csc
[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
 ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (7*C*(a + a*Cos[c + d*x])^3*Csc[c]*Se
c[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTa
n[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 0.106, size = 569, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/105*a^3*(120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+43*C)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)-4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(35
*A+52*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-105*A*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+175*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-147*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+65*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^4+(A+3\,C)a^3\cos\left(dx+c\right)^3+(3\,A+C)a^3\cos\left(dx+c\right)^2+3\,Aa^3\cos\left(\right.}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
 x)
```

3.146 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(5A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{8a^3(10A-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(35A-3}{}$$

```
[Out] (-4*a^3*(5*A - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*C)*E
llipticF[(c + d*x)/2, 2])/(3*d) - (8*a^3*(10*A - 3*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(3*d*Cos[c +
 d*x]^(3/2)) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(a*d*Sqrt[Cos[
c + d*x]]) - (2*(35*A - 3*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d)
```

Rubi [A] time = 0.58973, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3044, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(5A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{8a^3(10A-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(35A-3}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```



```
[Out] (-4*a^3*(5*A - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*C)*E
llipticF[(c + d*x)/2, 2])/(3*d) - (8*a^3*(10*A - 3*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(3*d*Cos[c +
 d*x]^(3/2)) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(a*d*Sqrt[Cos[
c + d*x]]) - (2*(35*A - 3*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d)
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^3\left(3aA-\frac{1}{2}a(5A-3C)\,c\right.}{\cos^{\frac{3}{2}}(c+dx)}}{3a}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{ad\sqrt{\cos(c+dx)}}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{ad\sqrt{\cos(c+dx)}}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{8a^3(10A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\cos(c+dx))^3\sin}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{8a^3(10A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\cos(c+dx))^3\sin}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(5A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{8a}{}$$

Mathematica [C] time = 6.53659, size = 909, normalized size = 4.31

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{12d} + \frac{\sec(c)(A\sin(c)+9A\sin(dx))\sec(c+dx)}{12d} - \frac{(5\cos(2c)}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-25*A +
18*C + 5*A*Cos[2*c] + 18*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (C*Cos[d*x]*Si
```

```
n[c])/(4*d) + (C*Cos[2*d*x]*Sin[2*c])/(40*d) + (C*Cos[c]*Sin[d*x])/(4*d) +
(A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c]
+ 9*A*Sin[d*x]))/(12*d) + (C*Cos[2*c]*Sin[2*d*x])/(40*d)) - (5*A*(a + a*Cos
[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) - (C*(a
+ a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) +
 (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (9*C*(a
 + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 0.115, size = 704, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -4/15*(24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A+13*C)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A+9*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(25*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*A*EllipticE(cos(1/2*
```

```
d*x+1/2*c),2^(1/2))+15*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*C*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+25*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-27*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^4+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^3+\left(3\,A+C\right)a^3\cos\left(dx+c\right)^2+3\,Aa^3\cos\left(dx\right.}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
```

```
cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
 x)
```

3.147 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{4a^3(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(21A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(11A+5C}{}$$

```
[Out] (-4*a^3*(9*A - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*C)*E
llipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(21*A + 5*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c +
 d*x]^(5/2)) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(5*a*d*Cos[c +
 d*x]^(3/2)) + (2*(11*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d*
Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.585176, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3044, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(21A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(11A+5C}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```



```
[Out] (-4*a^3*(9*A - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*C)*E
llipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(21*A + 5*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c +
 d*x]^(5/2)) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(5*a*d*Cos[c +
 d*x]^(3/2)) + (2*(11*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d*
Sqrt[Cos[c + d*x]])
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^3\left(3aA-\frac{1}{2}a(3A-5C)\,c\right.}{\cos^{\frac{5}{2}}(c+dx)}}{5a}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{5ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{5ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{5ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(21A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\cos(c+dx))^3\,\mathrm{si}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{4a^3(21A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\cos(c+dx))^3\,\mathrm{si}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{4a^3(9A-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{4a}{}$$

Mathematica [C] time = 6.57615, size = 905, normalized size = 4.25

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{20d} + \frac{\sec(c)(A\sin(c)+5A\sin(dx))\sec^2(c+dx)}{20d} + \frac{\sec(c)(5A}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-36*A +
5*C + 15*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (C*Cos[d*x]*Sin[c])/(12*d) + (
C*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Se
c[c]*Sec[c + d*x]^2*(A*Sin[c] + 5*A*Sin[d*x]))/(20*d) + (Sec[c]*Sec[c + d*x
]*(5*A*Sin[c] + 18*A*Sin[d*x] + 5*C*Sin[d*x]))/(20*d)) - (A*(a + a*Cos[c +
d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*C*(a + a
*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (9
*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{
-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Ta
n[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[
c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (C*(a +
 a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] time = 0.273, size = 939, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(40*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+60*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+108*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^4-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+100*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*A*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-100*C*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1
/2*c)^2+90*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*C*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5 + 3\,Ca^3\cos(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^3 + (3\,A+C)a^3\cos(dx+c)^2 + 3\,Aa^3\cos(\right.$$
$$\cos(dx+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c) + a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
 x)
```

3.148 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{4a^3(13A+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7A+5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(}{}$$

```
[Out] (-4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 35*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(53*A + 70*C)*Sin[c + d*x])/(10
5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d*Co
s[c + d*x]^(7/2)) + (12*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(35*a*d*
Cos[c + d*x]^(5/2)) + (2*(7*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])
/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.612531, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3044, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(13A+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7A+5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 35*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(53*A + 70*C)*Sin[c + d*x])/(10
5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d*Co
s[c + d*x]^(7/2)) + (12*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(35*a*d*
Cos[c + d*x]^(5/2)) + (2*(7*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])
/(15*d*Cos[c + d*x]^(3/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int\frac{(a+a\cos(c+dx))^3\left(3aA-\frac{1}{2}a(A-7C)\,\mathrm{co}\right.}{\cos^{\frac{7}{2}}(c+dx)}}{7a}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8a^3(53A+70C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{}$$

$$= \frac{8a^3(53A+70C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{}$$

$$= -\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(13A+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} +$$

Mathematica [C] time = 6.67996, size = 920, normalized size = 4.32

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{28d}+\frac{\sec(c)(5A\sin(c)+21A\sin(dx))\sec^3(c+dx)}{140d}+\frac{\sec(c}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-28*A -
25*C + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d
*x])/(28*d) + (Sec[c]*Sec[c + d*x]^3*(5*A*Sin[c] + 21*A*Sin[d*x]))/(140*d)
+ (Sec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 130*A*Sin[d*x] + 35*C*Sin[d*x]))/(4
20*d) + (Sec[c]*Sec[c + d*x]*(130*A*Sin[c] + 35*C*Sin[c] + 294*A*Sin[d*x] +
 315*C*Sin[d*x]))/(420*d)) - (13*A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (5*C*(a + a*Cos[c + d*x])^3*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (7*A*(a + a*Cos[c + d*x])^
3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (C*(a + a*Cos[c + d*x])^3*Csc[c
]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] time = 0.374, size = 1012, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/40*A/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+(3/8*A+1/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*C+
1/8*A)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^{3}\cos\left(dx+c\right)^{5}+3\,Ca^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+C\right)a^{3}\cos\left(dx+c\right)^{2}+3\,Aa^{3}\cos\left(\right.}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2),
 x)
```

3.149 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=246

$$\frac{4a^3(11A+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(16A+21C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(73A+63C)\sin}{315}$$

```
[Out] (-4*a^3*(17*A + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 21
*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(16*A + 21*C)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(17*A + 27*C)*Sin[c + d*x])/(15*d*Sqrt[
Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2)) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*
x]^(7/2)) + (2*(73*A + 63*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*
Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.636389, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3044, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(11A+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(16A+21C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(73A+63C)\sin}{315}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```


```
[Out] (-4*a^3*(17*A + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 21
*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(16*A + 21*C)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(17*A + 27*C)*Sin[c + d*x])/(15*d*Sqrt[
Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2)) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*
x]^(7/2)) + (2*(73*A + 63*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*
Cos[c + d*x]^(5/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^3\left(3aA+\frac{1}{2}a(A+9C)\cos\right.}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{21ad\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{21ad\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{21ad\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{8a^3(16A+21C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{4}{}$$

$$= \frac{8a^3(16A+21C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{4}{}$$

$$= \frac{4a^3(11A+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{8a^3(16A+21C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^3}{}$$

$$= -\frac{4a^3(17A+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(11A+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [C] time = 6.72268, size = 955, normalized size = 3.88

$$\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^5(c+dx)}{36d} + \frac{\sec(c)(7A\sin(c)+27A\sin(dx))\sec^4(c+dx)}{252d} + \frac{\sec(c)}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((17*A + 27
*C)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d) + (Se
c[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 27*A*Sin[d*x]))/(252*d) + (Sec[c]*Sec[c +
 d*x]^3*(135*A*Sin[c] + 238*A*Sin[d*x] + 63*C*Sin[d*x]))/(1260*d) + (Sec[c]
*Sec[c + d*x]^2*(238*A*Sin[c] + 63*C*Sin[c] + 330*A*Sin[d*x] + 315*C*Sin[d*
x]))/(1260*d) + (Sec[c]*Sec[c + d*x]*(110*A*Sin[c] + 105*C*Sin[c] + 238*A*S
in[d*x] + 378*C*Sin[d*x]))/(420*d)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (C*(a + a*Cos[c + d*x])^3*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (17*A*(a + a*Cos[c
+ d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
- Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d) + (9*C*(a + a*Cos[c + d*x
])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 0.394, size = 1246, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/5*(3/8*A+1/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(3/8*C+1/8*A)*(-1/6*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))+3/8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+1/8*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-
7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+3/8*A*(-1/56*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1
/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5+3\,Ca^3\cos(dx+c)^4+(A+3\,C)a^3\cos(dx+c)^3+(3\,A+C)a^3\cos(dx+c)^2+3\,Aa^3\cos(\cdots}{\cos(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2)
, x)
```

3.150 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{4a^3(105A+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(105A+143C)\sin(c+dx)}{231d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(35A+44C}{385d\cos^{\frac{5}{2}}}$$

```
[Out] (-4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(105*A + 143*
C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^3*(35*A + 44*C)*Sin[c + d*x])/
(385*d*Cos[c + d*x]^(5/2)) + (4*a^3*(105*A + 143*C)*Sin[c + d*x])/(231*d*Co
s[c + d*x]^(3/2)) + (4*a^3*(5*A + 7*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) +
 (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(33*a*d*Cos[c + d*x]^(9/2))
+ (2*(35*A + 33*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(231*d*Cos[c + d*
x]^(7/2))
```

Rubi [A] time = 0.670396, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3044, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^3(105A+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(105A+143C)\sin(c+dx)}{231d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(35A+44C}{385d\cos^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```


```
[Out] (-4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(105*A + 143*
C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^3*(35*A + 44*C)*Sin[c + d*x])/
(385*d*Cos[c + d*x]^(5/2)) + (4*a^3*(105*A + 143*C)*Sin[c + d*x])/(231*d*Co
s[c + d*x]^(3/2)) + (4*a^3*(5*A + 7*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) +
 (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(33*a*d*Cos[c + d*x]^(9/2))
+ (2*(35*A + 33*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(231*d*Cos[c + d*
x]^(7/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^3\left(3aA+\frac{1}{2}a(3A+11C)\right.}{\cos^{\frac{11}{2}}(c+dx)}}{11a}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{33ad\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{33ad\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{33ad\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{8a^3(35A+44C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4}{}$$

$$= \frac{8a^3(35A+44C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4}{}$$

$$= \frac{8a^3(35A+44C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^3(105A+143C)\sin(c+dx)}{231d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^3(5}{}$$

$$= -\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(105A+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}$$

Mathematica [C] time = 6.8029, size = 997, normalized size = 3.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/
2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((5*A + 7*C
)*Csc[c]*Sec[c])/(10*d) + (A*Sec[c]*Sec[c + d*x]^6*Sin[d*x])/(44*d) + (Sec[
```

```
c]*Sec[c + d*x]^5*(3*A*Sin[c] + 11*A*Sin[d*x]))/(132*d) + (Sec[c]*Sec[c + d
*x]^4*(77*A*Sin[c] + 126*A*Sin[d*x] + 33*C*Sin[d*x]))/(924*d) + (Sec[c]*Sec
[c + d*x]^3*(630*A*Sin[c] + 165*C*Sin[c] + 770*A*Sin[d*x] + 693*C*Sin[d*x])
)/(4620*d) + (Sec[c]*Sec[c + d*x]^2*(770*A*Sin[c] + 693*C*Sin[c] + 1050*A*S
in[d*x] + 1430*C*Sin[d*x]))/(4620*d) + (Sec[c]*Sec[c + d*x]*(525*A*Sin[c] +
 715*C*Sin[c] + 1155*A*Sin[d*x] + 1617*C*Sin[d*x]))/(2310*d)) - (5*A*(a + a
*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c]^2]) - (
13*C*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot
[c]^2]) + (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*S
qrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
 + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Si
n[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
+ (7*C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricP
FQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
+ Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*
Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2
))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 0.463, size = 1408, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/5*(3/
8*C+1/8*A)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1
```

```
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*A*(-1/352*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/154*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+15/
77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+3/8*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*C*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+3/8*A*(-1/144*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+(3
/8*A+1/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algori
```

```
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(13/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5 + 3\,Ca^3\cos(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^3 + (3\,A+C)a^3\cos(dx+c)^2 + 3\,Aa^3\cos(d}{\cos(dx+c)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
cos(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(13/2)
, x)
```

3.151 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{5(7A+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A+9C)\sin(c+dx}{7ad}$$

```
[Out] (-3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(7*A + 9*C)*Ellipti
cF[(c + d*x)/2, 2])/(21*a*d) + (5*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(21*a*d) - ((5*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((7*
A + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*a*d) - ((A + C)*Cos[c + d*x]^(
7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.216344, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2748, 2635, 2639, 2641}

$$\frac{5(7A+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A+9C)\sin(c+dx}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```


```
[Out] (-3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(7*A + 9*C)*Ellipti
cF[(c + d*x)/2, 2])/(21*a*d) + (5*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(21*a*d) - ((5*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((7*
A + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*a*d) - ((A + C)*Cos[c + d*x]^(
7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(5A+7C)+\frac{1}{2}a(7\ldots\right.}{a^2}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A+7C)\int\cos^{\frac{5}{2}}(c+dx)\,dx}{2a} + \frac{(7A+\ldots}{}$$

$$= -\frac{(5A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{(7A+9C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7ad} \ldots$$

$$= -\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(7A+9C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21ad} - \frac{(5A\ldots}{}$$

$$= -\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(7A+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad} + \frac{5(7A+9C)\sqrt{\ldots}}{}$$

Mathematica [C] time = 6.62403, size = 1219, normalized size = 6.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),
x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) -
(((21*I)/20)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
 (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(5*A + 5*C + 10*A*Cos[c] + 16
*C*Cos[c])*Csc[c])/(5*d) + ((28*A + 51*C)*Cos[d*x]*Sin[c])/(21*d) - (2*C*Co
s[2*d*x]*Sin[2*c])/(5*d) + (C*Cos[3*d*x]*Sin[3*c])/(7*d) + (2*Sec[c/2]*Sec[
c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + ((28*A + 51*C)*Cos[c]
*Sin[d*x])/(21*d) - (2*C*Cos[2*c]*Sin[2*d*x])/(5*d) + (C*Cos[3*c]*Sin[3*d*x
])/(7*d)))/(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (15*C*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(7*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.112, size = 295, normalized size = 1.5

$$-\frac{1}{105\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\,(17$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(175*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+225*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*C*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))-480*C*sin(1/2*d*x+1/2*c)^10+864*C*sin(1/2*d*x+1/2*c)
^8+(-280*A-888*C)*sin(1/2*d*x+1/2*c)^6+(630*A+930*C)*sin(1/2*d*x+1/2*c)^4+(
-245*A-321*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^4+A\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a*cos(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x
)
```

3.152 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A+7C)\sin(c+dx)}{5ad}$$

```
[Out] (3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A + 5*C)*EllipticF[
(c + d*x)/2, 2])/(3*a*d) - ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a*d) + ((5*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.187823, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2748, 2635, 2641, 2639}

$$-\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A+7C)\sin(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```



```
[Out] (3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A + 5*C)*EllipticF[
(c + d*x)/2, 2])/(3*a*d) - ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a*d) + ((5*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(3A+5C)+\frac{1}{2}a(\right.}{a^2}$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A+5C)\int\cos^{\frac{3}{2}}(c+dx)\,dx}{2a}+\frac{(5A+}{}$$

$$= -\frac{(3A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{(5A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}$$

$$= \frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(3A+5C)\sqrt{\text{cc}}}{}$$

Mathematica [C] time = 6.56582, size = 1170, normalized size = 7.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),
x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (
((21*I)/20)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(5*A + 5*C + 10*A*Cos[c] + 16
*C*Cos[c])*Csc[c])/(5*d) - (4*C*Cos[d*x]*Sin[c])/(3*d) + (2*C*Cos[2*d*x]*Si
n[2*c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x
)/2]))/d - (4*C*Cos[c]*Sin[d*x])/(3*d) + (2*C*Cos[2*c]*Sin[2*d*x])/(5*d)))/
(a + a*Cos[c + d*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a
*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (5*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.109, size = 276, normalized size = 1.7

$$\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(15\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*C*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))-48*C*sin(1/2*d*x+1/2*c)^8+56*C*sin(1/2*d*x+1/2*c)^6+(30*A+
30*C)*sin(1/2*d*x+1/2*c)^4+(-15*A-23*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a*cos(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x
)
```

3.153 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(3A+5C)\sin(c+dx)\sqrt{c}}{3ad}$$

```
[Out] -(((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((3*A + 5*C)*EllipticF[(c
+ d*x)/2, 2])/(3*a*d) + ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.17238, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2748, 2639, 2635, 2641}

$$\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(3A+5C)\sin(c+dx)\sqrt{c}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),x]
```



```
[Out] -(((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((3*A + 5*C)*EllipticF[(c
+ d*x)/2, 2])/(3*a*d) + ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\left(-\frac{1}{2}a(A+3C)+\frac{1}{2}a(3A\right.}{a^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A+3C)\int\sqrt{\cos(c+dx)}\,dx}{2a} + \frac{(3A+5}{}$$

$$= -\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(A+C)}{d}$$

$$= -\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(3A+5C)\sqrt{\cos(}}{3}$$

Mathematica [C] time = 6.49451, size = 1126, normalized size = 9.23

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x]),
x]
```

```
[Out] ((-I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (((3
*I)/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(A + C + 2*C*Cos[c])*Csc[c])/d + (4
*C*Cos[d*x]*Sin[c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2]
+ C*Sin[(d*x)/2]))/d + (4*C*Cos[c]*Sin[d*x])/(3*d)))/(a + a*Cos[c + d*x]) -
 (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1
+ Cot[c]^2]) - (5*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos
[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.11, size = 262, normalized size = 2.2

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\,\left(3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*C*sin(1/2*d*x+1/2*c)^6+(6*A+18*C)*sin(1/2*d*x+1/2*c)^4+(-3*A
-7*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x
)
```

3.154 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=83

$$\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] ((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A - C)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rubi [A] time = 0.154196, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3042, 2748, 2641, 2639}

$$\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A - C)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}a(A-C)+\frac{1}{2}a(A+3C)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{(A-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{(A+3C)\int \sqrt{\ldots}}{2\ldots}$$

$$= \frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sqrt{\cos(c+dx)}\ldots}{d(a+a\cos(c+\ldots}$$

Mathematica [C] time = 6.47345, size = 1095, normalized size = 13.19

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),
x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
```

```
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (((3*
I)/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c
/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A + C + 2*C*Cos[c])*Csc[c])/d - (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d))/(a + a*
Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c
+ d*x])*Sqrt[1 + Cot[c]^2]) + (C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d
*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.118, size = 247, normalized size = 3.

$$\frac{1}{ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A+2*C)*sin(1/2*d*x+1/2*c)^4+(-A-C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2+A\right)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^2 + a*co
s(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
 x)
```

3.155 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=113

$$-\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

```
[Out] -(((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - C)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((3*A + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A +
 C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.172951, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2748, 2636, 2639, 2641}

$$-\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```



```
[Out] -(((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - C)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((3*A + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A +
 C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A+C)-\frac{1}{2}a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{(A-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{(3A+C)\int \frac{}{\cos^{\frac{3}{2}}}}{2a}$$

$$= -\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c}$$

$$= -\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} -$$

Mathematica [C] time = 6.61113, size = 1128, normalized size = 9.98

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),
x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) -
((I/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((2*A + A*Cos[c] + C*Cos[c])*Csc[c/2]*
Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin
[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(a + a*Cos[c + d*x])
 + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]) - (C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.196, size = 316, normalized size = 2.8

$$-\frac{1}{ad}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A+C)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+C)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3 + a\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^3 + a*co
s(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
 x)
```

3.156 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A+3C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-($$

```
[Out] ((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A + 3*C)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- ((3*A + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.193421, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2748, 2636, 2641, 2639}

$$\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A+3C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-($$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```


```
[Out] ((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A + 3*C)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- ((3*A + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A+3C)-\frac{1}{2}a(3A+C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(3A+C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(5A+3C)\int}{2}$$

$$= \frac{(5A+3C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(3A+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+}$$

$$= \frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A+3C)\sin(c+d}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 6.89749, size = 1163, normalized size = 7.75

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),
x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (
(I/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c
/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(-(((2*A + A*Cos[c] + C*Cos[c])*Csc[c/2]
*Sec[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*S
in[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Se
c[c + d*x]*(A*Sin[c] - 3*A*Sin[d*x]))/(3*d)))/(a + a*Cos[c + d*x]) - (5*A*C
os[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + C
ot[c]^2]) - (C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*
x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 0.387, size = 486, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*A*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+
1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(A+C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^4 + a*co
s(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
 x)
```

3.157 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=192

$$-\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(5A+3C)\sin(c+dx}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A + 3*C)*EllipticF
[(c + d*x)/2, 2])/(3*a*d) + ((7*A + 5*C)*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^
(5/2)) - ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) + (3*(7*A +
5*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x])/(d*C
os[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.211847, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2748, 2636, 2639, 2641}

$$-\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(5A+3C)\sin(c+dx}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Cos[c + d*x])),x]
```


```
[Out] (-3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A + 3*C)*EllipticF
[(c + d*x)/2, 2])/(3*a*d) + ((7*A + 5*C)*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^
(5/2)) - ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) + (3*(7*A +
5*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x])/(d*C
os[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(7A+5C)-\frac{1}{2}a(5A+3C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(5A+3C)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} + \frac{(7A+5C)\ \ldots}{}$$

$$= \frac{(7A+5C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{(5A+3C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+\ldots}$$

$$= -\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(7A+5C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{(5A+3C)\sin(c+dx\ldots}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(7A+5C)\sin(c\ldots}{5ad\cos^{\frac{5}{2}}(c+\ldots}$$

<u>Mathematica</u> [C] time = 7.09504, size = 1207, normalized size = 6.29

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Cos[c + d*x])),
x]
```

```
[Out] (((-21*I)/20)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])
- (((3*I)/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
```

```
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
 (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((16*A + 10*C + 5*A*Cos[c] + 5*C
*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(
A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/
(5*d) + (4*Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] - 5*A*Sin[d*x]))/(15*d) - (4*S
ec[c]*Sec[c + d*x]*(5*A*Sin[c] - 24*A*Sin[d*x] - 15*C*Sin[d*x]))/(15*d)))/(
a + a*Cos[c + d*x]) + (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a
+ a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 0.365, size = 803, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-2/5*A/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)-2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(2*A+2*
C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
```

```
^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)+(-A-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^5+a\cos\left(dx+c\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^5 + a*co
s(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + A}{(a\cos{(dx+c)} + a)\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(7/2)),
 x)
```

3.158 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=196

$$-\frac{5(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4(5A+14C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A+3C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{4(5A+14C)\sin(c}{15}$$

```
[Out] (4*(5*A + 14*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(A + 3*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*a^2*d) + (4*(5*A + 14*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
- ((A + 3*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) -
((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.35062, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4(5A+14C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A+3C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{4(5A+14C)\sin(c}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (4*(5*A + 14*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(A + 3*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*a^2*d) + (4*(5*A + 14*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
- ((A + 3*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) -
((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}dx = -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(A+7C)+\frac{1}{2}a(5A+11C)\cos(c-}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(A+3C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+3C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} -$$

$$= -\frac{5(A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{4(5A+14C)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{15a^2d}$$

$$= \frac{4(5A+14C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(A+3C)\sqrt{c}}{}$$

Mathematica [C] time = 6.76216, size = 1248, normalized size = 6.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
2,x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (((
28*I)/5)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
```

```
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (
10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sq
rt[1 + Cot[c]^2]) + (10*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a
*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d
*x]]*((-8*(5*A + 10*C + 5*A*Cos[c] + 18*C*Cos[c])*Csc[c])/(5*d) - (16*C*Cos
[d*x]*Sin[c])/(3*d) + (4*C*Cos[2*d*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (8*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(A*Sin[(d*x)/2] + 2*C*Sin[(d*x)/2]))/d - (16*C*Cos[c]*Sin[d*x]
)/(3*d) + (4*C*Cos[2*c]*Sin[2*d*x])/(5*d) + (2*(A + C)*Sec[c/2 + (d*x)/2]^2
*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A] time = 0.133, size = 451, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*C*cos(1/2
*d*x+1/2*c)^10+352*C*cos(1/2*d*x+1/2*c)^8+120*A*cos(1/2*d*x+1/2*c)^6+50*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+120*A*cos(1/2*d*x+1/2*c)^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-120*C*cos(1/2*d*x+1/2*c)^6+150*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*cos(1/2*d*x+1/2*c)^3+336*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-190*A*cos(1/2*d*x+1/2*c)^4-266*C*cos(1/2*d*x+1/2*c)^4+75*A*cos(1/2*d
*x+1/2*c)^2+135*C*cos(1/2*d*x+1/2*c)^2-5*A-5*C)/a^2/cos(1/2*d*x+1/2*c)^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^2*cos(
d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2,
 x)
```

3.159 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{2(A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{2(A+5C)\sin(c+dx)\sqrt{}}{3a^2d}$$

```
[Out] -(((A + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (2*(A + 5*C)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + (2*(A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d) - ((A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
+ d*x])) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```

Rubi [A] time = 0.333059, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2977, 2748, 2639, 2635, 2641}

$$\frac{2(A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{2(A+5C)\sin(c+dx)\sqrt{}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] -(((A + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (2*(A + 5*C)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + (2*(A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d) - ((A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
+ d*x])) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(A-5C)+\frac{3}{2}a(A+3C)\cos(c+dx)\right)}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \cdots$$

$$= -\frac{(A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \cdots$$

$$= -\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{(A+7\cdots}{3\cdots}$$

$$= -\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{2(A+5C)\sqrt{\cos\cdots}}{}$$

Mathematica [C] time = 6.67069, size = 1209, normalized size = 7.51

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
2,x]
```

```
[Out] ((-I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - ((
(7*I)/2)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
```

```
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (
4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) - (20*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((4*(A + 3*C + 4*C*Cos[c])*Csc[c])/d + (8*C*Cos[d*x]*Sin[c])/(3*d) -
(2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) +
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/d + (8
*C*Cos[c]*Sin[d*x])/(3*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)
))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.138, size = 437, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C*cos(1/2*
d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3+6*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*cos(
1/2*d*x+1/2*c)^6+20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+42*C*c
os(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4-48*
C*cos(1/2*d*x+1/2*c)^4+9*A*cos(1/2*d*x+1/2*c)^2+21*C*cos(1/2*d*x+1/2*c)^2-A
-C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^2*cos(d*
x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2,
 x)
```

3.160 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{(A-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{4CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (4*C*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A - 5*C)*EllipticF[(c + d*x)/2,
 2])/(3*a^2*d) + ((A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[
c + d*x])^2)
```

Rubi [A] time = 0.287261, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2977, 2748, 2641, 2639}

$$\frac{(A-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{4CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (4*C*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A - 5*C)*EllipticF[(c + d*x)/2,
 2])/(3*a^2*d) + ((A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[
c + d*x])^2)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-C)+\frac{1}{2}a(A+7C)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,d}{3a^2}$$

$$= \frac{(A-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(A-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A}{}$$

$$= \frac{4CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(A-5C)\sqrt{\cos(c+dx)}\sin}{3a^2d(1+\cos(c+dx}$$

Mathematica [C] time = 6.53696, size = 814, normalized size = 6.46

$$\frac{2iC\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\sqrt{e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)+1}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}-\frac{2\,{}_2F_1(\cdots}{}\right.}{(\cos(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
2,x]
```

```
[Out] ((2*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2*
A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[
1 + Cot[c]^2]) + (10*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*
x]]*((-8*C*Cot[c/2])/d - (8*C*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d +
 (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d)
+ (2*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.137, size = 348, normalized size = 2.8

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2A\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\mathrm{Elliptic}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*C*cos(1/2*d*x+1/2*c)^6-10*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+38*C*cos(1/2*d*x+1/2*c)^4-3*A*co
s(1/2*d*x+1/2*c)^2-15*C*cos(1/2*d*x+1/2*c)^2+A+C)/a^2/cos(1/2*d*x+1/2*c)^3/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2 + 2*
a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{\cos(dx+c)}}{\left(a\cos(dx+c)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2,
 x)
```

3.161 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=125

$$\frac{2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(A + C)*EllipticF[(c + d*x
)/2, 2])/(3*a^2*d) - ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 +
Cos[c + d*x])) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[
c + d*x])^2)
```

Rubi [A] time = 0.299215, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2978, 2748, 2641, 2639}

$$\frac{2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] ((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(A + C)*EllipticF[(c + d*x
)/2, 2])/(3*a^2*d) - ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 +
Cos[c + d*x])) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[
c + d*x])^2)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{1}{2}a(5A-C)-\frac{1}{2}a(A-5C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+ \int$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(A}{}$$

$$= \frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-C)\sqrt{\cos(c+dx)}}{a^2d(1+\cos(c+}$$

Mathematica [C] time = 6.56965, size = 1176, normalized size = 9.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2
),x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - ((I
/2)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*
I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-
1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*A*C
os[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 +
 Cot[c]^2]) - (4*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[
c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*
((-4*(A - C)*Csc[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - C
*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Si
n[(d*x)/2]))/(3*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a +
 a*Cos[c + d*x])^2
```

Maple [B] time = 0.133, size = 419, normalized size = 3.4

$$\frac{1}{6\,a^{2}d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^{6}-4\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^6-4*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-16*A*cos(1/2*d*x+1/2*c)^4+20*C*cos(1/2*d*x+1/2*c)^4+3*A*co
s(1/2*d*x+1/2*c)^2-9*C*cos(1/2*d*x+1/2*c)^2+A+C)/a^2/cos(1/2*d*x+1/2*c)^3/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{3}+2\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3 + 2*
a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))
), x)
```

3.162 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{(5A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{4AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4A\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(}{3d\sqrt{\cos(}}$$

```
[Out] (-4*A*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - C)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (4*A*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((5*A - C
)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - ((A + C)*
Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.323075, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2978, 2748, 2636, 2639, 2641}

$$-\frac{(5A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{4AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4A\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(}{3d\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (-4*A*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - C)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (4*A*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((5*A - C
)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - ((A + C)*
Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A+C)-\frac{3}{2}a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(5A-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} +$$

$$= -\frac{(5A-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} +$$

$$= -\frac{(5A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4A\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(5A-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}$$

$$= -\frac{4AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4A\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{}{3a^2d}$$

Mathematica [C] time = 6.63269, size = 834, normalized size = 5.38

$$-\frac{2iA\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\sqrt{e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)+1}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)} - \frac{2\,{}_2F}{}\right.}{(\cos(c+dx)a+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2
),x]
```

```
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (1
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
```

```
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a
*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d
*x]]*((8*A*Cot[c/2]*Sec[c])/d + (8*A*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/
2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))
/(3*d) + (8*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*(A + C)*Sec[c/2 + (d*x)/
2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.152, size = 452, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)-48*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(43*A+C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(37*A+C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{4}+2\,a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4 + 2*
a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)
), x)
```

3.163 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{2(5A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{2(5A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] ((7*A + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(5*A + C)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) + (2*(5*A + C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(
3/2)) - ((7*A + C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((7*A + C)*Si
n[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A + C)*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.352324, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2978, 2748, 2636, 2641, 2639}

$$\frac{2(5A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{2(5A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] ((7*A + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(5*A + C)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) + (2*(5*A + C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(
3/2)) - ((7*A + C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((7*A + C)*Si
n[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A + C)*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A+C)-\frac{1}{2}a(5A-C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(7A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} +$$

$$= -\frac{(7A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} +$$

$$= \frac{2(5A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(7A+C)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(7A+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c$$

$$= \frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(5A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{2(5A+C)\sin(c+}{3a^2d\cos^{\frac{3}{2}}(c+dx}$$

<u>Mathematica</u> [C] time = 7.18574, size = 1245, normalized size = 6.59

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2
),x]
```

```
[Out] (((7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 +
 ((I/2)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
```

```
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) - (4*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a
*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d
*x]]*((-2*(4*A + 3*A*Cos[c] + C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d - (2*Se
c[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (4*S
ec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*A*Se
c[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 6*
A*Sin[d*x]))/(3*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a +
 a*Cos[c + d*x])^2
```

Maple [B] time = 0.357, size = 738, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A+
C)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c
)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+4
*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-8*A*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
```

```
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+4*A*(cos(1/2*d*x+1/2*c)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{5}+2\,a^{2}\cos\left(dx+c\right)^{4}+a^{2}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5 + 2*
a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)
), x)
```

3.164 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=250

$$-\frac{(13A+63C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A+33C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A+63C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{7(7A+33C}{}$$

```
[Out] (7*(7*A + 33*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A + 63*C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A + 63*C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + (7*(7*A + 33*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*a^
3*d) - ((A + C)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^
3) - (2*(A + 6*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d
*x])^2) - ((13*A + 63*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```

Rubi [A] time = 0.541373, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2977, 2748, 2635, 2641, 2639}

$$-\frac{(13A+63C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A+33C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A+63C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{7(7A+33C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (7*(7*A + 33*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A + 63*C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A + 63*C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + (7*(7*A + 33*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*a^
3*d) - ((A + C)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^
3) - (2*(A + 6*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d
*x])^2) - ((13*A + 63*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}dx = -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{1}{2}a(A-9C)+\frac{5}{2}a(A+3C)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(A+6C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(A+6C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(A+6C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= -\frac{(13A+63C)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} + \frac{7(7A+33C)\cos^{\frac{3}{2}}(c+dx)\sin(c}{30a^3d}$$

$$= \frac{7(7A+33C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A+63C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(13A+63C}{}$$

Mathematica [C] time = 7.0215, size = 1333, normalized size = 5.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 + (((231*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
```

```
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) + (42*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-4*(29*A + 99*C + 20*A*Cos[c] + 132*C*Cos[c])*Csc[c])/(5*d)
- (16*C*Cos[d*x]*Sin[c])/d + (8*C*Cos[2*d*x]*Sin[2*c])/(5*d) - (2*Sec[c/2]*
Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]
*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2] + 12*C*Sin[(d*x)/2]))/(15*d) - (4*S
ec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] + 99*C*Sin[(d*x)/2]))/(5*d) -
 (16*C*Cos[c]*Sin[d*x])/d + (8*C*Cos[2*c]*Sin[2*d*x])/(5*d) + (8*(7*A + 12*
C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^4*
Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 0.144, size = 479, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-192*C*cos(1/
2*d*x+1/2*c)^12+864*C*cos(1/2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+228*C*cos(1/2*d*x+1/2*c)^8+630*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*cos(1/2*d*x+1/2*c)^5+1386*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6-1590*C*cos(1/2*d*x+1/2*c)^6+264*A*cos
```

```
(1/2*d*x+1/2*c)^4+744*C*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2-57*C
*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{5}+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^5 + A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3,
 x)
```

3.165 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=209

$$\frac{(A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+119C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+11C)\sin(c-}{2a}$$

```
[Out] -((9*A + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 11*C)*Ellipti
cF[(c + d*x)/2, 2])/(2*a^3*d) + ((A + 11*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(2*a^3*d) - ((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c +
 d*x])^3) - (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x
])^2) - ((9*A + 119*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Co
s[c + d*x]))
```

Rubi [A] time = 0.494935, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2977, 2748, 2639, 2635, 2641}

$$\frac{(A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+119C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+11C)\sin(c-}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -((9*A + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 11*C)*Ellipti
cF[(c + d*x)/2, 2])/(2*a^3*d) + ((A + 11*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(2*a^3*d) - ((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c +
 d*x])^3) - (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x
])^2) - ((9*A + 119*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Co
s[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(3A-7C)+\frac{1}{2}a(3A+13C)\cos(c+\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \int \frac{\cos^{\frac{3}{2}}}{}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{(9A+}{3}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{(9A+}{3}$$

$$= -\frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+11C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{(A}{}$$

$$= -\frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{(A+11C)\sqrt{}}{}$$

Mathematica [C] time = 6.87104, size = 1296, normalized size = 6.2

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (((119*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
```

```
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])
^3*Sqrt[1 + Cot[c]^2]) - (22*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(
a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[
c + d*x]]*((4*(9*A + 59*C + 60*C*Cos[c])*Csc[c])/(5*d) + (16*C*Cos[d*x]*Sin
[c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)
/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2] + 19*C*Sin
[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] + 59*
C*Sin[(d*x)/2]))/(5*d) + (16*C*Cos[c]*Sin[d*x])/(3*d) - (4*(9*A + 19*C)*Sec
[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/
2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 0.142, size = 465, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*C*cos(1/
2*d*x+1/2*c)^10+108*A*cos(1/2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*c
os(1/2*d*x+1/2*c)^5+54*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468
*C*cos(1/2*d*x+1/2*c)^8+330*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+714*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/
2*c)^6-1058*C*cos(1/2*d*x+1/2*c)^6+114*A*cos(1/2*d*x+1/2*c)^4+474*C*cos(1/2
*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c)^2-47*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/
```

```
a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3,
 x)
```

3.166 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{(A-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-13C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+dx)\cos}{5d(a\cos(c+dx)}$$

```
[Out] -((A - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A - 13*C)*EllipticF[
(c + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) + (2*(A - 4*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1
5*a*d*(a + a*Cos[c + d*x])^2) + ((A - 13*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.480264, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2977, 2748, 2641, 2639}

$$\frac{(A-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-13C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+dx)\cos}{5d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -((A - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A - 13*C)*EllipticF[
(c + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) + (2*(A - 4*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1
5*a*d*(a + a*Cos[c + d*x])^2) + ((A - 13*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}dx = -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-C)+\frac{1}{2}a(A+11C)\cos(c+dx)\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(A-4C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(A-4C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(A-4C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+C)\cos^{\frac{5}{2}}(c}{5d(a+a}$$

Mathematica [C] time = 6.76445, size = 1271, normalized size = 7.14

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] ((-I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 + (
((49*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
```

```
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
- (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*
Sqrt[1 + Cot[c]^2]) + (26*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a
 + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
 + d*x]]*((-4*(-A + 29*C + 20*C*Cos[c])*Csc[c])/(5*d) + (4*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(A*Sin[(d*x)/2] - 29*C*Sin[(d*x)/2]))/(5*d) - (2*Sec[c/2]*Sec[c
/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[
c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] + 7*C*Sin[(d*x)/2]))/(15*d) + (8*(2*A +
7*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^
4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.148, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*C*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+578*C*cos(1/2*d*x+1/2*c)^
6-24*A*cos(1/2*d*x+1/2*c)^4-264*C*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2
*c)^2+37*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^3*cos(d*
x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3,
 x)
```

3.167 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}}{5d(a\cos(c+dx)+a}$$

```
[Out] ((A - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*C)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) + (2*(2*A - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Cos[c + d*x])^2) - ((A - 9*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.469161, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2977, 2978, 2748, 2641, 2639}

$$\frac{(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}}{5d(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] ((A - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*C)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) + (2*(2*A - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Cos[c + d*x])^2) - ((A - 9*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}dx = -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(7A-3C)-\frac{1}{2}a(A-9C)\cos(c+d\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(2A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(2A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(2A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+C)\cos^{\frac{3}{2}}(c+}{5d(a+a\cos}$$

Mathematica [C] time = 6.67485, size = 1259, normalized size = 6.99

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - ((
(9*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
```

```
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sq
rt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*
Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*((-4*(A - 9*C)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d
*x)/2] - 9*C*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin
[(d*x)/2] - 9*C*Sin[(d*x)/2]))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A
*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (4*(A - 9*C)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) + (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a
*Cos[c + d*x])^3
```

Maple [B] time = 0.148, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8-30*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+198*C*cos(1/2*d*x+1/2*c)^6+
6*A*cos(1/2*d*x+1/2*c)^4-114*C*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^
2+27*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3,
 x)
```

3.168 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=184

$$\frac{(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{2(3A-2C)\sin(c+dx)\sqrt{}}{15ad(a\cos(c+dx)}$$

```
[Out] ((9*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + C)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) - (2*(3*A - 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Cos[c + d*x])^2) - ((9*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.482234, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2978, 2748, 2641, 2639}

$$\frac{(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{2(3A-2C)\sin(c+dx)\sqrt{}}{15ad(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```


```
[Out] ((9*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + C)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) - (2*(3*A - 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Cos[c + d*x])^2) - ((9*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}dx = -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(9A-C)-\frac{1}{2}a(3A-7C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}dx}{5a^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(3A-2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(3A-2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(3A-2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= \frac{(9A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+C)\sqrt{\cos(c+dx}}{5d(a+a\cos(c}$$

Mathematica [C] time = 6.67097, size = 1265, normalized size = 6.88

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3
),x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
- ((I/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
```

```
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt
[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*((-4*(9*A - C)*Csc[c])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(3*A*Si
n[(d*x)/2] - 2*C*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*
A*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*
(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (8*(3*A - 2*C)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(15*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
 + a*Cos[c + d*x])^3
```

Maple [B] time = 0.15, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+1/2*c)^6+2
4*A*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
-17*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4 + 3*
a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))
), x)
```

3.169 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=219

$$-\frac{(13A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-C)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+\right.}$$

```
[Out] -((49*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - C)*EllipticF[
(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c
 + d*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d
*x])^3) - (2*(4*A - C)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^2) - ((13*A - C)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3
*Cos[c + d*x]))
```

Rubi [A] time = 0.523739, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2978, 2748, 2636, 2639, 2641}

$$-\frac{(13A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-C)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```



```
[Out] -((49*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - C)*EllipticF[
(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c
 + d*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d
*x])^3) - (2*(4*A - C)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^2) - ((13*A - C)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3
*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}+\frac{\int \frac{\frac{1}{2}a(11A+C)-\frac{5}{2}a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}-\frac{2(4A-C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}-\frac{2(4A-C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}-\frac{2(4A-C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= -\frac{(13A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(A+C)\sin(c}{5d\sqrt{\cos(c+dx)}(a+a}$$

$$= -\frac{(49A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-C)\sin(c}{10a^3d\sqrt{\cos(c+}}$$

Mathematica [C] time = 6.88519, size = 1301, normalized size = 5.94

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3
),x]
```

```
[Out] (((-49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 + ((I/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
```

```
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
+ (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3
*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a
 + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
 + d*x]]*((2*(20*A + 29*A*Cos[c] - C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(5
*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(
5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]
))/(15*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(11*A + C)*Sec[c/2 +
 (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5
*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.17, size = 685, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*si
```

```
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c
)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-C)*sin(1/2*
d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(817*A-
13*C)*sin(1/2*d*x+1/2*c)^6+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(124*A-C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(439*A-C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{5}+3\,a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5 + 3*
a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)
), x)
```

3.170 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=242

$$\frac{(11A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A+9C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(11A+C)\sin(c}{2a^3d\cos^{\frac{3}{2}}(c+}$$

```
[Out] ((119*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A + C)*Elliptic
F[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A + C)*Sin[c + d*x])/(2*a^3*d*Cos[c + d
*x]^(3/2)) - ((119*A + 9*C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - (
(A + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3) - (2*
A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((119*A
 + 9*C)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.523538, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 2978, 2748, 2636, 2641, 2639}

$$\frac{(11A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A+9C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(11A+C)\sin(c}{2a^3d\cos^{\frac{3}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```


```
[Out] ((119*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A + C)*Elliptic
F[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A + C)*Sin[c + d*x])/(2*a^3*d*Cos[c + d
*x]^(3/2)) - ((119*A + 9*C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - (
(A + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3) - (2*
A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((119*A
 + 9*C)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A+3C)-\frac{1}{2}a(7A-3C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= \frac{(11A+C)\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{(119A+9C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+ }{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= \frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(11A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{(11A+C)\sin(c}{2a^3d\cos^{\frac{3}{2}}(c+}$$

Mathematica [C] time = 7.42675, size = 1331, normalized size = 5.5

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3
),x]
```

```
[Out] (((119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
```

```
3 + (((9*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])
^3*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a
 + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
 + d*x]]*((-2*(60*A + 59*A*Cos[c] + 9*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(
5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/
(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(8*A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/
2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] + 9*C*Sin[(
d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (16*Sec[c]*
Sec[c + d*x]*(A*Sin[c] - 9*A*Sin[d*x]))/(3*d) - (8*(8*A + 3*C)*Sec[c/2 + (d
*x)/2]^2*Tan[c/2])/(15*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)
))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.19, size = 876, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*(12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(55*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-119*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(55*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*A*EllipticE(cos(1/2*d*
```

```
x+1/2*c),2^(1/2))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+24*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(55*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-119*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-9*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(55*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(119*A+9*C)*sin(1/2*d*x+1/2*c)^10+24*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(389*A+29*C)*sin(1/2*d*x+1/2*c)^8-10*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1111*A+81*C)*sin(1/2*d*x+1/2*c
)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1414*A+99*C)*si
n(1/2*d*x+1/2*c)^4-3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
343*A+23*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1
/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{6}+3\,a^{3}\cos\left(dx+c\right)^{5}+3\,a^{3}\cos\left(dx+c\right)^{4}+a^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6 + 3*
a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^3 \cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)
), x)
```

3.171 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=214

$$\frac{a(48A+35C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48A+35C)\sin(c+dx)\sqrt{\cos(c+}}{64d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (a*(48*A + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (a*(48*A + 35*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*d*Sqrt[a + a*Cos[c + d*x]]) + (a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(24*d
*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.474741, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3046, 2981, 2770, 2774, 216}

$$\frac{a(48A+35C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48A+35C)\sin(c+dx)\sqrt{\cos(c+}}{64d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (a*(48*A + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (a*(48*A + 35*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*d*Sqrt[a + a*Cos[c + d*x]]) + (a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(24*d
*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(4*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\int \cos^{\frac{3}{2}}(\ldots}{\ldots}$$

$$= \frac{aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos\ldots}}{4d}$$

$$= \frac{a(48A+35C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{aC\cos^{\frac{5}{2}}(c+d\ldots}{24d\sqrt{a+a\ldots}}$$

$$= \frac{a(48A+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a(48A+35C)\ldots}{96d\sqrt{\ldots}}$$

$$= \frac{a(48A+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a(48A+35C)\ldots}{96d\sqrt{\ldots}}$$

$$= \frac{\sqrt{a}(48A+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a(48A+35C)\sqrt{\ldots}}{64d\sqrt{a\ldots}}$$

Mathematica [A] time = 0.818245, size = 129, normalized size = 0.6

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(48A+35C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(2(4\ldots\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(48*A + 35*C)*ArcSi
n[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(144*A + 133*C + 2*(48*A
 + 53*C)*Cos[c + d*x] + 28*C*Cos[2*(c + d*x)] + 12*C*Cos[3*(c + d*x)])*Sin[
(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.228, size = 434, normalized size = 2.

$$\frac{(-1+\cos(dx+c))^4}{192\,d(\sin(dx+c))^8}\left(96\,A\sin(dx+c)(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+336\,A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(\ldots}{1+\cos\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^4*(96*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)+336*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)+384*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+48*C*sin(
d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+144*A*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)+56*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+70*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)+105*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+144*A
*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))+105*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c)))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^8/(cos(d*x
+c)/(1+cos(d*x+c)))^(7/2)
```

Maxima [B] time = 3.4734, size = 9933, normalized size = 46.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/768*(48*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
```

```
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - 1)))*A + (2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((156*(sin(4*d*x + 4*c)
^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 39*cos(4*d*x + 4*c)^2*si
n(4*d*x + 4*c) + 39*sin(4*d*x + 4*c)^3 + 156*(sin(4*d*x + 4*c)^3 + (cos(4*d
*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 39*(2*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 156*(sin(4*d*x + 4*c)^3 + (cos(
4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)
 + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x
+ 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 55*cos(4*d*x
+ 4*c) + 39)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4
*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(4*d*x + 4*c) + 55*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) - 39*cos(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) - 156*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) - (39*cos(4*d*x + 4*c)^3 + 4*(39*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x +
4*c) - 8)*sin(4*d*x + 4*c)^2 - 86*cos(4*d*x + 4*c)^2 + 55*cos(4*d*x + 4*c)
- 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (39*cos(4*d*x
 + 4*c) - 8)*sin(4*d*x + 4*c)^2 + 4*(39*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x
+ 4*c) - 8)*sin(4*d*x + 4*c)^2 + 70*cos(4*d*x + 4*c)^2 + 23*cos(4*d*x + 4*c
) - 8)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 8*cos(4*d*x
```

+ 4*c)^2 + (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*
c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*
d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(
4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 55*cos(4*d*x + 4*c) + 39)*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 55
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3
9*cos(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
4*(39*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 4
7*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - 39*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) - 4*(4*(39*cos(4*d*x + 4*c) - 8)*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (39*cos(4*d*x + 4*
c) - 8)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1)^(1/4)*((4*(11*sin(4*d*x + 4*c)^3 + 11*(cos(4*d*x + 4*
c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) - 24*(cos(4*d*x + 4*c)^2 +
sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 11*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 11*sin(4*d*x + 4*c)^3 + 4*(1
1*sin(4*d*x + 4*c)^3 + 11*(cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin
(4*d*x + 4*c) - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x +
4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(22*sin(4*d*x + 4*c)^3 + 2
2*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + 11*cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (48*cos(4*d*x
+ 4*c)^2 + 48*sin(4*d*x + 4*c)^2 - 37*cos(4*d*x + 4*c) - 11)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 11*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) - 2*(8*(11*sin(4*d*x + 4*c)^2 - 24*sin(4*d*x + 4*c)*sin(1
/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 11*(cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 22*sin(4*d*x + 4*c)^2 - 37*sin(4*d*x + 4
*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) - (24*cos(4*d*x + 4*c)^2 + 24*sin(4*d*x
+ 4*c)^2 + 11*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (11*cos
(4*d*x + 4*c)^3 + 4*(11*cos(4*d*x + 4*c)^3 + (11*cos(4*d*x + 4*c) + 24)*sin

```
(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)^2 - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x
 + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 37*cos(4*d*x + 4*c) + 24)*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + (11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 4*(
11*cos(4*d*x + 4*c)^3 + (11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 46*
cos(4*d*x + 4*c)^2 - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*
d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 59*c
os(4*d*x + 4*c) + 24)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 24*cos(4*d*x + 4*c)^2 + 2*(22*cos(4*d*x + 4*c)^3 + 2*(11*cos(4*d*x + 4*
c) + 24)*sin(4*d*x + 4*c)^2 + 26*cos(4*d*x + 4*c)^2 - (48*cos(4*d*x + 4*c)^
2 + 48*sin(4*d*x + 4*c)^2 - 37*cos(4*d*x + 4*c) - 11)*cos(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) - 11*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) - 48*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - (24*cos(4*d*x + 4*c)^2 + 24*sin(4*d*x +
4*c)^2 + 11*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) - 2*(8*((11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c) - 24*cos(1/4*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(11*cos(4*d*x + 4*c) + 24)*sin(4
*d*x + 4*c) - 37*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4
*d*x + 4*c) - 11*(cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 1
1*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*si
n(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 105*((4*(cos
(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*
d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x
 + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*c
os(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
```

+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
c))) + 1))) + 1) - (4(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d
*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(
cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos
(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c
))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2
(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (4*(cos(4*d*x + 4*c)^2 + sin
(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(
4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x
 + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x +
 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d
*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (4*(cos(4*d*x + 4*c)^2 + sin(4
*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*
d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + co
s(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x +
 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4

```
*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
 + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*C/(4*(cos(4*d*x + 4*c)^2
 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 +
2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos
(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4
*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*s
in(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))))/d
```

Fricas [A] time = 2.85074, size = 416, normalized size = 1.94

$$\frac{\left(48\, C \cos\left(dx+c\right)^{3} + 56\, C \cos\left(dx+c\right)^{2} + 2\, \left(48\, A + 35\, C\right) \cos\left(dx+c\right) + 144\, A + 105\, C\right) \sqrt{a \cos\left(dx+c\right) + a} \sqrt{\cos\left(dx+\right.}}{192\, \left(d \cos\left(dx+c\right) + d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/192*((48*C*cos(d*x + c)^3 + 56*C*cos(d*x + c)^2 + 2*(48*A + 35*C)*cos(d*x
 + c) + 144*A + 105*C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 3*((48*A + 35*C)*cos(d*x + c) + 48*A + 35*C)*sqrt(a)*arctan(sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c
) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.172 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right) d$

Optimal. Leaf size=169

$$\frac{\sqrt{a}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}{3d}$$

```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*d) + (a*(8*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos
[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(3*d)
```

Rubi [A] time = 0.386973, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3046, 2981, 2770, 2774, 216}

$$\frac{\sqrt{a}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*d) + (a*(8*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos
[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(3*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\int \sqrt{\cos}}{}$$

$$= \frac{aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}{3d}$$

$$= \frac{a(8A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{aC\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a+a\cos}}$$

$$= \frac{a(8A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{aC\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a+a\cos}}$$

$$= \frac{\sqrt{a}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(8A+5C)\sqrt{\cos(}}{8d\sqrt{a+a}}$$

Mathematica [A] time = 0.489632, size = 112, normalized size = 0.66

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(8A+5C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(24A\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(8*A + 5*C)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(24*A + 19*C + 10*C*Cos[c
+ d*x] + 4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.187, size = 362, normalized size = 2.1

$$-\frac{(-1+\cos(dx+c))^3}{24\,d\,(\sin(dx+c))^6}\left(24\,A\sin(dx+c)\,(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+48\,A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx}{1+\cos(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+8*C*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+24*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))+15*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(
d*x+c)^6/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
```

Maxima [B] time = 2.8723, size = 3663, normalized size = 21.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/96*(24*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
```

```
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (4*(cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))
*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*cos(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arctan2(-(cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))
 + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
, (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3
```

```
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin
(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + arctan
2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) - 1)))*C)/d
```

Fricas [A] time = 2.36609, size = 360, normalized size = 2.13

$$\frac{\left(8\,C\cos\left(dx+c\right)^{2}+10\,C\cos\left(dx+c\right)+24\,A+15\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(\left(8\,A+5\,C\right)\cos\left(d\right.\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*((8*C*cos(d*x + c)^2 + 10*C*cos(d*x + c) + 24*A + 15*C)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((8*A + 5*C)*cos(d*x + c) +
8*A + 5*C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

3.173 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{a}(8A+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{aC\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) + (a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*
x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.313866, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3046, 2981, 2774, 216}

$$\frac{\sqrt{a}(8A+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{aC\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) + (a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*
x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{1}{2}a(4A}{\sqrt{\cos(}}}{2}$$

$$= \frac{aC\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(}{2d}$$

$$= \frac{aC\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(}{2d}$$

$$= \frac{\sqrt{a}(8A+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{aC\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} +$$

Mathematica [A] time = 0.273462, size = 98, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+3C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(8*A + 3*C)*ArcSin[Sq
rt[2]*Sin[(c + d*x)/2]] + 2*C*Sqrt[Cos[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[
(3*(c + d*x))/2])))/(8*d)
```

Maple [A] time = 0.131, size = 202, normalized size = 1.6

$$\frac{(-1+\cos(dx+c))^2}{4\,d\,(\sin(dx+c))^4}\sqrt{a\,(1+\cos(dx+c))}\,(\cos(dx+c))^{\frac{3}{2}}\left(2\,C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,C\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*(2*C*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+8*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c))+3*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c)))/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] time = 2.30862, size = 1629, normalized size = 13.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*(16*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c
```

```
), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1)))*C)/d
```

Fricas [A] time = 2.24644, size = 316, normalized size = 2.55

$$\frac{(2\,C\cos\left(dx+c\right)+3\,C)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-((8\,A+3\,C)\cos\left(dx+c\right)+8\,A+3\,C)\sqrt{a}\,\mathrm{arctan}}{4\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*((2*C*cos(d*x + c) + 3*C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*s
in(d*x + c) - ((8*A + 3*C)*cos(d*x + c) + 8*A + 3*C)*sqrt(a)*arctan(sqrt(a*
```

```
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x +
c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)
), x)
```

3.174 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a(2A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a*
(2*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (
2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.319839, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3044, 2981, 2774, 216}

$$-\frac{a(2A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a*
(2*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (
2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{aA}{2}-\frac{1}{2}a(2A-C)\cos(c+\right.}{\sqrt{\cos(c+dx)}}}{a}$$

$$= -\frac{a(2A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a(2A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a(2A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + 2A$$

Mathematica [A] time = 0.24408, size = 100, normalized size = 0.85

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2A+C\cos(c+dx))+\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+\right.}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*C*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + C*Cos[c + d*x])*Sin[(c + d*x)/2
]))/(2*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.135, size = 166, normalized size = 1.4

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2}\sqrt{a(1+\cos(dx+c))}\left(C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\sin(dx+c)\sqrt{\frac{\cos(dx+}{1+\cos(dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(C*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d
*x+c))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.0864, size = 1202, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
```

```
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C + 8*A*(sqrt(2)*sqrt
(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A] time = 2.09799, size = 324, normalized size = 2.77

$$\frac{(C\cos(dx+c)+2A)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-\left(C\cos(dx+c)^2+C\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos\cdots}}{\cdots}\right.}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] ((C*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x
 + c) - (C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*
cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+C\cos^{2}\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(
3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

3.175 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*
A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.309065, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3044, 2980, 2774, 216}

$$\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*
A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{aA}{2}+\frac{3}{2}aC\cos(c+dx)\right.}{\cos^{\frac{3}{2}}(c+dx)}}{3a}$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+a}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+a}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.225038, size = 90, normalized size = 0.78

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2A\sin\left(\frac{3}{2}(c+dx)\right)+3\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*C*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*A*Sin[(3*(c + d*x))/2]))/(3*d*Cos[c
+ d*x]^(3/2))
```

Maple [A] time = 0.124, size = 127, normalized size = 1.1

$$-\frac{2}{3\,d\sin(dx+c)}\sqrt{a\,(1+\cos(dx+c))}\left(-3\,C\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(a*(1+cos(d*x+c)))^(1/2)*(-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos
(d*x+c)+2*A*cos(d*x+c)^2-A*cos(d*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 1.94595, size = 458, normalized size = 3.95

$$3\,C\sqrt{a}\arctan\left(\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/3*(3*C*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + 2*A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 2.15752, size = 338, normalized size = 2.91

$$\frac{2\left((2\,A\cos\left(dx+c\right)+A)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(C\cos\left(dx+c\right)^{3}+C\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\mathrm{arctan}\right.}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] 2/3*((2*A*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*(C*cos(d*x + c)^3 + C*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

3.176 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{2a(8A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*A*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(8*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
))
```

Rubi [A] time = 0.316309, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3044, 2980, 2771}

$$\frac{2a(8A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(8*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{aA}{2}+\frac{1}{2}a(2A+5C)\cos(\cdots\right.}{\cos^{\frac{5}{2}}(c+dx)}}{5a}$$

$$= \frac{2aA\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+d\cdots}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(8A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+\cdots}}$$

Mathematica [A] time = 0.275817, size = 73, normalized size = 0.59

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((8A+15C)\cos(2(c+dx))+8A\cos(c+dx)+14A+15C)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(14*A + 15*C + 8*A*Cos[c + d*x] + (8*A + 15*C)*
Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.107, size = 77, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(8A(\cos(dx+c))^2+15C(\cos(dx+c))^2+4A\cos(dx+c)+3A\right)}{15d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+15*C*cos(d*x+c)^2+4*A*cos(d*x+c)+
3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.6976, size = 454, normalized size = 3.69

$$\frac{2\left(\frac{15C\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}+\frac{A\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 2/15*(15*C*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + A*(15*sqrt(2)*sqr
t(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.7675, size = 211, normalized size = 1.72

$$\frac{2\left((8A+15C)\cos(dx+c)^2+4A\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{15\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*((8*A + 15*C)*cos(d*x + c)^2 + 4*A*cos(d*x + c) + 3*A)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2
), x)
```

3.177 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=168

$$\frac{2a(24A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\ldots$$

```
[Out] (2*a*A*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (4*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos
[c + d*x]^(7/2))
```

Rubi [A] time = 0.408091, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3044, 2980, 2772, 2771}

$$\frac{2a(24A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (4*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos
[c + d*x]^(7/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{aA}{2}+\frac{1}{2}a(4A+7C)\cos\right.}{\cos^{\frac{7}{2}}(c+dx)}}{7a}$$

$$= \frac{2aA\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(24A+35C)\sin(c+d}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}$$

$$= \frac{2aA\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(24A+35C)\sin(c+d}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(}}$$

Mathematica [A] time = 0.440184, size = 101, normalized size = 0.6

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(3(36A+35C)\cos(c+dx)+(24A+35C)\cos(2(c+dx))+24A\cos(3(c+dx))+5}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(54*A + 35*C + 3*(36*A + 35*C)*Cos[c + d*x] + (
24*A + 35*C)*Cos[2*(c + d*x)] + 24*A*Cos[3*(c + d*x)] + 35*C*Cos[3*(c + d*x
)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.112, size = 99, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(48A(\cos(dx+c))^3+70C(\cos(dx+c))^3+24A(\cos(dx+c))^2+35C(\cos(dx+c))^2+18A\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+70*C*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+35*C*cos(d*x+c)^2+18*A*cos(d*x+c)+15*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d
```

```
*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.69947, size = 641, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 2/105*(35*C*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*
sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 1)) + 3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/
d
```

Fricas [A] time = 1.37614, size = 261, normalized size = 1.55

$$\frac{2\left(2\left(24A+35C\right)\cos\left(dx+c\right)^{3}+\left(24A+35C\right)\cos\left(dx+c\right)^{2}+18A\cos\left(dx+c\right)+15A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+\right.}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(2*(24*A + 35*C)*cos(d*x + c)^3 + (24*A + 35*C)*cos(d*x + c)^2 + 18*A
*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x +
 c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

3.178 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{8a(16A+21C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(16A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a*A*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(16*A + 21*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (8*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Cos[c + d*x]]) + (16*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.463261, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3044, 2980, 2772, 2771}

$$\frac{8a(16A+21C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(16A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x
]
```



```
[Out] (2*a*A*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(16*A + 21*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (8*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Cos[c + d*x]]) + (16*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{aA}{2}+\frac{3}{2}a(2A+3C)\cos(c}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$= \frac{2aA\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+d}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}}$$

$$= \frac{2aA\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}}$$

$$= \frac{2aA\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.642179, size = 124, normalized size = 0.58

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(88A+63C)\cos(c+dx)+11(16A+21C)\cos(2(c+dx))+32A\cos(3(c+dx))+}{315d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(214*A + 189*C + 2*(88*A + 63*C)*Cos[c + d*x] +
 11*(16*A + 21*C)*Cos[2*(c + d*x)] + 32*A*Cos[3*(c + d*x)] + 42*C*Cos[3*(c
+ d*x)] + 32*A*Cos[4*(c + d*x)] + 42*C*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/
(315*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.127, size = 121, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(128A(\cos(dx+c))^4+168C(\cos(dx+c))^4+64A(\cos(dx+c))^3+84C(\cos(dx+c))^3+48A\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+168*C*cos(d*x+c)^4+64*A*cos(d*
x+c)^3+84*C*cos(d*x+c)^3+48*A*cos(d*x+c)^2+63*C*cos(d*x+c)^2+40*A*cos(d*x+c
)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.73003, size = 765, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 2/315*(21*C*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2
)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + A*(315*sqrt(2)*sqrt(a)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 735*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1206*sq
rt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 107*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos
(d*x + c) + 1)^11)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(1
1/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(
d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.49266, size = 308, normalized size = 1.45

$$\frac{2\left(8\left(16\,A+21\,C\right)\cos\left(dx+c\right)^{4}+4\left(16\,A+21\,C\right)\cos\left(dx+c\right)^{3}+3\left(16\,A+21\,C\right)\cos\left(dx+c\right)^{2}+40\,A\cos\left(dx+c\right)+35}{315\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/315*(8*(16*A + 21*C)*cos(d*x + c)^4 + 4*(16*A + 21*C)*cos(d*x + c)^3 + 3*
(16*A + 21*C)*cos(d*x + c)^2 + 40*A*cos(d*x + c) + 35*A)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(11/
2), x)
```

3.179 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=265

$$\frac{a^2(80A+67C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+133C)\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a\cos}}\right.}{128d}$$

```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^2*(176*A + 133*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(176*A + 133*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(80*A + 67*C)*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (3*a*C*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d) + (C*Cos[c + d*x]^(5/
2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.69548, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(80A+67C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+133C)\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a\cos}}\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^2*(176*A + 133*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(176*A + 133*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(80*A + 67*C)*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (3*a*C*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d) + (C*Cos[c + d*x]^(5/
2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\int \mathrm{c}}{}$$

$$= \frac{3aC\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{40d} + \frac{C\,\mathrm{c}}{}$$

$$= \frac{a^2(80A+67C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}} + \frac{3aC\cos^{\frac{5}{2}}(c}{}$$

$$= \frac{a^2(176A+133C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(80A}{2}$$

$$= \frac{a^2(176A+133C)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(176A}{}$$

$$= \frac{a^2(176A+133C)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(176A}{}$$

$$= \frac{a^{3/2}(176A+133C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^2(176A+}{12}$$

Mathematica [A] time = 1.51603, size = 147, normalized size = 0.55

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(176A+133C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]
^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(176*A + 133*C)*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2960*A + 2671*C +
2*(880*A + 1007*C)*Cos[c + d*x] + 4*(80*A + 181*C)*Cos[2*(c + d*x)] + 228*C
```

```
*Cos[3*(c + d*x)] + 48*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(3840*d)
```

Maple [B] time = 0.144, size = 507, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/1920/d*a*(-1+cos(d*x+c))^4*(640*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d
*x+c)^4*sin(d*x+c)+3040*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+6800*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3
84*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6+7040*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+912*C*sin(d*x+c)*cos(d*x
+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2640*A*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)+1064*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+1330*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+199
5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2640*A*cos(d*
x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+1995
*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/sin(d*x+c)^8/(cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)
```

Maxima [B] time = 3.67553, size = 6035, normalized size = 22.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/7680*(80*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
```

```
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*s
qrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*a
rctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
```

sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqr
t(a))*A + (10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin
(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((117*a*sin(4/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))) + 40*a*sin(3/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c))) + 117*a*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), co
s(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - (117*a*cos(4/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 40*a*cos(3/5*arctan2(sin(5*d*
x + 5*c), cos(5*d*x + 5*c))) - 117*a*cos(1/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c))) - 40*a)*sin(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)
))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + si
n(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(8*(a*cos(2/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d*x + 5*c) + a*sin(5*d*x + 5*c)*sin(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*a*cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(5*d*x + 5*c) + a*sin(5*d*x + 5*c))*co
s(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 5*(33*a*sin(4/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 33*a*sin(3/5*arctan2(sin(5*d*x +
5*c), cos(5*d*x + 5*c))) - 4*a*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))) - 100*a*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*cos(
1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 8*((a*cos(5*d*x + 5*c)
- a)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + (a*cos(5*d*x
+ 5*c) - a)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + a*cos(
5*d*x + 5*c) + 2*(a*cos(5*d*x + 5*c) - a)*cos(2/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))) - a)*sin(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
1)) + 5*(33*a*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 33*a*c
os(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 4*a*cos(2/5*arctan2(s
in(5*d*x + 5*c), cos(5*d*x + 5*c))) - 92*a*cos(1/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))) + 96*a)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*

```
c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
) + 1)))*sqrt(a) + 1995*(a*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +
 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x +
5*c), cos(5*d*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))),
cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c
), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))) + 1)^(1/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(
1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5
*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)
)) + 1))) + 1) - a*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(
5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin
(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*
arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5
*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
1)^(1/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arct
an2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(s
in(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c
), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))
) - 1) - a*arctan2((cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2
+ sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))) + 1)), (cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos
(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 1) + a*arctan2((cos(2/5*arctan2(s
in(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
) + 1)^(1/4)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)), (cos(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*
```

```
x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1
)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 2.34226, size = 505, normalized size = 1.91

$$\frac{\left(384\,Ca\cos\left(dx+c\right)^{4}+912\,Ca\cos\left(dx+c\right)^{3}+8\,(80\,A+133\,C)a\cos\left(dx+c\right)^{2}+10\,(176\,A+133\,C)a\cos\left(dx+c\right)+15\,(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] 1/1920*((384*C*a*cos(d*x + c)^4 + 912*C*a*cos(d*x + c)^3 + 8*(80*A + 133*C)
*a*cos(d*x + c)^2 + 10*(176*A + 133*C)*a*cos(d*x + c) + 15*(176*A + 133*C)*
a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*((176*A +
133*C)*a*cos(d*x + c) + (176*A + 133*C)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.180 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=218

$$\frac{a^2(16A+13C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{32d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(112A+75C)\sin(c+dx)\sqrt{\cos(c}}{64d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a^2*(112*A + 75*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*S
qrt[a + a*Cos[c + d*x]]) + (a^2*(16*A + 13*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(32*d*Sqrt[a + a*Cos[c + d*x]]) + (a*C*Cos[c + d*x]^(3/2)*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.626217, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(16A+13C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{32d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(112A+75C)\sin(c+dx)\sqrt{\cos(c}}{64d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a^2*(112*A + 75*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*S
qrt[a + a*Cos[c + d*x]]) + (a^2*(16*A + 13*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(32*d*Sqrt[a + a*Cos[c + d*x]]) + (a*C*Cos[c + d*x]^(3/2)*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{\int \sqrt{\cdots}}{\cdots}$$

$$= \frac{aC\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{8d} + \frac{C\cos\cdots}{\cdots}$$

$$= \frac{a^2(16A+13C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{32d\sqrt{a+a\cos(c+dx)}} + \frac{aC\cos^{\frac{3}{2}}(c+\cdots}{\cdots}$$

$$= \frac{a^2(112A+75C)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(16A+1\cdots}{32\cdots}$$

$$= \frac{a^2(112A+75C)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(16A+1\cdots}{32\cdots}$$

$$= \frac{a^{3/2}(112A+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(112A+75\cdots}{64d\cdots}$$

Mathematica [A] time = 0.862422, size = 128, normalized size = 0.59

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(112A+75C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}((32\cdots\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]
^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(112*A + 75*C)*ArcS
in[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(112*A + 95*C + (32*A +
 62*C)*Cos[c + d*x] + 20*C*Cos[2*(c + d*x)] + 4*C*Cos[3*(c + d*x)])*Sin[(c
+ d*x)/2]))/(128*d)
```

Maple [B] time = 0.115, size = 435, normalized size = 2.

$$-\frac{a\left(-1+\cos\left(dx+c\right)\right)^{3}}{64\,d\left(\sin\left(dx+c\right)\right)^{6}}\left(32\,A\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{3}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{5/2}+176\,A\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{2}\left(\frac{\cos}{1+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/64/d*a*(-1+cos(d*x+c))^3*(32*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)+176*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)+256*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+16*C*si
n(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+112*A*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)+40*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+50*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+75*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+112*
A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))+75*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^6/(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)
```

Maxima [B] time = 3.89161, size = 10855, normalized size = 49.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/256*(16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
```

```
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) -
 a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (2*(cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/
4)*((9*a*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 9*a*sin(4*d*x + 4*c)^3 + 36*
(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*s
in(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3
6*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 9*(2*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*
c) + a*sin(4*d*x + 4*c) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 36*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x
+ 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*
c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*
d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 - 9*a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 +
16*a*sin(4*d*x + 4*c)^2 - 25*a*cos(4*d*x + 4*c) + 9*a)*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 25*a*sin(4*d*x + 4*c))*sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) - 36*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))*sin(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
```

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*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)) - (9*a*cos(4*d*x + 4*c)^3 - 8*a*cos(4*d*x + 4*c)^2 + 4*(9*a*cos(4*
d*x + 4*c)^3 - 26*a*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4
*d*x + 4*c)^2 + 25*a*cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2
 + 4*(9*a*cos(4*d*x + 4*c)^3 + 10*a*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*
c) - 8*a)*sin(4*d*x + 4*c)^2 - 7*a*cos(4*d*x + 4*c) - 8*a)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*co
s(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 +
32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 9*a*cos(4*d*x + 4
*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 25*a*cos(4*d*x
 + 4*c) + 9*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64
*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) +
25*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
)*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(9*a*cos(4*d*x +
 4*c)^3 - 17*a*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x
+ 4*c)^2 + 8*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) - 9*(2*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) - 4*(4*(9*a*cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (9*a*cos(4*d*x + 4*c
) - 8*a)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 2*(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*((7*a*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 7*
a*sin(4*d*x + 4*c)^3 - 48*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*
a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^3 + 4*(7*a*sin(4*d*x + 4*c)^3 + 7*(a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x +
 4*c) + a)*sin(4*d*x + 4*c) - 68*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)
^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 7*a*cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 4*(7*a*
sin(4*d*x + 4*c)^3 + 48*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))*sin(4*d*x + 4*c) + (7*a*cos(4*d*x + 4*c)^2 + 14*a*cos(4*d*x + 4*c) + 19
*a)*sin(4*d*x + 4*c) - 68*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*
a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(14*a*sin(4*d
*x + 4*c)^3 + 7*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(
4*d*x + 4*c) + 14*(a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4
*c) - (136*a*cos(4*d*x + 4*c)^2 + 136*a*sin(4*d*x + 4*c)^2 - 129*a*cos(4*d*
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x + 4*c) - 7*a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(6*a*cos(4*d*x + 4*c)^2
 + 24*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) +
 a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 20*a*sin(4*d*x
 + 4*c)^2 - 129*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 8*(3*a*cos(4*d*x + 4*c)^2 + 10*a*sin(4*d*x + 4*c)^2 - 68*a*s
in(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*a*
cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 7*
(a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (68*a*cos(4*d*x
+ 4*c)^2 + 68*a*sin(4*d*x + 4*c)^2 + 7*a*cos(4*d*x + 4*c))*sin(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 1)) - (7*a*cos(4*d*x + 4*c)^3 - 48*(a*cos(4*d*x + 4*c)^2 + a*s
in(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^3 + 56*a*cos(4*d*x + 4*c)^2 + 4*(7*a*cos(4*d*x + 4*
c)^3 + 30*a*cos(4*d*x + 4*c)^2 + (7*a*cos(4*d*x + 4*c) + 44*a)*sin(4*d*x +
4*c)^2 - 93*a*cos(4*d*x + 4*c) - 44*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4
*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 56*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 7*(a*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c)^2 + 4*(7*a*cos(4*d*x + 4*c)
^3 + 70*a*cos(4*d*x + 4*c)^2 + 7*(a*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c
)^2 + 119*a*cos(4*d*x + 4*c) - 12*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c
)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) - 44*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x
 + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 7*a*sin(4*d*x + 4*c
)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(14*a*cos(4*d*x
+ 4*c)^3 + 92*a*cos(4*d*x + 4*c)^2 + 2*(7*a*cos(4*d*x + 4*c) + 53*a)*sin(4*
d*x + 4*c)^2 - 7*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 112*a*cos(4*d*x + 4*c) - (88*a*cos(4*d*x + 4*c)^2 + 88*a*si
n(4*d*x + 4*c)^2 - 81*a*cos(4*d*x + 4*c) - 7*a)*cos(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - (44*a*cos(4*d*x + 4*c)^2 + 44*a*sin(4*d*x + 4*c)^2 + 7*a*cos(4*d*x +
 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(96*a*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 81*a*c
os(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(4
4*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) -
 (7*a*cos(4*d*x + 4*c) + 53*a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) - 14*(a*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c)
 + 7*(a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arct
an2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 75*((a*cos(4*d*x + 4*c
)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c)

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 + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x
+ 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x +
 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*co
s(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1
/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1))) + 1) - (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x +
 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*
d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*s
in(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1))) - 1) - (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x
```

```
+ 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*
cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*
(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)) + 1) + (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(
4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a
*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*
cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*C/(4*(
cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin
(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*
d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.39606, size = 435, normalized size = 2.

$$\frac{\left(16\,Ca\cos\left(dx+c\right)^{3}+40\,Ca\cos\left(dx+c\right)^{2}+2\,(16\,A+25\,C)a\cos\left(dx+c\right)+(112\,A+75\,C)a\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos(}}{64\,(d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/64*((16*C*a*cos(d*x + c)^3 + 40*C*a*cos(d*x + c)^2 + 2*(16*A + 25*C)*a*co
s(d*x + c) + (112*A + 75*C)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) - ((112*A + 75*C)*a*cos(d*x + c) + (112*A + 75*C)*a)*sqrt(a)*a
rctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/
(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.181 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{a^{3/2}(24A+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+19C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{aC\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c}}{4d}$$

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^2*(24*A + 19*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) + (a*C*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rubi [A] time = 0.51752, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3046, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(24A+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+19C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{aC\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```


```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^2*(24*A + 19*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) + (a*C*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(3*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \frac{C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{\int \frac{(a+a\cos(c+dx))^{3/2}(}{\sqrt{}}}{}$$

$$= \frac{aC\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{C\sqrt{\cos(c+dx)}(a-}{}$$

$$= \frac{a^2(24A+19C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aC\sqrt{\cos(c+dx)}\sqrt{a+a\,c}}{4d}$$

$$= \frac{a^2(24A+19C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aC\sqrt{\cos(c+dx)}\sqrt{a+a\,c}}{4d}$$

$$= \frac{a^{3/2}(24A+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(24A+19C)\sqrt{\cos(c+dx}}{24d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.561469, size = 113, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(24A+11C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(24A}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(24*A + 11*C)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(24*A + 37*C + 22*C*Co
s[c + d*x] + 4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.169, size = 363, normalized size = 2.1

$$\frac{a(-1+\cos(dx+c))^2}{24d(\sin(dx+c))^4}\left(24A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+48A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx+}{1+\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+8*C*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+33*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+72*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))+33*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/sin
(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] time = 2.89824, size = 3707, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/96*(24*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
```

```
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
 (4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*
c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*
sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 3
3*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a*arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
```

```
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arctan2((co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4
)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 1.94062, size = 386, normalized size = 2.26

$$\frac{\left(8\,Ca\cos\left(dx+c\right)^{2}+22\,Ca\cos\left(dx+c\right)+3\,(8\,A+11\,C)a\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\,((24\,A+11}{24\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*((8*C*a*cos(d*x + c)^2 + 22*C*a*cos(d*x + c) + 3*(8*A + 11*C)*a)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((24*A + 11*C)*a*co
s(d*x + c) + (24*A + 11*C)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x +
c)), x)
```

3.182 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{a^{3/2}(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^2(8A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a(4A-C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\,c}}{2d}$$

```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) - (a^2*(8*A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]) - (a*(4*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(2*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]])
```

Rubi [A] time = 0.528851, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^2(8A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a(4A-C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\,c}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```


```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) - (a^2*(8*A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]) - (a*(4*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(2*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]])
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3aA}{2}-\frac{1}{2}a(4A}{\sqrt{\cos(c+dx)}}}{a}$$

$$= -\frac{a(4A-C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{2A(a+}{}$$

$$= -\frac{a^2(8A-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a(4A-C)\sqrt{\cos(c+dx)}\sqrt{}}{}$$

$$= -\frac{a^2(8A-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a(4A-C)\sqrt{\cos(c+dx)}\sqrt{}}{}$$

$$= \frac{a^{3/2}(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^2(8A-5C)\sqrt{\cos(c+dx)}\sin}{4d\sqrt{a+a\cos(c+dx}}$$

Mathematica [A] time = 0.546567, size = 119, normalized size = 0.68

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+7C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(8A+\right.}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(8*A + 7*C)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A + C + 7*C*Cos[c + d*x
] + C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.175, size = 325, normalized size = 1.9

$$-\frac{a(-1+\cos(dx+c))}{4d(\sin(dx+c))^2}\left(8A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+16A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx}{1+\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*a*(-1+cos(d*x+c))*(8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)+16*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+8
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*C*sin(d*x+c)*cos(d*x+c)^4
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))+7*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))*cos(d*x+c)^3)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/co
s(d*x+c)^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] time = 2.50609, size = 2805, normalized size = 16.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
```

```
+ 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C + 8*((a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arcta
n2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/d
```

Fricas [A] time = 1.95443, size = 401, normalized size = 2.29

$$\frac{\left(2\,Ca\cos\left(dx+c\right)^{2}+7\,Ca\cos\left(dx+c\right)+8\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(8\,A+7\,C\right)a\cos\left(dx+c\right)\right.}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*((2*C*a*cos(d*x + c)^2 + 7*C*a*cos(d*x + c) + 8*A*a)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - ((8*A + 7*C)*a*cos(d*x + c)^2 + (8
*A + 7*C)*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3
/2), x)
```

3.183 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$-\frac{a^2(8A-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\,\mathrm{si}}{}$$

```
[Out] (3*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (
a^2*(8*A - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
 + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.516975, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2981, 2774, 216}

$$-\frac{a^2(8A-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```



```
[Out] (3*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (
a^2*(8*A - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
 + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3aA}{2}-\frac{1}{2}a(2A}{\cos^{\frac{3}{2}}(c+dx)}}{3a}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c+dx))^{3/2}\sin(}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{a^2(8A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a^2(8A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{3a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(8A-3C)\sqrt{\cos(c+dx)}\sin(c+dx}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.517734, size = 116, normalized size = 0.72

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(20A\cos(c+dx)+4A+3C\cos(2(c+dx))+3C)+9\sqrt{2}C\sin^{-1}\right.}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sqrt[2]*C*ArcSin[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*C + 20*A*Cos[c + d*x] + 3*C
*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.138, size = 150, normalized size = 0.9

$$-\frac{a}{3\,d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-9\,C\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-9*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)+3*C*cos(d*x+c)^3+10*A*cos(d*x+c)^2-3*C*cos(d*x+c)^2-8*A*cos(d*x+c
)-2*A)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.09543, size = 1256, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(3*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C +
16*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*s
```

```
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [A] time = 1.56071, size = 383, normalized size = 2.38

$$\frac{\left(3\,Ca\cos\left(dx+c\right)^{2}+10\,Aa\cos\left(dx+c\right)+2\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-9\left(Ca\cos\left(dx+c\right)^{3}+C\right.}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*((3*C*a*cos(d*x + c)^2 + 10*A*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 9*(C*a*cos(d*x + c)^3 + C*a*cos(d
*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5
/2), x)
```

3.184 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}} + \frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} + \frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+ \cdots}{5 \cdots}$$

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(4*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3
/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
))
```

Rubi [A] time = 0.480942, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2980, 2774, 216}

$$\frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}} + \frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} + \frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+ \cdots}{5 \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(4*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3
/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
```

```
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{5}{2}aC\cos\right.}{\cos^{\frac{5}{2}}(c+dx)}}{5a}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{3/2}\sin(}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c}{5d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c}{5d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.697047, size = 121, normalized size = 0.74

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((6A+5C)\cos(2(c+dx))+6A\cos(c+dx)+8A+5C)+5\sqrt{2}C\right.}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(7/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(5*Sqrt[2]*C*ArcSin[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + (8*A + 5*C + 6*A*Cos[c + d*x] + (6*A
 + 5*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(5*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.15, size = 232, normalized size = 1.4

$$-\frac{2a}{5d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-5C\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)^2-5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+6*A*cos(d*x+c)
^3+5*C*cos(d*x+c)^3-3*A*cos(d*x+c)^2-5*C*cos(d*x+c)^2-2*A*cos(d*x+c)-A)/sin
(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.2052, size = 1642, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/10*(5*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
```

```
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(
a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4) + 8*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 1
0*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/
2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 1)))/d
```

Fricas [A] time = 1.61153, size = 390, normalized size = 2.39

$$\frac{2\left(\left((6A+5C)a\cos(dx+c)^2+3Aa\cos(dx+c)+Aa\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(Ca\cos(dx+\right.\right.}{5\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/5*(((6*A + 5*C)*a*cos(d*x + c)^2 + 3*A*a*cos(d*x + c) + A*a)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(C*a*cos(d*x + c)^4 + C*a*
cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7
/2), x)
```

3.185 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^2(4A+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(104A+175C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{6aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\ldots}{\ldots}$$

```
[Out] (2*a^2*(4*A + 5*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(104*A + 175*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]]) + (6*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.523874, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3044, 2975, 2980, 2771}

$$\frac{2a^2(4A+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(104A+175C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{6aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```


```
[Out] (2*a^2*(4*A + 5*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(104*A + 175*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]]) + (6*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{1}{2}a(2A}{\cos^{\frac{7}{2}}(c+dx)}}{7a}$$

$$= \frac{6aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{3/2}\sin(}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(4A+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{6aA\sqrt{a+a\cos(c+dx)}\sin(}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(4A+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(104A+175C)\sin(c}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.53584, size = 102, normalized size = 0.59

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((468A+525C)\cos(c+dx)+2(52A+35C)\cos(2(c+dx))+104A\cos(3(c+dx}{210d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(9/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(164*A + 70*C + (468*A + 525*C)*Cos[c + d*x]
+ 2*(52*A + 35*C)*Cos[2*(c + d*x)] + 104*A*Cos[3*(c + d*x)] + 175*C*Cos[3*(
c + d*x)])*Tan[(c + d*x)/2])/(210*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.104, size = 100, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(104A(\cos(dx+c))^3+175C(\cos(dx+c))^3+52A(\cos(dx+c))^2+35C(\cos(dx+c))^2+39}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+175*C*cos(d*x+c)^3+52*A*cos(
d*x+c)^2+35*C*cos(d*x+c)^2+39*A*cos(d*x+c)+15*A)*(a*(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.89633, size = 525, normalized size = 3.05

$$\frac{4\left(\frac{35\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)C}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}+\frac{\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{38\sqrt{2}a^{\frac{3}{2}}\,\mathrm{s}}{(\cos(dx\cdot}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\mathrm{si}}{(\mathrm{cos}}\right.}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 4/105*(35*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5)*C/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + (105*sqrt(2)*a^(3/2)*sin(d*x +
c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2
)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x
+ c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/
((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) +
 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.49635, size = 271, normalized size = 1.58

$$\frac{2\left((104\,A+175\,C)a\cos(dx+c)^{3}+(52\,A+35\,C)a\cos(dx+c)^{2}+39\,Aa\cos(dx+c)+15\,Aa\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(}}{105\left(d\cos(dx+c)^{5}+d\cos(dx+c)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*((104*A + 175*C)*a*cos(d*x + c)^3 + (52*A + 35*C)*a*cos(d*x + c)^2 +
39*A*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*s
in(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9
/2), x)
```

3.186 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=219

$$\frac{2a^2(136A+189C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+189C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a^2*(52*A + 63*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x
]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.6097, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2980, 2772, 2771}

$$\frac{2a^2(136A+189C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+189C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2)
,x]
```


```
[Out] (2*a^2*(52*A + 63*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x
]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{1}{2}a(4A+}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(52A+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(}{21d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(52A+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(136A+189C)\sin(c}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= \frac{2a^2(52A+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(136A+189C)\sin(c}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.718353, size = 123, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((748A+567C)\cos(c+dx)+(748A+882C)\cos(2(c+dx))+136A\cos(3(c+dx}{630d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(11/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(752*A + 693*C + (748*A + 567*C)*Cos[c + d*x]
 + (748*A + 882*C)*Cos[2*(c + d*x)] + 136*A*Cos[3*(c + d*x)] + 189*C*Cos[3*
(c + d*x)] + 136*A*Cos[4*(c + d*x)] + 189*C*Cos[4*(c + d*x)])*Tan[(c + d*x)
/2])/(630*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.117, size = 122, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(272A(\cos(dx+c))^4+378C(\cos(dx+c))^4+136A(\cos(dx+c))^3+189C(\cos(dx+c))^3+1}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+378*C*cos(d*x+c)^4+136*A*cos
(d*x+c)^3+189*C*cos(d*x+c)^3+102*A*cos(d*x+c)^2+63*C*cos(d*x+c)^2+85*A*cos(
d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.78124, size = 711, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 4/315*(63*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*C*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
+ (315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*a^(3/2
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 94*sqrt(2
)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.46071, size = 324, normalized size = 1.48

$$\frac{2\left(2\,(136\,A+189\,C)a\cos\left(dx+c\right)^{4}+(136\,A+189\,C)a\cos\left(dx+c\right)^{3}+3\,(34\,A+21\,C)a\cos\left(dx+c\right)^{2}+85\,Aa\cos\left(dx+\right.\right.}{315\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/315*(2*(136*A + 189*C)*a*cos(d*x + c)^4 + (136*A + 189*C)*a*cos(d*x + c)^
3 + 3*(34*A + 21*C)*a*cos(d*x + c)^2 + 85*A*a*cos(d*x + c) + 35*A*a)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*co
s(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(1
1/2), x)
```

3.187 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{8a^2(112A+143C)\sin(c+dx)}{1155d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(112A+143C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(28A+33C)\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^2*(28*A + 33*C)*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(112*A + 143*C)*Sin[c + d*x])/(385*d*Cos[c + d*x]^(5/2
)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sin[c + d*x])/(1155*d*
Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*(112*A + 143*C)*Sin[
c + d*x])/(1155*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqr
t[a + a*Cos[c + d*x]]*Sin[c + d*x])/(33*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 0.699736, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2980, 2772, 2771}

$$\frac{8a^2(112A+143C)\sin(c+dx)}{1155d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(112A+143C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(28A+33C)\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2)
,x]
```

```
[Out] (2*a^2*(28*A + 33*C)*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(112*A + 143*C)*Sin[c + d*x])/(385*d*Cos[c + d*x]^(5/2
)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sin[c + d*x])/(1155*d*
Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*(112*A + 143*C)*Sin[
c + d*x])/(1155*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqr
t[a + a*Cos[c + d*x]]*Sin[c + d*x])/(33*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{1}{2}a(6A}{\cos^{\frac{11}{2}}(c+dx)}}{11a}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{33d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{3/2}\sin(}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin}{33d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(112A+143C)\sin(}{385d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(112A+143C)\sin(}{385d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(112A+143C)\sin(}{385d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.801438, size = 146, normalized size = 0.55

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((4228A+4147C)\cos(c+dx)+2(728A+737C)\cos(2(c+dx))+1456A\cos(3(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(13/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(1652*A + 1188*C + (4228*A + 4147*C)*Cos[c +
d*x] + 2*(728*A + 737*C)*Cos[2*(c + d*x)] + 1456*A*Cos[3*(c + d*x)] + 1859*
C*Cos[3*(c + d*x)] + 224*A*Cos[4*(c + d*x)] + 286*C*Cos[4*(c + d*x)] + 224*
A*Cos[5*(c + d*x)] + 286*C*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(2310*d*Cos[
c + d*x]^(11/2))
```

Maple [A] time = 0.135, size = 144, normalized size = 0.5

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(896\,A\,(\cos(dx+c))^{5}+1144\,C\,(\cos(dx+c))^{5}+448\,A\,(\cos(dx+c))^{4}+572\,C\,(\cos(dx+c))^{4}+\right.}{1155\,d\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/1155/d*a*(-1+cos(d*x+c))*(896*A*cos(d*x+c)^5+1144*C*cos(d*x+c)^5+448*A*c
os(d*x+c)^4+572*C*cos(d*x+c)^4+336*A*cos(d*x+c)^3+429*C*cos(d*x+c)^3+280*A*
cos(d*x+c)^2+165*C*cos(d*x+c)^2+245*A*cos(d*x+c)+105*A)*(a*(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 1.9861, size = 837, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] 4/1155*(11*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(
2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*C*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 1)) + 7*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*
x + c) + 1) - 495*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105
6*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2
)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9 - 299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 + 46*sqrt(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x
```

```
+ c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.52965, size = 381, normalized size = 1.43

$$\frac{2\left(8\left(112A+143C\right)a\cos\left(dx+c\right)^{5}+4\left(112A+143C\right)a\cos\left(dx+c\right)^{4}+3\left(112A+143C\right)a\cos\left(dx+c\right)^{3}+5\left(56A+33\right.\right.}{1155\left(d\cos\left(dx+c\right)^{7}+d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] 2/1155*(8*(112*A + 143*C)*a*cos(d*x + c)^5 + 4*(112*A + 143*C)*a*cos(d*x +
c)^4 + 3*(112*A + 143*C)*a*cos(d*x + c)^3 + 5*(56*A + 33*C)*a*cos(d*x + c)^
2 + 245*A*a*cos(d*x + c) + 105*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(1
3/2), x)
```

3.188 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=312

$$\frac{a^3(136A+109C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(1304A+1015C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{768d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(24A+23C)\sin(c+dx)}{}$$

```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]])/(512*d) + (a^3*(1304*A + 1015*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(512*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1304*A + 1015*C)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(136*A + 109*C)*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(24*A
 + 23*C)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(96*d) +
 (a*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) +
(C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.912638, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(136A+109C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(1304A+1015C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{768d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(24A+23C)\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]])/(512*d) + (a^3*(1304*A + 1015*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(512*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1304*A + 1015*C)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(136*A + 109*C)*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(24*A
 + 23*C)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(96*d) +
 (a*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) +
(C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{6d} + \frac{\int c}{}$$

$$= \frac{aC\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{12d} + \frac{C}{}$$

$$= \frac{a^2(24A+23C)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c-}{96d}$$

$$= \frac{a^3(136A+109C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(24A-}{}$$

$$= \frac{a^3(1304A+1015C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{768d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(136}{}$$

$$= \frac{a^3(1304A+1015C)\sqrt{\cos(c+dx)}\sin(c+dx)}{512d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(130}{}$$

$$= \frac{a^3(1304A+1015C)\sqrt{\cos(c+dx)}\sin(c+dx)}{512d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(130}{}$$

$$= \frac{a^{5/2}(1304A+1015C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{512d} + \frac{a^3(1304A}{}$$

Mathematica [A] time = 2.46453, size = 170, normalized size = 0.54

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(1304A+1015C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]
^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(1304*A + 1015*
C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4648*A + 4193*C
 + (2896*A + 3234*C)*Cos[c + d*x] + 4*(184*A + 315*C)*Cos[2*(c + d*x)] + 96
*A*Cos[3*(c + d*x)] + 428*C*Cos[3*(c + d*x)] + 112*C*Cos[4*(c + d*x)] + 16*
C*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]))/(3072*d)
```

Maple [B] time = 0.162, size = 581, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/1536/d*a^2*(-1+cos(d*x+c))^4*(384*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin
(d*x+c)*cos(d*x+c)^5+2240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^4*
sin(d*x+c)+5936*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
+256*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^7+10600*A*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+896*C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6+10432*A*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+1392*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+3912*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
+1624*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2030*C*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3045*C*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3912*A*cos(d*x+c)^2*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3045*C*cos(d*x+c)^2*
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c
)^8
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.50217, size = 582, normalized size = 1.87

$$\frac{\left(256\,Ca^2\cos\left(dx+c\right)^5+896\,Ca^2\cos\left(dx+c\right)^4+48\left(8\,A+29\,C\right)a^2\cos\left(dx+c\right)^3+8\left(184\,A+203\,C\right)a^2\cos\left(dx+c\right)^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] 1/1536*((256*C*a^2*cos(d*x + c)^5 + 896*C*a^2*cos(d*x + c)^4 + 48*(8*A + 29
*C)*a^2*cos(d*x + c)^3 + 8*(184*A + 203*C)*a^2*cos(d*x + c)^2 + 2*(1304*A +
 1015*C)*a^2*cos(d*x + c) + 3*(1304*A + 1015*C)*a^2)*sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((1304*A + 1015*C)*a^2*cos(d*x + c)
+ (1304*A + 1015*C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.189 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right) d$

Optimal. Leaf size=265

$$\frac{a^3(1040A+787C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{960d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{240d}+\frac{a^{5/2}(400A}{}$$

```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^3*(400*A + 283*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1040*A + 787*C)*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(80*A + 79*C)*Cos[c + d*
x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(240*d) + (a*C*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.792657, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(1040A+787C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{960d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{240d}+\frac{a^{5/2}(400A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^3*(400*A + 283*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1040*A + 787*C)*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(80*A + 79*C)*Cos[c + d*
x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(240*d) + (a*C*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{\int}{}$$

$$= \frac{aC\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{8d} + \frac{C}{}$$

$$= \frac{a^2(80A+79C)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c}{240d}$$

$$= \frac{a^3(1040A+787C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(80A}{}$$

$$= \frac{a^3(400A+283C)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(1040}{}$$

$$= \frac{a^3(400A+283C)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(1040}{}$$

$$= \frac{a^{5/2}(400A+283C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^3(400A}{1}$$

Mathematica [A] time = 1.59037, size = 148, normalized size = 0.56

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(400A+283C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]
^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(400*A + 283*C
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6320*A + 5521*C
+ (2720*A + 3874*C)*Cos[c + d*x] + 4*(80*A + 331*C)*Cos[2*(c + d*x)] + 348*
```

```
C*Cos[3*(c + d*x)] + 48*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(3840*d)
```

Maple [B] time = 0.127, size = 509, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/1920/d*a^2*(-1+cos(d*x+c))^3*(640*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*co
s(d*x+c)^4*sin(d*x+c)+4000*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)+12080*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)+384*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6+14720*A*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+1392*C*sin(d*x+c)*c
os(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6000*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)+2264*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+2830*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+4245*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6000*A*
cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)+4245*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [B] time = 4.01267, size = 6151, normalized size = 23.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/7680*(80*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
```

```
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos
(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
```

3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)),
 (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)) - 1))*sqrt(a))*A + (10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2
*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((135*a^2*
sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*sin(3/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 135*a^2*sin(1/5*arctan2(sin(5*d*x
 + 5*c), cos(5*d*x + 5*c))))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
) + 1)) - (135*a^2*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 8
8*a^2*cos(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 135*a^2*cos(1/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 88*a^2)*sin(3/2*arctan2(si
n(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))) + 1)))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x
 + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(
1/4)*(8*(a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d
*x + 5*c) + a^2*sin(5*d*x + 5*c)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*
x + 5*c)))^2 + 2*a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*s
in(5*d*x + 5*c) + a^2*sin(5*d*x + 5*c))*cos(5/2*arctan2(sin(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c))) + 1)) - 5*(35*a^2*sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))) + 35*a^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 40
*a^2*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 248*a^2*sin(1/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*cos(1/2*arctan2(sin(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))) + 1)) - 8*(a^2*cos(5*d*x + 5*c) + (a^2*cos(5*d*x + 5*c)
 - a^2)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + (a^2*cos(5
*d*x + 5*c) - a^2)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 -
 a^2 + 2*(a^2*cos(5*d*x + 5*c) - a^2)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c))))*sin(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*
x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 5*
(35*a^2*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 35*a^2*cos(3
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 40*a^2*cos(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))) - 168*a^2*cos(1/5*arctan2(sin(5*d*x + 5*

```
c), cos(5*d*x + 5*c))) + 208*a^2)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x
 + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))) + 1)))*sqrt(a) + 4245*(a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c
), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(
cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*
x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c))) + 1)^(1/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)
))*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), co
s(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))) + 1))) + 1) - a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2
+ 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))),
 cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))) + 1)^(1/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos
(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*
arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x +
5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
))) + 1))) - 1) - a^2*arctan2((cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos
(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*sin(1/2*arctan
2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)), (cos(2/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
)^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*cos
(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*
arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 1) + a^2*arctan2((cos(
2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
```

```
1)), (cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))) + 1)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 2.43062, size = 525, normalized size = 1.98

$$\frac{\left(384\,Ca^2\cos\left(dx+c\right)^4+1392\,Ca^2\cos\left(dx+c\right)^3+8\,(80\,A+283\,C)a^2\cos\left(dx+c\right)^2+10\,(272\,A+283\,C)a^2\cos\left(dx+c\right)+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/1920*((384*C*a^2*cos(d*x + c)^4 + 1392*C*a^2*cos(d*x + c)^3 + 8*(80*A + 2
83*C)*a^2*cos(d*x + c)^2 + 10*(272*A + 283*C)*a^2*cos(d*x + c) + 15*(400*A
+ 283*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15
*((400*A + 283*C)*a^2*cos(d*x + c) + (400*A + 283*C)*a^2)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d
*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.190 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=218

$$\frac{a^{5/2}(304A+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+299C)\sin(c+dx)\sqrt{\cos(c+dx)}}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(16A+17C)\sin(c+dx)\sqrt{\cos}}{32d}$$

```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^3*(432*A + 299*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*
d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(16*A + 17*C)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]]*Sin[c + d*x])/(32*d) + (5*a*C*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.713289, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3046, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(304A+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+299C)\sin(c+dx)\sqrt{\cos(c+dx)}}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(16A+17C)\sin(c+dx)\sqrt{\cos}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```


```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^3*(432*A + 299*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*
d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(16*A + 17*C)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]]*Sin[c + d*x])/(32*d) + (5*a*C*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{4d} + \frac{\int \frac{(a+a\cos(c+dx))^{5/2}(}{\sqrt{}}}{}$$

$$= \frac{5aC\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{24d} + \frac{C\sqrt{\cos(c+dx)}}{}$$

$$= \frac{a^2(16A+17C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{32d} + \frac{5aC\sqrt{}}{}$$

$$= \frac{a^3(432A+299C)\sqrt{\cos(c+dx)}\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(16A+17C)\sqrt{\cos(c}}{}$$

$$= \frac{a^3(432A+299C)\sqrt{\cos(c+dx)}\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(16A+17C)\sqrt{\cos(c}}{}$$

$$= \frac{a^{5/2}(304A+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(432A+299C)\sqrt{\cos(c}}{192d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.945872, size = 131, normalized size = 0.6

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(304A+163C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(304*A + 163*C)
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(528*A + 581*C + (
96*A + 362*C)*Cos[c + d*x] + 92*C*Cos[2*(c + d*x)] + 12*C*Cos[3*(c + d*x)])
*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.119, size = 437, normalized size = 2.

$$\frac{a^2(-1+\cos(dx+c))^2}{192\,d\,(\sin(dx+c))^4}\left(96\,A\sin(dx+c)(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+720\,A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos}{1+\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(96*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)+720*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(5/2)+1152*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+48*C
*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+528*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+184*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+326*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+489*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+912*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+489*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)/cos(d*x+c)^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 4.46888, size = 11552, normalized size = 52.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/768*(48*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2
*d*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(
2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a
^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
```

*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (
10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*
c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x
 + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2
*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4
c) - 2(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 12*(
a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*si
n(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*a
^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) +
 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*
c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*
cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d
*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*
d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*(4*a^2*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)
^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 - 8*a^2*co
s(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 14*a^2*cos(4*d*x + 4*c)^2
+ 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*
c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (3*a

^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)
^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x
+ 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c
)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 +
2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*
a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
4*(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos(4*d*x
+ 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2)*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a
^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3*a^2*co
s(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(
(3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 3*a
^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) -
160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*
c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4*(3*a^2
*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) +
a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^
2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(3*
a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c)^2 + 6*a^2*cos(4*d*x +
4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*
d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*cos(4*d*x + 4*c)^2 - a^2*cos(
4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(4*d*x + 4*c)^2 + 320*a^2*sin(
4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*a^2)*sin(1/4*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^2*sin(4*d*x + 4*c)^2 - 317*a^

```
2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8
0*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c)
 + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*(10*a^2*
cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2 - 160*a^2*sin(4*d*x + 4*c)*s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 10*a^2*cos(4*d*x + 4*
c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 3*(a^2*cos(4*d*x
 + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (160*a^2*cos(4*d*x + 4*c)^
2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 120*a^2*cos(4*d*x + 4*c)^2 - 1
60*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c
) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 3*a^2*sin
(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(3*a
^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c)^2 - 197*a^2*cos(4*d*x + 4*c
) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 - 80*(a^
2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^
2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*si
n(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 126*a^2*cos(4*d*x + 4*c)^2
 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x +
 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2
*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*
d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*cos(4*d*x +
4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a^2*cos(4*d*x + 4*c) + 2*(3*a^
2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)^2 - (160*a^2*cos(4*d*x + 4*c
)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos(4*d*x + 4*c) - 3*a^2)*cos(1/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(4*d*x +
 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(
4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 6*(a^2
*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + a^
2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*
c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
```

```
 + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(
a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*
d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*
d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*c
os(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x +
 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^
2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c
))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2
(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)^2 + a^
```

```
2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 -
 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos
(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c
))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d
*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
+ 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*
c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*c
os(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x +
 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1
)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) -
1))*sqrt(a))*C/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x +
4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*
x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.44651, size = 464, normalized size = 2.13

$$\frac{\left(48\,Ca^2\cos\left(dx+c\right)^3+184\,Ca^2\cos\left(dx+c\right)^2+2\,(48\,A+163\,C)a^2\cos\left(dx+c\right)+3\,(176\,A+163\,C)a^2\right)\sqrt{a\cos\left(dx+c\right)+}}{192\,(d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/192*((48*C*a^2*cos(d*x + c)^3 + 184*C*a^2*cos(d*x + c)^2 + 2*(48*A + 163*
C)*a^2*cos(d*x + c) + 3*(176*A + 163*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))*sin(d*x + c) - 3*((304*A + 163*C)*a^2*cos(d*x + c) + (304*A +
 163*C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x +
c)), x)
```

3.191 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=222

$$\frac{5a^{5/2}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-49C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d}$$

```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) - (a^3*(24*A - 49*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) - (a^2*(8*A - 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) - (a*(6*A - C)*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.721012, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2976, 2981, 2774, 216}

$$\frac{5a^{5/2}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-49C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```


```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) - (a^3*(24*A - 49*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) - (a^2*(8*A - 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) - (a*(6*A - C)*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}-\frac{1}{2}a(6A-}{\sqrt{\cos(c+dx)}}}{a}$$

$$= -\frac{a(6A-C)\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2A(a+}{}$$

$$= -\frac{a^2(8A-3C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} - \frac{a(6A-}{}$$

$$= -\frac{a^3(24A-49C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(8A-3C)\sqrt{\cos(c+d}{}$$

$$= -\frac{a^3(24A-49C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(8A-3C)\sqrt{\cos(c+d}{}$$

$$= \frac{5a^{5/2}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} - \frac{a^3(24A-49C)\sqrt{\cos(c+dx)}}{24d\sqrt{a+a\cos(c+a}}$$

Mathematica [A] time = 0.905203, size = 142, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(8A+5C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(3(8}{48d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(8*A + 5*C)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(48*A + 17*C + 3*(8*A
 + 27*C)*Cos[c + d*x] + 17*C*Cos[2*(c + d*x)] + 2*C*Cos[3*(c + d*x)])*Sin[(
c + d*x)/2]))/(48*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.12, size = 399, normalized size = 1.8

$$-\frac{a^2(-1+\cos(dx+c))}{24d(\sin(dx+c))^2}\left(24A\sin(dx+c)(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+96A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos}{1+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(24*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)+96*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)+120*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+8*C*sin(
d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+48*A*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)+34*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+75*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)+120*A*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))+75*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*cos(d*x+c)^3)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.92025, size = 3966, normalized size = 17.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/96*((4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d
*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*
```

```
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
 - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c)
```

```
, cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)) - 1))*sqrt(a))*C + 24*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sqrt(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/d
```

Fricas [A] time = 1.99882, size = 471, normalized size = 2.12

$$\frac{\left(8\,Ca^{2}\cos\left(dx+c\right)^{3}+34\,Ca^{2}\cos\left(dx+c\right)^{2}+3\,(8\,A+25\,C)a^{2}\cos\left(dx+c\right)+48\,Aa^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,s}{24\left(d\cos\left(dx+c\right)^{2}+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*((8*C*a^2*cos(d*x + c)^3 + 34*C*a^2*cos(d*x + c)^2 + 3*(8*A + 25*C)*a^
2*cos(d*x + c) + 48*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c) - 15*((8*A + 5*C)*a^2*cos(d*x + c)^2 + (8*A + 5*C)*a^2*cos(d*x + c
))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(
d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3
/2), x)
```

3.192 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=218

$$\frac{a^{5/2}(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(56A-27C)\sin(c+dx)\sqrt{\cos(c+dx)}}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d}$$

```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) - (a^3*(56*A - 27*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[
a + a*Cos[c + d*x]]) - (a^2*(8*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
 d*x]]*Sin[c + d*x])/(2*d) + (10*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.723403, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3044, 2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(56A-27C)\sin(c+dx)\sqrt{\cos(c+dx)}}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) - (a^3*(56*A - 27*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[
a + a*Cos[c + d*x]]) - (a^2*(8*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
 d*x]]*Sin[c + d*x])/(2*d) + (10*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d*Cos[c + d*x]^(3/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}-\frac{1}{2}a(4A}{\cos^{\frac{3}{2}}(c+dx)}}{3a}$$

$$= \frac{10aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c+dx))^{5/2}\text{s}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{a^2(8A-C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{10aA(a}{}$$

$$= -\frac{a^3(56A-27C)\sqrt{\cos(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(8A-C)\sqrt{\cos(c+d}}{}$$

$$= -\frac{a^3(56A-27C)\sqrt{\cos(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(8A-C)\sqrt{\cos(c+d}}{}$$

$$= \frac{a^{5/2}(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(56A-27C)\sqrt{\cos(c+dx}}{12d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.768891, size = 141, normalized size = 0.65

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(6\sqrt{2}(8A+19C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\sin\left(\frac{1}{2}(c+dx)\right)((1}{48d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(6*Sqrt[2]*(8*A + 19*C)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(16*A + 33*C + (128*A
 + 9*C)*Cos[c + d*x] + 33*C*Cos[2*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sin[(c
 + d*x)/2]))/(48*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.109, size = 354, normalized size = 1.6

$$\frac{a^2}{12\,d\,(1+\cos(dx+c))}\sqrt{a\,(1+\cos(dx+c))}\left(24\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}A\,(\cos(dx+c))^2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c)))^(1/2)*(24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*A*cos
(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6
*C*sin(d*x+c)*cos(d*x+c)^3+57*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*C*cos(d*x+c
)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+24*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))*cos(d*x+c)+33*C*sin(d*x+c)*cos(d*x+c)^2+57*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*cos(d*x+c)+64*A*sin(d*x+c)*cos(d*x+c)+8*A*sin(d*x+c))*a^2/(1+
cos(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.6211, size = 3379, normalized size = 15.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/48*(3*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
```

```
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C + 8*(
30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)
*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((12*a^
2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 3
*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos(2*d*
x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a^2*co
s(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
 + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x +
 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A/(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.98088, size = 463, normalized size = 2.12

$$\frac{\left(6\,Ca^2\cos\left(dx+c\right)^3+33\,Ca^2\cos\left(dx+c\right)^2+64\,Aa^2\cos\left(dx+c\right)+8\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\cdots}{12\left(d\cos\left(dx+c\right)^3+d\cos\left(d\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/12*((6*C*a^2*cos(d*x + c)^3 + 33*C*a^2*cos(d*x + c)^2 + 64*A*a^2*cos(d*x
+ c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) -
3*((8*A + 19*C)*a^2*cos(d*x + c)^3 + (8*A + 19*C)*a^2*cos(d*x + c)^2)*sqrt(
a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)
)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5
/2), x)
```

3.193 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=210

$$-\frac{a^3(64A+15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (5*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (
a^3*(64*A + 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
 d*x]]) + (2*a^2*(8*A + 5*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sq
rt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Co
s[c + d*x]^(3/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[
c + d*x]^(5/2))
```

Rubi [A] time = 0.725002, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2981, 2774, 216}

$$-\frac{a^3(64A+15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```


```
[Out] (5*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (
a^3*(64*A + 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
 d*x]]) + (2*a^2*(8*A + 5*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sq
rt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Co
s[c + d*x]^(3/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[
c + d*x]^(5/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}-\frac{1}{2}a(2A-}{\cos^{\frac{5}{2}}(c+dx)}}{5a}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{5/2}\sin}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(8A+5C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx}{3d\cos^{\frac{3}{2}}(c}$$

$$= -\frac{a^3(64A+15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5C)\sqrt{a+a\cos}}{5d\sqrt{\cos(c}}$$

$$= -\frac{a^3(64A+15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5C)\sqrt{a+a\cos}}{5d\sqrt{\cos(}}$$

$$= \frac{5a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3(64A+15C)\sqrt{\cos(c+dx)}\sin(c+d}{15d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.89289, size = 141, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((112A+45C)\cos(c+dx)+4(43A+15C)\cos(2(c+dx))+19\right.}{120d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(300*Sqrt[2]*C*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(196*A + 60*C + (112*A + 45*C)
*Cos[c + d*x] + 4*(43*A + 15*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*S
in[(c + d*x)/2]))/(120*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.161, size = 245, normalized size = 1.2

$$-\frac{a^{2}}{15\,d\sin\left(dx+c\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-75\,C\sin\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{3/2}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-75*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))*cos(d*x+c)^2-75*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+15*C*cos(
d*x+c)^4+86*A*cos(d*x+c)^3+15*C*cos(d*x+c)^3-58*A*cos(d*x+c)^2-30*C*cos(d*x
+c)^2-22*A*cos(d*x+c)-6*A)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.20763, size = 1520, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/60*(15*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 +
```

```
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4) + 32*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x
 + c) + 1) - 35*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sq
rt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7
/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)))/d
```

Fricas [A] time = 1.70235, size = 451, normalized size = 2.15

$$\frac{\left(15\,Ca^2\cos\left(dx+c\right)^3+2\left(43\,A+15\,C\right)a^2\cos\left(dx+c\right)^2+28\,Aa^2\cos\left(dx+c\right)+6\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,\mathrm{s}\cdots}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*((15*C*a^2*cos(d*x + c)^3 + 2*(43*A + 15*C)*a^2*cos(d*x + c)^2 + 28*A*
a^2*cos(d*x + c) + 6*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - 75*(C*a^2*cos(d*x + c)^4 + C*a^2*cos(d*x + c)^3)*sqrt(a)*arctan
(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*co
s(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7
/2), x)
```

3.194 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{2a^2(8A+7C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+2a$$

```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(32*A + 49*C)*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(21*d
*Cos[c + d*x]^(3/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d
*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*C
os[c + d*x]^(7/2))
```

Rubi [A] time = 0.668005, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2980, 2774, 216}

$$\frac{2a^2(8A+7C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+2a$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```


```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(32*A + 49*C)*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(21*d
*Cos[c + d*x]^(3/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d
*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*C
os[c + d*x]^(7/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{7}{2}aC\cos\right.}{\cos^{\frac{7}{2}}(c+dx)}}{7a}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{5/2}\sin}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(8A+7C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx}{7d\cos^{\frac{5}{2}}(c}$$

$$= \frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+7C)\sqrt{a+a\cos(c+}}{21d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+7C)\sqrt{a+a\cos(c+}}{21d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 1.3308, size = 151, normalized size = 0.72

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((93A+84C)\cos(c+dx)+(23A+7C)\cos(2(c+dx))+23A\,c\right.}{84d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(84*Sqrt[2]*C*ArcSin[Sqrt[
2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 4*(29*A + 7*C + (93*A + 84*C)*Cos
[c + d*x] + (23*A + 7*C)*Cos[2*(c + d*x)] + 23*A*Cos[3*(c + d*x)] + 28*C*Co
s[3*(c + d*x)])*Sin[(c + d*x)/2]))/(84*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.103, size = 327, normalized size = 1.6

$$-\frac{2\,a^{2}}{21\,d\sin\left(dx+c\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-21\,C\sin\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{5/2}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-21*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))*cos(d*x+c)^3-42*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2-21*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+46*A*cos(d*x+c)^4+56*C*cos(d*x+c
)^4-23*A*cos(d*x+c)^3-49*C*cos(d*x+c)^3-11*A*cos(d*x+c)^2-7*C*cos(d*x+c)^2-
9*A*cos(d*x+c)-3*A)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 2.22742, size = 2214, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 1/42*(7*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 16*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.71152, size = 462, normalized size = 2.2

$$\frac{2\left(\left(2\,(23\,A+28\,C)a^{2}\cos\left(dx+c\right)^{3}+(23\,A+7\,C)a^{2}\cos\left(dx+c\right)^{2}+12\,Aa^{2}\cos\left(dx+c\right)+3\,Aa^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos}\cdots}{21\left(d\cos\left(dx+c\right)^{5}+d\cos\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/21*((2*(23*A + 28*C)*a^2*cos(d*x + c)^3 + (23*A + 7*C)*a^2*cos(d*x + c)^2
 + 12*A*a^2*cos(d*x + c) + 3*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 21*(C*a^2*cos(d*x + c)^5 + C*a^2*cos(d*x + c)^4)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9
/2), x)
```

3.195 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=219

$$\frac{2a^3(8A+11C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(64A+63C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(584A+903C)\sin(c+a}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}$$

```
[Out] (2*a^3*(8*A + 11*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^3*(584*A + 903*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 63*C)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (10*a*A*(a + a*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)
*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.719143, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3044, 2975, 2980, 2771}

$$\frac{2a^3(8A+11C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(64A+63C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(584A+903C)\sin(c+a}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2)
,x]
```


```
[Out] (2*a^3*(8*A + 11*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^3*(584*A + 903*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 63*C)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (10*a*A*(a + a*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)
*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a + a\cos(c + dx))^{5/2} \left(A + C\cos^2(c + dx)\right)}{\cos^{\frac{11}{2}}(c + dx)}\,dx = \frac{2A(a + a\cos(c + dx))^{5/2}\sin(c + dx)}{9d\cos^{\frac{9}{2}}(c + dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{1}{2}a(2A+}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$= \frac{10aA(a + a\cos(c + dx))^{3/2}\sin(c + dx)}{63d\cos^{\frac{7}{2}}(c + dx)} + \frac{2A(a + a\cos(c + dx))^{5/2}\sin}{9d\cos^{\frac{9}{2}}(c + dx)}$$

$$= \frac{2a^2(64A + 63C)\sqrt{a + a\cos(c + dx)}\sin(c + dx)}{315d\cos^{\frac{5}{2}}(c + dx)} + \frac{10aA(a + a\cos(c}{63d\cos}$$

$$= \frac{2a^3(8A + 11C)\sin(c + dx)}{15d\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}} + \frac{2a^2(64A + 63C)\sqrt{a + a\cos(c}}{315d\cos^{\frac{5}{2}}(c +}$$

$$= \frac{2a^3(8A + 11C)\sin(c + dx)}{15d\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}} + \frac{2a^3(584A + 903C)\sin(c +}{315d\sqrt{\cos(c + dx)}\sqrt{a + a\cos}}$$

Mathematica [A] time = 0.881527, size = 127, normalized size = 0.58

$$\frac{a^2\tan\left(\frac{1}{2}(c + dx)\right)\sqrt{a(\cos(c + dx) + 1)}(4(698A + 441C)\cos(c + dx) + 4(803A + 966C)\cos(2(c + dx)) + 584A\cos(3(c +}{1260d\cos^{\frac{9}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(2908*A + 2961*C + 4*(698*A + 441*C)*Cos[c
+ d*x] + 4*(803*A + 966*C)*Cos[2*(c + d*x)] + 584*A*Cos[3*(c + d*x)] + 588*
C*Cos[3*(c + d*x)] + 584*A*Cos[4*(c + d*x)] + 903*C*Cos[4*(c + d*x)])*Tan[(
c + d*x)/2])/(1260*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.114, size = 124, normalized size = 0.6

$$-\frac{2a^2(-1 + \cos(dx + c))\left(584A(\cos(dx + c))^4 + 903C(\cos(dx + c))^4 + 292A(\cos(dx + c))^3 + 294C(\cos(dx + c))^3 +}{315d\sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+903*C*cos(d*x+c)^4+292*A*c
os(d*x+c)^3+294*C*cos(d*x+c)^3+219*A*cos(d*x+c)^2+63*C*cos(d*x+c)^2+130*A*c
os(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.69634, size = 595, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 8/315*(21*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7)*C/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(7/2)) + (315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c)
 + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt
(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)
*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.53233, size = 339, normalized size = 1.55

$$\frac{2\left((584\,A+903\,C)a^2\cos(dx+c)^4+2\,(146\,A+147\,C)a^2\cos(dx+c)^3+3\,(73\,A+21\,C)a^2\cos(dx+c)^2+130\,Aa^2\cos\right.}{315\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/315*((584*A + 903*C)*a^2*cos(d*x + c)^4 + 2*(146*A + 147*C)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 21*C)*a^2*cos(d*x + c)^2 + 130*A*a^2*cos(d*x + c) + 35*A*
a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
1/2), x)
```

3.196 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{2a^3(568A+759C)\sin(c+dx)}{693d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(232A+297C)\sin(c+dx)}{693d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(32A+33C)\sin(c+dx)\sqrt{a\cos(c\cdots}}{231d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] (2*a^3*(232*A + 297*C)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(32*A + 33*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) + (10*a*A*(a +
a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 0.803021, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2980, 2772, 2771}

$$\frac{2a^3(568A+759C)\sin(c+dx)}{693d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(232A+297C)\sin(c+dx)}{693d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(32A+33C)\sin(c+dx)\sqrt{a\cos(c\cdots}}{231d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2)
,x]
```


```
[Out] (2*a^3*(232*A + 297*C)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(32*A + 33*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) + (10*a*A*(a +
a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{1}{2}a(4A}{\cos^{\frac{11}{2}}(c+dx)}}{11a}$$

$$= \frac{10aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{5/2}\,s}{11d\cos^{\frac{11}{2}}(c+dx}$$

$$= \frac{2a^2(32A+33C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)} + \frac{10aA(a+a\cos(}{99d\cos}$$

$$= \frac{2a^3(232A+297C)\sin(c+dx)}{693d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(32A+33C)\sqrt{a+a\cos}}{231d\cos^{\frac{7}{2}}(c}$$

$$= \frac{2a^3(232A+297C)\sin(c+dx)}{693d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(568A+759C)\sin(}{693d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c}}$$

$$= \frac{2a^3(232A+297C)\sin(c+dx)}{693d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(568A+759C)\sin(}{693d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c}}$$

Mathematica [A] time = 1.00817, size = 149, normalized size = 0.56

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(5014A+4983C)\cos(c+dx)+52(71A+66C)\cos(2(c+dx))+3692A\cos(3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(13/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3628*A + 2673*C + 2*(5014*A + 4983*C)*Cos[
c + d*x] + 52*(71*A + 66*C)*Cos[2*(c + d*x)] + 3692*A*Cos[3*(c + d*x)] + 45
87*C*Cos[3*(c + d*x)] + 568*A*Cos[4*(c + d*x)] + 759*C*Cos[4*(c + d*x)] + 5
68*A*Cos[5*(c + d*x)] + 759*C*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(2772*d*C
```

```
os[c + d*x]^(11/2))
```

Maple [A] time = 0.127, size = 146, normalized size = 0.6

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(1136\,A\left(\cos\left(dx+c\right)\right)^{5}+1518\,C\left(\cos\left(dx+c\right)\right)^{5}+568\,A\left(\cos\left(dx+c\right)\right)^{4}+759\,C\left(\cos\left(dx+c\right)\right)^{4}\right.}{693\,d\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/693/d*a^2*(-1+cos(d*x+c))*(1136*A*cos(d*x+c)^5+1518*C*cos(d*x+c)^5+568*A
*cos(d*x+c)^4+759*C*cos(d*x+c)^4+426*A*cos(d*x+c)^3+396*C*cos(d*x+c)^3+355*
A*cos(d*x+c)^2+99*C*cos(d*x+c)^2+224*A*cos(d*x+c)+63*A)*(a*(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 1.72205, size = 782, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] 8/693*(33*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*C*(sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + (693*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(2)*a^(5/2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3575*sq
rt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1300*sqrt(2)*a^(5/2)*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(
cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) +
```

```
1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.61442, size = 387, normalized size = 1.45

$$\frac{2\left(2\,(568\,A+759\,C)a^2\cos\left(dx+c\right)^5+(568\,A+759\,C)a^2\cos\left(dx+c\right)^4+6\,(71\,A+66\,C)a^2\cos\left(dx+c\right)^3+(355\,A+99\right.}{693\left(d\cos\left(dx+c\right)^7+d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] 2/693*(2*(568*A + 759*C)*a^2*cos(d*x + c)^5 + (568*A + 759*C)*a^2*cos(d*x +
 c)^4 + 6*(71*A + 66*C)*a^2*cos(d*x + c)^3 + (355*A + 99*C)*a^2*cos(d*x + c
)^2 + 224*A*a^2*cos(d*x + c) + 63*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
3/2), x)
```

3.197 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=313

$$\frac{8a^3(8368A+10439C)\sin(c+dx)}{45045d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(8368A+10439C)\sin(c+dx)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(2224A+2717C)\sin(c+d}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx}$$

```
[Out] (2*a^3*(2224*A + 2717*C)*Sin[c + d*x])/(9009*d*Cos[c + d*x]^(7/2)*Sqrt[a +
a*Cos[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Sin[c + d*x])/(15015*d*Cos[c +
 d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sin[c + d
*x])/(45045*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^3*(8368*
A + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1287
*d*Cos[c + d*x]^(9/2)) + (10*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
143*d*Cos[c + d*x]^(11/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/
(13*d*Cos[c + d*x]^(13/2))
```

Rubi [A] time = 0.91036, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2975, 2980, 2772, 2771}

$$\frac{8a^3(8368A+10439C)\sin(c+dx)}{45045d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(8368A+10439C)\sin(c+dx)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(2224A+2717C)\sin(c+d}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2)
,x]
```


```
[Out] (2*a^3*(2224*A + 2717*C)*Sin[c + d*x])/(9009*d*Cos[c + d*x]^(7/2)*Sqrt[a +
a*Cos[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Sin[c + d*x])/(15015*d*Cos[c +
 d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sin[c + d
*x])/(45045*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^3*(8368*
A + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1287
*d*Cos[c + d*x]^(9/2)) + (10*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
143*d*Cos[c + d*x]^(11/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/
(13*d*Cos[c + d*x]^(13/2))
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{13d\cos^{\frac{13}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{1}{2}a(6A}{\cos^{\frac{13}{2}}(c+dx)}}{13a}$$

$$= \frac{10aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{143d\cos^{\frac{11}{2}}(c+dx)} + \frac{2A(a+a\cos(c+dx))^{5/2}\,\mathrm{s}}{13d\cos^{\frac{13}{2}}(c+dx}$$

$$= \frac{2a^2(136A+143C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{1287d\cos^{\frac{9}{2}}(c+dx)} + \frac{10aA(a+a\,\mathrm{co}}{143d}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(136A+143C)\sqrt{a+a}}{1287d\cos}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(8368A+10439C)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(8368A+10439C)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(8368A+10439C)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

Mathematica [A] time = 0.911178, size = 171, normalized size = 0.55

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(1120(347A+286C)\cos(c+dx)+14(30334A+32747C)\cos(2(c+dx))+125}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(15/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(343612*A + 322751*C + 1120*(347*A + 286*C)
*Cos[c + d*x] + 14*(30334*A + 32747*C)*Cos[2*(c + d*x)] + 125520*A*Cos[3*(c
 + d*x)] + 141570*C*Cos[3*(c + d*x)] + 125520*A*Cos[4*(c + d*x)] + 156585*C
*Cos[4*(c + d*x)] + 16736*A*Cos[5*(c + d*x)] + 20878*C*Cos[5*(c + d*x)] + 1
6736*A*Cos[6*(c + d*x)] + 20878*C*Cos[6*(c + d*x)])*Tan[(c + d*x)/2])/(1801
80*d*Cos[c + d*x]^(13/2))
```

Maple [A] time = 0.148, size = 168, normalized size = 0.5

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(66944\,A\,(\cos(dx+c))^6+83512\,C\,(\cos(dx+c))^6+33472\,A\,(\cos(dx+c))^5+41756\,C\,(\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+83512*C*cos(d*x+c)^6+3
3472*A*cos(d*x+c)^5+41756*C*cos(d*x+c)^5+25104*A*cos(d*x+c)^4+31317*C*cos(d
*x+c)^4+20920*A*cos(d*x+c)^3+18590*C*cos(d*x+c)^3+18305*A*cos(d*x+c)^2+5005
*C*cos(d*x+c)^2+11970*A*cos(d*x+c)+3465*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x
+c)/cos(d*x+c)^(13/2)
```

Maxima [B] time = 3.60757, size = 906, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, al
gorithm="maxima")
```

```
[Out] 8/45045*(143*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqr
t(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*C*(sin(d*x + c
```

```
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)) + (45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) +
 1) - 165165*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*s
qrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x +
c) + 1)^11 + 88980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 -
11864*sqrt(2)*a^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^
10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.46632, size = 475, normalized size = 1.52

$$\frac{2\left(8\,(8368\,A+10439\,C)a^2\cos\left(dx+c\right)^6+4\,(8368\,A+10439\,C)a^2\cos\left(dx+c\right)^5+3\,(8368\,A+10439\,C)a^2\cos\left(dx+c\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, al
gorithm="fricas")
```

```
[Out] 2/45045*(8*(8368*A + 10439*C)*a^2*cos(d*x + c)^6 + 4*(8368*A + 10439*C)*a^2
*cos(d*x + c)^5 + 3*(8368*A + 10439*C)*a^2*cos(d*x + c)^4 + 10*(2092*A + 18
59*C)*a^2*cos(d*x + c)^3 + 35*(523*A + 143*C)*a^2*cos(d*x + c)^2 + 11970*A*
a^2*cos(d*x + c) + 3465*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c)/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
5/2), x)
```

3.198 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=226

$$-\frac{(8A+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((8*A + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*S
qrt[a]*d) + (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((8*A + 7*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - (C*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.749253, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3046, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(8A+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((8*A + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*S
qrt[a]*d) + (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((8*A + 7*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - (C*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
```

```
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(6A+5C)-\frac{1}{2}aC\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= -\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \int \frac{\sqrt{\cos(c+dx)}\left(-\right.}{}$$

$$= \frac{(8A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}}{3d\sqrt{}}$$

$$= \frac{(8A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}}{3d\sqrt{}}$$

$$= \frac{(8A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}}{3d\sqrt{}}$$

$$= -\frac{(8A+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(}}\right.}{\sqrt{a}d}$$

Mathematica [C] time = 2.10126, size = 349, normalized size = 1.54

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(24A-2C\cos(c+dx)+4C\cos(2(c+dx))+25C) - \frac{3i\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(((-3*I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))]*((-8*I)*A*d*x - (9*I)*C*d*x - (8*A + 9*C)*ArcSi
nh[E^(I*(c + d*x))] + 16*Sqrt[2]*(A + C)*Log[1 + E^(I*(c + d*x))] + 8*A*Log
[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + 9*C*Log[1 + Sqrt[1 + E^((2*I)*(c + d*
x))]] - 16*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))]] - 16*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + 4*Sqrt[Cos[c + d*x]]*(24*
A + 25*C - 2*C*Cos[c + d*x] + 4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*
d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.126, size = 429, normalized size = 1.9

$$-\frac{(-1+\cos(dx+c))^4}{24\,d\,(\sin(dx+c))^8\,a}\left(-24\,A\sin(dx+c)\,(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}-48\,A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx}{1+\cos(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^4*(-24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)-48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-8*C*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+24*A*2^(1/2)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+24*C*2^(1/2)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))-21
*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*cos(d*x+c
)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+27*C*co
s(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))
*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^8/(cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 29.4987, size = 555, normalized size = 2.46

$$\frac{\left(8\,C\cos\left(dx+c\right)^{2}-2\,C\cos\left(dx+c\right)+24\,A+21\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(8\,A+9\,C\right)\cos\left(\cdots\right.\right.}{24\left(ad\cos\left(dx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*((8*C*cos(d*x + c)^2 - 2*C*cos(d*x + c) + 24*A + 21*C)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((8*A + 9*C)*cos(d*x + c) + 8
*A + 9*C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))) - 24*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*arctan(
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a
), x)
```

3.199 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=183

$$\frac{(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{C\sin(c}{4d\sqrt{}}$$

```
[Out] ((8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - (C*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.564109, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3046, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{C\sin(c}{4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] ((8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - (C*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(4A+3C)-\frac{1}{2}aC\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a}$$

$$= -\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{-\frac{a^2C}{4}+\frac{1}{4}a^2(8A}{\sqrt{\cos(c+dx)}\sqrt{}}}{2}$$

$$= -\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + (-A-C)\int$$

$$= -\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A+C))\,S}{}$$

$$= \frac{(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c}}\right.}{\sqrt{a}d}$$

Mathematica [C] time = 1.58024, size = 344, normalized size = 1.88

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4C\left(\sin\left(\frac{3}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(8i\sqrt{2}(A+C)\log\left(1+e^{i(c+dx)}\right)-i(8A+7C)\sinh^{-1}\left(e\right.\right.}{}\right.}{8\sqrt{a(\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(8*A*d*x + 7*C*d*x - I*(8*A + 7*C)*ArcSinh[E^(I*(c + d
*x))] + (8*I)*Sqrt[2]*(A + C)*Log[1 + E^(I*(c + d*x))] + (8*I)*A*Log[1 + Sq
rt[1 + E^((2*I)*(c + d*x))]] + (7*I)*C*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))
]] - (8*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))]] - (8*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^(
```

```
(2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (4*C*Sqrt[Cos[c +
d*x]]*(-2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/d))/(8*Sqrt[a*(1 + Cos[
c + d*x])])
```

Maple [A] time = 0.138, size = 253, normalized size = 1.4

$$-\frac{(-1+\cos(dx+c))^3}{4\,d\,(\sin(dx+c))^6\,a}\left(2\,C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}+4\,C\arcsin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^3*(2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+4*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+4*C*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*2^(1/2)-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+8*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+7*C*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^
(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^6/(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 30.8548, size = 508, normalized size = 2.78

$$\frac{(2\,C\cos(dx+c)-C)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-((8\,A+7\,C)\cos(dx+c)+8\,A+7\,C)\sqrt{a}\arctan\Big(}{4\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*((2*C*cos(d*x + c) - C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - ((8*A + 7*C)*cos(d*x + c) + 8*A + 7*C)*sqrt(a)*arctan(sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 4*sqrt(2)*((A
+ C)*a*cos(d*x + c) + (A + C)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a
), x)
```

3.200 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -((C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d))
+ (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.386999, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3046, 2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d))
+ (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\frac{1}{2}a(2A+C)-\frac{1}{2}aC\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{C\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + (A+C)\int \frac{}{\sqrt{\cos(c+dx)}\sqrt{}}$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{C\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} - \frac{(2a(A}{}$$

$$= -\frac{C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{C\sqrt{}}{}$$

Mathematica [A] time = 0.230397, size = 104, normalized size = 0.78

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A+C)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)-\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-(Sqrt[2]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]) + 2*(A + C
)*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*C*Sqrt[Cos[c + d*x]]*Sin[
(c + d*x)/2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.117, size = 178, normalized size = 1.3

$$-\frac{(-1+\cos(dx+c))^2}{da(\sin(dx+c))^4}\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}-C\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+C\arcsin\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+C*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*2^(1/2)+C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/a/sin(d*x+c)^4/(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.6548, size = 447, normalized size = 3.36

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(C\cos\left(dx+c\right)+C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}((A+C)a\cos}{}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*C*sqrt(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x +
c) + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) - sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*arctan(sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt
(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+C\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d
*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))), x)
```

3.201 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.391715, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3044, 2982, 2782, 205, 2774, 216}

$$-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}aC\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + (-A-C)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2C)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx}}\right.}{ad}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{}{d\sqrt{}}$$

Mathematica [C] time = 3.43515, size = 235, normalized size = 1.74

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A+C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5\cos^2(c+dx)(\cos(c+dx)+2)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right)\right.\right.}{2\cos^{\frac{5}{2}}(c+dx)}\right.}{5d\sqrt{a(\cos(c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x
]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(5*C*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] - (2*Sin
[(c + d*x)/2])/Sqrt[Cos[c + d*x]]) + ((A + C)*Csc[(c + d*x)/2]^3*(5*Cos[c +
 d*x]^2*(2 + Cos[c + d*x])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*
Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - Hypergeometr
ic2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*S
in[c + d*x]^2))/(2*Cos[c + d*x]^(5/2))))/(5*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.132, size = 271, normalized size = 2.

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2 a}\left(2A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2} + 4A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+A*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*2^(1/2)*cos(d*x+c)^3+C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(
d*x+c)^3+2*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*cos(d*x+c)^3)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)/(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.6911, size = 513, normalized size = 3.8

$$\frac{2\sqrt{a\cos(dx+c)+a}A\sqrt{\cos(dx+c)}\sin(dx+c)-2\left(C\cos(dx+c)^2+C\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right.}{ad\cos(dx+c)^2+ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] (2*sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(C*cos(d*
x + c)^2 + C*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A
 + C)*a*cos(d*x + c))*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x +
```

```
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**
(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx + c)^2 + A}{\sqrt{a\cos(dx + c) + a}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/
2)), x)
```

3.202 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.347912, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2984, 12, 2782, 205}

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}a(2A+3C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + (A$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2a}{}$$

$$= \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

Mathematica [C] time = 6.74937, size = 556, normalized size = 4.09

$$\frac{2(A+C)\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-12\left(3\sin^4\left(\frac{c}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x
]]),x]
```

```
[Out] (-8*C*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2]^3)/(3*d*Sqrt[a*(1 + Cos[c + d*x
])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (2*(A + C)*Cot[c/2 + (d*x)/2]*Csc
[c/2 + (d*x)/2]^4*(-12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1
, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*
x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*S
in[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3
*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*
x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8
*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)
/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^
2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqr
```

```
t[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.136, size = 264, normalized size = 1.9

$$-\frac{1}{3\,da\,(1+\cos(dx+c))}\sqrt{a\,(1+\cos(dx+c))}\left(3\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\,(\cos(dx+c))^{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c)))^(1/2)*(3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*A*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+3*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*sin(d*x+c)*cos(d*x+c)-2*A*sin(d*x+c))/a/(
1+cos(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98603, size = 436, normalized size = 3.21

$$-\frac{2\,(A\cos(dx+c)-A)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{3\sqrt{2}\left((A+C)a\cos(dx+c)^{3}+(A+C)a\cos(dx+c)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\,c}}{2\,(}\right.}{\sqrt{a}}}{3\left(ad\cos(dx+c)^{3}+ad\cos(dx+c)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*(2*(A*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c) - 3*sqrt(2)*((A + C)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2
)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x +
c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^3
+ a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx+c)^2 + A}{\sqrt{a \cos(dx+c) + a} \cos(dx+c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/
2)), x)
```

3.203 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{2(13A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2A\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sin[c + d*x])/(15*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A + 15*C)*Sin[c + d*x])/(15*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.51097, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2984, 12, 2782, 205}

$$\frac{2(13A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2A\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sin[c + d*x])/(15*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A + 15*C)*Sin[c + d*x])/(15*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
```

```
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}a(4A+5C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + $$

$$= -\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}$$

Mathematica [C] time = 7.95202, size = 1765, normalized size = 9.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x
]]),x]
```

```
[Out] -((C*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])]*(
1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2))) - (2*(A + C)*Cot[c/2 + (d*x)/2]*Csc[c/2
 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210
105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (
d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyp
ergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12
+ 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500
*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*
```

```
x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hyperge
ometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[
{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*Sqrt[a*(1 + C
os[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]
^2)) + (C*Cos[c/2 + (d*x)/2]*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)
/2]^2)^(5/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (
2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])))/(15*d*Sqrt[a*(1 +
 Cos[c + d*x])])
```

Maple [B] time = 0.113, size = 418, normalized size = 2.3

$$-\frac{(\sin(dx+c))^2}{15\,da\,(-1+\cos(dx+c))\,(1+\cos(dx+c))^2}\sqrt{a\,(1+\cos(dx+c))}\left(15\,A\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arcsin\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(15*A*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+15*C*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*c
os(d*x+c)^3+30*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d
```

```
*x+c))/sin(d*x+c))*cos(d*x+c)^2+30*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+15*A*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+15*C*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)+26*A*sin(d*x+c)*cos(d*x+c)^2+30*C*sin(d*x+c)*cos(d*x+c)^2-2*A*
sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c))/a/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/co
s(d*x+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99614, size = 482, normalized size = 2.66

$$\frac{2\left((13\,A+15\,C)\cos\left(dx+c\right)^2-A\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{15\,\sqrt{2}\left((A+C)a\cos(dx+\right.}{}}{15\left(ad\cos\left(dx+c\right)^4+ad\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(2*((13*A + 15*C)*cos(d*x + c)^2 - A*cos(d*x + c) + 3*A)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A + C)*a*cos(d*x
 + c)^4 + (A + C)*a*cos(d*x + c)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(
a)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/
2)), x)
```

3.204 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=224

$$\frac{2(31A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2(43A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Cos[c + d
*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sin[c + d*x])/(35*d*Cos[c + d*x]
^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(31*A + 35*C)*Sin[c + d*x])/(105*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(43*A + 35*C)*Sin[c + d*x])
/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.68914, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3044, 2984, 12, 2782, 205}

$$\frac{2(31A+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2(43A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```



```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Cos[c + d
*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sin[c + d*x])/(35*d*Cos[c + d*x]
^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(31*A + 35*C)*Sin[c + d*x])/(105*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(43*A + 35*C)*Sin[c + d*x])
/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
```

```
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}a(6A+7C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2A\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+\cdots}}$$

Mathematica [C] time = 10.2408, size = 2490, normalized size = 11.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + a*Cos[c + d*x
]]),x]
```

```
[Out] (-2*C*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])
]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) + (2*(A + C)*Cot[c/2 + (d*x)/2]*Csc[c
/2 + (d*x)/2]^8*(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2]^4
 + 26785605*Sin[c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d*x)/2]^8 + 17767780
8*Sin[c/2 + (d*x)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^12 + 52080*Hypergeom
etric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
*Sin[c/2 + (d*x)/2]^12 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1
```

, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x
)/2]^12 + 213120160*Sin[c/2 + (d*x)/2]^14 - 168280*Hypergeometric2F1[2, 11/
2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x
)/2]^14 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 1214
97024*Sin[c/2 + (d*x)/2]^16 + 212520*Hypergeometric2F1[2, 11/2, 13/2, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 3360
*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 40125184*Sin[c/2 +
 (d*x)/2]^18 - 124320*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 2240*Hypergeometric
PFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 5840384*Sin[c/2 + (d*x)/2]^20 + 2
8000*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 560*HypergeometricPFQ[{2, 2, 2, 2, 1
1/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*
Sin[c/2 + (d*x)/2]^20 + 363825*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)] - 5336100*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)] + 34636140*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] - 131060160*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] + 320535600*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)] - 530671680*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 604296000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 468948480*ArcTanh[Sqrt[Sin[c/2 + (
d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 237726720*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 70963200*ArcTanh[Sqr
t[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^1
8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 9461760*ArcTan
h[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)
/2]^20*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1120*Cos[
(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[c/2 + (
d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[
c/2 + (d*x)/2]^2) + 280*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2},
{1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d
x)/2]^12(103 - 164*Sin[c/2 + (d*x)/2]^2 + 70*Sin[c/2 + (d*x)/2]^4)))/(404
25*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(9/2)*(-1 + 2*

```
Sin[c/2 + (d*x)/2]^2)) + (2*C*Cos[c/2 + (d*x)/2]*((5*Sin[c/2 + (d*x)/2])/(1
 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2) + 2*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2
 + (d*x)/2]^2)^(5/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(
3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))))/(105*d*S
qrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.123, size = 554, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/105/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4*(105*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^4+105*C*
(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2
)*cos(d*x+c)^4+315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3+315*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)
^2+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*2^(1/2)*cos(d*x+c)^2+105*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)+105*C*(cos(d*x+c)/(1+cos(d*x+c))
)^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)+86*A*sin(d*x+
c)*cos(d*x+c)^3+70*C*sin(d*x+c)*cos(d*x+c)^3-62*A*sin(d*x+c)*cos(d*x+c)^2-7
0*C*sin(d*x+c)*cos(d*x+c)^2+6*A*sin(d*x+c)*cos(d*x+c)-30*A*sin(d*x+c))/a/(-
1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03486, size = 532, normalized size = 2.38

$$-\frac{2\left((43\,A+35\,C)\cos\left(dx+c\right)^{3}-(31\,A+35\,C)\cos\left(dx+c\right)^{2}+3\,A\cos\left(dx+c\right)-15\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{105\left(ad\cos\left(dx+c\right)^{5}+ad\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/105*(2*((43*A + 35*C)*cos(d*x + c)^3 - (31*A + 35*C)*cos(d*x + c)^2 + 3*
A*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 105*sqrt(2)*((A + C)*a*cos(d*x + c)^5 + (A + C)*a*cos(d*x + c)^4)*ar
ctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(
(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^5 + a*
d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(9/
2)), x)
```

3.205 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=245

$$\frac{(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A+13C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+$$

```
[Out] ((8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*a
^(3/2)*d) - ((5*A + 13*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((2*A + 7*C)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((A +
2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.786682, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3042, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A+13C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2),
x]
```


```
[Out] ((8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*a
^(3/2)*d) - ((5*A + 13*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((2*A + 7*C)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((A +
2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(A+5C)+2a(A+2C)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A+2C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \int$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(5A+13C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}}\right.}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 2.40302, size = 370, normalized size = 1.51

$$\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(2A+3C\cos(c+dx)-C\cos(2(c+dx))+6C)+\frac{\sqrt{2}e^{\frac{1}{2}}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(8*A*d*x + 19*C*d*x - I*(8*A + 19*C)*ArcSinh[E^(I*(c
 + d*x))] + (2*I)*Sqrt[2]*(5*A + 13*C)*Log[1 + E^(I*(c + d*x))] + (8*I)*A*L
og[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + (19*I)*C*Log[1 + Sqrt[1 + E^((2*I)*
(c + d*x))]] - (10*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 +
E^((2*I)*(c + d*x))]] - (26*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*Sqrt[Cos
[c + d*x]]*(2*A + 6*C + 3*C*Cos[c + d*x] - C*Cos[2*(c + d*x)])*Sec[(c + d*x
)/2]*Tan[(c + d*x)/2]))/(4*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.16, size = 477, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(2*A*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2*C*cos
(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*A*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)+13*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+
5*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*sin(d*x+c)*cos(d*x+c
)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+19*C*si
n(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+4*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7*C*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^9/(cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [A] time = 88.8205, size = 678, normalized size = 2.77

$$\frac{\sqrt{2}\left((5A+13C)\cos(dx+c)^2+2(5A+13C)\cos(dx+c)+5A+13C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\left(2C\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A + 13*C)*cos(d*x + c)^2 + 2*(5*A + 13*C)*cos(d*x + c) + 5
*A + 13*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))/(sqrt(a)*sin(d*x + c))) + (2*C*cos(d*x + c)^2 - 3*C*cos(d*x + c) - 2*A -
 7*C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - ((8*A + 19
*C)*cos(d*x + c)^2 + 2*(8*A + 19*C)*cos(d*x + c) + 8*A + 19*C)*sqrt(a)*arct
an(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.206 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{(A+9C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A+3C)\sin(c+dx)\ldots}{2ad\sqrt{\ldots}}$$

```
[Out] (-3*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
+ ((A + 9*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((A + 3*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.57045, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3042, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(A+9C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A+3C)\sin(c+dx)\ldots}{2ad\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-3*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
+ ((A + 9*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((A + 3*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(A-3C)+a(A+3C)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,d}{2a^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \ldots$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \ldots$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \ldots$$

$$= -\frac{3C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(A+9C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \ldots$$

Mathematica [C] time = 2.0282, size = 238, normalized size = 1.27

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(A+2C\cos(c+dx)+3C)}{d} + \frac{i\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\sqrt{2}(A+9C)\tanh^{-1}\left(\frac{1-e^{i(c+\ldots}}{\sqrt{2}\sqrt{1+e^{2i(\ldots}}}\right.\ldots\right.}{d\sqrt{1+e^{2i(c+dx)}}}\ldots\right)}{2(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((I*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]*(6*C*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*(A + 9*C)*
ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 6*
C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]
) + (2*Sqrt[Cos[c + d*x]]*(A + 3*C + 2*C*Cos[c + d*x])*Sec[(c + d*x)/2]*Tan
```

```
[(c + d*x)/2])/d))/(2*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.148, size = 394, normalized size = 2.1

$$\frac{(-1+\cos(dx+c))^{3}}{4\,da^{2}\,(\sin(dx+c))^{7}}\sqrt{\cos(dx+c)}\sqrt{a\,(1+\cos(dx+c))}\left(2\,A\,(\cos(dx+c))^{3}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+2\,A\,(\cos(dx+c))^{2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+A*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-2*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)+9*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin
(d*x+c)*cos(d*x+c)^2+4*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+12*
C*sin(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c))+2*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*C*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/a^2/(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^{2}+A\right)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [A] time = 50.0278, size = 601, normalized size = 3.2

$$-\frac{\sqrt{2}\left((A+9\,C)\cos\left(dx+c\right)^2+2\,(A+9\,C)\cos\left(dx+c\right)+A+9\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\,(2\,C\cos\left(dx\right.}{4\left(a^2d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A + 9*C)*cos(d*x + c)^2 + 2*(A + 9*C)*cos(d*x + c) + A + 9*
C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*(2*C*cos(d*x + c) + A + 3*C)*sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))*sin(d*x + c) - 12*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c)
 + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.207 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(3A-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((3*A - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.410958, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3042, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((3*A - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(3A-C)+2aC\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-5C)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A-5C)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\right.}{2d}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(3A-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - $$

Mathematica [C] time = 1.88859, size = 227, normalized size = 1.57

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(A+C)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{d} - \frac{ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-\sqrt{2}(3A-5C)\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+4C\sin\right.}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x))]*(4*C*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(3*A - 5*C)*Arc
Tanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 4*C*A
rcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d
*x))]) - ((A + C)*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))
/(a*(1 + Cos[c + d*x]))^(3/2)
```

Maple [B] time = 0.127, size = 365, normalized size = 2.5

$$-\frac{(-1+\cos(dx+c))^2}{4da^2(\sin(dx+c))^5}\sqrt{a(1+\cos(dx+c))}\left(-2A(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2} - 2A(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-2*A*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)-2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)+2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3*A*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)-5*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d
*x+c)^2-8*C*sin(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)/cos(d*x+c))-2*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+2*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)/a^2/cos(d*x+c)^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
 c))), x)
```

Fricas [A] time = 49.1729, size = 579, normalized size = 3.99

$$-\frac{\sqrt{2}\left((3\,A-5\,C)\cos\left(dx+c\right)^{2}+2\,(3\,A-5\,C)\cos\left(dx+c\right)+3\,A-5\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\,\sqrt{a\cos}}{4\left(a^{2}d\cos\left(dx+c\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((3*A - 5*C)*cos(d*x + c)^2 + 2*(3*A - 5*C)*cos(d*x + c) + 3*
A - 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(sqrt(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A + C)*sqrt(cos(d*x
+ c))*sin(d*x + c) + 8*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*ar
ctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(
a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\cos^2(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c
+ d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx + c)^2 + A}{(a\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
 c))), x)
```

3.208 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{(7A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

```
[Out] -((7*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A + C)*Sin[c + d*x])/(
2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.37834, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3042, 2984, 12, 2782, 205}

$$-\frac{(7A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] -((7*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A + C)*Sin[c + d*x])/(
2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A+C)-a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(7A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+}$$

Mathematica [C] time = 3.83154, size = 434, normalized size = 2.86

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A-7C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx)}\right.\right.\right.}{2\cos^{\frac{3}{2}}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(30*(A + C)*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c
+ d*x]]] - 30*(A + C)*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] -
 (20*(A + C)*Sqrt[Cos[c + d*x]])/(-1 + Sin[(c + d*x)/2]) + (80*C*Sin[(c + d
*x)/2])/Sqrt[Cos[c + d*x]] - (20*(A + C)*Sqrt[Cos[c + d*x]])/(1 + Sin[(c +
d*x)/2]) + (5*(A + C)*(-1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(Cos[(
c + d*x)/4] + Sin[(c + d*x)/4])^2) - (5*(A + C)*(1 + 2*Sin[(c + d*x)/2]))/(
Sqrt[Cos[c + d*x]]*(-1 + Sin[(c + d*x)/2])) + ((A - 7*C)*Csc[(c + d*x)/2]^3
*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(1 - Cos[c + d*x] + ArcTanh[Sqr
t[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]
]) - 2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*S
in[(c + d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))/(2*Cos[c + d*x]^(3/2))))/(10*
d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.135, size = 341, normalized size = 2.2

$$-\frac{-1+\cos(dx+c)}{4\,d\,(\sin(dx+c))^3\,a^2}\left(-10\,A\,(\cos(dx+c))^4\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}-18\,A\,(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+2\,A\,(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(-10*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)-18*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)+7*A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)*cos(d*x+c)^3+18*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
-C*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+8*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+2*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+
c)))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
a^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.35114, size = 522, normalized size = 3.43

$$-\frac{\sqrt{2}\Big((7\,A-C)\cos(dx+c)^3+2\,(7\,A-C)\cos(dx+c)^2+(7\,A-C)\cos(dx+c)\Big)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c}}{2\left(a\cos(dx+c)^2+a\cos(dx\right.}\right.}{4\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(dx+c)^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((7*A - C)*cos(d*x + c)^3 + 2*(7*A - C)*cos(d*x + c)^2 + (7*A
 - C)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) -
 2*((5*A + C)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
3/2)), x)
```

3.209 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(11A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+}$$

```
[Out] ((11*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*
d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A + 3*C)*Sin[c + d*x
])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((19*A + 3*C)*Sin[
c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.549351, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3042, 2984, 12, 2782, 205}

$$\frac{(11A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] ((11*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*
d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A + 3*C)*Sin[c + d*x
])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((19*A + 3*C)*Sin[
c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
```

```
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A+3C)-2aA\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}$$

$$= \frac{(11A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c}$$

Mathematica [C] time = 6.83422, size = 1192, normalized size = 5.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] (8*C*Cos[c/2 + (d*x)/2]^3*Sin[c/2 + (d*x)/2])/(3*d*(a*(1 + Cos[c + d*x]))^(
3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - ((A + C)*Cos[c/2 + (d*x)/2]^3*(1
 - 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 + Sin[c/2 +
(d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A + C)*Cos[c/2 + (d*x)/2]
^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - Sin[c
/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (16*C*Cos[c/2 + (d*x)/
2]^3*Sin[c/2 + (d*x)/2])/(3*d*(a*(1 + Cos[c + d*x]))^(3/2)*Sqrt[1 - 2*Sin[c
/2 + (d*x)/2]^2]) - ((A + C)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 - 2*Sin[c/2
+ (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((
1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*S
```

```
in[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^
(3/2)) + ((A + C)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])
/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2
 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d
*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((
A - 7*C)*Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^2*(-12*Cos[(c + d*x)/2]^4*
HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[
c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2
, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^
8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2
+ (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*
(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (
d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*A
rcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*S
in[c/2 + (d*x)/2]^2))))/(63*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 +
 (d*x)/2]^2)^(7/2))
```

Maple [A] time = 0.167, size = 328, normalized size = 1.6

$$-\frac{1}{12\,da^2\sin(dx+c)(1+\cos(dx+c))}\sqrt{a(1+\cos(dx+c))}\left(33\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)\sqrt{2}(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/12/d*(a*(1+cos(d*x+c)))^(1/2)*(33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*s
in(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9*C*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1
/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9*C*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)-38*A*cos(d*x+c)^3-6*C*cos(d*x+c)^3+14*A*cos(d*x+c)^2+6*C*cos(d*x+c)^2+32*
A*cos(d*x+c)-8*A)/a^2/sin(d*x+c)/(1+cos(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.32374, size = 576, normalized size = 2.87

$$\frac{3\sqrt{2}\left((11A+3C)\cos(dx+c)^4 + 2(11A+3C)\cos(dx+c)^3 + (11A+3C)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{}}{2\left(a\cos(dx+c\right.}\right.}{12\left(a^2d\cos(dx+c)^4 + 2a^2d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*((11*A + 3*C)*cos(d*x + c)^4 + 2*(11*A + 3*C)*cos(d*x + c)^
3 + (11*A + 3*C)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos
(d*x + c))) - 2*((19*A + 3*C)*cos(d*x + c)^2 + 12*A*cos(d*x + c) - 4*A)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^
4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
5/2)), x)
```

3.210 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{(15A+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(13A+5C)\sin(c+dx)}{10ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}$$

```
[Out] -((15*A + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2
*d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)) + ((9*A + 5*C)*Sin[c + d*
x])/(10*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((13*A + 5*C)*Si
n[c + d*x])/(10*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + ((49*A +
 25*C)*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.739313, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3042, 2984, 12, 2782, 205}

$$-\frac{(15A+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(13A+5C)\sin(c+dx)}{10ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Cos[c + d*x])^(3/2)),
x]
```


```
[Out] -((15*A + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2
*d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)) + ((9*A + 5*C)*Sin[c + d*
x])/(10*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((13*A + 5*C)*Si
n[c + d*x])/(10*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + ((49*A +
 25*C)*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(9A+5C)-a(3A+C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(15A+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sin(c+d}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(}$$

Mathematica [C] time = 8.0837, size = 2422, normalized size = 9.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] (8*C*Cos[c/2 + (d*x)/2]^3*Sin[c/2 + (d*x)/2])/(5*d*(a*(1 + Cos[c + d*x]))^(
3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - ((A + C)*Cos[c/2 + (d*x)/2]^3*(1
 - 2*Sin[c/2 + (d*x)/2]))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 + Sin[c/2 +
 (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + ((A + C)*Cos[c/2 + (d*x)/2
]^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - Sin
[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (32*C*Cos[c/2 + (d*x
```

)/2]^3*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/(15*d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((A + C)*Cos[c/2 + (d*x)/2]^3*(105*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] - (4 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (19 + 29*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(15*d*(a*(1 + Cos[c + d*x]))^(3/2)) - ((A + C)*Cos[c/2 + (d*x)/2]^3*(105*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] - (4 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (19 - 29*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/(15*d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((-A + 7*C)*Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*(a*(1 +

```
Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (
d*x)/2]^2))
```

Maple [B] time = 0.116, size = 472, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/20/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(75*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3
+35*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+150*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+70*C*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(
d*x+c)*cos(d*x+c)^2+75*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)+35*C*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+
c)-98*A*cos(d*x+c)^4-50*C*cos(d*x+c)^4+26*A*cos(d*x+c)^3+10*C*cos(d*x+c)^3+
80*A*cos(d*x+c)^2+40*C*cos(d*x+c)^2-16*A*cos(d*x+c)+8*A)/a^2/(-1+cos(d*x+c)
)/(1+cos(d*x+c))^2/cos(d*x+c)^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.4252, size = 620, normalized size = 2.5

$$-\frac{5\sqrt{2}\left((15A+7C)\cos(dx+c)^5+2(15A+7C)\cos(dx+c)^4+(15A+7C)\cos(dx+c)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\cdots}{2\left(a\cos(dx+\cdots\right.}\right.\cdots}{20\left(a^2d\cos(dx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/20*(5*sqrt(2)*((15*A + 7*C)*cos(d*x + c)^5 + 2*(15*A + 7*C)*cos(d*x + c)
^4 + (15*A + 7*C)*cos(d*x + c)^3)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*co
s(d*x + c))) - 2*((49*A + 25*C)*cos(d*x + c)^3 + 4*(9*A + 5*C)*cos(d*x + c)
^2 - 4*A*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x
+ c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
7/2)), x)
```

3.211 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{(3A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(3A+115C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+$$

```
[Out] (-5*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
+ ((3*A + 115*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((A - 15*C)*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((3*A + 35*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.793073, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3042, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(3A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(3A+115C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2),
x]
```


```
[Out] (-5*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
+ ((3*A + 115*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((A - 15*C)*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((3*A + 35*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(3A-5C)+a(A+5C)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A-15C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A-15C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A-15C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A-15C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{5C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A+115C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} -$$

Mathematica [C] time = 2.17365, size = 256, normalized size = 1.08

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((7A+55C)\cos(c+dx)+3A+8C\cos(2(c+dx))+4}{8d(a(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]*(80*C*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*(3*A + 11
5*C)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
 - 80*C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x
))] + Sqrt[Cos[c + d*x]]*(3*A + 43*C + (7*A + 55*C)*Cos[c + d*x] + 8*C*Cos[
2*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*
x]))^(5/2))
```

Maple [B] time = 0.171, size = 583, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*(14*A*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+20*A*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)-8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3*
A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-20*A*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+115*C*2^(1/2)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+32*C*cos(d*x+c)^5*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c
)*cos(d*x+c)^2-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+115*C*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+160*C*sin(d*x+c)*cos(d*x
+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+78*C*
cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+160*C*sin(d*x+c)*cos(d*x+c)^
2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-40*C*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-70*C*cos(d*x+c)^2*(cos(d*x+c)/(1
```

```
+cos(d*x+c)))^(1/2))/sin(d*x+c)^11/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [A] time = 107.747, size = 772, normalized size = 3.26

$$-\frac{\sqrt{2}\left((3\,A+115\,C)\cos\left(dx+c\right)^3+3\,(3\,A+115\,C)\cos\left(dx+c\right)^2+3\,(3\,A+115\,C)\cos\left(dx+c\right)+3\,A+115\,C\right)\sqrt{a}\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A + 115*C)*cos(d*x + c)^3 + 3*(3*A + 115*C)*cos(d*x + c)
^2 + 3*(3*A + 115*C)*cos(d*x + c) + 3*A + 115*C)*sqrt(a)*arctan(sqrt(2)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(16*C*
cos(d*x + c)^2 + (7*A + 55*C)*cos(d*x + c) + 3*A + 35*C)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 160*(C*cos(d*x + c)^3 + 3*C*cos(d*
x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5
/2), x)
```

3.212 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{(5A-43C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(5A-11C}{16ac}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((5*A - 43*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A - 11*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.579869, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3042, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A-43C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(5A-11C}{16ac}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^(5/2),
x]
```



```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((5*A - 43*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A - 11*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(5A-3C)+4aC\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-11C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-11C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-11C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(5A-43C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(}{}$$

Mathematica [C] time = 1.91051, size = 244, normalized size = 1.27

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((A-15C)\cos(c+dx)+5A-11C) - \frac{i\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}}}{}\right.}{8d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))]*(32*C*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(5*A -
 43*C)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]
)] - 32*C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d
*x))] + Sqrt[Cos[c + d*x]]*(5*A - 11*C + (A - 15*C)*Cos[c + d*x])*Sec[(c +
```

```
d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.195, size = 553, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(2*A*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+12*A*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)+8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+5*
A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-12*A*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-43*C*2^(1/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+5*A*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
-43*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-64
*C*sin(d*x+c)*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))-30*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-64*C*si
n(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+8*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*C*cos(d*x+
c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/a^3/(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [A] time = 97.3025, size = 733, normalized size = 3.82

$$-\frac{\sqrt{2}\Big((5\,A-43\,C)\cos{(dx+c)}^{3}+3\,(5\,A-43\,C)\cos{(dx+c)}^{2}+3\,(5\,A-43\,C)\cos{(dx+c)}+5\,A-43\,C\Big)\sqrt{a}\arctan\Big(\frac{\sqrt{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((5*A - 43*C)*cos(d*x + c)^3 + 3*(5*A - 43*C)*cos(d*x + c)^2
 + 3*(5*A - 43*C)*cos(d*x + c) + 5*A - 43*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((A - 15*C
)*cos(d*x + c) + 5*A - 11*C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c) + 64*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) +
 C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin
(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos{(dx+c)}^{2}+A\right)\sqrt{\cos{(dx+c)}}}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5
/2), x)
```

3.213 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(19A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((19*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - 7*C)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.398808, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3042, 2978, 12, 2782, 205}

$$\frac{(19A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),
x]
```


```
[Out] ((19*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - 7*C)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(7A-C)-a(A-3C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \cdots$$

$$= \frac{(19A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+\cdots}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [C] time = 1.4649, size = 200, normalized size = 1.3

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{2}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((9A-7C)\cos(c+dx)+13A-3C)+\frac{i(19A+3C)e^{\frac{1}{2}i(c+d\cdots}}{\cdots}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(
5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(19*A + 3*C)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c + d*
x]]*(13*A - 3*C + (9*A - 7*C)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x
)/2])/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.13, size = 449, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(18*A*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)+44*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)+8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-19*A*2^(1/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-44*A*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)-3*C*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*sin(d*x+c)*cos(d*x+c)^3-19*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*si
n(d*x+c)*cos(d*x+c)^2-26*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-14*C*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*C*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+8*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+6*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)/a^3/cos(d*x+c)^(1/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.33245, size = 581, normalized size = 3.77

$$\frac{\sqrt{2}\left((19\,A+3\,C)\cos\left(dx+c\right)^{3}+3\,(19\,A+3\,C)\cos\left(dx+c\right)^{2}+3\,(19\,A+3\,C)\cos\left(dx+c\right)+19\,A+3\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)\cdots}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(\cdots\right)\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((19*A + 3*C)*cos(d*x + c)^3 + 3*(19*A + 3*C)*cos(d*x + c)^2
+ 3*(19*A + 3*C)*cos(d*x + c) + 19*A + 3*C)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
```

```
)^2 + a*cos(d*x + c))) - 2*((9*A - 7*C)*cos(d*x + c) + 13*A - 3*C)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
 c))), x)
```

3.214 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(49A+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{5(15A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-3C)\sin(c+dx}{16ad\sqrt{\cos(c+dx)}(a\cos(c+d}$$

```
[Out] (-5*(15*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(
4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 3*C)*Sin[c +
d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((49*A + C)*
Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.579113, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3042, 2978, 2984, 12, 2782, 205}

$$\frac{(49A+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{5(15A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-3C)\sin(c+dx}{16ad\sqrt{\cos(c+dx)}(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),
x]
```

```
[Out] (-5*(15*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(
4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 3*C)*Sin[c +
d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((49*A + C)*
Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
```

```
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A+C)-2a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{5(15A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sin(c+d}{4d\sqrt{\cos(c+dx)}(a+a\cos(}$$

Mathematica [C] time = 2.59879, size = 211, normalized size = 1.06

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(10(17A+C)\cos(c+dx)+(49A+C)\cos(2(c+dx))+113A+C)}{4\sqrt{\cos(c+dx)}} - \frac{5i(15A-C)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}{\sqrt{1+e^{2i(c+dx)}}}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(
5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-5*I)*(15*A - C)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2
*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((113*A + C +
 10*(17*A + C)*Cos[c + d*x] + (49*A + C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]
^3*Tan[(c + d*x)/2])/(4*Sqrt[Cos[c + d*x]])))/(4*d*(a*(1 + Cos[c + d*x]))^(
```

```
5/2))
```

Maple [B] time = 0.147, size = 479, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(-1+cos(d*x+c))^2*(-98*A*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)-268*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-136*A*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
2^(1/2)*sin(d*x+c)*cos(d*x+c)^4+204*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(5/2)-5*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x
+c)^4+75*A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)
^3+234*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-5*C*2^(1/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-2*C*cos(d*x+c)^5*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+64*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-8*C*cos(d*
x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*C*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/cos(d*x+c)^(5/2)/(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.08822, size = 648, normalized size = 3.26

$$-\frac{5\sqrt{2}\Big((15A-C)\cos(dx+c)^4+3(15A-C)\cos(dx+c)^3+3(15A-C)\cos(dx+c)^2+(15A-C)\cos(dx+c)\Big)\sqrt{a}\,\mathrm{ar}}{32\left(a^3d\cos(dx+c)^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/32*(5*sqrt(2)*((15*A - C)*cos(d*x + c)^4 + 3*(15*A - C)*cos(d*x + c)^3 +
 3*(15*A - C)*cos(d*x + c)^2 + (15*A - C)*cos(d*x + c))*sqrt(a)*arctan(1/2*
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((49*A + C)*cos(d*x + c)^2 + 5*(17*A
 + C)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(
3/2)), x)
```

3.215 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{5(19A+3C)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A+19C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((163*A + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A + C)*Sin[c + d
*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (5*(19*A + 3*
C)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (
(299*A + 27*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]])
```

Rubi [A] time = 0.769644, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3042, 2978, 2984, 12, 2782, 205}

$$\frac{5(19A+3C)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A+19C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),
x]
```


```
[Out] ((163*A + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A + C)*Sin[c + d
*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (5*(19*A + 3*
C)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (
(299*A + 27*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]])
```


Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A+3C)-a(3A-C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{(163A+19C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sin(c+dx}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}$$

Mathematica [C] time = 3.43762, size = 239, normalized size = 0.97

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((1537A+81C)\cos(c+dx)+2(503A+39C)\cos(2(c+dx))+299A\cos(3(c+dx))+878A+27C\cos(3(c+dx))+78}{8\cos^{\frac{3}{2}}(c+dx)}\right.}{12d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(
5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((3*I)*(163*A + 19*C)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^
((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((878*A +
78*C + (1537*A + 81*C)*Cos[c + d*x] + 2*(503*A + 39*C)*Cos[2*(c + d*x)] + 2
99*A*Cos[3*(c + d*x)] + 27*C*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(c +
d*x)/2])/(8*Cos[c + d*x]^(3/2))))/(12*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.111, size = 472, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(489*A*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x
+c)^3+57*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+978*A*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+114*C*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+489*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c)*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+57*C*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)-598*A*cos(d*x+c)^4-54*C*cos(d*x+c)^4-408*A*cos(d*x+c)^3-24*C*cos(
d*x+c)^3+686*A*cos(d*x+c)^2+78*C*cos(d*x+c)^2+384*A*cos(d*x+c)-64*A)/a^3/si
n(d*x+c)^3/(1+cos(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.07715, size = 713, normalized size = 2.9

$$\frac{3\sqrt{2}\left((163A+19C)\cos(dx+c)^5+3(163A+19C)\cos(dx+c)^4+3(163A+19C)\cos(dx+c)^3+(163A+19C)\cos\right.}{96\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*((163*A + 19*C)*cos(d*x + c)^5 + 3*(163*A + 19*C)*cos(d*x +
 c)^4 + 3*(163*A + 19*C)*cos(d*x + c)^3 + (163*A + 19*C)*cos(d*x + c)^2)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((299*A + 27*C)*cos
(d*x + c)^3 + (503*A + 39*C)*cos(d*x + c)^2 + 160*A*cos(d*x + c) - 32*A)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)
^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(
5/2)), x)
```

3.216 $\int \cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=92

$$\frac{B\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3B\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Bx}{8}+\frac{C\sin^5(c+dx)}{5d}-\frac{2C\sin^3(c+dx)}{3d}+\frac{C\sin(c+dx)}{d}$$

```
[Out] (3*B*x)/8 + (C*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*C*Sin[c + d*x]^3)/(3*d) + (C*Sin[c
 + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0909403, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3010, 2748, 2635, 8, 2633}

$$\frac{B\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3B\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Bx}{8}+\frac{C\sin^5(c+dx)}{5d}-\frac{2C\sin^3(c+dx)}{3d}+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*B*x)/8 + (C*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*C*Sin[c + d*x]^3)/(3*d) + (C*Sin[c
 + d*x]^5)/(5*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx &= \int \cos^4(c+dx)(B+C\cos(c+dx))\,dx\\ &= B\int \cos^4(c+dx)\,dx+C\int \cos^5(c+dx)\,dx\\ &= \frac{B\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3B)\int \cos^2(c+dx)\,dx-\frac{C\,\mathrm{Subs}}{}\\ &= \frac{C\sin(c+dx)}{d}+\frac{3B\cos(c+dx)\sin(c+dx)}{8d}+\frac{B\cos^3(c+dx)\sin}{4d}\\ &= \frac{3Bx}{8}+\frac{C\sin(c+dx)}{d}+\frac{3B\cos(c+dx)\sin(c+dx)}{8d}+\frac{B\cos^3(c+}{}\end{aligned}$$

Mathematica [A] time = 0.139672, size = 89, normalized size = 0.97

$$\frac{3B(c+dx)}{8d}+\frac{B\sin(2(c+dx))}{4d}+\frac{B\sin(4(c+dx))}{32d}+\frac{C\sin^5(c+dx)}{5d}-\frac{2C\sin^3(c+dx)}{3d}+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*B*(c + d*x))/(8*d) + (C*Sin[c + d*x])/d - (2*C*Sin[c + d*x]^3)/(3*d) + (
C*Sin[c + d*x]^5)/(5*d) + (B*Sin[2*(c + d*x)])/(4*d) + (B*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] time = 0.015, size = 70, normalized size = 0.8

$$\frac{1}{d}\left(\frac{C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.05114, size = 93, normalized size = 1.01

$$\frac{15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))B+32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)C}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B + 32*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C)/d
```

Fricas [A] time = 1.66314, size = 173, normalized size = 1.88

$$\frac{45Bdx+\left(24C\cos(dx+c)^4+30B\cos(dx+c)^3+32C\cos(dx+c)^2+45B\cos(dx+c)+64C\right)\sin(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/120*(45*B*d*x + (24*C*cos(d*x + c)^4 + 30*B*cos(d*x + c)^3 + 32*C*cos(d*x
 + c)^2 + 45*B*cos(d*x + c) + 64*C)*sin(d*x + c))/d
```

Sympy [A] time = 2.37452, size = 173, normalized size = 1.88

$$\begin{cases} \frac{3Bx\sin^4(c+dx)}{8} + \frac{3Bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Bx\cos^4(c+dx)}{8} + \frac{3B\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5B\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8C\sin^5(c+dx)}{15d} + \frac{4C\sin \\ x\left(B\cos(c) + C\cos^2(c)\right)\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*B*x*sin(c + d*x)**4/8 + 3*B*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*B*x*cos(c + d*x)**4/8 + 3*B*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*B*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*C*sin(c + d*x)**5/(15*d) + 4*C*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d) + C*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d,
0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c)**3, True))
```

Giac [A] time = 1.17141, size = 104, normalized size = 1.13

$$\frac{3}{8}Bx + \frac{C\sin(5dx + 5c)}{80d} + \frac{B\sin(4dx + 4c)}{32d} + \frac{5C\sin(3dx + 3c)}{48d} + \frac{B\sin(2dx + 2c)}{4d} + \frac{5C\sin(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 3/8*B*x + 1/80*C*sin(5*d*x + 5*c)/d + 1/32*B*sin(4*d*x + 4*c)/d + 5/48*C*si
n(3*d*x + 3*c)/d + 1/4*B*sin(2*d*x + 2*c)/d + 5/8*C*sin(d*x + c)/d
```

3.217 $\int \cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=76

$$-\frac{B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3C\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Cx}{8}$$

```
[Out] (3*C*x)/8 + (B*Sin[c + d*x])/d + (3*C*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (B*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0845783, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3010, 2748, 2633, 2635, 8}

$$-\frac{B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3C\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Cx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*x)/8 + (B*Sin[c + d*x])/d + (3*C*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (B*Sin[c + d*x]^3)/(3*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \int \cos^3(c+dx)(B+C\cos(c+dx))\,dx\\ &= B\int \cos^3(c+dx)\,dx + C\int \cos^4(c+dx)\,dx\\ &= \frac{C\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3C)\int \cos^2(c+dx)\,dx - \frac{B\,\mathrm{Subs}}{}\\ &= \frac{B\sin(c+dx)}{d}+\frac{3C\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin}{4d}\\ &= \frac{3Cx}{8}+\frac{B\sin(c+dx)}{d}+\frac{3C\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+}{}\end{aligned}$$

Mathematica [A] time = 0.0962518, size = 73, normalized size = 0.96

$$-\frac{B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)}{d}+\frac{3C(c+dx)}{8d}+\frac{C\sin(2(c+dx))}{4d}+\frac{C\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(c + d*x))/(8*d) + (B*Sin[c + d*x])/d - (B*Sin[c + d*x]^3)/(3*d) + (C*
Sin[2*(c + d*x)])/(4*d) + (C*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.013, size = 60, normalized size = 0.8

$$\frac{1}{d}\left(C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.26904, size = 77, normalized size = 1.01

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)B-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)C}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*C)/d
```

Fricas [A] time = 1.68527, size = 136, normalized size = 1.79

$$\frac{9\,Cdx+\left(6\,C\cos\left(dx+c\right)^{3}+8\,B\cos\left(dx+c\right)^{2}+9\,C\cos\left(dx+c\right)+16\,B\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/24*(9*C*d*x + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 9*C*cos(d*x + c)
 + 16*B)*sin(d*x + c))/d
```

Sympy [A] time = 1.37909, size = 150, normalized size = 1.97

$$\begin{cases}\frac{2B\sin^{3}(c+dx)}{3d}+\frac{B\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{3Cx\sin^{4}(c+dx)}{8}+\frac{3Cx\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3Cx\cos^{4}(c+dx)}{8}+\frac{3C\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{5C\sin(\\ x\left(B\cos(c)+C\cos^{2}(c)\right)\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((2*B*sin(c + d*x)**3/(3*d) + B*sin(c + d*x)*cos(c + d*x)**2/d + 3
*C*x*sin(c + d*x)**4/8 + 3*C*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*x*co
s(c + d*x)**4/8 + 3*C*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*sin(c + d*x)
*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c)**2, T
rue))
```

Giac [A] time = 1.18617, size = 84, normalized size = 1.11

$$\frac{3}{8}\,Cx + \frac{C\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{B\sin\left(3\,dx + 3\,c\right)}{12\,d} + \frac{C\sin\left(2\,dx + 2\,c\right)}{4\,d} + \frac{3\,B\sin\left(dx + c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 3/8*C*x + 1/32*C*sin(4*d*x + 4*c)/d + 1/12*B*sin(3*d*x + 3*c)/d + 1/4*C*sin
(2*d*x + 2*c)/d + 3/4*B*sin(d*x + c)/d
```

3.218 $\int \cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}-\frac{C\sin^3(c+dx)}{3d}+\frac{C\sin(c+dx)}{d}$$

```
[Out] (B*x)/2 + (C*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (C*Sin
[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0631004, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3010, 2748, 2635, 8, 2633}

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}-\frac{C\sin^3(c+dx)}{3d}+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (B*x)/2 + (C*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (C*Sin
[c + d*x]^3)/(3*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \int \cos^2(c+dx)(B+C\cos(c+dx))\,dx\\ &= B\int \cos^2(c+dx)\,dx + C\int \cos^3(c+dx)\,dx\\ &= \frac{B\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}B\int 1\,dx-\frac{C\,\text{Subst}\left(\int\left(1-x^2\right)\,dx\right.}{d}\\ &= \frac{Bx}{2}+\frac{C\sin(c+dx)}{d}+\frac{B\cos(c+dx)\sin(c+dx)}{2d}-\frac{C\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.06226, size = 57, normalized size = 1.06

$$\frac{B(c+dx)}{2d}+\frac{B\sin(2(c+dx))}{4d}-\frac{C\sin^3(c+dx)}{3d}+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (B*(c + d*x))/(2*d) + (C*Sin[c + d*x])/d - (C*Sin[c + d*x]^3)/(3*d) + (B*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.012, size = 49, normalized size = 0.9

$$\frac{1}{d}\left(\frac{C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.21927, size = 62, normalized size = 1.15

$$\frac{3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))B - 4\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)C}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B - 4*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*C)/d
```

Fricas [A] time = 1.5606, size = 105, normalized size = 1.94

$$\frac{3\,Bdx + \left(2\,C\cos(dx + c)^2 + 3\,B\cos(dx + c) + 4\,C\right)\sin(dx + c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*B*d*x + (2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 4*C)*sin(d*x + c))/
d
```

Sympy [A] time = 0.611654, size = 95, normalized size = 1.76

$$\begin{cases} \frac{Bx\sin^2(c+dx)}{2} + \frac{Bx\cos^2(c+dx)}{2} + \frac{B\sin(c+dx)\cos(c+dx)}{2d} + \frac{2C\sin^3(c+dx)}{3d} + \frac{C\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(B\cos(c) + C\cos^2(c)\right)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*x*sin(c + d*x)**2/2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*c
os(c + d*x)/(2*d) + 2*C*sin(c + d*x)**3/(3*d) + C*sin(c + d*x)*cos(c + d*x)
**2/d, Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.27177, size = 63, normalized size = 1.17

$$\frac{1}{2}Bx + \frac{C\sin(3\,dx + 3\,c)}{12\,d} + \frac{B\sin(2\,dx + 2\,c)}{4\,d} + \frac{3\,C\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*B*x + 1/12*C*sin(3*d*x + 3*c)/d + 1/4*B*sin(2*d*x + 2*c)/d + 3/4*C*sin(
d*x + c)/d
```

3.219 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=38

$$\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}$$

```
[Out] (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.023467, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2637, 2635, 8}

$$\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[B*Cos[c + d*x] + C*Cos[c + d*x]^2,x]
```

```
[Out] (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = B\int \cos(c+dx)\,dx + C\int \cos^2(c+dx)\,dx$$
$$= \frac{B\sin(c+dx)}{d} + \frac{C\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}C\int 1\,dx$$
$$= \frac{Cx}{2} + \frac{B\sin(c+dx)}{d} + \frac{C\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0614901, size = 35, normalized size = 0.92

$$\frac{4B\sin(c+dx)+C(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[B*Cos[c + d*x] + C*Cos[c + d*x]^2,x]
```

```
[Out] (4*B*Sin[c + d*x] + C*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.014, size = 40, normalized size = 1.1

$$\frac{B\sin(dx+c)}{d} + \frac{C}{d}\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(B*cos(d*x+c)+C*cos(d*x+c)^2,x)
```

```
[Out] B*sin(d*x+c)/d+C/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

Maxima [A] time = 1.26855, size = 47, normalized size = 1.24

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{4\,d} + \frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(B*cos(d*x+c)+C*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*C/d + B*sin(d*x + c)/d
```

Fricas [A] time = 1.5711, size = 72, normalized size = 1.89

$$\frac{Cdx + (C\cos(dx + c) + 2B)\sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(B*cos(d*x+c)+C*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(C*d*x + (C*cos(d*x + c) + 2*B)*sin(d*x + c))/d
```

Sympy [A] time = 0.352142, size = 63, normalized size = 1.66

$$B\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d \neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right) + C\left(\begin{cases}\frac{x\sin^2(c+dx)}{2} + \frac{x\cos^2(c+dx)}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0\\ x\cos^2(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(B*cos(d*x+c)+C*cos(d*x+c)**2,x)
```

```
[Out] B*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + C*Piecewise((x*
sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d),
Ne(d, 0)), (x*cos(c)**2, True))
```

Giac [A] time = 1.24177, size = 43, normalized size = 1.13

$$\frac{1}{4}C\left(2x + \frac{\sin(2dx + 2c)}{d}\right) + \frac{B\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(B*cos(d*x+c)+C*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*C*(2*x + sin(2*d*x + 2*c)/d) + B*sin(d*x + c)/d
```

3.220 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=15

$$Bx+\frac{C\sin(c+dx)}{d}$$

```
[Out] B*x + (C*Sin[c + d*x])/d
```

Rubi [A] time = 0.0296575, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3010, 2637}

$$Bx+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] B*x + (C*Sin[c + d*x])/d
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx &= \int (B+C\cos(c+dx))\,dx\\ &= Bx+C\int \cos(c+dx)\,dx\\ &= Bx+\frac{C\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0083274, size = 26, normalized size = 1.73

$$Bx + \frac{C\sin(c)\cos(dx)}{d} + \frac{C\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] B*x + (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.026, size = 21, normalized size = 1.4

$$\frac{C\sin(dx+c) + B(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(C*sin(d*x+c)+B*(d*x+c))
```

Maxima [A] time = 1.29137, size = 27, normalized size = 1.8

$$\frac{(dx+c)B + C\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*B + C*sin(d*x + c))/d
```

Fricas [A] time = 1.59613, size = 38, normalized size = 2.53

$$\frac{Bdx + C\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] (B*d*x + C*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\cos(c + dx))\cos(c + dx)\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x), x)
```

Giac [B] time = 1.34348, size = 53, normalized size = 3.53

$$\frac{(dx + c)B + \frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B + 2*C*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.221 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=16

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}+Cx$$

```
[Out] C*x + (B*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0475166, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3010, 2735, 3770}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}+Cx$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (B*ArcTanh[Sin[c + d*x]])/d
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (B+C\cos(c+dx))\sec(c+dx)\,dx$$
$$= Cx + B\int \sec(c+dx)\,dx$$
$$= Cx + \frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.0060913, size = 16, normalized size = 1.

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d} + Cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (B*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.035, size = 30, normalized size = 1.9

$$Cx + \frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Cc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] C*x+1/d*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*c
```

Maxima [B] time = 1.32661, size = 50, normalized size = 3.12

$$\frac{2(dx+c)C + B(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C + B*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [B] time = 1.67482, size = 95, normalized size = 5.94

$$\frac{2\,Cdx + B\log\left(\sin\left(dx+c\right)+1\right) - B\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*d*x + B*log(sin(d*x + c) + 1) - B*log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B + C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2, x)
```

Giac [B] time = 1.26151, size = 58, normalized size = 3.62

$$\frac{(dx+c)C + B\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)+1\right|\right) - B\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*C + B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

3.222 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{B\tan(c+dx)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/d + (B*Tan[c + d*x])/d
```

Rubi [A] time = 0.0623378, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3010, 2748, 3767, 8, 3770}

$$\frac{B\tan(c+dx)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/d + (B*Tan[c + d*x])/d
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= \int (B+C\cos(c+dx))\sec^2(c+dx)\,dx \\
&= B\int \sec^2(c+dx)\,dx + C\int \sec(c+dx)\,dx \\
&= \frac{C\tanh^{-1}(\sin(c+dx))}{d} - \frac{B\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} \\
&= \frac{C\tanh^{-1}(\sin(c+dx))}{d} + \frac{B\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.007535, size = 24, normalized size = 1.

$$\frac{B\tan(c+dx)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/d + (B*Tan[c + d*x])/d
```

Maple [A] time = 0.033, size = 32, normalized size = 1.3

$$\frac{B\tan(dx+c)}{d}+\frac{C\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] B*tan(d*x+c)/d+1/d*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.32283, size = 51, normalized size = 2.12

$$\frac{C(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,B\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(C*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*B*tan(d*x + c))/
d
```

Fricas [B] time = 1.65652, size = 162, normalized size = 6.75

$$\frac{C\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-C\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,B\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*cos(d*x + c)*log(sin(d*x + c) + 1) - C*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*B*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40596, size = 85, normalized size = 3.54

$$\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] (C*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*B*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.223 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=47

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}+\frac{C\tan(c+dx)}{d}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Tan[c + d*x])/d + (B*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rubi [A] time = 0.075895, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3010, 2748, 3768, 3770, 3767, 8}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}+\frac{C\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Tan[c + d*x])/d + (B*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx &= \int (B+C\cos(c+dx))\sec^3(c+dx)\,dx\\ &= B\int \sec^3(c+dx)\,dx + C\int \sec^2(c+dx)\,dx\\ &= \frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}B\int \sec(c+dx)\,dx-\frac{C\,\mathrm{Subst}(\int 1\,d\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)}{d}+\frac{B\sec(c+dx)\tan(c+d\end{aligned}$$

Mathematica [A] time = 0.0156961, size = 47, normalized size = 1.

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}+\frac{C\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Tan[c + d*x])/d + (B*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A] time = 0.034, size = 51, normalized size = 1.1

$$\frac{C\tan(dx+c)}{d}+\frac{B\sec(dx+c)\tan(dx+c)}{2d}+\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] C*tan(d*x+c)/d+1/2*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.36954, size = 78, normalized size = 1.66

$$-\frac{B\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4C\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(B*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*C*tan(d*x + c))/d
```

Fricas [A] time = 1.63899, size = 198, normalized size = 4.21

$$\frac{B\cos(dx+c)^2\log(\sin(dx+c)+1)-B\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2C\cos(dx+c)+B)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/4*(B*cos(d*x + c)^2*log(sin(d*x + c) + 1) - B*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*C*cos(d*x + c) + B)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32777, size = 142, normalized size = 3.02

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/2*(B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(B*tan(1/2*d*x + 1/2*c)^3 - 2*C*tan(1/2*d*x + 1/2*c)^3 + B*tan(1/
2*d*x + 1/2*c) + 2*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

3.224 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (C*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (B*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0800781, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3010, 2748, 3767, 3768, 3770}

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (C*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (B*Tan[c + d*x]^3)/(3*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int (B+C\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= B\int \sec^4(c+dx)\,dx + C\int \sec^3(c+dx)\,dx$$
$$= \frac{C\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}C\int \sec(c+dx)\,dx - \frac{B\,\mathrm{Subst}\left(\int (1$$
$$= \frac{C\tanh^{-1}(\sin(c+dx))}{2d} + \frac{B\tan(c+dx)}{d} + \frac{C\sec(c+dx)\tan(c+d$$

Mathematica [A] time = 0.151043, size = 60, normalized size = 0.95

$$\frac{B\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (B*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.039, size = 72, normalized size = 1.1

$$\frac{C\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{C\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{2\,B\tan(dx+c)}{3\,d}+\frac{B\tan(dx+c)(\sec(dx+c))^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/2*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*C*ln(sec(d*x+c)+tan(d*x+c))+2/3*B*tan(d
*x+c)/d+1/3/d*B*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.33846, size = 95, normalized size = 1.51

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)B-3\,C\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B - 3*C*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.67288, size = 236, normalized size = 3.75

$$\frac{3\,C\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,C\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,B\cos\left(dx+c\right)^{2}+3\,C\cos\left(dx+\right.\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*C*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*B*cos(d*x + c)^2 + 3*C*cos(d*x + c) + 2*B)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32756, size = 165, normalized size = 2.62

$$\frac{3\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,B}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/6*(3*C*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*C*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*B*tan(1/2*d*x + 1/2*c)^5 - 3*C*tan(1/2*d*x + 1/2*c)^5 - 4*
B*tan(1/2*d*x + 1/2*c)^3 + 6*B*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.225 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{3B\tanh^{-1}(\sin(c+dx))}{8d}+\frac{B\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3B\tan(c+dx)\sec(c+dx)}{8d}+\frac{C\tan^3(c+dx)}{3d}+\frac{C\tan(c+dx)}{d}$$

```
[Out] (3*B*ArcTanh[Sin[c + d*x]])/(8*d) + (C*Tan[c + d*x])/d + (3*B*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (C*Tan[c + d*
x]^3)/(3*d)
```

Rubi [A] time = 0.0913065, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3010, 2748, 3768, 3770, 3767}

$$\frac{3B\tanh^{-1}(\sin(c+dx))}{8d}+\frac{B\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3B\tan(c+dx)\sec(c+dx)}{8d}+\frac{C\tan^3(c+dx)}{3d}+\frac{C\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]])/(8*d) + (C*Tan[c + d*x])/d + (3*B*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (C*Tan[c + d*
x]^3)/(3*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (B+C\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= B\int \sec^5(c+dx)\,dx + C\int \sec^4(c+dx)\,dx$$
$$= \frac{B\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3B)\int \sec^3(c+dx)\,dx - \frac{C\,\text{Subst}}{}$$
$$= \frac{C\tan(c+dx)}{d} + \frac{3B\sec(c+dx)\tan(c+dx)}{8d} + \frac{B\sec^3(c+dx)\tan(}{4d}$$
$$= \frac{3B\tanh^{-1}(\sin(c+dx))}{8d} + \frac{C\tan(c+dx)}{d} + \frac{3B\sec(c+dx)\tan(c+}{8d}$$

Mathematica [A] time = 0.210047, size = 76, normalized size = 0.89

$$\frac{B\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3B\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d} + \frac{C\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*B*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (C*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.036, size = 92, normalized size = 1.1

$$\frac{2 C \tan (dx + c)}{3 d} + \frac{C \tan (dx + c) (\sec (dx + c))^{2}}{3 d} + \frac{B (\sec (dx + c))^{3} \tan (dx + c)}{4 d} + \frac{3 B \sec (dx + c) \tan (dx + c)}{8 d} + \frac{3 B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 2/3*C*tan(d*x+c)/d+1/3/d*C*tan(d*x+c)*sec(d*x+c)^2+1/4*B*sec(d*x+c)^3*tan(d
*x+c)/d+3/8*B*sec(d*x+c)*tan(d*x+c)/d+3/8/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.33661, size = 128, normalized size = 1.51

$$\frac{16 \left(\tan (dx + c)^{3} + 3 \tan (dx + c)\right) C - 3 B \left(\frac{2 \left(3 \sin(dx+c)^{3} - 5 \sin(dx+c)\right)}{\sin(dx+c)^{4} - 2 \sin(dx+c)^{2} + 1} - 3 \log (\sin (dx + c) + 1) + 3 \log (\sin (dx + c) - 1)\right)}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C - 3*B*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.67855, size = 266, normalized size = 3.13

$$\frac{9 B \cos (dx + c)^{4} \log (\sin (dx + c) + 1) - 9 B \cos (dx + c)^{4} \log (-\sin (dx + c) + 1) + 2 \left(16 C \cos (dx + c)^{3} + 9 B \cos (dx + \right.}{48 d \cos (dx + c)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(9*B*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*B*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*C*cos(d*x + c)^3 + 9*B*cos(d*x + c)^2 + 8*C*cos(d*x
 + c) + 6*B)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37822, size = 221, normalized size = 2.6

$$\frac{9\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/24*(9*B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*B*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(15*B*tan(1/2*d*x + 1/2*c)^7 - 24*C*tan(1/2*d*x + 1/2*c)^7 +
 9*B*tan(1/2*d*x + 1/2*c)^5 + 40*C*tan(1/2*d*x + 1/2*c)^5 + 9*B*tan(1/2*d*x
 + 1/2*c)^3 - 40*C*tan(1/2*d*x + 1/2*c)^3 + 15*B*tan(1/2*d*x + 1/2*c) + 24*
C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.226 $\int \cos^2(c+dx)(a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=125

$$-\frac{a(5B+4C)\sin^3(c+dx)}{15d}+\frac{a(5B+4C)\sin(c+dx)}{5d}+\frac{a(B+C)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(B+C)\sin(c+dx)\cos(c}{8d}$$

```
[Out] (3*a*(B + C)*x)/8 + (a*(5*B + 4*C)*Sin[c + d*x])/(5*d) + (3*a*(B + C)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(B + C)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*B + 4*C)*Sin[c + d*x]^3)
/(15*d)
```

Rubi [A] time = 0.218921, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2968, 3023, 2748, 2633, 2635, 8}

$$-\frac{a(5B+4C)\sin^3(c+dx)}{15d}+\frac{a(5B+4C)\sin(c+dx)}{5d}+\frac{a(B+C)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(B+C)\sin(c+dx)\cos(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)
,x]
```

```
[Out] (3*a*(B + C)*x)/8 + (a*(5*B + 4*C)*Sin[c + d*x])/(5*d) + (3*a*(B + C)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(B + C)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*B + 4*C)*Sin[c + d*x]^3)
/(15*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+a\cos(c+dx))(B+C\cos($$
$$= \int \cos^3(c+dx)\left(aB+(aB+aC)\cos(c+dx)+\right.$$
$$= \frac{aC\cos^4(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^3(c+$$
$$= \frac{aC\cos^4(c+dx)\sin(c+dx)}{5d}+(a(B+C))\int \mathrm{c}$$
$$= \frac{a(B+C)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aC\cos^4(}{}$$
$$= \frac{a(5B+4C)\sin(c+dx)}{5d}+\frac{3a(B+C)\cos(c+}{8d}$$
$$= \frac{3}{8}a(B+C)x+\frac{a(5B+4C)\sin(c+dx)}{5d}+\frac{3a(B}{}$$

Mathematica [A] time = 0.334915, size = 102, normalized size = 0.82

$$\frac{a(60(6B+5C)\sin(c+dx)+120(B+C)\sin(2(c+dx))+40B\sin(3(c+dx))+15B\sin(4(c+dx))+180Bdx+50C\sin(}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (a*(180*B*d*x + 180*C*d*x + 60*(6*B + 5*C)*Sin[c + d*x] + 120*(B + C)*Sin[2
*(c + d*x)] + 40*B*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] + 15*B*Sin[4*(c
 + d*x)] + 15*C*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.023, size = 128, normalized size = 1.

$$\frac{1}{d}\left(\frac{aC\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3a}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*C*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A] time = 1.17313, size = 167, normalized size = 1.34

$$-\frac{160\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Ba-32\left(3\,\sin\left(dx+c\right)^{5}-\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 15*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a - 32*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*C*a - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*C*a)/d
```

Fricas [A] time = 1.68487, size = 239, normalized size = 1.91

$$\frac{45\left(B+C\right)adx+\left(24\,Ca\cos\left(dx+c\right)^{4}+30\left(B+C\right)a\cos\left(dx+c\right)^{3}+8\left(5\,B+4\,C\right)a\cos\left(dx+c\right)^{2}+45\left(B+C\right)a\cos\left(dx+c\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/120*(45*(B + C)*a*d*x + (24*C*a*cos(d*x + c)^4 + 30*(B + C)*a*cos(d*x + c
)^3 + 8*(5*B + 4*C)*a*cos(d*x + c)^2 + 45*(B + C)*a*cos(d*x + c) + 16*(5*B
+ 4*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 2.65468, size = 338, normalized size = 2.7

$$\begin{cases}\frac{3Bax\sin^{4}(c+dx)}{8}+\frac{3Bax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3Bax\cos^{4}(c+dx)}{8}+\frac{3Ba\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{2Ba\sin^{3}(c+dx)}{3d}+\frac{5Ba\sin(c+dx)\cos^{3}(c+dx)}{8d}+\\ x\left(B\cos(c)+C\cos^{2}(c)\right)\left(a\cos(c)+a\right)\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 3*B*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 2*B*a*sin(c + d*x)**3/(3*d) + 5*B*a*sin(c + d*x)*cos(c + d*x)**3/(8*d)
+ B*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*a*x*sin(c + d*x)**4/8 + 3*C*a*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 8*C*a*sin(c
 + d*x)**5/(15*d) + 4*C*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*C*a*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + C*a*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2
)*(a*cos(c) + a)*cos(c)**2, True))
```

Giac [A] time = 1.32681, size = 151, normalized size = 1.21

$$\frac{3}{8}(Ba+Ca)x+\frac{Ca\sin(5dx+5c)}{80d}+\frac{(Ba+Ca)\sin(4dx+4c)}{32d}+\frac{(4Ba+5Ca)\sin(3dx+3c)}{48d}+\frac{(Ba+Ca)\sin(2dx+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 3/8*(B*a + C*a)*x + 1/80*C*a*sin(5*d*x + 5*c)/d + 1/32*(B*a + C*a)*sin(4*d*
x + 4*c)/d + 1/48*(4*B*a + 5*C*a)*sin(3*d*x + 3*c)/d + 1/4*(B*a + C*a)*sin(
2*d*x + 2*c)/d + 1/8*(6*B*a + 5*C*a)*sin(d*x + c)/d
```

3.227 $\int \cos(c+dx)(a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+d\right.$

Optimal. Leaf size=97

$$-\frac{a(B+C)\sin^3(c+dx)}{3d}+\frac{a(B+C)\sin(c+dx)}{d}+\frac{a(4B+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4B+3C)+\frac{aC\sin(c+dx)\,c}{4d}$$

```
[Out] (a*(4*B + 3*C)*x)/8 + (a*(B + C)*Sin[c + d*x])/d + (a*(4*B + 3*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(B +
 C)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.182049, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3029, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a(B+C)\sin^3(c+dx)}{3d}+\frac{a(B+C)\sin(c+dx)}{d}+\frac{a(4B+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4B+3C)+\frac{aC\sin(c+dx)\,c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

```
[Out] (a*(4*B + 3*C)*x)/8 + (a*(B + C)*Sin[c + d*x])/d + (a*(4*B + 3*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(B +
 C)*Sin[c + d*x]^3)/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+a\cos(c+dx))(B+C\cos(c+$$

$$= \int \cos^2(c+dx)\left(aB+(aB+aC)\cos(c+dx)+a\right.$$

$$= \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx)$$

$$= \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+(a(B+C))\int \cos$$

$$= \frac{a(4B+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aC\cos^3(c$$

$$= \frac{1}{8}a(4B+3C)x+\frac{a(B+C)\sin(c+dx)}{d}+\frac{a(4B+$$

Mathematica [A] time = 0.26561, size = 76, normalized size = 0.78

$$\frac{a(72(B+C)\sin(c+dx)+24(B+C)\sin(2(c+dx))+8B\sin(3(c+dx))+48Bdx+8C\sin(3(c+dx))+3C\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (a*(48*B*d*x + 36*C*d*x + 72*(B + C)*Sin[c + d*x] + 24*(B + C)*Sin[2*(c + d
*x)] + 8*B*Sin[3*(c + d*x)] + 8*C*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))
/(96*d)
```

Maple [A] time = 0.023, size = 107, normalized size = 1.1

$$\frac{1}{d}\left(aC\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ba\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aC\left(2+(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2
```

```
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.12658, size = 136, normalized size = 1.4

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba-24\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Ba+32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca-3}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 24*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a - 3*(12*d*x + 12*
c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a)/d
```

Fricas [A] time = 1.63477, size = 193, normalized size = 1.99

$$\frac{3\,(4\,B+3\,C)adx+\left(6\,Ca\cos(dx+c)^3+8\,(B+C)a\cos(dx+c)^2+3\,(4\,B+3\,C)a\cos(dx+c)+16\,(B+C)a\right)\sin(dx+}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/24*(3*(4*B + 3*C)*a*d*x + (6*C*a*cos(d*x + c)^3 + 8*(B + C)*a*cos(d*x + c
)^2 + 3*(4*B + 3*C)*a*cos(d*x + c) + 16*(B + C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.50394, size = 255, normalized size = 2.63

$$\begin{cases}\frac{Bax\sin^2(c+dx)}{2}+\frac{Bax\cos^2(c+dx)}{2}+\frac{2Ba\sin^3(c+dx)}{3d}+\frac{Ba\sin(c+dx)\cos^2(c+dx)}{d}+\frac{Ba\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Cax\sin^4(c+dx)}{8}+\frac{3Cax\sin^2(c+dx}{4} \\ x\left(B\cos(c)+C\cos^2(c)\right)(a\cos(c)+a)\cos(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 + 2*B*a*sin(c
+ d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(c + d*x)**2/d + B*a*sin(c + d*x)*cos
(c + d*x)/(2*d) + 3*C*a*x*sin(c + d*x)**4/8 + 3*C*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 3*C*a*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 2*C*a*sin(c + d*x)**3/(3*d) + 5*C*a*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + C*a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(B*cos(c) + C*co
s(c)**2)*(a*cos(c) + a)*cos(c), True))
```

Giac [A] time = 1.29581, size = 120, normalized size = 1.24

$$\frac{1}{8}(4Ba + 3Ca)x + \frac{Ca\sin(4dx + 4c)}{32d} + \frac{(Ba + Ca)\sin(3dx + 3c)}{12d} + \frac{(Ba + Ca)\sin(2dx + 2c)}{4d} + \frac{3(Ba + Ca)\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] 1/8*(4*B*a + 3*C*a)*x + 1/32*C*a*sin(4*d*x + 4*c)/d + 1/12*(B*a + C*a)*sin(
3*d*x + 3*c)/d + 1/4*(B*a + C*a)*sin(2*d*x + 2*c)/d + 3/4*(B*a + C*a)*sin(d
*x + c)/d
```

3.228 $\int (a+a\cos(c+dx)) \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=85

$$\frac{a(3B+C)\sin(c+dx)}{3d}+\frac{a(3B-C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}ax(B+C)+\frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3ad}$$

```
[Out] (a*(B + C)*x)/2 + (a*(3*B + C)*Sin[c + d*x])/(3*d) + (a*(3*B - C)*Cos[c + d
*x]*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.0774457, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3023, 2734}

$$\frac{a(3B+C)\sin(c+dx)}{3d}+\frac{a(3B-C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}ax(B+C)+\frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(B + C)*x)/2 + (a*(3*B + C)*Sin[c + d*x])/(3*d) + (a*(3*B - C)*Cos[c + d
*x]*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3ad}+\frac{\int(a+a\cos(c+dx))(2a}{}$$

$$=\frac{1}{2}a(B+C)x+\frac{a(3B+C)\sin(c+dx)}{3d}+\frac{a(3B-C)\cos(c+dx}{6d}$$

Mathematica [A] time = 0.174457, size = 65, normalized size = 0.76

$$\frac{a(3(4B+3C)\sin(c+dx)+3(B+C)\sin(2(c+dx))+6Bc+6Bdx+C\sin(3(c+dx))+6cC+6Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(6*B*c + 6*c*C + 6*B*d*x + 6*C*d*x + 3*(4*B + 3*C)*Sin[c + d*x] + 3*(B +
 C)*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.019, size = 85, normalized size = 1.

$$\frac{1}{d}\left(\frac{aC\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c))
```

Maxima [A] time = 1.10515, size = 107, normalized size = 1.26

$$\frac{3(2dx+2c+\sin(2dx+2c))Ba-4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca+3(2dx+2c+\sin(2dx+2c))Ca+12Ba\sin(}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 4*(sin(d*x + c)^3 - 3*sin(d*
x + c))*C*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a + 12*B*a*sin(d*x + c))
/d
```

Fricas [A] time = 1.58984, size = 146, normalized size = 1.72

$$\frac{3\,(B+C)adx+\left(2\,Ca\cos\left(dx+c\right)^{2}+3\,(B+C)a\cos\left(dx+c\right)+2\,(3\,B+2\,C)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(B + C)*a*d*x + (2*C*a*cos(d*x + c)^2 + 3*(B + C)*a*cos(d*x + c) + 2
*(3*B + 2*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.698973, size = 170, normalized size = 2.

$$\begin{cases}\frac{Bax\sin^{2}(c+dx)}{2}+\frac{Bax\cos^{2}(c+dx)}{2}+\frac{Ba\sin(c+dx)\cos(c+dx)}{2d}+\frac{Ba\sin(c+dx)}{d}+\frac{Cax\sin^{2}(c+dx)}{2}+\frac{Cax\cos^{2}(c+dx)}{2}+\frac{2Ca\sin^{3}(c+dx)}{3d}+\frac{Ca\sin(c\\x\left(B\cos(c)+C\cos^{2}(c)\right)(a\cos(c)+a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 + B*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/d + C*a*x*sin(c + d*x)**2/2 + C*
a*x*cos(c + d*x)**2/2 + 2*C*a*sin(c + d*x)**3/(3*d) + C*a*sin(c + d*x)*cos(
c + d*x)**2/d + C*a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(B*cos(c
) + C*cos(c)**2)*(a*cos(c) + a), True))
```

Giac [A] time = 1.33715, size = 92, normalized size = 1.08

$$\frac{1}{2}\left(Ba+Ca\right)x+\frac{Ca\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(Ba+Ca\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(4\,Ba+3\,Ca\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/2*(B*a + C*a)*x + 1/12*C*a*sin(3*d*x + 3*c)/d + 1/4*(B*a + C*a)*sin(2*d*x
 + 2*c)/d + 1/4*(4*B*a + 3*C*a)*sin(d*x + c)/d
```

3.229 $\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=47

$$\frac{a(B+C)\sin(c+dx)}{d}+\frac{1}{2}ax(2B+C)+\frac{aC\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (a*(2*B + C)*x)/2 + (a*(B + C)*Sin[c + d*x])/d + (a*C*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rubi [A] time = 0.0631725, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3029, 2734}

$$\frac{a(B+C)\sin(c+dx)}{d}+\frac{1}{2}ax(2B+C)+\frac{aC\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x
]
```


```
[Out] (a*(2*B + C)*x)/2 + (a*(B + C)*Sin[c + d*x])/d + (a*C*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int (a+a\cos(c+dx))(B+C\cos(c+dx))\,dx$$
$$=\frac{1}{2}a(2B+C)x+\frac{a(B+C)\sin(c+dx)}{d}+\frac{aC\cos(c}{}$$

Mathematica [A] time = 0.0981116, size = 44, normalized size = 0.94

$$\frac{a(4(B+C)\sin(c+dx)+4Bdx+C\sin(2(c+dx))+2cC+2Cdx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (a*(2*c*C + 4*B*d*x + 2*C*d*x + 4*(B + C)*Sin[c + d*x] + C*Sin[2*(c + d*x)]
))/(4*d)
```

Maple [A] time = 0.042, size = 57, normalized size = 1.2

$$\frac{1}{d}\left(aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\sin(dx+c)+aC\sin(dx+c)+Ba(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c)+a*C*sin(d
*x+c)+B*a*(d*x+c))
```

Maxima [A] time = 1.29986, size = 74, normalized size = 1.57

$$\frac{4(dx+c)Ba+(2dx+2c+\sin(2dx+2c))Ca+4Ba\sin(dx+c)+4Ca\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*a + 4*B*a*sin(d*x
 + c) + 4*C*a*sin(d*x + c))/d
```

Fricas [A] time = 1.61814, size = 99, normalized size = 2.11

$$\frac{(2B+C)adx + (Ca\cos(dx+c) + 2(B+C)a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="fricas")
```

```
[Out] 1/2*((2*B + C)*a*d*x + (C*a*cos(d*x + c) + 2*(B + C)*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int B\cos(c+dx)\sec(c+dx)\,dx + \int B\cos^2(c+dx)\sec(c+dx)\,dx + \int C\cos^2(c+dx)\sec(c+dx)\,dx + \int C\cos^3(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] a*(Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*se
c(c + d*x), x) + Integral(C*cos(c + d*x)**2*sec(c + d*x), x) + Integral(C*c
os(c + d*x)**3*sec(c + d*x), x))
```

Giac [B] time = 1.25378, size = 126, normalized size = 2.68

$$\frac{(2Ba+Ca)(dx+c) + \frac{2\left(2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="giac")
```

```
[Out] 1/2*((2*B*a + C*a)*(d*x + c) + 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 + C*a*tan(1/
2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) + 3*C*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.230 $\int (a+a\cos(c+dx)) \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=32

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+ax(B+C)+\frac{aC\sin(c+dx)}{d}$$

```
[Out] a*(B + C)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d
```

Rubi [A] time = 0.145278, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3029, 2968, 3023, 2735, 3770}

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+ax(B+C)+\frac{aC\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2
,x]
```

```
[Out] a*(B + C)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (a+a\cos(c+dx))(B+C\cos(c+dx))\sec(c+$$

$$= \int \left(aB+(aB+aC)\cos(c+dx)+aC\cos^2(c+d\right.$$

$$= \frac{aC\sin(c+dx)}{d} + \int (aB+a(B+C)\cos(c+dx))$$

$$= a(B+C)x + \frac{aC\sin(c+dx)}{d} + (aB)\int \sec(c+d$$

$$= a(B+C)x + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} + \frac{aC\sin(c}{d}$$

Mathematica [A] time = 0.0245105, size = 46, normalized size = 1.44

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{d} + aBx + \frac{aC\sin(c)\cos(dx)}{d} + \frac{aC\cos(c)\sin(dx)}{d} + aCx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] a*B*x + a*C*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*Cos[d*x]*Sin[c])/d + (
a*C*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.056, size = 56, normalized size = 1.8

$$Bax + aCx + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bac}{d} + \frac{aC\sin(dx+c)}{d} + \frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] B*a*x+a*C*x+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a*c+a*C*sin(d*x+c)/d+1/
d*a*C*c
```

Maxima [A] time = 1.2076, size = 78, normalized size = 2.44

$$\frac{2(dx+c)Ba + 2(dx+c)Ca + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Ca\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + 2*(d*x + c)*C*a + B*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*C*a*sin(d*x + c))/d
```

Fricas [A] time = 1.71491, size = 139, normalized size = 4.34

$$\frac{2(B+C)adx + Ba\log(\sin(dx+c)+1) - Ba\log(-\sin(dx+c)+1) + 2Ca\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(B + C)*a*d*x + B*a*log(sin(d*x + c) + 1) - B*a*log(-sin(d*x + c) +
1) + 2*C*a*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36079, size = 107, normalized size = 3.34

$$\frac{Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ba+Ca)(dx+c)+\frac{2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="giac")
```

```
[Out] (B*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + C*a)*(d*x + c) + 2*C*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.231 $\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=32

$$\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}+aCx$$

```
[Out] a*C*x + (a*(B + C)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rubi [A] time = 0.156156, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3029, 2968, 3021, 2735, 3770}

$$\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}+aCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3
,x]
```

```
[Out] a*C*x + (a*(B + C)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= \int (a+a\cos(c+dx))(B+C\cos(c+dx))\sec^2(c\\ &= \int \left(aB+(aB+aC)\cos(c+dx)+aC\cos^2(c+d\right.\\ &= \frac{aB\tan(c+dx)}{d}+\int (a(B+C)+aC\cos(c+dx)\\ &= aCx+\frac{aB\tan(c+dx)}{d}+(a(B+C))\int \sec(c+d\\ &= aCx+\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(}{d}\end{aligned}$$

Mathematica [A] time = 0.0185406, size = 43, normalized size = 1.34

$$\frac{aB\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+aCx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] a*C*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*
B*Tan[c + d*x])/d
```

Maple [A] time = 0.053, size = 65, normalized size = 2.

$$aCx+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba\tan(dx+c)}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] a*C*x+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/d+1/d*a*C*ln(sec(d*x
+c)+tan(d*x+c))+1/d*a*C*c
```

Maxima [B] time = 1.56645, size = 99, normalized size = 3.09

$$\frac{2(dx+c)Ca+Ba(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*B*a*tan(d*x + c))
/d
```

Fricas [B] time = 1.75611, size = 220, normalized size = 6.88

$$\frac{2Cadx\cos(dx+c)+(B+C)a\cos(dx+c)\log(\sin(dx+c)+1)-(B+C)a\cos(dx+c)\log(-\sin(dx+c)+1)+2Ba}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*a*d*x*cos(d*x + c) + (B + C)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (B + C)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*a*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45138, size = 113, normalized size = 3.53

$$\frac{(dx+c)Ca + (Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="giac")
```

```
[Out] ((d*x + c)*C*a + (B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + C*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.232 $\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*(B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(B + C)*Tan[c + d*x])/d + (a
*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.186527, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4
,x]
```

```
[Out] (a*(B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(B + C)*Tan[c + d*x])/d + (a
*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx)) \left(B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^4(c + dx)\, dx = \int (a + a\cos(c + dx))(B + C\cos(c + dx)) \sec^3($$
$$= \int \left(aB + (aB + aC)\cos(c + dx) + aC\cos^2(c +\right.$$
$$= \frac{aB\sec(c + dx)\tan(c + dx)}{2d} + \frac{1}{2}\int (2a(B + C)$$
$$= \frac{aB\sec(c + dx)\tan(c + dx)}{2d} + (a(B + C))\int \mathrm{se}$$
$$= \frac{a(B + 2C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB\sec(c + d}{}$$
$$= \frac{a(B + 2C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a(B + C)\tan}{d}$$

Mathematica [A] time = 0.0246544, size = 75, normalized size = 1.34

$$\frac{aB\tan(c + dx)}{d} + \frac{aB\tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB\tan(c + dx)\sec(c + dx)}{2d} + \frac{aC\tan(c + dx)}{d} + \frac{aC\tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (a*B*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Ta
n[c + d*x])/d + (a*C*Tan[c + d*x])/d + (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.055, size = 86, normalized size = 1.5

$$\frac{aC\tan(dx + c)}{d} + \frac{Ba\sec(dx + c)\tan(dx + c)}{2d} + \frac{Ba\ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{aC\ln(\sec(dx + c) + \tan(dx + c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*a*C*tan(d*x+c)+1/2*a*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+
tan(d*x+c))+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/d
```

Maxima [A] time = 1.12839, size = 128, normalized size = 2.29

$$-\frac{Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 2\,Ca(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) - 4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="maxima")
```

```
[Out] -1/4*(B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*B*a*tan(d*x + c) - 4*C*a*tan(d*x + c))/d
```

Fricas [A] time = 1.62916, size = 239, normalized size = 4.27

$$\frac{(B+2\,C)a\cos(dx+c)^2\log(\sin(dx+c)+1) - (B+2\,C)a\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\,(2\,(B+C)a\cos(dx+}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="fricas")
```

```
[Out] 1/4*((B + 2*C)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (B + 2*C)*a*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(B + C)*a*cos(d*x + c) + B*a)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29587, size = 167, normalized size = 2.98

$$\frac{(Ba+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="giac")
```

```
[Out] 1/2*((B*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + 2*C*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(B*a*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*tan(1
/2*d*x + 1/2*c)^3 - 3*B*a*tan(1/2*d*x + 1/2*c) - 2*C*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.233 $\int (a+a\cos(c+dx)) \left(B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx$

Optimal. Leaf size=86

$$\frac{a(2B+3C)\tan(c+dx)}{3d} + \frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(B+C)\tan(c+dx)\sec(c+dx)}{2d} + \frac{aB\tan(c+dx)\sec^2(c+a}{3d}$$

```
[Out] (a*(B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*B + 3*C)*Tan[c + d*x])/(3*d
) + (a*(B + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.209661, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3029, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a(2B+3C)\tan(c+dx)}{3d} + \frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(B+C)\tan(c+dx)\sec(c+dx)}{2d} + \frac{aB\tan(c+dx)\sec^2(c+a}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5
,x]
```

```
[Out] (a*(B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*B + 3*C)*Tan[c + d*x])/(3*d
) + (a*(B + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\cos(c + dx)) \left(B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^5(c + dx)\, dx = \int (a + a\cos(c + dx))(B + C\cos(c + dx)) \sec^4(c$$
$$= \int \left(aB + (aB + aC)\cos(c + dx) + aC\cos^2(c + d\right.$$
$$= \frac{aB\sec^2(c + dx)\tan(c + dx)}{3d} + \frac{1}{3}\int (3a(B + C) -$$
$$= \frac{aB\sec^2(c + dx)\tan(c + dx)}{3d} + (a(B + C))\int \sec$$
$$= \frac{a(B + C)\sec(c + dx)\tan(c + dx)}{2d} + \frac{aB\sec^2(c +}{}$$
$$= \frac{a(B + C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a(2B + 3C)\tan}{3d}$$

Mathematica [A] time = 0.312306, size = 56, normalized size = 0.65

$$\frac{a\left(3(B + C)\tanh^{-1}(\sin(c + dx)) + \tan(c + dx)\left(3(B + C)\sec(c + dx) + 2B\tan^2(c + dx) + 6(B + C)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] (a*(3*(B + C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(B + C) + 3*(B + C)*S
ec[c + d*x] + 2*B*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.059, size = 128, normalized size = 1.5

$$\frac{aC\tan(dx + c)\sec(dx + c)}{2d} + \frac{aC\ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{2Ba\tan(dx + c)}{3d} + \frac{Ba(\sec(dx + c))^2\tan(dx + c)}{3d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*B
*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d+1/d*a*C*tan(d*x+c)+1/2*a*B*
sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.29784, size = 171, normalized size = 1.99

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba-3\,Ba\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ca\left(\frac{2\,\sin(dx}{\sin(dx+c)}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, al
gorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*C*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*C*a*tan(d*x + c))/d
```

Fricas [A] time = 1.68686, size = 288, normalized size = 3.35

$$\frac{3\,(B+C)a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(B+C)a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(2\,B+3\,C)a\cos\left(\right.\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, al
gorithm="fricas")
```

```
[Out] 1/12*(3*(B + C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B + C)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*B + 3*C)*a*cos(d*x + c)^2 + 3*(B
+ C)*a*cos(d*x + c) + 2*B*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

[Out] Timed out

Giac [A] time = 1.64673, size = 208, normalized size = 2.42

$$\frac{3\,(Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-\cdots\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + C*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*B*a*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*tan
(1/2*d*x + 1/2*c)^5 - 4*B*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x + 1
/2*c)^3 + 9*B*a*tan(1/2*d*x + 1/2*c) + 9*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

3.234 $\int (a+a\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{a(B+C)\tan^3(c+dx)}{3d}+\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3B+4C)\tan(c+dx)\sec(c+dx}{8d}$$

```
[Out] (a*(3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(B + C)*Tan[c + d*x])/d +
(a*(3*B + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(B + C)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.216296, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{a(B+C)\tan^3(c+dx)}{3d}+\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3B+4C)\tan(c+dx)\sec(c+dx}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6
,x]
```

```
[Out] (a*(3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(B + C)*Tan[c + d*x])/d +
(a*(3*B + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(B + C)*Tan[c + d*x]^3)/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))\left(B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a + a\cos(c+dx))(B + C\cos(c+dx))\sec^5($$
$$= \int \left(aB + (aB + aC)\cos(c+dx) + aC\cos^2(c+\right.$$
$$= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (4a(B+C)$$
$$= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d} + (a(B+C))\int \mathrm{se}$$
$$= \frac{a(3B+4C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{aB\sec^3}{}$$
$$= \frac{a(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a(B+C)\,\mathrm{ta}}{d}$$

Mathematica [A] time = 0.359059, size = 77, normalized size = 0.73

$$\frac{a\left(3(3B+4C)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\left(8(B+C)(\cos(2(c+dx))+2)\sec(c+dx) + 6B\sec^2(c+d\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```

```
[Out] (a*(3*(3*B + 4*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*B + 12*C + 8*(B +
 C)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*B*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

Maple [A] time = 0.063, size = 171, normalized size = 1.6

$$\frac{2aC\tan(dx+c)}{3d} + \frac{aC\tan(dx+c)(\sec(dx+c))^2}{3d} + \frac{Ba(\sec(dx+c))^3\tan(dx+c)}{4d} + \frac{3Ba\sec(dx+c)\tan(dx+c)}{8d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 2/3/d*a*C*tan(d*x+c)+1/3/d*a*C*tan(d*x+c)*sec(d*x+c)^2+1/4*a*B*sec(d*x+c)^3
*tan(d*x+c)/d+3/8*a*B*sec(d*x+c)*tan(d*x+c)/d+3/8/d*B*a*ln(sec(d*x+c)+tan(d
*x+c))+1/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+
```

```
2/3*a*B*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.18509, size = 220, normalized size = 2.08

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca-3\,Ba\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin$$
$$48\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, al
gorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*C*a - 3*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*C*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.7344, size = 339, normalized size = 3.2

$$\frac{3\,(3\,B+4\,C)a\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(3\,B+4\,C)a\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16\,(B+C)a\cos\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, al
gorithm="fricas")
```

```
[Out] 1/48*(3*(3*B + 4*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*B + 4*C)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B + C)*a*cos(d*x + c)^3 +
3*(3*B + 4*C)*a*cos(d*x + c)^2 + 8*(B + C)*a*cos(d*x + c) + 6*B*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4386, size = 254, normalized size = 2.4

$$\frac{3\,(3\,Ba+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Ba+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ca\tan\left(\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a + 4*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*B*a + 4*C
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*B*a*tan(1/2*d*x + 1/2*c)^7 +
12*C*a*tan(1/2*d*x + 1/2*c)^7 - 49*B*a*tan(1/2*d*x + 1/2*c)^5 - 28*C*a*tan(
1/2*d*x + 1/2*c)^5 + 31*B*a*tan(1/2*d*x + 1/2*c)^3 + 52*C*a*tan(1/2*d*x + 1
/2*c)^3 - 39*B*a*tan(1/2*d*x + 1/2*c) - 36*C*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.235 $\int \cos(c+dx)(a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=160

$$-\frac{a^2(10B+9C)\sin^3(c+dx)}{15d}+\frac{a^2(10B+9C)\sin(c+dx)}{5d}+\frac{a^2(5B+6C)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7B+6C)\sin(c+}{8d}$$

```
[Out] (a^2*(7*B + 6*C)*x)/8 + (a^2*(10*B + 9*C)*Sin[c + d*x])/(5*d) + (a^2*(7*B +
 6*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*B + 6*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (C*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(10*B + 9*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.338862, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3029, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^2(10B+9C)\sin^3(c+dx)}{15d}+\frac{a^2(10B+9C)\sin(c+dx)}{5d}+\frac{a^2(5B+6C)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7B+6C)\sin(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)
,x]
```


```
[Out] (a^2*(7*B + 6*C)*x)/8 + (a^2*(10*B + 9*C)*Sin[c + d*x])/(5*d) + (a^2*(7*B +
 6*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*B + 6*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (C*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(10*B + 9*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos^2(c+dx)(a+a\cos(c+dx))^2(B+C\cos($$
$$= \frac{C\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx}{5d}$$
$$= \frac{C\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx}{5d}$$
$$= \frac{a^2(5B+6C)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{C\cos^3}{}$$
$$= \frac{a^2(5B+6C)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{C\cos^3}{}$$
$$= \frac{a^2(7B+6C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(5B+}{}$$
$$= \frac{1}{8}a^2(7B+6C)x + \frac{a^2(10B+9C)\sin(c+dx)}{5d} + $$

Mathematica [A] time = 0.324909, size = 104, normalized size = 0.65

$$\frac{a^2(60(12B+11C)\sin(c+dx)+240(B+C)\sin(2(c+dx))+80B\sin(3(c+dx))+15B\sin(4(c+dx))+420Bdx+90C\sin}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (a^2*(420*B*d*x + 360*C*d*x + 60*(12*B + 11*C)*Sin[c + d*x] + 240*(B + C)*S
in[2*(c + d*x)] + 80*B*Sin[3*(c + d*x)] + 90*C*Sin[3*(c + d*x)] + 15*B*Sin[
4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.024, size = 186, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) + 2\,a^2C\left(1/4\,\left((\cos(dx+c))^3+3/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+2/3*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/5*a^2*C*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(
d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.08221, size = 240, normalized size = 1.5

$$-\frac{320\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba^{2}-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Ba^{2}-120\left(2\,dx+2\,c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/480*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^2 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*C*a^2 + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 30*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2)/d
```

Fricas [A] time = 1.65918, size = 271, normalized size = 1.69

$$\frac{15\left(7\,B+6\,C\right)a^{2}dx+\left(24\,Ca^{2}\cos\left(dx+c\right)^{4}+30\left(B+2\,C\right)a^{2}\cos\left(dx+c\right)^{3}+8\left(10\,B+9\,C\right)a^{2}\cos\left(dx+c\right)^{2}+15\left(7\,B+6\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/120*(15*(7*B + 6*C)*a^2*d*x + (24*C*a^2*cos(d*x + c)^4 + 30*(B + 2*C)*a^2
*cos(d*x + c)^3 + 8*(10*B + 9*C)*a^2*cos(d*x + c)^2 + 15*(7*B + 6*C)*a^2*co
s(d*x + c) + 16*(10*B + 9*C)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 3.35701, size = 462, normalized size = 2.89

$$\begin{cases} \frac{3Ba^2x\sin^4(c+dx)}{8} + \frac{3Ba^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Ba^2x\sin^2(c+dx)}{2} + \frac{3Ba^2x\cos^4(c+dx)}{8} + \frac{Ba^2x\cos^2(c+dx)}{2} + \frac{3Ba^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{4Ba^2 \\ x\left(B\cos(c) + C\cos^2(c)\right)(a\cos(c)+a)^2\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*B*a**2*x*sin(c + d*x)**4/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + B*a**2*x*sin(c + d*x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/8 + B
*a**2*x*cos(c + d*x)**2/2 + 3*B*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4
*B*a**2*sin(c + d*x)**3/(3*d) + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 2*B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 3*C*a**2*x*sin(c + d*x)**4/4 + 3*C*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*C*a**2*x*cos(c + d*x)**4/4 + 8*C*a**2*sin(c + d*x)**5/(15*d
) + 4*C*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*C*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*C*a**2*sin(c + d*x)**3/(3*d) + C*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*C*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + C*a**2*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*(a*c
os(c) + a)**2*cos(c), True))
```

Giac [A] time = 1.64717, size = 185, normalized size = 1.16

$$\frac{Ca^2\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(7\,Ba^2 + 6\,Ca^2\right)x + \frac{\left(Ba^2 + 2\,Ca^2\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(8\,Ba^2 + 9\,Ca^2\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(Ba^2 + C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/80*C*a^2*sin(5*d*x + 5*c)/d + 1/8*(7*B*a^2 + 6*C*a^2)*x + 1/32*(B*a^2 + 2
*C*a^2)*sin(4*d*x + 4*c)/d + 1/48*(8*B*a^2 + 9*C*a^2)*sin(3*d*x + 3*c)/d +
1/2*(B*a^2 + C*a^2)*sin(2*d*x + 2*c)/d + 1/8*(12*B*a^2 + 11*C*a^2)*sin(d*x
+ c)/d
```

3.236 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=129

$$\frac{a^2(8B+7C)\sin(c+dx)}{6d}+\frac{a^2(8B+7C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8B+7C)+\frac{(4B-C)\sin(c+dx)(a\cos(c+dx)}{12d}$$

```
[Out] (a^2*(8*B + 7*C)*x)/8 + (a^2*(8*B + 7*C)*Sin[c + d*x])/(6*d) + (a^2*(8*B +
7*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*B - C)*(a + a*Cos[c + d*x])^2*
Sin[c + d*x])/(12*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.140573, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3023, 2751, 2644}

$$\frac{a^2(8B+7C)\sin(c+dx)}{6d}+\frac{a^2(8B+7C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8B+7C)+\frac{(4B-C)\sin(c+dx)(a\cos(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(8*B + 7*C)*x)/8 + (a^2*(8*B + 7*C)*Sin[c + d*x])/(6*d) + (a^2*(8*B +
7*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*B - C)*(a + a*Cos[c + d*x])^2*
Sin[c + d*x])/(12*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 \left(B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{\int (a+a\cos(c+dx))^2(3}{}$$
$$= \frac{(4B-C)(a+a\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{C(a+a\cos(c+}{4}$$
$$= \frac{1}{8}a^2(8B+7C)x + \frac{a^2(8B+7C)\sin(c+dx)}{6d} + \frac{a^2(8B+7C)\,c}{}$$

Mathematica [A] time = 0.330804, size = 86, normalized size = 0.67

$$\frac{a^2(24(7B+6C)\sin(c+dx) + 48(B+C)\sin(2(c+dx)) + 8B\sin(3(c+dx)) + 96Bdx + 16C\sin(3(c+dx)) + 3C\sin(4(c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(84*c*C + 96*B*d*x + 84*C*d*x + 24*(7*B + 6*C)*Sin[c + d*x] + 48*(B +
C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 16*C*Sin[3*(c + d*x)] + 3*C*Si
n[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.023, size = 154, normalized size = 1.2

$$\frac{1}{d}\left(a^2C\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a^2B\sin(dx+c) + \frac{2a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + 2a^2B(1/2\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*B*sin(d*x+c)+2/3*a
^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+
1/3*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.13956, size = 194, normalized size = 1.5

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ba^{2}-48\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^{2}+64\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ca^{2}-}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 48*(2*d*x + 2*c + sin(2
*d*x + 2*c))*B*a^2 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 3*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 - 24*(2*d*x + 2*c +
sin(2*d*x + 2*c))*C*a^2 - 96*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.65236, size = 213, normalized size = 1.65

$$\frac{3\left(8\,B+7\,C\right)a^{2}dx+\left(6\,Ca^{2}\cos\left(dx+c\right)^{3}+8\left(B+2\,C\right)a^{2}\cos\left(dx+c\right)^{2}+3\left(8\,B+7\,C\right)a^{2}\cos\left(dx+c\right)+8\left(5\,B+4\,C\right)a^{2}\right)\mathrm{s}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(8*B + 7*C)*a^2*d*x + (6*C*a^2*cos(d*x + c)^3 + 8*(B + 2*C)*a^2*cos
(d*x + c)^2 + 3*(8*B + 7*C)*a^2*cos(d*x + c) + 8*(5*B + 4*C)*a^2)*sin(d*x +
 c))/d
```

Sympy [A] time = 1.51834, size = 340, normalized size = 2.64

$$\begin{cases} Ba^{2}x\sin^{2}\left(c+dx\right)+Ba^{2}x\cos^{2}\left(c+dx\right)+\frac{2Ba^{2}\sin^{3}\left(c+dx\right)}{3d}+\frac{Ba^{2}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}+\frac{Ba^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{Ba^{2}\sin\left(c+dx\right)}{d}+ \\ x\left(B\cos\left(c\right)+C\cos^{2}\left(c\right)\right)\left(a\cos\left(c\right)+a\right)^{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*x*sin(c + d*x)**2 + B*a**2*x*cos(c + d*x)**2 + 2*B*a**2*s
in(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c
 + d*x)*cos(c + d*x)/d + B*a**2*sin(c + d*x)/d + 3*C*a**2*x*sin(c + d*x)**4
/8 + 3*C*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + C*a**2*x*sin(c + d*x)**
2/2 + 3*C*a**2*x*cos(c + d*x)**4/8 + C*a**2*x*cos(c + d*x)**2/2 + 3*C*a**2*
sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*C*a**2*sin(c + d*x)**3/(3*d) + 5*C*a
**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*C*a**2*sin(c + d*x)*cos(c + d*x)
**2/d + C*a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(B*cos(c) + C
*cos(c)**2)*(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.84607, size = 149, normalized size = 1.16

$$\frac{Ca^2\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(8\,Ba^2+7\,Ca^2\right)x+\frac{\left(Ba^2+2\,Ca^2\right)\sin(3\,dx+3\,c)}{12\,d}+\frac{\left(Ba^2+Ca^2\right)\sin(2\,dx+2\,c)}{2\,d}+\frac{\left(7\,Ba^2+6\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/32*C*a^2*sin(4*d*x + 4*c)/d + 1/8*(8*B*a^2 + 7*C*a^2)*x + 1/12*(B*a^2 + 2
*C*a^2)*sin(3*d*x + 3*c)/d + 1/2*(B*a^2 + C*a^2)*sin(2*d*x + 2*c)/d + 1/4*(
7*B*a^2 + 6*C*a^2)*sin(d*x + c)/d
```

3.237 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{2a^2(3B+2C)\sin(c+dx)}{3d}+\frac{a^2(3B+2C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(3B+2C)+\frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

```
[Out] (a^2*(3*B + 2*C)*x)/2 + (2*a^2*(3*B + 2*C)*Sin[c + d*x])/(3*d) + (a^2*(3*B
+ 2*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d)
```

Rubi [A] time = 0.124678, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3029, 2751, 2644}

$$\frac{2a^2(3B+2C)\sin(c+dx)}{3d}+\frac{a^2(3B+2C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(3B+2C)+\frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
,x]
```

```
[Out] (a^2*(3*B + 2*C)*x)/2 + (2*a^2*(3*B + 2*C)*Sin[c + d*x])/(3*d) + (a^2*(3*B
+ 2*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 \left(B\cos(c+dx) + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int (a + a\cos(c+dx))^2(B + C\cos(c+dx))\,dx$$
$$= \frac{C(a + a\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}(3B + 2C)$$
$$= \frac{1}{2}a^2(3B+2C)x + \frac{2a^2(3B+2C)\sin(c+dx)}{3d} + $$

Mathematica [A] time = 0.189075, size = 61, normalized size = 0.65

$$\frac{a^2(3(8B+7C)\sin(c+dx) + 3(B+2C)\sin(2(c+dx)) + 18Bdx + C\sin(3(c+dx)) + 12Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (a^2*(18*B*d*x + 12*C*d*x + 3*(8*B + 7*C)*Sin[c + d*x] + 3*(B + 2*C)*Sin[2*
(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.049, size = 116, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^2C\left(2 + (\cos(dx+c))^2\right)\sin(dx+c)}{3} + a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 2\,a^2C\,(1/2\,\cos(dx+c)\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(1/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*B*s
in(d*x+c)+a^2*C*sin(d*x+c)+a^2*B*(d*x+c))
```

Maxima [A] time = 1.05685, size = 149, normalized size = 1.59

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^{2}+12\,(dx+c)Ba^{2}-4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ca^{2}+6\,(2\,dx+2\,c+\sin\left(2\,dx+\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 12*(d*x + c)*B*a^2 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*C
*a^2 + 24*B*a^2*sin(d*x + c) + 12*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.64236, size = 165, normalized size = 1.76

$$\frac{3\,(3\,B+2\,C)a^{2}dx+\left(2\,Ca^{2}\cos\left(dx+c\right)^{2}+3\,(B+2\,C)a^{2}\cos\left(dx+c\right)+2\,(6\,B+5\,C)a^{2}\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] 1/6*(3*(3*B + 2*C)*a^2*d*x + (2*C*a^2*cos(d*x + c)^2 + 3*(B + 2*C)*a^2*cos(
d*x + c) + 2*(6*B + 5*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76221, size = 192, normalized size = 2.04

$$\frac{3\left(3\,Ba^2+2\,Ca^2\right)(dx+c)+\frac{2\left(9\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(3*B*a^2 + 2*C*a^2)*(d*x + c) + 2*(9*B*a^2*tan(1/2*d*x + 1/2*c)^5 +
6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 16*C*a^2
*tan(1/2*d*x + 1/2*c)^3 + 15*B*a^2*tan(1/2*d*x + 1/2*c) + 18*C*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.238 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=82

$$\frac{a^2(2B+3C)\sin(c+dx)}{2d}+\frac{a^2B\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x(4B+3C)+\frac{C\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

```
[Out] (a^2*(4*B + 3*C)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*B + 3*C)*
Sin[c + d*x])/(2*d) + (C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.272543, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^2(2B+3C)\sin(c+dx)}{2d}+\frac{a^2B\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x(4B+3C)+\frac{C\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^2,x]
```

```
[Out] (a^2*(4*B + 3*C)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*B + 3*C)*
Sin[c + d*x])/(2*d) + (C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (a+a\cos(c+dx))^2(B+C\cos(c+dx))\sec$$
$$= \frac{C\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{1}{2}\int (a$$
$$= \frac{C\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{1}{2}\int (2$$
$$= \frac{a^2(2B+3C)\sin(c+dx)}{2d} + \frac{C\left(a^2+a^2\cos(c+\right.}{2d}$$
$$= \frac{1}{2}a^2(4B+3C)x + \frac{a^2(2B+3C)\sin(c+dx)}{2d} +$$
$$= \frac{1}{2}a^2(4B+3C)x + \frac{a^2B\tanh^{-1}(\sin(c+dx))}{d} +$$

Mathematica [A] time = 0.177567, size = 96, normalized size = 1.17

$$\frac{a^2\left(4(B+2C)\sin(c+dx) - 4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4B\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right) + 8\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] (a^2*(8*B*d*x + 6*C*d*x - 4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(B + 2*C)*Sin[c + d*x] + C*S
in[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.065, size = 108, normalized size = 1.3

$$\frac{3\,a^2Cx}{2} + \frac{3\,a^2Cc}{2\,d} + \frac{a^2B\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\,\frac{a^2C\sin(dx+c)}{d} + 2\,a^2Bx + 2\,\frac{Ba^2c}{d} + \frac{a^2C\cos(dx+c)\sin(}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 3/2*a^2*C*x+3/2/d*a^2*C*c+1/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*C*sin
(d*x+c)+2*a^2*B*x+2/d*B*a^2*c+1/2/d*a^2*C*cos(d*x+c)*sin(d*x+c)+1/d*a^2*B*s
```

```
in(d*x+c)
```

Maxima [A] time = 1.07684, size = 136, normalized size = 1.66

$$\frac{8(dx+c)Ba^2 + (2dx+2c+\sin(2dx+2c))Ca^2 + 4(dx+c)Ca^2 + 2Ba^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) -}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] 1/4*(8*(d*x + c)*B*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 4*(d*x +
c)*C*a^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^
2*sin(d*x + c) + 8*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.71771, size = 194, normalized size = 2.37

$$\frac{(4B+3C)a^2dx + Ba^2\log(\sin(dx+c)+1) - Ba^2\log(-\sin(dx+c)+1) + \left(Ca^2\cos(dx+c) + 2(B+2C)a^2\right)\sin(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] 1/2*((4*B + 3*C)*a^2*d*x + B*a^2*log(sin(d*x + c) + 1) - B*a^2*log(-sin(d*x
 + c) + 1) + (C*a^2*cos(d*x + c) + 2*(B + 2*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,
x)
```

[Out] Timed out

Giac [A] time = 1.47208, size = 196, normalized size = 2.39

$$\frac{2 Ba^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 2 Ba^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \left(4 Ba^2 + 3 Ca^2\right)(dx + c) + \frac{2\left(2 Ba^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3\right.}{}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] 1/2*(2*B*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (4*B*a^2 + 3*C*a^2)*(d*x + c) + 2*(2*B*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)
+ 5*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.239 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{a^2(B-C)\sin(c+dx)}{d}+\frac{a^2(2B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}+a^2x(B+2C)$$

```
[Out] a^2*(B + 2*C)*x + (a^2*(2*B + C)*ArcTanh[Sin[c + d*x]])/d - (a^2*(B - C)*Si
n[c + d*x])/d + (B*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Rubi [A] time = 0.290653, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2975, 2968, 3023, 2735, 3770}

$$-\frac{a^2(B-C)\sin(c+dx)}{d}+\frac{a^2(2B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}+a^2x(B+2C)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^3,x]
```

```
[Out] a^2*(B + 2*C)*x + (a^2*(2*B + C)*ArcTanh[Sin[c + d*x]])/d - (a^2*(B - C)*Si
n[c + d*x])/d + (B*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int (a+a\cos(c+dx))^2(B+C\cos(c+dx))\sec^2($$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d} + \int (a+a$$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d} + \int \left(a^2(2\right.$$

$$= -\frac{a^2(B-C)\sin(c+dx)}{d} + \frac{B\left(a^2+a^2\cos(c+dx\right.}{d}$$

$$= a^2(B+2C)x - \frac{a^2(B-C)\sin(c+dx)}{d} + \frac{B\left(a^2+\right.}{}$$

$$= a^2(B+2C)x + \frac{a^2(2B+C)\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.31674, size = 143, normalized size = 1.93

$$\frac{a^2\left(B\tan(c+dx)-2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+Bc+Bdx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] (a^2*(B*c + 2*c*C + B*d*x + 2*C*d*x - 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*B*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*S
in[c + d*x] + B*Tan[c + d*x]))/d
```

Maple [A] time = 0.062, size = 107, normalized size = 1.5

$$a^2Bx + 2\,a^2Cx + 2\,\frac{a^2B\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2B\tan(dx+c)}{d} + \frac{Ba^2c}{d} + \frac{a^2C\sin(dx+c)}{d} + \frac{a^2C\ln(\sec(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] a^2*B*x+2*a^2*C*x+2/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*tan(d*x+c)/d+1/
d*B*a^2*c+1/d*a^2*C*sin(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*
C*c
```

Maxima [A] time = 1.08597, size = 142, normalized size = 1.92

$$\frac{2\,(dx+c)Ba^2+4\,(dx+c)Ca^2+2\,Ba^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)})+Ca^2(\log{(\sin{(dx+c)}+1)}-\log{(}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + 4*(d*x + c)*C*a^2 + 2*B*a^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*C*a^2*sin(d*x + c) + 2*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.72272, size = 278, normalized size = 3.76

$$\frac{2\,(B+2\,C)a^2dx\cos{(dx+c)}+(2\,B+C)a^2\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-(2\,B+C)a^2\cos{(dx+c)}\log{(-\sin{(dx+}}{2\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] 1/2*(2*(B + 2*C)*a^2*d*x*cos(d*x + c) + (2*B + C)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (2*B + C)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a^
2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56175, size = 209, normalized size = 2.82

$$\frac{\left(Ba^2+2\,Ca^2\right)(dx+c)+\left(2\,Ba^2+Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ba^2+Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Ba^2\tan\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] ((B*a^2 + 2*C*a^2)*(d*x + c) + (2*B*a^2 + C*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*B*a^2 + C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(B*a^2*
tan(1/2*d*x + 1/2*c)^3 - C*a^2*tan(1/2*d*x + 1/2*c)^3 + B*a^2*tan(1/2*d*x +
 1/2*c) + C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.240 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{a^2(3B+2C)\tan(c+dx)}{2d}+\frac{a^2(3B+4C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a^2Cx$$

```
[Out] a^2*C*x + (a^2*(3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(3*B + 2*C)*
Tan[c + d*x])/(2*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rubi [A] time = 0.299705, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2975, 2968, 3021, 2735, 3770}

$$\frac{a^2(3B+2C)\tan(c+dx)}{2d}+\frac{a^2(3B+4C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a^2Cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^4,x]
```


```
[Out] a^2*C*x + (a^2*(3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(3*B + 2*C)*
Tan[c + d*x])/(2*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int (a+a\cos(c+dx))^2(B+C\cos(c+dx))\sec$$
$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+d}{2d}$$
$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+d}{2d}$$
$$= \frac{a^2(3B+2C)\tan(c+dx)}{2d} + \frac{B\left(a^2+a^2\cos(c-\right.}{}$$
$$= a^2Cx + \frac{a^2(3B+2C)\tan(c+dx)}{2d} + \frac{B\left(a^2+a\right.}{}$$
$$= a^2Cx + \frac{a^2(3B+4C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a}{}$$

Mathematica [B] time = 1.43685, size = 277, normalized size = 3.15

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(2B+C)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(4*C*x - (2*(3*B + 4*C)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(3*B + 4*C)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + B/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
4*(2*B + C)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*B +
C)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))))/16
```

Maple [A] time = 0.063, size = 113, normalized size = 1.3

$$\frac{a^2C\tan(dx+c)}{d} + \frac{a^2B\sec(dx+c)\tan(dx+c)}{2d} + \frac{3a^2B\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 2\frac{a^2C\ln(\sec(dx+c)+\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*a^2*C*tan(d*x+c)+1/2*a^2*B*sec(d*x+c)*tan(d*x+c)/d+3/2/d*a^2*B*ln(sec(d
*x+c)+tan(d*x+c))+2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*B*tan(d*x+c)/d+
a^2*C*x+1/d*a^2*C*c
```

Maxima [A] time = 1.1399, size = 192, normalized size = 2.18

$$\frac{4\,(dx+c)Ca^2 - Ba^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Ba^2(\log\left(\sin\left(dx+c\right)+1\right) - \log(\sin}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a^2 - B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*B*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 4*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 8*B*a^2*tan(d*x + c) + 4*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.7488, size = 297, normalized size = 3.38

$$\frac{4\,Ca^2dx\cos\left(dx+c\right)^2 + (3\,B+4\,C)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - (3\,B+4\,C)a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] 1/4*(4*C*a^2*d*x*cos(d*x + c)^2 + (3*B + 4*C)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (3*B + 4*C)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(2*B + C)*a^2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69665, size = 208, normalized size = 2.36

$$\frac{2\,(dx+c)Ca^2+\left(3\,Ba^2+4\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(3\,Ba^2+4\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Ba^2\tan\left(\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*C*a^2 + (3*B*a^2 + 4*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (3*B*a^2 + 4*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*B*a^2*
tan(1/2*d*x + 1/2*c)^3 + 2*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^2*tan(1/2*d
*x + 1/2*c) - 2*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

3.241 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=113

$$\frac{a^2(5B+6C)\tan(c+dx)}{3d}+\frac{a^2(2B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4B+3C)\tan(c+dx)\sec(c+dx)}{6d}+\frac{B\tan(c+dx)\sec}{}$$

```
[Out] (a^2*(2*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(5*B + 6*C)*Tan[c + d*
x])/(3*d) + (a^2*(4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (B*(a^2 + a
^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.359873, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^2(5B+6C)\tan(c+dx)}{3d}+\frac{a^2(2B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4B+3C)\tan(c+dx)\sec(c+dx)}{6d}+\frac{B\tan(c+dx)\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^5,x]
```

```
[Out] (a^2*(2*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(5*B + 6*C)*Tan[c + d*
x])/(3*d) + (a^2*(4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (B*(a^2 + a
^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^2 \left(B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^5(c + dx)\, dx = \int (a + a\cos(c + dx))^2 (B + C\cos(c + dx))\sec^4($$

$$= \frac{B\left(a^2 + a^2\cos(c + dx)\right)\sec^2(c + dx)\tan(c + dx}{3d}$$

$$= \frac{B\left(a^2 + a^2\cos(c + dx)\right)\sec^2(c + dx)\tan(c + dx}{3d}$$

$$= \frac{a^2(4B + 3C)\sec(c + dx)\tan(c + dx)}{6d} + \frac{B\left(a^2 +\right.}{}$$

$$= \frac{a^2(4B + 3C)\sec(c + dx)\tan(c + dx)}{6d} + \frac{B\left(a^2 +\right.}{}$$

$$= \frac{a^2(2B + 3C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(4B + 3C}{}$$

$$= \frac{a^2(2B + 3C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(5B + 6C}{}$$

Mathematica [B] time = 5.72684, size = 451, normalized size = 3.99

$$a^2(\cos(c + dx) + 1)^2\sec^4\left(\frac{1}{2}(c + dx)\right)\left(\frac{4(5B+6C)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4(5B+6C)\sin\left(\frac{dx}{2}\right)}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^5,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*(2*B + 3*C)*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 6*(2*B + 3*C)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*B*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^3) + ((7*B + 3*C)*Cos[c/2] - (5*B + 3*C)*Sin[c/2])/((Cos[c/
2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(5*B + 6*C)*Si
n[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) +
 (2*B*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3) - ((7*B + 3*C)*Cos[c/2] + (5*B + 3*C)*Sin[c/2])/((Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*B + 6*C)*Sin[(d*x)/2]
)/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(48*d)
```

Maple [A] time = 0.066, size = 141, normalized size = 1.3

$$\frac{a^2C\sec(dx+c)\tan(dx+c)}{2d}+\frac{3a^2C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{5a^2B\tan(dx+c)}{3d}+\frac{a^2B(\sec(dx+c))^2\tan(dx}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/2/d*a^2*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+5/3
*a^2*B*tan(d*x+c)/d+1/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d+2/d*a^2*C*tan(d*x+c
)+a^2*B*sec(d*x+c)*tan(d*x+c)/d+1/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.27646, size = 235, normalized size = 2.08

$$4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2-6Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3Ca^2\left(\frac{2\sin}{\sin(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 - 6*B*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*C*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
12*B*a^2*tan(d*x + c) + 24*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.69348, size = 315, normalized size = 2.79

$$\frac{3(2B+3C)a^2\cos(dx+c)^3\log(\sin(dx+c)+1)-3(2B+3C)a^2\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2(5B+6\right.}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="fricas")
```

```
[Out] 1/12*(3*(2*B + 3*C)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*B + 3*C
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(5*B + 6*C)*a^2*cos(d*x
+ c)^2 + 3*(2*B + C)*a^2*cos(d*x + c) + 2*B*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.5839, size = 240, normalized size = 2.12

$$\frac{3\left(2\,Ba^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Ba^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Ca^2\,\mathrm{t}}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(2*B*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*B*a^2
+ 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a^2*tan(1/2*d*x + 1/
2*c)^5 + 9*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 16*B*a^2*tan(1/2*d*x + 1/2*c)^3 -
 24*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*tan(1/2*d*x + 1/2*c) + 15*C*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.242 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^2(4B+5C)\tan(c+dx)}{3d}+\frac{a^2(7B+8C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5B+4C)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7B+8C)\ta$$

```
[Out] (a^2*(7*B + 8*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*B + 5*C)*Tan[c + d*
x])/(3*d) + (a^2*(7*B + 8*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*B +
 4*C)*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.386232, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3029, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^2(4B+5C)\tan(c+dx)}{3d}+\frac{a^2(7B+8C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5B+4C)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7B+8C)\ta$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^6,x]
```


```
[Out] (a^2*(7*B + 8*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*B + 5*C)*Tan[c + d*
x])/(3*d) + (a^2*(7*B + 8*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*B +
 4*C)*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a+a\cos(c+dx))^2(B+C\cos(c+dx))\operatorname{sec}$$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+dx)\tan(c+}{4d}$$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+dx)\tan(c+}{4d}$$

$$= \frac{a^2(5B+4C)\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{B\left(a\right.}{}$$

$$= \frac{a^2(5B+4C)\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{B\left(a\right.}{}$$

$$= \frac{a^2(7B+8C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2(5B}{}$$

$$= \frac{a^2(7B+8C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(4B+}{}$$

Mathematica [A] time = 1.12752, size = 262, normalized size = 1.82

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(24(7B+8C)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^6,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(7*B + 8*C
)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(4*B + 5*C)*Sin[c] + 3*(15*B + 8*
```

```
C)*Sin[d*x] + 45*B*Sin[2*c + d*x] + 24*C*Sin[2*c + d*x] + 128*B*Sin[c + 2*d
*x] + 136*C*Sin[c + 2*d*x] - 24*C*Sin[3*c + 2*d*x] + 21*B*Sin[2*c + 3*d*x]
+ 24*C*Sin[2*c + 3*d*x] + 21*B*Sin[4*c + 3*d*x] + 24*C*Sin[4*c + 3*d*x] + 3
2*B*Sin[3*c + 4*d*x] + 40*C*Sin[3*c + 4*d*x])))/(768*d)
```

Maple [A] time = 0.07, size = 187, normalized size = 1.3

$$\frac{5\,a^2C\tan\left(dx+c\right)}{3\,d}+\frac{a^2C\tan\left(dx+c\right)\left(\sec\left(dx+c\right)\right)^2}{3\,d}+\frac{a^2B\left(\sec\left(dx+c\right)\right)^3\tan\left(dx+c\right)}{4\,d}+\frac{7\,a^2B\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 5/3/d*a^2*C*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/4*a^2*B*sec(d*
x+c)^3*tan(d*x+c)/d+7/8*a^2*B*sec(d*x+c)*tan(d*x+c)/d+7/8/d*a^2*B*ln(sec(d*
x+c)+tan(d*x+c))+1/d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+ta
n(d*x+c))+4/3*a^2*B*tan(d*x+c)/d+2/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.31683, size = 311, normalized size = 2.16

$$\frac{32\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^2+16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ca^2-3\,Ba^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a^2 - 3*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*C*a^2*tan(d*x +
 c))/d
```

Fricas [A] time = 1.70656, size = 362, normalized size = 2.51

$$\frac{3\,(7\,B+8\,C)a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(7\,B+8\,C)a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(4\,B+5\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(7*B + 8*C)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*B + 8*C
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(4*B + 5*C)*a^2*cos(d*x
+ c)^3 + 3*(7*B + 8*C)*a^2*cos(d*x + c)^2 + 8*(2*B + C)*a^2*cos(d*x + c) +
6*B*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41372, size = 286, normalized size = 1.99

$$3\left(7\,Ba^2+8\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Ba^2+8\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="giac")
```

```
[Out] 1/24*(3*(7*B*a^2 + 8*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*B*a^2
 + 8*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*B*a^2*tan(1/2*d*x +
1/2*c)^7 + 24*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*B*a^2*tan(1/2*d*x + 1/2*c)^
5 - 88*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 136
*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*B*a^2*tan(1/2*d*x + 1/2*c) - 72*C*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.243 $\int (a+a\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{a^2(9B+10C)\tan^3(c+dx)}{15d}+\frac{a^2(9B+10C)\tan(c+dx)}{5d}+\frac{a^2(6B+7C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(6B+5C)\tan(c+dx)}{20d}$$

```
[Out] (a^2*(6*B + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(9*B + 10*C)*Tan[c + d
*x])/(5*d) + (a^2*(6*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(6*B
+ 5*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*(a^2 + a^2*Cos[c + d*x])*Se
c[c + d*x]^4*Tan[c + d*x])/(5*d) + (a^2*(9*B + 10*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.393534, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2975, 2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{a^2(9B+10C)\tan^3(c+dx)}{15d}+\frac{a^2(9B+10C)\tan(c+dx)}{5d}+\frac{a^2(6B+7C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(6B+5C)\tan(c+dx)}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^7,x]
```

```
[Out] (a^2*(6*B + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(9*B + 10*C)*Tan[c + d
*x])/(5*d) + (a^2*(6*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(6*B
+ 5*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*(a^2 + a^2*Cos[c + d*x])*Se
c[c + d*x]^4*Tan[c + d*x])/(5*d) + (a^2*(9*B + 10*C)*Tan[c + d*x]^3)/(15*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int (a+a\cos(c+dx))^2(B+C\cos(c+dx))\sec$$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sec^4(c+dx)\tan(c+}{5d}$$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sec^4(c+dx)\tan(c+}{5d}$$

$$= \frac{a^2(6B+5C)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{B\left(a\right.}{}$$

$$= \frac{a^2(6B+5C)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{B\left(a\right.}{}$$

$$= \frac{a^2(6B+7C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(6B}{}$$

$$= \frac{a^2(6B+7C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(9B+}{}$$

Mathematica [A] time = 1.30457, size = 280, normalized size = 1.66

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(6B+7C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^7,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^5*(240*(6*B + 7*
C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(15*B + 14*C)*Sin[d*x] - 240*(B +
 2*C)*Sin[2*c + d*x] + 420*B*Sin[c + 2*d*x] + 330*C*Sin[c + 2*d*x] + 420*B*
Sin[3*c + 2*d*x] + 330*C*Sin[3*c + 2*d*x] + 720*B*Sin[2*c + 3*d*x] + 800*C*
```

```
Sin[2*c + 3*d*x] + 90*B*Sin[3*c + 4*d*x] + 105*C*Sin[3*c + 4*d*x] + 90*B*Si
n[5*c + 4*d*x] + 105*C*Sin[5*c + 4*d*x] + 144*B*Sin[4*c + 5*d*x] + 160*C*Si
n[4*c + 5*d*x])))/(7680*d)
```

Maple [A] time = 0.074, size = 235, normalized size = 1.4

$$\frac{a^2C\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{7a^2C\sec(dx+c)\tan(dx+c)}{8d}+\frac{7a^2C\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{6a^2B\tan(d\ldots}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/4/d*a^2*C*tan(d*x+c)*sec(d*x+c)^3+7/8/d*a^2*C*sec(d*x+c)*tan(d*x+c)+7/8/d
*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+6/5*a^2*B*tan(d*x+c)/d+1/5/d*a^2*B*tan(d*x
+c)*sec(d*x+c)^4+3/5*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d+4/3/d*a^2*C*tan(d*x+c)
+2/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/2*a^2*B*sec(d*x+c)^3*tan(d*x+c)/d+3/
4*a^2*B*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.2766, size = 375, normalized size = 2.22

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^2+80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2+160\left(\tan(dx+\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^2 +
80*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 160*(tan(d*x + c)^3 + 3*tan(d*
x + c))*C*a^2 - 30*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 15*C*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.72457, size = 421, normalized size = 2.49

$$\frac{15\,(6\,B+7\,C)a^2\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-15\,(6\,B+7\,C)a^2\cos{(dx+c)}^5\log{(-\sin{(dx+c)}+1)}+2\,\big(16\,(9\,B\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(6*B + 7*C)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(6*B +
7*C)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(9*B + 10*C)*a^2*cos
(d*x + c)^4 + 15*(6*B + 7*C)*a^2*cos(d*x + c)^3 + 8*(9*B + 10*C)*a^2*cos(d*
x + c)^2 + 30*(2*B + C)*a^2*cos(d*x + c) + 24*B*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**7,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69044, size = 332, normalized size = 1.96

$$\frac{15\left(6\,Ba^2+7\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(6\,Ba^2+7\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(90\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="giac")
```

```
[Out] 1/120*(15*(6*B*a^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*B*
a^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(90*B*a^2*tan(1/2*d*x
 + 1/2*c)^9 + 105*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 420*B*a^2*tan(1/2*d*x + 1/
2*c)^7 - 490*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*B*a^2*tan(1/2*d*x + 1/2*c)^
5 + 800*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 540*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 7
90*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 390*B*a^2*tan(1/2*d*x + 1/2*c) + 375*C*a^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.244 $\int \cos(c+dx)(a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=201

$$-\frac{a^3(19B+17C)\sin^3(c+dx)}{15d}+\frac{a^3(19B+17C)\sin(c+dx)}{5d}+\frac{a^3(22B+21C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3B+4C)\sin(c}{}$$

```
[Out] (a^3*(26*B + 23*C)*x)/16 + (a^3*(19*B + 17*C)*Sin[c + d*x])/(5*d) + (a^3*(2
6*B + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(22*B + 21*C)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((3*B + 4*C)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(19*B + 17*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.48687, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3029, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^3(19B+17C)\sin^3(c+dx)}{15d}+\frac{a^3(19B+17C)\sin(c+dx)}{5d}+\frac{a^3(22B+21C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3B+4C)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)
,x]
```


```
[Out] (a^3*(26*B + 23*C)*x)/16 + (a^3*(19*B + 17*C)*Sin[c + d*x])/(5*d) + (a^3*(2
6*B + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(22*B + 21*C)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((3*B + 4*C)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(19*B + 17*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+a\cos(c+dx))^3(B+C\cos$$
$$= \frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx}{6d}$$
$$= \frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx}{6d}$$
$$= \frac{aC\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx}{6d}$$
$$= \frac{a^3(22B+21C)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aC}{}$$
$$= \frac{a^3(22B+21C)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aC}{}$$
$$= \frac{a^3(26B+23C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^3(2}{}$$
$$= \frac{1}{16}a^3(26B+23C)x + \frac{a^3(19B+17C)\sin(c+a}{5d}$$

Mathematica [A] time = 0.414656, size = 130, normalized size = 0.65

$$\frac{a^3(120(23B+21C)\sin(c+dx)+15(64B+63C)\sin(2(c+dx))+340B\sin(3(c+dx))+90B\sin(4(c+dx))+12B\sin(5}{96}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (a^3*(1560*B*d*x + 1380*C*d*x + 120*(23*B + 21*C)*Sin[c + d*x] + 15*(64*B +
 63*C)*Sin[2*(c + d*x)] + 340*B*Sin[3*(c + d*x)] + 380*C*Sin[3*(c + d*x)] +
 90*B*Sin[4*(c + d*x)] + 135*C*Sin[4*(c + d*x)] + 12*B*Sin[5*(c + d*x)] + 3
6*C*Sin[5*(c + d*x)] + 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.027, size = 266, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^3C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3C\left(1/4\left((\cos(dx+c))^3+3/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3/5*a^3*C*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c)+a^3*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+
c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/5*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))
```

Maxima [A] time = 1.48159, size = 354, normalized size = 1.76

$$\frac{64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^3-960\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3+90(12dx+12c+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^3 + 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^3
- 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*C*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3)/d
```

Fricas [A] time = 1.7361, size = 332, normalized size = 1.65

$$\frac{15(26B+23C)a^3dx+\left(40Ca^3\cos(dx+c)^5+48(B+3C)a^3\cos(dx+c)^4+10(18B+23C)a^3\cos(dx+c)^3+16(19B\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/240*(15*(26*B + 23*C)*a^3*d*x + (40*C*a^3*cos(d*x + c)^5 + 48*(B + 3*C)*a
^3*cos(d*x + c)^4 + 10*(18*B + 23*C)*a^3*cos(d*x + c)^3 + 16*(19*B + 17*C)*
a^3*cos(d*x + c)^2 + 15*(26*B + 23*C)*a^3*cos(d*x + c) + 32*(19*B + 17*C)*a
^3)*sin(d*x + c))/d
```

Sympy [A] time = 6.1858, size = 699, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((9*B*a**3*x*sin(c + d*x)**4/8 + 9*B*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + B*a**3*x*sin(c + d*x)**2/2 + 9*B*a**3*x*cos(c + d*x)**4/8 + B
*a**3*x*cos(c + d*x)**2/2 + 8*B*a**3*sin(c + d*x)**5/(15*d) + 4*B*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 2*B*a**3*sin(c + d*x)**3/d + B*a**3*sin(c + d*x)*cos(c + d*x)**4/d +
 15*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*B*a**3*sin(c + d*x)*cos(c
 + d*x)**2/d + B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*C*a**3*x*sin(c +
d*x)**6/16 + 15*C*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*C*a**3*x*si
n(c + d*x)**4/8 + 15*C*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*C*a**3
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*C*a**3*x*cos(c + d*x)**6/16 + 9*C*
a**3*x*cos(c + d*x)**4/8 + 5*C*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8
*C*a**3*sin(c + d*x)**5/(5*d) + 5*C*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6
*d) + 4*C*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*C*a**3*sin(c + d*x)**3
*cos(c + d*x)/(8*d) + 2*C*a**3*sin(c + d*x)**3/(3*d) + 11*C*a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) + 3*C*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*C*
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d, Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*(a*cos(c) + a)**3*cos(c), True
))
```

Giac [A] time = 1.48267, size = 224, normalized size = 1.11

$$\frac{Ca^3 \sin(6\,dx+6\,c)}{192\,d} + \frac{1}{16}\left(26\,Ba^3 + 23\,Ca^3\right)x + \frac{\left(Ba^3 + 3\,Ca^3\right)\sin(5\,dx+5\,c)}{80\,d} + \frac{3\left(2\,Ba^3 + 3\,Ca^3\right)\sin(4\,dx+4\,c)}{64\,d} + \frac{(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/192*C*a^3*sin(6*d*x + 6*c)/d + 1/16*(26*B*a^3 + 23*C*a^3)*x + 1/80*(B*a^3
 + 3*C*a^3)*sin(5*d*x + 5*c)/d + 3/64*(2*B*a^3 + 3*C*a^3)*sin(4*d*x + 4*c)/
d + 1/48*(17*B*a^3 + 19*C*a^3)*sin(3*d*x + 3*c)/d + 1/64*(64*B*a^3 + 63*C*a
^3)*sin(2*d*x + 2*c)/d + 1/8*(23*B*a^3 + 21*C*a^3)*sin(d*x + c)/d
```

3.245 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=154

$$-\frac{a^3(15B+13C)\sin^3(c+dx)}{60d}+\frac{a^3(15B+13C)\sin(c+dx)}{5d}+\frac{3a^3(15B+13C)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15B+13C$$

```
[Out] (a^3*(15*B + 13*C)*x)/8 + (a^3*(15*B + 13*C)*Sin[c + d*x])/(5*d) + (3*a^3*(
15*B + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*B - C)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a
*d) - (a^3*(15*B + 13*C)*Sin[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.194015, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(15B+13C)\sin^3(c+dx)}{60d}+\frac{a^3(15B+13C)\sin(c+dx)}{5d}+\frac{3a^3(15B+13C)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15B+13C$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (a^3*(15*B + 13*C)*x)/8 + (a^3*(15*B + 13*C)*Sin[c + d*x])/(5*d) + (3*a^3*(
15*B + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*B - C)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a
*d) - (a^3*(15*B + 13*C)*Sin[c + d*x]^3)/(60*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3 \left(B\cos(c + dx) + C\cos^2(c + dx)\right) dx = \frac{C(a + a\cos(c + dx))^4 \sin(c + dx)}{5ad} + \frac{\int (a + a\cos(c + dx))^3 \ldots}{}$$
$$= \frac{(5B - C)(a + a\cos(c + dx))^3 \sin(c + dx)}{20d} + \frac{C(a + a\cos(c \ldots}{}$$
$$= \frac{(5B - C)(a + a\cos(c + dx))^3 \sin(c + dx)}{20d} + \frac{C(a + a\cos(c \ldots}{}$$
$$= \frac{1}{20}a^3(15B + 13C)x + \frac{(5B - C)(a + a\cos(c + dx))^3 \sin(c + \ldots}{20d}$$
$$= \frac{1}{20}a^3(15B + 13C)x + \frac{3a^3(15B + 13C)\sin(c + dx)}{20d} + \frac{3a^3(1\ldots}{}$$
$$= \frac{1}{8}a^3(15B + 13C)x + \frac{a^3(15B + 13C)\sin(c + dx)}{5d} + \frac{3a^3(15\ldots}{}$$

Mathematica [A] time = 0.431695, size = 108, normalized size = 0.7

$$\frac{a^3(60(26B + 23C)\sin(c + dx) + 480(B + C)\sin(2(c + dx)) + 120B\sin(3(c + dx)) + 15B\sin(4(c + dx)) + 900Bdx + 170\ldots}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(780*c*C + 900*B*d*x + 780*C*d*x + 60*(26*B + 23*C)*Sin[c + d*x] + 480
*(B + C)*Sin[2*(c + d*x)] + 120*B*Sin[3*(c + d*x)] + 170*C*Sin[3*(c + d*x)]
 + 15*B*Sin[4*(c + d*x)] + 45*C*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(
480*d)
```

Maple [A] time = 0.023, size = 223, normalized size = 1.5

$$\frac{1}{d}\left(a^3C\left(\frac{\cos(dx + c)\sin(dx + c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a^3B\sin(dx + c) + a^3C\left(2 + (\cos(dx + c))^2\right)\sin(dx + c) + 3\,a^3B\,(1/2\,\cos \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*sin(d*x+c)+a^3*C
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
```

```
*c)+3*a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^
3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)+a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3
/8*c))
```

Maxima [A] time = 1.20633, size = 288, normalized size = 1.87

$$-\frac{480\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^3-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Ba^3-360\left(2\,dx+2\,c+s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/480*(480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^3 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*C*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 45*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*C*a^3 - 480*B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.62282, size = 278, normalized size = 1.81

$$\frac{15\left(15\,B+13\,C\right)a^3dx+\left(24\,Ca^3\cos\left(dx+c\right)^4+30\left(B+3\,C\right)a^3\cos\left(dx+c\right)^3+8\left(15\,B+19\,C\right)a^3\cos\left(dx+c\right)^2+15\left(15\,B+\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(15*B + 13*C)*a^3*d*x + (24*C*a^3*cos(d*x + c)^4 + 30*(B + 3*C)*a
^3*cos(d*x + c)^3 + 8*(15*B + 19*C)*a^3*cos(d*x + c)^2 + 15*(15*B + 13*C)*a
^3*cos(d*x + c) + 8*(45*B + 38*C)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 3.37347, size = 532, normalized size = 3.45

$$\begin{cases} \frac{3Ba^3x\sin^4(c+dx)}{8} + \frac{3Ba^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Ba^3x\sin^2(c+dx)}{2} + \frac{3Ba^3x\cos^4(c+dx)}{8} + \frac{3Ba^3x\cos^2(c+dx)}{2} + \frac{3Ba^3\sin^3(c+dx)\cos(c+dx)}{8d} + \\ x\left(B\cos(c) + C\cos^2(c)\right)(a\cos(c) + a)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*B*a**3*x*sin(c + d*x)**4/8 + 3*B*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*B*a**3*x*sin(c + d*x)**2/2 + 3*B*a**3*x*cos(c + d*x)**4/8 +
 3*B*a**3*x*cos(c + d*x)**2/2 + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*B*a**3*sin(c + d*x)**3/d + 5*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 3*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*sin(c + d*x)*cos(c +
d*x)/(2*d) + B*a**3*sin(c + d*x)/d + 9*C*a**3*x*sin(c + d*x)**4/8 + 9*C*a**
3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + C*a**3*x*sin(c + d*x)**2/2 + 9*C*a*
*3*x*cos(c + d*x)**4/8 + C*a**3*x*cos(c + d*x)**2/2 + 8*C*a**3*sin(c + d*x)
**5/(15*d) + 4*C*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*C*a**3*sin(
c + d*x)**3*cos(c + d*x)/(8*d) + 2*C*a**3*sin(c + d*x)**3/d + C*a**3*sin(c
+ d*x)*cos(c + d*x)**4/d + 15*C*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3
*C*a**3*sin(c + d*x)*cos(c + d*x)**2/d + C*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.55717, size = 184, normalized size = 1.19

$$\frac{Ca^3\sin(5\,dx + 5\,c)}{80\,d} + \frac{1}{8}\left(15\,Ba^3 + 13\,Ca^3\right)x + \frac{\left(Ba^3 + 3\,Ca^3\right)\sin(4\,dx + 4\,c)}{32\,d} + \frac{\left(12\,Ba^3 + 17\,Ca^3\right)\sin(3\,dx + 3\,c)}{48\,d} + \frac{(Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/80*C*a^3*sin(5*d*x + 5*c)/d + 1/8*(15*B*a^3 + 13*C*a^3)*x + 1/32*(B*a^3 +
 3*C*a^3)*sin(4*d*x + 4*c)/d + 1/48*(12*B*a^3 + 17*C*a^3)*sin(3*d*x + 3*c)/
d + (B*a^3 + C*a^3)*sin(2*d*x + 2*c)/d + 1/8*(26*B*a^3 + 23*C*a^3)*sin(d*x
+ c)/d
```

3.246 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{a^3(4B+3C)\sin^3(c+dx)}{12d}+\frac{a^3(4B+3C)\sin(c+dx)}{d}+\frac{3a^3(4B+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4B+3C)+\frac{C\sin}{}$$

```
[Out] (5*a^3*(4*B + 3*C)*x)/8 + (a^3*(4*B + 3*C)*Sin[c + d*x])/d + (3*a^3*(4*B +
3*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(4*d) - (a^3*(4*B + 3*C)*Sin[c + d*x]^3)/(12*d)
```

Rubi [A] time = 0.166009, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(4B+3C)\sin^3(c+dx)}{12d}+\frac{a^3(4B+3C)\sin(c+dx)}{d}+\frac{3a^3(4B+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4B+3C)+\frac{C\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
,x]
```

```
[Out] (5*a^3*(4*B + 3*C)*x)/8 + (a^3*(4*B + 3*C)*Sin[c + d*x])/d + (3*a^3*(4*B +
3*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(4*d) - (a^3*(4*B + 3*C)*Sin[c + d*x]^3)/(12*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int (a+a\cos(c+dx))^3(B+C\cos(c+dx))\,dx$$
$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}(4B+3C)$$
$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}(4B+3C)$$
$$= \frac{1}{4}a^3(4B+3C)x + \frac{C(a+a\cos(c+dx))^3\sin(c+}{4d}$$
$$= \frac{1}{4}a^3(4B+3C)x + \frac{3a^3(4B+3C)\sin(c+dx)}{4d} +$$
$$= \frac{5}{8}a^3(4B+3C)x + \frac{a^3(4B+3C)\sin(c+dx)}{d} + \frac{3a}{}$$

Mathematica [A] time = 0.301944, size = 86, normalized size = 0.74

$$\frac{a^3(24(15B+13C)\sin(c+dx)+24(3B+4C)\sin(2(c+dx))+8B\sin(3(c+dx))+240Bdx+24C\sin(3(c+dx))+3C\sin}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (a^3*(240*B*d*x + 180*C*d*x + 24*(15*B + 13*C)*Sin[c + d*x] + 24*(3*B + 4*C
)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 24*C*Sin[3*(c + d*x)] + 3*C*Sin
[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.055, size = 176, normalized size = 1.5

$$\frac{1}{d}\left(a^3C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^3C\left(2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*
```

```
B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+3*a^3*B*sin(d*x+c)+a^3*C*sin(d*x+c)+a^3*B*(d*x+c))
```

Maxima [A] time = 1.06128, size = 225, normalized size = 1.94

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3-72(2dx+2c+\sin(2dx+2c))Ba^3-96(dx+c)Ba^3+96\left(\sin(dx+c)^3-3\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 72*(2*d*x + 2*c + sin(2
*d*x + 2*c))*B*a^3 - 96*(d*x + c)*B*a^3 + 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*C*a^3 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3
 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 - 288*B*a^3*sin(d*x + c) - 96*
C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.68229, size = 216, normalized size = 1.86

$$\frac{15(4B+3C)a^3dx+\left(6Ca^3\cos(dx+c)^3+8(B+3C)a^3\cos(dx+c)^2+9(4B+5C)a^3\cos(dx+c)+8(11B+9C)a^3\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] 1/24*(15*(4*B + 3*C)*a^3*d*x + (6*C*a^3*cos(d*x + c)^3 + 8*(B + 3*C)*a^3*co
s(d*x + c)^2 + 9*(4*B + 5*C)*a^3*cos(d*x + c) + 8*(11*B + 9*C)*a^3)*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49682, size = 238, normalized size = 2.05

$$\frac{15\left(4Ba^3+3Ca^3\right)(dx+c)+\frac{2\left(60Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+45Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+220Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+165Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+292Ba^3\tan(\frac{1}{2}d\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] 1/24*(15*(4*B*a^3 + 3*C*a^3)*(d*x + c) + 2*(60*B*a^3*tan(1/2*d*x + 1/2*c)^7
 + 45*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 165
*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 219*C*a^
3*tan(1/2*d*x + 1/2*c)^3 + 132*B*a^3*tan(1/2*d*x + 1/2*c) + 147*C*a^3*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.247 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{5a^3(B+C)\sin(c+dx)}{2d}+\frac{(3B+5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^3x(7B+5C)+\frac{a}{}$$

```
[Out] (a^3*(7*B + 5*C)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(B + C)*Si
n[c + d*x])/(2*d) + (a*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*B
 + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.381382, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^3(B+C)\sin(c+dx)}{2d}+\frac{(3B+5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^3x(7B+5C)+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^2,x]
```

```
[Out] (a^3*(7*B + 5*C)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(B + C)*Si
n[c + d*x])/(2*d) + (a*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*B
 + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (a+a\cos(c+dx))^3(B+C\cos(c+dx))\sec$$
$$= \frac{aC(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a$$
$$= \frac{aC(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3B+5}{}$$
$$= \frac{aC(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3B+5}{}$$
$$= \frac{5a^3(B+C)\sin(c+dx)}{2d}+\frac{aC(a+a\cos(c+d}{3d}$$
$$= \frac{1}{2}a^3(7B+5C)x+\frac{5a^3(B+C)\sin(c+dx)}{2d}+$$
$$= \frac{1}{2}a^3(7B+5C)x+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+$$

Mathematica [A] time = 0.253484, size = 113, normalized size = 1.02

$$\frac{a^3\left(9(4B+5C)\sin(c+dx)+3(B+3C)\sin(2(c+dx))-12B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12B\log\left(\sin\left(\frac{1}{2}(\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] (a^3*(42*B*d*x + 30*C*d*x - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(4*B + 5*C)*Sin[c + d*x]
 + 3*(B + 3*C)*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.078, size = 153, normalized size = 1.4

$$\frac{5a^3Cx}{2}+\frac{5a^3Cc}{2d}+\frac{a^3B\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{11a^3C\sin(dx+c)}{3d}+\frac{7a^3Bx}{2}+\frac{7a^3Bc}{2d}+\frac{3a^3C\cos(dx+c)\,\mathrm{s}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 5/2*a^3*C*x+5/2/d*a^3*C*c+1/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+11/3*a^3*C*si
n(d*x+c)/d+7/2*a^3*B*x+7/2/d*a^3*B*c+3/2/d*a^3*C*cos(d*x+c)*sin(d*x+c)+3*a^
3*B*sin(d*x+c)/d+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^3+1/2/d*a^3*B*cos(d*x+c)
*sin(d*x+c)
```

Maxima [A] time = 1.35925, size = 200, normalized size = 1.8

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^3+36\,(dx+c)Ba^3-4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ca^3+9\,(2\,dx+2\,c+\sin\left(2\,dx+2\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 36*(d*x + c)*B*a^3 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*C
*a^3 + 12*(d*x + c)*C*a^3 + 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 36*B*a^3*sin(d*x + c) + 36*C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.79279, size = 254, normalized size = 2.29

$$\frac{3\,(7\,B+5\,C)a^3dx+3\,Ba^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Ba^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ca^3\cos\left(dx+c\right)^2+3\,(B+3\,C)a^3\cos\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(7*B + 5*C)*a^3*d*x + 3*B*a^3*log(sin(d*x + c) + 1) - 3*B*a^3*log(-s
in(d*x + c) + 1) + (2*C*a^3*cos(d*x + c)^2 + 3*(B + 3*C)*a^3*cos(d*x + c) +
 2*(9*B + 11*C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63942, size = 243, normalized size = 2.19

$$\frac{6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(7\,Ba^3+5\,Ca^3\right)(dx+c)+\frac{2\left(15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*B*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(7*B*a^3 + 5*C*a^3)*(d*x + c) + 2*(15*B*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^3*tan(1/2*d*x + 1/
2*c)^3 + 40*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*B*a^3*tan(1/2*d*x + 1/2*c) +
33*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.248 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{a^3(3B+C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(2B-C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(6B+7C)+\frac{5a^3C\sin(c+dx)}{2d}+\frac{aB\ldots}{\ldots}$$

```
[Out] (a^3*(6*B + 7*C)*x)/2 + (a^3*(3*B + C)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*C*
Sin[c + d*x])/(2*d) - ((2*B - C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(2*
d) + (a*B*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.400305, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2975, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^3(3B+C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(2B-C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(6B+7C)+\frac{5a^3C\sin(c+dx)}{2d}+\frac{aB\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^3,x]
```


```
[Out] (a^3*(6*B + 7*C)*x)/2 + (a^3*(3*B + C)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*C*
Sin[c + d*x])/(2*d) - ((2*B - C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(2*
d) + (a*B*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int (a+a\cos(c+dx))^3(B+C\cos(c+dx))\sec^2($$

$$= \frac{aB(a+a\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+a$$

$$= -\frac{(2B-C)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} +$$

$$= -\frac{(2B-C)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} +$$

$$= \frac{5a^3C\sin(c+dx)}{2d} - \frac{(2B-C)\left(a^3+a^3\cos(c+a\right.}{2d}$$

$$= \frac{1}{2}a^3(6B+7C)x + \frac{5a^3C\sin(c+dx)}{2d} - \frac{(2B-C)}{}$$

$$= \frac{1}{2}a^3(6B+7C)x + \frac{a^3(3B+C)\tanh^{-1}(\sin(c+a}{d}$$

Mathematica [B] time = 1.71076, size = 272, normalized size = 2.47

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(B+3C)\sin(c)\cos(dx)}{d} + \frac{4(B+3C)\cos(c)\sin(dx)}{d} - \frac{4(3B+C)\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*(6*B + 7*C)*x - (4*(3*B + C
)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(3*B + C)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/d + (4*(B + 3*C)*Cos[d*x]*Sin[c])/d + (C*Cos[2*
d*x]*Sin[2*c])/d + (4*(B + 3*C)*Cos[c]*Sin[d*x])/d + (C*Cos[2*c]*Sin[2*d*x]
)/d + (4*B*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) + (4*B*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))))/32
```

Maple [A] time = 0.067, size = 145, normalized size = 1.3

$$\frac{a^3C\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3B\tan(dx+c)}{d}+\frac{7a^3Cx}{2}+\frac{7a^3Cc}{2d}+3\frac{a^3B\ln(\sec(dx+c)+\tan(dx+c))}{d}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*B*tan(d*x+c)+7/2*a^3*C*x+7/2/d*
a^3*C*c+3/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*sin(d*x+c)/d+3*a^3*B*x+
3/d*a^3*B*c+1/2/d*a^3*C*cos(d*x+c)*sin(d*x+c)+a^3*B*sin(d*x+c)/d
```

Maxima [A] time = 1.20951, size = 189, normalized size = 1.72

$$\frac{12(dx+c)Ba^3+(2dx+2c+\sin(2dx+2c))Ca^3+12(dx+c)Ca^3+6Ba^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 12*(d*x
+ c)*C*a^3 + 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*C*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c)
+ 12*C*a^3*sin(d*x + c) + 4*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.80824, size = 323, normalized size = 2.94

$$\frac{(6B+7C)a^3dx\cos(dx+c)+(3B+C)a^3\cos(dx+c)\log(\sin(dx+c)+1)-(3B+C)a^3\cos(dx+c)\log(-\sin(dx+}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] 1/2*((6*B + 7*C)*a^3*d*x*cos(d*x + c) + (3*B + C)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (3*B + C)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (C*a^3*
```

```
cos(d*x + c)^2 + 2*(B + 3*C)*a^3*cos(d*x + c) + 2*B*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41587, size = 259, normalized size = 2.35

$$-\frac{\frac{4 Ba^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-1} - \left(6 Ba^3 + 7 Ca^3\right)(dx + c) - 2\left(3 Ba^3 + Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) + 2\left(3 Ba^3 + Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] -1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*B*a^3
+ 7*C*a^3)*(d*x + c) - 2*(3*B*a^3 + C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(3*B*a^3 + C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x
 + 1/2*c) + 7*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.249 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{a^3(7B+6C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2B+C)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}-\frac{5a^3B\sin(c+dx)}{2d}+a^3x(B+3C)+\frac{aB}{}$$

```
[Out] a^3*(B + 3*C)*x + (a^3*(7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*B*
Sin[c + d*x])/(2*d) + ((2*B + C)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d +
 (a*B*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.425466, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2975, 2968, 3023, 2735, 3770}

$$\frac{a^3(7B+6C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2B+C)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}-\frac{5a^3B\sin(c+dx)}{2d}+a^3x(B+3C)+\frac{aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^4,x]
```



```
[Out] a^3*(B + 3*C)*x + (a^3*(7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*B*
Sin[c + d*x])/(2*d) + ((2*B + C)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d +
 (a*B*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \left(B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int (a + a\cos(c+dx))^3(B + C\cos(c+dx))\sec$$
$$= \frac{aB(a + a\cos(c+dx))^2\sec(c+dx)\tan(c+dx}{2d}$$
$$= \frac{(2B+C)\left(a^3 + a^3\cos(c+dx)\right)\tan(c+dx)}{d} +$$
$$= \frac{(2B+C)\left(a^3 + a^3\cos(c+dx)\right)\tan(c+dx)}{d} +$$
$$= -\frac{5a^3B\sin(c+dx)}{2d} + \frac{(2B+C)\left(a^3 + a^3\cos(c\right.}{d}$$
$$= a^3(B+3C)x - \frac{5a^3B\sin(c+dx)}{2d} + \frac{(2B+C)\left(\right.}{}$$
$$= a^3(B+3C)x + \frac{a^3(7B+6C)\tanh^{-1}(\sin(c+d}{2d}$$

Mathematica [A] time = 1.83738, size = 208, normalized size = 1.82

$$\frac{a^3\left(4(3B+C)\tan(c+dx) + \frac{B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{B}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 14B\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```

```
[Out] (a^3*(4*B*c + 12*c*C + 4*B*d*x + 12*C*d*x - 14*B*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 12*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 14*B*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*C*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + B/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - B/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 4*C*Sin[c + d*x] + 4*(3*B + C)*Tan[c + d*x]))/(4*d)
```

Maple [A] time = 0.071, size = 144, normalized size = 1.3

$$\frac{a^3C\tan(dx+c)}{d} + \frac{a^3B\sec(dx+c)\tan(dx+c)}{2d} + \frac{7a^3B\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 3\frac{a^3C\ln(\sec(dx+c)+\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*a^3*C*tan(d*x+c)+1/2/d*a^3*B*sec(d*x+c)*tan(d*x+c)+7/2/d*a^3*B*ln(sec(d
*x+c)+tan(d*x+c))+3/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^3*B*tan(d*x+c)+
3*a^3*C*x+3/d*a^3*C*c+a^3*C*sin(d*x+c)/d+a^3*B*x+1/d*a^3*B*c
```

Maxima [A] time = 1.07551, size = 223, normalized size = 1.96

$$\frac{4(dx+c)Ba^3+12(dx+c)Ca^3-Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6Ba^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+6Ca^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4Ca^3\sin(dx+c)+12Ba^3\tan(dx+c)+4Ca^3\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^3 + 12*(d*x + c)*C*a^3 - B*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*C*a^3*sin(d*x + c) + 12*B*a^3*tan(d*x + c)
 + 4*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.99004, size = 342, normalized size = 3.

$$\frac{4(B+3C)a^3dx\cos(dx+c)^2+(7B+6C)a^3\cos(dx+c)^2\log(\sin(dx+c)+1)-(7B+6C)a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(2Ca^3\cos(dx+c)^2+2(3B+C)a^3\cos(dx+c)+Ba^3\right)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] 1/4*(4*(B + 3*C)*a^3*d*x*cos(d*x + c)^2 + (7*B + 6*C)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (7*B + 6*C)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*C*a^3*cos(d*x + c)^2 + 2*(3*B + C)*a^3*cos(d*x + c) + B*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36995, size = 259, normalized size = 2.27

$$\frac{\frac{4\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 2\left(Ba^3 + 3\,Ca^3\right)(dx+c) + \left(7\,Ba^3 + 6\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(7\,Ba^3 + 6\,Ca^3\right)\log\left(\left|\tan\left(\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] 1/2*(4*C*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(B*a^3 +
 3*C*a^3)*(d*x + c) + (7*B*a^3 + 6*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - (7*B*a^3 + 6*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*B*a^3*tan
(1/2*d*x + 1/2*c)^3 + 2*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*B*a^3*tan(1/2*d*x
+ 1/2*c) - 2*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.250 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=125

$$\frac{5a^3(B+C)\tan(c+dx)}{2d}+\frac{a^3(5B+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5B+3C)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+$$

```
[Out] a^3*C*x + (a^3*(5*B + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(B + C)*Ta
n[c + d*x])/(2*d) + ((5*B + 3*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[
c + d*x])/(6*d) + (a*B*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d)
```

Rubi [A] time = 0.423576, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2975, 2968, 3021, 2735, 3770}

$$\frac{5a^3(B+C)\tan(c+dx)}{2d}+\frac{a^3(5B+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5B+3C)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^5,x]
```


```
[Out] a^3*C*x + (a^3*(5*B + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(B + C)*Ta
n[c + d*x])/(2*d) + ((5*B + 3*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[
c + d*x])/(6*d) + (a*B*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \left(B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int (a + a\cos(c+dx))^3(B + C\cos(c+dx))\sec^4($$

$$= \frac{aB(a + a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx}{3d}$$

$$= \frac{(5B+3C)\left(a^3 + a^3\cos(c+dx)\right)\sec(c+dx)\tan}{6d}$$

$$= \frac{(5B+3C)\left(a^3 + a^3\cos(c+dx)\right)\sec(c+dx)\tan}{6d}$$

$$= \frac{5a^3(B+C)\tan(c+dx)}{2d} + \frac{(5B+3C)\left(a^3 + a^3 c\right.}{}$$

$$= a^3Cx + \frac{5a^3(B+C)\tan(c+dx)}{2d} + \frac{(5B+3C)\left(\right.}{}$$

$$= a^3Cx + \frac{a^3(5B+7C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a}{}$$

<u>Mathematica</u> [B] time = 6.35884, size = 786, normalized size = 6.29

$$\frac{\sec^6\left(\frac{c}{2} + \frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(11B\sin\left(\frac{dx}{2}\right) + 9C\sin\left(\frac{dx}{2}\right)\right)}{24d\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)} + \frac{\sec^6\left(\frac{c}{2} + \frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(11B\sin\left(\frac{dx}{2}\right) + 9C\,\mathrm{s}\right.}{24d\left(\sin\left(\frac{c}{2}\right) + \cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2} + \frac{dx}{2}\right) + \cos\left(\frac{c}{2} + \right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^5,x]
```

```
[Out] (C*x*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/8 + ((-5*B - 7*C)*(a + a*
Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x
)/2]^6)/(16*d) + ((5*B + 7*C)*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(16*d) + (B*(a + a*Cos[c + d*x
])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (
d*x)/2]^6*(10*B*Cos[c/2] + 3*C*Cos[c/2] - 8*B*Sin[c/2] - 3*C*Sin[c/2]))/(96
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a
 + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*B*Sin[(d*x)/2] + 9*C*Sin[(d*x
)/2]))/(24*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)) + (B*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*C
os[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-10*B*Cos[c/2] - 3*C*Cos[c/2] - 8*B*Si
n[c/2] - 3*C*Sin[c/2]))/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
```

```
in[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*B*
Sin[(d*x)/2] + 9*C*Sin[(d*x)/2]))/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.074, size = 158, normalized size = 1.3

$$\frac{a^3C\sec(dx+c)\tan(dx+c)}{2d}+\frac{7a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{11a^3B\tan(dx+c)}{3d}+\frac{a^3B\tan(dx+c)(\sec(dx}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+7/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+11/
3/d*a^3*B*tan(d*x+c)+1/3/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2+3/d*a^3*C*tan(d*x+
c)+3/2/d*a^3*B*sec(d*x+c)*tan(d*x+c)+5/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+
a^3*C*x+1/d*a^3*C*c
```

Maxima [A] time = 1.36913, size = 286, normalized size = 2.29

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3+12(dx+c)Ca^3-9Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 12*(d*x + c)*C*a^3 - 9*B*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 3*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 18*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 36*B*a^3*tan(d*x + c) + 36*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.09021, size = 356, normalized size = 2.85

$$\frac{12\,Ca^3dx\cos(dx+c)^3+3(5B+7C)a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-3(5B+7C)a^3\cos(dx+c)^3\log(-\sin(dx}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="fricas")
```

```
[Out] 1/12*(12*C*a^3*d*x*cos(d*x + c)^3 + 3*(5*B + 7*C)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(5*B + 7*C)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(11*B + 9*C)*a^3*cos(d*x + c)^2 + 3*(3*B + C)*a^3*cos(d*x + c) + 2*B*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.54588, size = 255, normalized size = 2.04

$$\frac{6\,(dx+c)Ca^3 + 3\left(5\,Ba^3 + 7\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(5\,Ba^3 + 7\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(15\,Ba^3\,\mathrm{ta}\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*C*a^3 + 3*(5*B*a^3 + 7*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(5*B*a^3 + 7*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*B
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 36*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*B*a^3*tan(1/2*d
*x + 1/2*c) + 21*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

3.251 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=154

$$\frac{a^3(9B+11C)\tan(c+dx)}{3d}+\frac{5a^3(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27B+28C)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3B+2C)}{}$$

```
[Out] (5*a^3*(3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(9*B + 11*C)*Tan[c +
 d*x])/(3*d) + (a^3*(27*B + 28*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*B
 + 2*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*B*
(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.505596, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^3(9B+11C)\tan(c+dx)}{3d}+\frac{5a^3(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27B+28C)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3B+2C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^6,x]
```


```
[Out] (5*a^3*(3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(9*B + 11*C)*Tan[c +
 d*x])/(3*d) + (a^3*(27*B + 28*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*B
 + 2*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*B*
(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \left(B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a + a\cos(c+dx))^3 (B + C\cos(c+dx))\sec$$

$$= \frac{aB(a + a\cos(c+dx))^2\sec^3(c+dx)\tan(c+d}{4d}$$

$$= \frac{(3B+2C)\left(a^3 + a^3\cos(c+dx)\right)\sec^2(c+dx)}{6d}$$

$$= \frac{(3B+2C)\left(a^3 + a^3\cos(c+dx)\right)\sec^2(c+dx)}{6d}$$

$$= \frac{a^3(27B+28C)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(3B}{}$$

$$= \frac{a^3(27B+28C)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(3B}{}$$

$$= \frac{5a^3(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(27B}{}$$

$$= \frac{5a^3(3B+4C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(9B+}{}$$

Mathematica [A] time = 1.21608, size = 273, normalized size = 1.77

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(120(3B+4C)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^6,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(120*(3*B + 4*
C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(9*B + 11*C)*Sin[c] + (69*B + 36
*C)*Sin[d*x] + 69*B*Sin[2*c + d*x] + 36*C*Sin[2*c + d*x] + 264*B*Sin[c + 2*
d*x] + 280*C*Sin[c + 2*d*x] - 24*B*Sin[3*c + 2*d*x] - 72*C*Sin[3*c + 2*d*x]
 + 45*B*Sin[2*c + 3*d*x] + 36*C*Sin[2*c + 3*d*x] + 45*B*Sin[4*c + 3*d*x] +
36*C*Sin[4*c + 3*d*x] + 72*B*Sin[3*c + 4*d*x] + 88*C*Sin[3*c + 4*d*x])))/(1
```

536*d)

Maple [A] time = 0.076, size = 188, normalized size = 1.2

$$\frac{11\,a^{3}C\tan\left(dx+c\right)}{3\,d}+\frac{a^{3}C\tan\left(dx+c\right)\left(\sec\left(dx+c\right)\right)^{2}}{3\,d}+\frac{a^{3}B\tan\left(dx+c\right)\left(\sec\left(dx+c\right)\right)^{3}}{4\,d}+\frac{15\,a^{3}B\sec\left(dx+c\right)\tan\left(dx+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 11/3/d*a^3*C*tan(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan
(d*x+c)*sec(d*x+c)^3+15/8/d*a^3*B*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*B*ln(sec
(d*x+c)+tan(d*x+c))+3/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+5/2/d*a^3*C*ln(sec(d*
x+c)+tan(d*x+c))+3/d*a^3*B*tan(d*x+c)+1/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.27615, size = 363, normalized size = 2.36

$$\frac{48\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{3}+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca^{3}-3\,Ba^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="maxima")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a^3 - 3*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*a^3*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*B*a^3*tan(d*x + c) + 144*C*a^3*
tan(d*x + c))/d
```

Fricas [A] time = 1.75388, size = 366, normalized size = 2.38

$$\frac{15\,(3\,B+4\,C)a^3\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)}-15\,(3\,B+4\,C)a^3\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+1)}+2\left(8\,(9\,B+\right.}{48\,d\cos{(dx+c)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="fricas")
```

```
[Out] 1/48*(15*(3*B + 4*C)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(3*B + 4
*C)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(9*B + 11*C)*a^3*cos(d
*x + c)^3 + 9*(5*B + 4*C)*a^3*cos(d*x + c)^2 + 8*(3*B + C)*a^3*cos(d*x + c)
 + 6*B*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64964, size = 286, normalized size = 1.86

$$15\left(3\,Ba^3+4\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(3\,Ba^3+4\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="giac")
```

```
[Out] 1/24*(15*(3*B*a^3 + 4*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*B*a
^3 + 4*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*B*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 165*B*a^3*tan(1/2*d*x + 1/2*
c)^5 - 220*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*B*a^3*tan(1/2*d*x + 1/2*c)^3
+ 292*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 147*B*a^3*tan(1/2*d*x + 1/2*c) - 132*C
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.252 $\int (a+a\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=185

$$\frac{a^3(38B+45C)\tan(c+dx)}{15d}+\frac{a^3(13B+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43B+45C)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(13B$$

```
[Out] (a^3*(13*B + 15*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*B + 45*C)*Tan[c
+ d*x])/(15*d) + (a^3*(13*B + 15*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3
*(43*B + 45*C)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((7*B + 5*C)*(a^3 + a^
3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*B*(a + a*Cos[c + d
*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.533425, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3029, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^3(38B+45C)\tan(c+dx)}{15d}+\frac{a^3(13B+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43B+45C)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(13B$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^7,x]
```


```
[Out] (a^3*(13*B + 15*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*B + 45*C)*Tan[c
+ d*x])/(15*d) + (a^3*(13*B + 15*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3
*(43*B + 45*C)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((7*B + 5*C)*(a^3 + a^
3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*B*(a + a*Cos[c + d
*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int (a+a\cos(c+dx))^3(B+C\cos(c+dx))\sec$$

$$=\frac{aB(a+a\cos(c+dx))^2\sec^4(c+dx)\tan(c+d}{5d}$$

$$=\frac{(7B+5C)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)}{20d}$$

$$=\frac{(7B+5C)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)}{20d}$$

$$=\frac{a^3(43B+45C)\sec^2(c+dx)\tan(c+dx)}{60d}+\frac{(7}{}$$

$$=\frac{a^3(43B+45C)\sec^2(c+dx)\tan(c+dx)}{60d}+\frac{(7}{}$$

$$=\frac{a^3(13B+15C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3(}{}$$

$$=\frac{a^3(13B+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(38B}{}$$

Mathematica [A] time = 1.42365, size = 294, normalized size = 1.59

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(13B+15C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^7,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(13*B + 1
5*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(29*B + 30*C)*Sin[d*x] - 240*(3
*B + 5*C)*Sin[2*c + d*x] + 750*B*Sin[c + 2*d*x] + 570*C*Sin[c + 2*d*x] + 75
0*B*Sin[3*c + 2*d*x] + 570*C*Sin[3*c + 2*d*x] + 1520*B*Sin[2*c + 3*d*x] + 1
680*C*Sin[2*c + 3*d*x] - 120*C*Sin[4*c + 3*d*x] + 195*B*Sin[3*c + 4*d*x] +
225*C*Sin[3*c + 4*d*x] + 195*B*Sin[5*c + 4*d*x] + 225*C*Sin[5*c + 4*d*x] +
304*B*Sin[4*c + 5*d*x] + 360*C*Sin[4*c + 5*d*x])))/(15360*d)
```

Maple [A] time = 0.08, size = 234, normalized size = 1.3

$$\frac{a^3C\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{15\,a^3C\sec(dx+c)\tan(dx+c)}{8d}+\frac{15\,a^3C\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{38\,a^3B\tan}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+15/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+15/8
/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+38/15/d*a^3*B*tan(d*x+c)+1/5/d*a^3*B*tan
(d*x+c)*sec(d*x+c)^4+19/15/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2+3/d*a^3*C*tan(d*
x+c)+1/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3+
13/8/d*a^3*B*sec(d*x+c)*tan(d*x+c)+13/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.33351, size = 455, normalized size = 2.46

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3+240\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*C*a^3 - 45*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
```

```
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 180*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.72745, size = 431, normalized size = 2.33

$$\frac{15\,(13\,B+15\,C)a^3\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(13\,B+15\,C)a^3\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(13*B + 15*C)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(13*B
 + 15*C)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38*B + 45*C)*a^3
*cos(d*x + c)^4 + 15*(13*B + 15*C)*a^3*cos(d*x + c)^3 + 8*(19*B + 15*C)*a^3
*cos(d*x + c)^2 + 30*(3*B + C)*a^3*cos(d*x + c) + 24*B*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**7,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.60367, size = 332, normalized size = 1.79

$$\frac{15\left(13\,Ba^3+15\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(13\,Ba^3+15\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,Ba^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="giac")
```

```
[Out] 1/120*(15*(13*B*a^3 + 15*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(13
*B*a^3 + 15*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*B*a^3*tan(1/
2*d*x + 1/2*c)^9 + 225*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*B*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1050*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*B*a^3*tan(1/2*d*x +
1/2*c)^5 + 1920*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1830*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*B*a^3*tan(1/2*d*x + 1/2*c)
+ 735*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.253 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{(3B-4C)\sin^3(c+dx)}{3ad}-\frac{(3B-4C)\sin(c+dx)}{ad}+\frac{(B-C)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(B-C)\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] (3*(B - C)*x)/(2*a) - ((3*B - 4*C)*Sin[c + d*x])/(a*d) + (3*(B - C)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a*d) + ((B - C)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a +
 a*Cos[c + d*x])) + ((3*B - 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.26259, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2977, 2748, 2635, 8, 2633}

$$\frac{(3B-4C)\sin^3(c+dx)}{3ad}-\frac{(3B-4C)\sin(c+dx)}{ad}+\frac{(B-C)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(B-C)\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x
]),x]
```

```
[Out] (3*(B - C)*x)/(2*a) - ((3*B - 4*C)*Sin[c + d*x])/(a*d) + (3*(B - C)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a*d) + ((B - C)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a +
 a*Cos[c + d*x])) + ((3*B - 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$

$$= \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^2(c+dx)(3a(B-C)-a}{a^2}$$

$$= \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3B-4C)\int \cos^3(c+dx)\,dx}{a}$$

$$= \frac{3(B-C)\cos(c+dx)\sin(c+dx)}{2ad} + \frac{(B-C)\cos^3(c+dx)\sin(c+}{d(a+a\cos(c+dx))}$$

$$= \frac{3(B-C)x}{2a} - \frac{(3B-4C)\sin(c+dx)}{ad} + \frac{3(B-C)\cos(c+dx)\sin(c}{2ad}$$

Mathematica [B] time = 0.543616, size = 249, normalized size = 2.04

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(B-C)\cos\left(c+\frac{dx}{2}\right)-12B\sin\left(c+\frac{dx}{2}\right)-9B\sin\left(c+\frac{3dx}{2}\right)-9B\sin\left(2c+\frac{3dx}{2}\right)+3B\sin\left(2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c
 + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(B - C)*d*x*Cos[(d*x)/2] + 36*(B - C)*d*x*Co
s[c + (d*x)/2] - 60*B*Sin[(d*x)/2] + 69*C*Sin[(d*x)/2] - 12*B*Sin[c + (d*x)
/2] + 21*C*Sin[c + (d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 18*C*Sin[c + (3*d*x)
/2] - 9*B*Sin[2*c + (3*d*x)/2] + 18*C*Sin[2*c + (3*d*x)/2] + 3*B*Sin[2*c +
(5*d*x)/2] - 2*C*Sin[2*c + (5*d*x)/2] + 3*B*Sin[3*c + (5*d*x)/2] - 2*C*Sin[
3*c + (5*d*x)/2] + C*Sin[3*c + (7*d*x)/2] + C*Sin[4*c + (7*d*x)/2]))/(24*a*
d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.033, size = 281, normalized size = 2.3

$$-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{(\tan(1/2\,dx+c/2))^5 B}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^3}+5\,\frac{C(\tan(1/2\,dx+c/2))^5}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^3}-4\,\frac{(t}{ad\left((t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(tan(1/2*d*x+1
/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*B+5/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/
2*d*x+1/2*c)^5*C-4/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*B+16
/3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*C-1/a/d/(tan(1/2*d*x
+1/2*c)^2+1)^3*B*tan(1/2*d*x+1/2*c)+3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(
1/2*d*x+1/2*c)+3/a/d*arctan(tan(1/2*d*x+1/2*c))*B-3/a/d*arctan(tan(1/2*d*x+
1/2*c))*C
```

Maxima [B] time = 1.60894, size = 419, normalized size = 3.43

$$\frac{C\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\,\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3\,B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/3*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*B*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.65006, size = 242, normalized size = 1.98

$$\frac{9\,(B-C)dx\cos(dx+c)+9\,(B-C)dx+\left(2\,C\cos(dx+c)^3+(3\,B-C)\cos(dx+c)^2-(3\,B-7\,C)\cos(dx+c)-12\,B+1\right.}{6\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/6*(9*(B - C)*d*x*cos(d*x + c) + 9*(B - C)*d*x + (2*C*cos(d*x + c)^3 + (3*
B - C)*cos(d*x + c)^2 - (3*B - 7*C)*cos(d*x + c) - 12*B + 16*C)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 10.5512, size = 1166, normalized size = 9.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*B*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*
tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*B*d*x*tan(c/
2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*
a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*B*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 9*B*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*B*tan(c/2 + d*x/2)**7/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) - 36*B*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c
/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 42*B*tan(c/2 + d*x/2
)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c
/2 + d*x/2)**2 + 6*a*d) - 12*B*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*C*d*
x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)*
*4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*C*d*x*tan(c/2 + d*x/2)**4/(6*
a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x
/2)**2 + 6*a*d) - 27*C*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*C*d*x
/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 +
 d*x/2)**2 + 6*a*d) + 6*C*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 +
18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*C*tan
(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*C*tan(c/2 + d*x/2)**3/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) + 24*C*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
+ d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*(B*cos(c)
+ C*cos(c)**2)*cos(c)**2/(a*cos(c) + a), True))
```

<u>Giac</u> [A] time = 1.57862, size = 204, normalized size = 1.67

$$\frac{\frac{9\,(dx+c)(B-C)}{a}-\frac{6\left(B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-16\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,B\,\mathrm{ta}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/6*(9*(d*x + c)*(B - C)/a - 6*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/
2*c))/a - 2*(9*B*tan(1/2*d*x + 1/2*c)^5 - 15*C*tan(1/2*d*x + 1/2*c)^5 + 12*
B*tan(1/2*d*x + 1/2*c)^3 - 16*C*tan(1/2*d*x + 1/2*c)^3 + 3*B*tan(1/2*d*x +
1/2*c) - 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.254 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{2(B-C)\sin(c+dx)}{ad}+\frac{(B-C)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2B-3C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x(2B-3C)}{2a}$$

```
[Out] -((2*B - 3*C)*x)/(2*a) + (2*(B - C)*Sin[c + d*x])/(a*d) - ((2*B - 3*C)*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d) + ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.170956, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3029, 2977, 2734}

$$\frac{2(B-C)\sin(c+dx)}{ad}+\frac{(B-C)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2B-3C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x(2B-3C)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])
,x]
```

```
[Out] -((2*B - 3*C)*x)/(2*a) + (2*(B - C)*Sin[c + d*x])/(a*d) - ((2*B - 3*C)*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d) + ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos(c+dx)(2a(B-C)-a(2B\ldots}{a^2}$$
$$= -\frac{(2B-3C)x}{2a} + \frac{2(B-C)\sin(c+dx)}{ad} - \frac{(2B-3C)\cos(c+dx)\sin(c\ldots}{2ad}$$

Mathematica [A] time = 0.439461, size = 197, normalized size = 1.99

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-4dx(2B-3C)\cos\left(c+\frac{dx}{2}\right)+4B\sin\left(c+\frac{dx}{2}\right)+4B\sin\left(c+\frac{3dx}{2}\right)+4B\sin\left(2c+\frac{3dx}{2}\right)-4dx(2B-\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-4*(2*B - 3*C)*d*x*Cos[(d*x)/2] - 4*(2*B - 3*C)
*d*x*Cos[c + (d*x)/2] + 20*B*Sin[(d*x)/2] - 20*C*Sin[(d*x)/2] + 4*B*Sin[c +
 (d*x)/2] - 4*C*Sin[c + (d*x)/2] + 4*B*Sin[c + (3*d*x)/2] - 3*C*Sin[c + (3*
d*x)/2] + 4*B*Sin[2*c + (3*d*x)/2] - 3*C*Sin[2*c + (3*d*x)/2] + C*Sin[2*c +
 (5*d*x)/2] + C*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.03, size = 211, normalized size = 2.1

$$\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{C\,(\tan(1/2\,dx+c/2))^3}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^2}+2\,\frac{(\tan(1/2\,dx+c/2))^3\,B}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^2}-\frac{C}{ad}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(tan(1/2*d*x+1/
2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3+2/a/d/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2
*d*x+1/2*c)^3*B-1/a/d/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)+2/a/d
/(tan(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c)-2/a/d*arctan(tan(1/2*d*x+1
/2*c))*B+3/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.90977, size = 304, normalized size = 3.07

$$-\frac{C\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] -(C*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.59427, size = 204, normalized size = 2.06

$$-\frac{(2B-3C)dx\cos(dx+c)+(2B-3C)dx-\left(C\cos(dx+c)^2+(2B-C)\cos(dx+c)+4B-4C\right)\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] -1/2*((2*B - 3*C)*d*x*cos(d*x + c) + (2*B - 3*C)*d*x - (C*cos(d*x + c)^2 +
(2*B - C)*cos(d*x + c) + 4*B - 4*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 6.48015, size = 668, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-2*B*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*
tan(c/2 + d*x/2)**2 + 2*a*d) - 4*B*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 +
 d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*B*d*x/(2*a*d*tan(c/2 +
d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*B*tan(c/2 + d*x/2)**5/(2
*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 8*B*tan(c/2
 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d
) + 6*B*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2
)**2 + 2*a*d) + 3*C*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*
a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*C*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c
/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*C*d*x/(2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*C*tan(c/2 + d*x/2)**
5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*C*ta
n(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d) - 4*C*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 +
d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c)/(a*cos(c)
 + a), True))
```

Giac [A] time = 1.61571, size = 167, normalized size = 1.69

$$-\frac{\frac{(dx+c)(2B-3C)}{a}-\frac{2\left(B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algo
rithm="giac")
```

```
[Out] -1/2*((d*x + c)*(2*B - 3*C)/a - 2*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x +
 1/2*c))/a - 2*(2*B*tan(1/2*d*x + 1/2*c)^3 - 3*C*tan(1/2*d*x + 1/2*c)^3 + 2
*B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^2*a))/d
```

3.255 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=54

$$-\frac{(B-C)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(B-C)}{a}+\frac{C\sin(c+dx)}{ad}$$

```
[Out] ((B - C)*x)/a + (C*Sin[c + d*x])/(a*d) - ((B - C)*Sin[c + d*x])/(a*d*(1 + C
os[c + d*x]))
```

Rubi [A] time = 0.088818, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3023, 12, 2735, 2648}

$$-\frac{(B-C)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(B-C)}{a}+\frac{C\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((B - C)*x)/a + (C*Sin[c + d*x])/(a*d) - ((B - C)*Sin[c + d*x])/(a*d*(1 + C
os[c + d*x]))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{C\sin(c+dx)}{ad}+\frac{\int \frac{a(B-C)\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\ &= \frac{C\sin(c+dx)}{ad}+(B-C)\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx\\ &= \frac{(B-C)x}{a}+\frac{C\sin(c+dx)}{ad}+(-B+C)\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{(B-C)x}{a}+\frac{C\sin(c+dx)}{ad}-\frac{(B-C)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.23507, size = 126, normalized size = 2.33

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2dx(B-C)\cos\left(c+\frac{dx}{2}\right)+2dx(B-C)\cos\left(\frac{dx}{2}\right)-4B\sin\left(\frac{dx}{2}\right)+C\sin\left(c+\frac{dx}{2}\right)+C\sin\left(c+\frac{3dx}{2}\right.\right.}{2ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(2*(B - C)*d*x*Cos[(d*x)/2] + 2*(B - C)*d*x*Cos[
c + (d*x)/2] - 4*B*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2] + C*Sin[c + (d*x)/2] + C
*Sin[c + (3*d*x)/2] + C*Sin[2*c + (3*d*x)/2]))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.027, size = 108, normalized size = 2.

$$-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{C\tan(1/2\,dx+c/2)}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,B}{ad}-2\,\frac{\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d*C*tan(1/2*d*x+
1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/a/d*arctan(tan(1/2*d*x+1/2*c))*B-2/a/d*ar
ctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.87203, size = 193, normalized size = 3.57

$$-\frac{C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.58746, size = 147, normalized size = 2.72

$$\frac{(B-C)dx\cos(dx+c)+(B-C)dx+(C\cos(dx+c)-B+2C)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] ((B - C)*d*x*cos(d*x + c) + (B - C)*d*x + (C*cos(d*x + c) - B + 2*C)*sin(d*
x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 3.59394, size = 265, normalized size = 4.91

$$\begin{cases} \frac{Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{Bdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{Cdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{Cdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{C\tan^3}{ad\tan^2(} \\ \frac{x\left(B\cos(c)+C\cos^2(c)\right)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + B*d*
x/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*
x/2)**2 + a*d) - B*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) - C*d*x
*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - C*d*x/(a*d*tan(c/2 +
 d*x/2)**2 + a*d) + C*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) +
 3*C*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*(B*cos
(c) + C*cos(c)**2)/(a*cos(c) + a), True))
```

Giac [A] time = 1.6102, size = 105, normalized size = 1.94

$$\frac{\frac{(dx+c)(B-C)}{a} - \frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((d*x + c)*(B - C)/a - (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a
+ 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.256 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{(B-C)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Cx}{a}$$

```
[Out] (C*x)/a + ((B - C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.121369, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3029, 2735, 2648}

$$\frac{(B-C)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])
,x]
```

```
[Out] (C*x)/a + ((B - C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{B+C\cos(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{Cx}{a}-(-B+C)\int \frac{1}{a+a\cos(c+dx)}\,dx$$
$$= \frac{Cx}{a}+\frac{(B-C)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [B] time = 0.117477, size = 72, normalized size = 2.12

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2(B-C)\sin\left(\frac{dx}{2}\right)+Cdx\cos\left(c+\frac{dx}{2}\right)+Cdx\cos\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c +
 d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(C*d*x*Cos[(d*x)/2] + C*d*x*Cos[c + (d*x)/2] + 2
*(B - C)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.04, size = 56, normalized size = 1.7

$$\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*
d*x+1/2*c))*C
```

Maxima [B] time = 1.8655, size = 99, normalized size = 2.91

$$\frac{C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) + \frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] (C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + B*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.60494, size = 105, normalized size = 3.09

$$\frac{Cdx\cos(dx+c) + Cdx + (B-C)\sin(dx+c)}{ad\cos(dx+c) + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] (C*d*x*cos(d*x + c) + C*d*x + (B - C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\cos(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{C\cos^2(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(C*c
os(c + d*x)**2*sec(c + d*x)/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.82223, size = 58, normalized size = 1.71

$$\frac{\frac{(dx+c)C}{a}+\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, algo
rithm="giac")
```

```
[Out] ((d*x + c)*C/a + (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a)/d
```

3.257 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a*d) - ((B - C)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rubi [A] time = 0.165926, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3029, 2978, 12, 3770}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x
]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a*d) - ((B - C)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx &= \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx\\ &= -\frac{(B-C)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int aB\sec(c+dx)\,dx}{a^2}\\ &= -\frac{(B-C)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{B\int \sec(c+dx)\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.247445, size = 109, normalized size = 2.48

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((C-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+B\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c
 + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(B*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-B + C)*Sec[c/2]*Sin
[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.053, size = 78, normalized size = 1.8

$$-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*B*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*B*ln(tan
(1/2*d*x+1/2*c)+1)+1/a/d*C*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 1.21704, size = 134, normalized size = 3.05

$$\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{C\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] (B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + C*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.67426, size = 197, normalized size = 4.48

$$\frac{(B\cos(dx+c)+B)\log(\sin(dx+c)+1)-(B\cos(dx+c)+B)\log(-\sin(dx+c)+1)-2(B-C)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*((B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - (B*cos(d*x + c) + B)*log(
-sin(d*x + c) + 1) - 2*(B - C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx+\int \frac{C\cos^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(
C*cos(c + d*x)**2*sec(c + d*x)**2/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.57954, size = 96, normalized size = 2.18

$$\frac{\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - B*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a - (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a)/d
```

3.258 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{(2B-C)\tan(c+dx)}{ad}-\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] -(((B - C)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*B - C)*Tan[c + d*x])/(a*d) -
 ((B - C)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.239387, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2978, 2748, 3767, 8, 3770}

$$\frac{(2B-C)\tan(c+dx)}{ad}-\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x
]),x]
```

```
[Out] -(((B - C)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*B - C)*Tan[c + d*x])/(a*d) -
 ((B - C)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= -\frac{(B-C)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(2B-C)-a(B-C)\cos(c+dx))\,\mathrm{sec}}{a^2}$$
$$= -\frac{(B-C)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{(B-C)\int \sec(c+dx)\,dx}{a} + \frac{(2B-C)\int \mathrm{s}}{}$$
$$= -\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(B-C)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{(2B-C)}{}$$
$$= -\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad} + \frac{(2B-C)\tan(c+dx)}{ad} - \frac{(B-C)}{d(a+a}$$

Mathematica [B] time = 1.07494, size = 201, normalized size = 2.91

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((B-C)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\right.\right.\right.\right.}{ad(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c
 + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((B - C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((B -
 C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + (B*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
))))/(a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.054, size = 163, normalized size = 2.4

$$\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{B}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{C}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-1/a/d*B/(tan(1/2*d*x+
1/2*c)-1)+1/a/d*B*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C
-1/a/d*B/(tan(1/2*d*x+1/2*c)+1)-1/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)+1/a/d*ln(t
an(1/2*d*x+1/2*c)+1)*C
```

Maxima [B] time = 1.23708, size = 265, normalized size = 3.84

$$-\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right.}{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -(B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - C*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.68108, size = 320, normalized size = 4.64

$$-\frac{\left((B-C)\cos\left(dx+c\right)^{2}+(B-C)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((B-C)\cos\left(dx+c\right)^{2}+(B-C)\cos\left(dx+c\right)\right)\log}{2\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/2*(((B - C)*cos(d*x + c)^2 + (B - C)*cos(d*x + c))*log(sin(d*x + c) + 1)
 - ((B - C)*cos(d*x + c)^2 + (B - C)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
 2*((2*B - C)*cos(d*x + c) + B)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42072, size = 149, normalized size = 2.16

$$-\frac{\frac{(B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{(B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -((B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (B - C)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a - (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a +
2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.259 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{2(B-C)\tan(c+dx)}{ad}+\frac{(3B-2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3B-2C)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(B-C)\tan(c+dx)\text{se}}{d(a\cos(c+dx)+}$$

```
[Out] ((3*B - 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*(B - C)*Tan[c + d*x])/(a*d
) + ((3*B - 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((B - C)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.248659, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(B-C)\tan(c+dx)}{ad}+\frac{(3B-2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3B-2C)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(B-C)\tan(c+dx)\text{se}}{d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x
]),x]
```



```
[Out] ((3*B - 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*(B - C)*Tan[c + d*x])/(a*d
) + ((3*B - 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((B - C)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{(B+C\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(B-C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(3B-2C)-2a(B-C)\,\mathrm{c}}{a^2}$$

$$= -\frac{(B-C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(3B-2C)\int \sec^3(c+dx)\,dx}{a}$$

$$= \frac{(3B-2C)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(B-C)\sec(c+dx)\tan(c+}{d(a+a\cos(c+dx))}$$

$$= \frac{(3B-2C)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{2(B-C)\tan(c+dx)}{ad} + \frac{(3B-}{}$$

Mathematica [B] time = 3.11175, size = 289, normalized size = 2.7

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(4(C-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4(B-C)\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c
 + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(4*(-B + C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-6
*B + 4*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*B*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] - 4*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + B/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - B/(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 - (4*(B - C)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)))))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.064, size = 252, normalized size = 2.4

$$-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3\,B}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{C}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)+1/2/a/d*B/(tan(1/2*d
*x+1/2*c)-1)^2+3/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)-1/a/d/(tan(1/2*d*x+1/2*c)-1
)*C-3/2/a/d*B*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-1/2
/a/d*B/(tan(1/2*d*x+1/2*c)+1)^2+3/2/a/d*B/(tan(1/2*d*x+1/2*c)+1)-1/a/d/(tan
(1/2*d*x+1/2*c)+1)*C+3/2/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(tan(1/2*d*
x+1/2*c)+1)*C
```

Maxima [B] time = 1.31134, size = 381, normalized size = 3.56

$$-\frac{B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -1/2*(B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.6403, size = 385, normalized size = 3.6

$$\frac{\left((3B-2C)\cos(dx+c)^3+(3B-2C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((3B-2C)\cos(dx+c)^3+(3B-2C)\cos(\right.}{4\left(ad\cos(dx+c)^3+ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/4*(((3*B - 2*C)*cos(d*x + c)^3 + (3*B - 2*C)*cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - ((3*B - 2*C)*cos(d*x + c)^3 + (3*B - 2*C)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(4*(B - C)*cos(d*x + c)^2 + (B - 2*C)*cos(d*x + c) -
B)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51214, size = 212, normalized size = 1.98

$$\frac{\frac{(3B-2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(3B-2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2C\tan\left(\frac{1}{2}dx\right.\right.}{\left(\tan\left(\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((3*B - 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*B - 2*C)*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x +
 1/2*c))/a + 2*(3*B*tan(1/2*d*x + 1/2*c)^3 - 2*C*tan(1/2*d*x + 1/2*c)^3 - B
*tan(1/2*d*x + 1/2*c) + 2*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
- 1)^2*a))/d
```

3.260 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{(4B-3C)\tan^3(c+dx)}{3ad}+\frac{(4B-3C)\tan(c+dx)}{ad}-\frac{3(B-C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(B-C)\tan(c+dx)\sec(c+dx)}{2ad}-($$

```
[Out] (-3*(B - C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*B - 3*C)*Tan[c + d*x])/(a*
d) - (3*(B - C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((B - C)*Sec[c + d*x]^
2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*B - 3*C)*Tan[c + d*x]^3)/(3*
a*d)
```

Rubi [A] time = 0.264745, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2978, 2748, 3767, 3768, 3770}

$$\frac{(4B-3C)\tan^3(c+dx)}{3ad}+\frac{(4B-3C)\tan(c+dx)}{ad}-\frac{3(B-C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(B-C)\tan(c+dx)\sec(c+dx)}{2ad}-($$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/(a + a*Cos[c + d*x
]),x]
```


```
[Out] (-3*(B - C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*B - 3*C)*Tan[c + d*x])/(a*
d) - (3*(B - C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((B - C)*Sec[c + d*x]^
2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*B - 3*C)*Tan[c + d*x]^3)/(3*
a*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{(B+C\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int(a(4B-3C)-3a(B-C)\,\mathrm{co}}{a^2}$$

$$= -\frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(4B-3C)\int\sec^4(c+dx)\,dx}{a}$$

$$= -\frac{3(B-C)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(B-C)\sec^2(c+dx)\tan(c+}{d(a+a\cos(c+dx))}$$

$$= -\frac{3(B-C)\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(4B-3C)\tan(c+dx)}{ad} - \frac{3(B-}{}$$

Mathematica [B] time = 4.39558, size = 490, normalized size = 3.74

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left(-24B\sin\left(c-\frac{dx}{2}\right)-6B\sin\left(c+\frac{dx}{2}\right)-24B\sin\left(2c+\frac{dx}{2}\right)+21B\sin\left(c+\frac{3dx}{2}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/(a + a*Cos[c
 + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(144*(B - C)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c
]*Sec[c + d*x]^3*(6*(B + C)*Sin[(d*x)/2] + 3*(13*B - 9*C)*Sin[(3*d*x)/2] -
24*B*Sin[c - (d*x)/2] + 12*C*Sin[c - (d*x)/2] - 6*B*Sin[c + (d*x)/2] + 6*C*
Sin[c + (d*x)/2] - 24*B*Sin[2*c + (d*x)/2] + 24*C*Sin[2*c + (d*x)/2] + 21*B
*Sin[c + (3*d*x)/2] - 9*C*Sin[c + (3*d*x)/2] + 9*B*Sin[2*c + (3*d*x)/2] - 9
*C*Sin[2*c + (3*d*x)/2] - 9*B*Sin[3*c + (3*d*x)/2] + 9*C*Sin[3*c + (3*d*x)/
2] + 7*B*Sin[c + (5*d*x)/2] - 3*C*Sin[c + (5*d*x)/2] + B*Sin[2*c + (5*d*x)/
2] + 3*C*Sin[2*c + (5*d*x)/2] - 3*B*Sin[3*c + (5*d*x)/2] + 3*C*Sin[3*c + (5
*d*x)/2] - 9*B*Sin[4*c + (5*d*x)/2] + 9*C*Sin[4*c + (5*d*x)/2] + 16*B*Sin[2
*c + (7*d*x)/2] - 12*C*Sin[2*c + (7*d*x)/2] + 10*B*Sin[3*c + (7*d*x)/2] - 6
*C*Sin[3*c + (7*d*x)/2] + 6*B*Sin[4*c + (7*d*x)/2] - 6*C*Sin[4*c + (7*d*x)/
2])))/(48*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.066, size = 340, normalized size = 2.6

$$\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}+\frac{C}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{B}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-1/3/a/d*B/(tan(1/2*d*
x+1/2*c)-1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*C-1/a/d*B/(tan(1/2*d*x+1/2*c
)-1)^2+3/2/a/d*B*ln(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*
C-5/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/3/a/d
*B/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d*B/(tan(1/2*d*x+1/2*c)+1)^2-1/2/a/d/(tan(1
/2*d*x+1/2*c)+1)^2*C-3/2/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)+3/2/a/d*ln(tan(1/2*
d*x+1/2*c)+1)*C-5/2/a/d*B/(tan(1/2*d*x+1/2*c)+1)+3/2/a/d/(tan(1/2*d*x+1/2*c
)+1)*C
```

Maxima [B] time = 1.20924, size = 497, normalized size = 3.79

$$\frac{B\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3C\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin}{(\cos(}\right.}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\,s}{(\cos}}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/6*(B*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*C*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.76172, size = 419, normalized size = 3.2

$$-\frac{9\left((B-C)\cos(dx+c)^4+(B-C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left((B-C)\cos(dx+c)^4+(B-C)\cos(dx+c\right.}{12\left(ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/12*(9*((B - C)*cos(d*x + c)^4 + (B - C)*cos(d*x + c)^3)*log(sin(d*x + c)
 + 1) - 9*((B - C)*cos(d*x + c)^4 + (B - C)*cos(d*x + c)^3)*log(-sin(d*x +
c) + 1) - 2*(4*(4*B - 3*C)*cos(d*x + c)^3 + (7*B - 3*C)*cos(d*x + c)^2 - (B
 - 3*C)*cos(d*x + c) + 2*B)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
 + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34002, size = 246, normalized size = 1.88

$$-\frac{\frac{9(B-C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9(B-C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-9C\tan\left(\frac{1}{2}dx+\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/6*(9*(B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(B - C)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/a - 6*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/
2*c))/a + 2*(15*B*tan(1/2*d*x + 1/2*c)^5 - 9*C*tan(1/2*d*x + 1/2*c)^5 - 16*
B*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1/2*d*x + 1/2*c)^3 + 9*B*tan(1/2*d*x +
1/2*c) - 3*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.261 $\int \frac{\cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$\frac{4(2B-3C)\sin^3(c+dx)}{3a^2d}-\frac{4(2B-3C)\sin(c+dx)}{a^2d}+\frac{(7B-10C)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7B-10C)\sin(c+dx)\cos(}{2a^2d}$$

```
[Out] ((7*B - 10*C)*x)/(2*a^2) - (4*(2*B - 3*C)*Sin[c + d*x])/(a^2*d) + ((7*B - 1
0*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((7*B - 10*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((B - C)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(2*B - 3*C)*Sin[c + d*x]^3)/(3*a^2*
d)
```

Rubi [A] time = 0.396788, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2977, 2748, 2635, 8, 2633}

$$\frac{4(2B-3C)\sin^3(c+dx)}{3a^2d}-\frac{4(2B-3C)\sin(c+dx)}{a^2d}+\frac{(7B-10C)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7B-10C)\sin(c+dx)\cos(}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x
])^2,x]
```


```
[Out] ((7*B - 10*C)*x)/(2*a^2) - (4*(2*B - 3*C)*Sin[c + d*x])/(a^2*d) + ((7*B - 1
0*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((7*B - 10*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((B - C)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(2*B - 3*C)*Sin[c + d*x]^3)/(3*a^2*
d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos^4(c+dx)(B+C\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(B-C)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(4a(B-C)-3a(B-2C)\cos}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(7B-10C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(B-C)\cos^4(c+dx)\sin(}{3d(a+a\cos(c+dx)}$$

$$= \frac{(7B-10C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(B-C)\cos^4(c+dx)\sin(}{3d(a+a\cos(c+dx)}$$

$$= \frac{(7B-10C)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{(7B-10C)\cos^3(c+dx)\sin}{3a^2d(1+\cos(c+d}$$

$$= \frac{(7B-10C)x}{2a^2} - \frac{4(2B-3C)\sin(c+dx)}{a^2d} + \frac{(7B-10C)\cos(c+dx)}{2a^2d}$$

Mathematica [B] time = 0.607838, size = 369, normalized size = 2.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(7B-10C)\cos\left(c+\frac{dx}{2}\right)+147B\sin\left(c+\frac{dx}{2}\right)-239B\sin\left(c+\frac{3dx}{2}\right)-63B\sin\left(2c+\frac{3dx}{2}\right)-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*B - 10*C)*d*x*Cos[(d*x)/2] + 36*(7*B - 10
*C)*d*x*Cos[c + (d*x)/2] + 84*B*d*x*Cos[c + (3*d*x)/2] - 120*C*d*x*Cos[c +
(3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] - 120*C*d*x*Cos[2*c + (3*d*x)/2]
 - 381*B*Sin[(d*x)/2] + 516*C*Sin[(d*x)/2] + 147*B*Sin[c + (d*x)/2] - 156*C
*Sin[c + (d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 342*C*Sin[c + (3*d*x)/2] - 6
3*B*Sin[2*c + (3*d*x)/2] + 118*C*Sin[2*c + (3*d*x)/2] - 15*B*Sin[2*c + (5*d
*x)/2] + 30*C*Sin[2*c + (5*d*x)/2] - 15*B*Sin[3*c + (5*d*x)/2] + 30*C*Sin[3
*c + (5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] - 3*C*Sin[3*c + (7*d*x)/2] + 3*B
*Sin[4*c + (7*d*x)/2] - 3*C*Sin[4*c + (7*d*x)/2] + C*Sin[4*c + (9*d*x)/2] +
 C*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.033, size = 322, normalized size = 1.9

$$\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{9\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\,\frac{(\tan(1/2\,dx+c/2}{da^2\left((\tan(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*B*tan(1/2*d*x+1/2*c)+9/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(tan(1/2*d*x+1
/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*B+10/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan
(1/2*d*x+1/2*c)^5*C-8/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3
*B+40/3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(ta
n(1/2*d*x+1/2*c)^2+1)^3*B*tan(1/2*d*x+1/2*c)+6/d/a^2/(tan(1/2*d*x+1/2*c)^2+
1)^3*C*tan(1/2*d*x+1/2*c)+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B-10/d/a^2*arc
tan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 2.0116, size = 502, normalized size = 2.95

$$\frac{C\left(\frac{4\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-B\left(\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(C*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.70942, size = 387, normalized size = 2.28

$$\frac{3\,(7\,B-10\,C)dx\cos{(dx+c)}^2+6\,(7\,B-10\,C)dx\cos{(dx+c)}+3\,(7\,B-10\,C)dx+\left(2\,C\cos{(dx+c)}^4+(3\,B-2\,C)\cos{(}\right.}{6\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(7*B - 10*C)*d*x*cos(d*x + c)^2 + 6*(7*B - 10*C)*d*x*cos(d*x + c) +
3*(7*B - 10*C)*d*x + (2*C*cos(d*x + c)^4 + (3*B - 2*C)*cos(d*x + c)^3 - 6*(
B - 2*C)*cos(d*x + c)^2 - (43*B - 66*C)*cos(d*x + c) - 32*B + 48*C)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 28.4945, size = 1430, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,
x)
```

```
[Out] Piecewise((21*B*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*
a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63
*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*B*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4
 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x/(6*a**2*d*tan(c/2 +
 d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
+ 6*a**2*d) + B*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2
*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 18*B*t
an(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/
2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*tan(c/2 + d*x/2)**
5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 110*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(
c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) - 39*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
```

```
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
0*C*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*C*d*x*tan(c
/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*C*d*x*tan(c/2 + d*x/2)**
2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
- C*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*C*tan(c/2 + d*x
/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*C*tan(c/2 + d*x/2)**5/(6*a**2*d
*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 +
d*x/2)**2 + 6*a**2*d) + 160*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2
)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 63*C*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan
(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x
*(B*cos(c) + C*cos(c)**2)*cos(c)**3/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.4393, size = 259, normalized size = 1.52

$$\frac{\frac{3\,(dx+c)(7\,B-10\,C)}{a^2}-\frac{2\left(15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*B - 10*C)/a^2 - 2*(15*B*tan(1/2*d*x + 1/2*c)^5 - 30*C*t
an(1/2*d*x + 1/2*c)^5 + 24*B*tan(1/2*d*x + 1/2*c)^3 - 40*C*tan(1/2*d*x + 1/
2*c)^3 + 9*B*tan(1/2*d*x + 1/2*c) - 18*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 + 1)^3*a^2) + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*
x + 1/2*c)^3 - 21*B*a^4*tan(1/2*d*x + 1/2*c) + 27*C*a^4*tan(1/2*d*x + 1/2*c
))/a^6)/d
```

3.262 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{2(5B-8C)\sin(c+dx)}{3a^2d}+\frac{(5B-8C)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4B-7C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4B-7C)}{2a^2}+\frac{(B-$$

```
[Out] -((4*B - 7*C)*x)/(2*a^2) + (2*(5*B - 8*C)*Sin[c + d*x])/(3*a^2*d) - ((4*B -
 7*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((5*B - 8*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((B - C)*Cos[c + d*x]^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.36259, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3029, 2977, 2734}

$$\frac{2(5B-8C)\sin(c+dx)}{3a^2d}+\frac{(5B-8C)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4B-7C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4B-7C)}{2a^2}+\frac{(B-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x
])^2,x]
```

```
[Out] -((4*B - 7*C)*x)/(2*a^2) + (2*(5*B - 8*C)*Sin[c + d*x])/(3*a^2*d) - ((4*B -
 7*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((5*B - 8*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((B - C)*Cos[c + d*x]^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(3a(B-C)-a(2B-5C)\cos(}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(5B-8C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(B-C)\cos^3(c+dx)\sin(c+}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(4B-7C)x}{2a^2} + \frac{2(5B-8C)\sin(c+dx)}{3a^2d} - \frac{(4B-7C)\cos(c+dx)\,\mathrm{s}}{2a^2d}$$

Mathematica [B] time = 0.860781, size = 315, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(4B-7C)\cos\left(c+\frac{dx}{2}\right)-120B\sin\left(c+\frac{dx}{2}\right)+164B\sin\left(c+\frac{3dx}{2}\right)+36B\sin\left(2c+\frac{3dx}{2}\right)+12\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(4*B - 7*C)*d*x*Cos[(d*x)/2] - 36*(4*B - 7*
C)*d*x*Cos[c + (d*x)/2] - 48*B*d*x*Cos[c + (3*d*x)/2] + 84*C*d*x*Cos[c + (3
*d*x)/2] - 48*B*d*x*Cos[2*c + (3*d*x)/2] + 84*C*d*x*Cos[2*c + (3*d*x)/2] +
```

```
264*B*Sin[(d*x)/2] - 381*C*Sin[(d*x)/2] - 120*B*Sin[c + (d*x)/2] + 147*C*Si
n[c + (d*x)/2] + 164*B*Sin[c + (3*d*x)/2] - 239*C*Sin[c + (3*d*x)/2] + 36*B
*Sin[2*c + (3*d*x)/2] - 63*C*Sin[2*c + (3*d*x)/2] + 12*B*Sin[2*c + (5*d*x)/
2] - 15*C*Sin[2*c + (5*d*x)/2] + 12*B*Sin[3*c + (5*d*x)/2] - 15*C*Sin[3*c +
 (5*d*x)/2] + 3*C*Sin[3*c + (7*d*x)/2] + 3*C*Sin[4*c + (7*d*x)/2]))/(48*a^2
*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.031, size = 252, normalized size = 1.7

$$-\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\,\frac{C\,(\tan(1/2\,dx}{da^2\left((\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*B*tan(1/2*d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(tan(1/2*d*x+
1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan
(1/2*d*x+1/2*c)^3*B-3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)
+2/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c)-4/d/a^2*arctan(tan
(1/2*d*x+1/2*c))*B+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 2.03039, size = 382, normalized size = 2.6

$$-\frac{C\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(\cdots}{\cos(d\cdots}\right)}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
```

```
 1))/a^2) - B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

Fricas [A] time = 1.68669, size = 343, normalized size = 2.33

$$-\frac{3\,(4\,B-7\,C)dx\cos\left(dx+c\right)^{2}+6\,(4\,B-7\,C)dx\cos\left(dx+c\right)+3\,(4\,B-7\,C)dx-\left(3\,C\cos\left(dx+c\right)^{3}+6\,(B-C)\cos\left(dx+c\right.}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(4*B - 7*C)*d*x*cos(d*x + c)^2 + 6*(4*B - 7*C)*d*x*cos(d*x + c) + 3
*(4*B - 7*C)*d*x - (3*C*cos(d*x + c)^3 + 6*(B - C)*cos(d*x + c)^2 + (28*B -
 43*C)*cos(d*x + c) + 20*B - 32*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 17.4149, size = 848, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,
x)
```

```
[Out] Piecewise((-12*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 24*B*d*x*tan(c/2 + d*x/2)**2/(6*a
**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*
B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - B*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 13*B*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 +
d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 41*B*tan(c/2 + d*x/
2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) + 27*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 21*C*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/
```

```
2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*C*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) + 21*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) + C*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*C*tan(c/2 + d*x/2)**5/(6
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 7
1*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) - 39*C*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(B*cos(c) + C*
cos(c)**2)*cos(c)**2/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.39133, size = 221, normalized size = 1.5

$$-\frac{\frac{3\,(dx+c)(4\,B-7\,C)}{a^2}-\frac{6\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\cdots}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(4*B - 7*C)/a^2 - 6*(2*B*tan(1/2*d*x + 1/2*c)^3 - 5*C*tan
(1/2*d*x + 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) - 3*C*tan(1/2*d*x + 1/2*c))/
((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^
4*tan(1/2*d*x + 1/2*c)^3 - 15*B*a^4*tan(1/2*d*x + 1/2*c) + 21*C*a^4*tan(1/2
*d*x + 1/2*c))/a^6)/d
```

3.263 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=99

$$-\frac{(B-4C)\sin(c+dx)}{3a^2d}-\frac{(B-2C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(B-2C)}{a^2}+\frac{(B-C)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ((B - 2*C)*x)/a^2 - ((B - 4*C)*Sin[c + d*x])/(3*a^2*d) - ((B - 2*C)*Sin[c +
 d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.314939, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(B-4C)\sin(c+dx)}{3a^2d}-\frac{(B-2C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(B-2C)}{a^2}+\frac{(B-C)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])
^2,x]
```


```
[Out] ((B - 2*C)*x)/a^2 - ((B - 4*C)*Sin[c + d*x])/(3*a^2*d) - ((B - 2*C)*Sin[c +
 d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(2a(B-C)-a(B-4C)\cos(c+d}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{2a(B-C)\cos(c+dx)-a(B-4C)\cos^2(c+}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(B-4C)\sin(c+dx)}{3a^2d} + \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{3a^2(}{a}}{}$$

$$= -\frac{(B-4C)\sin(c+dx)}{3a^2d} + \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(B-2}{}$$

$$= \frac{(B-2C)x}{a^2} - \frac{(B-4C)\sin(c+dx)}{3a^2d} + \frac{(B-C)\cos^2(c+dx)\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(B-2C)x}{a^2} - \frac{(B-4C)\sin(c+dx)}{3a^2d} + \frac{(B-C)\cos^2(c+dx)\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

Mathematica [A] time = 0.679917, size = 137, normalized size = 1.38

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6\cos^3\left(\frac{1}{2}(c+dx)\right)(dx(B-2C)+C\sin(c+dx))+(B-C)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(B-C)\sec\left(\frac{c}{2}\right)\text{si}\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((B - C)*Sec[c/2]*Sin[(d*x)/2] - 2*(5*B - 8*C)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((B - 2*C)*d*x + C*
Sin[c + d*x]) + (B - C)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d
*x])^2)
```

Maple [A] time = 0.031, size = 149, normalized size = 1.5

$$\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{3B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{C\tan(1/2\,dx+c}{da^2\left((\tan(1/2\,dx+c/\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*B*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*C*tan(1/2*d*x+
1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B-4/d/a^
2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.99488, size = 258, normalized size = 2.61

$$\frac{C\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] 1/6*(C*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.6701, size = 294, normalized size = 2.97

$$\frac{3\,(B-2\,C)dx\cos{(dx+c)}^2+6\,(B-2\,C)dx\cos{(dx+c)}+3\,(B-2\,C)dx+\left(3\,C\cos{(dx+c)}^2-(5\,B-14\,C)\cos{(dx+c)}\right.}{3\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/3*(3*(B - 2*C)*d*x*cos(d*x + c)^2 + 6*(B - 2*C)*d*x*cos(d*x + c) + 3*(B -
 2*C)*d*x + (3*C*cos(d*x + c)^2 - (5*B - 14*C)*cos(d*x + c) - 4*B + 10*C)*s
```

```
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 10.3865, size = 415, normalized size = 4.19

$$\begin{cases} \frac{6Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{6Bdx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{8B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{9B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{12Cdx\tan}{6a^2d\tan^2\left(\frac{c}{2}\right.} \\ \frac{x\left(B\cos(c)+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((6*B*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 6*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + B*tan(c/2 + d*x
/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 8*B*tan(c/2 + d*x/2)**3/
(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 9*B*tan(c/2 + d*x/2)/(6*a**2*d*
tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*C*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*ta
n(c/2 + d*x/2)**2 + 6*a**2*d) - 12*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*
a**2*d) - C*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
 14*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*C*
tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(
B*cos(c) + C*cos(c)**2)*cos(c)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.39172, size = 161, normalized size = 1.63

$$\frac{\frac{6(dx+c)(B-2C)}{a^2} + \frac{12C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^2} + \frac{Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 9Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 15Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*(B - 2*C)/a^2 + 12*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2) + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x + 1/
2*c)^3 - 9*B*a^4*tan(1/2*d*x + 1/2*c) + 15*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)
/d
```

3.264 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{(2B-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Cx}{a^2}-\frac{(B-C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (C*x)/a^2 + ((2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((B -
 C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.110696, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3019, 2735, 2648}

$$\frac{(2B-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Cx}{a^2}-\frac{(B-C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (C*x)/a^2 + ((2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((B -
 C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-2a(B-C)-3aC\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= \frac{Cx}{a^2} - \frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2B-5C)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= \frac{Cx}{a^2} - \frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2B-5C)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [B] time = 0.337494, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-6B\sin\left(c+\frac{dx}{2}\right)+4B\sin\left(c+\frac{3dx}{2}\right)+6B\sin\left(\frac{dx}{2}\right)+12C\sin\left(c+\frac{dx}{2}\right)-10C\sin\left(c+\frac{3dx}{2}\right)+9Cd\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*C*d*x*Cos[(d*x)/2] + 9*C*d*x*Cos[c + (d*x)/
2] + 3*C*d*x*Cos[c + (3*d*x)/2] + 3*C*d*x*Cos[2*c + (3*d*x)/2] + 6*B*Sin[(d
*x)/2] - 18*C*Sin[(d*x)/2] - 6*B*Sin[c + (d*x)/2] + 12*C*Sin[c + (d*x)/2] +
 4*B*Sin[c + (3*d*x)/2] - 10*C*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.023, size = 97, normalized size = 1.4

$$-\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan(\tan(1/2\,dx}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^
2*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/
2*d*x+1/2*c))*C
```

Maxima [A] time = 1.86286, size = 162, normalized size = 2.31

$$-\frac{C\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{B\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - B*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.61882, size = 228, normalized size = 3.26

$$\frac{3\,Cdx\cos\left(dx+c\right)^2+6\,Cdx\cos\left(dx+c\right)+3\,Cdx+\left(\left(2\,B-5\,C\right)\cos\left(dx+c\right)+B-4\,C\right)\sin\left(dx+c\right)}{3\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*C*d*x*cos(d*x + c)^2 + 6*C*d*x*cos(d*x + c) + 3*C*d*x + ((2*B - 5*C)
*cos(d*x + c) + B - 4*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [A] time = 5.52192, size = 107, normalized size = 1.53

$$\begin{cases} -\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} + \frac{Cx}{a^2} + \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d} - \frac{3C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x\left(B\cos(c)+C\cos^2(c)\right)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-B*tan(c/2 + d*x/2)**3/(6*a**2*d) + B*tan(c/2 + d*x/2)/(2*a**2*d
) + C*x/a**2 + C*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*C*tan(c/2 + d*x/2)/(2*a
**2*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.42028, size = 116, normalized size = 1.66

$$\frac{\frac{6(dx+c)C}{a^2} - \frac{Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 3Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 9Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*C/a^2 - (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 3*B*a^4*tan(1/2*d*x + 1/2*c) + 9*C*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.265 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(B+2C)\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{(B-C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ((B - C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((B + 2*C)*Sin[c + d*
x])/(3*d*(a^2 + a^2*Cos[c + d*x]))
```

Rubi [A] time = 0.124032, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3029, 2750, 2648}

$$\frac{(B+2C)\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{(B-C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])
^2,x]
```

```
[Out] ((B - C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((B + 2*C)*Sin[c + d*
x])/(3*d*(a^2 + a^2*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{B+C\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$=\frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(B+2C)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$=\frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(B+2C)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [A] time = 0.17305, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((B+2C)\sin\left(c+\frac{3dx}{2}\right)+3(B+C)\sin\left(\frac{dx}{2}\right)-3C\sin\left(c+\frac{dx}{2}\right)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c +
 d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(B + C)*Sin[(d*x)/2] - 3*C*Sin[c + (d*x)/2] +
 (B + 2*C)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.043, size = 60, normalized size = 0.9

$$\frac{1}{2\,da^2}\left(\frac{B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3*B-1/3*C*tan(1/2*d*x+1/2*c)^3+B*tan(1/2*
d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.29957, size = 126, normalized size = 1.94

$$\frac{\frac{B\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{C\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] 1/6*(B*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + C*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.54849, size = 144, normalized size = 2.22

$$\frac{((B+2\,C)\cos(dx+c)+2\,B+C)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/3*((B + 2*C)*cos(d*x + c) + 2*B + C)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\cos(c+dx)\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx+\int\frac{C\cos^2(c+dx)\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x) + Integral(C*cos(c + d*x)**2*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c
+ d*x) + 1), x))/a**2
```

Giac [A] time = 1.41402, size = 81, normalized size = 1.25

$$\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] 1/6*(B*tan(1/2*d*x + 1/2*c)^3 - C*tan(1/2*d*x + 1/2*c)^3 + 3*B*tan(1/2*d*x
+ 1/2*c) + 3*C*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.266 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{(4B-C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*B - C)*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((B - C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.265789, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3029, 2978, 12, 3770}

$$-\frac{(4B-C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x
])^2,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*B - C)*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((B - C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(3aB-a(B-C)\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(4B-C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int 3a^2B\sec(c\cdots}{3a^4}$$

$$= -\frac{(4B-C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(B-C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{B\int\sec(c+d\cdots}{a^2}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(4B-C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(B-C)\sin(c\cdots}{3d(a+a\cos(c\cdots}$$

Mathematica [B] time = 0.492785, size = 170, normalized size = 2.15

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-C)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(B-C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(4B-C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^2\left(\frac{1}{2}(c+d\cdots\right.\right.}{3a^2d(\cos(c+dx\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c
 + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*B*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (B - C)*Sec[c/2]*
Sin[(d*x)/2] + 2*(4*B - C)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (B -
C)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.057, size = 119, normalized size = 1.5

$$-\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*B*tan(1/2*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2*B*ln(tan(1/2*
d*x+1/2*c)-1)+1/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.3611, size = 196, normalized size = 2.48

$$-\frac{B\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{C\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - C*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.70349, size = 338, normalized size = 4.28

$$\frac{3\left(B\cos\left(dx+c\right)^2+2\,B\cos\left(dx+c\right)+B\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(B\cos\left(dx+c\right)^2+2\,B\cos\left(dx+c\right)+B\right)\log\left(-\sin\left(dx\right.\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - 3*
(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*log(-sin(d*x + c) + 1) - 2*((4*B
- C)*cos(d*x + c) + 5*B - 2*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49198, size = 153, normalized size = 1.94

$$\frac{\frac{6\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*B*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x + 1/2
*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c) - 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.267 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$\frac{2(5B-2C)\tan(c+dx)}{3a^2d}-\frac{(2B-C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2B-C)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(B-C)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -(((2*B - C)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + (2*(5*B - 2*C)*Tan[c + d*x])
/(3*a^2*d) - ((2*B - C)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((B - C)
*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.379749, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2978, 2748, 3767, 8, 3770}

$$\frac{2(5B-2C)\tan(c+dx)}{3a^2d}-\frac{(2B-C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2B-C)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(B-C)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x
])^2,x]
```



```
[Out] -(((2*B - C)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + (2*(5*B - 2*C)*Tan[c + d*x])
/(3*a^2*d) - ((2*B - C)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((B - C)
*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(B-C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(a(4B-C)-2a(B-C)\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$=-\frac{(2B-C)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(B-C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \left(2a^2(5B-\right.}{}$$

$$=-\frac{(2B-C)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(B-C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(2(5B-2C))}{}$$

$$=-\frac{(2B-C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2B-C)\tan(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(B-}{3d(a+}$$

$$=-\frac{(2B-C)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2(5B-2C)\tan(c+dx)}{3a^2d}-\frac{(2B}{a^2d}$$

Mathematica [B] time = 1.53007, size = 264, normalized size = 2.47

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-C)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(B-C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left((2B-C)\left(\log\left(\cos\left(\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c
 + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((B - C)*Sec[c/2]*Sin[(d*x)/2] + 2*(7*B - 4*C)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((2*B - C)*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) + (B*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (B - C)
*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.06, size = 205, normalized size = 1.9

$$\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{B\ln(\tan(1/2\,dx+c}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*B*ln(tan(1/2*d
*x+1/2*c)-1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-1/d/a^2*B/(tan(1/2*d*x+1/2*
c)-1)-2/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C
-1/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.36401, size = 329, normalized size = 3.07

$$\frac{B\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{12\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{12\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}+\frac{12\,\sin(dx+c)}{\left(a^{2}-\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}\right)-C\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)}}{a^{2}}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - C*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

Fricas [B] time = 1.72181, size = 502, normalized size = 4.69

$$-\frac{3\left((2\,B-C)\cos\left(dx+c\right)^{3}+2\,(2\,B-C)\cos\left(dx+c\right)^{2}+(2\,B-C)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left((2\,B-C)\cos\left(d\right.\right.}{6\left(a^{2}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/6*(3*((2*B - C)*cos(d*x + c)^3 + 2*(2*B - C)*cos(d*x + c)^2 + (2*B - C)*
cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((2*B - C)*cos(d*x + c)^3 + 2*(2*B
- C)*cos(d*x + c)^2 + (2*B - C)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2
*(5*B - 2*C)*cos(d*x + c)^2 + (14*B - 5*C)*cos(d*x + c) + 3*B)*sin(d*x + c)
)/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74763, size = 209, normalized size = 1.95

$$-\frac{\frac{6\,(2\,B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,(2\,B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}-\frac{Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Ba^4}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/6*(6*(2*B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*B - C)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 - 1)*a^2) - (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 15*B*a^4*tan(1/2*d*x + 1/2*c) - 9*C*a^4*tan(1/2*d*x + 1/2*c))/
a^6)/d
```

3.268 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{2(8B-5C)\tan(c+dx)}{3a^2d}+\frac{(7B-4C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7B-4C)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8B-5C)\tan(c+dx)}{3a^2d(\cos(c+dx)}$$

```
[Out] ((7*B - 4*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*B - 5*C)*Tan[c + d*x]
)/(3*a^2*d) + ((7*B - 4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((8*B - 5
*C)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((B - C)*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.400943, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(8B-5C)\tan(c+dx)}{3a^2d}+\frac{(7B-4C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7B-4C)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8B-5C)\tan(c+dx)}{3a^2d(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x
])^2,x]
```



```
[Out] ((7*B - 4*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*B - 5*C)*Tan[c + d*x]
)/(3*a^2*d) + ((7*B - 4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((8*B - 5
*C)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((B - C)*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B+C\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(B-C)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(5B-2C)-3a(B-C)\cos(c+dx))\sec}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(8B-5C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(B-C)\sec(c+dx)\tan(c+}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(8B-5C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(B-C)\sec(c+dx)\tan(c+}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(7B-4C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(8B-5C)\sec(c+dx)\tan(c}{3a^2d(1+\cos(c+dx))}$$

$$= \frac{(7B-4C)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{2(8B-5C)\tan(c+dx)}{3a^2d} + \frac{(7B-}{}$$

Mathematica [B] time = 3.12033, size = 496, normalized size = 3.26

$$-\frac{96(7B-4C)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c
 + d*x])^2,x]
```

```
[Out] -(96*(7*B - 4*C)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]^2*(-14*(B - C)*Sin[(d*x)/2] + (97*B - 64*C)*Sin[(3*d*x)
/2] - 126*B*Sin[c - (d*x)/2] + 84*C*Sin[c - (d*x)/2] + 42*B*Sin[c + (d*x)/2
] - 42*C*Sin[c + (d*x)/2] - 98*B*Sin[2*c + (d*x)/2] + 56*C*Sin[2*c + (d*x)/
2] - 3*B*Sin[c + (3*d*x)/2] + 6*C*Sin[c + (3*d*x)/2] + 37*B*Sin[2*c + (3*d*
x)/2] - 34*C*Sin[2*c + (3*d*x)/2] - 63*B*Sin[3*c + (3*d*x)/2] + 36*C*Sin[3*
c + (3*d*x)/2] + 75*B*Sin[c + (5*d*x)/2] - 48*C*Sin[c + (5*d*x)/2] + 15*B*S
in[2*c + (5*d*x)/2] - 6*C*Sin[2*c + (5*d*x)/2] + 39*B*Sin[3*c + (5*d*x)/2]
- 30*C*Sin[3*c + (5*d*x)/2] - 21*B*Sin[4*c + (5*d*x)/2] + 12*C*Sin[4*c + (5
*d*x)/2] + 32*B*Sin[2*c + (7*d*x)/2] - 20*C*Sin[2*c + (7*d*x)/2] + 12*B*Sin
[3*c + (7*d*x)/2] - 6*C*Sin[3*c + (7*d*x)/2] + 20*B*Sin[4*c + (7*d*x)/2] -
14*C*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.067, size = 294, normalized size = 1.9

$$-\frac{B}{6\,d\,a^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{C}{6\,d\,a^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{7\,B}{2\,d\,a^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\,C}{2\,d\,a^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7\,B}{2\,d\,a^{2}}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^
2*B*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*ln(tan(1/
2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+5/2/d/a^2*B/(tan(1/2*d*x
+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^2*B/(tan(1/2*d*x+1/2*c)
-1)^2+5/2/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C+7
/2/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/
d/a^2*B/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.35129, size = 454, normalized size = 2.99

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}-\frac{2a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}\right)-C\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+}{(\cos(dx+c}}{a^{2}}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - C*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.76084, size = 564, normalized size = 3.71

$$\frac{3\left((7B-4C)\cos(dx+c)^4+2(7B-4C)\cos(dx+c)^3+(7B-4C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((7B-4C)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/12*(3*((7*B - 4*C)*cos(d*x + c)^4 + 2*(7*B - 4*C)*cos(d*x + c)^3 + (7*B -
 4*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*B - 4*C)*cos(d*x + c)^4
 + 2*(7*B - 4*C)*cos(d*x + c)^3 + (7*B - 4*C)*cos(d*x + c)^2)*log(-sin(d*x
+ c) + 1) - 2*(4*(8*B - 5*C)*cos(d*x + c)^3 + (43*B - 28*C)*cos(d*x + c)^2
+ 6*(B - C)*cos(d*x + c) - 3*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45803, size = 267, normalized size = 1.76

$$\frac{\frac{3(7B-4C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(7B-4C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2C\tan\left(\frac{1}{2}dx\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(7*B - 4*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*B - 4*C)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*B*tan(1/2*d*x + 1/2*c)^3 - 2*C
*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c) + 2*C*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (B*a^4*tan(1/2*d*x + 1/2*c)^3 -
C*a^4*tan(1/2*d*x + 1/2*c)^3 + 21*B*a^4*tan(1/2*d*x + 1/2*c) - 15*C*a^4*tan
(1/2*d*x + 1/2*c))/a^6)/d
```

3.269 $\int \frac{\cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{8(9B-19C)\sin(c+dx)}{15a^3d}+\frac{4(9B-19C)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6B-13C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6B-13C)}{2a^3}+$$

```
[Out] -((6*B - 13*C)*x)/(2*a^3) + (8*(9*B - 19*C)*Sin[c + d*x])/(15*a^3*d) - ((6*
B - 13*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((B - C)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((6*B - 11*C)*Cos[c + d*x]^3*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (4*(9*B - 19*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.534534, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3029, 2977, 2734}

$$\frac{8(9B-19C)\sin(c+dx)}{15a^3d}+\frac{4(9B-19C)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6B-13C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6B-13C)}{2a^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x
])^3,x]
```


```
[Out] -((6*B - 13*C)*x)/(2*a^3) + (8*(9*B - 19*C)*Sin[c + d*x])/(15*a^3*d) - ((6*
B - 13*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((B - C)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((6*B - 11*C)*Cos[c + d*x]^3*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (4*(9*B - 19*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos^4(c+dx)(B+C\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(B-C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(4a(B-C)-a(2B-7C)\cos}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(B-C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6B-11C)\cos^3(c+dx)\sin(}{15ad(a+a\cos(c+dx))}$$

$$= \frac{(B-C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6B-11C)\cos^3(c+dx)\sin(}{15ad(a+a\cos(c+dx))}$$

$$= -\frac{(6B-13C)x}{2a^3} + \frac{8(9B-19C)\sin(c+dx)}{15a^3d} - \frac{(6B-13C)\cos(c+}{2a^3d}$$

Mathematica [B] time = 0.833027, size = 435, normalized size = 2.25

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-600dx(6B-13C)\cos\left(c+\frac{dx}{2}\right)-4500B\sin\left(c+\frac{dx}{2}\right)+4860B\sin\left(c+\frac{3dx}{2}\right)-900B\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c
 + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(6*B - 13*C)*d*x*Cos[(d*x)/2] - 600*(6*B -
 13*C)*d*x*Cos[c + (d*x)/2] - 1800*B*d*x*Cos[c + (3*d*x)/2] + 3900*C*d*x*Co
s[c + (3*d*x)/2] - 1800*B*d*x*Cos[2*c + (3*d*x)/2] + 3900*C*d*x*Cos[2*c + (
3*d*x)/2] - 360*B*d*x*Cos[2*c + (5*d*x)/2] + 780*C*d*x*Cos[2*c + (5*d*x)/2]
 - 360*B*d*x*Cos[3*c + (5*d*x)/2] + 780*C*d*x*Cos[3*c + (5*d*x)/2] + 7020*B
*Sin[(d*x)/2] - 12760*C*Sin[(d*x)/2] - 4500*B*Sin[c + (d*x)/2] + 7560*C*Sin
[c + (d*x)/2] + 4860*B*Sin[c + (3*d*x)/2] - 9230*C*Sin[c + (3*d*x)/2] - 900
*B*Sin[2*c + (3*d*x)/2] + 930*C*Sin[2*c + (3*d*x)/2] + 1452*B*Sin[2*c + (5*
d*x)/2] - 2782*C*Sin[2*c + (5*d*x)/2] + 300*B*Sin[3*c + (5*d*x)/2] - 750*C*
Sin[3*c + (5*d*x)/2] + 60*B*Sin[3*c + (7*d*x)/2] - 105*C*Sin[3*c + (7*d*x)/
2] + 60*B*Sin[4*c + (7*d*x)/2] - 105*C*Sin[4*c + (7*d*x)/2] + 15*C*Sin[4*c
+ (9*d*x)/2] + 15*C*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.032, size = 292, normalized size = 1.5

$$\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{2\,C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,B}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*B+2/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*B*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3/(tan(1/2*d*x+1/2*c)
^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-7/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2
*d*x+1/2*c)^3+2/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c)-5/d/a
^3/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)-6/d/a^3*arctan(tan(1/2*d
*x+1/2*c))*B+13/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.49992, size = 435, normalized size = 2.25

$$-\frac{C\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3B\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(C*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*B*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 1.70379, size = 497, normalized size = 2.58

$$-\frac{15\,(6\,B-13\,C)dx\cos{(dx+c)}^3+45\,(6\,B-13\,C)dx\cos{(dx+c)}^2+45\,(6\,B-13\,C)dx\cos{(dx+c)}+15\,(6\,B-13\,C)dx-}{30\left(a^3d\cos{(dx+c)}^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/30*(15*(6*B - 13*C)*d*x*cos(d*x + c)^3 + 45*(6*B - 13*C)*d*x*cos(d*x + c
)^2 + 45*(6*B - 13*C)*d*x*cos(d*x + c) + 15*(6*B - 13*C)*d*x - (15*C*cos(d*
x + c)^4 + 15*(2*B - 3*C)*cos(d*x + c)^3 + (234*B - 479*C)*cos(d*x + c)^2 +
 3*(114*B - 239*C)*cos(d*x + c) + 144*B - 304*C)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 40.0983, size = 971, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,
x)
```

```
[Out] Piecewise((-180*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 360*B*d*x*tan(c/2 + d*x/2)**2
/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*
d) - 180*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)
**2 + 60*a**3*d) + 3*B*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 24*B*tan(c/2 + d*x/2)**7/(60
*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 198*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(
c/2 + d*x/2)**2 + 60*a**3*d) + 600*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2
 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*B*tan(c/2
+ d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 +
60*a**3*d) + 390*C*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*C*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3
*d) + 390*C*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) - 3*C*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4
+ 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*C*tan(c/2 + d*x/2)**7/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 388*C*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) - 1310*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c
/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*C*tan(c/
2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
+ 60*a**3*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c)**3/(a*cos(c) +
a)**3, True))
```

Giac [A] time = 1.35957, size = 270, normalized size = 1.4

$$-\frac{\frac{30\,(dx+c)(6\,B-13\,C)}{a^3}-\frac{60\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/60*(30*(d*x + c)*(6*B - 13*C)/a^3 - 60*(2*B*tan(1/2*d*x + 1/2*c)^3 - 7*C
*tan(1/2*d*x + 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) - 5*C*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5
 - 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*
C*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*B*a^12*tan(1/2*d*x + 1/2*c) - 465*C*a^1
```

```
2*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.270 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=147

$$-\frac{(7B-27C)\sin(c+dx)}{15a^3d}-\frac{(B-3C)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(B-3C)}{a^3}+\frac{(B-C)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4B-9C)\sin(c+}{15ad(a\cos(c}$$

```
[Out] ((B - 3*C)*x)/a^3 - ((7*B - 27*C)*Sin[c + d*x])/(15*a^3*d) + ((B - C)*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*B - 9*C)*Cos[c +
 d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((B - 3*C)*Sin[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.51487, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(7B-27C)\sin(c+dx)}{15a^3d}-\frac{(B-3C)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(B-3C)}{a^3}+\frac{(B-C)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4B-9C)\sin(c+}{15ad(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x
])^3,x]
```

```
[Out] ((B - 3*C)*x)/a^3 - ((7*B - 27*C)*Sin[c + d*x])/(15*a^3*d) + ((B - C)*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*B - 9*C)*Cos[c +
 d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((B - 3*C)*Sin[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(3a(B-C)-a(B-6C)\cos(c\text{-}}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4B-9C)\cos^2(c+dx)\sin(c+}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4B-9C)\cos^2(c+dx)\sin(c+}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(7B-27C)\sin(c+dx)}{15a^3d} + \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4I}{}$$

$$= -\frac{(7B-27C)\sin(c+dx)}{15a^3d} + \frac{(B-C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4I}{}$$

$$= \frac{(B-3C)x}{a^3} - \frac{(7B-27C)\sin(c+dx)}{15a^3d} + \frac{(B-C)\cos^3(c+dx)\sin(c}{5d(a+a\cos(c+dx))^{}}$$

$$= \frac{(B-3C)x}{a^3} - \frac{(7B-27C)\sin(c+dx)}{15a^3d} + \frac{(B-C)\cos^3(c+dx)\sin(c}{5d(a+a\cos(c+dx))^{}}$$

Mathematica [B] time = 0.861192, size = 361, normalized size = 2.46

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(300dx(B-3C)\cos\left(c+\frac{dx}{2}\right)+540B\sin\left(c+\frac{dx}{2}\right)-460B\sin\left(c+\frac{3dx}{2}\right)+180B\sin\left(2c+\frac{3dx}{2}\right)-12\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c
 + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(300*(B - 3*C)*d*x*Cos[(d*x)/2] + 300*(B - 3*C)*
d*x*Cos[c + (d*x)/2] + 150*B*d*x*Cos[c + (3*d*x)/2] - 450*C*d*x*Cos[c + (3*
d*x)/2] + 150*B*d*x*Cos[2*c + (3*d*x)/2] - 450*C*d*x*Cos[2*c + (3*d*x)/2] +
 30*B*d*x*Cos[2*c + (5*d*x)/2] - 90*C*d*x*Cos[2*c + (5*d*x)/2] + 30*B*d*x*C
os[3*c + (5*d*x)/2] - 90*C*d*x*Cos[3*c + (5*d*x)/2] - 740*B*Sin[(d*x)/2] +
1755*C*Sin[(d*x)/2] + 540*B*Sin[c + (d*x)/2] - 1125*C*Sin[c + (d*x)/2] - 46
0*B*Sin[c + (3*d*x)/2] + 1215*C*Sin[c + (3*d*x)/2] + 180*B*Sin[2*c + (3*d*x
)/2] - 225*C*Sin[2*c + (3*d*x)/2] - 128*B*Sin[2*c + (5*d*x)/2] + 363*C*Sin[
```

```
2*c + (5*d*x)/2] + 75*C*Sin[3*c + (5*d*x)/2] + 15*C*Sin[3*c + (7*d*x)/2] +
15*C*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.031, size = 189, normalized size = 1.3

$$-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,B}{4\,da^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*B-1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*B*tan
(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*C*tan(1/2*d*x+1/2*c
)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B-6/d/a^3*arc
tan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.96908, size = 312, normalized size = 2.12

$$\frac{3\,C\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-B\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/60*(3*C*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 1.67054, size = 429, normalized size = 2.92

$$\frac{15\,(B-3\,C)dx\cos\left(dx+c\right)^{3}+45\,(B-3\,C)dx\cos\left(dx+c\right)^{2}+45\,(B-3\,C)dx\cos\left(dx+c\right)+15\,(B-3\,C)dx+\left(15\,C\cos\left(dx\right.\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/15*(15*(B - 3*C)*d*x*cos(d*x + c)^3 + 45*(B - 3*C)*d*x*cos(d*x + c)^2 + 4
5*(B - 3*C)*d*x*cos(d*x + c) + 15*(B - 3*C)*d*x + (15*C*cos(d*x + c)^3 - (3
2*B - 117*C)*cos(d*x + c)^2 - 3*(17*B - 57*C)*cos(d*x + c) - 22*B + 72*C)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A] time = 25.2218, size = 502, normalized size = 3.41

$$\begin{cases}\frac{60Bdx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{60Bdx}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{3B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}+\frac{17B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}-\frac{85B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^{3}d}- \\ \frac{x\left(B\cos\left(c\right)+C\cos^{2}\left(c\right)\right)\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{3}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,
x)
```

```
[Out] Piecewise((60*B*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) + 60*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*B*tan(c
/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 17*B*tan(c/2 +
 d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 85*B*tan(c/2 + d*x
/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*B*tan(c/2 + d*x/2)
/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*C*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*C*d*x/(60*a**3*d*tan(c/
2 + d*x/2)**2 + 60*a**3*d) + 3*C*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d
*x/2)**2 + 60*a**3*d) - 27*C*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) + 225*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**
2 + 60*a**3*d) + 375*C*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c)**2/(a*cos(c) + a)**
```

```
3, True))
```

Giac [A] time = 1.52184, size = 209, normalized size = 1.42

$$\frac{\frac{60\,(dx+c)(B-3\,C)}{a^3}+\frac{120\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,Ba^{1}}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*(B - 3*C)/a^3 + 120*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*a^3) - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan(1/2
*d*x + 1/2*c)^5 - 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*C*a^12*tan(1/2*d*x
+ 1/2*c)^3 + 105*B*a^12*tan(1/2*d*x + 1/2*c) - 255*C*a^12*tan(1/2*d*x + 1/2
*c))/a^15)/d
```

3.271 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$\frac{(4B-29C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Cx}{a^3}+\frac{(B-C)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2B-7C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] (C*x)/a^3 + ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((2*B - 7*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((4*B -
29*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.338189, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3029, 2977, 2968, 3019, 2735, 2648}

$$\frac{(4B-29C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Cx}{a^3}+\frac{(B-C)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2B-7C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d*x])
^3,x]
```


```
[Out] (C*x)/a^3 + ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((2*B - 7*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((4*B -
29*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(2a(B-C)+5aC\cos(c+dx))}{(a+a\cos(c+dx))^2}\,d}{5a^2}$$

$$= \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{2a(B-C)\cos(c+dx)+5aC\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,d}{5a^2}$$

$$= \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2B-7C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \int$$

$$= \frac{Cx}{a^3} + \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2B-7C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{Cx}{a^3} + \frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2B-7C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

Mathematica [B] time = 0.53862, size = 241, normalized size = 2.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-60B\sin\left(c+\frac{dx}{2}\right)+40B\sin\left(c+\frac{3dx}{2}\right)-30B\sin\left(2c+\frac{3dx}{2}\right)+14B\sin\left(2c+\frac{5dx}{2}\right)+80B\sin\left(\frac{dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*C*d*x*Cos[(d*x)/2] + 150*C*d*x*Cos[c + (d
*x)/2] + 75*C*d*x*Cos[c + (3*d*x)/2] + 75*C*d*x*Cos[2*c + (3*d*x)/2] + 15*C
*d*x*Cos[2*c + (5*d*x)/2] + 15*C*d*x*Cos[3*c + (5*d*x)/2] + 80*B*Sin[(d*x)/
2] - 370*C*Sin[(d*x)/2] - 60*B*Sin[c + (d*x)/2] + 270*C*Sin[c + (d*x)/2] +
40*B*Sin[c + (3*d*x)/2] - 230*C*Sin[c + (3*d*x)/2] - 30*B*Sin[2*c + (3*d*x)
/2] + 90*C*Sin[2*c + (3*d*x)/2] + 14*B*Sin[2*c + (5*d*x)/2] - 64*C*Sin[2*c
+ (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.026, size = 137, normalized size = 1.2

$$\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{B}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{B}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*B+1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*B*tan(
1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/
2*c))*C
```

Maxima [A] time = 1.92624, size = 216, normalized size = 1.86

$$-\frac{C\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] -1/60*(C*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3)/d
```

Fricas [A] time = 1.64559, size = 351, normalized size = 3.03

$$\frac{15\,Cdx\cos(dx+c)^3+45\,Cdx\cos(dx+c)^2+45\,Cdx\cos(dx+c)+15\,Cdx+\left((7\,B-32\,C)\cos(dx+c)^2+3\,(2\,B-17\,C\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/15*(15*C*d*x*cos(d*x + c)^3 + 45*C*d*x*cos(d*x + c)^2 + 45*C*d*x*cos(d*x
+ c) + 15*C*d*x + ((7*B - 32*C)*cos(d*x + c)^2 + 3*(2*B - 17*C)*cos(d*x + c
```

```
) + 2*B - 22*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 15.3748, size = 151, normalized size = 1.3

$$\begin{cases} \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{Cx}{a^3} - \frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d} - \frac{7C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\left(B\cos(c)+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((B*tan(c/2 + d*x/2)**5/(20*a**3*d) - B*tan(c/2 + d*x/2)**3/(6*a**
3*d) + B*tan(c/2 + d*x/2)/(4*a**3*d) + C*x/a**3 - C*tan(c/2 + d*x/2)**5/(20
*a**3*d) + C*tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*C*tan(c/2 + d*x/2)/(4*a**3*
d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)*cos(c)/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.46601, size = 162, normalized size = 1.4

$$\frac{\frac{60(dx+c)C}{a^3} + \frac{3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ca^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 10Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 20Ca^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 15Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 105Ca^{12}\tan\left(\frac{1}{2}dx\right.}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*C/a^3 + (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan(
1/2*d*x + 1/2*c)^5 - 10*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*C*a^12*tan(1/2*d
*x + 1/2*c)^3 + 15*B*a^12*tan(1/2*d*x + 1/2*c) - 105*C*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

3.272 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(3B+7C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3B-8C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(B-C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -((B - C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((3*B - 8*C)*Sin[c +
 d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*B + 7*C)*Sin[c + d*x])/(15*d*(
a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.134152, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3019, 2750, 2648}

$$\frac{(3B+7C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3B-8C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(B-C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((B - C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((3*B - 8*C)*Sin[c +
 d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*B + 7*C)*Sin[c + d*x])/(15*d*(
a^3 + a^3*Cos[c + d*x]))
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{-3a(B-C)-5aC\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3B+7C)\int \frac{1}{a+a\cos(c+dx)}\,d}{15a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3B+7C)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] time = 0.328463, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-15(B+2C)\sin\left(c+\frac{dx}{2}\right)+15B\sin\left(c+\frac{3dx}{2}\right)+3B\sin\left(2c+\frac{5dx}{2}\right)+5(3B+8C)\sin\left(\frac{dx}{2}\right)+20C\,\mathrm{s}\right.}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*B + 8*C)*Sin[(d*x)/2] - 15*(B + 2*C)*Sin[c
 + (d*x)/2] + 15*B*Sin[c + (3*d*x)/2] + 20*C*Sin[c + (3*d*x)/2] - 15*C*Sin[
2*c + (3*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] + 7*C*Sin[2*c + (5*d*x)/2]))/(3
0*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.022, size = 64, normalized size = 0.6

$$\frac{1}{4\,d{a}^{3}}\left(\frac{-B+C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{2\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(-B+C)*tan(1/2*d*x+1/2*c)^5-2/3*C*tan(1/2*d*x+1/2*c)^3+B*tan
(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.1406, size = 155, normalized size = 1.52

$$\frac{\frac{C\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,B\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(C*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.52312, size = 227, normalized size = 2.23

$$\frac{\left((3\,B+7\,C)\cos\left(dx+c\right)^2+3\,(3\,B+2\,C)\cos\left(dx+c\right)+3\,B+2\,C\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((3*B + 7*C)*cos(d*x + c)^2 + 3*(3*B + 2*C)*cos(d*x + c) + 3*B + 2*C)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [A] time = 9.5851, size = 119, normalized size = 1.17

$$\begin{cases} -\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}+\frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}-\frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x\left(B\cos(c)+C\cos^2(c)\right)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-B*tan(c/2 + d*x/2)**5/(20*a**3*d) + B*tan(c/2 + d*x/2)/(4*a**3*
d) + C*tan(c/2 + d*x/2)**5/(20*a**3*d) - C*tan(c/2 + d*x/2)**3/(6*a**3*d) +
 C*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(B*cos(c) + C*cos(c)**2)/(a*c
os(c) + a)**3, True))
```

Giac [A] time = 1.48483, size = 101, normalized size = 0.99

$$-\frac{3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(3*B*tan(1/2*d*x + 1/2*c)^5 - 3*C*tan(1/2*d*x + 1/2*c)^5 + 10*C*tan(1
/2*d*x + 1/2*c)^3 - 15*B*tan(1/2*d*x + 1/2*c) - 15*C*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

3.273 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2B+3C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2B+3C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(B-C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] ((B - C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*B + 3*C)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*B + 3*C)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.15406, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3029, 2750, 2650, 2648}

$$\frac{(2B+3C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2B+3C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(B-C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d*x])
^3,x]
```

```
[Out] ((B - C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*B + 3*C)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*B + 3*C)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{B+C\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2B+3C)\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}$$

$$= \frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2B+3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2B+3C)\int \frac{1}{a \ldots}}{15 \ldots}$$

$$= \frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2B+3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2B+3C)\,\mathrm{s}\ldots}{15d\left(a^3+a^3\,\mathrm{c}\ldots\right.}$$

Mathematica [A] time = 0.258005, size = 96, normalized size = 0.94

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((2B+3C)\left(5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)+5(4B+3C)\sin\left(\frac{dx}{2}\right)-15C\sin\left(c+\frac{dx}{2}\right)\right)}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c +
 d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*B + 3*C)*Sin[(d*x)/2] - 15*C*Sin[c + (d*x)
/2] + (2*B + 3*C)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2])))/(30*a^3*d
*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.043, size = 64, normalized size = 0.6

$$\frac{1}{4\,da^3}\left(\frac{B-C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(B-C)*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3*B+B*tan(
1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.36874, size = 155, normalized size = 1.52

$$\frac{\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,C\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*C*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.57908, size = 227, normalized size = 2.23

$$\frac{\left((2\,B+3\,C)\cos\left(dx+c\right)^2+3\,(2\,B+3\,C)\cos\left(dx+c\right)+7\,B+3\,C\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/15*((2*B + 3*C)*cos(d*x + c)^2 + 3*(2*B + 3*C)*cos(d*x + c) + 7*B + 3*C)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx + \int \frac{C\cos^2(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2
+ 3*cos(c + d*x) + 1), x) + Integral(C*cos(c + d*x)**2*sec(c + d*x)/(cos(c
+ d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.61338, size = 101, normalized size = 0.99

$$\frac{3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] 1/60*(3*B*tan(1/2*d*x + 1/2*c)^5 - 3*C*tan(1/2*d*x + 1/2*c)^5 + 10*B*tan(1/
2*d*x + 1/2*c)^3 + 15*B*tan(1/2*d*x + 1/2*c) + 15*C*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

3.274 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{2(11B-C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7B-2C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(B-C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((B - C)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*B - 2*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 (2*(11*B - C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.402083, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3029, 2978, 12, 3770}

$$-\frac{2(11B-C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7B-2C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(B-C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c + d*x
])^3,x]
```


```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((B - C)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*B - 2*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 (2*(11*B - C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5aB-2a(B-C)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7B-2C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\left(15a^2B-a^2(\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7B-2C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11B-C\cdots}{15d\left(a^3+a\cdots\right.}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7B-2C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11B-C\cdots}{15d\left(a^3+a\cdots\right.}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(B-C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7B-2C)\sin\cdots}{15ad(a+a\cos(\cdots}$$

Mathematica [A] time = 0.904157, size = 197, normalized size = 1.68

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(75B\sin\left(c+\frac{dx}{2}\right)-95B\sin\left(c+\frac{3dx}{2}\right)+15B\sin\left(2c+\frac{3dx}{2}\right)-22B\sin\left(2c+\frac{5dx}{2}\right)-5(29B-4C)\sin\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c
 + d*x])^3,x]
```

```
[Out] (-240*B*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-5*(29*B
 - 4*C)*Sin[(d*x)/2] + 75*B*Sin[c + (d*x)/2] - 95*B*Sin[c + (3*d*x)/2] + 10
*C*Sin[c + (3*d*x)/2] + 15*B*Sin[2*c + (3*d*x)/2] - 22*B*Sin[2*c + (5*d*x)/
2] + 2*C*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.063, size = 159, normalized size = 1.4

$$-\frac{7B}{4da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{20da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{3da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^3}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -7/4/d/a^3*B*tan(1/2*d*x+1/2*c)-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/20/d/a^
3*C*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^3*C*tan(1
/2*d*x+1/2*c)^3+1/d/a^3*B*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a^3*B*ln(tan(1/2*d*x
+1/2*c)-1)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.04456, size = 252, normalized size = 2.15

$$-\frac{B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
```

```
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - C*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.74171, size = 481, normalized size = 4.11

$$\frac{15\left(B\cos\left(dx+c\right)^{3}+3\,B\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+B\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(B\cos\left(dx+c\right)^{3}+3\,B\cos\left(dx+c\right)\right.}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/30*(15*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*log
(sin(d*x + c) + 1) - 15*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*
x + c) + B)*log(-sin(d*x + c) + 1) - 2*(2*(11*B - C)*cos(d*x + c)^2 + 3*(17
*B - 2*C)*cos(d*x + c) + 32*B - 7*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.68955, size = 200, normalized size = 1.71

$$\frac{\frac{60\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{60\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}-\frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-10\,Ca^{12}\tan\left(\frac{1}{2}\,dx\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(60*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*B*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan(1/2*
d*x + 1/2*c)^5 + 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*C*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*B*a^12*tan(1/2*d*x + 1/2*c) - 15*C*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

3.275 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{2(36B-11C)\tan(c+dx)}{15a^3d}-\frac{(3B-C)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3B-C)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9B-4C)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(B-}{5d(a\,c}$$

```
[Out] -(((3*B - C)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*B - 11*C)*Tan[c + d*x
])/(15*a^3*d) - ((B - C)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*B
 - 4*C)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*B - C)*Tan[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.546017, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2978, 2748, 3767, 8, 3770}

$$\frac{2(36B-11C)\tan(c+dx)}{15a^3d}-\frac{(3B-C)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3B-C)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9B-4C)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(B-}{5d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c + d*x
])^3,x]
```


```
[Out] -(((3*B - C)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*B - 11*C)*Tan[c + d*x
])/(15*a^3*d) - ((B - C)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*B
 - 4*C)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*B - C)*Tan[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(6B-C)-3a(B-C)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9B-4C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\left(a^2(27B-7C\right.}{}}{}$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9B-4C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3B-C)\,\mathrm{t}}{d\left(a^3+a^3\,\mathrm{c}\right.}$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9B-4C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3B-C)\,\mathrm{t}}{d\left(a^3+a^3\,\mathrm{c}\right.}$$

$$= -\frac{(3B-C)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(B-C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9B-}{15ad(a}$$

$$= -\frac{(3B-C)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2(36B-11C)\tan(c+dx)}{15a^3d} - \frac{(B}{5d(}$$

Mathematica [B] time = 2.97625, size = 482, normalized size = 3.32

$$\frac{960(3B-C)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c
 + d*x])^3,x]
```

```
[Out] (960*(3*B - C)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Se
c[c]*Sec[c + d*x]*(-5*(51*B - 32*C)*Sin[(d*x)/2] + (567*B - 167*C)*Sin[(3*d
*x)/2] - 600*B*Sin[c - (d*x)/2] + 170*C*Sin[c - (d*x)/2] + 375*B*Sin[c + (d
*x)/2] - 170*C*Sin[c + (d*x)/2] - 480*B*Sin[2*c + (d*x)/2] + 160*C*Sin[2*c
+ (d*x)/2] - 60*B*Sin[c + (3*d*x)/2] + 75*C*Sin[c + (3*d*x)/2] + 402*B*Sin[
2*c + (3*d*x)/2] - 167*C*Sin[2*c + (3*d*x)/2] - 225*B*Sin[3*c + (3*d*x)/2]
+ 75*C*Sin[3*c + (3*d*x)/2] + 315*B*Sin[c + (5*d*x)/2] - 95*C*Sin[c + (5*d*
x)/2] + 30*B*Sin[2*c + (5*d*x)/2] + 15*C*Sin[2*c + (5*d*x)/2] + 240*B*Sin[3
*c + (5*d*x)/2] - 95*C*Sin[3*c + (5*d*x)/2] - 45*B*Sin[4*c + (5*d*x)/2] + 1
5*C*Sin[4*c + (5*d*x)/2] + 72*B*Sin[2*c + (7*d*x)/2] - 22*C*Sin[2*c + (7*d*
```

```
x)/2] + 15*B*Sin[3*c + (7*d*x)/2] + 57*B*Sin[4*c + (7*d*x)/2] - 22*C*Sin[4*
c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.065, size = 245, normalized size = 1.7

$$\frac{B}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{2\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{3\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,B}{4\,d a^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*B-1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*B*tan
(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)+3/d/a^3*B*ln(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C-1/d/a^3*B/(tan(1/2*d*x+1/2*c)-1)-
3/d/a^3*B*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a
^3*B/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.04081, size = 386, normalized size = 2.66

$$\frac{3\,B\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-C\left(\frac{105\,\mathrm{si}}{\cos(d}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - C*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 1.70249, size = 670, normalized size = 4.62

$$-\frac{15\left((3B-C)\cos(dx+c)^4+3(3B-C)\cos(dx+c)^3+3(3B-C)\cos(dx+c)^2+(3B-C)\cos(dx+c)\right)\log(\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/30*(15*((3*B - C)*cos(d*x + c)^4 + 3*(3*B - C)*cos(d*x + c)^3 + 3*(3*B -
 C)*cos(d*x + c)^2 + (3*B - C)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((3
*B - C)*cos(d*x + c)^4 + 3*(3*B - C)*cos(d*x + c)^3 + 3*(3*B - C)*cos(d*x +
 c)^2 + (3*B - C)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(36*B - 11*C)
*cos(d*x + c)^3 + 3*(57*B - 17*C)*cos(d*x + c)^2 + (117*B - 32*C)*cos(d*x +
 c) + 15*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7693, size = 257, normalized size = 1.77

$$-\frac{\frac{60(3B-C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60(3B-C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{120B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}-\frac{3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ca^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3}{}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/60*(60*(3*B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(3*B - C)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 - 1)*a^3) - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*ta
n(1/2*d*x + 1/2*c)^5 + 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*C*a^12*tan(1/2
*d*x + 1/2*c)^3 + 255*B*a^12*tan(1/2*d*x + 1/2*c) - 105*C*a^12*tan(1/2*d*x
+ 1/2*c))/a^15)/d
```

3.276 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{8(19B-9C)\tan(c+dx)}{15a^3d}+\frac{(13B-6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13B-6C)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19B-9C)\tan(c}{15d\left(a^3\cos(c\right.}$$

```
[Out] ((13*B - 6*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (8*(19*B - 9*C)*Tan[c + d*
x])/(15*a^3*d) + ((13*B - 6*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((B -
 C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*B - 6*C)
*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (4*(19*B - 9*
C)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.565793, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{8(19B-9C)\tan(c+dx)}{15a^3d}+\frac{(13B-6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13B-6C)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19B-9C)\tan(c}{15d\left(a^3\cos(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c + d*x
])^3,x]
```



```
[Out] ((13*B - 6*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (8*(19*B - 9*C)*Tan[c + d*
x])/(15*a^3*d) + ((13*B - 6*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((B -
 C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*B - 6*C)
*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (4*(19*B - 9*
C)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{(B+C\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$=-\frac{(B-C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{(a(7B-2C)-4a(B-C)\cos(c+dx))\sec}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$=-\frac{(B-C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(11B-6C)\sec(c+dx)\tan(c}{15ad(a+a\cos(c+dx))^2}$$

$$=-\frac{(B-C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(11B-6C)\sec(c+dx)\tan(c}{15ad(a+a\cos(c+dx))^2}$$

$$=-\frac{(B-C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(11B-6C)\sec(c+dx)\tan(c}{15ad(a+a\cos(c+dx))^2}$$

$$=\frac{(13B-6C)\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{(B-C)\sec(c+dx)\tan(c+}{5d(a+a\cos(c+dx))^3}$$

$$=\frac{(13B-6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{8(19B-9C)\tan(c+dx)}{15a^3d}+\frac{(13}{}$$

Mathematica [B] time = 4.72756, size = 610, normalized size = 3.11

$$-\frac{1920(13B-6C)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c
 + d*x])^3,x]
```

```
[Out] -(1920*(13*B - 6*C)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/
2]*Sec[c]*Sec[c + d*x]^2*((-1235*B + 870*C)*Sin[(d*x)/2] + 5*(761*B - 366*C
)*Sin[(3*d*x)/2] - 4329*B*Sin[c - (d*x)/2] + 2094*C*Sin[c - (d*x)/2] + 1989
*B*Sin[c + (d*x)/2] - 1314*C*Sin[c + (d*x)/2] - 3575*B*Sin[2*c + (d*x)/2] +
 1650*C*Sin[2*c + (d*x)/2] - 475*B*Sin[c + (3*d*x)/2] + 450*C*Sin[c + (3*d*
x)/2] + 2005*B*Sin[2*c + (3*d*x)/2] - 1230*C*Sin[2*c + (3*d*x)/2] - 2275*B*
Sin[3*c + (3*d*x)/2] + 1050*C*Sin[3*c + (3*d*x)/2] + 2673*B*Sin[c + (5*d*x)
/2] - 1278*C*Sin[c + (5*d*x)/2] + 105*B*Sin[2*c + (5*d*x)/2] + 90*C*Sin[2*c
 + (5*d*x)/2] + 1593*B*Sin[3*c + (5*d*x)/2] - 918*C*Sin[3*c + (5*d*x)/2] -
975*B*Sin[4*c + (5*d*x)/2] + 450*C*Sin[4*c + (5*d*x)/2] + 1325*B*Sin[2*c +
```

```
(7*d*x)/2] - 630*C*Sin[2*c + (7*d*x)/2] + 255*B*Sin[3*c + (7*d*x)/2] - 60*C
*Sin[3*c + (7*d*x)/2] + 875*B*Sin[4*c + (7*d*x)/2] - 480*C*Sin[4*c + (7*d*x
)/2] - 195*B*Sin[5*c + (7*d*x)/2] + 90*C*Sin[5*c + (7*d*x)/2] + 304*B*Sin[3
*c + (9*d*x)/2] - 144*C*Sin[3*c + (9*d*x)/2] + 90*B*Sin[4*c + (9*d*x)/2] -
30*C*Sin[4*c + (9*d*x)/2] + 214*B*Sin[5*c + (9*d*x)/2] - 114*C*Sin[5*c + (9
*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.07, size = 334, normalized size = 1.7

$$-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31\,B}{4\,da^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*B+1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*B*ta
n(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)-13/2/d/a^3*B*ln(tan(1/2*d*
x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C+7/2/d/a^3*B/(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^3*B/(tan(1/2*d*x+1/2*c)-1)^
2+7/2/d/a^3*B/(tan(1/2*d*x+1/2*c)+1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*C+13/2/
d/a^3*B*ln(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a
^3*B/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.05891, size = 509, normalized size = 2.6

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-3\,C\left(\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
```

```
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*C*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 1.70459, size = 756, normalized size = 3.86

$$\frac{15\left((13\,B-6\,C)\cos\left(dx+c\right)^{5}+3\,(13\,B-6\,C)\cos\left(dx+c\right)^{4}+3\,(13\,B-6\,C)\cos\left(dx+c\right)^{3}+(13\,B-6\,C)\cos\left(dx+c\right)^{2}\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/60*(15*((13*B - 6*C)*cos(d*x + c)^5 + 3*(13*B - 6*C)*cos(d*x + c)^4 + 3*(
13*B - 6*C)*cos(d*x + c)^3 + (13*B - 6*C)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - 15*((13*B - 6*C)*cos(d*x + c)^5 + 3*(13*B - 6*C)*cos(d*x + c)^4 + 3*
(13*B - 6*C)*cos(d*x + c)^3 + (13*B - 6*C)*cos(d*x + c)^2)*log(-sin(d*x + c
) + 1) - 2*(16*(19*B - 9*C)*cos(d*x + c)^4 + 3*(239*B - 114*C)*cos(d*x + c)
^3 + (479*B - 234*C)*cos(d*x + c)^2 + 15*(3*B - 2*C)*cos(d*x + c) - 15*B)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.442, size = 315, normalized size = 1.61

$$\frac{30\,(13\,B-6\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(13\,B-6\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{60\left(7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,C\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(30*(13*B - 6*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13*B - 6
*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*B*tan(1/2*d*x + 1/2*c)^3
 - 2*C*tan(1/2*d*x + 1/2*c)^3 - 5*B*tan(1/2*d*x + 1/2*c) + 2*C*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*B*a^12*tan(1/2*d*x + 1/
2*c)^5 - 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^12*tan(1/2*d*x + 1/2*c)^3
 - 30*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*B*a^12*tan(1/2*d*x + 1/2*c) - 255
*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.277 $\int \sqrt{a+a\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=101

$$\frac{2(5B-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5B+7C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] (2*a*(5*B + 7*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(5*B -
2*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.132336, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3023, 2751, 2646}

$$\frac{2(5B-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5B+7C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*a*(5*B + 7*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(5*B -
2*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{2\int\sqrt{a+a\cos(c+}}{}$$

$$=\frac{2(5B-2C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2C(a+a\cos}{}$$

$$=\frac{2a(5B+7C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2(5B-2C)\sqrt{a+a\cos(c+dx)}}{15d}$$

Mathematica [A] time = 0.240121, size = 64, normalized size = 0.63

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(5B+4C)\cos(c+dx)+20B+3C\cos(2(c+dx))+19C)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(20*B + 19*C + 2*(5*B + 4*C)*Cos[c + d*x] + 3*C
*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.06, size = 83, normalized size = 0.8

$$\frac{2a\sqrt{2}}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12C(\sin(1/2dx+c/2))^4+(-10B-20C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+15B+15C\right)\frac{}{\sqrt{a\left(\cos\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*C*sin(1/2*d*x+1/2*c)^4+(-1
0*B-20*C)*sin(1/2*d*x+1/2*c)^2+15*B+15*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

Maxima [A] time = 1.86105, size = 119, normalized size = 1.18

$$\frac{10\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}+\left(3\,\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\,\sqrt{2}\sin\left(\frac{1}{2}\,dx\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*
sqrt(a) + (3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.56997, size = 170, normalized size = 1.68

$$\frac{2\left(3\,C\cos\left(dx+c\right)^{2}+\left(5\,B+4\,C\right)\cos\left(dx+c\right)+10\,B+8\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/15*(3*C*cos(d*x + c)^2 + (5*B + 4*C)*cos(d*x + c) + 10*B + 8*C)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.278 $\int (a+a\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=138

$$\frac{8a^2(21B+19C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7B-2C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21B+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

```
[Out] (8*a^2*(21*B + 19*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(21*B + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*B - 2*
C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a + a*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.188278, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3023, 2751, 2647, 2646}

$$\frac{8a^2(21B+19C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7B-2C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21B+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (8*a^2*(21*B + 19*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(21*B + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*B - 2*
C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a + a*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*a*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{2C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}+\frac{2\int(a+a\cos(c+$$
$$=\frac{2(7B-2C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2C(a+a$$
$$=\frac{2a(21B+19C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d}+\frac{2(7B-$$
$$=\frac{8a^2(21B+19C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(21B+19C)\sqrt{a+a\cos}}{105d}$$

Mathematica [A] time = 0.393073, size = 81, normalized size = 0.59

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((252B+253C)\cos(c+dx)+6(7B+13C)\cos(2(c+dx))+546B+15C\cos(3(c}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(546*B + 494*C + (252*B + 253*C)*Cos[c + d*x]
 + 6*(7*B + 13*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2
```

```
])/(210*d)
```

Maple [A] time = 0.065, size = 104, normalized size = 0.8

$$\frac{4\,a^2\sqrt{2}}{105\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-60\,C\,(\sin(1/2\,dx+c/2))^6+(42\,B+168\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+(-105\,B-175\,C)\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-60*C*sin(1/2*d*x+1/2*c)^6
+(42*B+168*C)*sin(1/2*d*x+1/2*c)^4+(-105*B-175*C)*sin(1/2*d*x+1/2*c)^2+105*
B+105*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.86838, size = 166, normalized size = 1.2

$$\frac{42\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}+\left(15\,\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}a\,\mathrm{si}\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/420*(42*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c
) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + (15*sqrt(2)*a*sin(7/2*d*
x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.57489, size = 236, normalized size = 1.71

$$\frac{2\left(15\,Ca\cos(dx+c)^3+3\,(7\,B+13\,C)a\cos(dx+c)^2+(63\,B+52\,C)a\cos(dx+c)+2\,(63\,B+52\,C)a\right)\sqrt{a\cos(dx+c)+}}{105\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/105*(15*C*a*cos(d*x + c)^3 + 3*(7*B + 13*C)*a*cos(d*x + c)^2 + (63*B + 52
*C)*a*cos(d*x + c) + 2*(63*B + 52*C)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.279 $\int (a+a\cos(c+dx))^{5/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=175

$$\frac{16a^2(15B+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(15B+13C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(9B-2C)\sin(c+dx)(a\cos(c+dx)+\dots}{63d}$$

```
[Out] (64*a^3*(15*B + 13*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(15*B + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(15
*B + 13*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(9*B - 2*C
)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*C*(a + a*Cos[c + d*x
])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.222564, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3023, 2751, 2647, 2646}

$$\frac{16a^2(15B+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(15B+13C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(9B-2C)\sin(c+dx)(a\cos(c+dx)+\dots}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (64*a^3*(15*B + 13*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(15*B + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(15
*B + 13*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(9*B - 2*C
)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*C*(a + a*Cos[c + d*x
])^(7/2)*Sin[c + d*x])/(9*a*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad}+\frac{2\int(a+a\cos(c+\cdots}{\cdots}$$

$$=\frac{2(9B-2C)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d}+\frac{2C(a+a\cdots}{\cdots}$$

$$=\frac{2a(15B+13C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d}+\frac{2(9B\cdots}{\cdots}$$

$$=\frac{16a^2(15B+13C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d}+\frac{2a(15\cdots}{\cdots}$$

$$=\frac{64a^3(15B+13C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2(15B+13C)\sqrt{a+a\cdots}}{315\cdots}$$

Mathematica [A] time = 0.690572, size = 105, normalized size = 0.6

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((3030B+3116C)\cos(c+dx)+8(90B+127C)\cos(2(c+dx))+90B\cos(3(c+\cdots}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(6240*B + 5653*C + (3030*B + 3116*C)*Cos[c
+ d*x] + 8*(90*B + 127*C)*Cos[2*(c + d*x)] + 90*B*Cos[3*(c + d*x)] + 260*C*
Cos[3*(c + d*x)] + 35*C*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.063, size = 123, normalized size = 0.7

$$\frac{8\,a^3\sqrt{2}}{315\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\,C\,(\sin(1/2\,dx+c/2))^8+(-90\,B-540\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(315\,B+819\,C)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*C*sin(1/2*d*x+1/2*c)^8
+(-90*B-540*C)*sin(1/2*d*x+1/2*c)^6+(315*B+819*C)*sin(1/2*d*x+1/2*c)^4+(-42
0*B-630*C)*sin(1/2*d*x+1/2*c)^2+315*B+315*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

Maxima [A] time = 1.94693, size = 232, normalized size = 1.33

$$\frac{30\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}+\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/2520*(30*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x
 + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c))*B*sqrt(a) + (35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)
*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sq
rt(a))/d
```

Fricas [A] time = 1.61618, size = 302, normalized size = 1.73

$$\frac{2 \left(35 \, C a^{2} \cos \left(dx + c\right)^{4} + 5 \, {\left(9 \, B + 26 \, C\right)} a^{2} \cos \left(dx + c\right)^{3} + 3 \, {\left(60 \, B + 73 \, C\right)} a^{2} \cos \left(dx + c\right)^{2} + {\left(345 \, B + 292 \, C\right)} a^{2} \cos \left(dx + \cdots\right.}{315 \, {\left(d \cos \left(dx + c\right) + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*C*a^2*cos(d*x + c)^4 + 5*(9*B + 26*C)*a^2*cos(d*x + c)^3 + 3*(60*
B + 73*C)*a^2*cos(d*x + c)^2 + (345*B + 292*C)*a^2*cos(d*x + c) + 2*(345*B
+ 292*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos \left(dx + c\right)^{2} + B \cos \left(dx + c\right)\right) \left(a \cos \left(dx + c\right) + a\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c) + a)^(5/2), x
)
```

3.280 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

```
[Out] -((Sqrt[2]*(B - C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(3*B - 2*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.15125, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3023, 2751, 2649, 206}

$$\frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(B - C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(3*B - 2*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{2\int \frac{\frac{aC}{2}+\frac{1}{2}a(3B-2C)\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}+(-B+C)\int \frac{}{\sqrt{a}}$$

$$= \frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{(2(B-C))\,\mathrm{Subs}}{}$$

$$= -\frac{\sqrt{2}(B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\,\mathrm{c}}}{}$$

Mathematica [A] time = 0.167282, size = 78, normalized size = 0.66

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3(B-C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6B\sin\left(\frac{1}{2}(c+dx)\right)-4C\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*(B - C)*ArcTanh[Sin[(c + d*x)/2]] + 6*B*Sin[(c + d*
x)/2] - 4*C*Sin[(c + d*x)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.102, size = 194, normalized size = 1.6

$$\frac{\sqrt{2}}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4C\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^2+6B\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*B*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-3*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a))*a+3*C*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.67185, size = 400, normalized size = 3.39

$$\frac{4\left(C\cos\left(dx+c\right)+3\,B-C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)-\frac{3\sqrt{2}\left(\left(B-C\right)a\cos\left(dx+c\right)+\left(B-C\right)a\right)\log\left(-\frac{\cos\left(dx+c\right)^2-\frac{2\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{\sqrt{a}}}{\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1}\right.}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/6*(4*(C*cos(d*x + c) + 3*B - C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 3
*sqrt(2)*((B - C)*a*cos(d*x + c) + (B - C)*a)*log(-(cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.83651, size = 153, normalized size = 1.3

$$\frac{\frac{3\sqrt{2}(B-C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\sqrt{2}(3Ba-2Ca)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\sqrt{2}Ba\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/3*(3*sqrt(2)*(B - C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(sqrt(2)*(3*B*a - 2*C*a)*tan(1/2*d*x +
 1/2*c)^2 + 3*sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 +
 a)^(3/2))/d
```

3.281 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3B-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(B-C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2C\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((3*B - 7*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2)) + (2*C*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.151733, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3019, 2751, 2649, 206}

$$\frac{(3B-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(B-C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2C\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((3*B - 7*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2)) + (2*C*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(B-C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{2}a(B-C)-2aC\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{(3B-7C)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}}{4a}$$

$$= -\frac{(B-C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{(3B-7C)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\right.}{2ad}$$

$$= \frac{(3B-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(B-C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+}{ad\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.438449, size = 104, normalized size = 0.88

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(B-4C\cos(c+dx)-5C)-(3B-7C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-((3*B - 7*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5) + Cos[(c + d*x
)/2]^3*(B - 5*C - 4*C*Cos[c + d*x])*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*
x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] time = 0.112, size = 256, normalized size = 2.2

$$\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2))^2}+2a}{\cos(1/2dx+c/2)}\right)a(\cos(1/2dx+c/2))^2B-7C\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(}{\cos(1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)*ln(2*(2*a^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x
+1/2*c)^2*B-7*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2
*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+8*C*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/si
n(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.9454, size = 493, normalized size = 4.18

$$-\frac{\sqrt{2}\left((3B-7C)\cos(dx+c)^2+2(3B-7C)\cos(dx+c)+3B-7C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/8*(sqrt(2)*((3*B - 7*C)*cos(d*x + c)^2 + 2*(3*B - 7*C)*cos(d*x + c) + 3*
B - 7*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(4*C*cos(d*x + c) - B + 5*C)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.88909, size = 177, normalized size = 1.5

$$-\frac{\frac{\left(\frac{\sqrt{2}\left(Ba^2-Ca^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{\sqrt{2}\left(Ba^2-9\,Ca^2\right)}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{\sqrt{2}(3\,B-7\,C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/4*((sqrt(2)*(B*a^2 - C*a^2)*tan(1/2*d*x + 1/2*c)^2/a^3 + sqrt(2)*(B*a^2
- 9*C*a^2)/a^3)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + s
qrt(2)*(3*B - 7*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a)))/a^(3/2))/d
```

3.282 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5B-13C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(B-C)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((5*B + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((5*B - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rubi [A] time = 0.171919, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3019, 2750, 2649, 206}

$$\frac{(5B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5B-13C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(B-C)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] ((5*B + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((5*B - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```


Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(B-C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{5}{2}a(B-C)-4aC\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5B-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(5B+19C)\int \frac{1}{\sqrt{a+a\cos}}$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5B-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(5B+19C)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(5B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(B-C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5B-13C}{16ad(a+a\,c}$$

Mathematica [A] time = 0.584768, size = 87, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left((5B-13C)\cos(c+dx)+B-9C\right)+2(5B+19C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*B + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (B - 9*C + (
5*B - 13*C)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^
(3/2))
```

Maple [B] time = 0.105, size = 292, normalized size = 2.3

$$\frac{1}{32d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5B\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+2\,a}{\cos\left(1/2\,dx+c/2\right)}\right)\left(\cos\left(1/2\,dx+c/2\right)\right)^4a+19\,C\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*B*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a+19*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+5*B*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-13*C*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.94646, size = 586, normalized size = 4.65

$$\frac{\sqrt{2}\left((5B+19C)\cos(dx+c)^3+3(5B+19C)\cos(dx+c)^2+3(5B+19C)\cos(dx+c)+5B+19C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c}{\cdots}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/64*(sqrt(2)*((5*B + 19*C)*cos(d*x + c)^3 + 3*(5*B + 19*C)*cos(d*x + c)^2
+ 3*(5*B + 19*C)*cos(d*x + c) + 5*B + 19*C)*sqrt(a)*log(-(a*cos(d*x + c)^2
- 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*B - 13*C)*cos(d*x +
 c) + B - 9*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.13397, size = 181, normalized size = 1.44

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Ba^5-Ca^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}-\frac{\sqrt{2}\left(3\,Ba^5-11\,Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(5\,B+19\,C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(B*a^5 - C*a^5)*tan(1/
2*d*x + 1/2*c)^2/a^8 - sqrt(2)*(3*B*a^5 - 11*C*a^5)/a^8)*tan(1/2*d*x + 1/2*
c) + sqrt(2)*(5*B + 19*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.283 $\int \cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=111

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10C\sin(c+d}{2}$$

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*C*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*B*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.107145, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3010, 2748, 2635, 2639, 2641}

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10C\sin(c+d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*C*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*B*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx &= \int \cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))\,dx \\
&= B\int \cos^{\frac{5}{2}}(c+dx)\,dx + C\int \cos^{\frac{7}{2}}(c+dx)\,dx \\
&= \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5} \\
&= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10C\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2B\cos^{\frac{3}{2}}(c}{} \\
&= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10C\sqrt{\cos(c+dx)}\,s}{21d}
\end{aligned}
$$

Mathematica [A] time = 0.495779, size = 77, normalized size = 0.69

$$
\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(42B\cos(c+dx)+15C\cos(2(c+dx))+65C)+126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+50CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{105d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (126*B*EllipticE[(c + d*x)/2, 2] + 50*C*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*C + 42*B*Cos[c + d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]
```

```
)/(105*d)
```

Maple [A] time = 0.128, size = 290, normalized size = 2.6

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}\cos\left(1/2\,dx+c/2\right)+\left(-168\,B-360\,C\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*B
-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.284 $\int \sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (6*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0881781, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3010, 2748, 2635, 2641, 2639}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (6*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```


Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx &= \int \cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))\,dx\\ &= B\int \cos^{\frac{3}{2}}(c+dx)\,dx+C\int \cos^{\frac{5}{2}}(c+dx)\,dx\\ &= \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{3}\\ &= \frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sin}{3d}\end{aligned}$$

Mathematica [A] time = 0.200131, size = 66, normalized size = 0.76

$$\frac{2\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5B+3C\cos(c+dx))+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+9CE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(9*C*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 0.122, size = 262, normalized size = 3.

$$-\frac{2}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}\cos\left(1/2\,dx+c/2\right)+\left(20\,B+24\,C\right)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(20*B+24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-10*B-6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.285 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0786756, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3010, 2748, 2639, 2635, 2641}

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= \int \sqrt{\cos(c+dx)}(B+C\cos(c+dx))\,dx\\ &= B\int \sqrt{\cos(c+dx)}\,dx + C\int \cos^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}C\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.107101, size = 53, normalized size = 0.87

$$\frac{2\left(3BE\left(\frac{1}{2}(c+dx)\middle|2\right)+C\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*B*EllipticE[(c + d*x)/2, 2] + C*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

Maple [B] time = 0.297, size = 229, normalized size = 3.8

$$\frac{2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-4\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+3\,B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)+B\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

3.286 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0662762, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3010, 2748, 2641, 2639}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + C\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A] time = 0.0632098, size = 35, normalized size = 1.

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Maple [A] time = 0.115, size = 152, normalized size = 4.3

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(B\mathrm{EllipticF}\left(\right.\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)+B}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c) + B)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

3.287 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/d + (2
*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0746286, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3010, 2748, 2636, 2639, 2641}

$$-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/d + (2
*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + C\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - B\int \sqrt{\cos(c+dx)}\,dx \\
&= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.137267, size = 51, normalized size = 0.89

$$\frac{2\left(-BE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{B\sin(c+dx)}{\sqrt{\cos(c+dx)}}+CF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(-(B*EllipticE[(c + d*x)/2, 2]) + C*EllipticF[(c + d*x)/2, 2] + (B*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 0.124, size = 148, normalized size = 2.6

$$-2\,{\frac{B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}{\rm EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)-2\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\right.\right.}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,d\right.\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2*(B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{C\cos\left(dx+c\right)+B}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c) + B)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

3.288 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*C*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 0.0843538, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3010, 2748, 2636, 2641, 2639}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*C*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + C\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - C\int \sqrt{\cos(c+d}$$

$$= -\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C\sin(c+dx}{d\sqrt{\cos(c+dx}}$$

Mathematica [A] time = 0.399362, size = 65, normalized size = 0.78

$$\frac{\frac{2\sin(c+dx)(B+3C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} + 2BF\left(\frac{1}{2}(c+dx)\middle|2\right) - 6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*C*EllipticE[(c + d*x)/2, 2] + 2*B*EllipticF[(c + d*x)/2, 2] + (2*(B + 3
*C*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] time = 0.277, size = 397, normalized size = 4.8

$$\frac{2}{3d}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,B\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{EllipticF}\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*B*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-12*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+6*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)+B}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c) + B)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

3.289 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6B\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*C*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0988952, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3010, 2748, 2636, 2639, 2641}

$$-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6B\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*C*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \int \frac{B+C\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\ &= B\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + C\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}C\int \frac{}{\sqrt{\cos}}\\ &= \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6B\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} -\\ &= -\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C\sin(c+dx}{3d\cos^{\frac{3}{2}}(c+d}\end{aligned}$$

Mathematica [A] time = 0.296194, size = 95, normalized size = 0.86

$$\frac{9B\sin(2(c+dx))+6B\tan(c+dx)-18B\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10C\sin(c+dx)+10C\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-18*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*C*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*C*Sin[c + d*x] + 9*B*Sin[2*(c + d*x)] + 6
*B*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 0.352, size = 502, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*B/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\cos(dx+c)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)+B}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c) + B)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

3.290 $\int \cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=132

$$\frac{(6A+5C)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6A+5C)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6A+5C)+\frac{B\sin^5(c+dx)}{5d}-\frac{2B\sin^3(c+}{3d}$$

```
[Out] ((6*A + 5*C)*x)/16 + (B*Sin[c + d*x])/d + ((6*A + 5*C)*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) + ((6*A + 5*C)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (C*Cos[c
 + d*x]^5*Sin[c + d*x])/(6*d) - (2*B*Sin[c + d*x]^3)/(3*d) + (B*Sin[c + d*x
]^5)/(5*d)
```

Rubi [A] time = 0.114, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3023, 2748, 2635, 8, 2633}

$$\frac{(6A+5C)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6A+5C)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6A+5C)+\frac{B\sin^5(c+dx)}{5d}-\frac{2B\sin^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] ((6*A + 5*C)*x)/16 + (B*Sin[c + d*x])/d + ((6*A + 5*C)*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) + ((6*A + 5*C)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (C*Cos[c
 + d*x]^5*Sin[c + d*x])/(6*d) - (2*B*Sin[c + d*x]^3)/(3*d) + (B*Sin[c + d*x
]^5)/(5*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx &= \frac{C\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^4(c+dx)(6A+5C+\\ &= \frac{C\cos^5(c+dx)\sin(c+dx)}{6d}+B\int \cos^5(c+dx)\,dx+\frac{1}{6}(6A+\\ &= \frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{C\cos^5(c+dx)\sin(c\\ &= \frac{B\sin(c+dx)}{d}+\frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(6A+\\ &= \frac{1}{16}(6A+5C)x+\frac{B\sin(c+dx)}{d}+\frac{(6A+5C)\cos(c+dx)\sin\end{aligned}$$

Mathematica [A] time = 0.275709, size = 102, normalized size = 0.77

$$\frac{5((48A+45C)\sin(2(c+dx))+(6A+9C)\sin(4(c+dx))+72Ac+72Adx+C\sin(6(c+dx))+60cC+60Cdx)+192}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (960*B*Sin[c + d*x] - 640*B*Sin[c + d*x]^3 + 192*B*Sin[c + d*x]^5 + 5*(72*A
*c + 60*c*C + 72*A*d*x + 60*C*d*x + (48*A + 45*C)*Sin[2*(c + d*x)] + (6*A +
 9*C)*Sin[4*(c + d*x)] + C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.015, size = 115, normalized size = 0.9

$$\frac{1}{d}\left(C\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{B\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+1/5*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.01, size = 155, normalized size = 1.17

$$\frac{30(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))A+64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)B-5\left(\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] 1/960*(30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A + 64*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B - 5*(4*sin(2*d*x +
 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C)/d
```

Fricas [A] time = 1.88558, size = 244, normalized size = 1.85

$$\frac{15(6A+5C)dx+\left(40C\cos(dx+c)^5+48B\cos(dx+c)^4+10(6A+5C)\cos(dx+c)^3+64B\cos(dx+c)^2+15(6A-\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/240*(15*(6*A + 5*C)*d*x + (40*C*cos(d*x + c)^5 + 48*B*cos(d*x + c)^4 + 10
*(6*A + 5*C)*cos(d*x + c)^3 + 64*B*cos(d*x + c)^2 + 15*(6*A + 5*C)*cos(d*x
+ c) + 128*B)*sin(d*x + c))/d
```

Sympy [A] time = 4.59171, size = 321, normalized size = 2.43

$$\begin{cases} \frac{3Ax\sin^4(c+dx)}{8} + \frac{3Ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Ax\cos^4(c+dx)}{8} + \frac{3A\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5A\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8B\sin^5(c+dx)}{15d} + \frac{4B}{} \\ x\left(A + B\cos(c) + C\cos^2(c)\right)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*x*sin(c + d*x)**4/8 + 3*A*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*A*x*cos(c + d*x)**4/8 + 3*A*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*sin(c + d*x)**5/(15*d) + 4*B*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d) + B*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*x
*sin(c + d*x)**6/16 + 15*C*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*C*x*si
n(c + d*x)**2*cos(c + d*x)**4/16 + 5*C*x*cos(c + d*x)**6/16 + 5*C*sin(c + d
*x)**5*cos(c + d*x)/(16*d) + 5*C*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11
*C*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos
(c)**2)*cos(c)**4, True))
```

Giac [A] time = 1.20374, size = 149, normalized size = 1.13

$$\frac{1}{16}(6A + 5C)x + \frac{C\sin(6dx + 6c)}{192d} + \frac{B\sin(5dx + 5c)}{80d} + \frac{(2A + 3C)\sin(4dx + 4c)}{64d} + \frac{5B\sin(3dx + 3c)}{48d} + \frac{(16A +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*A + 5*C)*x + 1/192*C*sin(6*d*x + 6*c)/d + 1/80*B*sin(5*d*x + 5*c)/d
 + 1/64*(2*A + 3*C)*sin(4*d*x + 4*c)/d + 5/48*B*sin(3*d*x + 3*c)/d + 1/64*(
16*A + 15*C)*sin(2*d*x + 2*c)/d + 5/8*B*sin(d*x + c)/d
```

3.291 $\int \cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=113

$$-\frac{(5A+4C)\sin^3(c+dx)}{15d}+\frac{(5A+4C)\sin(c+dx)}{5d}+\frac{B\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3B\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Bx}{8}+\frac{C}{}$$

```
[Out] (3*B*x)/8 + ((5*A + 4*C)*Sin[c + d*x])/(5*d) + (3*B*Cos[c + d*x]*Sin[c + d*
x])/(8*d) + (B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (C*Cos[c + d*x]^4*Sin[c
 + d*x])/(5*d) - ((5*A + 4*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.107283, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3023, 2748, 2633, 2635, 8}

$$-\frac{(5A+4C)\sin^3(c+dx)}{15d}+\frac{(5A+4C)\sin(c+dx)}{5d}+\frac{B\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3B\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Bx}{8}+\frac{C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*B*x)/8 + ((5*A + 4*C)*Sin[c + d*x])/(5*d) + (3*B*Cos[c + d*x]*Sin[c + d*
x])/(8*d) + (B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (C*Cos[c + d*x]^4*Sin[c
 + d*x])/(5*d) - ((5*A + 4*C)*Sin[c + d*x]^3)/(15*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^4(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^3(c+dx)(5A+4C+$$
$$= \frac{C\cos^4(c+dx)\sin(c+dx)}{5d} + B\int \cos^4(c+dx)\,dx + \frac{1}{5}(5A+$$
$$= \frac{B\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{C\cos^4(c+dx)\sin(c+dx)}{5d} + \frac{1}{4}$$
$$= \frac{(5A+4C)\sin(c+dx)}{5d} + \frac{3B\cos(c+dx)\sin(c+dx)}{8d} + \frac{B\cos}{}$$
$$= \frac{3Bx}{8} + \frac{(5A+4C)\sin(c+dx)}{5d} + \frac{3B\cos(c+dx)\sin(c+dx)}{8d}$$

Mathematica [A] time = 0.176897, size = 87, normalized size = 0.77

$$\frac{60(6A+5C)\sin(c+dx)+40A\sin(3(c+dx))+120B\sin(2(c+dx))+15B\sin(4(c+dx))+180Bc+180Bdx+50C\sin}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x] + 120*B*Sin[2*(c + d*x)]
 + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] + 15*B*Sin[4*(c + d*x)] +
6*C*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.013, size = 89, normalized size = 0.8

$$\frac{1}{d}\left(\frac{C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*(2+cos(d*x+c)^2)*sin(d*
x+c))
```

Maxima [A] time = 1.10748, size = 120, normalized size = 1.06

$$-\frac{160\left(\sin(dx+c)^3-3\sin(dx+c)\right)A-15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))B-32\left(3\sin(dx+c)^5-1\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A - 15*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c
)^3 + 15*sin(d*x + c))*C)/d
```

Fricas [A] time = 1.96333, size = 194, normalized size = 1.72

$$\frac{45Bdx+\left(24C\cos(dx+c)^4+30B\cos(dx+c)^3+8(5A+4C)\cos(dx+c)^2+45B\cos(dx+c)+80A+64C\right)\sin(dx}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/120*(45*B*d*x + (24*C*cos(d*x + c)^4 + 30*B*cos(d*x + c)^3 + 8*(5*A + 4*C
)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 80*A + 64*C)*sin(d*x + c))/d
```

Sympy [A] time = 2.39523, size = 209, normalized size = 1.85

$$\begin{cases} \frac{2A\sin^3(c+dx)}{3d} + \frac{A\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Bx\sin^4(c+dx)}{8} + \frac{3Bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Bx\cos^4(c+dx)}{8} + \frac{3B\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5B\sin \\ x\left(A + B\cos(c) + C\cos^2(c)\right)\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((2*A*sin(c + d*x)**3/(3*d) + A*sin(c + d*x)*cos(c + d*x)**2/d + 3
*B*x*sin(c + d*x)**4/8 + 3*B*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*x*co
s(c + d*x)**4/8 + 3*B*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*B*sin(c + d*x)
*cos(c + d*x)**3/(8*d) + 8*C*sin(c + d*x)**5/(15*d) + 4*C*sin(c + d*x)**3*c
os(c + d*x)**2/(3*d) + C*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A +
 B*cos(c) + C*cos(c)**2)*cos(c)**3, True))
```

Giac [A] time = 1.2193, size = 120, normalized size = 1.06

$$\frac{3}{8}Bx + \frac{C\sin(5dx + 5c)}{80d} + \frac{B\sin(4dx + 4c)}{32d} + \frac{(4A + 5C)\sin(3dx + 3c)}{48d} + \frac{B\sin(2dx + 2c)}{4d} + \frac{(6A + 5C)\sin(dx +}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 3/8*B*x + 1/80*C*sin(5*d*x + 5*c)/d + 1/32*B*sin(4*d*x + 4*c)/d + 1/48*(4*A
 + 5*C)*sin(3*d*x + 3*c)/d + 1/4*B*sin(2*d*x + 2*c)/d + 1/8*(6*A + 5*C)*sin
(d*x + c)/d
```

3.292 $\int \cos^2(c+dx) \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=88

$$\frac{(4A+3C)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}x(4A+3C) - \frac{B\sin^3(c+dx)}{3d} + \frac{B\sin(c+dx)}{d} + \frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

```
[Out] ((4*A + 3*C)*x)/8 + (B*Sin[c + d*x])/d + ((4*A + 3*C)*Cos[c + d*x]*Sin[c +
d*x])/(8*d) + (C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (B*Sin[c + d*x]^3)/(3
*d)
```

Rubi [A] time = 0.0945887, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3023, 2748, 2635, 8, 2633}

$$\frac{(4A+3C)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}x(4A+3C) - \frac{B\sin^3(c+dx)}{3d} + \frac{B\sin(c+dx)}{d} + \frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x)/8 + (B*Sin[c + d*x])/d + ((4*A + 3*C)*Cos[c + d*x]*Sin[c +
d*x])/(8*d) + (C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (B*Sin[c + d*x]^3)/(3
*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx)(4A+3C+$$

$$= \frac{C\cos^3(c+dx)\sin(c+dx)}{4d}+B\int \cos^3(c+dx)\,dx+\frac{1}{4}(4A+$$

$$= \frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+$$

$$= \frac{1}{8}(4A+3C)x+\frac{B\sin(c+dx)}{d}+\frac{(4A+3C)\cos(c+dx)\sin($$

Mathematica [A] time = 0.153774, size = 70, normalized size = 0.8

$$\frac{24(A+C)\sin(2(c+dx))+48Ac+48Adx-32B\sin^3(c+dx)+96B\sin(c+dx)+3C\sin(4(c+dx))+36cC+36Cdx}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 96*B*Sin[c + d*x] - 32*B*Sin[c + d
*x]^3 + 24*(A + C)*Sin[2*(c + d*x)] + 3*C*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.013, size = 84, normalized size = 1.

$$\frac{1}{d}\left(C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+A\left(\frac{\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*(
2+cos(d*x+c)^2)*sin(d*x+c)+A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.967407, size = 104, normalized size = 1.18

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)B+3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A - 32*(sin(d*x + c)^3 - 3*sin(d*
x + c))*B + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C)/d
```

Fricas [A] time = 1.9633, size = 163, normalized size = 1.85

$$\frac{3\,(4\,A+3\,C)dx+\left(6\,C\cos(dx+c)^3+8\,B\cos(dx+c)^2+3\,(4\,A+3\,C)\cos(dx+c)+16\,B\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/24*(3*(4*A + 3*C)*d*x + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A
 + 3*C)*cos(d*x + c) + 16*B)*sin(d*x + c))/d
```

Sympy [A] time = 1.29187, size = 197, normalized size = 2.24

$$\begin{cases} \frac{Ax\sin^2(c+dx)}{2} + \frac{Ax\cos^2(c+dx)}{2} + \frac{A\sin(c+dx)\cos(c+dx)}{2d} + \frac{2B\sin^3(c+dx)}{3d} + \frac{B\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Cx\sin^4(c+dx)}{8} + \frac{3Cx\sin^2(c+dx)\cos^2(c+dx)}{4} \\ x\left(A + B\cos(c) + C\cos^2(c)\right)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*x*sin(c + d*x)**2/2 + A*x*cos(c + d*x)**2/2 + A*sin(c + d*x)*c
os(c + d*x)/(2*d) + 2*B*sin(c + d*x)**3/(3*d) + B*sin(c + d*x)*cos(c + d*x)
**2/d + 3*C*x*sin(c + d*x)**4/8 + 3*C*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 3*C*x*cos(c + d*x)**4/8 + 3*C*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*sin
(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)
*cos(c)**2, True))
```

Giac [A] time = 1.17457, size = 95, normalized size = 1.08

$$\frac{1}{8}(4A + 3C)x + \frac{C\sin(4dx + 4c)}{32d} + \frac{B\sin(3dx + 3c)}{12d} + \frac{(A + C)\sin(2dx + 2c)}{4d} + \frac{3B\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*A + 3*C)*x + 1/32*C*sin(4*d*x + 4*c)/d + 1/12*B*sin(3*d*x + 3*c)/d +
 1/4*(A + C)*sin(2*d*x + 2*c)/d + 3/4*B*sin(d*x + c)/d
```

3.293 $\int \cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=69

$$\frac{(3A+2C)\sin(c+dx)}{3d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] (B*x)/2 + ((3*A + 2*C)*Sin[c + d*x])/(3*d) + (B*Cos[c + d*x]*Sin[c + d*x])/
(2*d) + (C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0470357, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {3023, 2734}

$$\frac{(3A+2C)\sin(c+dx)}{3d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (B*x)/2 + ((3*A + 2*C)*Sin[c + d*x])/(3*d) + (B*Cos[c + d*x]*Sin[c + d*x])/
(2*d) + (C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\int \cos(c+dx)(3A+2C+3B$$
$$= \frac{Bx}{2}+\frac{(3A+2C)\sin(c+dx)}{3d}+\frac{B\cos(c+dx)\sin(c+dx)}{2d}+$$

Mathematica [A] time = 0.0937808, size = 53, normalized size = 0.77

$$\frac{3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+6Bc+6Bdx+C\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + C*Si
n[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.014, size = 57, normalized size = 0.8

$$\frac{1}{d}\left(\frac{C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+A\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+A*sin(d*x+c))
```

Maxima [A] time = 0.988847, size = 74, normalized size = 1.07

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)C+12\,A\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B - 4*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*C + 12*A*sin(d*x + c))/d
```

Fricas [A] time = 1.87581, size = 113, normalized size = 1.64

$$\frac{3\,Bdx + \left(2\,C\cos\left(dx+c\right)^2 + 3\,B\cos\left(dx+c\right) + 6\,A + 4\,C\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*B*d*x + (2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sin(d*x
+ c))/d
```

Sympy [A] time = 0.60504, size = 107, normalized size = 1.55

$$\begin{cases} \frac{A\sin\left(c+dx\right)}{d} + \frac{Bx\sin^2\left(c+dx\right)}{2} + \frac{Bx\cos^2\left(c+dx\right)}{2} + \frac{B\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} + \frac{2C\sin^3\left(c+dx\right)}{3d} + \frac{C\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d} & \text{for}\ d \neq 0 \\ x\left(A + B\cos\left(c\right) + C\cos^2\left(c\right)\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*sin(c + d*x)/d + B*x*sin(c + d*x)**2/2 + B*x*cos(c + d*x)**2/2
 + B*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*sin(c + d*x)**3/(3*d) + C*sin(c
+ d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*cos(c)
, True))
```

Giac [A] time = 1.19979, size = 72, normalized size = 1.04

$$\frac{1}{2}\,Bx + \frac{C\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{B\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{\left(4\,A+3\,C\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*B*x + 1/12*C*sin(3*d*x + 3*c)/d + 1/4*B*sin(2*d*x + 2*c)/d + 1/4*(4*A +
 3*C)*sin(d*x + c)/d
```

3.294 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=41

$$Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}$$

```
[Out] A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0246001, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2637, 2635, 8}

$$Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Cos[c + d*x] + C*Cos[c + d*x]^2,x]
```

```
[Out] A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx = Ax + B\int \cos(c+dx)\,dx + C\int \cos^2(c+dx)\,dx$$
$$= Ax + \frac{B\sin(c+dx)}{d} + \frac{C\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}C\int 1\,dx$$
$$= Ax + \frac{Cx}{2} + \frac{B\sin(c+dx)}{d} + \frac{C\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0443743, size = 55, normalized size = 1.34

$$Ax + \frac{B\sin(c)\cos(dx)}{d} + \frac{B\cos(c)\sin(dx)}{d} + \frac{C(c+dx)}{2d} + \frac{C\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Cos[c + d*x] + C*Cos[c + d*x]^2,x]
```

```
[Out] A*x + (C*(c + d*x))/(2*d) + (B*Cos[d*x]*Sin[c])/d + (B*Cos[c]*Sin[d*x])/d +
 (C*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.003, size = 43, normalized size = 1.1

$$Ax + \frac{B\sin(dx+c)}{d} + \frac{C}{d}\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A+B*cos(d*x+c)+C*cos(d*x+c)^2,x)
```

```
[Out] A*x+B*sin(d*x+c)/d+C/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

Maxima [A] time = 1.00092, size = 51, normalized size = 1.24

$$Ax + \frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))C}{4\,d} + \frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*cos(d*x+c)+C*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] A*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*C/d + B*sin(d*x + c)/d
```

Fricas [A] time = 1.84824, size = 82, normalized size = 2.

$$\frac{(2A+C)dx + (C\cos(dx+c) + 2B)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*cos(d*x+c)+C*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*A + C)*d*x + (C*cos(d*x + c) + 2*B)*sin(d*x + c))/d
```

Sympy [A] time = 0.277103, size = 66, normalized size = 1.61

$$Ax + B\left(\begin{cases} \frac{\sin(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}\right) + C\left(\begin{cases} \frac{x\sin^2(c+dx)}{2} + \frac{x\cos^2(c+dx)}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x\cos^2(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*cos(d*x+c)+C*cos(d*x+c)**2,x)
```

```
[Out] A*x + B*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + C*Piecewi
se((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(
2*d), Ne(d, 0)), (x*cos(c)**2, True))
```

Giac [A] time = 1.18851, size = 47, normalized size = 1.15

$$\frac{1}{4}C\left(2x + \frac{\sin(2dx+2c)}{d}\right) + Ax + \frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*cos(d*x+c)+C*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*C*(2*x + sin(2*d*x + 2*c)/d) + A*x + B*sin(d*x + c)/d
```

3.295 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{A\tanh^{-1}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}$$

```
[Out] B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d
```

Rubi [A] time = 0.0521309, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3023, 2735, 3770}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx &= \frac{C\sin(c+dx)}{d}+\int (A+B\cos(c+dx))\sec(c+dx)\,dx\\
&= Bx+\frac{C\sin(c+dx)}{d}+A\int \sec(c+dx)\,dx\\
&= Bx+\frac{A\tanh^{-1}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0198298, size = 38, normalized size = 1.41

$$
\frac{A\tanh^{-1}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c)\cos(dx)}{d}+\frac{C\cos(c)\sin(dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d
*x])/d
```

Maple [A] time = 0.032, size = 41, normalized size = 1.5

$$
Bx+\frac{A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bc}{d}+\frac{C\sin(dx+c)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] B*x+1/d*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*c+C*sin(d*x+c)/d
```

Maxima [A] time = 0.973572, size = 49, normalized size = 1.81

$$
\frac{(dx+c)B+A\log(\sec(dx+c)+\tan(dx+c))+C\sin(dx+c)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*B + A*log(sec(d*x + c) + tan(d*x + c)) + C*sin(d*x + c))/d
```

Fricas [A] time = 1.89053, size = 120, normalized size = 4.44

$$\frac{2\,Bdx + A\log\left(\sin\left(dx+c\right)+1\right) - A\log\left(-\sin\left(dx+c\right)+1\right) + 2\,C\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*d*x + A*log(sin(d*x + c) + 1) - A*log(-sin(d*x + c) + 1) + 2*C*sin
(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A + B\cos\left(c+dx\right) + C\cos^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Giac [B] time = 1.21668, size = 95, normalized size = 3.52

$$\frac{(dx+c)B + A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B + A*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 2*C*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.296 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{A\tan(c+dx)}{d}+\frac{B\tanh^{-1}(\sin(c+dx))}{d}+Cx$$

```
[Out] C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d
```

Rubi [A] time = 0.0540409, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3021, 2735, 3770}

$$\frac{A\tan(c+dx)}{d}+\frac{B\tanh^{-1}(\sin(c+dx))}{d}+Cx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A\tan(c+dx)}{d}+\int (B+C\cos(c+dx))\sec(c+dx)\,dx$$
$$= Cx+\frac{A\tan(c+dx)}{d}+B\int \sec(c+dx)\,dx$$
$$= Cx+\frac{B\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0185488, size = 27, normalized size = 1.

$$\frac{A\tan(c+dx)}{d}+\frac{B\tanh^{-1}(\sin(c+dx))}{d}+Cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d
```

Maple [A] time = 0.035, size = 41, normalized size = 1.5

$$Cx+\frac{A\tan(dx+c)}{d}+\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] C*x+A*tan(d*x+c)/d+1/d*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*c
```

Maxima [A] time = 0.972369, size = 62, normalized size = 2.3

$$\frac{2(dx+c)C+B(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2A\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(d*x + c)*C + B*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*
A*tan(d*x + c))/d
```

Fricas [B] time = 1.94841, size = 193, normalized size = 7.15

$$\frac{2\,Cdx\cos\left(dx+c\right)+B\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-B\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,A\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*C*d*x*cos(d*x + c) + B*cos(d*x + c)*log(sin(d*x + c) + 1) - B*cos(d*
x + c)*log(-sin(d*x + c) + 1) + 2*A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2, x)
```

Giac [B] time = 1.26282, size = 95, normalized size = 3.52

$$\frac{(dx+c)C+B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*C + B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*A*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.297 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}+\frac{B\tan(c+dx)}{d}$$

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (A*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0790041, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3021, 2748, 3767, 8, 3770}

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}+\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (A*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= \frac{A\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (2B+(A+2C)\cos(c+dx)) \\ &= \frac{A\sec(c+dx)\tan(c+dx)}{2d}+B\int \sec^2(c+dx)\,dx+\frac{1}{2}(A+2C \\ &= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\sec(c+dx)\tan(c+dx)}{2d}- \\ &= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)}{d}+\frac{A\sec(c+d}{}\end{aligned}$$

Mathematica [A] time = 0.0202146, size = 59, normalized size = 1.16

$$\frac{A\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}+\frac{B\tan(c+dx)}{d}+\frac{C\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(2*d) + (C*ArcTanh[Sin[c + d*x]])/d + (B*Tan[c +
d*x])/d + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.038, size = 70, normalized size = 1.4

$$\frac{A\sec(dx+c)\tan(dx+c)}{2d}+\frac{A\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{B\tan(dx+c)}{d}+\frac{C\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*A*ln(sec(d*x+c)+tan(d*x+c))+B*tan(d*x+c
)/d+1/d*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00499, size = 111, normalized size = 2.18

$$-\frac{A\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2\,C(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(A*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 2*C*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) -
4*B*tan(d*x + c))/d
```

Fricas [A] time = 2.02768, size = 220, normalized size = 4.31

$$\frac{(A+2\,C)\cos(dx+c)^2\log(\sin(dx+c)+1)-(A+2\,C)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(2\,B\cos(dx+c)+A)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas
")
```

```
[Out] 1/4*((A + 2*C)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*C)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*(2*B*cos(d*x + c) + A)*sin(d*x + c))/(d*cos(
d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27152, size = 153, normalized size = 3.

$$\frac{(A+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(A+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+A\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*((A + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A + 2*C)*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) + 2*(A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2
*c)^3 + A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^2)/d
```

3.298 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=78

$$\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*A + 3*C)*Tan[c + d*x])/(3*d) + (B*Sec
[c + d*x]*Tan[c + d*x])/(2*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.10269, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*A + 3*C)*Tan[c + d*x])/(3*d) + (B*Sec
[c + d*x]*Tan[c + d*x])/(2*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx &= \frac{A\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (3B+(2A+3C)\cos(c+dx \\ &= \frac{A\sec^2(c+dx)\tan(c+dx)}{3d}+B\int \sec^3(c+dx)\,dx+\frac{1}{3}(2A+3 \\ &= \frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{A\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{2}B \\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{B\sec(c+\end{aligned}$$

Mathematica [A] time = 0.204667, size = 51, normalized size = 0.65

$$\frac{\tan(c+dx)\left(2A\tan^2(c+dx)+6(A+C)+3B\sec(c+dx)\right)+3B\tanh^{-1}(\sin(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + C) + 3*B*Sec[c + d*x] + 2
*A*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.038, size = 83, normalized size = 1.1

$$\frac{2A\tan(dx+c)}{3d}+\frac{A(\sec(dx+c))^2\tan(dx+c)}{3d}+\frac{B\sec(dx+c)\tan(dx+c)}{2d}+\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 2/3*A*tan(d*x+c)/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2*B*sec(d*x+c)*tan(d*x
+c)/d+1/2/d*B*ln(sec(d*x+c)+tan(d*x+c))+C*tan(d*x+c)/d
```

Maxima [A] time = 0.976953, size = 107, normalized size = 1.37

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)A-3B\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12C\tan(dx+}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima
")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A - 3*B*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*C*tan(d*x
 + c))/d
```

Fricas [A] time = 1.96641, size = 250, normalized size = 3.21

$$\frac{3B\cos(dx+c)^3\log(\sin(dx+c)+1)-3B\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2(2A+3C)\cos(dx+c)^2+3B\cos\right.}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas
")
```

```
[Out] 1/12*(3*B*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*B*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)
*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33062, size = 219, normalized size = 2.81

$$\frac{3\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,A}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*B*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 6*
C*tan(1/2*d*x + 1/2*c)^5 - 4*A*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x +
1/2*c)^3 + 6*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.299 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}+$$

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (B*Tan[c + d*x])/d + ((3*A + 4*
C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (B*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.103897, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3021, 2748, 3767, 3768, 3770}

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```



```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (B*Tan[c + d*x])/d + ((3*A + 4*
C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (B*Tan[c + d*x]^3)/(3*d)
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx &= \frac{A\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (4B + (3A+4C)\cos(c+d\\ &= \frac{A\sec^3(c+dx)\tan(c+dx)}{4d} + B\int \sec^4(c+dx)\,dx + \frac{1}{4}(3A+\\ &= \frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{A\sec^3(c+dx)\tan(c+\\ &= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{B\tan(c+dx)}{d} + \frac{(3A+4C)\end{aligned}$$

Mathematica [A] time = 0.256505, size = 71, normalized size = 0.73

$$\frac{\tan(c+dx)\left(3(3A+4C)\sec(c+dx) + 6A\sec^3(c+dx) + 8B\left(\tan^2(c+dx)+3\right)\right) + 3(3A+4C)\tanh^{-1}(\sin(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(3*A + 4*C)*Sec[c +
d*x] + 6*A*Sec[c + d*x]^3 + 8*B*(3 + Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.04, size = 130, normalized size = 1.3

$$\frac{A(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3A\sec(dx+c)\tan(dx+c)}{8d}+\frac{3A\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2B\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/4*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*A*ln(se
c(d*x+c)+tan(d*x+c))+2/3*B*tan(d*x+c)/d+1/3/d*B*tan(d*x+c)*sec(d*x+c)^2+1/2
*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00276, size = 188, normalized size = 1.94

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)B-3A\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima
")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B - 3*A*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 2.00681, size = 306, normalized size = 3.15

$$\frac{3(3A+4C)\cos(dx+c)^4\log(\sin(dx+c)+1)-3(3A+4C)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16B\cos(dx+\right.}{48d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas
")
```

```
[Out] 1/48*(3*(3*A + 4*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*C)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C
)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32384, size = 311, normalized size = 3.21

$$\frac{3\,(3\,A+4\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,A+4\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*A + 4*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A + 4*C)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*tan(1/2*d*x + 1/2*c)^7 - 24*B*tan(
1/2*d*x + 1/2*c)^7 + 12*C*tan(1/2*d*x + 1/2*c)^7 + 9*A*tan(1/2*d*x + 1/2*c)
^5 + 40*B*tan(1/2*d*x + 1/2*c)^5 - 12*C*tan(1/2*d*x + 1/2*c)^5 + 9*A*tan(1/
2*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^
3 + 15*A*tan(1/2*d*x + 1/2*c) + 24*B*tan(1/2*d*x + 1/2*c) + 12*C*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.300 $\int \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=122

$$\frac{(4A + 5C)\tan^3(c + dx)}{15d} + \frac{(4A + 5C)\tan(c + dx)}{5d} + \frac{A\tan(c + dx)\sec^4(c + dx)}{5d} + \frac{3B\tanh^{-1}(\sin(c + dx))}{8d} + \frac{B\tan(c +$$

```
[Out] (3*B*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*A + 5*C)*Tan[c + d*x])/(5*d) + (3*B
*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*A + 5*C)*Tan[c + d*x]^3)/(15*d
)
```

Rubi [A] time = 0.118319, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3021, 2748, 3768, 3770, 3767}

$$\frac{(4A + 5C)\tan^3(c + dx)}{15d} + \frac{(4A + 5C)\tan(c + dx)}{5d} + \frac{A\tan(c + dx)\sec^4(c + dx)}{5d} + \frac{3B\tanh^{-1}(\sin(c + dx))}{8d} + \frac{B\tan(c +$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] (3*B*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*A + 5*C)*Tan[c + d*x])/(5*d) + (3*B
*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*A + 5*C)*Tan[c + d*x]^3)/(15*d
)
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx &= \frac{A\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (5B+(4A+5C)\cos(c+dx\\ &= \frac{A\sec^4(c+dx)\tan(c+dx)}{5d}+B\int \sec^5(c+dx)\,dx+\frac{1}{5}(4A+5\\ &= \frac{B\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{A\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{4}(\\ &= \frac{(4A+5C)\tan(c+dx)}{5d}+\frac{3B\sec(c+dx)\tan(c+dx)}{8d}+\frac{B\sec^3}{}\\ &= \frac{3B\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4A+5C)\tan(c+dx)}{5d}+\frac{3B\sec(c}{}\end{aligned}$$

Mathematica [A] time = 0.560577, size = 80, normalized size = 0.66

$$\frac{\tan(c+dx)\left(8\left(5(2A+C)\tan^2(c+dx)+3A\tan^4(c+dx)+15(A+C)\right)+30B\sec^3(c+dx)+45B\sec(c+dx)\right)+45B\,\mathrm{ta}}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (45*B*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(45*B*Sec[c + d*x] + 30*B*Sec[c
+ d*x]^3 + 8*(15*(A + C) + 5*(2*A + C)*Tan[c + d*x]^2 + 3*A*Tan[c + d*x]^4)
))/(120*d)
```

Maple [A] time = 0.042, size = 144, normalized size = 1.2

$$\frac{8\,A\tan\left(dx+c\right)}{15\,d}+\frac{A\left(\sec\left(dx+c\right)\right)^{4}\tan\left(dx+c\right)}{5\,d}+\frac{4\,A\left(\sec\left(dx+c\right)\right)^{2}\tan\left(dx+c\right)}{15\,d}+\frac{B\left(\sec\left(dx+c\right)\right)^{3}\tan\left(dx+c\right)}{4\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 8/15*A*tan(d*x+c)/d+1/5*A*sec(d*x+c)^4*tan(d*x+c)/d+4/15*A*sec(d*x+c)^2*tan
(d*x+c)/d+1/4*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8*B*sec(d*x+c)*tan(d*x+c)/d+3/8
/d*B*ln(sec(d*x+c)+tan(d*x+c))+2/3*C*tan(d*x+c)/d+1/3/d*C*tan(d*x+c)*sec(d*
x+c)^2
```

Maxima [A] time = 0.988888, size = 171, normalized size = 1.4

$$\frac{16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)A+80\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)C-15\,B\left(\frac{2\left(3\,\sin(dx+c)^{3}\right.}{\sin(dx+c)^{4}-2\,\mathrm{s}}\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima
")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A + 80*(
tan(d*x + c)^3 + 3*tan(d*x + c))*C - 15*B*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
+ c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 2.01564, size = 329, normalized size = 2.7

$$\frac{45\,B\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-45\,B\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(4\,A+5\,C\right)\cos\left(dx+c\right)^{4}+4\right.}{240\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas
")
```

```
[Out] 1/240*(45*B*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*B*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 2*(16*(4*A + 5*C)*cos(d*x + c)^4 + 45*B*cos(d*x + c)^3
 + 8*(4*A + 5*C)*cos(d*x + c)^2 + 30*B*cos(d*x + c) + 24*A)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26249, size = 332, normalized size = 2.72

$$\frac{45\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+120\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(45*B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*B*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(120*A*tan(1/2*d*x + 1/2*c)^9 - 75*B*tan(1/2*d*x + 1/2*c)
^9 + 120*C*tan(1/2*d*x + 1/2*c)^9 - 160*A*tan(1/2*d*x + 1/2*c)^7 + 30*B*tan
(1/2*d*x + 1/2*c)^7 - 320*C*tan(1/2*d*x + 1/2*c)^7 + 464*A*tan(1/2*d*x + 1/
2*c)^5 + 400*C*tan(1/2*d*x + 1/2*c)^5 - 160*A*tan(1/2*d*x + 1/2*c)^3 - 30*B
*tan(1/2*d*x + 1/2*c)^3 - 320*C*tan(1/2*d*x + 1/2*c)^3 + 120*A*tan(1/2*d*x
+ 1/2*c) + 75*B*tan(1/2*d*x + 1/2*c) + 120*C*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^5)/d
```

3.301 $\int \cos^2(c+dx)(a+a\cos(c+dx))\left(A + B\cos(c + dx) + C\cos^2\right.$

Optimal. Leaf size=143

$$-\frac{a(5A + 5B + 4C)\sin^3(c + dx)}{15d} + \frac{a(5A + 5B + 4C)\sin(c + dx)}{5d} + \frac{a(4A + 3(B + C))\sin(c + dx)\cos(c + dx)}{8d} + \frac{1}{8}ax(4A$$

```
[Out] (a*(4*A + 3*(B + C))*x)/8 + (a*(5*A + 5*B + 4*C)*Sin[c + d*x])/(5*d) + (a*(
4*A + 3*(B + C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(B + C)*Cos[c + d*x]
^3*Sin[c + d*x])/(4*d) + (a*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A
+ 5*B + 4*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.228879, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{a(5A + 5B + 4C)\sin^3(c + dx)}{15d} + \frac{a(5A + 5B + 4C)\sin(c + dx)}{5d} + \frac{a(4A + 3(B + C))\sin(c + dx)\cos(c + dx)}{8d} + \frac{1}{8}ax(4A$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (a*(4*A + 3*(B + C))*x)/8 + (a*(5*A + 5*B + 4*C)*Sin[c + d*x])/(5*d) + (a*(
4*A + 3*(B + C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(B + C)*Cos[c + d*x]
^3*Sin[c + d*x])/(4*d) + (a*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A
+ 5*B + 4*C)*Sin[c + d*x]^3)/(15*d)
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{aC\cos^4(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^2(c$$

$$= \frac{a(B+C)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aC\,\mathrm{co}}{}$$

$$= \frac{a(B+C)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aC\,\mathrm{co}}{}$$

$$= \frac{a(4A+3(B+C))\cos(c+dx)\sin(c+dx)}{8d}$$

$$= \frac{1}{8}a(4A+3(B+C))x+\frac{a(5A+5B+4C)\,\mathrm{s}}{5d}$$

Mathematica [A] time = 0.44178, size = 93, normalized size = 0.65

$$\frac{a\left(-160(A+B+2C)\sin^3(c+dx)+480(A+B+C)\sin(c+dx)+15(4(4A+3(B+C))(c+dx)+8(A+B+C)\sin(2(c+dx)) \cdots\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (a*(480*(A + B + C)*Sin[c + d*x] - 160*(A + B + 2*C)*Sin[c + d*x]^3 + 96*C*
Sin[c + d*x]^5 + 15*(4*(4*A + 3*(B + C))*(c + d*x) + 8*(A + B + C)*Sin[2*(c
 + d*x)] + (B + C)*Sin[4*(c + d*x)])))/(480*d)
```

Maple [A] time = 0.026, size = 173, normalized size = 1.2

$$\frac{1}{d}\left(\frac{aC\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3a}{8}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*C*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.0114, size = 224, normalized size = 1.57

$$-\frac{160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-120(2dx+2c+\sin(2dx+2c))Aa+160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 120*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 15*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a - 32*(3*sin(d*x + c)^5
- 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a - 15*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*C*a)/d
```

Fricas [A] time = 1.87631, size = 282, normalized size = 1.97

$$\frac{15\,(4\,A+3\,B+3\,C)adx+\left(24\,Ca\cos\left(dx+c\right)^{4}+30\,(B+C)a\cos\left(dx+c\right)^{3}+8\,(5\,A+5\,B+4\,C)a\cos\left(dx+c\right)^{2}+15\,(4\,A}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(4*A + 3*B + 3*C)*a*d*x + (24*C*a*cos(d*x + c)^4 + 30*(B + C)*a*c
os(d*x + c)^3 + 8*(5*A + 5*B + 4*C)*a*cos(d*x + c)^2 + 15*(4*A + 3*B + 3*C)
*a*cos(d*x + c) + 16*(5*A + 5*B + 4*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 2.95936, size = 428, normalized size = 2.99

$$\begin{cases} \frac{Aax\sin^{2}(c+dx)}{2}+\frac{Aax\cos^{2}(c+dx)}{2}+\frac{2Aa\sin^{3}(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bax\sin^{4}(c+dx)}{8}+\frac{3Bax\sin^{2}(c+dx)}{4} \\ x\left(a\cos(c)+a\right)\left(A+B\cos(c)+C\cos^{2}(c)\right)\cos^{2}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + 2*A*a*sin(c
+ d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + A*a*sin(c + d*x)*cos
(c + d*x)/(2*d) + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 3*B*a*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 2*B*a*sin(c + d*x)**3/(3*d) + 5*B*a*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*a*x*sin(c + d*x)**4/8 + 3
*C*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 8*C*
a*sin(c + d*x)**5/(15*d) + 4*C*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*
C*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + C*a*sin(c + d*x)*cos(c + d*x)**4/d
```

```
 + 5*C*a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + a)*(
A + B*cos(c) + C*cos(c)**2)*cos(c)**2, True))
```

Giac [A] time = 1.26807, size = 174, normalized size = 1.22

$$\frac{1}{8}(4Aa + 3Ba + 3Ca)x + \frac{Ca\sin(5dx + 5c)}{80d} + \frac{(Ba + Ca)\sin(4dx + 4c)}{32d} + \frac{(4Aa + 4Ba + 5Ca)\sin(3dx + 3c)}{48d} + \frac{(A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/8*(4*A*a + 3*B*a + 3*C*a)*x + 1/80*C*a*sin(5*d*x + 5*c)/d + 1/32*(B*a + C
*a)*sin(4*d*x + 4*c)/d + 1/48*(4*A*a + 4*B*a + 5*C*a)*sin(3*d*x + 3*c)/d +
1/4*(A*a + B*a + C*a)*sin(2*d*x + 2*c)/d + 1/8*(6*A*a + 6*B*a + 5*C*a)*sin(
d*x + c)/d
```

3.302 $\int \cos(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(\right.$

Optimal. Leaf size=118

$$\frac{a(3A+2(B+C))\sin(c+dx)}{3d}+\frac{a(4A+4B+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+4B+3C)+\frac{a(B+C)\sin(c+dx)}{3d}$$

```
[Out] (a*(4*A + 4*B + 3*C)*x)/8 + (a*(3*A + 2*(B + C))*Sin[c + d*x])/(3*d) + (a*(
4*A + 4*B + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(B + C)*Cos[c + d*x]
^2*Sin[c + d*x])/(3*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.138148, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3033, 3023, 2734}

$$\frac{a(3A+2(B+C))\sin(c+dx)}{3d}+\frac{a(4A+4B+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+4B+3C)+\frac{a(B+C)\sin(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (a*(4*A + 4*B + 3*C)*x)/8 + (a*(3*A + 2*(B + C))*Sin[c + d*x])/(3*d) + (a*(
4*A + 4*B + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(B + C)*Cos[c + d*x]
^2*Sin[c + d*x])/(3*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos(c$$

$$= \frac{a(B+C)\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{aC\,\mathrm{co}}{}$$

$$= \frac{1}{8}a(4A+4B+3C)x+\frac{a(3A+2(B+C))\,\mathrm{s}}{3d}$$

Mathematica [A] time = 0.395352, size = 96, normalized size = 0.81

$$\frac{a(24(4A+3(B+C))\sin(c+dx)+24(A+B+C)\sin(2(c+dx))+48Adx+8B\sin(3(c+dx))+48Bc+48Bdx+8C\,\mathrm{si}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (a*(48*B*c + 24*c*C + 48*A*d*x + 48*B*d*x + 36*C*d*x + 24*(4*A + 3*(B + C))
*Sin[c + d*x] + 24*(A + B + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 8*
C*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.024, size = 141, normalized size = 1.2

$$\frac{1}{d}\left(aC\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ba\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aC\left(2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*
x+1/2*c)+a*A*sin(d*x+c))
```

Maxima [A] time = 0.987671, size = 178, normalized size = 1.51

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba+24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba-32\left(\sin\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*B*a + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 32*(sin(d*x + c)^
3 - 3*sin(d*x + c))*C*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*C*a + 96*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.89475, size = 221, normalized size = 1.87

$$\frac{3\,(4\,A+4\,B+3\,C)adx+\left(6\,Ca\cos(dx+c)^3+8\,(B+C)a\cos(dx+c)^2+3\,(4\,A+4\,B+3\,C)a\cos(dx+c)+8\,(3\,A+2\,B\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/24*(3*(4*A + 4*B + 3*C)*a*d*x + (6*C*a*cos(d*x + c)^3 + 8*(B + C)*a*cos(d
*x + c)^2 + 3*(4*A + 4*B + 3*C)*a*cos(d*x + c) + 8*(3*A + 2*B + 2*C)*a)*sin
(d*x + c))/d
```

Sympy [A] time = 1.42431, size = 320, normalized size = 2.71

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{Aa\sin(c+dx)}{d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{2Ba\sin^3(c+dx)}{3d} + \frac{Ba\sin(\\ x(a\cos(c)+a)\left(A+B\cos(c)+C\cos^2(c)\right)\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(
c + d*x)**2/d + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*C*a*x*sin(c + d*x)*
*4/8 + 3*C*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/
8 + 3*C*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*C*a*sin(c + d*x)**3/(3*d)
+ 5*C*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a*sin(c + d*x)*cos(c + d*x)*
*2/d, Ne(d, 0)), (x*(a*cos(c) + a)*(A + B*cos(c) + C*cos(c)**2)*cos(c), Tru
e))
```

Giac [A] time = 1.23747, size = 138, normalized size = 1.17

$$\frac{1}{8}(4Aa+4Ba+3Ca)x + \frac{Ca\sin(4dx+4c)}{32d} + \frac{(Ba+Ca)\sin(3dx+3c)}{12d} + \frac{(Aa+Ba+Ca)\sin(2dx+2c)}{4d} + \frac{(4Aa+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/8*(4*A*a + 4*B*a + 3*C*a)*x + 1/32*C*a*sin(4*d*x + 4*c)/d + 1/12*(B*a + C
*a)*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a + C*a)*sin(2*d*x + 2*c)/d + 1/4*(4*
A*a + 3*B*a + 3*C*a)*sin(d*x + c)/d
```

3.303 $\int (a+a\cos(c+dx)) \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=91

$$\frac{a(3A+3B+C)\sin(c+dx)}{3d}+\frac{1}{2}ax(2A+B+C)+\frac{a(3B-C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3ad}$$

```
[Out] (a*(2*A + B + C)*x)/2 + (a*(3*A + 3*B + C)*Sin[c + d*x])/(3*d) + (a*(3*B -
C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x
])/(3*a*d)
```

Rubi [A] time = 0.0831877, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3023, 2734}

$$\frac{a(3A+3B+C)\sin(c+dx)}{3d}+\frac{1}{2}ax(2A+B+C)+\frac{a(3B-C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{C\sin(c+dx)(a\cos(c+dx)+a)^2}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(2*A + B + C)*x)/2 + (a*(3*A + 3*B + C)*Sin[c + d*x])/(3*d) + (a*(3*B -
C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x
])/(3*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3ad}+\frac{\int(a+a\cos(c+d}{}$$
$$=\frac{1}{2}a(2A+B+C)x+\frac{a(3A+3B+C)\sin(c+dx)}{3d}+\frac{a(}{}$$

Mathematica [A] time = 0.228186, size = 65, normalized size = 0.71

$$\frac{a(3(4A+4B+3C)\sin(c+dx)+12Adx+3(B+C)\sin(2(c+dx))+6Bdx+C\sin(3(c+dx))+6Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(12*A*d*x + 6*B*d*x + 6*C*d*x + 3*(4*A + 4*B + 3*C)*Sin[c + d*x] + 3*(B
+ C)*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.022, size = 102, normalized size = 1.1

$$\frac{1}{d}\left(\frac{aC\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c)+B*
a*sin(d*x+c)+a*A*(d*x+c))
```

Maxima [A] time = 0.988821, size = 132, normalized size = 1.45

$$\frac{12(dx+c)Aa+3(2dx+2c+\sin(2dx+2c))Ba-4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca+3(2dx+2c+\sin(2dx+2c}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 4*(sin(d*
x + c)^3 - 3*sin(d*x + c))*C*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a + 1
2*A*a*sin(d*x + c) + 12*B*a*sin(d*x + c))/d
```

Fricas [A] time = 2.11899, size = 162, normalized size = 1.78

$$\frac{3\,(2\,A+B+C)adx+\left(2\,Ca\cos\left(dx+c\right)^2+3\,(B+C)a\cos\left(dx+c\right)+2\,(3\,A+3\,B+2\,C)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(2*A + B + C)*a*d*x + (2*C*a*cos(d*x + c)^2 + 3*(B + C)*a*cos(d*x +
c) + 2*(3*A + 3*B + 2*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.703342, size = 189, normalized size = 2.08

$$\begin{cases} Aax+\frac{Aa\sin\left(c+dx\right)}{d}+\frac{Bax\sin^2\left(c+dx\right)}{2}+\frac{Bax\cos^2\left(c+dx\right)}{2}+\frac{Ba\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Ba\sin\left(c+dx\right)}{d}+\frac{Cax\sin^2\left(c+dx\right)}{2}+\frac{Cax\cos^2\left(c+dx\right)}{2}+\frac{2C \\ x\left(a\cos\left(c\right)+a\right)\left(A+B\cos\left(c\right)+C\cos^2\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise(((A*a*x + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*a*x*cos
(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/d +
 C*a*x*sin(c + d*x)**2/2 + C*a*x*cos(c + d*x)**2/2 + 2*C*a*sin(c + d*x)**3/
(3*d) + C*a*sin(c + d*x)*cos(c + d*x)**2/d + C*a*sin(c + d*x)*cos(c + d*x)/
(2*d), Ne(d, 0)), (x*(a*cos(c) + a)*(A + B*cos(c) + C*cos(c)**2), True))
```

Giac [A] time = 1.19219, size = 103, normalized size = 1.13

$$\frac{1}{2}\left(2\,Aa+Ba+Ca\right)x+\frac{Ca\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(Ba+Ca\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(4\,Aa+4\,Ba+3\,Ca\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*A*a + B*a + C*a)*x + 1/12*C*a*sin(3*d*x + 3*c)/d + 1/4*(B*a + C*a)*s
in(2*d*x + 2*c)/d + 1/4*(4*A*a + 4*B*a + 3*C*a)*sin(d*x + c)/d
```

3.304 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{1}{2}ax(2A+2B+C)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(B+C)\sin(c+dx)}{d}+\frac{aC\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (a*(2*A + 2*B + C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*(B + C)*Sin[c
+ d*x])/d + (a*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.135698, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3033, 3023, 2735, 3770}

$$\frac{1}{2}ax(2A+2B+C)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(B+C)\sin(c+dx)}{d}+\frac{aC\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```

```
[Out] (a*(2*A + 2*B + C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*(B + C)*Sin[c
+ d*x])/d + (a*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{aC\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2aA+\right.$$
$$= \frac{a(B+C)\sin(c+dx)}{d}+\frac{aC\cos(c+dx)\sin}{2d}$$
$$= \frac{1}{2}a(2A+2B+C)x+\frac{a(B+C)\sin(c+dx)}{d}$$
$$= \frac{1}{2}a(2A+2B+C)x+\frac{aA\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A] time = 0.134322, size = 59, normalized size = 0.94

$$\frac{a\left(4A\tanh^{-1}(\sin(c+dx))+4Adx+4(B+C)\sin(c+dx)+4Bdx+C\sin(2(c+dx))+2cC+2Cdx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```

```
[Out] (a*(2*c*C + 4*A*d*x + 4*B*d*x + 2*C*d*x + 4*A*ArcTanh[Sin[c + d*x]] + 4*(B
+ C)*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.051, size = 100, normalized size = 1.6

$$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+Bax+\frac{Bac}{d}+\frac{aC\sin(dx+c)}{d}+aAx+\frac{Aac}{d}+\frac{Ba\sin(dx+c)}{d}+\frac{aC\cos(dx+c)\sin}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+B*a*x+1/d*B*a*c+a*C*sin(d*x+c)/d+a*A*x+1/
d*A*a*c+1/d*B*a*sin(d*x+c)+1/2*a*C*cos(d*x+c)*sin(d*x+c)/d+1/2*a*C*x+1/2/d*
a*C*c
```

Maxima [A] time = 0.979101, size = 111, normalized size = 1.76

$$\frac{4(dx+c)Aa+4(dx+c)Ba+(2dx+2c+\sin(2dx+2c))Ca+4Aa\log(\sec(dx+c)+\tan(dx+c))+4Ba\sin(dx+c}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a + 4*(d*x + c)*B*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*C
*a + 4*A*a*log(sec(d*x + c) + tan(d*x + c)) + 4*B*a*sin(d*x + c) + 4*C*a*si
n(d*x + c))/d
```

Fricas [A] time = 2.34583, size = 184, normalized size = 2.92

$$\frac{(2A+2B+C)adx+Aa\log(\sin(dx+c)+1)-Aa\log(-\sin(dx+c)+1)+(Ca\cos(dx+c)+2(B+C)a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] 1/2*((2*A + 2*B + C)*a*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x +
 c) + 1) + (C*a*cos(d*x + c) + 2*(B + C)*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^{2}\left(c+dx\right)\sec\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*sec(
c + d*x), x) + Integral(C*cos(c + d*x)**2*sec(c + d*x), x) + Integral(C*cos
(c + d*x)**3*sec(c + d*x), x))
```

Giac [B] time = 1.25072, size = 177, normalized size = 2.81

$$\frac{2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(2\,Aa+2\,Ba+Ca)(dx+c)+\frac{2\left(2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (2*A*a + 2*B*a + C*a)*(d*x + c) + 2*(2*B*a*tan(1/2*d*x + 1/
2*c)^3 + C*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) + 3*C*a*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.305 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=46

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+ax(B+C)+\frac{aC\sin(c+dx)}{d}$$

```
[Out] a*(B + C)*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d +
(a*A*Tan[c + d*x])/d
```

Rubi [A] time = 0.136131, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3031, 3023, 2735, 3770}

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+ax(B+C)+\frac{aC\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```



```
[Out] a*(B + C)*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d +
(a*A*Tan[c + d*x])/d
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{aA\tan(c+dx)}{d}-\int\left(-a(A+B)-a(B+\right.$$
$$=\frac{aC\sin(c+dx)}{d}+\frac{aA\tan(c+dx)}{d}-\int(-a$$
$$=a(B+C)x+\frac{aC\sin(c+dx)}{d}+\frac{aA\tan(c+}{d}$$
$$=a(B+C)x+\frac{a(A+B)\tanh^{-1}(\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0289937, size = 71, normalized size = 1.54

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+aBx+\frac{aC\sin(c)\cos(dx)}{d}+\frac{aC\cos(c)\sin(dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```

```
[Out] a*B*x + a*C*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])
/d + (a*C*Cos[d*x]*Sin[c])/d + (a*C*Cos[c]*Sin[d*x])/d + (a*A*Tan[c + d*x])
```

```
/d
```

Maple [A] time = 0.063, size = 88, normalized size = 1.9

$$Bax + aCx + \frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{aA\tan(dx+c)}{d} + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bac}{d} + \frac{aC\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] B*a*x+a*C*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+a*A*tan(d*x+c)/d+1/d*B*a*ln(s
ec(d*x+c)+tan(d*x+c))+1/d*B*a*c+a*C*sin(d*x+c)/d+1/d*a*C*c
```

Maxima [A] time = 0.997236, size = 124, normalized size = 2.7

$$\frac{2(dx+c)Ba + 2(dx+c)Ca + Aa(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + 2*(d*x + c)*C*a + A*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2
*C*a*sin(d*x + c) + 2*A*a*tan(d*x + c))/d
```

Fricas [A] time = 2.28719, size = 257, normalized size = 5.59

$$\frac{2(B+C)adx\cos(dx+c) + (A+B)a\cos(dx+c)\log(\sin(dx+c)+1) - (A+B)a\cos(dx+c)\log(-\sin(dx+c)+1) +}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] 1/2*(2*(B + C)*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c)
 + 1) - (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a*cos(d*x + c)
 + A*a)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.24842, size = 178, normalized size = 3.87

$$\frac{(Ba+Ca)(dx+c)+(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] ((B*a + C*a)*(d*x + c) + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)
^3 - C*a*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + C*a*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.306 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{a(A+2(B+C))\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+aCx$$

```
[Out] a*C*x + (a*(A + 2*(B + C))*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c
+ d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.157749, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3031, 3021, 2735, 3770}

$$\frac{a(A+2(B+C))\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+aCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```


```
[Out] a*C*x + (a*(A + 2*(B + C))*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c
+ d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{aA\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\int\left(-2a(A\right.$$

$$=\frac{a(A+B)\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\text{ta}}{2d}$$

$$=aCx+\frac{a(A+B)\tan(c+dx)}{d}+\frac{aA\sec(c}{}$$

$$=aCx+\frac{a(A+2(B+C))\tanh^{-1}(\sin(c+d}{2d}$$

Mathematica [A] time = 0.0344218, size = 92, normalized size = 1.48

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```

```
[Out] a*C*x + (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*B*ArcTanh[Sin[c + d*x]])/d +
 (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d + (a*B*Tan[c + d*x])/
```

```
d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.066, size = 117, normalized size = 1.9

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Ba\tan(dx+c)}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan
(d*x+c)/d+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+a*A*tan(d*x+c)/d+1/d*B*a*ln(sec
(d*x+c)+tan(d*x+c))+a*C*x+1/d*a*C*c
```

Maxima [B] time = 1.01262, size = 176, normalized size = 2.84

$$\frac{4(dx+c)Ca-Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+2Ba(\log(\sin(dx+c)+1)-\log(\sin(dx}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a - A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*B*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 2*C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 4*A*a*tan(d*x + c) + 4*B*a*tan(d*x + c))/d
```

Fricas [A] time = 2.03274, size = 292, normalized size = 4.71

$$\frac{4Cadx\cos(dx+c)^2+(A+2B+2C)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(A+2B+2C)a\cos(dx+c)^2\log(-\sin(dx}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] 1/4*(4*C*a*d*x*cos(d*x + c)^2 + (A + 2*B + 2*C)*a*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (A + 2*B + 2*C)*a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(
2*(A + B)*a*cos(d*x + c) + A*a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.30916, size = 190, normalized size = 3.06

$$\frac{2\,(dx+c)Ca + (Aa + 2\,Ba + 2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Aa + 2\,Ba + 2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(Aa\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + (A*a + 2*B*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (A*a + 2*B*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*ta
n(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c)^3 - 3*A*a*tan(1/2*d*x + 1
/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.307 $\int (a+a\cos(c+dx)) \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx)\, dx$

Optimal. Leaf size=91

$$\frac{a(2A+3(B+C))\tan(c+dx)}{3d} + \frac{a(A+B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d} + \frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] (a*(A + B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*(B + C))*Tan[c
+ d*x])/(3*d) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.211266, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3031, 3021, 2748, 3767, 8, 3770}

$$\frac{a(2A+3(B+C))\tan(c+dx)}{3d} + \frac{a(A+B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d} + \frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```



```
[Out] (a*(A + B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*(B + C))*Tan[c
+ d*x])/(3*d) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}\int \big(-3a(A$$

$$= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d} + \frac{aA\sec}{}$$

$$= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d} + \frac{aA\sec}{}$$

$$= \frac{a(A+B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(A}{}$$

$$= \frac{a(A+B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(2A}{}$$

Mathematica [A] time = 0.367292, size = 60, normalized size = 0.66

$$\frac{a\left(3(A+B+2C)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(3(A+B)\sec(c+dx) + 6(A+B+C) + 2A\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```

```
[Out] (a*(3*(A + B + 2*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B + C) + 3
*(A + B)*Sec[c + d*x] + 2*A*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.068, size = 160, normalized size = 1.8

$$\frac{2\,aA\tan(dx+c)}{3\,d} + \frac{aA\,(\sec(dx+c))^2\tan(dx+c)}{3\,d} + \frac{Ba\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a*B*sec(d*x+c)*t
an(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*C*tan(d*x+c)+1/2*a*A*
sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/
d+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01358, size = 219, normalized size = 2.41

$$4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa-3\,Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Ba\left(\frac{2\sin(d}{\sin(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 6*C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a*t
an(d*x + c) + 12*C*a*tan(d*x + c))/d
```

Fricas [A] time = 1.98741, size = 312, normalized size = 3.43

$$\frac{3(A+B+2C)a\cos(dx+c)^3\log(\sin(dx+c)+1)-3(A+B+2C)a\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2(2A\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] 1/12*(3*(A + B + 2*C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B + 2
*C)*a*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*B + 3*C)*a*cos(
d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.2607, size = 277, normalized size = 3.04

$$3(Aa+Ba+2Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3(Aa+Ba+2Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(3Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+3B\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(A*a + B*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*
a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*
c)^5 + 3*B*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*tan(1/2*d*x + 1/2*c)^5 - 4*A*a*
tan(1/2*d*x + 1/2*c)^3 - 12*B*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c) + 6*C
*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.308 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\mathrm{se}$ $dx)\,dx$

Optimal. Leaf size=125

$$-\frac{a(-3(A+B+C)+A+B)\tan(c+dx)}{3d}+\frac{a(3A+4(B+C))\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4(B+C))\tan(c+dx)\sec}{8d}$$

```
[Out] (a*(3*A + 4*(B + C))*ArcTanh[Sin[c + d*x]])/(8*d) - (a*(A + B - 3*(A + B +
C))*Tan[c + d*x])/(3*d) + (a*(3*A + 4*(B + C))*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (a*(A + B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c + d*x]^3*
Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.253843, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {3031, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{a(-3(A+B+C)+A+B)\tan(c+dx)}{3d}+\frac{a(3A+4(B+C))\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4(B+C))\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^5,x]
```


```
[Out] (a*(3*A + 4*(B + C))*ArcTanh[Sin[c + d*x]])/(8*d) - (a*(A + B - 3*(A + B +
C))*Tan[c + d*x])/(3*d) + (a*(3*A + 4*(B + C))*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (a*(A + B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c + d*x]^3*
Tan[c + d*x])/(4*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}\int \left(-4a\right.$$
$$= \frac{a(A+B)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{aA\,\mathrm{s}}{}$$
$$= \frac{a(A+B)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{aA\,\mathrm{s}}{}$$
$$= \frac{a(3A+4(B+C))\sec(c+dx)\tan(c+dx)}{8d}$$
$$= \frac{a(3A+4(B+C))\tanh^{-1}(\sin(c+dx))}{8d} -$$

Mathematica [A] time = 0.592432, size = 84, normalized size = 0.67

$$\frac{a\left(3(3A+4(B+C))\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left((A+B)\tan^2(c+dx)+3(A+B+C)\right)+3(3A+4(B+C))\mathrm{s}\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```

```
[Out] (a*(3*(3*A + 4*(B + C))*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(3*A + 4*(B
 + C))*Sec[c + d*x] + 6*A*Sec[c + d*x]^3 + 8*(3*(A + B + C) + (A + B)*Tan[c
 + d*x]^2))))/(24*d)
```

Maple [A] time = 0.072, size = 223, normalized size = 1.8

$$\frac{aA\left(\sec(dx+c)\right)^3\tan(dx+c)}{4d} + \frac{3aA\sec(dx+c)\tan(dx+c)}{8d} + \frac{3aA\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{2Ba\tan(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A
*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*B*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*
x+c)/d+1/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+
2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a*B*sec(d*x+c)*t
```

```
an(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*C*tan(d*x+c)
```

Maxima [A] time = 1.02244, size = 294, normalized size = 2.35

$$16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba-3\,Aa\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)) - 12*C*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*C*a*tan(d*x + c))/d
```

Fricas [A] time = 2.04004, size = 375, normalized size = 3.

$$\frac{3\,(3\,A+4\,B+4\,C)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+4\,B+4\,C)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\right.}{48\,d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(3*A + 4*B + 4*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A +
 4*B + 4*C)*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(2*A + 2*B + 3*C
)*a*cos(d*x + c)^3 + 3*(3*A + 4*B + 4*C)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos
(d*x + c) + 6*A*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.28072, size = 343, normalized size = 2.74

$$3\,(3\,Aa+4\,Ba+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Ba+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a + 4*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A
*a + 4*B*a + 4*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a*tan(1/2*d
*x + 1/2*c)^7 + 12*B*a*tan(1/2*d*x + 1/2*c)^7 + 12*C*a*tan(1/2*d*x + 1/2*c)
^7 - 49*A*a*tan(1/2*d*x + 1/2*c)^5 - 28*B*a*tan(1/2*d*x + 1/2*c)^5 - 60*C*a
*tan(1/2*d*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*
x + 1/2*c)^3 + 84*C*a*tan(1/2*d*x + 1/2*c)^3 - 39*A*a*tan(1/2*d*x + 1/2*c)
- 36*B*a*tan(1/2*d*x + 1/2*c) - 36*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^4)/d
```

3.309 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2\right.$

Optimal. Leaf size=213

$$-\frac{a^2(10A+9B+8C)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B+8C)\sin(c+dx)}{5d}+\frac{a^2(10A+12B+9C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^2}{}$$

```
[Out] (a^2*(14*A + 12*B + 11*C)*x)/16 + (a^2*(10*A + 9*B + 8*C)*Sin[c + d*x])/(5*
d) + (a^2*(14*A + 12*B + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(10
*A + 12*B + 9*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + (C*Cos[c + d*x]^3*(a
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + ((3*B + C)*Cos[c + d*x]^3*(a^2 +
 a^2*Cos[c + d*x])*Sin[c + d*x])/(15*d) - (a^2*(10*A + 9*B + 8*C)*Sin[c + d
*x]^3)/(15*d)
```

Rubi [A] time = 0.503067, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3045, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^2(10A+9B+8C)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B+8C)\sin(c+dx)}{5d}+\frac{a^2(10A+12B+9C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```


```
[Out] (a^2*(14*A + 12*B + 11*C)*x)/16 + (a^2*(10*A + 9*B + 8*C)*Sin[c + d*x])/(5*
d) + (a^2*(14*A + 12*B + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(10
*A + 12*B + 9*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + (C*Cos[c + d*x]^3*(a
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + ((3*B + C)*Cos[c + d*x]^3*(a^2 +
 a^2*Cos[c + d*x])*Sin[c + d*x])/(15*d) - (a^2*(10*A + 9*B + 8*C)*Sin[c + d
*x]^3)/(15*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+}{6d}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+}{6d}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+}{6d}$$

$$= \frac{a^2(10A+12B+9C)\cos^3(c+dx)\sin(c+}{40d}$$

$$= \frac{a^2(10A+12B+9C)\cos^3(c+dx)\sin(c+}{40d}$$

$$= \frac{a^2(14A+12B+11C)\cos(c+dx)\sin(c+}{16d}$$

$$= \frac{1}{16}a^2(14A+12B+11C)x + \frac{a^2(10A+9B}{}$$

Mathematica [A] time = 0.712005, size = 171, normalized size = 0.8

$$\frac{a^2(120(12A+11B+10C)\sin(c+dx)+15(32A+32B+31C)\sin(2(c+dx))+160A\sin(3(c+dx))+30A\sin(4(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (a^2*(720*B*c + 420*c*C + 840*A*d*x + 720*B*d*x + 660*C*d*x + 120*(12*A + 1
1*B + 10*C)*Sin[c + d*x] + 15*(32*A + 32*B + 31*C)*Sin[2*(c + d*x)] + 160*A
*Sin[3*(c + d*x)] + 180*B*Sin[3*(c + d*x)] + 200*C*Sin[3*(c + d*x)] + 30*A*
```

```
Sin[4*(c + d*x)] + 60*B*Sin[4*(c + d*x)] + 75*C*Sin[4*(c + d*x)] + 12*B*Sin
[5*(c + d*x)] + 24*C*Sin[5*(c + d*x)] + 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.029, size = 304, normalized size = 1.4

$$\frac{1}{d}\left(Aa^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^2B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^2*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x
+3/8*c)+2/3*A*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*B*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/5*a^2*C*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c)+A*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/8*d*x+3/8*c)+1/5*a^2*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*
C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+
5/16*c))
```

Maxima [A] time = 1.00709, size = 400, normalized size = 1.88

$$-\frac{640\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-30\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^2-240\,(2\,dx+2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/960*(640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 30*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 240*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^2 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 60*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 128*(3*sin(d*x + c)^5 - 10*s
in(d*x + c)^3 + 15*sin(d*x + c))*C*a^2 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x -
 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^2 - 30*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2)/d
```

Fricas [A] time = 1.97651, size = 373, normalized size = 1.75

$$\frac{15\,(14\,A+12\,B+11\,C)a^2dx+\left(40\,Ca^2\cos\left(dx+c\right)^5+48\,(B+2\,C)a^2\cos\left(dx+c\right)^4+10\,(6\,A+12\,B+11\,C)a^2\cos\left(dx+c\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(14*A + 12*B + 11*C)*a^2*d*x + (40*C*a^2*cos(d*x + c)^5 + 48*(B +
 2*C)*a^2*cos(d*x + c)^4 + 10*(6*A + 12*B + 11*C)*a^2*cos(d*x + c)^3 + 16*(
10*A + 9*B + 8*C)*a^2*cos(d*x + c)^2 + 15*(14*A + 12*B + 11*C)*a^2*cos(d*x
+ c) + 32*(10*A + 9*B + 8*C)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 6.16607, size = 821, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Piecewise(((3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/8 + A
*a**2*x*cos(c + d*x)**2/2 + 3*A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4
*A*a**2*sin(c + d*x)**3/(3*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 2*A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 3*B*a**2*x*sin(c + d*x)**4/4 + 3*B*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/4 + 8*B*a**2*sin(c + d*x)**5/(15*d
) + 4*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + B*a**2*
sin(c + d*x)*cos(c + d*x)**2/d + 5*C*a**2*x*sin(c + d*x)**6/16 + 15*C*a**2*
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*C*a**2*x*sin(c + d*x)**4/8 + 15*C*
a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*C*a**2*x*sin(c + d*x)**2*cos(
c + d*x)**2/4 + 5*C*a**2*x*cos(c + d*x)**6/16 + 3*C*a**2*x*cos(c + d*x)**4/
8 + 5*C*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 16*C*a**2*sin(c + d*x)**
5/(15*d) + 5*C*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*C*a**2*sin(c
```

```
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*C*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 11*C*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*C*a**2*sin(c + d*x)*
cos(c + d*x)**4/d + 5*C*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)),
 (x*(a*cos(c) + a)**2*(A + B*cos(c) + C*cos(c)**2)*cos(c)**2, True))
```

Giac [A] time = 1.24183, size = 265, normalized size = 1.24

$$\frac{Ca^2\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(14\,Aa^2+12\,Ba^2+11\,Ca^2\right)x+\frac{\left(Ba^2+2\,Ca^2\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{\left(2\,Aa^2+4\,Ba^2+5\,Ca^2\right)\sin(}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/192*C*a^2*sin(6*d*x + 6*c)/d + 1/16*(14*A*a^2 + 12*B*a^2 + 11*C*a^2)*x +
1/80*(B*a^2 + 2*C*a^2)*sin(5*d*x + 5*c)/d + 1/64*(2*A*a^2 + 4*B*a^2 + 5*C*a
^2)*sin(4*d*x + 4*c)/d + 1/48*(8*A*a^2 + 9*B*a^2 + 10*C*a^2)*sin(3*d*x + 3*
c)/d + 1/64*(32*A*a^2 + 32*B*a^2 + 31*C*a^2)*sin(2*d*x + 2*c)/d + 1/8*(12*A
*a^2 + 11*B*a^2 + 10*C*a^2)*sin(d*x + c)/d
```

3.310 $\int \cos(c+dx)(a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(\right.$

Optimal. Leaf size=181

$$\frac{a^2(8A+7B+6C)\sin(c+dx)}{6d} + \frac{a^2(8A+7B+6C)\sin(c+dx)\cos(c+dx)}{24d} + \frac{1}{8}a^2x(8A+7B+6C) + \frac{(20A-5B+6C)\,\mathrm{s}}{}$$

```
[Out] (a^2*(8*A + 7*B + 6*C)*x)/8 + (a^2*(8*A + 7*B + 6*C)*Sin[c + d*x])/(6*d) +
(a^2*(8*A + 7*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((20*A - 5*B + 6
*C)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + (C*Cos[c + d*x]^2*(a + a*
Cos[c + d*x])^2*Sin[c + d*x])/(5*d) + ((5*B + 2*C)*(a + a*Cos[c + d*x])^3*S
in[c + d*x])/(20*a*d)
```

Rubi [A] time = 0.335672, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3045, 2968, 3023, 2751, 2644}

$$\frac{a^2(8A+7B+6C)\sin(c+dx)}{6d} + \frac{a^2(8A+7B+6C)\sin(c+dx)\cos(c+dx)}{24d} + \frac{1}{8}a^2x(8A+7B+6C) + \frac{(20A-5B+6C)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] (a^2*(8*A + 7*B + 6*C)*x)/8 + (a^2*(8*A + 7*B + 6*C)*Sin[c + d*x])/(6*d) +
(a^2*(8*A + 7*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((20*A - 5*B + 6
*C)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + (C*Cos[c + d*x]^2*(a + a*
Cos[c + d*x])^2*Sin[c + d*x])/(5*d) + ((5*B + 2*C)*(a + a*Cos[c + d*x])^3*S
in[c + d*x])/(20*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx &= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^2\sin(c\\
&= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^2\sin(c\\
&= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^2\sin(c\\
&= \frac{(20A-5B+6C)(a+a\cos(c+dx))^2\sin\\
&= \frac{1}{8}a^2(8A+7B+6C)x+\frac{a^2(8A+7B+6C}{6d}\end{aligned}$$

Mathematica [A] time = 0.613454, size = 132, normalized size = 0.73

$$\frac{a^2(60(14A+12B+11C)\sin(c+dx)+240(A+B+C)\sin(2(c+dx))+40A\sin(3(c+dx))+480Adx+80B\sin(3(c+d$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (a^2*(420*B*c + 240*c*C + 480*A*d*x + 420*B*d*x + 360*C*d*x + 60*(14*A + 12
*B + 11*C)*Sin[c + d*x] + 240*(A + B + C)*Sin[2*(c + d*x)] + 40*A*Sin[3*(c
+ d*x)] + 80*B*Sin[3*(c + d*x)] + 90*C*Sin[3*(c + d*x)] + 15*B*Sin[4*(c + d
*x)] + 30*C*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.026, size = 247, normalized size = 1.4

$$\frac{1}{d}\left(Aa^2\sin(dx+c)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+2\,Aa^2\,(1/2\,\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^2*sin(d*x+c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a
^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+2*A*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+2/3*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*C*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*a^2*(2+cos(d*x+c)^2)*sin(d*x+
c)+a^2*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*a
^2*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.01848, size = 319, normalized size = 1.76

$$-\frac{160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 240*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 15*(1
2*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 120*(2*d*x +
2*c + sin(2*d*x + 2*c))*B*a^2 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 +
15*sin(d*x + c))*C*a^2 + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 30*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 - 480*A*a^2*si
n(d*x + c))/d
```

Fricas [A] time = 2.05661, size = 305, normalized size = 1.69

$$\frac{15\,(8\,A+7\,B+6\,C)a^2dx+\left(24\,Ca^2\cos\left(dx+c\right)^4+30\,(B+2\,C)a^2\cos\left(dx+c\right)^3+8\,(5\,A+10\,B+9\,C)a^2\cos\left(dx+c\right)^2+\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(8*A + 7*B + 6*C)*a^2*d*x + (24*C*a^2*cos(d*x + c)^4 + 30*(B + 2*
C)*a^2*cos(d*x + c)^3 + 8*(5*A + 10*B + 9*C)*a^2*cos(d*x + c)^2 + 15*(8*A +
 7*B + 6*C)*a^2*cos(d*x + c) + 8*(25*A + 20*B + 18*C)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 3.38498, size = 570, normalized size = 3.15

$$\begin{cases} Aa^2x\sin^2\left(c+dx\right)+Aa^2x\cos^2\left(c+dx\right)+\frac{2Aa^2\sin^3\left(c+dx\right)}{3d}+\frac{Aa^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{Aa^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{Aa^2\sin\left(c+dx\right)}{d} \\ x\left(a\cos\left(c\right)+a\right)^2\left(A+B\cos\left(c\right)+C\cos^2\left(c\right)\right)\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2 + A*a**2*x*cos(c + d*x)**2 + 2*A*a**2*s
in(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c
 + d*x)*cos(c + d*x)/d + A*a**2*sin(c + d*x)/d + 3*B*a**2*x*sin(c + d*x)**4
/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**2*x*sin(c + d*x)**
2/2 + 3*B*a**2*x*cos(c + d*x)**4/8 + B*a**2*x*cos(c + d*x)**2/2 + 3*B*a**2*
sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*B*a**2*sin(c + d*x)**3/(3*d) + 5*B*a
**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*B*a**2*sin(c + d*x)*cos(c + d*x)
```

```
**2/d + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*C*a**2*x*sin(c + d*x)**4
/4 + 3*C*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*C*a**2*x*cos(c + d*x)
**4/4 + 8*C*a**2*sin(c + d*x)**5/(15*d) + 4*C*a**2*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + 3*C*a**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 2*C*a**2*sin(
c + d*x)**3/(3*d) + C*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*a**2*sin(c
+ d*x)*cos(c + d*x)**3/(4*d) + C*a**2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d,
 0)), (x*(a*cos(c) + a)**2*(A + B*cos(c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.21977, size = 216, normalized size = 1.19

$$\frac{Ca^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(8\,Aa^2+7\,Ba^2+6\,Ca^2\right)x+\frac{\left(Ba^2+2\,Ca^2\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(4\,Aa^2+8\,Ba^2+9\,Ca^2\right)\sin(3\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/80*C*a^2*sin(5*d*x + 5*c)/d + 1/8*(8*A*a^2 + 7*B*a^2 + 6*C*a^2)*x + 1/32*
(B*a^2 + 2*C*a^2)*sin(4*d*x + 4*c)/d + 1/48*(4*A*a^2 + 8*B*a^2 + 9*C*a^2)*s
in(3*d*x + 3*c)/d + 1/2*(A*a^2 + B*a^2 + C*a^2)*sin(2*d*x + 2*c)/d + 1/8*(1
4*A*a^2 + 12*B*a^2 + 11*C*a^2)*sin(d*x + c)/d
```

3.311 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$

Optimal. Leaf size=138

$$\frac{a^2(12A+8B+7C)\sin(c+dx)}{6d}+\frac{a^2(12A+8B+7C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(12A+8B+7C)+\frac{(4B-C)\sin(}{}$$

```
[Out] (a^2*(12*A + 8*B + 7*C)*x)/8 + (a^2*(12*A + 8*B + 7*C)*Sin[c + d*x])/(6*d)
+ (a^2*(12*A + 8*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*B - C)*(a
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c
 + d*x])/(4*a*d)
```

Rubi [A] time = 0.174789, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3023, 2751, 2644}

$$\frac{a^2(12A+8B+7C)\sin(c+dx)}{6d}+\frac{a^2(12A+8B+7C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(12A+8B+7C)+\frac{(4B-C)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```



```
[Out] (a^2*(12*A + 8*B + 7*C)*x)/8 + (a^2*(12*A + 8*B + 7*C)*Sin[c + d*x])/(6*d)
+ (a^2*(12*A + 8*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*B - C)*(a
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c
 + d*x])/(4*a*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4ad}+\frac{\int(a+a\cos(c+d}{}$$
$$= \frac{(4B-C)(a+a\cos(c+dx))^2\sin(c+dx)}{12d}+\frac{C(a+a\cos}{}$$
$$= \frac{1}{8}a^2(12A+8B+7C)x+\frac{a^2(12A+8B+7C)\sin(c+d}{6d}$$

Mathematica [A] time = 0.386641, size = 94, normalized size = 0.68

$$\frac{a^2(24(8A+7B+6C)\sin(c+dx)+24(A+2(B+C))\sin(2(c+dx))+144Adx+8B\sin(3(c+dx))+96Bdx+16C\sin(3}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*(144*A*d*x + 96*B*d*x + 84*C*d*x + 24*(8*A + 7*B + 6*C)*Sin[c + d*x] +
 24*(A + 2*(B + C))*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 16*C*Sin[3*(c
 + d*x)] + 3*C*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.024, size = 203, normalized size = 1.5

$$\frac{1}{d}\left(a^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2a^2C\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*
a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*B*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a^
2*sin(d*x+c)+a^2*B*sin(d*x+c)+A*a^2*(d*x+c))
```

Maxima [A] time = 1.00831, size = 257, normalized size = 1.86

$$24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+96\,(dx+c)Aa^2-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2+48\,(2\,dx+2\,c+\sin(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 96*(d*x + c)*A*a^2 - 32*(
sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 48*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*B*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 3*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 + 24*(2*d*x + 2*c + sin(2*d*x +
 2*c))*C*a^2 + 192*A*a^2*sin(d*x + c) + 96*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.94683, size = 239, normalized size = 1.73

$$\frac{3\,(12\,A+8\,B+7\,C)a^2dx+\left(6\,Ca^2\cos(dx+c)^3+8\,(B+2\,C)a^2\cos(dx+c)^2+3\,(4\,A+8\,B+7\,C)a^2\cos(dx+c)+8\,(6}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/24*(3*(12*A + 8*B + 7*C)*a^2*d*x + (6*C*a^2*cos(d*x + c)^3 + 8*(B + 2*C)*
a^2*cos(d*x + c)^2 + 3*(4*A + 8*B + 7*C)*a^2*cos(d*x + c) + 8*(6*A + 5*B +
4*C)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 1.64916, size = 420, normalized size = 3.04

$$\begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{Aa^2x\cos^2(c+dx)}{2} + Aa^2x + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Aa^2\sin(c+dx)}{d} + Ba^2x\sin^2(c+dx) + Ba^2x\cos^2(c+dx) + \\ x(a\cos(c)+a)^2\left(A+B\cos(c)+C\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*x + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*sin(c + d*x)/d + B*a
**2*x*sin(c + d*x)**2 + B*a**2*x*cos(c + d*x)**2 + 2*B*a**2*sin(c + d*x)**3
/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c + d*x)*cos(c
+ d*x)/d + B*a**2*sin(c + d*x)/d + 3*C*a**2*x*sin(c + d*x)**4/8 + 3*C*a**2*
x*sin(c + d*x)**2*cos(c + d*x)**2/4 + C*a**2*x*sin(c + d*x)**2/2 + 3*C*a**2
*x*cos(c + d*x)**4/8 + C*a**2*x*cos(c + d*x)**2/2 + 3*C*a**2*sin(c + d*x)**
3*cos(c + d*x)/(8*d) + 4*C*a**2*sin(c + d*x)**3/(3*d) + 5*C*a**2*sin(c + d*
x)*cos(c + d*x)**3/(8*d) + 2*C*a**2*sin(c + d*x)*cos(c + d*x)**2/d + C*a**2
*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**2*(A + B*co
s(c) + C*cos(c)**2), True))
```

Giac [A] time = 1.20714, size = 174, normalized size = 1.26

$$\frac{Ca^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(12\,Aa^2 + 8\,Ba^2 + 7\,Ca^2\right)x + \frac{\left(Ba^2 + 2\,Ca^2\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(Aa^2 + 2\,Ba^2 + 2\,Ca^2\right)\sin(2\,dx+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/32*C*a^2*sin(4*d*x + 4*c)/d + 1/8*(12*A*a^2 + 8*B*a^2 + 7*C*a^2)*x + 1/12
*(B*a^2 + 2*C*a^2)*sin(3*d*x + 3*c)/d + 1/4*(A*a^2 + 2*B*a^2 + 2*C*a^2)*sin
(2*d*x + 2*c)/d + 1/4*(8*A*a^2 + 7*B*a^2 + 6*C*a^2)*sin(d*x + c)/d
```

3.312 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ s… $dx)\,dx$

Optimal. Leaf size=120

$$\frac{a^2(2A+3B+2C)\sin(c+dx)}{2d} + \frac{1}{2}a^2x(4A+3B+2C) + \frac{a^2A\tanh^{-1}(\sin(c+dx))}{d} + \frac{(3B+2C)\sin(c+dx)\left(a^2\cos(c+\ldots\right.}{6d}$$

```
[Out] (a^2*(4*A + 3*B + 2*C)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A +
 3*B + 2*C)*Sin[c + d*x])/(2*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(
3*d) + ((3*B + 2*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.343351, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3045, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^2(2A+3B+2C)\sin(c+dx)}{2d} + \frac{1}{2}a^2x(4A+3B+2C) + \frac{a^2A\tanh^{-1}(\sin(c+dx))}{d} + \frac{(3B+2C)\sin(c+dx)\left(a^2\cos(c+\ldots\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```


```
[Out] (a^2*(4*A + 3*B + 2*C)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A +
 3*B + 2*C)*Sin[c + d*x])/(2*d) + (C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(
3*d) + ((3*B + 2*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{\int (a -}{}$$
$$= \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{(3B +}{}$$
$$= \frac{C(a+a\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{(3B +}{}$$
$$= \frac{a^2(2A+3B+2C)\sin(c+dx)}{2d} + \frac{C(a+a}{}$$
$$= \frac{1}{2}a^2(4A+3B+2C)x + \frac{a^2(2A+3B+2C}{2d}$$
$$= \frac{1}{2}a^2(4A+3B+2C)x + \frac{a^2A\tanh^{-1}(\sin(c}{d}$$

Mathematica [A] time = 0.311171, size = 121, normalized size = 1.01

$$\frac{a^2\left(3(4A+8B+7C)\sin(c+dx)-12A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (a^2*(24*A*d*x + 18*B*d*x + 12*C*d*x - 12*A*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 12*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(4*A + 8*B +
7*C)*Sin[c + d*x] + 3*(B + 2*C)*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12
*d)
```

Maple [A] time = 0.059, size = 181, normalized size = 1.5

$$\frac{Aa^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{3a^2Bx}{2} + \frac{3a^2Bc}{2d} + \frac{5a^2C\sin(dx+c)}{3d} + 2Aa^2x + 2\frac{Aa^2c}{d} + 2\frac{a^2B\sin(dx+c)}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+3/2*a^2*B*x+3/2/d*B*a^2*c+5/3/d*a^2*C*s
in(d*x+c)+2*A*a^2*x+2/d*A*a^2*c+2/d*a^2*B*sin(d*x+c)+1/d*a^2*C*cos(d*x+c)*s
in(d*x+c)+a^2*C*x+1/d*a^2*C*c+1/d*A*a^2*sin(d*x+c)+1/2/d*a^2*B*cos(d*x+c)*s
in(d*x+c)+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^2
```

Maxima [A] time = 0.992002, size = 207, normalized size = 1.72

$$\frac{24\,(dx+c)Aa^2+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+12\,(dx+c)Ba^2-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^2+6\,(2\,dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/12*(24*(d*x + c)*A*a^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 12*(d
*x + c)*B*a^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 6*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*C*a^2 + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 12*
A*a^2*sin(d*x + c) + 24*B*a^2*sin(d*x + c) + 12*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 2.05267, size = 269, normalized size = 2.24

$$\frac{3\,(4\,A+3\,B+2\,C)a^2dx+3\,Aa^2\log(\sin(dx+c)+1)-3\,Aa^2\log(-\sin(dx+c)+1)+\left(2\,Ca^2\cos(dx+c)^2+3\,(B+2\,C\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(4*A + 3*B + 2*C)*a^2*d*x + 3*A*a^2*log(sin(d*x + c) + 1) - 3*A*a^2*
log(-sin(d*x + c) + 1) + (2*C*a^2*cos(d*x + c)^2 + 3*(B + 2*C)*a^2*cos(d*x
+ c) + 2*(3*A + 6*B + 5*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.30885, size = 317, normalized size = 2.64

$$\frac{6\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(4\,Aa^2+3\,Ba^2+2\,Ca^2\right)(dx+c)+\frac{2\left(6\,Aa^2\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="giac")
```

```
[Out] 1/6*(6*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(4*A*a^2 + 3*B*a^2 + 2*C*a^2)*(d*x + c) + 2*(6*A*a^2*
tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d
*x + 1/2*c)^5 + 12*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a^2*tan(1/2*d*x + 1/
2*c)^3 + 16*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 1
5*B*a^2*tan(1/2*d*x + 1/2*c) + 18*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

3.313 $\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec$ $dx)\,dx$

Optimal. Leaf size=121

$$-\frac{a^2(2A-2B-3C)\sin(c+dx)}{2d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x(2A+4B+3C)-\frac{(2A-C)\sin(c+dx)\left(a^2\cos\right.}{2d}$$

```
[Out] (a^2*(2*A + 4*B + 3*C)*x)/2 + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^
2*(2*A - 2*B - 3*C)*Sin[c + d*x])/(2*d) - ((2*A - C)*(a^2 + a^2*Cos[c + d*x
])*Sin[c + d*x])/(2*d) + (A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.383819, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2976, 2968, 3023, 2735, 3770}

$$-\frac{a^2(2A-2B-3C)\sin(c+dx)}{2d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x(2A+4B+3C)-\frac{(2A-C)\sin(c+dx)\left(a^2\cos\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (a^2*(2*A + 4*B + 3*C)*x)/2 + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^
2*(2*A - 2*B - 3*C)*Sin[c + d*x])/(2*d) - ((2*A - C)*(a^2 + a^2*Cos[c + d*x
])*Sin[c + d*x])/(2*d) + (A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\tan(c+dx)}{d} + \int(a$$

$$= -\frac{(2A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+d}{2d}$$

$$= -\frac{(2A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+d}{2d}$$

$$= -\frac{a^2(2A-2B-3C)\sin(c+dx)}{2d} - \frac{(2A-C}{}$$

$$= \frac{1}{2}a^2(2A+4B+3C)x - \frac{a^2(2A-2B-3C)}{2d}$$

$$= \frac{1}{2}a^2(2A+4B+3C)x + \frac{a^2(2A+B)\tanh^{-}}{d}$$

Mathematica [A] time = 0.579331, size = 174, normalized size = 1.44

$$\frac{a^2\left(4A\tan(c+dx)-8A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+4Ac+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (a^2*(4*A*c + 8*B*c + 6*c*C + 4*A*d*x + 8*B*d*x + 6*C*d*x - 8*A*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - 4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + 8*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*B*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 4*(B + 2*C)*Sin[c + d*x] + C*Sin[2*(c + d*x)] + 4*A*
Tan[c + d*x]))/(4*d)
```

Maple [A] time = 0.072, size = 160, normalized size = 1.3

$$\frac{Aa^2\tan(dx+c)}{d} + \frac{a^2B\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{3a^2Cx}{2} + \frac{3a^2Cc}{2d} + 2\frac{Aa^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*a^2*tan(d*x+c)+1/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+3/2*a^2*C*x+3/2/d*
a^2*C*c+2/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*B*x+2/d*B*a^2*c+2/d*a^2*C
*sin(d*x+c)+A*a^2*x+1/d*A*a^2*c+1/d*a^2*B*sin(d*x+c)+1/2/d*a^2*C*cos(d*x+c)
*sin(d*x+c)
```

Maxima [A] time = 1.03136, size = 204, normalized size = 1.69

$$\frac{4\,(dx+c)Aa^2+8\,(dx+c)Ba^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^2+4\,(dx+c)Ca^2+4\,Aa^2(\log(\sin(dx+c)+1)-\log(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^2 + 8*(d*x + c)*B*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c
))*C*a^2 + 4*(d*x + c)*C*a^2 + 4*A*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B
*a^2*sin(d*x + c) + 8*C*a^2*sin(d*x + c) + 4*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 2.03978, size = 331, normalized size = 2.74

$$\frac{(2\,A+4\,B+3\,C)a^2dx\cos(dx+c)+(2\,A+B)a^2\cos(dx+c)\log(\sin(dx+c)+1)-(2\,A+B)a^2\cos(dx+c)\log(-\sin\ldots}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="fricas")
```

```
[Out] 1/2*((2*A + 4*B + 3*C)*a^2*d*x*cos(d*x + c) + (2*A + B)*a^2*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - (2*A + B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (
C*a^2*cos(d*x + c)^2 + 2*(B + 2*C)*a^2*cos(d*x + c) + 2*A*a^2)*sin(d*x + c)
)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19574, size = 267, normalized size = 2.21

$$-\frac{\frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\left(2\,Aa^2+4\,Ba^2+3\,Ca^2\right)(dx+c)-2\left(2\,Aa^2+Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(2\,Aa^2+Ba^2\right)\mathrm{lo}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="giac")
```

```
[Out] -1/2*(4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (2*A*a^2
+ 4*B*a^2 + 3*C*a^2)*(d*x + c) - 2*(2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) + 2*(2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(
2*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*t
an(1/2*d*x + 1/2*c) + 5*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 + 1)^2)/d
```

3.314 $\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) s$ $dx) \, dx$

Optimal. Leaf size=123

$$-\frac{a^2(3A+2B-2C)\sin(c+dx)}{2d}+\frac{a^2(3A+4B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+B)\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

```
[Out] a^2*(B + 2*C)*x + (a^2*(3*A + 4*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (a^
2*(3*A + 2*B - 2*C)*Sin[c + d*x])/(2*d) + ((A + B)*(a^2 + a^2*Cos[c + d*x])
*Tan[c + d*x])/d + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rubi [A] time = 0.392462, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2975, 2968, 3023, 2735, 3770}

$$-\frac{a^2(3A+2B-2C)\sin(c+dx)}{2d}+\frac{a^2(3A+4B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+B)\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] a^2*(B + 2*C)*x + (a^2*(3*A + 4*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (a^
2*(3*A + 2*B - 2*C)*Sin[c + d*x])/(2*d) + ((A + B)*(a^2 + a^2*Cos[c + d*x])
*Tan[c + d*x])/d + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^2 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^3(c + dx)\,dx = \frac{A(a + a\cos(c + dx))^2\sec(c + dx)\tan(c \cdot}{2d}$$
$$= \frac{(A + B)\left(a^2 + a^2\cos(c + dx)\right)\tan(c + dx}{d}$$
$$= \frac{(A + B)\left(a^2 + a^2\cos(c + dx)\right)\tan(c + dx}{d}$$
$$= -\frac{a^2(3A + 2B - 2C)\sin(c + dx)}{2d} + \frac{(A + \cdot}{}$$
$$= a^2(B + 2C)x - \frac{a^2(3A + 2B - 2C)\sin(c +}{2d}$$
$$= a^2(B + 2C)x + \frac{a^2(3A + 4B + 2C)\tanh^{-}}{2d}$$

Mathematica [B] time = 1.44448, size = 259, normalized size = 2.11

$$\frac{a^2(\cos(c + dx) + 1)^2\sec^4\left(\frac{1}{2}(c + dx)\right)\left(-2(3A + 4B + 2C)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2(3A + 4B + 2C)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(4*(B + 2*C)*(c + d*x) - 2*(3*
A + 4*B + 2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(3*A + 4*B + 2*
C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^2 + (4*(2*A + B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]) - A/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(2*A + B)*Sin[(c
+ d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*C*Sin[c + d*x]))/(16*d
)
```

Maple [A] time = 0.076, size = 166, normalized size = 1.4

$$\frac{Aa^2\sec(dx + c)\tan(dx + c)}{2d} + \frac{3Aa^2\ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{a^2B\tan(dx + c)}{d} + \frac{a^2C\ln(\sec(dx + c) + \tan(\cdot}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^2*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2
*B*tan(d*x+c)/d+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a^2*tan(d*x+c)+2/
d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*C*x+2/d*a^2*C*c+a^2*B*x+1/d*B*a^2*c
+1/d*a^2*C*sin(d*x+c)
```

Maxima [A] time = 1.01207, size = 259, normalized size = 2.11

$$\frac{4\,(dx+c)Ba^2 + 8\,(dx+c)Ca^2 - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Aa^2(\log\left(\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 + 8*(d*x + c)*C*a^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 4*C*a^2*sin(d*x + c) + 8*A*a^2*tan(d*x + c) + 4*B*a^2*tan(d*x
+ c))/d
```

Fricas [A] time = 2.05396, size = 358, normalized size = 2.91

$$\frac{4\,(B+2\,C)a^2dx\cos\left(dx+c\right)^2 + (3\,A+4\,B+2\,C)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - (3\,A+4\,B+2\,C)a^2\cos\left(dx+c\right)^2}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="fricas")
```

```
[Out] 1/4*(4*(B + 2*C)*a^2*d*x*cos(d*x + c)^2 + (3*A + 4*B + 2*C)*a^2*cos(d*x + c
)^2*log(sin(d*x + c) + 1) - (3*A + 4*B + 2*C)*a^2*cos(d*x + c)^2*log(-sin(d
*x + c) + 1) + 2*(2*C*a^2*cos(d*x + c)^2 + 2*(2*A + B)*a^2*cos(d*x + c) + A
*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.288, size = 275, normalized size = 2.24

$$\frac{\frac{4\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(Ba^2+2\,Ca^2\right)(dx+c)+\left(3\,Aa^2+4\,Ba^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(3\,Aa^2+4\,Ba^2+2\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="giac")
```

```
[Out] 1/2*(4*C*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(B*a^2 +
 2*C*a^2)*(d*x + c) + (3*A*a^2 + 4*B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1
/2*c) + 1)) - (3*A*a^2 + 4*B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(3*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 -
5*A*a^2*tan(1/2*d*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^2)/d
```

3.315 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec$ $dx) \, dx$

Optimal. Leaf size=134

$$\frac{a^2(2A+3B+2C)\tan(c+dx)}{2d} + \frac{a^2(2A+3B+4C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2A+3B)\tan(c+dx)\sec(c+dx)\left(a^2\cos(c\right.}{6d}$$

```
[Out] a^2*C*x + (a^2*(2*A + 3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A +
 3*B + 2*C)*Tan[c + d*x])/(2*d) + ((2*A + 3*B)*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*d)
```

Rubi [A] time = 0.38518, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2975, 2968, 3021, 2735, 3770}

$$\frac{a^2(2A+3B+2C)\tan(c+dx)}{2d} + \frac{a^2(2A+3B+4C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2A+3B)\tan(c+dx)\sec(c+dx)\left(a^2\cos(c\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] a^2*C*x + (a^2*(2*A + 3*B + 4*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A +
 3*B + 2*C)*Tan[c + d*x])/(2*d) + ((2*A + 3*B)*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{(2A+3B)\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx}{6d}$$

$$= \frac{(2A+3B)\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx}{6d}$$

$$= \frac{a^2(2A+3B+2C)\tan(c+dx)}{2d} + \frac{(2A+3}{}$$

$$= a^2Cx + \frac{a^2(2A+3B+2C)\tan(c+dx)}{2d} + $$

$$= a^2Cx + \frac{a^2(2A+3B+4C)\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [B] time = 5.1629, size = 315, normalized size = 2.35

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6(2A+3B+4C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\tan(c+dx)\sec^2(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(12*c*C + 12*C*d*x - 6*(2*A +
3*B + 4*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*A*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + 18*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
24*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (7*A)/(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^2 + (3*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - (7*A)/
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (3*B)/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + 2*(7*A + 6*B + 3*C - A*Cos[c + d*x] + (5*A + 6*B + 3*C)*Cos[
2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x]))/(48*d)
```

Maple [A] time = 0.078, size = 193, normalized size = 1.4

$$\frac{5\,Aa^2\tan(dx+c)}{3\,d} + \frac{Aa^2\tan(dx+c)(\sec(dx+c))^2}{3\,d} + \frac{a^2B\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{3\,a^2B\ln(\sec(dx+c)+\tan(dx-}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 5/3/d*A*a^2*tan(d*x+c)+1/3/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+1/2*a^2*B*sec(d*
x+c)*tan(d*x+c)/d+3/2/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*tan(d*x+c
)+1/d*A*a^2*sec(d*x+c)*tan(d*x+c)+1/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2
*B*tan(d*x+c)/d+2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+a^2*C*x+1/d*a^2*C*c
```

Maxima [A] time = 1.0195, size = 302, normalized size = 2.25

$$4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^2+12\left(dx+c\right)Ca^2-6\,Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 12*(d*x + c)*C*a^2 - 6*A*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 3*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^2*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 12*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 12*A*a^2*tan(d*x + c) + 24*B*a^2*tan(d*x + c) + 12*C*a^2*tan(d*x + c))
/d
```

Fricas [A] time = 2.16061, size = 379, normalized size = 2.83

$$\frac{12\,Ca^2dx\cos\left(dx+c\right)^3+3\left(2\,A+3\,B+4\,C\right)a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,A+3\,B+4\,C\right)a^2\cos\left(dx+c\right)^3\,\mathrm{l}}{12\,d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="fricas")
```

```
[Out] 1/12*(12*C*a^2*d*x*cos(d*x + c)^3 + 3*(2*A + 3*B + 4*C)*a^2*cos(d*x + c)^3*
log(sin(d*x + c) + 1) - 3*(2*A + 3*B + 4*C)*a^2*cos(d*x + c)^3*log(-sin(d*x
```

```
 + c) + 1) + 2*(2*(5*A + 6*B + 3*C)*a^2*cos(d*x + c)^2 + 3*(2*A + B)*a^2*co
s(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27702, size = 338, normalized size = 2.52

$$\frac{6\,(dx+c)Ca^2+3\left(2\,Aa^2+3\,Ba^2+4\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^2+3\,Ba^2+4\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*C*a^2 + 3*(2*A*a^2 + 3*B*a^2 + 4*C*a^2)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) - 3*(2*A*a^2 + 3*B*a^2 + 4*C*a^2)*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*
c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2
4*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2
*tan(1/2*d*x + 1/2*c) + 15*B*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.316 $\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^5(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{a^2(4A+5B+6C)\tan(c+dx)}{3d}+\frac{a^2(7A+8B+12C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(11A+16B+12C)\tan(c+dx)\sec(c+dx)}{24d}$$

```
[Out] (a^2*(7*A + 8*B + 12*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*A + 5*B + 6*
C)*Tan[c + d*x])/(3*d) + (a^2*(11*A + 16*B + 12*C)*Sec[c + d*x]*Tan[c + d*x
])/(24*d) + ((A + 2*B)*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x]
)/(6*d) + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.474555, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3043, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^2(4A+5B+6C)\tan(c+dx)}{3d}+\frac{a^2(7A+8B+12C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(11A+16B+12C)\tan(c+dx)\sec(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```


```
[Out] (a^2*(7*A + 8*B + 12*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*A + 5*B + 6*
C)*Tan[c + d*x])/(3*d) + (a^2*(11*A + 16*B + 12*C)*Sec[c + d*x]*Tan[c + d*x
])/(24*d) + ((A + 2*B)*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x]
)/(6*d) + (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^2\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{(A+2B)\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+}{6d}$$
$$= \frac{(A+2B)\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+}{6d}$$
$$= \frac{a^2(11A+16B+12C)\sec(c+dx)\tan(c}{24d}$$
$$= \frac{a^2(11A+16B+12C)\sec(c+dx)\tan(c}{24d}$$
$$= \frac{a^2(7A+8B+12C)\tanh^{-1}(\sin(c+dx))}{8d}$$
$$= \frac{a^2(7A+8B+12C)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [B] time = 3.94815, size = 404, normalized size = 2.52

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{16(4A+5B+6C)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{16(4A+5B+6C)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}+\frac{29A+28B+12C}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*(7*A + 8*B + 12*C)*Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(7*A + 8*B + 12*C)*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + (3*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + (2
9*A + 28*B + 12*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (8*(2*A + B)*S
```

```
in[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (16*(4*A + 5*B +
 6*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (3*A)/(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (8*(2*A + B)*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^3 + (-29*A - 4*(7*B + 3*C))/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 + (16*(4*A + 5*B + 6*C)*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])))/(192*d)
```

Maple [A] time = 0.082, size = 246, normalized size = 1.5

$$\frac{Aa^2\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{7Aa^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{7Aa^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{5a^2B\tan(d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^2*tan(d*x+c)*sec(d*x+c)^3+7/8/d*A*a^2*sec(d*x+c)*tan(d*x+c)+7/8/d
*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+5/3*a^2*B*tan(d*x+c)/d+1/3*a^2*B*sec(d*x+c
)^2*tan(d*x+c)/d+1/2/d*a^2*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*C*ln(sec(d*x+c
)+tan(d*x+c))+4/3/d*A*a^2*tan(d*x+c)+2/3/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+a^
2*B*sec(d*x+c)*tan(d*x+c)/d+1/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*C*t
an(d*x+c)
```

Maxima [B] time = 1.04679, size = 427, normalized size = 2.67

$$32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2-3Aa^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2 - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*C*a^2*(2*sin(d*
```

```
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)
) + 24*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*B*a^2*tan
(d*x + c) + 96*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 2.03521, size = 397, normalized size = 2.48

$$\frac{3\,(7\,A+8\,B+12\,C)a^2\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)}-3\,(7\,A+8\,B+12\,C)a^2\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+1)}}{48\,d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="fricas")
```

```
[Out] 1/48*(3*(7*A + 8*B + 12*C)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*
A + 8*B + 12*C)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(4*A + 5*B
 + 6*C)*a^2*cos(d*x + c)^3 + 3*(7*A + 8*B + 4*C)*a^2*cos(d*x + c)^2 + 8*(2*
A + B)*a^2*cos(d*x + c) + 6*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2658, size = 392, normalized size = 2.45

$$3\left(7\,Aa^2+8\,Ba^2+12\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Aa^2+8\,Ba^2+12\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="giac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2 + 12*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 3*(7*A*a^2 + 8*B*a^2 + 12*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2
1*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 36*C*a^2
*tan(1/2*d*x + 1/2*c)^7 - 77*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 88*B*a^2*tan(1/
2*d*x + 1/2*c)^5 - 132*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 136*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 156*C*a^2*tan(1/2*d*x + 1/2
*c)^3 - 75*A*a^2*tan(1/2*d*x + 1/2*c) - 72*B*a^2*tan(1/2*d*x + 1/2*c) - 60*
C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.317 $\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{s}$ $dx)\,dx$

Optimal. Leaf size=196

$$\frac{a^2(18A+20B+25C)\tan(c+dx)}{15d}+\frac{a^2(6A+7B+8C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(18A+25B+20C)\tan(c+dx)\sec^2(}{60d}$$

```
[Out] (a^2*(6*A + 7*B + 8*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(18*A + 20*B + 2
5*C)*Tan[c + d*x])/(15*d) + (a^2*(6*A + 7*B + 8*C)*Sec[c + d*x]*Tan[c + d*x
])/(8*d) + (a^2*(18*A + 25*B + 20*C)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) +
((2*A + 5*B)*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) +
 (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.512652, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3043, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^2(18A+20B+25C)\tan(c+dx)}{15d}+\frac{a^2(6A+7B+8C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(18A+25B+20C)\tan(c+dx)\sec^2(}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```


```
[Out] (a^2*(6*A + 7*B + 8*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(18*A + 20*B + 2
5*C)*Tan[c + d*x])/(15*d) + (a^2*(6*A + 7*B + 8*C)*Sec[c + d*x]*Tan[c + d*x
])/(8*d) + (a^2*(18*A + 25*B + 20*C)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) +
((2*A + 5*B)*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) +
 (A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^2\sec^4(c+dx)\tan(c}{5d}$$

$$=\frac{(2A+5B)\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+}{20d}$$

$$=\frac{(2A+5B)\left(a^2+a^2\cos(c+dx)\right)\sec^3(c+}{20d}$$

$$=\frac{a^2(18A+25B+20C)\sec^2(c+dx)\tan(c}{60d}$$

$$=\frac{a^2(18A+25B+20C)\sec^2(c+dx)\tan(c}{60d}$$

$$=\frac{a^2(6A+7B+8C)\sec(c+dx)\tan(c+d}{8d}$$

$$=\frac{a^2(6A+7B+8C)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [B] time = 5.30991, size = 502, normalized size = 2.56

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{16(18A+20B+25C)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{2(39A+20(2B+C))\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{16(18A+20B+25C)\sin\left(\frac{1}{2}(c+dx\right.}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-30*(6*A + 7*B + 8*C)*Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] + 30*(6*A + 7*B + 8*C)*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + (3*(12*A + 5*B))/(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^4 + (129*A + 145*B + 140*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 +
(12*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (2*(39*A
+ 20*(2*B + C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 +
 (16*(18*A + 20*B + 25*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]) + (12*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 -
(3*(12*A + 5*B))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (2*(39*A + 20*(2
*B + C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (-129*
A - 5*(29*B + 28*C))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (16*(18*A +
20*B + 25*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(960
*d)
```

Maple [A] time = 0.088, size = 315, normalized size = 1.6

$$\frac{6\,Aa^2\tan(dx+c)}{5\,d}+\frac{Aa^2\tan(dx+c)(\sec(dx+c))^4}{5\,d}+\frac{3\,Aa^2\tan(dx+c)(\sec(dx+c))^2}{5\,d}+\frac{a^2B(\sec(dx+c))^3\tan(dx+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 6/5/d*A*a^2*tan(d*x+c)+1/5/d*A*a^2*tan(d*x+c)*sec(d*x+c)^4+3/5/d*A*a^2*tan(
d*x+c)*sec(d*x+c)^2+1/4*a^2*B*sec(d*x+c)^3*tan(d*x+c)/d+7/8*a^2*B*sec(d*x+c
)*tan(d*x+c)/d+7/8/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+5/3/d*a^2*C*tan(d*x+c)
+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*A*a^2*tan(d*x+c)*sec(d*x+c)^3+3/
4/d*A*a^2*sec(d*x+c)*tan(d*x+c)+3/4/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a
^2*B*tan(d*x+c)/d+2/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d+1/d*a^2*C*sec(d*x+c)*
tan(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01797, size = 486, normalized size = 2.48

$$16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^2+80\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2+160\left(\tan(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 +
80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 160*(tan(d*x + c)^3 + 3*tan(d*
x + c))*B*a^2 + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 - 30*A*a^2*(2*(3
*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 15*B*a^2*(2*(3*sin(d*
x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(
sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*B*a^2*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*C*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 240*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 2.09134, size = 463, normalized size = 2.36

$$15\,(6\,A+7\,B+8\,C)a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(6\,A+7\,B+8\,C)a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(6*A + 7*B + 8*C)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(
6*A + 7*B + 8*C)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(18*A + 2
0*B + 25*C)*a^2*cos(d*x + c)^4 + 15*(6*A + 7*B + 8*C)*a^2*cos(d*x + c)^3 +
8*(9*A + 10*B + 5*C)*a^2*cos(d*x + c)^2 + 30*(2*A + B)*a^2*cos(d*x + c) + 2
4*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
6,x)
```

[Out] Timed out

Giac [A] time = 1.2344, size = 460, normalized size = 2.35

$$15\left(6\,Aa^2+7\,Ba^2+8\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(6\,Aa^2+7\,Ba^2+8\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(90\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="giac")
```

```
[Out] 1/120*(15*(6*A*a^2 + 7*B*a^2 + 8*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(6*A*a^2 + 7*B*a^2 + 8*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(
90*A*a^2*tan(1/2*d*x + 1/2*c)^9 + 105*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 120*C*
a^2*tan(1/2*d*x + 1/2*c)^9 - 420*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 490*B*a^2*t
an(1/2*d*x + 1/2*c)^7 - 560*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*A*a^2*tan(1/
2*d*x + 1/2*c)^5 + 800*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 1120*C*a^2*tan(1/2*d*
x + 1/2*c)^5 - 540*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 790*B*a^2*tan(1/2*d*x + 1
/2*c)^3 - 1040*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 390*A*a^2*tan(1/2*d*x + 1/2*c
) + 375*B*a^2*tan(1/2*d*x + 1/2*c) + 360*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.318 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos\right.$

Optimal. Leaf size=265

$$-\frac{a^3(133A+119B+108C)\sin^3(c+dx)}{105d}+\frac{a^3(133A+119B+108C)\sin(c+dx)}{35d}+\frac{a^3(154A+147B+129C)\sin(c+dx)}{280d}$$

```
[Out] (a^3*(26*A + 23*B + 21*C)*x)/16 + (a^3*(133*A + 119*B + 108*C)*Sin[c + d*x]
)/(35*d) + (a^3*(26*A + 23*B + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a
^3*(154*A + 147*B + 129*C)*Cos[c + d*x]^3*Sin[c + d*x])/(280*d) + (C*Cos[c
+ d*x]^3*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) + ((7*B + 3*C)*Cos[c +
d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(42*a*d) + ((3*A + 4*B + 3*
C)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d) - (a^3*(133
*A + 119*B + 108*C)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.678233, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3045, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^3(133A+119B+108C)\sin^3(c+dx)}{105d}+\frac{a^3(133A+119B+108C)\sin(c+dx)}{35d}+\frac{a^3(154A+147B+129C)\sin(c+dx)}{280d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```


```
[Out] (a^3*(26*A + 23*B + 21*C)*x)/16 + (a^3*(133*A + 119*B + 108*C)*Sin[c + d*x]
)/(35*d) + (a^3*(26*A + 23*B + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a
^3*(154*A + 147*B + 129*C)*Cos[c + d*x]^3*Sin[c + d*x])/(280*d) + (C*Cos[c
+ d*x]^3*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) + ((7*B + 3*C)*Cos[c +
d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(42*a*d) + ((3*A + 4*B + 3*
C)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d) - (a^3*(133
*A + 119*B + 108*C)*Sin[c + d*x]^3)/(105*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c}{7d}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c}{7d}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c}{7d}$$

$$= \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c}{7d}$$

$$= \frac{a^3(154A+147B+129C)\cos^3(c+dx)\,\mathrm{s}}{280d}$$

$$= \frac{a^3(154A+147B+129C)\cos^3(c+dx)\,\mathrm{s}}{280d}$$

$$= \frac{a^3(26A+23B+21C)\cos(c+dx)\sin(c}{16d}$$

$$= \frac{1}{16}a^3(26A+23B+21C)x + \frac{a^3(133A+}{}$$

Mathematica [A] time = 1.06557, size = 204, normalized size = 0.77

$$\frac{a^3(105(184A+168B+155C)\sin(c+dx)+105(64A+63B+61C)\sin(2(c+dx))+2380A\sin(3(c+dx))+630A\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (a^3*(9660*B*c + 5460*c*C + 10920*A*d*x + 9660*B*d*x + 8820*C*d*x + 105*(18
4*A + 168*B + 155*C)*Sin[c + d*x] + 105*(64*A + 63*B + 61*C)*Sin[2*(c + d*x
)] + 2380*A*Sin[3*(c + d*x)] + 2660*B*Sin[3*(c + d*x)] + 2835*C*Sin[3*(c +
d*x)] + 630*A*Sin[4*(c + d*x)] + 945*B*Sin[4*(c + d*x)] + 1155*C*Sin[4*(c +
 d*x)] + 84*A*Sin[5*(c + d*x)] + 252*B*Sin[5*(c + d*x)] + 399*C*Sin[5*(c +
d*x)] + 35*B*Sin[6*(c + d*x)] + 105*C*Sin[6*(c + d*x)] + 15*C*Sin[7*(c + d*
x)]))/(6720*d)
```

Maple [A] time = 0.029, size = 427, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x
+3/8*c)+A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*co
s(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c)+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8
*d*x+3/8*c)+3/5*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*
C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+
5/16*c)+1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/6
*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c
)+1/7*a^3*C*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c
))
```

Maxima [A] time = 1.01304, size = 574, normalized size = 2.17

$$\frac{448\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{3}-6720\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}+630\,(12\,dx+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/6720*(448*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3
- 6720*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 630*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 1680*(2*d*x + 2*c + sin(2*d*x + 2
*c))*A*a^3 + 1344*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*
B*a^3 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*B*a^3 - 2240*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 63
0*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 192*(5*si
n(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*C*a
^3 + 1344*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^3 -
105*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*C*a^3 + 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*C*a^3)/d
```

Fricas [A] time = 2.14476, size = 454, normalized size = 1.71

$$\frac{105\,(26\,A + 23\,B + 21\,C)a^3 dx + \left(240\,Ca^3\cos\left(dx + c\right)^6 + 280\,(B + 3\,C)a^3\cos\left(dx + c\right)^5 + 48\,(7\,A + 21\,B + 27\,C)a^3\cos(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/1680*(105*(26*A + 23*B + 21*C)*a^3*d*x + (240*C*a^3*cos(d*x + c)^6 + 280*
(B + 3*C)*a^3*cos(d*x + c)^5 + 48*(7*A + 21*B + 27*C)*a^3*cos(d*x + c)^4 +
70*(18*A + 23*B + 21*C)*a^3*cos(d*x + c)^3 + 16*(133*A + 119*B + 108*C)*a^3
*cos(d*x + c)^2 + 105*(26*A + 23*B + 21*C)*a^3*cos(d*x + c) + 32*(133*A + 1
19*B + 108*C)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 11.1243, size = 1149, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Piecewise((9*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 9*A*a**3*x*cos(c + d*x)**4/8 + A
```

```
*a**3*x*cos(c + d*x)**2/2 + 8*A*a**3*sin(c + d*x)**5/(15*d) + 4*A*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 2*A*a**3*sin(c + d*x)**3/d + A*a**3*sin(c + d*x)*cos(c + d*x)**4/d +
 15*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*A*a**3*sin(c + d*x)*cos(c
 + d*x)**2/d + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a**3*x*sin(c +
d*x)**6/16 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*B*a**3*x*si
n(c + d*x)**4/8 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*B*a**3
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*B*a**3*x*cos(c + d*x)**6/16 + 9*B*
a**3*x*cos(c + d*x)**4/8 + 5*B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8
*B*a**3*sin(c + d*x)**5/(5*d) + 5*B*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6
*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*B*a**3*sin(c + d*x)**3
*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/(3*d) + 11*B*a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) + 3*B*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*B*
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d + 15*C*a**3*x*sin(c + d*x)**6/16 + 45*C*a**3*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 3*C*a**3*x*sin(c + d*x)**4/8 + 45*C*a**3*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 3*C*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 15*C*a**3
*x*cos(c + d*x)**6/16 + 3*C*a**3*x*cos(c + d*x)**4/8 + 16*C*a**3*sin(c + d*
x)**7/(35*d) + 8*C*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 15*C*a**3*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + 8*C*a**3*sin(c + d*x)**5/(5*d) + 2*C*a
**3*sin(c + d*x)**3*cos(c + d*x)**4/d + 5*C*a**3*sin(c + d*x)**3*cos(c + d*
x)**3/(2*d) + 4*C*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*C*a**3*sin(c +
 d*x)**3*cos(c + d*x)/(8*d) + C*a**3*sin(c + d*x)*cos(c + d*x)**6/d + 33*C*
a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 3*C*a**3*sin(c + d*x)*cos(c + d*
x)**4/d + 5*C*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos
(c) + a)**3*(A + B*cos(c) + C*cos(c)**2)*cos(c)**2, True))
```

Giac [A] time = 1.24902, size = 309, normalized size = 1.17

$$\frac{Ca^3 \sin(7\,dx+7\,c)}{448\,d} + \frac{1}{16}\left(26\,Aa^3 + 23\,Ba^3 + 21\,Ca^3\right)x + \frac{\left(Ba^3 + 3\,Ca^3\right)\sin(6\,dx+6\,c)}{192\,d} + \frac{\left(4\,Aa^3 + 12\,Ba^3 + 19\,Ca^3\right)\sin}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/448*C*a^3*sin(7*d*x + 7*c)/d + 1/16*(26*A*a^3 + 23*B*a^3 + 21*C*a^3)*x +
1/192*(B*a^3 + 3*C*a^3)*sin(6*d*x + 6*c)/d + 1/320*(4*A*a^3 + 12*B*a^3 + 19
*C*a^3)*sin(5*d*x + 5*c)/d + 1/64*(6*A*a^3 + 9*B*a^3 + 11*C*a^3)*sin(4*d*x
+ 4*c)/d + 1/192*(68*A*a^3 + 76*B*a^3 + 81*C*a^3)*sin(3*d*x + 3*c)/d + 1/64
*(64*A*a^3 + 63*B*a^3 + 61*C*a^3)*sin(2*d*x + 2*c)/d + 1/64*(184*A*a^3 + 16
```

```
8*B*a^3 + 155*C*a^3)*sin(d*x + c)/d
```

3.319 $\int \cos(c+dx)(a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(\right.$

Optimal. Leaf size=207

$$-\frac{a^3(30A+26B+23C)\sin^3(c+dx)}{120d}+\frac{a^3(30A+26B+23C)\sin(c+dx)}{10d}+\frac{3a^3(30A+26B+23C)\sin(c+dx)\cos(c+d}{80d}$$

```
[Out] (a^3*(30*A + 26*B + 23*C)*x)/16 + (a^3*(30*A + 26*B + 23*C)*Sin[c + d*x])/(
10*d) + (3*a^3*(30*A + 26*B + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + ((3
0*A - 6*B + 7*C)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(120*d) + (C*Cos[c +
d*x]^2*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(6*d) + ((2*B + C)*(a + a*Cos[c
 + d*x])^4*Sin[c + d*x])/(10*a*d) - (a^3*(30*A + 26*B + 23*C)*Sin[c + d*x]^
3)/(120*d)
```

Rubi [A] time = 0.396215, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3045, 2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(30A+26B+23C)\sin^3(c+dx)}{120d}+\frac{a^3(30A+26B+23C)\sin(c+dx)}{10d}+\frac{3a^3(30A+26B+23C)\sin(c+dx)\cos(c+d}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] (a^3*(30*A + 26*B + 23*C)*x)/16 + (a^3*(30*A + 26*B + 23*C)*Sin[c + d*x])/(
10*d) + (3*a^3*(30*A + 26*B + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + ((3
0*A - 6*B + 7*C)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(120*d) + (C*Cos[c +
d*x]^2*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(6*d) + ((2*B + C)*(a + a*Cos[c
 + d*x])^4*Sin[c + d*x])/(10*a*d) - (a^3*(30*A + 26*B + 23*C)*Sin[c + d*x]^
3)/(120*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^3\sin(c+}{6d}$$
$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^3\sin(c+}{6d}$$
$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^3\sin(c+}{6d}$$
$$= \frac{(30A-6B+7C)(a+a\cos(c+dx))^3\sin(c}{120d}$$
$$= \frac{(30A-6B+7C)(a+a\cos(c+dx))^3\sin(c}{120d}$$
$$= \frac{1}{40}a^3(30A+26B+23C)x+\frac{(30A-6B+}{}$$
$$= \frac{1}{40}a^3(30A+26B+23C)x+\frac{3a^3(30A+26}{}$$
$$= \frac{1}{16}a^3(30A+26B+23C)x+\frac{a^3(30A+26B}{}$$

Mathematica [A] time = 0.629833, size = 171, normalized size = 0.83

$$\frac{a^3(120(26A+23B+21C)\sin(c+dx)+15(64A+64B+63C)\sin(2(c+dx))+240A\sin(3(c+dx))+30A\sin(4(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (a^3*(1560*B*c + 900*c*C + 1800*A*d*x + 1560*B*d*x + 1380*C*d*x + 120*(26*A
 + 23*B + 21*C)*Sin[c + d*x] + 15*(64*A + 64*B + 63*C)*Sin[2*(c + d*x)] + 2
```

```
40*A*Sin[3*(c + d*x)] + 340*B*Sin[3*(c + d*x)] + 380*C*Sin[3*(c + d*x)] + 3
0*A*Sin[4*(c + d*x)] + 90*B*Sin[4*(c + d*x)] + 135*C*Sin[4*(c + d*x)] + 12*
B*Sin[5*(c + d*x)] + 36*C*Sin[5*(c + d*x)] + 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.03, size = 364, normalized size = 1.8

$$\frac{1}{d}\left(Aa^3\sin(dx+c)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+3\,Aa^3\,(1/2\ \mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a
^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*C*(1/4*(cos(d*x+c)^3+3/2*co
s(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*a^3*C*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c)+a^3*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+
c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.02349, size = 478, normalized size = 2.31

$$-\frac{960\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^3-30\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3-720\,(2\,dx+2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/960*(960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 30*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^3 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^3 + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 90*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 240*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c
```

```
))*C*a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*C*a^3 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 9
0*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 - 960*A*a^3
*sin(d*x + c))/d
```

Fricas [A] time = 1.96045, size = 378, normalized size = 1.83

$$\frac{15\,(30\,A+26\,B+23\,C)a^3dx+\left(40\,Ca^3\cos\left(dx+c\right)^5+48\,(B+3\,C)a^3\cos\left(dx+c\right)^4+10\,(6\,A+18\,B+23\,C)a^3\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(30*A + 26*B + 23*C)*a^3*d*x + (40*C*a^3*cos(d*x + c)^5 + 48*(B +
 3*C)*a^3*cos(d*x + c)^4 + 10*(6*A + 18*B + 23*C)*a^3*cos(d*x + c)^3 + 16*(
15*A + 19*B + 17*C)*a^3*cos(d*x + c)^2 + 15*(30*A + 26*B + 23*C)*a^3*cos(d*
x + c) + 16*(45*A + 38*B + 34*C)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 6.3139, size = 932, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 +
 3*A*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*A*a**3*sin(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**3*sin(c + d*x)*cos(c +
d*x)/(2*d) + A*a**3*sin(c + d*x)/d + 9*B*a**3*x*sin(c + d*x)**4/8 + 9*B*a**
3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**3*x*sin(c + d*x)**2/2 + 9*B*a*
*3*x*cos(c + d*x)**4/8 + B*a**3*x*cos(c + d*x)**2/2 + 8*B*a**3*sin(c + d*x)
**5/(15*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**3*sin(
c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/d + B*a**3*sin(c
```

```
+ d*x)*cos(c + d*x)**4/d + 15*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3
*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + B*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d) + 5*C*a**3*x*sin(c + d*x)**6/16 + 15*C*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 9*C*a**3*x*sin(c + d*x)**4/8 + 15*C*a**3*x*sin(c + d*x)**2*cos
(c + d*x)**4/16 + 9*C*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*C*a**3*x
*cos(c + d*x)**6/16 + 9*C*a**3*x*cos(c + d*x)**4/8 + 5*C*a**3*sin(c + d*x)*
*5*cos(c + d*x)/(16*d) + 8*C*a**3*sin(c + d*x)**5/(5*d) + 5*C*a**3*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 4*C*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d
+ 9*C*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*C*a**3*sin(c + d*x)**3/(3
*d) + 11*C*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 3*C*a**3*sin(c + d*x)
*cos(c + d*x)**4/d + 15*C*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a**3*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*(A + B*cos(
c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.18807, size = 265, normalized size = 1.28

$$\frac{Ca^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(30\,Aa^3+26\,Ba^3+23\,Ca^3\right)x+\frac{\left(Ba^3+3\,Ca^3\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{\left(2\,Aa^3+6\,Ba^3+9\,Ca^3\right)\sin}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/192*C*a^3*sin(6*d*x + 6*c)/d + 1/16*(30*A*a^3 + 26*B*a^3 + 23*C*a^3)*x +
1/80*(B*a^3 + 3*C*a^3)*sin(5*d*x + 5*c)/d + 1/64*(2*A*a^3 + 6*B*a^3 + 9*C*a
^3)*sin(4*d*x + 4*c)/d + 1/48*(12*A*a^3 + 17*B*a^3 + 19*C*a^3)*sin(3*d*x +
3*c)/d + 1/64*(64*A*a^3 + 64*B*a^3 + 63*C*a^3)*sin(2*d*x + 2*c)/d + 1/8*(26
*A*a^3 + 23*B*a^3 + 21*C*a^3)*sin(d*x + c)/d
```

3.320 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) d$

Optimal. Leaf size=166

$$-\frac{a^3(20A+15B+13C)\sin^3(c+dx)}{60d}+\frac{a^3(20A+15B+13C)\sin(c+dx)}{5d}+\frac{3a^3(20A+15B+13C)\sin(c+dx)\cos(c+dx}{40d}$$

```
[Out] (a^3*(20*A + 15*B + 13*C)*x)/8 + (a^3*(20*A + 15*B + 13*C)*Sin[c + d*x])/(5
*d) + (3*a^3*(20*A + 15*B + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*B
 - C)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])
^4*Sin[c + d*x])/(5*a*d) - (a^3*(20*A + 15*B + 13*C)*Sin[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.226378, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(20A+15B+13C)\sin^3(c+dx)}{60d}+\frac{a^3(20A+15B+13C)\sin(c+dx)}{5d}+\frac{3a^3(20A+15B+13C)\sin(c+dx)\cos(c+dx}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(20*A + 15*B + 13*C)*x)/8 + (a^3*(20*A + 15*B + 13*C)*Sin[c + d*x])/(5
*d) + (3*a^3*(20*A + 15*B + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*B
 - C)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])
^4*Sin[c + d*x])/(5*a*d) - (a^3*(20*A + 15*B + 13*C)*Sin[c + d*x]^3)/(60*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{5ad} + \frac{\int (a+a\cos(c+dx)}{}$$
$$= \frac{(5B-C)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{C(a+a\cos}{}$$
$$= \frac{(5B-C)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{C(a+a\cos}{}$$
$$= \frac{1}{20}a^3(20A+15B+13C)x + \frac{(5B-C)(a+a\cos(c+dx}{20d}$$
$$= \frac{1}{20}a^3(20A+15B+13C)x + \frac{3a^3(20A+15B+13C)\sin}{20d}$$
$$= \frac{1}{8}a^3(20A+15B+13C)x + \frac{a^3(20A+15B+13C)\sin(}{5d}$$

Mathematica [A] time = 0.481844, size = 129, normalized size = 0.78

$$\frac{a^3(60(30A+26B+23C)\sin(c+dx)+120(3A+4(B+C))\sin(2(c+dx))+40A\sin(3(c+dx))+1200Adx+120B\sin(3}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^3*(1200*A*d*x + 900*B*d*x + 780*C*d*x + 60*(30*A + 26*B + 23*C)*Sin[c +
d*x] + 120*(3*A + 4*(B + C))*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 120
*B*Sin[3*(c + d*x)] + 170*C*Sin[3*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 45*C
*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.027, size = 295, normalized size = 1.8

$$\frac{1}{d}\left(\frac{a^3C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^3B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*a^3*C*(1/4*(cos(d*x
```

```
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*a^3*(2+cos(d*x+c)^2)*
sin(d*x+c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(2+cos(d*x+c)^2)*sin(d*x
+c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*(1/2*cos(d*x+
c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+3*A*a^3*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c))
```

Maxima [A] time = 1.00335, size = 381, normalized size = 2.3

$$-\frac{160\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}-360\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-480\left(dx+c\right)Aa^{3}+480\left(\sin\left(dx+c\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 360*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a^3 - 480*(d*x + c)*A*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d
*x + c))*B*a^3 - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*B*a^3 - 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 32*(3*sin(d*x + c)^5
- 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^3 + 480*(sin(d*x + c)^3 - 3*sin(
d*x + c))*C*a^3 - 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*C*a^3 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 - 1440*A*a^3*sin(d*x +
 c) - 480*B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.92371, size = 315, normalized size = 1.9

$$\frac{15\left(20\,A+15\,B+13\,C\right)a^{3}dx+\left(24\,Ca^{3}\cos\left(dx+c\right)^{4}+30\left(B+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+8\left(5\,A+15\,B+19\,C\right)a^{3}\cos\left(dx+\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(20*A + 15*B + 13*C)*a^3*d*x + (24*C*a^3*cos(d*x + c)^4 + 30*(B +
 3*C)*a^3*cos(d*x + c)^3 + 8*(5*A + 15*B + 19*C)*a^3*cos(d*x + c)^2 + 15*(1
2*A + 15*B + 13*C)*a^3*cos(d*x + c) + 8*(55*A + 45*B + 38*C)*a^3)*sin(d*x +
 c))/d
```

Sympy [A] time = 3.46027, size = 658, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**2/2 + A*
a**3*x + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*A*a**3*sin(c + d*x)/d +
 3*B*a**3*x*sin(c + d*x)**4/8 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*B*a**3*x*sin(c + d*x)**2/2 + 3*B*a**3*x*cos(c + d*x)**4/8 + 3*B*a**3*
x*cos(c + d*x)**2/2 + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**
3*sin(c + d*x)**3/d + 5*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*B*a**
3*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d)
 + B*a**3*sin(c + d*x)/d + 9*C*a**3*x*sin(c + d*x)**4/8 + 9*C*a**3*x*sin(c
+ d*x)**2*cos(c + d*x)**2/4 + C*a**3*x*sin(c + d*x)**2/2 + 9*C*a**3*x*cos(c
 + d*x)**4/8 + C*a**3*x*cos(c + d*x)**2/2 + 8*C*a**3*sin(c + d*x)**5/(15*d)
 + 4*C*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*C*a**3*sin(c + d*x)**
3*cos(c + d*x)/(8*d) + 2*C*a**3*sin(c + d*x)**3/d + C*a**3*sin(c + d*x)*cos
(c + d*x)**4/d + 15*C*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*C*a**3*si
n(c + d*x)*cos(c + d*x)**2/d + C*a**3*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d
, 0)), (x*(a*cos(c) + a)**3*(A + B*cos(c) + C*cos(c)**2), True))
```

Giac [A] time = 1.14808, size = 220, normalized size = 1.33

$$\frac{Ca^3\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(20\,Aa^3+15\,Ba^3+13\,Ca^3\right)x+\frac{\left(Ba^3+3\,Ca^3\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(4\,Aa^3+12\,Ba^3+17\,Ca^3\right)\sin(}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/80*C*a^3*sin(5*d*x + 5*c)/d + 1/8*(20*A*a^3 + 15*B*a^3 + 13*C*a^3)*x + 1/
32*(B*a^3 + 3*C*a^3)*sin(4*d*x + 4*c)/d + 1/48*(4*A*a^3 + 12*B*a^3 + 17*C*a
^3)*sin(3*d*x + 3*c)/d + 1/4*(3*A*a^3 + 4*B*a^3 + 4*C*a^3)*sin(2*d*x + 2*c)
/d + 1/8*(30*A*a^3 + 26*B*a^3 + 23*C*a^3)*sin(d*x + c)/d
```

3.321 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ s dx) dx

Optimal. Leaf size=162

$$\frac{5a^3(4A+4B+3C)\sin(c+dx)}{8d} + \frac{(12A+20B+15C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{24d} + \frac{1}{8}a^3x(28A+20B+15C) + $$

```
[Out] (a^3*(28*A + 20*B + 15*C)*x)/8 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(
4*A + 4*B + 3*C)*Sin[c + d*x])/(8*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*
x])/(4*d) + ((4*B + 3*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*d)
+ ((12*A + 20*B + 15*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.477943, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3045, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^3(4A+4B+3C)\sin(c+dx)}{8d} + \frac{(12A+20B+15C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{24d} + \frac{1}{8}a^3x(28A+20B+15C) + $$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (a^3*(28*A + 20*B + 15*C)*x)/8 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(
4*A + 4*B + 3*C)*Sin[c + d*x])/(8*d) + (C*(a + a*Cos[c + d*x])^3*Sin[c + d*
x])/(4*d) + ((4*B + 3*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*d)
+ ((12*A + 20*B + 15*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{\int(a-$$

$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{(4B+$$

$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{(4B+$$

$$= \frac{C(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{(4B+$$

$$= \frac{5a^3(4A+4B+3C)\sin(c+dx)}{8d} + \frac{C(a+$$

$$= \frac{1}{8}a^3(28A+20B+15C)x + \frac{5a^3(4A+4B$$

$$= \frac{1}{8}a^3(28A+20B+15C)x + \frac{a^3A\tanh^{-1}(s}{d}$$

Mathematica [A] time = 0.479983, size = 147, normalized size = 0.91

$$\frac{a^3\left(24(12A+15B+13C)\sin(c+dx)+24(A+3B+4C)\sin(2(c+dx))-96A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (a^3*(336*A*d*x + 240*B*d*x + 180*C*d*x - 96*A*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 96*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(12*A + 1
5*B + 13*C)*Sin[c + d*x] + 24*(A + 3*B + 4*C)*Sin[2*(c + d*x)] + 8*B*Sin[3*
(c + d*x)] + 24*C*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.067, size = 251, normalized size = 1.6

$$\frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{5a^3Bx}{2} + \frac{5a^3Bc}{2d} + 3\frac{a^3C\sin(dx+c)}{d} + \frac{7Aa^3x}{2} + \frac{7Aa^3c}{2d} + \frac{11a^3B\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+5/2*a^3*B*x+5/2/d*a^3*B*c+3*a^3*C*sin(d
*x+c)/d+7/2*A*a^3*x+7/2/d*A*a^3*c+11/3*a^3*B*sin(d*x+c)/d+15/8/d*a^3*C*cos(
d*x+c)*sin(d*x+c)+15/8*a^3*C*x+15/8/d*a^3*C*c+3*a^3*A*sin(d*x+c)/d+3/2/d*a^
3*B*cos(d*x+c)*sin(d*x+c)+1/d*C*sin(d*x+c)*cos(d*x+c)^2*a^3+1/2/d*A*a^3*cos
(d*x+c)*sin(d*x+c)+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^3+1/4/d*a^3*C*sin(d*x+
c)*cos(d*x+c)^3
```

Maxima [A] time = 1.01984, size = 315, normalized size = 1.94

$$\frac{24\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^3 + 288\,(dx + c)Aa^3 - 32\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ba^3 + 72\,(2\,dx + 2\,c + \sin(2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 288*(d*x + c)*A*a^3 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 72*(2*d*x + 2*c + sin(2*d*x + 2*c
))*B*a^3 + 96*(d*x + c)*B*a^3 - 96*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3
+ 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 + 72*(2*d
*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 96*A*a^3*log(sec(d*x + c) + tan(d*x +
c)) + 288*A*a^3*sin(d*x + c) + 288*B*a^3*sin(d*x + c) + 96*C*a^3*sin(d*x +
c))/d
```

Fricas [A] time = 1.93959, size = 336, normalized size = 2.07

$$\frac{3\,(28\,A + 20\,B + 15\,C)a^3dx + 12\,Aa^3\log(\sin(dx + c) + 1) - 12\,Aa^3\log(-\sin(dx + c) + 1) + \left(6\,Ca^3\cos(dx + c)^3 + 8\,(B}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="fricas")
```

```
[Out] 1/24*(3*(28*A + 20*B + 15*C)*a^3*d*x + 12*A*a^3*log(sin(d*x + c) + 1) - 12*
A*a^3*log(-sin(d*x + c) + 1) + (6*C*a^3*cos(d*x + c)^3 + 8*(B + 3*C)*a^3*co
```

```
s(d*x + c)^2 + 3*(4*A + 12*B + 15*C)*a^3*cos(d*x + c) + 8*(9*A + 11*B + 9*C
)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.26506, size = 386, normalized size = 2.38

$$24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(28\,Aa^3+20\,Ba^3+15\,Ca^3\right)(dx+c)+\frac{2(60}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="giac")
```

```
[Out] 1/24*(24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(28*A*a^3 + 20*B*a^3 + 15*C*a^3)*(d*x + c) + 2*(60
*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 45*C*a^3*
tan(1/2*d*x + 1/2*c)^7 + 204*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 220*B*a^3*tan(1
/2*d*x + 1/2*c)^5 + 165*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 228*A*a^3*tan(1/2*d*
x + 1/2*c)^3 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 219*C*a^3*tan(1/2*d*x + 1
/2*c)^3 + 84*A*a^3*tan(1/2*d*x + 1/2*c) + 132*B*a^3*tan(1/2*d*x + 1/2*c) +
147*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.322 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ se $dx)\,dx$

Optimal. Leaf size=156

$$-\frac{(6A-3B-5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^3x(6A+7B+5C)-\frac{(3A-C}{}$$

```
[Out] (a^3*(6*A + 7*B + 5*C)*x)/2 + (a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*
a^3*(B + C)*Sin[c + d*x])/(2*d) - ((3*A - C)*(a^2 + a^2*Cos[c + d*x])^2*Sin
[c + d*x])/(3*a*d) - ((6*A - 3*B - 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*
x])/(6*d) + (A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.509521, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2976, 2968, 3023, 2735, 3770}

$$-\frac{(6A-3B-5C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^3x(6A+7B+5C)-\frac{(3A-C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```


```
[Out] (a^3*(6*A + 7*B + 5*C)*x)/2 + (a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*
a^3*(B + C)*Sin[c + d*x])/(2*d) - ((3*A - C)*(a^2 + a^2*Cos[c + d*x])^2*Sin
[c + d*x])/(3*a*d) - ((6*A - 3*B - 5*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*
x])/(6*d) + (A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\tan(c+dx)}{d} + \frac{\int (a \cdots}{}$$
$$= -\frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+\cdots}{3ad}$$
$$= -\frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+\cdots}{3ad}$$
$$= -\frac{(3A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+\cdots}{3ad}$$
$$= \frac{5a^3(B+C)\sin(c+dx)}{2d} - \frac{(3A-C)\left(a^2+\cdots\right.}{}$$
$$= \frac{1}{2}a^3(6A+7B+5C)x + \frac{5a^3(B+C)\sin(c\cdots}{2d}$$
$$= \frac{1}{2}a^3(6A+7B+5C)x + \frac{a^3(3A+B)\tanh^{\cdots}}{d}$$

Mathematica [A] time = 0.958279, size = 227, normalized size = 1.46

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(6(6A+7B+5C)(c+dx)+3(4A+12B+15C)\sin(c+dx)-12(3A+B)\log\left(\cos\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(6*(6*A + 7*B + 5*C)*(c + d*x)
 - 12*(3*A + B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*(3*A + B)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*A*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]) + (12*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]) + 3*(4*A + 12*B + 15*C)*Sin[c + d*x] + 3*(B + 3*C)*Sin[2*(c
 + d*x)] + C*Sin[3*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.082, size = 221, normalized size = 1.4

$$\frac{Aa^3\tan(dx+c)}{d} + \frac{a^3B\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{5a^3Cx}{2} + \frac{5a^3Cc}{2d} + 3\frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{7\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*a^3*tan(d*x+c)+1/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+5/2*a^3*C*x+5/2/d*
a^3*C*c+3/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+7/2*a^3*B*x+7/2/d*a^3*B*c+11/3*
a^3*C*sin(d*x+c)/d+3*A*a^3*x+3/d*A*a^3*c+3*a^3*B*sin(d*x+c)/d+3/2/d*a^3*C*c
os(d*x+c)*sin(d*x+c)+a^3*A*sin(d*x+c)/d+1/2/d*a^3*B*cos(d*x+c)*sin(d*x+c)+1
/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^3
```

Maxima [A] time = 0.999814, size = 284, normalized size = 1.82

$$\frac{36\,(dx+c)Aa^3 + 3\,(2\,dx + 2\,c + \sin(2\,dx+2\,c))Ba^3 + 36\,(dx+c)Ba^3 - 4\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)Ca^3 + 9\,(2\,dx\,+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="maxima")
```

```
[Out] 1/12*(36*(d*x + c)*A*a^3 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 36*(d
*x + c)*B*a^3 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 9*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*C*a^3 + 12*(d*x + c)*C*a^3 + 18*A*a^3*(log(sin(d*x + c)
 + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 12*A*a^3*sin(d*x + c) + 36*B*a^3*sin(d*x + c) + 36*C*a^3*si
n(d*x + c) + 12*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.12494, size = 398, normalized size = 2.55

$$\frac{3\,(6\,A + 7\,B + 5\,C)a^3dx\cos(dx+c) + 3\,(3\,A+B)a^3\cos(dx+c)\log(\sin(dx+c)+1) - 3\,(3\,A+B)a^3\cos(dx+c)\log(}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(6*A + 7*B + 5*C)*a^3*d*x*cos(d*x + c) + 3*(3*A + B)*a^3*cos(d*x + c
)*log(sin(d*x + c) + 1) - 3*(3*A + B)*a^3*cos(d*x + c)*log(-sin(d*x + c) +
```

```
1) + (2*C*a^3*cos(d*x + c)^3 + 3*(B + 3*C)*a^3*cos(d*x + c)^2 + 2*(3*A + 9*
B + 11*C)*a^3*cos(d*x + c) + 6*A*a^3)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26877, size = 379, normalized size = 2.43

$$-\frac{\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\left(6\,Aa^3+7\,Ba^3+5\,Ca^3\right)(dx+c) - 6\left(3\,Aa^3+Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 6\left(3\,Aa^3+Ba^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="giac")
```

```
[Out] -1/6*(12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(6*A*a
^3 + 7*B*a^3 + 5*C*a^3)*(d*x + c) - 6*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) + 6*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*C*
a^3*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^3*tan
(1/2*d*x + 1/2*c)^3 + 40*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x
 + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c) + 33*C*a^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.323 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{a^3(7A+6B+2C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{(4A+2B-C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d} + \frac{(3A+2B)\tan(c+dx)\left(a^2\right.}{2ad}$$

```
[Out] (a^3*(2*A + 6*B + 7*C)*x)/2 + (a^3*(7*A + 6*B + 2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) - (5*a^3*(A - C)*Sin[c + d*x])/(2*d) - ((4*A + 2*B - C)*(a^3 + a^3*C
os[c + d*x])*Sin[c + d*x])/(2*d) + ((3*A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*
Tan[c + d*x])/(2*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rubi [A] time = 0.533527, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3043, 2975, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^3(7A+6B+2C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{(4A+2B-C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d} + \frac{(3A+2B)\tan(c+dx)\left(a^2\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```



```
[Out] (a^3*(2*A + 6*B + 7*C)*x)/2 + (a^3*(7*A + 6*B + 2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) - (5*a^3*(A - C)*Sin[c + d*x])/(2*d) - ((4*A + 2*B - C)*(a^3 + a^3*C
os[c + d*x])*Sin[c + d*x])/(2*d) + ((3*A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*
Tan[c + d*x])/(2*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^3\sec(c+dx)\tan(c}{2d}$$

$$=\frac{(3A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\tan(c+}{2ad}$$

$$=-\frac{(4A+2B-C)\left(a^3+a^3\cos(c+dx)\right)\sin}{2d}$$

$$=-\frac{(4A+2B-C)\left(a^3+a^3\cos(c+dx)\right)\sin}{2d}$$

$$=-\frac{5a^3(A-C)\sin(c+dx)}{2d}-\frac{(4A+2B-C}{}$$

$$=\frac{1}{2}a^3(2A+6B+7C)x-\frac{5a^3(A-C)\sin(c}{2d}$$

$$=\frac{1}{2}a^3(2A+6B+7C)x+\frac{a^3(7A+6B+2C}{}$$

Mathematica [A] time = 1.47749, size = 256, normalized size = 1.46

$$a^3\left(2(2A+6B+7C)(c+dx)-2(7A+6B+2C)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+2(7A+6B+2C)\log\left(\sin\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (a^3*(2*(2*A + 6*B + 7*C)*(c + d*x) - 2*(7*A + 6*B + 2*C)*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + 2*(7*A + 6*B + 2*C)*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*(3*A + B)*Sin[
(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - A/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2 + (4*(3*A + B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]) + 4*(B + 3*C)*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.086, size = 219, normalized size = 1.3

$$\frac{Aa^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{7\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a^3B\tan(dx+c)}{d}+\frac{a^3C\ln(\sec(dx+c)+\tan(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+7/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d
*a^3*B*tan(d*x+c)+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^3*tan(d*x+c)+
3/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+7/2*a^3*C*x+7/2/d*a^3*C*c+3*a^3*B*x+3/d
*a^3*B*c+3*a^3*C*sin(d*x+c)/d+A*a^3*x+1/d*A*a^3*c+a^3*B*sin(d*x+c)/d+1/2/d*
a^3*C*cos(d*x+c)*sin(d*x+c)
```

Maxima [A] time = 1.02162, size = 320, normalized size = 1.83

$$\frac{4\,(dx+c)Aa^3+12\,(dx+c)Ba^3+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^3+12\,(dx+c)Ca^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^3 + 12*(d*x + c)*B*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*
c))*C*a^3 + 12*(d*x + c)*C*a^3 - A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x +
c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) + 2*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 4*B*a^3*sin(d*x + c) + 12*C*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c) + 4*B
*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.15055, size = 410, normalized size = 2.34

$$\frac{2\,(2\,A+6\,B+7\,C)a^3dx\cos\left(dx+c\right)^2+(7\,A+6\,B+2\,C)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(7\,A+6\,B+2\,C)a^3\cos}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*A + 6*B + 7*C)*a^3*d*x*cos(d*x + c)^2 + (7*A + 6*B + 2*C)*a^3*cos
(d*x + c)^2*log(sin(d*x + c) + 1) - (7*A + 6*B + 2*C)*a^3*cos(d*x + c)^2*lo
g(-sin(d*x + c) + 1) + 2*(C*a^3*cos(d*x + c)^3 + 2*(B + 3*C)*a^3*cos(d*x +
c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24958, size = 378, normalized size = 2.16

$$\frac{\left(2\,Aa^3+6\,Ba^3+7\,Ca^3\right)(dx+c)+\left(7\,Aa^3+6\,Ba^3+2\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(7\,Aa^3+6\,Ba^3+2\,Ca^3\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="giac")
```

```
[Out] 1/2*((2*A*a^3 + 6*B*a^3 + 7*C*a^3)*(d*x + c) + (7*A*a^3 + 6*B*a^3 + 2*C*a^3
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (7*A*a^3 + 6*B*a^3 + 2*C*a^3)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a^3*
tan(1/2*d*x + 1/2*c)^7 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^3*tan(1/2*d
*x + 1/2*c)^5 + 3*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^3*tan(1/2*d*x + 1/2*
c)^3 + 9*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^3*tan(1/2*d*x + 1/2*c) - 4*B*
a^3*tan(1/2*d*x + 1/2*c) - 7*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^4 - 1)^2)/d
```

3.324 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)$ s dx) dx

Optimal. Leaf size=169

$$\frac{a^3(5A + 7B + 6C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{(5A + 6B + 3C)\tan(c + dx)\left(a^3\cos(c + dx) + a^3\right)}{3d} - \frac{5a^3(A + B)\sin(c + dx)}{2d}$$

```
[Out] a^3*(B + 3*C)*x + (a^3*(5*A + 7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*
a^3*(A + B)*Sin[c + d*x])/(2*d) + ((5*A + 6*B + 3*C)*(a^3 + a^3*Cos[c + d*x
])*Tan[c + d*x])/(3*d) + ((A + B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*T
an[c + d*x])/(2*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x
])/(3*d)
```

Rubi [A] time = 0.571831, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2975, 2968, 3023, 2735, 3770}

$$\frac{a^3(5A + 7B + 6C)\tanh^{-1}(\sin(c + dx))}{2d} + \frac{(5A + 6B + 3C)\tan(c + dx)\left(a^3\cos(c + dx) + a^3\right)}{3d} - \frac{5a^3(A + B)\sin(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```


```
[Out] a^3*(B + 3*C)*x + (a^3*(5*A + 7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*
a^3*(A + B)*Sin[c + d*x])/(2*d) + ((5*A + 6*B + 3*C)*(a^3 + a^3*Cos[c + d*x
])*Tan[c + d*x])/(3*d) + ((A + B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*T
an[c + d*x])/(2*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x
])/(3*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c}{3d}$$

$$= \frac{(A+B)\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx}{2ad}$$

$$= \frac{(5A+6B+3C)\left(a^3+a^3\cos(c+dx)\right)\tan}{3d}$$

$$= \frac{(5A+6B+3C)\left(a^3+a^3\cos(c+dx)\right)\tan}{3d}$$

$$= -\frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{(5A+6B+3}{}$$

$$= a^3(B+3C)x - \frac{5a^3(A+B)\sin(c+dx)}{2d} +$$

$$= a^3(B+3C)x + \frac{a^3(5A+7B+6C)\tanh^{-}}{2d}$$

Mathematica [B] time = 4.25868, size = 354, normalized size = 2.09

$$a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(11A+9B+3C)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4(11A+9B+3C)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} - 6(5A+7B+6C)\log\left(\cos\left(\frac{1}{2}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(12*(B + 3*C)*(c + d*x) - 6*(5
*A + 7*B + 6*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(5*A + 7*B + 6
*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (10*A + 3*B)/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^2 + (2*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^3 + (4*(11*A + 9*B + 3*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]) + (2*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3 + (-10*A - 3*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(11*
A + 9*B + 3*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12
*C*Sin[c + d*x]))/(96*d)
```

Maple [A] time = 0.088, size = 226, normalized size = 1.3

$$\frac{11\,Aa^{3}\tan\left(dx+c\right)}{3\,d}+\frac{Aa^{3}\tan\left(dx+c\right)\left(\sec\left(dx+c\right)\right)^{2}}{3\,d}+\frac{a^{3}B\sec\left(dx+c\right)\tan\left(dx+c\right)}{2\,d}+\frac{7\,a^{3}B\ln\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 11/3/d*A*a^3*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^3*B*sec
(d*x+c)*tan(d*x+c)+7/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*C*tan(d*x+
c)+3/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+
3/d*a^3*B*tan(d*x+c)+3/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*x+3/d*a^3*
C*c+a^3*B*x+1/d*a^3*B*c+a^3*C*sin(d*x+c)/d
```

Maxima [A] time = 1.01589, size = 370, normalized size = 2.19

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}+12\left(dx+c\right)Ba^{3}+36\left(dx+c\right)Ca^{3}-9\,Aa^{3}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*a^3 + 36*(
d*x + c)*C*a^3 - 9*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin
(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 18*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 12*C*a^3*sin(d*x + c) + 36*A*a^3*tan(d*x + c) + 36*B*a^3*tan(d*x
+ c) + 12*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.07646, size = 425, normalized size = 2.51

$$\frac{12\left(B+3\,C\right)a^{3}dx\cos\left(dx+c\right)^{3}+3\left(5\,A+7\,B+6\,C\right)a^{3}\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,A+7\,B+6\,C\right)a^{3}\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="fricas")
```

```
[Out] 1/12*(12*(B + 3*C)*a^3*d*x*cos(d*x + c)^3 + 3*(5*A + 7*B + 6*C)*a^3*cos(d*x
 + c)^3*log(sin(d*x + c) + 1) - 3*(5*A + 7*B + 6*C)*a^3*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(6*C*a^3*cos(d*x + c)^3 + 2*(11*A + 9*B + 3*C)*a^3*c
os(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + 2*A*a^3)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28721, size = 389, normalized size = 2.3

$$\frac{\frac{12\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(Ba^3+3\,Ca^3\right)(dx+c)+3\left(5\,Aa^3+7\,Ba^3+6\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(5\,Aa^3+7\,Ba^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="giac")
```

```
[Out] 1/6*(12*C*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(B*a^3
+ 3*C*a^3)*(d*x + c) + 3*(5*A*a^3 + 7*B*a^3 + 6*C*a^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 3*(5*A*a^3 + 7*B*a^3 + 6*C*a^3)*log(abs(tan(1/2*d*x + 1/2*
c) - 1)) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*
c)^5 + 6*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 3
```

```
6*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3
*tan(1/2*d*x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c) + 6*C*a^3*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.325 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{s}$ $dx)\,dx$

Optimal. Leaf size=183

$$\frac{5a^3(3A+4(B+C))\tan(c+dx)}{8d}+\frac{a^3(15A+20B+28C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(15A+20B+12C)\tan(c+dx)\sec(c}{24d}$$

```
[Out] a^3*C*x + (a^3*(15*A + 20*B + 28*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(
3*A + 4*(B + C))*Tan[c + d*x])/(8*d) + ((15*A + 20*B + 12*C)*(a^3 + a^3*Cos
[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A + 4*B)*(a^2 + a^2*Cos[
c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(12*a*d) + (A*(a + a*Cos[c + d*x])
^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.553734, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2975, 2968, 3021, 2735, 3770}

$$\frac{5a^3(3A+4(B+C))\tan(c+dx)}{8d}+\frac{a^3(15A+20B+28C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(15A+20B+12C)\tan(c+dx)\sec(c}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] a^3*C*x + (a^3*(15*A + 20*B + 28*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(
3*A + 4*(B + C))*Tan[c + d*x])/(8*d) + ((15*A + 20*B + 12*C)*(a^3 + a^3*Cos
[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A + 4*B)*(a^2 + a^2*Cos[
c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(12*a*d) + (A*(a + a*Cos[c + d*x])
^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^3\sec^3(c+dx)\tan(c}{4d}$$

$$=\frac{(3A+4B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^2(c}{12ad}$$

$$=\frac{(15A+20B+12C)\left(a^3+a^3\cos(c+dx)\right)}{24d}$$

$$=\frac{(15A+20B+12C)\left(a^3+a^3\cos(c+dx)\right)}{24d}$$

$$=\frac{5a^3(3A+4(B+C))\tan(c+dx)}{8d}+\frac{(15A}{}$$

$$=a^3Cx+\frac{5a^3(3A+4(B+C))\tan(c+dx)}{8d}$$

$$=a^3Cx+\frac{a^3(15A+20B+28C)\tanh^{-1}(\mathrm{si}}{8d}$$

Mathematica [B] time = 6.18326, size = 793, normalized size = 4.33

$$\frac{\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(9A\sin\left(\frac{1}{2}(c+dx)\right)+11B\sin\left(\frac{1}{2}(c+dx)\right)+9C\sin\left(\frac{1}{2}(c+dx)\right)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (C*(c + d*x)*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/(8*d) + ((-15*A -
 20*B - 28*C)*(a + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]]*Sec[c/2 + (d*x)/2]^6)/(64*d) + ((15*A + 20*B + 28*C)*(a + a*Cos[c + d*x]
)^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(64*d) +
 (A*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/(128*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^4) + ((57*A + 40*B + 12*C)*(a + a*Cos[c + d*x])^3*Sec[c/
2 + (d*x)/2]^6)/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (A*(a + a
*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/(128*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^4) + ((-57*A - 40*B - 12*C)*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)
/2]^6)/(384*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((a + a*Cos[c + d*
```

```
x])^3*Sec[c/2 + (d*x)/2]^6*(3*A*Sin[(c + d*x)/2] + B*Sin[(c + d*x)/2]))/(48
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c
/2 + (d*x)/2]^6*(3*A*Sin[(c + d*x)/2] + B*Sin[(c + d*x)/2]))/(48*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/
2]^6*(9*A*Sin[(c + d*x)/2] + 11*B*Sin[(c + d*x)/2] + 9*C*Sin[(c + d*x)/2]))
/(24*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((a + a*Cos[c + d*x])^3*Sec
[c/2 + (d*x)/2]^6*(9*A*Sin[(c + d*x)/2] + 11*B*Sin[(c + d*x)/2] + 9*C*Sin[(
c + d*x)/2]))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.098, size = 262, normalized size = 1.4

$$\frac{Aa^3\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{15\,Aa^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{15\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{11\,a^3B\,\mathrm{tan}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+15/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+15/8
/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*a^3*B*tan(d*x+c)+1/3/d*a^3*B*tan(
d*x+c)*sec(d*x+c)^2+1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+7/2/d*a^3*C*ln(sec(d*
x+c)+tan(d*x+c))+3/d*A*a^3*tan(d*x+c)+1/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/2
/d*a^3*B*sec(d*x+c)*tan(d*x+c)+5/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^
3*C*tan(d*x+c)+a^3*C*x+1/d*a^3*C*c
```

Maxima [B] time = 1.0297, size = 494, normalized size = 2.7

$$\frac{48\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3+16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^3+48\,(dx+c)Ca^3-3\,Aa^3\left(\frac{2\left(3\,\sin(dx+c)^3-\right.}{\sin(dx+c)^4-2\,\mathrm{si}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="maxima")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^3 + 48*(d*x + c)*C*a^3 - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
```

```
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 - 12*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) + 24*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 72*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^
3*tan(d*x + c) + 144*B*a^3*tan(d*x + c) + 144*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.16669, size = 447, normalized size = 2.44

$$\frac{48\,Ca^3dx\cos(dx+c)^4 + 3\,(15\,A+20\,B+28\,C)a^3\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\,(15\,A+20\,B+28\,C)a^3\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="fricas")
```

```
[Out] 1/48*(48*C*a^3*d*x*cos(d*x + c)^4 + 3*(15*A + 20*B + 28*C)*a^3*cos(d*x + c)
^4*log(sin(d*x + c) + 1) - 3*(15*A + 20*B + 28*C)*a^3*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(8*(9*A + 11*B + 9*C)*a^3*cos(d*x + c)^3 + 3*(15*A + 1
2*B + 4*C)*a^3*cos(d*x + c)^2 + 8*(3*A + B)*a^3*cos(d*x + c) + 6*A*a^3)*sin
(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26833, size = 406, normalized size = 2.22

$$24(dx+c)Ca^3+3\left(15\,Aa^3+20\,Ba^3+28\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(15\,Aa^3+20\,Ba^3+28\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*C*a^3 + 3*(15*A*a^3 + 20*B*a^3 + 28*C*a^3)*log(abs(tan(1
/2*d*x + 1/2*c) + 1)) - 3*(15*A*a^3 + 20*B*a^3 + 28*C*a^3)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(45*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*B*a^3*tan(1/2*
d*x + 1/2*c)^7 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 165*A*a^3*tan(1/2*d*x +
1/2*c)^5 - 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 204*C*a^3*tan(1/2*d*x + 1/2*c
)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 +
 228*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 147*A*a^3*tan(1/2*d*x + 1/2*c) - 132*B*
a^3*tan(1/2*d*x + 1/2*c) - 84*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

3.326 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ s $dx)\,dx$

Optimal. Leaf size=212

$$\frac{a^3(38A+45B+55C)\tan(c+dx)}{15d}+\frac{a^3(13A+15B+20C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(109A+135B+140C)\tan(c+dx}{120d}$$

```
[Out] (a^3*(13*A + 15*B + 20*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 45*B
+ 55*C)*Tan[c + d*x])/(15*d) + (a^3*(109*A + 135*B + 140*C)*Sec[c + d*x]*Ta
n[c + d*x])/(120*d) + ((11*A + 15*B + 10*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c
+ d*x]^2*Tan[c + d*x])/(30*d) + ((3*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec
[c + d*x]^3*Tan[c + d*x])/(20*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]
^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.637081, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3043, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^3(38A+45B+55C)\tan(c+dx)}{15d}+\frac{a^3(13A+15B+20C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(109A+135B+140C)\tan(c+dx}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```


```
[Out] (a^3*(13*A + 15*B + 20*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 45*B
+ 55*C)*Tan[c + d*x])/(15*d) + (a^3*(109*A + 135*B + 140*C)*Sec[c + d*x]*Ta
n[c + d*x])/(120*d) + ((11*A + 15*B + 10*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c
+ d*x]^2*Tan[c + d*x])/(30*d) + ((3*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec
[c + d*x]^3*Tan[c + d*x])/(20*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]
^4*Tan[c + d*x])/(5*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
```

```
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c}{5d}$$

$$=\frac{(3A+5B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c}{20ad}$$

$$=\frac{(11A+15B+10C)\left(a^3+a^3\cos(c+dx)\right)}{30d}$$

$$=\frac{(11A+15B+10C)\left(a^3+a^3\cos(c+dx)\right)}{30d}$$

$$=\frac{a^3(109A+135B+140C)\sec(c+dx)\tan}{120d}$$

$$=\frac{a^3(109A+135B+140C)\sec(c+dx)\tan}{120d}$$

$$=\frac{a^3(13A+15B+20C)\tanh^{-1}(\sin(c+dx}{8d}$$

$$=\frac{a^3(13A+15B+20C)\tanh^{-1}(\sin(c+dx}{8d}$$

Mathematica [B] time = 6.19492, size = 931, normalized size = 4.39

$$\frac{(-13A-15B-20C)(\cos(c+dx)a+a)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d}+\frac{(13A+15B+20C)(\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] ((-13*A - 15*B - 20*C)*(a + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(64*d) + ((13*A + 15*B + 20*C)*(a + a*Cos
[c + d*x])^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c/2 + (d*x)/2]^6)
/(64*d) + ((17*A + 5*B)*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/(640*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + ((274*A + 285*B + 200*C)*(a + a
*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/(1920*d*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])^2) + (A*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(c + d*x)
/2])/(160*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5) + (A*(a + a*Cos[c + d*
x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(c + d*x)/2])/(160*d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^5) + ((-17*A - 5*B)*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2
]^6)/(640*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + ((-274*A - 285*B - 2
00*C)*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/(1920*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(79
*A*Sin[(c + d*x)/2] + 60*B*Sin[(c + d*x)/2] + 20*C*Sin[(c + d*x)/2]))/(960*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/
2 + (d*x)/2]^6*(79*A*Sin[(c + d*x)/2] + 60*B*Sin[(c + d*x)/2] + 20*C*Sin[(c
 + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((a + a*Cos[
c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(38*A*Sin[(c + d*x)/2] + 45*B*Sin[(c + d*x
)/2] + 55*C*Sin[(c + d*x)/2]))/(120*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(38*A*Sin[(c + d*x)/2] + 4
5*B*Sin[(c + d*x)/2] + 55*C*Sin[(c + d*x)/2]))/(120*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]))
```

Maple [A] time = 0.095, size = 316, normalized size = 1.5

$$\frac{38\,Aa^3\tan(dx+c)}{15\,d}+\frac{Aa^3\tan(dx+c)(\sec(dx+c))^4}{5\,d}+\frac{19\,Aa^3\tan(dx+c)(\sec(dx+c))^2}{15\,d}+\frac{a^3B\tan(dx+c)(\sec(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 38/15/d*A*a^3*tan(d*x+c)+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+19/15/d*A*a^3*
tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3+15/8/d*a^3*B*se
c(d*x+c)*tan(d*x+c)+15/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*a^3*C*tan
(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+3/4/d*A*a^3*tan(d*x+c)*sec(d*x+
c)^3+13/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+
c))+3/d*a^3*B*tan(d*x+c)+1/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^3*C*sec(
```

```
d*x+c)*tan(d*x+c)+5/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [B] time = 1.06863, size = 602, normalized size = 2.84

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+240\left(\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 45*A*a^3*(2*(
3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1)
- 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 15*B*a^3*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*A*a^3*(2*sin(d*x + c)/(s
in(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 180*B
*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)) - 180*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*C*a^3*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x + c) + 720*C*a^3*tan(d*x + c))/
d
```

Fricas [A] time = 2.04784, size = 477, normalized size = 2.25

$$\frac{15\,(13\,A+15\,B+20\,C)a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(13\,A+15\,B+20\,C)a^3\cos(dx+c)^5\log(-\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(13*A + 15*B + 20*C)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 1
5*(13*A + 15*B + 20*C)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38
*A + 45*B + 55*C)*a^3*cos(d*x + c)^4 + 15*(13*A + 15*B + 12*C)*a^3*cos(d*x
```

```
+ c)^3 + 8*(19*A + 15*B + 5*C)*a^3*cos(d*x + c)^2 + 30*(3*A + B)*a^3*cos(d*
x + c) + 24*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2301, size = 460, normalized size = 2.17

$$\frac{15\left(13\,Aa^3+15\,Ba^3+20\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(13\,Aa^3+15\,Ba^3+20\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="giac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3 + 20*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 15*(13*A*a^3 + 15*B*a^3 + 20*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(195*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 +
 300*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 1050
*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 1400*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*
a^3*tan(1/2*d*x + 1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 2560*C*a^3
*tan(1/2*d*x + 1/2*c)^5 - 1330*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 1830*B*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 2120*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/
2*d*x + 1/2*c) + 735*B*a^3*tan(1/2*d*x + 1/2*c) + 660*C*a^3*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.327 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) s$ $dx)\, dx$

Optimal. Leaf size=244

$$\frac{a^3(34A+38B+45C)\tan(c+dx)}{15d}+\frac{a^3(23A+26B+30C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(73A+86B+90C)\tan(c+dx)\,s}{120d}$$

```
[Out] (a^3*(23*A + 26*B + 30*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(34*A + 38*B
 + 45*C)*Tan[c + d*x])/(15*d) + (a^3*(23*A + 26*B + 30*C)*Sec[c + d*x]*Tan[
c + d*x])/(16*d) + (a^3*(73*A + 86*B + 90*C)*Sec[c + d*x]^2*Tan[c + d*x])/(
120*d) + ((31*A + 42*B + 30*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[
c + d*x])/(120*d) + ((A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Ta
n[c + d*x])/(10*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x
])/(6*d)
```

Rubi [A] time = 0.703489, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3043, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^3(34A+38B+45C)\tan(c+dx)}{15d}+\frac{a^3(23A+26B+30C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(73A+86B+90C)\tan(c+dx)\,s}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^7,x]
```


```
[Out] (a^3*(23*A + 26*B + 30*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(34*A + 38*B
 + 45*C)*Tan[c + d*x])/(15*d) + (a^3*(23*A + 26*B + 30*C)*Sec[c + d*x]*Tan[
c + d*x])/(16*d) + (a^3*(73*A + 86*B + 90*C)*Sec[c + d*x]^2*Tan[c + d*x])/(
120*d) + ((31*A + 42*B + 30*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[
c + d*x])/(120*d) + ((A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Ta
n[c + d*x])/(10*a*d) + (A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x
])/(6*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^7(c + dx)\,dx = \frac{A(a + a\cos(c + dx))^3\sec^5(c + dx)\tan(c +}{6d}$$

$$= \frac{(A + 2B)\left(a^2 + a^2\cos(c + dx)\right)^2\sec^4(c + d}{10ad}$$

$$= \frac{(31A + 42B + 30C)\left(a^3 + a^3\cos(c + dx)\right)s}{120d}$$

$$= \frac{(31A + 42B + 30C)\left(a^3 + a^3\cos(c + dx)\right)s}{120d}$$

$$= \frac{a^3(73A + 86B + 90C)\sec^2(c + dx)\tan(c -}{120d}$$

$$= \frac{a^3(73A + 86B + 90C)\sec^2(c + dx)\tan(c -}{120d}$$

$$= \frac{a^3(23A + 26B + 30C)\sec(c + dx)\tan(c +}{16d}$$

$$= \frac{a^3(23A + 26B + 30C)\tanh^{-1}(\sin(c + dx)}{16d}$$

Mathematica [A] time = 1.88904, size = 265, normalized size = 1.09

$$-\frac{a^3(\cos(c + dx) + 1)^3\sec^6\left(\frac{1}{2}(c + dx)\right)\sec^6(c + dx)\left(240(23A + 26B + 30C)\cos^6(c + dx)\left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^7,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(240*(23*A + 2
6*B + 30*C)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2*(2275*A + 1890*B + 1590*C + 16*(3
44*A + 328*B + 315*C)*Cos[c + d*x] + 20*(115*A + 114*B + 102*C)*Cos[2*(c +
d*x)] + 1904*A*Cos[3*(c + d*x)] + 2128*B*Cos[3*(c + d*x)] + 2280*C*Cos[3*(c
 + d*x)] + 345*A*Cos[4*(c + d*x)] + 390*B*Cos[4*(c + d*x)] + 450*C*Cos[4*(c
 + d*x)] + 272*A*Cos[5*(c + d*x)] + 304*B*Cos[5*(c + d*x)] + 360*C*Cos[5*(c
 + d*x)])*Sin[c + d*x]))/(30720*d)
```

Maple [A] time = 0.098, size = 385, normalized size = 1.6

$$\frac{Aa^3\tan(dx+c)(\sec(dx+c))^5}{6d}+\frac{23\,Aa^3\tan(dx+c)(\sec(dx+c))^3}{24\,d}+\frac{23\,Aa^3\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{23\,Aa^3\ln(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/6/d*A*a^3*tan(d*x+c)*sec(d*x+c)^5+23/24/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+2
3/16/d*A*a^3*sec(d*x+c)*tan(d*x+c)+23/16/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+
38/15/d*a^3*B*tan(d*x+c)+1/5/d*a^3*B*tan(d*x+c)*sec(d*x+c)^4+19/15/d*a^3*B*
tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+15/8/d*a^3*C*se
c(d*x+c)*tan(d*x+c)+15/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+34/15/d*A*a^3*ta
n(d*x+c)+3/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+17/15/d*A*a^3*tan(d*x+c)*sec(d
*x+c)^2+3/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3+13/8/d*a^3*B*sec(d*x+c)*tan(d*x
+c)+13/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^3*C*tan(d*x+c)+1/d*a^3*C*t
an(d*x+c)*sec(d*x+c)^2
```

Maxima [B] time = 1.08061, size = 755, normalized size = 3.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="maxima")
```

```
[Out] 1/480*(96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
160*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 32*(3*tan(d*x + c)^5 + 10*tan
(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 480*(tan(d*x + c)^3 + 3*tan(d*x + c)
)*B*a^3 + 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 5*A*a^3*(2*(15*sin(
d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d
*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d
*x + c) - 1)) - 90*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 90*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 30*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B
*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)) - 360*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
```

```
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.09937, size = 540, normalized size = 2.21

$$\frac{15\,(23\,A + 26\,B + 30\,C)a^3 \cos{(dx + c)}^6 \log{(\sin{(dx + c)} + 1)} - 15\,(23\,A + 26\,B + 30\,C)a^3 \cos{(dx + c)}^6 \log{(-\sin{(dx + c)}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="fricas")
```

```
[Out] 1/480*(15*(23*A + 26*B + 30*C)*a^3*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 1
5*(23*A + 26*B + 30*C)*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(3
4*A + 38*B + 45*C)*a^3*cos(d*x + c)^5 + 15*(23*A + 26*B + 30*C)*a^3*cos(d*x
 + c)^4 + 16*(17*A + 19*B + 15*C)*a^3*cos(d*x + c)^3 + 10*(23*A + 18*B + 6*
C)*a^3*cos(d*x + c)^2 + 48*(3*A + B)*a^3*cos(d*x + c) + 40*A*a^3)*sin(d*x +
 c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27767, size = 529, normalized size = 2.17

$$\frac{15\left(23\,Aa^3 + 26\,Ba^3 + 30\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(23\,Aa^3 + 26\,Ba^3 + 30\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="giac")
```

```
[Out] 1/240*(15*(23*A*a^3 + 26*B*a^3 + 30*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 15*(23*A*a^3 + 26*B*a^3 + 30*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(345*A*a^3*tan(1/2*d*x + 1/2*c)^11 + 390*B*a^3*tan(1/2*d*x + 1/2*c)^11
 + 450*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 1955*A*a^3*tan(1/2*d*x + 1/2*c)^9 -
2210*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 2550*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 455
4*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 5148*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 5940*C
*a^3*tan(1/2*d*x + 1/2*c)^7 - 5814*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 5988*B*a^
3*tan(1/2*d*x + 1/2*c)^5 - 7500*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4190*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 5130*C*a^3*tan(
1/2*d*x + 1/2*c)^3 - 1575*A*a^3*tan(1/2*d*x + 1/2*c) - 1530*B*a^3*tan(1/2*d
*x + 1/2*c) - 1470*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^6)/d
```

3.328 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4 \left(A + B\cos(c + dx) + C\cos^2\right.$

Optimal. Leaf size=304

$$-\frac{a^4(252A + 227B + 208C)\sin^3(c + dx)}{105d} + \frac{a^4(252A + 227B + 208C)\sin(c + dx)}{35d} + \frac{a^4(2408A + 2208B + 2007C)\sin(c +}{2240d}$$

```
[Out] (a^4*(392*A + 352*B + 323*C)*x)/128 + (a^4*(252*A + 227*B + 208*C)*Sin[c +
d*x])/(35*d) + (a^4*(392*A + 352*B + 323*C)*Cos[c + d*x]*Sin[c + d*x])/(128
*d) + (a^4*(2408*A + 2208*B + 2007*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2240*d)
 + (a*(2*B + C)*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(14*d)
+ (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(8*d) + ((56*A + 8
0*B + 61*C)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(336*d)
 + (7*(8*A + 8*B + 7*C)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x
])/(120*d) - (a^4*(252*A + 227*B + 208*C)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.862627, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3045, 2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^4(252A + 227B + 208C)\sin^3(c + dx)}{105d} + \frac{a^4(252A + 227B + 208C)\sin(c + dx)}{35d} + \frac{a^4(2408A + 2208B + 2007C)\sin(c +}{2240d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```


```
[Out] (a^4*(392*A + 352*B + 323*C)*x)/128 + (a^4*(252*A + 227*B + 208*C)*Sin[c +
d*x])/(35*d) + (a^4*(392*A + 352*B + 323*C)*Cos[c + d*x]*Sin[c + d*x])/(128
*d) + (a^4*(2408*A + 2208*B + 2007*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2240*d)
 + (a*(2*B + C)*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(14*d)
+ (C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(8*d) + ((56*A + 8
0*B + 61*C)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(336*d)
 + (7*(8*A + 8*B + 7*C)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x
])/(120*d) - (a^4*(252*A + 227*B + 208*C)*Sin[c + d*x]^3)/(105*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
```

```
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^3(c+dx)(a+a\cos(c+dx))^4\sin(c+}{8d}$$

$$= \frac{a(2B+C)\cos^3(c+dx)(a+a\cos(c+dx))}{14d}$$

$$= \frac{a(2B+C)\cos^3(c+dx)(a+a\cos(c+dx))}{14d}$$

$$= \frac{a(2B+C)\cos^3(c+dx)(a+a\cos(c+dx))}{14d}$$

$$= \frac{a(2B+C)\cos^3(c+dx)(a+a\cos(c+dx))}{14d}$$

$$= \frac{a^4(2408A+2208B+2007C)\cos^3(c+dx}{2240d}$$

$$= \frac{a^4(2408A+2208B+2007C)\cos^3(c+dx}{2240d}$$

$$= \frac{a^4(392A+352B+323C)\cos(c+dx)\sin}{128d}$$

$$= \frac{1}{128}a^4(392A+352B+323C)x+\frac{a^4(252A}{}$$

Mathematica [A] time = 1.42378, size = 237, normalized size = 0.78

$$\frac{a^4(1680(352A+323B+300C)\sin(c+dx)+1680(127A+124B+120C)\sin(2(c+dx))+80640A\sin(3(c+dx))+2520}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (a^4*(295680*B*c + 164640*c*C + 329280*A*d*x + 295680*B*d*x + 271320*C*d*x
+ 1680*(352*A + 323*B + 300*C)*Sin[c + d*x] + 1680*(127*A + 124*B + 120*C)*
Sin[2*(c + d*x)] + 80640*A*Sin[3*(c + d*x)] + 87920*B*Sin[3*(c + d*x)] + 91
840*C*Sin[3*(c + d*x)] + 25200*A*Sin[4*(c + d*x)] + 33600*B*Sin[4*(c + d*x)
] + 39480*C*Sin[4*(c + d*x)] + 5376*A*Sin[5*(c + d*x)] + 10416*B*Sin[5*(c +
 d*x)] + 14784*C*Sin[5*(c + d*x)] + 560*A*Sin[6*(c + d*x)] + 2240*B*Sin[6*(
c + d*x)] + 4480*C*Sin[6*(c + d*x)] + 240*B*Sin[7*(c + d*x)] + 960*C*Sin[7*
(c + d*x)] + 105*C*Sin[8*(c + d*x)]))/(107520*d)
```

Maple [B] time = 0.032, size = 577, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+a^4*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x
+3/8*c)+4/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*B*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c)+6/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*
a^4*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*
d*x+5/16*c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*
B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+
5/16*c)+4/7*a^4*C*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c)+A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+
c)+5/16*d*x+5/16*c)+1/7*a^4*B*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d
*x+c)^2)*sin(d*x+c)+a^4*C*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x
+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [B] time = 1.07853, size = 782, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/107520*(28672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*
a^4 - 560*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^4 - 143360*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 2
0160*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 26880*
(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 3072*(5*sin(d*x + c)^7 - 21*sin(d*
x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*B*a^4 + 43008*(3*sin(d*x +
c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 2240*(4*sin(2*d*x + 2*c
)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 - 358
40*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 13440*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 12288*(5*sin(d*x + c)^7 - 21*sin(d*
x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*C*a^4 + 28672*(3*sin(d*x +
c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^4 - 35*(128*sin(2*d*x + 2*c
)^3 - 840*d*x - 840*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin
(2*d*x + 2*c))*C*a^4 - 3360*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4
*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^4 + 3360*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*C*a^4)/d
```

Fricas [A] time = 2.0633, size = 536, normalized size = 1.76

$$\frac{105\,(392\,A+352\,B+323\,C)a^4dx+\left(1680\,Ca^4\cos\left(dx+c\right)^7+1920\,(B+4\,C)a^4\cos\left(dx+c\right)^6+280\,(8\,A+32\,B+55\,C)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/13440*(105*(392*A + 352*B + 323*C)*a^4*d*x + (1680*C*a^4*cos(d*x + c)^7 +
 1920*(B + 4*C)*a^4*cos(d*x + c)^6 + 280*(8*A + 32*B + 55*C)*a^4*cos(d*x +
c)^5 + 1536*(7*A + 12*B + 13*C)*a^4*cos(d*x + c)^4 + 70*(328*A + 352*B + 32
3*C)*a^4*cos(d*x + c)^3 + 128*(252*A + 227*B + 208*C)*a^4*cos(d*x + c)^2 +
105*(392*A + 352*B + 323*C)*a^4*cos(d*x + c) + 256*(252*A + 227*B + 208*C)*
a^4)*sin(d*x + c))/d
```

Sympy [A] time = 19.1669, size = 1640, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Piecewise((5*A*a**4*x*sin(c + d*x)**6/16 + 15*A*a**4*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 9*A*a**4*x*sin(c + d*x)**4/4 + 15*A*a**4*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 9*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a**4
*x*sin(c + d*x)**2/2 + 5*A*a**4*x*cos(c + d*x)**6/16 + 9*A*a**4*x*cos(c + d
*x)**4/4 + A*a**4*x*cos(c + d*x)**2/2 + 5*A*a**4*sin(c + d*x)**5*cos(c + d*
x)/(16*d) + 32*A*a**4*sin(c + d*x)**5/(15*d) + 5*A*a**4*sin(c + d*x)**3*cos
(c + d*x)**3/(6*d) + 16*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*
a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) +
11*A*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*A*a**4*sin(c + d*x)*cos(c
 + d*x)**4/d + 15*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*A*a**4*sin(
c + d*x)*cos(c + d*x)**2/d + A*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a
**4*x*sin(c + d*x)**6/4 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 3
*B*a**4*x*sin(c + d*x)**4/2 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/4
 + 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*B*a**4*x*cos(c + d*x)**6/
4 + 3*B*a**4*x*cos(c + d*x)**4/2 + 16*B*a**4*sin(c + d*x)**7/(35*d) + 8*B*a
**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 5*B*a**4*sin(c + d*x)**5*cos(c
+ d*x)/(4*d) + 16*B*a**4*sin(c + d*x)**5/(5*d) + 2*B*a**4*sin(c + d*x)**3*c
os(c + d*x)**4/d + 10*B*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 8*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a**4*sin(c + d*x)**3*cos(c + d*x
)/(2*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c + d*x)*cos(c + d*x)
**6/d + 11*B*a**4*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 6*B*a**4*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + B*a**4*
sin(c + d*x)*cos(c + d*x)**2/d + 35*C*a**4*x*sin(c + d*x)**8/128 + 35*C*a**
4*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*C*a**4*x*sin(c + d*x)**6/8 + 10
5*C*a**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 45*C*a**4*x*sin(c + d*x)**4
*cos(c + d*x)**2/8 + 3*C*a**4*x*sin(c + d*x)**4/8 + 35*C*a**4*x*sin(c + d*x
)**2*cos(c + d*x)**6/32 + 45*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 3
*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 35*C*a**4*x*cos(c + d*x)**8/1
28 + 15*C*a**4*x*cos(c + d*x)**6/8 + 3*C*a**4*x*cos(c + d*x)**4/8 + 35*C*a*
*4*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 64*C*a**4*sin(c + d*x)**7/(35*d)
+ 385*C*a**4*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 32*C*a**4*sin(c + d*
x)**5*cos(c + d*x)**2/(5*d) + 15*C*a**4*sin(c + d*x)**5*cos(c + d*x)/(8*d)
+ 32*C*a**4*sin(c + d*x)**5/(15*d) + 511*C*a**4*sin(c + d*x)**3*cos(c + d*x
)**5/(384*d) + 8*C*a**4*sin(c + d*x)**3*cos(c + d*x)**4/d + 5*C*a**4*sin(c
+ d*x)**3*cos(c + d*x)**3/d + 16*C*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*
```

```
d) + 3*C*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 93*C*a**4*sin(c + d*x)*c
os(c + d*x)**7/(128*d) + 4*C*a**4*sin(c + d*x)*cos(c + d*x)**6/d + 33*C*a**
4*sin(c + d*x)*cos(c + d*x)**5/(8*d) + 4*C*a**4*sin(c + d*x)*cos(c + d*x)**
4/d + 5*C*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c)
+ a)**4*(A + B*cos(c) + C*cos(c)**2)*cos(c)**2, True))
```

Giac [A] time = 1.25972, size = 352, normalized size = 1.16

$$\frac{Ca^4\sin(8\,dx+8\,c)}{1024\,d}+\frac{1}{128}\left(392\,Aa^4+352\,Ba^4+323\,Ca^4\right)x+\frac{\left(Ba^4+4\,Ca^4\right)\sin(7\,dx+7\,c)}{448\,d}+\frac{\left(Aa^4+4\,Ba^4+8\,Ca^4\right)\sin}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1024*C*a^4*sin(8*d*x + 8*c)/d + 1/128*(392*A*a^4 + 352*B*a^4 + 323*C*a^4)
*x + 1/448*(B*a^4 + 4*C*a^4)*sin(7*d*x + 7*c)/d + 1/192*(A*a^4 + 4*B*a^4 +
8*C*a^4)*sin(6*d*x + 6*c)/d + 1/320*(16*A*a^4 + 31*B*a^4 + 44*C*a^4)*sin(5*
d*x + 5*c)/d + 1/128*(30*A*a^4 + 40*B*a^4 + 47*C*a^4)*sin(4*d*x + 4*c)/d +
1/192*(144*A*a^4 + 157*B*a^4 + 164*C*a^4)*sin(3*d*x + 3*c)/d + 1/64*(127*A*
a^4 + 124*B*a^4 + 120*C*a^4)*sin(2*d*x + 2*c)/d + 1/64*(352*A*a^4 + 323*B*a
^4 + 300*C*a^4)*sin(d*x + c)/d
```

3.329 $\int \cos(c+dx)(a+a\cos(c+dx))^4 \left(A + B\cos(c + dx) + C\cos^2\right.$

Optimal. Leaf size=243

$$-\frac{2a^4(56A+49B+44C)\sin^3(c+dx)}{105d}+\frac{4a^4(56A+49B+44C)\sin(c+dx)}{35d}+\frac{a^4(56A+49B+44C)\sin(c+dx)\cos^3(c}{280d}$$

```
[Out] (a^4*(56*A + 49*B + 44*C)*x)/16 + (4*a^4*(56*A + 49*B + 44*C)*Sin[c + d*x])
/(35*d) + (27*a^4*(56*A + 49*B + 44*C)*Cos[c + d*x]*Sin[c + d*x])/(560*d) +
 (a^4*(56*A + 49*B + 44*C)*Cos[c + d*x]^3*Sin[c + d*x])/(280*d) + ((42*A -
7*B + 8*C)*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(210*d) + (C*Cos[c + d*x]^2
*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(7*d) + ((7*B + 4*C)*(a + a*Cos[c + d
*x])^5*Sin[c + d*x])/(42*a*d) - (2*a^4*(56*A + 49*B + 44*C)*Sin[c + d*x]^3)
/(105*d)
```

Rubi [A] time = 0.454472, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3045, 2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{2a^4(56A+49B+44C)\sin^3(c+dx)}{105d}+\frac{4a^4(56A+49B+44C)\sin(c+dx)}{35d}+\frac{a^4(56A+49B+44C)\sin(c+dx)\cos^3(c}{280d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] (a^4*(56*A + 49*B + 44*C)*x)/16 + (4*a^4*(56*A + 49*B + 44*C)*Sin[c + d*x])
/(35*d) + (27*a^4*(56*A + 49*B + 44*C)*Cos[c + d*x]*Sin[c + d*x])/(560*d) +
 (a^4*(56*A + 49*B + 44*C)*Cos[c + d*x]^3*Sin[c + d*x])/(280*d) + ((42*A -
7*B + 8*C)*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(210*d) + (C*Cos[c + d*x]^2
*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(7*d) + ((7*B + 4*C)*(a + a*Cos[c + d
*x])^5*Sin[c + d*x])/(42*a*d) - (2*a^4*(56*A + 49*B + 44*C)*Sin[c + d*x]^3)
/(105*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
```

```
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^4\sin(c}{7d}$$

$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^4\sin(c}{7d}$$

$$= \frac{C\cos^2(c+dx)(a+a\cos(c+dx))^4\sin(c}{7d}$$

$$= \frac{(42A-7B+8C)(a+a\cos(c+dx))^4\sin}{210d}$$

$$= \frac{(42A-7B+8C)(a+a\cos(c+dx))^4\sin}{210d}$$

$$= \frac{1}{70}a^4(56A+49B+44C)x + \frac{(42A-7B}{}$$

$$= \frac{1}{70}a^4(56A+49B+44C)x + \frac{2a^4(56A+}{}$$

$$= \frac{2}{35}a^4(56A+49B+44C)x + \frac{4a^4(56A+}{}$$

$$= \frac{1}{16}a^4(56A+49B+44C)x + \frac{4a^4(56A+}{}$$

Mathematica [A] time = 0.940435, size = 204, normalized size = 0.84

$$\frac{a^4(105(392A+352B+323C)\sin(c+dx)+105(128A+127B+124C)\sin(2(c+dx))+4060A\sin(3(c+dx))+840A\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (a^4*(20580*B*c + 11760*c*C + 23520*A*d*x + 20580*B*d*x + 18480*C*d*x + 105
*(392*A + 352*B + 323*C)*Sin[c + d*x] + 105*(128*A + 127*B + 124*C)*Sin[2*(
c + d*x)] + 4060*A*Sin[3*(c + d*x)] + 5040*B*Sin[3*(c + d*x)] + 5495*C*Sin[
3*(c + d*x)] + 840*A*Sin[4*(c + d*x)] + 1575*B*Sin[4*(c + d*x)] + 2100*C*Si
n[4*(c + d*x)] + 84*A*Sin[5*(c + d*x)] + 336*B*Sin[5*(c + d*x)] + 651*C*Sin
[5*(c + d*x)] + 35*B*Sin[6*(c + d*x)] + 140*C*Sin[6*(c + d*x)] + 15*C*Sin[7
*(c + d*x)]))/(6720*d)
```

Maple [B] time = 0.033, size = 490, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a
^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+4/3*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*C*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)
+6*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*a
^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*a^4*B*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/
8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)+a^4*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/7*a^4*C*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.0405, size = 652, normalized size = 2.68

$$\frac{448\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{4}-13440\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}+840\,(12\,dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/6720*(448*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4
- 13440*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 840*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 6720*(2*d*x + 2*c + sin(2*d*x +
2*c))*A*a^4 + 1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^4 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48
*sin(2*d*x + 2*c))*B*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 1
260*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 1680*(2
*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*C*a^4 + 2688*(3*sin(d*x + c)^5
 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^4 - 140*(4*sin(2*d*x + 2*c)^3 -
 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^4 - 2240*(si
n(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 840*(12*d*x + 12*c + sin(4*d*x + 4*c
) + 8*sin(2*d*x + 2*c))*C*a^4 + 6720*A*a^4*sin(d*x + c))/d
```

Fricas [A] time = 2.15902, size = 454, normalized size = 1.87

$$\frac{105\,(56\,A+49\,B+44\,C)a^4dx+\left(240\,Ca^4\cos\left(dx+c\right)^6+280\,(B+4\,C)a^4\cos\left(dx+c\right)^5+48\,(7\,A+28\,B+48\,C)a^4\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/1680*(105*(56*A + 49*B + 44*C)*a^4*d*x + (240*C*a^4*cos(d*x + c)^6 + 280*
(B + 4*C)*a^4*cos(d*x + c)^5 + 48*(7*A + 28*B + 48*C)*a^4*cos(d*x + c)^4 +
70*(24*A + 41*B + 44*C)*a^4*cos(d*x + c)^3 + 16*(238*A + 252*B + 227*C)*a^4
*cos(d*x + c)^2 + 105*(56*A + 49*B + 44*C)*a^4*cos(d*x + c) + 16*(581*A + 5
04*B + 454*C)*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 11.4942, size = 1258, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/2 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2 + 2*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/2 + 2*A
*a**4*x*cos(c + d*x)**2 + 8*A*a**4*sin(c + d*x)**5/(15*d) + 4*A*a**4*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 4*A*a**4*sin(c + d*x)**3/d + A*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5
*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 6*A*a**4*sin(c + d*x)*cos(c +
d*x)**2/d + 2*A*a**4*sin(c + d*x)*cos(c + d*x)/d + A*a**4*sin(c + d*x)/d +
5*B*a**4*x*sin(c + d*x)**6/16 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2
/16 + 9*B*a**4*x*sin(c + d*x)**4/4 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**4/16 + 9*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + B*a**4*x*sin(c +
d*x)**2/2 + 5*B*a**4*x*cos(c + d*x)**6/16 + 9*B*a**4*x*cos(c + d*x)**4/4 +
B*a**4*x*cos(c + d*x)**2/2 + 5*B*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
 32*B*a**4*sin(c + d*x)**5/(15*d) + 5*B*a**4*sin(c + d*x)**3*cos(c + d*x)**
3/(6*d) + 16*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**4*sin(c
+ d*x)**3*cos(c + d*x)/(4*d) + 8*B*a**4*sin(c + d*x)**3/(3*d) + 11*B*a**4*s
in(c + d*x)*cos(c + d*x)**5/(16*d) + 4*B*a**4*sin(c + d*x)*cos(c + d*x)**4/
d + 15*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*B*a**4*sin(c + d*x)*co
s(c + d*x)**2/d + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*C*a**4*x*sin(c
 + d*x)**6/4 + 15*C*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 3*C*a**4*x*s
in(c + d*x)**4/2 + 15*C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/4 + 3*C*a**4
*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*C*a**4*x*cos(c + d*x)**6/4 + 3*C*a**
4*x*cos(c + d*x)**4/2 + 16*C*a**4*sin(c + d*x)**7/(35*d) + 8*C*a**4*sin(c +
 d*x)**5*cos(c + d*x)**2/(5*d) + 5*C*a**4*sin(c + d*x)**5*cos(c + d*x)/(4*d
) + 16*C*a**4*sin(c + d*x)**5/(5*d) + 2*C*a**4*sin(c + d*x)**3*cos(c + d*x)
**4/d + 10*C*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 8*C*a**4*sin(c +
d*x)**3*cos(c + d*x)**2/d + 3*C*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 2
*C*a**4*sin(c + d*x)**3/(3*d) + C*a**4*sin(c + d*x)*cos(c + d*x)**6/d + 11*
C*a**4*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 6*C*a**4*sin(c + d*x)*cos(c + d
*x)**4/d + 5*C*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + C*a**4*sin(c + d*x
)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**4*(A + B*cos(c) + C*cos(
c)**2)*cos(c), True))
```

Giac [A] time = 1.25046, size = 309, normalized size = 1.27

$$\frac{Ca^4 \sin(7\,dx + 7\,c)}{448\,d} + \frac{1}{16}\left(56\,Aa^4 + 49\,Ba^4 + 44\,Ca^4\right)x + \frac{\left(Ba^4 + 4\,Ca^4\right)\sin(6\,dx + 6\,c)}{192\,d} + \frac{\left(4\,Aa^4 + 16\,Ba^4 + 31\,Ca^4\right)\sin}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
```

```
algorithm="giac")
```

```
[Out] 1/448*C*a^4*sin(7*d*x + 7*c)/d + 1/16*(56*A*a^4 + 49*B*a^4 + 44*C*a^4)*x +
1/192*(B*a^4 + 4*C*a^4)*sin(6*d*x + 6*c)/d + 1/320*(4*A*a^4 + 16*B*a^4 + 31
*C*a^4)*sin(5*d*x + 5*c)/d + 1/64*(8*A*a^4 + 15*B*a^4 + 20*C*a^4)*sin(4*d*x
 + 4*c)/d + 1/192*(116*A*a^4 + 144*B*a^4 + 157*C*a^4)*sin(3*d*x + 3*c)/d +
1/64*(128*A*a^4 + 127*B*a^4 + 124*C*a^4)*sin(2*d*x + 2*c)/d + 1/64*(392*A*a
^4 + 352*B*a^4 + 323*C*a^4)*sin(d*x + c)/d
```

3.330 $\int (a+a\cos(c+dx))^4 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) d$

Optimal. Leaf size=200

$$-\frac{2a^4(10A+8B+7C)\sin^3(c+dx)}{15d} + \frac{4a^4(10A+8B+7C)\sin(c+dx)}{5d} + \frac{a^4(10A+8B+7C)\sin(c+dx)\cos^3(c+dx)}{40d} +$$

```
[Out] (7*a^4*(10*A + 8*B + 7*C)*x)/16 + (4*a^4*(10*A + 8*B + 7*C)*Sin[c + d*x])/(
5*d) + (27*a^4*(10*A + 8*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*
(10*A + 8*B + 7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + ((6*B - C)*(a + a*
Cos[c + d*x])^4*Sin[c + d*x])/(30*d) + (C*(a + a*Cos[c + d*x])^5*Sin[c + d*
x])/(6*a*d) - (2*a^4*(10*A + 8*B + 7*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.279667, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{2a^4(10A+8B+7C)\sin^3(c+dx)}{15d} + \frac{4a^4(10A+8B+7C)\sin(c+dx)}{5d} + \frac{a^4(10A+8B+7C)\sin(c+dx)\cos^3(c+dx)}{40d} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (7*a^4*(10*A + 8*B + 7*C)*x)/16 + (4*a^4*(10*A + 8*B + 7*C)*Sin[c + d*x])/(
5*d) + (27*a^4*(10*A + 8*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*
(10*A + 8*B + 7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + ((6*B - C)*(a + a*
Cos[c + d*x])^4*Sin[c + d*x])/(30*d) + (C*(a + a*Cos[c + d*x])^5*Sin[c + d*
x])/(6*a*d) - (2*a^4*(10*A + 8*B + 7*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C(a+a\cos(c+dx))^5\sin(c+dx)}{6ad} + \frac{\int (a+a\cos(c+d\cdots}{}$$
$$= \frac{(6B-C)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{C(a+a\cos\cdots}{}$$
$$= \frac{(6B-C)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{C(a+a\cos\cdots}{}$$
$$= \frac{1}{10}a^4(10A+8B+7C)x + \frac{(6B-C)(a+a\cos(c+dx))^4\cdots}{30d}$$
$$= \frac{1}{10}a^4(10A+8B+7C)x + \frac{2a^4(10A+8B+7C)\sin(c+\cdots}{5d}$$
$$= \frac{2}{5}a^4(10A+8B+7C)x + \frac{4a^4(10A+8B+7C)\sin(c+\cdots}{5d}$$
$$= \frac{7}{16}a^4(10A+8B+7C)x + \frac{4a^4(10A+8B+7C)\sin(c+\cdots}{5d}$$

Mathematica [A] time = 0.534772, size = 163, normalized size = 0.82

$$\frac{a^4(120(56A+49B+44C)\sin(c+dx)+15(112A+128B+127C)\sin(2(c+dx))+320A\sin(3(c+dx))+30A\sin(4(c+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^4*(4200*A*d*x + 3360*B*d*x + 2940*C*d*x + 120*(56*A + 49*B + 44*C)*Sin[c
 + d*x] + 15*(112*A + 128*B + 127*C)*Sin[2*(c + d*x)] + 320*A*Sin[3*(c + d*
x)] + 580*B*Sin[3*(c + d*x)] + 720*C*Sin[3*(c + d*x)] + 30*A*Sin[4*(c + d*x
)] + 120*B*Sin[4*(c + d*x)] + 225*C*Sin[4*(c + d*x)] + 12*B*Sin[5*(c + d*x)
] + 48*C*Sin[5*(c + d*x)] + 5*C*Sin[6*(c + d*x)]))/(960*d)
```

Maple [B] time = 0.028, size = 416, normalized size = 2.1

$$\frac{1}{d}\left(a^4C\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a^4B\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(a^4*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^4*B*(1/4*(cos(d*x+c)^3+
3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*a^4*C*(1/4*(cos(d*x+c)^3+3/2*co
s(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+2
*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)+6*
A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^4*B*(1/2*cos(d*x+c)*sin
(d*x+c)+1/2*d*x+1/2*c)+a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*A*
a^4*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [B] time = 1.04867, size = 540, normalized size = 2.7

$$-\frac{1280\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}-30\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{4}-1440\left(2\,dx+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/960*(1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 30*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 1440*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a^4 - 960*(d*x + c)*A*a^4 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x +
 c)^3 + 15*sin(d*x + c))*B*a^4 + 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a
^4 - 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 96
0*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d
*x + c)^3 + 15*sin(d*x + c))*C*a^4 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*
c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*a^4 + 1280*(sin(d*x + c)^3
- 3*sin(d*x + c))*C*a^4 - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*C*a^4 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 - 3840*A*a^4*
sin(d*x + c) - 960*B*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.92585, size = 378, normalized size = 1.89

$$\frac{105\left(10\,A+8\,B+7\,C\right)a^{4}dx+\left(40\,Ca^{4}\cos\left(dx+c\right)^{5}+48\left(B+4\,C\right)a^{4}\cos\left(dx+c\right)^{4}+10\left(6\,A+24\,B+41\,C\right)a^{4}\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(105*(10*A + 8*B + 7*C)*a^4*d*x + (40*C*a^4*cos(d*x + c)^5 + 48*(B +
4*C)*a^4*cos(d*x + c)^4 + 10*(6*A + 24*B + 41*C)*a^4*cos(d*x + c)^3 + 32*(1
0*A + 17*B + 18*C)*a^4*cos(d*x + c)^2 + 15*(54*A + 56*B + 49*C)*a^4*cos(d*x
 + c) + 16*(100*A + 83*B + 72*C)*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 6.61921, size = 1005, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/8 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/8 + 3
*A*a**4*x*cos(c + d*x)**2 + A*a**4*x + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) + 5*A*a**4*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + 4*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**4*sin(c + d*
x)*cos(c + d*x)/d + 4*A*a**4*sin(c + d*x)/d + 3*B*a**4*x*sin(c + d*x)**4/2
+ 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 2*B*a**4*x*sin(c + d*x)**2 +
 3*B*a**4*x*cos(c + d*x)**4/2 + 2*B*a**4*x*cos(c + d*x)**2 + 8*B*a**4*sin(c
 + d*x)**5/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*B*a**4*sin(c + d*x)**3/d + B*a**4
*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*
d) + 6*B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*B*a**4*sin(c + d*x)*cos(c
+ d*x)/d + B*a**4*sin(c + d*x)/d + 5*C*a**4*x*sin(c + d*x)**6/16 + 15*C*a**
4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*C*a**4*x*sin(c + d*x)**4/4 + 15*
C*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*C*a**4*x*sin(c + d*x)**2*co
s(c + d*x)**2/2 + C*a**4*x*sin(c + d*x)**2/2 + 5*C*a**4*x*cos(c + d*x)**6/1
6 + 9*C*a**4*x*cos(c + d*x)**4/4 + C*a**4*x*cos(c + d*x)**2/2 + 5*C*a**4*si
n(c + d*x)**5*cos(c + d*x)/(16*d) + 32*C*a**4*sin(c + d*x)**5/(15*d) + 5*C*
a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 16*C*a**4*sin(c + d*x)**3*cos(
c + d*x)**2/(3*d) + 9*C*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*C*a**4*
sin(c + d*x)**3/(3*d) + 11*C*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*C
*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*C*a**4*sin(c + d*x)*cos(c + d*x)*
*3/(4*d) + 4*C*a**4*sin(c + d*x)*cos(c + d*x)**2/d + C*a**4*sin(c + d*x)*co
s(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**4*(A + B*cos(c) + C*cos(c)*
*2), True))
```

Giac [A] time = 1.29142, size = 265, normalized size = 1.32

$$\frac{Ca^4\sin(6\,dx+6\,c)}{192\,d}+\frac{7}{16}\left(10\,Aa^4+8\,Ba^4+7\,Ca^4\right)x+\frac{\left(Ba^4+4\,Ca^4\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{\left(2\,Aa^4+8\,Ba^4+15\,Ca^4\right)\sin(4}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/192*C*a^4*sin(6*d*x + 6*c)/d + 7/16*(10*A*a^4 + 8*B*a^4 + 7*C*a^4)*x + 1/
80*(B*a^4 + 4*C*a^4)*sin(5*d*x + 5*c)/d + 1/64*(2*A*a^4 + 8*B*a^4 + 15*C*a^
4)*sin(4*d*x + 4*c)/d + 1/48*(16*A*a^4 + 29*B*a^4 + 36*C*a^4)*sin(3*d*x + 3
*c)/d + 1/64*(112*A*a^4 + 128*B*a^4 + 127*C*a^4)*sin(2*d*x + 2*c)/d + 1/8*(
56*A*a^4 + 49*B*a^4 + 44*C*a^4)*sin(d*x + c)/d
```

3.331 $\int (a+a\cos(c+dx))^4 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ se dx) dx

Optimal. Leaf size=195

$$\frac{a^4(40A+35B+28C)\sin(c+dx)}{8d} + \frac{(20A+35B+28C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{60d} + \frac{(32A+35B+28C)\sin(c -}{24}$$

```
[Out] (a^4*(48*A + 35*B + 28*C)*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (a^4*(40
*A + 35*B + 28*C)*Sin[c + d*x])/(8*d) + (a*(5*B + 4*C)*(a + a*Cos[c + d*x])
^3*Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + (
(20*A + 35*B + 28*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + ((32
*A + 35*B + 28*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.603097, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3045, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^4(40A+35B+28C)\sin(c+dx)}{8d} + \frac{(20A+35B+28C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{60d} + \frac{(32A+35B+28C)\sin(c -}{24}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```


```
[Out] (a^4*(48*A + 35*B + 28*C)*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (a^4*(40
*A + 35*B + 28*C)*Sin[c + d*x])/(8*d) + (a*(5*B + 4*C)*(a + a*Cos[c + d*x])
^3*Sin[c + d*x])/(20*d) + (C*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + (
(20*A + 35*B + 28*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + ((32
*A + 35*B + 28*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
```

```
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{\int (a+$$
$$= \frac{a(5B+4C)(a+a\cos(c+dx))^3\sin(c+dx)}{20d}$$
$$= \frac{a(5B+4C)(a+a\cos(c+dx))^3\sin(c+dx)}{20d}$$
$$= \frac{a(5B+4C)(a+a\cos(c+dx))^3\sin(c+dx)}{20d}$$
$$= \frac{a(5B+4C)(a+a\cos(c+dx))^3\sin(c+dx)}{20d}$$
$$= \frac{a^4(40A+35B+28C)\sin(c+dx)}{8d} + \frac{a(5B$$
$$= \frac{1}{8}a^4(48A+35B+28C)x + \frac{a^4(40A+35B$$
$$= \frac{1}{8}a^4(48A+35B+28C)x + \frac{a^4A\tanh^{-1}(\sin}{d}$$

Mathematica [A] time = 0.744236, size = 182, normalized size = 0.93

$$\frac{a^4\left(60(54A+56B+49C)\sin(c+dx)+120(4A+7B+8C)\sin(2(c+dx))+40A\sin(3(c+dx))-480A\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (a^4*(2880*A*d*x + 2100*B*d*x + 1680*C*d*x - 480*A*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 480*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*(54*
A + 56*B + 49*C)*Sin[c + d*x] + 120*(4*A + 7*B + 8*C)*Sin[2*(c + d*x)] + 40
*A*Sin[3*(c + d*x)] + 160*B*Sin[3*(c + d*x)] + 290*C*Sin[3*(c + d*x)] + 15*
B*Sin[4*(c + d*x)] + 60*C*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.075, size = 320, normalized size = 1.6

$$\frac{7a^4Cx}{2} + \frac{7a^4C\cos(dx+c)\sin(dx+c)}{2d} + \frac{35a^4Bx}{8} + \frac{27a^4B\cos(dx+c)\sin(dx+c)}{8d} + 6Aa^4x + 2\frac{Aa^4\cos(dx+c)\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 7/2*a^4*C*x+7/2/d*a^4*C*cos(d*x+c)*sin(d*x+c)+35/8*a^4*B*x+27/8/d*a^4*B*cos
(d*x+c)*sin(d*x+c)+6*A*a^4*x+2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+1/d*a^4*C*sin(
d*x+c)*cos(d*x+c)^3+1/4/d*a^4*B*sin(d*x+c)*cos(d*x+c)^3+4/3/d*B*cos(d*x+c)^
2*sin(d*x+c)*a^4+20/3/d*a^4*B*sin(d*x+c)+1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^
4+20/3/d*A*a^4*sin(d*x+c)+83/15/d*a^4*C*sin(d*x+c)+1/5/d*a^4*C*sin(d*x+c)*c
os(d*x+c)^4+34/15/d*a^4*C*sin(d*x+c)*cos(d*x+c)^2+7/2/d*a^4*C*c+35/8/d*a^4*
B*c+6/d*A*a^4*c+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.03721, size = 439, normalized size = 2.25

$$-\frac{160\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^4-480\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^4-1920\left(dx+c\right)Aa^4+640\left(\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="maxima")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 480*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a^4 - 1920*(d*x + c)*A*a^4 + 640*(sin(d*x + c)^3 - 3*sin(
d*x + c))*B*a^4 - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*B*a^4 - 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 480*(d*x + c)*B*a^4
- 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^4 + 960*(
sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 - 60*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*C*a^4 - 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4
 - 480*A*a^4*log(sec(d*x + c) + tan(d*x + c)) - 2880*A*a^4*sin(d*x + c) - 1
920*B*a^4*sin(d*x + c) - 480*C*a^4*sin(d*x + c))/d
```

Fricas [A] time = 2.12322, size = 408, normalized size = 2.09

$$\frac{15\left(48\,A+35\,B+28\,C\right)a^4dx+60\,Aa^4\log\left(\sin\left(dx+c\right)+1\right)-60\,Aa^4\log\left(-\sin\left(dx+c\right)+1\right)+\left(24\,Ca^4\cos\left(dx+c\right)^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(48*A + 35*B + 28*C)*a^4*d*x + 60*A*a^4*log(sin(d*x + c) + 1) - 6
0*A*a^4*log(-sin(d*x + c) + 1) + (24*C*a^4*cos(d*x + c)^4 + 30*(B + 4*C)*a^
4*cos(d*x + c)^3 + 8*(5*A + 20*B + 34*C)*a^4*cos(d*x + c)^2 + 15*(16*A + 27
*B + 28*C)*a^4*cos(d*x + c) + 8*(100*A + 100*B + 83*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.25558, size = 455, normalized size = 2.33

$$\frac{120\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-120\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\left(48\,Aa^4+35\,Ba^4+28\,Ca^4\right)(dx+c)+\frac{2(\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="giac")
```

```
[Out] 1/120*(120*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 120*A*a^4*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 15*(48*A*a^4 + 35*B*a^4 + 28*C*a^4)*(d*x + c) + 2
*(600*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 420
*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 2720*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 2450*B*
a^4*tan(1/2*d*x + 1/2*c)^7 + 1960*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 4720*A*a^4
*tan(1/2*d*x + 1/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 3584*C*a^4*ta
n(1/2*d*x + 1/2*c)^5 + 3680*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 3950*B*a^4*tan(1
/2*d*x + 1/2*c)^3 + 3160*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 1080*A*a^4*tan(1/2*
```

```
d*x + 1/2*c) + 1395*B*a^4*tan(1/2*d*x + 1/2*c) + 1500*C*a^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.332 $\int (a+a\cos(c+dx))^4 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)$ se dx) dx

Optimal. Leaf size=196

$$\frac{5a^4(4A + 8B + 7C)\sin(c + dx)}{8d} - \frac{(12A - 4B - 7C)\sin(c + dx)\left(a^2\cos(c + dx) + a^2\right)^2}{12d} - \frac{(12A - 32B - 35C)\sin(c + dx)\,(}{24d}$$

```
[Out] (a^4*(52*A + 48*B + 35*C)*x)/8 + (a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d +
(5*a^4*(4*A + 8*B + 7*C)*Sin[c + d*x])/(8*d) - (a*(4*A - C)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(4*d) - ((12*A - 4*B - 7*C)*(a^2 + a^2*Cos[c + d*x])^
2*Sin[c + d*x])/(12*d) - ((12*A - 32*B - 35*C)*(a^4 + a^4*Cos[c + d*x])*Sin
[c + d*x])/(24*d) + (A*(a + a*Cos[c + d*x])^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.679115, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^4(4A + 8B + 7C)\sin(c + dx)}{8d} - \frac{(12A - 4B - 7C)\sin(c + dx)\left(a^2\cos(c + dx) + a^2\right)^2}{12d} - \frac{(12A - 32B - 35C)\sin(c + dx)\,(}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```



```
[Out] (a^4*(52*A + 48*B + 35*C)*x)/8 + (a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d +
(5*a^4*(4*A + 8*B + 7*C)*Sin[c + d*x])/(8*d) - (a*(4*A - C)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(4*d) - ((12*A - 4*B - 7*C)*(a^2 + a^2*Cos[c + d*x])^
2*Sin[c + d*x])/(12*d) - ((12*A - 32*B - 35*C)*(a^4 + a^4*Cos[c + d*x])*Sin
[c + d*x])/(24*d) + (A*(a + a*Cos[c + d*x])^4*Tan[c + d*x])/d
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\tan(c+dx)}{d} + \frac{\int (a\ldots}{}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx\ldots}{4d}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx\ldots}{4d}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx\ldots}{4d}$$

$$= -\frac{a(4A-C)(a+a\cos(c+dx))^3\sin(c+dx\ldots}{4d}$$

$$= \frac{5a^4(4A+8B+7C)\sin(c+dx)}{8d} - \frac{a(4A-\ldots}{}$$

$$= \frac{1}{8}a^4(52A+48B+35C)x + \frac{5a^4(4A+8B\ldots}{}$$

$$= \frac{1}{8}a^4(52A+48B+35C)x + \frac{a^4(4A+B)\tan\ldots}{}$$

Mathematica [A] time = 1.87439, size = 246, normalized size = 1.26

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\Big(12(52A+48B+35C)(c+dx)+24(16A+27B+28C)\sin(c+dx)+24(A+4B+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(12*(52*A + 48*B + 35*C)*(c +
d*x) - 96*(4*A + B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*(4*A + B)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (96*A*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) + (96*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]) + 24*(16*A + 27*B + 28*C)*Sin[c + d*x] + 24*(A + 4*B +
7*C)*Sin[2*(c + d*x)] + 8*(B + 4*C)*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]
))/(1536*d)
```

Maple [A] time = 0.09, size = 289, normalized size = 1.5

$$\frac{13\,Aa^4x}{2}+\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4C\sin(dx+c)(\cos(dx+c))^3}{4d}+\frac{35\,a^4Cx}{8}+\frac{27\,a^4C\cos(dx+c)\sin(dx+c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 13/2*A*a^4*x+1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+1/4/d*a^4*C*sin(d*x+c)*cos(d
*x+c)^3+35/8*a^4*C*x+27/8/d*a^4*C*cos(d*x+c)*sin(d*x+c)+6*a^4*B*x+2/d*a^4*B
*cos(d*x+c)*sin(d*x+c)+1/3/d*B*cos(d*x+c)^2*sin(d*x+c)*a^4+20/3/d*a^4*B*sin
(d*x+c)+4/3/d*a^4*C*sin(d*x+c)*cos(d*x+c)^2+20/3/d*a^4*C*sin(d*x+c)+1/d*A*a
^4*tan(d*x+c)+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^4*sin(d*x+c)+35/8
/d*a^4*C*c+6/d*a^4*B*c+13/2/d*A*a^4*c+4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.0366, size = 392, normalized size = 2.

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+576\,(dx+c)Aa^4-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^4+96\,(2\,dx+2\,c+\sin(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 576*(d*x + c)*A*a^4 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 96*(2*d*x + 2*c + sin(2*d*x + 2*c
))*B*a^4 + 384*(d*x + c)*B*a^4 - 128*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^
4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4 + 144*(
2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 96*(d*x + c)*C*a^4 + 192*A*a^4*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*B*a^4*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1)) + 384*A*a^4*sin(d*x + c) + 576*B*a^4*sin(d*x +
c) + 384*C*a^4*sin(d*x + c) + 96*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.21651, size = 466, normalized size = 2.38

$$\frac{3\,(52\,A+48\,B+35\,C)a^4dx\cos(dx+c)+12\,(4\,A+B)a^4\cos(dx+c)\log(\sin(dx+c)+1)-12\,(4\,A+B)a^4\cos(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="fricas")
```

```
[Out] 1/24*(3*(52*A + 48*B + 35*C)*a^4*d*x*cos(d*x + c) + 12*(4*A + B)*a^4*cos(d*
x + c)*log(sin(d*x + c) + 1) - 12*(4*A + B)*a^4*cos(d*x + c)*log(-sin(d*x +
 c) + 1) + (6*C*a^4*cos(d*x + c)^4 + 8*(B + 4*C)*a^4*cos(d*x + c)^3 + 3*(4*
A + 16*B + 27*C)*a^4*cos(d*x + c)^2 + 32*(3*A + 5*B + 5*C)*a^4*cos(d*x + c)
 + 24*A*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31086, size = 448, normalized size = 2.29

$$-\frac{\frac{48\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\left(52\,Aa^4+48\,Ba^4+35\,Ca^4\right)(dx+c)-24\left(4\,Aa^4+Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+24\left(4\,Aa^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="giac")
```

```
[Out] -1/24*(48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(52*A
*a^4 + 48*B*a^4 + 35*C*a^4)*(d*x + c) - 24*(4*A*a^4 + B*a^4)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) + 24*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(84*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7
 + 105*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 276*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 42
```

```
4*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 385*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 300*A*a
^4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 511*C*a^4*ta
n(1/2*d*x + 1/2*c)^3 + 108*A*a^4*tan(1/2*d*x + 1/2*c) + 216*B*a^4*tan(1/2*d
*x + 1/2*c) + 279*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^
4)/d
```

3.333 $\int (a+a\cos(c+dx))^4 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \text{se}$ $dx)\,dx$

Optimal. Leaf size=206

$$-\frac{5a^4(A-B-2C)\sin(c+dx)}{2d}+\frac{a^4(13A+8B+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(15A+6B-2C)\sin(c+dx)\left(a^2\cos(c+dx)\right.}{6d}$$

```
[Out] (a^4*(8*A + 13*B + 12*C)*x)/2 + (a^4*(13*A + 8*B + 2*C)*ArcTanh[Sin[c + d*x
]])/(2*d) - (5*a^4*(A - B - 2*C)*Sin[c + d*x])/(2*d) - ((15*A + 6*B - 2*C)*
(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) - ((18*A + 3*B - 8*C)*(a^4 +
 a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d) + (a*(2*A + B)*(a + a*Cos[c + d*x])^
3*Tan[c + d*x])/d + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2
*d)
```

Rubi [A] time = 0.685534, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3043, 2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(A-B-2C)\sin(c+dx)}{2d}+\frac{a^4(13A+8B+2C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(15A+6B-2C)\sin(c+dx)\left(a^2\cos(c+dx)\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```


```
[Out] (a^4*(8*A + 13*B + 12*C)*x)/2 + (a^4*(13*A + 8*B + 2*C)*ArcTanh[Sin[c + d*x
]])/(2*d) - (5*a^4*(A - B - 2*C)*Sin[c + d*x])/(2*d) - ((15*A + 6*B - 2*C)*
(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) - ((18*A + 3*B - 8*C)*(a^4 +
 a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d) + (a*(2*A + B)*(a + a*Cos[c + d*x])^
3*Tan[c + d*x])/d + (A*(a + a*Cos[c + d*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2
*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec(c+dx)\tan(c+}{2d}$$

$$= \frac{a(2A+B)(a+a\cos(c+dx))^3\tan(c+dx)}{d}$$

$$= -\frac{(15A+6B-2C)\left(a^2+a^2\cos(c+dx)\right)^2 \mathrm{s}}{6d}$$

$$= -\frac{(15A+6B-2C)\left(a^2+a^2\cos(c+dx)\right)^2 \mathrm{s}}{6d}$$

$$= -\frac{(15A+6B-2C)\left(a^2+a^2\cos(c+dx)\right)^2 \mathrm{s}}{6d}$$

$$= -\frac{5a^4(A-B-2C)\sin(c+dx)}{2d} - \frac{(15A+6}{}$$

$$= \frac{1}{2}a^4(8A+13B+12C)x - \frac{5a^4(A-B-2C}{2d}$$

$$= \frac{1}{2}a^4(8A+13B+12C)x + \frac{a^4(13A+8B+}{}$$

Mathematica [A] time = 3.66794, size = 299, normalized size = 1.45

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(6(8A+13B+12C)(c+dx)+3(4A+16B+27C)\sin(c+dx)-6(13A+8B+2C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(6*(8*A + 13*B + 12*C)*(c + d*
x) - 6*(13*A + 8*B + 2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(13*
A + 8*B + 2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*A)/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])^2 + (12*(4*A + B)*Sin[(c + d*x)/2])/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]) - (3*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 +
 (12*(4*A + B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 3*
(4*A + 16*B + 27*C)*Sin[c + d*x] + 3*(B + 4*C)*Sin[2*(c + d*x)] + C*Sin[3*(
c + d*x)]))/(192*d)
```

Maple [A] time = 0.095, size = 280, normalized size = 1.4

$$\frac{Aa^4 \sec(dx+c)\tan(dx+c)}{2d} + \frac{13\,Aa^4 \ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{a^4 B\tan(dx+c)}{d} + \frac{a^4 C\ln(\sec(dx+c)+\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+13/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/
d*a^4*B*tan(d*x+c)+1/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^4*tan(d*x+c)
+4/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*C*x+6/d*a^4*C*c+13/2*a^4*B*x+13/
2/d*a^4*B*c+20/3/d*a^4*C*sin(d*x+c)+4*A*a^4*x+4/d*A*a^4*c+4/d*a^4*B*sin(d*x
+c)+2/d*a^4*C*cos(d*x+c)*sin(d*x+c)+1/d*A*a^4*sin(d*x+c)+1/2/d*a^4*B*cos(d*
x+c)*sin(d*x+c)+1/3/d*a^4*C*sin(d*x+c)*cos(d*x+c)^2
```

Maxima [A] time = 1.10959, size = 400, normalized size = 1.94

$$\frac{48(dx+c)Aa^4 + 3(2dx+2c+\sin(2dx+2c))Ba^4 + 72(dx+c)Ba^4 - 4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Ca^4 + 12(2dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="maxima")
```

```
[Out] 1/12*(48*(d*x + c)*A*a^4 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 72*(d
*x + c)*B*a^4 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 12*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*C*a^4 + 48*(d*x + c)*C*a^4 - 3*A*a^4*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*A
*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*B*a^4*(log(sin(d*
x + c) + 1) - log(sin(d*x + c) - 1)) + 6*C*a^4*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 12*A*a^4*sin(d*x + c) + 48*B*a^4*sin(d*x + c) + 72*C*
a^4*sin(d*x + c) + 48*A*a^4*tan(d*x + c) + 12*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.18024, size = 486, normalized size = 2.36

$$\frac{6\,(8\,A+13\,B+12\,C)a^4dx\cos{(dx+c)}^2+3\,(13\,A+8\,B+2\,C)a^4\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-3\,(13\,A+8\,B+2\,C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(8*A + 13*B + 12*C)*a^4*d*x*cos(d*x + c)^2 + 3*(13*A + 8*B + 2*C)*a
^4*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(13*A + 8*B + 2*C)*a^4*cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) + 2*(2*C*a^4*cos(d*x + c)^4 + 3*(B + 4*C)*a^4
*cos(d*x + c)^3 + 2*(3*A + 12*B + 20*C)*a^4*cos(d*x + c)^2 + 6*(4*A + B)*a^
4*cos(d*x + c) + 3*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2468, size = 468, normalized size = 2.27

$$3\left(8\,Aa^4+13\,Ba^4+12\,Ca^4\right)(dx+c)+3\left(13\,Aa^4+8\,Ba^4+2\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(13\,Aa^4+8\,Ba^4+2\,C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(8*A*a^4 + 13*B*a^4 + 12*C*a^4)*(d*x + c) + 3*(13*A*a^4 + 8*B*a^4 +
2*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(13*A*a^4 + 8*B*a^4 + 2*C*a
^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*(7*A*a^4*tan(1/2*d*x + 1/2*c)^3
+ 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) - 2*B*a^4*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(6*A*a^4*tan(1/2*d*
x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*C*a^4*tan(1/2*d*x + 1/2
*c)^5 + 12*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 +
 76*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4*
tan(1/2*d*x + 1/2*c) + 54*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

3.334 $\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec$ dx) dx

Optimal. Leaf size=219

$$-\frac{5a^4(2A+B-C)\sin(c+dx)}{2d}+\frac{a^4(12A+13B+8C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(22A+18B+3C)\sin(c+dx)\left(a^4\cos(c+d\right.}{6d}$$

```
[Out] (a^4*(2*A + 8*B + 13*C)*x)/2 + (a^4*(12*A + 13*B + 8*C)*ArcTanh[Sin[c + d*x
]])/(2*d) - (5*a^4*(2*A + B - C)*Sin[c + d*x])/(2*d) - ((22*A + 18*B + 3*C)
*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d) + ((16*A + 15*B + 6*C)*(a^2 +
 a^2*Cos[c + d*x])^2*Tan[c + d*x])/(6*d) + (a*(4*A + 3*B)*(a + a*Cos[c + d*
x])^3*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.713735, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3043, 2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(2A+B-C)\sin(c+dx)}{2d}+\frac{a^4(12A+13B+8C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(22A+18B+3C)\sin(c+dx)\left(a^4\cos(c+d\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```


```
[Out] (a^4*(2*A + 8*B + 13*C)*x)/2 + (a^4*(12*A + 13*B + 8*C)*ArcTanh[Sin[c + d*x
]])/(2*d) - (5*a^4*(2*A + B - C)*Sin[c + d*x])/(2*d) - ((22*A + 18*B + 3*C)
*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d) + ((16*A + 15*B + 6*C)*(a^2 +
 a^2*Cos[c + d*x])^2*Tan[c + d*x])/(6*d) + (a*(4*A + 3*B)*(a + a*Cos[c + d*
x])^3*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^4\sec^2(c+dx)\tan(c+}{3d}$$

$$=\frac{a(4A+3B)(a+a\cos(c+dx))^3\sec(c+dx}{6d}$$

$$=\frac{(16A+15B+6C)\left(a^2+a^2\cos(c+dx)\right)^2 \text{t}}{6d}$$

$$=-\frac{(22A+18B+3C)\left(a^4+a^4\cos(c+dx)\right)}{6d}$$

$$=-\frac{(22A+18B+3C)\left(a^4+a^4\cos(c+dx)\right)}{6d}$$

$$=-\frac{5a^4(2A+B-C)\sin(c+dx)}{2d}-\frac{(22A+1}{}$$

$$=\frac{1}{2}a^4(2A+8B+13C)x-\frac{5a^4(2A+B-C)}{2d}$$

$$=\frac{1}{2}a^4(2A+8B+13C)x+\frac{a^4(12A+13B+}{}$$

Mathematica [A] time = 5.69424, size = 354, normalized size = 1.62

$$a^4\left(6(2A+8B+13C)(c+dx)+\frac{4(20A+3(4B+C))\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4(20A+3(4B+C))\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-6(12A+13B+8C)\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (a^4*(6*(2*A + 8*B + 13*C)*(c + d*x) - 6*(12*A + 13*B + 8*C)*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 6*(12*A + 13*B + 8*C)*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + (13*A + 3*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (
2*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (4*(20*A +
3*(4*B + C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*A
*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (-13*A - 3*B)/
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(20*A + 3*(4*B + C))*Sin[(c +
d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12*(B + 4*C)*Sin[c + d*x]
+ 3*C*Sin[2*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.099, size = 279, normalized size = 1.3

$$\frac{20\,Aa^4\tan(dx+c)}{3d}+\frac{Aa^4\tan(dx+c)(\sec(dx+c))^2}{3d}+\frac{a^4B\sec(dx+c)\tan(dx+c)}{2d}+\frac{13\,a^4B\ln(\sec(dx+c)+\tan(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 20/3/d*A*a^4*tan(d*x+c)+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec
(d*x+c)*tan(d*x+c)+13/2/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*C*tan(d*x
+c)+2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+6/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d
*a^4*B*tan(d*x+c)+4/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+13/2*a^4*C*x+13/2/d*a
^4*C*c+4*a^4*B*x+4/d*a^4*B*c+4/d*a^4*C*sin(d*x+c)+A*a^4*x+1/d*A*a^4*c+1/d*a
^4*B*sin(d*x+c)+1/2/d*a^4*C*cos(d*x+c)*sin(d*x+c)
```

Maxima [A] time = 1.02482, size = 432, normalized size = 1.97

$$\frac{4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+12\,(dx+c)Aa^4+48\,(dx+c)Ba^4+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^4+72\,(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 12*(d*x + c)*A*a^4 + 48*(
d*x + c)*B*a^4 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 72*(d*x + c)*C*
a^4 - 12*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
 + log(sin(d*x + c) - 1)) - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(sin(d*x + c)
 + 1) - log(sin(d*x + c) - 1)) + 36*B*a^4*(log(sin(d*x + c) + 1) - log(sin(
d*x + c) - 1)) + 24*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 12*B*a^4*sin(d*x + c) + 48*C*a^4*sin(d*x + c) + 72*A*a^4*tan(d*x + c) + 48
*B*a^4*tan(d*x + c) + 12*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.29112, size = 487, normalized size = 2.22

$$\frac{6\,(2\,A+8\,B+13\,C)a^4dx\cos{(dx+c)}^3+3\,(12\,A+13\,B+8\,C)a^4\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(12\,A+13\,B+8\,C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(2*A + 8*B + 13*C)*a^4*d*x*cos(d*x + c)^3 + 3*(12*A + 13*B + 8*C)*a
^4*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(12*A + 13*B + 8*C)*a^4*cos(d*x
 + c)^3*log(-sin(d*x + c) + 1) + 2*(3*C*a^4*cos(d*x + c)^4 + 6*(B + 4*C)*a^
4*cos(d*x + c)^3 + 2*(20*A + 12*B + 3*C)*a^4*cos(d*x + c)^2 + 3*(4*A + B)*a
^4*cos(d*x + c) + 2*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28613, size = 468, normalized size = 2.14

$$\frac{3\left(2\,Aa^4+8\,Ba^4+13\,Ca^4\right)(dx+c)+3\left(12\,Aa^4+13\,Ba^4+8\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(12\,Aa^4+13\,Ba^4+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(2*A*a^4 + 8*B*a^4 + 13*C*a^4)*(d*x + c) + 3*(12*A*a^4 + 13*B*a^4 +
8*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(12*A*a^4 + 13*B*a^4 + 8*C*
a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 6*(2*B*a^4*tan(1/2*d*x + 1/2*c)^3
 + 7*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c) + 9*C*a^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 2*(30*A*a^4*tan(1/2*
d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^4*tan(1/2*d*x + 1/
2*c)^5 - 76*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^4*tan(1/2*d*x + 1/2*c)^3
- 12*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^
4*tan(1/2*d*x + 1/2*c) + 6*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^3)/d
```

3.335 $\int (a+a\cos(c+dx))^4 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)$ se dx) dx

Optimal. Leaf size=217

$$-\frac{5a^4(7A+8B+4C)\sin(c+dx)}{8d}+\frac{a^4(35A+48B+52C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(35A+44B+36C)\tan(c+dx)\left(a^4\cos\right.}{12d}$$

```
[Out] a^4*(B + 4*C)*x + (a^4*(35*A + 48*B + 52*C)*ArcTanh[Sin[c + d*x]])/(8*d) -
(5*a^4*(7*A + 8*B + 4*C)*Sin[c + d*x])/(8*d) + ((35*A + 44*B + 36*C)*(a^4 +
 a^4*Cos[c + d*x])*Tan[c + d*x])/(12*d) + ((7*A + 8*B + 4*C)*(a^2 + a^2*Cos
[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A + B)*(a + a*Cos[c + d
*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c
 + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.742734, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2975, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(7A+8B+4C)\sin(c+dx)}{8d}+\frac{a^4(35A+48B+52C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(35A+44B+36C)\tan(c+dx)\left(a^4\cos\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```


```
[Out] a^4*(B + 4*C)*x + (a^4*(35*A + 48*B + 52*C)*ArcTanh[Sin[c + d*x]])/(8*d) -
(5*a^4*(7*A + 8*B + 4*C)*Sin[c + d*x])/(8*d) + ((35*A + 44*B + 36*C)*(a^4 +
 a^4*Cos[c + d*x])*Tan[c + d*x])/(12*d) + ((7*A + 8*B + 4*C)*(a^2 + a^2*Cos
[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A + B)*(a + a*Cos[c + d
*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c
 + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^3(c+dx)\tan(c+}{4d}$$

$$= \frac{a(A+B)(a+a\cos(c+dx))^3\sec^2(c+dx)}{3d}$$

$$= \frac{(7A+8B+4C)\left(a^2+a^2\cos(c+dx)\right)^2\sec}{8d}$$

$$= \frac{(35A+44B+36C)\left(a^4+a^4\cos(c+dx)\right)}{12d}$$

$$= \frac{(35A+44B+36C)\left(a^4+a^4\cos(c+dx)\right)}{12d}$$

$$= -\frac{5a^4(7A+8B+4C)\sin(c+dx)}{8d} + \frac{(35A}{}$$

$$= a^4(B+4C)x - \frac{5a^4(7A+8B+4C)\sin(c+}{8d}$$

$$= a^4(B+4C)x + \frac{a^4(35A+48B+52C)\tanh}{8d}$$

Mathematica [B] time = 6.19332, size = 838, normalized size = 3.86

$$\frac{(B+4C)(c+dx)(\cos(c+dx)a+a)^4\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d} + \frac{(-35A-48B-52C)(\cos(c+dx)a+a)^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] ((B + 4*C)*(c + d*x)*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(16*d) +
((-35*A - 48*B - 52*C)*(a + a*Cos[c + d*x])^4*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]]*Sec[c/2 + (d*x)/2]^8)/(128*d) + ((35*A + 48*B + 52*C)*(a + a*Co
s[c + d*x])^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c/2 + (d*x)/2]^8
)/(128*d) + (A*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(256*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^4) + ((97*A + 52*B + 12*C)*(a + a*Cos[c + d*x
```

```
])^4*Sec[c/2 + (d*x)/2]^8)/(768*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
- (A*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(256*d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^4) + ((-97*A - 52*B - 12*C)*(a + a*Cos[c + d*x])^4*Sec[
c/2 + (d*x)/2]^8)/(768*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((a + a
*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(4*A*Sin[(c + d*x)/2] + B*Sin[(c + d*
x)/2]))/(96*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((a + a*Cos[c + d*
x])^4*Sec[c/2 + (d*x)/2]^8*(4*A*Sin[(c + d*x)/2] + B*Sin[(c + d*x)/2]))/(96
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((a + a*Cos[c + d*x])^4*Sec[c
/2 + (d*x)/2]^8*(5*A*Sin[(c + d*x)/2] + 5*B*Sin[(c + d*x)/2] + 3*C*Sin[(c +
 d*x)/2]))/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((a + a*Cos[c + d
*x])^4*Sec[c/2 + (d*x)/2]^8*(5*A*Sin[(c + d*x)/2] + 5*B*Sin[(c + d*x)/2] +
3*C*Sin[(c + d*x)/2]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (C*(a
 + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[c + d*x])/(16*d)
```

Maple [A] time = 0.098, size = 294, normalized size = 1.4

$$\frac{Aa^4\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{27\,Aa^4\sec(dx+c)\tan(dx+c)}{8d}+\frac{35\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{20\,a^4B\,t}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+27/8/d*A*a^4*sec(d*x+c)*tan(d*x+c)+35/8
/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*a^4*B*tan(d*x+c)+1/3/d*a^4*B*tan(
d*x+c)*sec(d*x+c)^2+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+13/2/d*a^4*C*ln(sec(d
*x+c)+tan(d*x+c))+20/3/d*A*a^4*tan(d*x+c)+4/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)
^2+2/d*a^4*B*sec(d*x+c)*tan(d*x+c)+6/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*
a^4*C*tan(d*x+c)+4*a^4*C*x+4/d*a^4*C*c+a^4*B*x+1/d*a^4*B*c+1/d*a^4*C*sin(d*
x+c)
```

Maxima [B] time = 1.06335, size = 562, normalized size = 2.59

$$\frac{64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^4+48\,(dx+c)Ba^4+192\,(dx+c)Ca^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^4 + 48*(d*x + c)*B*a^4 + 192*(d*x + c)*C*a^4 - 3*A*a^4*(2*
(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1)
 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*A*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 - 48*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 12*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 96*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 144*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
48*C*a^4*sin(d*x + c) + 192*A*a^4*tan(d*x + c) + 288*B*a^4*tan(d*x + c) + 1
92*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.49951, size = 493, normalized size = 2.27

$$48\,(B+4\,C)a^4dx\cos\left(dx+c\right)^4+3\,(35\,A+48\,B+52\,C)a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(35\,A+48\,B+52\,C)a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="fricas")
```

```
[Out] 1/48*(48*(B + 4*C)*a^4*d*x*cos(d*x + c)^4 + 3*(35*A + 48*B + 52*C)*a^4*cos(
d*x + c)^4*log(sin(d*x + c) + 1) - 3*(35*A + 48*B + 52*C)*a^4*cos(d*x + c)^
4*log(-sin(d*x + c) + 1) + 2*(24*C*a^4*cos(d*x + c)^4 + 32*(5*A + 5*B + 3*C
)*a^4*cos(d*x + c)^3 + 3*(27*A + 16*B + 4*C)*a^4*cos(d*x + c)^2 + 8*(4*A +
B)*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29461, size = 458, normalized size = 2.11

$$\frac{\frac{48\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+24\left(Ba^4+4\,Ca^4\right)(dx+c)+3\left(35\,Aa^4+48\,Ba^4+52\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(35\,Aa^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="giac")
```

```
[Out] 1/24*(48*C*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 24*(B*a^
4 + 4*C*a^4)*(d*x + c) + 3*(35*A*a^4 + 48*B*a^4 + 52*C*a^4)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - 3*(35*A*a^4 + 48*B*a^4 + 52*C*a^4)*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*
d*x + 1/2*c)^7 + 84*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 276*C*a^4*tan(1/2*d*x + 1/2*c
)^5 + 511*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 +
 300*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*A*a^4*tan(1/2*d*x + 1/2*c) - 216*B*
a^4*tan(1/2*d*x + 1/2*c) - 108*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^4)/d
```

3.336 $\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec$ $dx)\,dx$

Optimal. Leaf size=225

$$\frac{a^4(28A+35B+40C)\tan(c+dx)}{8d}+\frac{a^4(28A+35B+48C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(28A+35B+20C)\tan(c+dx)\sec^2(}{60d}$$

```
[Out] a^4*C*x + (a^4*(28*A + 35*B + 48*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(28
*A + 35*B + 40*C)*Tan[c + d*x])/(8*d) + ((28*A + 35*B + 32*C)*(a^4 + a^4*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((28*A + 35*B + 20*C)*(a^2
+ a^2*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + (a*(4*A + 5*B)*
(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + a*Cos[
c + d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.691009, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3043, 2975, 2968, 3021, 2735, 3770}

$$\frac{a^4(28A+35B+40C)\tan(c+dx)}{8d}+\frac{a^4(28A+35B+48C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(28A+35B+20C)\tan(c+dx)\sec^2(}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```


```
[Out] a^4*C*x + (a^4*(28*A + 35*B + 48*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(28
*A + 35*B + 40*C)*Tan[c + d*x])/(8*d) + ((28*A + 35*B + 32*C)*(a^4 + a^4*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((28*A + 35*B + 20*C)*(a^2
+ a^2*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + (a*(4*A + 5*B)*
(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + a*Cos[
c + d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^4(c+dx)\tan(c+}{5d}$$
$$= \frac{a(4A+5B)(a+a\cos(c+dx))^3\sec^3(c+d}{20d}$$
$$= \frac{(28A+35B+20C)\left(a^2+a^2\cos(c+dx)\right)^2}{60d}$$
$$= \frac{(28A+35B+32C)\left(a^4+a^4\cos(c+dx)\right)}{24d}$$
$$= \frac{(28A+35B+32C)\left(a^4+a^4\cos(c+dx)\right)}{24d}$$
$$= \frac{a^4(28A+35B+40C)\tan(c+dx)}{8d} + \frac{(28A}{}$$
$$= a^4Cx + \frac{a^4(28A+35B+40C)\tan(c+dx)}{8d}$$
$$= a^4Cx + \frac{a^4(28A+35B+48C)\tanh^{-1}(\sin}{8d}$$

Mathematica [B] time = 6.20683, size = 971, normalized size = 4.32

$$\frac{C(c+dx)(\cos(c+dx)a+a)^4\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d} + \frac{(-28A-35B-48C)(\cos(c+dx)a+a)^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (C*(c + d*x)*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(16*d) + ((-28*A
- 35*B - 48*C)*(a + a*Cos[c + d*x])^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sec[c/2 + (d*x)/2]^8)/(128*d) + ((28*A + 35*B + 48*C)*(a + a*Cos[c + d*
x])^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c/2 + (d*x)/2]^8)/(128*d
) + ((22*A + 5*B)*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(1280*d*(Cos
```

```
[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + ((559*A + 485*B + 260*C)*(a + a*Cos[
c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(3840*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^2) + (A*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(c + d*x)/2])/
(320*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5) + (A*(a + a*Cos[c + d*x])^4
*Sec[c/2 + (d*x)/2]^8*Sin[(c + d*x)/2])/(320*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^5) + ((-22*A - 5*B)*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/
(1280*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + ((-559*A - 485*B - 260*C
)*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8)/(3840*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2) + ((a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(139*A*
Sin[(c + d*x)/2] + 80*B*Sin[(c + d*x)/2] + 20*C*Sin[(c + d*x)/2]))/(1920*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((a + a*Cos[c + d*x])^4*Sec[c/2
+ (d*x)/2]^8*(139*A*Sin[(c + d*x)/2] + 80*B*Sin[(c + d*x)/2] + 20*C*Sin[(c
+ d*x)/2]))/(1920*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((a + a*Cos[
c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(83*A*Sin[(c + d*x)/2] + 100*B*Sin[(c + d*
x)/2] + 100*C*Sin[(c + d*x)/2]))/(240*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])) + ((a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(83*A*Sin[(c + d*x)/2] +
 100*B*Sin[(c + d*x)/2] + 100*C*Sin[(c + d*x)/2]))/(240*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.101, size = 331, normalized size = 1.5

$$\frac{83\,Aa^4\tan(dx+c)}{15\,d}+\frac{Aa^4\tan(dx+c)(\sec(dx+c))^4}{5\,d}+\frac{34\,Aa^4\tan(dx+c)(\sec(dx+c))^2}{15\,d}+\frac{a^4B\tan(dx+c)(\sec(d}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 83/15/d*A*a^4*tan(d*x+c)+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+34/15/d*A*a^4*
tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3+27/8/d*a^4*B*se
c(d*x+c)*tan(d*x+c)+35/8/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*a^4*C*tan
(d*x+c)+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+1/d*A*a^4*tan(d*x+c)*sec(d*x+c)
^3+7/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+7/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+
20/3/d*a^4*B*tan(d*x+c)+4/3/d*a^4*B*tan(d*x+c)*sec(d*x+c)^2+2/d*a^4*C*sec(d
*x+c)*tan(d*x+c)+6/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+a^4*C*x+1/d*a^4*C*c
```

Maxima [B] time = 1.03938, size = 670, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 320*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 240*(d*x + c)
*C*a^4 - 60*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 240*A
*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)) - 360*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 240*C*a^4*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*a^4*(
log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 480*C*a^4*(log(sin(d*x + c
) + 1) - log(sin(d*x + c) - 1)) + 240*A*a^4*tan(d*x + c) + 960*B*a^4*tan(d*
x + c) + 1440*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.16467, size = 521, normalized size = 2.32

$$\frac{240\,Ca^4dx\cos(dx+c)^5+15\,(28\,A+35\,B+48\,C)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(28\,A+35\,B+48\,C)a^4\cos(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="fricas")
```

```
[Out] 1/240*(240*C*a^4*d*x*cos(d*x + c)^5 + 15*(28*A + 35*B + 48*C)*a^4*cos(d*x +
 c)^5*log(sin(d*x + c) + 1) - 15*(28*A + 35*B + 48*C)*a^4*cos(d*x + c)^5*lo
g(-sin(d*x + c) + 1) + 2*(8*(83*A + 100*B + 100*C)*a^4*cos(d*x + c)^4 + 15*
(28*A + 27*B + 16*C)*a^4*cos(d*x + c)^3 + 8*(34*A + 20*B + 5*C)*a^4*cos(d*x
 + c)^2 + 30*(4*A + B)*a^4*cos(d*x + c) + 24*A*a^4)*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24557, size = 475, normalized size = 2.11

$$\frac{120\,(dx+c)Ca^4 + 15\left(28\,Aa^4 + 35\,Ba^4 + 48\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(28\,Aa^4 + 35\,Ba^4 + 48\,Ca^4\right)\log\left(\left|\mathrm{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="giac")
```

```
[Out] 1/120*(120*(d*x + c)*C*a^4 + 15*(28*A*a^4 + 35*B*a^4 + 48*C*a^4)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 15*(28*A*a^4 + 35*B*a^4 + 48*C*a^4)*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 525*B*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 600*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 1960*A*a^4*tan(1/
2*d*x + 1/2*c)^7 - 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 2720*C*a^4*tan(1/2*d
*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 4480*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 + 4720*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 3160*A*a^4*tan(1/2*d*x + 1
/2*c)^3 - 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3680*C*a^4*tan(1/2*d*x + 1/2*
c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c) + 1395*B*a^4*tan(1/2*d*x + 1/2*c) +
1080*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.337 $\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{se}$ $dx)\,dx$

Optimal. Leaf size=253

$$\frac{a^4(72A+83B+100C)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B+10C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(417A+488B+550C)\tan(c+dx)}{240d}$$

```
[Out] (7*a^4*(7*A + 8*B + 10*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(72*A + 83*B
 + 100*C)*Tan[c + d*x])/(15*d) + (a^4*(417*A + 488*B + 550*C)*Sec[c + d*x]*
Tan[c + d*x])/(240*d) + ((43*A + 52*B + 50*C)*(a^4 + a^4*Cos[c + d*x])*Sec[
c + d*x]^2*Tan[c + d*x])/(60*d) + ((37*A + 48*B + 30*C)*(a^2 + a^2*Cos[c +
d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (a*(2*A + 3*B)*(a + a*Cos[c
+ d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (A*(a + a*Cos[c + d*x])^4*S
ec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.833446, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3043, 2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^4(72A+83B+100C)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B+10C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(417A+488B+550C)\tan(c+dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^7,x]
```


```
[Out] (7*a^4*(7*A + 8*B + 10*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(72*A + 83*B
 + 100*C)*Tan[c + d*x])/(15*d) + (a^4*(417*A + 488*B + 550*C)*Sec[c + d*x]*
Tan[c + d*x])/(240*d) + ((43*A + 52*B + 50*C)*(a^4 + a^4*Cos[c + d*x])*Sec[
c + d*x]^2*Tan[c + d*x])/(60*d) + ((37*A + 48*B + 30*C)*(a^2 + a^2*Cos[c +
d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (a*(2*A + 3*B)*(a + a*Cos[c
+ d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (A*(a + a*Cos[c + d*x])^4*S
ec[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^4\sec^5(c+dx)\tan(c+}{6d}$$

$$= \frac{a(2A+3B)(a+a\cos(c+dx))^3\sec^4(c+d}{15d}$$

$$= \frac{(37A+48B+30C)\left(a^2+a^2\cos(c+dx)\right)^2}{120d}$$

$$= \frac{(43A+52B+50C)\left(a^4+a^4\cos(c+dx)\right)}{60d}$$

$$= \frac{(43A+52B+50C)\left(a^4+a^4\cos(c+dx)\right)}{60d}$$

$$= \frac{a^4(417A+488B+550C)\sec(c+dx)\tan(}{240d}$$

$$= \frac{a^4(417A+488B+550C)\sec(c+dx)\tan(}{240d}$$

$$= \frac{7a^4(7A+8B+10C)\tanh^{-1}(\sin(c+dx))}{16d}$$

$$= \frac{7a^4(7A+8B+10C)\tanh^{-1}(\sin(c+dx))}{16d}$$

Mathematica [A] time = 2.05746, size = 265, normalized size = 1.05

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(840(7A+8B+10C)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^7,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(840*(7*A + 8*
B + 10*C)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - (4165*A + 3480*B + 2670*C + 16*(672*A
 + 643*B + 620*C)*Cos[c + d*x] + 20*(229*A + 216*B + 174*C)*Cos[2*(c + d*x)
] + 4032*A*Cos[3*(c + d*x)] + 4408*B*Cos[3*(c + d*x)] + 4640*C*Cos[3*(c + d
*x)] + 735*A*Cos[4*(c + d*x)] + 840*B*Cos[4*(c + d*x)] + 810*C*Cos[4*(c + d
*x)] + 576*A*Cos[5*(c + d*x)] + 664*B*Cos[5*(c + d*x)] + 800*C*Cos[5*(c + d
*x)])*Sin[c + d*x]))/(30720*d)
```

Maple [A] time = 0.106, size = 385, normalized size = 1.5

$$\frac{20\,a^4C\tan(dx+c)}{3\,d}+\frac{83\,a^4B\tan(dx+c)}{15\,d}+\frac{a^4B\tan(dx+c)(\sec(dx+c))^4}{5\,d}+\frac{34\,a^4B\tan(dx+c)(\sec(dx+c))^2}{15\,d}+\frac{A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 20/3/d*a^4*C*tan(d*x+c)+83/15/d*a^4*B*tan(d*x+c)+1/5/d*a^4*B*tan(d*x+c)*sec
(d*x+c)^4+34/15/d*a^4*B*tan(d*x+c)*sec(d*x+c)^2+1/6/d*A*a^4*tan(d*x+c)*sec(
d*x+c)^5+41/24/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+49/16/d*A*a^4*sec(d*x+c)*tan
(d*x+c)+1/4/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+27/8/d*a^4*C*sec(d*x+c)*tan(d*x
+c)+1/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3+7/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)+24/
5/d*A*a^4*tan(d*x+c)+4/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+12/5/d*A*a^4*tan(d
*x+c)*sec(d*x+c)^2+4/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+7/2/d*a^4*B*ln(sec(d
*x+c)+tan(d*x+c))+35/8/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+49/16/d*A*a^4*ln(s
ec(d*x+c)+tan(d*x+c))
```

Maxima [B] time = 1.06076, size = 871, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 32*(3*tan(d*x + c)^5 + 10*ta
n(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d*x + c
))*B*a^4 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 - 5*A*a^4*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 180*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 120*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4
 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1
)) - 30*C*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12
0*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 480*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 720*C*a^4*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*C*a^
4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 480*B*a^4*tan(d*x + c)
+ 1920*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.09994, size = 539, normalized size = 2.13

$$\frac{105\,(7\,A + 8\,B + 10\,C)a^4\cos{(dx + c)}^6\log{(\sin{(dx + c)} + 1)} - 105\,(7\,A + 8\,B + 10\,C)a^4\cos{(dx + c)}^6\log{(-\sin{(dx + c)} +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="fricas")
```

```
[Out] 1/480*(105*(7*A + 8*B + 10*C)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 10
5*(7*A + 8*B + 10*C)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(72*
A + 83*B + 100*C)*a^4*cos(d*x + c)^5 + 15*(49*A + 56*B + 54*C)*a^4*cos(d*x
+ c)^4 + 32*(18*A + 17*B + 10*C)*a^4*cos(d*x + c)^3 + 10*(41*A + 24*B + 6*C
)*a^4*cos(d*x + c)^2 + 48*(4*A + B)*a^4*cos(d*x + c) + 40*A*a^4)*sin(d*x +
c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33507, size = 529, normalized size = 2.09

$$105\left(7\,Aa^4+8\,Ba^4+10\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(7\,Aa^4+8\,Ba^4+10\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="giac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4 + 10*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 105*(7*A*a^4 + 8*B*a^4 + 10*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
 2*(735*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 +
 1050*C*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 4
760*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 5950*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702
*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 13860*
C*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 13488*B
*a^4*tan(1/2*d*x + 1/2*c)^5 - 16860*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a
^4*tan(1/2*d*x + 1/2*c)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 10690*C*a^4
*tan(1/2*d*x + 1/2*c)^3 - 3105*A*a^4*tan(1/2*d*x + 1/2*c) - 3000*B*a^4*tan(
1/2*d*x + 1/2*c) - 2790*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^6)/d
```

3.338 $\int (a+a\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{se}$ $dx)\,dx$

Optimal. Leaf size=287

$$\frac{a^4(454A+504B+581C)\tan(c+dx)}{105d}+\frac{a^4(44A+49B+56C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(988A+1113B+1232C)\tan(c}{840d}$$

```
[Out] (a^4*(44*A + 49*B + 56*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(454*A + 504
*B + 581*C)*Tan[c + d*x])/(105*d) + (a^4*(44*A + 49*B + 56*C)*Sec[c + d*x]*
Tan[c + d*x])/(16*d) + (a^4*(988*A + 1113*B + 1232*C)*Sec[c + d*x]^2*Tan[c
+ d*x])/(840*d) + ((436*A + 511*B + 504*C)*(a^4 + a^4*Cos[c + d*x])*Sec[c +
 d*x]^3*Tan[c + d*x])/(840*d) + ((16*A + 21*B + 14*C)*(a^2 + a^2*Cos[c + d*
x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(70*d) + (a*(4*A + 7*B)*(a + a*Cos[c + d
*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(42*d) + (A*(a + a*Cos[c + d*x])^4*Sec[
c + d*x]^6*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.868061, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3043, 2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^4(454A+504B+581C)\tan(c+dx)}{105d}+\frac{a^4(44A+49B+56C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(988A+1113B+1232C)\tan(c}{840d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^8,x]
```


```
[Out] (a^4*(44*A + 49*B + 56*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(454*A + 504
*B + 581*C)*Tan[c + d*x])/(105*d) + (a^4*(44*A + 49*B + 56*C)*Sec[c + d*x]*
Tan[c + d*x])/(16*d) + (a^4*(988*A + 1113*B + 1232*C)*Sec[c + d*x]^2*Tan[c
+ d*x])/(840*d) + ((436*A + 511*B + 504*C)*(a^4 + a^4*Cos[c + d*x])*Sec[c +
 d*x]^3*Tan[c + d*x])/(840*d) + ((16*A + 21*B + 14*C)*(a^2 + a^2*Cos[c + d*
x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(70*d) + (a*(4*A + 7*B)*(a + a*Cos[c + d
*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(42*d) + (A*(a + a*Cos[c + d*x])^4*Sec[
c + d*x]^6*Tan[c + d*x])/(7*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^4 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^8(c + dx)\,dx = \frac{A(a + a\cos(c + dx))^4\sec^6(c + dx)\tan(c}{7d}$$

$$= \frac{a(4A + 7B)(a + a\cos(c + dx))^3\sec^5(c +}{42d}$$

$$= \frac{(16A + 21B + 14C)\left(a^2 + a^2\cos(c + dx)\right)}{70d}$$

$$= \frac{(436A + 511B + 504C)\left(a^4 + a^4\cos(c +\right.}{840d}$$

$$= \frac{(436A + 511B + 504C)\left(a^4 + a^4\cos(c +\right.}{840d}$$

$$= \frac{a^4(988A + 1113B + 1232C)\sec^2(c + dx)}{840d}$$

$$= \frac{a^4(988A + 1113B + 1232C)\sec^2(c + dx)}{840d}$$

$$= \frac{a^4(44A + 49B + 56C)\sec(c + dx)\tan(c}{16d}$$

$$= \frac{a^4(44A + 49B + 56C)\tanh^{-1}(\sin(c + dx}{16d}$$

Mathematica [A] time = 3.44124, size = 298, normalized size = 1.04

$$-\frac{a^4(\cos(c + dx) + 1)^4\sec^8\left(\frac{1}{2}(c + dx)\right)\sec^7(c + dx)\left(3360(44A + 49B + 56C)\cos^7(c + dx)\left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^8,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^7*(3360*(44*A +
49*B + 56*C)*Cos[c + d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2*(80384*A + 75264*B + 72016*C + 7
0*(1444*A + 1291*B + 1128*C)*Cos[c + d*x] + 8*(12746*A + 12936*B + 12859*C)
*Cos[2*(c + d*x)] + 35420*A*Cos[3*(c + d*x)] + 37205*B*Cos[3*(c + d*x)] + 3
6120*C*Cos[3*(c + d*x)] + 29056*A*Cos[4*(c + d*x)] + 32256*B*Cos[4*(c + d*x
)] + 35504*C*Cos[4*(c + d*x)] + 4620*A*Cos[5*(c + d*x)] + 5145*B*Cos[5*(c +
 d*x)] + 5880*C*Cos[5*(c + d*x)] + 3632*A*Cos[6*(c + d*x)] + 4032*B*Cos[6*(
```

```
c + d*x)] + 4648*C*Cos[6*(c + d*x)])*Sin[c + d*x]))/(860160*d)
```

Maple [A] time = 0.114, size = 454, normalized size = 1.6

$$\frac{454\,Aa^4\tan(dx+c)}{105\,d}+\frac{Aa^4\tan(dx+c)(\sec(dx+c))^6}{7\,d}+\frac{48\,Aa^4\tan(dx+c)(\sec(dx+c))^4}{35\,d}+\frac{227\,Aa^4\tan(dx+c)(\mathrm{se}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x)
```

```
[Out] 454/105/d*A*a^4*tan(d*x+c)+1/7/d*A*a^4*tan(d*x+c)*sec(d*x+c)^6+48/35/d*A*a^
4*tan(d*x+c)*sec(d*x+c)^4+227/105/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/5/d*a^4
*C*tan(d*x+c)*sec(d*x+c)^4+34/15/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+24/5/d*a^4
*B*tan(d*x+c)+4/5/d*a^4*B*tan(d*x+c)*sec(d*x+c)^4+12/5/d*a^4*B*tan(d*x+c)*s
ec(d*x+c)^2+1/6/d*a^4*B*tan(d*x+c)*sec(d*x+c)^5+41/24/d*a^4*B*tan(d*x+c)*se
c(d*x+c)^3+49/16/d*a^4*B*sec(d*x+c)*tan(d*x+c)+2/3/d*A*a^4*tan(d*x+c)*sec(d
*x+c)^5+11/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+11/4/d*A*a^4*sec(d*x+c)*tan(d*
x+c)+1/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+7/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+49
/16/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+11/4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c)
)+7/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+83/15/d*a^4*C*tan(d*x+c)
```

Maxima [B] time = 1.0551, size = 987, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x
, algorithm="maxima")
```

```
[Out] 1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*A*a^4 + 1344*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*A*a^4 + 1120*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 896*(3*tan(d
*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 4480*(tan(d*x + c)
^3 + 3*tan(d*x + c))*B*a^4 + 224*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15
*tan(d*x + c))*C*a^4 + 6720*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 - 140*A
*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x
+ c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1
) + 15*log(sin(d*x + c) - 1)) - 35*B*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x
```

```
 + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x +
 c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*A*a
^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^
2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 1260*B*a^4*(2
*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1
) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*C*a^4*(2*(3*si
n(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*
log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*B*a^4*(2*sin(d*x + c
)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3
360*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) + 3360*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.09729, size = 617, normalized size = 2.15

$$\frac{105\,(44\,A+49\,B+56\,C)a^4\cos{(dx+c)}^7\log{(\sin{(dx+c)}+1)}-105\,(44\,A+49\,B+56\,C)a^4\cos{(dx+c)}^7\log{(-\sin{(dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x
, algorithm="fricas")
```

```
[Out] 1/3360*(105*(44*A + 49*B + 56*C)*a^4*cos(d*x + c)^7*log(sin(d*x + c) + 1) -
 105*(44*A + 49*B + 56*C)*a^4*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 2*(16
*(454*A + 504*B + 581*C)*a^4*cos(d*x + c)^6 + 105*(44*A + 49*B + 56*C)*a^4*
cos(d*x + c)^5 + 16*(227*A + 252*B + 238*C)*a^4*cos(d*x + c)^4 + 70*(44*A +
 41*B + 24*C)*a^4*cos(d*x + c)^3 + 48*(48*A + 28*B + 7*C)*a^4*cos(d*x + c)^
2 + 280*(4*A + B)*a^4*cos(d*x + c) + 240*A*a^4)*sin(d*x + c))/(d*cos(d*x +
c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
8,x)
```

[Out] Timed out

Giac [A] time = 1.28688, size = 598, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(44*A*a^4 + 49*B*a^4 + 56*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 105*(44*A*a^4 + 49*B*a^4 + 56*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(4620*A*a^4*tan(1/2*d*x + 1/2*c)^13 + 5145*B*a^4*tan(1/2*d*x + 1/2*
c)^13 + 5880*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 30800*A*a^4*tan(1/2*d*x + 1/2*
c)^11 - 34300*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 39200*C*a^4*tan(1/2*d*x + 1/2
*c)^11 + 87164*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 97069*B*a^4*tan(1/2*d*x + 1/2
*c)^9 + 110936*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 135168*A*a^4*tan(1/2*d*x + 1/
2*c)^7 - 150528*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 172032*C*a^4*tan(1/2*d*x + 1
/2*c)^7 + 126084*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 134099*B*a^4*tan(1/2*d*x +
1/2*c)^5 + 159656*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 58800*A*a^4*tan(1/2*d*x +
1/2*c)^3 - 73220*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 86240*C*a^4*tan(1/2*d*x + 1
/2*c)^3 + 22260*A*a^4*tan(1/2*d*x + 1/2*c) + 21735*B*a^4*tan(1/2*d*x + 1/2*
c) + 21000*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.339 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{(3A-4B+4C)\sin^3(c+dx)}{3ad}-\frac{(3A-4B+4C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(4A-4B+5C)\text{s}}{}$$

```
[Out] (3*(4*A - 4*B + 5*C)*x)/(8*a) - ((3*A - 4*B + 4*C)*Sin[c + d*x])/(a*d) + (3
*(4*A - 4*B + 5*C)*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((4*A - 4*B + 5*C)*
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - ((A - B + C)*Cos[c + d*x]^4*Sin[c +
d*x])/(d*(a + a*Cos[c + d*x])) + ((3*A - 4*B + 4*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.23375, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3041, 2748, 2633, 2635, 8}

$$\frac{(3A-4B+4C)\sin^3(c+dx)}{3ad}-\frac{(3A-4B+4C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(4A-4B+5C)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x]),x]
```


```
[Out] (3*(4*A - 4*B + 5*C)*x)/(8*a) - ((3*A - 4*B + 4*C)*Sin[c + d*x])/(a*d) + (3
*(4*A - 4*B + 5*C)*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((4*A - 4*B + 5*C)*
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - ((A - B + C)*Cos[c + d*x]^4*Sin[c +
d*x])/(d*(a + a*Cos[c + d*x])) + ((3*A - 4*B + 4*C)*Sin[c + d*x]^3)/(3*a*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^3(c+dx)(-a(3$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-4B+4C)\int c}{a}$$

$$= \frac{(4A-4B+5C)\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{(A-B+C)\cos^4(}{d(a+a\,c}$$

$$= -\frac{(3A-4B+4C)\sin(c+dx)}{ad}+\frac{3(4A-4B+5C)\cos(c+dx)}{8ad}$$

$$= \frac{3(4A-4B+5C)x}{8a}-\frac{(3A-4B+4C)\sin(c+dx)}{ad}+\frac{3(4A-4}{}$$

Mathematica [B] time = 0.762329, size = 393, normalized size = 2.26

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(72dx(4A-4B+5C)\cos\left(c+\frac{dx}{2}\right)+72dx(4A-4B+5C)\cos\left(\frac{dx}{2}\right)-96A\sin\left(c+\frac{dx}{2}\right)-72A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(72*(4*A - 4*B + 5*C)*d*x*Cos[(d*x)/2] + 72*(4*A
 - 4*B + 5*C)*d*x*Cos[c + (d*x)/2] - 480*A*Sin[(d*x)/2] + 552*B*Sin[(d*x)/2
] - 552*C*Sin[(d*x)/2] - 96*A*Sin[c + (d*x)/2] + 168*B*Sin[c + (d*x)/2] - 1
68*C*Sin[c + (d*x)/2] - 72*A*Sin[c + (3*d*x)/2] + 144*B*Sin[c + (3*d*x)/2]
- 120*C*Sin[c + (3*d*x)/2] - 72*A*Sin[2*c + (3*d*x)/2] + 144*B*Sin[2*c + (3
*d*x)/2] - 120*C*Sin[2*c + (3*d*x)/2] + 24*A*Sin[2*c + (5*d*x)/2] - 16*B*Si
n[2*c + (5*d*x)/2] + 40*C*Sin[2*c + (5*d*x)/2] + 24*A*Sin[3*c + (5*d*x)/2]
- 16*B*Sin[3*c + (5*d*x)/2] + 40*C*Sin[3*c + (5*d*x)/2] + 8*B*Sin[3*c + (7*
d*x)/2] - 5*C*Sin[3*c + (7*d*x)/2] + 8*B*Sin[4*c + (7*d*x)/2] - 5*C*Sin[4*c
 + (7*d*x)/2] + 3*C*Sin[4*c + (9*d*x)/2] + 3*C*Sin[5*c + (9*d*x)/2]))/(192*
a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.039, size = 526, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+
1/2*c)-3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*A-25/4/a/d/(ta
n(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*C+5/a/d/(tan(1/2*d*x+1/2*c)^2+
1)^4*tan(1/2*d*x+1/2*c)^7*B-7/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/
2*c)^5*A-115/12/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*C+31/3/
a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*B-5/a/d/(tan(1/2*d*x+1/
2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*A-109/12/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*ta
n(1/2*d*x+1/2*c)^3*C+25/3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)
^3*B-1/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*A*tan(1/2*d*x+1/2*c)-7/4/a/d/(tan(1/2
*d*x+1/2*c)^2+1)^4*C*tan(1/2*d*x+1/2*c)+3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^4*B*
tan(1/2*d*x+1/2*c)+3/a/d*arctan(tan(1/2*d*x+1/2*c))*A-3/a/d*arctan(tan(1/2*
```

```
d*x+1/2*c))*B+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.54274, size = 709, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -1/12*(C*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 4*B*((9*sin(d*x +
 c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*x + c)/(a*
(cos(d*x + c) + 1))) + 12*A*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.91629, size = 344, normalized size = 1.98

$$\frac{9\,(4\,A-4\,B+5\,C)dx\cos\left(dx+c\right)+9\,(4\,A-4\,B+5\,C)dx+\left(6\,C\cos\left(dx+c\right)^{4}+2\,(4\,B-C)\cos\left(dx+c\right)^{3}+(12\,A-4\,B+\right.}{24\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] 1/24*(9*(4*A - 4*B + 5*C)*d*x*cos(d*x + c) + 9*(4*A - 4*B + 5*C)*d*x + (6*C
*cos(d*x + c)^4 + 2*(4*B - C)*cos(d*x + c)^3 + (12*A - 4*B + 13*C)*cos(d*x
+ c)^2 - (12*A - 28*B + 19*C)*cos(d*x + c) - 48*A + 64*B - 64*C)*sin(d*x +
```

```
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 19.3042, size = 2688, normalized size = 15.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x
)
```

```
[Out] Piecewise((36*A*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*
d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) + 144*A*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) + 216*A*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d
*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*
d*tan(c/2 + d*x/2)**2 + 24*a*d) + 144*A*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan
(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 36*A*d*x/(24*a*d*tan(c/2 + d*x/2)
**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan
(c/2 + d*x/2)**2 + 24*a*d) - 24*A*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x
/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*
tan(c/2 + d*x/2)**2 + 24*a*d) - 168*A*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 +
 d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*
a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 312*A*tan(c/2 + d*x/2)**5/(24*a*d*tan(c
/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 +
 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 216*A*tan(c/2 + d*x/2)**3/(24*a*d*t
an(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)*
*4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 48*A*tan(c/2 + d*x/2)/(24*a*d*t
an(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)*
*4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 36*B*d*x*tan(c/2 + d*x/2)**8/(2
4*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 144*B*d*x*tan(c/2 + d*x/
2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*ta
n(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 216*B*d*x*tan(c/
2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 14
4*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 144*B*d*
x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)
**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) -
36*B*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d
*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 24*B*tan(c/2
```

```
+ d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*
a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 216*B*tan(
c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 +
144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 392*B*
tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**
6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 29
6*B*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/
2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
+ 96*B*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/
2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
+ 45*C*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2
 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 2
4*a*d) + 180*C*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
)**2 + 24*a*d) + 270*C*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8
+ 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2
 + d*x/2)**2 + 24*a*d) + 180*C*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*
x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d
*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*C*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*
a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*
x/2)**2 + 24*a*d) - 24*C*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 +
96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 +
 d*x/2)**2 + 24*a*d) - 246*C*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**
8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c
/2 + d*x/2)**2 + 24*a*d) - 374*C*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/
2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*t
an(c/2 + d*x/2)**2 + 24*a*d) - 314*C*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 +
d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a
*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 66*C*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 +
d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a
*d*tan(c/2 + d*x/2)**2 + 24*a*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2
)*cos(c)**3/(a*cos(c) + a), True))
```

Giac [A] time = 1.17125, size = 336, normalized size = 1.93

$$\frac{\frac{9\,(dx+c)(4\,A-4\,B+5\,C)}{a} - \frac{24\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a} - \frac{2\left(36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-60\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+75\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x,
```

```
algorithm="giac")

[Out] 1/24*(9*(d*x + c)*(4*A - 4*B + 5*C)/a - 24*(A*tan(1/2*d*x + 1/2*c) - B*tan(
1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*(36*A*tan(1/2*d*x + 1/2*c)
^7 - 60*B*tan(1/2*d*x + 1/2*c)^7 + 75*C*tan(1/2*d*x + 1/2*c)^7 + 84*A*tan(1
/2*d*x + 1/2*c)^5 - 124*B*tan(1/2*d*x + 1/2*c)^5 + 115*C*tan(1/2*d*x + 1/2*
c)^5 + 60*A*tan(1/2*d*x + 1/2*c)^3 - 100*B*tan(1/2*d*x + 1/2*c)^3 + 109*C*t
an(1/2*d*x + 1/2*c)^3 + 12*A*tan(1/2*d*x + 1/2*c) - 36*B*tan(1/2*d*x + 1/2*
c) + 21*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.340 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{(3A-3B+4C)\sin^3(c+dx)}{3ad}+\frac{(3A-3B+4C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A-3B+3C)\text{s}}{}$$

```
[Out] -((2*A - 3*B + 3*C)*x)/(2*a) + ((3*A - 3*B + 4*C)*Sin[c + d*x])/(a*d) - ((2
*A - 3*B + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d
*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - ((3*A - 3*B + 4*C)*Sin[c + d
*x]^3)/(3*a*d)
```

Rubi [A] time = 0.211874, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3041, 2748, 2635, 8, 2633}

$$-\frac{(3A-3B+4C)\sin^3(c+dx)}{3ad}+\frac{(3A-3B+4C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A-3B+3C)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x]),x]
```

```
[Out] -((2*A - 3*B + 3*C)*x)/(2*a) + ((3*A - 3*B + 4*C)*Sin[c + d*x])/(a*d) - ((2
*A - 3*B + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d
*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - ((3*A - 3*B + 4*C)*Sin[c + d
*x]^3)/(3*a*d)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx=-\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int\cos^2(c+dx)(-a$$
$$=-\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A-3B+3C)\int}{a}$$
$$=-\frac{(2A-3B+3C)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A-B+C)\cos}{d(a+a}$$
$$=-\frac{(2A-3B+3C)x}{2a}+\frac{(3A-3B+4C)\sin(c+dx)}{ad}-\frac{(2A-}{}$$

Mathematica [B] time = 0.79562, size = 307, normalized size = 2.21

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-12dx(2A-3B+3C)\cos\left(c+\frac{dx}{2}\right)-12dx(2A-3B+3C)\cos\left(\frac{dx}{2}\right)+12A\sin\left(c+\frac{dx}{2}\right)+12A\,s\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-12*(2*A - 3*B + 3*C)*d*x*Cos[(d*x)/2] - 12*(2*
A - 3*B + 3*C)*d*x*Cos[c + (d*x)/2] + 60*A*Sin[(d*x)/2] - 60*B*Sin[(d*x)/2]
 + 69*C*Sin[(d*x)/2] + 12*A*Sin[c + (d*x)/2] - 12*B*Sin[c + (d*x)/2] + 21*C
*Sin[c + (d*x)/2] + 12*A*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 18*C
*Sin[c + (3*d*x)/2] + 12*A*Sin[2*c + (3*d*x)/2] - 9*B*Sin[2*c + (3*d*x)/2]
+ 18*C*Sin[2*c + (3*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] - 2*C*Sin[2*c + (5*d
*x)/2] + 3*B*Sin[3*c + (5*d*x)/2] - 2*C*Sin[3*c + (5*d*x)/2] + C*Sin[3*c +
(7*d*x)/2] + C*Sin[4*c + (7*d*x)/2]))/(24*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.036, size = 420, normalized size = 3.

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\frac{(\tan(1/2\,dx+c/2))^5B}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)^3}+2\frac{A(\tan(1/2\,dx+c/2)}{ad\left((\tan(1/2\,dx+c/2))^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)-3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*B+2/a/d/(tan(1/
2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*A+5/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3
*tan(1/2*d*x+1/2*c)^5*C-4/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)
^3*B+4/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*A+16/3/a/d/(tan(
1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*C-1/a/d/(tan(1/2*d*x+1/2*c)^2+1)
^3*B*tan(1/2*d*x+1/2*c)+2/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*
c)+3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)-2/a/d*arctan(tan(1
/2*d*x+1/2*c))*A+3/a/d*arctan(tan(1/2*d*x+1/2*c))*B-3/a/d*arctan(tan(1/2*d*
x+1/2*c))*C
```

Maple [B] time = 1.48633, size = 540, normalized size = 3.88

$$\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{}{c}\right.}{a}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] 1/3*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*B*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*A*(
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d
*x + c) + 1))))/d
```

Fricas [A] time = 1.94933, size = 288, normalized size = 2.07

$$-\frac{3\,(2\,A-3\,B+3\,C)dx\cos\left(dx+c\right)+3\,(2\,A-3\,B+3\,C)dx-\left(2\,C\cos\left(dx+c\right)^{3}+(3\,B-C)\cos\left(dx+c\right)^{2}+(6\,A-3\,B+}{6\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(2*A - 3*B + 3*C)*d*x*cos(d*x + c) + 3*(2*A - 3*B + 3*C)*d*x - (2*C
*cos(d*x + c)^3 + (3*B - C)*cos(d*x + c)^2 + (6*A - 3*B + 7*C)*cos(d*x + c)
 + 12*A - 12*B + 16*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 11.7455, size = 1739, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x
)
```

```
[Out] Piecewise((-6*A*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d
*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 18*A*d*x*tan(c
/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18
*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 18*A*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan
(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2
+ 6*a*d) - 6*A*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
+ 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*A*tan(c/2 + d*x/2)**7/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 30*A*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(
c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 42*A*tan(c/2 + d*x/
2)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(
c/2 + d*x/2)**2 + 6*a*d) + 18*A*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6
 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*B*d
*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)
**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*B*d*x*tan(c/2 + d*x/2)**4/(6
*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*
x/2)**2 + 6*a*d) + 27*B*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*B*d*
x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2
+ d*x/2)**2 + 6*a*d) - 6*B*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 36*B*ta
n(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 42*B*tan(c/2 + d*x/2)**3/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) - 12*B*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*C*d*x*tan(c/2 + d*x/
2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(
c/2 + d*x/2)**2 + 6*a*d) - 27*C*d*x*tan(c/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*
x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d)
- 27*C*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
+ d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*C*d*x/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a
*d) + 6*C*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 +
 d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*C*tan(c/2 + d*x/2)**5
/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 +
 d*x/2)**2 + 6*a*d) + 50*C*tan(c/2 + d*x/2)**3/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 24*C*ta
n(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18
*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**
2)*cos(c)**2/(a*cos(c) + a), True))
```

Giac [A] time = 1.19232, size = 279, normalized size = 2.01

$$-\frac{\frac{3\,(dx+c)(2\,A-3\,B+3\,C)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(2*A - 3*B + 3*C)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1
/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*(6*A*tan(1/2*d*x + 1/2*c)^5
 - 9*B*tan(1/2*d*x + 1/2*c)^5 + 15*C*tan(1/2*d*x + 1/2*c)^5 + 12*A*tan(1/2*
d*x + 1/2*c)^3 - 12*B*tan(1/2*d*x + 1/2*c)^3 + 16*C*tan(1/2*d*x + 1/2*c)^3
+ 6*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c) + 9*C*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.341 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{(A-2B+2C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(2A-2B+3C)\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x(2A-$$

```
[Out] ((2*A - 2*B + 3*C)*x)/(2*a) - ((A - 2*B + 2*C)*Sin[c + d*x])/(a*d) + ((2*A
 - 2*B + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d*x]
^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.122544, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {3041, 2734}

$$-\frac{(A-2B+2C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(2A-2B+3C)\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x(2A-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d
*x]),x]
```


```
[Out] ((2*A - 2*B + 3*C)*x)/(2*a) - ((A - 2*B + 2*C)*Sin[c + d*x])/(a*d) + ((2*A
 - 2*B + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d*x]
^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos(c+dx)(-a(A$$

$$= \frac{(2A-2B+3C)x}{2a} - \frac{(A-2B+2C)\sin(c+dx)}{ad} + \frac{(2A-2B}{}$$

Mathematica [A] time = 0.465781, size = 213, normalized size = 1.94

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4dx(2A-2B+3C)\cos\left(c+\frac{dx}{2}\right)+4dx(2A-2B+3C)\cos\left(\frac{dx}{2}\right)-16A\sin\left(\frac{dx}{2}\right)+4B\sin\left(c+\frac{d}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(2*A - 2*B + 3*C)*d*x*Cos[(d*x)/2] + 4*(2*A -
 2*B + 3*C)*d*x*Cos[c + (d*x)/2] - 16*A*Sin[(d*x)/2] + 20*B*Sin[(d*x)/2] -
20*C*Sin[(d*x)/2] + 4*B*Sin[c + (d*x)/2] - 4*C*Sin[c + (d*x)/2] + 4*B*Sin[c
 + (3*d*x)/2] - 3*C*Sin[c + (3*d*x)/2] + 4*B*Sin[2*c + (3*d*x)/2] - 3*C*Sin
[2*c + (3*d*x)/2] + C*Sin[2*c + (5*d*x)/2] + C*Sin[3*c + (5*d*x)/2]))/(8*a*
d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.033, size = 248, normalized size = 2.3

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{C\left(\tan(1/2\,dx+c/2)\right)^3}{ad\left(\left(\tan(1/2\,dx+c/2)\right)^2+1\right)^2}+2\,\frac{(\tan(1/2\,dx+c/2}{ad\left((\tan(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+
1/2*c)-3/a/d/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3+2/a/d/(tan(1
/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-1/a/d/(tan(1/2*d*x+1/2*c)^2+1)^
2*C*tan(1/2*d*x+1/2*c)+2/a/d/(tan(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c
)+2/a/d*arctan(tan(1/2*d*x+1/2*c))*A-2/a/d*arctan(tan(1/2*d*x+1/2*c))*B+3/a
/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.50071, size = 369, normalized size = 3.35

$$-\frac{C\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -(C*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - A*(2*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.97678, size = 227, normalized size = 2.06

$$\frac{(2A-2B+3C)dx\cos(dx+c)+(2A-2B+3C)dx+\left(C\cos(dx+c)^2+(2B-C)\cos(dx+c)-2A+4B-4C\right)\sin}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*((2*A - 2*B + 3*C)*d*x*cos(d*x + c) + (2*A - 2*B + 3*C)*d*x + (C*cos(d*
x + c)^2 + (2*B - C)*cos(d*x + c) - 2*A + 4*B - 4*C)*sin(d*x + c))/(a*d*cos
```

```
(d*x + c) + a*d)
```

Sympy [A] time = 6.83402, size = 993, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*A*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*t
an(c/2 + d*x/2)**2 + 2*a*d) + 4*A*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 +
d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*A*d*x/(2*a*d*tan(c/2 + d
*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*A*tan(c/2 + d*x/2)**5/(2*
a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*A*tan(c/2
+ d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d)
 - 2*A*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)
**2 + 2*a*d) - 2*B*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a
*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*B*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*B*d*x/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*B*tan(c/2 + d*x/2)**5
/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 8*B*tan(
c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*
a*d) + 6*B*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*
x/2)**2 + 2*a*d) + 3*C*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 +
 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*C*d*x*tan(c/2 + d*x/2)**2/(2*a*d*ta
n(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*C*d*x/(2*a*d*tan
(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*C*tan(c/2 + d*x/2
)**5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*C
*tan(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2
 + 2*a*d) - 4*C*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2
 + d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*cos(c)/(a
*cos(c) + a), True))
```

Giac [A] time = 1.16703, size = 186, normalized size = 1.69

$$\frac{\frac{(dx+c)(2A-2B+3C)}{a} - \frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} + \frac{2\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((d*x + c)*(2*A - 2*B + 3*C)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*
d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*(2*B*tan(1/2*d*x + 1/2*c)^3 -
3*C*tan(1/2*d*x + 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.342 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{(A-B+C)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(B-C)}{a}+\frac{C\sin(c+dx)}{ad}$$

```
[Out] ((B - C)*x)/a + (C*Sin[c + d*x])/(a*d) + ((A - B + C)*Sin[c + d*x])/(a*d*(1
 + Cos[c + d*x]))
```

Rubi [A] time = 0.112187, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3023, 2735, 2648}

$$\frac{(A-B+C)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(B-C)}{a}+\frac{C\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```


```
[Out] ((B - C)*x)/a + (C*Sin[c + d*x])/(a*d) + ((A - B + C)*Sin[c + d*x])/(a*d*(1
 + Cos[c + d*x]))
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{C\sin(c+dx)}{ad}+\frac{\int \frac{aA+a(B-C)\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}$$
$$=\frac{(B-C)x}{a}+\frac{C\sin(c+dx)}{ad}+(A-B+C)\int \frac{1}{a+a\cos(c+dx)}\,dx$$
$$=\frac{(B-C)x}{a}+\frac{C\sin(c+dx)}{ad}+\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [B] time = 0.280063, size = 136, normalized size = 2.52

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4A\sin\left(\frac{dx}{2}\right)+2dx(B-C)\cos\left(c+\frac{dx}{2}\right)+2dx(B-C)\cos\left(\frac{dx}{2}\right)-4B\sin\left(\frac{dx}{2}\right)+C\sin\left(c+\frac{dx}{2}\right)+C}{2ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(2*(B - C)*d*x*Cos[(d*x)/2] + 2*(B - C)*d*x*Cos[
c + (d*x)/2] + 4*A*Sin[(d*x)/2] - 4*B*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2] + C*S
in[c + (d*x)/2] + C*Sin[c + (3*d*x)/2] + C*Sin[2*c + (3*d*x)/2]))/(2*a*d*(1
 + Cos[c + d*x]))
```

Maple [B] time = 0.027, size = 125, normalized size = 2.3

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{C\tan(1/2\,dx+c/2)}{ad\left((\tan(1/2\,dx+c/2))^2+1\right)}+2\,\frac{\arctan(\tan(1/2\,dx+c/2}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)+2/a/d*C*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/a/d*arctan(tan(
```

```
1/2*d*x+1/2*c))*B-2/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.4741, size = 223, normalized size = 4.13

$$-\frac{C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-\frac{A\sin(dx+c}{a(\cos(dx+c)+}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -(C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))) - A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.90474, size = 153, normalized size = 2.83

$$\frac{(B-C)dx\cos(dx+c)+(B-C)dx+(C\cos(dx+c)+A-B+2C)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] ((B - C)*d*x*cos(d*x + c) + (B - C)*d*x + (C*cos(d*x + c) + A - B + 2*C)*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 3.58236, size = 330, normalized size = 6.11

$$\begin{cases}\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{Bdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{Cdx\tan}{ad\tan^2(} \\ \frac{x\left(A+B\cos(c)+C\cos^2(c)\right)}{a\cos(c)+a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) + A*tan(c/
2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) + B*d*x*tan(c/2 + d*x/2)**2/(a*d
*tan(c/2 + d*x/2)**2 + a*d) + B*d*x/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*tan
(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*tan(c/2 + d*x/2)/(a*d*
tan(c/2 + d*x/2)**2 + a*d) - C*d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2
)**2 + a*d) - C*d*x/(a*d*tan(c/2 + d*x/2)**2 + a*d) + C*tan(c/2 + d*x/2)**3
/(a*d*tan(c/2 + d*x/2)**2 + a*d) + 3*C*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/
2)**2 + a*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)/(a*cos(c) + a), Tr
ue))
```

Giac [A] time = 1.19073, size = 119, normalized size = 2.2

$$\frac{\frac{(dx+c)(B-C)}{a}+\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] ((d*x + c)*(B - C)/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c) + C
*tan(1/2*d*x + 1/2*c))/a + 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 + 1)*a))/d
```

3.343 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{a}$$

```
[Out] (C*x)/a + (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B + C)*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.119369, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3041, 2735, 3770}

$$-\frac{(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d
*x]),x]
```

```
[Out] (C*x)/a + (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B + C)*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int (aA+aC\cos(c+dx))\sec(c+d}{a^2}$$

$$= \frac{Cx}{a}-\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{A\int \sec(c+dx)\,dx}{a}$$

$$= \frac{Cx}{a}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [B] time = 0.540545, size = 163, normalized size = 3.2

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\right.\right.}{ad(\cos(c+dx)+1)(2A+2B\cos(c+dx)+C\cos(2(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos
[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*(Cos[(c + d*x)/
2]*(C*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]]) - (A - B + C)*Sec[c/2]*Sin[(d*x)/2]))/(a*d*(1 + C
os[c + d*x])*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.055, size = 115, normalized size = 2.3

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*A*ln(tan
(1/2*d*x+1/2*c)+1)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)+2/
a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.48401, size = 197, normalized size = 3.86

$$\frac{C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) + A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) + \frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] (C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*si
n(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.97946, size = 247, normalized size = 4.84

$$\frac{2\,Cdx\cos\left(dx+c\right) + 2\,Cdx + \left(A\cos\left(dx+c\right) + A\right)\log\left(\sin\left(dx+c\right)+1\right) - \left(A\cos\left(dx+c\right) + A\right)\log\left(-\sin\left(dx+c\right)+1\right) -}{2\left(ad\cos\left(dx+c\right) + ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*d*x*cos(d*x + c) + 2*C*d*x + (A*cos(d*x + c) + A)*log(sin(d*x + c)
 + 1) - (A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*(A - B + C)*sin(d*x
 + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{C\cos^2(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(C*cos(c + d*x)**2*sec(c + d*x
)/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.19232, size = 124, normalized size = 2.43

$$\frac{\frac{(dx+c)C}{a} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] ((d*x + c)*C/a + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c) +
C*tan(1/2*d*x + 1/2*c))/a)/d
```

3.344 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{(2A-B+C)\tan(c+dx)}{ad}-\frac{(A-B+C)\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] -(((A - B)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A - B + C)*Tan[c + d*x])/(a*
d) - ((A - B + C)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.173298, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3041, 2748, 3767, 8, 3770}

$$\frac{(2A-B+C)\tan(c+dx)}{ad}-\frac{(A-B+C)\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c +
 d*x]),x]
```

```
[Out] -(((A - B)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A - B + C)*Tan[c + d*x])/(a*
d) - ((A - B + C)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int (a(2A-B+C)-a(A-B)\cos}{a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-B)\int \sec(c+dx)\,dx}{a}+\frac{(2A}{}$$

$$= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B+C)\tan(c+dx)}{d(a+a\cos(c+dx))}-$$

$$= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(2A-B+C)\tan(c+dx)}{ad}-$$

Mathematica [B] time = 1.30215, size = 256, normalized size = 3.61

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(\sec\left(\frac{c}{2}\right)(A-B+C)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((A\right.\right.}{ad(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*C
os[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*
((A - B + C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((A - B)*(Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) +
 (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a*d*(1 + C
os[c + d*x])*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.062, size = 180, normalized size = 2.5

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{A}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{B}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d
*B*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1/a/d*A*ln(tan(1
/2*d*x+1/2*c)+1)+1/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.04874, size = 294, normalized size = 4.14

$$-\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\right.}{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - C*sin(d*x + c)/(a*(cos(d*x
+ c) + 1)))/d
```

Fricas [A] time = 1.88036, size = 325, normalized size = 4.58

$$-\frac{\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1)
 - ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
 2*((2*A - B + C)*cos(d*x + c) + A)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d
*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{C\cos^2(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c)),x
)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(C*cos(c + d*x)**2*sec(c
 + d*x)**2/(cos(c + d*x) + 1), x))/a
```

Giac [A] time = 1.24518, size = 163, normalized size = 2.3

$$-\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] -((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c) + C*t
an(1/2*d*x + 1/2*c))/a + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
- 1)*a))/d
```

3.345 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{(2A-2B+C)\tan(c+dx)}{ad}+\frac{(3A-2B+2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B+2C)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B$$

```
[Out] ((3*A - 2*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((2*A - 2*B + C)*Tan[c
+ d*x])/(a*d) + ((3*A - 2*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A
 - B + C)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.201625, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2748, 3768, 3770, 3767, 8}

$$-\frac{(2A-2B+C)\tan(c+dx)}{ad}+\frac{(3A-2B+2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B+2C)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c +
 d*x]),x]
```

```
[Out] ((3*A - 2*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((2*A - 2*B + C)*Tan[c
+ d*x])/(a*d) + ((3*A - 2*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A
 - B + C)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int(a(3A-2B+2C$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(2A-2B+C)\int \mathrm{se}}{a}$$

$$= \frac{(3A-2B+2C)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B+C)\sec(}{d(a+a\,\mathrm{c}}$$

$$= \frac{(3A-2B+2C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(2A-2B+C)\tan(}{ad}$$

Mathematica [B] time = 1.44026, size = 256, normalized size = 2.19

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-4(A-B+C)\tan\left(\frac{1}{2}(c+dx)\right)-2(3A-2B+2C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(3A-2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*C
os[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(-2*(3*A - 2*B + 2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 2*(3*A - 2*B + 2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + A/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*(-A + B)*Sin[(c + d*x)/2])/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]) - A/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^2 + (4*(-A + B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) -
 4*(A - B + C)*Tan[(c + d*x)/2]))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.066, size = 311, normalized size = 2.7

$$-\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3\,A}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+
1/2*c)+1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2+3/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)-
1/a/d*B/(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*B*l
n(tan(1/2*d*x+1/2*c)-1)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-1/2/a/d*A/(tan(1/2
*d*x+1/2*c)+1)^2+3/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1/a/d*B/(tan(1/2*d*x+1/2*
c)+1)+3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)+1
/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [B] time = 1.01616, size = 481, normalized size = 4.11

$$-\frac{A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(d}{\cos(dx}\right.}{a}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -1/2*(A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 2
*C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 1.96998, size = 428, normalized size = 3.66

$$\frac{\left((3A-2B+2C)\cos(dx+c)^3+(3A-2B+2C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((3A-2B+2C)\cos(dx+c\right.}{4\left(ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] 1/4*(((3*A - 2*B + 2*C)*cos(d*x + c)^3 + (3*A - 2*B + 2*C)*cos(d*x + c)^2)*
log(sin(d*x + c) + 1) - ((3*A - 2*B + 2*C)*cos(d*x + c)^3 + (3*A - 2*B + 2*
C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(2*(2*A - 2*B + C)*cos(d*x +
c)^2 + (A - 2*B)*cos(d*x + c) - A)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.22098, size = 235, normalized size = 2.01

$$\frac{\frac{(3A-2B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(3A-2B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3A\tan \cdots\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/2*((3*A - 2*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*A - 2*B +
2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*t
an(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*(3*A*tan(1/2*d*x + 1/2*
c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*
x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.346 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{(4A-3B+3C)\tan^3(c+dx)}{3ad}+\frac{(4A-3B+3C)\tan(c+dx)}{ad}-\frac{(3A-3B+2C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(3A-3B+2C)}{}$$

```
[Out] -((3*A - 3*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A - 3*B + 3*C)*Tan
[c + d*x])/(a*d) - ((3*A - 3*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) -
((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A
- 3*B + 3*C)*Tan[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.216843, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3041, 2748, 3767, 3768, 3770}

$$\frac{(4A-3B+3C)\tan^3(c+dx)}{3ad}+\frac{(4A-3B+3C)\tan(c+dx)}{ad}-\frac{(3A-3B+2C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(3A-3B+2C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c +
 d*x]),x]
```

```
[Out] -((3*A - 3*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A - 3*B + 3*C)*Tan
[c + d*x])/(a*d) - ((3*A - 3*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) -
((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A
- 3*B + 3*C)*Tan[c + d*x]^3)/(3*a*d)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int(a(4A-3B+3C)$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-3B+2C)\int s}{a}$$

$$= -\frac{(3A-3B+2C)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B+C)\sec^2}{d(a+a}$$

$$= -\frac{(3A-3B+2C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(4A-3B+3C)\tan}{ad}$$

Mathematica [B] time = 3.87529, size = 351, normalized size = 2.37

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(12(A-B+C)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{4(5A-3B+3C)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4(5A-3B+3C)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}+6(3A-3B+2C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*C
os[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(6*(3*A - 3*B + 2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 6*(3*A - 3*B + 2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (-2
*A + 3*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*A*Sin[(c + d*x)/2])/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (4*(5*A - 3*B + 3*C)*Sin[(c + d*x
)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*A*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3 + (2*A - 3*B)/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (4*(5*A - 3*B + 3*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]) + 12*(A - B + C)*Tan[(c + d*x)/2]))/(6*a*d*(1 + Cos[c + d*
x]))
```

Maple [B] time = 0.07, size = 442, normalized size = 3.

$$\frac{A}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{A}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)-1/3/a/d*A/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2+1
/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)^2+3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-3/2/a/
d*B*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-5/2/a/d*A/(ta
n(1/2*d*x+1/2*c)-1)+3/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)-1/a/d/(tan(1/2*d*x+1/2
*c)-1)*C-1/3/a/d*A/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d*A/(tan(1/2*d*x+1/2*c)+1)^
2-1/2/a/d*B/(tan(1/2*d*x+1/2*c)+1)^2-3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)+3/2
/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C-5/2/a/d*A/
(tan(1/2*d*x+1/2*c)+1)+3/2/a/d*B/(tan(1/2*d*x+1/2*c)+1)-1/a/d/(tan(1/2*d*x+
1/2*c)+1)*C
```

Maxima [B] time = 1.03175, size = 655, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] 1/6*(A*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - 6*C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x +
 c) + 1))))/d
```

Fricas [A] time = 1.99522, size = 489, normalized size = 3.3

$$-\frac{3\left((3A-3B+2C)\cos(dx+c)^4+(3A-3B+2C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((3A-3B+2C)\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -1/12*(3*((3*A - 3*B + 2*C)*cos(d*x + c)^4 + (3*A - 3*B + 2*C)*cos(d*x + c)
^3)*log(sin(d*x + c) + 1) - 3*((3*A - 3*B + 2*C)*cos(d*x + c)^4 + (3*A - 3*
B + 2*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(4*(4*A - 3*B + 3*C)*co
s(d*x + c)^3 + (7*A - 3*B + 6*C)*cos(d*x + c)^2 - (A - 3*B)*cos(d*x + c) +
2*A)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.24097, size = 328, normalized size = 2.22

$$-\frac{\frac{3(3A-3B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3(3A-3B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2(\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] -1/6*(3*(3*A - 3*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*(3*A - 3
*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c)
- B*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*(15*A*tan(1/2*d*x
+ 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 - 16*A
*tan(1/2*d*x + 1/2*c)^3 + 12*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x +
1/2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.347 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$-\frac{(5A-8B+12C)\sin^3(c+dx)}{3a^2d}+\frac{(5A-8B+12C)\sin(c+dx)}{a^2d}-\frac{(4A-7B+10C)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B}{}$$

```
[Out] -((4*A - 7*B + 10*C)*x)/(2*a^2) + ((5*A - 8*B + 12*C)*Sin[c + d*x])/(a^2*d)
 - ((4*A - 7*B + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((4*A - 7*B +
 10*C)*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B
+ C)*Cos[c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) - ((5*A - 8*
B + 12*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.379138, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2977, 2748, 2635, 8, 2633}

$$-\frac{(5A-8B+12C)\sin^3(c+dx)}{3a^2d}+\frac{(5A-8B+12C)\sin(c+dx)}{a^2d}-\frac{(4A-7B+10C)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^2,x]
```


```
[Out] -((4*A - 7*B + 10*C)*x)/(2*a^2) + ((5*A - 8*B + 12*C)*Sin[c + d*x])/(a^2*d)
 - ((4*A - 7*B + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((4*A - 7*B +
 10*C)*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B
+ C)*Cos[c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) - ((5*A - 8*
B + 12*C)*Sin[c + d*x]^3)/(3*a^2*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(-a(A-4B+}{a+a}}{}$$

$$= -\frac{(4A-7B+10C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos}{3d(a+}$$

$$= -\frac{(4A-7B+10C)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos}{3d(a+}$$

$$= -\frac{(4A-7B+10C)\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{(4A-7B+10C}{3a^2d(}$$

$$= -\frac{(4A-7B+10C)x}{2a^2} + \frac{(5A-8B+12C)\sin(c+dx)}{a^2d} - \frac{(4A-}{}$$

Mathematica [B] time = 0.915014, size = 481, normalized size = 2.6

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(4A-7B+10C)\cos\left(c+\frac{dx}{2}\right)-36dx(4A-7B+10C)\cos\left(\frac{dx}{2}\right)-120A\sin\left(c+\frac{dx}{2}\right)+164\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(4*A - 7*B + 10*C)*d*x*Cos[(d*x)/2] - 36*(4
*A - 7*B + 10*C)*d*x*Cos[c + (d*x)/2] - 48*A*d*x*Cos[c + (3*d*x)/2] + 84*B*
d*x*Cos[c + (3*d*x)/2] - 120*C*d*x*Cos[c + (3*d*x)/2] - 48*A*d*x*Cos[2*c +
(3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] - 120*C*d*x*Cos[2*c + (3*d*x)/2]
 + 264*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] + 516*C*Sin[(d*x)/2] - 120*A*Sin
[c + (d*x)/2] + 147*B*Sin[c + (d*x)/2] - 156*C*Sin[c + (d*x)/2] + 164*A*Sin
[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 342*C*Sin[c + (3*d*x)/2] + 36*
A*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 118*C*Sin[2*c + (3*d*x
)/2] + 12*A*Sin[2*c + (5*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*C*Sin[2*c
 + (5*d*x)/2] + 12*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[3*c + (5*d*x)/2] + 30*
C*Sin[3*c + (5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] - 3*C*Sin[3*c + (7*d*x)/2
] + 3*B*Sin[4*c + (7*d*x)/2] - 3*C*Sin[4*c + (7*d*x)/2] + C*Sin[4*c + (9*d*
x)/2] + C*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.036, size = 482, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^
2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2
*d*x+1/2*c)+9/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)
^3*tan(1/2*d*x+1/2*c)^5*A-5/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/
2*c)^5*B+10/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*C+4/d/a^2
/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*A-8/d/a^2/(tan(1/2*d*x+1/2
*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*B+40/3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*C*t
an(1/2*d*x+1/2*c)^3+2/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*c)
-3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^3*B*tan(1/2*d*x+1/2*c)+6/d/a^2/(tan(1/2*d
*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A+
7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B-10/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.47181, size = 657, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] 1/6*(C*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2) + A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2 + 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 2.01982, size = 437, normalized size = 2.36

$$-\frac{3\,(4\,A-7\,B+10\,C)dx\cos{(dx+c)}^2+6\,(4\,A-7\,B+10\,C)dx\cos{(dx+c)}+3\,(4\,A-7\,B+10\,C)dx-\left(2\,C\cos{(dx+c)}^4\right.}{6\left(a^2d\cos{(dx+c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] -1/6*(3*(4*A - 7*B + 10*C)*d*x*cos(d*x + c)^2 + 6*(4*A - 7*B + 10*C)*d*x*co
s(d*x + c) + 3*(4*A - 7*B + 10*C)*d*x - (2*C*cos(d*x + c)^4 + (3*B - 2*C)*c
os(d*x + c)^3 + 6*(A - B + 2*C)*cos(d*x + c)^2 + (28*A - 43*B + 66*C)*cos(d
*x + c) + 20*A - 32*B + 48*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 30.9776, size = 2134, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
2,x)
```

```
[Out] Piecewise((-12*A*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
6*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 36*A*d*x*tan(c
/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*A*d*x/(6*a**2*d*tan(c/2
+ d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) - A*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**
2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 12*A*
tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x
/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 54*A*tan(c/2 + d*x/2)*
*5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*
d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 68*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(
c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) + 27*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 2
```

```
1*B*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*B*d*x*tan(c
/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*B*d*x*tan(c/2 + d*x/2)**
2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
+ B*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 18*B*tan(c/2 + d*x
/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*tan(c/2 + d*x/2)**5/(6*a**2*d*
tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d
*x/2)**2 + 6*a**2*d) - 110*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)
**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - 39*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(
c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*C*d*x*tan(
c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)*
*4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*C*d*x*tan(c/2 + d*x/2)*
*4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*
d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*C*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*
tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d
*x/2)**2 + 6*a**2*d) - 30*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*t
an(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - C*tan(c/2
+ d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 +
 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*C*tan(c/2 + d*x/2)**7/(6*a*
*2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/
2 + d*x/2)**2 + 6*a**2*d) + 138*C*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d
*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 +
6*a**2*d) + 160*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a*
*2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*C
*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2
)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(A + B*cos(
c) + C*cos(c)**2)*cos(c)**3/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.15369, size = 359, normalized size = 1.94

$$-\frac{\frac{3\,(dx+c)(4\,A-7\,B+10\,C)}{a^2}-\frac{2\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,C\tan\left(\frac{1}{2}\,d\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(4*A - 7*B + 10*C)/a^2 - 2*(6*A*tan(1/2*d*x + 1/2*c)^5 -
15*B*tan(1/2*d*x + 1/2*c)^5 + 30*C*tan(1/2*d*x + 1/2*c)^5 + 12*A*tan(1/2*d*
x + 1/2*c)^3 - 24*B*tan(1/2*d*x + 1/2*c)^3 + 40*C*tan(1/2*d*x + 1/2*c)^3 +
6*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c) + 18*C*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3
- B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*ta
n(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2*d*x + 1/2*c) - 27*C*a^4*tan(1/2*d*x +
 1/2*c))/a^6)/d
```

3.348 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$-\frac{2(2A-5B+8C)\sin(c+dx)}{3a^2d}-\frac{(2A-5B+8C)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(2A-4B+7C)\sin(c+dx)\cos(c+dx)}{2a^2d}+$$

```
[Out] ((2*A - 4*B + 7*C)*x)/(2*a^2) - (2*(2*A - 5*B + 8*C)*Sin[c + d*x])/(3*a^2*d
) + ((2*A - 4*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((2*A - 5*B +
 8*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B +
 C)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.316792, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3041, 2977, 2734}

$$-\frac{2(2A-5B+8C)\sin(c+dx)}{3a^2d}-\frac{(2A-5B+8C)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(2A-4B+7C)\sin(c+dx)\cos(c+dx)}{2a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^2,x]
```

```
[Out] ((2*A - 4*B + 7*C)*x)/(2*a^2) - (2*(2*A - 5*B + 8*C)*Sin[c + d*x])/(3*a^2*d
) + ((2*A - 4*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((2*A - 5*B +
 8*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B +
 C)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(3a(B-C)+\cdots}{a+a\,\mathrm{c}\cdots}}{3\cdots}$$
$$= -\frac{(2A-5B+8C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos\cdots}{3d(a+a\cdots}$$
$$= \frac{(2A-4B+7C)x}{2a^2} - \frac{2(2A-5B+8C)\sin(c+dx)}{3a^2d} + \frac{(2A-4\cdots}{\cdots}$$

Mathematica [B] time = 0.844568, size = 385, normalized size = 2.41

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(2A-4B+7C)\cos\left(c+\frac{dx}{2}\right)+36dx(2A-4B+7C)\cos\left(\frac{dx}{2}\right)+96A\sin\left(c+\frac{dx}{2}\right)-80A\sin\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(2*A - 4*B + 7*C)*d*x*Cos[(d*x)/2] + 36*(2*A
 - 4*B + 7*C)*d*x*Cos[c + (d*x)/2] + 24*A*d*x*Cos[c + (3*d*x)/2] - 48*B*d*x
*Cos[c + (3*d*x)/2] + 84*C*d*x*Cos[c + (3*d*x)/2] + 24*A*d*x*Cos[2*c + (3*d
*x)/2] - 48*B*d*x*Cos[2*c + (3*d*x)/2] + 84*C*d*x*Cos[2*c + (3*d*x)/2] - 14
4*A*Sin[(d*x)/2] + 264*B*Sin[(d*x)/2] - 381*C*Sin[(d*x)/2] + 96*A*Sin[c + (
d*x)/2] - 120*B*Sin[c + (d*x)/2] + 147*C*Sin[c + (d*x)/2] - 80*A*Sin[c + (3
*d*x)/2] + 164*B*Sin[c + (3*d*x)/2] - 239*C*Sin[c + (3*d*x)/2] + 36*B*Sin[2
*c + (3*d*x)/2] - 63*C*Sin[2*c + (3*d*x)/2] + 12*B*Sin[2*c + (5*d*x)/2] - 1
5*C*Sin[2*c + (5*d*x)/2] + 12*B*Sin[3*c + (5*d*x)/2] - 15*C*Sin[3*c + (5*d*
x)/2] + 3*C*Sin[3*c + (7*d*x)/2] + 3*C*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1
+ Cos[c + d*x])^2)
```

Maple [B] time = 0.033, size = 309, normalized size = 1.9

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*
d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^
2*C*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2
*c)^3*B-3/d/a^2/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2/(ta
n(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2
*c))*A-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B+7/d/a^2*arctan(tan(1/2*d*x+1/2*
c))*C
```

Maxima [B] time = 1.5328, size = 475, normalized size = 2.97

$$-\frac{C\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(\ldots}{\cos(\ldots}\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] -1/6*(C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))) + A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 1.91712, size = 383, normalized size = 2.39

$$\frac{3\,(2\,A-4\,B+7\,C)dx\cos\left(dx+c\right)^{2}+6\,(2\,A-4\,B+7\,C)dx\cos\left(dx+c\right)+3\,(2\,A-4\,B+7\,C)dx+\left(3\,C\cos\left(dx+c\right)^{3}+6\,(\cdots\right.}{6\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A - 4*B + 7*C)*d*x*cos(d*x + c)^2 + 6*(2*A - 4*B + 7*C)*d*x*cos(d
*x + c) + 3*(2*A - 4*B + 7*C)*d*x + (3*C*cos(d*x + c)^3 + 6*(B - C)*cos(d*x
 + c)^2 - (10*A - 28*B + 43*C)*cos(d*x + c) - 8*A + 20*B - 32*C)*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 20.0107, size = 1261, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
2,x)
```

```
[Out] Piecewise((6*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 12*A*d*x*tan(c/2 + d*x/2)**2/(6*a**
```

```
2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 6*A*d
*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d
) + A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2
 + d*x/2)**2 + 6*a**2*d) - 7*A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/
2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 17*A*tan(c/2 + d*x/2)**
3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
 - 9*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) - 12*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d
*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 24*B*d*x*tan(c/2 + d
*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*
a**2*d) - 12*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/
2)**2 + 6*a**2*d) - B*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 1
2*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 13*B*tan(c/2 + d*x/2)**5/(6*a**2
*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 41*B*t
an(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/
2)**2 + 6*a**2*d) + 27*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 1
2*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*C*d*x*tan(c/2 + d*x/2)**4/(6*
a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42
*C*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/
2 + d*x/2)**2 + 6*a**2*d) + 21*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**
2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + C*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c
/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*C*tan(c/2 +
 d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 +
6*a**2*d) - 71*C*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**
2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 39*C*tan(c/2 + d*x/2)/(6*a**2*d*tan(c
/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(
A + B*cos(c) + C*cos(c)**2)*cos(c)**2/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.15979, size = 267, normalized size = 1.67

$$\frac{\frac{3\,(dx+c)(2\,A-4\,B+7\,C)}{a^2}+\frac{6\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*(2*A - 4*B + 7*C)/a^2 + 6*(2*B*tan(1/2*d*x + 1/2*c)^3 - 5*
C*tan(1/2*d*x + 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) - 3*C*tan(1/2*d*x + 1/2
```

```
*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
 B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^4*tan(
1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c) - 21*C*a^4*tan(1/2*d*x + 1
/2*c))/a^6)/d
```

3.349 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=103

$$\frac{(A-B+4C)\sin(c+dx)}{3a^2d}-\frac{(B-2C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(B-2C)}{a^2}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ((B - 2*C)*x)/a^2 + ((A - B + 4*C)*Sin[c + d*x])/(3*a^2*d) - ((B - 2*C)*Sin
[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.258534, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3041, 2968, 3023, 12, 2735, 2648}

$$\frac{(A-B+4C)\sin(c+dx)}{3a^2d}-\frac{(B-2C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(B-2C)}{a^2}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d
*x])^2,x]
```

```
[Out] ((B - 2*C)*x)/a^2 + ((A - B + 4*C)*Sin[c + d*x])/(3*a^2*d) - ((B - 2*C)*Sin
[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(a(A+2B-2\cdots}{a+a\cdots}}{\cdots}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{a(A+2B-2C)\cos(c+d\cdots}{a+a\cdots}}{\cdots}$$

$$= \frac{(A-B+4C)\sin(c+dx)}{3a^2d} - \frac{(A-B+C)\cos^2(c+dx)\sin(c+\cdots}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B+4C)\sin(c+dx)}{3a^2d} - \frac{(A-B+C)\cos^2(c+dx)\sin(c+\cdots}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(B-2C)x}{a^2} + \frac{(A-B+4C)\sin(c+dx)}{3a^2d} - \frac{(A-B+C)\cos^2(c\cdots}{3d(a+a\cos\cdots}$$

$$= \frac{(B-2C)x}{a^2} + \frac{(A-B+4C)\sin(c+dx)}{3a^2d} - \frac{(A-B+C)\cos^2(c\cdots}{3d(a+a\cos\cdots}$$

Mathematica [B] time = 0.67245, size = 275, normalized size = 2.67

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-12A\sin\left(c+\frac{dx}{2}\right)+8A\sin\left(c+\frac{3dx}{2}\right)+12A\sin\left(\frac{dx}{2}\right)+18dx(B-2C)\cos\left(c+\frac{dx}{2}\right)+24B\sin\left(c\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(18*(B - 2*C)*d*x*Cos[(d*x)/2] + 18*(B - 2*C)*d*
x*Cos[c + (d*x)/2] + 6*B*d*x*Cos[c + (3*d*x)/2] - 12*C*d*x*Cos[c + (3*d*x)/
2] + 6*B*d*x*Cos[2*c + (3*d*x)/2] - 12*C*d*x*Cos[2*c + (3*d*x)/2] + 12*A*Si
n[(d*x)/2] - 36*B*Sin[(d*x)/2] + 66*C*Sin[(d*x)/2] - 12*A*Sin[c + (d*x)/2]
+ 24*B*Sin[c + (d*x)/2] - 30*C*Sin[c + (d*x)/2] + 8*A*Sin[c + (3*d*x)/2] -
20*B*Sin[c + (3*d*x)/2] + 41*C*Sin[c + (3*d*x)/2] + 9*C*Sin[2*c + (3*d*x)/2
] + 3*C*Sin[2*c + (5*d*x)/2] + 3*C*Sin[3*c + (5*d*x)/2]))/(12*a^2*d*(1 + Co
s[c + d*x])^2)
```

Maple [A] time = 0.033, size = 187, normalized size = 1.8

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^
2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2
*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*C*tan(1/2*d*x+1/2*c)/(ta
n(1/2*d*x+1/2*c)^2+1)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B-4/d/a^2*arctan(t
an(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.54286, size = 317, normalized size = 3.08

$$\frac{C\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c}{\cos(dx+c}\right.}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(C*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) + A*(3*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.94044, size = 308, normalized size = 2.99

$$\frac{3(B-2C)dx\cos(dx+c)^2+6(B-2C)dx\cos(dx+c)+3(B-2C)dx+\left(3C\cos(dx+c)^2+(2A-5B+14C)\cos(dx\right.}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/3*(3*(B - 2*C)*d*x*cos(d*x + c)^2 + 6*(B - 2*C)*d*x*cos(d*x + c) + 3*(B -
 2*C)*d*x + (3*C*cos(d*x + c)^2 + (2*A - 5*B + 14*C)*cos(d*x + c) + A - 4*B
 + 10*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)
```

Sympy [A] time = 11.2336, size = 536, normalized size = 5.2

$$\begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{2A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{3A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{6Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{6Bdx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{B\,\mathrm{ta}}{6a^2d\,\mathrm{tan}} \\ \frac{x\left(A+B\cos(c)+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x
)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
 + 2*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 3*A*
tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 6*B*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 6*B*d*x/(6*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) + B*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2
+ d*x/2)**2 + 6*a**2*d) - 8*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) - 9*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a
**2*d) - 12*C*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - 12*C*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - C*tan(c/2 + d*x
/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 14*C*tan(c/2 + d*x/2)**3
/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*C*tan(c/2 + d*x/2)/(6*a**2*
d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**
2)*cos(c)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.17297, size = 204, normalized size = 1.98

$$\frac{\frac{6\,(dx+c)(B-2\,C)}{a^2} + \frac{12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2} - \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*(B - 2*C)/a^2 + 12*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/
2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*
a^4*tan(1/2*d*x + 1/2*c) - 15*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.350 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$\frac{(A+2B-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Cx}{a^2}+\frac{(A-B+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (C*x)/a^2 + ((A + 2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + (
(A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.119789, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3019, 2735, 2648}

$$\frac{(A+2B-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Cx}{a^2}+\frac{(A-B+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (C*x)/a^2 + ((A + 2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + (
(A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-a(A+2B-2C)-3aC\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= \frac{Cx}{a^2} + \frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A+2B-5C)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= \frac{Cx}{a^2} + \frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A+2B-5C)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [B] time = 0.406985, size = 175, normalized size = 2.43

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(2A\sin\left(c+\frac{3dx}{2}\right)+6A\sin\left(\frac{dx}{2}\right)-6B\sin\left(c+\frac{dx}{2}\right)+4B\sin\left(c+\frac{3dx}{2}\right)+6B\sin\left(\frac{dx}{2}\right)+12C\sin\left(c\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*C*d*x*Cos[(d*x)/2] + 9*C*d*x*Cos[c + (d*x)/
2] + 3*C*d*x*Cos[c + (3*d*x)/2] + 3*C*d*x*Cos[2*c + (3*d*x)/2] + 6*A*Sin[(d
*x)/2] + 6*B*Sin[(d*x)/2] - 18*C*Sin[(d*x)/2] - 6*B*Sin[c + (d*x)/2] + 12*C
*Sin[c + (d*x)/2] + 2*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2] - 10*C*
Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.026, size = 135, normalized size = 1.9

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*B*tan(1/2*
d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c)
)*C
```

Maxima [B] time = 1.46162, size = 221, normalized size = 3.07

$$-\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/6*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - A*(3*sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - B*(3
*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2
)/d
```

Fricas [A] time = 1.88977, size = 242, normalized size = 3.36

$$\frac{3\,Cdx\cos(dx+c)^2+6\,Cdx\cos(dx+c)+3\,Cdx+((A+2\,B-5\,C)\cos(dx+c)+2\,A+B-4\,C)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/3*(3*C*d*x*cos(d*x + c)^2 + 6*C*d*x*cos(d*x + c) + 3*C*d*x + ((A + 2*B -
5*C)*cos(d*x + c) + 2*A + B - 4*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 5.7179, size = 148, normalized size = 2.06

$$\begin{cases} \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}+\frac{Cx}{a^2}+\frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x\left(A+B\cos(c)+C\cos^2(c)\right)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d)
 - B*tan(c/2 + d*x/2)**3/(6*a**2*d) + B*tan(c/2 + d*x/2)/(2*a**2*d) + C*x/a
**2 + C*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*C*tan(c/2 + d*x/2)/(2*a**2*d), N
e(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.15094, size = 157, normalized size = 2.18

$$\frac{\frac{6(dx+c)C}{a^2}+\frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/6*(6*(d*x + c)*C/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) +
3*B*a^4*tan(1/2*d*x + 1/2*c) - 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.351 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{(4A-B-2C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*A - B - 2*C)*Sin[c + d*x])/(3*a^2*d
*(1 + Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rubi [A] time = 0.214822, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3041, 2978, 12, 3770}

$$-\frac{(4A-B-2C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B+C)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d
*x])^2,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*A - B - 2*C)*Sin[c + d*x])/(3*a^2*d
*(1 + Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3aA-a(A-B-2C)\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(4A-B-2C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int 3a}{}$$

$$= -\frac{(4A-B-2C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{A\int}{}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(4A-B-2C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B}{3d(a+}$$

Mathematica [B] time = 0.849884, size = 221, normalized size = 2.66

$$-\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\left(\tan\left(\frac{c}{2}\right)(A-B+C)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)(A-B+C)\sin\right.}{3a^2d(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos
[c + d*x])^2,x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*(6*A*Cos[(c +
d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + (A - B + C)*Sec[c/2]*Sin[(d*x)/2] + 2*(4*A - B - 2*C
)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A - B + C)*Cos[(c + d*x)/2]*T
an[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2*(2*A + C + 2*B*Cos[c + d*x] + C*Cos
[2*(c + d*x)]))
```

Maple [A] time = 0.057, size = 157, normalized size = 1.9

$$\frac{B}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,A}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{d a^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{d a^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{A}{6\,d a^2}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*A*tan(1/2*d*x+1/2*c)+1/d/a^2*A*ln(
tan(1/2*d*x+1/2*c)+1)-1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)-1/6/d/a^2*tan(1/2*
d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2
*c)^3+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 0.996903, size = 257, normalized size = 3.1

$$-\frac{A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{B\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}-\frac{C\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) + 1)
```

```
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - C*(3*sin(d*x + c)/(cos(d*x + c
) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 2.0472, size = 351, normalized size = 4.23

$$\frac{3\left(A\cos\left(dx+c\right)^2+2\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(A\cos\left(dx+c\right)^2+2\,A\cos\left(dx+c\right)+A\right)\log\left(-\sin\left(d\right.\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 3*
(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*((4*A
- B - 2*C)*cos(d*x + c) + 5*A - 2*B - C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec\left(c+dx\right)}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx+\int \frac{C\cos^2\left(c+dx\right)\sec\left(c+dx\right)}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**2,x
)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + Integ
ral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)
+ Integral(C*cos(c + d*x)**2*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x)
 + 1), x))/a**2
```

Giac [A] time = 1.1866, size = 194, normalized size = 2.34

$$\frac{\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ba^4}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*A*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 3*B*a
^4*tan(1/2*d*x + 1/2*c) - 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.352 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{(10A-4B+C)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B+C)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -(((2*A - B)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + ((10*A - 4*B + C)*Tan[c + d*
x])/(3*a^2*d) - ((2*A - B)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A -
 B + C)*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.34481, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2748, 3767, 8, 3770}

$$\frac{(10A-4B+C)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B+C)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c +
 d*x])^2,x]
```

```
[Out] -(((2*A - B)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + ((10*A - 4*B + C)*Tan[c + d*
x])/(3*a^2*d) - ((2*A - B)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A -
 B + C)*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(4A-B+C)-a(2A-2B-C)\cos(c+dx))}{a+a\cos(c+dx)}}{3a^2}$$
$$= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(a^2(1\right.}{}$$
$$= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(2A-B}{}$$
$$= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A}{3a}$$
$$= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{(10A-4B+C)\tan(c+dx}{3a^2d}$$

Mathematica [B] time = 1.88892, size = 321, normalized size = 2.94

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(\tan\left(\frac{c}{2}\right)(A-B+C)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)(A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*C
os[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]*Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*
((A - B + C)*Sec[c/2]*Sin[(d*x)/2] + 2*(7*A - 4*B + C)*Cos[(c + d*x)/2]^2*S
ec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((2*A - B)*(Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A*Sin[
d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (A - B + C)*Cos[(c
+ d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2*(2*A + C + 2*B*Cos[c + d
*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.065, size = 243, normalized size = 2.2

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{5\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{3\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*
d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1
)-1/d/a^2*B*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)-2/d/a
^2*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a^2*A/
(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.02929, size = 387, normalized size = 3.55

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c}{(\cos(dx+c)}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2) + C*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/a^2)/d
```

Fricas [A] time = 1.97958, size = 514, normalized size = 4.72

$$-\frac{3\left((2A-B)\cos(dx+c)^3+2(2A-B)\cos(dx+c)^2+(2A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((2A-B)\cos\right.}{6(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] -1/6*(3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A - B)*cos(d*x + c)^2 + (2*A - B)*
cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A
- B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*((
10*A - 4*B + C)*cos(d*x + c)^2 + (14*A - 5*B + 2*C)*cos(d*x + c) + 3*A)*sin
(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20588, size = 251, normalized size = 2.3

$$-\frac{\frac{6\,(2\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,(2\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(6*(2*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*A - B)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) -
 9*B*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.353 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$-\frac{2(8A-5B+2C)\tan(c+dx)}{3a^2d}+\frac{(7A-4B+2C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B+2C)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8$$

```
[Out] ((7*A - 4*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*A - 5*B + 2*C)*
Tan[c + d*x])/(3*a^2*d) + ((7*A - 4*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((8*A - 5*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c
+ d*x])) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x]
)^2)
```

Rubi [A] time = 0.360768, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3041, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(8A-5B+2C)\tan(c+dx)}{3a^2d}+\frac{(7A-4B+2C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B+2C)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c +
 d*x])^2,x]
```



```
[Out] ((7*A - 4*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*A - 5*B + 2*C)*
Tan[c + d*x])/(3*a^2*d) + ((7*A - 4*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((8*A - 5*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c
+ d*x])) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x]
)^2)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(5A-2B+2C)-3a(A}{a+a}}{}$$

$$= -\frac{(8A-5B+2C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sec}{3d(a+a}$$

$$= -\frac{(8A-5B+2C)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sec}{3d(a+a}$$

$$= \frac{(7A-4B+2C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(8A-5B+2C)}{3a^2d(1}$$

$$= \frac{(7A-4B+2C)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{2(8A-5B+2C)\tan}{3a^2d}$$

Mathematica [B] time = 6.17683, size = 578, normalized size = 3.5

$$-\frac{2(7A-4B+2C)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a\cos(c+dx)+a)^2} + \frac{2(7A-4B+2C)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{1}{2}(\right.\right.}{d(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*C
os[c + d*x])^2,x]
```

```
[Out] (-2*(7*A - 4*B + 2*C)*Cos[c/2 + (d*x)/2]^4*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]])/(d*(a + a*Cos[c + d*x])^2) + (2*(7*A - 4*B + 2*C)*Cos[c/2 + (d*x)
/2]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(d*(a + a*Cos[c + d*x])^2)
+ (A*Cos[c/2 + (d*x)/2]^4)/(d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2) - (A*Cos[c/2 + (d*x)/2]^4)/(d*(a + a*Cos[c + d*x])^2*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (4*Cos[c/2 + (d*x)/2]^4*(2*A*Sin[(c
 + d*x)/2] - B*Sin[(c + d*x)/2]))/(d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) - (4*Cos[c/2 + (d*x)/2]^4*(2*A*Sin[(c + d*x)/2] - B
*Sin[(c + d*x)/2]))/(d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])) - (2*Cos[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^3*(A*Sin[(c + d*x)/2]
- B*Sin[(c + d*x)/2] + C*Sin[(c + d*x)/2]))/(3*d*(a + a*Cos[c + d*x])^2) -
(4*Cos[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]*(10*A*Sin[(c + d*x)/2] - 7*B*Sin[(
c + d*x)/2] + 4*C*Sin[(c + d*x)/2]))/(3*d*(a + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.075, size = 373, normalized size = 2.3

$$-\frac{A}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\,B}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^
2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2
*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)+5/2/d/a^2*A/(tan(1/2*d*x+1/2*c)-
1)-1/d/a^2*B/(tan(1/2*d*x+1/2*c)-1)-7/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)+2/
d/a^2*B*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a
^2*A/(tan(1/2*d*x+1/2*c)-1)^2+5/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2*B/
(tan(1/2*d*x+1/2*c)+1)+7/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^2*B*ln(ta
n(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^2*A/(tan(1/2
*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.01204, size = 582, normalized size = 3.53

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+\cdots}}{a^2}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))) + C*((9*sin(d*x + c)/(cos(d*x + c) + 1) +
sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c)
```

```
 + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Fricas [A] time = 1.9805, size = 630, normalized size = 3.82

$$\frac{3\left((7A-4B+2C)\cos(dx+c)^4+2(7A-4B+2C)\cos(dx+c)^3+(7A-4B+2C)\cos(dx+c)^2\right)\log(\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] 1/12*(3*((7*A - 4*B + 2*C)*cos(d*x + c)^4 + 2*(7*A - 4*B + 2*C)*cos(d*x + c
)^3 + (7*A - 4*B + 2*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*A - 4
*B + 2*C)*cos(d*x + c)^4 + 2*(7*A - 4*B + 2*C)*cos(d*x + c)^3 + (7*A - 4*B
+ 2*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*(8*A - 5*B + 2*C)*cos(
d*x + c)^3 + (43*A - 28*B + 10*C)*cos(d*x + c)^2 + 6*(A - B)*cos(d*x + c) -
 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*
cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24511, size = 317, normalized size = 1.92

$$\frac{3(7A-4B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(7A-4B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(7*A - 4*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A -
4*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*A*tan(1/2*d*x + 1/
2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x +
 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 2
1*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan(1/2*d*x + 1/2*c) + 9*C*a^4*tan(
1/2*d*x + 1/2*c))/a^6)/d
```

3.354 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$\frac{(12A-8B+5C)\tan^3(c+dx)}{3a^2d}+\frac{(12A-8B+5C)\tan(c+dx)}{a^2d}-\frac{(10A-7B+4C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B+$$

```
[Out] -((10*A - 7*B + 4*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + ((12*A - 8*B + 5*C)
*Tan[c + d*x])/(a^2*d) - ((10*A - 7*B + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((10*A - 7*B + 4*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos
[c + d*x])) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c +
 d*x])^2) + ((12*A - 8*B + 5*C)*Tan[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.387534, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2748, 3767, 3768, 3770}

$$\frac{(12A-8B+5C)\tan^3(c+dx)}{3a^2d}+\frac{(12A-8B+5C)\tan(c+dx)}{a^2d}-\frac{(10A-7B+4C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c +
 d*x])^2,x]
```



```
[Out] -((10*A - 7*B + 4*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + ((12*A - 8*B + 5*C)
*Tan[c + d*x])/(a^2*d) - ((10*A - 7*B + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((10*A - 7*B + 4*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos
[c + d*x])) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c +
 d*x])^2) + ((12*A - 8*B + 5*C)*Tan[c + d*x]^3)/(3*a^2*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(3a(2A-B+C)-a(4A}{a}}{}$$

$$= -\frac{(10A-7B+4C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)}{3d(a}$$

$$= -\frac{(10A-7B+4C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)}{3d(a}$$

$$= -\frac{(10A-7B+4C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(10A-7B+}{3a^2}$$

$$= -\frac{(10A-7B+4C)\tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{(12A-8B+5C)}{a^2d}$$

Mathematica [B] time = 6.21015, size = 763, normalized size = 3.93

$$\frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(11A\sin\left(\frac{1}{2}(c+dx)\right)-6B\sin\left(\frac{1}{2}(c+dx)\right)+3C\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d(a\cos(c+dx)+a)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(11A\sin\left(\frac{1}{2}(c+dx)\right)\right.}{3d(a\cos(c+dx)+a)^2\left(\mathrm{si}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*C
os[c + d*x])^2,x]
```

```
[Out] (2*(10*A - 7*B + 4*C)*Cos[c/2 + (d*x)/2]^4*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]])/(d*(a + a*Cos[c + d*x])^2) - (2*(10*A - 7*B + 4*C)*Cos[c/2 + (d*x
)/2]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(d*(a + a*Cos[c + d*x])^2)
 + ((-5*A + 3*B)*Cos[c/2 + (d*x)/2]^4)/(3*d*(a + a*Cos[c + d*x])^2*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^2) + (2*A*Cos[c/2 + (d*x)/2]^4*Sin[(c + d*x)/
2])/(3*d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) +
(2*A*Cos[c/2 + (d*x)/2]^4*Sin[(c + d*x)/2])/(3*d*(a + a*Cos[c + d*x])^2*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((5*A - 3*B)*Cos[c/2 + (d*x)/2]^4)/
(3*d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*C
os[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^3*(A*Sin[(c + d*x)/2] - B*Sin[(c + d*x
)/2] + C*Sin[(c + d*x)/2]))/(3*d*(a + a*Cos[c + d*x])^2) + (4*Cos[c/2 + (d*
x)/2]^4*(11*A*Sin[(c + d*x)/2] - 6*B*Sin[(c + d*x)/2] + 3*C*Sin[(c + d*x)/2
]))/(3*d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4
*Cos[c/2 + (d*x)/2]^4*(11*A*Sin[(c + d*x)/2] - 6*B*Sin[(c + d*x)/2] + 3*C*S
in[(c + d*x)/2]))/(3*d*(a + a*Cos[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])) + (4*Cos[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]*(13*A*Sin[(c + d*x)/2]
 - 10*B*Sin[(c + d*x)/2] + 7*C*Sin[(c + d*x)/2]))/(3*d*(a + a*Cos[c + d*x])
```

```
^2)
```

Maple [B] time = 0.078, size = 506, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*
d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)-3/2/d/a^2*A/(tan(1/2*d*x+1/2*c)-1
)^2+1/2/d/a^2*B/(tan(1/2*d*x+1/2*c)-1)^2+5/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)
-7/2/d/a^2*B*ln(tan(1/2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-5/
d/a^2*A/(tan(1/2*d*x+1/2*c)-1)+5/2/d/a^2*B/(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(
tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)^3+3/2/d/a^2*A/(t
an(1/2*d*x+1/2*c)+1)^2-1/2/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)^2-5/d/a^2*A*ln(ta
n(1/2*d*x+1/2*c)+1)+7/2/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^2*ln(tan(1/2
*d*x+1/2*c)+1)*C-5/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)+5/2/d/a^2*B/(tan(1/2*d*x+
1/2*c)+1)-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)+
1)^3
```

Maxima [B] time = 1.04391, size = 765, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
```

```
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) + C*((15*sin(d*x + c)/
(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 2.02162, size = 689, normalized size = 3.55

$$-\frac{3\left((10\,A-7\,B+4\,C)\cos\left(dx+c\right)^{5}+2\,(10\,A-7\,B+4\,C)\cos\left(dx+c\right)^{4}+(10\,A-7\,B+4\,C)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] -1/12*(3*((10*A - 7*B + 4*C)*cos(d*x + c)^5 + 2*(10*A - 7*B + 4*C)*cos(d*x
+ c)^4 + (10*A - 7*B + 4*C)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((10*
A - 7*B + 4*C)*cos(d*x + c)^5 + 2*(10*A - 7*B + 4*C)*cos(d*x + c)^4 + (10*A
 - 7*B + 4*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(4*(12*A - 8*B + 5
*C)*cos(d*x + c)^4 + (66*A - 43*B + 28*C)*cos(d*x + c)^3 + 6*(2*A - B + C)*
cos(d*x + c)^2 - (2*A - 3*B)*cos(d*x + c) + 2*A)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21765, size = 409, normalized size = 2.11

$$-\frac{\frac{3\,(10\,A-7\,B+4\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\,(10\,A-7\,B+4\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(3*(10*A - 7*B + 4*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(10*A
 - 7*B + 4*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(30*A*tan(1/2*d*x
+ 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 - 40*
A*tan(1/2*d*x + 1/2*c)^3 + 24*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x +
 1/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x
 + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 +
 27*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*tan(1/2*d*x + 1/2*c) + 15*C*a^4*t
an(1/2*d*x + 1/2*c))/a^6)/d
```

3.355 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{4(9A-19B+34C)\sin^3(c+dx)}{15a^3d}+\frac{4(9A-19B+34C)\sin(c+dx)}{5a^3d}-\frac{(6A-13B+23C)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6$$

```
[Out] -((6*A - 13*B + 23*C)*x)/(2*a^3) + (4*(9*A - 19*B + 34*C)*Sin[c + d*x])/(5*
a^3*d) - ((6*A - 13*B + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A -
B + C)*Cos[c + d*x]^5*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((3*A -
8*B + 13*C)*Cos[c + d*x]^4*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
((6*A - 13*B + 23*C)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d
*x])) - (4*(9*A - 19*B + 34*C)*Sin[c + d*x]^3)/(15*a^3*d)
```

Rubi [A] time = 0.55628, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2977, 2748, 2635, 8, 2633}

$$-\frac{4(9A-19B+34C)\sin^3(c+dx)}{15a^3d}+\frac{4(9A-19B+34C)\sin(c+dx)}{5a^3d}-\frac{(6A-13B+23C)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^3,x]
```


```
[Out] -((6*A - 13*B + 23*C)*x)/(2*a^3) + (4*(9*A - 19*B + 34*C)*Sin[c + d*x])/(5*
a^3*d) - ((6*A - 13*B + 23*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A -
B + C)*Cos[c + d*x]^5*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((3*A -
8*B + 13*C)*Cos[c + d*x]^4*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
((6*A - 13*B + 23*C)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d
*x])) - (4*(9*A - 19*B + 34*C)*Sin[c + d*x]^3)/(15*a^3*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
```

```
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}dx = -\frac{(A-B+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^4(c+dx)(5a(B-C)}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A-8B+13C)}{15ad(a+}$$

$$= -\frac{(A-B+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A-8B+13C)}{15ad(a+}$$

$$= -\frac{(A-B+C)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A-8B+13C)}{15ad(a+}$$

$$= -\frac{(6A-13B+23C)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A-B+C)}{5d(a}$$

$$= -\frac{(6A-13B+23C)x}{2a^3} + \frac{4(9A-19B+34C)\sin(c+dx)}{5a^3d} -$$

Mathematica [B] time = 1.9533, size = 663, normalized size = 2.8

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-600dx(6A-13B+23C)\cos\left(c+\frac{dx}{2}\right)-600dx(6A-13B+23C)\cos\left(\frac{dx}{2}\right)-4500A\sin\left(c+\frac{dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(6*A - 13*B + 23*C)*d*x*Cos[(d*x)/2] - 600
*(6*A - 13*B + 23*C)*d*x*Cos[c + (d*x)/2] - 1800*A*d*x*Cos[c + (3*d*x)/2] +
 3900*B*d*x*Cos[c + (3*d*x)/2] - 6900*C*d*x*Cos[c + (3*d*x)/2] - 1800*A*d*x
*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (3*d*x)/2] - 6900*C*d*x*Cos[2*
c + (3*d*x)/2] - 360*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*
x)/2] - 1380*C*d*x*Cos[2*c + (5*d*x)/2] - 360*A*d*x*Cos[3*c + (5*d*x)/2] +
780*B*d*x*Cos[3*c + (5*d*x)/2] - 1380*C*d*x*Cos[3*c + (5*d*x)/2] + 7020*A*S
in[(d*x)/2] - 12760*B*Sin[(d*x)/2] + 20410*C*Sin[(d*x)/2] - 4500*A*Sin[c +
(d*x)/2] + 7560*B*Sin[c + (d*x)/2] - 11110*C*Sin[c + (d*x)/2] + 4860*A*Sin[
c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] + 15380*C*Sin[c + (3*d*x)/2] - 9
00*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] - 380*C*Sin[2*c + (3
*d*x)/2] + 1452*A*Sin[2*c + (5*d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 4777
*C*Sin[2*c + (5*d*x)/2] + 300*A*Sin[3*c + (5*d*x)/2] - 750*B*Sin[3*c + (5*d
```

```
*x)/2] + 1625*C*Sin[3*c + (5*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 105*B*Si
n[3*c + (7*d*x)/2] + 230*C*Sin[3*c + (7*d*x)/2] + 60*A*Sin[4*c + (7*d*x)/2]
 - 105*B*Sin[4*c + (7*d*x)/2] + 230*C*Sin[4*c + (7*d*x)/2] + 15*B*Sin[4*c +
 (9*d*x)/2] - 20*C*Sin[4*c + (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2] - 20*C*
Sin[5*c + (9*d*x)/2] + 5*C*Sin[5*c + (11*d*x)/2] + 5*C*Sin[6*c + (11*d*x)/2
]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 0.037, size = 542, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/20/d/
a^3*C*tan(1/2*d*x+1/2*c)^5-1/2/d/a^3*tan(1/2*d*x+1/2*c)^3*A+2/3/d/a^3*tan(1
/2*d*x+1/2*c)^3*B-5/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan(1/2*d*x
+1/2*c)-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)+49/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d
/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*c)^5-7/d/a^3/(tan(1/2*d*x
+1/2*c)^2+1)^3*B*tan(1/2*d*x+1/2*c)^5+17/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*C
*tan(1/2*d*x+1/2*c)^5+4/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)
^3*A-12/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*B+76/3/d/a^3/
(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)^3+2/d/a^3/(tan(1/2*d*x+1/2*
c)^2+1)^3*A*tan(1/2*d*x+1/2*c)-5/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*B*tan(1/2
*d*x+1/2*c)+11/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)-6/d/a^
3*arctan(tan(1/2*d*x+1/2*c))*A+13/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B-23/d/a
^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.51038, size = 738, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] 1/60*(C*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
```

```
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 3*A*(40*sin(d*x + c)/((a^3
+ a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*
x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^3))/d
```

Fricas [A] time = 1.98132, size = 613, normalized size = 2.59

$$-\frac{15\,(6\,A-13\,B+23\,C)dx\cos{(dx+c)}^3+45\,(6\,A-13\,B+23\,C)dx\cos{(dx+c)}^2+45\,(6\,A-13\,B+23\,C)dx\cos{(dx+c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] -1/30*(15*(6*A - 13*B + 23*C)*d*x*cos(d*x + c)^3 + 45*(6*A - 13*B + 23*C)*d
*x*cos(d*x + c)^2 + 45*(6*A - 13*B + 23*C)*d*x*cos(d*x + c) + 15*(6*A - 13*
B + 23*C)*d*x - (10*C*cos(d*x + c)^5 + 15*(B - C)*cos(d*x + c)^4 + 5*(6*A -
 9*B + 19*C)*cos(d*x + c)^3 + (234*A - 479*B + 869*C)*cos(d*x + c)^2 + 3*(1
14*A - 239*B + 429*C)*cos(d*x + c) + 144*A - 304*B + 544*C)*sin(d*x + c))/(
a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*
d)
```

Sympy [A] time = 78.0989, size = 2373, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
3,x)
```

```
[Out] Piecewise((-180*A*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 +
180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d
) - 540*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3
*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 540*
A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c
/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*A*d*x/(6
0*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*A*tan(c/2 + d*x/2)**11/(60*a**3*d*tan(
c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x
/2)**2 + 60*a**3*d) - 21*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)*
*6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
**3*d) + 174*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**
3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 798
*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2
+ d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 975*A*tan(c/2 +
 d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4
+ 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*A*tan(c/2 + d*x/2)/(60*
a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*ta
n(c/2 + d*x/2)**2 + 60*a**3*d) + 390*B*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*t
an(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 +
d*x/2)**2 + 60*a**3*d) + 1170*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2
+ d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)*
*2 + 60*a**3*d) + 1170*B*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2
)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) + 390*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 +
d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*B*tan(c/2 + d*x
/2)**11/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 1
80*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 31*B*tan(c/2 + d*x/2)**9/(60*a
**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) - 354*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/
2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) - 1698*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)*
*6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
**3*d) - 2075*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a*
*3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 76
5*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 +
d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 690*C*d*x*tan(c/2
 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**
4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 2070*C*d*x*tan(c/2 + d*x/
2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180
*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 2070*C*d*x*tan(c/2 + d*x/2)**2/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 690*C*d*x/(60*a**3*d*tan(c/2 + d*x/2)**
6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a*
*3*d) + 3*C*tan(c/2 + d*x/2)**11/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*
```

```
d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 41*C*
tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d
*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 594*C*tan(c/2 + d*
x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 1
80*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3078*C*tan(c/2 + d*x/2)**5/(60
*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*t
an(c/2 + d*x/2)**2 + 60*a**3*d) + 3675*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 1395*C*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)*
*6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
**3*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*cos(c)**4/(a*cos(c) + a)
**3, True))
```

Giac [A] time = 1.29128, size = 432, normalized size = 1.82

$$-\frac{\frac{30\,(dx+c)(6\,A-13\,B+23\,C)}{a^3}-\frac{20\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+51\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+76\,C\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -1/60*(30*(d*x + c)*(6*A - 13*B + 23*C)/a^3 - 20*(6*A*tan(1/2*d*x + 1/2*c)^
5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 51*C*tan(1/2*d*x + 1/2*c)^5 + 12*A*tan(1/
2*d*x + 1/2*c)^3 - 36*B*tan(1/2*d*x + 1/2*c)^3 + 76*C*tan(1/2*d*x + 1/2*c)^
3 + 6*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c) + 33*C*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1
/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5
 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 50
*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^
12*tan(1/2*d*x + 1/2*c) + 735*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.356 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{2(11A-36B+76C)\sin(c+dx)}{15a^3d}-\frac{(11A-36B+76C)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A-6B+13C)\sin(c+dx)\cos(c+}{2a^3d}$$

```
[Out] ((2*A - 6*B + 13*C)*x)/(2*a^3) - (2*(11*A - 36*B + 76*C)*Sin[c + d*x])/(15*
a^3*d) + ((2*A - 6*B + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B
 + C)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((A - 6*B
 + 11*C)*Cos[c + d*x]^3*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((1
1*A - 36*B + 76*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*
x]))
```

Rubi [A] time = 0.50102, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3041, 2977, 2734}

$$-\frac{2(11A-36B+76C)\sin(c+dx)}{15a^3d}-\frac{(11A-36B+76C)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A-6B+13C)\sin(c+dx)\cos(c+}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^3,x]
```

```
[Out] ((2*A - 6*B + 13*C)*x)/(2*a^3) - (2*(11*A - 36*B + 76*C)*Sin[c + d*x])/(15*
a^3*d) + ((2*A - 6*B + 13*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B
 + C)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((A - 6*B
 + 11*C)*Cos[c + d*x]^3*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((1
1*A - 36*B + 76*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*
x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
```

```
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(a(A+4B\cdots}{(a+\cdots}}{\cdots}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(A-6B+11C)\,\mathrm{co}\cdots}{15ad(a+a\cdots}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(A-6B+11C)\,\mathrm{co}\cdots}{15ad(a+a\cdots}$$

$$= \frac{(2A-6B+13C)x}{2a^3} - \frac{2(11A-36B+76C)\sin(c+dx)}{15a^3d} + \frac{(2\cdots}{\cdots}$$

Mathematica [B] time = 0.952584, size = 565, normalized size = 2.73

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(2A-6B+13C)\cos\left(c+\frac{dx}{2}\right)+600dx(2A-6B+13C)\cos\left(\frac{dx}{2}\right)+2160A\sin\left(c+\frac{dx}{2}\right)-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(2*A - 6*B + 13*C)*d*x*Cos[(d*x)/2] + 600*(
2*A - 6*B + 13*C)*d*x*Cos[c + (d*x)/2] + 600*A*d*x*Cos[c + (3*d*x)/2] - 180
0*B*d*x*Cos[c + (3*d*x)/2] + 3900*C*d*x*Cos[c + (3*d*x)/2] + 600*A*d*x*Cos[
2*c + (3*d*x)/2] - 1800*B*d*x*Cos[2*c + (3*d*x)/2] + 3900*C*d*x*Cos[2*c + (
3*d*x)/2] + 120*A*d*x*Cos[2*c + (5*d*x)/2] - 360*B*d*x*Cos[2*c + (5*d*x)/2]
 + 780*C*d*x*Cos[2*c + (5*d*x)/2] + 120*A*d*x*Cos[3*c + (5*d*x)/2] - 360*B*
d*x*Cos[3*c + (5*d*x)/2] + 780*C*d*x*Cos[3*c + (5*d*x)/2] - 2960*A*Sin[(d*x
)/2] + 7020*B*Sin[(d*x)/2] - 12760*C*Sin[(d*x)/2] + 2160*A*Sin[c + (d*x)/2]
 - 4500*B*Sin[c + (d*x)/2] + 7560*C*Sin[c + (d*x)/2] - 1840*A*Sin[c + (3*d*
x)/2] + 4860*B*Sin[c + (3*d*x)/2] - 9230*C*Sin[c + (3*d*x)/2] + 720*A*Sin[2
*c + (3*d*x)/2] - 900*B*Sin[2*c + (3*d*x)/2] + 930*C*Sin[2*c + (3*d*x)/2] -
 512*A*Sin[2*c + (5*d*x)/2] + 1452*B*Sin[2*c + (5*d*x)/2] - 2782*C*Sin[2*c
+ (5*d*x)/2] + 300*B*Sin[3*c + (5*d*x)/2] - 750*C*Sin[3*c + (5*d*x)/2] + 60
*B*Sin[3*c + (7*d*x)/2] - 105*C*Sin[3*c + (7*d*x)/2] + 60*B*Sin[4*c + (7*d*
x)/2] - 105*C*Sin[4*c + (7*d*x)/2] + 15*C*Sin[4*c + (9*d*x)/2] + 15*C*Sin[5
*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.037, size = 369, normalized size = 1.8

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{2\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/20/d
/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A-1/2/d/a^3*tan(
1/2*d*x+1/2*c)^3*B+2/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan(1/2*d*x
+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-31/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d
```

```
/a^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-7/d/a^3/(tan(1/2*d*x
+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3-5/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^2*C*
tan(1/2*d*x+1/2*c)+2/d/a^3/(tan(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c)+
2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B+1
3/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.56879, size = 555, normalized size = 2.68

$$-\frac{C\left(\frac{60\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3\,B\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] -1/60*(C*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*B*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3) + A*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3
- 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 2.03804, size = 556, normalized size = 2.69

$$\frac{15\,(2\,A-6\,B+13\,C)dx\cos(dx+c)^3+45\,(2\,A-6\,B+13\,C)dx\cos(dx+c)^2+45\,(2\,A-6\,B+13\,C)dx\cos(dx+c)+1\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/30*(15*(2*A - 6*B + 13*C)*d*x*cos(d*x + c)^3 + 45*(2*A - 6*B + 13*C)*d*x*
cos(d*x + c)^2 + 45*(2*A - 6*B + 13*C)*d*x*cos(d*x + c) + 15*(2*A - 6*B + 1
3*C)*d*x + (15*C*cos(d*x + c)^4 + 15*(2*B - 3*C)*cos(d*x + c)^3 - (64*A - 2
34*B + 479*C)*cos(d*x + c)^2 - 3*(34*A - 114*B + 239*C)*cos(d*x + c) - 44*A
 + 144*B - 304*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 40.07, size = 1445, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 120*A*d*x*tan(c/2 + d*x/2)**2/(
60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 60*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
 + 60*a**3*d) - 3*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 14*A*tan(c/2 + d*x/2)**7/(60*a*
*3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 68
*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2
+ d*x/2)**2 + 60*a**3*d) - 190*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d
*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*A*tan(c/2 + d*
x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a
**3*d) - 180*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120
*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 360*B*d*x*tan(c/2 + d*x/2)**2/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 180*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
 + 60*a**3*d) + 3*B*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 24*B*tan(c/2 + d*x/2)**7/(60*a*
*3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 19
8*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2
 + d*x/2)**2 + 60*a**3*d) + 600*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 +
d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*B*tan(c/2 + d
*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*
a**3*d) + 390*C*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*C*d*x*tan(c/2 + d*x/2)**2/(
60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 390*C*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**
```

```
2 + 60*a**3*d) - 3*C*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 1
20*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*C*tan(c/2 + d*x/2)**7/(60*a
**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3
88*C*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/
2 + d*x/2)**2 + 60*a**3*d) - 1310*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2
+ d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*C*tan(c/2 +
 d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*cos(c)**3/(a*cos(c) +
 a)**3, True))
```

Giac [A] time = 1.2032, size = 340, normalized size = 1.64

$$\frac{\frac{30\,(dx+c)(2\,A-6\,B+13\,C)}{a^3}+\frac{60\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(30*(d*x + c)*(2*A - 6*B + 13*C)/a^3 + 60*(2*B*tan(1/2*d*x + 1/2*c)^3
- 7*C*tan(1/2*d*x + 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) - 5*C*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 -
 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 40*C
*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12
*tan(1/2*d*x + 1/2*c) + 465*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.357 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{(2A-7B+27C)\sin(c+dx)}{15a^3d}-\frac{(B-3C)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(B-3C)}{a^3}-\frac{(A-B+C)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(A+4B-}{15}$$

```
[Out] ((B - 3*C)*x)/a^3 + ((2*A - 7*B + 27*C)*Sin[c + d*x])/(15*a^3*d) - ((A - B
+ C)*Cos[c + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((A + 4*B
- 9*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((B -
 3*C)*Sin[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.47639, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3041, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{(2A-7B+27C)\sin(c+dx)}{15a^3d}-\frac{(B-3C)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(B-3C)}{a^3}-\frac{(A-B+C)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(A+4B-}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^3,x]
```


```
[Out] ((B - 3*C)*x)/a^3 + ((2*A - 7*B + 27*C)*Sin[c + d*x])/(15*a^3*d) - ((A - B
+ C)*Cos[c + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((A + 4*B
- 9*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((B -
 3*C)*Sin[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(a(2A+3B-}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B-9C)\cos^2}{15ad(a+a\,c}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B-9C)\cos^2}{15ad(a+a\,c}$$

$$= \frac{(2A-7B+27C)\sin(c+dx)}{15a^3d} - \frac{(A-B+C)\cos^3(c+dx)\sin(}{5d(a+a\cos(c+dx))^3}$$

$$= \frac{(2A-7B+27C)\sin(c+dx)}{15a^3d} - \frac{(A-B+C)\cos^3(c+dx)\sin(}{5d(a+a\cos(c+dx))^3}$$

$$= \frac{(B-3C)x}{a^3} + \frac{(2A-7B+27C)\sin(c+dx)}{15a^3d} - \frac{(A-B+C)\cos}{5d(a+a}$$

$$= \frac{(B-3C)x}{a^3} + \frac{(2A-7B+27C)\sin(c+dx)}{15a^3d} - \frac{(A-B+C)\cos}{5d(a+a}$$

Mathematica [B] time = 1.00057, size = 423, normalized size = 2.78

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-120A\sin\left(c+\frac{dx}{2}\right)+80A\sin\left(c+\frac{3dx}{2}\right)-60A\sin\left(2c+\frac{3dx}{2}\right)+28A\sin\left(2c+\frac{5dx}{2}\right)+160A\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(300*(B - 3*C)*d*x*Cos[(d*x)/2] + 300*(B - 3*C)*
d*x*Cos[c + (d*x)/2] + 150*B*d*x*Cos[c + (3*d*x)/2] - 450*C*d*x*Cos[c + (3*
d*x)/2] + 150*B*d*x*Cos[2*c + (3*d*x)/2] - 450*C*d*x*Cos[2*c + (3*d*x)/2] +
 30*B*d*x*Cos[2*c + (5*d*x)/2] - 90*C*d*x*Cos[2*c + (5*d*x)/2] + 30*B*d*x*C
os[3*c + (5*d*x)/2] - 90*C*d*x*Cos[3*c + (5*d*x)/2] + 160*A*Sin[(d*x)/2] -
740*B*Sin[(d*x)/2] + 1755*C*Sin[(d*x)/2] - 120*A*Sin[c + (d*x)/2] + 540*B*S
in[c + (d*x)/2] - 1125*C*Sin[c + (d*x)/2] + 80*A*Sin[c + (3*d*x)/2] - 460*B
```

```
*Sin[c + (3*d*x)/2] + 1215*C*Sin[c + (3*d*x)/2] - 60*A*Sin[2*c + (3*d*x)/2]
 + 180*B*Sin[2*c + (3*d*x)/2] - 225*C*Sin[2*c + (3*d*x)/2] + 28*A*Sin[2*c +
 (5*d*x)/2] - 128*B*Sin[2*c + (5*d*x)/2] + 363*C*Sin[2*c + (5*d*x)/2] + 75*
C*Sin[3*c + (5*d*x)/2] + 15*C*Sin[3*c + (7*d*x)/2] + 15*C*Sin[4*c + (7*d*x)
/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.033, size = 247, normalized size = 1.6

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{3\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/20/d/
a^3*C*tan(1/2*d*x+1/2*c)^5-1/6/d/a^3*tan(1/2*d*x+1/2*c)^3*A+1/3/d/a^3*tan(1
/2*d*x+1/2*c)^3*B-1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(1/2*d*x+
1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a
^3*C*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/a^3*arctan(tan(1/2*d*x
+1/2*c))*B-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.55138, size = 398, normalized size = 2.62

$$\frac{3\,C\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-B\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] 1/60*(3*C*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
```

```
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
A*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.91543, size = 454, normalized size = 2.99

$$\frac{15\,(B-3\,C)dx\cos{(dx+c)}^3+45\,(B-3\,C)dx\cos{(dx+c)}^2+45\,(B-3\,C)dx\cos{(dx+c)}+15\,(B-3\,C)dx+\left(15\,C\cos{(dx}\right.}{15\left(a^3d\cos{(dx+c)}^3+3\,a^3d\cos{(dx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/15*(15*(B - 3*C)*d*x*cos(d*x + c)^3 + 45*(B - 3*C)*d*x*cos(d*x + c)^2 + 4
5*(B - 3*C)*d*x*cos(d*x + c) + 15*(B - 3*C)*d*x + (15*C*cos(d*x + c)^3 + (7
*A - 32*B + 117*C)*cos(d*x + c)^2 + 3*(2*A - 17*B + 57*C)*cos(d*x + c) + 2*
A - 22*B + 72*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 24.1584, size = 665, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
3,x)
```

```
[Out] Piecewise((3*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3
*d) - 7*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 5*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 15*A
*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 60*B*d*x*ta
n(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 60*B*d*x/(6
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*B*tan(c/2 + d*x/2)**7/(60*a**
3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 17*B*tan(c/2 + d*x/2)**5/(60*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) - 85*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(
c/2 + d*x/2)**2 + 60*a**3*d) - 105*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 +
d*x/2)**2 + 60*a**3*d) - 180*C*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 +
```

```
d*x/2)**2 + 60*a**3*d) - 180*C*d*x/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**
3*d) + 3*C*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 27*C*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 22
5*C*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*C
*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (
x*(A + B*cos(c) + C*cos(c)**2)*cos(c)**2/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.22639, size = 274, normalized size = 1.8

$$\frac{\frac{60\,(dx+c)(B-3\,C)}{a^3}+\frac{120\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,Ba^{12}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*(B - 3*C)/a^3 + 120*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*a^3) + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2
*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 10*A*a^12*tan(1/2*d*x +
 1/2*c)^3 + 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*C*a^12*tan(1/2*d*x + 1/2*
c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/2*c) + 2
55*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.358 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{(6A+4B-29C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Cx}{a^3}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(3A+2B-7C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] (C*x)/a^3 - ((A - B + C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d
*x])^3) - ((3*A + 2*B - 7*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2)
+ ((6*A + 4*B - 29*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.281472, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3041, 2968, 3019, 2735, 2648}

$$\frac{(6A+4B-29C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Cx}{a^3}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(3A+2B-7C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d
*x])^3,x]
```

```
[Out] (C*x)/a^3 - ((A - B + C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d
*x])^3) - ((3*A + 2*B - 7*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2)
+ ((6*A + 4*B - 29*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(a(3A+2B-2C}{(a+a\cos(c+}}{5a^2}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{a(3A+2B-2C)\cos(c+dx}{(a+a\cos(c+}}{5a^2}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A+2B-7C)\sin(}{15ad(a+a\cos(c+}$$

$$= \frac{Cx}{a^3} - \frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A+2B-7C)}{15ad(a+a\cos}$$

$$= \frac{Cx}{a^3} - \frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(3A+2B-7C)}{15ad(a+a\cos}$$

Mathematica [B] time = 0.738556, size = 289, normalized size = 2.35

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-30A\sin\left(c+\frac{dx}{2}\right)+30A\sin\left(c+\frac{3dx}{2}\right)+6A\sin\left(2c+\frac{5dx}{2}\right)+30A\sin\left(\frac{dx}{2}\right)-60B\sin\left(c+\frac{dx}{2}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*C*d*x*Cos[(d*x)/2] + 150*C*d*x*Cos[c + (d
*x)/2] + 75*C*d*x*Cos[c + (3*d*x)/2] + 75*C*d*x*Cos[2*c + (3*d*x)/2] + 15*C
*d*x*Cos[2*c + (5*d*x)/2] + 15*C*d*x*Cos[3*c + (5*d*x)/2] + 30*A*Sin[(d*x)/
2] + 80*B*Sin[(d*x)/2] - 370*C*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2] - 60*B*
Sin[c + (d*x)/2] + 270*C*Sin[c + (d*x)/2] + 30*A*Sin[c + (3*d*x)/2] + 40*B*
Sin[c + (3*d*x)/2] - 230*C*Sin[c + (3*d*x)/2] - 30*B*Sin[2*c + (3*d*x)/2] +
 90*C*Sin[2*c + (3*d*x)/2] + 6*A*Sin[2*c + (5*d*x)/2] + 14*B*Sin[2*c + (5*d
*x)/2] - 64*C*Sin[2*c + (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.029, size = 175, normalized size = 1.4

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{3\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/20/d
/a^3*C*tan(1/2*d*x+1/2*c)^5-1/6/d/a^3*tan(1/2*d*x+1/2*c)^3*B+1/3/d/a^3*C*ta
n(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(1/2*d*x+1/2*c)+1/4/d/a^3*B*tan(1/2*d*x+1
/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.53845, size = 277, normalized size = 2.25

$$-\frac{C\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}-\frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin}{(\cos}\right.}{a^3}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(C*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3 - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*
x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.88688, size = 375, normalized size = 3.05

$$\frac{15\,Cdx\cos(dx+c)^3+45\,Cdx\cos(dx+c)^2+45\,Cdx\cos(dx+c)+15\,Cdx+\left((3\,A+7\,B-32\,C)\cos(dx+c)^2+3\,(3\,A\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/15*(15*C*d*x*cos(d*x + c)^3 + 45*C*d*x*cos(d*x + c)^2 + 45*C*d*x*cos(d*x
+ c) + 15*C*d*x + ((3*A + 7*B - 32*C)*cos(d*x + c)^2 + 3*(3*A + 2*B - 17*C)
*cos(d*x + c) + 3*A + 2*B - 22*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
```

```
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 14.4187, size = 192, normalized size = 1.56

$$\begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{Cx}{a^3} - \frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d} - \frac{7C\tan\left(\frac{c}{2}+\right.}{4a^3d} \\ \frac{x\left(A+B\cos(c)+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x
)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)/(4*a**3*
d) + B*tan(c/2 + d*x/2)**5/(20*a**3*d) - B*tan(c/2 + d*x/2)**3/(6*a**3*d) +
 B*tan(c/2 + d*x/2)/(4*a**3*d) + C*x/a**3 - C*tan(c/2 + d*x/2)**5/(20*a**3*
d) + C*tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*C*tan(c/2 + d*x/2)/(4*a**3*d), Ne
(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*cos(c)/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.18034, size = 207, normalized size = 1.68

$$\frac{\frac{60(dx+c)C}{a^3} - \frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 3Ca^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 10Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 20Ca^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 15Aa^{12}\tan\left(\frac{1}{2}dx+\right.}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*C/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 10*B*a^12*tan(1/2*d*
x + 1/2*c)^3 - 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^12*tan(1/2*d*x + 1
/2*c) - 15*B*a^12*tan(1/2*d*x + 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a
^15)/d
```

3.359 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$\frac{(2A+3B+7C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B-8C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B - 8*C
)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 3*B + 7*C)*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.132216, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3019, 2750, 2648}

$$\frac{(2A+3B+7C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B-8C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B - 8*C
)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 3*B + 7*C)*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{-a(2A+3B-3C)-5aC\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2A+3B+7C)\int}{15a}$$
$$= \frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(2A+3B+7C)\,\mathrm{si}}{15d\left(a^3+a^3\cos(\right.}$$

Mathematica [A] time = 0.401919, size = 164, normalized size = 1.5

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(5(4A+3B+8C)\sin\left(\frac{dx}{2}\right)+10A\sin\left(c+\frac{3dx}{2}\right)+2A\sin\left(2c+\frac{5dx}{2}\right)-15(B+2C)\sin\left(c+\frac{dx}{2}\right)+1\right.}{30a^3d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*A + 3*B + 8*C)*Sin[(d*x)/2] - 15*(B + 2*C)
*Sin[c + (d*x)/2] + 10*A*Sin[c + (3*d*x)/2] + 15*B*Sin[c + (3*d*x)/2] + 20*
C*Sin[c + (3*d*x)/2] - 15*C*Sin[2*c + (3*d*x)/2] + 2*A*Sin[2*c + (5*d*x)/2]
 + 3*B*Sin[2*c + (5*d*x)/2] + 7*C*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos
[c + d*x])^3)
```

Maple [A] time = 0.024, size = 113, normalized size = 1.

$$\frac{1}{4da^3}\left(\frac{A}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{2C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*A*tan(1/2*d*x+1/2*c)^5-1/5*B*tan(1/2*d*x+1/2*c)^5+1/5*C*tan(
1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3*A-2/3*C*tan(1/2*d*x+1/2*c)^3+A*ta
n(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.0728, size = 242, normalized size = 2.22

$$\frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + C*(15*sin(d*x + c)
/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c)/(cos(d*x + c) + 1) - s
in(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.77728, size = 251, normalized size = 2.3

$$\frac{\left((2A+3B+7C)\cos(dx+c)^2+3(2A+3B+2C)\cos(dx+c)+7A+3B+2C\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/15*((2*A + 3*B + 7*C)*cos(d*x + c)^2 + 3*(2*A + 3*B + 2*C)*cos(d*x + c) +
 7*A + 3*B + 2*C)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 9.51399, size = 180, normalized size = 1.65

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{C\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{C\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{fo} \\ \frac{x\left(A+B\cos(c)+C\cos^2(c)\right)}{(a\cos(c)+a)^3} & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) - B*tan(c/2 + d*x/2)**5/(20*a**3*d) +
B*tan(c/2 + d*x/2)/(4*a**3*d) + C*tan(c/2 + d*x/2)**5/(20*a**3*d) - C*tan(c
/2 + d*x/2)**3/(6*a**3*d) + C*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A
 + B*cos(c) + C*cos(c)**2)/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.2074, size = 155, normalized size = 1.42

$$\frac{3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 10A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 10C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 +}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 3*C*tan(1/2
*d*x + 1/2*c)^5 + 10*A*tan(1/2*d*x + 1/2*c)^3 - 10*C*tan(1/2*d*x + 1/2*c)^3
 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c) + 15*C*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.360 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=124

$$-\frac{(22A-2B-3C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B-3C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*
Cos[c + d*x])^3) - ((7*A - 2*B - 3*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c +
d*x])^2) - ((22*A - 2*B - 3*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x])
)
```

Rubi [A] time = 0.352924, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3041, 2978, 12, 3770}

$$-\frac{(22A-2B-3C)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B-3C)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B+C)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d
*x])^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*
Cos[c + d*x])^3) - ((7*A - 2*B - 3*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c +
d*x])^2) - ((22*A - 2*B - 3*C)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x])
)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{(5aA-a(2A-2B-3C)\cos(c+dx))\sec(c+dx}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(7A-2B-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{\int}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(7A-2B-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{(22}{15}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(7A-2B-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{(22}{15}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(7A-2B}{15ad(a+}$$

Mathematica [B] time = 1.63551, size = 276, normalized size = 2.23

$$-\frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(5(29A-4B-3C)\sin\left(\frac{dx}{2}\right)+15(C-5A)\sin\left(c+\frac{dx}{2}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos
[c + d*x])^3,x]
```

```
[Out] -((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*(240*A*Cos[(c + d*x)/2]^6*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + Cos[(c + d*x)/2]*Sec[c/2]*(5*(29*A - 4*B - 3*C)*Sin[(d*x)/2] + 15*(-5
*A + C)*Sin[c + (d*x)/2] + 95*A*Sin[c + (3*d*x)/2] - 10*B*Sin[c + (3*d*x)/2
] - 15*C*Sin[c + (3*d*x)/2] - 15*A*Sin[2*c + (3*d*x)/2] + 22*A*Sin[2*c + (5
*d*x)/2] - 2*B*Sin[2*c + (5*d*x)/2] - 3*C*Sin[2*c + (5*d*x)/2])))/(15*a^3*d
*(1 + Cos[c + d*x])^3*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.059, size = 197, normalized size = 1.6

$$\frac{B}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7\,A}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{da^3}\ln\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*B*tan(1/2*d*x+1/2*c)-7/4/d/a^3*A*tan(1/2*d*x+1/2*c)-1/20/d/a^3*A*
tan(1/2*d*x+1/2*c)^5+1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a^3*A*ln(tan(1/
2*d*x+1/2*c)-1)-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A+1/20/d/a^3*B*tan(1/2*d*x+1
/2*c)^5-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/6/d/a^3*tan(1/2*d*x+1/2*c)^3*B+
1/4/d/a^3*C*tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.02899, size = 313, normalized size = 2.52

$$-\frac{A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right.}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 3*C*(5*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 1.98019, size = 505, normalized size = 4.07

$$\frac{15\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}+3\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx\right.\right.}{30\left(a^{3}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/30*(15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*log
(sin(d*x + c) + 1) - 15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*
x + c) + A)*log(-sin(d*x + c) + 1) - 2*((22*A - 2*B - 3*C)*cos(d*x + c)^2 +
 3*(17*A - 2*B - 3*C)*cos(d*x + c) + 32*A - 7*B - 3*C)*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**3,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.25786, size = 243, normalized size = 1.96

$$\frac{\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Aa^{12}\tan\left(\frac{1}{2}\,c\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(60*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x +
1/2*c)^3 - 10*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*
c) - 15*B*a^12*tan(1/2*d*x + 1/2*c) - 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.361 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=150

$$\frac{2(36A-11B+C)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-4B-C)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-$$

```
[Out] -(((3*A - B)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*A - 11*B + C)*Tan[c +
 d*x])/(15*a^3*d) - ((A - B + C)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
 - ((9*A - 4*B - C)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*A -
 B)*Tan[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.516727, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2748, 3767, 8, 3770}

$$\frac{2(36A-11B+C)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-4B-C)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c +
 d*x])^3,x]
```



```
[Out] -(((3*A - B)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*A - 11*B + C)*Tan[c +
 d*x])/(15*a^3*d) - ((A - B + C)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
 - ((9*A - 4*B - C)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*A -
 B)*Tan[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}dx = -\frac{(A-B+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(6A-B+C)-a(3A-3B-2C)\cos(c+dx)}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B-C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \int$$

$$= -\frac{(A-B+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B-C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3}{d(}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B-C)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3}{d(}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B+C)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} -$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2(36A-11B+C)\tan(c+}{15a^3d}$$

Mathematica [B] time = 6.34686, size = 839, normalized size = 5.59

$$\frac{\frac{16(3A-B)\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(\cos(c+dx)+1)^3(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))} - \frac{16(3A-B)\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(A\sec\right.}{d(\cos(c+dx)+1)^3(2A+C+2B\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*C
os[c + d*x])^3,x]
```

```
[Out] ((16*(3*A - B)*Cos[c/2 + (d*x)/2]^6*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] -
 Sin[c/2 + (d*x)/2]]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2))/(d*(1 + Cos[c
 + d*x])^3*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (16*(3*A -
B)*Cos[c/2 + (d*x)/2]^6*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (
d*x)/2]]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2))/(d*(1 + Cos[c + d*x])^3*(
2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) + (Cos[c/2 + (d*x)/2]*Cos
[c + d*x]*Sec[c/2]*Sec[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*(-255*A*S
in[(d*x)/2] + 160*B*Sin[(d*x)/2] - 20*C*Sin[(d*x)/2] + 567*A*Sin[(3*d*x)/2]
 - 167*B*Sin[(3*d*x)/2] + 22*C*Sin[(3*d*x)/2] - 600*A*Sin[c - (d*x)/2] + 17
0*B*Sin[c - (d*x)/2] - 10*C*Sin[c - (d*x)/2] + 375*A*Sin[c + (d*x)/2] - 170
*B*Sin[c + (d*x)/2] + 10*C*Sin[c + (d*x)/2] - 480*A*Sin[2*c + (d*x)/2] + 16
0*B*Sin[2*c + (d*x)/2] - 20*C*Sin[2*c + (d*x)/2] - 60*A*Sin[c + (3*d*x)/2]
+ 75*B*Sin[c + (3*d*x)/2] + 402*A*Sin[2*c + (3*d*x)/2] - 167*B*Sin[2*c + (3
```

```
*d*x)/2] + 22*C*Sin[2*c + (3*d*x)/2] - 225*A*Sin[3*c + (3*d*x)/2] + 75*B*Si
n[3*c + (3*d*x)/2] + 315*A*Sin[c + (5*d*x)/2] - 95*B*Sin[c + (5*d*x)/2] + 1
0*C*Sin[c + (5*d*x)/2] + 30*A*Sin[2*c + (5*d*x)/2] + 15*B*Sin[2*c + (5*d*x)
/2] + 240*A*Sin[3*c + (5*d*x)/2] - 95*B*Sin[3*c + (5*d*x)/2] + 10*C*Sin[3*c
 + (5*d*x)/2] - 45*A*Sin[4*c + (5*d*x)/2] + 15*B*Sin[4*c + (5*d*x)/2] + 72*
A*Sin[2*c + (7*d*x)/2] - 22*B*Sin[2*c + (7*d*x)/2] + 2*C*Sin[2*c + (7*d*x)/
2] + 15*A*Sin[3*c + (7*d*x)/2] + 57*A*Sin[4*c + (7*d*x)/2] - 22*B*Sin[4*c +
 (7*d*x)/2] + 2*C*Sin[4*c + (7*d*x)/2]))/(60*d*(1 + Cos[c + d*x])^3*(2*A +
C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])))/a^3
```

Maple [B] time = 0.072, size = 303, normalized size = 2.

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{3\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/20/d/
a^3*C*tan(1/2*d*x+1/2*c)^5+1/2/d/a^3*tan(1/2*d*x+1/2*c)^3*A-1/3/d/a^3*tan(1
/2*d*x+1/2*c)^3*B+1/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan(1/2*d*x
+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)+3/d/a
^3*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*B*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*A/
(tan(1/2*d*x+1/2*c)-1)-3/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*B*ln(tan(
1/2*d*x+1/2*c)+1)-1/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.04952, size = 473, normalized size = 3.15

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-B\left(\frac{105\,\mathrm{si}}{\cos(}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3) + C*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/
d
```

Fricas [A] time = 2.34524, size = 691, normalized size = 4.61

$$-\frac{15\left((3A-B)\cos(dx+c)^4+3(3A-B)\cos(dx+c)^3+3(3A-B)\cos(dx+c)^2+(3A-B)\cos(dx+c)\right)\log(\sin(dx\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] -1/30*(15*((3*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A -
 B)*cos(d*x + c)^2 + (3*A - B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((3
*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A - B)*cos(d*x +
 c)^2 + (3*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(36*A - 11*B
+ C)*cos(d*x + c)^3 + 3*(57*A - 17*B + 2*C)*cos(d*x + c)^2 + (117*A - 32*B
+ 7*C)*cos(d*x + c) + 15*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26901, size = 323, normalized size = 2.15

$$-\frac{\frac{60\,(3\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,(3\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -1/60*(60*(3*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(3*A - B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*ta
n(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*
d*x + 1/2*c)^3 - 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 10*C*a^12*tan(1/2*d*x +
 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/2*
c) + 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.362 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=210

$$-\frac{2(76A-36B+11C)\tan(c+dx)}{15a^3d}+\frac{(13A-6B+2C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B+2C)\tan(c+dx)\sec(c+dx)}{2a^3d}$$

```
[Out] ((13*A - 6*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(76*A - 36*B + 11
*C)*Tan[c + d*x])/(15*a^3*d) + ((13*A - 6*B + 2*C)*Sec[c + d*x]*Tan[c + d*x
])/(2*a^3*d) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c +
d*x])^3) - ((11*A - 6*B + C)*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[
c + d*x])^2) - ((76*A - 36*B + 11*C)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3
+ a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.556505, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3041, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(76A-36B+11C)\tan(c+dx)}{15a^3d}+\frac{(13A-6B+2C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B+2C)\tan(c+dx)\sec(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c +
 d*x])^3,x]
```


```
[Out] ((13*A - 6*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(76*A - 36*B + 11
*C)*Tan[c + d*x])/(15*a^3*d) + ((13*A - 6*B + 2*C)*Sec[c + d*x]*Tan[c + d*x
])/(2*a^3*d) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c +
d*x])^3) - ((11*A - 6*B + C)*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[
c + d*x])^2) - ((76*A - 36*B + 11*C)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3
+ a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
```

```
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{(a(7A-2B+2C)-a(4A-}{(a+a\,c}}{}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(11A-6B+C)\sec(}{15ad(a+a\,co}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(11A-6B+C)\sec(}{15ad(a+a\,co}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(11A-6B+C)\sec(}{15ad(a+a\,co}$$

$$= \frac{(13A-6B+2C)\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{(A-B+C)\sec(}{5d(a+a\,c}$$

$$= \frac{(13A-6B+2C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{2(76A-36B+11C)}{15a^3d}$$

Mathematica [A] time = 1.45282, size = 206, normalized size = 0.98

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-4(107A-57B+22C)\tan\left(\frac{1}{2}(c+dx)\right)-96(A-B+C)\sin^6\left(\frac{1}{2}(c+dx)\right)\csc^5(c+dx)-16(17A-12}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*C
os[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(-30*(13*A - 6*B + 2*C)*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 16*(17*A - 12*B
+ 7*C)*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 96*(A - B + C)*Csc[c + d*x]^5*Si
n[(c + d*x)/2]^6 - 4*(107*A - 57*B + 22*C)*Tan[(c + d*x)/2] - 60*(3*A - B)*
Tan[c + d*x] + 30*A*Sec[c + d*x]*Tan[c + d*x]))/(15*a^3*d*(1 + Cos[c + d*x]
)^3)
```

Maple [B] time = 0.08, size = 433, normalized size = 2.1

$$-\frac{A}{20\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\frac{B}{20\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{C}{20\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{2\,A}{3\,d a^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{B}{2\,d a^{3}}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/20/d
/a^3*C*tan(1/2*d*x+1/2*c)^5-2/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*tan(
1/2*d*x+1/2*c)^3*B-1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*tan(1/2*d*
x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)+7/2
/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*B/(tan(1/2*d*x+1/2*c)-1)-13/2/d/a^3
*A*ln(tan(1/2*d*x+1/2*c)-1)+3/d/a^3*B*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(t
an(1/2*d*x+1/2*c)-1)*C+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^2+7/2/d/a^3*A/(ta
n(1/2*d*x+1/2*c)+1)-1/d/a^3*B/(tan(1/2*d*x+1/2*c)+1)+13/2/d/a^3*A*ln(tan(1/
2*d*x+1/2*c)+1)-3/d/a^3*B*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1
/2*c)+1)*C-1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.09989, size = 666, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3) + C*((105*sin(d*x + c)/(co
s(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
```

```
3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 2.32358, size = 849, normalized size = 4.04

$$15\left((13A-6B+2C)\cos(dx+c)^5+3(13A-6B+2C)\cos(dx+c)^4+3(13A-6B+2C)\cos(dx+c)^3+(13A-6B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/60*(15*((13*A - 6*B + 2*C)*cos(d*x + c)^5 + 3*(13*A - 6*B + 2*C)*cos(d*x
+ c)^4 + 3*(13*A - 6*B + 2*C)*cos(d*x + c)^3 + (13*A - 6*B + 2*C)*cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 15*((13*A - 6*B + 2*C)*cos(d*x + c)^5 + 3*(1
3*A - 6*B + 2*C)*cos(d*x + c)^4 + 3*(13*A - 6*B + 2*C)*cos(d*x + c)^3 + (13
*A - 6*B + 2*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*(76*A - 36*B
+ 11*C)*cos(d*x + c)^4 + 3*(239*A - 114*B + 34*C)*cos(d*x + c)^3 + (479*A -
 234*B + 64*C)*cos(d*x + c)^2 + 15*(3*A - 2*B)*cos(d*x + c) - 15*A)*sin(d*x
 + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c
)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33398, size = 389, normalized size = 1.85

$$\frac{30(13A-6B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{30(13A-6B+2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(7A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(30*(13*A - 6*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13
*A - 6*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*A*tan(1/2*d*
x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c) + 2*B*
tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(
1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x
 + 1/2*c)^5 + 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^12*tan(1/2*d*x + 1/
2*c)^3 + 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c)
 - 255*B*a^12*tan(1/2*d*x + 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.363 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=246

$$\frac{4(34A-19B+9C)\tan^3(c+dx)}{15a^3d}+\frac{4(34A-19B+9C)\tan(c+dx)}{5a^3d}-\frac{(23A-13B+6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(23A-$$

```
[Out] -((23*A - 13*B + 6*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (4*(34*A - 19*B +
9*C)*Tan[c + d*x])/(5*a^3*d) - ((23*A - 13*B + 6*C)*Sec[c + d*x]*Tan[c + d*
x])/(2*a^3*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Cos[c
 + d*x])^3) - ((13*A - 8*B + 3*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a +
 a*Cos[c + d*x])^2) - ((23*A - 13*B + 6*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*
d*(a^3 + a^3*Cos[c + d*x])) + (4*(34*A - 19*B + 9*C)*Tan[c + d*x]^3)/(15*a^
3*d)
```

Rubi [A] time = 0.57554, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2748, 3767, 3768, 3770}

$$\frac{4(34A-19B+9C)\tan^3(c+dx)}{15a^3d}+\frac{4(34A-19B+9C)\tan(c+dx)}{5a^3d}-\frac{(23A-13B+6C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(23A-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c +
 d*x])^3,x]
```


```
[Out] -((23*A - 13*B + 6*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (4*(34*A - 19*B +
9*C)*Tan[c + d*x])/(5*a^3*d) - ((23*A - 13*B + 6*C)*Sec[c + d*x]*Tan[c + d*
x])/(2*a^3*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Cos[c
 + d*x])^3) - ((13*A - 8*B + 3*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a +
 a*Cos[c + d*x])^2) - ((23*A - 13*B + 6*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*
d*(a^3 + a^3*Cos[c + d*x])) + (4*(34*A - 19*B + 9*C)*Tan[c + d*x]^3)/(15*a^
3*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
```

```
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(8A-3B+3C)-5a(A-}{(a+a\,\mathrm{co}}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(13A-8B+3C)\,\mathrm{se}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(13A-8B+3C)\,\mathrm{se}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(13A-8B+3C)\,\mathrm{se}}{15ad(a+a}$$

$$= -\frac{(23A-13B+6C)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A-B+C)\,\mathrm{se}}{5d(a+}$$

$$= -\frac{(23A-13B+6C)\tanh^{-1}(\sin(c+dx))}{2a^3d} + \frac{4(34A-19B+9\mathrm{C}}{5a^3\mathrm{d}}$$

Mathematica [A] time = 0.989298, size = 270, normalized size = 1.1

$$\frac{960(23A-13B+6C)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*C
os[c + d*x])^3,x]
```

```
[Out] (960*(23*A - 13*B + 6*C)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*Cos[(c + d*x)/2]
*(4321*A - 2331*B + 1146*C + (7814*A - 4274*B + 2124*C)*Cos[c + d*x] + 8*(6
91*A - 381*B + 186*C)*Cos[2*(c + d*x)] + 3098*A*Cos[3*(c + d*x)] - 1718*B*C
os[3*(c + d*x)] + 828*C*Cos[3*(c + d*x)] + 1287*A*Cos[4*(c + d*x)] - 717*B*
Cos[4*(c + d*x)] + 342*C*Cos[4*(c + d*x)] + 272*A*Cos[5*(c + d*x)] - 152*B*
Cos[5*(c + d*x)] + 72*C*Cos[5*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2])/
(240*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 0.085, size = 566, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*B/(tan(1/2*d*x+1/2*c)-1)^2-1/2/d/a^3*B/(tan(1/2*d*x+1/2*c)+1)^2+7
/2/d/a^3*B/(tan(1/2*d*x+1/2*c)-1)+7/2/d/a^3*B/(tan(1/2*d*x+1/2*c)+1)+13/2/d
/a^3*B*ln(tan(1/2*d*x+1/2*c)+1)-13/2/d/a^3*B*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a
^3/(tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^3-1/d/a^3/(t
an(1/2*d*x+1/2*c)+1)*C-1/3/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)^3+3/d/a^3*ln(tan(
1/2*d*x+1/2*c)-1)*C-2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^2-3/d/a^3*ln(tan(1/2*d
*x+1/2*c)+1)*C+2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)^2-17/2/d/a^3*A/(tan(1/2*d*x
+1/2*c)-1)-17/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)+23/2/d/a^3*A*ln(tan(1/2*d*x+
1/2*c)-1)-23/2/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)-2/3/d/a^3*tan(1/2*d*x+1/2*c
)^3*B-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/2
0/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+5/6/d/a^3*
tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+49/4/d/a^3*A*tan(1/
2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 1.09323, size = 851, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] 1/60*(A*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) - 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 - 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) + 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 690*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 690*l
og(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3) - B*(60*(5*sin(d*x + c)/(cos(d
*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (4
65*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 390*log(sin(d*x + c)/(cos(d
```

```
*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3) +
 3*C*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^3))/d
```

Fricas [A] time = 2.11088, size = 910, normalized size = 3.7

$$-\frac{15\left((23\,A-13\,B+6\,C)\cos\left(dx+c\right)^{6}+3\,(23\,A-13\,B+6\,C)\cos\left(dx+c\right)^{5}+3\,(23\,A-13\,B+6\,C)\cos\left(dx+c\right)^{4}+(23\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] -1/60*(15*((23*A - 13*B + 6*C)*cos(d*x + c)^6 + 3*(23*A - 13*B + 6*C)*cos(d
*x + c)^5 + 3*(23*A - 13*B + 6*C)*cos(d*x + c)^4 + (23*A - 13*B + 6*C)*cos(
d*x + c)^3)*log(sin(d*x + c) + 1) - 15*((23*A - 13*B + 6*C)*cos(d*x + c)^6
+ 3*(23*A - 13*B + 6*C)*cos(d*x + c)^5 + 3*(23*A - 13*B + 6*C)*cos(d*x + c)
^4 + (23*A - 13*B + 6*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*(34
*A - 19*B + 9*C)*cos(d*x + c)^5 + 3*(429*A - 239*B + 114*C)*cos(d*x + c)^4
+ (869*A - 479*B + 234*C)*cos(d*x + c)^3 + 5*(19*A - 9*B + 6*C)*cos(d*x + c
)^2 - 15*(A - B)*cos(d*x + c) + 10*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6 +
 3*a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + a^3*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24013, size = 481, normalized size = 1.96

$$-\frac{\frac{30\,(23\,A-13\,B+6\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(23\,A-13\,B+6\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{20\left(51\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -1/60*(30*(23*A - 13*B + 6*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(
23*A - 13*B + 6*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 20*(51*A*tan(1/
2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5
 - 76*A*tan(1/2*d*x + 1/2*c)^3 + 36*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2
*d*x + 1/2*c)^3 + 33*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c) + 6
*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3) - (3*A*a^12*t
an(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*
d*x + 1/2*c)^5 + 50*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^12*tan(1/2*d*x +
 1/2*c)^3 + 30*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 735*A*a^12*tan(1/2*d*x + 1/2
*c) - 465*B*a^12*tan(1/2*d*x + 1/2*c) + 255*C*a^12*tan(1/2*d*x + 1/2*c))/a^
15)/d
```

3.364 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=245

$$-\frac{8(20A-83B+216C)\sin(c+dx)}{105a^4d}-\frac{(10A-52B+129C)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{4(20A-83B+216C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1}$$

```
[Out] ((2*A - 8*B + 21*C)*x)/(2*a^4) - (8*(20*A - 83*B + 216*C)*Sin[c + d*x])/(10
5*a^4*d) + ((2*A - 8*B + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((10*
A - 52*B + 129*C)*Cos[c + d*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x]
)^2) - (4*(20*A - 83*B + 216*C)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1
+ Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Co
s[c + d*x])^4) + ((B - 2*C)*Cos[c + d*x]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[
c + d*x])^3)
```

Rubi [A] time = 0.691098, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3041, 2977, 2734}

$$-\frac{8(20A-83B+216C)\sin(c+dx)}{105a^4d}-\frac{(10A-52B+129C)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{4(20A-83B+216C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^4,x]
```


```
[Out] ((2*A - 8*B + 21*C)*x)/(2*a^4) - (8*(20*A - 83*B + 216*C)*Sin[c + d*x])/(10
5*a^4*d) + ((2*A - 8*B + 21*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((10*
A - 52*B + 129*C)*Cos[c + d*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x]
)^2) - (4*(20*A - 83*B + 216*C)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1
+ Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Co
s[c + d*x])^4) + ((B - 2*C)*Cos[c + d*x]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[
c + d*x])^3)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^4(c+dx)(a(2A+5B}{(a}}{}$$

$$= -\frac{(A-B+C)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(B-2C)\cos^4(c+}{5ad(a+a\cos}$$

$$= -\frac{(10A-52B+129C)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+}{7d}$$

$$= -\frac{(10A-52B+129C)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+}{7d}$$

$$= \frac{(2A-8B+21C)x}{2a^4} - \frac{8(20A-83B+216C)\sin(c+dx)}{105a^4d} + \frac{(}{}$$

Mathematica [A] time = 2.79694, size = 299, normalized size = 1.22

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(210\cos^7\left(\frac{1}{2}(c+dx)\right)(2dx(2A-8B+21C)+4(B-4C)\sin(c+dx)+C\sin(2(c+dx)))+4\tan\left(\frac{c}{2}\right)(16\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^4,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(15*(A - B + C)*Sec[c/2]*Sin[(d*x)/2] - 6*(25*A - 32*B
+ 39*C)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 4*(160*A - 286*B + 447*C
)*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] - 8*(260*A - 764*B + 1653*C)*Cos
[(c + d*x)/2]^6*Sec[c/2]*Sin[(d*x)/2] + 210*Cos[(c + d*x)/2]^7*(2*(2*A - 8*
B + 21*C)*d*x + 4*(B - 4*C)*Sin[c + d*x] + C*Sin[2*(c + d*x)]) + 15*(A - B
+ C)*Cos[(c + d*x)/2]*Tan[c/2] - 6*(25*A - 32*B + 39*C)*Cos[(c + d*x)/2]^3*
Tan[c/2] + 4*(160*A - 286*B + 447*C)*Cos[(c + d*x)/2]^5*Tan[c/2]))/(105*a^4
*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.034, size = 429, normalized size = 1.8

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,B}{40\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+1/56/d/
a^4*tan(1/2*d*x+1/2*c)^7*C-1/8/d/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*ta
n(1/2*d*x+1/2*c)^5-9/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*tan(1/2*d*
x+1/2*c)^3*A-23/24/d/a^4*tan(1/2*d*x+1/2*c)^3*B+13/8/d/a^4*C*tan(1/2*d*x+1/
2*c)^3-15/8/d/a^4*A*tan(1/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-111/
8/d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x
+1/2*c)^3*B-9/d/a^4/(tan(1/2*d*x+1/2*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)^3+2/d/a
^4/(tan(1/2*d*x+1/2*c)^2+1)^2*B*tan(1/2*d*x+1/2*c)-7/d/a^4/(tan(1/2*d*x+1/2
*c)^2+1)^2*C*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*A-8/d/a^
4*arctan(tan(1/2*d*x+1/2*c))*B+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.57059, size = 640, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] -1/840*(3*C*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4) + 5*A*((315*sin(d*x + c)/(cos(d
*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arct
an(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 2.11227, size = 730, normalized size = 2.98

$$\frac{105\,(2\,A-8\,B+21\,C)dx\cos\left(dx+c\right)^{4}+420\,(2\,A-8\,B+21\,C)dx\cos\left(dx+c\right)^{3}+630\,(2\,A-8\,B+21\,C)dx\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/210*(105*(2*A - 8*B + 21*C)*d*x*cos(d*x + c)^4 + 420*(2*A - 8*B + 21*C)*d
*x*cos(d*x + c)^3 + 630*(2*A - 8*B + 21*C)*d*x*cos(d*x + c)^2 + 420*(2*A -
8*B + 21*C)*d*x*cos(d*x + c) + 105*(2*A - 8*B + 21*C)*d*x + (105*C*cos(d*x
+ c)^5 + 210*(B - 2*C)*cos(d*x + c)^4 - 4*(130*A - 592*B + 1509*C)*cos(d*x
+ c)^3 - 4*(310*A - 1318*B + 3411*C)*cos(d*x + c)^2 - (1070*A - 4472*B + 11
619*C)*cos(d*x + c) - 320*A + 1328*B - 3456*C)*sin(d*x + c))/(a^4*d*cos(d*x
 + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x
 + c) + a^4*d)
```

Sympy [A] time = 99.8879, size = 1624, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
4,x)
```

```
[Out] Piecewise((840*A*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 +
1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 1680*A*d*x*tan(c/2 + d*x/2)
**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840
*a**4*d) + 840*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2
+ d*x/2)**2 + 840*a**4*d) + 15*A*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 +
 d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 75*A*tan(c/2 +
 d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**
2 + 840*a**4*d) + 190*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4
 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 910*A*tan(c/2 + d*x/2)**
5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a
**4*d) - 2765*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*
a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1575*A*tan(c/2 + d*x/2)/(840*a**
4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3
360*B*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d
*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 6720*B*d*x*tan(c/2 + d*x/2)**2/(840*a*
*4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) -
3360*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**
2 + 840*a**4*d) - 15*B*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 + d*x/2)**4
 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 117*B*tan(c/2 + d*x/2)**
9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a
**4*d) - 526*B*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a
**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 3682*B*tan(c/2 + d*x/2)**5/(840*a
**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) +
 11165*B*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*
tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*B*tan(c/2 + d*x/2)/(840*a**4*d*tan
(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 8820*C*d
*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/
2 + d*x/2)**2 + 840*a**4*d) + 17640*C*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*t
an(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 8820*C
*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 84
0*a**4*d) + 15*C*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 + d*x/2)**4 + 168
0*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 159*C*tan(c/2 + d*x/2)**9/(840
*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d)
 + 1002*C*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d
```

```
*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 9114*C*tan(c/2 + d*x/2)**5/(840*a**4*d
*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 2950
5*C*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c
/2 + d*x/2)**2 + 840*a**4*d) - 17535*C*tan(c/2 + d*x/2)/(840*a**4*d*tan(c/2
 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x
*(A + B*cos(c) + C*cos(c)**2)*cos(c)**4/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.25714, size = 408, normalized size = 1.67

$$\frac{\frac{420\,(dx+c)(2\,A-8\,B+21\,C)}{a^4}+\frac{840\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\,\mathrm{t}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(420*(d*x + c)*(2*A - 8*B + 21*C)/a^4 + 840*(2*B*tan(1/2*d*x + 1/2*c)
^3 - 9*C*tan(1/2*d*x + 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) - 7*C*tan(1/2*d*
x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x +
 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*
c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*tan(1/2*d*x + 1/2*c)^
5 - 189*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 -
 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 1365*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 1
575*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/2*d*x + 1/2*c) - 11655*
C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.365 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=195

$$\frac{(6A-55B+244C)\sin(c+dx)}{105a^4d}+\frac{(3A+25B-88C)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(B-4C)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(B-4C)}{a^4}-\frac{(A$$

```
[Out] ((B - 4*C)*x)/a^4 + ((6*A - 55*B + 244*C)*Sin[c + d*x])/(105*a^4*d) + ((3*A
 + 25*B - 88*C)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^
2) - ((B - 4*C)*Sin[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos
[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((2*A + 5*B - 12*C
)*Cos[c + d*x]^3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.648229, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3041, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{(6A-55B+244C)\sin(c+dx)}{105a^4d}+\frac{(3A+25B-88C)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(B-4C)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(B-4C)}{a^4}-\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^4,x]
```


```
[Out] ((B - 4*C)*x)/a^4 + ((6*A - 55*B + 244*C)*Sin[c + d*x])/(105*a^4*d) + ((3*A
 + 25*B - 88*C)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^
2) - ((B - 4*C)*Sin[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos
[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((2*A + 5*B - 12*C
)*Cos[c + d*x]^3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}dx = -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(a(3A+4B-}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(2A+5B-12C)\,\mathrm{co}}{35ad(a+a}$$

$$= \frac{(3A+25B-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\,\mathrm{co}}{7d(a+}$$

$$= \frac{(3A+25B-88C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\,\mathrm{co}}{7d(a+}$$

$$= \frac{(6A-55B+244C)\sin(c+dx)}{105a^4d} + \frac{(3A+25B-88C)\cos^2(c-}{105a^4d(1+\cos(c}$$

$$= \frac{(6A-55B+244C)\sin(c+dx)}{105a^4d} + \frac{(3A+25B-88C)\cos^2(c-}{105a^4d(1+\cos(c}$$

$$= \frac{(B-4C)x}{a^4} + \frac{(6A-55B+244C)\sin(c+dx)}{105a^4d} + \frac{(3A+25B-}{105a}$$

$$= \frac{(B-4C)x}{a^4} + \frac{(6A-55B+244C)\sin(c+dx)}{105a^4d} + \frac{(3A+25B-}{105a}$$

Mathematica [B] time = 1.10958, size = 571, normalized size = 2.93

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-2520A\sin\left(c+\frac{dx}{2}\right)+1764A\sin\left(c+\frac{3dx}{2}\right)-1260A\sin\left(2c+\frac{3dx}{2}\right)+588A\sin\left(2c+\frac{5dx}{2}\right)-420\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(7350*(B - 4*C)*d*x*Cos[(d*x)/2] + 7350*(B - 4*C
)*d*x*Cos[c + (d*x)/2] + 4410*B*d*x*Cos[c + (3*d*x)/2] - 17640*C*d*x*Cos[c
```

```
+ (3*d*x)/2] + 4410*B*d*x*Cos[2*c + (3*d*x)/2] - 17640*C*d*x*Cos[2*c + (3*d
*x)/2] + 1470*B*d*x*Cos[2*c + (5*d*x)/2] - 5880*C*d*x*Cos[2*c + (5*d*x)/2]
+ 1470*B*d*x*Cos[3*c + (5*d*x)/2] - 5880*C*d*x*Cos[3*c + (5*d*x)/2] + 210*B
*d*x*Cos[3*c + (7*d*x)/2] - 840*C*d*x*Cos[3*c + (7*d*x)/2] + 210*B*d*x*Cos[
4*c + (7*d*x)/2] - 840*C*d*x*Cos[4*c + (7*d*x)/2] + 2520*A*Sin[(d*x)/2] - 1
9880*B*Sin[(d*x)/2] + 60830*C*Sin[(d*x)/2] - 2520*A*Sin[c + (d*x)/2] + 1652
0*B*Sin[c + (d*x)/2] - 46130*C*Sin[c + (d*x)/2] + 1764*A*Sin[c + (3*d*x)/2]
 - 14280*B*Sin[c + (3*d*x)/2] + 46116*C*Sin[c + (3*d*x)/2] - 1260*A*Sin[2*c
 + (3*d*x)/2] + 7560*B*Sin[2*c + (3*d*x)/2] - 18060*C*Sin[2*c + (3*d*x)/2]
+ 588*A*Sin[2*c + (5*d*x)/2] - 5600*B*Sin[2*c + (5*d*x)/2] + 19292*C*Sin[2*
c + (5*d*x)/2] - 420*A*Sin[3*c + (5*d*x)/2] + 1680*B*Sin[3*c + (5*d*x)/2] -
 2100*C*Sin[3*c + (5*d*x)/2] + 144*A*Sin[3*c + (7*d*x)/2] - 1040*B*Sin[3*c
+ (7*d*x)/2] + 3791*C*Sin[3*c + (7*d*x)/2] + 735*C*Sin[4*c + (7*d*x)/2] + 1
05*C*Sin[4*c + (9*d*x)/2] + 105*C*Sin[5*c + (9*d*x)/2]))/(1680*a^4*d*(1 + C
os[c + d*x])^4)
```

Maple [A] time = 0.034, size = 307, normalized size = 1.6

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/56/d
/a^4*tan(1/2*d*x+1/2*c)^7*C+3/40/d/a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*t
an(1/2*d*x+1/2*c)^5+7/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*tan(1/2*d*x
+1/2*c)^3*A+11/24/d/a^4*tan(1/2*d*x+1/2*c)^3*B-23/24/d/a^4*C*tan(1/2*d*x+1/
2*c)^3+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+49/8/
d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*C*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)
^2+1)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B-8/d/a^4*arctan(tan(1/2*d*x+1/2*c
))*C
```

Maxima [A] time = 1.57601, size = 481, normalized size = 2.47

$$\frac{C\left(\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5\,B\left(\frac{315\,\sin(dx+c}{\cos(dx+c)+1}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] 1/840*(C*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4) + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.99902, size = 606, normalized size = 3.11

$$\frac{105\,(B-4\,C)dx\cos\left(dx+c\right)^{4}+420\,(B-4\,C)dx\cos\left(dx+c\right)^{3}+630\,(B-4\,C)dx\cos\left(dx+c\right)^{2}+420\,(B-4\,C)dx\cos\left(dx+\right.}{105\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/105*(105*(B - 4*C)*d*x*cos(d*x + c)^4 + 420*(B - 4*C)*d*x*cos(d*x + c)^3
+ 630*(B - 4*C)*d*x*cos(d*x + c)^2 + 420*(B - 4*C)*d*x*cos(d*x + c) + 105*(
B - 4*C)*d*x + (105*C*cos(d*x + c)^4 + 4*(9*A - 65*B + 296*C)*cos(d*x + c)^
3 + (39*A - 620*B + 2636*C)*cos(d*x + c)^2 + (24*A - 535*B + 2236*C)*cos(d*
x + c) + 6*A - 160*B + 664*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d
*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 58.6102, size = 746, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
4,x)
```

```
[Out] Piecewise((-15*A*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*
a**4*d) + 48*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a*
*4*d) - 42*A*tan(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4
*d) + 105*A*tan(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d)
+ 840*B*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*
d) + 840*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 15*B*tan(c/2
 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 90*B*tan(c/2 +
 d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 280*B*tan(c/2 +
d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1190*B*tan(c/2 +
d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1575*B*tan(c/2 +
d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*C*d*x*tan(c/2 +
 d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*C*d*x/(840*
a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*C*tan(c/2 + d*x/2)**9/(840*a*
*4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*C*tan(c/2 + d*x/2)**7/(840*a**
4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*C*tan(c/2 + d*x/2)**5/(840*a**4
*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*C*tan(c/2 + d*x/2)**3/(840*a**4
*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*C*tan(c/2 + d*x/2)/(840*a**4*d*
tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**
2)*cos(c)**3/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.24765, size = 344, normalized size = 1.76

$$\frac{\frac{840\,(dx+c)(B-4\,C)}{a^4}+\frac{1680\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)*(B - 4*C)/a^4 + 1680*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*ta
n(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 - 63*A*a^24*tan(1/2
*d*x + 1/2*c)^5 + 105*B*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*C*a^24*tan(1/2*d*
x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2*d*x +
 1/2*c)^3 + 805*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/
2*c) + 1575*B*a^24*tan(1/2*d*x + 1/2*c) - 5145*C*a^24*tan(1/2*d*x + 1/2*c))
```

/a^28)/d

3.366 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=164

$$\frac{(16A+12B-215C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A+6B-55C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Cx}{a^4}-\frac{(A-B+C)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(4A+$$

```
[Out] (C*x)/a^4 - ((8*A + 6*B - 55*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^2) + ((16*A + 12*B - 215*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) -
 ((A - B + C)*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + (
(4*A + 3*B - 10*C)*Cos[c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x]
)^3)
```

Rubi [A] time = 0.478351, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2977, 2968, 3019, 2735, 2648}

$$\frac{(16A+12B-215C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(8A+6B-55C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Cx}{a^4}-\frac{(A-B+C)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(4A+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^4,x]
```


```
[Out] (C*x)/a^4 - ((8*A + 6*B - 55*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^2) + ((16*A + 12*B - 215*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) -
 ((A - B + C)*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + (
(4*A + 3*B - 10*C)*Cos[c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x]
)^3)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(a(4A+3\ldots}{(a+a\cos\ldots}}{7\ldots}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A+3B-10C)\ldots}{35ad(a+\ldots}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A+3B-10C)\ldots}{35ad(a+\ldots}$$

$$= -\frac{(8A+6B-55C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^3(c+dx)\ldots}{7d(a+a\cos(c+d\ldots}$$

$$= \frac{Cx}{a^4} - \frac{(8A+6B-55C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^3(c+\ldots}{7d(a+a\cos(\ldots}$$

$$= \frac{Cx}{a^4} - \frac{(8A+6B-55C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^3(c+\ldots}{7d(a+a\cos(\ldots}$$

Mathematica [B] time = 0.901815, size = 405, normalized size = 2.47

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-350A\sin\left(c+\frac{dx}{2}\right)+336A\sin\left(c+\frac{3dx}{2}\right)-210A\sin\left(2c+\frac{3dx}{2}\right)+182A\sin\left(2c+\frac{5dx}{2}\right)+26A\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*C*d*x*Cos[(d*x)/2] + 3675*C*d*x*Cos[c +
(d*x)/2] + 2205*C*d*x*Cos[c + (3*d*x)/2] + 2205*C*d*x*Cos[2*c + (3*d*x)/2]
+ 735*C*d*x*Cos[2*c + (5*d*x)/2] + 735*C*d*x*Cos[3*c + (5*d*x)/2] + 105*C*d
*x*Cos[3*c + (7*d*x)/2] + 105*C*d*x*Cos[4*c + (7*d*x)/2] + 560*A*Sin[(d*x)/
2] + 1260*B*Sin[(d*x)/2] - 9940*C*Sin[(d*x)/2] - 350*A*Sin[c + (d*x)/2] - 1
260*B*Sin[c + (d*x)/2] + 8260*C*Sin[c + (d*x)/2] + 336*A*Sin[c + (3*d*x)/2]
 + 882*B*Sin[c + (3*d*x)/2] - 7140*C*Sin[c + (3*d*x)/2] - 210*A*Sin[2*c + (
3*d*x)/2] - 630*B*Sin[2*c + (3*d*x)/2] + 3780*C*Sin[2*c + (3*d*x)/2] + 182*
A*Sin[2*c + (5*d*x)/2] + 294*B*Sin[2*c + (5*d*x)/2] - 2800*C*Sin[2*c + (5*d
*x)/2] - 210*B*Sin[3*c + (5*d*x)/2] + 840*C*Sin[3*c + (5*d*x)/2] + 26*A*Sin
[3*c + (7*d*x)/2] + 72*B*Sin[3*c + (7*d*x)/2] - 520*C*Sin[3*c + (7*d*x)/2])
)/(13440*a^4*d)
```

Maple [A] time = 0.032, size = 255, normalized size = 1.6

$$\frac{A}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,B}{40\,d a^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+1/56/d/
a^4*tan(1/2*d*x+1/2*c)^7*C-1/40/d/a^4*A*tan(1/2*d*x+1/2*c)^5+3/40/d/a^4*B*t
an(1/2*d*x+1/2*c)^5-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5-1/24/d/a^4*tan(1/2*d*x
+1/2*c)^3*A-1/8/d/a^4*tan(1/2*d*x+1/2*c)^3*B+11/24/d/a^4*C*tan(1/2*d*x+1/2*
c)^3+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)-15/8/d/a
^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.57301, size = 386, normalized size = 2.35

$$-\frac{5\,C\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{A\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(d}{(\cos(dx+}\right.}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] -1/840*(5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 2.00151, size = 512, normalized size = 3.12

$$\frac{105\,Cdx\cos\left(dx+c\right)^{4}+420\,Cdx\cos\left(dx+c\right)^{3}+630\,Cdx\cos\left(dx+c\right)^{2}+420\,Cdx\cos\left(dx+c\right)+105\,Cdx+\left((13\,A+36\right.}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/105*(105*C*d*x*cos(d*x + c)^4 + 420*C*d*x*cos(d*x + c)^3 + 630*C*d*x*cos(
d*x + c)^2 + 420*C*d*x*cos(d*x + c) + 105*C*d*x + ((13*A + 36*B - 260*C)*co
s(d*x + c)^3 + (52*A + 39*B - 620*C)*cos(d*x + c)^2 + (32*A + 24*B - 535*C)
*cos(d*x + c) + 8*A + 6*B - 160*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [A] time = 35.8602, size = 279, normalized size = 1.7

$$\begin{cases} \frac{A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}-\frac{B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}+\frac{3B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+ \\ \frac{x\left(A+B\cos\left(c\right)+C\cos^{2}\left(c\right)\right)\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a*
*4*d) - A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d) -
 B*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*B*tan(c/2 + d*x/2)**5/(40*a**4*d) -
B*tan(c/2 + d*x/2)**3/(8*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d) + C*x/a**4
 + C*tan(c/2 + d*x/2)**7/(56*a**4*d) - C*tan(c/2 + d*x/2)**5/(8*a**4*d) + 1
1*C*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*C*tan(c/2 + d*x/2)/(8*a**4*d), Ne(
d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*cos(c)**2/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.22266, size = 297, normalized size = 1.81

$$\frac{\frac{840\,(dx+c)C}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+63\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,Ca^{24}\tan(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)*C/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*A*a^24*tan(1
/2*d*x + 1/2*c)^5 + 63*B*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*C*a^24*tan(1/2*d
*x + 1/2*c)^5 - 35*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*x +
 1/2*c)^3 + 385*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^24*tan(1/2*d*x + 1/
2*c) + 105*B*a^24*tan(1/2*d*x + 1/2*c) - 1575*C*a^24*tan(1/2*d*x + 1/2*c))/
a^28)/d
```

3.367 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=148

$$\frac{(8A+13B+36C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}+\frac{(23A-2B-54C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{(6A+B-8C}{35ad(a\cos(}$$

```
[Out] ((23*A - 2*B - 54*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + ((8*A
 + 13*B + 36*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)
*Cos[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((6*A + B - 8*
C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.348283, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3041, 2968, 3019, 2750, 2648}

$$\frac{(8A+13B+36C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}+\frac{(23A-2B-54C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{(6A+B-8C}{35ad(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d
*x])^4,x]
```

```
[Out] ((23*A - 2*B - 54*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + ((8*A
 + 13*B + 36*C)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)
*Cos[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((6*A + B - 8*
C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(a(5A+2B-}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{a(5A+2B-2C)\cos(c+}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(6A+B-8C)\sin(}{35ad(a+a\cos(c+}$$

$$= \frac{(23A-2B-54C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^2(c+dx)\sin}{7d(a+a\cos(c+dx)}$$

$$= \frac{(23A-2B-54C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^2(c+dx)\sin}{7d(a+a\cos(c+dx)}$$

Mathematica [A] time = 0.512895, size = 239, normalized size = 1.61

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(4A+5B+18C)\sin\left(c+\frac{dx}{2}\right)+70(2A+4B+9C)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+56A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(2*A + 4*B + 9*C)*Sin[(d*x)/2] - 35*(4*A + 5
*B + 18*C)*Sin[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d
*x)/2] + 441*C*Sin[c + (3*d*x)/2] - 105*B*Sin[2*c + (3*d*x)/2] - 315*C*Sin[
2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*x)/2] +
147*C*Sin[2*c + (5*d*x)/2] - 105*C*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*
d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2] + 36*C*Sin[3*c + (7*d*x)/2]))/(420*a^4*
d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.027, size = 108, normalized size = 0.7

$$\frac{1}{8\,da^4}\left(\frac{-A+B-C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,C-A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A-B-3\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B-C)*tan(1/2*d*x+1/2*c)^7+1/5*(3*C-A-B)*tan(1/2*d*x+1/2*
c)^5+1/3*(A-B-3*C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+
1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.02245, size = 350, normalized size = 2.36

$$\frac{\frac{A\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1}+\frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{B\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1}-\frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3C\left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1}-\frac{35 \sin}{(\cos(dx}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x,
algorithm="maxima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 + 3*C*(35*sin(d*x + c)/(cos(d*x
+ c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.98035, size = 343, normalized size = 2.32

$$\frac{\left((8A+13B+36C)\cos(dx+c)^3+(32A+52B+39C)\cos(dx+c)^2+4(13A+8B+6C)\cos(dx+c)+13A+8B\right.}{105\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] 1/105*((8*A + 13*B + 36*C)*cos(d*x + c)^3 + (32*A + 52*B + 39*C)*cos(d*x +
c)^2 + 4*(13*A + 8*B + 6*C)*cos(d*x + c) + 13*A + 8*B + 6*C)*sin(d*x + c)/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
```

```
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 24.0661, size = 267, normalized size = 1.8

$$\begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} - \\ \frac{x\left(A+B\cos(c)+C\cos^2(c)\right)\cos(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x
)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a
**4*d) + A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
+ B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) - B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d) - C*tan(c/
2 + d*x/2)**7/(56*a**4*d) + 3*C*tan(c/2 + d*x/2)**5/(40*a**4*d) - C*tan(c/2
 + d*x/2)**3/(8*a**4*d) + C*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x*(A +
 B*cos(c) + C*cos(c)**2)*cos(c)/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.16399, size = 231, normalized size = 1.56

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 21\,B\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 15*C*ta
n(1/2*d*x + 1/2*c)^7 + 21*A*tan(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2
*c)^5 - 63*C*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*c)^3 + 35*B*ta
n(1/2*d*x + 1/2*c)^3 + 105*C*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1
/2*c) - 105*B*tan(1/2*d*x + 1/2*c) - 105*C*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.368 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=148

$$\frac{(6A+8B+13C)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(6A+8B+13C)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B-11C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B+C)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] ((A - B + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B - 11*
C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((6*A + 8*B + 13*C)*Sin[
c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) + ((6*A + 8*B + 13*C)*Sin[c +
d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rubi [A] time = 0.175611, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3019, 2750, 2650, 2648}

$$\frac{(6A+8B+13C)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(6A+8B+13C)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B-11C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B+C)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((A - B + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B - 11*
C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((6*A + 8*B + 13*C)*Sin[
c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) + ((6*A + 8*B + 13*C)*Sin[c +
d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{-a(3A+4B-4C)-7aC\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B-11C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(6A+8B+13C}{}$$

$$= \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B-11C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(6A+8B+13C}{105d\left(a^2+a^2 c\right.}$$

$$= \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B-11C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(6A+8B+13C}{105d\left(a^2+a^2 c\right.}$$

Mathematica [A] time = 0.457853, size = 208, normalized size = 1.41

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(70(3A+2B+4C)\sin\left(\frac{dx}{2}\right)+126A\sin\left(c+\frac{3dx}{2}\right)+42A\sin\left(2c+\frac{5dx}{2}\right)+6A\sin\left(3c+\frac{7dx}{2}\right)-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(3*A + 2*B + 4*C)*Sin[(d*x)/2] - 35*(4*B + 5
*C)*Sin[c + (d*x)/2] + 126*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2]
+ 168*C*Sin[c + (3*d*x)/2] - 105*C*Sin[2*c + (3*d*x)/2] + 42*A*Sin[2*c + (5
*d*x)/2] + 56*B*Sin[2*c + (5*d*x)/2] + 91*C*Sin[2*c + (5*d*x)/2] + 6*A*Sin[
3*c + (7*d*x)/2] + 8*B*Sin[3*c + (7*d*x)/2] + 13*C*Sin[3*c + (7*d*x)/2]))/(
420*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.025, size = 106, normalized size = 0.7

$$\frac{1}{8\,da^4}\left(\frac{A-B+C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A-C-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,A+B-C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B+C)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-C-B)*tan(1/2*d*x+1/2*c
)^5+1/3*(3*A+B-C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1
/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.0392, size = 350, normalized size = 2.36

$$\frac{\frac{B\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{C\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin}{(\cos(d}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + C*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x
+ c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.78031, size = 343, normalized size = 2.32

$$\frac{\left((6\,A+8\,B+13\,C)\cos\left(dx+c\right)^{3}+4\,(6\,A+8\,B+13\,C)\cos\left(dx+c\right)^{2}+(39\,A+52\,B+32\,C)\cos\left(dx+c\right)+36\,A+13\,B}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/105*((6*A + 8*B + 13*C)*cos(d*x + c)^3 + 4*(6*A + 8*B + 13*C)*cos(d*x + c
)^2 + (39*A + 52*B + 32*C)*cos(d*x + c) + 36*A + 13*B + 8*C)*sin(d*x + c)/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 16.4384, size = 264, normalized size = 1.78

$$\begin{cases} \frac{A\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}+\frac{3A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}+\frac{A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}-\frac{B\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{B\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}+\frac{B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d}+ \\ \frac{x\left(A+B\cos\left(c\right)+C\cos^{2}\left(c\right)\right)}{\left(a\cos\left(c\right)+a\right)^{4}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40*
a**4*d) + A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
- B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) + B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d) + C*tan(c/
2 + d*x/2)**7/(56*a**4*d) - C*tan(c/2 + d*x/2)**5/(40*a**4*d) - C*tan(c/2 +
 d*x/2)**3/(24*a**4*d) + C*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x*(A +
B*cos(c) + C*cos(c)**2)/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.17779, size = 231, normalized size = 1.56

$$15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 15*C*tan
(1/2*d*x + 1/2*c)^7 + 63*A*tan(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*
c)^5 - 21*C*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2*c)^3 + 35*B*ta
n(1/2*d*x + 1/2*c)^3 - 35*C*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/
2*c) + 105*B*tan(1/2*d*x + 1/2*c) + 105*C*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.369 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=157

$$-\frac{2(80A-3B-4C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B-8C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B-4C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 6*B - 8*C)*Sin[c + d*x])/(105*
a^4*d*(1 + Cos[c + d*x])^2) - (2*(80*A - 3*B - 4*C)*Sin[c + d*x])/(105*a^4*
d*(1 + Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x]
)^4) - ((10*A - 3*B - 4*C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.492343, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3041, 2978, 12, 3770}

$$-\frac{2(80A-3B-4C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B-8C)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B-4C)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d
*x])^4,x]
```


```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 6*B - 8*C)*Sin[c + d*x])/(105*
a^4*d*(1 + Cos[c + d*x])^2) - (2*(80*A - 3*B - 4*C)*Sin[c + d*x])/(105*a^4*
d*(1 + Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x]
)^4) - ((10*A - 3*B - 4*C)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(7aA-a(3A-3B-4C)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B-4C)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \int$$

$$= -\frac{(55A-6B-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(1}{}$$

$$= -\frac{(55A-6B-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(1}{}$$

$$= -\frac{(55A-6B-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(1}{}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-6B-8C)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-}{7d(}$$

Mathematica [B] time = 2.54935, size = 334, normalized size = 2.13

$$-\frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(70(49A-3B-2C)\sin\left(\frac{dx}{2}\right)-70(31A-2C)\sin\left(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos
[c + d*x])^4,x]
```

```
[Out] -((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*(6720*A*Cos[(c + d*x)/2]^8*(Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(70*(49*A - 3*B - 2*C)*Sin[(d*x)/2] - 70*(
31*A - 2*C)*Sin[c + (d*x)/2] + 2625*A*Sin[c + (3*d*x)/2] - 126*B*Sin[c + (3
*d*x)/2] - 168*C*Sin[c + (3*d*x)/2] - 735*A*Sin[2*c + (3*d*x)/2] + 1015*A*S
in[2*c + (5*d*x)/2] - 42*B*Sin[2*c + (5*d*x)/2] - 56*C*Sin[2*c + (5*d*x)/2]
 - 105*A*Sin[3*c + (5*d*x)/2] + 160*A*Sin[3*c + (7*d*x)/2] - 6*B*Sin[3*c +
(7*d*x)/2] - 8*C*Sin[3*c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4*(2
*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.067, size = 277, normalized size = 1.8

$$\frac{B}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{15\,A}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*B*tan(1/2*d*x+1/2*c)-15/8/d/a^4*A*tan(1/2*d*x+1/2*c)-1/56/d/a^4*t
an(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/56/d/a^4*tan(1/2*
d*x+1/2*c)^7*C-1/8/d/a^4*A*tan(1/2*d*x+1/2*c)^5+1/d/a^4*A*ln(tan(1/2*d*x+1/
2*c)+1)-1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-11/24/d/a^4*tan(1/2*d*x+1/2*c)^3
*A+3/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-1/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+1/8/
d/a^4*tan(1/2*d*x+1/2*c)^3*B+1/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*C*
tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 1.41307, size = 423, normalized size = 2.69

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-\frac{C\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c}{(\cos(dx+c)+1}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x,
algorithm="maxima")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - C*(105*sin(d*x + c
)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 -
 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 1.9907, size = 660, normalized size = 4.2

$$\frac{105\left(A\cos(dx+c)^4+4\,A\cos(dx+c)^3+6\,A\cos(dx+c)^2+4\,A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-105\left(A\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] 1/210*(105*(A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*
A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 105*(A*cos(d*x + c)^4 + 4*A*cos
(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + A)*log(-sin(d*x + c)
+ 1) - 2*(2*(80*A - 3*B - 4*C)*cos(d*x + c)^3 + (535*A - 24*B - 32*C)*cos(d
*x + c)^2 + (620*A - 39*B - 52*C)*cos(d*x + c) + 260*A - 36*B - 13*C)*sin(d
*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
 c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**4,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.33225, size = 335, normalized size = 2.13

$$\frac{\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4} - \frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4} - \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 105\,Aa^{24}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] 1/840*(840*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2
*d*x + 1/2*c)^5 - 63*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 21*C*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*x + 1
/2*c)^3 - 35*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*
c) - 105*B*a^24*tan(1/2*d*x + 1/2*c) - 105*C*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

3.370 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=185

$$\frac{2(332A-80B+3C)\tan(c+dx)}{105a^4d}-\frac{(88A-25B-3C)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+}{a^4d(\cos(c+dx)+}$$

```
[Out] -(((4*A - B)*ArcTanh[Sin[c + d*x]])/(a^4*d)) + (2*(332*A - 80*B + 3*C)*Tan[
c + d*x])/(105*a^4*d) - ((88*A - 25*B - 3*C)*Tan[c + d*x])/(105*a^4*d*(1 +
Cos[c + d*x])^2) - ((4*A - B)*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((
A - B + C)*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((12*A - 5*B - 2*C)
*Tan[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.717059, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2748, 3767, 8, 3770}

$$\frac{2(332A-80B+3C)\tan(c+dx)}{105a^4d}-\frac{(88A-25B-3C)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+}{a^4d(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c +
 d*x])^4,x]
```

```
[Out] -(((4*A - B)*ArcTanh[Sin[c + d*x]])/(a^4*d)) + (2*(332*A - 80*B + 3*C)*Tan[
c + d*x])/(105*a^4*d) - ((88*A - 25*B - 3*C)*Tan[c + d*x])/(105*a^4*d*(1 +
Cos[c + d*x])^2) - ((4*A - B)*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((
A - B + C)*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((12*A - 5*B - 2*C)
*Tan[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(8A-B+C)-a(4A-4B-3C)\cos(c+dx)}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B-2C)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} +$$

$$= -\frac{(88A-25B-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{(88A-25B-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{(88A-25B-3C)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} -$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(88A-25B-3C)\tan(c+}{105a^4d(1+\cos(c+dx))^2}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{2(332A-80B+3C)\tan(c}{105a^4d}$$

Mathematica [B] time = 6.40249, size = 1190, normalized size = 6.43

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*C
os[c + d*x])^4,x]
```

```
[Out] ((32*(4*A - B)*Cos[c/2 + (d*x)/2]^8*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] -
 Sin[c/2 + (d*x)/2]]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2))/(d*(1 + Cos[c
 + d*x])^4*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (32*(4*A -
B)*Cos[c/2 + (d*x)/2]^8*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (
d*x)/2]]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2))/(d*(1 + Cos[c + d*x])^4*(
2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) + (4*Cos[c/2 + (d*x)/2]^2
*Cos[c + d*x]^2*Sec[c/2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*(A*Sin[c/2
] - B*Sin[c/2] + C*Sin[c/2]))/(7*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[
c + d*x] + C*Cos[2*c + 2*d*x])) + (8*Cos[c/2 + (d*x)/2]^4*Cos[c + d*x]^2*Se
c[c/2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*(17*A*Sin[c/2] - 10*B*Sin[c/
2] + 3*C*Sin[c/2]))/(35*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[c + d*x]
+ C*Cos[2*c + 2*d*x])) + (16*Cos[c/2 + (d*x)/2]^6*Cos[c + d*x]^2*Sec[c/2]*(
```

```
C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*(139*A*Sin[c/2] - 55*B*Sin[c/2] + 6*
C*Sin[c/2]))/(105*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[c + d*x] + C*Co
s[2*c + 2*d*x])) + (4*Cos[c/2 + (d*x)/2]*Cos[c + d*x]^2*Sec[c/2]*(C + B*Sec
[c + d*x] + A*Sec[c + d*x]^2)*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x
)/2]))/(7*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c +
2*d*x])) + (8*Cos[c/2 + (d*x)/2]^3*Cos[c + d*x]^2*Sec[c/2]*(C + B*Sec[c + d
*x] + A*Sec[c + d*x]^2)*(17*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2] + 3*C*Sin[(d
*x)/2]))/(35*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c
 + 2*d*x])) + (32*Cos[c/2 + (d*x)/2]^7*Cos[c + d*x]^2*Sec[c/2]*(C + B*Sec[c
 + d*x] + A*Sec[c + d*x]^2)*(559*A*Sin[(d*x)/2] - 160*B*Sin[(d*x)/2] + 6*C*
Sin[(d*x)/2]))/(105*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[c + d*x] + C*
Cos[2*c + 2*d*x])) + (16*Cos[c/2 + (d*x)/2]^5*Cos[c + d*x]^2*Sec[c/2]*(C +
B*Sec[c + d*x] + A*Sec[c + d*x]^2)*(139*A*Sin[(d*x)/2] - 55*B*Sin[(d*x)/2]
+ 6*C*Sin[(d*x)/2]))/(105*d*(1 + Cos[c + d*x])^4*(2*A + C + 2*B*Cos[c + d*x
] + C*Cos[2*c + 2*d*x])) + (32*A*Cos[c/2 + (d*x)/2]^8*Cos[c + d*x]*Sec[c]*(
C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x])/(d*(1 + Cos[c + d*x])^4*(2
*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])))/a^4
```

Maple [B] time = 0.077, size = 363, normalized size = 2.

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+1/56/d/
a^4*tan(1/2*d*x+1/2*c)^7*C+7/40/d/a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*ta
n(1/2*d*x+1/2*c)^5+3/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4*tan(1/2*d*
x+1/2*c)^3*A-11/24/d/a^4*tan(1/2*d*x+1/2*c)^3*B+1/8/d/a^4*C*tan(1/2*d*x+1/2
*c)^3+49/8/d/a^4*A*tan(1/2*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+1/8/d
/a^4*C*tan(1/2*d*x+1/2*c)+4/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4*ln(tan
(1/2*d*x+1/2*c)-1)*B-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-4/d/a^4*A*ln(tan(1/2*
d*x+1/2*c)+1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4*A/(tan(1/2*d*x+1/2
*c)+1)
```

Maxima [B] time = 1.32959, size = 555, normalized size = 3.

$$A\left(\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) + 3*C*(35*sin(d*x + c)/(cos(d
*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 2.07873, size = 873, normalized size = 4.72

$$-\frac{105\left((4A-B)\cos(dx+c)^5+4(4A-B)\cos(dx+c)^4+6(4A-B)\cos(dx+c)^3+4(4A-B)\cos(dx+c)^2+(4A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] -1/210*(105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*x + c)^4 + 6*(4*A
 - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A - B)*cos(d*x + c))
*log(sin(d*x + c) + 1) - 105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*
x + c)^4 + 6*(4*A - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A -
 B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(332*A - 80*B + 3*C)*cos(d*
x + c)^4 + (2236*A - 535*B + 24*C)*cos(d*x + c)^3 + (2636*A - 620*B + 39*C)
*cos(d*x + c)^2 + 4*(296*A - 65*B + 9*C)*cos(d*x + c) + 105*A)*sin(d*x + c)
```

```
)/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 +
 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29042, size = 392, normalized size = 2.12

$$-\frac{\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] -1/840*(840*(4*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(4*A - B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 1680*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a
^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*
tan(1/2*d*x + 1/2*c)^5 - 105*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 63*C*a^24*tan(
1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2
*d*x + 1/2*c)^3 + 105*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d
*x + 1/2*c) - 1575*B*a^24*tan(1/2*d*x + 1/2*c) + 105*C*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

3.371 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=248

$$-\frac{8(216A-83B+20C)\tan(c+dx)}{105a^4d}+\frac{(21A-8B+2C)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B+2C)\tan(c+dx)\sec(c+dx)}{2a^4d}$$

```
[Out] ((21*A - 8*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(216*A - 83*B + 2
0*C)*Tan[c + d*x])/(105*a^4*d) + ((21*A - 8*B + 2*C)*Sec[c + d*x]*Tan[c + d
*x])/(2*a^4*d) - ((129*A - 52*B + 10*C)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4
*d*(1 + Cos[c + d*x])^2) - (4*(216*A - 83*B + 20*C)*Sec[c + d*x]*Tan[c + d*
x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x]
)/(7*d*(a + a*Cos[c + d*x])^4) - ((2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*a
*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.761248, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3041, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{8(216A-83B+20C)\tan(c+dx)}{105a^4d}+\frac{(21A-8B+2C)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B+2C)\tan(c+dx)\sec(c+dx)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c +
 d*x])^4,x]
```


```
[Out] ((21*A - 8*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(216*A - 83*B + 2
0*C)*Tan[c + d*x])/(105*a^4*d) + ((21*A - 8*B + 2*C)*Sec[c + d*x]*Tan[c + d
*x])/(2*a^4*d) - ((129*A - 52*B + 10*C)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4
*d*(1 + Cos[c + d*x])^2) - (4*(216*A - 83*B + 20*C)*Sec[c + d*x]*Tan[c + d*
x])/(105*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x]
)/(7*d*(a + a*Cos[c + d*x])^4) - ((2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*a
*d*(a + a*Cos[c + d*x])^3)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(9A-2B+2C)-a(5A-5}{(a+a}}{}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2A-B)\sec(c+dx)}{5ad(a+a\cos(c}$$

$$= -\frac{(129A-52B+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)}{7d(a}$$

$$= -\frac{(129A-52B+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)}{7d(a}$$

$$= -\frac{(129A-52B+10C)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)}{7d(a}$$

$$= \frac{(21A-8B+2C)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(129A-52B+1}{105a^4d}$$

$$= \frac{(21A-8B+2C)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{8(216A-83B+20C}{105a^4d}$$

Mathematica [A] time = 1.47684, size = 271, normalized size = 1.09

$$-\frac{13440(21A-8B+2C)\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*C
os[c + d*x])^4,x]
```

```
[Out] -(13440*(21*A - 8*B + 2*C)*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*Cos[(c + d*x)/
2]*(58161*A - 22888*B + 5290*C + 8*(12813*A - 4994*B + 1130*C)*Cos[c + d*x]
 + 60*(1177*A - 456*B + 106*C)*Cos[2*(c + d*x)] + 35928*A*Cos[3*(c + d*x)]
- 13864*B*Cos[3*(c + d*x)] + 3280*C*Cos[3*(c + d*x)] + 11619*A*Cos[4*(c + d
*x)] - 4472*B*Cos[4*(c + d*x)] + 1070*C*Cos[4*(c + d*x)] + 1728*A*Cos[5*(c
```

```
+ d*x)] - 664*B*Cos[5*(c + d*x)] + 160*C*Cos[5*(c + d*x)])*Sec[c + d*x]^2*S
in[(c + d*x)/2])/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [B] time = 0.086, size = 493, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/56/d
/a^4*tan(1/2*d*x+1/2*c)^7*C-9/40/d/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*
tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4*tan(1/2*d*
x+1/2*c)^3*A+23/24/d/a^4*tan(1/2*d*x+1/2*c)^3*B-11/24/d/a^4*C*tan(1/2*d*x+1
/2*c)^3-111/8/d/a^4*A*tan(1/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-15
/8/d/a^4*C*tan(1/2*d*x+1/2*c)+9/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/(t
an(1/2*d*x+1/2*c)-1)*B-21/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)+4/d/a^4*ln(tan
(1/2*d*x+1/2*c)-1)*B-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^4*A/(tan(1/
2*d*x+1/2*c)-1)^2+9/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4/(tan(1/2*d*x+1
/2*c)+1)*B+21/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-4/d/a^4*ln(tan(1/2*d*x+1/2
*c)+1)*B+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)
+1)^2
```

Maxima [B] time = 1.4234, size = 751, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
```

```
1)/a^4) - B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)
+ 5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 1
68*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [A] time = 2.16491, size = 1062, normalized size = 4.28

$$\frac{105\left((21\,A-8\,B+2\,C)\cos\left(dx+c\right)^{6}+4\,(21\,A-8\,B+2\,C)\cos\left(dx+c\right)^{5}+6\,(21\,A-8\,B+2\,C)\cos\left(dx+c\right)^{4}+4\,(21\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/420*(105*((21*A - 8*B + 2*C)*cos(d*x + c)^6 + 4*(21*A - 8*B + 2*C)*cos(d*
x + c)^5 + 6*(21*A - 8*B + 2*C)*cos(d*x + c)^4 + 4*(21*A - 8*B + 2*C)*cos(d
*x + c)^3 + (21*A - 8*B + 2*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*
((21*A - 8*B + 2*C)*cos(d*x + c)^6 + 4*(21*A - 8*B + 2*C)*cos(d*x + c)^5 +
6*(21*A - 8*B + 2*C)*cos(d*x + c)^4 + 4*(21*A - 8*B + 2*C)*cos(d*x + c)^3 +
 (21*A - 8*B + 2*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(16*(216*A -
 83*B + 20*C)*cos(d*x + c)^5 + (11619*A - 4472*B + 1070*C)*cos(d*x + c)^4 +
 4*(3411*A - 1318*B + 310*C)*cos(d*x + c)^3 + 4*(1509*A - 592*B + 130*C)*co
s(d*x + c)^2 + 210*(2*A - B)*cos(d*x + c) - 105*A)*sin(d*x + c))/(a^4*d*cos
(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25808, size = 458, normalized size = 1.85

$$\frac{420\,(21\,A-8\,B+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(21\,A-8\,B+2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,A\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(420*(21*A - 8*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*
(21*A - 8*B + 2*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*A*tan(1/
2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 7*A*tan(1/2*d*x + 1/2*c) +
2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24
*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(
1/2*d*x + 1/2*c)^7 + 189*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 105*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d
*x + 1/2*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 385*C*a^24*tan(1/2*d*x
+ 1/2*c)^3 + 11655*A*a^24*tan(1/2*d*x + 1/2*c) - 5145*B*a^24*tan(1/2*d*x +
1/2*c) + 1575*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.372 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=287

$$\frac{4(454A-216B+83C)\tan^3(c+dx)}{105a^4d}+\frac{4(454A-216B+83C)\tan(c+dx)}{35a^4d}-\frac{(44A-21B+8C)\tanh^{-1}(\sin(c+dx))}{2a^4d}-$$

```
[Out] -((44*A - 21*B + 8*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + (4*(454*A - 216*B
+ 83*C)*Tan[c + d*x])/(35*a^4*d) - ((44*A - 21*B + 8*C)*Sec[c + d*x]*Tan[c
+ d*x])/(2*a^4*d) - ((178*A - 87*B + 31*C)*Sec[c + d*x]^2*Tan[c + d*x])/(10
5*a^4*d*(1 + Cos[c + d*x])^2) - ((44*A - 21*B + 8*C)*Sec[c + d*x]^2*Tan[c +
 d*x])/(3*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d
*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((16*A - 9*B + 2*C)*Sec[c + d*x]^2*Tan[
c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (4*(454*A - 216*B + 83*C)*Tan[c
 + d*x]^3)/(105*a^4*d)
```

Rubi [A] time = 0.798291, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2748, 3767, 3768, 3770}

$$\frac{4(454A-216B+83C)\tan^3(c+dx)}{105a^4d}+\frac{4(454A-216B+83C)\tan(c+dx)}{35a^4d}-\frac{(44A-21B+8C)\tanh^{-1}(\sin(c+dx))}{2a^4d}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c +
 d*x])^4,x]
```


```
[Out] -((44*A - 21*B + 8*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + (4*(454*A - 216*B
+ 83*C)*Tan[c + d*x])/(35*a^4*d) - ((44*A - 21*B + 8*C)*Sec[c + d*x]*Tan[c
+ d*x])/(2*a^4*d) - ((178*A - 87*B + 31*C)*Sec[c + d*x]^2*Tan[c + d*x])/(10
5*a^4*d*(1 + Cos[c + d*x])^2) - ((44*A - 21*B + 8*C)*Sec[c + d*x]^2*Tan[c +
 d*x])/(3*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d
*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((16*A - 9*B + 2*C)*Sec[c + d*x]^2*Tan[
c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (4*(454*A - 216*B + 83*C)*Tan[c
 + d*x]^3)/(105*a^4*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(10A-3B+3C)-a(6A}{(a+}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(16A-9B+2C)\,\mathrm{se}}{35ad(a+a}$$

$$= -\frac{(178A-87B+31C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C}{7d(}$$

$$= -\frac{(178A-87B+31C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C}{7d(}$$

$$= -\frac{(178A-87B+31C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C}{7d(}$$

$$= -\frac{(44A-21B+8C)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(178A-87B+}{105a}$$

$$= -\frac{(44A-21B+8C)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{4(454A-216B+}{35}$$

Mathematica [A] time = 1.65462, size = 304, normalized size = 1.06

$$\frac{26880(44A-21B+8C)\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*C
os[c + d*x])^4,x]
```

```
[Out] (26880*(44*A - 21*B + 8*C)*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*Cos[(c + d*x)/
2]*(217696*A - 102504*B + 39952*C + 14*(28252*A - 13353*B + 5224*C)*Cos[c +
 d*x] + 56*(5218*A - 2472*B + 961*C)*Cos[2*(c + d*x)] + 173316*A*Cos[3*(c +
 d*x)] - 82239*B*Cos[3*(c + d*x)] + 31832*C*Cos[3*(c + d*x)] + 79264*A*Cos[
4*(c + d*x)] - 37656*B*Cos[4*(c + d*x)] + 14528*C*Cos[4*(c + d*x)] + 24436*
A*Cos[5*(c + d*x)] - 11619*B*Cos[5*(c + d*x)] + 4472*C*Cos[5*(c + d*x)] + 3
632*A*Cos[6*(c + d*x)] - 1728*B*Cos[6*(c + d*x)] + 664*C*Cos[6*(c + d*x)])*
```

```
Sec[c + d*x]^3*Sin[(c + d*x)/2])/(3360*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [B] time = 0.092, size = 626, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)^2*B-1/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2*B+9
/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)*B+9/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B-21/2/d
/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B+21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a
^4/(tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^3-1/d/a^4/(t
an(1/2*d*x+1/2*c)+1)*C-1/3/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)^3+4/d/a^4*ln(tan(
1/2*d*x+1/2*c)-1)*C-5/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^2-4/d/a^4*ln(tan(1/2
*d*x+1/2*c)+1)*C+5/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)^2-1/56/d/a^4*tan(1/2*d*
x+1/2*c)^7*B-9/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4*tan(1/2*d*x+1/2*c
)^3*B-111/8/d/a^4*B*tan(1/2*d*x+1/2*c)-13/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-13
/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-22/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)+22/d/a^
4*A*ln(tan(1/2*d*x+1/2*c)-1)+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*t
an(1/2*d*x+1/2*c)^7*C+11/40/d/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*C*tan(1
/2*d*x+1/2*c)^5+59/24/d/a^4*tan(1/2*d*x+1/2*c)^3*A+23/24/d/a^4*C*tan(1/2*d*
x+1/2*c)^3+209/8/d/a^4*A*tan(1/2*d*x+1/2*c)+49/8/d/a^4*C*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 1.40792, size = 932, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] 1/840*(A*(560*(27*sin(d*x + c)/(cos(d*x + c) + 1) - 62*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 39*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4 - 3*a^4*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (21945*sin(d*x + c)/(cos(d*x
+ c) + 1) + 2065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 231*sin(d*x + c)^5/(
```

```
cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 18480*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 18480*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^4) - 3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(c
os(d*x + c) + 1) + 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 294
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/a^4) + C*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c
) + 1) + 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a^4))/d
```

Fricas [A] time = 2.18403, size = 1135, normalized size = 3.95

$$-\frac{105\left((44\,A-21\,B+8\,C)\cos\left(dx+c\right)^{7}+4\,(44\,A-21\,B+8\,C)\cos\left(dx+c\right)^{6}+6\,(44\,A-21\,B+8\,C)\cos\left(dx+c\right)^{5}+4\,(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] -1/420*(105*((44*A - 21*B + 8*C)*cos(d*x + c)^7 + 4*(44*A - 21*B + 8*C)*cos
(d*x + c)^6 + 6*(44*A - 21*B + 8*C)*cos(d*x + c)^5 + 4*(44*A - 21*B + 8*C)*
cos(d*x + c)^4 + (44*A - 21*B + 8*C)*cos(d*x + c)^3)*log(sin(d*x + c) + 1)
- 105*((44*A - 21*B + 8*C)*cos(d*x + c)^7 + 4*(44*A - 21*B + 8*C)*cos(d*x +
 c)^6 + 6*(44*A - 21*B + 8*C)*cos(d*x + c)^5 + 4*(44*A - 21*B + 8*C)*cos(d*
x + c)^4 + (44*A - 21*B + 8*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(
16*(454*A - 216*B + 83*C)*cos(d*x + c)^6 + (24436*A - 11619*B + 4472*C)*cos
(d*x + c)^5 + 4*(7184*A - 3411*B + 1318*C)*cos(d*x + c)^4 + 4*(3196*A - 150
9*B + 592*C)*cos(d*x + c)^3 + 70*(14*A - 6*B + 3*C)*cos(d*x + c)^2 - 35*(4*
A - 3*B)*cos(d*x + c) + 70*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^7 + 4*a^4*d
*cos(d*x + c)^6 + 6*a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + a^4*d*c
os(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2168, size = 549, normalized size = 1.91

$$-\frac{\frac{420\,(44\,A-21\,B+8\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(44\,A-21\,B+8\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{280\left(78\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\,\mathrm{t}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] -1/840*(420*(44*A - 21*B + 8*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 42
0*(44*A - 21*B + 8*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 280*(78*A*ta
n(1/2*d*x + 1/2*c)^5 - 27*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*
c)^5 - 124*A*tan(1/2*d*x + 1/2*c)^3 + 48*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*ta
n(1/2*d*x + 1/2*c)^3 + 54*A*tan(1/2*d*x + 1/2*c) - 21*B*tan(1/2*d*x + 1/2*c
) + 6*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4) - (15*A*
a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*
tan(1/2*d*x + 1/2*c)^7 + 231*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 189*B*a^24*tan
(1/2*d*x + 1/2*c)^5 + 147*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 2065*A*a^24*tan(1
/2*d*x + 1/2*c)^3 - 1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 805*C*a^24*tan(1/2
*d*x + 1/2*c)^3 + 21945*A*a^24*tan(1/2*d*x + 1/2*c) - 11655*B*a^24*tan(1/2*
d*x + 1/2*c) + 5145*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.373 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=239

$$\frac{2a(99A+88B+80C)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4(99A+88B+80C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+88B}{}$$

```
[Out] (4*a*(99*A + 88*B + 80*C)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*a*(99*A + 88*B + 80*C)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*a*(11*B + C)*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a +
a*Cos[c + d*x]]) - (8*(99*A + 88*B + 80*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(3465*d) + (2*C*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(11*d) + (4*(99*A + 88*B + 80*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])
/(1155*a*d)
```

Rubi [A] time = 0.546978, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3045, 2981, 2770, 2759, 2751, 2646}

$$\frac{2a(99A+88B+80C)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4(99A+88B+80C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+88B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (4*a*(99*A + 88*B + 80*C)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*a*(99*A + 88*B + 80*C)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*a*(11*B + C)*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a +
a*Cos[c + d*x]]) - (8*(99*A + 88*B + 80*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(3465*d) + (2*C*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(11*d) + (4*(99*A + 88*B + 80*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])
/(1155*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+}{11d}$$

$$= \frac{2a(11B+C)\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2C}{}$$

$$= \frac{2a(99A+88B+80C)\cos^3(c+dx)\sin(c+}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a(99A+88B+80C)\cos^3(c+dx)\sin(c+}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a(99A+88B+80C)\cos^3(c+dx)\sin(c+}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{4a(99A+88B+80C)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a(99}{}$$

Mathematica [A] time = 1.25331, size = 145, normalized size = 0.61

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(9306A+8272B+9095C)\cos(c+dx)+8(594A+913B+830C)\cos(2(c+dx))\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(30096*A + 29062*B + 26420*C + 2*(9306*A + 8272
*B + 9095*C)*Cos[c + d*x] + 8*(594*A + 913*B + 830*C)*Cos[2*(c + d*x)] + 19
80*A*Cos[3*(c + d*x)] + 1760*B*Cos[3*(c + d*x)] + 3175*C*Cos[3*(c + d*x)] +
 770*B*Cos[4*(c + d*x)] + 700*C*Cos[4*(c + d*x)] + 315*C*Cos[5*(c + d*x)])*
Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 0.071, size = 152, normalized size = 0.6

$$\frac{2a\sqrt{2}}{3465d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-10080\,C\,(\sin(1/2\,dx+c/2))^{10}+(6160\,B+30800\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}+(-3960\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 2/3465*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-10080*C*sin(1/2*d*x+1/2*c)
^10+(6160*B+30800*C)*sin(1/2*d*x+1/2*c)^8+(-3960*A-15840*B-39600*C)*sin(1/2
*d*x+1/2*c)^6+(8316*A+16632*B+27720*C)*sin(1/2*d*x+1/2*c)^4+(-6930*A-9240*B
-11550*C)*sin(1/2*d*x+1/2*c)^2+3465*A+3465*B+3465*C)*2^(1/2)/(a*cos(1/2*d*x
+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.80573, size = 320, normalized size = 1.34

$$\frac{396\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+22\left(35\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] 1/55440*(396*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*
c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) + 22*(35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/
2*c) + 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 1890*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + 5*(63*sqrt(2)*sin(11/2*d*x
 + 11/2*c) + 77*sqrt(2)*sin(9/2*d*x + 9/2*c) + 495*sqrt(2)*sin(7/2*d*x + 7/
2*c) + 693*sqrt(2)*sin(5/2*d*x + 5/2*c) + 2310*sqrt(2)*sin(3/2*d*x + 3/2*c)
 + 6930*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.91414, size = 363, normalized size = 1.52

$$\frac{2\left(315\,C\cos(dx+c)^{5}+35\,(11\,B+10\,C)\cos(dx+c)^{4}+5\,(99\,A+88\,B+80\,C)\cos(dx+c)^{3}+6\,(99\,A+88\,B+80\,C)\,c\right.}{3465\,(d\cos(dx\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*C*cos(d*x + c)^5 + 35*(11*B + 10*C)*cos(d*x + c)^4 + 5*(99*A +
88*B + 80*C)*cos(d*x + c)^3 + 6*(99*A + 88*B + 80*C)*cos(d*x + c)^2 + 8*(99
*A + 88*B + 80*C)*cos(d*x + c) + 1584*A + 1408*B + 1280*C)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
cos(d*x + c)^3, x)
```

3.374 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=193

$$\frac{2(21A+18B+16C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+18B+16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a(21A}{45}$$

```
[Out] (2*a*(21*A + 18*B + 16*C)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(9*B + C)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]])
- (4*(21*A + 18*B + 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) +
(2*C*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(21*A
 + 18*B + 16*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.466658, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3045, 2981, 2759, 2751, 2646}

$$\frac{2(21A+18B+16C)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+18B+16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a(21A}{45}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*a*(21*A + 18*B + 16*C)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(9*B + C)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]])
- (4*(21*A + 18*B + 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) +
(2*C*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(21*A
 + 18*B + 16*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c}{9d}$$
$$= \frac{2a(9B+C)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$
$$= \frac{2a(9B+C)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$
$$= \frac{2a(9B+C)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} - \frac{4}{}$$
$$= \frac{2a(21A+18B+16C)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a(9}{}$$

Mathematica [A] time = 0.693961, size = 114, normalized size = 0.59

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((672A+94(9B+8C))\cos(c+dx)+4(63A+54B+83C)\cos(2(c+dx))+1596A}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(1596*A + 1368*B + 1321*C + (672*A + 94*(9*B +
8*C))*Cos[c + d*x] + 4*(63*A + 54*B + 83*C)*Cos[2*(c + d*x)] + 90*B*Cos[3*(
c + d*x)] + 80*C*Cos[3*(c + d*x)] + 35*C*Cos[4*(c + d*x)])*Tan[(c + d*x)/2]
)/(1260*d)
```

Maple [A] time = 0.074, size = 130, normalized size = 0.7

$$\frac{2a\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560C(\sin(1/2dx+c/2))^8+(-360B-1440C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(252A+756B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*C*sin(1/2*d*x+1/2*c)^8+(
-360*B-1440*C)*sin(1/2*d*x+1/2*c)^6+(252*A+756*B+1512*C)*sin(1/2*d*x+1/2*c)
^4+(-420*A-630*B-840*C)*sin(1/2*d*x+1/2*c)^2+315*A+315*B+315*C)*2^(1/2)/(a*
cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.07014, size = 262, normalized size = 1.36

$$\frac{84\left(3\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+18\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] 1/2520*(84*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
 + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 18*(5*sqrt(2)*sin(7/2*d*x +
 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + (35*sqrt(2)*sin(9/2*d*x + 9
/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c) + 252*sqrt(2)*sin(5/2*d*x + 5/2*c)
+ 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 1890*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*s
qrt(a))/d
```

Fricas [A] time = 1.90968, size = 298, normalized size = 1.54

$$\frac{2\left(35\,C\cos\left(dx+c\right)^{4}+5\left(9\,B+8\,C\right)\cos\left(dx+c\right)^{3}+3\left(21\,A+18\,B+16\,C\right)\cos\left(dx+c\right)^{2}+4\left(21\,A+18\,B+16\,C\right)\cos\left(dx\right.\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*C*cos(d*x + c)^4 + 5*(9*B + 8*C)*cos(d*x + c)^3 + 3*(21*A + 18*B
+ 16*C)*cos(d*x + c)^2 + 4*(21*A + 18*B + 16*C)*cos(d*x + c) + 168*A + 144*
B + 128*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
cos(d*x + c)^2, x)
```

3.375 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos$

Optimal. Leaf size=147

$$\frac{2(35A-14B+18C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(35A+49B+27C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7B+C)\sin(c+dx)(a\cos(c}{35ad}$$

```
[Out] (2*a*(35*A + 49*B + 27*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*(35*A - 14*B + 18*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2
*C*Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (2*(7*B +
C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.347413, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3045, 2968, 3023, 2751, 2646}

$$\frac{2(35A-14B+18C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(35A+49B+27C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7B+C)\sin(c+dx)(a\cos(c}{35ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (2*a*(35*A + 49*B + 27*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*(35*A - 14*B + 18*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2
*C*Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (2*(7*B +
C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+}{7d}$$

$$= \frac{2C\cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+}{7d}$$

$$= \frac{2C\cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+}{7d}$$

$$= \frac{2(35A-14B+18C)\sqrt{a+a\cos(c+dx)}\sin}{105d}$$

$$= \frac{2a(35A+49B+27C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(35A}{}$$

Mathematica [A] time = 0.407407, size = 86, normalized size = 0.59

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((140A+112B+141C)\cos(c+dx)+280A+6(7B+6C)\cos(2(c+dx))+266B+1}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(280*A + 266*B + 228*C + (140*A + 112*B + 141*C
)*Cos[c + d*x] + 6*(7*B + 6*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Ta
n[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.069, size = 108, normalized size = 0.7

$$\frac{2a\sqrt{2}}{105d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-120C(\sin(1/2dx+c/2))^6+(84B+252C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+(-70A-140B-21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 2/105*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-120*C*sin(1/2*d*x+1/2*c)^6+
(84*B+252*C)*sin(1/2*d*x+1/2*c)^4+(-70*A-140*B-210*C)*sin(1/2*d*x+1/2*c)^2+
105*A+105*B+105*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.26533, size = 205, normalized size = 1.39

$$140\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+14\left(3\,\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\,\sqrt{2}\sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/420*(140*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*
A*sqrt(a) + 14*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/
2*c) + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + 3*(5*sqrt(2)*sin(7/2*d*
x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*
c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.80892, size = 238, normalized size = 1.62

$$\frac{2\left(15\,C\cos\left(dx+c\right)^{3}+3\,(7\,B+6\,C)\cos\left(dx+c\right)^{2}+(35\,A+28\,B+24\,C)\cos\left(dx+c\right)+70\,A+56\,B+48\,C\right)\sqrt{a\cos\left(dx\right.}}{105\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/105*(15*C*cos(d*x + c)^3 + 3*(7*B + 6*C)*cos(d*x + c)^2 + (35*A + 28*B +
24*C)*cos(d*x + c) + 70*A + 56*B + 48*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.376 $\int \sqrt{a + a\cos(c+dx)} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$

Optimal. Leaf size=104

$$\frac{2a(15A+5B+7C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}} + \frac{2(5B-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d} + \frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] (2*a*(15*A + 5*B + 7*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
(5*B - 2*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.148088, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3023, 2751, 2646}

$$\frac{2a(15A+5B+7C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}} + \frac{2(5B-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d} + \frac{2C\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(15*A + 5*B + 7*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
(5*B - 2*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{2\int\sqrt{a+a\cos(}{}$$
$$= \frac{2(5B-2C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2C(a+a}{}$$
$$= \frac{2a(15A+5B+7C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2(5B-2C)\sqrt{a+a\,c}}{1}$$

Mathematica [A] time = 0.171688, size = 67, normalized size = 0.64

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(30A+2(5B+4C)\cos(c+dx)+20B+3C\cos(2(c+dx))+19C)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(30*A + 20*B + 19*C + 2*(5*B + 4*C)*Cos[c + d*x
] + 3*C*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.07, size = 86, normalized size = 0.8

$$\frac{2\,a\sqrt{2}}{15\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12\,C\,(\sin(1/2\,dx+c/2))^4+(-10\,B-20\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+15\,A+15\,B+15\,C\right)\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*C*sin(1/2*d*x+1/2*c)^4+(-1
0*B-20*C)*sin(1/2*d*x+1/2*c)^2+15*A+15*B+15*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

Maxima [A] time = 2.13371, size = 143, normalized size = 1.38

$$\frac{60\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}+\left(3\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sqrt{2}\sin\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] 1/30*(60*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + 10*(sqrt(2)*sin(3/2*d*x +
 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + (3*sqrt(2)*sin(5/2*d*
x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 30*sqrt(2)*sin(1/2*d*x + 1/2*
c))*C*sqrt(a))/d
```

Fricas [A] time = 1.85852, size = 180, normalized size = 1.73

$$\frac{2\left(3\,C\cos\left(dx+c\right)^{2}+\left(5\,B+4\,C\right)\cos\left(dx+c\right)+15\,A+10\,B+8\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/15*(3*C*cos(d*x + c)^2 + (5*B + 4*C)*cos(d*x + c) + 15*A + 10*B + 8*C)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.377 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(3B+C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*(3*B + C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.274986, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3045, 2981, 2773, 206}

$$\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(3B+C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*(3*B + C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2\int\sqrt{a}\ldots}{\ldots}$$

$$=\frac{2a(3B+C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c\ldots}}{3\ldots}$$

$$=\frac{2a(3B+C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c\ldots}}{3\ldots}$$

$$=\frac{2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a(3B+C\ldots}{3d\sqrt{a+a\ldots}}$$

Mathematica [A] time = 0.186021, size = 84, normalized size = 0.84

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)(3B+C\cos(c+dx)+2C)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*A*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]] + 2*(3*B + 2*C + C*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] time = 0.197, size = 272, normalized size = 2.7

$$\frac{1}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4C\sqrt{2}\sqrt{a(\sin(1/2\,dx+c/2))^2}\sqrt{a}(\sin(1/2\,dx+c/2))^2+3A\ln\left(4\frac{a\sqrt{2}\cos(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/3/a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+3*A*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+6*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+6*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.91459, size = 77, normalized size = 0.77

$$\frac{6\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)C\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="maxima")
```

```
[Out] 1/3*(6*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*sin(3/2*d*x + 3/2*
c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 2.04989, size = 383, normalized size = 3.83

$$\frac{3\left(A\cos\left(dx+c\right)+A\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(C\cos\left(dx+c\right)+3\,B+\right.}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(C*cos(d*x + c) + 3*B + 2*C)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.35958, size = 263, normalized size = 2.63

$$\frac{\frac{3\,Aa^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(3\,\sqrt{2}Ba^2+3\,\sqrt{2}Ca^2+\left(3\,\sqrt{2}Ba^2+\sqrt{2}Ca^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="giac")
```

```
[Out] 1/3*(3*A*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))
/abs(a) + 2*(3*sqrt(2)*B*a^2 + 3*sqrt(2)*C*a^2 + (3*sqrt(2)*B*a^2 + sqrt(2)
*C*a^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c
)^2 + a)^(3/2))/d
```

3.378 $\int \sqrt{a + a\cos(c + dx)} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) dx) \, dx$

Optimal. Leaf size=98

$$\frac{\sqrt{a}(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} - \frac{a(A-2C)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}} + \frac{A\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d - (a*(A - 2*C)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a +
 a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.305684, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {3043, 2981, 2773, 206}

$$\frac{\sqrt{a}(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} - \frac{a(A-2C)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}} + \frac{A\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d - (a*(A - 2*C)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a +
 a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d}+\frac{\int\sqrt{\ldots}}{\ldots}$$

$$= -\frac{a(A-2C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{A\sqrt{a+a\cos(\ldots}}{\ldots}$$

$$= -\frac{a(A-2C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{A\sqrt{a+a\cos(\ldots}}{\ldots}$$

$$= \frac{\sqrt{a}(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a(\ldots}{d\ldots}$$

Mathematica [A] time = 0.280807, size = 95, normalized size = 0.97

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(A+2B)\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx\ldots\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(A + 2*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 2*C*Cos[c + d*x])
*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.201, size = 694, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-2*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a))*a-2*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*a-4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-4*B*ln(-4/(-2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-8*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+A*ln(-4/(-2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2
)*cos(1/2*d*x+1/2*c)+2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*C*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/
2)/d
```

Maxima [B] time = 1.98674, size = 987, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="maxima")
```

```
[Out] 1/4*(8*sqrt(2)*C*sqrt(a)*sin(1/2*d*x + 1/2*c) - (4*sqrt(2)*cos(5/2*d*x + 5/
2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4
*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
qrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c
) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
4*sqrt(2)*sin(3/2*d*x + 3/2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 2.25244, size = 435, normalized size = 4.44

$$\frac{\left((A+2\,B)\cos\left(dx+c\right)^{2}+(A+2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((A + 2*B)*cos(d*x + c)^2 + (A + 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
```

```
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2
*C*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)
^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.11934, size = 396, normalized size = 4.04

$$\frac{\frac{4\sqrt{2}Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} + \left(A\sqrt{a}+2B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)-\left(A\sqrt{a}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*C*a*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
+ (A*sqrt(a) + 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (A*sqrt(a) + 2*B*sqr
t(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(3/2) - A*a^(5/2))/((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.379 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{a}(3A+4B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] (Sqrt[a]*(3*A + 4*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(4*d) + (a*(A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
 + (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.347986, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {3043, 2980, 2773, 206}

$$\frac{\sqrt{a}(3A+4B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] (Sqrt[a]*(3*A + 4*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(4*d) + (a*(A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
 + (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{a(A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+}{}$$

$$= \frac{a(A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sqrt{a+a\cos(c+}{}$$

$$= \frac{\sqrt{a}(3A+4B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}$$

Mathematica [A] time = 0.544685, size = 111, normalized size = 0.95

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(3A+4B+8C)\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(3*A +
 4*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*A + (3*
A + 4*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.215, size = 1376, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(3*A*ln(-4/(-2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))+4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+4*B*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4
-4*(3*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+4*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))*a+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+3*A*ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
```

```
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*a))*a+10*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a+8*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+8*C*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 20.8168, size = 4525, normalized size = 38.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
cos(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
```

)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*sin(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*

```
cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 1
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1) - 4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(
2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3
/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2
*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))
*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1))/d
```

Fricas [A] time = 3.04453, size = 477, normalized size = 4.08

$$\frac{\left((3\,A+4\,B+8\,C)\cos\left(dx+c\right)^{3}+(3\,A+4\,B+8\,C)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*(((3*A + 4*B + 8*C)*cos(d*x + c)^3 + (3*A + 4*B + 8*C)*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 4*((3*A + 4*B)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.05681, size = 653, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*A*sqrt(a) + 4*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (
3*A*sqrt(a) + 4*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(
2)*(5*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6
*A*a^(3/2) - 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^6*B*a^(3/2) + 19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*A*a^(5/2) + 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(5/2) - 17*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(7/2) - 36*(sqrt(a)*tan(1/2*d*
```

```
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(7/2) + A*a^(9/2) +
4*B*a^(9/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^2*a + a^2)^2)/d
```

3.380 $\int \sqrt{a+a\cos(c+dx)} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=163

$$\frac{a(5A+6B+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(5A+6B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{A\,\mathrm{t}}{}$$

```
[Out] (Sqrt[a]*(5*A + 6*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(8*d) + (a*(5*A + 6*B + 8*C)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c
+ d*x]]) + (a*(A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c +
 d*x]]) + (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.424553, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2980, 2772, 2773, 206}

$$\frac{a(5A+6B+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(5A+6B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{A\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```


```
[Out] (Sqrt[a]*(5*A + 6*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(8*d) + (a*(5*A + 6*B + 8*C)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c
+ d*x]]) + (a*(A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c +
 d*x]]) + (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c}{3d}$$

$$= \frac{a(A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{A}{}$$

$$= \frac{a(5A+6B+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(A+6}{12}$$

$$= \frac{a(5A+6B+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(A+6}{12}$$

$$= \frac{\sqrt{a}(5A+6B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right.}{8d}$$

Mathematica [A] time = 1.15084, size = 138, normalized size = 0.85

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(5A+6B+8C)\cos(2(c+dx))+4(5A+6B)\cos(c\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(5*A
 + 6*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (31*A + 18
*B + 24*C + 4*(5*A + 6*B)*Cos[c + d*x] + 3*(5*A + 6*B + 8*C)*Cos[2*(c + d*x
)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.234, size = 1897, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(5*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))+6*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+6*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a))+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)
^6+12*(10*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+12*B*a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+15*A*ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+18*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a))*a+18*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-2*(80*A*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+96*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+45*A*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+45*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+54*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+54*B*ln(-4/(-2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+72*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a+72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/
2*d*x+1/2*c)^2+66*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+15*A*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+60*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+18
*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+18*B*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1

```
/2)+24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Maxima [B] time = 22.1747, size = 7733, normalized size = 47.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="maxima")
```

```
[Out] -1/96*((120*(sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*co
s(13/2*d*x + 13/2*c) - 8*(15*sin(11/2*d*x + 11/2*c) + 50*sin(9/2*d*x + 9/2*
c) + 42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x + 3/2
*c))*cos(6*d*x + 6*c) + 360*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(11/2*
d*x + 11/2*c) + 1200*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(9/2*d*x + 9/
2*c) - 24*(42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x
 + 3/2*c))*cos(4*d*x + 4*c) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*co
s(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)
^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2
*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2
```

+ 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*si
n(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x +
4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) + 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^
2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*s
in(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2
)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 120*(cos(6*d*x
+ 6*c) + 3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(13/2*d*x + 13/2*c
) + 8*(15*cos(11/2*d*x + 11/2*c) + 50*cos(9/2*d*x + 9/2*c) + 42*cos(7/2*d*x
 + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*
c) - 120*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(11/2*d*x + 11/2*
c) - 400*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(9/2*d*x + 9/2*c)
 + 24*(42*cos(7/2*d*x + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3
/2*c))*sin(4*d*x + 4*c) - 336*(3*cos(2*d*x + 2*c) + 1)*sin(7/2*d*x + 7/2*c)
 - 24*(3*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c) + 1008*cos(7/2*d*x + 7/
2*c)*sin(2*d*x + 2*c) + 72*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 120*cos(
3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 120*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + 120*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 40*sin(3/2*d*x + 3/2*c))
*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*s
qrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*
x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) - 6*(3*(log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d

*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 12*
(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x
 + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 2*(6*(lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x +
2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*sin(5/2*d*x + 5/2*c) - 2*
sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4
c) - 4(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*sin(1/2*d*x + 1/2*c) -
3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1

/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 24*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt

```
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x
+ 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))*C*sqrt(a)/(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 3.11808, size = 532, normalized size = 3.26

$$\frac{3\left((5A+6B+8C)\cos(dx+c)^4+(5A+6B+8C)\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*((5*A + 6*B + 8*C)*cos(d*x + c)^4 + (5*A + 6*B + 8*C)*cos(d*x + c)^
3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(3*(5*A + 6*B + 8*C)*cos(d*x + c)^2 + 2*(5*A + 6*B)*cos(
d*x + c) + 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 +
d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.11785, size = 1162, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*A*sqrt(a) + 6*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))
- 3*(5*A*sqrt(a) + 6*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*
sqrt(2)*(63*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^10*A*a^(3/2) - 30*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^10*B*a^(3/2) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(3/2) - 369*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(5/2) + 66*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(5/2) - 888*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(5/2) + 1
638*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A
*a^(7/2) + 756*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*B*a^(7/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^6*C*a^(7/2) - 1074*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(9/2) - 732*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(9/2) - 1776*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(9/2) + 171*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(11
/2) + 138*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*B*a^(11/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2*C*a^(11/2) - 13*A*a^(13/2) - 6*B*a^(13/2) - 24*C*a^(13/2))
/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
a^2)^3)/d
```

3.381 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a(35A+40B+48C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(35A+40B+48C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(35A+40B+48C)\tan(c+dx)\sec(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(35*A + 40*B + 48*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(64*d) + (a*(35*A + 40*B + 48*C)*Tan[c + d*x])/(64*d*Sqrt[a +
a*Cos[c + d*x]]) + (a*(35*A + 40*B + 48*C)*Sec[c + d*x]*Tan[c + d*x])/(96*d
*Sqrt[a + a*Cos[c + d*x]]) + (a*(A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(24*
d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d)
```

Rubi [A] time = 0.508531, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2980, 2772, 2773, 206}

$$\frac{a(35A+40B+48C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(35A+40B+48C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(35A+40B+48C)\tan(c+dx)\sec(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^5,x]
```



```
[Out] (Sqrt[a]*(35*A + 40*B + 48*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(64*d) + (a*(35*A + 40*B + 48*C)*Tan[c + d*x])/(64*d*Sqrt[a +
a*Cos[c + d*x]]) + (a*(35*A + 40*B + 48*C)*Sec[c + d*x]*Tan[c + d*x])/(96*d
*Sqrt[a + a*Cos[c + d*x]]) + (a*(A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(24*
d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+}{4d}$$
$$= \frac{a(A+8B)\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{A}{}$$
$$= \frac{a(35A+40B+48C)\sec(c+dx)\tan(c+}{96d\sqrt{a+a\cos(c+dx)}}$$
$$= \frac{a(35A+40B+48C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a(35}{}$$
$$= \frac{a(35A+40B+48C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a(35}{}$$
$$= \frac{\sqrt{a}(35A+40B+48C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a+a\cos(}}\right.}{64d}$$

Mathematica [A] time = 1.4722, size = 168, normalized size = 0.8

$$\frac{\sqrt{a(\cos(c+dx)+1)}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)((539A+616B+432C)\cos(c+dx)+4(35A+40B+48C)\cos(2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^5,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(35*A + 40*B + 48*C)*ArcTanh[Sqrt[2]
*Sin[(c + d*x)/2]]*Sec[(c + d*x)/2] + ((332*A + 160*B + 192*C + (539*A + 61
6*B + 432*C)*Cos[c + d*x] + 4*(35*A + 40*B + 48*C)*Cos[2*(c + d*x)] + 105*A
*Cos[3*(c + d*x)] + 120*B*Cos[3*(c + d*x)] + 144*C*Cos[3*(c + d*x)])*Sec[c
+ d*x]^4*Tan[(c + d*x)/2])/2))/(384*d)
```

Maple [B] time = 0.262, size = 2370, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)

[Out] 1/24*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(35*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+35*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+40*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+40*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+48*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+48*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(35*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+40*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+70*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+70*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+80*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+80*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+96*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+96*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(385*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+440*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+528*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+315*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+315*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+360*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+360*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+432*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+432*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(511*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+584*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+624*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+210*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+210*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24

```
0*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+240*B*ln(-4/(-2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2
)*cos(1/2*d*x+1/2*c)+2*a))*a+288*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a))*a+288*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/
2*c)^2+558*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+105*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+105*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+528*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+120*B*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+120*B*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a+480*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+144*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+144*C*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.76776, size = 601, normalized size = 2.88

$$\frac{3\left((35\,A+40\,B+48\,C)\cos\left(dx+c\right)^{5}+(35\,A+40\,B+48\,C)\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+}}{\cos(dx+c)^{3}+\cos(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="fricas")
```

```
[Out] 1/768*(3*((35*A + 40*B + 48*C)*cos(d*x + c)^5 + (35*A + 40*B + 48*C)*cos(d*
x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(3*(35*A + 40*B + 48*C)*cos(d*x + c)^3 + 2*(35*A
+ 40*B + 48*C)*cos(d*x + c)^2 + 8*(7*A + 8*B)*cos(d*x + c) + 48*A)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.09345, size = 1494, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*A*sqrt(a) + 40*B*sqrt(a) + 48*C*sqrt(a))*log(abs((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3
))) - 3*(35*A*sqrt(a) + 40*B*sqrt(a) + 48*C*sqrt(a))*log(abs((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)
)) - 4*sqrt(2)*(279*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^14*A*a^(3/2) - 504*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^14*B*a^(3/2) + 240*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(3/2) + 285*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*a^(5/2) + 5976*(sqrt(
```

a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*a^(5/2) - 1968*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(5/2) - 4605*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(7/2) - 31320*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(7/2) - 2640*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(7/2) + 37281*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(9/2) + 90168*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(9/2) + 41616*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(9/2) - 35643*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(11/2) - 66024*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(11/2) - 42288*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(11/2) + 9175*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(13/2) + 16904*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(13/2) + 12528*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(13/2) - 1311*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(15/2) - 1992*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(15/2) - 1392*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(15/2) + 43*A*a^(17/2) + 104*B*a^(17/2) + 48*C*a^(17/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d

3.382 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=243

$$\frac{2a^2(99A+110B+84C)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(429A+374B+336C)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}+\frac{2(429A+374B+336C)\sin}{1}$$

```
[Out] (2*a^2*(429*A + 374*B + 336*C)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]
]) + (2*a^2*(99*A + 110*B + 84*C)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[
a + a*Cos[c + d*x]]) - (4*a*(429*A + 374*B + 336*C)*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(3465*d) + (2*a*(11*B + 3*C)*Cos[c + d*x]^3*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(99*d) + (2*(429*A + 374*B + 336*C)*(a + a*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(1155*d) + (2*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.700156, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3045, 2976, 2981, 2759, 2751, 2646}

$$\frac{2a^2(99A+110B+84C)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(429A+374B+336C)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}+\frac{2(429A+374B+336C)\sin}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*a^2*(429*A + 374*B + 336*C)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]
]) + (2*a^2*(99*A + 110*B + 84*C)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[
a + a*Cos[c + d*x]]) - (4*a*(429*A + 374*B + 336*C)*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(3465*d) + (2*a*(11*B + 3*C)*Cos[c + d*x]^3*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(99*d) + (2*(429*A + 374*B + 336*C)*(a + a*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(1155*d) + (2*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(11*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin}{11d}$$

$$= \frac{2a(11B+3C)\cos^3(c+dx)\sqrt{a+a\cos(c}}{99d}$$

$$= \frac{2a^2(99A+110B+84C)\cos^3(c+dx)\sin}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2(99A+110B+84C)\cos^3(c+dx)\sin}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2(99A+110B+84C)\cos^3(c+dx)\sin}{693d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2(429A+374B+336C)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 1.2803, size = 145, normalized size = 0.6

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((33396A+35156B+34734C)\cos(c+dx)+8(1287A+1507B+1743C)\cos(2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(65208*A + 59158*B + 55482*C + (33396*A + 351
56*B + 34734*C)*Cos[c + d*x] + 8*(1287*A + 1507*B + 1743*C)*Cos[2*(c + d*x)
] + 1980*A*Cos[3*(c + d*x)] + 3740*B*Cos[3*(c + d*x)] + 4935*C*Cos[3*(c + d
*x)] + 770*B*Cos[4*(c + d*x)] + 1470*C*Cos[4*(c + d*x)] + 315*C*Cos[5*(c +
d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 0.067, size = 154, normalized size = 0.6

$$\frac{4\,a^2\sqrt{2}}{3465\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-5040\,C\,(\sin(1/2\,dx+c/2))^{10}+(3080\,B+18480\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8+(-1980\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 4/3465*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-5040*C*sin(1/2*d*x+1/2*c
)^10+(3080*B+18480*C)*sin(1/2*d*x+1/2*c)^8+(-1980*A-9900*B-27720*C)*sin(1/2
*d*x+1/2*c)^6+(5544*A+12474*B+22176*C)*sin(1/2*d*x+1/2*c)^4+(-5775*A-8085*B
-10395*C)*sin(1/2*d*x+1/2*c)^2+3465*A+3465*B+3465*C)*2^(1/2)/(a*cos(1/2*d*x
+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.23586, size = 340, normalized size = 1.4

$$\frac{132\left(15\,\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+175\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+735\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] 1/55440*(132*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x
+ 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x +
 1/2*c))*A*sqrt(a) + 22*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*
sin(7/2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + 21*
(15*sqrt(2)*a*sin(11/2*d*x + 11/2*c) + 55*sqrt(2)*a*sin(9/2*d*x + 9/2*c) +
165*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 429*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 9
90*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 3630*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*
sqrt(a))/d
```

Fricas [A] time = 2.18888, size = 392, normalized size = 1.61

$$\frac{2\left(315\,Ca\cos(dx+c)^5+35\,(11\,B+21\,C)a\cos(dx+c)^4+5\,(99\,A+187\,B+168\,C)a\cos(dx+c)^3+3\,(429\,A+374\,B\right.}{3465\,(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*C*a*cos(d*x + c)^5 + 35*(11*B + 21*C)*a*cos(d*x + c)^4 + 5*(99*
A + 187*B + 168*C)*a*cos(d*x + c)^3 + 3*(429*A + 374*B + 336*C)*a*cos(d*x +
 c)^2 + 4*(429*A + 374*B + 336*C)*a*cos(d*x + c) + 8*(429*A + 374*B + 336*C
)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.383 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2} \left(A + B\cos(c + dx) + C \text{co}\right.$

Optimal. Leaf size=187

$$\frac{8a^2(63A + 57B + 47C)\sin(c + dx)}{315d\sqrt{a\cos(c + dx) + a}} + \frac{2(63A - 18B + 22C)\sin(c + dx)(a\cos(c + dx) + a)^{3/2}}{315d} + \frac{2a(63A + 57B + 47C)\text{si}}{3}$$

```
[Out] (8*a^2*(63*A + 57*B + 47*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
+ (2*a*(63*A + 57*B + 47*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d)
+ (2*(63*A - 18*B + 22*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*d)
+ (2*C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d) + (2*(
3*B + C)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(21*a*d)
```

Rubi [A] time = 0.415421, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3045, 2968, 3023, 2751, 2647, 2646}

$$\frac{8a^2(63A + 57B + 47C)\sin(c + dx)}{315d\sqrt{a\cos(c + dx) + a}} + \frac{2(63A - 18B + 22C)\sin(c + dx)(a\cos(c + dx) + a)^{3/2}}{315d} + \frac{2a(63A + 57B + 47C)\text{si}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```



```
[Out] (8*a^2*(63*A + 57*B + 47*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
+ (2*a*(63*A + 57*B + 47*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d)
+ (2*(63*A - 18*B + 22*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*d)
+ (2*C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d) + (2*(
3*B + C)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(21*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\sin}{9d}$$

$$= \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\sin}{9d}$$

$$= \frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\sin}{9d}$$

$$= \frac{2(63A-18B+22C)(a+a\cos(c+dx))}{315d}$$

$$= \frac{2a(63A+57B+47C)\sqrt{a+a\cos(c+dx}}{315d}$$

$$= \frac{8a^2(63A+57B+47C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.755938, size = 113, normalized size = 0.6

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(756A+759B+799C)\cos(c+dx)+4(63A+117B+137C)\cos(2(c+dx))+3}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(3276*A + 2964*B + 2689*C + 2*(756*A + 759*B
+ 799*C)*Cos[c + d*x] + 4*(63*A + 117*B + 137*C)*Cos[2*(c + d*x)] + 90*B*Co
s[3*(c + d*x)] + 170*C*Cos[3*(c + d*x)] + 35*C*Cos[4*(c + d*x)])*Tan[(c + d
*x)/2])/(1260*d)
```

Maple [A] time = 0.066, size = 132, normalized size = 0.7

$$\frac{4a^2\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280C(\sin(1/2dx+c/2))^8+(-180B-900C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(126A+504B+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*C*sin(1/2*d*x+1/2*c)^8
+(-180*B-900*C)*sin(1/2*d*x+1/2*c)^6+(126*A+504*B+1134*C)*sin(1/2*d*x+1/2*c
)^4+(-315*A-525*B-735*C)*sin(1/2*d*x+1/2*c)^2+315*A+315*B+315*C)*2^(1/2)/(a
*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.21449, size = 277, normalized size = 1.48

$$\frac{252\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+6\left(15\,\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/2520*(252*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 6*(15*sqrt(2)*a*sin(7/
2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*
d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + (35*sqrt(2)*
a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 378*sqrt(2)*a
*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 3780*sqrt(2)*
a*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.89954, size = 320, normalized size = 1.71

$$\frac{2\left(35\,Ca\cos\left(dx+c\right)^{4}+5\,(9\,B+17\,C)a\cos\left(dx+c\right)^{3}+3\,(21\,A+39\,B+34\,C)a\cos\left(dx+c\right)^{2}+(189\,A+156\,B+136\,C)a\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/315*(35*C*a*cos(d*x + c)^4 + 5*(9*B + 17*C)*a*cos(d*x + c)^3 + 3*(21*A +
39*B + 34*C)*a*cos(d*x + c)^2 + (189*A + 156*B + 136*C)*a*cos(d*x + c) + 2*
(189*A + 156*B + 136*C)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
```

```
+ c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*cos(d*x + c), x)
```

3.384 $\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=144

$$\frac{8a^2(35A+21B+19C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a(35A+21B+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2(7B-2C)\sin(c+dx)(a\cos\ldots}{35d}$$

```
[Out] (8*a^2*(35*A + 21*B + 19*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]])
+ (2*a*(35*A + 21*B + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d)
+ (2*(7*B - 2*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a
+ a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.197376, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3023, 2751, 2647, 2646}

$$\frac{8a^2(35A+21B+19C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a(35A+21B+19C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2(7B-2C)\sin(c+dx)(a\cos\ldots}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (8*a^2*(35*A + 21*B + 19*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]])
+ (2*a*(35*A + 21*B + 19*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d)
+ (2*(7*B - 2*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a
+ a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}+\frac{2\int(a+a\,\mathrm{co}}{}$$
$$= \frac{2(7B-2C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2C(}{}$$
$$= \frac{2a(35A+21B+19C)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{105d}$$
$$= \frac{8a^2(35A+21B+19C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(35A+21}{}$$

Mathematica [A] time = 0.348632, size = 87, normalized size = 0.6

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((140A+252B+253C)\cos(c+dx)+700A+6(7B+13C)\cos(2(c+dx))+546}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(700*A + 546*B + 494*C + (140*A + 252*B + 253
*C)*Cos[c + d*x] + 6*(7*B + 13*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])
```

```
*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.223, size = 110, normalized size = 0.8

$$\frac{4a^2\sqrt{2}}{105d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-60C\left(\sin\left(1/2dx+c/2\right)\right)^6+\left(42B+168C\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\left(-35A-105B-17\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-60*C*sin(1/2*d*x+1/2*c)^6
+(42*B+168*C)*sin(1/2*d*x+1/2*c)^4+(-35*A-105*B-175*C)*sin(1/2*d*x+1/2*c)^2
+105*A+105*B+105*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.63633, size = 215, normalized size = 1.49

$$\frac{140\left(\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+9\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+42\left(\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+20\sqrt{2}a\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] 1/420*(140*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*
c))*A*sqrt(a) + 42*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + (15*sqrt(2)*a*s
in(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin
(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.94392, size = 257, normalized size = 1.78

$$\frac{2\left(15Ca\cos\left(dx+c\right)^3+3\left(7B+13C\right)a\cos\left(dx+c\right)^2+\left(35A+63B+52C\right)a\cos\left(dx+c\right)+\left(175A+126B+104C\right)a\right)\sqrt{}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/105*(15*C*a*cos(d*x + c)^3 + 3*(7*B + 13*C)*a*cos(d*x + c)^2 + (35*A + 63
*B + 52*C)*a*cos(d*x + c) + (175*A + 126*B + 104*C)*a)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.385 $\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=142

$$\frac{2a^2(15A+20B+12C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(5B+3C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2C}{}$$

```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(15*A + 20*B + 12*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a*(5*B + 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a +
 a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.427034, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3045, 2976, 2981, 2773, 206}

$$\frac{2a^2(15A+20B+12C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(5B+3C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```


```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(15*A + 20*B + 12*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a*(5*B + 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a +
 a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2\ldots}{\ldots}$$

$$= \frac{2a(5B+3C)\sqrt{a+a\cos(c+dx)}\sin(c+d\ldots}{15d}$$

$$= \frac{2a^2(15A+20B+12C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a(\ldots}{\ldots}$$

$$= \frac{2a^2(15A+20B+12C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a(\ldots}{\ldots}$$

$$= \frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2(15A\ldots}{15\ldots}$$

Mathematica [A] time = 0.420754, size = 105, normalized size = 0.74

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(30A+2(5B+9C)\cos(c+dx)+50B+3C\cos(2(c+dx))+39C)\ldots\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*A*ArcTanh[Sqrt[2
]*Sin[(c + d*x)/2]] + (30*A + 50*B + 39*C + 2*(5*B + 9*C)*Cos[c + d*x] + 3*
C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d)
```

Maple [B] time = 0.188, size = 335, normalized size = 2.4

$$\frac{1}{15d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24C\sqrt{2}\sqrt{a(\sin(1/2dx+c/2))^2}\sqrt{a}(\sin(1/2dx+c/2))^4-20\sqrt{a(\sin(1/2dx+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/15*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*2^(1/2)*(B+3*C)*sin(1/2*d*x+1/2*c)^2+30*A*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a))*a+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+60*
B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+60*C*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2
)/d
```

Maxima [A] time = 1.8286, size = 126, normalized size = 0.89

$$\frac{10\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}+3\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*B*sqrt(a) + 3*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x +
 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 2.02315, size = 456, normalized size = 3.21

$$\frac{15\,(Aa\cos\left(dx+c\right)+Aa)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3\,Ca\cos\left(dx+c\right)^2\right.}{30\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(A*a*cos(d*x + c) + A*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x +
 c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*C*a*cos(d*x + c)^2 + (
5*B + 9*C)*a*cos(d*x + c) + (15*A + 25*B + 18*C)*a)*sqrt(a*cos(d*x + c) + a
```

```
)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.80781, size = 343, normalized size = 2.42

$$\frac{\frac{15\,Aa^{\frac{5}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(15\,\sqrt{2}Aa^{4}+30\,\sqrt{2}Ba^{4}+30\,\sqrt{2}Ca^{4}+\left(30\,\sqrt{2}Aa^{4}+50\,\sqrt{2}Ba^{4}+30\,\sqrt{2}Ca^{4}+\left(15\,\sqrt{2}Aa^{4}+20 \cdots\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a \cdots\right.}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="giac")
```

```
[Out] 1/15*(15*A*a^(5/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a
))/abs(a) + 2*(15*sqrt(2)*A*a^4 + 30*sqrt(2)*B*a^4 + 30*sqrt(2)*C*a^4 + (30
*sqrt(2)*A*a^4 + 50*sqrt(2)*B*a^4 + 30*sqrt(2)*C*a^4 + (15*sqrt(2)*A*a^4 +
20*sqrt(2)*B*a^4 + 12*sqrt(2)*C*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.386 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx) \, dx$

Optimal. Leaf size=144

$$-\frac{a^2(3A-6B-8C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+$$

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(3*A - 6*B - 8*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
 - (a*(3*A - 2*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (A*(a + a*
Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.50009, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2976, 2981, 2773, 206}

$$-\frac{a^2(3A-6B-8C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```


```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(3*A - 6*B - 8*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
 - (a*(3*A - 2*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (A*(a + a*
Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^{3/2}\tan(c+dx)}{d}+$$

$$=-\frac{a(3A-2C)\sqrt{a+a\cos(c+dx)}\sin(c+}{3d}$$

$$=-\frac{a^2(3A-6B-8C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}-\frac{a(3A}{}$$

$$=-\frac{a^2(3A-6B-8C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}-\frac{a(3A}{}$$

$$=\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-$$

Mathematica [A] time = 0.50809, size = 118, normalized size = 0.82

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(3A+2(3B+5C)\cos(c+dx)+C\cos(2(c+dx))+\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^2,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*(3*A
 + 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(3*A + C + 2*(3*
B + 5*C)*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Maple [B] time = 0.211, size = 781, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-2*(12*B*a^(1/2
```

```
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+28*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+9*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+9*A*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6*B*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*a))*a+6*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*si
n(1/2*d*x+1/2*c)^2+6*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+9*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+12*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+6
*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+6*B*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a+24*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 2.07661, size = 1828, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))
*C*sqrt(a) - 3*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(
2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
```

```
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2
*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x
+ 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/
2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt
(2)*a)*sin(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/2*d*x + 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/
2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 2.21874, size = 502, normalized size = 3.49

$$\frac{3\left((3A+2B)a\cos(dx+c)^2+(3A+2B)a\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(d}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{12\left(d\cos(dx+c)^2+d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqrt(a)
*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
```

```
^2)) + 4*(2*C*a*cos(d*x + c)^2 + 2*(3*B + 5*C)*a*cos(d*x + c) + 3*A*a)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.12146, size = 460, normalized size = 3.19

$$3\left(3\,Aa^{\frac{3}{2}}+2\,Ba^{\frac{3}{2}}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-3\left(3\,Aa^{\frac{3}{2}}+2\,Ba^{\frac{3}{2}}\right)\log\left(\left|\left(\sqrt{\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*A*a^(3/2) + 2*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(3*A*a^(3/2
) + 2*B*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 12*sqrt(2)*(3*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2) - A*a^(7/2))
/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
a^2) + 4*(3*sqrt(2)*B*a^3 + 6*sqrt(2)*C*a^3 + (3*sqrt(2)*B*a^3 + 4*sqrt(2)*
C*a^3)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)
^2 + a)^(3/2))/d
```

3.387 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \ldots dx) \, dx$

Optimal. Leaf size=159

$$-\frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+12B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(3A+4B)\tan(c+dx)\sqrt{a\cos(c+dx}}{4d}$$

```
[Out] (a^(3/2)*(7*A + 12*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/(4*d) - (a^2*(5*A + 4*B - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos
[c + d*x]]) + (a*(3*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) +
 (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.504224, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2975, 2981, 2773, 206}

$$-\frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+12B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(3A+4B)\tan(c+dx)\sqrt{a\cos(c+dx}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```


```
[Out] (a^(3/2)*(7*A + 12*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/(4*d) - (a^2*(5*A + 4*B - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos
[c + d*x]]) + (a*(3*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) +
 (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan}{2d}$$

$$= \frac{a(3A+4B)\sqrt{a+a\cos(c+dx)}\tan(c+}{4d}$$

$$= -\frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a(3A}{}$$

$$= -\frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a(3A}{}$$

$$= \frac{a^{3/2}(7A+12B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a+a\cos(c}}\right.}{4d}$$

Mathematica [A] time = 0.750991, size = 127, normalized size = 0.8

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((7A+4B)\cos(c+dx)+2(A+2C\cos(2(c+dx))\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^3,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(7*A
 + 12*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*((7*A +
 4*B)*Cos[c + d*x] + 2*(A + 2*C + 2*C*Cos[2*(c + d*x)]))*Sin[(c + d*x)/2]))
/(8*d)
```

Maple [B] time = 0.229, size = 1453, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(16*C*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+12*B*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a))*a+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*
(7*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*A*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+12*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+8*C*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+18*A*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+12*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+8*B*a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+16*C
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1
/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1
/2*c)^2)^(1/2)/d
```

Maxima [B] time = 2.83579, size = 4508, normalized size = 28.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="maxima")
```

```
[Out] -1/16*((12*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*c
)^2*sin(3/2*d*x + 3/2*c) + 12*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 4
8*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) + 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos(3/2
*d*x + 3/2*c)*sin(2*d*x + 2*c) - 24*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
 - 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 1
2*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 48*(
a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 4*(12*a*c
os(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 20*a*sin(7/2*d*x + 7/2*c) - 21*a*sin
(5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 7*(sqrt(2)
*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(
2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*
d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 2) - 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)
^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*
(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*
cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x +
 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2
*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
```

3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(13/2*d*x + 13/2*c) - 12*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(11/2*d*x + 11/2*c) - 48*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(12*a*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 20*a*cos(7/2*d*x + 7/2*c) + 21*a*cos(5/2*d*x + 5/2*c) + 9*a*cos(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c) - 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*x + 7/2*c) - 84*(2*a*cos(2*d*x + 2*c) + a)*sin(5/2*d*x + 5/2*c) - 24*a*sin(3/2*d*x + 3/2*c) - 4*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 56*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) + 4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +

```
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*sin(5/2*d*x + 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2
)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 3.12972, size = 521, normalized size = 3.28

$$\frac{\left((7A+12B+8C)a\cos(dx+c)^3+(7A+12B+8C)a\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(c}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos(dx+c)^3+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*(((7*A + 12*B + 8*C)*a*cos(d*x + c)^3 + (7*A + 12*B + 8*C)*a*cos(d*x +
 c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*
x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(8*C*a*cos(d*x + c)^2 + (7*A + 4*B)*a*cos(d*x + c) +
 2*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*
x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.07358, size = 702, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="giac")
```

```
[Out] 1/8*(16*sqrt(2)*C*a^2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a) + (7*A*a^(3/2) + 12*B*a^(3/2) + 8*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) -
(7*A*a^(3/2) + 12*B*a^(3/2) + 8*C*a^(3/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqr
t(2)*(7*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^6*A*a^(5/2) + 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^6*B*a^(5/2) - 95*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^4*A*a^(7/2) - 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(7/2) + 53*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(9/2) + 36*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(9/2) - 5*A*a^(11/
2) - 4*B*a^(11/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2*a + a^2)^2)/d
```

3.388 $\int (a+a\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^4(c+dx)\,dx$

Optimal. Leaf size=165

$$\frac{a^2(19A+30B+24C)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+14B+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(A+2B)\tan(c+dx)\sec(c+d\ldots}{4d}$$

```
[Out] (a^(3/2)*(11*A + 14*B + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(8*d) + (a^2*(19*A + 30*B + 24*C)*Tan[c + d*x])/(24*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a*(A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[
c + d*x])/(4*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d)
```

Rubi [A] time = 0.558928, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2975, 2980, 2773, 206}

$$\frac{a^2(19A+30B+24C)\tan(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+14B+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(A+2B)\tan(c+dx)\sec(c+d\ldots}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```


```
[Out] (a^(3/2)*(11*A + 14*B + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(8*d) + (a^2*(19*A + 30*B + 24*C)*Tan[c + d*x])/(24*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a*(A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[
c + d*x])/(4*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan}{3d}$$

$$= \frac{a(A+2B)\sqrt{a+a\cos(c+dx)}\sec(c+d}{4d}$$

$$= \frac{a^2(19A+30B+24C)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a(}{}$$

$$= \frac{a^2(19A+30B+24C)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a(}{}$$

$$= \frac{a^{3/2}(11A+14B+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a+a\cos}}\right.}{8d}$$

Mathematica [A] time = 1.24961, size = 139, normalized size = 0.84

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(11A+14B+8C)\cos(2(c+dx))+4(11A+6B)\cos\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^4,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(1
1*A + 14*B + 24*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (49*A
 + 42*B + 24*C + 4*(11*A + 6*B)*Cos[c + d*x] + 3*(11*A + 14*B + 8*C)*Cos[2*
(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.237, size = 1897, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(11*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+11*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+14*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+14*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(22*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+28*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+33*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+42*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+72*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-2*(176*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+192*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+99*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+99*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+126*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+126*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+216*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+216*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+126*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+33*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+108*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+42*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+72*C*ln(4/(2*cos(1/2*d*x+1

```
/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*a))*a+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+72*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*
c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/
2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.01146, size = 558, normalized size = 3.38

$$\frac{3\left((11\,A+14\,B+24\,C)a\cos\left(dx+c\right)^{4}+(11\,A+14\,B+24\,C)a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)}}{\cos(dx+c)^{3}+\cos}\right.}{96\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*((11*A + 14*B + 24*C)*a*cos(d*x + c)^4 + (11*A + 14*B + 24*C)*a*cos
(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*
cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(3*(11*A + 14*B + 8*C)*a*cos(d*x + c)^2 + 2*(1
1*A + 6*B)*a*cos(d*x + c) + 8*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(
d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.47062, size = 1162, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(11*A*a^(3/2) + 14*B*a^(3/2) + 24*C*a^(3/2))*log(abs((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)
)) - 3*(11*A*a^(3/2) + 14*B*a^(3/2) + 24*C*a^(3/2))*log(abs((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^10*A*a^(5/2) + 42*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^10*B*a^(5/2) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(5/2) - 303*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(7/2) - 822*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(7/2) - 888*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(7
/2) + 2394*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*A*a^(9/2) + 3780*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^6*B*a^(9/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(9/2) - 1806*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(11/2) - 2508*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(11/2) - 1776*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(11
/2) + 309*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*A*a^(13/2) + 498*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2*B*a^(13/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(13/2) - 19*A*a^(15/2) - 30*B*a^(15/2) - 2
4*C*a^(15/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*a + a^2)^3)/d
```

3.389 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ ... $dx)\, dx$

Optimal. Leaf size=215

$$\frac{a^2(75A+88B+112C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+88B+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(39A+56B+48C)\tan(c+}{96d\sqrt{a\cos(c+dx}}$$

```
[Out] (a^(3/2)*(75*A + 88*B + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Co
s[c + d*x]]])/(64*d) + (a^2*(75*A + 88*B + 112*C)*Tan[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (a^2*(39*A + 56*B + 48*C)*Sec[c + d*x]*Tan[c + d*x])
/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(3*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c +
 d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.641821, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2975, 2980, 2772, 2773, 206}

$$\frac{a^2(75A+88B+112C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+88B+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(39A+56B+48C)\tan(c+}{96d\sqrt{a\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```

```
[Out] (a^(3/2)*(75*A + 88*B + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Co
s[c + d*x]]])/(64*d) + (a^2*(75*A + 88*B + 112*C)*Tan[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (a^2*(39*A + 56*B + 48*C)*Sec[c + d*x]*Tan[c + d*x])
/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(3*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (A*(a + a*Cos[c + d*x])^(3/2)*Sec[c +
 d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan}{4d}$$

$$=\frac{a(3A+8B)\sqrt{a+a\cos(c+dx)}\sec^2(c+}{24d}$$

$$=\frac{a^2(39A+56B+48C)\sec(c+dx)\tan(}{96d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^2(75A+88B+112C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+$$

$$=\frac{a^2(75A+88B+112C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+$$

$$=\frac{a^{3/2}(75A+88B+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a+a\cos}}\right.}{64d}$$

Mathematica [A] time = 2.04978, size = 174, normalized size = 0.81

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((1155A+1048B+1008C)\cos(c+dx)+4(75A+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*(7
5*A + 88*B + 112*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (492
*A + 352*B + 192*C + (1155*A + 1048*B + 1008*C)*Cos[c + d*x] + 4*(75*A + 88
*B + 48*C)*Cos[2*(c + d*x)] + 225*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(c + d*x
```

```
)] + 336*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d)
```

Maple [B] time = 0.27, size = 2370, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(75*A*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+75*A*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a))+88*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+88*B
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+112*C*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))+112*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)))*si
n(1/2*d*x+1/2*c)^8-48*(75*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
88*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+112*C*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+150*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+150*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+176*B*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+176*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a))*a+224*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+224*
C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(82
5*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+968*B*a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+1104*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+675*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+675*A*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+792*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+792*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
```

```
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1008*
C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1008*C*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(1095*A*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1208*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+1200*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+450*A*ln(-
4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+450*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a))*a+528*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+
528*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+672*C*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+672*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a))*a)*sin(1/2*d*x+1/2*c)^2+225*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*a))*a+1086*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+225*A*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+264*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a))*a+1008*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+264*
B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+336*C*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))*a+864*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+336*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+
1/2*c)-2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/sin(1/2*d*x+1/2*c)/(a*co
s(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.17135, size = 622, normalized size = 2.89

$$\frac{3\left((75\,A + 88\,B + 112\,C)a\cos\left(dx + c\right)^{5} + (75\,A + 88\,B + 112\,C)a\cos\left(dx + c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3} - 7\,a\cos(dx+c)^{2} - 4\,\sqrt{a\cos(dx+}}{\cos(dx+c)^{3} + \mathrm{co}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="fricas")
```

```
[Out] 1/768*(3*((75*A + 88*B + 112*C)*a*cos(d*x + c)^5 + (75*A + 88*B + 112*C)*a*
cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)) + 4*(3*(75*A + 88*B + 112*C)*a*cos(d*x + c)^3 +
 2*(75*A + 88*B + 48*C)*a*cos(d*x + c)^2 + 8*(15*A + 8*B)*a*cos(d*x + c) +
48*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*
x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.6308, size = 1494, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="giac")
```

```
[Out] 1/384*(3*(75*A*a^(3/2) + 88*B*a^(3/2) + 112*C*a^(3/2))*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) +
3))) - 3*(75*A*a^(3/2) + 88*B*a^(3/2) + 112*C*a^(3/2))*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) -
3))) + 4*sqrt(2)*(225*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^14*A*a^(5/2) + 264*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^14*B*a^(5/2) + 336*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(5/2) - 6261*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*a^(7/2) - 4008*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*a^(7/
2) - 8592*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^12*C*a^(7/2) + 35925*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^10*A*a^(9/2) + 33960*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(9/2) + 70032*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(9/2) - 127449*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(11/2) -
131784*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
8*B*a^(11/2) - 208080*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^8*C*a^(11/2) + 101667*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(13/2) + 108312*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(13/2) + 154608*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(13/2) -
26079*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4
*A*a^(15/2) - 29432*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^4*B*a^(15/2) - 44208*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(15/2) + 3303*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(17/2) + 3384*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(17/2) + 5424*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(17
/2) - 147*A*a^(19/2) - 152*B*a^(19/2) - 240*C*a^(19/2))/((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.390 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=263

$$\frac{a^2(133A+150B+176C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(133A+150B+176C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(67A+90B+80C)\tan(}{240d\sqrt{a\cos(c-}}$$

```
[Out] (a^(3/2)*(133*A + 150*B + 176*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/(128*d) + (a^2*(133*A + 150*B + 176*C)*Tan[c + d*x])/(128*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(133*A + 150*B + 176*C)*Sec[c + d*x]*Tan[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(67*A + 90*B + 80*C)*Sec[
c + d*x]^2*Tan[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(3*A + 10*B)
*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (A*(a + a*C
os[c + d*x])^(3/2)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.762709, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2975, 2980, 2772, 2773, 206}

$$\frac{a^2(133A+150B+176C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(133A+150B+176C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(67A+90B+80C)\tan(}{240d\sqrt{a\cos(c-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^6,x]
```

```
[Out] (a^(3/2)*(133*A + 150*B + 176*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/(128*d) + (a^2*(133*A + 150*B + 176*C)*Tan[c + d*x])/(128*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(133*A + 150*B + 176*C)*Sec[c + d*x]*Tan[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(67*A + 90*B + 80*C)*Sec[
c + d*x]^2*Tan[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(3*A + 10*B)
*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (A*(a + a*C
os[c + d*x])^(3/2)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(}{5d}$$

$$=\frac{a(3A+10B)\sqrt{a+a\cos(c+dx)}\sec^3(c+}{40d}$$

$$=\frac{a^2(67A+90B+80C)\sec^2(c+dx)\tan(c}{240d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^2(133A+150B+176C)\sec(c+dx)\tan}{192d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^2(133A+150B+176C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+$$

$$=\frac{a^2(133A+150B+176C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+$$

$$=\frac{a^{3/2}(133A+150B+176C)\tanh^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{a+a}}\right.}{128d}$$

Mathematica [A] time = 3.14001, size = 208, normalized size = 0.79

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(1273A+1070B+880C)\cos(c+dx)+4(3059A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^6,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]*(
133*A + 150*B + 176*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (
13313*A + 11550*B + 10480*C + 12*(1273*A + 1070*B + 880*C)*Cos[c + d*x] + 4
*(3059*A + 3450*B + 3280*C)*Cos[2*(c + d*x)] + 2660*A*Cos[3*(c + d*x)] + 30
00*B*Cos[3*(c + d*x)] + 3520*C*Cos[3*(c + d*x)] + 1995*A*Cos[4*(c + d*x)] +
 2250*B*Cos[4*(c + d*x)] + 2640*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(153
60*d)
```

Maple [B] time = 0.289, size = 2843, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/120*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(13
3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+133*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))+150*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)+150*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+176*C*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))+176*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a)))*sin(1/2*d*x+1/2*c)^10+240*(266*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+300*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+352*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+665*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+665*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+750
*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+750*B*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a))*a+880*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a+880*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2
*c)^8-80*(1862*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2100*B*a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2464*C*2^(1/2)*(a*sin(1/2*d*x+1/
```

2*c)^2)^(1/2)*a^(1/2)+1995*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
2^(1/2)(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a+1995*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2250*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2250*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a+2640*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2640*C*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(1702
4*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+19200*B*a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+21760*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+9975*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+9975*A*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+11250*B*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a+11250*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a+13200*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+13200*C*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-10*(6004*A*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+6552*B*a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+6848*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+1995*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1995*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2250*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+2250*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2640*C*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2640*C*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+11370*A*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+1995*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a))*a+1995*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+10860*B*a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2250*B*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a))*a+2250*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1

```
0080*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2640*C*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2640*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^5/(2*cos(1/2*d*x+1/2*c)-2^(1/2)
)^5/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.22125, size = 703, normalized size = 2.67

$$\frac{15\left((133\,A+150\,B+176\,C)a\cos\left(dx+c\right)^{6}+(133\,A+150\,B+176\,C)a\cos\left(dx+c\right)^{5}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{}}{\cos(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="fricas")
```

```
[Out] 1/7680*(15*((133*A + 150*B + 176*C)*a*cos(d*x + c)^6 + (133*A + 150*B + 176
*C)*a*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(15*(133*A + 150*B + 176*C)*a*cos(d*x
 + c)^4 + 10*(133*A + 150*B + 176*C)*a*cos(d*x + c)^3 + 8*(133*A + 150*B +
80*C)*a*cos(d*x + c)^2 + 48*(19*A + 10*B)*a*cos(d*x + c) + 384*A*a)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.83888, size = 1827, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="giac")
```

```
[Out] 1/3840*(15*(133*A*a^(3/2) + 150*B*a^(3/2) + 176*C*a^(3/2))*log(abs((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2
) + 3))) - 15*(133*A*a^(3/2) + 150*B*a^(3/2) + 176*C*a^(3/2))*log(abs((sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqr
t(2) - 3))) + 4*sqrt(2)*(1995*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^18*A*a^(5/2) + 2250*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*a^(5/2) + 2640*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*a^(5/2) - 38505*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*A*a^(7/2)
 - 76110*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^16*B*a^(7/2) - 55920*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^16*C*a^(7/2) + 561660*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(9/2) + 737160*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*a^(9/2) + 582720*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(9/2) -
 2684100*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^12*A*a^(11/2) - 3492600*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^12*B*a^(11/2) - 3395520*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(11/2) + 7371738*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(13/2) + 902286
0*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*
a^(13/2) + 9329760*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
```

```
*c)^2 + a))^10*C*a^(13/2) - 6407470*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(15/2) - 7635300*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(15/2) - 8110880*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(15/2) +
 2176620*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^6*A*a^(17/2) + 2614440*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^6*B*a^(17/2) + 2882880*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(17/2) - 399860*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(19/2) - 460440*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(19/2
) - 498880*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^4*C*a^(19/2) + 34035*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*A*a^(21/2) + 41850*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(21/2) + 42960*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(21/2) - 1201*A*a^(23/2
) - 1470*B*a^(23/2) - 1520*C*a^(23/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^5)/d
```

3.391 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=294

$$\frac{2a^3(2717A+2522B+2224C)\sin(c+dx)\cos^3(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(143A+182B+136C)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+}}{1287d}$$

```
[Out] (2*a^3*(10439*A + 9230*B + 8368*C)*Sin[c + d*x])/(6435*d*Sqrt[a + a*Cos[c +
 d*x]]) + (2*a^3*(2717*A + 2522*B + 2224*C)*Cos[c + d*x]^3*Sin[c + d*x])/(9
009*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a^2*(10439*A + 9230*B + 8368*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(45045*d) + (2*a^2*(143*A + 182*B + 136*C)
*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1287*d) + (2*a*(104
39*A + 9230*B + 8368*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15015*d)
+ (2*a*(13*B + 5*C)*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])
/(143*d) + (2*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13
*d)
```

Rubi [A] time = 0.959381, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3045, 2976, 2981, 2759, 2751, 2646}

$$\frac{2a^3(2717A+2522B+2224C)\sin(c+dx)\cos^3(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(143A+182B+136C)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+}}{1287d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*a^3*(10439*A + 9230*B + 8368*C)*Sin[c + d*x])/(6435*d*Sqrt[a + a*Cos[c +
 d*x]]) + (2*a^3*(2717*A + 2522*B + 2224*C)*Cos[c + d*x]^3*Sin[c + d*x])/(9
009*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a^2*(10439*A + 9230*B + 8368*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(45045*d) + (2*a^2*(143*A + 182*B + 136*C)
*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1287*d) + (2*a*(104
39*A + 9230*B + 8368*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15015*d)
+ (2*a*(13*B + 5*C)*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])
/(143*d) + (2*C*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13
*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\sin}{13d}$$

$$= \frac{2a(13B+5C)\cos^3(c+dx)(a+a\cos(c+}{143d}$$

$$= \frac{2a^2(143A+182B+136C)\cos^3(c+dx)}{1287d}$$

$$= \frac{2a^3(2717A+2522B+2224C)\cos^3(c+}{9009d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^3(2717A+2522B+2224C)\cos^3(c+}{9009d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^3(2717A+2522B+2224C)\cos^3(c+}{9009d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^3(10439A+9230B+8368C)\sin(c+}{6435d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 1.76446, size = 180, normalized size = 0.61

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(4(445588A+454285B+453146C)\cos(c+dx)+(581152A+676000B+74651}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3233516*A + 2980640*B + 2798182*C + 4*(445
588*A + 454285*B + 453146*C)*Cos[c + d*x] + (581152*A + 676000*B + 746519*C
)*Cos[2*(c + d*x)] + 148720*A*Cos[3*(c + d*x)] + 225550*B*Cos[3*(c + d*x)]
+ 287060*C*Cos[3*(c + d*x)] + 20020*A*Cos[4*(c + d*x)] + 58240*B*Cos[4*(c +
 d*x)] + 94010*C*Cos[4*(c + d*x)] + 8190*B*Cos[5*(c + d*x)] + 23940*C*Cos[5
*(c + d*x)] + 3465*C*Cos[6*(c + d*x)])*Tan[(c + d*x)/2])/(720720*d)
```

Maple [A] time = 0.077, size = 176, normalized size = 0.6

$$\frac{8\,a^3\sqrt{2}}{45045\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(55440\,C\,(\sin(1/2\,dx+c/2))^{12}+(-32760\,B-262080\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}+(2002\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 8/45045*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(55440*C*sin(1/2*d*x+1/2*
c)^12+(-32760*B-262080*C)*sin(1/2*d*x+1/2*c)^10+(20020*A+140140*B+520520*C)
*sin(1/2*d*x+1/2*c)^8+(-77220*A-244530*B-566280*C)*sin(1/2*d*x+1/2*c)^6+(11
7117*A+225225*B+369369*C)*sin(1/2*d*x+1/2*c)^4+(-90090*A-120120*B-150150*C)
*sin(1/2*d*x+1/2*c)^2+45045*A+45045*B+45045*C)*2^(1/2)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

Maxima [A] time = 3.26115, size = 448, normalized size = 1.52

$$\frac{572\left(35\,\sqrt{2}a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+225\,\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+756\,\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+2100\,\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] 1/1441440*(572*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7
/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*s
in(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 13
0*(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*x + 9/
2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5/2*d*x
```

```
 + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c))*B*sqrt(a) + (3465*sqrt(2)*a^2*sin(13/2*d*x + 13/2*c) + 204
75*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 70070*sqrt(2)*a^2*sin(9/2*d*x + 9/2
*c) + 193050*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 459459*sqrt(2)*a^2*sin(5/2*
d*x + 5/2*c) + 1066065*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 3783780*sqrt(2)*a
^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 2.00845, size = 498, normalized size = 1.69

$$\frac{2\left(3465\,Ca^2\cos\left(dx+c\right)^6+315\,(13\,B+38\,C)a^2\cos\left(dx+c\right)^5+35\,(143\,A+416\,B+523\,C)a^2\cos\left(dx+c\right)^4+5\,(3718\,A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*C*a^2*cos(d*x + c)^6 + 315*(13*B + 38*C)*a^2*cos(d*x + c)^5 +
 35*(143*A + 416*B + 523*C)*a^2*cos(d*x + c)^4 + 5*(3718*A + 4615*B + 4184*
C)*a^2*cos(d*x + c)^3 + 3*(10439*A + 9230*B + 8368*C)*a^2*cos(d*x + c)^2 +
4*(10439*A + 9230*B + 8368*C)*a^2*cos(d*x + c) + 8*(10439*A + 9230*B + 8368
*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.392 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos\right.$

Optimal. Leaf size=229

$$\frac{16a^2(165A+143B+125C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3465d}+\frac{64a^3(165A+143B+125C)\sin(c+dx)}{3465d\sqrt{a\cos(c+dx)+a}}+\frac{2(99A-22B+2}{}$$

```
[Out] (64*a^3*(165*A + 143*B + 125*C)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c + d*
x]]) + (16*a^2*(165*A + 143*B + 125*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3465*d) + (2*a*(165*A + 143*B + 125*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(1155*d) + (2*(99*A - 22*B + 26*C)*(a + a*Cos[c + d*x])^(5/2)*Sin[
c + d*x])/(693*d) + (2*C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(11*d) + (2*(11*B + 5*C)*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99
*a*d)
```

Rubi [A] time = 0.493746, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3045, 2968, 3023, 2751, 2647, 2646}

$$\frac{16a^2(165A+143B+125C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3465d}+\frac{64a^3(165A+143B+125C)\sin(c+dx)}{3465d\sqrt{a\cos(c+dx)+a}}+\frac{2(99A-22B+2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (64*a^3*(165*A + 143*B + 125*C)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c + d*
x]]) + (16*a^2*(165*A + 143*B + 125*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3465*d) + (2*a*(165*A + 143*B + 125*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(1155*d) + (2*(99*A - 22*B + 26*C)*(a + a*Cos[c + d*x])^(5/2)*Sin[
c + d*x])/(693*d) + (2*C*Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(11*d) + (2*(11*B + 5*C)*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99
*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\sin(}{11d}$$

$$=\frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\sin(}{11d}$$

$$=\frac{2C\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\sin(}{11d}$$

$$=\frac{2(99A-22B+26C)(a+a\cos(c+dx))^{5/}}{693d}$$

$$=\frac{2a(165A+143B+125C)(a+a\cos(c+d}{1155d}$$

$$=\frac{16a^2(165A+143B+125C)\sqrt{a+a\cos(c}}{3465d}$$

$$=\frac{64a^3(165A+143B+125C)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 1.24694, size = 147, normalized size = 0.64

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((66660A+68552B+69890C)\cos(c+dx)+16(990A+1397B+1625C)\cos(2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(137280*A + 124366*B + 114640*C + (66660*A
+ 68552*B + 69890*C)*Cos[c + d*x] + 16*(990*A + 1397*B + 1625*C)*Cos[2*(c +
 d*x)] + 1980*A*Cos[3*(c + d*x)] + 5720*B*Cos[3*(c + d*x)] + 8675*C*Cos[3*(
c + d*x)] + 770*B*Cos[4*(c + d*x)] + 2240*C*Cos[4*(c + d*x)] + 315*C*Cos[5*
(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 0.08, size = 154, normalized size = 0.7

$$\frac{8a^3\sqrt{2}}{3465d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-2520C(\sin(1/2dx+c/2))^{10}+(1540B+10780C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8+(-990A-5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 8/3465*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-2520*C*sin(1/2*d*x+1/2*c
)^10+(1540*B+10780*C)*sin(1/2*d*x+1/2*c)^8+(-990*A-5940*B-18810*C)*sin(1/2*
d*x+1/2*c)^6+(3465*A+9009*B+17325*C)*sin(1/2*d*x+1/2*c)^4+(-4620*A-6930*B-9
240*C)*sin(1/2*d*x+1/2*c)^2+3465*A+3465*B+3465*C)*2^(1/2)/(a*cos(1/2*d*x+1/
2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.14184, size = 381, normalized size = 1.66

$$\frac{660\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/55440*(660*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d
*x + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2
*d*x + 1/2*c))*A*sqrt(a) + 22*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sq
rt(2)*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 210
0*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))
*B*sqrt(a) + 5*(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin
(9/2*d*x + 9/2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^
2*sin(5/2*d*x + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt
(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.93397, size = 413, normalized size = 1.8

$$\frac{2\left(315\,Ca^2\cos\left(dx+c\right)^5+35\left(11\,B+32\,C\right)a^2\cos\left(dx+c\right)^4+5\left(99\,A+286\,B+355\,C\right)a^2\cos\left(dx+c\right)^3+3\left(660\,A+80\right.}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/3465*(315*C*a^2*cos(d*x + c)^5 + 35*(11*B + 32*C)*a^2*cos(d*x + c)^4 + 5*
(99*A + 286*B + 355*C)*a^2*cos(d*x + c)^3 + 3*(660*A + 803*B + 710*C)*a^2*c
os(d*x + c)^2 + (3795*A + 3212*B + 2840*C)*a^2*cos(d*x + c) + 2*(3795*A + 3
212*B + 2840*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c)
+ d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)*cos(d*x + c), x)
```

3.393 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right.$

Optimal. Leaf size=184

$$\frac{16a^2(21A+15B+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(21A+15B+13C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a(21A+15B+13C)}{}$$

```
[Out] (64*a^3*(21*A + 15*B + 13*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
 + (16*a^2*(21*A + 15*B + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315
*d) + (2*a*(21*A + 15*B + 13*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1
05*d) + (2*(9*B - 2*C)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2
*C*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.245148, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3023, 2751, 2647, 2646}

$$\frac{16a^2(21A+15B+13C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(21A+15B+13C)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a(21A+15B+13C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (64*a^3*(21*A + 15*B + 13*C)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
 + (16*a^2*(21*A + 15*B + 13*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315
*d) + (2*a*(21*A + 15*B + 13*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1
05*d) + (2*(9*B - 2*C)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2
*C*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \frac{2C(a + a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int (a + a\cos(}{}$$

$$= \frac{2(9B-2C)(a + a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2C(a}{}$$

$$= \frac{2a(21A+15B+13C)(a + a\cos(c+dx))^{3/2}\sin(c+d}{105d}$$

$$= \frac{16a^2(21A+15B+13C)\sqrt{a + a\cos(c+dx)}\sin(c+d}{315d}$$

$$= \frac{64a^3(21A+15B+13C)\sin(c+dx)}{315d\sqrt{a + a\cos(c+dx)}} + \frac{16a^2(21A+1}{}$$

Mathematica [A] time = 0.658205, size = 114, normalized size = 0.62

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((2352A+3030B+3116C)\cos(c+dx)+4(63A+180B+254C)\cos(2(c+dx))}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(7476*A + 6240*B + 5653*C + (2352*A + 3030*
B + 3116*C)*Cos[c + d*x] + 4*(63*A + 180*B + 254*C)*Cos[2*(c + d*x)] + 90*B
*Cos[3*(c + d*x)] + 260*C*Cos[3*(c + d*x)] + 35*C*Cos[4*(c + d*x)])*Tan[(c
+ d*x)/2])/(1260*d)
```

Maple [A] time = 0.065, size = 132, normalized size = 0.7

$$\frac{8\,a^3\sqrt{2}}{315\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\,C\,(\sin(1/2\,dx+c/2))^8+(-90\,B-540\,C)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+(63\,A+315\,B+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*C*sin(1/2*d*x+1/2*c)^8
+(-90*B-540*C)*sin(1/2*d*x+1/2*c)^6+(63*A+315*B+819*C)*sin(1/2*d*x+1/2*c)^4
+(-210*A-420*B-630*C)*sin(1/2*d*x+1/2*c)^2+315*A+315*B+315*C)*2^(1/2)/(a*co
s(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 2.30506, size = 311, normalized size = 1.69

$$\frac{84\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+30\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] 1/2520*(84*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 30*(3*sqrt(2)
*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(2
)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a
) + (35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*d*x + 7/
```

```
2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(3/2*d*x
+ 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 1.7568, size = 331, normalized size = 1.8

$$\frac{2\left(35\,Ca^2\cos\left(dx+c\right)^4+5\,(9\,B+26\,C)a^2\cos\left(dx+c\right)^3+3\,(21\,A+60\,B+73\,C)a^2\cos\left(dx+c\right)^2+(294\,A+345\,B+292}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/315*(35*C*a^2*cos(d*x + c)^4 + 5*(9*B + 26*C)*a^2*cos(d*x + c)^3 + 3*(21*
A + 60*B + 73*C)*a^2*cos(d*x + c)^2 + (294*A + 345*B + 292*C)*a^2*cos(d*x +
 c) + (903*A + 690*B + 584*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d
*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
), x)
```

3.394 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ ... $dx)\,dx$

Optimal. Leaf size=182

$$\frac{2a^3(245A+224B+160C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2(35A+56B+40C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d} + \frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos}}\right.}{d}$$

```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(245*A + 224*B + 160*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]
]) + (2*a^2*(35*A + 56*B + 40*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(10
5*d) + (2*a*(7*B + 5*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (
2*C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.661587, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3045, 2976, 2981, 2773, 206}

$$\frac{2a^3(245A+224B+160C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2(35A+56B+40C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d} + \frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```

```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(245*A + 224*B + 160*C)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]
]) + (2*a^2*(35*A + 56*B + 40*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(10
5*d) + (2*a*(7*B + 5*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (
2*C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
```

```
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{}$$

$$= \frac{2a(7B+5C)(a+a\cos(c+dx))^{3/2}\sin(c+}{35d}$$

$$= \frac{2a^2(35A+56B+40C)\sqrt{a+a\cos(c+dx)}}{105d}$$

$$= \frac{2a^3(245A+224B+160C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a^3(245A+224B+160C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(245}{1}$$

Mathematica [A] time = 0.734871, size = 127, normalized size = 0.7

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((140A+392B+505C)\cos(c+dx)+1120A+6(7B+20C)\text{co}\right.}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(420*Sqrt[2]*A*ArcTanh[Sqr
t[2]*Sin[(c + d*x)/2]] + 2*(1120*A + 1246*B + 1040*C + (140*A + 392*B + 505
*C)*Cos[c + d*x] + 6*(7*B + 20*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])
*Sin[(c + d*x)/2]))/(420*d)
```

Maple [B] time = 0.201, size = 377, normalized size = 2.1

$$\frac{1}{105\,d}a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-240\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^6+168\,\sqrt{a\,(\sin(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/105*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*C*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*2^(1/2)*(B+5*C)*sin(1/2*d*x+1/2*c)^4-140*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*2^(1/2)*(A+4*B+8*C)*sin(1/2*d*x+1/2*c
)^2+630*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+105*A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+105*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+840*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+840*C*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*
d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.87061, size = 188, normalized size = 1.03

$$\frac{14\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}+5\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+\cdots\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
+ 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + 5*(3*sqrt(2)*a
^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))
/d
```

Fricas [A] time = 2.16476, size = 540, normalized size = 2.97

$$\frac{105\left(Aa^2\cos\left(dx+c\right)+Aa^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(15\,Ca^2\cos\left(dx\cdots\right.\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="fricas")
```

```
[Out] 1/210*(105*(A*a^2*cos(d*x + c) + A*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a
*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin
(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(15*C*a^2*cos(d*x +
 c)^3 + 3*(7*B + 20*C)*a^2*cos(d*x + c)^2 + (35*A + 98*B + 115*C)*a^2*cos(d
*x + c) + (280*A + 301*B + 230*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.93448, size = 397, normalized size = 2.18

$$\frac{\frac{105 A a^{\frac{7}{2}} \log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{2\left(315\sqrt{2}Aa^6+420\sqrt{2}Ba^6+420\sqrt{2}Ca^6+\left(875\sqrt{2}Aa^6+980\sqrt{2}Ba^6+700\sqrt{2}Ca^6+\left(805\sqrt{2}\cdots\right.\right.\right.}{\cdots}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="giac")
```

```
[Out] 1/105*(105*A*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6
```

```
*a))/abs(a) + 2*(315*sqrt(2)*A*a^6 + 420*sqrt(2)*B*a^6 + 420*sqrt(2)*C*a^6
+ (875*sqrt(2)*A*a^6 + 980*sqrt(2)*B*a^6 + 700*sqrt(2)*C*a^6 + (805*sqrt(2)
*A*a^6 + 784*sqrt(2)*B*a^6 + 560*sqrt(2)*C*a^6 + (245*sqrt(2)*A*a^6 + 224*s
qrt(2)*B*a^6 + 160*sqrt(2)*C*a^6)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2
*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)
^2 + a)^(7/2))/d
```

3.395 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ ... $dx)\,dx$

Optimal. Leaf size=184

$$\frac{a^3(15A+70B+64C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(15A-10B-16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin\cdots}{\sqrt{a\cos\cdots}}\right.}{d}$$

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a^3*(15*A + 70*B + 64*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*
x]]) - (a^2*(15*A - 10*B - 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15
*d) - (a*(5*A - 2*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (A*(a
 + a*Cos[c + d*x])^(5/2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.67613, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2976, 2981, 2773, 206}

$$\frac{a^3(15A+70B+64C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(15A-10B-16C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin\cdots}{\sqrt{a\cos\cdots}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```


```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a^3*(15*A + 70*B + 64*C)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*
x]]) - (a^2*(15*A - 10*B - 16*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15
*d) - (a*(5*A - 2*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (A*(a
 + a*Cos[c + d*x])^(5/2)*Tan[c + d*x])/d
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^{5/2}\tan(c+dx)}{d}+\frac{\int(}{}$$

$$=-\frac{a(5A-2C)(a+a\cos(c+dx))^{3/2}\sin(c}{5d}$$

$$=-\frac{a^2(15A-10B-16C)\sqrt{a+a\cos(c+dx}}{15d}$$

$$=\frac{a^3(15A+70B+64C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}-\frac{a^2(1}{}$$

$$=\frac{a^3(15A+70B+64C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}-\frac{a^2(1}{}$$

$$=\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+$$

Mathematica [A] time = 0.783709, size = 145, normalized size = 0.79

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((60A+160B+181C)\cos(c+dx)+30A+2(5B+}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(30*Sqrt[2]*(
5*A + 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(30*A + 10*B
+ 28*C + (60*A + 160*B + 181*C)*Cos[c + d*x] + 2*(5*B + 14*C)*Cos[2*(c + d*
x)] + 3*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(60*d)
```

Maple [B] time = 0.24, size = 846, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*C*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+16*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*2^(1/2)*(5*B+23*C)*sin(1/2*d*x+1/2*c)^4-10*(1
2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+40*B*a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+64*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+15*A*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a))*a+6*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1
/2*c)^2+90*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))*a+180*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+30*B*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+30*B*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a))*a+240*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1
/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 3.18903, size = 11036, normalized size = 59.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(42*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a) - 5*(1449*sqrt(
2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d
*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
+ 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 60*sqrt(2)*a^2*s
in(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^
```

2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*cos(5/2*d*x
+ 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*
c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 69*sqrt
(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 198*sqrt(2)*a^2*sin(1/2*d*x
 + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 198*sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 12
*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c)^2
 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c))*sin(2*
d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*
sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sq
rt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(5/2*d
*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1/2*d*
x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(13
/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c)
 + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)
 + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(
5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*c
os(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x +
2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x +
2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*
sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c
))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x
 + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x
 + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a
^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x +
2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*
d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin
(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)
*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x
+ 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(2*
d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 18
9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) - 21*(60*sqrt(2)*a^2*sin(1/2*d*x
+ 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) - 315*(a^2*cos(
1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 +
 a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c

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)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*
sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(
1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x +
 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2
*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*
x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*
c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/2*d*x + 1/2*c)^2 +
 a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d
*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2
*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(
5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)
^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a
^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2
*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos
(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2
*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d
*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x
 + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1
/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*
sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*
sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 +
 (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)
^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2
*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2
+ a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*
sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*
cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d
*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
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+ 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 35*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(13/2*d*x + 13/2*c) + 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(11/2*d*x + 11/2*c) + 7*(9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 - (5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 - (5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 4*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*sin(1

/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(9/2*d*x + 9/2*c) - 390*(sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*
a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*c
os(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(
2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5
/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2
*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin
(7/2*d*x + 7/2*c) - 21*(69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 189*sqrt(2)
*a^2*sin(1/2*d*x + 1/2*c)^2 + 69*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^
2)*cos(5/2*d*x + 5/2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*
d*x + 1/2*c) - 6*sqrt(2)*a^2)*cos(2*d*x + 2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/
2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*sin(2*d*x + 2*c)^2 + 1
2*sqrt(2)*a^2 + 138*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - sq
rt(2)*a^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x +
 1/2*c))*cos(5/2*d*x + 5/2*c) + (69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 - 50
*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 189*sqrt(2)*a^2*si
n(1/2*d*x + 1/2*c)^2 + 24*sqrt(2)*a^2)*cos(2*d*x + 2*c) - 10*(5*sqrt(2)*a^2
*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c) + 105*(1
2*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*
sin(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^
2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) - 252*(5*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c) - 135
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 +
 (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(
2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 +
 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sq
rt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(
2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*
x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d
*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqr
t(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x +
1/2*c))*sin(5/2*d*x + 5/2*c))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 63*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*
x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/
2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/
2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*
sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*

```
d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(
1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*
sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x
 + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + s
qrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*
d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*
d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(5/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1260*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2
+ sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)
*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x +
 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) +
 sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2
+ sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*c
os(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*s
in(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) +
 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2
*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(5/2*d*x +
 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2
*c)^2 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*
sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + cos(1/2*d
*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) +
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + cos(1/2*d*x + 1/2*c)^2 + 2*(cos(2*
d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)
 + 2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(5/2*
d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2))/d
```

Fricas [A] time = 2.24833, size = 579, normalized size = 3.15

$$\frac{15\left((5A+2B)a^2\cos(dx+c)^2+(5A+2B)a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{60\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sq
rt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x
 + c)^2)) + 4*(6*C*a^2*cos(d*x + c)^3 + 2*(5*B + 14*C)*a^2*cos(d*x + c)^2 +
 2*(15*A + 40*B + 43*C)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.2091, size = 539, normalized size = 2.93

$$15\left(5\,Aa^{\frac{5}{2}}+2\,Ba^{\frac{5}{2}}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-15\left(5\,Aa^{\frac{5}{2}}+2\,Ba^{\frac{5}{2}}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*(5*A*a^(5/2) + 2*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(5*A*a^(
5/2) + 2*B*a^(5/2))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
```

```
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 60*sqrt(2)*(3*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(7/2) - A*a^(9/
2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4
- 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
 + a^2) + 4*(15*sqrt(2)*A*a^5 + 45*sqrt(2)*B*a^5 + 60*sqrt(2)*C*a^5 + (30*s
qrt(2)*A*a^5 + 80*sqrt(2)*B*a^5 + 80*sqrt(2)*C*a^5 + (15*sqrt(2)*A*a^5 + 35
*sqrt(2)*B*a^5 + 32*sqrt(2)*C*a^5)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.396 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=199

$$-\frac{a^3(27A-12B-56C)\sin(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(21A+12B-8C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{12d}+\frac{a^{5/2}(19A+20B+8C)\tanh}{4d}$$

```
[Out] (a^(5/2)*(19*A + 20*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(4*d) - (a^3*(27*A - 12*B - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a +
a*Cos[c + d*x]]) - (a^2*(21*A + 12*B - 8*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(12*d) + (a*(5*A + 4*B)*(a + a*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(4
*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.701436, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2975, 2976, 2981, 2773, 206}

$$-\frac{a^3(27A-12B-56C)\sin(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(21A+12B-8C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{12d}+\frac{a^{5/2}(19A+20B+8C)\tanh}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```

```
[Out] (a^(5/2)*(19*A + 20*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(4*d) - (a^3*(27*A - 12*B - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a +
a*Cos[c + d*x]]) - (a^2*(21*A + 12*B - 8*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(12*d) + (a*(5*A + 4*B)*(a + a*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(4
*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^3(c + dx)\,dx = \frac{A(a + a\cos(c + dx))^{5/2}\sec(c + dx)\tan}{2d}$$

$$= \frac{a(5A + 4B)(a + a\cos(c + dx))^{3/2}\tan(c}{4d}$$

$$= -\frac{a^2(21A + 12B - 8C)\sqrt{a + a\cos(c + d}}{12d}$$

$$= -\frac{a^3(27A - 12B - 56C)\sin(c + dx)}{12d\sqrt{a + a\cos(c + dx)}} - \frac{a}{}$$

$$= -\frac{a^3(27A - 12B - 56C)\sin(c + dx)}{12d\sqrt{a + a\cos(c + dx)}} - \frac{a}{}$$

$$= \frac{a^{5/2}(19A + 20B + 8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{a+a\cos}}\right.}{4d}$$

Mathematica [A] time = 1.07885, size = 153, normalized size = 0.77

$$\frac{a^2\sec\left(\frac{1}{2}(c + dx)\right)\sec^2(c + dx)\sqrt{a(\cos(c + dx) + 1)}\left(4\sin\left(\frac{1}{2}(c + dx)\right)(3(11A + 4B + 2C)\cos(c + dx) + 6A + 4(3B + 8C}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(6*Sqrt[2]*
(19*A + 20*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 4*(6
*A + 12*B + 32*C + 3*(11*A + 4*B + 2*C)*Cos[c + d*x] + 4*(3*B + 8*C)*Cos[2*
```

```
(c + d*x)] + 2*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.244, size = 1512, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-128*C*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+4*(48*B*a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+176*C*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+57*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+57*
A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+60*B*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a))*a+60*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-4*(33*A*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+60*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+152*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+57*A*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+57*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2*a))*a+60*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+60*B*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24*C*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a))*a+24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a)*sin(1/2*d*x+1/2*c)^2+78*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+57*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+57*A*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a))*a+72*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+60*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+60*B*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+144*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24*C*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^
2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.08879, size = 591, normalized size = 2.97

$$\frac{3\left((19\,A+20\,B+8\,C)a^2\cos\left(dx+c\right)^3+(19\,A+20\,B+8\,C)a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c}}{\cos(dx+c)^3+\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*((19*A + 20*B + 8*C)*a^2*cos(d*x + c)^3 + (19*A + 20*B + 8*C)*a^2*c
os(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(
a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*(8*C*a^2*cos(d*x + c)^3 + 8*(3*B + 8*C)*a^2*
cos(d*x + c)^2 + 3*(11*A + 4*B)*a^2*cos(d*x + c) + 6*A*a^2)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.50227, size = 761, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(19*A*a^(5/2) + 20*B*a^(5/2) + 8*C*a^(5/2))*log(abs((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))
) - 3*(19*A*a^(5/2) + 20*B*a^(5/2) + 8*C*a^(5/2))*log(abs((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))
+ 16*(3*sqrt(2)*B*a^4 + 9*sqrt(2)*C*a^4 + (3*sqrt(2)*B*a^4 + 7*sqrt(2)*C*a^
4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 +
 a)^(3/2) + 12*sqrt(2)*(19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^6*A*a^(7/2) + 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(7/2) - 171*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(9/2) - 76*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(9/2) + 89*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(11/2) +
 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B
*a^(11/2) - 9*A*a^(13/2) - 4*B*a^(13/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.397 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=207

$$-\frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(31A+42B+24C)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{a^{5/2}(25A+38B+40C)\text{ t}}{8d}$$

```
[Out] (a^(5/2)*(25*A + 38*B + 40*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(8*d) - (a^3*(49*A + 54*B - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a^2*(31*A + 42*B + 24*C)*Sqrt[a + a*Cos[c + d*x]]*Tan[
c + d*x])/(24*d) + (a*(5*A + 6*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*T
an[c + d*x])/(12*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c +
d*x])/(3*d)
```

Rubi [A] time = 0.726982, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2975, 2981, 2773, 206}

$$-\frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(31A+42B+24C)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{a^{5/2}(25A+38B+40C)\text{ t}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```


```
[Out] (a^(5/2)*(25*A + 38*B + 40*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(8*d) - (a^3*(49*A + 54*B - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a^2*(31*A + 42*B + 24*C)*Sqrt[a + a*Cos[c + d*x]]*Tan[
c + d*x])/(24*d) + (a*(5*A + 6*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*T
an[c + d*x])/(12*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c +
d*x])/(3*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\tan}{3d}$$

$$= \frac{a(5A+6B)(a+a\cos(c+dx))^{3/2}\sec(c}{12d}$$

$$= \frac{a^2(31A+42B+24C)\sqrt{a+a\cos(c+d}}{24d}$$

$$= -\frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + $$

$$= -\frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{a^{5/2}(25A+38B+40C)\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a+a\cos}}\right.}{8d}$$

Mathematica [A] time = 1.51506, size = 156, normalized size = 0.75

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(17A+6(B+3C))\cos(c+dx)+3(25A+22B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*
(25*A + 38*B + 40*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (91
*A + 66*B + 24*C + 4*(17*A + 6*(B + 3*C))*Cos[c + d*x] + 3*(25*A + 22*B + 8
*C)*Cos[2*(c + d*x)] + 24*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.257, size = 1925, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*(32*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+25*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+25*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+38*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+38*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+40*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+40*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+12*(50*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+44*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+112*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+114*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+120*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+120*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-736*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-450*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-576*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-684*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-684*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-768*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-720*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-720*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+234*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+156*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*

```
2*c)^2)^(1/2)+114*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+144*C*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+120*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a+120*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2
*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.11268, size = 606, normalized size = 2.93

$$\frac{3\left((25\,A+38\,B+40\,C)a^2\cos{(dx+c)}^4+(25\,A+38\,B+40\,C)a^2\cos{(dx+c)}^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(d}}{\cos(dx+c)^3+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*((25*A + 38*B + 40*C)*a^2*cos(d*x + c)^4 + (25*A + 38*B + 40*C)*a^2
*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqr
t(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)) + 4*(48*C*a^2*cos(d*x + c)^3 + 3*(25*A + 22*B
+ 8*C)*a^2*cos(d*x + c)^2 + 2*(17*A + 6*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.84894, size = 1210, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="giac")
```

```
[Out] 1/48*(96*sqrt(2)*C*a^3*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a) + 3*(25*A*a^(5/2) + 38*B*a^(5/2) + 40*C*a^(5/2))*log(abs((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)
)) - 3*(25*A*a^(5/2) + 38*B*a^(5/2) + 40*C*a^(5/2))*log(abs((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(75*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^10*A*a^(7/2) + 114*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^10*B*a^(7/2) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(7/2) - 1125*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(9/2) - 1710*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(9/2) - 88
8*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a
^(9/2) + 6174*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^6*A*a^(11/2) + 6804*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^6*B*a^(11/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(11/2) - 4314*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(13/2) - 4284*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(13/2) - 1
776*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C
*a^(13/2) + 807*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
```

```
^2 + a))^2*A*a^(15/2) + 858*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*B*a^(15/2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(15/2) - 49*A*a^(17/2) - 54*B*a^(17/
2) - 24*C*a^(17/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*a + a^2)^3)/d
```

3.398 $\int (a+a\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=215

$$\frac{a^3(299A+392B+432C)\tan(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+200B+304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+24B+16C)\tan}{}$$

```
[Out] (a^(5/2)*(163*A + 200*B + 304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/(64*d) + (a^3*(299*A + 392*B + 432*C)*Tan[c + d*x])/(192*d*
Sqrt[a + a*Cos[c + d*x]]) + (a^2*(17*A + 24*B + 16*C)*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]*Tan[c + d*x])/(32*d) + (a*(5*A + 8*B)*(a + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (A*(a + a*Cos[c + d*x])^(5/2)*
Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.789524, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2975, 2980, 2773, 206}

$$\frac{a^3(299A+392B+432C)\tan(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+200B+304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+24B+16C)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```

```
[Out] (a^(5/2)*(163*A + 200*B + 304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/(64*d) + (a^3*(299*A + 392*B + 432*C)*Tan[c + d*x])/(192*d*
Sqrt[a + a*Cos[c + d*x]]) + (a^2*(17*A + 24*B + 16*C)*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]*Tan[c + d*x])/(32*d) + (a*(5*A + 8*B)*(a + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (A*(a + a*Cos[c + d*x])^(5/2)*
Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\tan(}{4d}$$

$$= \frac{a(5A+8B)(a+a\cos(c+dx))^{3/2}\sec^2(c-}{24d}$$

$$= \frac{a^2(17A+24B+16C)\sqrt{a+a\cos(c+dx)}}{32d}$$

$$= \frac{a^3(299A+392B+432C)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{a^3(299A+392B+432C)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{a^{5/2}(163A+200B+304C)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.}{64d}$$

Mathematica [A] time = 1.95556, size = 176, normalized size = 0.82

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((2203A+2056B+1584C)\cos(c+dx)+4(163A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^5,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*
(163*A + 200*B + 304*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 +
(844*A + 544*B + 192*C + (2203*A + 2056*B + 1584*C)*Cos[c + d*x] + 4*(163*A
 + 136*B + 48*C)*Cos[2*(c + d*x)] + 489*A*Cos[3*(c + d*x)] + 600*B*Cos[3*(c
 + d*x)] + 528*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d)
```

Maple [B] time = 0.29, size = 2369, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(163*A
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+163*A*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a))+200*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+2
00*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+304*C*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))+304*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))
)*sin(1/2*d*x+1/2*c)^8-48*(163*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+200*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+176*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+326*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+326*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+400*B*ln
(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+400*B*ln(4/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a))*a+608*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a+608*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^6
+8*(1793*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2072*B*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+1680*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)+1467*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1467
*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1800*B*ln(-4/(-2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2
)*cos(1/2*d*x+1/2*c)+2*a))*a+1800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*a))*a+2736*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2736*C*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-3912*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-9212*A*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-3912*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
-4800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
```

```
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-9632*B*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-4800*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a-7296*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-7104*C*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-7296*C*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+489*A*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*a))*a+2094*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+489*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+600*B*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1872*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+600*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+912*C*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1248*C*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+912*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/
2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.09616, size = 649, normalized size = 3.02

$$\frac{3\left((163\,A + 200\,B + 304\,C)a^2\cos\left(dx+c\right)^5 + (163\,A + 200\,B + 304\,C)a^2\cos\left(dx+c\right)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\,\sqrt{}}{\cos(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="fricas")
```

```
[Out] 1/768*(3*((163*A + 200*B + 304*C)*a^2*cos(d*x + c)^5 + (163*A + 200*B + 304
*C)*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
- 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(163*A + 200*B + 176*C)*a^2*cos(
d*x + c)^3 + 2*(163*A + 136*B + 48*C)*a^2*cos(d*x + c)^2 + 8*(23*A + 8*B)*a
^2*cos(d*x + c) + 48*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d
*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.12499, size = 1494, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="giac")
```

```
[Out] 1/384*(3*(163*A*a^(5/2) + 200*B*a^(5/2) + 304*C*a^(5/2))*log(abs((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2)
+ 3))) - 3*(163*A*a^(5/2) + 200*B*a^(5/2) + 304*C*a^(5/2))*log(abs((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2
) - 3))) + 4*sqrt(2)*(489*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^14*A*a^(7/2) + 600*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*a^(7/2) + 912*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(7/2) - 10269*(sqrt(a)*ta
```

n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*a^(9/2) - 126
00*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B
a^(9/2) - 19152(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^12*C*a^(9/2) + 69885*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^10*A*a^(11/2) + 103992*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(11/2) + 137424*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(11/2) - 259
233*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A
a^(13/2) - 339864(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^8*B*a^(13/2) - 374544*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(13/2) + 209979*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(15/2) + 262920*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(15/2) + 266
928*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C
a^(15/2) - 55511(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*A*a^(17/2) - 73640*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*B*a^(17/2) - 75888*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(17/2) + 6687*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(19/2) + 8808*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(19/
2) + 9456*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*C*a^(19/2) - 299*A*a^(21/2) - 392*B*a^(21/2) - 432*C*a^(21/2))/((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/
d

3.399 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ ... $dx)\, dx$

Optimal. Leaf size=261

$$\frac{a^3(283A+326B+400C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}} + \frac{a^{5/2}(283A+326B+400C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d} + \frac{a^2(79A+110B+80C)\,\mathrm{t}}{}$$

```
[Out] (a^(5/2)*(283*A + 326*B + 400*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/(128*d) + (a^3*(283*A + 326*B + 400*C)*Tan[c + d*x])/(128*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(787*A + 950*B + 1040*C)*Sec[c + d*x]*Tan
[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(79*A + 110*B + 80*C)*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(240*d) + (a*(A + 2*B)*
(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (A*(a + a*C
os[c + d*x])^(5/2)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.902485, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2975, 2980, 2772, 2773, 206}

$$\frac{a^3(283A+326B+400C)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}} + \frac{a^{5/2}(283A+326B+400C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d} + \frac{a^2(79A+110B+80C)\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^6,x]
```


```
[Out] (a^(5/2)*(283*A + 326*B + 400*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/(128*d) + (a^3*(283*A + 326*B + 400*C)*Tan[c + d*x])/(128*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(787*A + 950*B + 1040*C)*Sec[c + d*x]*Tan
[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(79*A + 110*B + 80*C)*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(240*d) + (a*(A + 2*B)*
(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (A*(a + a*C
os[c + d*x])^(5/2)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\tan}{5d}$$

$$= \frac{a(A+2B)(a+a\cos(c+dx))^{3/2}\sec^3(c}{8d}$$

$$= \frac{a^2(79A+110B+80C)\sqrt{a+a\cos(c+}}{240d}$$

$$= \frac{a^3(787A+950B+1040C)\sec(c+dx)}{960d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(283A+326B+400C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(283A+326B+400C)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{5/2}(283A+326B+400C)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{128d}$$

Mathematica [A] time = 2.81305, size = 210, normalized size = 0.8

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(2343A+1950B+1360C)\cos(c+dx)+4(6509\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^6,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]
*(283*A + 326*B + 400*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 +
 (24863*A + 22030*B + 20560*C + 12*(2343*A + 1950*B + 1360*C)*Cos[c + d*x]
+ 4*(6509*A + 6730*B + 6640*C)*Cos[2*(c + d*x)] + 5660*A*Cos[3*(c + d*x)] +
 6520*B*Cos[3*(c + d*x)] + 5440*C*Cos[3*(c + d*x)] + 4245*A*Cos[4*(c + d*x)
] + 4890*B*Cos[4*(c + d*x)] + 6000*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(
15360*d)
```

Maple [B] time = 0.296, size = 2843, normalized size = 10.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/120*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(28
3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+283*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))+326*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)+326*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+400*C*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))+400*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a)))*sin(1/2*d*x+1/2*c)^10+240*(566*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+652*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+800*C*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1415*A*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+1415*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1
630*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1630*B*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a+2000*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d
*x+1/2*c)^8-80*(3962*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4564*
B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+5344*C*2^(1/2)*(a*sin(1/2*
```

d*x+1/2*c)^2)^(1/2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+4890*B*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a))*a+6000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6
000*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8
*(36224*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+40960*B*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+44800*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+21225*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+21
225*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+24450*B*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+24450*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a))*a+30000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+30000*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-10*(12
556*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+13400*B*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+13376*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4245*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+4890*B*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)*a+6000*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6000*C*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+22230*A*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a))*a+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+209
40*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+4890*B*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a))*a+4890*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c

```
)+2*a))*a+18720*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+6000*C*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+6000*C*ln(-4/(-2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^5/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))^5/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.19313, size = 722, normalized size = 2.77

$$\frac{15\left((283\,A+326\,B+400\,C)a^2\cos\left(dx+c\right)^6+(283\,A+326\,B+400\,C)a^2\cos\left(dx+c\right)^5\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4}{\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="fricas")
```

```
[Out] 1/7680*(15*((283*A + 326*B + 400*C)*a^2*cos(d*x + c)^6 + (283*A + 326*B + 4
00*C)*a^2*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(15*(283*A + 326*B + 400*C)*a^2*c
os(d*x + c)^4 + 10*(283*A + 326*B + 272*C)*a^2*cos(d*x + c)^3 + 8*(283*A +
230*B + 80*C)*a^2*cos(d*x + c)^2 + 48*(29*A + 10*B)*a^2*cos(d*x + c) + 384*
A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x
 + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.3759, size = 1827, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="giac")
```

```
[Out] 1/3840*(15*(283*A*a^(5/2) + 326*B*a^(5/2) + 400*C*a^(5/2))*log(abs((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2
) + 3))) - 15*(283*A*a^(5/2) + 326*B*a^(5/2) + 400*C*a^(5/2))*log(abs((sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqr
t(2) - 3))) + 4*sqrt(2)*(4245*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^18*A*a^(7/2) + 4890*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*a^(7/2) + 6000*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*a^(7/2) - 114615*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*A*a^(9/2
) - 132030*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^16*B*a^(9/2) - 162000*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^16*C*a^(9/2) + 1298820*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*a^(11/2) + 1319880*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*a^(11/2) + 1801920
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a
^(11/2) - 6176700*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^12*A*a^(13/2) - 6888120*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^12*B*a^(13/2) - 9764160*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(13/2) + 16394598*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(15/2
) + 18352620*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^10*B*a^(15/2) + 24060960*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
```

```
/2*d*x + 1/2*c)^2 + a))^10*C*a^(15/2) - 14042770*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(17/2) - 15746180*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(17/2) - 1
9910240*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^8*C*a^(17/2) + 4791060*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^6*A*a^(19/2) + 5497320*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(19/2) + 7135680*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(19/2) - 860300*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(21/2
) - 959320*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^4*B*a^(21/2) - 1268800*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*C*a^(21/2) + 75885*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(23/2) + 84810*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(23/2) + 111600*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(23/2)
 - 2671*A*a^(25/2) - 2990*B*a^(25/2) - 3920*C*a^(25/2))/((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^5)/d
```

3.400 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right.$ $dx)\,dx$

Optimal. Leaf size=311

$$\frac{a^3(1015A+1132B+1304C)\tan(c+dx)}{512d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(1015A+1132B+1304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{512d}+\frac{a^2(115A+156B+}{}$$

```
[Out] (a^(5/2)*(1015*A + 1132*B + 1304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
 a*Cos[c + d*x]]])/(512*d) + (a^3*(1015*A + 1132*B + 1304*C)*Tan[c + d*x])/
(512*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1015*A + 1132*B + 1304*C)*Sec[c +
d*x]*Tan[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(545*A + 628*B +
 680*C)*Sec[c + d*x]^2*Tan[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^
2*(115*A + 156*B + 120*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d
*x])/(480*d) + (a*(5*A + 12*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*Ta
n[c + d*x])/(60*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*Tan[c + d
*x])/(6*d)
```

Rubi [A] time = 0.966408, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2975, 2980, 2772, 2773, 206}

$$\frac{a^3(1015A+1132B+1304C)\tan(c+dx)}{512d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(1015A+1132B+1304C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{512d}+\frac{a^2(115A+156B+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^7,x]
```


```
[Out] (a^(5/2)*(1015*A + 1132*B + 1304*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
 a*Cos[c + d*x]]])/(512*d) + (a^3*(1015*A + 1132*B + 1304*C)*Tan[c + d*x])/
(512*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1015*A + 1132*B + 1304*C)*Sec[c +
d*x]*Tan[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(545*A + 628*B +
 680*C)*Sec[c + d*x]^2*Tan[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^
2*(115*A + 156*B + 120*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d
*x])/(480*d) + (a*(5*A + 12*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*Ta
n[c + d*x])/(60*d) + (A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*Tan[c + d
*x])/(6*d)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{A(a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\tan}{6d}$$

$$=\frac{a(5A+12B)(a+a\cos(c+dx))^{3/2}\sec^4}{60d}$$

$$=\frac{a^2(115A+156B+120C)\sqrt{a+a\cos(c}}{480d}$$

$$=\frac{a^3(545A+628B+680C)\sec^2(c+dx)}{960d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^3(1015A+1132B+1304C)\sec(c+d}{768d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^3(1015A+1132B+1304C)\tan(c+d}{512d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^3(1015A+1132B+1304C)\tan(c+d}{512d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{a^{5/2}(1015A+1132B+1304C)\tanh^{-1}\Big(}{512d}$$

Mathematica [A] time = 3.94586, size = 242, normalized size = 0.78

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((321370A+303048B+283920C)\cos(c+dx)+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^7,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^6*(120*Sqrt[2
]*(1015*A + 1132*B + 1304*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]
^6 + (137060*A + 112464*B + 93600*C + (321370*A + 303048*B + 283920*C)*Cos[
c + d*x] + 16*(8555*A + 8444*B + 7480*C)*Cos[2*(c + d*x)] + 108605*A*Cos[3*
(c + d*x)] + 121124*B*Cos[3*(c + d*x)] + 127240*C*Cos[3*(c + d*x)] + 20300*
A*Cos[4*(c + d*x)] + 22640*B*Cos[4*(c + d*x)] + 26080*C*Cos[4*(c + d*x)] +
15225*A*Cos[5*(c + d*x)] + 16980*B*Cos[5*(c + d*x)] + 19560*C*Cos[5*(c + d*
x)])*Sin[(c + d*x)/2]))/(122880*d)
```

Maple [B] time = 0.411, size = 3316, normalized size = 10.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/240*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(960*a*(101
5*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+1015*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))+1132*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
)+1132*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+1304*C*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+1304*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a)))*sin(1/2*d*x+1/2*c)^12-960*(1015*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)+1132*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+1304*
C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3045*A*ln(4/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a))*a+3045*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a+3396*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+3396*B*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
```

2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3912*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a))*a+3912*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*s
in(1/2*d*x+1/2*c)^10+80*(34510*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+38488*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+44336*C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+45675*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a))*a+45675*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+
50940*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+50940*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+58680*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))*a+58680*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(
1/2*d*x+1/2*c)^8-96*(33495*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+37356*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+42520*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+25375*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a+25375*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2830
0*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+28300*B*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+32600*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))*a+32600*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*
d*x+1/2*c)^6+12*(162980*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+18
0304*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+198560*C*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+76125*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))*a+76125*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+84900
*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+84900*B*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+97800*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a))*a+97800*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d
*x+1/2*c)^4-20*(31897*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3400
4*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+35176*C*2^(1/2)*(a*sin(1

```
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9135*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a))*a+9135*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+10188*B*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+10188*B*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a+11736*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))
*a+11736*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2
*c)^2+15225*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+92430*A*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+15225*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a))*a+16980*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a+88920*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+16980*B*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+19560*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a+83760*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+195
60*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2
^(1/2))^6/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^6/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*
x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^7,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.25491, size = 810, normalized size = 2.6

$$15\left((1015\,A+1132\,B+1304\,C)a^2\cos(dx+c)^7+(1015\,A+1132\,B+1304\,C)a^2\cos(dx+c)^6\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^7,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*((1015*A + 1132*B + 1304*C)*a^2*cos(d*x + c)^7 + (1015*A + 1132
*B + 1304*C)*a^2*cos(d*x + c)^6)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x +
c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(15*(1015*A + 1132*B + 130
4*C)*a^2*cos(d*x + c)^5 + 10*(1015*A + 1132*B + 1304*C)*a^2*cos(d*x + c)^4
+ 8*(1015*A + 1132*B + 920*C)*a^2*cos(d*x + c)^3 + 48*(145*A + 116*B + 40*C
)*a^2*cos(d*x + c)^2 + 128*(35*A + 12*B)*a^2*cos(d*x + c) + 1280*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.81421, size = 2159, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^7,x, algorithm="giac")
```

[Out] 1/15360*(15*(1015*A*a^(5/2) + 1132*B*a^(5/2) + 1304*C*a^(5/2))*log(abs((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sq
rt(2) + 3))) - 15*(1015*A*a^(5/2) + 1132*B*a^(5/2) + 1304*C*a^(5/2))*log(ab
s((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a
*(2*sqrt(2) - 3))) + 4*sqrt(2)*(15225*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^22*A*a^(7/2) + 16980*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^22*B*a^(7/2) + 19560*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^22*C*a^(7/2) - 5
02425*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2
0*A*a^(9/2) - 560340*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^20*B*a^(9/2) - 645480*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^20*C*a^(9/2) + 6518495*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*a^(11/2) + 7963020*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*a^(11/2)
 + 8467800*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^18*C*a^(11/2) - 49683495*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^16*A*a^(13/2) - 56336940*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*B*a^(13/2) - 59757720*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*a^(13/2) + 1
91286330*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^14*A*a^(15/2) + 219014472*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^14*B*a^(15/2) + 244004880*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*a^(15/2) - 418895130*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*a^(17/2) -
474348232*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^12*B*a^(17/2) - 531000080*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^12*C*a^(17/2) + 374587230*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*a^(19/2) + 421769112*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*a^(19/2) +
 473308080*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^10*C*a^(19/2) - 154254030*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^8*A*a^(21/2) - 174597720*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(21/2) - 198757680*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(21/2) + 3
5939005*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^6*A*a^(23/2) + 40114980*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^6*B*a^(23/2) + 45352200*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(23/2) - 4649085*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(25/2) - 5273124*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(2
5/2) - 5884680*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^4*C*a^(25/2) + 324435*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*A*a^(27/2) + 367644*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(27/2) + 411000*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(27/2) - 9435*A*

```
a^(29/2) - 10684*B*a^(29/2) - 11960*C*a^(29/2))/((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^6)/d
```

3.401 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=254

$$\frac{2(21A-3B+19C)\sin(c+dx)\cos^2(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(21A-93B+29C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315ad}+\frac{4(147A-111B+1}{315d\sqrt{a\cos(c}}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(Sqrt[a]*d)) + (4*(147*A - 111*B + 143*C)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(21*A - 3*B + 19*C)*Cos[c + d*x]^2*Sin[c
 + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(9*B - C)*Cos[c + d*x]^3*Sin
[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^4*Sin[c + d*
x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(21*A - 93*B + 29*C)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(315*a*d)
```

Rubi [A] time = 0.837927, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3045, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(21A-3B+19C)\sin(c+dx)\cos^2(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(21A-93B+29C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315ad}+\frac{4(147A-111B+1}{315d\sqrt{a\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos
[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(Sqrt[a]*d)) + (4*(147*A - 111*B + 143*C)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(21*A - 3*B + 19*C)*Cos[c + d*x]^2*Sin[c
 + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(9*B - C)*Cos[c + d*x]^3*Sin
[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^4*Sin[c + d*
x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(21*A - 93*B + 29*C)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(315*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^3(c+dx)\left(\frac{1}{2}a(9A+8C)+\frac{1}{2}a(9B-C\ldots\right.}{\sqrt{a+a\cos(c+dx)}}}{9a}$$

$$= \frac{2(9B-C)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^4(c+dx)\sin(c+\ldots}{9d\sqrt{a+a\cos(c+dx\ldots}}$$

$$= \frac{2(21A-3B+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(9B-C)\cos\ldots}{63d\sqrt{a\ldots}}$$

$$= \frac{2(21A-3B+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(9B-C)\cos\ldots}{63d\sqrt{a\ldots}}$$

$$= \frac{2(21A-3B+19C)\cos^2(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(9B-C)\cos\ldots}{63d\sqrt{a\ldots}}$$

$$= \frac{4(147A-111B+143C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(21A-3B+19C)\mathrm{co}\ldots}{105d\sqrt{a+a\ldots}}$$

$$= \frac{4(147A-111B+143C)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(21A-3B+19C)\mathrm{co}\ldots}{105d\sqrt{a+a\ldots}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(147A-111B\ldots}{315d\sqrt{a+\ldots}}$$

Mathematica [A] time = 0.78595, size = 144, normalized size = 0.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)(-2(84A-507B+131C)\cos(c+dx)+4(63A-9B+92C)\cos(2(c+dx))+2436A+\ldots\right.}{1260d\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a +
 a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-2520*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + 2*(2436*A
- 1068*B + 2389*C - 2*(84*A - 507*B + 131*C)*Cos[c + d*x] + 4*(63*A - 9*B +
 92*C)*Cos[2*(c + d*x)] + 90*B*Cos[3*(c + d*x)] - 10*C*Cos[3*(c + d*x)] + 3
5*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(1260*d*Sqrt[a*(1 + Cos[c + d*x])]
)
```

Maple [A] time = 0.15, size = 392, normalized size = 1.5

$$\frac{1}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(1120\,C\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{a}\left(\sin\left(1/2\,dx+c/2\right)\right)^8-720\,\sqrt{2}\sqrt{a\left(\sin\left(1\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/315*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1120*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-720*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(B+3*C)*sin(1/2*d*x+1/2*c)^6+504*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*B+4*C)*sin(1/2*d*x+1/2*c)^4-420*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*B+2*C)*sin(1/2*d*x+1/2*
c)^2-315*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+a))*a*A+315*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a))*a*B-315*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C+630*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+630*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin
(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maple [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.99137, size = 582, normalized size = 2.29

$$\frac{4\left(35\,C\cos\left(dx+c\right)^{4}+5\,(9\,B-C)\cos\left(dx+c\right)^{3}+3\,(21\,A-3\,B+19\,C)\cos\left(dx+c\right)^{2}-(21\,A-93\,B+29\,C)\cos\left(dx+c\right)}{630}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 1/630*(4*(35*C*cos(d*x + c)^4 + 5*(9*B - C)*cos(d*x + c)^3 + 3*(21*A - 3*B
+ 19*C)*cos(d*x + c)^2 - (21*A - 93*B + 29*C)*cos(d*x + c) + 273*A - 129*B
+ 257*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 315*sqrt(2)*((A - B + C)*a
*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 +
2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.0079, size = 363, normalized size = 1.43

$$\frac{\frac{315\left(\sqrt{2}A-\sqrt{2}B+\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\left(315\sqrt{2}Aa^{4}+315\sqrt{2}Ca^{4}+\left(1050\sqrt{2}Aa^{4}-420\sqrt{2}Ba^{4}+840\sqrt{2}Ca^{4}+\left(1512\sqrt{2}A\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] 1/315*(315*(sqrt(2)*A - sqrt(2)*B + sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x
 + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(315*sqrt(2)*A
*a^4 + 315*sqrt(2)*C*a^4 + (1050*sqrt(2)*A*a^4 - 420*sqrt(2)*B*a^4 + 840*sq
rt(2)*C*a^4 + (1512*sqrt(2)*A*a^4 - 756*sqrt(2)*B*a^4 + 1638*sqrt(2)*C*a^4
+ (1134*sqrt(2)*A*a^4 - 612*sqrt(2)*B*a^4 + 936*sqrt(2)*C*a^4 + (357*sqrt(2
)*A*a^4 - 276*sqrt(2)*B*a^4 + 383*sqrt(2)*C*a^4)*tan(1/2*d*x + 1/2*c)^2)*ta
n(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1
/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2))/d
```

3.402 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2(35A-7B+31C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{4(35A-49B+37C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{a\cos}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(Sqrt[a]*d) - (4*(35*A - 49*B + 37*C)*Sin[c + d*x])/(105*d*Sq
rt[a + a*Cos[c + d*x]]) + (2*(7*B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*S
qrt[a + a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*(35*A - 7*B + 31*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(105*a*d)
```

Rubi [A] time = 0.611791, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3045, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(35A-7B+31C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{4(35A-49B+37C)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{a\cos}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos
[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(Sqrt[a]*d) - (4*(35*A - 49*B + 37*C)*Sin[c + d*x])/(105*d*Sq
rt[a + a*Cos[c + d*x]]) + (2*(7*B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*S
qrt[a + a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*(35*A - 7*B + 31*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(105*a*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(\frac{1}{2}a(7A+6C)+\frac{1}{2}a(7B-C\right.}{\sqrt{a+a\cos(c+dx)}}}{7a}$$

$$= \frac{2(7B-C)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sin(c+}{7d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{2(7B-C)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sin(c+}{7d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{2(7B-C)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^3(c+dx)\sin(c+}{7d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{4(35A-49B+37C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7B-C)\cos^2(c+dx)\,s}{35d\sqrt{a+a\cos(c-}}$$

$$= -\frac{4(35A-49B+37C)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7B-C)\cos^2(c+dx)\,s}{35d\sqrt{a+a\cos(c-}}$$

$$= \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(35A-49B+3}{105d\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.637913, size = 118, normalized size = 0.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(420(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)((140A-28B+169C)\cos(c+dx)-140A\right.}{210d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a +
 a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(420*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + 2*(-140*A +
406*B - 178*C + (140*A - 28*B + 169*C)*Cos[c + d*x] + 6*(7*B - C)*Cos[2*(c
+ d*x)] + 15*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(210*d*Sqrt[a*(1 + Cos[
c + d*x])])
```

Maple [A] time = 0.143, size = 324, normalized size = 1.6

$$\frac{1}{105\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-240\,C\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{a}\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+168\,\sqrt{2}\sqrt{a\left(\sin\left(1\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/105*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(B+2*C)*sin(1/2*d*x+1/2*c)^4-140*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+B+2*C)*sin(1/2*d*x+1/2*c)^2+105*2^
(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a
*A-105*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+a))*a*B+105*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a))*a*C+210*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/
a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.01942, size = 524, normalized size = 2.52

$$\frac{4\left(15\,C\cos\left(dx+c\right)^{3}+3\,(7\,B-C)\cos\left(dx+c\right)^{2}+(35\,A-7\,B+31\,C)\cos\left(dx+c\right)-35\,A+91\,B-43\,C\right)\sqrt{a\cos\left(dx+c\right)}}{210\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 1/210*(4*(15*C*cos(d*x + c)^3 + 3*(7*B - C)*cos(d*x + c)^2 + (35*A - 7*B +
31*C)*cos(d*x + c) - 35*A + 91*B - 43*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c) + 105*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(
d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x
+ c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.87054, size = 271, normalized size = 1.3

$$-\frac{\frac{105\,\sqrt{2}(A-B+C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}-\frac{2\left(105\,\sqrt{2}Ba^{3}-\left(\left(\sqrt{2}\left(70\,Aa^{3}-119\,Ba^{3}+92\,Ca^{3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+7\,\sqrt{2}\left(20\,Aa^{3}-37\,Ba^{3}\right.\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}\right.\right.}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(2)*(A - B + C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) - 2*(105*sqrt(2)*B*a^3 - ((sqrt(2
)*(70*A*a^3 - 119*B*a^3 + 92*C*a^3)*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(2)*(20*
A*a^3 - 37*B*a^3 + 16*C*a^3))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(2*A*a^3
- 7*B*a^3 + 4*C*a^3))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1
/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

3.403 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=164

$$\frac{2(15A-10B+14C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(5B-C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(Sqrt[a]*d)) + (2*(15*A - 10*B + 14*C)*Sin[c + d*x])/(15*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a +
 a*Cos[c + d*x]]) + (2*(5*B - C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15
*a*d)
```

Rubi [A] time = 0.380234, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3045, 2968, 3023, 2751, 2649, 206}

$$\frac{2(15A-10B+14C)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(5B-C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + a*Cos[c
 + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(Sqrt[a]*d)) + (2*(15*A - 10*B + 14*C)*Sin[c + d*x])/(15*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*C*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a +
 a*Cos[c + d*x]]) + (2*(5*B - C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15
*a*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
```

```
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos(c+dx)\left(\frac{1}{2}a(5A+4C)+\frac{1}{2}a(5B-C)\cos\right.}{\sqrt{a+a\cos(c+dx)}}}{5a}$$

$$= \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}a(5A+4C)\cos(c+dx)+\frac{1}{2}a(5B-C)\,c}{\sqrt{a+a\cos(c+dx)}}}{5a}$$

$$= \frac{2C\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5B-C)\sqrt{a+a\cos(c+dx)}\,\mathrm{si}}{15ad}$$

$$= \frac{2(15A-10B+14C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^2(c+dx)\sin(c+dx}{5d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(15A-10B+14C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2C\cos^2(c+dx)\sin(c+dx}{5d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(15A-10B+1}{15d\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.367138, size = 98, normalized size = 0.6

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)(30A+2(5B-C)\cos(c+dx)-10B+3C\cos(2(c+dx))+29C)-15(A-B+C)\tanh\right.}{15d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + a
*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-15*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + (30*A - 10
*B + 29*C + 2*(5*B - C)*Cos[c + d*x] + 3*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/
2]))/(15*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.133, size = 306, normalized size = 1.9

$$\frac{1}{15\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,C\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^4-20\,\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-20*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(B+C)*sin(1/2*d*x+1/2*c)^2-15*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+15*2^(1/2)*
ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-15*
2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))
*a*C+30*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+30*C*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*
x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.00587, size = 466, normalized size = 2.84

$$\frac{4\left(3\,C\cos\left(dx+c\right)^{2}+\left(5\,B-C\right)\cos\left(dx+c\right)+15\,A-5\,B+13\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)+\frac{15\,\sqrt{2}((A-B+C)a\cos(dx+}{}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/30*(4*(3*C*cos(d*x + c)^2 + (5*B - C)*cos(d*x + c) + 15*A - 5*B + 13*C)*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c) + 15*sqrt(2)*((A - B + C)*a*cos(d*x +
```

```
c) + (A - B + C)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/
2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.03628, size = 255, normalized size = 1.55

$$\frac{\frac{15\left(\sqrt{2}A-\sqrt{2}B+\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\left(15\,\sqrt{2}Aa^{2}+15\,\sqrt{2}Ca^{2}+\left(30\,\sqrt{2}Aa^{2}-10\,\sqrt{2}Ba^{2}+20\,\sqrt{2}Ca^{2}+\left(15\,\sqrt{2}Aa^{2}-10\,\sqrt{2}Ba\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\right.}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/15*(15*(sqrt(2)*A - sqrt(2)*B + sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x +
 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(15*sqrt(2)*A*a^
2 + 15*sqrt(2)*C*a^2 + (30*sqrt(2)*A*a^2 - 10*sqrt(2)*B*a^2 + 20*sqrt(2)*C*
a^2 + (15*sqrt(2)*A*a^2 - 10*sqrt(2)*B*a^2 + 17*sqrt(2)*C*a^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1
/2*c)^2 + a)^(5/2))/d
```

3.404 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(Sqrt[a]*d) + (2*(3*B - 2*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.158269, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3023, 2751, 2649, 206}

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(Sqrt[a]*d) + (2*(3*B - 2*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*C*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{2\int \frac{\frac{1}{2}a(3A+C)+\frac{1}{2}a(3B-2C)\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}+(A-B+C)$$

$$= \frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}-\frac{(2(A-B+C}{}$$

$$= \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2C\sqrt{a}}{}$$

Mathematica [A] time = 0.143693, size = 79, normalized size = 0.67

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(3(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6B\sin\left(\frac{1}{2}(c+dx)\right)-4C\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],
x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(3*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + 6*B*Sin[(c +
 d*x)/2] - 4*C*Sin[(c + d*x)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.128, size = 233, normalized size = 2.

$$\frac{\sqrt{2}}{3\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-4\,C\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^{2}}\,(\sin(1/2\,dx+c/2))^{2}+3\,A\ln\left(4\,\frac{\sqrt{a}\sqrt{a\,(\sin(1/2}{\cos(1/2\,}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+3*A*ln(4/cos(1/2*d*x+1
/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-3*B*ln(4/cos(1/2*d*x+1/
2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+6*B*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+3*C*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2
)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.94138, size = 410, normalized size = 3.47

$$\frac{4\,(C\cos(dx+c)+3\,B-C)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)+\frac{3\,\sqrt{2}((A-B+C)a\cos(dx+c)+(A-B+C)a)\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\,\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sqrt{a}}}{\cos(dx+c)^{2}+2\,\mathrm{co}}\right.}{\sqrt{a}}}{6\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/6*(4*(C*cos(d*x + c) + 3*B - C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 3
*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*x + c)^2
- 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c)
- 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.97408, size = 154, normalized size = 1.31

$$-\frac{\frac{3\sqrt{2}(A-B+C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}-\frac{2\left(\sqrt{2}(3Ba-2Ca)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\sqrt{2}Ba\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/3*(3*sqrt(2)*(A - B + C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) - 2*(sqrt(2)*(3*B*a - 2*C*a)*tan(1/2*
d*x + 1/2*c)^2 + 3*sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c
)^2 + a)^(3/2))/d
```

3.405 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(Sqrt[a]*d) + (2*C*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]
])
```

Rubi [A] time = 0.314117, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3045, 2985, 2649, 206, 2773}

$$-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]],x]
```


```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(Sqrt[a]*d) + (2*C*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]
])
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{aA}{2}+\frac{1}{2}a(B-C)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{a}$$
$$= \frac{2C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{A\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,d}{a}$$
$$= \frac{2C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2A)\,\text{Subst}\left(\int \frac{1}{a-x^2}dx, x, -\frac{a\sin(c+dx}{\sqrt{a+a\cos(c+}}\right.}{d}$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\,\text{si}}{\sqrt{2}\sqrt{a+}}\right.}{\sqrt{a}d}$$

Mathematica [A] time = 0.3299, size = 86, normalized size = 0.73

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + a
*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-((A - B + C)*ArcTanh[Sin[(c + d*x)/2]]) + Sqrt[2]*A*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*C*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Co
s[c + d*x])])
```

Maple [B] time = 0.242, size = 337, normalized size = 2.9

$$-\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+a}{\cos(1/2\,dx+c/2)}\right)aA-\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2}}{\cos(1/2\,dx+c/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-2^(1/2)*ln(4/cos(1
/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+2^(1/2)*ln(4/
cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-A*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a-2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.37988, size = 624, normalized size = 5.29

$$\frac{(A\cos(dx+c)+A)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos(dx+c)+a}C\sin}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*C*sin(d*
x + c) + sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d
*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x +
```

```
c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\sqrt{a\left(\cos\left(c + dx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**(1/
2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(a*(cos(
c + d*x) + 1)), x)
```

Giac [B] time = 3.35405, size = 277, normalized size = 2.35

$$\frac{\frac{4\sqrt{2}C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} + \frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}+C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a} + \frac{2A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2 + \cdots\right.\right.}{\sqrt{a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*C*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) +
sqrt(2)*(A*sqrt(a) - B*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + 2*A*log(abs((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))
)/sqrt(a) - 2*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/sqrt(a))/d
```

3.406 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=120

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -(((A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqr
t[a]*d)) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c
 + d*x]])
```

Rubi [A] time = 0.351239, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3043, 2985, 2649, 206, 2773}

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + a*Cos
[c + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqr
t[a]*d)) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c
 + d*x]])
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-2B)+\frac{1}{2}a(A+2C)\cos(c+dx)\right)\sec(c+dx}{\sqrt{a+a\cos(c+dx)}}}{a}$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(A-2B)\int\sqrt{a+a\cos(c+dx)}\sec(c+}{2a}$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(A-2B)\,\mathrm{Subst}\left(\int\frac{1}{a-x^2}\,dx, x, -\frac{a\sin(}{\sqrt{a+a\,\mathrm{co}}}\right.}{d}$$

$$= -\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\right.}{\sqrt{a}d}$$

Mathematica [A] time = 0.509859, size = 96, normalized size = 0.8

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(A-2B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a +
 a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(A - 2
*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sec[c + d*x]*Sin[(c + d*x)/2]))
/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.249, size = 893, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-2*A*2^(1/2)*ln(4/c
os(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+2*B*ln(4/cos(
1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*2^(1/2)-2*C*2^(1
```

```
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))-2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-2*B*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-2*2^(1/
2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+
2*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a
))*a*C+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*
a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+2*B*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2
*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.86782, size = 710, normalized size = 5.92

$$-\frac{\left((A-2\,B)\cos\left(dx+c\right)^2+(A-2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(ad\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] -1/4*(((A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos
(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*s
qrt(a*cos(d*x + c) + a)*A*sin(d*x + c) - 2*sqrt(2)*((A - B + C)*a*cos(d*x +
 c)^2 + (A - B + C)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)
^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.30905, size = 440, normalized size = 3.67

$$-\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}+C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a}+\frac{\left(A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + (A*sqrt(a) - 2*B*sqrt
```

```
(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3)))/a - (A*sqrt(a) - 2*B*sqrt(a))*log(abs((sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(
2) - 3)))/a - 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.407 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{(7A-4B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+}{2d\sqrt{a\,\mathrm{co}}}$$

```
[Out] ((7*A - 4*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Tan[c + d*x])/(4*d
*Sqrt[a + a*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*
Cos[c + d*x]])
```

Rubi [A] time = 0.540153, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2984, 2985, 2649, 206, 2773}

$$\frac{(7A-4B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{A\tan(c+}{2d\sqrt{a\,\mathrm{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + a*Cos
[c + d*x]],x]
```



```
[Out] ((7*A - 4*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Tan[c + d*x])/(4*d
*Sqrt[a + a*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*
Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-4B)+\frac{1}{2}a(3A+4C)\cos(c+dx)\right)\text{s}}{\sqrt{a+a\cos(c+dx)}}}{2a}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}a^2\right.}{}}{}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + (-A+$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(2(A-}{}$$

$$= \frac{(7A-4B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B+C)\tan}{}$$

Mathematica [A] time = 0.485583, size = 118, normalized size = 0.7

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-8(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(7A-4B+8C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c\right.\right.}{4d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a +
 a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-8*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(7*A
- 4*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(-A + 4*B +
 2*A*Sec[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.317, size = 1750, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a*(8*A*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-8*B*ln(4/
cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*2^(1/2)+8*C*
2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))
-7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-7*A*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+4*B
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-8*C*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a))-8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(
1/2*d*x+1/2*c)^4+4*(8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+a))*a*A+A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a))*a*B-4*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+8*2^(1/2)*ln(4/c
os(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-7*A*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+4*B*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-8*C*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*a))*a-8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*si
n(1/2*d*x+1/2*c)^2-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+a))*a*A+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*A*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a))*a-4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+8*B*a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+8*C*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)+2*a))*a+8*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a
^(3/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/si
n(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.39277, size = 779, normalized size = 4.61

$$\frac{\left((7A-4B+8C)\cos(dx+c)^3+(7A-4B+8C)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 1/16*(((7*A - 4*B + 8*C)*cos(d*x + c)^3 + (7*A - 4*B + 8*C)*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) - 4*((A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c) + 8*sqrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*
x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/
sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.36356, size = 745, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*(A*sqrt(a) - B*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + (7*A*sqrt(a) - 4*B*s
qrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a - (7*A*sqrt(a) - 4*B*sqrt(
a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a - 4*sqrt(2)*(17*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a)
 - 57*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4
*A*a^(3/2) + 76*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*B*a^(3/2) + 19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*A*a^(5/2) - 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(5/2) - 3*A*a^(7/2) + 4*B*a^(7/2))/((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2
)/d
```

3.408 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{(7A-2B+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{(9A-14B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A$$

```
[Out] -((9*A - 14*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B + 8*C)*Tan[c +
d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - 6*B)*Sec[c + d*x]*Tan[c + d*x]
)/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.736983, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2984, 2985, 2649, 206, 2773}

$$\frac{(7A-2B+8C)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{(9A-14B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + a*Cos
[c + d*x]],x]
```


```
[Out] -((9*A - 14*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B + 8*C)*Tan[c +
d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - 6*B)*Sec[c + d*x]*Tan[c + d*x]
)/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{A\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{\int\frac{\left(-\frac{1}{2}a(A-6B)+\frac{1}{2}a(5A+6C)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}}{3a}$$

$$=-\frac{(A-6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}+\frac{A\sec^2(c+dx)\tan(c+}{3d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{(7A-2B+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}-\frac{(A-6B)\sec(c+dx)\tan(c+}{12d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{(7A-2B+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}-\frac{(A-6B)\sec(c+dx)\tan(c+}{12d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{(7A-2B+8C)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}-\frac{(A-6B)\sec(c+dx)\tan(c+}{12d\sqrt{a+a\cos(c+dx)}}$$

$$=-\frac{(9A-14B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B+C)}{}$$

Mathematica [A] time = 1.36725, size = 147, normalized size = 0.69

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(48(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\sqrt{2}(9A-14B+8C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c\right.\right.}{24d\sqrt{a(\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a +
 a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(48*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] - 3*Sqrt[2]*(9*
A - 14*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (37*A - 6*B + 24*C - 4*
(A - 6*B)*Cos[c + d*x] + 3*(7*A - 2*B + 8*C)*Cos[2*(c + d*x)])*Sec[c + d*x]
^3*Sin[(c + d*x)/2]))/(24*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.311, size = 2374, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(16*A*2^(1/2)*
ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-16*B*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*2^(1/2)+1
6*C*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+a))-9*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-9*A*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a))+14*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*
a))+14*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))-8*C*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))-8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))
)*sin(1/2*d*x+1/2*c)^6+12*(48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+14*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+a))*a*B-4*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+48*2
^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*
a*C+16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-27*A*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-27*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a))*a+42*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a-24*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a)*sin(1/
2*d*x+1/2*c)^4-2*(144*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+a))*a*A+80*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-144*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+a))*a*B+144*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a))*a*C+96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-8
1*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
```

```
/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-81*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a))*a+126*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a+126*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-72*C*ln(-4/(-2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-72*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*
c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+48*2^(1/2)*ln(4/cos(1/2*
d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-27*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+54*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-27*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+12*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+42*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-24*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+48*C*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)-24*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^
3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.5617, size = 838, normalized size = 3.93

$$\frac{3\left((9\,A-14\,B+8\,C)\cos{(dx+c)}^{4}+(9\,A-14\,B+8\,C)\cos{(dx+c)}^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\mathrm{co}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*((9*A - 14*B + 8*C)*cos(d*x + c)^4 + (9*A - 14*B + 8*C)*cos(d*x + c
)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 +
cos(d*x + c)^2)) + 4*(3*(7*A - 2*B + 8*C)*cos(d*x + c)^2 - 2*(A - 6*B)*cos(
d*x + c) + 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) + 48*sqrt(2)*((A - B
+ C)*a*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2
- 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c)
- 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 +
a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.67871, size = 1253, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/
2),x, algorithm="giac")

[Out] -1/48*(24*sqrt(2)*(A*sqrt(a) - B*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + 3*(9*A*sqrt(a) -
14*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a - 3*(9*A*sqrt(a) - 1
4*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a - 4*sqrt(2)*(165*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(a
) - 102*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^10*B*sqrt(a) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^10*C*sqrt(a) - 1323*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^8*A*a^(3/2) + 954*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(3/2) - 888*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(3/2) + 3906*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(5/2) - 22
68*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*
a^(5/2) + 3024*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*C*a^(5/2) - 2118*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*A*a^(7/2) + 1044*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(7/2) - 1776*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(7/2) + 393*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(9/2) - 222*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(9/
2) + 360*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2*C*a^(9/2) - 31*A*a^(11/2) + 18*B*a^(11/2) - 24*C*a^(11/2))/((sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.409 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=259

$$-\frac{(21A-56B+16C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{(107A-72B+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{a\cos(c+}}\right.}{\sqrt{a}d}$$

```
[Out] ((107*A - 72*B + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((21*A - 56*B + 16*C)*T
an[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + ((43*A - 8*B + 48*C)*Sec[c +
 d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - 8*B)*Sec[c + d*
x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.934182, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3043, 2984, 2985, 2649, 206, 2773}

$$-\frac{(21A-56B+16C)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{(107A-72B+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{a\cos(c+}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[a + a*Cos
[c + d*x]],x]
```

```
[Out] ((107*A - 72*B + 112*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((21*A - 56*B + 16*C)*T
an[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + ((43*A - 8*B + 48*C)*Sec[c +
 d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - 8*B)*Sec[c + d*
x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (A*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
```

```
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec^3(c+dx)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-8B)+\frac{1}{2}a(7A+8C)\cos(c+dx\right.}{\sqrt{a+a\cos(c+dx)}}}{4a}$$

$$= -\frac{(A-8B)\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec^3(c+dx)\tan(c}{4d\sqrt{a+a\cos(c+}}$$

$$= \frac{(43A-8B+48C)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} - \frac{(A-8B)\sec^2(}{24d\sqrt{a+}}$$

$$= -\frac{(21A-56B+16C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{(43A-8B+48C)\sec(}{96d\sqrt{a+a\,\text{co}}}$$

$$= -\frac{(21A-56B+16C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{(43A-8B+48C)\sec(}{96d\sqrt{a+a\,\text{co}}}$$

$$= -\frac{(21A-56B+16C)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{(43A-8B+48C)\sec(}{96d\sqrt{a+a\,\text{co}}}$$

$$= \frac{(107A-72B+112C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{a}d} - \frac{\sqrt{2}(A-B+}{}$$

Mathematica [A] time = 2.7992, size = 200, normalized size = 0.77

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)((221A-760B+144C)\cos(c+dx)-4(43A-8B+48C)\cos(2(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[a +
 a*Cos[c + d*x]],x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(768*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]
]*Cos[c + d*x]^4 - 6*Sqrt[2]*(107*A - 72*B + 112*C)*ArcTanh[Sqrt[2]*Sin[(c
```

```
+ d*x)/2]]*Cos[c + d*x]^4 + (-364*A + 32*B - 192*C + (221*A - 760*B + 144*C
)*Cos[c + d*x] - 4*(43*A - 8*B + 48*C)*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d
*x)] - 168*B*Cos[3*(c + d*x)] + 48*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(
384*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.335, size = 2997, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/24*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-48*a*(-128*A*2^(1
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+128
*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*2^(1
/2)-128*C*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+a))+107*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x
+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+107*A*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-72*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))-72*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+112*C*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))+112*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8+48*(-256*2^(1/2)*ln(4/cos(1/2*d*x+1/2*
c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-21*A*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+256*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+56*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-256*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a))*a*C-16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
214*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+214*A*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a-144*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a))*a-144*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+224*C*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+224*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
```

^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6-8*(-1152*2^(1/2)*ln(4/cos(1/2*d*x+1
/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-103*A*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1152*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+488*B*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-1152*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+a))*a*C-48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+963*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+963*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-648*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*a-648*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1008*C
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+1008*C*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+4*(-768*2^(1/2)*ln(4/cos(1
/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-25*A*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+768*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*
(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+536*B*a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-768*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C+48*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+642*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+642*A*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-432*B*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*a))*a-432*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6
72*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a+672*C*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+384*2^(1/2)*ln(4/cos(1/
2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-384*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+384*2
^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*
a*C-321*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-321*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-126*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+216*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+216*B*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-432*B*a^(1/2)*2^(1/2)*(a*sin(1/

```
2*d*x+1/2*c)^2)^(1/2)-336*C*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
-336*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*a-96*C*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))^4/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.90721, size = 910, normalized size = 3.51

$$\frac{3\left((107A-72B+112C)\cos(dx+c)^5+(107A-72B+112C)\cos(dx+c)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c}}{\cos(dx+c)^3+\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 1/768*(3*((107*A - 72*B + 112*C)*cos(d*x + c)^5 + (107*A - 72*B + 112*C)*co
s(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x
+ c)^3 + cos(d*x + c)^2)) - 4*(3*(21*A - 56*B + 16*C)*cos(d*x + c)^3 - 2*(4
3*A - 8*B + 48*C)*cos(d*x + c)^2 + 8*(A - 8*B)*cos(d*x + c) - 48*A)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c) + 384*sqrt(2)*((A - B + C)*a*cos(d*x + c)^5
+ (A - B + C)*a*cos(d*x + c)^4)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 +
```

```
2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5/(a+a*cos(d*x+c))**
(1/2),x)

[Out] Timed out
```

Giac [B] time = 3.95099, size = 1585, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+a*cos(d*x+c))^(1/
2),x, algorithm="giac")

[Out] 1/384*(192*sqrt(2)*(A*sqrt(a) - B*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + 3*(107*A*sqrt(a)
 - 72*B*sqrt(a) + 112*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a - 3*(107*A*sqrt
(a) - 72*B*sqrt(a) + 112*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a - 4*sqrt(2)*
(1599*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^1
4*A*sqrt(a) - 1320*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^14*B*sqrt(a) + 816*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(a) - 18219*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*a^(3/2) + 18504*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*a^(3/2) - 12528*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*a^(3
/2) + 91467*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^10*A*a^(5/2) - 96072*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^10*B*a^(5/2) + 64752*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*a^(5/2) - 177735*(sqrt(a)*tan(1/2*d*x
```

+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a^(7/2) + 215016*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*a^(7/2) - 124848*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*a^(7/2) + 100413*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(9/2) - 136056*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(9/2) + 70032*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(9/2) - 26881*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(11/2) + 36056*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(11/2) - 19152*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*a^(11/2) + 3321*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(13/2) - 4632*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(13/2) + 2640*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(13/2) - 205*A*a^(15/2) + 248*B*a^(15/2) - 144*C*a^(15/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d

3.410 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{(245A-273B+397C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}+\frac{(11A-15B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\text{si}}{2d(a\text{ co}}$$

```
[Out] ((11*A - 15*B + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^4*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((455*A - 651*B + 799*C)*Sin[c + d
*x])/(105*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((35*A - 63*B + 67*C)*Cos[c + d*x
]^2*Sin[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((7*A - 7*B + 11*C)*C
os[c + d*x]^3*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((245*A - 2
73*B + 397*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(210*a^2*d)
```

Rubi [A] time = 0.872587, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3041, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{(245A-273B+397C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}+\frac{(11A-15B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\text{si}}{2d(a\text{ co}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^(3/2),x]
```


```
[Out] ((11*A - 15*B + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^4*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((455*A - 651*B + 799*C)*Sin[c + d
*x])/(105*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((35*A - 63*B + 67*C)*Cos[c + d*x
]^2*Sin[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((7*A - 7*B + 11*C)*C
os[c + d*x]^3*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((245*A - 2
73*B + 397*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(210*a^2*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(-2a(A-\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A-7B+11C)\cdots}{14ad\sqrt{a\cdots}}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(35A-63B+67C\cdots}{70ad\sqrt{a\cdots}}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(35A-63B+67C\cdots}{70ad\sqrt{a\cdots}}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(35A-63B+67C\cdots}{70ad\sqrt{a\cdots}}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(455A-651B+7\cdots}{105ad\sqrt{a+a\cdots}}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(455A-651B+7\cdots}{105ad\sqrt{a+a\cdots}}$$

$$= \frac{(11A-15B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C\cdots}{2d(a\cdots}$$

Mathematica [A] time = 1.50264, size = 180, normalized size = 0.65

$$\frac{\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(6(140A-273B+277C)\cos(c+dx)-4(35A-21B+64C)\cos(2(c+dx))+1190A-}{105d\left(\sin^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^(3/2),x]
```

```
[Out] (-105*(11*A - 15*B + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (
Cos[(c + d*x)/2]^3*(1190*A - 1974*B + 2161*C + 6*(140*A - 273*B + 277*C)*Co
s[c + d*x] - 4*(35*A - 21*B + 64*C)*Cos[2*(c + d*x)] - 42*B*Cos[3*(c + d*x)
] + 18*C*Cos[3*(c + d*x)] - 15*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2])/2)/(10
5*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] time = 0.16, size = 577, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/420/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(960*C*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-96*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(7*B+17*C)*sin(1/2*d*x+1/2*c)^6+112*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(5*A+6*B+16*C)*sin(1/2*d*x+1/2*c)^4-
35*2^(1/2)*(33*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+a))*a-8*A*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-45*B*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+48*B*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)+57*C*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+a))*a-16*C*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))*sin(1/
2*d*x+1/2*c)^2+1155*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a))*a*A-1575*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+1995*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*C-945*A*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+1785*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)-1785*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d*
x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.00992, size = 709, normalized size = 2.56

$$\frac{105\sqrt{2}\Big((11\,A-15\,B+19\,C)\cos\left(dx+c\right)^2+2\,(11\,A-15\,B+19\,C)\cos\left(dx+c\right)+11\,A-15\,B+19\,C\Big)\sqrt{a}\log\Big(-\frac{a\cos(dx+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] 1/840*(105*sqrt(2)*((11*A - 15*B + 19*C)*cos(d*x + c)^2 + 2*(11*A - 15*B +
19*C)*cos(d*x + c) + 11*A - 15*B + 19*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2
*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) -
 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(60*C*cos(d*x + c)^4 + 12*
(7*B - 3*C)*cos(d*x + c)^3 + 28*(5*A - 3*B + 7*C)*cos(d*x + c)^2 - 12*(35*A
 - 63*B + 67*C)*cos(d*x + c) - 665*A + 1029*B - 1201*C)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.83865, size = 412, normalized size = 1.49

$$-\frac{\frac{105\left(11\sqrt{2}A-15\sqrt{2}B+19\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\left(\frac{105\left(\sqrt{2}Aa^{5}-\sqrt{2}Ba^{5}+\sqrt{2}Ca^{5}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{3}}+\frac{4\left(455\sqrt{2}Aa^{5}-693\sqrt{2}Ba^{5}+\cdots\right.}{a^{3}}\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] -1/420*(105*(11*sqrt(2)*A - 15*sqrt(2)*B + 19*sqrt(2)*C)*log(abs(-sqrt(a)*t
an(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((((10
5*(sqrt(2)*A*a^5 - sqrt(2)*B*a^5 + sqrt(2)*C*a^5)*tan(1/2*d*x + 1/2*c)^2/a^
3 + 4*(455*sqrt(2)*A*a^5 - 693*sqrt(2)*B*a^5 + 877*sqrt(2)*C*a^5)/a^3)*tan(
1/2*d*x + 1/2*c)^2 + 14*(305*sqrt(2)*A*a^5 - 453*sqrt(2)*B*a^5 + 517*sqrt(2
)*C*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 140*(25*sqrt(2)*A*a^5 - 39*sqrt(2)*B
*a^5 + 47*sqrt(2)*C*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 105*(9*sqrt(2)*A*a^5
 - 17*sqrt(2)*B*a^5 + 17*sqrt(2)*C*a^5)/a^3)*tan(1/2*d*x + 1/2*c)/(a*tan(1/
2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

3.411 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{(15A-35B+39C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}-\frac{(7A-11B+15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c}{2d(a\cos(c}$$

```
[Out] -((7*A - 11*B + 15*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^3*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((45*A - 65*B + 93*C)*Sin[c + d*x]
)/(15*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((5*A - 5*B + 9*C)*Cos[c + d*x]^2*Sin
[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((15*A - 35*B + 39*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(30*a^2*d)
```

Rubi [A] time = 0.669542, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3041, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{(15A-35B+39C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}-\frac{(7A-11B+15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c}{2d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^(3/2),x]
```


```
[Out] -((7*A - 11*B + 15*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^3*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((45*A - 65*B + 93*C)*Sin[c + d*x]
)/(15*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((5*A - 5*B + 9*C)*Cos[c + d*x]^2*Sin
[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((15*A - 35*B + 39*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(30*a^2*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
```

```
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(-a(A-3\right.}{}}{}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-5B+9C)\,\mathrm{co}}{10ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-5B+9C)\,\mathrm{co}}{10ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-5B+9C)\,\mathrm{co}}{10ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(45A-65B+93C}{15ad\sqrt{a+a\,\mathrm{co}}}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(45A-65B+93C}{15ad\sqrt{a+a\,\mathrm{co}}}$$

$$= -\frac{(7A-11B+15C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C}{2d(a}$$

Mathematica [A] time = 0.722273, size = 153, normalized size = 0.67

$$\frac{15(7A-11B+15C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(3(20A-20B+39C)}{15d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^(3/2),x]
```

```
[Out] (15*(7*A - 11*B + 15*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - Cos[
(c + d*x)/2]^3*(75*A - 85*B + 141*C + 3*(20*A - 20*B + 39*C)*Cos[c + d*x] +
 2*(5*B - 3*C)*Cos[2*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/(
15*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [B] time = 0.159, size = 533, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/60*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6-16*2^(1/2)*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*(5*B+6*C)*sin(1/2*d*x+1/2*c)^4+5*2^(1/2)*(24*A*a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-21*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+a))*a-8*B*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+33*B*ln(4/
cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+48*C*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-45*C*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)*sin(1/2*d*x+1/2*c)^2+105*2^(1/2)*ln(4/co
s(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-165*2^(1/2
)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+2
25*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a))*a*C-135*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+135*B*a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-255*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*
d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.98861, size = 641, normalized size = 2.8

$$15\sqrt{2}\Big((7A-11B+15C)\cos(dx+c)^2+2(7A-11B+15C)\cos(dx+c)+7A-11B+15C\Big)\sqrt{a}\log\Big(-\frac{a\cos(dx+c)^2+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*sqrt(2)*((7*A - 11*B + 15*C)*cos(d*x + c)^2 + 2*(7*A - 11*B + 15*
C)*cos(d*x + c) + 7*A - 11*B + 15*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqr
t(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(12*C*cos(d*x + c)^3 + 4*(5*B
- 3*C)*cos(d*x + c)^2 + 12*(5*A - 5*B + 9*C)*cos(d*x + c) + 75*A - 95*B + 1
47*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.59542, size = 308, normalized size = 1.34

$$\frac{\frac{15\sqrt{2}(7A-11B+15C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3+Ca^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2}+\frac{\sqrt{2}(165Aa^3-245Ba^3+381Ca^3)}{a^2}\right)\tan\left(\frac{1}{2}dx+\right.\right.\right.\right.}{\left(a\,\mathrm{ta}\right.}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] 1/60*(15*sqrt(2)*(7*A - 11*B + 15*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c)
+ sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + (((15*sqrt(2)*(A*a^3 - B*a
^3 + C*a^3)*tan(1/2*d*x + 1/2*c)^2/a^2 + sqrt(2)*(165*A*a^3 - 245*B*a^3 + 3
81*C*a^3)/a^2)*tan(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*(57*A*a^3 - 73*B*a^3 + 10
5*C*a^3)/a^2)*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*(9*A*a^3 - 9*B*a^3 + 17*C
*a^3)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.412 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{(3A-3B+7C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{(3A-7B+11C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)}{2d(a\cos(c+dx)+\cdots}$$

```
[Out] ((3*A - 7*B + 11*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^2*Sin[c + d*
x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((3*A - 9*B + 13*C)*Sin[c + d*x])/(3
*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((3*A - 3*B + 7*C)*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(6*a^2*d)
```

Rubi [A] time = 0.396036, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2968, 3023, 2751, 2649, 206}

$$\frac{(3A-3B+7C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{(3A-7B+11C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)}{2d(a\cos(c+dx)+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d
*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B + 11*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^2*Sin[c + d*
x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((3*A - 9*B + 13*C)*Sin[c + d*x])/(3
*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((3*A - 3*B + 7*C)*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(6*a^2*d)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(B-C)+\ldots\right.}{\sqrt{a+a\ldots}}}{\ldots}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{2a(B-C)\cos(c+dx)+\frac{1}{2}\ldots}{\sqrt{a+a\ldots}}}{\ldots}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-3B+7C)\sqrt{a\ldots}}{\ldots}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A-9B+13C)\sin\ldots}{3ad\sqrt{a+a\cos(\ldots}}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A-9B+13C)\sin\ldots}{3ad\sqrt{a+a\cos(\ldots}}$$

$$= \frac{(3A-7B+11C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\cos\ldots}{2d(a+a\ldots}$$

Mathematica [A] time = 0.47891, size = 131, normalized size = 0.72

$$\frac{-\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(-3A+12(B-C)\cos(c+dx)+15B+2C\cos(2(c+dx))-17C)-3(3A-7B+11C\ldots}{3d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^(3/2),x]
```

```
[Out] (-3*(3*A - 7*B + 11*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - Cos[(
c + d*x)/2]^3*(-3*A + 15*B - 17*C + 12*(B - C)*Cos[c + d*x] + 2*C*Cos[2*(c
+ d*x)])*Sin[(c + d*x)/2])/(3*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c +
 d*x)/2]^2))
```

Maple [B] time = 0.151, size = 407, normalized size = 2.3

$$\frac{1}{12d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(16C\sqrt{2}\sqrt{a(\sin(1/2dx+c/2))^2}\sqrt{a}(\cos(1/2dx+c/2))^4+9A\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2)\ldots}}{\cos(1/2dx+c/2)\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+9*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-21*2^(1/
2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*
a*cos(1/2*d*x+1/2*c)^2*B+33*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+24*B*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-40*C*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-3*A*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-3*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c
)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.9775, size = 575, normalized size = 3.18

$$\frac{3\sqrt{2}\left((3A-7B+11C)\cos(dx+c)^2+2(3A-7B+11C)\cos(dx+c)+3A-7B+11C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos\ldots}}{\cos(\ldots}\right.}{24\left(a^2d\cos(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*((3*A - 7*B + 11*C)*cos(d*x + c)^2 + 2*(3*A - 7*B + 11*C)*c
os(d*x + c) + 3*A - 7*B + 11*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*
```

```
sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*C*cos(d*x + c)^2 + 12*(B - C)*co
s(d*x + c) - 3*A + 15*B - 19*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2
*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/
2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.28762, size = 262, normalized size = 1.45

$$-\frac{\frac{3\left(3\sqrt{2}A-7\sqrt{2}B+11\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\frac{3\left(\sqrt{2}Aa-\sqrt{2}Ba+\sqrt{2}Ca\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a}+\frac{2\left(3\sqrt{2}Aa-15\sqrt{2}Ba+23\sqrt{2}Ca\right)}{a}\right)\tan\left(\ldots\right.}{\ldots\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)\ldots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] -1/12*(3*(3*sqrt(2)*A - 7*sqrt(2)*B + 11*sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/
2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((3*(sqrt(2
)*A*a - sqrt(2)*B*a + sqrt(2)*C*a)*tan(1/2*d*x + 1/2*c)^2/a + 2*(3*sqrt(2)*
A*a - 15*sqrt(2)*B*a + 23*sqrt(2)*C*a)/a)*tan(1/2*d*x + 1/2*c)^2 + 3*(sqrt(
2)*A*a - 9*sqrt(2)*B*a + 9*sqrt(2)*C*a)/a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*
d*x + 1/2*c)^2 + a)^(3/2))/d
```

3.413 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{(A+3B-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2C\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((A + 3*B - 7*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B + C)*Sin[c + d*x])/(2*d*(a + a*Co
s[c + d*x])^(3/2)) + (2*C*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.161838, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3019, 2751, 2649, 206}

$$\frac{(A+3B-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2C\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B - 7*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B + C)*Sin[c + d*x])/(2*d*(a + a*Co
s[c + d*x])^(3/2)) + (2*C*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(A+3B-3C)-2aC\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{(A+3B-7C)\int \frac{\cdots}{\sqrt{a+a\,\cdots}}}{4a}$$

$$= \frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+3B-7C)\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{(A+3B-7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\cdots}{ad\sqrt{\cdots}}$$

Mathematica [A] time = 0.501908, size = 83, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(A-B+4C\cos(c+dx)+5C)+(A+3B-7C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2
),x]
```

```
[Out] ((A + 3*B - 7*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B + 5*C
+ 4*C*Cos[c + d*x])*Tan[(c + d*x)/2])/(2*a*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.143, size = 334, normalized size = 2.8

$$\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+2\,a}{\cos\left(1/2\,dx+c/2\right)}\right)\sqrt{2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a+3\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2\right.\right.}}{\cos\left(1/2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1
/2*c)^2*a+3*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos
(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2*B-7*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+8
*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+A*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-B*a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.93488, size = 514, normalized size = 4.28

$$-\frac{\sqrt{2}\left((A+3\,B-7\,C)\cos\left(dx+c\right)^2+2\,(A+3\,B-7\,C)\cos\left(dx+c\right)+A+3\,B-7\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^2+2}\right.}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((A + 3*B - 7*C)*cos(d*x + c)^2 + 2*(A + 3*B - 7*C)*cos(d*x +
 c) + A + 3*B - 7*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(4*C*cos(d*x + c) + A - B + 5*C)*sqrt(a*cos(d*
x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^
2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.3979, size = 194, normalized size = 1.62

$$\frac{\frac{\left(\frac{\sqrt{2}\left(Aa^2-Ba^2+Ca^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{\sqrt{2}\left(Aa^2-Ba^2+9\,Ca^2\right)}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}-\frac{\sqrt{2}(A+3\,B-7\,C)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] 1/4*((sqrt(2)*(A*a^2 - B*a^2 + C*a^2)*tan(1/2*d*x + 1/2*c)^2/a^3 + sqrt(2)*
(A*a^2 - B*a^2 + 9*C*a^2)/a^3)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a) - sqrt(2)*(A + 3*B - 7*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c)
 + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2))/d
```

3.414 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{(5A-B-3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - B - 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d*x])/(2*d*(a + a*
Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.348043, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3041, 2985, 2649, 206, 2773}

$$-\frac{(5A-B-3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d
*x])^(3/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - B - 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d*x])/(2*d*(a + a*
Cos[c + d*x])^(3/2))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2aA-\frac{1}{2}a(A-B-3C)\cos(c+dx)\right)\sec(c+dx}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\cos(c+dx)}\sec(c+d}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(}{\sqrt{a+a\,\text{co}}}\right.}{ad}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B-3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{a+a\,\text{co}}}\right.}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 0.940872, size = 135, normalized size = 1.03

$$\frac{(A-B+C)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(5A-B-3C)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sqrt{2}A\cos^5\left(\frac{1}{2}\right.}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos
[c + d*x])^(3/2),x]
```

```
[Out] ((5*A - B - 3*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 4*Sqrt[2]*A
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (A - B + C)*Cos[(c
+ d*x)/2]^3*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c
+ d*x)/2]^2))
```

Maple [B] time = 0.292, size = 453, normalized size = 3.5

$$-\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5A\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2))^2}+2a}{\cos(1/2dx+c/2)}\right)\sqrt{2}(\cos(1/2dx+c/2))^2a-\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin}}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*co
s(1/2*d*x+1/2*c)^2*a-2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2*B-3*C*ln(2*(2*a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2
*c)^2*a-4*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^
2*a-4*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a
+A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-B*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.27389, size = 767, normalized size = 5.85

$$-\frac{\sqrt{2}\left((5\,A-B-3\,C)\cos\left(dx+c\right)^2+2\,(5\,A-B-3\,C)\cos\left(dx+c\right)+5\,A-B-3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\sqrt{a\cos(dx+c)+}}{\cos(dx+c)^2+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((5*A - B - 3*C)*cos(d*x + c)^2 + 2*(5*A - B - 3*C)*cos(d*x +
 c) + 5*A - B - 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt
(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)
```

```
*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(A - B + C)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a\left(\cos\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**(3/
2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(a*(cos(c +
d*x) + 1))**(3/2), x)
```

Giac [B] time = 5.0664, size = 305, normalized size = 2.33

$$\frac{\frac{\sqrt{2}\left(5A\sqrt{a}-B\sqrt{a}-3C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a^2}+\frac{8A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^{\frac{3}{2}}}-\frac{8A\,\mathrm{lo}}{}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(5*A*sqrt(a) - B*sqrt(a) - 3*C*sqrt(a))*log((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 8*A*log(abs((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sq
rt(2) + 3)))/a^(3/2) - 8*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) - 2*sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a + sqrt(2)*C*a)*tan(1/2*d
*x + 1/2*c)/a^3)/d
```

3.415 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{(9A-5B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A-B+C)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B+C\cdots}{2d(a\cos(c\cdots}$$

```
[Out] -(((3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(3/2)*d)) + ((9*A - 5*B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Tan[c + d*x])/(
2*d*(a + a*Cos[c + d*x])^(3/2)) + ((3*A - B + C)*Tan[c + d*x])/(2*a*d*Sqrt[
a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.567165, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2984, 2985, 2649, 206, 2773}

$$\frac{(9A-5B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A-B+C)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B+C\cdots}{2d(a\cos(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(3/2)*d)) + ((9*A - 5*B + C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Tan[c + d*x])/(
2*d*(a + a*Cos[c + d*x])^(3/2)) + ((3*A - B + C)*Tan[c + d*x])/(2*a*d*Sqrt[
a + a*Cos[c + d*x]])
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(3A-B+C)-\frac{1}{2}a(3A-3B-C)\cos(c+\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B+C)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int}{}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B+C)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{(3}{}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B+C)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{(3}{}$$

$$= -\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B+C)\tanh^{-}}{2\sqrt{2}a^{3}}$$

Mathematica [A] time = 1.59583, size = 196, normalized size = 1.13

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(2(9A-5B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sqrt{2}(3A}{}\right.}{d(a(\cos(c+dx)+1))^{3/2}(2A+2B\cos(c+dx)+C\cos(2(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*
(2*(9*A - 5*B + C)*ArcTanh[Sin[(c + d*x)/2]] + (4*Sqrt[2]*(3*A - 2*B)*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 - 2*(3*A - B + C + 2*A*Sec[
c + d*x])*Sin[(c + d*x)/2])/(-1 + Sin[(c + d*x)/2]^2)))/(d*(a*(1 + Cos[c +
d*x]))^(3/2)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.287, size = 1222, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-10*B*2
^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*
c))*cos(1/2*d*x+1/2*c)^4*a+2*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(-4*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(
2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+8*B*ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+8*B*ln(-4*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1
/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-9*A*ln(2*(2*a^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^
2*a+5*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d
*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2*B-C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+6*A*a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+6*A*ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a+6*A*ln(-4*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)
/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-2*B*a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-4*B*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-4*B*ln(-4*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/
2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+2*C*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-C*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(
a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.71867, size = 900, normalized size = 5.2

$$\frac{\sqrt{2}\left((9\,A-5\,B+C)\cos\left(dx+c\right)^{3}+2\,(9\,A-5\,B+C)\cos\left(dx+c\right)^{2}+(9\,A-5\,B+C)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((9*A - 5*B + C)*cos(d*x + c)^3 + 2*(9*A - 5*B + C)*cos(d*x +
c)^2 + (9*A - 5*B + C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqr
t(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*((3*A - 2*B)*cos(d*x + c)^3 +
2*(3*A - 2*B)*cos(d*x + c)^2 + (3*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((
3*A - B + C)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^
2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.26917, size = 518, normalized size = 2.99

$$-\frac{\frac{\sqrt{2}\left(9\,A\sqrt{a}-5\,B\sqrt{a}+C\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a^{2}}+\frac{4\left(3\,A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{\ldots}\right.\right.\right.}{a^{2}}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2)*(9*A*sqrt(a) - 5*B*sqrt(a) + C*sqrt(a))*log((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 4*(3*A*sqrt(a)
- 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^2 - 4*(3*A*sqrt(a) - 2*B*sqrt(a))
*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))/a^2 - 16*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/(((sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a) - 2
*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a + sqrt(2)*C*
a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.416 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=232

$$\frac{(19A-12B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B+5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B+2C)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((19*A - 12*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*a^(3/2)*d) - ((13*A - 9*B + 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A - 6*B + 2*C)
*Tan[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d*x]
*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((2*A - B + C)*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.779912, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2984, 2985, 2649, 206, 2773}

$$\frac{(19A-12B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B+5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B+2C)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c +
 d*x])^(3/2),x]
```



```
[Out] ((19*A - 12*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*a^(3/2)*d) - ((13*A - 9*B + 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A - 6*B + 2*C)
*Tan[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d*x]
*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((2*A - B + C)*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \int \frac{\left(2a(2A-B+C)-\frac{1}{2}a(5A\right.}{\sqrt{a}}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B+C)\sec(c}{2ad\sqrt{a+a}}$$

$$= -\frac{(7A-6B+2C)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec(c+dx)\tan}{2d(a+a\cos(c+dx))}$$

$$= -\frac{(7A-6B+2C)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec(c+dx)\tan}{2d(a+a\cos(c+dx))}$$

$$= -\frac{(7A-6B+2C)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec(c+dx)\tan}{2d(a+a\cos(c+dx))}$$

$$= \frac{(19A-12B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B+5C}{}$$

Mathematica [A] time = 1.54388, size = 186, normalized size = 0.8

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)((7A-6B+2C)\cos(2(c+dx))+(6A-8B)\cos(c+dx)+3A-6B+2C)-\sqrt{2}(19A-12B+8C)\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}(}{\sin^2\left(\frac{1}{2}(c+dx)\right)-1}\right.}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(-2*(13*A - 9*B + 5*C)*ArcTanh[Sin[(c + d*x)/2]] + (-(S
qrt[2]*(19*A - 12*B + 8*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/
2]^2) + ((3*A - 6*B + 2*C + (6*A - 8*B)*Cos[c + d*x] + (7*A - 6*B + 2*C)*Co
s[2*(c + d*x)])*Sec[c + d*x]^2*Sin[(c + d*x)/2])/2)/(-1 + Sin[(c + d*x)/2]^
2)))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.316, size = 2261, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c
)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a+48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*a))*cos(1/2*d*x+1/2*c)^6*a+12*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(
1/2)))*cos(1/2*d*x+1/2*c)^2*a-48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a))*cos(1/2*d*x+1/2*c)^4*a-48*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
)*cos(1/2*d*x+1/2*c)^4*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*c
os(1/2*d*x+1/2*c)^2*a-8*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1
/2*d*x+1/2*c)^2*a-8*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*
d*x+1/2*c)^2*a+32*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*
x+1/2*c)^4*a+32*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1
/2*c)^4*a-32*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1
/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*
c)^6*a-32*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)
^6*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*
a-76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a+
76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+76*
A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-19*A*
ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-19*A*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(
```

```
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-2*B*a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-72*B*2^
(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c
))*cos(1/2*d*x+1/2*c)^6*a-24*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)*cos(1/2*d*x+1/2*c)^4+104*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a+40*C*ln(2*(2*a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2
*d*x+1/2*c)^6*a+28*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2
*d*x+1/2*c)^4+72*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+16*B*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-18*2^(1/2)*ln(2*(2*a^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2
*B-104*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/
2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-40*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+8*C*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+26*A*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2
)*cos(1/2*d*x+1/2*c)^2*a+10*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-22*A*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-8*C*a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2)/a^(5/2)/cos(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin
(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.16986, size = 999, normalized size = 4.31

$$\frac{2\sqrt{2}\left((13\,A-9\,B+5\,C)\cos(dx+c)^4+2\,(13\,A-9\,B+5\,C)\cos(dx+c)^3+(13\,A-9\,B+5\,C)\cos(dx+c)^2\right)\sqrt{a}\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*((13*A - 9*B + 5*C)*cos(d*x + c)^4 + 2*(13*A - 9*B + 5*C)*c
os(d*x + c)^3 + (13*A - 9*B + 5*C)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(
d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B + 8*
C)*cos(d*x + c)^4 + 2*(19*A - 12*B + 8*C)*cos(d*x + c)^3 + (19*A - 12*B + 8
*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*
sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)) - 4*((7*A - 6*B + 2*C)*cos(d*x + c)^2 + (3*
A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*
cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.67105, size = 819, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(13*A*sqrt(a) - 9*B*sqrt(a) + 5*C*sqrt(a))*log((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + (19*A*sqrt(a)
 - 12*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
```

```
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^2 - (19*A*sqrt(a)
 - 12*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^2 - 2*sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a + sqrt(2)*C*a)*tan(1/2*d
*x + 1/2*c)/a^3 - 4*sqrt(2)*(29*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a) - 133*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 76*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(3/2) + 55*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/
2) - 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*B*a^(5/2) - 7*A*a^(7/2) + 4*B*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a))/d
```

3.417 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{(47A-38B+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(21A-14B+12C)\tan(c}{8ad\sqrt{a\cos(c+dx)+}}$$

```
[Out] -((47*A - 38*B + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*a^(3/2)*d) + ((17*A - 13*B + 9*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((21*A - 14*B +
 12*C)*Tan[c + d*x])/(8*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((13*A - 12*B + 6*C
)*Sec[c + d*x]*Tan[c + d*x])/(12*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B +
C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((5*A -
3*B + 3*C)*Sec[c + d*x]^2*Tan[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.992175, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2984, 2985, 2649, 206, 2773}

$$-\frac{(47A-38B+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(21A-14B+12C)\tan(c}{8ad\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*Cos[c +
 d*x])^(3/2),x]
```


```
[Out] -((47*A - 38*B + 24*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*a^(3/2)*d) + ((17*A - 13*B + 9*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((21*A - 14*B +
 12*C)*Tan[c + d*x])/(8*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((13*A - 12*B + 6*C
)*Sec[c + d*x]*Tan[c + d*x])/(12*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B +
C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((5*A -
3*B + 3*C)*Sec[c + d*x]^2*Tan[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
```

```
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(5A-3B+3C)-\frac{1}{2}a(7A\right.}{\sqrt{a}}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-3B+3C)\sec}{6ad\sqrt{a+a}}$$

$$= -\frac{(13A-12B+6C)\sec(c+dx)\tan(c+dx)}{12ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec}{2d(a+a}$$

$$= \frac{(21A-14B+12C)\tan(c+dx)}{8ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A-12B+6C)\sec(c+}{12ad\sqrt{a+a\cos(c}}$$

$$= \frac{(21A-14B+12C)\tan(c+dx)}{8ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A-12B+6C)\sec(c+}{12ad\sqrt{a+a\cos(c}}$$

$$= \frac{(21A-14B+12C)\tan(c+dx)}{8ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A-12B+6C)\sec(c+}{12ad\sqrt{a+a\cos(c}}$$

$$= -\frac{(47A-38B+24C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A-13B+}{}$$

Mathematica [A] time = 2.63368, size = 223, normalized size = 0.79

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(12(17A-13B+9C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{3\sqrt{2}(47A-38B+24C)\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{4}\sin(}{}\right.}{12d(a(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + a*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(12*(17*A - 13*B + 9*C)*ArcTanh[Sin[(c + d*x)/2]] + (3*
Sqrt[2]*(47*A - 38*B + 24*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x
)/2]^2 - ((106*A - 36*B + 48*C + 3*(55*A - 26*B + 36*C)*Cos[c + d*x] + (74*
A - 36*B + 48*C)*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)] - 42*B*Cos[3*(c +
 d*x)] + 36*C*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2])/4)/(-1 + S
in[(c + d*x)/2]^2)))/(12*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.351, size = 2993, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-576*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-1128*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-1128*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)
-2^(1/2)))*cos(1/2*d*x+1/2*c)^8*a-576*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-
2^(1/2)))*cos(1/2*d*x+1/2*c)^8*a+912*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2
^(1/2)))*cos(1/2*d*x+1/2*c)^8*a+912*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*a))*cos(1/2*d*x+1/2*c)^8*a-1368*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(
1/2)))*cos(1/2*d*x+1/2*c)^6*a-1368*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a))*cos(1/2*d*x+1/2*c)^6*a-114*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/
2)))*cos(1/2*d*x+1/2*c)^2*a+684*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
))*cos(1/2*d*x+1/2*c)^4*a+684*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
)*cos(1/2*d*x+1/2*c)^4*a-114*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
cos(1/2*d*x+1/2*c)^2*a+72*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos
(1/2*d*x+1/2*c)^2*a+72*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2
```

)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1
/2*d*x+1/2*c)^2*a-432*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/
2*d*x+1/2*c)^4*a-432*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1
/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*
d*x+1/2*c)^4*a+864*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*
x+1/2*c)^6*a+864*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x
+1/2*c)^6*a+1692*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+
1/2*c)^6*a+1692*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+
1/2*c)^6*a-846*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x
+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/
2*c)^4*a-846*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2
*c)^4*a+141*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*
c)^2*a+141*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)
^2*a+12*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-12*C*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-12*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-1248*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a-336*B*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+1872*B*2^(1/2)*ln(2*(2*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)
^6*a+432*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c
)^4+864*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1
/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a+1632*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a+50
4*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-244
8*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))
*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-1296*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-608*A*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-936*B*2^(
1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c)
)*cos(1/2*d*x+1/2*c)^4*a-156*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*cos(1/2*d*x+1/2*c)^2+156*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2*B+1224*A*2^(1/2)*ln
(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1
/2*d*x+1/2*c)^4*a+648*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+288*C*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-336*C*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-204*A*ln(2*(2*a^(1/2)*(a*

```
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/
2*c)^2*a-108*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*
d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+218*A*a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+120*C*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2)/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2
*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 4.26224, size = 1069, normalized size = 3.76

$$12\sqrt{2}\Big((17A-13B+9C)\cos(dx+c)^5+2(17A-13B+9C)\cos(dx+c)^4+(17A-13B+9C)\cos(dx+c)^3\Big)\sqrt{a}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] 1/96*(12*sqrt(2)*((17*A - 13*B + 9*C)*cos(d*x + c)^5 + 2*(17*A - 13*B + 9*C
)*cos(d*x + c)^4 + (17*A - 13*B + 9*C)*cos(d*x + c)^3)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*((47*A - 38*
B + 24*C)*cos(d*x + c)^5 + 2*(47*A - 38*B + 24*C)*cos(d*x + c)^4 + (47*A -
38*B + 24*C)*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(21*A - 14*B + 12*C)*cos(d*
x + c)^3 + (37*A - 18*B + 24*C)*cos(d*x + c)^2 - 6*(A - 2*B)*cos(d*x + c) +
 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*
```

```
d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+a*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.93169, size = 1328, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] -1/48*(6*sqrt(2)*(17*A*sqrt(a) - 13*B*sqrt(a) + 9*C*sqrt(a))*log((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 3*(47*A*
sqrt(a) - 38*B*sqrt(a) + 24*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^2 - 3*(47
*A*sqrt(a) - 38*B*sqrt(a) + 24*C*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^2 - 12
*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a + sqrt(2)*C*
a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(339*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(a) - 174*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(a) + 72*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(a) -
 3165*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8
*A*a^(3/2) + 1842*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^8*B*a^(3/2) - 888*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^8*C*a^(3/2) + 9198*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*a^(5/2) - 5292*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^(5/2) + 3024*(sqrt(a)*t
```

an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*a^(5/2) - 493
8*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a
^(7/2) + 2820*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^4*B*a^(7/2) - 1776*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*C*a^(7/2) + 975*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(9/2) - 582*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(9/2) + 360*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*a^(9/2) - 73*A*a^(1
1/2) + 42*B*a^(11/2) - 24*C*a^(11/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3*a))/d

3.418 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{(45A-85B+157C)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(195A-475B+787C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}+\frac{(465A-985B}{120a^2d\sqrt{a}}$$

```
[Out] -((75*A - 163*B + 283*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^4*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 13*B + 21*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((465*A - 985*B +
 1729*C)*Sin[c + d*x])/(120*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((45*A - 85*B
 + 157*C)*Cos[c + d*x]^2*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
- ((195*A - 475*B + 787*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(240*a^3*
d)
```

Rubi [A] time = 0.897723, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3041, 2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{(45A-85B+157C)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(195A-475B+787C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}+\frac{(465A-985B}{120a^2d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^(5/2),x]
```


```
[Out] -((75*A - 163*B + 283*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^4*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 13*B + 21*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((465*A - 985*B +
 1729*C)*Sin[c + d*x])/(120*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((45*A - 85*B
 + 157*C)*Cos[c + d*x]^2*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
- ((195*A - 475*B + 787*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(240*a^3*
d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(B-C)\right.}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B+21C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B+21C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B+21C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B+21C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B+21C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B+21C)}{16ad(a+}$$

$$= -\frac{(75A-163B+283C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B}{4}$$

Mathematica [A] time = 1.58186, size = 152, normalized size = 0.55

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(5(255A-479B+887C)\cos(c+dx)+16(15A-25B+52C)\cos(2(c+dx))+975A+40B\cos(3(c+}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^(5/2),x]
```

```
[Out] (-30*(75*A - 163*B + 283*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 +
(975*A - 1895*B + 3491*C + 5*(255*A - 479*B + 887*C)*Cos[c + d*x] + 16*(15*
A - 25*B + 52*C)*Cos[2*(c + d*x)] + 40*B*Cos[3*(c + d*x)] - 40*C*Cos[3*(c +
 d*x)] + 12*C*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(240*a*d*(a*(1 + Cos[c +
```

```
d*x]))^(3/2))
```

Maple [B] time = 0.171, size = 617, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/480/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(768*C*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^8+640*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-2176*C*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-1125*A*2^(1/2)*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/
2*d*x+1/2*c)^4*a+2445*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-4245*C*2^(1/2)*ln(2*(2*
a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x
+1/2*c)^4*a+960*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4-2560*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4+5248*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4+315*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)^2-435*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)^2+555*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)^2-30*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+30*B*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-30*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.04391, size = 809, normalized size = 2.92

$$\frac{15\sqrt{2}\left((75A-163B+283C)\cos(dx+c)^3+3(75A-163B+283C)\cos(dx+c)^2+3(75A-163B+283C)\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] 1/960*(15*sqrt(2)*((75*A - 163*B + 283*C)*cos(d*x + c)^3 + 3*(75*A - 163*B
+ 283*C)*cos(d*x + c)^2 + 3*(75*A - 163*B + 283*C)*cos(d*x + c) + 75*A - 16
3*B + 283*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(96*C*cos(d*x + c)^4 + 160*(B - C)*cos(d*x + c)^3 + 32
*(15*A - 25*B + 49*C)*cos(d*x + c)^2 + 5*(255*A - 503*B + 911*C)*cos(d*x +
c) + 735*A - 1495*B + 2671*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.27863, size = 414, normalized size = 1.49

$$\frac{\frac{15\left(75\sqrt{2}A-163\sqrt{2}B+283\sqrt{2}C\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{5}{2}}}-\frac{\left(\left(\left(15\left(\frac{2\left(\sqrt{2}Aa^2-\sqrt{2}Ba^2+\sqrt{2}Ca^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2}-\frac{13\sqrt{2}Aa^2-21\sqrt{2}Ba^2+29}{a^2}\right.\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] 1/480*(15*(75*sqrt(2)*A - 163*sqrt(2)*B + 283*sqrt(2)*C)*log(abs(-sqrt(a)*t
an(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2) - (((15*
(2*(sqrt(2)*A*a^2 - sqrt(2)*B*a^2 + sqrt(2)*C*a^2)*tan(1/2*d*x + 1/2*c)^2/a
^2 - (13*sqrt(2)*A*a^2 - 21*sqrt(2)*B*a^2 + 29*sqrt(2)*C*a^2)/a^2)*tan(1/2*
d*x + 1/2*c)^2 - (1725*sqrt(2)*A*a^2 - 3685*sqrt(2)*B*a^2 + 6733*sqrt(2)*C*
a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 5*(549*sqrt(2)*A*a^2 - 1133*sqrt(2)*B*a^
2 + 1973*sqrt(2)*C*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 15*(83*sqrt(2)*A*a^2
- 155*sqrt(2)*B*a^2 + 291*sqrt(2)*C*a^2)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1
/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.419 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=227

$$\frac{(15A-39B+95C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{(21A-93B+197C)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A-75B+163C)\tanh^{-1}\left(\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((19*A - 75*B + 163*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^3*Sin[c
+ d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((A - 9*B + 17*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((21*A - 93*B + 197*C)
*Sin[c + d*x])/(24*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((15*A - 39*B + 95*C)*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.686808, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3041, 2977, 2968, 3023, 2751, 2649, 206}

$$\frac{(15A-39B+95C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{(21A-93B+197C)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A-75B+163C)\tanh^{-1}\left(\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c +
 d*x])^(5/2),x]
```


```
[Out] ((19*A - 75*B + 163*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^3*Sin[c
+ d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((A - 9*B + 17*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((21*A - 93*B + 197*C)
*Sin[c + d*x])/(24*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((15*A - 39*B + 95*C)*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(48*a^3*d)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(a(A+3B\right.}{(a}}{}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A-9B+17C)\,\mathrm{co}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A-9B+17C)\,\mathrm{co}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A-9B+17C)\,\mathrm{co}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A-9B+17C)\,\mathrm{co}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(A-9B+17C)\,\mathrm{co}}{16ad(a+a}$$

$$= \frac{(19A-75B+163C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+}{4d(}$$

Mathematica [A] time = 1.16389, size = 126, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((-39A+255B-479C)\cos(c+dx)-27A+16(3B-5C)\cos(2(c+dx))+195B+8C\cos(3(c+dx))-}{48ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*C
os[c + d*x])^(5/2),x]
```

```
[Out] (6*(19*A - 75*B + 163*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-2
7*A + 195*B - 379*C + (-39*A + 255*B - 479*C)*Cos[c + d*x] + 16*(3*B - 5*C)
*Cos[2*(c + d*x)] + 8*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(48*a*d*(a*(1 +
 Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.157, size = 512, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+57*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-225*B*
2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2
*c))*cos(1/2*d*x+1/2*c)^4*a+489*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+192*B*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-512*C*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-39*A*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+63*B*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-87*C*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+6*A*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-6*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+6*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c
)^3/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 2.0934, size = 743, normalized size = 3.27

$$3\sqrt{2}\Big((19A-75B+163C)\cos(dx+c)^3+3(19A-75B+163C)\cos(dx+c)^2+3(19A-75B+163C)\cos(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] 1/192*(3*sqrt(2)*((19*A - 75*B + 163*C)*cos(d*x + c)^3 + 3*(19*A - 75*B + 1
63*C)*cos(d*x + c)^2 + 3*(19*A - 75*B + 163*C)*cos(d*x + c) + 19*A - 75*B +
 163*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1)) + 4*(32*C*cos(d*x + c)^3 + 32*(3*B - 5*C)*cos(d*x + c)^2 - (39*
A - 255*B + 503*C)*cos(d*x + c) - 27*A + 147*B - 299*C)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.23447, size = 311, normalized size = 1.37

$$\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5+Ca^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6}-\frac{\sqrt{2}\left(7Aa^5-15Ba^5+23Ca^5\right)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}\left(15Aa^5-75Ba^5+167Ca^5\right)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{3\sqrt{2}\left(11Aa^5-83Ba^5+155Ca^5\right)}{a^6}\right)\tan}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] 1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a^5)*tan(1/2*d*x + 1/2*c)^2/a^6 - s
qrt(2)*(7*A*a^5 - 15*B*a^5 + 23*C*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt
(2)*(15*A*a^5 - 75*B*a^5 + 167*C*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(
2)*(11*A*a^5 - 83*B*a^5 + 155*C*a^5)/a^6)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d
*x + 1/2*c)^2 + a)^(3/2) - 3*sqrt(2)*(19*A - 75*B + 163*C)*log(abs(-sqrt(a)
*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.420 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$\frac{(A-B+9C)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(5A+19B-75C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(3A}{16a}$$

```
[Out] ((5*A + 19*B - 75*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^2*Sin[c +
d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((3*A + 5*B - 13*C)*Sin[c + d*x])/
(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((A - B + 9*C)*Sin[c + d*x])/(4*a^2*d
*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.407442, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2968, 3019, 2751, 2649, 206}

$$\frac{(A-B+9C)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(5A+19B-75C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B+C)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(3A}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos[c + d
*x])^(5/2),x]
```



```
[Out] ((5*A + 19*B - 75*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^2*Sin[c +
d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((3*A + 5*B - 13*C)*Sin[c + d*x])/
(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((A - B + 9*C)*Sin[c + d*x])/(4*a^2*d
*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A+B-C}{(a+a\,c}}{}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{2a(A+B-C)\cos(c+dx}{(a+a\,c}}{}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(3A+5B-13C)\,\mathrm{si}}{16ad(a+a\cos(c}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(3A+5B-13C)\,\mathrm{si}}{16ad(a+a\cos(c}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(3A+5B-13C)\,\mathrm{si}}{16ad(a+a\cos(c}$$

$$= \frac{(5A+19B-75C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\,\mathrm{c}}{4d(a+}$$

Mathematica [A] time = 0.799489, size = 107, normalized size = 0.6

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((5A-13B+85C)\cos(c+dx)+A-9B+16C\cos(2(c+dx))+65C)+2(5A+19B-75C)\cos^3\left(\frac{1}{2}(c\right.}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 19*B - 75*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A -
9*B + 65*C + (5*A - 13*B + 85*C)*Cos[c + d*x] + 16*C*Cos[2*(c + d*x)])*Tan[
(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.158, size = 442, normalized size = 2.5

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5\,A\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^4\,a+19\,B\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a+19*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-75*C*2^(1/2)*ln(2*(2*a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c
)^4*a+64*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c
)^4+5*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2
-13*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2
1*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*B*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))
/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.03941, size = 674, normalized size = 3.77

$$-\frac{\sqrt{2}\left((5\,A+19\,B-75\,C)\cos\left(dx+c\right)^{3}+3\,(5\,A+19\,B-75\,C)\cos\left(dx+c\right)^{2}+3\,(5\,A+19\,B-75\,C)\cos\left(dx+c\right)+5\,A+\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*((5*A + 19*B - 75*C)*cos(d*x + c)^3 + 3*(5*A + 19*B - 75*C)*
cos(d*x + c)^2 + 3*(5*A + 19*B - 75*C)*cos(d*x + c) + 5*A + 19*B - 75*C)*sq
rt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
) - 4*(32*C*cos(d*x + c)^2 + (5*A - 13*B + 85*C)*cos(d*x + c) + A - 9*B + 4
9*C)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/
2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.85058, size = 285, normalized size = 1.59

$$-\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Aa^{6}-\sqrt{2}Ba^{6}+\sqrt{2}Ca^{6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{8}}-\frac{\sqrt{2}Aa^{6}-9\sqrt{2}Ba^{6}+17\sqrt{2}Ca^{6}}{a^{8}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{3\sqrt{2}Aa^{6}-11\sqrt{2}Ba^{6}+83\sqrt{2}Ca^{6}}{a^{8}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}+\frac{\left(5\sqrt{2}A+19\sqrt{2}B-75\sqrt{}\right.}{}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] -1/32*(((2*(sqrt(2)*A*a^6 - sqrt(2)*B*a^6 + sqrt(2)*C*a^6)*tan(1/2*d*x + 1/
2*c)^2/a^8 - (sqrt(2)*A*a^6 - 9*sqrt(2)*B*a^6 + 17*sqrt(2)*C*a^6)/a^8)*tan(
1/2*d*x + 1/2*c)^2 - (3*sqrt(2)*A*a^6 - 11*sqrt(2)*B*a^6 + 83*sqrt(2)*C*a^6
)/a^8)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + (5*sqrt(2)
*A + 19*sqrt(2)*B - 75*sqrt(2)*C)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.421 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{(3A+5B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A+5B-13C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B+C)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] ((3*A + 5*B + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B + C)*Sin[c + d*x])/(4*d*(a +
a*Cos[c + d*x])^(5/2)) + ((3*A + 5*B - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*C
os[c + d*x])^(3/2))
```

Rubi [A] time = 0.17685, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3019, 2750, 2649, 206}

$$\frac{(3A+5B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A+5B-13C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B+C)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B + 19*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B + C)*Sin[c + d*x])/(4*d*(a +
a*Cos[c + d*x])^(5/2)) + ((3*A + 5*B - 13*C)*Sin[c + d*x])/(16*a*d*(a + a*C
os[c + d*x])^(3/2))
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3A+5B-5C)-4aC\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3A+5B+19C \ldots}{\ldots}$$

$$= \frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B-13C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(3A+5B+19C \ldots}{\ldots}$$

$$= \frac{(3A+5B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \ldots$$

Mathematica [A] time = 0.567004, size = 96, normalized size = 0.72

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((3A+5B-13C)\cos(c+dx)+7A+B-9C)+2(3A+5B+19C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c \ldots\right.\right.}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (2*(3*A + 5*B + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (7*A +
 B - 9*C + (3*A + 5*B - 13*C)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1
 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.217, size = 407, normalized size = 3.1

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3\,A\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^4\,a+5\,B\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^4*a+5*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+19*C*2^(1/2)*ln(2*(2*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)
^4*a+3*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^
2+5*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-1
3*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*B*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))
/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.02877, size = 635, normalized size = 4.77

$$\frac{\sqrt{2}\left((3A+5B+19C)\cos(dx+c)^3+3(3A+5B+19C)\cos(dx+c)^2+3(3A+5B+19C)\cos(dx+c)+3A+5B\right.}{64\left(a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B + 19*C)*cos(d*x + c)^3 + 3*(3*A + 5*B + 19*C)*cos
(d*x + c)^2 + 3*(3*A + 5*B + 19*C)*cos(d*x + c) + 3*A + 5*B + 19*C)*sqrt(a)
*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*
x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4
*((3*A + 5*B - 13*C)*cos(d*x + c) + 7*A + B - 9*C)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.63535, size = 200, normalized size = 1.5

$$\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5+Ca^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{\sqrt{2}\left(5Aa^5+3Ba^5-11Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\sqrt{2}(3A+5B+19C)\log\left(\left|-\right.\right.}{}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a^5)
*tan(1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(5*A*a^5 + 3*B*a^5 - 11*C*a^5)/a^8)*t
an(1/2*d*x + 1/2*c) - sqrt(2)*(3*A + 5*B + 19*C)*log(abs(-sqrt(a)*tan(1/2*d
*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.422 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(43A-3B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B-5C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B+C}{4d(a\cos(c}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 3*B - 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*(a
+ a*Cos[c + d*x])^(5/2)) - ((11*A - 3*B - 5*C)*Sin[c + d*x])/(16*a*d*(a + a
*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.520945, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3041, 2978, 2985, 2649, 206, 2773}

$$-\frac{(43A-3B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B-5C)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B+C}{4d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos[c + d
*x])^(5/2),x]
```


```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 3*B - 5*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*(a
+ a*Cos[c + d*x])^(5/2)) - ((11*A - 3*B - 5*C)*Sin[c + d*x])/(16*a*d*(a + a
*Cos[c + d*x])^(3/2))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4aA-\frac{1}{2}a(3A-3B-5C)\cos(c+dx)\right)\sec(\ldots}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B-5C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \ldots$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B-5C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \ldots$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B-5C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \ldots$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{2}\sqrt{a}\ldots}\right)}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 1.75182, size = 200, normalized size = 1.17

$$-\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(A\sec(c+dx)+B+C\cos(c+dx))\left(2(43A-3B-5C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin\left(\frac{1}{2}(\ldots\right.}{\ldots}\right.}{4d(a(\cos(c+dx)+1))^{5/2}(2A+2B\cos(c+dx)+C\cos(2\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + a*Cos
[c + d*x])^(5/2),x]
```

```
[Out] -(Cos[(c + d*x)/2]^5*Cos[c + d*x]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*(2*
(43*A - 3*B - 5*C)*ArcTanh[Sin[(c + d*x)/2]] + (-64*Sqrt[2]*A*ArcTanh[Sqrt[
2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^4 + (15*A - 7*B - C + (11*A - 3*B - 5
*C)*Cos[c + d*x])*Sin[(c + d*x)/2])/(-1 + Sin[(c + d*x)/2]^2)^2))/(4*d*(a*(
1 + Cos[c + d*x]))^(5/2)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.273, size = 560, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A*2^(
1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c)
)*cos(1/2*d*x+1/2*c)^4*a-3*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-5*C*2^(1/2)*ln(2*(
2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d
*x+1/2*c)^4*a-32*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+
1/2*c)^4*a-32*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/
2*c)^4*a+11*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)^2-3*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)^2-5*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2
+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*B*a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.37823, size = 948, normalized size = 5.54

$$-\frac{\sqrt{2}\left((43\,A-3\,B-5\,C)\cos\left(dx+c\right)^{3}+3\,(43\,A-3\,B-5\,C)\cos\left(dx+c\right)^{2}+3\,(43\,A-3\,B-5\,C)\cos\left(dx+c\right)+43\,A-3\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*((43*A - 3*B - 5*C)*cos(d*x + c)^3 + 3*(43*A - 3*B - 5*C)*co
s(d*x + c)^2 + 3*(43*A - 3*B - 5*C)*cos(d*x + c) + 43*A - 3*B - 5*C)*sqrt(a
)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d
*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
32*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*((11*A - 3*B - 5*C)*cos(d*x + c) + 15*A - 7*B - C)*sqrt(a*cos(d*x +
c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^
3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+a*cos(d*x+c))**(5/
2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.11978, size = 360, normalized size = 2.11

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5+Ca^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}\left(13\,Aa^5-5\,Ba^5-3\,Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}\left(43\,A\sqrt{a}-3\,B\sqrt{a}-5\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a
^5)*tan(1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(13*A*a^5 - 5*B*a^5 - 3*C*a^5)/a^8
```

```
)*tan(1/2*d*x + 1/2*c) - sqrt(2)*(43*A*sqrt(a) - 3*B*sqrt(a) - 5*C*sqrt(a))
*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)
/a^3 - 64*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(5/2) + 64*A*log(abs((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
)/a^(5/2))/d
```

3.423 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(35A-11B+3C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(115A-43B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-($$

```
[Out] -(((5*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(5/2)*d)) + ((115*A - 43*B + 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Tan[c + d
*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((15*A - 7*B - C)*Tan[c + d*x])/(16
*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((35*A - 11*B + 3*C)*Tan[c + d*x])/(16*a
^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.80223, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3041, 2978, 2984, 2985, 2649, 206, 2773}

$$\frac{(35A-11B+3C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(115A-43B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-($$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*Cos[c +
 d*x])^(5/2),x]
```


```
[Out] -(((5*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(5/2)*d)) + ((115*A - 43*B + 3*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Tan[c + d
*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((15*A - 7*B - C)*Tan[c + d*x])/(16
*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((35*A - 11*B + 3*C)*Tan[c + d*x])/(16*a
^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(a(5A-B+C)-\frac{1}{2}a(5A-5B-3C)\cos(c+\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B-C)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B+3C)\,\text{ta}}{16\sqrt{}}$$

Mathematica [A] time = 3.77043, size = 189, normalized size = 0.87

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)(2(55A-15B+7C)\cos(c+dx)+(35A-11B+3C)\cos(2(c+dx))+67A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + a*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]*(4*(115*A - 43*B + 3*C)*ArcTanh[Sin[(c + d*x
)/2]]*Cos[(c + d*x)/2]^4*Cos[c + d*x] - 64*Sqrt[2]*(5*A - 2*B)*ArcTanh[Sqrt
[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^4*Cos[c + d*x] + (67*A - 11*B + 3*C
+ 2*(55*A - 15*B + 7*C)*Cos[c + d*x] + (35*A - 11*B + 3*C)*Cos[2*(c + d*x)]
)*Sin[(c + d*x)/2]))/(32*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.317, size = 1327, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-86*B
*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/
2*c))*cos(1/2*d*x+1/2*c)^6*a+6*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(-4*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-
2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a+64*B*ln(-4*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)
/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a+64*B*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a-115*A*2^(1/2)*ln(2*(2*
a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x
+1/2*c)^4*a+43*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-3*C*2^(1/2)*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*
a+70*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+
80*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+80
*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-22*B*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-32*B*ln
```

```
(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-32*B*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+6*C*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-15*A*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+7*B*a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+C*a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+2*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1
/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.83463, size = 1112, normalized size = 5.12

$$\frac{\sqrt{2}\Big((115\,A-43\,B+3\,C)\cos\left(dx+c\right)^4+3\,(115\,A-43\,B+3\,C)\cos\left(dx+c\right)^3+3\,(115\,A-43\,B+3\,C)\cos\left(dx+c\right)^2+(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((115*A - 43*B + 3*C)*cos(d*x + c)^4 + 3*(115*A - 43*B + 3*C)
*cos(d*x + c)^3 + 3*(115*A - 43*B + 3*C)*cos(d*x + c)^2 + (115*A - 43*B + 3
*C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) - 16*((5*A - 2*B)*cos(d*x + c)^4 + 3*(5*A - 2*B)*cos(
```

```
d*x + c)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2 + (5*A - 2*B)*cos(d*x + c))*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)) + 4*((35*A - 11*B + 3*C)*cos(d*x + c)^2 + (55*A - 15*B + 7*C)*cos(d*
x + c) + 16*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4
 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+a*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.59757, size = 575, normalized size = 2.65

$$2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5+Ca^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{\sqrt{2}\left(21\,Aa^5-13\,Ba^5+5\,Ca^5\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\sqrt{2}\left(115\,A\sqrt{a}-43\,B\sqrt{a}+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] 1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a^
5)*tan(1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(21*A*a^5 - 13*B*a^5 + 5*C*a^5)/a^8
)*tan(1/2*d*x + 1/2*c) - sqrt(2)*(115*A*sqrt(a) - 43*B*sqrt(a) + 3*C*sqrt(a
))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
2)/a^3 - 32*(5*A*sqrt(a) - 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^3 + 32*(
5*A*sqrt(a) - 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^3 + 128*sqrt(2)*(3*(s
```

```
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(
a) - A*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*a + a^2)*a^2))/d
```

3.424 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$-\frac{(63A-35B+11C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A-20B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B+43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{a\cos(c}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((39*A - 20*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*a^(5/2)*d) - ((219*A - 115*B + 43*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((63*A - 35*B
 + 11*C)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*S
ec[c + d*x]*Tan[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A - 11*B
+ 3*C)*Sec[c + d*x]*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((3
1*A - 15*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d
*x]])
```

Rubi [A] time = 1.01811, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3041, 2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{(63A-35B+11C)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A-20B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B+43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{a\cos(c}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*Cos[c +
 d*x])^(5/2),x]
```


```
[Out] ((39*A - 20*B + 8*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*a^(5/2)*d) - ((219*A - 115*B + 43*C)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((63*A - 35*B
 + 11*C)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*S
ec[c + d*x]*Tan[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A - 11*B
+ 3*C)*Sec[c + d*x]*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((3
1*A - 15*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d
*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \int \frac{\left(2a(3A-B+C)-\frac{1}{2}a(7A\right.}{(a+a}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B+3C)s}{16ad(a+a}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B+3C)s}{16ad(a+a}$$

$$= -\frac{(63A-35B+11C)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec(c+dx)}{4d(a+a\cos(c+}$$

$$= -\frac{(63A-35B+11C)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec(c+dx)}{4d(a+a\cos(c+}$$

$$= -\frac{(63A-35B+11C)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\sec(c+dx)}{4d(a+a\cos(c+}$$

$$= \frac{(39A-20B+8C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B}{}$$

Mathematica [A] time = 5.2668, size = 248, normalized size = 0.89

$$\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-\frac{1}{8}\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)((269A-169B+33C)\cos(c+dx)+10(19A-11B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + a*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^6*Sec[c + d*x]^2*(-((219*A - 115*B + 43*C)*ArcTanh[Sin[(c
 + d*x)/2]]*Cos[(c + d*x)/2]^9*Cos[c + d*x]^2) + 4*Sqrt[2]*(39*A - 20*B + 8
*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^9*Cos[c + d*x]^2 - (
Cos[(c + d*x)/2]^5*(158*A - 110*B + 30*C + (269*A - 169*B + 33*C)*Cos[c + d
*x] + 10*(19*A - 11*B + 3*C)*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)] - 35*
B*Cos[3*(c + d*x)] + 11*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/8))/(8*a*d*(a
*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.343, size = 2366, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/8*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-128*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-
2^(1/2)))*cos(1/2*d*x+1/2*c)^8*a-128*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2
^(1/2)))*cos(1/2*d*x+1/2*c)^8*a+320*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a
^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^
(1/2)))*cos(1/2*d*x+1/2*c)^8*a+320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a))*cos(1/2*d*x+1/2*c)^8*a-320*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/
2)))*cos(1/2*d*x+1/2*c)^6*a-320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
))*cos(1/2*d*x+1/2*c)^6*a+80*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*
cos(1/2*d*x+1/2*c)^4*a+80*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*co
s(1/2*d*x+1/2*c)^4*a-32*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(
1/2*d*x+1/2*c)^4*a-32*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2
*d*x+1/2*c)^4*a+128*C*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d
*x+1/2*c)^6*a+128*C*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*
x+1/2*c)^6*a+624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+
1/2*c)^6*a+624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1
/2*c)^6*a-156*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a))*cos(1/2*d*x+1/2
*c)^4*a-156*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/
```

```
2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*
c)^4*a-2*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)-460*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a-140*B*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+460*B*2^(1/2)*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*
a+100*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4
+172*C*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*
d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a+876*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a+252*A*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-876*A*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1
/2)*cos(1/2*d*x+1/2*c)^6*a-172*C*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-188*A*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-115*B*2^(1/2)*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(
1/2*d*x+1/2*c)^4*a-11*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)^2+219*A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a+43*C*2^(1/2)*ln(2*(2*a^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/
2*c)^4*a+44*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/
2*c)^6-36*C*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*
c)^4+19*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)
^2+3*C*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2)
/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/
2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 4.9397, size = 1218, normalized size = 4.35

$$\frac{\sqrt{2}\Big((219\,A - 115\,B + 43\,C)\cos{(dx+c)}^{5} + 3\,(219\,A - 115\,B + 43\,C)\cos{(dx+c)}^{4} + 3\,(219\,A - 115\,B + 43\,C)\cos{(dx+c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((219*A - 115*B + 43*C)*cos(d*x + c)^5 + 3*(219*A - 115*B + 4
3*C)*cos(d*x + c)^4 + 3*(219*A - 115*B + 43*C)*cos(d*x + c)^3 + (219*A - 11
5*B + 43*C)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((39*A - 20*B + 8*C)*cos(d*x + c)^5 + 3
*(39*A - 20*B + 8*C)*cos(d*x + c)^4 + 3*(39*A - 20*B + 8*C)*cos(d*x + c)^3
+ (39*A - 20*B + 8*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d
*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((63*A - 35*B + 11*C)
*cos(d*x + c)^3 + 5*(19*A - 11*B + 3*C)*cos(d*x + c)^2 + 4*(5*A - 4*B)*cos(
d*x + c) - 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+a*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.92407, size = 876, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a
^5)*tan(1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(29*A*a^5 - 21*B*a^5 + 13*C*a^5)/a
^8)*tan(1/2*d*x + 1/2*c) - sqrt(2)*(219*A*sqrt(a) - 115*B*sqrt(a) + 43*C*sq
rt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2)/a^3 - 8*(39*A*sqrt(a) - 20*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2
) + 3)))/a^3 + 8*(39*A*sqrt(a) - 20*B*sqrt(a) + 8*C*sqrt(a))*log(abs((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt
(2) - 3)))/a^3 + 32*sqrt(2)*(41*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a) - 209*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 76*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(3/2) + 91*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/
2) - 36*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*B*a^(5/2) - 11*A*a^(7/2) + 4*B*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a^2))/d
```

3.425 $\int \cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=123

$$\frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A + 5*C)*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2
*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(7*d)
```

Rubi [A] time = 0.116629, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3023, 2748, 2635, 2641, 2639}

$$\frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A + 5*C)*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2
*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(7*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx &= \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(7A+\right.\\ &= \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+B\int \cos^{\frac{5}{2}}(c+dx)\,dx+\frac{1}{7}(7A\\ &= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2B\cos^{\frac{3}{2}}(c+dx)\sin}{5d}\\ &= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7A}{}\end{aligned}$$

Mathematica [A] time = 0.54562, size = 86, normalized size = 0.7

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(70A+42B\cos(c+dx)+15C\cos(2(c+dx))+65C)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+126BE}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2]
 + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x] + 15*C*Cos[2*(c + d*
x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 0.161, size = 342, normalized size = 2.8

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}\cos\left(1/2\,dx+c/2\right)+\left(-168\,B-360\,C\right)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.426 $\int \sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=93

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.100584, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3023, 2748, 2639, 2635, 2641}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int\sqrt{\cos(c+dx)}\left(\frac{1}{2}(5A+\right.$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+B\int\cos^{\frac{3}{2}}(c+dx)\,dx+\frac{1}{5}(5$$

$$= \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

$$= \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{c}}{}$$

Mathematica [A] time = 0.237066, size = 72, normalized size = 0.77

$$\frac{2\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(5B+3C\cos(c+dx))+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2]
 + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 0.21, size = 308, normalized size = 3.3

$$\frac{2}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(24\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}\cos\left(1/2\,dx+c/2\right)+\left(-20\,B-24\,C\right)\left(\sin\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.427 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=65

$$\frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A + C)*EllipticF[(c + d*x)/2, 2])
/(3*d) + (2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0865771, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3023, 2748, 2641, 2639}

$$\frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A + C)*EllipticF[(c + d*x)/2, 2])
/(3*d) + (2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}(3A+C)+\frac{3}{2}B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+B\int \sqrt{\cos(c+dx)}\,dx+\frac{1}{3}(3A+C)\int \frac{}{\sqrt{\cos}}$$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sqrt{\cos(c+dx)}\sin(c}{3d}$$

Mathematica [A] time = 0.125737, size = 57, normalized size = 0.88

$$\frac{2\left((3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3BE\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*B*EllipticE[(c + d*x)/2, 2] + (3*A + C)*EllipticF[(c + d*x)/2, 2] + C
*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 0.156, size = 274, normalized size = 4.2

$$-\frac{2}{3\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+3\,A\sqrt{\left(\sin\left(1/2\,dx+c/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

3.428 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/
d + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0881948, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3021, 2748, 2641, 2639}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/
d + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{B}{2}-\frac{1}{2}(A-C)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+(-A+C)\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.186119, size = 54, normalized size = 0.89

$$\frac{2\left((C-A)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*((-A + C)*EllipticE[(c + d*x)/2, 2] + B*EllipticF[(c + d*x)/2, 2] + (A*S
in[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 0.184, size = 194, normalized size = 3.2

$$-2\,\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)-2\,A\cos(1/2\,dx+c/2)\,(\sin(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

3.429 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(A + 3*C)*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.101688, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3021, 2748, 2636, 2639, 2641}

$$\frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(A + 3*C)*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3B}{2}+\frac{1}{2}(A+3C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\frac{1}{3}(A+3C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-B\int \sqrt{c\\ &= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{d}\end{aligned}$$

Mathematica [A] time = 0.522094, size = 69, normalized size = 0.79

$$\frac{\frac{2\sin(c+dx)(A+3B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}+2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2] + 2*(A + 3*C)*EllipticF[(c + d*x)/2, 2] + (
2*(A + 3*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] time = 0.376, size = 500, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

3.430 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos
[c + d*x]])
```

Rubi [A] time = 0.11977, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3021, 2748, 2636, 2641, 2639}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos
[c + d*x]])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\frac{5B}{2}+\frac{1}{2}(3A+5C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{5}(3A+5C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{1}{3}B\int \frac{}{\sqrt{c}}\\ &= -\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\cdots\end{aligned}$$

Mathematica [A] time = 0.453496, size = 112, normalized size = 0.91

$$\frac{-6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9A\sin(2(c+dx))+6A\tan(c+dx)+10B\sin(c+dx)+10B\cos^{\frac{3}{2}}(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c +
 d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 9*A*Sin[2*(c +
d*x)] + 15*C*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 0.509, size = 799, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^3*(36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2
*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

3.431 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=211

$$\frac{10a(11A+11B+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2a(9A+7(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(11A+11B+9C)\sin(c+dx)\cos^{\frac{5}{2}}(c}{77d}$$

```
[Out] (2*a*(9*A + 7*(B + C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(11*A + 11
*B + 9*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*a*(11*A + 11*B + 9*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*a*(9*A + 7*(B + C))*Cos[c + d*x
]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(11*A + 11*B + 9*C)*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(77*d) + (2*a*(B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
+ (2*a*C*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.287296, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3033, 3023, 2748, 2635, 2639, 2641}

$$\frac{10a(11A+11B+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2a(9A+7(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(11A+11B+9C)\sin(c+dx)\cos^{\frac{5}{2}}(c}{77d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*a*(9*A + 7*(B + C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(11*A + 11
*B + 9*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*a*(11*A + 11*B + 9*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*a*(9*A + 7*(B + C))*Cos[c + d*x
]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(11*A + 11*B + 9*C)*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(77*d) + (2*a*(B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
+ (2*a*C*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
```

```
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2aC\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{2}{11}\int \text{co}$$
$$= \frac{2a(B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2a}{}$$
$$= \frac{2a(B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2a}{}$$
$$= \frac{2a(9A+7(B+C))\cos^{\frac{3}{2}}(c+dx)\sin(c+}{45d}$$
$$= \frac{2a(9A+7(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10}{}$$
$$= \frac{2a(9A+7(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{}{}$$

Mathematica [C] time = 6.43207, size = 1344, normalized size = 6.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((9*A + 7*B
 + 7*C)*Cot[c])/(15*d) + ((506*A + 506*B + 435*C)*Cos[d*x]*Sin[c])/(1848*d)
 + ((18*A + 19*B + 19*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + ((44*A + 44*B + 57*
C)*Cos[3*d*x]*Sin[3*c])/(1232*d) + ((B + C)*Cos[4*d*x]*Sin[4*c])/(72*d) + (
C*Cos[5*d*x]*Sin[5*c])/(176*d) + ((506*A + 506*B + 435*C)*Cos[c]*Sin[d*x])/
(1848*d) + ((18*A + 19*B + 19*C)*Cos[2*c]*Sin[2*d*x])/(180*d) + ((44*A + 44
*B + 57*C)*Cos[3*c]*Sin[3*d*x])/(1232*d) + ((B + C)*Cos[4*c]*Sin[4*d*x])/(7
2*d) + (C*Cos[5*c]*Sin[5*d*x])/(176*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (
d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (15*C*(1 + Cos[c + d*
```

```
x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(77*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[
c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
- Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (7*B*(1 + Cos[c + d*x])
*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d) - (7*C*(1 + Cos[c + d*x])*Csc[c]*S
ec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]]))/(30*d))
```

Maple [B] time = 0.2, size = 543, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(20160*C*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B-62720*C)*sin(1/2*d*x+1/2
*c)^10*cos(1/2*d*x+1/2*c)+(7920*A+32560*B+81520*C)*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-17424*A-34672*B-57712*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(14784*A+19712*B+24332*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(
-4026*A-4488*B-4638*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+825*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
```

```
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+825*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+675*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(
d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{5}+(B+C)a\cos\left(dx+c\right)^{4}+(A+B)a\cos\left(dx+c\right)^{3}+Aa\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^5 + (B + C)*a*cos(d*x + c)^4 + (A + B)*a*cos(d*x
 + c)^3 + A*a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.432 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=177

$$\frac{2a(7A+5(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(9A+9B+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}$$

```
[Out] (2*a*(9*A + 9*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*A + 5*(B
 + C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*(B + C))*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(9*A + 9*B + 7*C)*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(45*d) + (2*a*(B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
+ (2*a*C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.267401, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7A+5(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(9A+9B+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*a*(9*A + 9*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*A + 5*(B
 + C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*(B + C))*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(9*A + 9*B + 7*C)*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(45*d) + (2*a*(B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
+ (2*a*C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \cos$$

$$= \frac{2a(B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2a}{}$$

$$= \frac{2a(B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2a}{}$$

$$= \frac{2a(7A+5(B+C))\sqrt{\cos(c+dx)}\sin(c+}{21d}$$

$$= \frac{2a(9A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a}{}$$

Mathematica [C] time = 6.35771, size = 1292, normalized size = 7.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((9*A + 9*B
 + 7*C)*Cot[c])/(15*d) + ((28*A + 23*B + 23*C)*Cos[d*x]*Sin[c])/(84*d) + ((
18*A + 18*B + 19*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + ((B + C)*Cos[3*d*x]*Sin[
3*c])/(28*d) + (C*Cos[4*d*x]*Sin[4*c])/(72*d) + ((28*A + 23*B + 23*C)*Cos[c
]*Sin[d*x])/(84*d) + ((18*A + 18*B + 19*C)*Cos[2*c]*Sin[2*d*x])/(180*d) + (
(B + C)*Cos[3*c]*Sin[3*d*x])/(28*d) + (C*Cos[4*c]*Sin[4*d*x])/(72*d)) - (A*
(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) -
(5*B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^
2]) - (5*C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + C
```

```
ot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*S
qrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
 + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Si
n[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
 - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (7*C*(
1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1
/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sq
rt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
 - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d))
```

Maple [B] time = 0.206, size = 512, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*C*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(720*B+2960*C)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-504*A-1584*B-3152*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(924*A+1344*B+1792*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-336
*A-366*B-408*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18
9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d
```

```
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^4+(B+C)a\cos\left(dx+c\right)^3+(A+B)a\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^4 + (B + C)*a*cos(d*x + c)^3 + (A + B)*a*cos(d*x
 + c)^2 + A*a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

3.433 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=144

$$\frac{2a(7A+7B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+7B+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

```
[Out] (2*a*(5*A + 3*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 7*B +
 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 7*B + 5*C)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d) + (2*a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.228459, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(7A+7B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+7B+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*a*(5*A + 3*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 7*B +
 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 7*B + 5*C)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d) + (2*a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int\sqrt{c}$$
$$=\frac{2a(B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a}{}$$
$$=\frac{2a(B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a}{}$$
$$=\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a}{}$$
$$=\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a}{}$$

Mathematica [C] time = 6.31434, size = 1240, normalized size = 8.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((5*A + 3*B
 + 3*C)*Cot[c])/(5*d) + ((28*A + 28*B + 23*C)*Cos[d*x]*Sin[c])/(84*d) + ((B
 + C)*Cos[2*d*x]*Sin[2*c])/(10*d) + (C*Cos[3*d*x]*Sin[3*c])/(28*d) + ((28*A
 + 28*B + 23*C)*Cos[c]*Sin[d*x])/(84*d) + ((B + C)*Cos[2*c]*Sin[2*d*x])/(10
*d) + (C*Cos[3*c]*Sin[3*d*x])/(28*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (5*C*(1 + Cos[c + d*x])*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (A*(1 + Cos[c + d*x])*C
sc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
```

```
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[
c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[
Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2]]))/(10*d) - (3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*
x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(10*d))
```

Maple [B] time = 0.183, size = 481, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*C*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-528*C)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(140*A+308*B+448*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-70*A-112*B-122*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
```

```
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^3+(B+C)a\cos\left(dx+c\right)^2+(A+B)a\cos\left(dx+c\right)+Aa\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.434 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{2a(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+5B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aC\sin}{}$$

```
[Out] (2*a*(5*A + 5*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + B + C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(B + C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.196872, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3033, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+5B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aC\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```

```
[Out] (2*a*(5*A + 5*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + B + C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(B + C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \frac{\frac{5aA}{2}+\frac{1}{2}a(5A+5\ldots}{\ldots}$$

$$= \frac{2a(B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2aC\cos^{\frac{3}{2}}(c+dx\ldots}{5d}$$

$$= \frac{2a(B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2aC\cos^{\frac{3}{2}}(c+dx\ldots}{5d}$$

$$= \frac{2a(5A+5B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+B+C)F\ldots}{3d}$$

Mathematica [C] time = 6.36202, size = 1186, normalized size = 11.08

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq rt[Cos[c + d*x]],x]

[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((5*A + 5*B + 3*C)*Cot[c])/(5*d) + ((B + C)*Cos[d*x]*Sin[c])/(3*d) + (C*Cos[2*d*x]*Sin [2*c])/(10*d) + ((B + C)*Cos[c]*Sin[d*x])/(3*d) + (C*Cos[2*c]*Sin[2*d*x])/(10*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]] *Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d *x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot [c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/ 4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyperge ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcT an[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]] *Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^ 2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{ -1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]* Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Ta n[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/S qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4 }, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S qrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt [Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d *x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c] *Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))

Maple [B] time = 0.191, size = 447, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(20*B+44*C)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*B-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+(B+C)a\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

3.435 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2a(3A+3B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (-2*a*(A - B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + 3*B + C)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) +
 (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.201303, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3031, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+3B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```



```
[Out] (-2*a*(A - B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + 3*B + C)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) +
 (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{-\frac{1}{2}a(A+B)+\frac{1}{2}a(A-B-C)}{\sqrt{\cos}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{4}{3}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - (a$$

$$= -\frac{2a(A-B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(3A+3B+C)I}{3d}$$

Mathematica [C] time = 6.44175, size = 1173, normalized size = 11.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((-2*A + B
+ C + B*Cos[2*c] + C*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (C*Cos[d*x]*Sin[c])/(
3*d) + (C*Cos[c]*Sin[d*x])/(3*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d) - (A
*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (
B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) -
(C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2])
 + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-
1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*T
an[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan
[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c
]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sq
rt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (B*(1 + C
os[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt
[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (C*(1 + Cos[c + d*x])
*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
```

```
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.207, size = 380, normalized size = 3.8

$$-\frac{2a}{3d}\left(4C\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+3A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*a*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

3.436 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{2a(A+3(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*a*(A + B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*(B + C))*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
 + (2*a*(A + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.218942, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3031, 3021, 2748, 2641, 2639}

$$\frac{2a(A+3(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```

```
[Out] (-2*a*(A + B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*(B + C))*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
 + (2*a*(A + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{1}{2}a(A+3(B+C}{\cos^{\frac{3}{2}}}$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{4}{3}\int \frac{-\frac{1}{4}a(}{}$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-(a(A+B-$$

$$= -\frac{2a(A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+3(B+C))F}{3d}$$

Mathematica [C] time = 6.46908, size = 1180, normalized size = 11.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((-2*A - 2*
B + C + C*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x
])/(3*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(
3*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Co
t[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + C
ot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 +
Cot[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sq
rt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
+ (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin
[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) +
 (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (C*(1 + Cos
[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
```

```
d*x + ArcTan[Tan[c]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.431, size = 515, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*A*(-1/6*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x
+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+\left(B+C\right)a\cos\left(dx+c\right)^{2}+\left(A+B\right)a\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

3.437 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{2a(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+5(B+C))\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a(A+B)\sin}{3d\cos^{\frac{3}{2}}(c}$$

```
[Out] (-2*a*(3*A + 5*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B + 3*
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)) + (2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A
+ 5*(B + C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.254223, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+5(B+C))\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a(A+B)\sin}{3d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B + 3*
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)) + (2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A
+ 5*(B + C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
```

```
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{1}{2}a(3A+5(B+\cdots}{\cos^{\frac{5}{2}}\cdots}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a\cdots}{\cdots}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a(A+B\cdots$$

$$= \frac{2a(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \cdots$$

$$= -\frac{2a(3A+5(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B+3C\cdots}{3\cdots}$$

Mathematica [C] time = 6.54121, size = 1228, normalized size = 8.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*B
+ 5*C)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (S
ec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (
Sec[c]*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 9*A*Sin[d*x] + 15*B*Sin[d*x]
 + 15*C*Sin[d*x]))/(15*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
```

```
Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2
+ (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[
c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]))/(10*d) + (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^
2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
+ Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[
1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(
Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(2*d) + (C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d)
)
```

Maple [B] time = 0.582, size = 739, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*B+1/2*C)*(-(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/10*A/
```

```
(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+(B+C)a\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

3.438 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{2a(5A+7(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(3A+3B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7(B+C))\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+3B}{5d\sqrt{}}$$

```
[Out] (-2*a*(3*A + 3*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*(B
 + C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Cos[c
+ d*x]^(7/2)) + (2*a*(A + B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*
(5*A + 7*(B + C))*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 3*B
 + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.270646, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(5A+7(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(3A+3B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7(B+C))\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+3B}{5d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```



```
[Out] (-2*a*(3*A + 3*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*(B
 + C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Cos[c
+ d*x]^(7/2)) + (2*a*(A + B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*
(5*A + 7*(B + C))*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 3*B
 + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
```

```
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B)-\frac{1}{2}a(5A+7(B+}{\cos^{\frac{7}{2}}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a}{}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(a(3A+$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(5A+7}{21d}$$

$$= -\frac{2a(3A+3B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7(B+}{}$$

Mathematica [C] time = 6.61621, size = 1284, normalized size = 7.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 3*B
+ 5*C)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(7*d) + (S
ec[c]*Sec[c + d*x]^3*(5*A*Sin[c] + 7*A*Sin[d*x] + 7*B*Sin[d*x]))/(35*d) + (
Sec[c]*Sec[c + d*x]^2*(21*A*Sin[c] + 21*B*Sin[c] + 25*A*Sin[d*x] + 35*B*Sin
[d*x] + 35*C*Sin[d*x]))/(105*d) + (Sec[c]*Sec[c + d*x]*(25*A*Sin[c] + 35*B*
Sin[c] + 35*C*Sin[c] + 63*A*Sin[d*x] + 63*B*Sin[d*x] + 105*C*Sin[d*x]))/(10
5*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 +
 Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[
1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
```

```
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sq
rt[1 + Cot[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((H
ypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + T
an[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c
]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
)/(10*d) + (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sq
rt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
+ (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin
[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
+ (C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 0.817, size = 849, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(-1/5
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2
*B+1/2*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*C*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/5*(1/2*A+1/2*B)/
(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+(B+C)a\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

3.439 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=251

$$\frac{4a^2(66A+55B+50C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^2(9A+8B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(99A+121B+89C)\sin(c+dx)\cos}{693d}$$

```
[Out] (4*a^2*(9*A + 8*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(66*A +
 55*B + 50*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^2*(66*A + 55*B + 50
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^2*(9*A + 8*B + 7*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a^2*(99*A + 121*B + 89*C)*Cos[c +
 d*x]^(5/2)*Sin[c + d*x])/(693*d) + (2*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(11*d) + (2*(11*B + 4*C)*Cos[c + d*x]^(5/2)*(a^2 + a^
2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```

Rubi [A] time = 0.537206, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3045, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(66A+55B+50C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^2(9A+8B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(99A+121B+89C)\sin(c+dx)\cos}{693d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```



```
[Out] (4*a^2*(9*A + 8*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(66*A +
 55*B + 50*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^2*(66*A + 55*B + 50
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^2*(9*A + 8*B + 7*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a^2*(99*A + 121*B + 89*C)*Cos[c +
 d*x]^(5/2)*Sin[c + d*x])/(693*d) + (2*C*Cos[c + d*x]^(5/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(11*d) + (2*(11*B + 4*C)*Cos[c + d*x]^(5/2)*(a^2 + a^
2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c}{11d}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c}{11d}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c}{11d}$$

$$= \frac{2a^2(99A+121B+89C)\cos^{\frac{5}{2}}(c+dx)\sin}{693d}$$

$$= \frac{2a^2(99A+121B+89C)\cos^{\frac{5}{2}}(c+dx)\sin}{693d}$$

$$= \frac{4a^2(66A+55B+50C)\sqrt{\cos(c+dx)}\sin(}{231d}$$

$$= \frac{4a^2(9A+8B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a}{}$$

Mathematica [C] time = 6.40997, size = 1374, normalized size = 5.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((9*A + 8*
B + 7*C)*Cot[c])/(15*d) + ((1122*A + 1012*B + 941*C)*Cos[d*x]*Sin[c])/(3696
*d) + ((36*A + 37*B + 38*C)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((44*A + 88*B +
101*C)*Cos[3*d*x]*Sin[3*c])/(2464*d) + ((B + 2*C)*Cos[4*d*x]*Sin[4*c])/(144
*d) + (C*Cos[5*d*x]*Sin[5*c])/(352*d) + ((1122*A + 1012*B + 941*C)*Cos[c]*S
in[d*x])/(3696*d) + ((36*A + 37*B + 38*C)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((
44*A + 88*B + 101*C)*Cos[3*c]*Sin[3*d*x])/(2464*d) + ((B + 2*C)*Cos[4*c]*Si
n[4*d*x])/(144*d) + (C*Cos[5*c]*Sin[5*d*x])/(352*d)) - (2*A*(a + a*Cos[c +
d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a
*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (
50*C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*d*Sqrt[1 + Co
t[c]^2]) - (3*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10
*d) - (4*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqr
t[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
 (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[
c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d) -
 (7*C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d)
```

Maple [A] time = 0.189, size = 545, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*B-37520*C)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(3960*A+20240*B+57040*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-11484*A-26048*B-46192*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(12474*A+17248*B+22022*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-3861*A-4257*B-4563*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2079*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+990*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1848*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+825*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+750
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^5+(B+2C)a^2\cos(dx+c)^4+(A+2B+C)a^2\cos(dx+c)^3+(2A+B)a^2\cos(dx+c)^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^5 + (B + 2*C)*a^2*cos(d*x + c)^4 + (A + 2*B +
C)*a^2*cos(d*x + c)^3 + (2*A + B)*a^2*cos(d*x + c)^2 + A*a^2*cos(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.440 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=215

$$\frac{4a^2(7A+6B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(21A+27B+19C)\sin(c+dx)\cos^{\frac{3}{2}}(c}{105d}$$

```
[Out] (4*a^2*(12*A + 9*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A +
 6*B + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 6*B + 5*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a^2*(21*A + 27*B + 19*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*C*Cos[c + d*x]^(3/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(9*d) + (2*(9*B + 4*C)*Cos[c + d*x]^(3/2)*(a^2 + a^2*
Cos[c + d*x])*Sin[c + d*x])/(63*d)
```

Rubi [A] time = 0.504427, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(7A+6B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(21A+27B+19C)\sin(c+dx)\cos^{\frac{3}{2}}(c}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (4*a^2*(12*A + 9*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A +
 6*B + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 6*B + 5*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a^2*(21*A + 27*B + 19*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*C*Cos[c + d*x]^(3/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(9*d) + (2*(9*B + 4*C)*Cos[c + d*x]^(3/2)*(a^2 + a^2*
Cos[c + d*x])*Sin[c + d*x])/(63*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
```

```
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(}{9d}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(}{9d}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(}{9d}$$

$$= \frac{2a^2(21A+27B+19C)\cos^{\frac{3}{2}}(c+dx)\sin(}{105d}$$

$$= \frac{2a^2(21A+27B+19C)\cos^{\frac{3}{2}}(c+dx)\sin(}{105d}$$

$$= \frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + $$

$$= \frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + $$

Mathematica [C] time = 6.3569, size = 1322, normalized size = 6.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((12*A + 9
*B + 8*C)*Cot[c])/(15*d) + ((56*A + 51*B + 46*C)*Cos[d*x]*Sin[c])/(168*d) +
 ((18*A + 36*B + 37*C)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((B + 2*C)*Cos[3*d*x]
*Sin[3*c])/(56*d) + (C*Cos[4*d*x]*Sin[4*c])/(144*d) + ((56*A + 51*B + 46*C)
*Cos[c]*Sin[d*x])/(168*d) + ((18*A + 36*B + 37*C)*Cos[2*c]*Sin[2*d*x])/(360
*d) + ((B + 2*C)*Cos[3*c]*Sin[3*d*x])/(56*d) + (C*Cos[4*c]*Sin[4*d*x])/(144
*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
 Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d
*Sqrt[1 + Cot[c]^2]) - (5*C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2
 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*T
an[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]]))/(5*d) - (3*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*
x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(10*d) - (4*C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]
^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(15*d)
```

Maple [B] time = 0.188, size = 514, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(360*B+1840*C)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-252*A-1044*B-2368*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(672*A+1134*B+1568*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-27
3*A-351*B-387*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-252*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
89*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4+(B+2\,C)a^2\cos\left(dx+c\right)^3+(A+2\,B+C)a^2\cos\left(dx+c\right)^2+(2\,A+B)a^2\cos\left(dx+c\right)+Aa\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sq
rt(cos(d*x + c)), x)
```

3.441 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=179

$$\frac{4a^2(14A+7B+6C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+4B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+49B+33C)\sin(c+dx)\sqrt{\cos(c}}{105d}$$

```
[Out] (4*a^2*(5*A + 4*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(14*A +
7*B + 6*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 49*B + 33*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^2*Sin[c + d*x])/(7*d) + (2*(7*B + 4*C)*Sqrt[Cos[c + d*x]]*(a^2
 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.477012, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(14A+7B+6C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+4B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+49B+33C)\sin(c+dx)\sqrt{\cos(c}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```



```
[Out] (4*a^2*(5*A + 4*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(14*A +
7*B + 6*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 49*B + 33*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^2*Sin[c + d*x])/(7*d) + (2*(7*B + 4*C)*Sqrt[Cos[c + d*x]]*(a^2
 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2\int}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7}{}$$

$$= \frac{2a^2(35A+49B+33C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C}{}$$

$$= \frac{2a^2(35A+49B+33C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C}{}$$

$$= \frac{4a^2(5A+4B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(14A+7B}{}$$

Mathematica [C] time = 6.45446, size = 1270, normalized size = 7.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((5*A + 4*
B + 3*C)*Cot[c])/(5*d) + ((28*A + 56*B + 51*C)*Cos[d*x]*Sin[c])/(168*d) + (
(B + 2*C)*Cos[2*d*x]*Sin[2*c])/(20*d) + (C*Cos[3*d*x]*Sin[3*c])/(56*d) + ((
28*A + 56*B + 51*C)*Cos[c]*Sin[d*x])/(168*d) + ((B + 2*C)*Cos[2*c]*Sin[2*d*
x])/(20*d) + (C*Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*A*(a + a*Cos[c + d*x])^2*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
```

```
Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d
*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*C*(a + a*
Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (A*
(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (2*B*(a + a
*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) - (3*C*(a + a*Cos[c
 + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [B] time = 0.212, size = 483, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-84*B-348*C)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(70*A+224*B+378*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
```

```
)+(-35*A-91*B-117*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-84*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ca^{2}\cos\left(dx+c\right)^{4}+(B+2\,C)a^{2}\cos\left(dx+c\right)^{3}+(A+2\,B+C)a^{2}\cos\left(dx+c\right)^{2}+(2\,A+B)a^{2}\cos\left(dx+c\right)+Aa^{2}}{\sqrt{\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sq
rt(cos(d*x + c)), x)
```

3.442 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{4a^2(3A+2B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(15A-5B-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}$$

```
[Out] (4*a^2*(5*B + 4*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + 2*B + C
)*EllipticF[(c + d*x)/2, 2])/(3*d) - (2*a^2*(15*A - 5*B - 7*C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) - (2*(5*A - C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*
x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.479432, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(3A+2B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(15A-5B-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```


```
[Out] (4*a^2*(5*B + 4*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + 2*B + C
)*EllipticF[(c + d*x)/2, 2])/(3*d) - (2*a^2*(15*A - 5*B - 7*C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) - (2*(5*A - C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*
x])*Sin[c + d*x])/(5*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-C)\sqrt{\cos(}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-C)\sqrt{\cos(}}{}$$

$$= -\frac{2a^2(15A-5B-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(}{}$$

$$= -\frac{2a^2(15A-5B-7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A(}{}$$

$$= \frac{4a^2(5B+4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(3A+2B+C)F\left(\right.}{3d}$$

Mathematica [C] time = 6.54976, size = 1039, normalized size = 6.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-5*A + 1
0*B + 8*C + 5*A*Cos[2*c] + 10*B*Cos[2*c] + 8*C*Cos[2*c])*Csc[c]*Sec[c])/(20
*d) + ((B + 2*C)*Cos[d*x]*Sin[c])/(6*d) + (C*Cos[2*d*x]*Sin[2*c])/(20*d) +
((B + 2*C)*Cos[c]*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d)
+ (C*Cos[2*c]*Sin[2*d*x])/(20*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperg
```

```
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
+ (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(a + a*Cos[c + d*x])^2
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c +
d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (2*C*(a + a*Cos[c + d*x])^
2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [B] time = 0.233, size = 595, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^2*(-12*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*B+16*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A+5*B+13*C)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+10*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca^{2}\cos\left(dx+c\right)^{4}+\left(B+2\,C\right)a^{2}\cos\left(dx+c\right)^{3}+\left(A+2\,B+C\right)a^{2}\cos\left(dx+c\right)^{2}+\left(2\,A+B\right)a^{2}\cos\left(dx+c\right)+Aa^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(3/2), x)
```

3.443 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{4a^2(2A+3B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(5A+3B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(4A+3B)\sin(c+dx)\left(a^2\cos(c+\right.}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B + 2*C)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) - (2*a^2*(5*A + 3*B - C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c +
 d*x]^(3/2)) + (2*(4*A + 3*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]])
```

Rubi [A] time = 0.471599, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(2A+3B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(5A+3B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(4A+3B)\sin(c+dx)\left(a^2\cos(c+\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```


```
[Out] (-4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B + 2*C)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) - (2*a^2*(5*A + 3*B - C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c +
 d*x]^(3/2)) + (2*(4*A + 3*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\int\frac{(a+a\cos(c+dx))}{}}{}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(4A+3B)\left(a^2\right.}{3a}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(4A+3B)\left(a^2\right.}{3a}$$

$$=-\frac{2a^2(5A+3B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2A(a}{}$$

$$=-\frac{2a^2(5A+3B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2A(a}{}$$

$$=-\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(2A+3B+2C)F}{3d}$$

Mathematica [C] time = 6.62385, size = 1025, normalized size = 5.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-4*A - B
 + 2*C + B*Cos[2*c] + 2*C*Cos[2*c])*Csc[c]*Sec[c])/(4*d) + (C*Cos[d*x]*Sin[
c])/(6*d) + (C*Cos[c]*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/
```

```
(6*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(6*d
)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
 Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqr
t[1 + Cot[c]^2]) - (2*C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x
)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2
]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sq
rt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]))/(2*d) - (C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(
(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
 Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]))/(2*d)
```

Maple [B] time = 0.486, size = 800, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+4*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
```

```
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6*C*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
sin(1/2*d*x+1/2*c)^2+4*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-4*C*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+7*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+3*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^{2}\cos\left(dx+c\right)^{4}+\left(B+2\,C\right)a^{2}\cos\left(dx+c\right)^{3}+\left(A+2\,B+C\right)a^{2}\cos\left(dx+c\right)^{2}+\left(2\,A+B\right)a^{2}\cos\left(dx+c\right)+Aa^{2}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(5
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(5/2), x)
```

3.444 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{4a^2(A+2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(17A+25B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(4A+5B)\,s}{}$$

```
[Out] (-4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B + 3*
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(17*A + 25*B + 15*C)*Sin[c + d
*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/
(5*d*Cos[c + d*x]^(5/2)) + (2*(4*A + 5*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c +
d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.487078, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(A+2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(17A+25B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(4A+5B)\,s}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```



```
[Out] (-4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B + 3*
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(17*A + 25*B + 15*C)*Sin[c + d
*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/
(5*d*Cos[c + d*x]^(5/2)) + (2*(4*A + 5*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c +
d*x])/(15*d*Cos[c + d*x]^(3/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(4A+5B)\left(a^2\right.}{15}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(4A+5B)\left(a^2\right.}{15}$$

$$= \frac{2a^2(17A+25B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c}{5d\cos}$$

$$= \frac{2a^2(17A+25B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\cos(c}{5d\cos}$$

$$= -\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+2B+3C)F}{3d}$$

Mathematica [C] time = 6.72394, size = 1041, normalized size = 5.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((-16*A -
20*B - 5*C + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + (A*Sec[c]*Sec[c + d*x]^3
```

```
*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 10*A*Sin[d*x] + 5*
B*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(10*A*Sin[c] + 5*B*Sin[c] + 24*A
*Sin[d*x] + 30*B*Sin[d*x] + 15*C*Sin[d*x]))/(30*d)) - (A*(a + a*Cos[c + d*x
])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Co
s[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(a
 + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) +
(2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ
[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Co
s[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))
/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) + (B*(a
+ a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d)
```

Maple [B] time = 0.597, size = 906, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(1/2*A+1/4*B)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/4*A+1/2*B+1/4*C)*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/20*A/(8*sin(
1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*
d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-
8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^{2}\cos\left(dx+c\right)^{4}+\left(B+2\,C\right)a^{2}\cos\left(dx+c\right)^{3}+\left(A+2\,B+C\right)a^{2}\cos\left(dx+c\right)^{2}+\left(2\,A+B\right)a^{2}\cos\left(dx+c\right)+Aa^{2}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(7
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(7/2), x)
```

3.445 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=215

$$\frac{4a^2(6A+7B+14C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(}{}$$

```
[Out] (-4*a^2*(3*A + 4*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A +
7*B + 14*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 49*B + 35*C)
*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^2*(3*A + 4*B + 5*C)*Sin[c
+ d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d*Cos[c + d*x]^(7/2)) + (2*(4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c
+ d*x])/(35*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.524024, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^2(6A+7B+14C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```


```
[Out] (-4*a^2*(3*A + 4*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A +
7*B + 14*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 49*B + 35*C)
*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^2*(3*A + 4*B + 5*C)*Sin[c
+ d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d*Cos[c + d*x]^(7/2)) + (2*(4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c
+ d*x])/(35*d*Cos[c + d*x]^(5/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+d}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(4A+7B)\left(a\right.}{3}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(4A+7B)\left(a\right.}{3}$$

$$= \frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(}{7d\,\mathrm{co}}$$

$$= \frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(}{7d\,\mathrm{co}}$$

$$= \frac{4a^2(6A+7B+14C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^2(33A+4}{105}$$

$$= -\frac{4a^2(3A+4B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7}{}$$

Mathematica [C] time = 6.82619, size = 1310, normalized size = 6.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((3*A + 4*B
 + 5*C)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) +
(Sec[c]*Sec[c + d*x]^3*(5*A*Sin[c] + 14*A*Sin[d*x] + 7*B*Sin[d*x]))/(70*d)
+ (Sec[c]*Sec[c + d*x]^2*(42*A*Sin[c] + 21*B*Sin[c] + 60*A*Sin[d*x] + 70*B*
Sin[d*x] + 35*C*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(60*A*Sin[c] + 70
*B*Sin[c] + 35*C*Sin[c] + 126*A*Sin[d*x] + 168*B*Sin[d*x] + 210*C*Sin[d*x])
```

```
)/(210*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2
}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*
Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(3*d*Sqrt[1 + Cot[c]^2]) - (2*C*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (3*A*(a + a*Cos[c + d*x])^2*Csc[c]*Se
c[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTa
n[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]))/(10*d) + (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2
 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*T
an[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]]))/(5*d) + (C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)
/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(2*d)
```

Maple [B] time = 0.807, size = 932, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/4*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/
21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+(1/4*A+1/2*B+1/4*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/4*
B+1/2*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*
c)^2-1)-1/5*(1/2*A+1/4*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos(dx+c)^4 + (B+2C)a^2\cos(dx+c)^3 + (A+2B+C)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(9
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(9/2), x)
```

3.446 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=251

$$\frac{4a^2(5A+6B+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+6B+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(19$$

```
[Out] (-4*a^2*(8*A + 9*B + 12*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A
+ 6*B + 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(19*A + 27*B + 21*C
)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)) + (4*a^2*(5*A + 6*B + 7*C)*Sin[c
 + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (4*a^2*(8*A + 9*B + 12*C)*Sin[c + d*x]
)/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d*Cos[c + d*x]^(9/2)) + (2*(4*A + 9*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(63*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.551878, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3043, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(5A+6B+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+6B+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(19$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```


```
[Out] (-4*a^2*(8*A + 9*B + 12*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A
+ 6*B + 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(19*A + 27*B + 21*C
)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)) + (4*a^2*(5*A + 6*B + 7*C)*Sin[c
 + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (4*a^2*(8*A + 9*B + 12*C)*Sin[c + d*x]
)/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d*Cos[c + d*x]^(9/2)) + (2*(4*A + 9*B)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(63*d*Cos[c + d*x]^(7/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(4A+9B)\left(a^2\right.}{63}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(4A+9B)\left(a^2\right.}{63}$$

$$= \frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c}{9d\cos}$$

$$= \frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\cos(c}{9d\cos}$$

$$= \frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(5A+6B+}{21d\cos^{\frac{3}{2}}}$$

$$= -\frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(5A+6B}{}$$

Mathematica [C] time = 6.93691, size = 1364, normalized size = 5.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((8*A + 9*B
 + 12*C)*Csc[c]*Sec[c])/(15*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(18*d)
+ (Sec[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 18*A*Sin[d*x] + 9*B*Sin[d*x]))/(126*
d) + (Sec[c]*Sec[c + d*x]^3*(90*A*Sin[c] + 45*B*Sin[c] + 112*A*Sin[d*x] + 1
26*B*Sin[d*x] + 63*C*Sin[d*x]))/(630*d) + (Sec[c]*Sec[c + d*x]*(25*A*Sin[c]
 + 30*B*Sin[c] + 35*C*Sin[c] + 56*A*Sin[d*x] + 63*B*Sin[d*x] + 84*C*Sin[d*x
```

```
]))/(105*d) + (Sec[c]*Sec[c + d*x]^2*(112*A*Sin[c] + 126*B*Sin[c] + 63*C*Si
n[c] + 150*A*Sin[d*x] + 180*B*Sin[d*x] + 210*C*Sin[d*x]))/(630*d)) - (5*A*(
a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]
) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 +
Cot[c]^2]) - (C*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sq
rt[1 + Cot[c]^2]) + (4*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(15*d) + (3*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(10*d) + (2*C*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hyperge
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d
)
```

Maple [B] time = 1.128, size = 1181, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*((1/2*A+1/
4*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))+(1/4*B+1/2*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(-1/144
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-
1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1
/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+1
/4*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)-1/5*(1/4*A+1/2*B+1/4*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4
+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos(dx+c)^4 + (B+2C)a^2\cos(dx+c)^3 + (A+2B+C)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{11}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/co
s(d*x + c)^(11/2), x)
```

3.447 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=303

$$\frac{4a^3(121A+105B+95C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(221A+195B+175C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{20a^3(286A+273B+236C)}{9009}$$

```
[Out] (4*a^3*(221*A + 195*B + 175*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (4*a^3*
(121*A + 105*B + 95*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A +
 105*B + 95*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(221*A + 1
95*B + 175*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(585*d) + (20*a^3*(286*A + 2
73*B + 236*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(9009*d) + (2*C*Cos[c + d*x]
^(5/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(13*d) + (2*(13*B + 6*C)*Cos[c
+ d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(143*a*d) + (2*(143*A
 + 195*B + 145*C)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])
/(1287*d)
```

Rubi [A] time = 0.734794, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3045, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(121A+105B+95C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(221A+195B+175C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{20a^3(286A+273B+236C)}{9009}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (4*a^3*(221*A + 195*B + 175*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (4*a^3*
(121*A + 105*B + 95*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A +
 105*B + 95*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(221*A + 1
95*B + 175*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(585*d) + (20*a^3*(286*A + 2
73*B + 236*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(9009*d) + (2*C*Cos[c + d*x]
^(5/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(13*d) + (2*(13*B + 6*C)*Cos[c
+ d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(143*a*d) + (2*(143*A
 + 195*B + 145*C)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])
/(1287*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin}{13d}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin}{13d}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin}{13d}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin}{13d}$$

$$= \frac{20a^3(286A+273B+236C)\cos^{\frac{5}{2}}(c+d}{9009d}$$

$$= \frac{20a^3(286A+273B+236C)\cos^{\frac{5}{2}}(c+d}{9009d}$$

$$= \frac{4a^3(121A+105B+95C)\sqrt{\cos(c+dx)}}{231d}$$

$$= \frac{4a^3(221A+195B+175C)E\left(\frac{1}{2}(c+dx)\right.}{195d}$$

Mathematica [C] time = 6.4697, size = 1426, normalized size = 4.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((221*A +
195*B + 175*C)*Cot[c])/(390*d) + ((2134*A + 1953*B + 1811*C)*Cos[d*x]*Sin[c
])/(7392*d) + ((7592*A + 7800*B + 7825*C)*Cos[2*d*x]*Sin[2*c])/(74880*d) +
((132*A + 189*B + 215*C)*Cos[3*d*x]*Sin[3*c])/(4928*d) + ((13*A + 39*B + 59
*C)*Cos[4*d*x]*Sin[4*c])/(3744*d) + ((B + 3*C)*Cos[5*d*x]*Sin[5*c])/(704*d)
 + (C*Cos[6*d*x]*Sin[6*c])/(1664*d) + ((2134*A + 1953*B + 1811*C)*Cos[c]*Si
n[d*x])/(7392*d) + ((7592*A + 7800*B + 7825*C)*Cos[2*c]*Sin[2*d*x])/(74880*
d) + ((132*A + 189*B + 215*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + ((13*A + 39*B
 + 59*C)*Cos[4*c]*Sin[4*d*x])/(3744*d) + ((B + 3*C)*Cos[5*c]*Sin[5*d*x])/(7
04*d) + (C*Cos[6*c]*Sin[6*d*x])/(1664*d)) - (11*A*(a + a*Cos[c + d*x])^3*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c +
d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c]^2]) - (95*C*(a +
 a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(462*d*Sqrt[1 + Cot[c]^2])
- (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d) - (B
*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
```

```
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (35*C*(a +
 a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(156*d)
```

Maple [A] time = 0.275, size = 576, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(131040*B+1058400*C)*sin(1/2*
d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-80080*A-567840*B-2122400*C)*sin(1/2*d*x+
1/2*c)^10*cos(1/2*d*x+1/2*c)+(314600*A+1004640*B+2331040*C)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-487916*A-939120*B-1535860*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(386386*A+510510*B+633710*C)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-105534*A-114660*B-121230*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)-51051*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+23595*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-45045*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+20475*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-40425*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+18525*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^3 \cos\left(dx+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{6}+\left(B+3\,C\right)a^{3}\cos\left(dx+c\right)^{5}+\left(A+3\,B+3\,C\right)a^{3}\cos\left(dx+c\right)^{4}+\left(3\,A+3\,B+C\right)a^{3}\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^6 + (B + 3*C)*a^3*cos(d*x + c)^5 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^4 + (3*A + 3*B + C)*a^3*cos(d*x + c)^3 + (3*A + B)*a^
3*cos(d*x + c)^2 + A*a^3*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.448 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=267

$$\frac{4a^3(143A+121B+105C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(21A+17B+15C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(264A+253B+210C)\sin(c}{1155d}$$

```
[Out] (4*a^3*(21*A + 17*B + 15*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(143
*A + 121*B + 105*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(143*A + 12
1*B + 105*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(264*A + 253
*B + 210*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1155*d) + (2*C*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d) + (2*(11*B + 6*C)*Cos[c +
d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d) + (2*(99*A + 1
43*B + 105*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(69
3*d)
```

Rubi [A] time = 0.685037, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(143A+121B+105C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(21A+17B+15C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(264A+253B+210C)\sin(c}{1155d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```



```
[Out] (4*a^3*(21*A + 17*B + 15*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(143
*A + 121*B + 105*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(143*A + 12
1*B + 105*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(264*A + 253
*B + 210*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1155*d) + (2*C*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d) + (2*(11*B + 6*C)*Cos[c +
d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d) + (2*(99*A + 1
43*B + 105*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(69
3*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$= \frac{4a^3(264A+253B+210C)\cos^{\frac{3}{2}}(c+dx)}{1155d}$$

$$= \frac{4a^3(264A+253B+210C)\cos^{\frac{3}{2}}(c+dx)}{1155d}$$

$$= \frac{4a^3(21A+17B+15C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

$$= \frac{4a^3(21A+17B+15C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Mathematica [C] time = 6.41759, size = 1374, normalized size = 5.15

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((21*A + 17*B + 15*C)*Cot[c])/(30*d) + ((2354*A + 2134*B + 1953*C)*Cos[d*x]*Sin[c])/(7392*d) + ((54*A + 73*B + 75*C)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((44*A + 132*B + 189*C)*Cos[3*d*x]*Sin[3*c])/(4928*d) + ((B + 3*C)*Cos[4*d*x]*Sin[4*c])/(288*d) + (C*Cos[5*d*x]*Sin[5*c])/(704*d) + ((2354*A + 2134*B + 1953*C)*Cos[c]*Sin[d*x])/(7392*d) + ((54*A + 73*B + 75*C)*Cos[2*c]*Sin[2*d*x])/(720*d) + ((44*A + 132*B + 189*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + ((B + 3*C)*Cos[4*c]*Sin[4*d*x])/(288*d) + (C*Cos[5*c]*Sin[5*d*x])/(704*d)) - (13*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (11*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (5*C*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c]^2]) - (7*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (17*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d) - (C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]

```
 + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Si
n[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [A] time = 0.232, size = 545, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*B-43680*C)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(3960*A+24200*B+77280*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-14256*A-37532*B-72240*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(19866*A+29722*B+39270*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-6864*A-8118*B-8820*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2145*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-4851*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1815*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3927*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1575*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
465*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^3\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^5+(B+3\,C)a^3\cos\left(dx+c\right)^4+(A+3\,B+3\,C)a^3\cos\left(dx+c\right)^3+(3\,A+3\,B+C)a^3\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.449 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=231

$$\frac{4a^3(21A+13B+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(27A+21B+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(42A+41B+32C)\sin(c+dx)\sqrt{}}{105d}$$

```
[Out] (4*a^3*(27*A + 21*B + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21*
A + 13*B + 11*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B +
32*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a
 + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*(3*B + 2*C)*Sqrt[Cos[c + d*x]
]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d) + (2*(63*A + 99*B + 73*
C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d)
```

Rubi [A] time = 0.659689, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(21A+13B+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(27A+21B+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(42A+41B+32C)\sin(c+dx)\sqrt{}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```



```
[Out] (4*a^3*(27*A + 21*B + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21*
A + 13*B + 11*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B +
32*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Cos[c + d*x]]*(a
 + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*(3*B + 2*C)*Sqrt[Cos[c + d*x]
]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d) + (2*(63*A + 99*B + 73*
C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
```

```
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{2\int}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{2(3}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{2(3}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{2(3}{}$$

$$= \frac{4a^3(42A+41B+32C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C}{}$$

$$= \frac{4a^3(42A+41B+32C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C}{}$$

$$= \frac{4a^3(27A+21B+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(21A+}{}$$

Mathematica [C] time = 6.52194, size = 1322, normalized size = 5.72

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((27*A + 2
1*B + 17*C)*Cot[c])/(30*d) + ((84*A + 107*B + 97*C)*Cos[d*x]*Sin[c])/(336*d
) + ((18*A + 54*B + 73*C)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((B + 3*C)*Cos[3*d
```

```
*x]*Sin[3*c])/(112*d) + (C*Cos[4*d*x]*Sin[4*c])/(288*d) + ((84*A + 107*B +
97*C)*Cos[c]*Sin[d*x])/(336*d) + ((18*A + 54*B + 73*C)*Cos[2*c]*Sin[2*d*x])
/(720*d) + ((B + 3*C)*Cos[3*c]*Sin[3*d*x])/(112*d) + (C*Cos[4*c]*Sin[4*d*x]
)/(288*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sq
rt[1 + Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(42*d*Sqrt[1 + Cot[c]^2]) - (11*C*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2
]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
an[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (9*A*(a + a*Cos[c + d*x])^3*Csc[c
]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (7*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec
[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]))/(20*d) - (17*C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2
 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*T
an[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]]))/(60*d)
```

Maple [A] time = 0.319, size = 514, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(360*B+2200*C)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-252*A-1296*B-3412*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(882*A+1806*B+2702*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-37
8*A-624*B-738*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-567*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+195*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
441*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+165*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos\left(dx+c\right)^5+(B+3\,C)a^3\cos\left(dx+c\right)^4+(A+3\,B+3\,C)a^3\cos\left(dx+c\right)^3+(3\,A+3\,B+C)a^3\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sq
rt(cos(d*x + c)), x)
```

3.450 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=229

$$\frac{4a^3(35A+21B+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(5A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(35A-42B-41C)\sin(c+dx)\sqrt{\cos}}{105d}$$

```
[Out] (4*a^3*(5*A + 9*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A +
21*B + 13*C)*EllipticF[(c + d*x)/2, 2])/(21*d) - (4*a^3*(35*A - 42*B - 41*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*(7*A - C)*Sqrt[Cos[c + d*x]]*(a^2 +
 a^2*Cos[c + d*x])^2*Sin[c + d*x])/(7*a*d) - (2*(35*A - 7*B - 11*C)*Sqrt[Co
s[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.672339, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(35A+21B+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(5A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(35A-42B-41C)\sin(c+dx)\sqrt{\cos}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```


```
[Out] (4*a^3*(5*A + 9*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A +
21*B + 13*C)*EllipticF[(c + d*x)/2, 2])/(21*d) - (4*a^3*(35*A - 42*B - 41*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*(7*A - C)*Sqrt[Cos[c + d*x]]*(a^2 +
 a^2*Cos[c + d*x])^2*Sin[c + d*x])/(7*a*d) - (2*(35*A - 7*B - 11*C)*Sqrt[Co
s[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(7A-C)\sqrt{\cos(}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(7A-C)\sqrt{\cos(}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(7A-C)\sqrt{\cos(}}{}$$

$$= -\frac{4a^3(35A-42B-41C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2}{}$$

$$= -\frac{4a^3(35A-42B-41C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2}{}$$

$$= \frac{4a^3(5A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(35A+21B}{}$$

Mathematica [C] time = 6.66481, size = 1313, normalized size = 5.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((5*A + 18
*B + 14*C + 15*A*Cos[2*c] + 18*B*Cos[2*c] + 14*C*Cos[2*c])*Csc[c]*Sec[c])/(
40*d) + ((28*A + 84*B + 107*C)*Cos[d*x]*Sin[c])/(336*d) + ((B + 3*C)*Cos[2*
d*x]*Sin[2*c])/(40*d) + (C*Cos[3*d*x]*Sin[3*c])/(112*d) + ((28*A + 84*B + 1
07*C)*Cos[c]*Sin[d*x])/(336*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(4*d) + (
(B + 3*C)*Cos[2*c]*Sin[2*d*x])/(40*d) + (C*Cos[3*c]*Sin[3*d*x])/(112*d)) -
(5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[
c]^2]) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1
 + Cot[c]^2]) - (13*C*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
42*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)
/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(4*d) - (9*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(20*d) - (7*C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(20*d)
```

Maple [B] time = 0.339, size = 727, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/105*a^3*(120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(7*B+36*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+21*B+43*C)*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(70*A+63*B+104*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+175*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)-105*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+105*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-189*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+65*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5 + (B+3C)a^3\cos(dx+c)^4 + (A+3B+3C)a^3\cos(dx+c)^3 + (3A+3B+C)a^3\cos(dx+\right.$$
$$\overline{\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(3/2), x)
```

3.451 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=227

$$\frac{4a^3(5A+5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(5A-5B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(20A+5B-6C)\sin(c+dx)\sqrt{\cos(c+}}{15d}$$

```
[Out] (-4*a^3*(5*A - 5*B - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A +
5*B + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(20*A + 5*B - 6*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(2*A + B)*(a^2 + a^2*Cos[c + d*x])^2*Si
n[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - (2*(35*A + 15*B - 3*C)*Sqrt[Cos[c +
d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.661095, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3043, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(5A+5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(5A-5B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(20A+5B-6C)\sin(c+dx)\sqrt{\cos(c+}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```


```
[Out] (-4*a^3*(5*A - 5*B - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A +
5*B + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(20*A + 5*B - 6*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(2*A + B)*(a^2 + a^2*Cos[c + d*x])^2*Si
n[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - (2*(35*A + 15*B - 3*C)*Sqrt[Cos[c +
d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(2A+B)\left(a^2+\right.}{ac}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(2A+B)\left(a^2+\right.}{ac}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(2A+B)\left(a^2+\right.}{ac}$$

$$= -\frac{4a^3(20A+5B-6C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= -\frac{4a^3(20A+5B-6C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= -\frac{4a^3(5A-5B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(5A+5B}{}$$

Mathematica [C] time = 6.76402, size = 1297, normalized size = 5.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-25*A +
5*B + 18*C + 5*A*Cos[2*c] + 15*B*Cos[2*c] + 18*C*Cos[2*c])*Csc[c]*Sec[c])/(
40*d) + ((B + 3*C)*Cos[d*x]*Sin[c])/(12*d) + (C*Cos[2*d*x]*Sin[2*c])/(40*d)
 + ((B + 3*C)*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/
(12*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(12
*d) + (C*Cos[2*c]*Sin[2*d*x])/(40*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*
```

```
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^
3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) - (C*(a + a*Cos[c +
 d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (A*(a + a*
Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S
qrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
 ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (B*(a + a*Cos[c +
d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (9*C*(a + a*Cos[c + d*x])^
3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 0.688, size = 950, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-24*C*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+96*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+50*A*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2+30*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-90*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+50*B*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*sin(1/2*d*x+1/2*c)^2-30*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-5
0*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+30*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*s
in(1/2*d*x+1/2*c)^2-54*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-78*C
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+50*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-25*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+20*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+18*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5+(B+3C)a^3\cos(dx+c)^4+(A+3B+3C)a^3\cos(dx+c)^3+(3A+3B+C)a^3\cos(dx+\right.}{\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(5/2), x)
```

3.452 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{4a^3(3A+5(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(21A+20B+5C)\sin(c+dx)\sqrt{\cos(c+}}{15d}$$

```
[Out] (-4*a^3*(9*A + 5*B - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A +
5*(B + C))*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(21*A + 20*B + 5*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2)) + (2*(33*A + 35*B + 15*C)*(a^3 +
 a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.677917, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(3A+5(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(21A+20B+5C)\sin(c+dx)\sqrt{\cos(c+}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```



```
[Out] (-4*a^3*(9*A + 5*B - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A +
5*(B + C))*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(21*A + 20*B + 5*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2)) + (2*(33*A + 35*B + 15*C)*(a^3 +
 a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6A+5B)\left(a^2\right.}{15}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6A+5B)\left(a^2\right.}{15}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6A+5B)\left(a^2\right.}{15}$$

$$= -\frac{4a^3(21A+20B+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= -\frac{4a^3(21A+20B+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= -\frac{4a^3(9A+5B-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(3A+5(B}{}$$

Mathematica [C] time = 6.80837, size = 1298, normalized size = 5.64

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-36*A -
25*B + 5*C + 5*B*Cos[2*c] + 15*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (C*Cos[d
*x]*Sin[c])/(12*d) + (C*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^3*
Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 15*A*Sin[d*x] + 5*B
*Sin[d*x]))/(60*d) + (Sec[c]*Sec[c + d*x]*(15*A*Sin[c] + 5*B*Sin[c] + 54*A*
Sin[d*x] + 45*B*Sin[d*x] + 15*C*Sin[d*x]))/(60*d)) - (A*(a + a*Cos[c + d*x]
)^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos
[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) - (5*C*(
a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2])
 + (9*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (B
*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (C*(a + a*
Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S
qrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
 ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] time = 0.893, size = 1328, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-120*C*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+90*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-20*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+60*B*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^4+100*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-6
0*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-60*C*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^4+60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
4+108*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+100*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^4-100*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1
/2*c)^2-60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*A*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-100*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^2+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+190*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^
```

```
8*cos(1/2*d*x+1/2*c)-180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^{3}\cos\left(dx+c\right)^{5}+(B+3\,C)a^{3}\cos\left(dx+c\right)^{4}+(A+3\,B+3\,C)a^{3}\cos\left(dx+c\right)^{3}+(3\,A+3\,B+C)a^{3}\cos\left(dx+\right.}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(7/2), x)
```

3.453 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=231

$$\frac{4a^3(13A+21B+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7A+9B+5C)\sin(c+dx)\left(a^3\cos(c\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4*a^3*(7*A + 9*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A +
 21*B + 35*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(106*A + 147*B + 1
40*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^
3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(6*A + 7*B)*(a^2 + a^2*Cos[c
+ d*x])^2*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(7*A + 9*B + 5*C)*
(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.668971, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(13A+21B+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7A+9B+5C)\sin(c+dx)\left(a^3\cos(c\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```


```
[Out] (-4*a^3*(7*A + 9*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A +
 21*B + 35*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(106*A + 147*B + 1
40*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^
3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(6*A + 7*B)*(a^2 + a^2*Cos[c
+ d*x])^2*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(7*A + 9*B + 5*C)*
(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+d}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(6A+7B)\left(a\right.}{3}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(6A+7B)\left(a\right.}{3}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(6A+7B)\left(a\right.}{3}$$

$$= \frac{4a^3(106A+147B+140C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\,c}{7}$$

$$= \frac{4a^3(106A+147B+140C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}} + \frac{2A(a+a\,c}{7}$$

$$= -\frac{4a^3(7A+9B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(13A+2}{}$$

Mathematica [C] time = 6.89867, size = 1317, normalized size = 5.7

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]

[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-28*A -
36*B - 25*C + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (A*Sec[c]*Sec[c + d*x]^
4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c + d*x]^3*(5*A*Sin[c] + 21*A*Sin[d*x] + 7
*B*Sin[d*x]))/(140*d) + (Sec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 21*B*Sin[c] +
130*A*Sin[d*x] + 105*B*Sin[d*x] + 35*C*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c
+ d*x]*(130*A*Sin[c] + 105*B*Sin[c] + 35*C*Sin[c] + 294*A*Sin[d*x] + 378*B*
Sin[d*x] + 315*C*Sin[d*x]))/(420*d)) - (13*A*(a + a*Cos[c + d*x])^3*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^3
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*C*(a + a*Cos[c
+ d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (7*A*(a +
a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (9*B*(a + a*Co
s[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (C*(a + a*Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)

Maple [B] time = 0.933, size = 1097, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/8*A*(-1/56*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A
+3/8*B+1/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*A+3/8*B+3/8*C)
*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-
1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca^{3}\cos\left(dx+c\right)^{5}+(B+3\,C)a^{3}\cos\left(dx+c\right)^{4}+(A+3\,B+3\,C)a^{3}\cos\left(dx+c\right)^{3}+(3\,A+3\,B+C)a^{3}\cos\left(dx+c\right.}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(9/2), x)
```

3.454 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=267

$$\frac{4a^3(11A+13B+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (-4*a^3*(17*A + 21*B + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11
*A + 13*B + 21*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(32*A + 41*B +
 42*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(17*A + 21*B + 27*
C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(2*A + 3*B)*(a^2 + a^2*Cos[c + d*
x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*x]^(7/2)) + (2*(73*A + 99*B + 63*C)*(
a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.70224, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(11A+13B+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```


```
[Out] (-4*a^3*(17*A + 21*B + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11
*A + 13*B + 21*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(32*A + 41*B +
 42*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(17*A + 21*B + 27*
C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(2*A + 3*B)*(a^2 + a^2*Cos[c + d*
x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*x]^(7/2)) + (2*(73*A + 99*B + 63*C)*(
a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+d}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(2A+3B)\left(a\right.}{2}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(2A+3B)\left(a\right.}{2}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(2A+3B)\left(a\right.}{2}$$

$$= \frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(}{9d\,\mathrm{co}}$$

$$= \frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\cos(}{9d\,\mathrm{co}}$$

$$= \frac{4a^3(11A+13B+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^3(32A+}{10}$$

$$= -\frac{4a^3(17A+21B+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(11A}{}$$

Mathematica [C] time = 6.95827, size = 1364, normalized size = 5.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((17*A + 21
*B + 27*C)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d
```

) + (Sec[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 27*A*Sin[d*x] + 9*B*Sin[d*x]))/(25
2*d) + (Sec[c]*Sec[c + d*x]^3*(135*A*Sin[c] + 45*B*Sin[c] + 238*A*Sin[d*x]
+ 189*B*Sin[d*x] + 63*C*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]^2*(238*A
*Sin[c] + 189*B*Sin[c] + 63*C*Sin[c] + 330*A*Sin[d*x] + 390*B*Sin[d*x] + 31
5*C*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]*(110*A*Sin[c] + 130*B*Sin[c]
 + 105*C*Sin[c] + 238*A*Sin[d*x] + 294*B*Sin[d*x] + 378*C*Sin[d*x]))/(420*d
)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1
 + Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
42*d*Sqrt[1 + Cot[c]^2]) - (C*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c
/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]))/(60*d) + (7*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
(d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]
]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(20*d) + (9*C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)
/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(20*d)

Maple [B] time = 1.089, size = 1262, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+(3/8*A+1/8*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+(1/8*A+3/8*B+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+1/8*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2
+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))))+(1/8*B+3/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/5*(3/8*A+3/8*B+1/8*C)/(8*sin(1/2*d*x+1/2*c)^
6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos\left(dx+c\right)^5+(B+3\,C)a^3\cos\left(dx+c\right)^4+(A+3\,B+3\,C)a^3\cos\left(dx+c\right)^3+(3\,A+3\,B+C)a^3\cos\left(dx+c\right.}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(11/2), x)
```

3.455 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{4a^3(105A+121B+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(105A+121B+143C)\sin(c\cdot}{231d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4*a^3*(15*A + 17*B + 21*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(10
5*A + 121*B + 143*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(210*A + 2
53*B + 264*C)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(5/2)) + (4*a^3*(105*A + 1
21*B + 143*C)*Sin[c + d*x])/(231*d*Cos[c + d*x]^(3/2)) + (4*a^3*(15*A + 17*
B + 21*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x
])^3*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + (2*(6*A + 11*B)*(a^2 + a^2*
Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d*Cos[c + d*x]^(9/2)) + (2*(105*A + 143
*B + 99*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)
)
```

Rubi [A] time = 0.748762, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3043, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^3(105A+121B+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(105A+121B+143C)\sin(c\cdot}{231d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(13/2),x]
```


```
[Out] (-4*a^3*(15*A + 17*B + 21*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(10
5*A + 121*B + 143*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(210*A + 2
53*B + 264*C)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(5/2)) + (4*a^3*(105*A + 1
21*B + 143*C)*Sin[c + d*x])/(231*d*Cos[c + d*x]^(3/2)) + (4*a^3*(15*A + 17*
B + 21*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x
])^3*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + (2*(6*A + 11*B)*(a^2 + a^2*
Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d*Cos[c + d*x]^(9/2)) + (2*(105*A + 143
*B + 99*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)
)
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+d}{}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2(6A+11B)\left(\right.}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2(6A+11B)\left(\right.}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2(6A+11B)\left(\right.}{}$$

$$= \frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\,c}{11}$$

$$= \frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\,c}{11}$$

$$= \frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^3(105A}{2}$$

$$= -\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(105}{}$$

Mathematica [C] time = 7.11775, size = 1418, normalized size = 4.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((15*A + 17
*B + 21*C)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^6*Sin[d*x])/(44*d
```

```
) + (Sec[c]*Sec[c + d*x]^5*(9*A*Sin[c] + 33*A*Sin[d*x] + 11*B*Sin[d*x]))/(3
96*d) + (Sec[c]*Sec[c + d*x]^4*(231*A*Sin[c] + 77*B*Sin[c] + 378*A*Sin[d*x]
 + 297*B*Sin[d*x] + 99*C*Sin[d*x]))/(2772*d) + (Sec[c]*Sec[c + d*x]*(525*A*
Sin[c] + 605*B*Sin[c] + 715*C*Sin[c] + 1155*A*Sin[d*x] + 1309*B*Sin[d*x] +
1617*C*Sin[d*x]))/(2310*d) + (Sec[c]*Sec[c + d*x]^3*(1890*A*Sin[c] + 1485*B
*Sin[c] + 495*C*Sin[c] + 2310*A*Sin[d*x] + 2618*B*Sin[d*x] + 2079*C*Sin[d*x
]))/(13860*d) + (Sec[c]*Sec[c + d*x]^2*(2310*A*Sin[c] + 2618*B*Sin[c] + 207
9*C*Sin[c] + 3150*A*Sin[d*x] + 3630*B*Sin[d*x] + 4290*C*Sin[d*x]))/(13860*d
)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1
+ Cot[c]^2]) - (11*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(4
2*d*Sqrt[1 + Cot[c]^2]) - (13*C*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c
/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]))/(4*d) + (17*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
(d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]
]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(60*d) + (7*C*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)
/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(20*d)
```

Maple [B] time = 1.316, size = 1424, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*((3/8*A+3
/8*B+1/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+(1/8*B+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A
+1/8*B)*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3
-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))+1/8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1
/2*d*x+1/2*c)^2-1)-1/5*(1/8*A+3/8*B+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1
/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*
A*(-1/352*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/15
4*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
```

```
-1/2+cos(1/2*d*x+1/2*c)^2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^{3}\cos\left(dx+c\right)^{5}+\left(B+3\,C\right)a^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,B+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+3\,B+C\right)a^{3}\cos\left(dx+c\right.}{\cos\left(dx+c\right)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/co
s(d*x + c)^(13/2), x)
```

3.456 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{5(7A-7B+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{3(5A-7B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A-7$$

```
[Out] (-3*(5*A - 7*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(7*A - 7*B +
9*C)*EllipticF[(c + d*x)/2, 2])/(21*a*d) + (5*(7*A - 7*B + 9*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*a*d) - ((5*A - 7*B + 7*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(5*a*d) + ((7*A - 7*B + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*
a*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
))
```

Rubi [A] time = 0.252536, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2748, 2635, 2639, 2641}

$$\frac{5(7A-7B+9C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{3(5A-7B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A-7$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x]),x]
```


```
[Out] (-3*(5*A - 7*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(7*A - 7*B +
9*C)*EllipticF[(c + d*x)/2, 2])/(21*a*d) + (5*(7*A - 7*B + 9*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*a*d) - ((5*A - 7*B + 7*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(5*a*d) + ((7*A - 7*B + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*
a*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}dx = -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A-7B+7C)\int c}{2a}$$

$$= -\frac{(5A-7B+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{(7A-7B+9C}{}$$

$$= -\frac{3(5A-7B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(7A-7B+9C)\sqrt{\cos}}{21ad}$$

$$= -\frac{3(5A-7B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(7A-7B+9C)F\left(\frac{1}{2}(\right.}{21ad}$$

<u>Mathematica</u> [C] time = 6.85507, size = 1752, normalized size = 8.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
(((21*I)/20)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
```

```
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) -
 (((21*I)/20)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])
+ (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(5*A - 5*B + 5*C + 10*A*Cos[
c] - 16*B*Cos[c] + 16*C*Cos[c])*Csc[c])/(5*d) + ((28*A - 28*B + 51*C)*Cos[d
*x]*Sin[c])/(21*d) + (2*(B - C)*Cos[2*d*x]*Sin[2*c])/(5*d) + (C*Cos[3*d*x]*
Sin[3*c])/(7*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d
*x)/2] + C*Sin[(d*x)/2]))/d + ((28*A - 28*B + 51*C)*Cos[c]*Sin[d*x])/(21*d)
 + (2*(B - C)*Cos[2*c]*Sin[2*d*x])/(5*d) + (C*Cos[3*c]*Sin[3*d*x])/(7*d)))/
(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a
 + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (15*C*Cos[c/2 +
(d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.313, size = 341, normalized size = 1.6

$$-\frac{1}{105\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(175*
```

```
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-175*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+225*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*C*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-480*C*sin(1/2*d*x+1/2*c)^10+(336*B+864*
C)*sin(1/2*d*x+1/2*c)^8+(-280*A-392*B-888*C)*sin(1/2*d*x+1/2*c)^6+(630*A-21
0*B+930*C)*sin(1/2*d*x+1/2*c)^4+(-245*A+161*B-321*C)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^4+B\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(
d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos
(d*x + c) + a), x)
```

3.457 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=174

$$-\frac{(3A-5B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(5A-5B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A-5}{}$$

```
[Out] (3*(5*A - 5*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A - 5*B + 5*C
)*EllipticF[(c + d*x)/2, 2])/(3*a*d) - ((3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d) + ((5*A - 5*B + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*a*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c
+ d*x]))
```

Rubi [A] time = 0.233607, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2748, 2635, 2641, 2639}

$$-\frac{(3A-5B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(5A-5B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A-5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x]),x]
```


```
[Out] (3*(5*A - 5*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A - 5*B + 5*C
)*EllipticF[(c + d*x)/2, 2])/(3*a*d) - ((3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d) + ((5*A - 5*B + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*a*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c
+ d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \cos^{\frac{3}{2}}(c+dx)\left(-\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-5B+5C)\int}{2}$$

$$= -\frac{(3A-5B+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{(5A-5B+7}{}$$

$$= \frac{3(5A-5B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(3A-5B+5C)F\left(\frac{1}{2}(c\right.}{3ad}$$

Mathematica [C] time = 6.76384, size = 1697, normalized size = 9.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + ((
(21*I)/20)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (
Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(5*A - 5*B + 5*C + 10*A*Cos[c]
 - 10*B*Cos[c] + 16*C*Cos[c])*Csc[c])/(5*d) + (4*(B - C)*Cos[d*x]*Sin[c])/(
3*d) + (2*C*Cos[2*d*x]*Sin[2*c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*(B - C)*Cos[c]*Sin[
d*x])/(3*d) + (2*C*Cos[2*c]*Sin[2*d*x])/(5*d)))/(a + a*Cos[c + d*x]) + (A*C
os[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
```

```
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot
[c]^2]) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]) + (5*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3
*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.261, size = 319, normalized size = 1.8

$$\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.(15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))-48*C*sin(1/2*d*x+1/2*c)^8+(40*B+56*C)*sin(1/2*d
*x+1/2*c)^6+(30*A-90*B+30*C)*sin(1/2*d*x+1/2*c)^4+(-15*A+35*B-23*C)*sin(1/2
*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*
x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a), x)
```

3.458 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{(3A-3B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(3A-3B+$$

```
[Out] -(((A - 3*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((3*A - 3*B + 5*C)*E
llipticF[(c + d*x)/2, 2])/(3*a*d) + ((3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*a*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rubi [A] time = 0.20333, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2748, 2639, 2635, 2641}

$$\frac{(3A-3B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(3A-3B+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x]),x]
```

```
[Out] -(((A - 3*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((3*A - 3*B + 5*C)*E
llipticF[(c + d*x)/2, 2])/(3*a*d) + ((3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*a*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int\sqrt{\cos(c+dx)}\Big(}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-3B+3C)\int}{2}$$

$$= -\frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-3B+5C)\sqrt{\cos(c}}{3ad}$$

$$= -\frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-3B+5C)F\left(\frac{1}{2}(c\right.}{3ad}$$

Mathematica [C] time = 6.60917, size = 1644, normalized size = 12.27

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] ((-I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (((3
*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (((3*
I)/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c
/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(-A + B - C + 2*B*Cos[c] - 2*C*Cos[
c])*Csc[c])/d + (4*C*Cos[d*x]*Sin[c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2
]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*C*Cos[c]*Sin[d
*x])/(3*d)))/(a + a*Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*C
sc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (5*C*Cos[c/
```

```
2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]
^2])
```

Maple [A] time = 0.272, size = 300, normalized size = 2.2

$$-\frac{1}{3ad}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-8*C*sin(1/2*d*x+1/2*c)^6+(6*A-6*B+18*C)*sin(1/2*d*x+1
/2*c)^4+(-3*A+3*B-7*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a), x)
```

3.459 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=90

$$\frac{(A+B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] ((A - B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A + B - C)*EllipticF[(c
 + d*x)/2, 2])/(a*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rubi [A] time = 0.183167, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {3041, 2748, 2641, 2639}

$$\frac{(A+B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])),x]
```

```
[Out] ((A - B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A + B - C)*EllipticF[(c
 + d*x)/2, 2])/(a*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx &= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int\frac{\frac{1}{2}a(A+B-C)+\frac{1}{2}a(A-B+3C)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}\\ &= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{(A+B-C)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{(A\ \ldots}{\ldots}\\ &= \frac{(A-B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\ldots}{d(a\ldots}\end{aligned}$$

Mathematica [C] time = 6.59649, size = 1607, normalized size = 17.86

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])),x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
```

```
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - ((I/4
)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (((3*I)/4)
*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c/2 +
(d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B + C + 2*C*Cos[c])*Csc[c])/d - (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/
2]))/d))/(a + a*Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
an[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (C*Cos[c/2 + (d
*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [A] time = 0.415, size = 281, normalized size = 3.1

$$\frac{1}{ad}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(A\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))+(2*A-2*B+2*C)*sin(1/2*d*x+1/2*c)^4+(-A+B-C)*sin(1/2*d*x+1/2*c)^2)/a
/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/cos(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

3.460 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=125

$$-\frac{(A-B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

```
[Out] -(((3*A - B + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - B - C)*EllipticF
[(c + d*x)/2, 2])/(a*d) + ((3*A - B + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*
x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
))
```

Rubi [A] time = 0.207155, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2748, 2636, 2639, 2641}

$$-\frac{(A-B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[
c + d*x])),x]
```



```
[Out] -(((3*A - B + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - B - C)*EllipticF
[(c + d*x)/2, 2])/(a*d) + ((3*A - B + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*
x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}a(3A-B+C)-\frac{1}{2}a(A-B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}-\frac{(A-B-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{(3A-$$

$$= -\frac{(A-B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B+}{d\sqrt{\cos(c+dx}$$

$$= -\frac{(3A-B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B}{ad\sqrt{}}$$

Mathematica [C] time = 6.77573, size = 1642, normalized size = 13.14

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - ((I/4
)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((2*A + A*Cos[c] - B*Cos[c] + C*Cos[c])*Csc
[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] -
 B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d
))/(a + a*Cos[c + d*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a
+ a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hy
```

```
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (C*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 0.485, size = 353, normalized size = 2.8

$$-\frac{1}{ad}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B+C)*sin(1/2*d*x+1/2*c)^4
+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B+C)*sin(1/2*d*x
+1/2*c)^2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

3.461 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=165

$$\frac{(5A-3B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B+3C}{3ad\cos^{\frac{3}{2}}(}$$

```
[Out] ((3*A - 3*B + C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B + 3*C)*Elli
pticF[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 3*B + 3*C)*Sin[c + d*x])/(3*a*d*Co
s[c + d*x]^(3/2)) - ((3*A - 3*B + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.228716, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2748, 2636, 2641, 2639}

$$\frac{(5A-3B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B+3C}{3ad\cos^{\frac{3}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[
c + d*x])),x]
```

```
[Out] ((3*A - 3*B + C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B + 3*C)*Elli
pticF[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 3*B + 3*C)*Sin[c + d*x])/(3*a*d*Co
s[c + d*x]^(3/2)) - ((3*A - 3*B + C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
```

```
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B+3C)-\frac{1}{2}a(3A-3B+C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(3A-3B+C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(5A}{}$$

$$= \frac{(5A-3B+3C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(3A-3B+C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B+C}{d\cos^{\frac{3}{2}}(c+dx)(}$$

$$= \frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-}{3}$$

Mathematica [C] time = 7.12781, size = 1686, normalized size = 10.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + 
a*Cos[c + d*x])),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + ((
```

```
I/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c/
2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(-(((2*A - 2*B + A*Cos[c] - B*Cos[c] + C*
Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Si
n[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]
^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 3*A*Sin[d*x] + 3*B*
Sin[d*x]))/(3*d)))/(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d
*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (
C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 +
Cot[c]^2])
```

Maple [B] time = 0.686, size = 494, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*A*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-2*A+2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
```

```
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(A-B+C)*(cos(1/2*d*x+1/2*c)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4 + a\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

3.462 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=210

$$-\frac{(5A-5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(7A-5B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(5A-5B+}{3ad\,\mathrm{cc}}$$

```
[Out] (-3*(7*A - 5*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A - 5*B + 3*
C)*EllipticF[(c + d*x)/2, 2])/(3*a*d) + ((7*A - 5*B + 5*C)*Sin[c + d*x])/(5
*a*d*Cos[c + d*x]^(5/2)) - ((5*A - 5*B + 3*C)*Sin[c + d*x])/(3*a*d*Cos[c +
d*x]^(3/2)) + (3*(7*A - 5*B + 5*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.24693, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2748, 2636, 2639, 2641}

$$-\frac{(5A-5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(7A-5B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(5A-5B+}{3ad\,\mathrm{cc}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Cos[
c + d*x])),x]
```


```
[Out] (-3*(7*A - 5*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A - 5*B + 3*
C)*EllipticF[(c + d*x)/2, 2])/(3*a*d) + ((7*A - 5*B + 5*C)*Sin[c + d*x])/(5
*a*d*Cos[c + d*x]^(5/2)) - ((5*A - 5*B + 3*C)*Sin[c + d*x])/(3*a*d*Cos[c +
d*x]^(3/2)) + (3*(7*A - 5*B + 5*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
```

```
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(7A-5B+5C)-\frac{1}{2}a(5A-5B+3C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,d}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(5A-5B+3C)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} + \frac{(7A}{}$$

$$= \frac{(7A-5B+5C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{(5A-5B+3C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(A-B+}{d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{(5A-5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(7A-5B+5C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{(5A-5B}{3ad}$$

$$= -\frac{3(7A-5B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(5A-5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(7A}{}$$

Mathematica [C] time = 7.47686, size = 1745, normalized size = 8.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
a*Cos[c + d*x])),x]
```

```
[Out] (((-21*I)/20)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])
 + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
```

```
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) -
 (((3*I)/4)*C*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((16*A - 10*B + 10*C + 5*A*Cos[c]
 - 5*B*Cos[c] + 5*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (2*Sec[c/2]*S
ec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (
4*A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]^2*(3*A*S
in[c] - 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) - (4*Sec[c]*Sec[c + d*x]*(5*A*
Sin[c] - 5*B*Sin[c] - 24*A*Sin[d*x] + 15*B*Sin[d*x] - 15*C*Sin[d*x]))/(15*d
)))/(a + a*Cos[c + d*x]) + (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[
c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (C*Cos[c/2 +
 (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 1.003, size = 812, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-2*A+2*B)*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(2*A-2*B+2*C)*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*A/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+(-A+B-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{5}+a\cos\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c) + a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos
(d*x + c)^(7/2)), x)
```

3.463 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=214

$$-\frac{5(A-2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(20A-35B+56C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A-2B+3C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{(20\cdots}{\cdots}$$

```
[Out] ((20*A - 35*B + 56*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(A - 2*B +
3*C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B + 3*C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(3*a^2*d) + ((20*A - 35*B + 56*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(15*a^2*d) - ((A - 2*B + 3*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.40253, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(A-2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(20A-35B+56C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A-2B+3C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{(20\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^2,x]
```

```
[Out] ((20*A - 35*B + 56*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(A - 2*B +
3*C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B + 3*C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(3*a^2*d) + ((20*A - 35*B + 56*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(15*a^2*d) - ((A - 2*B + 3*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}dx = -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(A-\right.}{}}{}$$

$$= -\frac{(A-2B+3C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos}{3d(a+a}$$

$$= -\frac{(A-2B+3C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos}{3d(a+a}$$

$$= -\frac{5(A-2B+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{(20A-35B+5}{}$$

$$= \frac{(20A-35B+56C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(A-2B+3C)F\left(\frac{1}{2}(c\right.}{3a^2d}$$

Mathematica [C] time = 7.06069, size = 1789, normalized size = 8.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^2,x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (((
7*I)/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
```

```
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
(28*I)/5)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 +
(10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*S
qrt[1 + Cot[c]^2]) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a
+ a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (10*C*Cos[c/2 + (d*x)/2]^4*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*
x)/2]^4*Sqrt[Cos[c + d*x]]*((-4*(10*A - 15*B + 20*C + 10*A*Cos[c] - 20*B*Co
s[c] + 36*C*Cos[c])*Csc[c])/(5*d) + (8*(B - 2*C)*Cos[d*x]*Sin[c])/(3*d) + (
4*C*Cos[2*d*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d
*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d
*x)/2]*(2*A*Sin[(d*x)/2] - 3*B*Sin[(d*x)/2] + 4*C*Sin[(d*x)/2]))/d + (8*(B
- 2*C)*Cos[c]*Sin[d*x])/(3*d) + (4*C*Cos[2*c]*Sin[2*d*x])/(5*d) + (2*(A - B
 + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A] time = 0.272, size = 491, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*Ellipt
```

```
icF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+75*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*C*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*cos(1/2*d*x+1/2*c)+96*C*sin(1/2*d*x+1/2*c)^10+(-80*B-128*C)*sin(1/2*d*
x+1/2*c)^8+(-120*A+380*B-328*C)*sin(1/2*d*x+1/2*c)^6+(170*A-420*B+526*C)*si
n(1/2*d*x+1/2*c)^4+(-55*A+125*B-171*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*
x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^4+B\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(
d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^2, x)
```

3.464 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=180

$$\frac{(2A-5B+10C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(2A-5$$

```
[Out] -(((A - 4*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A - 5*B + 10*C
)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B + 10*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d) - ((A - 4*B + 7*C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.375277, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(2A-5B+10C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(2A-5$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^2,x]
```


```
[Out] -(((A - 4*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A - 5*B + 10*C
)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B + 10*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d) - ((A - 4*B + 7*C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}dx = -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(A+5\right.}{a\cdot}}{}$$

$$= -\frac{(A-4B+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos}{3d(a+a}$$

$$= -\frac{(A-4B+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos}{3d(a+a}$$

$$= -\frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B+10C)\sqrt{\cos(c}}{3a^2d}$$

$$= -\frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B+10C)F\left(\frac{1}{2}(c+\right.}{3a^2d}$$

Mathematica [C] time = 6.89518, size = 1741, normalized size = 9.67

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^2,x]
```

```
[Out] ((-I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
```

```
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (((7*
I)/2)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*A
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*C
os[c + d*x])^2*Sqrt[1 + Cot[c]^2]) - (20*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/
2]^4*Sqrt[Cos[c + d*x]]*((-4*(-A + 2*B - 3*C + 2*B*Cos[c] - 4*C*Cos[c])*Csc
[c])/d + (8*C*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/d + (8
*C*Cos[c]*Sin[d*x])/(3*d) - (2*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(
3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.302, size = 472, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*
```

```
C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+21*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*C*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+21*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)+16*C*sin(1/2*d*x+1/2*c)^8+(-12*A+24*B-76*C)*sin(1/2*d*x+1/2*c
)^6+(16*A-34*B+84*C)*sin(1/2*d*x+1/2*c)^4+(-5*A+11*B-25*C)*sin(1/2*d*x+1/2*
c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*
x + c))/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^2, x)
```

3.465 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=139

$$\frac{(A+2B-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A+2B-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(B-4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B+C)\sin}{3d(a\cos}$$

```
[Out] -(((B - 4*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((A + 2*B - 5*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) + ((A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.34334, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2977, 2748, 2641, 2639}

$$\frac{(A+2B-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A+2B-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(B-4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B+C)\sin}{3d(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^2,x]
```


```
[Out] -(((B - 4*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((A + 2*B - 5*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) + ((A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A+B\right.}{a+}}{}$$

$$= \frac{(A+2B-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos^{\frac{3}{2}}}{3d(a+a\,c}$$

$$= \frac{(A+2B-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos^{\frac{3}{2}}}{3d(a+a\,c}$$

$$= -\frac{(B-4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}$$

Mathematica [C] time = 6.72864, size = 1347, normalized size = 9.69

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^2,x]
```

```
[Out] ((-I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
2*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2*A*
Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
```

```
- ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
+ Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos
[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (10*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]
^4*Sqrt[Cos[c + d*x]]*((-4*(-B + 2*C + 2*C*Cos[c])*Csc[c])/d + (4*Sec[c/2]*
Sec[c/2 + (d*x)/2]*(B*Sin[(d*x)/2] - 2*C*Sin[(d*x)/2]))/d + (2*Sec[c/2]*Sec
[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d)
 + (2*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x
])^2
```

Maple [B] time = 0.288, size = 507, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-24*C*cos(1/2*d*x+1/2*c)^6-10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^3-24*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*
cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4+38*C*cos(1/2*d*x+1/2*c)^4-3*
A*cos(1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2-15*C*cos(1/2*d*x+1/2*c)^2+A
-B+C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^2, x)
```

3.466 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{(2A+B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B+C)\sin(c+dx)}{3d(a\cos(c+dx)}$$

```
[Out] ((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A + B + 2*C)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) - ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d
*(1 + Cos[c + d*x])) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.347399, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2978, 2748, 2641, 2639}

$$\frac{(2A+B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B+C)\sin(c+dx)}{3d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^2),x]
```


```
[Out] ((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A + B + 2*C)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) - ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d
*(1 + Cos[c + d*x])) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(5A+B-C)-\frac{1}{2}a(A-B-5C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} d}{3a^2}$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(A-C)\sqrt{\cos(c}}{a^2d(1+}$$

Mathematica [C] time = 6.70715, size = 1342, normalized size = 10.09

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^2),x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - ((I
/2)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*
I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-
1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*A*C
os[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
```

```
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 +
 Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(3*d*(a + a*Cos[
c + d*x])^2*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4
*Sqrt[Cos[c + d*x]]*((-4*(A - C)*Csc[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]
*(A*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A
*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (2*(A - B + C)*Se
c[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.289, size = 507, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-12*C*cos(1/2*d*x+1/2*c)^6-4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(
1/2*d*x+1/2*c)^3-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*A*co
s(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+20*C*cos(1/2*d*x+1/2*c)^4+3*A*c
os(1/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2-9*C*cos(1/2*d*x+1/2*c)^2+A-B+C
)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{3}+2\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*s
qrt(cos(d*x + c))), x)
```

3.467 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=175

$$-\frac{(5A-2B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-2B-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -(((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - ((5*A - 2*B - C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) + ((4*A - B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c
+ d*x]]) - ((5*A - 2*B - C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 +
Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*
Cos[c + d*x])^2)
```

Rubi [A] time = 0.376358, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2978, 2748, 2636, 2639, 2641}

$$-\frac{(5A-2B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-2B-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[
c + d*x])^2),x]
```


```
[Out] -(((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - ((5*A - 2*B - C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) + ((4*A - B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c
+ d*x]]) - ((5*A - 2*B - C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 +
Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*
Cos[c + d*x])^2)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A-B+C)-\frac{3}{2}a(A-B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(5A-2B-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(5A-2B-C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(5A-2B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(5A-2B-C}{3a^2d\sqrt{\cos(c+dx}}$$

$$= -\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin}{a^2d\sqrt{\cos(c}}$$

Mathematica [C] time = 6.85917, size = 1380, normalized size = 7.89

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^2),x]
```

```
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
```

```
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (10*A
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^2*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]
^4*Sqrt[Cos[c + d*x]]*((2*(2*A + 2*A*Cos[c] - B*Cos[c])*Csc[c/2]*Sec[c/2]*S
ec[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - B*Sin[(d*x)/2
]))/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] +
 C*Sin[(d*x)/2]))/(3*d) + (8*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*(A - B
+ C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.749, size = 563, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(2*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B+C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B+C)*sin(1/2*d*x+1/2*c)^2)/sin(
1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x
+1/2*c)^2-1)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{4}+2\,a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*c
os(d*x + c)^(3/2)), x)
```

3.468 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{(10A-5B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{(10A-5B+}{3a^2d\,\mathrm{co}}$$

```
[Out] ((7*A - 4*B + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((10*A - 5*B + 2*C)*E
llipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((10*A - 5*B + 2*C)*Sin[c + d*x])/(3*
a^2*d*Cos[c + d*x]^(3/2)) - ((7*A - 4*B + C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[
c + d*x]]) - ((7*A - 4*B + C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
+ Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.402163, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2978, 2748, 2636, 2641, 2639}

$$\frac{(10A-5B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{(10A-5B+}{3a^2d\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[
c + d*x])^2),x]
```


```
[Out] ((7*A - 4*B + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((10*A - 5*B + 2*C)*E
llipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((10*A - 5*B + 2*C)*Sin[c + d*x])/(3*
a^2*d*Cos[c + d*x]^(3/2)) - ((7*A - 4*B + C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[
c + d*x]]) - ((7*A - 4*B + C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
+ Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^2)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
```

```
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A-B+C)-\frac{1}{2}a(5A-5B-C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= -\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= \frac{(10A-5B+2C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(7A-4B+C)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(7A-4B}{3a^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(10A-5B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(10A}{}$$

Mathematica [C] time = 7.43594, size = 1782, normalized size = 8.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Cos[c + d*x])^2),x]
```

```
[Out] (((7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 -
 ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
```

```
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
I/2)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (20*A
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*C
os[c + d*x])^2*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2
]^4*Sqrt[Cos[c + d*x]]*((-2*(4*A - 2*B + 3*A*Cos[c] - 2*B*Cos[c] + C*Cos[c]
)*Csc[c/2]*Sec[c/2]*Sec[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*
x)/2] - 2*B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (8*A*Sec[
c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 6*A*
Sin[d*x] + 3*B*Sin[d*x]))/(3*d) - (2*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c
/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [B] time = 0.905, size = 751, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(4*A*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(A-B+C)*(2*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4
-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+(-8*A+4*B)*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(4*A-2*B)*(cos(1/2
*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{5}+2\,a^{2}\cos\left(dx+c\right)^{4}+a^{2}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^5 + 2*a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*c
os(d*x + c)^(5/2)), x)
```

3.469 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=273

$$-\frac{(13A-33B+63C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B+33C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-33B+63C)\sin(c+dx)\cos^{\frac{5}{2}}(c+a}{10d\left(a^3\cos(c+dx)+a^3\right)}$$

```
[Out] (7*(7*A - 17*B + 33*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 33*
B + 63*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A - 33*B + 63*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(6*a^3*d) + (7*(7*A - 17*B + 33*C)*Cos[c + d*x
]^(3/2)*Sin[c + d*x])/(30*a^3*d) - ((A - B + C)*Cos[c + d*x]^(9/2)*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((2*A - 7*B + 12*C)*Cos[c + d*x]^(7/2)
*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 33*B + 63*C)*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.602229, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2977, 2748, 2635, 2641, 2639}

$$-\frac{(13A-33B+63C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B+33C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-33B+63C)\sin(c+dx)\cos^{\frac{5}{2}}(c+a}{10d\left(a^3\cos(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^3,x]
```


```
[Out] (7*(7*A - 17*B + 33*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 33*
B + 63*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A - 33*B + 63*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(6*a^3*d) + (7*(7*A - 17*B + 33*C)*Cos[c + d*x
]^(3/2)*Sin[c + d*x])/(30*a^3*d) - ((A - B + C)*Cos[c + d*x]^(9/2)*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((2*A - 7*B + 12*C)*Cos[c + d*x]^(7/2)
*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 33*B + 63*C)*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{1}{2}a(A+9\right.}{(a+}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B+12C)\,\mathrm{c}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B+12C)\,\mathrm{c}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B+12C)\,\mathrm{c}}{15ad(a+a}$$

$$= -\frac{(13A-33B+63C)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} + \frac{7(7A-17B}{}$$

$$= \frac{7(7A-17B+33C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B+63C)F\left(\right.}{6a^3d}$$

Mathematica [C] time = 7.36806, size = 1888, normalized size = 6.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (((119*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
```

```
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 + (((231*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*
d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/
E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3
*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(
1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c +
 d*x])^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c
+ d*x])^3*Sqrt[1 + Cot[c]^2]) - (22*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (42*C*Cos[c/2 + (d*x)/2]
^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[
c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((-4*(29*A - 59*B + 99*C + 20*A*Cos[c]
- 60*B*Cos[c] + 132*C*Cos[c])*Csc[c])/(5*d) + (16*(B - 3*C)*Cos[d*x]*Sin[c]
)/(3*d) + (8*C*Cos[2*d*x]*Sin[2*c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*S
ec[c/2 + (d*x)/2]^3*(14*A*Sin[(d*x)/2] - 19*B*Sin[(d*x)/2] + 24*C*Sin[(d*x)
/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] - 59*B*Sin
[(d*x)/2] + 99*C*Sin[(d*x)/2]))/(5*d) + (16*(B - 3*C)*Cos[c]*Sin[d*x])/(3*d
) + (8*C*Cos[2*c]*Sin[2*d*x])/(5*d) + (4*(14*A - 19*B + 24*C)*Sec[c/2 + (d*
x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5
*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.315, size = 666, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-192*C*cos(1/
2*d*x+1/2*c)^12-160*B*cos(1/2*d*x+1/2*c)^10+864*C*cos(1/2*d*x+1/2*c)^10+348
*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-468*B*cos(1/2*d*x+1/
2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-714*B*cos(1/2*d
*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+228*C*cos(1/2*d*x+1/2*c)^8+630*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+1386*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^
6-1590*C*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1/2*d*x+
1/2*c)^4+744*C*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*
d*x+1/2*c)^2-57*C*cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^
5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{5}+B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^5 + B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(cos(
d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
 a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos
(d*x + c) + a)^3, x)
```

3.470 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{(3A-13B+33C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B+119C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}$$

```
[Out] -((9*A - 49*B + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B
 + 33*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((3*A - 13*B + 33*C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[
c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((B - 2*C)*Cos[c + d*x]^(5/2)*Sin[
c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((9*A - 49*B + 119*C)*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.574788, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(3A-13B+33C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B+119C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^3,x]
```


```
[Out] -((9*A - 49*B + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B
 + 33*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((3*A - 13*B + 33*C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[
c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((B - 2*C)*Cos[c + d*x]^(5/2)*Sin[
c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((9*A - 49*B + 119*C)*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
```

```
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(3A+\right.}{(}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(B-2C)\cos^{\frac{5}{2}}(c+}{3ad(a+a\cos}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(B-2C)\cos^{\frac{5}{2}}(c+}{3ad(a+a\cos}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(B-2C)\cos^{\frac{5}{2}}(c+}{3ad(a+a\cos}$$

$$= -\frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B+33C)\sqrt{}}{6}$$

$$= -\frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B+33C)F(}{6a^3d}$$

Mathematica [C] time = 7.12713, size = 1841, normalized size = 7.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 + (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
```

```
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 - (((119*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c +
d*x])^3 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) - (22*C*Cos[c/2 + (d*x)/2]^6
*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/
2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((-4*(-9*A + 29*B - 59*C + 20*B*Cos[c] -
60*C*Cos[c])*Csc[c])/(5*d) + (16*C*Cos[d*x]*Sin[c])/(3*d) + (2*Sec[c/2]*Sec
[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d)
 - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2] - 14*B*Sin[(d*x)/2] +
 19*C*Sin[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)
/2] - 29*B*Sin[(d*x)/2] + 59*C*Sin[(d*x)/2]))/(5*d) + (16*C*Cos[c]*Sin[d*x]
)/(3*d) - (4*(9*A - 14*B + 19*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2
*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.314, size = 638, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*C*cos(1/
2*d*x+1/2*c)^10+108*A*cos(1/2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*c
os(1/2*d*x+1/2*c)^5+54*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348
*B*cos(1/2*d*x+1/2*c)^8-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*C*cos(1/2*d*x+1/
2*c)^8+330*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+714*C*cos(1/2*d
*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(
1/2*d*x+1/2*c)^6-1058*C*cos(1/2*d*x+1/2*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264
*B*cos(1/2*d*x+1/2*c)^4+474*C*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c)^
2+37*B*cos(1/2*d*x+1/2*c)^2-47*C*cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{4}+B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(
d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
 a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^3, x)
```

3.471 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{(A+3B-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+3B-13C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B}{}$$

```
[Out] -((A + 9*B - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B - 13*C
)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B - 8*C)*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((A + 3*B - 13*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.530205, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2977, 2748, 2641, 2639}

$$\frac{(A+3B-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+3B-13C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^3,x]
```


```
[Out] -((A + 9*B - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B - 13*C
)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B - 8*C)*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((A + 3*B - 13*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
```

```
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}dx = -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A+B\right.}{(a+}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\text{co}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\text{co}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\text{co}}{15ad(a+a}$$

$$= -\frac{(A+9B-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B-13C)F\left(\frac{1}{2}(c+\right.}{6a^3d}$$

Mathematica [C] time = 7.08224, size = 1809, normalized size = 9.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] ((-I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (
((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
```

```
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 +
 (((49*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^
3*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a
+ a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (26*C*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (
d*x)/2]^6*Sqrt[Cos[c + d*x]]*((-4*(-A - 9*B + 29*C + 20*C*Cos[c])*Csc[c])/(
5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2] -
29*C*Sin[(d*x)/2]))/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2
] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2
]^3*(4*A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2] + 14*C*Sin[(d*x)/2]))/(15*d) + (4*
(4*A - 9*B + 14*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B + C)*S
ec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.298, size = 624, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
```

```
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*cos(1/2*d*x+1/2*c)^5-294*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6+578*C*cos(1/2*d*x
+1/2*c)^6-24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4-264*C*cos(1/
2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-27*B*cos(1/2*d*x+1/2*c)^2+37*C*cos
(1/2*d*x+1/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*
x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^3, x)
```

3.472 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=191

$$\frac{(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B+C)\mathrm{si}}{5d(a\,\mathrm{co}}$$

```
[Out] ((A - B - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B + 3*C)*Ellip
ticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A + B - 6*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((A - B - 9*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.524232, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2977, 2978, 2748, 2641, 2639}

$$\frac{(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B+C)\mathrm{si}}{5d(a\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^3,x]
```

```
[Out] ((A - B - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B + 3*C)*Ellip
ticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A + B - 6*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((A - B - 9*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(7A+\right.}{(a}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A+B-6C)\sqrt{\text{co}}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A+B-6C)\sqrt{\text{co}}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A+B-6C)\sqrt{\text{co}}}{15ad(a+a}$$

$$= \frac{(A-B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{6a^3d}$$

Mathematica [C] time = 6.89809, size = 1799, normalized size = 9.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - ((
I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
```

```
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (((9
*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (2
*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt
[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]
^6*Sqrt[Cos[c + d*x]]*((-4*(A - B - 9*C)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] - 9*C*Sin[(d*x)/2]))/(5*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2] - 9*C*S
in[(d*x)/2]))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B
*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (4*(A + 4*B - 9*C)*Sec[c/2 + (d*x)
/2]^2*Tan[c/2])/(15*d) + (2*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d
)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.376, size = 624, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8-30*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*cos(1/2*d*x+1/2*c)^5-54*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+198*C*cos(1/2*d*x+1/2*c
)^6+6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4-114*C*cos(1/2*d*x+1/
2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*B*cos(1/2*d*x+1/2*c)^2+27*C*cos(1/2*d*x+
1/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^3, x)
```

3.473 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B-4C}{15ad(}$$

```
[Out] ((9*A + B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B + C)*Ellip
ticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((6*A - B - 4*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((9*A + B - C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.530449, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3041, 2978, 2748, 2641, 2639}

$$\frac{(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B-4C}{15ad(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^3),x]
```


```
[Out] ((9*A + B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B + C)*Ellip
ticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((6*A - B - 4*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((9*A + B - C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(9A+B-C)-\frac{1}{2}a(3A-3B-7C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B-4C)\sqrt{\cos(c+dx)}\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B-4C)\sqrt{\cos(c+dx)}\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B-4C)\sqrt{\cos(c+dx)}\sin(c}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(9A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B+C}{5d(}$$

Mathematica [C] time = 6.90769, size = 1802, normalized size = 9.34

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^3),x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
+ ((I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
```

```
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
((I/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (2
*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1
 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^3*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]
^6*Sqrt[Cos[c + d*x]]*((-4*(9*A + B - C)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/
2 + (d*x)/2]^3*(6*A*Sin[(d*x)/2] - B*Sin[(d*x)/2] - 4*C*Sin[(d*x)/2]))/(15*
d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2] - C*
Sin[(d*x)/2]))/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B
*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (4*(6*A - B - 4*C)*Sec[c/2 + (d*x)
/2]^2*Tan[c/2])/(15*d) - (2*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d
)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.39, size = 624, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
```

```
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+1/2*c
)^6+24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2*d*x+1/2
*c)^4+3*A*cos(1/2*d*x+1/2*c)^2+7*B*cos(1/2*d*x+1/2*c)^2-17*C*cos(1/2*d*x+1/
2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x +
```

```
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*s
qrt(cos(d*x + c))), x)
```

3.474 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{(13A-3B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B-C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B-}{6d\sqrt{\cos(c+dx)}\left(a^3\right.}$$

```
[Out] -((49*A - 9*B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B - C
)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - 9*B - C)*Sin[c + d*x])/(1
0*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]]*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B - 2*C)*Sin[c + d*x])/(15*a*d*Sq
rt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B - C)*Sin[c + d*x])/
(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.580356, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2978, 2748, 2636, 2639, 2641}

$$-\frac{(13A-3B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B-C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B-}{6d\sqrt{\cos(c+dx)}\left(a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[
c + d*x])^3),x]
```


```
[Out] -((49*A - 9*B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B - C
)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - 9*B - C)*Sin[c + d*x])/(1
0*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]]*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B - 2*C)*Sin[c + d*x])/(15*a*d*Sq
rt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B - C)*Sin[c + d*x])/
(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
```

```
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A-B+C)-\frac{5}{2}a(A-B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(13A-3B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9B-C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A}{5d\sqrt{\cos(}}$$

$$= -\frac{(49A-9B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A}{1}$$

Mathematica [C] time = 7.21339, size = 1841, normalized size = 7.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^3),x]
```

```
[Out] (((-49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 + (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
```

```
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 + ((I/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^3*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a
 + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (
d*x)/2]^6*Sqrt[Cos[c + d*x]]*((2*(20*A + 29*A*Cos[c] - 9*B*Cos[c] - C*Cos[c
])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*S
in[(d*x)/2] - 9*B*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2] - 6*B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(1
5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] +
C*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(11*A -
 6*B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B + C)*Sec[c/2 +
(d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 0.367, size = 793, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*
B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*A*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(49*A-9*B-C)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(817*A-147*B-13*C)*sin(1/2*d*x+1/2*c)^6+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(248*A-43*B-2*C)*sin(1/2*
d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69
*B-C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{5}+3\,a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x +
c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*c
os(d*x + c)^(3/2)), x)
```

3.475 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=270

$$\frac{(33A-13B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A-49B+9C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33$$

```
[Out] ((119*A - 49*B + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B
 + 3*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B + 3*C)*Sin[c +
 d*x])/(6*a^3*d*Cos[c + d*x]^(3/2)) - ((119*A - 49*B + 9*C)*Sin[c + d*x])/(
10*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Cos[c + d*
x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((119*A - 49*B + 9*C)*Sin[c + d*x])/(30*
d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.61494, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3041, 2978, 2748, 2636, 2641, 2639}

$$\frac{(33A-13B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A-49B+9C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[
c + d*x])^3),x]
```


```
[Out] ((119*A - 49*B + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B
 + 3*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B + 3*C)*Sin[c +
 d*x])/(6*a^3*d*Cos[c + d*x]^(3/2)) - ((119*A - 49*B + 9*C)*Sin[c + d*x])/(
10*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Cos[c + d*
x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((119*A - 49*B + 9*C)*Sin[c + d*x])/(30*
d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B+3C)-\frac{1}{2}a(7A-7B-3C)\cos(c+dx}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= \frac{(33A-13B+3C)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{(119A-49B+9C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A}{5d\cos^{\frac{3}{2}}(}$$

$$= \frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} +$$

Mathematica [C] time = 7.93584, size = 1883, normalized size = 6.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Cos[c + d*x])^3),x]
```

```
[Out] (((119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
```

```
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 - (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 + (((9*I)/10)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I
)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
+ E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d
*x])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c/2 + (d*x)/2]^6*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2
 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((-2*(60*A - 20*B + 59*A*Cos[c] - 29*B*Cos
[c] + 9*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (
d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (4*Se
c[c/2]*Sec[c/2 + (d*x)/2]^3*(16*A*Sin[(d*x)/2] - 11*B*Sin[(d*x)/2] + 6*C*Si
n[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 2
9*B*Sin[(d*x)/2] + 9*C*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]^2*S
in[d*x])/(3*d) + (16*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 9*A*Sin[d*x] + 3*B*Sin
[d*x]))/(3*d) - (4*(16*A - 11*B + 6*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d
) - (2*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*
x])^3
```

Maple [B] time = 1.267, size = 1040, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(8*A*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(4*A-2*B)*(2*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)
^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+(-24*A+8*B)*(-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(12*A-4*B)*(cos
(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(A-B+C)*(1/5*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+4/5*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+18/5*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-8/5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18
/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{6}+3\,a^{3}\cos\left(dx+c\right)^{5}+3\,a^{3}\cos\left(dx+c\right)^{4}+a^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^6 + 3*a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + a^3*cos(d*x +
c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*c
os(d*x + c)^(5/2)), x)
```

3.476 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=227

$$\frac{a(48A+40B+35C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48A+40B+35C)\,\mathrm{s}}{64d\sqrt{a\cos}}$$

```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(64*d) + (a*(48*A + 40*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(48*A + 40*B + 35*C)*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*B + C)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(
5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.52814, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3045, 2981, 2770, 2774, 216}

$$\frac{a(48A+40B+35C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48A+40B+35C)\,\mathrm{s}}{64d\sqrt{a\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```


```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(64*d) + (a*(48*A + 40*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(48*A + 40*B + 35*C)*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*B + C)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(
5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
```

```
+ 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c}{4d}$$

$$= \frac{a(8B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{C}{}$$

$$= \frac{a(48A+40B+35C)\cos^{\frac{3}{2}}(c+dx)\sin(c}{96d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a(48A+40B+35C)\sqrt{\cos(c+dx)}\sin(c}{64d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a(48A+40B+35C)\sqrt{\cos(c+dx)}\sin(c}{64d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{a}(48A+40B+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{a+a\cos(c+}}\right.}{64d}$$

Mathematica [A] time = 0.89565, size = 144, normalized size = 0.63

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(48A+40B+35C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+}\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] +
 C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(48*A + 40*B + 35*C
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(144*A + 152*B +
133*C + 2*(48*A + 40*B + 53*C)*Cos[c + d*x] + 4*(8*B + 7*C)*Cos[2*(c + d*x)
] + 12*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.146, size = 622, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^4*(96*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)+336*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)+64*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+384*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+144*B*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+48*C*sin(d*x+c)*cos(d*x+c)^5*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)+200*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+56*C
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+120*B*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+70*C*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+105*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+144*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))+120*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+105*C*cos(d*x+c)^2*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(3/2)*(a*(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)^8/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 7.3838, size = 467, normalized size = 2.06

$$\frac{\left(48\,C\cos\left(dx+c\right)^{3}+8\,(8\,B+7\,C)\cos\left(dx+c\right)^{2}+2\,(48\,A+40\,B+35\,C)\cos\left(dx+c\right)+144\,A+120\,B+105\,C\right)\sqrt{a\cos\left(\right.}}{192\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*((48*C*cos(d*x + c)^3 + 8*(8*B + 7*C)*cos(d*x + c)^2 + 2*(48*A + 40*B
 + 35*C)*cos(d*x + c) + 144*A + 120*B + 105*C)*sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))*sin(d*x + c) - 3*((48*A + 40*B + 35*C)*cos(d*x + c) + 48*A
+ 40*B + 35*C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.477 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=179

$$\frac{\sqrt{a}(8A+6B+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+6B+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a(6B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+}{12d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]])/(8*d) + (a*(8*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d
*Sqrt[a + a*Cos[c + d*x]]) + (a*(6*B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/
(12*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.430604, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3045, 2981, 2770, 2774, 216}

$$\frac{\sqrt{a}(8A+6B+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+6B+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a(6B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+}{12d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```


```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]])/(8*d) + (a*(8*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d
*Sqrt[a + a*Cos[c + d*x]]) + (a*(6*B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/
(12*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+}{3d}$$

$$= \frac{a(6B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\,c}{}$$

$$= \frac{a(8A+6B+5C)\sqrt{\cos(c+dx)}\sin(c+dx}{8d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a(8A+6B+5C)\sqrt{\cos(c+dx)}\sin(c+dx}{8d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{a}(8A+6B+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}$$

Mathematica [A] time = 0.499931, size = 124, normalized size = 0.69

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(8A+6B+5C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(24\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] +
 C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(8*A + 6*B + 5*C)*A
rcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(24*A + 18*B + 19*C
+ 2*(6*B + 5*C)*Cos[c + d*x] + 4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48
*d)
```

Maple [B] time = 0.11, size = 514, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
+12*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+24*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+30*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)+8*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+18*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+
10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*C*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*cos(d*x+c)^2*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+18*B*cos(d*x+c)^2
*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*C*cos(d
*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*co
s(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/s
in(d*x+c)^6
```

<u>Maxima</u> [B] time = 3.33372, size = 5090, normalized size = 28.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
```

+ 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + 6*(2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x
+ 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(
2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + si
n(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2
*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*
sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B + (4*
(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)
*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3*arctan2

(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arctan2(-(co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))
 + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1
) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2

```
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*C)/d
```

Fricas [A] time = 4.56575, size = 398, normalized size = 2.22

$$\frac{\left(8\,C\cos\left(dx+c\right)^{2}+2\left(6\,B+5\,C\right)\cos\left(dx+c\right)+24\,A+18\,B+15\,C\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(\left(8\right.\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*C*cos(d*x + c)^2 + 2*(6*B + 5*C)*cos(d*x + c) + 24*A + 18*B + 15*C
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((8*A + 6*B
+ 5*C)*cos(d*x + c) + 8*A + 6*B + 5*C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.478 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{a}(8A+4B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]])/(4*d) + (a*(4*B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.355226, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3045, 2981, 2774, 216}

$$\frac{\sqrt{a}(8A+4B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]])/(4*d) + (a*(4*B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\int \frac{\sqrt{a}}{-}}{}$$

$$= \frac{a(4B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}}{}$$

$$= \frac{a(4B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}}{}$$

$$= \frac{\sqrt{a}(8A+4B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(4B+C}{4d}$$

Mathematica [A] time = 0.359461, size = 103, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+4B+3C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(4B$$
$$8d$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(8*A + 4*B + 3*C)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*B + 3*C + 2*C*Cos[c
 + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.151, size = 328, normalized size = 2.5

$$\frac{(-1+\cos(dx+c))^2}{4\,d\,(\sin(dx+c))^4}\left(4\,B\cos(dx+c)\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}+4\,B\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}+2\,C\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*C*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+3*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+
c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [B] time = 2.75042, size = 2695, normalized size = 20.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + cos(d*x + c)) + 4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B +
 (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) -
 (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
```

```
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
1)))*C)/d
```

Fricas [A] time = 4.57627, size = 340, normalized size = 2.6

$$\frac{(2\,C\cos(dx+c)+4\,B+3\,C)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-((8\,A+4\,B+3\,C)\cos(dx+c)+8\,A+4\,B}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*C*cos(d*x + c) + 4*B + 3*C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - ((8*A + 4*B + 3*C)*cos(d*x + c) + 8*A + 4*B + 3*C)*sqrt
(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c
))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
sqrt(cos(d*x + c)), x)
```

3.479 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{a(2A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a}(2B+C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*(2*B + C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d - (a*(2*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*
x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.357451, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3043, 2981, 2774, 216}

$$-\frac{a(2A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a}(2B+C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*B + C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d - (a*(2*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*
x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx}}{}}{}$$

$$= -\frac{a(2A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\,\mathrm{co}}}{d\sqrt{}}$$

$$= -\frac{a(2A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\,\mathrm{co}}}{d\sqrt{}}$$

$$= \frac{\sqrt{a}(2B+C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a(2A-C)\sqrt{\cos(}}{d\sqrt{a+a}}$$

Mathematica [A] time = 0.30405, size = 104, normalized size = 0.86

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2A+C\cos(c+dx))+\sqrt{2}(2B+C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{c}\right.}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*B + C)*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + C*Cos[c + d*x])*Sin[(c
+ d*x)/2]))/(2*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.136, size = 210, normalized size = 1.7

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2}\sqrt{a(1+\cos(dx+c))}\left(C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\sin(dx+c)\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(C*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+2*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos
(d*x+c)+C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.37409, size = 1397, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C + 8*A
*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*
(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A] time = 2.69673, size = 346, normalized size = 2.86

$$\frac{(C\cos(dx+c)+2A)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-\left((2B+C)\cos(dx+c)^2+(2B+C)\cos(dx+c)\right)}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] ((C*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x
```

```
 + c) - ((2*B + C)*cos(d*x + c)^2 + (2*B + C)*cos(d*x + c))*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos
(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cos(c+dx)+1)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2
)/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
cos(d*x + c)^(3/2), x)
```

3.480 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.342526, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3043, 2980, 2774, 216}

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```


```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}(\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c\cdots}}{3d\cos^{\frac{3}{2}}\cdots}$$

$$= \frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c\cdots}}{3d\cos^{\frac{3}{2}}\cdots}$$

$$= \frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a(A+3B)\sin(c\cdots}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos\cdots}}$$

Mathematica [A] time = 0.399244, size = 105, normalized size = 0.88

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((2A+3B)\cos(c+dx)+A)+3\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*C*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (2*A + 3*B)*Cos[c + d*x])*Sin[(
c + d*x)/2]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.127, size = 147, normalized size = 1.2

$$-\frac{2}{3\,d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-3\,C\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] -2/3/d*(a*(1+cos(d*x+c)))^(1/2)*(-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos
(d*x+c)+2*A*cos(d*x+c)^2+3*B*cos(d*x+c)^2-A*cos(d*x+c)-3*B*cos(d*x+c)-A)/si
n(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.02027, size = 587, normalized size = 4.89

$$\frac{3\,C\sqrt{a}\arctan\left(\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*C*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + 6*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - s
qrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + 2*A*(
3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1
)))/d
```

Fricas [A] time = 2.19633, size = 348, normalized size = 2.9

$$\frac{2\left(\left((2\,A+3\,B)\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(C\cos\left(dx+c\right)^{3}+C\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,a}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 3*(C*cos(d*x + c)^3 + C*cos(d*x + c)^2)*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos
(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
cos(d*x + c)^(5/2), x)
```

3.481 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{2a(8A+10B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*a*(A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*(8*A + 10*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```

Rubi [A] time = 0.368801, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3043, 2980, 2771}

$$\frac{2a(8A+10B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(7/2),x]
```


```
[Out] (2*a*(A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*(8*A + 10*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}}{\cdots}}{\cdots}$$

$$= \frac{2a(A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos\cdots}}{5d\cos\cdots}$$

$$= \frac{2a(A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(8A+10\cdots}{15d\sqrt{\cos(c+\cdots}}$$

Mathematica [A] time = 0.370711, size = 85, normalized size = 0.65

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((8A+10B+15C)\cos(2(c+dx))+2(4A+5B)\cos(c+dx)+14A+10B+15C)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(14*A + 10*B + 15*C + 2*(4*A + 5*B)*Cos[c + d*x
] + (8*A + 10*B + 15*C)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d
*x]^(5/2))
```

Maple [A] time = 0.109, size = 97, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(8A(\cos(dx+c))^{2}+10B(\cos(dx+c))^{2}+15C(\cos(dx+c))^{2}+4A\cos(dx+c)+5B\cos(dx+$$
$$15\,d\sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)
,x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+15*C*cos(d*x+c)
^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/c
os(d*x+c)^(5/2)
```

Maxima [B] time = 1.78187, size = 707, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*C*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + 5*B*(3*sqrt(2)*sq
rt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + A*(15*s
qrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(a)*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)
```

```
^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.86929, size = 231, normalized size = 1.78

$$\frac{2\left((8A+10B+15C)\cos(dx+c)^2+(4A+5B)\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{15\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*((8*A + 10*B + 15*C)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4
+ d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
cos(d*x + c)^(7/2), x)
```

3.482 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{2a(24A+28B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a*(A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Cos[c + d*x]]) + (4*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.439264, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3043, 2980, 2772, 2771}

$$\frac{2a(24A+28B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(9/2),x]
```


```
[Out] (2*a*(A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Cos[c + d*x]]) + (4*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int\frac{\sqrt{a+a\cos(c+dx)}}{}}{}$$

$$= \frac{2a(A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos}}{7d\cos}$$

$$= \frac{2a(A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(24A+28}{105d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2a(A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(24A+28}{105d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A] time = 0.595181, size = 121, normalized size = 0.68

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(3(36A+42B+35C)\cos(c+dx)+(24A+28B+35C)\cos(2(c+dx))+24A\cos(}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(54*A + 28*B + 35*C + 3*(36*A + 42*B + 35*C)*Co
s[c + d*x] + (24*A + 28*B + 35*C)*Cos[2*(c + d*x)] + 24*A*Cos[3*(c + d*x)]
+ 28*B*Cos[3*(c + d*x)] + 35*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(105*d*C
os[c + d*x]^(7/2))
```

Maple [A] time = 0.133, size = 130, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(48A(\cos(dx+c))^3+56B(\cos(dx+c))^3+70C(\cos(dx+c))^3+24A(\cos(dx+c))^2+28B(}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
,x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+70*C*cos(d*x+
c)^3+24*A*cos(d*x+c)^2+28*B*cos(d*x+c)^2+35*C*cos(d*x+c)^2+18*A*cos(d*x+c)+
21*B*cos(d*x+c)+15*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.81096, size = 959, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 2/105*(35*C*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*
sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 1)) + 7*B*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
 25*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7
/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 3*A*(35*sqrt(2)*sqrt(
a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)
*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x
 + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 2.03539, size = 292, normalized size = 1.64

$$\frac{2\left(2\,(24\,A+28\,B+35\,C)\cos\left(dx+c\right)^{3}+(24\,A+28\,B+35\,C)\cos\left(dx+c\right)^{2}+3\,(6\,A+7\,B)\cos\left(dx+c\right)+15\,A\right)\sqrt{a\cos\left(\right.}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(2*(24*A + 28*B + 35*C)*cos(d*x + c)^3 + (24*A + 28*B + 35*C)*cos(d*x
 + c)^2 + 3*(6*A + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
cos(d*x + c)^(9/2), x)
```

3.483 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=226

$$\frac{8a(16A+18B+21C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(16A+18B+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+18B+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a*(A + 9*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(315*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*(16*A + 18*B + 21*C)*Sin
[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.518175, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3043, 2980, 2772, 2771}

$$\frac{8a(16A+18B+21C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(16A+18B+21C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+18B+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(11/2),x]
```


```
[Out] (2*a*(A + 9*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(315*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*(16*A + 18*B + 21*C)*Sin
[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2A\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\cos(c+dx)}(}{}}{}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(}}{9d\cos}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+18B}{105d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+18B}{105d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+18B}{105d\cos^{\frac{5}{2}}(c+}$$

Mathematica [A] time = 0.854243, size = 155, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(88A+99B+63C)\cos(c+dx)+11(16A+18B+21C)\cos(2(c+dx))+32A\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(214*A + 162*B + 189*C + 2*(88*A + 99*B + 63*C)
*Cos[c + d*x] + 11*(16*A + 18*B + 21*C)*Cos[2*(c + d*x)] + 32*A*Cos[3*(c +
d*x)] + 36*B*Cos[3*(c + d*x)] + 42*C*Cos[3*(c + d*x)] + 32*A*Cos[4*(c + d*x
)] + 36*B*Cos[4*(c + d*x)] + 42*C*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(315*
d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.129, size = 163, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(128A(\cos(dx+c))^4+144B(\cos(dx+c))^4+168C(\cos(dx+c))^4+64A(\cos(dx+c))^3+72B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2
),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+168*C*cos(d
*x+c)^4+64*A*cos(d*x+c)^3+72*B*cos(d*x+c)^3+84*C*cos(d*x+c)^3+48*A*cos(d*x+
c)^2+54*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+40*A*cos(d*x+c)+45*B*cos(d*x+c)+35
*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.81562, size = 1145, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 2/315*(21*C*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2
)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 9*B*(35*sqrt(2)*sqrt(a)*sin(d*x
 + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2
)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + 1)) + A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c
) + 1) - 735*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqr
t(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 107*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 +
 sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.9939, size = 348, normalized size = 1.54

$$\frac{2\left(8\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{4}+4\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{3}+3\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{2}+5\,(8\right.}{315\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*(16*A + 18*B + 21*C)*cos(d*x + c)^4 + 4*(16*A + 18*B + 21*C)*cos(d
*x + c)^3 + 3*(16*A + 18*B + 21*C)*cos(d*x + c)^2 + 5*(8*A + 9*B)*cos(d*x +
 c) + 35*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos
(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/cos(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/cos(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
cos(d*x + c)^(11/2), x)
```

3.484 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\,\mathrm{c}\right.$

Optimal. Leaf size=283

$$\frac{a^2(80A+90B+67C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(176A+150B+133C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+15}{}$$

```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(128*d) + (a^2*(176*A + 150*B + 133*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(176*A + 150*B + 133*C)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(
80*A + 90*B + 67*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[
c + d*x]]) + (a*(10*B + 3*C)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(40*d) + (C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rubi [A] time = 0.751687, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(80A+90B+67C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(176A+150B+133C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+15}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(128*d) + (a^2*(176*A + 150*B + 133*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(176*A + 150*B + 133*C)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(
80*A + 90*B + 67*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[
c + d*x]]) + (a*(10*B + 3*C)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(40*d) + (C*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,\mathrm{si}}{5d}$$

$$= \frac{a(10B+3C)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(}}{40d}$$

$$= \frac{a^2(80A+90B+67C)\cos^{\frac{5}{2}}(c+dx)\,\mathrm{si}}{240d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2(176A+150B+133C)\cos^{\frac{3}{2}}(c+dx}{192d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2(176A+150B+133C)\sqrt{\cos(c+dx}}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2(176A+150B+133C)\sqrt{\cos(c+dx}}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{3/2}(176A+150B+133C)\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.}{128d}$$

Mathematica [A] time = 1.57473, size = 170, normalized size = 0.6

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(176A+150B+133C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\mathrm{co}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
 + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(176*A + 150*B +
 133*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2960*A + 2
850*B + 2671*C + 2*(880*A + 930*B + 1007*C)*Cos[c + d*x] + 4*(80*A + 150*B
+ 181*C)*Cos[2*(c + d*x)] + 120*B*Cos[3*(c + d*x)] + 228*C*Cos[3*(c + d*x)]
 + 48*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(3840*d)
```

Maple [B] time = 0.136, size = 731, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x)
```

```
[Out] 1/1920/d*a*(-1+cos(d*x+c))^4*(640*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d
*x+c)^4*sin(d*x+c)+3040*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+480*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+68
00*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+1680*B*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+384*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6+7040*A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)+2700*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+912*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+2640*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3750*B*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1064*C*sin(d*x+c)*cos(d*
x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2250*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)+1330*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+1995*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+2640*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))+2250*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))+1995*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3
/2)/sin(d*x+c)^8/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 6.84455, size = 574, normalized size = 2.03

$$\frac{\left(384\,Ca\cos\left(dx+c\right)^{4}+48\,(10\,B+19\,C)a\cos\left(dx+c\right)^{3}+8\,(80\,A+150\,B+133\,C)a\cos\left(dx+c\right)^{2}+10\,(176\,A+150\,B+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 1/1920*((384*C*a*cos(d*x + c)^4 + 48*(10*B + 19*C)*a*cos(d*x + c)^3 + 8*(80
*A + 150*B + 133*C)*a*cos(d*x + c)^2 + 10*(176*A + 150*B + 133*C)*a*cos(d*x
 + c) + 15*(176*A + 150*B + 133*C)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))*sin(d*x + c) - 15*((176*A + 150*B + 133*C)*a*cos(d*x + c) + (176*A +
 150*B + 133*C)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.485 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=233

$$\frac{a^2(48A+56B+39C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+88B+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(112A+88B+75}{64d\sqrt{a}}$$

```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(64*d) + (a^2*(112*A + 88*B + 75*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(48*A + 56*B + 39*C)*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*B + 3*C)*C
os[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d) + (C*Cos[c
+ d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.64485, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(48A+56B+39C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+88B+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(112A+88B+75}{64d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]])/(64*d) + (a^2*(112*A + 88*B + 75*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(48*A + 56*B + 39*C)*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*B + 3*C)*C
os[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d) + (C*Cos[c
+ d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
```

```
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,\mathrm{s}}{4d}$$

$$= \frac{a(8B+3C)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c}}{24d}$$

$$= \frac{a^2(48A+56B+39C)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{96d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2(112A+88B+75C)\sqrt{\cos(c+dx)}\,\mathrm{s}}{64d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2(112A+88B+75C)\sqrt{\cos(c+dx)}\,\mathrm{s}}{64d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{3/2}(112A+88B+75C)\sin^{-1}\left(\frac{\sqrt{a}\,\mathrm{si}}{\sqrt{a+a\,\mathrm{c}}}\right.}{64d}$$

Mathematica [A] time = 0.950164, size = 145, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(112A+88B+75C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c}\right.}{384}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
 + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(112*A + 88*B + 7
5*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(336*A + 296*B
 + 285*C + 2*(48*A + 88*B + 93*C)*Cos[c + d*x] + 4*(8*B + 15*C)*Cos[2*(c +
d*x)] + 12*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.112, size = 623, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x)
```

```
[Out] -1/192/d*a*(-1+cos(d*x+c))^3*(96*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)+528*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)+64*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+768*A
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+240*B*cos(d*x+c)^3
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+48*C*sin(d*x+c)*cos(d*x+c)^5*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+336*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(5/2)+440*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1
20*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+264*B*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+150*C*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+225*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+336*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+264*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+225*C*cos(d*x+c)^2*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 6.81902, size = 493, normalized size = 2.12

$$\left(48\,Ca\cos\left(dx+c\right)^{3}+8\,(8\,B+15\,C)a\cos\left(dx+c\right)^{2}+2\,(48\,A+88\,B+75\,C)a\cos\left(dx+c\right)+3\,(112\,A+88\,B+75\,C)a\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 1/192*((48*C*a*cos(d*x + c)^3 + 8*(8*B + 15*C)*a*cos(d*x + c)^2 + 2*(48*A +
 88*B + 75*C)*a*cos(d*x + c) + 3*(112*A + 88*B + 75*C)*a)*sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((112*A + 88*B + 75*C)*a*cos(d*
x + c) + (112*A + 88*B + 75*C)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.486 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{a^{3/2}(24A+14B+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+30B+19C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a(2B+C)\sin(c+dx)\sqrt{}}{}$$

```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(8*d) + (a^2*(24*A + 30*B + 19*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(2*B + C)*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.565572, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3045, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(24A+14B+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+30B+19C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a(2B+C)\sin(c+dx)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```


```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(8*d) + (a^2*(24*A + 30*B + 19*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(2*B + C)*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
```

```
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + -$$

$$= \frac{a(2B+C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{a^2(24A+30B+19C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a(}{}$$

$$= \frac{a^2(24A+30B+19C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a(}{}$$

$$= \frac{a^{3/2}(24A+14B+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(24}{}$$

Mathematica [A] time = 0.73166, size = 125, normalized size = 0.69

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(24A+14B+11C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+d}\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(24*A + 14*B + 11
*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(24*A + 42*B +
37*C + 2*(6*B + 11*C)*Cos[c + d*x] + 4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]
))/(48*d)
```

Maple [B] time = 0.117, size = 515, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)+12*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+24*A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+54*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)+8*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+42*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
+22*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*C*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+72*A*cos(d*x+c)^2*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+42*B*cos(d*x+c)^
2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+33*C*cos(
d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(
a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^4/cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)
```

Maxima [B] time = 3.38719, size = 5162, normalized size = 28.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
```

+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + 6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + (4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4

)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))),
(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(c
os(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1))) - 1) - a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*

```
c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*
cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 4.49186, size = 428, normalized size = 2.36

$$\frac{\left(8\,Ca\cos\left(dx+c\right)^{2}+2\,(6\,B+11\,C)a\cos\left(dx+c\right)+3\,(8\,A+14\,B+11\,C)a\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{24\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*C*a*cos(d*x + c)^2 + 2*(6*B + 11*C)*a*cos(d*x + c) + 3*(8*A + 14*B
 + 11*C)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((
24*A + 14*B + 11*C)*a*cos(d*x + c) + (24*A + 14*B + 11*C)*a)*sqrt(a)*arctan
(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*co
s(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/sqrt(cos(d*x + c)), x)
```

3.487 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{a^{3/2}(8A+12B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^2(8A-4B-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a(4A-C)\sin(c+dx)\sqrt{\cos(c}}{2d}$$

```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(4*d) - (a^2*(8*A - 4*B - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]) - (a*(4*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c +
 d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.580265, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(8A+12B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^2(8A-4B-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a(4A-C)\sin(c+dx)\sqrt{\cos(c}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/(4*d) - (a^2*(8*A - 4*B - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]) - (a*(4*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c +
 d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= -\frac{a(4A-C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+}{2d}$$

$$= -\frac{a^2(8A-4B-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a(4A}{}$$

$$= -\frac{a^2(8A-4B-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a(4A}{}$$

$$= \frac{a^{3/2}(8A+12B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^2(8A}{}$$

Mathematica [A] time = 0.603579, size = 127, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+12B+7C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(\right.}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(8*A + 12*B + 7*C)*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A + C + (4*B + 7
*C)*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Cos[c +
 d*x]])
```

Maple [B] time = 0.111, size = 443, normalized size = 2.5

$$-\frac{a(-1+\cos(dx+c))}{4d(\sin(dx+c))^2}\left(8A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+16A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx+}{1+\cos(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/4/d*a*(-1+cos(d*x+c))*(8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)+16*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+4
*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+4*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+2*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+7*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*
cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)+12*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(
d*x+c)^3+7*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*cos(d*x+c)^3)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
sin(d*x+c)^2/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.87151, size = 3887, normalized size = 21.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
```

```
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*C + 8*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/d
```

Fricas [A] time = 4.50707, size = 431, normalized size = 2.38

$$\frac{\left(2\,Ca\cos\left(dx+c\right)^2+\left(4\,B+7\,C\right)a\cos\left(dx+c\right)+8\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(8\,A+12\,B+7\right.\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*C*a*cos(d*x + c)^2 + (4*B + 7*C)*a*cos(d*x + c) + 8*A*a)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - ((8*A + 12*B + 7*C)*a*cos(
d*x + c)^2 + (8*A + 12*B + 7*C)*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2
+ d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(3/2), x)
```

3.488 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=171

$$-\frac{a^2(8A+6B-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(A+B)\sin(c+dx)\sqrt{a\cos(c+}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^2*(8*A + 6*B - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a*(A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2))
```

Rubi [A] time = 0.576854, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2981, 2774, 216}

$$-\frac{a^2(8A+6B-3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(A+B)\sin(c+dx)\sqrt{a\cos(c+}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^2*(8*A + 6*B - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a*(A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2))
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(}{}}{}$$

$$= \frac{2a(A+B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2A(a+}{}$$

$$= -\frac{a^2(8A+6B-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a(}{}$$

$$= -\frac{a^2(8A+6B-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a(}{}$$

$$= \frac{a^{3/2}(2B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(8A+6B}{3a}$$

Mathematica [A] time = 0.740273, size = 128, normalized size = 0.75

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(5A+3B)\cos(c+dx)+4A+3C\cos(2(c+dx))+3C)+3\sqrt{2}\right.}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(2*B + 3*C)*ArcSi
n[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*C + 4*(5*A + 3*B)
*Cos[c + d*x] + 3*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Cos[c + d*x]^
(3/2))
```

Maple [B] time = 0.102, size = 302, normalized size = 1.8

$$-\frac{a}{3\,d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-6\,B\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] -1/3/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-9*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))*cos(d*x+c)+3*C*cos(d*x+c)^3+10*A*cos(d*x+c)^2+6*B*cos(d*x+c)^2-3
*C*cos(d*x+c)^2-8*A*cos(d*x+c)-6*B*cos(d*x+c)-2*A)/sin(d*x+c)/cos(d*x+c)^(3
/2)
```

Maxima [B] time = 2.5132, size = 2599, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
```

```
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C +
6*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
)*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4) + 16*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt
(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [A] time = 2.57701, size = 423, normalized size = 2.47

$$\frac{\left(3\,Ca\cos\left(dx+c\right)^{2}+2\,(5\,A+3\,B)a\cos\left(dx+c\right)+2\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(\left(2\,B+3\,C\right)a\right.}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*C*a*cos(d*x + c)^2 + 2*(5*A + 3*B)*a*cos(d*x + c) + 2*A*a)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((2*B + 3*C)*a*cos(d*x
 + c)^3 + (2*B + 3*C)*a*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(
d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(5/2), x)
```

3.489 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A}{}$$

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(12*A + 20*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.530172, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2980, 2774, 216}

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(12*A + 20*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
5*d*Cos[c + d*x]^(5/2))
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c}{}}{}$$

$$= \frac{2a(3A+5B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(}{}$$

$$= \frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a(3A+5B}{}$$

$$= \frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a(3A+5B}{}$$

$$= \frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(12A+20B+1}{15d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [A] time = 0.849999, size = 134, normalized size = 0.78

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((18A+25B+15C)\cos(2(c+dx))+2(9A+5B)\cos(c+dx)+\right.}{30d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sqrt[2]*C*ArcSin[Sqrt[2]
*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(24*A + 25*B + 15*C + 2*(9*A + 5*
B)*Cos[c + d*x] + (18*A + 25*B + 15*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))
/(30*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.106, size = 263, normalized size = 1.5

$$-\frac{2a}{15d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-15C\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x)
```

```
[Out] -2/15/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))
*cos(d*x+c)^2-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+18*A*cos(d*
x+c)^3+25*B*cos(d*x+c)^3+15*C*cos(d*x+c)^3-9*A*cos(d*x+c)^2-20*B*cos(d*x+c)
^2-15*C*cos(d*x+c)^2-6*A*cos(d*x+c)-5*B*cos(d*x+c)-3*A)/sin(d*x+c)/cos(d*x+
c)^(5/2)
```

Maxima [B] time = 2.17908, size = 1808, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*(15*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
+ 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*
(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4) + 40*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
 5*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*B/((sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + 24*(5*sqrt(2)*a^(3
/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 2.23066, size = 419, normalized size = 2.44

$$\frac{2\left(\left((18\,A+25\,B+15\,C)a\cos\left(dx+c\right)^{2}+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right.}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(((18*A + 25*B + 15*C)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) +
 3*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*(C*a*
cos(d*x + c)^4 + C*a*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x
 + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(7/2), x)
```

3.490 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{2a^2(4A+6B+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(104A+126B+175C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a(3A+7B)\sin(c+dx)\sqrt{a\cos(c+dx}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*a^2*(4*A + 6*B + 5*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + (2*a^2*(104*A + 126*B + 175*C)*Sin[c + d*x])/(105*d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(3*A + 7*B)*Sqrt[a + a*Cos[c
+ d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.579203, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3043, 2975, 2980, 2771}

$$\frac{2a^2(4A+6B+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(104A+126B+175C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a(3A+7B)\sin(c+dx)\sqrt{a\cos(c+dx}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```


```
[Out] (2*a^2*(4*A + 6*B + 5*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + (2*a^2*(104*A + 126*B + 175*C)*Sin[c + d*x])/(105*d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(3*A + 7*B)*Sqrt[a + a*Cos[c
+ d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
```

```
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(}{}}{}$$

$$= \frac{2a(3A+7B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(}{}$$

$$= \frac{2a^2(4A+6B+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(3A+7B}{}$$

$$= \frac{2a^2(4A+6B+5C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(104A+}{105d\sqrt{\cos(}}$$

Mathematica [A] time = 0.703005, size = 122, normalized size = 0.66

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((468A+462B+525C)\cos(c+dx)+2(52A+63B+35C)\cos(2(c+dx))+104}{210d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(164*A + 126*B + 70*C + (468*A + 462*B + 525*
C)*Cos[c + d*x] + 2*(52*A + 63*B + 35*C)*Cos[2*(c + d*x)] + 104*A*Cos[3*(c
+ d*x)] + 126*B*Cos[3*(c + d*x)] + 175*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2]
)/(210*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.114, size = 131, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(104A(\cos(dx+c))^3+126B(\cos(dx+c))^3+175C(\cos(dx+c))^3+52A(\cos(dx+c))^2+6\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+175*C*cos
(d*x+c)^3+52*A*cos(d*x+c)^2+63*B*cos(d*x+c)^2+35*C*cos(d*x+c)^2+39*A*cos(d*
x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(
7/2)
```

Maxima [B] time = 1.75735, size = 815, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 4/105*(35*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5)*C/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + 21*(5*sqrt(2)*a^(3/2)*sin(d*x +
 c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^
(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + (105*sqrt(2)*a^(3/2)*sin(d*x + c)/(c
os(d*x + c) + 1) - 245*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(
3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 2.04511, size = 305, normalized size = 1.66

$$\frac{2\left((104\,A+126\,B+175\,C)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B+35\,C)a\cos\left(dx+c\right)^{2}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)+15\,Aa\right)\sqrt{}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*((104*A + 126*B + 175*C)*a*cos(d*x + c)^3 + (52*A + 63*B + 35*C)*a*co
s(d*x + c)^2 + 3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(9/2), x)
```

3.491 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{2a^2(136A+156B+189C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+156B+189C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^2*(52*A + 72*B + 63*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sin[c + d*x])/(315*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(136*A + 156*B + 189*C)
*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) +
 (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.679072, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2980, 2772, 2771}

$$\frac{2a^2(136A+156B+189C)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+156B+189C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(11/2),x]
```


```
[Out] (2*a^2*(52*A + 72*B + 63*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sin[c + d*x])/(315*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(136*A + 156*B + 189*C)
*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) +
 (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2a(A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a+}{}$$

$$= \frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(A+3B)}{}$$

$$= \frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(136A+}{315d\cos^{\frac{3}{2}}(}$$

$$= \frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(136A+}{315d\cos^{\frac{3}{2}}(}$$

Mathematica [A] time = 0.968068, size = 157, normalized size = 0.68

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((748A+81(8B+7C))\cos(c+dx)+(748A+858B+882C)\cos(2(c+dx))+136}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(752*A + 702*B + 693*C + (748*A + 81*(8*B + 7
*C))*Cos[c + d*x] + (748*A + 858*B + 882*C)*Cos[2*(c + d*x)] + 136*A*Cos[3*
(c + d*x)] + 156*B*Cos[3*(c + d*x)] + 189*C*Cos[3*(c + d*x)] + 136*A*Cos[4*
(c + d*x)] + 156*B*Cos[4*(c + d*x)] + 189*C*Cos[4*(c + d*x)])*Tan[(c + d*x)
/2])/(630*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.125, size = 164, normalized size = 0.7

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(272\,A\,(\cos(dx+c))^4+312\,B\,(\cos(dx+c))^4+378\,C\,(\cos(dx+c))^4+136\,A\,(\cos(dx+c))^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+378*C*cos
(d*x+c)^4+136*A*cos(d*x+c)^3+156*B*cos(d*x+c)^3+189*C*cos(d*x+c)^3+102*A*co
s(d*x+c)^2+117*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+85*A*cos(d*x+c)+45*B*cos(d*
x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.79064, size = 1064, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 4/315*(63*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*C*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
+ 3*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 1)) + (315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) +
1) - 840*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)
*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x
```

```
+ c) + 1)^9 - 94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 2.41062, size = 369, normalized size = 1.59

$$\frac{2\left(2\,(136\,A+156\,B+189\,C)a\cos\left(dx+c\right)^{4}+(136\,A+156\,B+189\,C)a\cos\left(dx+c\right)^{3}+3\,(34\,A+39\,B+21\,C)a\cos\left(dx+\right.}{315\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(2*(136*A + 156*B + 189*C)*a*cos(d*x + c)^4 + (136*A + 156*B + 189*C)
*a*cos(d*x + c)^3 + 3*(34*A + 39*B + 21*C)*a*cos(d*x + c)^2 + 5*(17*A + 9*B
)*a*cos(d*x + c) + 35*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(11/2), x)
```

3.492 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{8a^2(336A+374B+429C)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(336A+374B+429C)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^2*(84*A + 110*B + 99*C)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)*Sqrt[a
 + a*Cos[c + d*x]]) + (2*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(1155*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*
C)*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16
*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]) + (2*a*(3*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 0.763366, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2980, 2772, 2771}

$$\frac{8a^2(336A+374B+429C)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(336A+374B+429C)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(13/2),x]
```


```
[Out] (2*a^2*(84*A + 110*B + 99*C)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)*Sqrt[a
 + a*Cos[c + d*x]]) + (2*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(1155*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*
C)*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16
*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]) + (2*a*(3*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(11*d*Cos[c + d*x]^(11/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2a(3A+11B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(3A+11}{}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(336A+}{1155d\cos^{\frac{5}{2}}(}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(336A+}{1155d\cos^{\frac{5}{2}}(}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(336A+}{1155d\cos^{\frac{5}{2}}(}$$

Mathematica [A] time = 1.04132, size = 187, normalized size = 0.66

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((12684A+12386B+12441C)\cos(c+dx)+(4368A+4862B+4422C)\cos(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(4956*A + 4114*B + 3564*C + (12684*A + 12386*
B + 12441*C)*Cos[c + d*x] + (4368*A + 4862*B + 4422*C)*Cos[2*(c + d*x)] + 4
```

```
368*A*Cos[3*(c + d*x)] + 4862*B*Cos[3*(c + d*x)] + 5577*C*Cos[3*(c + d*x)]
+ 672*A*Cos[4*(c + d*x)] + 748*B*Cos[4*(c + d*x)] + 858*C*Cos[4*(c + d*x)]
+ 672*A*Cos[5*(c + d*x)] + 748*B*Cos[5*(c + d*x)] + 858*C*Cos[5*(c + d*x)])
*Tan[(c + d*x)/2])/(6930*d*Cos[c + d*x]^(11/2))
```

Maple [A] time = 0.102, size = 197, normalized size = 0.7

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(2688\,A\,(\cos(dx+c))^5+2992\,B\,(\cos(dx+c))^5+3432\,C\,(\cos(dx+c))^5+1344\,A\,(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2
),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(2688*A*cos(d*x+c)^5+2992*B*cos(d*x+c)^5+3432*C
*cos(d*x+c)^5+1344*A*cos(d*x+c)^4+1496*B*cos(d*x+c)^4+1716*C*cos(d*x+c)^4+1
008*A*cos(d*x+c)^3+1122*B*cos(d*x+c)^3+1287*C*cos(d*x+c)^3+840*A*cos(d*x+c)
^2+935*B*cos(d*x+c)^2+495*C*cos(d*x+c)^2+735*A*cos(d*x+c)+385*B*cos(d*x+c)+
315*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 1.81685, size = 1251, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(13/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(33*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(
2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*C*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 1)) + 11*(315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d
*x + c) + 1) - 840*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13
```

```
44*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/
2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9
/(cos(d*x + c) + 1)^9 - 94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 1)) + 21*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqr
t(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)*a^
(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 + 1)))/d
```

Fricas [A] time = 2.38625, size = 440, normalized size = 1.55

$$\frac{2\left(8\,(336\,A+374\,B+429\,C)a\cos\left(dx+c\right)^{5}+4\,(336\,A+374\,B+429\,C)a\cos\left(dx+c\right)^{4}+3\,(336\,A+374\,B+429\,C)a\cos}{3465\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(13/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(8*(336*A + 374*B + 429*C)*a*cos(d*x + c)^5 + 4*(336*A + 374*B + 429
*C)*a*cos(d*x + c)^4 + 3*(336*A + 374*B + 429*C)*a*cos(d*x + c)^3 + 5*(168*
A + 187*B + 99*C)*a*cos(d*x + c)^2 + 35*(21*A + 11*B)*a*cos(d*x + c) + 315*
A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(13/2), x)
```

3.493 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=333

$$\frac{a^3(680A+628B+545C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{960d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(1304A+1132B+1015C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{768d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(120A}{}$$

```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
a*Cos[c + d*x]]])/(512*d) + (a^3*(1304*A + 1132*B + 1015*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(512*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1304*A + 1132*B
+ 1015*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]])
 + (a^3*(680*A + 628*B + 545*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(960*d*Sqr
t[a + a*Cos[c + d*x]]) + (a^2*(120*A + 156*B + 115*C)*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(480*d) + (a*(12*B + 5*C)*Cos[c + d*x]
^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(60*d) + (C*Cos[c + d*x]^(5
/2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.989966, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(680A+628B+545C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{960d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(1304A+1132B+1015C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{768d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(120A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
a*Cos[c + d*x]]])/(512*d) + (a^3*(1304*A + 1132*B + 1015*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(512*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1304*A + 1132*B
+ 1015*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]])
 + (a^3*(680*A + 628*B + 545*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(960*d*Sqr
t[a + a*Cos[c + d*x]]) + (a^2*(120*A + 156*B + 115*C)*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(480*d) + (a*(12*B + 5*C)*Cos[c + d*x]
^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(60*d) + (C*Cos[c + d*x]^(5
/2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\sin}{6d}$$

$$= \frac{a(12B+5C)\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+}{60d}$$

$$= \frac{a^2(120A+156B+115C)\cos^{\frac{5}{2}}(c+dx)\sqrt{}}{480d}$$

$$= \frac{a^3(680A+628B+545C)\cos^{\frac{5}{2}}(c+dx)}{960d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(1304A+1132B+1015C)\cos^{\frac{3}{2}}(c+}{768d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(1304A+1132B+1015C)\sqrt{\cos(c+}}{512d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(1304A+1132B+1015C)\sqrt{\cos(c+}}{512d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{5/2}(1304A+1132B+1015C)\sin^{-1}\left(\frac{}{\sqrt{}}\right.}{512d}$$

Mathematica [A] time = 2.16488, size = 205, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(1304A+1132B+1015C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
 + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(1304*A + 1132
*B + 1015*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(23240
*A + 22084*B + 20965*C + 2*(7240*A + 7748*B + 8085*C)*Cos[c + d*x] + 4*(920
*A + 1324*B + 1575*C)*Cos[2*(c + d*x)] + 480*A*Cos[3*(c + d*x)] + 1392*B*Co
s[3*(c + d*x)] + 2140*C*Cos[3*(c + d*x)] + 192*B*Cos[4*(c + d*x)] + 560*C*C
os[4*(c + d*x)] + 80*C*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

Maple [B] time = 0.146, size = 841, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x)
```

```
[Out] 1/7680/d*a^2*(-1+cos(d*x+c))^4*(1920*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*si
n(d*x+c)*cos(d*x+c)^5+11200*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^
4*sin(d*x+c)+1536*B*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)+29680*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+7104*B
*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1280*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^7+53000*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+14624*B*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)+4480*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)^6+52160*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(5/2)+20376*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
+6960*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19560*A*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+28300*B*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8120*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)+16980*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+10150*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)+15225*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1956
0*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))+16980*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))+15225*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/sin(d*
x+c)^8/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 6.93179, size = 679, normalized size = 2.04

$$\frac{\left(1280\,Ca^2\cos\left(dx+c\right)^5+128\left(12\,B+35\,C\right)a^2\cos\left(dx+c\right)^4+48\left(40\,A+116\,B+145\,C\right)a^2\cos\left(dx+c\right)^3+8\left(920\,A+113\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 1/7680*((1280*C*a^2*cos(d*x + c)^5 + 128*(12*B + 35*C)*a^2*cos(d*x + c)^4 +
 48*(40*A + 116*B + 145*C)*a^2*cos(d*x + c)^3 + 8*(920*A + 1132*B + 1015*C)
*a^2*cos(d*x + c)^2 + 10*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c) + 15*(
1304*A + 1132*B + 1015*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) - 15*((1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c) + (1304*A +
1132*B + 1015*C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d
*x+c)**2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.494 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=281

$$\frac{a^3(1040A+950B+787C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{960d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+110B+79C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}{240d}$$

```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(128*d) + (a^3*(400*A + 326*B + 283*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1040*A + 950*B + 787*C
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*
(80*A + 110*B + 79*C)*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(240*d) + (a*(2*B + C)*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(8*d) + (C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(5*d)
```

Rubi [A] time = 0.881471, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(1040A+950B+787C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{960d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+110B+79C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```



```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(128*d) + (a^3*(400*A + 326*B + 283*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(1040*A + 950*B + 787*C
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*
(80*A + 110*B + 79*C)*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(240*d) + (a*(2*B + C)*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(8*d) + (C*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(5*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\sin}{5d}$$

$$= \frac{a(2B+C)\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}{8d}$$

$$= \frac{a^2(80A+110B+79C)\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}{240d}$$

$$= \frac{a^3(1040A+950B+787C)\cos^{\frac{3}{2}}(c+dx}{960d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(400A+326B+283C)\sqrt{\cos(c+dx)}}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^3(400A+326B+283C)\sqrt{\cos(c+dx)}}{128d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^{5/2}(400A+326B+283C)\sin^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{a+a}}\right.}{128d}$$

Mathematica [A] time = 1.70187, size = 171, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(400A+326B+283C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
 + C*Cos[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(400*A + 326*B
 + 283*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6320*A +
 5810*B + 5521*C + (2720*A + 3620*B + 3874*C)*Cos[c + d*x] + 4*(80*A + 230*
B + 331*C)*Cos[2*(c + d*x)] + 120*B*Cos[3*(c + d*x)] + 348*C*Cos[3*(c + d*x
)] + 48*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(3840*d)
```

Maple [B] time = 0.128, size = 733, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x)
```

```
[Out] -1/1920/d*a^2*(-1+cos(d*x+c))^3*(640*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*co
s(d*x+c)^4*sin(d*x+c)+4000*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)+480*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
+12080*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2320*B*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+384*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6+14720*A*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+5100*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)+1392*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+6000*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+8150*B*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2264*C*sin(d*x+c)*
cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4890*B*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2830*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)+4245*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+6000*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)/cos(d*x+c))+4890*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4245*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^(1/2)/sin(d*x+c)^6/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 5.84095, size = 593, normalized size = 2.11

$$\left(384\, C a^2 \cos\left(dx+c\right)^4 + 48\left(10\, B + 29\, C\right) a^2 \cos\left(dx+c\right)^3 + 8\left(80\, A + 230\, B + 283\, C\right) a^2 \cos\left(dx+c\right)^2 + 10\left(272\, A + 326\, B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 1/1920*((384*C*a^2*cos(d*x + c)^4 + 48*(10*B + 29*C)*a^2*cos(d*x + c)^3 + 8
*(80*A + 230*B + 283*C)*a^2*cos(d*x + c)^2 + 10*(272*A + 326*B + 283*C)*a^2
*cos(d*x + c) + 15*(400*A + 326*B + 283*C)*a^2)*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c) - 15*((400*A + 326*B + 283*C)*a^2*cos(d*x + c
) + (400*A + 326*B + 283*C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.495 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=233

$$\frac{a^{5/2}(304A+200B+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+392B+299C)\sin(c+dx)\sqrt{\cos(c+dx)}}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(16A+24B}{}$$

```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(64*d) + (a^3*(432*A + 392*B + 299*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(16*A + 24*B + 17*C)*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(32*d) + (a*(8*B + 5
*C)*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C
*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.78535, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3045, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(304A+200B+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+392B+299C)\sin(c+dx)\sqrt{\cos(c+dx)}}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(16A+24B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```


```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(64*d) + (a^3*(432*A + 392*B + 299*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(16*A + 24*B + 17*C)*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(32*d) + (a*(8*B + 5
*C)*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C
*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{4d} + \cdots$$

$$= \frac{a(8B+5C)\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+\cdots}{24d}$$

$$= \frac{a^2(16A+24B+17C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d\cdots}}{32d}$$

$$= \frac{a^3(432A+392B+299C)\sqrt{\cos(c+dx)}\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{a^3(432A+392B+299C)\sqrt{\cos(c+dx)}\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \cdots$$

$$= \frac{a^{5/2}(304A+200B+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \cdots$$

Mathematica [A] time = 1.06692, size = 146, normalized size = 0.63

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(304A+200B+163C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(\cdots}\right.}{38\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(304*A + 200*B
+ 163*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(528*A + 6
32*B + 581*C + (96*A + 272*B + 362*C)*Cos[c + d*x] + 4*(8*B + 23*C)*Cos[2*(
c + d*x)] + 12*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.124, size = 625, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(96*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)+720*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(5/2)+64*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1152
*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+336*B*cos(d*x+c)
^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+48*C*sin(d*x+c)*cos(d*x+c)^
5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+528*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)+872*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
+184*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+600*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+326*C*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*C*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+912*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+600*B*cos(d*x+c)^2*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+489*C*cos(d*x+c)^2*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(
1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 5.81653, size = 520, normalized size = 2.23

$$\frac{\left(48\,Ca^2\cos\left(dx+c\right)^3+8\,(8\,B+23\,C)a^2\cos\left(dx+c\right)^2+2\,(48\,A+136\,B+163\,C)a^2\cos\left(dx+c\right)+3\,(176\,A+200\,B+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*((48*C*a^2*cos(d*x + c)^3 + 8*(8*B + 23*C)*a^2*cos(d*x + c)^2 + 2*(48
*A + 136*B + 163*C)*a^2*cos(d*x + c) + 3*(176*A + 200*B + 163*C)*a^2)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((304*A + 200*B + 1
63*C)*a^2*cos(d*x + c) + (304*A + 200*B + 163*C)*a^2)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x +
 c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/sqrt(cos(d*x + c)), x)
```

3.496 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=231

$$\frac{a^{5/2}(40A+38B+25C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-54B-49C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-2B-3C)\sin}{}$$

```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(8*d) - (a^3*(24*A - 54*B - 49*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) - (a^2*(8*A - 2*B - 3*C)*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (a*(6*A - C)*Sqrt[Cos[c
 + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c
 + d*x])^(5/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.793108, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(40A+38B+25C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-54B-49C)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-2B-3C)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```



```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]])/(8*d) - (a^3*(24*A - 54*B - 49*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) - (a^2*(8*A - 2*B - 3*C)*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (a*(6*A - C)*Sqrt[Cos[c
 + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c
 + d*x])^(5/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c}{}}{}$$

$$= -\frac{a(6A-C)\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(}{3d}$$

$$= -\frac{a^2(8A-2B-3C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx}}{4d}$$

$$= -\frac{a^3(24A-54B-49C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{a^3(24A-54B-49C)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} -$$

$$= \frac{a^{5/2}(40A+38B+25C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} - \frac{a^3(}{}$$

Mathematica [A] time = 1.06524, size = 156, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(40A+38B+25C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+\right.\right.}{48d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(40*A + 38*B +
25*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(48*A + 6*B +
 17*C + 3*(8*A + 22*B + 27*C)*Cos[c + d*x] + (6*B + 17*C)*Cos[2*(c + d*x)]
+ 2*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.12, size = 553, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(24*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)+96*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)+12*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+120*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+78*B*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*C*sin(d*x+c)*cos(d*x+c)^5*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)+66*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+34*C*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*C*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+120*A*cos(d*x+c)^3*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+114*B*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3+75*C*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3)*(a*(1+c
os(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)
```

Maxima [B] time = 3.44707, size = 5457, normalized size = 23.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
```

```
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + (4*
(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c)
 - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*
a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
```

1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a
^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1
))*sqrt(a))*C + 24*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*sin(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(

```
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sqrt(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/d
```

Fricas [A] time = 3.92781, size = 517, normalized size = 2.24

$$\frac{\left(8\,Ca^2\cos(dx+c)^3 + 2\,(6\,B+17\,C)a^2\cos(dx+c)^2 + 3\,(8\,A+22\,B+25\,C)a^2\cos(dx+c) + 48\,Aa^2\right)\sqrt{a\cos(dx+c)+}}{24\,(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*C*a^2*cos(d*x + c)^3 + 2*(6*B + 17*C)*a^2*cos(d*x + c)^2 + 3*(8*A
+ 22*B + 25*C)*a^2*cos(d*x + c) + 48*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))*sin(d*x + c) - 3*((40*A + 38*B + 25*C)*a^2*cos(d*x + c)^2 + (4
0*A + 38*B + 25*C)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(3/2), x)
```

3.497 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=233

$$\frac{a^{5/2}(8A+20B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(56A+12B-27C)\sin(c+dx)\sqrt{\cos(c+dx)}}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A+4B-C)\sin(c$$

```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/(4*d) - (a^3*(56*A + 12*B - 27*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(12*d*Sqrt[a + a*Cos[c + d*x]]) - (a^2*(8*A + 4*B - C)*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*(5*A + 3*B)*(a + a*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.798877, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3043, 2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(8A+20B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(56A+12B-27C)\sin(c+dx)\sqrt{\cos(c+dx)}}{12d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A+4B-C)\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```


```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/(4*d) - (a^3*(56*A + 12*B - 27*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(12*d*Sqrt[a + a*Cos[c + d*x]]) - (a^2*(8*A + 4*B - C)*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*(5*A + 3*B)*(a + a*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + a*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
```

```
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(\ldots}{\ldots}}{\ldots}$$

$$= \frac{2a(5A+3B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A\ldots}{\ldots}$$

$$= -\frac{a^2(8A+4B-C)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\ldots}{2d}$$

$$= -\frac{a^3(56A+12B-27C)\sqrt{\cos(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} - \ldots$$

$$= -\frac{a^3(56A+12B-27C)\sqrt{\cos(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} - \ldots$$

$$= \frac{a^{5/2}(8A+20B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(5\ldots}{\ldots}$$

Mathematica [A] time = 1.07375, size = 156, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(6\sqrt{2}(8A+20B+19C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\sin\left(\frac{1}{2}(c+\ldots\right.\right.}{48d\cos^{\frac{3}{2}}(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(6*Sqrt[2]*(8*A + 20*B + 1
9*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(16*A + 12*B +
 33*C + (128*A + 48*B + 9*C)*Cos[c + d*x] + 3*(4*B + 11*C)*Cos[2*(c + d*x)]
 + 3*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 0.117, size = 514, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c)))^(1/2)*(24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*A*cos
(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6
0*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6*C*sin(d*x+c)*cos(d*x+c)^3+57*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*A*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+12
*B*sin(d*x+c)*cos(d*x+c)^2+60*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*B*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+33*C*sin(
d*x+c)*cos(d*x+c)^2+57*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*C*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+64*A*sin(d*x+c)*
cos(d*x+c)+24*B*sin(d*x+c)*cos(d*x+c)+8*A*sin(d*x+c))*a^2/(1+cos(d*x+c))/co
s(d*x+c)^(3/2)
```

Maxima [B] time = 3.12935, size = 4690, normalized size = 20.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C + 12*
(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
```

```
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 8
*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4) + 8*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(2*d*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c)
+ 4*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c)
- a^2 + 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 +
 a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
```

```
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - 1))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1))/d
```

Fricas [A] time = 3.97738, size = 508, normalized size = 2.18

$$\frac{\left(6\,Ca^2\cos\left(dx+c\right)^3+3\,(4\,B+11\,C)a^2\cos\left(dx+c\right)^2+8\,(8\,A+3\,B)a^2\cos\left(dx+c\right)+8\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx\right.}}{12\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*((6*C*a^2*cos(d*x + c)^3 + 3*(4*B + 11*C)*a^2*cos(d*x + c)^2 + 8*(8*A
+ 3*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c) - 3*((8*A + 20*B + 19*C)*a^2*cos(d*x + c)^3 + (8*A + 20*B
+ 19*C)*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(5/2), x)
```

3.498 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{a^3(64A+70B+15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+10B+5C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(2B+5C}{}$$

```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^3*(64*A + 70*B + 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqr
t[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 10*B + 5*C)*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*(A + B)*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*(a + a*Cos[c + d*x])^(
5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.808836, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2981, 2774, 216}

$$-\frac{a^3(64A+70B+15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+10B+5C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(2B+5C}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```



```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^3*(64*A + 70*B + 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqr
t[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 10*B + 5*C)*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*(A + B)*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*(a + a*Cos[c + d*x])^(
5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(}{}}{}$$

$$= \frac{2a(A+B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(}{}$$

$$= \frac{2a^2(8A+10B+5C)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a^3(64A+70B+15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{a^3(64A+70B+15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{a^{5/2}(2B+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3(64A+70}{1}$$

Mathematica [A] time = 1.3299, size = 156, normalized size = 0.7

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((112A+40B+45C)\cos(c+dx)+4(43A+40B+15C)\cos(\right.}{120d\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(60*Sqrt[2]*(2*B + 5*C)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(196*A + 160*B + 60*C
 + (112*A + 40*B + 45*C)*Cos[c + d*x] + 4*(43*A + 40*B + 15*C)*Cos[2*(c + d
*x)] + 15*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(120*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.105, size = 479, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x)
```

```
[Out] -1/15/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))*cos(d*x+c)^2-60*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)-30*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))-75*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)^2-75*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+15*C*cos(d*x+c)^4+
86*A*cos(d*x+c)^3+80*B*cos(d*x+c)^3+15*C*cos(d*x+c)^3-58*A*cos(d*x+c)^2-70*
B*cos(d*x+c)^2-30*C*cos(d*x+c)^2-22*A*cos(d*x+c)-10*B*cos(d*x+c)-6*A)/sin(d
*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.7966, size = 3401, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*(15*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
```

2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4) + 10*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(2*d*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d
*x + 2*c) + 4*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x
+ 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d
*x + 2*c) - a^2 + 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*sqrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^
2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*s
in(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x

```
+ 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) + 32*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x +
c) + 1) - 35*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(
2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)))/d
```

Fricas [A] time = 2.37737, size = 501, normalized size = 2.25

$$\frac{\left(15\,Ca^2\cos\left(dx+c\right)^3+2\left(43\,A+40\,B+15\,C\right)a^2\cos\left(dx+c\right)^2+2\left(14\,A+5\,B\right)a^2\cos\left(dx+c\right)+6\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+}}{15\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*((15*C*a^2*cos(d*x + c)^3 + 2*(43*A + 40*B + 15*C)*a^2*cos(d*x + c)^2
+ 2*(14*A + 5*B)*a^2*cos(d*x + c) + 6*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))*sin(d*x + c) - 15*((2*B + 5*C)*a^2*cos(d*x + c)^4 + (2*B + 5*
C)*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(7/2), x)
```

3.499 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=222

$$\frac{2a^2(40A+56B+35C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(160A+224B+245C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{a\cos(c+}}\right.}{d}$$

```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(160*A + 224*B + 245*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]) + (2*a^2*(40*A + 56*B + 35*C)*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 7*B)*(a + a*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.724433, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2980, 2774, 216}

$$\frac{2a^2(40A+56B+35C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(160A+224B+245C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{a\cos(c+}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```

```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(160*A + 224*B + 245*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]) + (2*a^2*(40*A + 56*B + 35*C)*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 7*B)*(a + a*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + a*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2a(5A+7B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(}{}$$

$$= \frac{2a^2(40A+56B+35C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^3(160A+224B+245C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(40A+}{}$$

$$= \frac{2a^3(160A+224B+245C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(40A+}{}$$

$$= \frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(160A+224B+2}{105d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [A] time = 1.55055, size = 172, normalized size = 0.77

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((930A+987B+840C)\cos(c+dx)+2(115A+98B+35C)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(420*Sqrt[2]*C*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 2*(290*A + 196*B + 70*C + (930*A
 + 987*B + 840*C)*Cos[c + d*x] + 2*(115*A + 98*B + 35*C)*Cos[2*(c + d*x)] +
 230*A*Cos[3*(c + d*x)] + 301*B*Cos[3*(c + d*x)] + 280*C*Cos[3*(c + d*x)])*
Sin[(c + d*x)/2]))/(420*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.107, size = 369, normalized size = 1.7

$$-\frac{2a^2}{105\,d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(-105\,C\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x)
```

```
[Out] -2/105/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-105*C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))*cos(d*x+c)^3-210*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2-105*
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+230*A*cos(d*x+c)^4+301*B*cos
(d*x+c)^4+280*C*cos(d*x+c)^4-115*A*cos(d*x+c)^3-203*B*cos(d*x+c)^3-245*C*co
s(d*x+c)^3-55*A*cos(d*x+c)^2-77*B*cos(d*x+c)^2-35*C*cos(d*x+c)^2-45*A*cos(d
*x+c)-21*B*cos(d*x+c)-15*A)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 2.23129, size = 2415, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 1/210*(35*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a
^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) +
```

```
3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*
c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^
2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
 + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*s
qrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
) + 112*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^
(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)) + 80*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c)
 + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1
)))/d
```

Fricas [A] time = 2.08914, size = 502, normalized size = 2.26

$$\frac{2\left(\left((230\,A+301\,B+280\,C)a^2\cos(dx+c)^3+(115\,A+98\,B+35\,C)a^2\cos(dx+c)^2+3\,(20\,A+7\,B)a^2\cos(dx+c)+1\right.\right.}{105\left(d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(((230*A + 301*B + 280*C)*a^2*cos(d*x + c)^3 + (115*A + 98*B + 35*C)*
a^2*cos(d*x + c)^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 105*(C*a^2*cos(d*x + c)^5
+ C*a^2*cos(d*x + c)^4)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(9/2), x)
```

3.500 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2a^3(8A+10B+11C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(64A+90B+63C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(584A+690B+90}{315d\sqrt{\cos(c+dx)}\sqrt{a\,c}}$$

```
[Out] (2*a^3*(8*A + 10*B + 11*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a +
a*Cos[c + d*x]]) + (2*a^3*(584*A + 690*B + 903*C)*Sin[c + d*x])/(315*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 90*B + 63*C)*Sqrt
[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*a*(5*A +
 9*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) +
(2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.808779, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3043, 2975, 2980, 2771}

$$\frac{2a^3(8A+10B+11C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(64A+90B+63C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(584A+690B+90}{315d\sqrt{\cos(c+dx)}\sqrt{a\,c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(11/2),x]
```


```
[Out] (2*a^3*(8*A + 10*B + 11*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a +
a*Cos[c + d*x]]) + (2*a^3*(584*A + 690*B + 903*C)*Sin[c + d*x])/(315*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 90*B + 63*C)*Sqrt
[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*a*(5*A +
 9*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) +
(2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(}{}}{}$$

$$= \frac{2a(5A+9B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2a^2(64A+90B+63C)\sqrt{a+a\cos(c+dx)}\sin(c+}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^3(8A+10B+11C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(64A+}{}$$

$$= \frac{2a^3(8A+10B+11C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(584A+}{315d\sqrt{\cos(}}$$

Mathematica [A] time = 1.21635, size = 158, normalized size = 0.68

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(2(1396A+1215B+882C)\cos(c+dx)+4(803A+870B+966C)\cos(2(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(2908*A + 2790*B + 2961*C + 2*(1396*A + 121
5*B + 882*C)*Cos[c + d*x] + 4*(803*A + 870*B + 966*C)*Cos[2*(c + d*x)] + 58
4*A*Cos[3*(c + d*x)] + 690*B*Cos[3*(c + d*x)] + 588*C*Cos[3*(c + d*x)] + 58
4*A*Cos[4*(c + d*x)] + 690*B*Cos[4*(c + d*x)] + 903*C*Cos[4*(c + d*x)])*Tan
[(c + d*x)/2])/(1260*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.123, size = 166, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(584A(\cos(dx+c))^4+690B(\cos(dx+c))^4+903C(\cos(dx+c))^4+292A(\cos(dx+c))^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+903*C*c
os(d*x+c)^4+292*A*cos(d*x+c)^3+345*B*cos(d*x+c)^3+294*C*cos(d*x+c)^3+219*A*
cos(d*x+c)^2+180*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+130*A*cos(d*x+c)+45*B*cos
(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.73398, size = 921, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 8/315*(21*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7)*C/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(7/2)) + 15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x +
c) + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(
2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(co
s(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 1)) + (315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.82521, size = 379, normalized size = 1.62

$$\frac{2\left(\left(584\,A+690\,B+903\,C\right)a^{2}\cos\left(dx+c\right)^{4}+\left(292\,A+345\,B+294\,C\right)a^{2}\cos\left(dx+c\right)^{3}+3\left(73\,A+60\,B+21\,C\right)a^{2}\cos\left(\right.\right.}{315\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*((584*A + 690*B + 903*C)*a^2*cos(d*x + c)^4 + (292*A + 345*B + 294*C)
*a^2*cos(d*x + c)^3 + 3*(73*A + 60*B + 21*C)*a^2*cos(d*x + c)^2 + 5*(26*A +
 9*B)*a^2*cos(d*x + c) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(11/2), x)
```

3.501 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2a^3(2840A+3212B+3795C)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(32A+44B+33C)\sin(c+}{231d\cos^{\frac{7}{2}}(c}$$

```
[Out] (2*a^3*(1160*A + 1364*B + 1485*C)*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(5/2)*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sin[c + d*x])
/(3465*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(2840*A + 32
12*B + 3795*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])
/(231*d*Cos[c + d*x]^(7/2)) + (2*a*(5*A + 11*B)*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*S
in[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 0.895296, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2980, 2772, 2771}

$$\frac{2a^3(2840A+3212B+3795C)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(32A+44B+33C)\sin(c+}{231d\cos^{\frac{7}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(13/2),x]
```


```
[Out] (2*a^3*(1160*A + 1364*B + 1485*C)*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(5/2)*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sin[c + d*x])
/(3465*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(2840*A + 32
12*B + 3795*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])
/(231*d*Cos[c + d*x]^(7/2)) + (2*a*(5*A + 11*B)*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*S
in[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2a(5A+11B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2a^2(32A+44B+33C)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{231d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(32A}{}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(284}{3465d}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(284}{3465d}$$

Mathematica [A] time = 0.934449, size = 190, normalized size = 0.67

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}((50140A+49654B+49830C)\cos(c+dx)+4(4615A+4642B+4290C)\cos(2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(18140*A + 15356*B + 13365*C + (50140*A + 4
9654*B + 49830*C)*Cos[c + d*x] + 4*(4615*A + 4642*B + 4290*C)*Cos[2*(c + d*
x)] + 18460*A*Cos[3*(c + d*x)] + 20878*B*Cos[3*(c + d*x)] + 22935*C*Cos[3*(
c + d*x)] + 2840*A*Cos[4*(c + d*x)] + 3212*B*Cos[4*(c + d*x)] + 3795*C*Cos[
4*(c + d*x)] + 2840*A*Cos[5*(c + d*x)] + 3212*B*Cos[5*(c + d*x)] + 3795*C*C
os[5*(c + d*x)])*Tan[(c + d*x)/2])/(13860*d*Cos[c + d*x]^(11/2))
```

Maple [A] time = 0.104, size = 199, normalized size = 0.7

$$-\frac{2 a^2 (-1+\cos(dx+c)) \left(5680 A (\cos(dx+c))^5 + 6424 B (\cos(dx+c))^5 + 7590 C (\cos(dx+c))^5 + 2840 A (\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2
),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+7590
*C*cos(d*x+c)^5+2840*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+3795*C*cos(d*x+c)^4
+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3+1980*C*cos(d*x+c)^3+1775*A*cos(d*x
+c)^2+1430*B*cos(d*x+c)^2+495*C*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*
x+c)+315*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 1.79716, size = 1170, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(13/2),x, algorithm="maxima")
```

```
[Out] 8/3465*(165*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2
)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*C*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 11*(315*sqrt
```

```
(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572
*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
+ 5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(2)*a^(
5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1300*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a^(5/2)
*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(co
s(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + s
in(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.85883, size = 462, normalized size = 1.63

$$\frac{2\left(2\,(2840\,A+3212\,B+3795\,C)a^2\cos{(dx+c)}^5+(2840\,A+3212\,B+3795\,C)a^2\cos{(dx+c)}^4+3\,(710\,A+803\,B+660}{346}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(13/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(2*(2840*A + 3212*B + 3795*C)*a^2*cos(d*x + c)^5 + (2840*A + 3212*B
+ 3795*C)*a^2*cos(d*x + c)^4 + 3*(710*A + 803*B + 660*C)*a^2*cos(d*x + c)^3
 + 5*(355*A + 286*B + 99*C)*a^2*cos(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d
*x + c) + 315*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x +
c)/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(13/2), x)
```

3.502 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=334

$$\frac{8a^3(8368A+9230B+10439C)\sin(c+dx)}{45045d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(8368A+9230B+10439C)\sin(c+dx)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(2224A+2522B+2717\ldots}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\,\mathrm{co}\ldots}}$$

```
[Out] (2*a^3*(2224*A + 2522*B + 2717*C)*Sin[c + d*x])/(9009*d*Cos[c + d*x]^(7/2)*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x]
)/(15015*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(8368*A +
9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[
c + d*x]]) + (16*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1287*d*Cos[c + d*x]^(9/2)) + (2*a*(
5*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(143*d*Cos[c + d*x]^(1
1/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13*d*Cos[c + d*x]^(1
3/2))
```

Rubi [A] time = 0.989349, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2975, 2980, 2772, 2771}

$$\frac{8a^3(8368A+9230B+10439C)\sin(c+dx)}{45045d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(8368A+9230B+10439C)\sin(c+dx)}{15015d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(2224A+2522B+2717\ldots}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\,\mathrm{co}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(15/2),x]
```

```
[Out] (2*a^3*(2224*A + 2522*B + 2717*C)*Sin[c + d*x])/(9009*d*Cos[c + d*x]^(7/2)*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x]
)/(15015*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(8368*A +
9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[
c + d*x]]) + (16*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(1287*d*Cos[c + d*x]^(9/2)) + (2*a*(
5*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(143*d*Cos[c + d*x]^(1
1/2)) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13*d*Cos[c + d*x]^(1
```

```
3/2))
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{2A(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{13d\cos^{\frac{13}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2a(5A+13B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{143d\cos^{\frac{11}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2a^2(136A+182B+143C)\sqrt{a+a\cos(c+dx)}\sin(c}{1287d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(13}{}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(83}{1501}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(83}{1501}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(83}{1501}$$

Mathematica [A] time = 1.21634, size = 224, normalized size = 0.67

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}(70(5552A+5083B+4576C)\cos(c+dx)+14(30334A+31850B+32747C)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(15/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(343612*A + 325910*B + 322751*C + 70*(5552*
A + 5083*B + 4576*C)*Cos[c + d*x] + 14*(30334*A + 31850*B + 32747*C)*Cos[2*
(c + d*x)] + 125520*A*Cos[3*(c + d*x)] + 138450*B*Cos[3*(c + d*x)] + 141570
*C*Cos[3*(c + d*x)] + 125520*A*Cos[4*(c + d*x)] + 138450*B*Cos[4*(c + d*x)]
 + 156585*C*Cos[4*(c + d*x)] + 16736*A*Cos[5*(c + d*x)] + 18460*B*Cos[5*(c
+ d*x)] + 20878*C*Cos[5*(c + d*x)] + 16736*A*Cos[6*(c + d*x)] + 18460*B*Cos
[6*(c + d*x)] + 20878*C*Cos[6*(c + d*x)])*Tan[(c + d*x)/2])/(180180*d*Cos[c
 + d*x]^(13/2))
```

Maple [A] time = 0.11, size = 232, normalized size = 0.7

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(66944\,A\,(\cos(dx+c))^6+73840\,B\,(\cos(dx+c))^6+83512\,C\,(\cos(dx+c))^6+33472\,A\,(\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2
),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+73840*B*cos(d*x+c)^6+8
3512*C*cos(d*x+c)^6+33472*A*cos(d*x+c)^5+36920*B*cos(d*x+c)^5+41756*C*cos(d
*x+c)^5+25104*A*cos(d*x+c)^4+27690*B*cos(d*x+c)^4+31317*C*cos(d*x+c)^4+2092
0*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18590*C*cos(d*x+c)^3+18305*A*cos(d*x+
c)^2+14560*B*cos(d*x+c)^2+5005*C*cos(d*x+c)^2+11970*A*cos(d*x+c)+4095*B*cos
(d*x+c)+3465*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(13/2)
```

Maxima [B] time = 1.80642, size = 1355, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(15/2),x, algorithm="maxima")
```

```
[Out] 8/45045*(143*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqr
t(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*C*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)) + 65*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c)
+ 1) - 2310*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt
(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^1
1 + 200*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*B*(sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(1
3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + (45045*sq
rt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*sqrt(2)*a^(5/2)*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 289185*s
qrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 88980*sqrt(2)*a^(5/2
)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqrt(2)*a^(5/2)*sin(d*x + c
)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/(
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)/(cos(d*x + c) +
 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.94444, size = 547, normalized size = 1.64

$$\frac{2\left(8\,(8368\,A + 9230\,B + 10439\,C)a^2\cos{(dx+c)}^6 + 4\,(8368\,A + 9230\,B + 10439\,C)a^2\cos{(dx+c)}^5 + 3\,(8368\,A + 9230\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(15/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(8*(8368*A + 9230*B + 10439*C)*a^2*cos(d*x + c)^6 + 4*(8368*A + 923
0*B + 10439*C)*a^2*cos(d*x + c)^5 + 3*(8368*A + 9230*B + 10439*C)*a^2*cos(d
```

```
*x + c)^4 + 5*(4184*A + 4615*B + 3718*C)*a^2*cos(d*x + c)^3 + 35*(523*A + 4
16*B + 143*C)*a^2*cos(d*x + c)^2 + 315*(38*A + 13*B)*a^2*cos(d*x + c) + 346
5*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*
x + c)^8 + d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(15/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(15/2), x)
```

3.503 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=241

$$-\frac{(8A-14B+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A-2B+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{2}\sqrt{\cos(c+d}}\right.}{\sqrt{a}d}$$

```
[Out] -((8*A - 14*B + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((8*A - 2*
B + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
((6*B - C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]])
 + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.845151, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3045, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(8A-14B+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A-2B+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{2}\sqrt{\cos(c+d}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + a
*Cos[c + d*x]],x]
```



```
[Out] -((8*A - 14*B + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((8*A - 2*
B + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
((6*B - C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]])
 + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(6A+5C)+\frac{1}{2}a(6B-C)\right.}{\sqrt{a+a\cos(c+dx)}}}{3a}$$

$$= \frac{(6B-C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+}{3d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(8A-2B+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{(6B-C)\cos^{\frac{3}{2}}(c}{12d\sqrt{a+a}}$$

$$= \frac{(8A-2B+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{(6B-C)\cos^{\frac{3}{2}}(c}{12d\sqrt{a+a}}$$

$$= \frac{(8A-2B+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{(6B-C)\cos^{\frac{3}{2}}(c}{12d\sqrt{a+a}}$$

$$= -\frac{(8A-14B+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B+C)\,\mathrm{ta}}{}$$

Mathematica [C] time = 2.92385, size = 449, normalized size = 1.86

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(24A+2(6B-C)\cos(c+dx)-6B+4C\cos(2(c+dx))+25C)+\frac{3\sqrt{2}e^{\frac{1}{2}i(c}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*((3*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(-8*A*d*x + 14*B*d*x - 9*C*d*x + I*(8*A - 14*B + 9*C
)*ArcSinh[E^(I*(c + d*x))] - (16*I)*Sqrt[2]*(A - B + C)*Log[1 + E^(I*(c + d
*x))] - (8*I)*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + (14*I)*B*Log[1 + S
qrt[1 + E^((2*I)*(c + d*x))]] - (9*I)*C*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x)
)]] + (16*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))]] - (16*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 +
E^((2*I)*(c + d*x))]] + (16*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + 4*Sqrt[Cos
[c + d*x]]*(24*A - 6*B + 25*C + 2*(6*B - C)*Cos[c + d*x] + 4*C*Cos[2*(c + d
*x)])*Sin[(c + d*x)/2]))/(48*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.125, size = 613, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^4*(-24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)-48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)-12*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-24*A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-6*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)-8*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2
*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*2^(1/2)*c
os(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))-24*B*2^(1/2)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*cos(d*x+c)^2+24*C*2^(1/2)*cos(d*x+c)^2*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-21*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+24*A*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))-42*B*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))+27*C*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(cos
(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^8/a
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a
*cos(d*x + c) + a), x)
```

3.504 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{(8A-4B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(4B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((8*A - 4*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((4*B - C)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.625072, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3045, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A-4B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(4B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + a
*Cos[c + d*x]],x]
```


```
[Out] ((8*A - 4*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((4*B - C)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (C*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
```

```
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(4A+3C)+\frac{1}{2}a(4B-C)\right.}{\sqrt{a+a\cos(c+dx)}}}{2a}$$

$$= \frac{(4B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+}{2d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(4B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+}{2d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(4B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+}{2d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{(8A-4B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B+C)\tan}{}$$

Mathematica [C] time = 2.15324, size = 431, normalized size = 2.21

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(4B+2C\cos(c+dx)-C)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(8i\sqrt{2}(A-B+C)\log\left(1+e^{i(c+dx)}\right)-i(8A-4B+7C)\,\mathrm{s}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(8*A*d*x - 4*B*d*x + 7*C*d*x - I*(8*A - 4*B + 7*C)*Arc
Sinh[E^(I*(c + d*x))] + (8*I)*Sqrt[2]*(A - B + C)*Log[1 + E^(I*(c + d*x))]
+ (8*I)*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (4*I)*B*Log[1 + Sqrt[1 +
 E^((2*I)*(c + d*x))]] + (7*I)*C*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (
```

```
8*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x
))]] + (8*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))]] - (8*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E
^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (4*Sqrt[Cos[c +
d*x]]*(4*B - C + 2*C*Cos[c + d*x])*Sin[(c + d*x)/2])/d))/(8*Sqrt[a*(1 + Cos
[c + d*x])])
```

Maple [B] time = 0.157, size = 421, normalized size = 2.2

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)^{3}}{4\,d\left(\sin\left(dx+c\right)\right)^{6}a}\left(4\,B\cos\left(dx+c\right)\sin\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{3/2}+4\,B\sin\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{3/2}+2\,C\,\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^3*(4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*C*sin(d*x+c)*
cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*2^(1/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))-4*B*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))+4*C*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-C*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x+c)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-4*B*arctan(sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+7*C*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c))*(a*(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/sin(d*x+c)^6/(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(a
*cos(d*x + c) + a), x)
```

3.505 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((2*B - C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a
]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (C*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.439583, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3045, 2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]]),x]
```


```
[Out] ((2*B - C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a
]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (C*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\frac{1}{2}a(2A+C)+\frac{1}{2}a(2B-C)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2B-C)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + (A-B+C)\int$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2B-C)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+d}}\right.}{ad}$$

$$= \frac{(2B-C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos}}\right.}{\sqrt{a}d}$$

Mathematica [A] time = 0.281917, size = 112, normalized size = 0.79

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B+C)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}(2B-C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos}\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(Sqrt[2]*(2*B - C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(
A - B + C)*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*C*Sqrt[Cos[c + d
*x]]*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.118, size = 240, normalized size = 1.7

$$-\frac{(-1+\cos(dx+c))^2}{da(\sin(dx+c))^4}\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}-B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}+C\arcsin\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-B*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+C*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*2^(1/2)-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2*B*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+C*arctan(sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c))
)^(1/2)/a/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 86.897, size = 477, normalized size = 3.38

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(2\,B-C\right)\cos\left(dx+c\right)+2\,B-C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*C*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*B - C)*co
s(d*x + c) + 2*B - C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A -
B + C)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(a*(cos(c + d*x) + 1
))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))), x)
```

3.506 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.424536, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3043, 2982, 2782, 205, 2774, 216}

$$-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]),x]
```

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}aC\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + (-A+B-C)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 5.10152, size = 266, normalized size = 1.93

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{2}(A-B+C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(\sin^4\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[
a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(5*C*Cos[c + d*x]^2*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*
x)/2]]*Sqrt[Cos[c + d*x]] - 2*Sin[(c + d*x)/2]) + 10*B*Cos[c + d*x]^2*Sin[(
c + d*x)/2] - ((A - B + C)*Csc[(c + d*x)/2]^3*(-5*Cos[c + d*x]^2*(2 + Cos[c
 + d*x])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2
)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) + Hypergeometric2F1[2, 5/2, 7/2,
 -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]^2))/2)
)/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.135, size = 303, normalized size = 2.2

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2a}\left(2A\sin(dx+c)(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+4A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+A*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*2^(1/2)*cos(d*x+c)^3-B*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(
d*x+c)^3+C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3+2*C*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3)*(
a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 67.5655, size = 524, normalized size = 3.8

$$\frac{2\sqrt{a\cos(dx+c)+a}A\sqrt{\cos(dx+c)}\sin(dx+c) - 2\left(C\cos(dx+c)^2 + C\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right.}{ad\cos(dx+c)^2 + ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] (2*sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(C*cos(d*
x + c)^2 + C*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B + C)*a*cos(d*x + c)^2
```

```
+ (A - B + C)*a*cos(d*x + c))*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*co
s(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(a*(cos(c + d*x) + 1
))*cos(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{\sqrt{a\cos(dx + c) + a}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*cos(d*x + c)^(3/2)), x)
```

3.507 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.377353, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2984, 12, 2782, 205}

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + a*
Cos[c + d*x]]),x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(2A+3C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} -$$

$$= \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+d}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}$$

Mathematica [C] time = 6.78888, size = 701, normalized size = 4.9

$$2(A-B+C)\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-12\left(3\,\mathrm{si}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[
a + a*Cos[c + d*x]]),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - (8*C*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d
*x)/2]^3)/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2
)) + (8*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d
*x])]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (2*(A - B + C)*Cot[c/2 + (d*x)/2]
*Csc[c/2 + (d*x)/2]^4*(-12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}
, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 +
 (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 -
 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2
 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 +
 (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2
```

```
 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (
d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)
/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(63*d
*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.161, size = 379, normalized size = 2.7

$$-\frac{1}{3\,da\,(1+\cos(dx+c))}\sqrt{a\,(1+\cos(dx+c))}\left(3\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\,(\cos(dx+c))^{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-3\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c)))^(1/2)*(3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)^2*2^(1/2
)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*sin(d
*x+c)*cos(d*x+c)-6*B*sin(d*x+c)*cos(d*x+c)-2*A*sin(d*x+c))/a/(1+cos(d*x+c))
/cos(d*x+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08113, size = 458, normalized size = 3.2

$$-\frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{3\sqrt{2}\left((A-B+C)a\cos(dx+c)^3+(A-B+C)a\cos(dx+c)^2\right)}{\sqrt{a}}}{3\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 3*sqrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*
a*cos(d*x + c)^2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d
*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*cos(d*x + c)^(5/2)), x)
```

3.508 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=191

$$\frac{2(13A-5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A - 5*B + 15*C)*Si
n[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.559269, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2984, 12, 2782, 205}

$$\frac{2(13A-5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + a*
Cos[c + d*x]]),x]
```



```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A - 5*B + 15*C)*Si
n[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+\frac{1}{2}a(4A+5C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}$$

Mathematica [C] time = 8.00007, size = 1950, normalized size = 10.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[
a + a*Cos[c + d*x]]),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(5*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (C*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x
)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (
16*B*Cos[c/2 + (d*x)/2]*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3
/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/(15*d*Sqrt[
a*(1 + Cos[c + d*x])]) - (2*(A - B + C)*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/
2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c
/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^1
0 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hypergeometr
icPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
```

```
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hy
pergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeo
metric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F
1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2
 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 567
00*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/
2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 2
91060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin
[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
- 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/
2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2
+ (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*Sqrt[a*(1 + Cos[c + d*
x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (C
*Cos[c/2 + (d*x)/2]*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5
/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2
 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])))/(15*d*Sqrt[a*(1 + Cos[c +
d*x])])
```

Maple [B] time = 0.119, size = 601, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/15/d*sin(d*x+c)^2*(a*(1+cos(d*x+c)))^(1/2)*(15*A*cos(d*x+c)^3*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-15*B*cos
(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+15*C*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+30*A*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-30*B*cos(d*x+c)^2*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+30*
C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))+15*A*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))-15*B*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+15*C*cos(d*x+c)*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+26*A
*sin(d*x+c)*cos(d*x+c)^2-10*B*sin(d*x+c)*cos(d*x+c)^2+30*C*sin(d*x+c)*cos(d
*x+c)^2-2*A*sin(d*x+c)*cos(d*x+c)+10*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)
)/a/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.094, size = 512, normalized size = 2.68

$$\frac{2\left((13\,A-5\,B+15\,C)\cos\left(dx+c\right)^2-(A-5\,B)\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{15\sqrt{2}}{\cdots}}{15\left(ad\cos\left(dx+c\right)^4+ad\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*((13*A - 5*B + 15*C)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A
- B + C)*a*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x + c)^3)*arctan(1/2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2
 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^
3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*cos(d*x + c)^(7/2)), x)
```

3.509 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=237

$$\frac{2(31A-7B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,\mathrm{c}}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Cos[c
 + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sin[c
 + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(43*A - 9
1*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
])
```

Rubi [A] time = 0.759988, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3043, 2984, 12, 2782, 205}

$$\frac{2(31A-7B+35C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,\mathrm{c}}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + a*
Cos[c + d*x]]),x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Cos[c
 + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sin[c
 + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(43*A - 9
1*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
])
```


Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-7B)+\frac{1}{2}a(6A+7C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+d}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a}}$$

Mathematica [C] time = 10.3501, size = 2716, normalized size = 11.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[
a + a*Cos[c + d*x]]),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(7*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) - (2*C*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d
*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
 + (2*(A - B + C)*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^8*(363825*Sin[c/2 +
 (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2]^4 + 26785605*Sin[c/2 + (d*x)/2]^6
- 86790165*Sin[c/2 + (d*x)/2]^8 + 177677808*Sin[c/2 + (d*x)/2]^10 - 2392830
44*Sin[c/2 + (d*x)/2]^12 + 52080*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 +
```

```
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 560*Hype
rgeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 213120160*Sin[c/2 + (d*
x)/2]^14 - 168280*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 2240*HypergeometricPFQ[
{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 121497024*Sin[c/2 + (d*x)/2]^16 + 212
520*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 3360*HypergeometricPFQ[{2, 2, 2, 2, 1
1/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*
Sin[c/2 + (d*x)/2]^16 + 40125184*Sin[c/2 + (d*x)/2]^18 - 124320*Hypergeomet
ric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
in[c/2 + (d*x)/2]^18 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1,
 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)
/2]^18 - 5840384*Sin[c/2 + (d*x)/2]^20 + 28000*Hypergeometric2F1[2, 11/2, 1
3/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]
^20 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 363825*Ar
cTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 5336100*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 34636140*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 131060160*ArcTanh[
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2
]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 320535600*Ar
cTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (
d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 530671
680*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c
/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] +
 604296000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] - 468948480*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] + 237726720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin
[c/2 + (d*x)/2]^2)] - 70963200*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^18*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)] + 9461760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^20*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1120*Cos[(c + d*x)/2]^6*HypergeometricPFQ[
{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 280*Cos[(c +
d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2}, {1, 13/2}, Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(103 - 164*Sin[c/2 + (d
```

```
*x)/2]^2 + 70*Sin[c/2 + (d*x)/2]^4)))/(40425*d*Sqrt[a*(1 + Cos[c + d*x])]*(
1 - 2*Sin[c/2 + (d*x)/2]^2)^(9/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (8*B*Cos
[c/2 + (d*x)/2]*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)
+ 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (
d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])))/(35*d*Sqrt[a*(1 + Cos[c + d*x]
)]) + (2*C*Cos[c/2 + (d*x)/2]*((5*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x
)/2]^2)^(7/2) + 2*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2
) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 +
 (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))))/(105*d*Sqrt[a*(1 + Cos[c +
d*x])])
```

Maple [B] time = 0.137, size = 805, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/105/d*sin(d*x+c)^4*(a*(1+cos(d*x+c)))^(1/2)*(105*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^4-105*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*B*cos(d*x+c)^4*arcsin((-1+cos(d*x+c
))/sin(d*x+c))+105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^4+315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3-315*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)*B*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c
))+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*2^(1/2)*cos(d*x+c)^3+315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2-315*2^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))+315*C*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*c
os(d*x+c)^2+105*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*2^(1/2)*cos(d*x+c)-105*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*B*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+105*C*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)+86*A*s
in(d*x+c)*cos(d*x+c)^3-182*B*sin(d*x+c)*cos(d*x+c)^3+70*C*sin(d*x+c)*cos(d*
x+c)^3-62*A*sin(d*x+c)*cos(d*x+c)^2+14*B*sin(d*x+c)*cos(d*x+c)^2-70*C*sin(d
*x+c)*cos(d*x+c)^2+6*A*sin(d*x+c)*cos(d*x+c)-42*B*sin(d*x+c)*cos(d*x+c)-30*
A*sin(d*x+c))/a/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12084, size = 571, normalized size = 2.41

$$-\frac{2\left((43\,A-91\,B+35\,C)\cos\left(dx+c\right)^{3}-(31\,A-7\,B+35\,C)\cos\left(dx+c\right)^{2}+3\,(A-7\,B)\cos\left(dx+c\right)-15\,A\right)\sqrt{a\cos\left(dx+\right.}}{105\left(ad\cos\left(dx+c\right)^{5}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(2*((43*A - 91*B + 35*C)*cos(d*x + c)^3 - (31*A - 7*B + 35*C)*cos(d*
x + c)^2 + 3*(A - 7*B)*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))*sin(d*x + c) - 105*sqrt(2)*((A - B + C)*a*cos(d*x + c)^5 + (A
- B + C)*a*cos(d*x + c)^4)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt
(a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*cos(d*x + c)^(9/2)), x)
```

3.510 $\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{(8aA-4aB-4Ab+7bB)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4aB+4Ab-bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(a-b)(A-B)\tan^{-1}}{}$$

```
[Out] ((8*a*A - 4*A*b - 4*a*B + 7*b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(4*Sqrt[a]*d) - (Sqrt[2]*(a - b)*(A - B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]
*d) + ((4*A*b + 4*a*B - b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Cos[c + d*x]]) + (b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*C
os[c + d*x]])
```

Rubi [A] time = 0.753215, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3045, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8aA-4aB-4Ab+7bB)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4aB+4Ab-bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(a-b)(A-B)\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^
2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((8*a*A - 4*A*b - 4*a*B + 7*b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]])/(4*Sqrt[a]*d) - (Sqrt[2]*(a - b)*(A - B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]
*d) + ((4*A*b + 4*a*B - b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Cos[c + d*x]]) + (b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*C
os[c + d*x]])
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
```

```
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(4aA\cdots\right.}{\cdots}$$

$$= \frac{(4Ab+4aB-bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{bB\cdots}{\cdots}$$

$$= \frac{(4Ab+4aB-bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{bB\cdots}{\cdots}$$

$$= \frac{(4Ab+4aB-bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{bB\cdots}{\cdots}$$

$$= \frac{(8aA-4Ab-4aB+7bB)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \cdots$$

Mathematica [C] time = 2.92695, size = 540, normalized size = 2.54

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}(4aB+4Ab+2bB\cos(c+dx)-bB)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(8i\sqrt{2}(a-b)(A-B)\log\left(1+e^{i(c+dx)}\right)-i(8aA\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c +
 d*x]^2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(8*a*A*d*x - 4*A*b*d*x - 4*a*B*d*x + 7*b*B*d*x - I*(8*
```

```
a*A - 4*A*b - 4*a*B + 7*b*B)*ArcSinh[E^(I*(c + d*x))] + (8*I)*Sqrt[2]*(a -
b)*(A - B)*Log[1 + E^(I*(c + d*x))] + (8*I)*a*A*Log[1 + Sqrt[1 + E^((2*I)*(
c + d*x))]] - (4*I)*A*b*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (4*I)*a*B*
Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + (7*I)*b*B*Log[1 + Sqrt[1 + E^((2*I
)*(c + d*x))]] - (8*I)*Sqrt[2]*a*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))]] + (8*I)*Sqrt[2]*A*b*Log[1 - E^(I*(c + d*x)) + Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] + (8*I)*Sqrt[2]*a*B*Log[1 - E^(I*(c + d*
x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] - (8*I)*Sqrt[2]*b*B*Log[1 - E^
(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]]))/(d*Sqrt[1 + E^((2*
I)*(c + d*x))]) + (4*Sqrt[Cos[c + d*x]]*(4*A*b + 4*a*B - b*B + 2*b*B*Cos[c
+ d*x])*Sin[(c + d*x)/2])/d))/(8*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.104, size = 571, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d
*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^3*(4*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
sin(d*x+c)*b+4*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*a+
4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*b+2*B*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*sin(d*x+c)*a+4*A*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a
-4*A*cos(d*x+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*b-4*B*cos(d*x+c)
*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a+4*B*cos(d*x+c)*2^(1/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*b-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)*b+8*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))*a-4*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))*b-4*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*a+7*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*b)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a
+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(cos(d*
x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.511 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=260

$$\frac{(8A-12B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A-9B+13C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\,c}{2d(a\cos(c+dx)+}$$

```
[Out] ((8*A - 12*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (
(A - B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2
)) - ((2*A - 6*B + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*d*Sqrt[a + a*
Cos[c + d*x]]) + ((A - B + 2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.86877, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3041, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A-12B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A-9B+13C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\,c}{2d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^(3/2),x]
```


```
[Out] ((8*A - 12*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (
(A - B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2
)) - ((2*A - 6*B + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*d*Sqrt[a + a*
Cos[c + d*x]]) + ((A - B + 2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqr
t[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
```

```
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(A-\right.}{\sqrt{}}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-B+2C)\cos^{\frac{3}{2}}}{2ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A-6B+7C)\sqrt{c}}{4ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A-6B+7C)\sqrt{c}}{4ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A-6B+7C)\sqrt{c}}{4ad\sqrt{a+}}$$

$$= \frac{(8A-12B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(5A-9B+13C)\,t}{}$$

Mathematica [C] time = 3.21137, size = 462, normalized size = 1.78

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(-2A+(4B-3C)\cos(c+dx)+6B+C\cos(2(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(8*A*d*x - 12*B*d*x + 19*C*d*x - I*(8*A - 12*B + 19*
C)*ArcSinh[E^(I*(c + d*x))] + (2*I)*Sqrt[2]*(5*A - 9*B + 13*C)*Log[1 + E^(I
*(c + d*x))] + (8*I)*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (12*I)*B*Lo
g[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + (19*I)*C*Log[1 + Sqrt[1 + E^((2*I)*(
c + d*x))]] - (10*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E
^((2*I)*(c + d*x))]] + (18*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*S
qrt[1 + E^((2*I)*(c + d*x))]] - (26*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) +
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*
Sqrt[Cos[c + d*x]]*(-2*A + 6*B - 6*C + (4*B - 3*C)*Cos[c + d*x] + C*Cos[2*(
c + d*x)])*Sec[(c + d*x)/2]*Tan[(c + d*x)/2]))/(4*d*(a*(1 + Cos[c + d*x]))^
(3/2))
```

Maple [B] time = 0.109, size = 685, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(2*A*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)-4*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-2*A*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-6*B*cos(d*x+c)^3*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)-2*C*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*A*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-2*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)-9*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))+4*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)+13*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+5
*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)
^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-12*B*sin
(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+19*C*sin(d*x+c)*
cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)+4*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7*C*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^9/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2
```

```
)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^(3/2), x)
```

3.512 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(A-5B+9C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+d}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((2*B - 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3
/2)*d) + ((A - 5*B + 9*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((A - B
+ 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.626733, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3041, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(A-5B+9C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+d}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^(3/2),x]
```


```
[Out] ((2*B - 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3
/2)*d) + ((A - 5*B + 9*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((A - B
+ 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
```

```
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(A+3\right.}{\sqrt{a}}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-B+3C)\sqrt{\cos}}{2ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-B+3C)\sqrt{\cos}}{2ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-B+3C)\sqrt{\cos}}{2ad\sqrt{a+a}}$$

$$= \frac{(2B-3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B+9C)\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{2\sqrt{2}a^3}$$

Mathematica [C] time = 2.40446, size = 413, normalized size = 2.04

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(A-B+2C\cos(c+dx)+3C)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-i\sqrt{2}(A-5B+9C)\log\left(1+e^{i(c}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(4*B*d*x - 6*C*d*x - (2*I)*(2*B - 3*C)*ArcSinh[E^(I*
(c + d*x))] - I*Sqrt[2]*(A - 5*B + 9*C)*Log[1 + E^(I*(c + d*x))] + (4*I)*B*
Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (6*I)*C*Log[1 + Sqrt[1 + E^((2*I)*
(c + d*x))]] + I*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2
```

```
*I)*(c + d*x))]] - (5*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))]] + (9*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) + Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (2*S
qrt[Cos[c + d*x]]*(A - B + 3*C + 2*C*Cos[c + d*x])*Sec[(c + d*x)/2]*Tan[(c
+ d*x)/2])/d))/(2*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.153, size = 542, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2*B*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
-5*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+9*C
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+4*C*cos
(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-8*B*sin(d*x+c)*cos(d*x+c)^2*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*B*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+12*C*sin(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/cos(d*x+c))-6*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))/a^2/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^(3/2), x)
```

3.513 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{(3A+B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((3*A + B - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.433656, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3041, 2982, 2782, 205, 2774, 216}

$$\frac{(3A+B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(3/2)),x]
```


```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((3*A + B - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(3A+B-C)+2aC\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+B-5C)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\,c}}}{4a}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A+B-5C)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\right.}{2d}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(3A+B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right.}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 3.08252, size = 366, normalized size = 2.46

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2(A-B+C)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-i\sqrt{2}(3A+B-5C)\log\left(1+e^{i(c+dx)}\right)+3i\sqrt{2}\right.}{}\right.}{2(a(\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(4*C*d*x - (4*I)*C*ArcSinh[E^(I*(c + d*x))] - I*Sqrt
[2]*(3*A + B - 5*C)*Log[1 + E^(I*(c + d*x))] + (4*I)*C*Log[1 + Sqrt[1 + E^(
(2*I)*(c + d*x))]] + (3*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt
[1 + E^((2*I)*(c + d*x))]] + I*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))]] - (5*I)*Sqrt[2]*C*Log[1 - E^(I*(c + d*x)) +
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))])
- (2*(A - B + C)*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))/
(2*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.142, size = 460, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-2*A*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)-2*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)+2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*B*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)
*sin(d*x+c)*cos(d*x+c)^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+B*sin(d*x+c)
*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-5*C*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-2*B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)-8*C*sin(d*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-2*C*cos(d*x+c)^3*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+2*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)/a^2/cos(d*x+c)^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*sqrt(cos(d*x + c))), x)
```

3.514 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{(7A-3B-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A-B+C)\sin(c+dx}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)}$$

```
[Out] -((7*A - 3*B - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d
*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B + C)*S
in[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.418086, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3041, 2984, 12, 2782, 205}

$$-\frac{(7A-3B-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A-B+C)\sin(c+dx}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[
c + d*x])^(3/2)),x]
```

```
[Out] -((7*A - 3*B - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d
*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B + C)*S
in[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A-B+C)-a(A-B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d}$$

$$= -\frac{(7A-3B-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\sin(}{2d\sqrt{\cos(c+dx)}(a+a\,c}$$

Mathematica [C] time = 4.64982, size = 455, normalized size = 2.83

$$\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A+3B-7C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c}\right.\right.\right.}{2\cos^{\frac{3}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(30*(A - B + C)*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Co
s[c + d*x]]] - 30*(A - B + C)*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[Cos[c +
d*x]]] - (20*(A - B + C)*Sqrt[Cos[c + d*x]])/(-1 + Sin[(c + d*x)/2]) + (80*
C*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]] - (20*(A - B + C)*Sqrt[Cos[c + d*x]]
)/(1 + Sin[(c + d*x)/2]) + (5*(A - B + C)*(-1 + 2*Sin[(c + d*x)/2]))/(Sqrt[
Cos[c + d*x]]*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2) - (5*(A - B + C)*(1
+ 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(-1 + Sin[(c + d*x)/2])) + ((A +
 3*B - 7*C)*Csc[(c + d*x)/2]^3*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(
1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c
+ d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c
 + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))
/(2*Cos[c + d*x]^(3/2))))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.149, size = 438, normalized size = 2.7

$$\frac{-1+\cos(dx+c)}{4\,d\,(\sin(dx+c))^3\,a^2}\left(10\,A\,(\cos(dx+c))^4\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+18\,A\,(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}-2\,A\,(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*(10*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+
18*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2*A*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)-2*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)-7*A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-
18*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3*B*cos(d*x+c)^3*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)+C*2^(1/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)+2*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*C*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)-2*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^3/(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/a^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.68446, size = 552, normalized size = 3.43

$$-\frac{\sqrt{2}\left((7\,A-3\,B-C)\cos(dx+c)^3+2\,(7\,A-3\,B-C)\cos(dx+c)^2+(7\,A-3\,B-C)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos}}{2(}\right.}{4\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((7*A - 3*B - C)*cos(d*x + c)^3 + 2*(7*A - 3*B - C)*cos(d*x +
 c)^2 + (7*A - 3*B - C)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 +
a*cos(d*x + c))) - 2*((5*A - B + C)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(
d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(3/2)), x)
```

3.515 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{(11A-7B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B+C)\sin(c+}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+}$$

```
[Out] ((11*A - 7*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c +
 d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B +
3*C)*Sin[c + d*x])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((
19*A - 15*B + 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])
```

Rubi [A] time = 0.598038, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3041, 2984, 12, 2782, 205}

$$\frac{(11A-7B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B+C)\sin(c+}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[
c + d*x])^(3/2)),x]
```



```
[Out] ((11*A - 7*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c +
 d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B +
3*C)*Sin[c + d*x])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((
19*A - 15*B + 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
```

```
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B+3C)-2a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B+3C)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{(11A-7B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\sin}{2d\cos^{\frac{3}{2}}(c+dx)(a+a}$$

Mathematica [C] time = 6.87591, size = 1207, normalized size = 5.67

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (8*C*Cos[c/2 + (d*x)/2]^3*Sin[c/2 + (d*x)/2])/(3*d*(a*(1 + Cos[c + d*x]))^(
3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - ((A - B + C)*Cos[c/2 + (d*x)/2]^
3*(1 - 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 + Sin[c/
2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A - B + C)*Cos[c/2 +
(d*x)/2]^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1
 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (16*C*Cos[c/2
+ (d*x)/2]^3*Sin[c/2 + (d*x)/2])/(3*d*(a*(1 + Cos[c + d*x]))^(3/2)*Sqrt[1 -
 2*Sin[c/2 + (d*x)/2]^2]) - ((A - B + C)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1
- 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 + Sin[c/2 +
(d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*
```

```
Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos
[c + d*x]))^(3/2)) + ((A - B + C)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 + 2*Sin
[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2
])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1
- 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*
x]))^(3/2)) + ((A + 3*B - 7*C)*Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^2*(-
12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 +
(d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hyperge
ometric2F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2)
)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^
4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Si
n[c/2 + (d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^
4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2
 + (d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (
d*x)/2]^2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(63*d*(a*(1 + Cos[c + d*x]))^
(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.115, size = 471, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -1/12/d*(a*(1+cos(d*x+c)))^(1/2)*(33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*s
in(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*B*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)*cos(d*x+c)^2+9*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1
/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*A*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)-21*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+9*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-38*A*cos(d*x+
c)^3+30*B*cos(d*x+c)^3-6*C*cos(d*x+c)^3+14*A*cos(d*x+c)^2-6*B*cos(d*x+c)^2+
6*C*cos(d*x+c)^2+32*A*cos(d*x+c)-24*B*cos(d*x+c)-8*A)/a^2/sin(d*x+c)/(1+cos
(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.43174, size = 618, normalized size = 2.9

$$\frac{3\sqrt{2}\left((11A-7B+3C)\cos(dx+c)^4+2(11A-7B+3C)\cos(dx+c)^3+(11A-7B+3C)\cos(dx+c)^2\right)\sqrt{a}\arctan}{12\left(a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*((11*A - 7*B + 3*C)*cos(d*x + c)^4 + 2*(11*A - 7*B + 3*C)*c
os(d*x + c)^3 + (11*A - 7*B + 3*C)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(
d*x + c)^2 + a*cos(d*x + c))) - 2*((19*A - 15*B + 3*C)*cos(d*x + c)^2 + 12*
(A - B)*cos(d*x + c) - 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x +
 c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(5/2)), x)
```

3.516 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{(15A-11B+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(39A-35B+15C)\sin(c+dx)}{30ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(9A-5B+5C)\,\text{si}}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a\,\text{c}}}$$

```
[Out] -((15*A - 11*B + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c
 + d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B
+ 5*C)*Sin[c + d*x])/(10*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) -
 ((39*A - 35*B + 15*C)*Sin[c + d*x])/(30*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + ((147*A - 95*B + 75*C)*Sin[c + d*x])/(30*a*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.820798, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3041, 2984, 12, 2782, 205}

$$-\frac{(15A-11B+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(39A-35B+15C)\sin(c+dx)}{30ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(9A-5B+5C)\,\text{si}}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a\,\text{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Cos[
c + d*x])^(3/2)),x]
```


```
[Out] -((15*A - 11*B + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c
 + d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B
+ 5*C)*Sin[c + d*x])/(10*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) -
 ((39*A - 35*B + 15*C)*Sin[c + d*x])/(30*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + ((147*A - 95*B + 75*C)*Sin[c + d*x])/(30*a*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(9A-5B+5C)-a(3A-3B+C)\cos(c+dx}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}$$

$$= -\frac{(15A-11B+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)}{2d\cos^{\frac{5}{2}}(c+dx)(a}$$

Mathematica [C] time = 8.11606, size = 2437, normalized size = 9.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (8*C*Cos[c/2 + (d*x)/2]^3*Sin[c/2 + (d*x)/2])/(5*d*(a*(1 + Cos[c + d*x]))^(
3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - ((A - B + C)*Cos[c/2 + (d*x)/2]^
3*(1 - 2*Sin[c/2 + (d*x)/2]))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 + Sin[c
/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + ((A - B + C)*Cos[c/2 +
 (d*x)/2]^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2)*
(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (32*C*Cos[c/
```

2 + (d*x)/2]^3*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*
Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/(15*d*(a*(1 + Cos[c
+ d*x]))^(3/2)) + ((A - B + C)*Cos[c/2 + (d*x)/2]^3*(105*ArcTan[(1 - 2*Sin[
c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] - (4 + 3*Sin[c/2 + (d*x)/
2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (19 + 2
9*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/
2]^2]) + (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(
15*d*(a*(1 + Cos[c + d*x]))^(3/2)) - ((A - B + C)*Cos[c/2 + (d*x)/2]^3*(105
*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] - (4 -
 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(3/2)) + (19 - 29*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 -
 2*Sin[c/2 + (d*x)/2]^2]) + (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[
c/2 + (d*x)/2])))/(15*d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((-A - 3*B + 7*C)*C
ot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*(4725*Sin[c/2 + (d*x)/2]^2 - 48825
*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x
)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10
- 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 51
8760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*S
in[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2
+ (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d
*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2

```
]^2)))/(675*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/
2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [B] time = 0.127, size = 683, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -1/60/d*sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(225*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^
3-165*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*B*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+450*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*s
in(d*x+c)*cos(d*x+c)^2-330*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+210*C*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*c
os(d*x+c)^2+225*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)-165*2^(1/2)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+105
*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*sin(d*x+c)*cos(d*x+c)-294*A*cos(d*x+c)^4+190*B*cos(d*x+c)^4-150*C*cos(
d*x+c)^4+78*A*cos(d*x+c)^3-70*B*cos(d*x+c)^3+30*C*cos(d*x+c)^3+240*A*cos(d*
x+c)^2-160*B*cos(d*x+c)^2+120*C*cos(d*x+c)^2-48*A*cos(d*x+c)+40*B*cos(d*x+c
)+24*A)/a^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 2.14684, size = 684, normalized size = 2.6

$$-\frac{15\sqrt{2}\left((15A-11B+7C)\cos(dx+c)^5+2(15A-11B+7C)\cos(dx+c)^4+(15A-11B+7C)\cos(dx+c)^3\right)\sqrt{a}\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(2)*((15*A - 11*B + 7*C)*cos(d*x + c)^5 + 2*(15*A - 11*B + 7*
C)*cos(d*x + c)^4 + (15*A - 11*B + 7*C)*cos(d*x + c)^3)*sqrt(a)*arctan(1/2*
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((147*A - 95*B + 75*C)*cos(d*x + c)^
3 + 12*(9*A - 5*B + 5*C)*cos(d*x + c)^2 - 4*(3*A - 5*B)*cos(d*x + c) + 12*A
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(7/2)), x)
```

3.517 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{(3A-11B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(3A-43B+115C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2B-5C)\sin^{-1}\left(\frac{-}{\sqrt{}}\right.}{a^{5/2}d}$$

```
[Out] ((2*B - 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5
/2)*d) + ((3*A - 43*B + 115*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B
+ C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((
A + 7*B - 15*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x
])^(3/2)) + ((3*A - 11*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d
*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.87189, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3041, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-11B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(3A-43B+115C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2B-5C)\sin^{-1}\left(\frac{-}{\sqrt{}}\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^(5/2),x]
```


```
[Out] ((2*B - 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5
/2)*d) + ((3*A - 43*B + 115*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B
+ C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((
A + 7*B - 15*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x
])^(3/2)) + ((3*A - 11*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d
*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(3A\right.}{(a}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B-15C)\,c}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B-15C)\,c}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B-15C)\,c}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B-15C)\,c}{16ad(a+}$$

$$= \frac{(2B-5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B+115C)\tan^{-}}{16\sqrt{}}$$

Mathematica [C] time = 4.12897, size = 434, normalized size = 1.71

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((7A-15B+55C)\cos(c+dx)+3A-11B+8C\cos(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(32*B*d*x - 80*C*d*x - (16*I)*(2*B - 5*C)*ArcSinh[E^
(I*(c + d*x))] - I*Sqrt[2]*(3*A - 43*B + 115*C)*Log[1 + E^(I*(c + d*x))] +
(32*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (80*I)*C*Log[1 + Sqrt[1 +
 E^((2*I)*(c + d*x))]] + (3*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))]] - (43*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) +
 Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] + (115*I)*Sqrt[2]*C*Log[1 - E^(I*(c
 + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c +
```

```
d*x))] + Sqrt[Cos[c + d*x]]*(3*A - 11*B + 43*C + (7*A - 15*B + 55*C)*Cos[c
+ d*x] + 8*C*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(
a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.11, size = 881, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^5*(14*A*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+20*A*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)-8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-30
*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*A*2^(1/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-20*A*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(5/2)-43*B*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2
^(1/2)*sin(d*x+c)-22*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+115*C
*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+32*C*co
s(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*A*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)-43*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
-64*B*sin(d*x+c)*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c))+30*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+115*
C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+160*C*
sin(d*x+c)*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))+78*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-64*B*sin(d
*x+c)*cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))+22*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+160*C*sin(d*x+c)*
cos(d*x+c)^2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)-40*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-70*C*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^11/(cos(d*x+c)/(1+cos(d*x+c)))^(
7/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^(5/2), x)
```

3.518 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(5A+3B-43C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((5*A + 3*B - 43*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A + 3*B
- 11*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)
)
```

Rubi [A] time = 0.630439, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3041, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A+3B-43C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + a*Cos
[c + d*x])^(5/2),x]
```


```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((5*A + 3*B - 43*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A + 3*B
- 11*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)
)
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}a(5A\right.}{(a+a}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A+3B-11C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A+3B-11C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A+3B-11C)}{16ad(a+}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(5A+3B-43C)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{\cos}}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 3.16345, size = 385, normalized size = 1.92

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((A+7B-15C)\cos(c+dx)+5A+3B-11C)+\frac{\sqrt{2}e^{\frac{1}{2}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(32*C*d*x - (32*I)*C*ArcSinh[E^(I*(c + d*x))] - I*Sq
```

```
rt[2]*(5*A + 3*B - 43*C)*Log[1 + E^(I*(c + d*x))] + (32*I)*C*Log[1 + Sqrt[1
 + E^((2*I)*(c + d*x))]] + (5*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))]] + (3*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x))
+ Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] - (43*I)*Sqrt[2]*C*Log[1 - E^(I*(c
 + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c +
d*x))] + Sqrt[Cos[c + d*x]]*(5*A + 3*B - 11*C + (A + 7*B - 15*C)*Cos[c + d*
x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)
)
```

Maple [B] time = 0.1, size = 747, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] -1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(2*A*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+12*A*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)+8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+14
*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5*A*2^(1/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-12*A*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(5/2)+3*B*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^
(1/2)*sin(d*x+c)+6*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-43*C*2^
(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+5*A*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2-10*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)+3*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))-14*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-64
*C*sin(d*x+c)*cos(d*x+c)^3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))-43*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*
cos(d*x+c)^2-30*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*B*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-64*C*sin(d*x+c)*cos(d*x+c)^2*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*C*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))/a^3/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)\sqrt{\cos{(dx+c)}}}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos
(d*x + c) + a)^(5/2), x)
```

3.519 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=163

$$\frac{(19A+5B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B+C)\sin(c+dx}{4d(a\cos(c+dx)}$$

```
[Out] ((19*A + 5*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - B - 7
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.441338, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3041, 2978, 12, 2782, 205}

$$\frac{(19A+5B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B+C)\sin(c+dx}{4d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(5/2)),x]
```

```
[Out] ((19*A + 5*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - B - 7
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(7A+B-C)-a(A-B-3C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,d}{4a^2}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B-7C)\sqrt{\cos(c+dx)}\sin(}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B-7C)\sqrt{\cos(c+dx)}\sin(}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B-7C)\sqrt{\cos(c+dx)}\sin(}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(19A+5B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\sqrt{\cos(c+}}{4d(a+a\cos(c}$$

Mathematica [C] time = 1.88878, size = 209, normalized size = 1.28

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{2}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((9A-B-7C)\cos(c+dx)+13A-5B-3C)+\frac{i(19}{}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(19*A + 5*B + 3*C)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^
((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[
c + d*x]]*(13*A - 5*B - 3*C + (9*A - B - 7*C)*Cos[c + d*x])*Sec[(c + d*x)/2
]^3*Tan[(c + d*x)/2])/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.144, size = 643, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(18*A*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)+44*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)+8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2*B*cos(d*x+c)^4*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-44*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(5/2)-19*A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+
c)^3-10*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-5*B*cos(d*x+c)^3*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)-3*C*2^(1/2)*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-26*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)-19*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos
(d*x+c)^2+2*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-5*B*sin(d*x+c)
*cos(d*x+c)^2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-14*C*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^
(1/2)*sin(d*x+c)*cos(d*x+c)^2+10*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)+8*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6*C*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/a^3/cos(
d*x+c)^(1/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.3118, size = 626, normalized size = 3.84

$$\frac{\sqrt{2}\left((19\,A + 5\,B + 3\,C)\cos\left(dx + c\right)^{3} + 3\,(19\,A + 5\,B + 3\,C)\cos\left(dx + c\right)^{2} + 3\,(19\,A + 5\,B + 3\,C)\cos\left(dx + c\right) + 19\,A + 5\,B}{32\,\left(a^{3}d\cos\left(dx + c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((19*A + 5*B + 3*C)*cos(d*x + c)^3 + 3*(19*A + 5*B + 3*C)*cos
(d*x + c)^2 + 3*(19*A + 5*B + 3*C)*cos(d*x + c) + 19*A + 5*B + 3*C)*sqrt(a)
*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin
(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((9*A - B - 7*C)*cos(d*x
 + c) + 13*A - 5*B - 3*C)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*sqrt(cos(d*x + c))), x)
```

3.520 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=211

$$\frac{(49A-9B+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B-3C)\sin}{16ad\sqrt{\cos(c+dx)}(a\cos(}$$

```
[Out] -((75*A - 19*B - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[
c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 5*
B - 3*C)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2
)) + ((49*A - 9*B + C)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]])
```

Rubi [A] time = 0.639784, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3041, 2978, 2984, 12, 2782, 205}

$$\frac{(49A-9B+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B-3C)\sin}{16ad\sqrt{\cos(c+dx)}(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Cos[
c + d*x])^(5/2)),x]
```


```
[Out] -((75*A - 19*B - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[
c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 5*
B - 3*C)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2
)) + ((49*A - 9*B + C)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
```

```
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B+C)-2a(A-B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+d$$

$$= -\frac{(75A-19B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\,s}{4d\sqrt{\cos(c+dx)}(a+$$

Mathematica [C] time = 2.44642, size = 225, normalized size = 1.07

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(2(85A-13B+5C)\cos(c+dx)+(49A-9B+C)\cos(2(c+dx))+113A-9B+C)}{4\sqrt{\cos(c+dx)}} - \frac{i(75A-19B-5C)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(}}}{}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B - 5*C)*E^((I/2)*(c + d*x))*Sqrt[(1
+ E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
```

```
[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((113*
A - 9*B + C + 2*(85*A - 13*B + 5*C)*Cos[c + d*x] + (49*A - 9*B + C)*Cos[2*(
c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2])/(4*Sqrt[Cos[c + d*x]])))/(4
*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.111, size = 675, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))^2*(98*A*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)+268*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+136*A*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-18*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)-75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c)*cos(
d*x+c)^4-204*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+19*B*cos(d*x+
c)^4*2^(1/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-26*B*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
2^(1/2)*sin(d*x+c)*cos(d*x+c)^4-75*A*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*sin(d*x+c)*cos(d*x+c)^3-234*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)+19*B*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*sin(d*x+c
)+18*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*C*cos(d*x+c)^5*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*C*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)*cos(d*x+c)^3-64*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+26*B*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*C*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)-10*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(a*(
1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.45177, size = 711, normalized size = 3.37

$$-\frac{\sqrt{2}\left((75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{4}+3\,(75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{2}+(75\,A\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((75*A - 19*B - 5*C)*cos(d*x + c)^4 + 3*(75*A - 19*B - 5*C)*
cos(d*x + c)^3 + 3*(75*A - 19*B - 5*C)*cos(d*x + c)^2 + (75*A - 19*B - 5*C)
*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((
49*A - 9*B + C)*cos(d*x + c)^2 + (85*A - 13*B + 5*C)*cos(d*x + c) + 32*A)*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*cos(d*x + c)^(3/2)), x)
```

3.521 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{(95A-39B+15C)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B+19C)\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{\cos(c+}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((163*A - 75*B + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin
[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9
*B + C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)
) + ((95*A - 39*B + 15*C)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a
 + a*Cos[c + d*x]]) - ((299*A - 147*B + 27*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.849588, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3041, 2978, 2984, 12, 2782, 205}

$$\frac{(95A-39B+15C)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B+19C)\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{\cos(c+}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Cos[
c + d*x])^(5/2)),x]
```


```
[Out] ((163*A - 75*B + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin
[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9
*B + C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)
) + ((95*A - 39*B + 15*C)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a
 + a*Cos[c + d*x]]) - ((299*A - 147*B + 27*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A-3B+3C)-a(3A-3B-C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B+C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{(163A-75B+19C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)}{4d\cos^{\frac{3}{2}}(c+dx)(a+}$$

Mathematica [C] time = 4.07997, size = 262, normalized size = 1.

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((1537A-825B+81C)\cos(c+dx)+2(503A-255B+39C)\cos(2(c+dx))+299A\cos(3(c+dx))+878A-147B\cos}{8\cos^{\frac{3}{2}}(c+dx)}\right.}{12d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((3*I)*(163*A - 75*B + 19*C)*E^((I/2)*(c + d*x))*Sqrt[
(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((8
78*A - 510*B + 78*C + (1537*A - 825*B + 81*C)*Cos[c + d*x] + 2*(503*A - 255
*B + 39*C)*Cos[2*(c + d*x)] + 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x
)] + 27*C*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2])/(8*Cos[c +
 d*x]^(3/2))))/(12*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.123, size = 683, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] -1/96/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(-489*A*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d
*x+c)^3+225*B*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-57*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3-978*
A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+450*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-114*C*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-489*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+225*B*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c
)-57*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+598*A*cos(d*x+c)^4-294*B*cos(d*x+c)^4+54*C*c
os(d*x+c)^4+408*A*cos(d*x+c)^3-216*B*cos(d*x+c)^3+24*C*cos(d*x+c)^3-686*A*c
os(d*x+c)^2+318*B*cos(d*x+c)^2-78*C*cos(d*x+c)^2-384*A*cos(d*x+c)+192*B*cos
(d*x+c)+64*A)/a^3/sin(d*x+c)^3/(1+cos(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.155, size = 784, normalized size = 3.

$$3\sqrt{2}\big((163\,A - 75\,B + 19\,C)\cos(dx+c)^5 + 3\,(163\,A - 75\,B + 19\,C)\cos(dx+c)^4 + 3\,(163\,A - 75\,B + 19\,C)\cos(dx+c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*((163*A - 75*B + 19*C)*cos(d*x + c)^5 + 3*(163*A - 75*B + 1
9*C)*cos(d*x + c)^4 + 3*(163*A - 75*B + 19*C)*cos(d*x + c)^3 + (163*A - 75*
B + 19*C)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) +
a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x +
c))) - 2*((299*A - 147*B + 27*C)*cos(d*x + c)^3 + (503*A - 255*B + 39*C)*co
s(d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x + c) - 32*A)*sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x +
 c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*cos(d*x + c)^(5/2)), x)
```

3.522 $\int \cos^2(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=131

$$\frac{a(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3C)+\frac{aC\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{b(5A+4C)\sin^3(c+dx)}{15d}+\frac{b(5A+$$

```
[Out] (a*(4*A + 3*C)*x)/8 + (b*(5*A + 4*C)*Sin[c + d*x])/(5*d) + (a*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (b*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (b*(5*A + 4*C)*Sin[c + d*x]^3)/(
15*d)
```

Rubi [A] time = 0.195374, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3034, 3023, 2748, 2635, 8, 2633}

$$\frac{a(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3C)+\frac{aC\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{b(5A+4C)\sin^3(c+dx)}{15d}+\frac{b(5A+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (a*(4*A + 3*C)*x)/8 + (b*(5*A + 4*C)*Sin[c + d*x])/(5*d) + (a*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (b*C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (b*(5*A + 4*C)*Sin[c + d*x]^3)/(
15*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{bC\cos^4(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^2(c+dx)\left(5aA+b\right.$$
$$= \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{bC\cos^4(c+dx)\sin(c+dx}{5d}$$
$$= \frac{aC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{bC\cos^4(c+dx)\sin(c+dx}{5d}$$
$$= \frac{a(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aC\cos^3(c+dx)\sin}{4d}$$
$$= \frac{1}{8}a(4A+3C)x+\frac{b(5A+4C)\sin(c+dx)}{5d}+\frac{a(4A+3C)\,c}{}$$

Mathematica [A] time = 0.303863, size = 89, normalized size = 0.68

$$\frac{15a(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))-160b(A+2C)\sin^3(c+dx)+480b(A+C)\sin(c}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (480*b*(A + C)*Sin[c + d*x] - 160*b*(A + 2*C)*Sin[c + d*x]^3 + 96*b*C*Sin[c
 + d*x]^5 + 15*a*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*
Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.017, size = 117, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Cb\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+aC\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*b*(2+cos(d*x+c)^2)*
sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00642, size = 153, normalized size = 1.17

$$\frac{120(2dx+2c+\sin(2dx+2c))Aa+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Ca-160\left(\sin(dx+c)^3-3\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 15*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a - 160*(sin(d*x + c)^3 - 3*sin(d*x + c
```

```
))*A*b + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b)/d
```

Fricas [A] time = 1.40283, size = 242, normalized size = 1.85

$$\frac{15\,(4\,A+3\,C)adx+\left(24\,Cb\cos\left(dx+c\right)^{4}+30\,Ca\cos\left(dx+c\right)^{3}+8\,(5\,A+4\,C)b\cos\left(dx+c\right)^{2}+15\,(4\,A+3\,C)a\cos\left(dx\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(4*A + 3*C)*a*d*x + (24*C*b*cos(d*x + c)^4 + 30*C*a*cos(d*x + c)^
3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 15*(4*A + 3*C)*a*cos(d*x + c) + 16*(5*
A + 4*C)*b)*sin(d*x + c))/d
```

Sympy [A] time = 3.11007, size = 279, normalized size = 2.13

$$\begin{cases}\frac{Aax\sin^{2}(c+dx)}{2}+\frac{Aax\cos^{2}(c+dx)}{2}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Ab\sin^{3}(c+dx)}{3d}+\frac{Ab\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{3Cax\sin^{4}(c+dx)}{8}+\frac{3Cax\sin^{2}(c+\ldots}{\ldots} \\ x\left(A+C\cos^{2}(c)\right)(a+b\cos(c))\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + 2*A*b*sin(c + d*x)**3/(3*d) + A*b*sin(c + d*x)*co
s(c + d*x)**2/d + 3*C*a*x*sin(c + d*x)**4/8 + 3*C*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 3*C*a*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 5*C*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*C*b*sin(c + d*x)**5
/(15*d) + 4*C*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*b*sin(c + d*x)*co
s(c + d*x)**4/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*(a + b*cos(c))*cos(c)**2,
True))
```

Giac [A] time = 1.54285, size = 147, normalized size = 1.12

$$\frac{1}{8}\,(4\,Aa+3\,Ca)x+\frac{Cb\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{Ca\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(4\,Ab+5\,Cb)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(Aa+Ca)\sin\left(2\,dx+2\ldots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(4*A*a + 3*C*a)*x + 1/80*C*b*sin(5*d*x + 5*c)/d + 1/32*C*a*sin(4*d*x +
4*c)/d + 1/48*(4*A*b + 5*C*b)*sin(3*d*x + 3*c)/d + 1/4*(A*a + C*a)*sin(2*d*
x + 2*c)/d + 1/8*(6*A*b + 5*C*b)*sin(d*x + c)/d
```

3.523 $\int \cos(c+dx)(a+b\cos(c+dx)) \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=108

$$\frac{a(3A+2C)\sin(c+dx)}{3d} + \frac{aC\sin(c+dx)\cos^2(c+dx)}{3d} + \frac{b(4A+3C)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}bx(4A+3C) + \frac{bC\sin}{}$$

```
[Out] (b*(4*A + 3*C)*x)/8 + (a*(3*A + 2*C)*Sin[c + d*x])/(3*d) + (b*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) +
 (b*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.107872, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3034, 3023, 2734}

$$\frac{a(3A+2C)\sin(c+dx)}{3d} + \frac{aC\sin(c+dx)\cos^2(c+dx)}{3d} + \frac{b(4A+3C)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}bx(4A+3C) + \frac{bC\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b*(4*A + 3*C)*x)/8 + (a*(3*A + 2*C)*Sin[c + d*x])/(3*d) + (b*(4*A + 3*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) +
 (b*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx = \frac{bC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos(c+dx)\left(4aA+b(4A\right.$$

$$= \frac{aC\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{bC\cos^3(c+dx)\sin(c+dx)}{4d}$$

$$= \frac{1}{8}b(4A+3C)x+\frac{a(3A+2C)\sin(c+dx)}{3d}+\frac{b(4A+3C)\cos}{}$$

Mathematica [A] time = 0.210545, size = 84, normalized size = 0.78

$$\frac{24a(4A+3C)\sin(c+dx)+8aC\sin(3(c+dx))+24b(A+C)\sin(2(c+dx))+48Abc+48Abdx+3bC\sin(4(c+dx))+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (48*A*b*c + 36*b*c*C + 48*A*b*d*x + 36*b*C*d*x + 24*a*(4*A + 3*C)*Sin[c + d
*x] + 24*b*(A + C)*Sin[2*(c + d*x)] + 8*a*C*Sin[3*(c + d*x)] + 3*b*C*Sin[4*
(c + d*x)])/(96*d)
```

Maple [A] time = 0.016, size = 96, normalized size = 0.9

$$\frac{1}{d}\left(Cb\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{aC\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ab\left(\frac{\cos(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+a*A*sin(d*x+c))
```

Maxima [A] time = 1.00369, size = 122, normalized size = 1.13

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ca-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ab-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a - 24*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*b - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*
C*b - 96*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.40914, size = 189, normalized size = 1.75

$$\frac{3\left(4\,A+3\,C\right)bdx+\left(6\,Cb\cos\left(dx+c\right)^{3}+8\,Ca\cos\left(dx+c\right)^{2}+3\left(4\,A+3\,C\right)b\cos\left(dx+c\right)+8\left(3\,A+2\,C\right)a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A + 3*C)*b*d*x + (6*C*b*cos(d*x + c)^3 + 8*C*a*cos(d*x + c)^2 +
3*(4*A + 3*C)*b*cos(d*x + c) + 8*(3*A + 2*C)*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.74869, size = 226, normalized size = 2.09

$$\begin{cases} \frac{Aa\sin\left(c+dx\right)}{d}+\frac{Abx\sin^{2}\left(c+dx\right)}{2}+\frac{Abx\cos^{2}\left(c+dx\right)}{2}+\frac{Ab\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Ca\sin^{3}\left(c+dx\right)}{3d}+\frac{Ca\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}+\frac{3Cbx\sin^{4}\left(c+dx\right)}{8} \\ x\left(A+C\cos^{2}\left(c\right)\right)\left(a+b\cos\left(c\right)\right)\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d + A*b*x*sin(c + d*x)**2/2 + A*b*x*cos(c + d*x
)**2/2 + A*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*a*sin(c + d*x)**3/(3*d)
+ C*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*b*x*sin(c + d*x)**4/8 + 3*C*b*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*b*x*cos(c + d*x)**4/8 + 3*C*b*sin(c
 + d*x)**3*cos(c + d*x)/(8*d) + 5*C*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), N
e(d, 0)), (x*(A + C*cos(c)**2)*(a + b*cos(c))*cos(c), True))
```

Giac [A] time = 1.57608, size = 116, normalized size = 1.07

$$\frac{1}{8}(4\,Ab+3\,Cb)x+\frac{Cb\sin(4\,dx+4\,c)}{32\,d}+\frac{Ca\sin(3\,dx+3\,c)}{12\,d}+\frac{(Ab+Cb)\sin(2\,dx+2\,c)}{4\,d}+\frac{(4\,Aa+3\,Ca)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*b + 3*C*b)*x + 1/32*C*b*sin(4*d*x + 4*c)/d + 1/12*C*a*sin(3*d*x +
3*c)/d + 1/4*(A*b + C*b)*sin(2*d*x + 2*c)/d + 1/4*(4*A*a + 3*C*a)*sin(d*x +
 c)/d
```

3.524 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=96

$$-\frac{\left(a^2C-b^2(3A+2C)\right)\sin(c+dx)}{3bd}+\frac{1}{2}ax(2A+C)+\frac{C\sin(c+dx)(a+b\cos(c+dx))^2}{3bd}-\frac{aC\sin(c+dx)\cos(c+dx)}{6d}$$

```
[Out] (a*(2*A + C)*x)/2 - ((a^2*C - b^2*(3*A + 2*C))*Sin[c + d*x])/(3*b*d) - (a*C
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(3*b*d)
```

Rubi [A] time = 0.0736837, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3024, 2734}

$$-\frac{\left(a^2C-b^2(3A+2C)\right)\sin(c+dx)}{3bd}+\frac{1}{2}ax(2A+C)+\frac{C\sin(c+dx)(a+b\cos(c+dx))^2}{3bd}-\frac{aC\sin(c+dx)\cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a*(2*A + C)*x)/2 - ((a^2*C - b^2*(3*A + 2*C))*Sin[c + d*x])/(3*b*d) - (a*C
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(3*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{C(a+b\cos(c+dx))^2\sin(c+dx)}{3bd} + \frac{\int (a+b\cos(c+dx))(b(3A+2C)-}{3b}$$
$$= \frac{1}{2}a(2A+C)x - \frac{\left(a^2C-b^2(3A+2C)\right)\sin(c+dx)}{3bd} - \frac{aC\cos(c+dx)\sin(}{6d}$$

Mathematica [A] time = 0.116301, size = 64, normalized size = 0.67

$$\frac{12aAdx+3aC\sin(2(c+dx))+6acC+6aCdx+3b(4A+3C)\sin(c+dx)+bC\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*a*c*C + 12*a*A*d*x + 6*a*C*d*x + 3*b*(4*A + 3*C)*Sin[c + d*x] + 3*a*C*Si
n[2*(c + d*x)] + b*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.016, size = 68, normalized size = 0.7

$$\frac{1}{d}\left(\frac{Cb\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+aA(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*C*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+A*b*sin(d*x+c)+a*A*(d*x+c))
```

Maxima [A] time = 0.989892, size = 90, normalized size = 0.94

$$\frac{12(dx+c)Aa+3(2dx+2c+\sin(2dx+2c))Ca-4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Cb+12Ab\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a - 4*(sin(d*
x + c)^3 - 3*sin(d*x + c))*C*b + 12*A*b*sin(d*x + c))/d
```

Fricas [A] time = 1.3702, size = 140, normalized size = 1.46

$$\frac{3\,(2\,A+C)adx+\left(2\,Cb\cos\left(dx+c\right)^{2}+3\,Ca\cos\left(dx+c\right)+2\,(3\,A+2\,C)b\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A + C)*a*d*x + (2*C*b*cos(d*x + c)^2 + 3*C*a*cos(d*x + c) + 2*(3*
A + 2*C)*b)*sin(d*x + c))/d
```

Sympy [A] time = 0.664865, size = 121, normalized size = 1.26

$$\begin{cases} Aax+\frac{Ab\sin\left(c+dx\right)}{d}+\frac{Cax\sin^{2}\left(c+dx\right)}{2}+\frac{Cax\cos^{2}\left(c+dx\right)}{2}+\frac{Ca\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Cb\sin^{3}\left(c+dx\right)}{3d}+\frac{Cb\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d} & \text{for } d \neq \\ x\left(A+C\cos^{2}\left(c\right)\right)\left(a+b\cos\left(c\right)\right) & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x + A*b*sin(c + d*x)/d + C*a*x*sin(c + d*x)**2/2 + C*a*x*cos
(c + d*x)**2/2 + C*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*b*sin(c + d*x)**
3/(3*d) + C*b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + C*cos(c)**
2)*(a + b*cos(c)), True))
```

Giac [A] time = 1.32369, size = 86, normalized size = 0.9

$$\frac{1}{2}\,(2\,Aa+Ca)x+\frac{Cb\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{Ca\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(4\,Ab+3\,Cb)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + C*a)*x + 1/12*C*b*sin(3*d*x + 3*c)/d + 1/4*C*a*sin(2*d*x + 2*c
)/d + 1/4*(4*A*b + 3*C*b)*sin(d*x + c)/d
```

3.525 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}+\frac{1}{2}bx(2A+C)+\frac{bC\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (b*(2*A + C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d +
(b*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.115879, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3034, 3023, 2735, 3770}

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}+\frac{1}{2}bx(2A+C)+\frac{bC\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (b*(2*A + C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Sin[c + d*x])/d +
(b*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx &= \frac{bC\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\int\left(2aA+b(2A+C)\cos(c+\right.\\ &= \frac{aC\sin(c+dx)}{d}+\frac{bC\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\int(2aA+b\\ &= \frac{1}{2}b(2A+C)x+\frac{aC\sin(c+dx)}{d}+\frac{bC\cos(c+dx)\sin(c+dx)}{2d}\\ &= \frac{1}{2}b(2A+C)x+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c+dx)}{d}+\end{aligned}$$

Mathematica [A] time = 0.130634, size = 73, normalized size = 1.26

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\sin(c)\cos(dx)}{d}+\frac{aC\cos(c)\sin(dx)}{d}+Abx+\frac{bC(c+dx)}{2d}+\frac{bC\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] A*b*x + (b*C*(c + d*x))/(2*d) + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*Cos[d*
x]*Sin[c])/d + (a*C*Cos[c]*Sin[d*x])/d + (b*C*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.037, size = 77, normalized size = 1.3

$$Abx+\frac{Abc}{d}+\frac{Cb\cos(dx+c)\sin(dx+c)}{2d}+\frac{bCx}{2}+\frac{Cbc}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] A*b*x+1/d*A*b*c+1/2*b*C*cos(d*x+c)*sin(d*x+c)/d+1/2*b*C*x+1/2/d*C*b*c+1/d*a
*A*ln(sec(d*x+c)+tan(d*x+c))+a*C*sin(d*x+c)/d
```

Maxima [A] time = 0.979164, size = 85, normalized size = 1.47

$$\frac{4\,(dx+c)Ab+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Cb+4\,Aa\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+4\,Ca\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*A*b + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*b + 4*A*a*log(sec
(d*x + c) + tan(d*x + c)) + 4*C*a*sin(d*x + c))/d
```

Fricas [A] time = 1.4418, size = 167, normalized size = 2.88

$$\frac{(2\,A+C)bdx+Aa\log\left(\sin\left(dx+c\right)+1\right)-Aa\log\left(-\sin\left(dx+c\right)+1\right)+\left(Cb\cos\left(dx+c\right)+2\,Ca\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*((2*A + C)*b*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c) +
1) + (C*b*cos(d*x + c) + 2*C*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+C\cos^{2}\left(c+dx\right)\right)\left(a+b\cos\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 1.52131, size = 171, normalized size = 2.95

$$\frac{2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(2\,Ab+Cb)(dx+c)+\frac{2\left(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Cb\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (2*A*b + C*b)*(d*x + c) + 2*(2*C*a*tan(1/2*d*x + 1/2*c)^3 -
 C*b*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.526 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=42

$$\frac{aA\tan(c+dx)}{d}+aCx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin(c+dx)}{d}$$

```
[Out] a*C*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (b*C*Sin[c + d*x])/d + (a*A*Tan[c +
 d*x])/d
```

Rubi [A] time = 0.106029, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3032, 3023, 2735, 3770}

$$\frac{aA\tan(c+dx)}{d}+aCx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] a*C*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (b*C*Sin[c + d*x])/d + (a*A*Tan[c +
 d*x])/d
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{aA\tan(c+dx)}{d}+\int \left(Ab+aC\cos(c+dx)+bC\cos^2(c+d\right.$$
$$= \frac{bC\sin(c+dx)}{d}+\frac{aA\tan(c+dx)}{d}+\int (Ab+aC\cos(c+dx$$
$$= aCx+\frac{bC\sin(c+dx)}{d}+\frac{aA\tan(c+dx)}{d}+(Ab)\int \sec(c+$$
$$= aCx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin(c+dx)}{d}+\frac{aA\tan}{d}$$

Mathematica [A] time = 0.0194474, size = 54, normalized size = 1.29

$$\frac{aA\tan(c+dx)}{d}+aCx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin(c)\cos(dx)}{d}+\frac{bC\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] a*C*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (b*C*Cos[d*x]*Sin[c])/d + (b*C*Cos[
c]*Sin[d*x])/d + (a*A*Tan[c + d*x])/d
```

Maple [A] time = 0.042, size = 57, normalized size = 1.4

$$aCx + \frac{Ab \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{A \tan(dx+c)\, a}{d} + \frac{Cb \sin(dx+c)}{d} + \frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] a*C*x+1/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+a*A*tan(d*x+c)/d+b*C*sin(d*x+c)/d+1
/d*a*C*c
```

Maxima [A] time = 0.991217, size = 80, normalized size = 1.9

$$\frac{2\,(dx+c)Ca + Ab(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,Cb\sin(dx+c) + 2\,Aa\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + A*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*C*b*sin(d*x + c) + 2*A*a*tan(d*x + c))/d
```

Fricas [B] time = 1.4363, size = 232, normalized size = 5.52

$$\frac{2\,Cadx\cos(dx+c) + Ab\cos(dx+c)\log(\sin(dx+c)+1) - Ab\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,(Cb\cos(dx+c}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*C*a*d*x*cos(d*x + c) + A*b*cos(d*x + c)*log(sin(d*x + c) + 1) - A*b*
cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*b*cos(d*x + c) + A*a)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49484, size = 158, normalized size = 3.76

$$\frac{(dx+c)Ca + Ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - Ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + Aa\tan\left(\right.\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((d*x + c)*C*a + A*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*b*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - C*b*tan(1/2*d*x + 1
/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^4 - 1))/d
```

3.527 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{Ab\tan(c+dx)}{d}+bCx$$

```
[Out] b*C*x + (a*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*b*Tan[c + d*x])/d +
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.13253, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3032, 3021, 2735, 3770}

$$\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{Ab\tan(c+dx)}{d}+bCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] b*C*x + (a*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*b*Tan[c + d*x])/d +
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2Ab+a(A+2C)\cos(c\right.\\ &= \frac{Ab\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a(A\\ &= bCx+\frac{Ab\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}(a\\ &= bCx+\frac{a(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{Ab\tan(c+dx)}{d}+\end{aligned}$$

Mathematica [A] time = 0.0229607, size = 67, normalized size = 1.16

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{Ab\tan(c+dx)}{d}+bCx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] b*C*x + (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*ArcTanh[Sin[c + d*x]])/d +
 (A*b*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.045, size = 85, normalized size = 1.5

$$\frac{Ab\tan(dx+c)}{d}+bCx+\frac{Cbc}{d}+\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aC\ln(\sec(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] A*b*tan(d*x+c)/d+b*C*x+1/d*C*b*c+1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*
ln(sec(d*x+c)+tan(d*x+c))+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00517, size = 128, normalized size = 2.21

$$\frac{4(dx+c)Cb-Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+2Ca(\log(\sin(dx+c)+1)-\log(\sin(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*C*b - A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 4*A*b*tan(d*x + c))/d
```

Fricas [A] time = 1.42517, size = 267, normalized size = 4.6

$$\frac{4Cbdx\cos(dx+c)^2+(A+2C)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(A+2C)a\cos(dx+c)^2\log(-\sin(dx+c)+1)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*C*b*d*x*cos(d*x + c)^2 + (A + 2*C)*a*cos(d*x + c)^2*log(sin(d*x + c)
 + 1) - (A + 2*C)*a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*b*cos(d*
```

```
x + c) + A*a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34295, size = 178, normalized size = 3.07

$$\frac{2\,(dx+c)Cb+(Aa+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,A\cdots\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*C*b + (A*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
A*a + 2*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*
c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(
1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.528 $\int(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{a(2A+3C)\tan(c+dx)}{3d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{b(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{Ab\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*C)*Tan[c + d*x])/(3
*d) + (A*b*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 0.172846, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3032, 3021, 2748, 3767, 8, 3770}

$$\frac{a(2A+3C)\tan(c+dx)}{3d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{b(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{Ab\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*C)*Tan[c + d*x])/(3
*d) + (A*b*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \left(3Ab+a(2A+3C)\cos\right.$$

$$= \frac{Ab\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}$$

$$= \frac{Ab\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}$$

$$= \frac{b(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{Ab\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{b(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(2A+3C)\tan(c+dx)}{3d}$$

Mathematica [A] time = 0.323874, size = 59, normalized size = 0.69

$$\frac{\tan(c+dx)\left(2aA\tan^2(c+dx)+6a(A+C)+3Ab\sec(c+dx)\right)+3b(A+2C)\tanh^{-1}(\sin(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (3*b*(A + 2*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*(A + C) + 3*A*b*Se
c[c + d*x] + 2*a*A*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.048, size = 108, normalized size = 1.3

$$\frac{Ab\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Cb\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{2A\tan(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/2*A*b*sec(d*x+c)*tan(d*x+c)/d+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*b
*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*
x+c)/d+1/d*a*C*tan(d*x+c)
```

Maxima [A] time = 1.01852, size = 144, normalized size = 1.67

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa-3Ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6Cb(\log(\sin}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*b*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*b*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*C*a*tan(d*x + c))/d
```

Fricas [A] time = 1.51209, size = 285, normalized size = 3.31

$$\frac{3\,(A+2\,C)b\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(A+2\,C)b\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(2\,A+3\,C)a\cos\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(A + 2*C)*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + 2*C)*b*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*C)*a*cos(d*x + c)^2 + 3
*A*b*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58034, size = 248, normalized size = 2.88

$$\frac{3\,(Ab+2\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ab+2\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(A*b + 2*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*b + 2*C*b)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a
```

```
*tan(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x
+ 1/2*c)^3 - 12*C*a*tan(1/2*d*x + 1/2*c)^3 + 6*A*a*tan(1/2*d*x + 1/2*c) + 6
*C*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^3)/d
```

3.529 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{a(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{b(2A+3C)\tan}{3d}$$

```
[Out] (a*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(2*A + 3*C)*Tan[c + d*x])/
(3*d) + (a*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*b*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.190869, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3032, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{b(2A+3C)\tan}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(2*A + 3*C)*Tan[c + d*x])/
(3*d) + (a*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*b*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \left(4Ab+a(3A+4C)\cos\right.$$
$$= \frac{Ab\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{Ab\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{a(3A+4C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{Ab\sec^2(c+dx)\tan}{3d}$$
$$= \frac{a(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(2A+3C)\tan(c+dx)}{3d}$$

Mathematica [A] time = 0.446148, size = 80, normalized size = 0.68

$$\frac{\tan(c+dx)\left(3a(3A+4C)\sec(c+dx)+6aA\sec^3(c+dx)+8b\left(A\tan^2(c+dx)+3(A+C)\right)\right)+3a(3A+4C)\tanh^{-1}(\sin}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*(3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(3*A + 4*C)*Sec[
c + d*x] + 6*a*A*Sec[c + d*x]^3 + 8*b*(3*(A + C) + A*Tan[c + d*x]^2)))/(24*
d)
```

Maple [A] time = 0.047, size = 149, normalized size = 1.3

$$\frac{2\,Ab\tan(dx+c)}{3\,d}+\frac{Ab(\sec(dx+c))^2\tan(dx+c)}{3\,d}+\frac{Cb\tan(dx+c)}{d}+\frac{aA(\sec(dx+c))^3\tan(dx+c)}{4\,d}+\frac{3\,aA\sec(dx+}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 2/3*A*b*tan(d*x+c)/d+1/3*A*b*sec(d*x+c)^2*tan(d*x+c)/d+1/d*C*b*tan(d*x+c)+1
/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A*
ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(
d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.992948, size = 205, normalized size = 1.75

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ab-3\,Aa\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)\cdots}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b - 3*A*a*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*C*b*tan(d*x +
 c))/d
```

Fricas [A] time = 1.41702, size = 335, normalized size = 2.86

$$\frac{3\,(3\,A+4\,C)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+4\,C)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(2\,A+3\,C\cdots\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A + 4*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*C)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(2*A + 3*C)*b*cos(d*x + c)^3
 + 3*(3*A + 4*C)*a*cos(d*x + c)^2 + 8*A*b*cos(d*x + c) + 6*A*a)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2762, size = 410, normalized size = 3.5

$$\frac{3\,(3\,Aa+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ca\tan(\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a + 4*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*C
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 +
 12*C*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 - 24*C*b*tan
(1/2*d*x + 1/2*c)^7 + 9*A*a*tan(1/2*d*x + 1/2*c)^5 - 12*C*a*tan(1/2*d*x + 1
/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 72*C*b*tan(1/2*d*x + 1/2*c)^5 + 9
*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/
2*d*x + 1/2*c)^3 - 72*C*b*tan(1/2*d*x + 1/2*c)^3 + 15*A*a*tan(1/2*d*x + 1/2
*c) + 12*C*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 24*C*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.530 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=140

$$\frac{a(4A+5C)\tan^3(c+dx)}{15d}+\frac{a(4A+5C)\tan(c+dx)}{5d}+\frac{aA\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{b(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(4*A + 5*C)*Tan[c + d*x])/
(5*d) + (b*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*b*Sec[c + d*x]
^3*Tan[c + d*x])/(4*d) + (a*A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(4*A
+ 5*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.203593, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3032, 3021, 2748, 3767, 3768, 3770}

$$\frac{a(4A+5C)\tan^3(c+dx)}{15d}+\frac{a(4A+5C)\tan(c+dx)}{5d}+\frac{aA\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{b(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(4*A + 5*C)*Tan[c + d*x])/
(5*d) + (b*(3*A + 4*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*b*Sec[c + d*x]
^3*Tan[c + d*x])/(4*d) + (a*A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(4*A
+ 5*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + b\cos(c + dx)) \left(A + C\cos^2(c + dx)\right) \sec^6(c + dx)\, dx = \frac{aA\sec^4(c + dx)\tan(c + dx)}{5d} + \frac{1}{5}\int \left(5Ab + a(4A + 5C)\, c\right.$$
$$= \frac{Ab\sec^3(c + dx)\tan(c + dx)}{4d} + \frac{aA\sec^4(c + dx)\tan(c + d}{5d}$$
$$= \frac{Ab\sec^3(c + dx)\tan(c + dx)}{4d} + \frac{aA\sec^4(c + dx)\tan(c + d}{5d}$$
$$= \frac{b(3A + 4C)\sec(c + dx)\tan(c + dx)}{8d} + \frac{Ab\sec^3(c + dx)\, ta}{4d}$$
$$= \frac{b(3A + 4C)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(4A + 5C)\tan(c + d}{5d}$$

Mathematica [A] time = 0.78606, size = 96, normalized size = 0.69

$$\frac{\tan(c + dx)\left(8a\left(5(2A + C)\tan^2(c + dx) + 3A\tan^4(c + dx) + 15(A + C)\right) + 15b(3A + 4C)\sec(c + dx) + 30Ab\sec^3(c +\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (15*b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*b*(3*A + 4*C)*Se
c[c + d*x] + 30*A*b*Sec[c + d*x]^3 + 8*a*(15*(A + C) + 5*(2*A + C)*Tan[c +
d*x]^2 + 3*A*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.05, size = 192, normalized size = 1.4

$$\frac{Ab\left(\sec\left(dx + c\right)\right)^3\tan\left(dx + c\right)}{4\,d} + \frac{3\,Ab\sec\left(dx + c\right)\tan\left(dx + c\right)}{8\,d} + \frac{3\,Ab\ln\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{8\,d} + \frac{Cb\tan\left(dx + \right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/4*A*b*sec(d*x+c)^3*tan(d*x+c)/d+3/8*A*b*sec(d*x+c)*tan(d*x+c)/d+3/8/d*A*b
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*C*b*tan(d*x+c)*sec(d*x+c)+1/2/d*C*b*ln(sec
(d*x+c)+tan(d*x+c))+8/15*a*A*tan(d*x+c)/d+1/5*a*A*sec(d*x+c)^4*tan(d*x+c)/d
+4/15*a*A*sec(d*x+c)^2*tan(d*x+c)/d+2/3/d*a*C*tan(d*x+c)+1/3/d*a*C*tan(d*x+
c)*sec(d*x+c)^2
```

Maxima [A] time = 1.00314, size = 236, normalized size = 1.69

$$\frac{16\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)Aa+80\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Ca-15\,Ab\left(\frac{2\left(3\sin(dx+c)\ldots}{\sin(dx+c)^{4}-\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a + 80
*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a - 15*A*b*(2*(3*sin(d*x + c)^3 - 5*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) - 60*C*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.44778, size = 389, normalized size = 2.78

$$\frac{15\,(3\,A+4\,C)b\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,(3\,A+4\,C)b\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(4\,A+5\ldots\right.}{240\,d\cos\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*A + 4*C)*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*A + 4*
C)*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(4*A + 5*C)*a*cos(d*x +
c)^4 + 15*(3*A + 4*C)*b*cos(d*x + c)^3 + 8*(4*A + 5*C)*a*cos(d*x + c)^2 + 3
0*A*b*cos(d*x + c) + 24*A*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50369, size = 451, normalized size = 3.22

$$15\,(3\,Ab+4\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(3\,Ab+4\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+120\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*A*b + 4*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*b +
4*C*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a*tan(1/2*d*x + 1/2*c)
^9 + 120*C*a*tan(1/2*d*x + 1/2*c)^9 - 75*A*b*tan(1/2*d*x + 1/2*c)^9 - 60*C*
b*tan(1/2*d*x + 1/2*c)^9 - 160*A*a*tan(1/2*d*x + 1/2*c)^7 - 320*C*a*tan(1/2
*d*x + 1/2*c)^7 + 30*A*b*tan(1/2*d*x + 1/2*c)^7 + 120*C*b*tan(1/2*d*x + 1/2
*c)^7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*C*a*tan(1/2*d*x + 1/2*c)^5 - 1
60*A*a*tan(1/2*d*x + 1/2*c)^3 - 320*C*a*tan(1/2*d*x + 1/2*c)^3 - 30*A*b*tan
(1/2*d*x + 1/2*c)^3 - 120*C*b*tan(1/2*d*x + 1/2*c)^3 + 120*A*a*tan(1/2*d*x
+ 1/2*c) + 120*C*a*tan(1/2*d*x + 1/2*c) + 75*A*b*tan(1/2*d*x + 1/2*c) + 60*
C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.531 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=214

$$\frac{\left(2a^2C+b^2(6A+5C)\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{\left(a^2(8A+6C)+b^2(6A+5C)\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(a^2(8A+\right.$$

```
[Out] ((b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*x)/16 + (2*a*b*(5*A + 4*C)*Sin[c + d*x
])/(5*d) + ((b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*Cos[c + d*x]*Sin[c + d*x])/
(16*d) + ((2*a^2*C + b^2*(6*A + 5*C))*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) +
 (a*b*C*Cos[c + d*x]^4*Sin[c + d*x])/(15*d) + (C*Cos[c + d*x]^3*(a + b*Cos[
c + d*x])^2*Sin[c + d*x])/(6*d) - (2*a*b*(5*A + 4*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.489375, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3050, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{\left(2a^2C+b^2(6A+5C)\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{\left(a^2(8A+6C)+b^2(6A+5C)\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(a^2(8A+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] ((b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*x)/16 + (2*a*b*(5*A + 4*C)*Sin[c + d*x
])/(5*d) + ((b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*Cos[c + d*x]*Sin[c + d*x])/
(16*d) + ((2*a^2*C + b^2*(6*A + 5*C))*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) +
 (a*b*C*Cos[c + d*x]^4*Sin[c + d*x])/(15*d) + (C*Cos[c + d*x]^3*(a + b*Cos[
c + d*x])^2*Sin[c + d*x])/(6*d) - (2*a*b*(5*A + 4*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
```

```
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{6d}+\frac{1}{6}\int \cos$$
$$= \frac{abC\cos^4(c+dx)\sin(c+dx)}{15d}+\frac{C\cos^3(c+dx)(a+b\cos($$
$$= \frac{\left(2a^2C+b^2(6A+5C)\right)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{abC\cos}{}$$
$$= \frac{\left(2a^2C+b^2(6A+5C)\right)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{abC\cos}{}$$
$$= \frac{\left(b^2(6A+5C)+a^2(8A+6C)\right)\cos(c+dx)\sin(c+dx)}{16d}+$$
$$= \frac{1}{16}\left(b^2(6A+5C)+a^2(8A+6C)\right)x+\frac{2ab(5A+4C)\sin(c}{5d}$$

Mathematica [A] time = 0.7054, size = 160, normalized size = 0.75

$$\frac{60(c+dx)\left(a^2(8A+6C)+b^2(6A+5C)\right)+15\left(16a^2(A+C)+b^2(16A+15C)\right)\sin(2(c+dx))+15\left(2a^2C+2Ab^2+3b^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (60*(b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*(c + d*x) + 240*a*b*(6*A + 5*C)*Sin
[c + d*x] + 15*(16*a^2*(A + C) + b^2*(16*A + 15*C))*Sin[2*(c + d*x)] + 40*a
*b*(4*A + 5*C)*Sin[3*(c + d*x)] + 15*(2*A*b^2 + 2*a^2*C + 3*b^2*C)*Sin[4*(c
 + d*x)] + 24*a*b*C*Sin[5*(c + d*x)] + 5*b^2*C*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.02, size = 209, normalized size = 1.

$$\frac{1}{d}\left(Ab^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2C\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+b^2
*C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c)+2/3*a*A*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2/5*a*b*C*(8/3+cos(d*x+c)^4+
4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.01576, size = 273, normalized size = 1.28

$$\frac{240\,(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right))Aa^2 + 30\,(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) + 8\,\sin\left(2\,dx + 2\,c\right))Ca^2 - 640\,\left(\sin\left(dx + c\right)^3 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 30*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*A*a*b + 128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*C*a*b + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^2 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*C*b^2)/d
```

Fricas [A] time = 1.49532, size = 383, normalized size = 1.79

$$\frac{15\left(2\,(4\,A + 3\,C)a^2 + (6\,A + 5\,C)b^2\right)dx + \left(40\,Cb^2\cos\left(dx + c\right)^5 + 96\,Cab\cos\left(dx + c\right)^4 + 32\,(5\,A + 4\,C)ab\cos\left(dx + c\right)^2 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(2*(4*A + 3*C)*a^2 + (6*A + 5*C)*b^2)*d*x + (40*C*b^2*cos(d*x + c
)^5 + 96*C*a*b*cos(d*x + c)^4 + 32*(5*A + 4*C)*a*b*cos(d*x + c)^2 + 10*(6*C
*a^2 + (6*A + 5*C)*b^2)*cos(d*x + c)^3 + 64*(5*A + 4*C)*a*b + 15*(2*(4*A +
3*C)*a^2 + (6*A + 5*C)*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 6.52721, size = 592, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*A*a*b*sin(c + d*x)**3/(3*d) + 2*A*a*b*
sin(c + d*x)*cos(c + d*x)**2/d + 3*A*b**2*x*sin(c + d*x)**4/8 + 3*A*b**2*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*b**2*x*cos(c + d*x)**4/8 + 3*A*b**2
*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*b**2*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + 3*C*a**2*x*sin(c + d*x)**4/8 + 3*C*a**2*x*sin(c + d*x)**2*cos(c +
d*x)**2/4 + 3*C*a**2*x*cos(c + d*x)**4/8 + 3*C*a**2*sin(c + d*x)**3*cos(c +
 d*x)/(8*d) + 5*C*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 16*C*a*b*sin(c
+ d*x)**5/(15*d) + 8*C*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*C*a*b*
sin(c + d*x)*cos(c + d*x)**4/d + 5*C*b**2*x*sin(c + d*x)**6/16 + 15*C*b**2*
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*C*b**2*x*sin(c + d*x)**2*cos(c +
d*x)**4/16 + 5*C*b**2*x*cos(c + d*x)**6/16 + 5*C*b**2*sin(c + d*x)**5*cos(c
 + d*x)/(16*d) + 5*C*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*b**2
*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a +
b*cos(c))**2*cos(c)**2, True))
```

Giac [A] time = 1.41274, size = 247, normalized size = 1.15

$$\frac{Cb^2 \sin(6\,dx + 6\,c)}{192\,d} + \frac{Cab \sin(5\,dx + 5\,c)}{40\,d} + \frac{1}{16}\left(8\,Aa^2 + 6\,Ca^2 + 6\,Ab^2 + 5\,Cb^2\right)x + \frac{\left(2\,Ca^2 + 2\,Ab^2 + 3\,Cb^2\right)\sin(4\,dx +}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/192*C*b^2*sin(6*d*x + 6*c)/d + 1/40*C*a*b*sin(5*d*x + 5*c)/d + 1/16*(8*A*
a^2 + 6*C*a^2 + 6*A*b^2 + 5*C*b^2)*x + 1/64*(2*C*a^2 + 2*A*b^2 + 3*C*b^2)*s
in(4*d*x + 4*c)/d + 1/24*(4*A*a*b + 5*C*a*b)*sin(3*d*x + 3*c)/d + 1/64*(16*
A*a^2 + 16*C*a^2 + 16*A*b^2 + 15*C*b^2)*sin(2*d*x + 2*c)/d + 1/4*(6*A*a*b +
 5*C*a*b)*sin(d*x + c)/d
```

3.532 $\int \cos(c+dx)(a+b\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=178

$$\frac{\left(5a^2(3A+2C)+2b^2(5A+4C)\right)\sin(c+dx)}{15d}+\frac{\left(2a^2C+b^2(5A+4C)\right)\sin(c+dx)\cos^2(c+dx)}{15d}+\frac{ab(4A+3C)\sin(c+}{4d}$$

```
[Out] (a*b*(4*A + 3*C)*x)/4 + ((5*a^2*(3*A + 2*C) + 2*b^2*(5*A + 4*C))*Sin[c + d*
x])/(15*d) + (a*b*(4*A + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((2*a^2*C
+ b^2*(5*A + 4*C))*Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (a*b*C*Cos[c + d*x
]^3*Sin[c + d*x])/(10*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(5*d)
```

Rubi [A] time = 0.296465, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3050, 3033, 3023, 2734}

$$\frac{\left(5a^2(3A+2C)+2b^2(5A+4C)\right)\sin(c+dx)}{15d}+\frac{\left(2a^2C+b^2(5A+4C)\right)\sin(c+dx)\cos^2(c+dx)}{15d}+\frac{ab(4A+3C)\sin(c+}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a*b*(4*A + 3*C)*x)/4 + ((5*a^2*(3*A + 2*C) + 2*b^2*(5*A + 4*C))*Sin[c + d*
x])/(15*d) + (a*b*(4*A + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((2*a^2*C
+ b^2*(5*A + 4*C))*Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (a*b*C*Cos[c + d*x
]^3*Sin[c + d*x])/(10*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(5*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
```

```
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^2(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{5d}+\frac{1}{5}\int \cos(\ldots$$

$$= \frac{abC\cos^3(c+dx)\sin(c+dx)}{10d}+\frac{C\cos^2(c+dx)(a+b\cos(c\ldots}{5d}$$

$$= \frac{\left(2a^2C+b^2(5A+4C)\right)\cos^2(c+dx)\sin(c+dx)}{15d}+\frac{abC\cos\ldots}{}$$

$$= \frac{1}{4}ab(4A+3C)x+\frac{\left(5a^2(3A+2C)+2b^2(5A+4C)\right)\sin(c+\ldots}{15d}$$

Mathematica [A] time = 0.427378, size = 126, normalized size = 0.71

$$\frac{30\left(a^2(8A+6C)+b^2(6A+5C)\right)\sin(c+dx)+5\left(4a^2C+4Ab^2+5b^2C\right)\sin(3(c+dx))+60ab(4A+3C)(c+dx)+120}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (60*a*b*(4*A + 3*C)*(c + d*x) + 30*(b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*Sin[
c + d*x] + 120*a*b*(A + C)*Sin[2*(c + d*x)] + 5*(4*A*b^2 + 4*a^2*C + 5*b^2*
C)*Sin[3*(c + d*x)] + 15*a*b*C*Sin[4*(c + d*x)] + 3*b^2*C*Sin[5*(c + d*x)])
/(240*d)
```

Maple [A] time = 0.017, size = 158, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Ab^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{b^2C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+2aAb(1/2\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+1/5*b^2*C*(8/3+cos(d*x+c)^4+4/3*
cos(d*x+c)^2)*sin(d*x+c)+2*a*A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
2*a*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^2*
sin(d*x+c)+1/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.00523, size = 208, normalized size = 1.17

$$-\frac{80\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca^2-120(2dx+2c+\sin(2dx+2c))Aab-15(12dx+12c+\sin(4dx+4c)+8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/240*(80*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 120*(2*d*x + 2*c + sin
(2*d*x + 2*c))*A*a*b - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
 2*c))*C*a*b + 80*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^2 - 16*(3*sin(d*x +
 c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^2 - 240*A*a^2*sin(d*x + c)
)/d
```

Fricas [A] time = 1.57553, size = 300, normalized size = 1.69

$$\frac{15\,(4\,A+3\,C)abdx+\left(12\,Cb^2\cos\left(dx+c\right)^4+30\,Cab\cos\left(dx+c\right)^3+15\,(4\,A+3\,C)ab\cos\left(dx+c\right)+20\,(3\,A+2\,C)a^2+8}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*(4*A + 3*C)*a*b*d*x + (12*C*b^2*cos(d*x + c)^4 + 30*C*a*b*cos(d*x
+ c)^3 + 15*(4*A + 3*C)*a*b*cos(d*x + c) + 20*(3*A + 2*C)*a^2 + 8*(5*A + 4*
C)*b^2 + 4*(5*C*a^2 + (5*A + 4*C)*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.98397, size = 350, normalized size = 1.97

$$\begin{cases}\frac{Aa^2\sin\left(c+dx\right)}{d}+Aabx\sin^2\left(c+dx\right)+Aabx\cos^2\left(c+dx\right)+\frac{Aab\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{2Ab^2\sin^3\left(c+dx\right)}{3d}+\frac{Ab^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\\ x\left(A+C\cos^2\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^2\cos\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*x*sin(c + d*x)**2 + A*a*b*x*cos(c
+ d*x)**2 + A*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*A*b**2*sin(c + d*x)**3/(3
*d) + A*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 2*C*a**2*sin(c + d*x)**3/(3*d
) + C*a**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*a*b*x*sin(c + d*x)**4/4 + 3
*C*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*C*a*b*x*cos(c + d*x)**4/4 +
3*C*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 5*C*a*b*sin(c + d*x)*cos(c + d
*x)**3/(4*d) + 8*C*b**2*sin(c + d*x)**5/(15*d) + 4*C*b**2*sin(c + d*x)**3*c
os(c + d*x)**2/(3*d) + C*b**2*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x
*(A + C*cos(c)**2)*(a + b*cos(c))**2*cos(c), True))
```

Giac [A] time = 1.50016, size = 192, normalized size = 1.08

$$\frac{Cb^2 \sin(5\,dx+5\,c)}{80\,d} + \frac{Cab \sin(4\,dx+4\,c)}{16\,d} + \frac{1}{4}\,(4\,Aab+3\,Cab)x + \frac{\left(4\,Ca^2+4\,Ab^2+5\,Cb^2\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{(Aab+C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/80*C*b^2*sin(5*d*x + 5*c)/d + 1/16*C*a*b*sin(4*d*x + 4*c)/d + 1/4*(4*A*a*
b + 3*C*a*b)*x + 1/48*(4*C*a^2 + 4*A*b^2 + 5*C*b^2)*sin(3*d*x + 3*c)/d + 1/
2*(A*a*b + C*a*b)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^2 + 6*C*a^2 + 6*A*b^2 + 5
*C*b^2)*sin(d*x + c)/d
```

3.533 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=161

$$\frac{a\left(a^2(-C)+12Ab^2+8b^2C\right)\sin(c+dx)}{6bd}-\frac{\left(2a^2C-3b^2(4A+3C)\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2(2A+C)+b^2(4A+\right.$$

```
[Out] ((4*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/8 + (a*(12*A*b^2 - a^2*C + 8*b^2*C)
*Sin[c + d*x])/(6*b*d) - ((2*a^2*C - 3*b^2*(4*A + 3*C))*Cos[c + d*x]*Sin[c
+ d*x])/(24*d) - (a*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(12*b*d) + (C*(a
 + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.214536, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3024, 2753, 2734}

$$\frac{a\left(a^2(-C)+12Ab^2+8b^2C\right)\sin(c+dx)}{6bd}-\frac{\left(2a^2C-3b^2(4A+3C)\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2(2A+C)+b^2(4A+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] ((4*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/8 + (a*(12*A*b^2 - a^2*C + 8*b^2*C)
*Sin[c + d*x])/(6*b*d) - ((2*a^2*C - 3*b^2*(4*A + 3*C))*Cos[c + d*x]*Sin[c
+ d*x])/(24*d) - (a*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(12*b*d) + (C*(a
 + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right) dx = \frac{C(a+b\cos(c+dx))^3\sin(c+dx)}{4bd} + \frac{\int (a+b\cos(c+dx))^2(b(4A+3C}{4b}$$

$$= -\frac{aC(a+b\cos(c+dx))^2\sin(c+dx)}{12bd} + \frac{C(a+b\cos(c+dx))^3\sin(c+}{4bd}$$

$$= \frac{1}{8}\left(4a^2(2A+C)+b^2(4A+3C)\right)x + \frac{a\left(12Ab^2-a^2C+8b^2C\right)\sin(c+}{6bd}$$

Mathematica [A] time = 0.367917, size = 106, normalized size = 0.66

$$\frac{12(c+dx)\left(4a^2(2A+C)+b^2(4A+3C)\right)+24\left(C\left(a^2+b^2\right)+Ab^2\right)\sin(2(c+dx))+48ab(4A+3C)\sin(c+dx)+16abC}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (12*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*(c + d*x) + 48*a*b*(4*A + 3*C)*Sin[
c + d*x] + 24*(A*b^2 + (a^2 + b^2)*C)*Sin[2*(c + d*x)] + 16*a*b*C*Sin[3*(c
+ d*x)] + 3*b^2*C*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.019, size = 140, normalized size = 0.9

$$\frac{1}{d}\left(b^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2abC\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(b^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3
*a*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*A*b*sin(d*x+c)+
A*a^2*(d*x+c))
```

Maxima [A] time = 0.979583, size = 176, normalized size = 1.09

$$\frac{96\,(dx+c)Aa^2+24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^2-64\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Cab+24\,(2\,dx+2\,c+\sin(2\,d}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*A*a^2 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 - 64*(
sin(d*x + c)^3 - 3*sin(d*x + c))*C*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*A*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*b^2 +
 192*A*a*b*sin(d*x + c))/d
```

Fricas [A] time = 1.57037, size = 248, normalized size = 1.54

$$\frac{3\left(4\,(2\,A+C)a^2+(4\,A+3\,C)b^2\right)dx+\left(6\,Cb^2\cos(dx+c)^3+16\,Cab\cos(dx+c)^2+16\,(3\,A+2\,C)ab+3\left(4\,Ca^2+(4\,A+\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*(2*A + C)*a^2 + (4*A + 3*C)*b^2)*d*x + (6*C*b^2*cos(d*x + c)^3 +
 16*C*a*b*cos(d*x + c)^2 + 16*(3*A + 2*C)*a*b + 3*(4*C*a^2 + (4*A + 3*C)*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.60649, size = 309, normalized size = 1.92

$$\begin{cases} Aa^2x+\frac{2Aab\sin(c+dx)}{d}+\frac{Ab^2x\sin^2(c+dx)}{2}+\frac{Ab^2x\cos^2(c+dx)}{2}+\frac{Ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{Ca^2x\sin^2(c+dx)}{2}+\frac{Ca^2x\cos^2(c+dx)}{2}+\frac{Ca^2\sin(c+}{} \\ x\left(A+C\cos^2(c)\right)(a+b\cos(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*x + 2*A*a*b*sin(c + d*x)/d + A*b**2*x*sin(c + d*x)**2/2 +
 A*b**2*x*cos(c + d*x)**2/2 + A*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + C*a*
*2*x*sin(c + d*x)**2/2 + C*a**2*x*cos(c + d*x)**2/2 + C*a**2*sin(c + d*x)*c
os(c + d*x)/(2*d) + 4*C*a*b*sin(c + d*x)**3/(3*d) + 2*C*a*b*sin(c + d*x)*co
s(c + d*x)**2/d + 3*C*b**2*x*sin(c + d*x)**4/8 + 3*C*b**2*x*sin(c + d*x)**2
*cos(c + d*x)**2/4 + 3*C*b**2*x*cos(c + d*x)**4/8 + 3*C*b**2*sin(c + d*x)**
3*cos(c + d*x)/(8*d) + 5*C*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0
)), (x*(A + C*cos(c)**2)*(a + b*cos(c))**2, True))
```

Giac [A] time = 1.28016, size = 157, normalized size = 0.98

$$\frac{Cb^2\sin(4\,dx+4\,c)}{32\,d}+\frac{Cab\sin(3\,dx+3\,c)}{6\,d}+\frac{1}{8}\left(8\,Aa^2+4\,Ca^2+4\,Ab^2+3\,Cb^2\right)x+\frac{\left(Ca^2+Ab^2+Cb^2\right)\sin(2\,dx+2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/32*C*b^2*sin(4*d*x + 4*c)/d + 1/6*C*a*b*sin(3*d*x + 3*c)/d + 1/8*(8*A*a^2
 + 4*C*a^2 + 4*A*b^2 + 3*C*b^2)*x + 1/4*(C*a^2 + A*b^2 + C*b^2)*sin(2*d*x +
 2*c)/d + 1/2*(4*A*a*b + 3*C*a*b)*sin(d*x + c)/d
```

3.534 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{\left(2C\left(a^2+b^2\right)+3Ab^2\right)\sin(c+dx)}{3d}+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+abx(2A+C)+\frac{abC\sin(c+dx)\cos(c+dx)}{3d}+\frac{C\sin(c+$$

```
[Out] a*b*(2*A + C)*x + (a^2*A*ArcTanh[Sin[c + d*x]])/d + ((3*A*b^2 + 2*(a^2 + b^
2)*C)*Sin[c + d*x])/(3*d) + (a*b*C*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (C*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.278564, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3050, 3033, 3023, 2735, 3770}

$$\frac{\left(2C\left(a^2+b^2\right)+3Ab^2\right)\sin(c+dx)}{3d}+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+abx(2A+C)+\frac{abC\sin(c+dx)\cos(c+dx)}{3d}+\frac{C\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] a*b*(2*A + C)*x + (a^2*A*ArcTanh[Sin[c + d*x]])/d + ((3*A*b^2 + 2*(a^2 + b^
2)*C)*Sin[c + d*x])/(3*d) + (a*b*C*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (C*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (a+b\cos(c+dx))$$
$$= \frac{abC\cos(c+dx)\sin(c+dx)}{3d} + \frac{C(a+b\cos(c+dx))^2\sin(c+}{3d}$$
$$= \frac{\left(3Ab^2+2\left(a^2+b^2\right)C\right)\sin(c+dx)}{3d} + \frac{abC\cos(c+dx)\sin(c}{3d}$$
$$= ab(2A+C)x + \frac{\left(3Ab^2+2\left(a^2+b^2\right)C\right)\sin(c+dx)}{3d} + \frac{abC\,c}{}$$
$$= ab(2A+C)x + \frac{a^2A\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(3Ab^2+2\left(a^2+\right.\right.}{}$$

Mathematica [A] time = 0.248575, size = 145, normalized size = 1.41

$$\frac{3\left(4a^2C+4Ab^2+3b^2C\right)\sin(c+dx) - 12a^2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (24*a*A*b*c + 12*a*b*c*C + 24*a*A*b*d*x + 12*a*b*C*d*x - 12*a^2*A*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2*A*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 3*(4*A*b^2 + 4*a^2*C + 3*b^2*C)*Sin[c + d*x] + 6*a*b*C*Sin[2*(c
+ d*x)] + b^2*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.044, size = 137, normalized size = 1.3

$$\frac{Ab^2\sin(dx+c)}{d} + \frac{C\sin(dx+c)(\cos(dx+c))^2b^2}{3d} + \frac{2b^2C\sin(dx+c)}{3d} + 2aAbx + 2\frac{Aabc}{d} + \frac{abC\cos(dx+c)\sin(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*A*b^2*sin(d*x+c)+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*b^2+2/3*b^2*C*sin(d*x+
c)/d+2*a*A*b*x+2/d*A*a*b*c+a*b*C*cos(d*x+c)*sin(d*x+c)/d+a*b*C*x+1/d*a*b*C*
c+1/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*sin(d*x+c)
```

Maxima [A] time = 0.987378, size = 142, normalized size = 1.38

$$\frac{12\,(dx+c)Aab + 3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Cab - 2\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)Cb^2 + 6\,Aa^2\log(\sec(dx+c) + \cdots}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/6*(12*(d*x + c)*A*a*b + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b - 2*(sin
(d*x + c)^3 - 3*sin(d*x + c))*C*b^2 + 6*A*a^2*log(sec(d*x + c) + tan(d*x +
c)) + 6*C*a^2*sin(d*x + c) + 6*A*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.5569, size = 250, normalized size = 2.43

$$\frac{6\,(2\,A + C)abdx + 3\,Aa^2\log(\sin(dx+c)+1) - 3\,Aa^2\log(-\sin(dx+c)+1) + 2\left(Cb^2\cos(dx+c)^2 + 3\,Cab\cos(dx+\cdots\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] 1/6*(6*(2*A + C)*a*b*d*x + 3*A*a^2*log(sin(d*x + c) + 1) - 3*A*a^2*log(-sin
(d*x + c) + 1) + 2*(C*b^2*cos(d*x + c)^2 + 3*C*a*b*cos(d*x + c) + 3*C*a^2 +
 (3*A + 2*C)*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37585, size = 346, normalized size = 3.36

$$3\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(2\,Aab+Cab)(dx+c)+\frac{2\left(3\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*A*a*b + C*a*b)*(d*x + c) + 2*(3*C*a^2*tan(1/2*d*x
+ 1/2*c)^5 - 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^2*tan(1/2*d*x + 1/2*c)^
5 + 3*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*b
^2*tan(1/2*d*x + 1/2*c)^3 + 2*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^2*tan(1/
2*d*x + 1/2*c) + 3*C*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c
) + 3*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.535 $\int (a+b\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=109

$$\frac{1}{2}x\left(C\left(2a^2+b^2\right)+2Ab^2\right)-\frac{2ab(A-C)\sin(c+dx)}{d}+\frac{2aAb\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

```
[Out] ((2*A*b^2 + (2*a^2 + b^2)*C)*x)/2 + (2*a*A*b*ArcTanh[Sin[c + d*x]])/d - (2*
a*b*(A - C)*Sin[c + d*x])/d - (b^2*(2*A - C)*Cos[c + d*x]*Sin[c + d*x])/(2*
d) + (A*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.314727, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3048, 3033, 3023, 2735, 3770}

$$\frac{1}{2}x\left(C\left(2a^2+b^2\right)+2Ab^2\right)-\frac{2ab(A-C)\sin(c+dx)}{d}+\frac{2aAb\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] ((2*A*b^2 + (2*a^2 + b^2)*C)*x)/2 + (2*a*A*b*ArcTanh[Sin[c + d*x]])/d - (2*
a*b*(A - C)*Sin[c + d*x])/d - (b^2*(2*A - C)*Cos[c + d*x]*Sin[c + d*x])/(2*
d) + (A*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{A(a+b\cos(c+dx))^2\tan(c+dx)}{d}+\int (a+b\cos(c+dx))$$

$$=-\frac{b^2(2A-C)\cos(c+dx)\sin(c+dx)}{2d}+\frac{A(a+b\cos(c+dx}{d}$$

$$=-\frac{2ab(A-C)\sin(c+dx)}{d}-\frac{b^2(2A-C)\cos(c+dx)\sin(c+}{2d}$$

$$=\frac{1}{2}\left(2Ab^2+\left(2a^2+b^2\right)C\right)x-\frac{2ab(A-C)\sin(c+dx)}{d}-\frac{b^2(}{}$$

$$=\frac{1}{2}\left(2Ab^2+\left(2a^2+b^2\right)C\right)x+\frac{2aAb\tanh^{-1}(\sin(c+dx))}{d}-$$

Mathematica [A] time = 0.713461, size = 132, normalized size = 1.21

$$\frac{2(c+dx)\left(C\left(2a^2+b^2\right)+2Ab^2\right)+\tan(c+dx)\left(4a^2A+b^2C\cos(2(c+dx))+b^2C\right)-8aAb\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*(2*A*b^2 + (2*a^2 + b^2)*C)*(c + d*x) - 8*a*A*b*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 8*a*A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*b
*C*Sin[c + d*x] + (4*a^2*A + b^2*C + b^2*C*Cos[2*(c + d*x)])*Tan[c + d*x])/
(4*d)
```

Maple [A] time = 0.051, size = 120, normalized size = 1.1

$$Ab^2x+\frac{Ab^2c}{d}+\frac{b^2C\cos(dx+c)\sin(dx+c)}{2d}+\frac{b^2Cx}{2}+\frac{b^2Cc}{2d}+2\frac{aAb\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{abC\sin(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] A*b^2*x+1/d*A*b^2*c+1/2/d*b^2*C*cos(d*x+c)*sin(d*x+c)+1/2*b^2*C*x+1/2/d*b^2
*C*c+2/d*a*A*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*C*sin(d*x+c)+1/d*A*a^2*tan
(d*x+c)+a^2*C*x+1/d*a^2*C*c
```

Maxima [A] time = 1.01696, size = 134, normalized size = 1.23

$$\frac{4(dx+c)Ca^2+4(dx+c)Ab^2+(2dx+2c+\sin(2dx+2c))Cb^2+4Aab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a^2 + 4*(d*x + c)*A*b^2 + (2*d*x + 2*c + sin(2*d*x + 2*c
))*C*b^2 + 4*A*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*C*a*
```

```
b*sin(d*x + c) + 4*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.46055, size = 309, normalized size = 2.83

$$\frac{2\,Aab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-2\,Aab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ca^2+\left(2\,A+C\right)b^2\right)dx\cos\left(dx+\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] 1/2*(2*A*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 2*A*a*b*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (2*C*a^2 + (2*A + C)*b^2)*d*x*cos(d*x + c) + (C*b^2*co
s(d*x + c)^2 + 4*C*a*b*cos(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.67502, size = 236, normalized size = 2.17

$$\frac{4\,Aab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,Aab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\left(2\,Ca^2+2\,Ab^2+Cb^2\right)\left(dx+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] 1/2*(4*A*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*A*a*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (2*C*a^2 + 2*A*b^2 + C*b^2)*(d*x + c) + 2*(4*C*a*b*tan(1/2*d*x + 1/2*c
)^3 - C*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*a*b*tan(1/2*d*x + 1/2*c) + C*b^2*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.536 $\int (a+b\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{\left(a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aAb\tan(c+dx)}{d}+\frac{A\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}+2abCx$$

```
[Out] 2*a*b*C*x + ((2*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*
(A - 2*C)*Sin[c + d*x])/(2*d) + (a*A*b*Tan[c + d*x])/d + (A*(a + b*Cos[c +
d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.312713, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3048, 3031, 3023, 2735, 3770}

$$\frac{\left(a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aAb\tan(c+dx)}{d}+\frac{A\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}+2abCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] 2*a*b*C*x + ((2*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*
(A - 2*C)*Sin[c + d*x])/(2*d) + (a*A*b*Tan[c + d*x])/d + (A*(a + b*Cos[c +
d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (a+$$

$$= \frac{aAb\tan(c+dx)}{d} + \frac{A(a+b\cos(c+dx))^2\sec(c+dx)\tan(}{2d}$$

$$= -\frac{b^2(A-2C)\sin(c+dx)}{2d} + \frac{aAb\tan(c+dx)}{d} + \frac{A(a+b\,co}{}$$

$$= 2abCx - \frac{b^2(A-2C)\sin(c+dx)}{2d} + \frac{aAb\tan(c+dx)}{d} + \frac{A(}{}$$

$$= 2abCx + \frac{\left(2Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b^2(}{}$$

Mathematica [B] time = 1.28264, size = 249, normalized size = 2.42

$$\frac{-2\left(a^2(A+2C)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(a^2(A+2C)+2Ab^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (8*a*b*C*(c + d*x) - 2*(2*A*b^2 + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 2*(2*A*b^2 + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (8*a*A*b*Sin
[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^2*A)/(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2 + (8*a*A*b*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]) + 4*b^2*C*Sin[c + d*x])/(4*d)
```

Maple [A] time = 0.053, size = 133, normalized size = 1.3

$$\frac{Ab^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^2C\sin(dx+c)}{d} + 2\,\frac{aAb\tan(dx+c)}{d} + 2\,abCx + 2\,\frac{Cabc}{d} + \frac{Aa^2\sec(dx+c)\tan(}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^2*C*sin(d*x+c)/d+2*a*A*b*tan(d*x+c)/d
+2*a*b*C*x+2/d*a*b*C*c+1/2/d*A*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^2*ln(sec
```

```
(d*x+c)+tan(d*x+c))+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.0088, size = 189, normalized size = 1.83

$$\frac{8\,(dx+c)Cab - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Ca^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\mathrm{si}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] 1/4*(8*(d*x + c)*C*a*b - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*C*b^2*sin(d*x + c) + 8*A*a*b*tan(d*x + c))/d
```

Fricas [A] time = 1.60001, size = 347, normalized size = 3.37

$$\frac{8\,Cabdx\cos\left(dx+c\right)^2 + \left((A+2\,C)a^2 + 2\,Ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left((A+2\,C)a^2 + 2\,Ab^2\right)\cos\left(dx+c\right)^2}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] 1/4*(8*C*a*b*d*x*cos(d*x + c)^2 + ((A + 2*C)*a^2 + 2*A*b^2)*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - ((A + 2*C)*a^2 + 2*A*b^2)*cos(d*x + c)^2*log(-sin(d
*x + c) + 1) + 2*(2*C*b^2*cos(d*x + c)^2 + 4*A*a*b*cos(d*x + c) + A*a^2)*si
n(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2545, size = 255, normalized size = 2.48

$$\frac{4\,(dx+c)Cab+\frac{4\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\left(Aa^2+2\,Ca^2+2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(Aa^2+2\,Ca^2+2\,Ab^2\right)\log\left(\left|\tan\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] 1/2*(4*(d*x + c)*C*a*b + 4*C*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)
^2 + 1) + (A*a^2 + 2*C*a^2 + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
(A*a^2 + 2*C*a^2 + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*t
an(1/2*d*x + 1/2*c)^3 - 4*A*a*b*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x
+ 1/2*c) + 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.537 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{\left(a^2(2A+3C)+2Ab^2\right)\tan(c+dx)}{3d}+\frac{ab(A+2C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aAb\tan(c+dx)\sec(c+dx)}{3d}+\frac{A\tan(c+dx)}{}$$

```
[Out] b^2*C*x + (a*b*(A + 2*C)*ArcTanh[Sin[c + d*x]])/d + ((2*A*b^2 + a^2*(2*A +
3*C))*Tan[c + d*x])/(3*d) + (a*A*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (A*(a
 + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.314508, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3048, 3031, 3021, 2735, 3770}

$$\frac{\left(a^2(2A+3C)+2Ab^2\right)\tan(c+dx)}{3d}+\frac{ab(A+2C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aAb\tan(c+dx)\sec(c+dx)}{3d}+\frac{A\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] b^2*C*x + (a*b*(A + 2*C)*ArcTanh[Sin[c + d*x]])/d + ((2*A*b^2 + a^2*(2*A +
3*C))*Tan[c + d*x])/(3*d) + (a*A*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (A*(a
 + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a$$

$$= \frac{aAb\sec(c+dx)\tan(c+dx)}{3d}+\frac{A(a+b\cos(c+dx))^2\sec}{3d}$$

$$= \frac{\left(2Ab^2+a^2(2A+3C)\right)\tan(c+dx)}{3d}+\frac{aAb\sec(c+dx)\tan}{3d}$$

$$= b^2Cx+\frac{\left(2Ab^2+a^2(2A+3C)\right)\tan(c+dx)}{3d}+\frac{aAb\sec(c}{}$$

$$= b^2Cx+\frac{ab(A+2C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(2Ab^2+a^2(2\right.}{}$$

Mathematica [A] time = 0.439717, size = 76, normalized size = 0.68

$$\frac{3\tan(c+dx)\left(a^2(A+C)+aAb\sec(c+dx)+Ab^2\right)+a^2A\tan^3(c+dx)+3ab(A+2C)\tanh^{-1}(\sin(c+dx))+3b^2Cdx}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (3*b^2*C*d*x + 3*a*b*(A + 2*C)*ArcTanh[Sin[c + d*x]] + 3*(A*b^2 + a^2*(A +
C) + a*A*b*Sec[c + d*x])*Tan[c + d*x] + a^2*A*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.052, size = 145, normalized size = 1.3

$$\frac{Ab^2\tan(dx+c)}{d}+b^2Cx+\frac{Cb^2c}{d}+\frac{aAb\sec(dx+c)\tan(dx+c)}{d}+\frac{aAb\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{abC\ln(\mathrm{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*A*b^2*tan(d*x+c)+b^2*C*x+1/d*b^2*C*c+a*A*b*sec(d*x+c)*tan(d*x+c)/d+1/d*
a*A*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*A
*a^2*tan(d*x+c)+1/3/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+1/d*a^2*C*tan(d*x+c)
```

Maxima [A] time = 1.01538, size = 184, normalized size = 1.64

$$\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+6(dx+c)Cb^2-3Aab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 6*(d*x + c)*C*b^2 - 3*A*a*
b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 6*C*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*
C*a^2*tan(d*x + c) + 6*A*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.45657, size = 347, normalized size = 3.1

$$\frac{6Cb^2dx\cos(dx+c)^3+3(A+2C)ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3(A+2C)ab\cos(dx+c)^3\log(-\sin(dx+c)}{6d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(6*C*b^2*d*x*cos(d*x + c)^3 + 3*(A + 2*C)*a*b*cos(d*x + c)^3*log(sin(d*
x + c) + 1) - 3*(A + 2*C)*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(3*
A*a*b*cos(d*x + c) + A*a^2 + ((2*A + 3*C)*a^2 + 3*A*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

[Out] Timed out

Giac [B] time = 1.40049, size = 354, normalized size = 3.16

$$3\,(dx+c)Cb^2+3\,(Aab+2\,Cab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aab+2\,Cab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] 1/3*(3*(d*x + c)*C*b^2 + 3*(A*a*b + 2*C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 3*(A*a*b + 2*C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*
tan(1/2*d*x + 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*a*b*tan(1/2*d
*x + 1/2*c)^5 + 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*A*a^2*tan(1/2*d*x + 1/2*
c)^3 - 6*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 6*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*
A*a^2*tan(1/2*d*x + 1/2*c) + 3*C*a^2*tan(1/2*d*x + 1/2*c) + 3*A*a*b*tan(1/2
*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
3)/d
```

3.538 $\int (a+b\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=154

$$\frac{\left(a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(a^2(3A+4C)+2Ab^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab(2A+3C)\,\mathrm{ta}}{3d}$$

```
[Out] ((4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*
(2*A + 3*C)*Tan[c + d*x])/(3*d) + ((2*A*b^2 + a^2*(3*A + 4*C))*Sec[c + d*x]
*Tan[c + d*x])/(8*d) + (a*A*b*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (A*(a +
b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.42167, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(a^2(3A+4C)+2Ab^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab(2A+3C)\,\mathrm{ta}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] ((4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*
(2*A + 3*C)*Tan[c + d*x])/(3*d) + ((2*A*b^2 + a^2*(3*A + 4*C))*Sec[c + d*x]
*Tan[c + d*x])/(8*d) + (a*A*b*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (A*(a +
b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (a+$$
$$= \frac{aAb\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{A(a+b\cos(c+dx))^2\sec^3}{4d}$$
$$= \frac{\left(2Ab^2+a^2(3A+4C)\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{aAb\sec}{}$$
$$= \frac{\left(2Ab^2+a^2(3A+4C)\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{aAb\sec}{}$$
$$= \frac{\left(4b^2(A+2C)+a^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(2A}{}$$
$$= \frac{\left(4b^2(A+2C)+a^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab(}{}$$

Mathematica [A] time = 0.618409, size = 107, normalized size = 0.69

$$\frac{3\left(a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(a^2(3A+4C)+4Ab^2\right)\sec(c+dx)+6a^2A\sec^3(c+\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (3*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]
*(3*(4*A*b^2 + a^2*(3*A + 4*C))*Sec[c + d*x] + 6*a^2*A*Sec[c + d*x]^3 + 16*
a*b*(3*(A + C) + A*Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.059, size = 229, normalized size = 1.5

$$\frac{Ab^2\sec(dx+c)\tan(dx+c)}{2d} + \frac{Ab^2\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{b^2C\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{4aAb\tan(}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/2/d*A*b^2*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d
*b^2*C*ln(sec(d*x+c)+tan(d*x+c))+4/3*a*A*b*tan(d*x+c)/d+2/3*a*A*b*sec(d*x+c
)^2*tan(d*x+c)/d+2/d*a*b*C*tan(d*x+c)+1/4/d*A*a^2*tan(d*x+c)*sec(d*x+c)^3+3
/8/d*A*a^2*sec(d*x+c)*tan(d*x+c)+3/8/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/
d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.0178, size = 313, normalized size = 2.03

$$32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aab-3\,Aa^{2}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a*b - 3*A*a^2*(2*(3*sin(d*x +
c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 24*C*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*C
*a*b*tan(d*x + c))/d
```

Fricas [A] time = 1.66438, size = 424, normalized size = 2.75

$$3\left((3\,A+4\,C)a^{2}+4\,(A+2\,C)b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left((3\,A+4\,C)a^{2}+4\,(A+2\,C)b^{2}\right)\cos\left(dx+c\right)^{4}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/48*(3*((3*A + 4*C)*a^2 + 4*(A + 2*C)*b^2)*cos(d*x + c)^4*log(sin(d*x + c)
 + 1) - 3*((3*A + 4*C)*a^2 + 4*(A + 2*C)*b^2)*cos(d*x + c)^4*log(-sin(d*x +
 c) + 1) + 2*(16*(2*A + 3*C)*a*b*cos(d*x + c)^3 + 16*A*a*b*cos(d*x + c) + 6
*A*a^2 + 3*((3*A + 4*C)*a^2 + 4*A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos
```

```
(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)

[Out] Timed out
```

Giac [B] time = 1.4491, size = 575, normalized size = 3.73

$$3\left(3\,Aa^2+4\,Ca^2+4\,Ab^2+8\,Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Aa^2+4\,Ca^2+4\,Ab^2+8\,Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
giac")

[Out] 1/24*(3*(3*A*a^2 + 4*C*a^2 + 4*A*b^2 + 8*C*b^2)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) - 3*(3*A*a^2 + 4*C*a^2 + 4*A*b^2 + 8*C*b^2)*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) + 2*(15*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^2*tan(1/2*d*x + 1
/2*c)^7 - 48*A*a*b*tan(1/2*d*x + 1/2*c)^7 - 48*C*a*b*tan(1/2*d*x + 1/2*c)^7
 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 12*C*
a^2*tan(1/2*d*x + 1/2*c)^5 + 80*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 144*C*a*b*ta
n(1/2*d*x + 1/2*c)^5 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*
x + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2
*c)^3 - 144*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3
+ 15*A*a^2*tan(1/2*d*x + 1/2*c) + 12*C*a^2*tan(1/2*d*x + 1/2*c) + 48*A*a*b*
tan(1/2*d*x + 1/2*c) + 48*C*a*b*tan(1/2*d*x + 1/2*c) + 12*A*b^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.539 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=187

$$\frac{\left(2a^2(4A+5C)+5b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{\left(a^2(4A+5C)+2Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{15d}+\frac{ab(3A+4C)\tanh^{-1}}{4d}$$

```
[Out] (a*b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(4*d) + ((5*b^2*(2*A + 3*C) + 2*a^2
*(4*A + 5*C))*Tan[c + d*x])/(15*d) + (a*b*(3*A + 4*C)*Sec[c + d*x]*Tan[c +
d*x])/(4*d) + ((2*A*b^2 + a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15
*d) + (a*A*b*Sec[c + d*x]^3*Tan[c + d*x])/(10*d) + (A*(a + b*Cos[c + d*x])^
2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.446714, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3048, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(2a^2(4A+5C)+5b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{\left(a^2(4A+5C)+2Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{15d}+\frac{ab(3A+4C)\tanh^{-1}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] (a*b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(4*d) + ((5*b^2*(2*A + 3*C) + 2*a^2
*(4*A + 5*C))*Tan[c + d*x])/(15*d) + (a*b*(3*A + 4*C)*Sec[c + d*x]*Tan[c +
d*x])/(4*d) + ((2*A*b^2 + a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15
*d) + (a*A*b*Sec[c + d*x]^3*Tan[c + d*x])/(10*d) + (A*(a + b*Cos[c + d*x])^
2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a$$
$$= \frac{aAb\sec^3(c+dx)\tan(c+dx)}{10d}+\frac{A(a+b\cos(c+dx))^2\,\mathrm{se}}{5d}$$
$$= \frac{\left(2Ab^2+a^2(4A+5C)\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{aAb\,\mathrm{s}}{}$$
$$= \frac{\left(2Ab^2+a^2(4A+5C)\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{aAb\,\mathrm{s}}{}$$
$$= \frac{ab(3A+4C)\sec(c+dx)\tan(c+dx)}{4d}+\frac{\left(2Ab^2+a^2(4A\right.}{}$$
$$= \frac{ab(3A+4C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\left(5b^2(2A+3C)+2a\right.}{}$$

Mathematica [A] time = 1.09355, size = 115, normalized size = 0.61

$$\frac{\tan(c+dx)\left(20\left(a^2(2A+C)+Ab^2\right)\tan^2(c+dx)+60\left(a^2+b^2\right)(A+C)+12a^2A\tan^4(c+dx)+15ab(3A+4C)\sec(c\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (15*a*b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(60*(a^2 + b^2)*(A
 + C) + 15*a*b*(3*A + 4*C)*Sec[c + d*x] + 30*a*A*b*Sec[c + d*x]^3 + 20*(A*b
^2 + a^2*(2*A + C))*Tan[c + d*x]^2 + 12*a^2*A*Tan[c + d*x]^4))/(60*d)
```

Maple [A] time = 0.056, size = 257, normalized size = 1.4

$$\frac{2\,Ab^{2}\tan\left(dx+c\right)}{3\,d}+\frac{Ab^{2}\tan\left(dx+c\right)\left(\sec\left(dx+c\right)\right)^{2}}{3\,d}+\frac{b^{2}C\tan\left(dx+c\right)}{d}+\frac{aAb\left(\sec\left(dx+c\right)\right)^{3}\tan\left(dx+c\right)}{2\,d}+\frac{3\,aAb\sec}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 2/3/d*A*b^2*tan(d*x+c)+1/3/d*A*b^2*tan(d*x+c)*sec(d*x+c)^2+1/d*b^2*C*tan(d*
x+c)+1/2*a*A*b*sec(d*x+c)^3*tan(d*x+c)/d+3/4*a*A*b*sec(d*x+c)*tan(d*x+c)/d+
3/4/d*a*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*b*C*tan(d*x+c)*sec(d*x+c)+1/d*a
*b*C*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*A*a^2*tan(d*x+c)+1/5/d*A*a^2*tan(d*x+
c)*sec(d*x+c)^4+4/15/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+2/3/d*a^2*C*tan(d*x+c)
+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.00044, size = 292, normalized size = 1.56

$$\frac{8\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Aa^{2}+40\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca^{2}+40\left(\tan\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 + 4
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 + 40*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*A*b^2 - 15*A*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 60*C*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)) + 120*C*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.47705, size = 455, normalized size = 2.43

$$\frac{15\left(3\,A+4\,C\right)ab\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,A+4\,C\right)ab\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(15\left(3\,A+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(3*A + 4*C)*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*A +
4*C)*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(15*(3*A + 4*C)*a*b*cos(
d*x + c)^3 + 4*(2*(4*A + 5*C)*a^2 + 5*(2*A + 3*C)*b^2)*cos(d*x + c)^4 + 30*
A*a*b*cos(d*x + c) + 12*A*a^2 + 4*((4*A + 5*C)*a^2 + 5*A*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28288, size = 718, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] 1/60*(15*(3*A*a*b + 4*C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*a
*b + 4*C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a^2*tan(1/2*d*x
+ 1/2*c)^9 + 60*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*A*a*b*tan(1/2*d*x + 1/2*c
)^9 - 60*C*a*b*tan(1/2*d*x + 1/2*c)^9 + 60*A*b^2*tan(1/2*d*x + 1/2*c)^9 + 6
0*C*b^2*tan(1/2*d*x + 1/2*c)^9 - 80*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 160*C*a^
2*tan(1/2*d*x + 1/2*c)^7 + 30*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 120*C*a*b*tan(
1/2*d*x + 1/2*c)^7 - 160*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 240*C*b^2*tan(1/2*d
*x + 1/2*c)^7 + 232*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 200*C*a^2*tan(1/2*d*x +
1/2*c)^5 + 200*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 360*C*b^2*tan(1/2*d*x + 1/2*c
)^5 - 80*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 160*C*a^2*tan(1/2*d*x + 1/2*c)^3 -
30*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 120*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 160*A*
```

```
b^2*tan(1/2*d*x + 1/2*c)^3 - 240*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 60*A*a^2*ta
n(1/2*d*x + 1/2*c) + 60*C*a^2*tan(1/2*d*x + 1/2*c) + 75*A*a*b*tan(1/2*d*x +
 1/2*c) + 60*C*a*b*tan(1/2*d*x + 1/2*c) + 60*A*b^2*tan(1/2*d*x + 1/2*c) + 6
0*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.540 $\int \cos(c+dx)(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=264

$$\frac{a\left(5a^2(3A+2C)+6b^2(5A+4C)\right)\sin(c+dx)}{15d}+\frac{b\left(6a^2C+5b^2(6A+5C)\right)\sin(c+dx)\cos^3(c+dx)}{120d}+\frac{a\left(C\left(a^2+12b^2\right)\right.}{}$$

```
[Out] (b*(6*a^2*(4*A + 3*C) + b^2*(6*A + 5*C))*x)/16 + (a*(5*a^2*(3*A + 2*C) + 6*
b^2*(5*A + 4*C))*Sin[c + d*x])/(15*d) + (b*(6*a^2*(4*A + 3*C) + b^2*(6*A +
5*C))*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(15*A*b^2 + (a^2 + 12*b^2)*C)*
Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (b*(6*a^2*C + 5*b^2*(6*A + 5*C))*Cos[
c + d*x]^3*Sin[c + d*x])/(120*d) + (a*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])
^2*Sin[c + d*x])/(10*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*Sin[c +
d*x])/(6*d)
```

Rubi [A] time = 0.539327, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3050, 3049, 3033, 3023, 2734}

$$\frac{a\left(5a^2(3A+2C)+6b^2(5A+4C)\right)\sin(c+dx)}{15d}+\frac{b\left(6a^2C+5b^2(6A+5C)\right)\sin(c+dx)\cos^3(c+dx)}{120d}+\frac{a\left(C\left(a^2+12b^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] (b*(6*a^2*(4*A + 3*C) + b^2*(6*A + 5*C))*x)/16 + (a*(5*a^2*(3*A + 2*C) + 6*
b^2*(5*A + 4*C))*Sin[c + d*x])/(15*d) + (b*(6*a^2*(4*A + 3*C) + b^2*(6*A +
5*C))*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(15*A*b^2 + (a^2 + 12*b^2)*C)*
Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (b*(6*a^2*C + 5*b^2*(6*A + 5*C))*Cos[
c + d*x]^3*Sin[c + d*x])/(120*d) + (a*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])
^2*Sin[c + d*x])/(10*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*Sin[c +
d*x])/(6*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
```

```
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right) dx = \frac{C\cos^2(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{6d} + \frac{1}{6}\int \cos$$

$$= \frac{aC\cos^2(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{10d} + \frac{C\cos^2}{}$$

$$= \frac{b\left(6a^2C+5b^2(6A+5C)\right)\cos^3(c+dx)\sin(c+dx)}{120d} + \frac{aC}{}$$

$$= \frac{a\left(15Ab^2+\left(a^2+12b^2\right)C\right)\cos^2(c+dx)\sin(c+dx)}{15d} + \frac{b\left(\right.}{}$$

$$= \frac{1}{16}b\left(6a^2(4A+3C)+b^2(6A+5C)\right)x + \frac{a\left(5a^2(3A+2C)\right.}{}$$

Mathematica [A] time = 0.687322, size = 252, normalized size = 0.95

$$\frac{120a\left(a^2(8A+6C)+3b^2(6A+5C)\right)\sin(c+dx)+15b\left(48a^2(A+C)+b^2(16A+15C)\right)\sin(2(c+dx))+1440a^2Abc+1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (1440*a^2*A*b*c + 360*A*b^3*c + 1080*a^2*b*c*C + 300*b^3*c*C + 1440*a^2*A*b
*d*x + 360*A*b^3*d*x + 1080*a^2*b*C*d*x + 300*b^3*C*d*x + 120*a*(3*b^2*(6*A
 + 5*C) + a^2*(8*A + 6*C))*Sin[c + d*x] + 15*b*(48*a^2*(A + C) + b^2*(16*A
+ 15*C))*Sin[2*(c + d*x)] + 240*a*A*b^2*Sin[3*(c + d*x)] + 80*a^3*C*Sin[3*(
c + d*x)] + 300*a*b^2*C*Sin[3*(c + d*x)] + 30*A*b^3*Sin[4*(c + d*x)] + 90*a
^2*b*C*Sin[4*(c + d*x)] + 45*b^3*C*Sin[4*(c + d*x)] + 36*a*b^2*C*Sin[5*(c +
 d*x)] + 5*b^3*C*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.022, size = 249, normalized size = 0.9

$$\frac{1}{d}\left(Ab^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+Cb^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+C*b
^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c)+a*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3/5*C*a*b^2*(8/3+cos(d*x+c)^4+
4/3*cos(d*x+c)^2)*sin(d*x+c)+3*A*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)+3*a^2*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c
)+A*a^3*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.02193, size = 328, normalized size = 1.24

$$-\frac{320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca^3-720\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Aa^2b-90\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/960*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 720*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a^2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*
x + 2*c))*C*a^2*b + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 - 192*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a*b^2 - 30*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^3 + 5*(4*sin(2*d*x + 2*
c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*b^3 - 96
0*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.58444, size = 450, normalized size = 1.7

$$\frac{15\left(6\left(4\,A+3\,C\right)a^2b+\left(6\,A+5\,C\right)b^3\right)dx+\left(40\,Cb^3\cos(dx+c)^5+144\,Cab^2\cos(dx+c)^4+80\left(3\,A+2\,C\right)a^3+96\left(5\,A+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(6*(4*A + 3*C)*a^2*b + (6*A + 5*C)*b^3)*d*x + (40*C*b^3*cos(d*x +
 c)^5 + 144*C*a*b^2*cos(d*x + c)^4 + 80*(3*A + 2*C)*a^3 + 96*(5*A + 4*C)*a*
b^2 + 10*(18*C*a^2*b + (6*A + 5*C)*b^3)*cos(d*x + c)^3 + 16*(5*C*a^3 + 3*(5
*A + 4*C)*a*b^2)*cos(d*x + c)^2 + 15*(6*(4*A + 3*C)*a^2*b + (6*A + 5*C)*b^3
```

```
)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 7.44937, size = 668, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)/d + 3*A*a**2*b*x*sin(c + d*x)**2/2 + 3*A*a**
2*b*x*cos(c + d*x)**2/2 + 3*A*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*
a*b**2*sin(c + d*x)**3/d + 3*A*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*
b**3*x*sin(c + d*x)**4/8 + 3*A*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3
*A*b**3*x*cos(c + d*x)**4/8 + 3*A*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) +
 5*A*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*C*a**3*sin(c + d*x)**3/(3*
d) + C*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 9*C*a**2*b*x*sin(c + d*x)**4/8
 + 9*C*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*C*a**2*b*x*cos(c + d*
x)**4/8 + 9*C*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*C*a**2*b*sin(c
 + d*x)*cos(c + d*x)**3/(8*d) + 8*C*a*b**2*sin(c + d*x)**5/(5*d) + 4*C*a*b*
*2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*C*a*b**2*sin(c + d*x)*cos(c + d*x)
**4/d + 5*C*b**3*x*sin(c + d*x)**6/16 + 15*C*b**3*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 15*C*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*C*b**3*x*c
os(c + d*x)**6/16 + 5*C*b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*C*b**3
*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*b**3*sin(c + d*x)*cos(c + d*x
)**5/(16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a + b*cos(c))**3*cos(c), True
))
```

Giac [A] time = 1.20546, size = 292, normalized size = 1.11

$$\frac{Cb^3 \sin(6\,dx + 6\,c)}{192\,d} + \frac{3\,Cab^2 \sin(5\,dx + 5\,c)}{80\,d} + \frac{1}{16}\left(24\,Aa^2b + 18\,Ca^2b + 6\,Ab^3 + 5\,Cb^3\right)x + \frac{\left(6\,Ca^2b + 2\,Ab^3 + 3\,Cb^3\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/192*C*b^3*sin(6*d*x + 6*c)/d + 3/80*C*a*b^2*sin(5*d*x + 5*c)/d + 1/16*(24
*A*a^2*b + 18*C*a^2*b + 6*A*b^3 + 5*C*b^3)*x + 1/64*(6*C*a^2*b + 2*A*b^3 +
```

```
3*C*b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*C*a^3 + 12*A*a*b^2 + 15*C*a*b^2)*sin(
3*d*x + 3*c)/d + 1/64*(48*A*a^2*b + 48*C*a^2*b + 16*A*b^3 + 15*C*b^3)*sin(2
*d*x + 2*c)/d + 1/8*(8*A*a^3 + 6*C*a^3 + 18*A*a*b^2 + 15*C*a*b^2)*sin(d*x +
 c)/d
```

3.541 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=225

$$-\frac{\left(-4a^2b^2(20A+13C)+3a^4C-4b^4(5A+4C)\right)\sin(c+dx)}{30bd}-\frac{\left(3a^2C-4b^2(5A+4C)\right)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}-$$

```
[Out] (a*(4*a^2*(2*A + C) + 3*b^2*(4*A + 3*C))*x)/8 - ((3*a^4*C - 4*b^4*(5*A + 4*
C) - 4*a^2*b^2*(20*A + 13*C))*Sin[c + d*x])/(30*b*d) + (a*(100*A*b^2 - 6*a^
2*C + 71*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(120*d) - ((3*a^2*C - 4*b^2*(5*A
 + 4*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) - (a*C*(a + b*Cos[c
+ d*x])^3*Sin[c + d*x])/(20*b*d) + (C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/
(5*b*d)
```

Rubi [A] time = 0.340941, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3024, 2753, 2734}

$$-\frac{\left(-4a^2b^2(20A+13C)+3a^4C-4b^4(5A+4C)\right)\sin(c+dx)}{30bd}-\frac{\left(3a^2C-4b^2(5A+4C)\right)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a*(4*a^2*(2*A + C) + 3*b^2*(4*A + 3*C))*x)/8 - ((3*a^4*C - 4*b^4*(5*A + 4*
C) - 4*a^2*b^2*(20*A + 13*C))*Sin[c + d*x])/(30*b*d) + (a*(100*A*b^2 - 6*a^
2*C + 71*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(120*d) - ((3*a^2*C - 4*b^2*(5*A
 + 4*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) - (a*C*(a + b*Cos[c
+ d*x])^3*Sin[c + d*x])/(20*b*d) + (C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/
(5*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx=\frac{C(a+b\cos(c+dx))^4\sin(c+dx)}{5bd}+\frac{\int(a+b\cos(c+dx))^3(b(5A+4C)}{5b}$$
$$=-\frac{aC(a+b\cos(c+dx))^3\sin(c+dx)}{20bd}+\frac{C(a+b\cos(c+dx))^4\sin(c+dx}{5bd}$$
$$=-\frac{\left(3a^2C-4b^2(5A+4C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60bd}-\frac{aC(a+b\,c}{}$$
$$=\frac{1}{8}a\left(4a^2(2A+C)+3b^2(4A+3C)\right)x-\frac{\left(3a^4C-4b^4(5A+4C)-4a^2b^2(\right.}{30bd}$$

Mathematica [A] time = 0.682024, size = 160, normalized size = 0.71

$$\frac{60a(c+dx)\left(4a^2(2A+C)+3b^2(4A+3C)\right)+60b\left(6a^2(4A+3C)+b^2(6A+5C)\right)\sin(c+dx)+10b\left(12a^2C+4Ab^2+5b\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (60*a*(4*a^2*(2*A + C) + 3*b^2*(4*A + 3*C))*(c + d*x) + 60*b*(6*a^2*(4*A +
3*C) + b^2*(6*A + 5*C))*Sin[c + d*x] + 120*a*(3*A*b^2 + (a^2 + 3*b^2)*C)*Si
n[2*(c + d*x)] + 10*b*(4*A*b^2 + 12*a^2*C + 5*b^2*C)*Sin[3*(c + d*x)] + 45*
a*b^2*C*Sin[4*(c + d*x)] + 6*b^3*C*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.018, size = 201, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Cb^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3Cab^2\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*C*a*b^2*(1/
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*b^3*(2+cos(
d*x+c)^2)*sin(d*x+c)+a^2*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a*A*b^2*(1/2*cos
(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+
1/2*c)+3*A*a^2*b*sin(d*x+c)+A*a^3*(d*x+c))
```

Maxima [A] time = 1.01009, size = 262, normalized size = 1.16

$$\frac{480(dx+c)Aa^3+120(2dx+2c+\sin(2dx+2c))Ca^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca^2b+360(2dx+2c+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(480*(d*x + c)*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 - 4
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b + 360*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*A*a*b^2 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c
))*C*a*b^2 - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^3 + 32*(3*sin(d*x +
c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^3 + 1440*A*a^2*b*sin(d*x +
c))/d
```

Fricas [A] time = 1.43832, size = 367, normalized size = 1.63

$$\frac{15\left(4(2A+C)a^3+3(4A+3C)ab^2\right)dx+\left(24Cb^3\cos(dx+c)^4+90Cab^2\cos(dx+c)^3+120(3A+2C)a^2b+16(5A\ldots\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(4*(2*A + C)*a^3 + 3*(4*A + 3*C)*a*b^2)*d*x + (24*C*b^3*cos(d*x +
 c)^4 + 90*C*a*b^2*cos(d*x + c)^3 + 120*(3*A + 2*C)*a^2*b + 16*(5*A + 4*C)*
b^3 + 8*(15*C*a^2*b + (5*A + 4*C)*b^3)*cos(d*x + c)^2 + 15*(4*C*a^3 + 3*(4*
A + 3*C)*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.88849, size = 440, normalized size = 1.96

$$\begin{cases} Aa^3x + \frac{3Aa^2b\sin(c+dx)}{d} + \frac{3Aab^2x\sin^2(c+dx)}{2} + \frac{3Aab^2x\cos^2(c+dx)}{2} + \frac{3Aab^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Ab^3\sin^3(c+dx)}{3d} + \frac{Ab^3\sin(c+dx)\cos^2(c+dx)}{d} \\ x\left(A + C\cos^2(c)\right)(a + b\cos(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**3*x + 3*A*a**2*b*sin(c + d*x)/d + 3*A*a*b**2*x*sin(c + d*x)
**2/2 + 3*A*a*b**2*x*cos(c + d*x)**2/2 + 3*A*a*b**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 2*A*b**3*sin(c + d*x)**3/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x
)**2/d + C*a**3*x*sin(c + d*x)**2/2 + C*a**3*x*cos(c + d*x)**2/2 + C*a**3*s
in(c + d*x)*cos(c + d*x)/(2*d) + 2*C*a**2*b*sin(c + d*x)**3/d + 3*C*a**2*b*
sin(c + d*x)*cos(c + d*x)**2/d + 9*C*a*b**2*x*sin(c + d*x)**4/8 + 9*C*a*b**
2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*C*a*b**2*x*cos(c + d*x)**4/8 + 9*
C*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*C*a*b**2*sin(c + d*x)*cos(
c + d*x)**3/(8*d) + 8*C*b**3*sin(c + d*x)**5/(15*d) + 4*C*b**3*sin(c + d*x)
**3*cos(c + d*x)**2/(3*d) + C*b**3*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)
), (x*(A + C*cos(c)**2)*(a + b*cos(c))**3, True))
```

Giac [A] time = 1.19548, size = 235, normalized size = 1.04

$$\frac{Cb^3\sin(5\,dx + 5\,c)}{80\,d} + \frac{3\,Cab^2\sin(4\,dx + 4\,c)}{32\,d} + \frac{1}{8}\left(8\,Aa^3 + 4\,Ca^3 + 12\,Aab^2 + 9\,Cab^2\right)x + \frac{\left(12\,Ca^2b + 4\,Ab^3 + 5\,Cb^3\right)\sin}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/80*C*b^3*sin(5*d*x + 5*c)/d + 3/32*C*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*A*
a^3 + 4*C*a^3 + 12*A*a*b^2 + 9*C*a*b^2)*x + 1/48*(12*C*a^2*b + 4*A*b^3 + 5*
C*b^3)*sin(3*d*x + 3*c)/d + 1/4*(C*a^3 + 3*A*a*b^2 + 3*C*a*b^2)*sin(2*d*x +
 2*c)/d + 1/8*(24*A*a^2*b + 18*C*a^2*b + 6*A*b^3 + 5*C*b^3)*sin(d*x + c)/d
```

3.542 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=167

$$\frac{a\left(C\left(a^2+4b^2\right)+6Ab^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2C+b^2(4A+3C)\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2(2A+C)+b^2(4A\right.$$

```
[Out] (b*(12*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/8 + (a^3*A*ArcTanh[Sin[c + d*x]]
)/d + (a*(6*A*b^2 + (a^2 + 4*b^2)*C)*Sin[c + d*x])/(2*d) + (b*(2*a^2*C + b^
2*(4*A + 3*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*(a + b*Cos[c + d*x])
^2*Sin[c + d*x])/(4*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.542427, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3050, 3049, 3033, 3023, 2735, 3770}

$$\frac{a\left(C\left(a^2+4b^2\right)+6Ab^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2C+b^2(4A+3C)\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2(2A+C)+b^2(4A\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```



```
[Out] (b*(12*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/8 + (a^3*A*ArcTanh[Sin[c + d*x]]
)/d + (a*(6*A*b^2 + (a^2 + 4*b^2)*C)*Sin[c + d*x])/(2*d) + (b*(2*a^2*C + b^
2*(4*A + 3*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*C*(a + b*Cos[c + d*x])
^2*Sin[c + d*x])/(4*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
```

))

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (a+b\cos(c+dx$$
$$= \frac{aC(a+b\cos(c+dx))^2\sin(c+dx)}{4d} + \frac{C(a+b\cos(c+dx))}{4d}$$
$$= \frac{b\left(2a^2C+b^2(4A+3C)\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aC(a+}{}$$
$$= \frac{a\left(6Ab^2+\left(a^2+4b^2\right)C\right)\sin(c+dx)}{2d} + \frac{b\left(2a^2C+b^2(4A+\right.}{}$$
$$= \frac{1}{8}b\left(12a^2(2A+C)+b^2(4A+3C)\right)x + \frac{a\left(6Ab^2+\left(a^2+4b\right.\right.}{2d}$$
$$= \frac{1}{8}b\left(12a^2(2A+C)+b^2(4A+3C)\right)x + \frac{a^3A\tanh^{-1}(\sin(c}{d}$$

Mathematica [A] time = 0.561259, size = 180, normalized size = 1.08

$$\frac{4b(c+dx)\left(12a^2(2A+C)+b^2(4A+3C)\right)+8a\left(4a^2C+12Ab^2+9b^2C\right)\sin(c+dx)+8b\left(C\left(3a^2+b^2\right)+Ab^2\right)\sin(2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (4*b*(12*a^2*(2*A + C) + b^2*(4*A + 3*C))*(c + d*x) - 32*a^3*A*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 32*a^3*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 8*a*(12*A*b^2 + 4*a^2*C + 9*b^2*C)*Sin[c + d*x] + 8*b*(A*b^2 + (3*a
^2 + b^2)*C)*Sin[2*(c + d*x)] + 8*a*b^2*C*Sin[3*(c + d*x)] + b^3*C*Sin[4*(c
 + d*x)])/(32*d)
```

Maple [A] time = 0.049, size = 252, normalized size = 1.5

$$\frac{Ab^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{Ab^3x}{2} + \frac{Ab^3c}{2d} + \frac{Cb^3\sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{3Cb^3\cos(dx+c)\sin(dx+c)}{8d} + \frac{3b^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/2/d*A*b^3*cos(d*x+c)*sin(d*x+c)+1/2*A*b^3*x+1/2/d*A*b^3*c+1/4/d*C*b^3*sin
(d*x+c)*cos(d*x+c)^3+3/8/d*C*b^3*cos(d*x+c)*sin(d*x+c)+3/8*b^3*C*x+3/8/d*C*
b^3*c+3/d*a*A*b^2*sin(d*x+c)+1/d*C*sin(d*x+c)*cos(d*x+c)^2*a*b^2+2/d*C*a*b^
2*sin(d*x+c)+3*A*a^2*b*x+3/d*A*a^2*b*c+3/2/d*a^2*b*C*cos(d*x+c)*sin(d*x+c)+
3/2*a^2*b*C*x+3/2/d*a^2*b*C*c+1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*C*sin
(d*x+c)/d
```

Maxima [A] time = 0.99887, size = 225, normalized size = 1.35

$$\frac{96\,(dx+c)Aa^2b+24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ca^2b-32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Cab^2+8\,(2\,dx+2\,c+\sin(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/32*(96*(d*x + c)*A*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2*b -
32*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a*b^2 + 8*(2*d*x + 2*c + sin(2*d*x +
 2*c))*A*b^3 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*b^
3 + 32*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 32*C*a^3*sin(d*x + c) + 96*
A*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.60282, size = 354, normalized size = 2.12

$$\frac{4\,Aa^3\log\left(\sin\left(dx+c\right)+1\right)-4\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(12\,(2\,A+C)a^2b+(4\,A+3\,C)b^3\right)dx+\left(2\,Cb^3\cos\left(dx+c\right)^3\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] 1/8*(4*A*a^3*log(sin(d*x + c) + 1) - 4*A*a^3*log(-sin(d*x + c) + 1) + (12*(
2*A + C)*a^2*b + (4*A + 3*C)*b^3)*d*x + (2*C*b^3*cos(d*x + c)^3 + 8*C*a*b^2
*cos(d*x + c)^2 + 8*C*a^3 + 8*(3*A + 2*C)*a*b^2 + (12*C*a^2*b + (4*A + 3*C)
*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33711, size = 679, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] 1/8*(8*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (24*A*a^2*b + 12*C*a^2*b + 4*A*b^3 + 3*C*b^3)*(d*x + c)
 + 2*(8*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 +
24*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 4*A
*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*C*a^3*tan
(1/2*d*x + 1/2*c)^5 - 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*a*b^2*tan(1/
2*d*x + 1/2*c)^5 + 40*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*b^3*tan(1/2*d*x
+ 1/2*c)^5 + 3*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 24*C*a^3*tan(1/2*d*x + 1/2*c)
^3 + 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*A*a*b^2*tan(1/2*d*x + 1/2*c)^3
+ 40*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*
b^3*tan(1/2*d*x + 1/2*c)^3 + 8*C*a^3*tan(1/2*d*x + 1/2*c) + 12*C*a^2*b*tan(
1/2*d*x + 1/2*c) + 24*A*a*b^2*tan(1/2*d*x + 1/2*c) + 24*C*a*b^2*tan(1/2*d*x
 + 1/2*c) + 4*A*b^3*tan(1/2*d*x + 1/2*c) + 5*C*b^3*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.543 $\int (a+b\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=167

$$-\frac{b\left(a^2(6A-8C)-b^2(3A+2C)\right)\sin(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2C+6Ab^2+3b^2C\right)+\frac{3a^2Ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{ab^2(6A-5C}{}$$

```
[Out] (a*(6*A*b^2 + 2*a^2*C + 3*b^2*C)*x)/2 + (3*a^2*A*b*ArcTanh[Sin[c + d*x]])/d
 - (b*(a^2*(6*A - 8*C) - b^2*(3*A + 2*C))*Sin[c + d*x])/(3*d) - (a*b^2*(6*A
 - 5*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) - (b*(3*A - C)*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(3*d) + (A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.503616, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\left(a^2(6A-8C)-b^2(3A+2C)\right)\sin(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2C+6Ab^2+3b^2C\right)+\frac{3a^2Ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{ab^2(6A-5C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] (a*(6*A*b^2 + 2*a^2*C + 3*b^2*C)*x)/2 + (3*a^2*A*b*ArcTanh[Sin[c + d*x]])/d
 - (b*(a^2*(6*A - 8*C) - b^2*(3*A + 2*C))*Sin[c + d*x])/(3*d) - (a*b^2*(6*A
 - 5*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) - (b*(3*A - C)*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(3*d) + (A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+b\cos(c+dx))^2$$
$$= -\frac{b(3A-C)(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{A(a+b\cos}{}$$
$$= -\frac{ab^2(6A-5C)\cos(c+dx)\sin(c+dx)}{6d} - \frac{b(3A-C)(a+b}{}$$
$$= -\frac{b\left(a^2(6A-8C)-b^2(3A+2C)\right)\sin(c+dx)}{3d} - \frac{ab^2(6A-5}{}$$
$$= \frac{1}{2}a\left(6Ab^2+2a^2C+3b^2C\right)x - \frac{b\left(a^2(6A-8C)-b^2(3A+2\right.}{3d}$$
$$= \frac{1}{2}a\left(6Ab^2+2a^2C+3b^2C\right)x + \frac{3a^2Ab\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.866054, size = 185, normalized size = 1.11

$$\frac{3b\left(3C\left(4a^2+b^2\right)+4Ab^2\right)\sin(c+dx)-36a^2Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+36a^2Ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (36*a*A*b^2*c + 12*a^3*c*C + 18*a*b^2*c*C + 36*a*A*b^2*d*x + 12*a^3*C*d*x +
 18*a*b^2*C*d*x - 36*a^2*A*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 36*
a^2*A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*b*(4*A*b^2 + 3*(4*a^2
+ b^2)*C)*Sin[c + d*x] + 9*a*b^2*C*Sin[2*(c + d*x)] + b^3*C*Sin[3*(c + d*x)
] + 12*a^3*A*Tan[c + d*x])/(12*d)
```

Maple [A] time = 0.055, size = 183, normalized size = 1.1

$$\frac{Ab^3\sin(dx+c)}{d} + \frac{C\sin(dx+c)(\cos(dx+c))^2b^3}{3d} + \frac{2Cb^3\sin(dx+c)}{3d} + 3aAb^2x + 3\frac{Aab^2c}{d} + \frac{3Cab^2\cos(dx+c)\sin(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*b^3*sin(d*x+c)+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*b^3+2/3/d*C*b^3*sin(d*
x+c)+3*a*A*b^2*x+3/d*A*a*b^2*c+3/2/d*C*a*b^2*cos(d*x+c)*sin(d*x+c)+3/2*a*b^
2*C*x+3/2/d*C*a*b^2*c+3/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*C*sin
(d*x+c)+1/d*A*a^3*tan(d*x+c)+a^3*C*x+1/d*a^3*C*c
```

Maxima [A] time = 1.02213, size = 190, normalized size = 1.14

$$\frac{12\,(dx+c)Ca^3+36\,(dx+c)Aab^2+9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Cab^2-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Cb^3+18\,Aa}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(d*x + c)*C*a^3 + 36*(d*x + c)*A*a*b^2 + 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*C*a*b^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*b^3 + 18*A*a^2*b*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*C*a^2*b*sin(d*x + c) +
 12*A*b^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.59462, size = 394, normalized size = 2.36

$$\frac{9\,Aa^2b\cos(dx+c)\log(\sin(dx+c)+1)-9\,Aa^2b\cos(dx+c)\log(-\sin(dx+c)+1)+3\left(2\,Ca^3+3\,(2\,A+C)ab^2\right)dx\,c}{6\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] 1/6*(9*A*a^2*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 9*A*a^2*b*cos(d*x + c)*
log(-sin(d*x + c) + 1) + 3*(2*C*a^3 + 3*(2*A + C)*a*b^2)*d*x*cos(d*x + c) +
 (2*C*b^3*cos(d*x + c)^3 + 9*C*a*b^2*cos(d*x + c)^2 + 6*A*a^3 + 2*(9*C*a^2*
b + (3*A + 2*C)*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7086, size = 413, normalized size = 2.47

$$\frac{18\,Aa^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,Aa^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+3\left(2\,Ca^3+6\,Aab^2+3\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] 1/6*(18*A*a^2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*A*a^2*b*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*
c)^2 - 1) + 3*(2*C*a^3 + 6*A*a*b^2 + 3*C*a*b^2)*(d*x + c) + 2*(18*C*a^2*b*t
an(1/2*d*x + 1/2*c)^5 - 9*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^3*tan(1/2*
d*x + 1/2*c)^5 + 6*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b*tan(1/2*d*x +
1/2*c)^3 + 12*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^3*tan(1/2*d*x + 1/2*c)^3
 + 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 9*C*a*b^2*tan(1/2*d*x + 1/2*c) + 6*A*b
^3*tan(1/2*d*x + 1/2*c) + 6*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

3.544 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=168

$$\frac{a\left(a^2(A+2C)+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}bx\left(C\left(6a^2+b^2\right)+2Ab^2\right)-\frac{3ab^2(3A-2C)\sin(c+dx)}{2d}+\frac{3Ab\tan(c+$$

```
[Out] (b*(2*A*b^2 + (6*a^2 + b^2)*C)*x)/2 + (a*(6*A*b^2 + a^2*(A + 2*C))*ArcTanh[
Sin[c + d*x]])/(2*d) - (3*a*b^2*(3*A - 2*C)*Sin[c + d*x])/(2*d) - (b^3*(4*A
 - C)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*A*b*(a + b*Cos[c + d*x])^2*Tan[
c + d*x])/(2*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Rubi [A] time = 0.579688, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3047, 3033, 3023, 2735, 3770}

$$\frac{a\left(a^2(A+2C)+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}bx\left(C\left(6a^2+b^2\right)+2Ab^2\right)-\frac{3ab^2(3A-2C)\sin(c+dx)}{2d}+\frac{3Ab\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] (b*(2*A*b^2 + (6*a^2 + b^2)*C)*x)/2 + (a*(6*A*b^2 + a^2*(A + 2*C))*ArcTanh[
Sin[c + d*x]])/(2*d) - (3*a*b^2*(3*A - 2*C)*Sin[c + d*x])/(2*d) - (b^3*(4*A
 - C)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*A*b*(a + b*Cos[c + d*x])^2*Tan[
c + d*x])/(2*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (a+$$
$$= \frac{3Ab(a+b\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{A(a+b\cos(c+d}{}$$
$$= -\frac{b^3(4A-C)\cos(c+dx)\sin(c+dx)}{2d} + \frac{3Ab(a+b\cos(c}{}$$
$$= -\frac{3ab^2(3A-2C)\sin(c+dx)}{2d} - \frac{b^3(4A-C)\cos(c+dx)\sin}{2d}$$
$$= \frac{1}{2}b\left(2Ab^2+\left(6a^2+b^2\right)C\right)x - \frac{3ab^2(3A-2C)\sin(c+dx)}{2d}$$
$$= \frac{1}{2}b\left(2Ab^2+\left(6a^2+b^2\right)C\right)x + \frac{a\left(6Ab^2+a^2(A+2C)\right)\tan}{2d}$$

Mathematica [A] time = 1.51221, size = 285, normalized size = 1.7

$$2b(c+dx)\left(C\left(6a^2+b^2\right)+2Ab^2\right) - 2a\left(a^2(A+2C)+6Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2a\left(a^2(A+2C)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(2*A*b^2 + (6*a^2 + b^2)*C)*(c + d*x) - 2*a*(6*A*b^2 + a^2*(A + 2*C))*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(6*A*b^2 + a^2*(A + 2*C))*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*A)/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + (12*a^2*A*b*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]) - (a^3*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (12*a^2*A*b*Si
n[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12*a*b^2*C*Sin[c +
d*x] + b^3*C*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.06, size = 196, normalized size = 1.2

$$Ab^3x + \frac{Ab^3c}{d} + \frac{Cb^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{b^3Cx}{2} + \frac{Cb^3c}{2d} + 3\frac{aAb^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\frac{Cab^2\sin(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] A*b^3*x+1/d*A*b^3*c+1/2/d*C*b^3*cos(d*x+c)*sin(d*x+c)+1/2*b^3*C*x+1/2/d*C*b
^3*c+3/d*a*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/d*C*a*b^2*sin(d*x+c)+3/d*A*a^2
*b*tan(d*x+c)+3*a^2*b*C*x+3/d*a^2*b*C*c+1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1
/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01112, size = 242, normalized size = 1.44

$$\frac{12\,(dx+c)Ca^2b+4\,(dx+c)Ab^3+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Cb^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] 1/4*(12*(d*x + c)*C*a^2*b + 4*(d*x + c)*A*b^3 + (2*d*x + 2*c + sin(2*d*x +
2*c))*C*b^3 - A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
 + 1) + log(sin(d*x + c) - 1)) + 2*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 12*C*a*b^2*sin(d*x + c) + 12*A*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 1.58239, size = 419, normalized size = 2.49

$$\frac{2\left(6\,Ca^2b+(2\,A+C)b^3\right)dx\cos{(dx+c)}^2+\left((A+2\,C)a^3+6\,Aab^2\right)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-\left((A+2\,C)a^3+6\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] 1/4*(2*(6*C*a^2*b + (2*A + C)*b^3)*d*x*cos(d*x + c)^2 + ((A + 2*C)*a^3 + 6*
A*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - ((A + 2*C)*a^3 + 6*A*a*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(C*b^3*cos(d*x + c)^3 + 6*C*a*b^
```

```
2*cos(d*x + c)^2 + 6*A*a^2*b*cos(d*x + c) + A*a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30672, size = 520, normalized size = 3.1

$$\frac{\left(6\,Ca^2b + 2\,Ab^3 + Cb^3\right)(dx + c) + \left(Aa^3 + 2\,Ca^3 + 6\,Aab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Aa^3 + 2\,Ca^3 + 6\,Aab^2\right)\log\left(\left|\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] 1/2*((6*C*a^2*b + 2*A*b^3 + C*b^3)*(d*x + c) + (A*a^3 + 2*C*a^3 + 6*A*a*b^2
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a^3 + 2*C*a^3 + 6*A*a*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^7 - 6*A*a^2*b*
tan(1/2*d*x + 1/2*c)^7 + 6*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - C*b^3*tan(1/2*d
*x + 1/2*c)^7 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b*tan(1/2*d*x + 1/
2*c)^5 - 6*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*C*b^3*tan(1/2*d*x + 1/2*c)^5
+ 3*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 6*C*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*b^3*tan(1/2*d*x + 1/2*c)^3 + A*a^3*tan(1/
2*d*x + 1/2*c) + 6*A*a^2*b*tan(1/2*d*x + 1/2*c) + 6*C*a*b^2*tan(1/2*d*x + 1
/2*c) + C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.545 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=163

$$\frac{a\left(a^2(2A+3C)+3Ab^2\right)\tan(c+dx)}{3d}+\frac{b\left(3a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec^2(c+dx)(a+b}{3d}$$

```
[Out] 3*a*b^2*C*x + (b*(2*A*b^2 + 3*a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) -
 (b^3*(5*A - 6*C)*Sin[c + d*x])/(6*d) + (a*(3*A*b^2 + a^2*(2*A + 3*C))*Tan[
c + d*x])/(3*d) + (A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2
*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.535914, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3047, 3031, 3023, 2735, 3770}

$$\frac{a\left(a^2(2A+3C)+3Ab^2\right)\tan(c+dx)}{3d}+\frac{b\left(3a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec^2(c+dx)(a+b}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] 3*a*b^2*C*x + (b*(2*A*b^2 + 3*a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) -
 (b^3*(5*A - 6*C)*Sin[c + d*x])/(6*d) + (a*(3*A*b^2 + a^2*(2*A + 3*C))*Tan[
c + d*x])/(3*d) + (A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2
*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a+$$
$$= \frac{Ab(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{A(a+b$$
$$= \frac{a\left(3Ab^2+a^2(2A+3C)\right)\tan(c+dx)}{3d}+\frac{Ab(a+b\cos(c+a$$
$$= -\frac{b^3(5A-6C)\sin(c+dx)}{6d}+\frac{a\left(3Ab^2+a^2(2A+3C)\right)\tan($$
$$= 3ab^2Cx-\frac{b^3(5A-6C)\sin(c+dx)}{6d}+\frac{a\left(3Ab^2+a^2(2A+3\right.}{3d}$$
$$= 3ab^2Cx+\frac{b\left(2Ab^2+3a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2d}-$$

Mathematica [B] time = 4.29332, size = 377, normalized size = 2.31

$$\frac{\frac{4a\left(a^2(2A+3C)+9Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4a\left(a^2(2A+3C)+9Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-6b\left(3a^2(A+2C)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (36*a*b^2*C*(c + d*x) - 6*b*(2*A*b^2 + 3*a^2*(A + 2*C))*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 6*b*(2*A*b^2 + 3*a^2*(A + 2*C))*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] + (a^2*A*(a + 9*b))/(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^2 + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3
 + (4*a*(9*A*b^2 + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - S
in[(c + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3 - (a^2*A*(a + 9*b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4
*a*(9*A*b^2 + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]) + 12*b^3*C*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.062, size = 195, normalized size = 1.2

$$\frac{Ab^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cb^3\sin(dx+c)}{d}+3\frac{aAb^2\tan(dx+c)}{d}+3ab^2Cx+3\frac{Cab^2c}{d}+\frac{3Aa^2b\sec(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*b^3*sin(d*x+c)+3/d*a*A*b^2*tan(d*
x+c)+3*a*b^2*C*x+3/d*C*a*b^2*c+3/2/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*
a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+2/3/d
*A*a^3*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/d*a^3*C*tan(d*x+c)
```

Maxima [A] time = 1.03364, size = 244, normalized size = 1.5

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+36(dx+c)Cab^2-9Aa^2b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 36*(d*x + c)*C*a*b^2 - 9*
A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 18*C*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
 1)) + 6*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*C*b^3*s
in(d*x + c) + 12*C*a^3*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.52551, size = 440, normalized size = 2.7

$$\frac{36\,Cab^2dx\cos(dx+c)^3+3\left(3(A+2C)a^2b+2Ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(3(A+2C)a^2b+2Ab^3\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] 1/12*(36*C*a*b^2*d*x*cos(d*x + c)^3 + 3*(3*(A + 2*C)*a^2*b + 2*A*b^3)*cos(d
*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*(A + 2*C)*a^2*b + 2*A*b^3)*cos(d*x +
 c)^3*log(-sin(d*x + c) + 1) + 2*(6*C*b^3*cos(d*x + c)^3 + 9*A*a^2*b*cos(d*
x + c) + 2*A*a^3 + 2*((2*A + 3*C)*a^3 + 9*A*a*b^2)*cos(d*x + c)^2)*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32864, size = 435, normalized size = 2.67

$$\frac{18\,(dx+c)Cab^2+\frac{12\,Cb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+3\left(3\,Aa^2b+6\,Ca^2b+2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Aa^2b+6\,Ca^2b+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] 1/6*(18*(d*x + c)*C*a*b^2 + 12*C*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/
2*c)^2 + 1) + 3*(3*A*a^2*b + 6*C*a^2*b + 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 3*(3*A*a^2*b + 6*C*a^2*b + 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^3*tan(1/2*d*x + 1/2*c)^
5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 -
4*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b
^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 6*C*a^3*tan(1/2*
d*x + 1/2*c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/
```

```
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.546 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=182

$$\frac{b\left(a^2(4A+6C)+Ab^2\right)\tan(c+dx)}{2d}+\frac{a\left(a^2(3A+4C)+12b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(a^2(3A+4C)+2Ab^2\right)\text{t}}{8d}$$

```
[Out] b^3*C*x + (a*(12*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8
*d) + (b*(A*b^2 + a^2*(4*A + 6*C))*Tan[c + d*x])/(2*d) + (a*(2*A*b^2 + a^2*
(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*b*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^2*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x
]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.599905, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3047, 3031, 3021, 2735, 3770}

$$\frac{b\left(a^2(4A+6C)+Ab^2\right)\tan(c+dx)}{2d}+\frac{a\left(a^2(3A+4C)+12b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(a^2(3A+4C)+2Ab^2\right)\text{t}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] b^3*C*x + (a*(12*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8
*d) + (b*(A*b^2 + a^2*(4*A + 6*C))*Tan[c + d*x])/(2*d) + (a*(2*A*b^2 + a^2*
(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (A*b*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^2*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x
]^3*Tan[c + d*x])/(4*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{A(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (a+$$
$$=\frac{Ab(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{4d}+\frac{A(a+b}{}$$
$$=\frac{a\left(2Ab^2+a^2(3A+4C)\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{Ab(a+}{}$$
$$=\frac{b\left(Ab^2+a^2(4A+6C)\right)\tan(c+dx)}{2d}+\frac{a\left(2Ab^2+a^2(3A+\right.}{}$$
$$=b^3Cx+\frac{b\left(Ab^2+a^2(4A+6C)\right)\tan(c+dx)}{2d}+\frac{a\left(2Ab^2+a\right.}{}$$
$$=b^3Cx+\frac{a\left(12b^2(A+2C)+a^2(3A+4C)\right)\tanh^{-1}(\sin(c+d}{8d}$$

Mathematica [A] time = 0.942651, size = 127, normalized size = 0.7

$$\frac{a\left(a^2(3A+4C)+12b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(a\left(a^2(3A+4C)+12Ab^2\right)\sec(c+dx)+8b\left(3a^2(A-\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (8*b^3*C*d*x + a*(12*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]]
 + (8*b*(A*b^2 + 3*a^2*(A + C)) + a*(12*A*b^2 + a^2*(3*A + 4*C))*Sec[c + d*
x] + 2*a^3*A*Sec[c + d*x]^3)*Tan[c + d*x] + 8*a^2*A*b*Tan[c + d*x]^3)/(8*d)
```

Maple [A] time = 0.061, size = 267, normalized size = 1.5

$$\frac{Ab^3\tan(dx+c)}{d}+b^3Cx+\frac{Cb^3c}{d}+\frac{3\,aAb^2\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{3\,aAb^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+3\,\frac{Cab^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/d*A*b^3*tan(d*x+c)+b^3*C*x+1/d*C*b^3*c+3/2/d*a*A*b^2*sec(d*x+c)*tan(d*x+c
)+3/2/d*a*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x
+c))+2/d*A*a^2*b*tan(d*x+c)+1/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)^2+3/d*a^2*b*C
*tan(d*x+c)+1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^3*sec(d*x+c)*tan(
d*x+c)+3/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^3*C*sec(d*x+c)*tan(d*x
+c)+1/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02143, size = 352, normalized size = 1.93

$$16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2b+16(dx+c)Cb^3-Aa^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2*b + 16*(d*x + c)*C*b^3 - A
*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c
)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 4*C*a^3*(2*
sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c
) - 1)) - 12*A*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) + 24*C*a*b^2*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 48*C*a^2*b*tan(d*x + c) + 16*A*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.55796, size = 485, normalized size = 2.66

$$16\,Cb^3dx\cos(dx+c)^4+\left((3\,A+4\,C)a^3+12\,(A+2\,C)ab^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-\left((3\,A+4\,C)a^3+12\,(A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/16*(16*C*b^3*d*x*cos(d*x + c)^4 + ((3*A + 4*C)*a^3 + 12*(A + 2*C)*a*b^2)*
cos(d*x + c)^4*log(sin(d*x + c) + 1) - ((3*A + 4*C)*a^3 + 12*(A + 2*C)*a*b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*A*a^2*b*cos(d*x + c) + 2*A*
a^3 + 8*((2*A + 3*C)*a^2*b + A*b^3)*cos(d*x + c)^3 + ((3*A + 4*C)*a^3 + 12*
A*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37252, size = 710, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] 1/8*(8*(d*x + c)*C*b^3 + (3*A*a^3 + 4*C*a^3 + 12*A*a*b^2 + 24*C*a*b^2)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - (3*A*a^3 + 4*C*a^3 + 12*A*a*b^2 + 24*C*a*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 4*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*C
*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*A*b^3
*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 4*C*a^3*tan(1/2*
d*x + 1/2*c)^5 + 40*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*b^3*tan(1/2*d*x + 1/
2*c)^5 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^3*tan(1/2*d*x + 1/2*c)^3 -
40*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 72*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*
A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*
tan(1/2*d*x + 1/2*c) + 4*C*a^3*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b*tan(1/2*d*
x + 1/2*c) + 24*C*a^2*b*tan(1/2*d*x + 1/2*c) + 12*A*a*b^2*tan(1/2*d*x + 1/2
*c) + 8*A*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.547 $\int (a+b\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=227

$$\frac{a\left(2a^2(4A+5C)+15b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{b\left(3a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(2a^2(4A\right.}{}$$

```
[Out] (b*(4*b^2*(A + 2*C) + 3*a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (a*
(15*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Tan[c + d*x])/(15*d) + (3*b*(2*A*b
^2 + 5*a^2*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a*(3*A*b^2 + 2
*a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (3*A*b*(a + b*Cos[c
 + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*Cos[c + d*x])^3*
Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.71411, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3048, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{a\left(2a^2(4A+5C)+15b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{b\left(3a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(2a^2(4A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] (b*(4*b^2*(A + 2*C) + 3*a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (a*
(15*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Tan[c + d*x])/(15*d) + (3*b*(2*A*b
^2 + 5*a^2*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a*(3*A*b^2 + 2
*a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (3*A*b*(a + b*Cos[c
 + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*Cos[c + d*x])^3*
Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a$$

$$= \frac{3Ab(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{A(a$$

$$= \frac{a\left(3Ab^2+2a^2(4A+5C)\right)\sec^2(c+dx)\tan(c+dx)}{30d}+\frac{3A$$

$$= \frac{3b\left(2Ab^2+5a^2(3A+4C)\right)\sec(c+dx)\tan(c+dx)}{40d}+\frac{a\big(}{}$$

$$= \frac{3b\left(2Ab^2+5a^2(3A+4C)\right)\sec(c+dx)\tan(c+dx)}{40d}+\frac{a\big(}{}$$

$$= \frac{b\left(4b^2(A+2C)+3a^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$

$$= \frac{b\left(4b^2(A+2C)+3a^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Mathematica [A] time = 2.24835, size = 150, normalized size = 0.66

$$\frac{15b\left(3a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8a\left(5\left(a^2(2A+C)+3Ab^2\right)\tan^2(c+dx)+15\left(a\right.\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (15*b*(4*b^2*(A + 2*C) + 3*a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]] + Tan[c +
 d*x]*(15*b*(4*A*b^2 + 3*a^2*(3*A + 4*C))*Sec[c + d*x] + 90*a^2*A*b*Sec[c +
 d*x]^3 + 8*a*(15*(a^2 + 3*b^2)*(A + C) + 5*(3*A*b^2 + a^2*(2*A + C))*Tan[c
 + d*x]^2 + 3*a^2*A*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.066, size = 338, normalized size = 1.5

$$\frac{Ab^3 \sec(dx+c)\tan(dx+c)}{2d} + \frac{Ab^3 \ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{Cb^3 \ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\frac{aAb^2 \tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/2/d*A*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d
*C*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*A*b^2*tan(d*x+c)+1/d*a*A*b^2*tan(d*x
+c)*sec(d*x+c)^2+3/d*C*a*b^2*tan(d*x+c)+3/4/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)
^3+9/8/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+9/8/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+
c))+3/2/d*a^2*b*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x
+c))+8/15/d*A*a^3*tan(d*x+c)+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*a
^3*tan(d*x+c)*sec(d*x+c)^2+2/3/d*a^3*C*tan(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*se
c(d*x+c)^2
```

Maxima [A] time = 1.01956, size = 400, normalized size = 1.76

$$\frac{16\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)Aa^3 + 80\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ca^3 + 240\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d*
x + c))*A*a*b^2 - 45*A*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
```

```
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 180*C*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 60*A*b^3*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*C*b^3*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 720*C*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.59986, size = 552, normalized size = 2.43

$$\frac{15\left(3\,(3\,A+4\,C)a^2b+4\,(A+2\,C)b^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,(3\,A+4\,C)a^2b+4\,(A+2\,C)b^3\right)\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(3*(3*A + 4*C)*a^2*b + 4*(A + 2*C)*b^3)*cos(d*x + c)^5*log(sin(d*
x + c) + 1) - 15*(3*(3*A + 4*C)*a^2*b + 4*(A + 2*C)*b^3)*cos(d*x + c)^5*log
(-sin(d*x + c) + 1) + 2*(90*A*a^2*b*cos(d*x + c) + 8*(2*(4*A + 5*C)*a^3 + 1
5*(2*A + 3*C)*a*b^2)*cos(d*x + c)^4 + 24*A*a^3 + 15*(3*(3*A + 4*C)*a^2*b +
4*A*b^3)*cos(d*x + c)^3 + 8*((4*A + 5*C)*a^3 + 15*A*a*b^2)*cos(d*x + c)^2)*
sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5271, size = 886, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] 1/120*(15*(9*A*a^2*b + 12*C*a^2*b + 4*A*b^3 + 8*C*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(9*A*a^2*b + 12*C*a^2*b + 4*A*b^3 + 8*C*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 120*C*a^3*
tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 180*C*a^2*b*t
an(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 360*C*a*b^2*ta
n(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 - 160*A*a^3*tan(1/2*
d*x + 1/2*c)^7 - 320*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 90*A*a^2*b*tan(1/2*d*x
+ 1/2*c)^7 + 360*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*A*a*b^2*tan(1/2*d*x +
 1/2*c)^7 - 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*A*b^3*tan(1/2*d*x + 1
/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 400*C*a^3*tan(1/2*d*x + 1/2*c)
^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 2160*C*a*b^2*tan(1/2*d*x + 1/2*c
)^5 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 320*C*a^3*tan(1/2*d*x + 1/2*c)^3 -
 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 360*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9
60*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 1
20*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/2*d*x + 1/2*c) + 120*C*a^
3*tan(1/2*d*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x + 1/2*c) + 180*C*a^2*b*tan
(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c) + 360*C*a*b^2*tan(1/2*
d*x + 1/2*c) + 60*A*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
5)/d
```

3.548 $\int (a+b\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=273

$$\frac{b\left(6a^2(4A+5C)+5b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{a\left(a^2(5A+6C)+6b^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a\left(5a^2(5A+\right.}{}$$

```
[Out] (a*(6*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*ArcTanh[Sin[c + d*x]])/(16*d) + (b
*(5*b^2*(2*A + 3*C) + 6*a^2*(4*A + 5*C))*Tan[c + d*x])/(15*d) + (a*(6*b^2*(
3*A + 4*C) + a^2*(5*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (b*(A*b^2
 + 3*a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (a*(6*A*b^2 + 5
*a^2*(5*A + 6*C))*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (A*b*(a + b*Cos[c
+ d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (A*(a + b*Cos[c + d*x])^3*S
ec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.786249, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3048, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b\left(6a^2(4A+5C)+5b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{a\left(a^2(5A+6C)+6b^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a\left(5a^2(5A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```


```
[Out] (a*(6*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*ArcTanh[Sin[c + d*x]])/(16*d) + (b
*(5*b^2*(2*A + 3*C) + 6*a^2*(4*A + 5*C))*Tan[c + d*x])/(15*d) + (a*(6*b^2*(
3*A + 4*C) + a^2*(5*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (b*(A*b^2
 + 3*a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (a*(6*A*b^2 + 5
*a^2*(5*A + 6*C))*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (A*b*(a + b*Cos[c
+ d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (A*(a + b*Cos[c + d*x])^3*S
ec[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
```

```
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}\int (a+$$

$$= \frac{Ab(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{10d} + \frac{A(a+b}{}$$

$$= \frac{a\left(6Ab^2+5a^2(5A+6C)\right)\sec^3(c+dx)\tan(c+dx)}{120d} + \frac{Ab(a}{}$$

$$= \frac{b\left(Ab^2+3a^2(4A+5C)\right)\sec^2(c+dx)\tan(c+dx)}{15d} + \frac{a\left(6A\right.}{}$$

$$= \frac{b\left(Ab^2+3a^2(4A+5C)\right)\sec^2(c+dx)\tan(c+dx)}{15d} + \frac{a\left(6A\right.}{}$$

$$= \frac{a\left(6b^2(3A+4C)+a^2(5A+6C)\right)\sec(c+dx)\tan(c+dx)}{16d}$$

$$= \frac{a\left(6b^2(3A+4C)+a^2(5A+6C)\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{b}{}$$

Mathematica [A] time = 1.66517, size = 184, normalized size = 0.67

$$\frac{15a\left(a^2(5A+6C)+6b^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(16b\left(5\left(3a^2(2A+C)+Ab^2\right)\tan^2(c+dx)+15\left(3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] (15*a*(6*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*ArcTanh[Sin[c + d*x]] + Tan[c +
 d*x]*(15*a*(6*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*Sec[c + d*x] + 10*a*(18*A
*b^2 + a^2*(5*A + 6*C))*Sec[c + d*x]^3 + 40*a^3*A*Sec[c + d*x]^5 + 16*b*(15
*(3*a^2 + b^2)*(A + C) + 5*(A*b^2 + 3*a^2*(2*A + C))*Tan[c + d*x]^2 + 9*a^2
*A*Tan[c + d*x]^4)))/(240*d)
```

Maple [A] time = 0.064, size = 430, normalized size = 1.6

$$\frac{2\,Ab^3\tan(dx+c)}{3\,d} + \frac{Ab^3\tan(dx+c)(\sec(dx+c))^2}{3\,d} + \frac{Cb^3\tan(dx+c)}{d} + \frac{3\,aAb^2\tan(dx+c)(\sec(dx+c))^3}{4\,d} + \frac{9\,aAb^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 2/3/d*A*b^3*tan(d*x+c)+1/3/d*A*b^3*tan(d*x+c)*sec(d*x+c)^2+1/d*C*b^3*tan(d*
x+c)+3/4/d*a*A*b^2*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a*A*b^2*sec(d*x+c)*tan(d*x
+c)+9/8/d*a*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*C*a*b^2*tan(d*x+c)*sec(d*
x+c)+3/2/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/5/d*A*a^2*b*tan(d*x+c)+3/5/d
*A*a^2*b*tan(d*x+c)*sec(d*x+c)^4+4/5/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)^2+2/d*
a^2*b*C*tan(d*x+c)+1/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)^2+1/6/d*A*a^3*tan(d*x+
c)*sec(d*x+c)^5+5/24/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+5/16/d*A*a^3*sec(d*x+c
)*tan(d*x+c)+5/16/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^3*C*tan(d*x+c)*
sec(d*x+c)^3+3/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+3/8/d*a^3*C*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 1.05218, size = 521, normalized size = 1.91

$$\frac{96\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Aa^{2}b+480\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca^{2}b+160\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
maxima")
```

```
[Out] 1/480*(96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2*b
+ 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b + 160*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*b^3 - 5*A*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33
*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) -
 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 30*C*a^3*(2*(3*sin(
d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 90*A*a*b^2*(2*(3*sin(d*x +
 c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin
(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 360*C*a*b^2*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*C*
b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.55409, size = 636, normalized size = 2.33

$$\frac{15\left(\left(5\,A+6\,C\right)a^{3}+6\left(3\,A+4\,C\right)ab^{2}\right)\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-15\left(\left(5\,A+6\,C\right)a^{3}+6\left(3\,A+4\,C\right)ab^{2}\right)\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
fricas")
```

```
[Out] 1/480*(15*((5*A + 6*C)*a^3 + 6*(3*A + 4*C)*a*b^2)*cos(d*x + c)^6*log(sin(d*
x + c) + 1) - 15*((5*A + 6*C)*a^3 + 6*(3*A + 4*C)*a*b^2)*cos(d*x + c)^6*log
(-sin(d*x + c) + 1) + 2*(16*(6*(4*A + 5*C)*a^2*b + 5*(2*A + 3*C)*b^3)*cos(d
*x + c)^5 + 144*A*a^2*b*cos(d*x + c) + 15*((5*A + 6*C)*a^3 + 6*(3*A + 4*C)*
a*b^2)*cos(d*x + c)^4 + 40*A*a^3 + 16*(3*(4*A + 5*C)*a^2*b + 5*A*b^3)*cos(d
*x + c)^3 + 10*((5*A + 6*C)*a^3 + 18*A*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64299, size = 1258, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
giac")
```

```
[Out] 1/240*(15*(5*A*a^3 + 6*C*a^3 + 18*A*a*b^2 + 24*C*a*b^2)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 15*(5*A*a^3 + 6*C*a^3 + 18*A*a*b^2 + 24*C*a*b^2)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*A*a^3*tan(1/2*d*x + 1/2*c)^11 + 150*C*a
^3*tan(1/2*d*x + 1/2*c)^11 - 720*A*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 720*C*a^
2*b*tan(1/2*d*x + 1/2*c)^11 + 450*A*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 360*C*a
*b^2*tan(1/2*d*x + 1/2*c)^11 - 240*A*b^3*tan(1/2*d*x + 1/2*c)^11 - 240*C*b^
3*tan(1/2*d*x + 1/2*c)^11 + 25*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 210*C*a^3*tan
(1/2*d*x + 1/2*c)^9 + 1680*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 2640*C*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 - 630*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 1080*C*a*b^2*ta
```

n(1/2*d*x + 1/2*c)^9 + 880*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 1200*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 2400*C*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 3744*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 180*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 1440*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 2400*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2640*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 1080*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 880*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 1200*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 165*A*a^3*tan(1/2*d*x + 1/2*c) + 150*C*a^3*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b*tan(1/2*d*x + 1/2*c) + 450*A*a*b^2*tan(1/2*d*x + 1/2*c) + 360*C*a*b^2*tan(1/2*d*x + 1/2*c) + 240*A*b^3*tan(1/2*d*x + 1/2*c) + 240*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

3.549 $\int \cos(c+dx)(a+b\cos(c+dx))^4 \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=345

$$\frac{\left(84a^2b^2(5A+4C)+35a^4(3A+2C)+8b^4(7A+6C)\right)\sin(c+dx)}{105d}+\frac{\left(2a^2C+b^2(7A+6C)\right)\sin(c+dx)\cos^2(c+dx)(a+}{35d}$$

```
[Out] (a*b*(b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*x)/4 + ((35*a^4*(3*A + 2*C) + 84*a
^2*b^2*(5*A + 4*C) + 8*b^4*(7*A + 6*C))*Sin[c + d*x])/(105*d) + (a*b*(b^2*(
6*A + 5*C) + a^2*(8*A + 6*C))*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((4*a^4*C
+ 4*b^4*(7*A + 6*C) + 3*a^2*b^2*(63*A + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])
/(105*d) + (a*b*(126*A*b^2 + 6*a^2*C + 103*b^2*C)*Cos[c + d*x]^3*Sin[c + d*
x])/(210*d) + ((2*a^2*C + b^2*(7*A + 6*C))*Cos[c + d*x]^2*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(35*d) + (2*a*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*S
in[c + d*x])/(21*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*Sin[c + d*x]
)/(7*d)
```

Rubi [A] time = 0.863321, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3050, 3049, 3033, 3023, 2734}

$$\frac{\left(84a^2b^2(5A+4C)+35a^4(3A+2C)+8b^4(7A+6C)\right)\sin(c+dx)}{105d}+\frac{\left(2a^2C+b^2(7A+6C)\right)\sin(c+dx)\cos^2(c+dx)(a+}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a*b*(b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*x)/4 + ((35*a^4*(3*A + 2*C) + 84*a
^2*b^2*(5*A + 4*C) + 8*b^4*(7*A + 6*C))*Sin[c + d*x])/(105*d) + (a*b*(b^2*(
6*A + 5*C) + a^2*(8*A + 6*C))*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((4*a^4*C
+ 4*b^4*(7*A + 6*C) + 3*a^2*b^2*(63*A + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])
/(105*d) + (a*b*(126*A*b^2 + 6*a^2*C + 103*b^2*C)*Cos[c + d*x]^3*Sin[c + d*
x])/(210*d) + ((2*a^2*C + b^2*(7*A + 6*C))*Cos[c + d*x]^2*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(35*d) + (2*a*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*S
in[c + d*x])/(21*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*Sin[c + d*x]
)/(7*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
```

```
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
```

```
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)dx = \frac{C\cos^2(c+dx)(a+b\cos(c+dx))^4\sin(c+dx)}{7d}+\frac{1}{7}\int \cos($$

$$= \frac{2aC\cos^2(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{21d}+\frac{C\cos^2(}{}$$

$$= \frac{\left(2a^2C+b^2(7A+6C)\right)\cos^2(c+dx)(a+b\cos(c+dx))^2\sin}{35d}$$

$$= \frac{ab\left(126Ab^2+6a^2C+103b^2C\right)\cos^3(c+dx)\sin(c+dx)}{210d}+$$

$$= \frac{\left(4a^4C+4b^4(7A+6C)+3a^2b^2(63A+50C)\right)\cos^2(c+dx)}{105d}$$

$$= \frac{1}{4}ab\left(b^2(6A+5C)+a^2(8A+6C)\right)x+\frac{\left(35a^4(3A+2C)+8\right.}{}$$

Mathematica [A] time = 0.842598, size = 351, normalized size = 1.02

$$\frac{420ab\left(16a^2(A+C)+b^2(16A+15C)\right)\sin(2(c+dx))+105\left(48a^2b^2(6A+5C)+16a^4(4A+3C)+5b^4(8A+7C)\right)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (13440*a^3*A*b*c + 10080*a*A*b^3*c + 10080*a^3*b*c*C + 8400*a*b^3*c*C + 134
40*a^3*A*b*d*x + 10080*a*A*b^3*d*x + 10080*a^3*b*C*d*x + 8400*a*b^3*C*d*x +
 105*(16*a^4*(4*A + 3*C) + 48*a^2*b^2*(6*A + 5*C) + 5*b^4*(8*A + 7*C))*Sin[
c + d*x] + 420*a*b*(16*a^2*(A + C) + b^2*(16*A + 15*C))*Sin[2*(c + d*x)] +
3360*a^2*A*b^2*Sin[3*(c + d*x)] + 700*A*b^4*Sin[3*(c + d*x)] + 560*a^4*C*Si
n[3*(c + d*x)] + 4200*a^2*b^2*C*Sin[3*(c + d*x)] + 735*b^4*C*Sin[3*(c + d*x
)] + 840*a*A*b^3*Sin[4*(c + d*x)] + 840*a^3*b*C*Sin[4*(c + d*x)] + 1260*a*b
^3*C*Sin[4*(c + d*x)] + 84*A*b^4*Sin[5*(c + d*x)] + 504*a^2*b^2*C*Sin[5*(c
+ d*x)] + 147*b^4*C*Sin[5*(c + d*x)] + 140*a*b^3*C*Sin[6*(c + d*x)] + 15*b^
4*C*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.021, size = 332, normalized size = 1.

$$\frac{1}{d}\left(\frac{Ab^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+\frac{Cb^4\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/7*C*b^4*(16
/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+4*a*A*b^3*(1/
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*C*a*b^3*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+2
*a^2*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+6/5*a^2*b^2*C*(8/3+cos(d*x+c)^4+4/3*
cos(d*x+c)^2)*sin(d*x+c)+4*A*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+4*a^3*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*
a^4*sin(d*x+c)+1/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.998762, size = 444, normalized size = 1.29

$$-\frac{560\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca^4-1680(2dx+2c+\sin(2dx+2c))Aa^3b-210(12dx+12c+\sin(4dx+4c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/1680*(560*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 - 1680*(2*d*x + 2*c +
sin(2*d*x + 2*c))*A*a^3*b - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*C*a^3*b + 3360*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 - 6
72*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^2*b^2 - 210
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^3 + 35*(4*si
n(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c)
)*C*a*b^3 - 112*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*
b^4 + 48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin
(d*x + c))*C*b^4 - 1680*A*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.62317, size = 595, normalized size = 1.72

$$\frac{105\left(2(4A+3C)a^3b+(6A+5C)ab^3\right)dx+\left(60Cb^4\cos(dx+c)^6+280Cab^3\cos(dx+c)^5+140(3A+2C)a^4+336(5\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/420*(105*(2*(4*A + 3*C)*a^3*b + (6*A + 5*C)*a*b^3)*d*x + (60*C*b^4*cos(d*
x + c)^6 + 280*C*a*b^3*cos(d*x + c)^5 + 140*(3*A + 2*C)*a^4 + 336*(5*A + 4*
C)*a^2*b^2 + 32*(7*A + 6*C)*b^4 + 12*(42*C*a^2*b^2 + (7*A + 6*C)*b^4)*cos(d
*x + c)^4 + 70*(6*C*a^3*b + (6*A + 5*C)*a*b^3)*cos(d*x + c)^3 + 4*(35*C*a^4
 + 42*(5*A + 4*C)*a^2*b^2 + 4*(7*A + 6*C)*b^4)*cos(d*x + c)^2 + 105*(2*(4*A
 + 3*C)*a^3*b + (6*A + 5*C)*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 13.1851, size = 850, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**4*sin(c + d*x)/d + 2*A*a**3*b*x*sin(c + d*x)**2 + 2*A*a**3*
b*x*cos(c + d*x)**2 + 2*A*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*A*a**2*b**
2*sin(c + d*x)**3/d + 6*A*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a*
b**3*x*sin(c + d*x)**4/2 + 3*A*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3
*A*a*b**3*x*cos(c + d*x)**4/2 + 3*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 5*A*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*A*b**4*sin(c + d*x)*
*5/(15*d) + 4*A*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**4*sin(c +
 d*x)*cos(c + d*x)**4/d + 2*C*a**4*sin(c + d*x)**3/(3*d) + C*a**4*sin(c + d
*x)*cos(c + d*x)**2/d + 3*C*a**3*b*x*sin(c + d*x)**4/2 + 3*C*a**3*b*x*sin(c
 + d*x)**2*cos(c + d*x)**2 + 3*C*a**3*b*x*cos(c + d*x)**4/2 + 3*C*a**3*b*si
n(c + d*x)**3*cos(c + d*x)/(2*d) + 5*C*a**3*b*sin(c + d*x)*cos(c + d*x)**3/
(2*d) + 16*C*a**2*b**2*sin(c + d*x)**5/(5*d) + 8*C*a**2*b**2*sin(c + d*x)**
3*cos(c + d*x)**2/d + 6*C*a**2*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*a*
b**3*x*sin(c + d*x)**6/4 + 15*C*a*b**3*x*sin(c + d*x)**4*cos(c + d*x)**2/4
+ 15*C*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/4 + 5*C*a*b**3*x*cos(c + d*
x)**6/4 + 5*C*a*b**3*sin(c + d*x)**5*cos(c + d*x)/(4*d) + 10*C*a*b**3*sin(c
 + d*x)**3*cos(c + d*x)**3/(3*d) + 11*C*a*b**3*sin(c + d*x)*cos(c + d*x)**5
```

```
/(4*d) + 16*C*b**4*sin(c + d*x)**7/(35*d) + 8*C*b**4*sin(c + d*x)**5*cos(c
+ d*x)**2/(5*d) + 2*C*b**4*sin(c + d*x)**3*cos(c + d*x)**4/d + C*b**4*sin(c
 + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*(a + b*cos(c))**
4*cos(c), True))
```

Giac [A] time = 1.26344, size = 392, normalized size = 1.14

$$\frac{Cb^4\sin(7\,dx+7\,c)}{448\,d}+\frac{Cab^3\sin(6\,dx+6\,c)}{48\,d}+\frac{1}{4}\left(8\,Aa^3b+6\,Ca^3b+6\,Aab^3+5\,Cab^3\right)x+\frac{\left(24\,Ca^2b^2+4\,Ab^4+7\,Cb^4\right)\mathrm{s}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/448*C*b^4*sin(7*d*x + 7*c)/d + 1/48*C*a*b^3*sin(6*d*x + 6*c)/d + 1/4*(8*A
*a^3*b + 6*C*a^3*b + 6*A*a*b^3 + 5*C*a*b^3)*x + 1/320*(24*C*a^2*b^2 + 4*A*b
^4 + 7*C*b^4)*sin(5*d*x + 5*c)/d + 1/16*(2*C*a^3*b + 2*A*a*b^3 + 3*C*a*b^3)
*sin(4*d*x + 4*c)/d + 1/192*(16*C*a^4 + 96*A*a^2*b^2 + 120*C*a^2*b^2 + 20*A
*b^4 + 21*C*b^4)*sin(3*d*x + 3*c)/d + 1/16*(16*A*a^3*b + 16*C*a^3*b + 16*A*
a*b^3 + 15*C*a*b^3)*sin(2*d*x + 2*c)/d + 1/64*(64*A*a^4 + 48*C*a^4 + 288*A*
a^2*b^2 + 240*C*a^2*b^2 + 40*A*b^4 + 35*C*b^4)*sin(d*x + c)/d
```

3.550 $\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=301

$$-\frac{a\left(-a^2b^2(190A+121C)+4a^4C-32b^4(5A+4C)\right)\sin(c+dx)}{60bd}-\frac{\left(4a^2C-5b^2(6A+5C)\right)\sin(c+dx)(a+b\cos(c+dx))}{120bd}$$

```
[Out] ((8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*x)/16 - (a*(4
*a^4*C - 32*b^4*(5*A + 4*C) - a^2*b^2*(190*A + 121*C))*Sin[c + d*x])/(60*b*
d) - ((8*a^4*C - 15*b^4*(6*A + 5*C) - 2*a^2*b^2*(130*A + 89*C))*Cos[c + d*x
]*Sin[c + d*x])/(240*d) + (a*(70*A*b^2 - 4*a^2*C + 53*b^2*C)*(a + b*Cos[c +
 d*x])^2*Sin[c + d*x])/(120*b*d) - ((4*a^2*C - 5*b^2*(6*A + 5*C))*(a + b*Co
s[c + d*x])^3*Sin[c + d*x])/(120*b*d) - (a*C*(a + b*Cos[c + d*x])^4*Sin[c +
 d*x])/(30*b*d) + (C*(a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.528148, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3024, 2753, 2734}

$$-\frac{a\left(-a^2b^2(190A+121C)+4a^4C-32b^4(5A+4C)\right)\sin(c+dx)}{60bd}-\frac{\left(4a^2C-5b^2(6A+5C)\right)\sin(c+dx)(a+b\cos(c+dx))}{120bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] ((8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*x)/16 - (a*(4
*a^4*C - 32*b^4*(5*A + 4*C) - a^2*b^2*(190*A + 121*C))*Sin[c + d*x])/(60*b*
d) - ((8*a^4*C - 15*b^4*(6*A + 5*C) - 2*a^2*b^2*(130*A + 89*C))*Cos[c + d*x
]*Sin[c + d*x])/(240*d) + (a*(70*A*b^2 - 4*a^2*C + 53*b^2*C)*(a + b*Cos[c +
 d*x])^2*Sin[c + d*x])/(120*b*d) - ((4*a^2*C - 5*b^2*(6*A + 5*C))*(a + b*Co
s[c + d*x])^3*Sin[c + d*x])/(120*b*d) - (a*C*(a + b*Cos[c + d*x])^4*Sin[c +
 d*x])/(30*b*d) + (C*(a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right) dx = \frac{C(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int (a+b\cos(c+dx))^4(b(6A+5C}{6b}$$

$$= -\frac{aC(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} + \frac{C(a+b\cos(c+dx))^5\sin(c+}{6bd}$$

$$= -\frac{\left(4a^2C-5b^2(6A+5C)\right)(a+b\cos(c+dx))^3\sin(c+dx)}{120bd} - \frac{aC(a+b}{}$$

$$= \frac{a\left(70Ab^2-4a^2C+53b^2C\right)(a+b\cos(c+dx))^2\sin(c+dx)}{120bd} - \frac{\left(4a^2C\right.}{}$$

$$= \frac{1}{16}\left(8a^4(2A+C)+12a^2b^2(4A+3C)+b^4(6A+5C)\right)x - \frac{a\left(4a^4C-3\right.}{}$$

Mathematica [A] time = 0.841072, size = 301, normalized size = 1.

$$\frac{480ab\left(a^2(8A+6C)+b^2(6A+5C)\right)\sin(c+dx)+15\left(96a^2b^2(A+C)+16a^4C+b^4(16A+15C)\right)\sin(2(c+dx))+288}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (960*a^4*A*c + 2880*a^2*A*b^2*c + 360*A*b^4*c + 480*a^4*c*C + 2160*a^2*b^2*
c*C + 300*b^4*c*C + 960*a^4*A*d*x + 2880*a^2*A*b^2*d*x + 360*A*b^4*d*x + 48
0*a^4*C*d*x + 2160*a^2*b^2*C*d*x + 300*b^4*C*d*x + 480*a*b*(b^2*(6*A + 5*C)
```

```
 + a^2*(8*A + 6*C))*Sin[c + d*x] + 15*(16*a^4*C + 96*a^2*b^2*(A + C) + b^4*
(16*A + 15*C))*Sin[2*(c + d*x)] + 320*a*A*b^3*Sin[3*(c + d*x)] + 320*a^3*b*
C*Sin[3*(c + d*x)] + 400*a*b^3*C*Sin[3*(c + d*x)] + 30*A*b^4*Sin[4*(c + d*x
)] + 180*a^2*b^2*C*Sin[4*(c + d*x)] + 45*b^4*C*Sin[4*(c + d*x)] + 48*a*b^3*
C*Sin[5*(c + d*x)] + 5*b^4*C*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.02, size = 294, normalized size = 1.

$$\frac{1}{d}\left(Cb^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{4\,Cab^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4/5*C*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
+A*b^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*a^2*b
^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a*A*b
^3*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a^3*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^
2*A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*C*(1/2*cos(d*x+c)*sin
(d*x+c)+1/2*d*x+1/2*c)+4*A*a^3*b*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [A] time = 1.01342, size = 382, normalized size = 1.27

$$\frac{960\,(dx+c)Aa^4+240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^4-1280\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^3b+1440\,(2\,dx+2\,c+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/960*(960*(d*x + c)*A*a^4 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 - 1
280*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3*b + 1440*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^2*b^2 + 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
+ 2*c))*C*a^2*b^2 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^3 + 256*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a*b^3 + 30*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^4 - 5*(4*sin(2*d*x + 2
*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*b^4 + 3
```

```
840*A*a^3*b*sin(d*x + c))/d
```

Fricas [A] time = 1.63747, size = 504, normalized size = 1.67

$$\frac{15\left(8\,(2\,A+C)a^4+12\,(4\,A+3\,C)a^2b^2+(6\,A+5\,C)b^4\right)dx+\left(40\,Cb^4\cos\left(dx+c\right)^5+192\,Cab^3\cos\left(dx+c\right)^4+320\,(3\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*(2*A + C)*a^4 + 12*(4*A + 3*C)*a^2*b^2 + (6*A + 5*C)*b^4)*d*x
+ (40*C*b^4*cos(d*x + c)^5 + 192*C*a*b^3*cos(d*x + c)^4 + 320*(3*A + 2*C)*a
^3*b + 128*(5*A + 4*C)*a*b^3 + 10*(36*C*a^2*b^2 + (6*A + 5*C)*b^4)*cos(d*x
+ c)^3 + 64*(5*C*a^3*b + (5*A + 4*C)*a*b^3)*cos(d*x + c)^2 + 15*(8*C*a^4 +
12*(4*A + 3*C)*a^2*b^2 + (6*A + 5*C)*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 8.08065, size = 748, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**4*x + 4*A*a**3*b*sin(c + d*x)/d + 3*A*a**2*b**2*x*sin(c + d
*x)**2 + 3*A*a**2*b**2*x*cos(c + d*x)**2 + 3*A*a**2*b**2*sin(c + d*x)*cos(c
 + d*x)/d + 8*A*a*b**3*sin(c + d*x)**3/(3*d) + 4*A*a*b**3*sin(c + d*x)*cos(
c + d*x)**2/d + 3*A*b**4*x*sin(c + d*x)**4/8 + 3*A*b**4*x*sin(c + d*x)**2*c
os(c + d*x)**2/4 + 3*A*b**4*x*cos(c + d*x)**4/8 + 3*A*b**4*sin(c + d*x)**3*
cos(c + d*x)/(8*d) + 5*A*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a**4*x
*sin(c + d*x)**2/2 + C*a**4*x*cos(c + d*x)**2/2 + C*a**4*sin(c + d*x)*cos(c
 + d*x)/(2*d) + 8*C*a**3*b*sin(c + d*x)**3/(3*d) + 4*C*a**3*b*sin(c + d*x)*
cos(c + d*x)**2/d + 9*C*a**2*b**2*x*sin(c + d*x)**4/4 + 9*C*a**2*b**2*x*sin
(c + d*x)**2*cos(c + d*x)**2/2 + 9*C*a**2*b**2*x*cos(c + d*x)**4/4 + 9*C*a*
*2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*C*a**2*b**2*sin(c + d*x)*co
s(c + d*x)**3/(4*d) + 32*C*a*b**3*sin(c + d*x)**5/(15*d) + 16*C*a*b**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*C*a*b**3*sin(c + d*x)*cos(c + d*x)**4
/d + 5*C*b**4*x*sin(c + d*x)**6/16 + 15*C*b**4*x*sin(c + d*x)**4*cos(c + d*
```

```
x)**2/16 + 15*C*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*C*b**4*x*cos(
c + d*x)**6/16 + 5*C*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*C*b**4*si
n(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*b**4*sin(c + d*x)*cos(c + d*x)**
5/(16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*(a + b*cos(c))**4, True))
```

Giac [A] time = 1.68198, size = 333, normalized size = 1.11

$$\frac{Cb^4\sin(6\,dx+6\,c)}{192\,d}+\frac{Cab^3\sin(5\,dx+5\,c)}{20\,d}+\frac{1}{16}\left(16\,Aa^4+8\,Ca^4+48\,Aa^2b^2+36\,Ca^2b^2+6\,Ab^4+5\,Cb^4\right)x+\frac{\left(12\,Ca^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/192*C*b^4*sin(6*d*x + 6*c)/d + 1/20*C*a*b^3*sin(5*d*x + 5*c)/d + 1/16*(16
*A*a^4 + 8*C*a^4 + 48*A*a^2*b^2 + 36*C*a^2*b^2 + 6*A*b^4 + 5*C*b^4)*x + 1/6
4*(12*C*a^2*b^2 + 2*A*b^4 + 3*C*b^4)*sin(4*d*x + 4*c)/d + 1/12*(4*C*a^3*b +
 4*A*a*b^3 + 5*C*a*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*C*a^4 + 96*A*a^2*b^2
+ 96*C*a^2*b^2 + 16*A*b^4 + 15*C*b^4)*sin(2*d*x + 2*c)/d + 1/2*(8*A*a^3*b +
 6*C*a^3*b + 6*A*a*b^3 + 5*C*a*b^3)*sin(d*x + c)/d
```

3.551 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=227

$$\frac{\left(a^2b^2(85A+56C)+6a^4C+2b^4(5A+4C)\right)\sin(c+dx)}{15d}+\frac{ab\left(6a^2C+40Ab^2+29b^2C\right)\sin(c+dx)\cos(c+dx)}{30d}+\frac{(3a^2C}{}$$

```
[Out] (a*b*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/2 + (a^4*A*ArcTanh[Sin[c + d*x]
])/d + ((6*a^4*C + 2*b^4*(5*A + 4*C) + a^2*b^2*(85*A + 56*C))*Sin[c + d*x])
/(15*d) + (a*b*(40*A*b^2 + 6*a^2*C + 29*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(
30*d) + ((3*a^2*C + b^2*(5*A + 4*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
15*d) + (a*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(5*d) + (C*(a + b*Cos[c +
 d*x])^4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.792801, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3050, 3049, 3033, 3023, 2735, 3770}

$$\frac{\left(a^2b^2(85A+56C)+6a^4C+2b^4(5A+4C)\right)\sin(c+dx)}{15d}+\frac{ab\left(6a^2C+40Ab^2+29b^2C\right)\sin(c+dx)\cos(c+dx)}{30d}+\frac{(3a^2C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] (a*b*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/2 + (a^4*A*ArcTanh[Sin[c + d*x]
])/d + ((6*a^4*C + 2*b^4*(5*A + 4*C) + a^2*b^2*(85*A + 56*C))*Sin[c + d*x])
/(15*d) + (a*b*(40*A*b^2 + 6*a^2*C + 29*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(
30*d) + ((3*a^2*C + b^2*(5*A + 4*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
15*d) + (a*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(5*d) + (C*(a + b*Cos[c +
 d*x])^4*Sin[c + d*x])/(5*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
```

```
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+b\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}\int (a+b\cos(c+dx$$

$$= \frac{aC(a+b\cos(c+dx))^3\sin(c+dx)}{5d}+\frac{C(a+b\cos(c+dx))}{5d}$$

$$= \frac{\left(3a^2C+b^2(5A+4C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{15d}+$$

$$= \frac{ab\left(40Ab^2+6a^2C+29b^2C\right)\cos(c+dx)\sin(c+dx)}{30d}+\frac{(3}{}$$

$$= \frac{\left(6a^4C+2b^4(5A+4C)+a^2b^2(85A+56C)\right)\sin(c+dx)}{15d}+$$

$$= \frac{1}{2}ab\left(4a^2(2A+C)+b^2(4A+3C)\right)x+\frac{\left(6a^4C+2b^4(5A+\right.}{}$$

$$= \frac{1}{2}ab\left(4a^2(2A+C)+b^2(4A+3C)\right)x+\frac{a^4A\tanh^{-1}(\sin(c}{d}$$

Mathematica [A] time = 1.03833, size = 226, normalized size = 1.

$$\frac{120ab(c+dx)\left(4a^2(2A+C)+b^2(4A+3C)\right)+240ab\left(C\left(a^2+b^2\right)+Ab^2\right)\sin(2(c+dx))+30\left(12a^2b^2(4A+3C)+8a^4C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (120*a*b*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*(c + d*x) - 240*a^4*A*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] + 240*a^4*A*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + 30*(8*a^4*C + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*Sin[c
+ d*x] + 240*a*b*(A*b^2 + (a^2 + b^2)*C)*Sin[2*(c + d*x)] + 5*b^2*(4*A*b^2
+ 24*a^2*C + 5*b^2*C)*Sin[3*(c + d*x)] + 30*a*b^3*C*Sin[4*(c + d*x)] + 3*b^
4*C*Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.054, size = 364, normalized size = 1.6

$$\frac{A\sin(dx+c)(\cos(dx+c))^2 b^4}{3d}+\frac{2Ab^4\sin(dx+c)}{3d}+\frac{8Cb^4\sin(dx+c)}{15d}+\frac{Cb^4\sin(dx+c)(\cos(dx+c))^4}{5d}+\frac{4Cb^4\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*b^4+2/3/d*A*b^4*sin(d*x+c)+8/15/d*C*b^4*sin
(d*x+c)+1/5/d*C*b^4*sin(d*x+c)*cos(d*x+c)^4+4/15/d*C*b^4*sin(d*x+c)*cos(d*x
+c)^2+2/d*a*A*b^3*cos(d*x+c)*sin(d*x+c)+2*a*A*b^3*x+2/d*a*A*b^3*c+1/d*C*a*b
^3*sin(d*x+c)*cos(d*x+c)^3+3/2/d*C*a*b^3*cos(d*x+c)*sin(d*x+c)+3/2*a*b^3*C*
x+3/2/d*C*a*b^3*c+6/d*a^2*A*b^2*sin(d*x+c)+2/d*C*cos(d*x+c)^2*sin(d*x+c)*a^
2*b^2+4/d*a^2*b^2*C*sin(d*x+c)+4*A*a^3*b*x+4/d*A*a^3*b*c+2/d*a^3*b*C*cos(d*
x+c)*sin(d*x+c)+2*a^3*b*C*x+2/d*a^3*b*C*c+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c
))+1/d*a^4*C*sin(d*x+c)
```

Maxima [A] time = 1.00588, size = 313, normalized size = 1.38

$$\frac{480(dx+c)Aa^3b+120(2dx+2c+\sin(2dx+2c))Ca^3b-240\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca^2b^2+120(2dx+2c+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/120*(480*(d*x + c)*A*a^3*b + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3*b
 - 240*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b^2 + 120*(2*d*x + 2*c + sin
(2*d*x + 2*c))*A*a*b^3 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*C*a*b^3 - 40*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^4 + 8*(3*sin(d*
x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^4 + 120*A*a^4*log(sec(d
*x + c) + tan(d*x + c)) + 120*C*a^4*sin(d*x + c) + 720*A*a^2*b^2*sin(d*x +
c))/d
```

Fricas [A] time = 1.62882, size = 473, normalized size = 2.08

$$\frac{15Aa^4\log(\sin(dx+c)+1)-15Aa^4\log(-\sin(dx+c)+1)+15\left(4(2A+C)a^3b+(4A+3C)ab^3\right)dx+\left(6Cb^4\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*A*a^4*log(sin(d*x + c) + 1) - 15*A*a^4*log(-sin(d*x + c) + 1) + 15
*(4*(2*A + C)*a^3*b + (4*A + 3*C)*a*b^3)*d*x + (6*C*b^4*cos(d*x + c)^4 + 30
*C*a*b^3*cos(d*x + c)^3 + 30*C*a^4 + 60*(3*A + 2*C)*a^2*b^2 + 4*(5*A + 4*C)
*b^4 + 2*(30*C*a^2*b^2 + (5*A + 4*C)*b^4)*cos(d*x + c)^2 + 15*(4*C*a^3*b +
(4*A + 3*C)*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.78789, size = 1017, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] 1/30*(30*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 30*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 15*(8*A*a^3*b + 4*C*a^3*b + 4*A*a*b^3 + 3*C*a*b^3)*(
d*x + c) + 2*(30*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 60*C*a^3*b*tan(1/2*d*x + 1/
2*c)^9 + 180*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 180*C*a^2*b^2*tan(1/2*d*x +
 1/2*c)^9 - 60*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 75*C*a*b^3*tan(1/2*d*x + 1/
2*c)^9 + 30*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 30*C*b^4*tan(1/2*d*x + 1/2*c)^9
+ 120*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 120*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 7
20*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 480*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7
- 120*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 30*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 +
```

```
80*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 40*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 180*C*a
^4*tan(1/2*d*x + 1/2*c)^5 + 1080*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 600*C*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 100*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 116*C*b^
4*tan(1/2*d*x + 1/2*c)^5 + 120*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 120*C*a^3*b*t
an(1/2*d*x + 1/2*c)^3 + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 480*C*a^2*b^
2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 30*C*a*b^3*
tan(1/2*d*x + 1/2*c)^3 + 80*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 40*C*b^4*tan(1/2
*d*x + 1/2*c)^3 + 30*C*a^4*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b*tan(1/2*d*x +
1/2*c) + 180*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 180*C*a^2*b^2*tan(1/2*d*x + 1
/2*c) + 60*A*a*b^3*tan(1/2*d*x + 1/2*c) + 75*C*a*b^3*tan(1/2*d*x + 1/2*c) +
 30*A*b^4*tan(1/2*d*x + 1/2*c) + 30*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^5)/d
```

3.552 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=229

$$-\frac{ab\left(a^2(12A-19C)-8b^2(3A+2C)\right)\sin(c+dx)}{6d}-\frac{b^2\left(a^2(24A-26C)-3b^2(4A+3C)\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x($$

```
[Out] ((8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*x)/8 + (4*a^3*A*b*ArcTa
nh[Sin[c + d*x]])/d - (a*b*(a^2*(12*A - 19*C) - 8*b^2*(3*A + 2*C))*Sin[c +
d*x])/(6*d) - (b^2*(a^2*(24*A - 26*C) - 3*b^2*(4*A + 3*C))*Cos[c + d*x]*Sin
[c + d*x])/(24*d) - (a*b*(12*A - 7*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/
(12*d) - (b*(4*A - C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (A*(a +
b*Cos[c + d*x])^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.802126, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3049, 3033, 3023, 2735, 3770}

$$-\frac{ab\left(a^2(12A-19C)-8b^2(3A+2C)\right)\sin(c+dx)}{6d}-\frac{b^2\left(a^2(24A-26C)-3b^2(4A+3C)\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] ((8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*x)/8 + (4*a^3*A*b*ArcTa
nh[Sin[c + d*x]])/d - (a*b*(a^2*(12*A - 19*C) - 8*b^2*(3*A + 2*C))*Sin[c +
d*x])/(6*d) - (b^2*(a^2*(24*A - 26*C) - 3*b^2*(4*A + 3*C))*Cos[c + d*x]*Sin
[c + d*x])/(24*d) - (a*b*(12*A - 7*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/
(12*d) - (b*(4*A - C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (A*(a +
b*Cos[c + d*x])^4*Tan[c + d*x])/d
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\tan(c+dx)}{d} + \int (a+b\cos(c+dx)$$

$$= -\frac{b(4A-C)(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{A(a+b\cos}{}$$

$$= -\frac{ab(12A-7C)(a+b\cos(c+dx))^2\sin(c+dx)}{12d} - \frac{b(4A-}{}$$

$$= -\frac{b^2\left(a^2(24A-26C)-3b^2(4A+3C)\right)\cos(c+dx)\sin(c+}{24d}$$

$$= -\frac{ab\left(a^2(12A-19C)-8b^2(3A+2C)\right)\sin(c+dx)}{6d} - \frac{b^2\left(a\right.}{}$$

$$= \frac{1}{8}\left(8a^4C+24a^2b^2(2A+C)+b^4(4A+3C)\right)x - \frac{ab\left(a^2(12\right.}{}$$

$$= \frac{1}{8}\left(8a^4C+24a^2b^2(2A+C)+b^4(4A+3C)\right)x + \frac{4a^3Ab\,\mathrm{ta}}{}$$

Mathematica [A] time = 1.34126, size = 274, normalized size = 1.2

$$\frac{12(c+dx)\left(24a^2b^2(2A+C)+8a^4C+b^4(4A+3C)\right)+96ab\left(4a^2C+4Ab^2+3b^2C\right)\sin(c+dx)+24b^2\left(C\left(6a^2+b^2\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (12*(8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*(c + d*x) - 384*a^3*
A*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 384*a^3*A*b*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + (96*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]) + (96*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]) + 96*a*b*(4*A*b^2 + 4*a^2*C + 3*b^2*C)*Sin[c + d*x] + 24*b^2*(A*
b^2 + (6*a^2 + b^2)*C)*Sin[2*(c + d*x)] + 32*a*b^3*C*Sin[3*(c + d*x)] + 3*b
^4*C*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.061, size = 296, normalized size = 1.3

$$\frac{Ab^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{Ab^4x}{2}+\frac{Ab^4c}{2d}+\frac{Cb^4\sin(dx+c)(\cos(dx+c))^3}{4d}+\frac{3Cb^4\cos(dx+c)\sin(dx+c)}{8d}+\frac{3b^4C}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/2/d*A*b^4*cos(d*x+c)*sin(d*x+c)+1/2*A*b^4*x+1/2/d*A*b^4*c+1/4/d*C*b^4*sin
(d*x+c)*cos(d*x+c)^3+3/8/d*C*b^4*cos(d*x+c)*sin(d*x+c)+3/8*b^4*C*x+3/8/d*C*
b^4*c+4/d*a*A*b^3*sin(d*x+c)+4/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a*b^3+8/3/d*C*
a*b^3*sin(d*x+c)+6*a^2*A*b^2*x+6/d*A*a^2*b^2*c+3/d*a^2*b^2*C*cos(d*x+c)*sin
(d*x+c)+3*a^2*b^2*C*x+3/d*a^2*b^2*C*c+4/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))
+4/d*a^3*b*C*sin(d*x+c)+1/d*A*a^4*tan(d*x+c)+a^4*C*x+1/d*a^4*C*c
```

Maxima [A] time = 1.07936, size = 275, normalized size = 1.2

$$96(dx+c)Ca^4+576(dx+c)Aa^2b^2+144(2dx+2c+\sin(2dx+2c))Ca^2b^2-128\left(\sin(dx+c)^3-3\sin(dx+c)\right)Cab^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] 1/96*(96*(d*x + c)*C*a^4 + 576*(d*x + c)*A*a^2*b^2 + 144*(2*d*x + 2*c + sin
(2*d*x + 2*c))*C*a^2*b^2 - 128*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a*b^3 +
24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 + 3*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*C*b^4 + 192*A*a^3*b*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 384*C*a^3*b*sin(d*x + c) + 384*A*a*b^3*sin(d*x + c) +
 96*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.69908, size = 504, normalized size = 2.2

$$48Aa^3b\cos(dx+c)\log(\sin(dx+c)+1)-48Aa^3b\cos(dx+c)\log(-\sin(dx+c)+1)+3\left(8Ca^4+24(2A+C)a^2b^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] 1/24*(48*A*a^3*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 48*A*a^3*b*cos(d*x +
c)*log(-sin(d*x + c) + 1) + 3*(8*C*a^4 + 24*(2*A + C)*a^2*b^2 + (4*A + 3*C)
*b^4)*d*x*cos(d*x + c) + (6*C*b^4*cos(d*x + c)^4 + 32*C*a*b^3*cos(d*x + c)^
3 + 24*A*a^4 + 3*(24*C*a^2*b^2 + (4*A + 3*C)*b^4)*cos(d*x + c)^2 + 32*(3*C*
a^3*b + (3*A + 2*C)*a*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.71155, size = 753, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] 1/24*(96*A*a^3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 96*A*a^3*b*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1) + 3*(8*C*a^4 + 48*A*a^2*b^2 + 24*C*a^2*b^2 + 4*A*b^4 + 3*C*b^4)*
(d*x + c) + 2*(96*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^7 + 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*C*a*b^3*tan(1/2*d*x +
1/2*c)^7 - 12*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*C*b^4*tan(1/2*d*x + 1/2*c)^
7 + 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^
5 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^5
 - 12*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 9*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*C
*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 288*A
*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*A*b
```

```
^4*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 96*C*a^3*b*tan
(1/2*d*x + 1/2*c) + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*A*a*b^3*tan(1/2*
d*x + 1/2*c) + 96*C*a*b^3*tan(1/2*d*x + 1/2*c) + 12*A*b^4*tan(1/2*d*x + 1/2
*c) + 15*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.553 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=219

$$-\frac{b^2\left(a^2(39A-34C)-2b^2(3A+2C)\right)\sin(c+dx)}{6d}+\frac{a^2\left(a^2(A+2C)+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+2abx\left(C\left(2a^2+b^2\right.\right.$$

```
[Out] 2*a*b*(2*A*b^2 + (2*a^2 + b^2)*C)*x + (a^2*(12*A*b^2 + a^2*(A + 2*C))*ArcTa
nh[Sin[c + d*x]])/(2*d) - (b^2*(a^2*(39*A - 34*C) - 2*b^2*(3*A + 2*C))*Sin[
c + d*x])/(6*d) - (a*b^3*(9*A - 4*C)*Cos[c + d*x]*Sin[c + d*x])/(3*d) - (b^
2*(15*A - 2*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + (2*A*b*(a + b*C
os[c + d*x])^3*Tan[c + d*x])/d + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

Rubi [A] time = 0.902528, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3047, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b^2\left(a^2(39A-34C)-2b^2(3A+2C)\right)\sin(c+dx)}{6d}+\frac{a^2\left(a^2(A+2C)+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+2abx\left(C\left(2a^2+b^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] 2*a*b*(2*A*b^2 + (2*a^2 + b^2)*C)*x + (a^2*(12*A*b^2 + a^2*(A + 2*C))*ArcTa
nh[Sin[c + d*x]])/(2*d) - (b^2*(a^2*(39*A - 34*C) - 2*b^2*(3*A + 2*C))*Sin[
c + d*x])/(6*d) - (a*b^3*(9*A - 4*C)*Cos[c + d*x]*Sin[c + d*x])/(3*d) - (b^
2*(15*A - 2*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + (2*A*b*(a + b*C
os[c + d*x])^3*Tan[c + d*x])/d + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a+$$

$$= \frac{2Ab(a+b\cos(c+dx))^3\tan(c+dx)}{d}+\frac{A(a+b\cos(c+a}{}$$

$$= -\frac{b^2(15A-2C)(a+b\cos(c+dx))^2\sin(c+dx)}{6d}+\frac{2Ab(a}{}$$

$$= -\frac{ab^3(9A-4C)\cos(c+dx)\sin(c+dx)}{3d}-\frac{b^2(15A-2C)(}{}$$

$$= -\frac{b^2\left(a^2(39A-34C)-2b^2(3A+2C)\right)\sin(c+dx)}{6d}-\frac{ab^3(}{}$$

$$= 2ab\left(2Ab^2+\left(2a^2+b^2\right)C\right)x-\frac{b^2\left(a^2(39A-34C)-2b^2(}{6d}$$

$$= 2ab\left(2Ab^2+\left(2a^2+b^2\right)C\right)x+\frac{a^2\left(12Ab^2+a^2(A+2C)\right)}{2d}$$

Mathematica [A] time = 3.09963, size = 323, normalized size = 1.47

$$24ab(c+dx)\left(C\left(2a^2+b^2\right)+2Ab^2\right)+3b^2\left(3C\left(8a^2+b^2\right)+4Ab^2\right)\sin(c+dx)-6a^2\left(a^2(A+2C)+12Ab^2\right)\log\left(\cos\left(\frac{1}{2}(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (24*a*b*(2*A*b^2 + (2*a^2 + b^2)*C)*(c + d*x) - 6*a^2*(12*A*b^2 + a^2*(A +
2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*a^2*(12*A*b^2 + a^2*(A +
 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*a^4*A)/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^2 + (48*a^3*A*b*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]) - (3*a^4*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (
48*a^3*A*b*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 3*b^2*
(4*A*b^2 + 3*(8*a^2 + b^2)*C)*Sin[c + d*x] + 12*a*b^3*C*Sin[2*(c + d*x)] +
b^4*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.065, size = 259, normalized size = 1.2

$$\frac{Ab^4\sin(dx+c)}{d}+\frac{Cb^4\sin(dx+c)(\cos(dx+c))^2}{3d}+\frac{2Cb^4\sin(dx+c)}{3d}+4aAb^3x+4\frac{Aab^3c}{d}+2\frac{Cab^3\cos(dx+c)\sin(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*b^4*sin(d*x+c)+1/3/d*C*b^4*sin(d*x+c)*cos(d*x+c)^2+2/3/d*C*b^4*sin(d*
x+c)+4*a*A*b^3*x+4/d*a*A*b^3*c+2/d*C*a*b^3*cos(d*x+c)*sin(d*x+c)+2*a*b^3*C*
x+2/d*C*a*b^3*c+6/d*a^2*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*C*sin(d
*x+c)+4/d*A*a^3*b*tan(d*x+c)+4*a^3*b*C*x+4/d*a^3*b*C*c+1/2/d*A*a^4*sec(d*x+
c)*tan(d*x+c)+1/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*C*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 1.0631, size = 298, normalized size = 1.36

$$\frac{48(dx+c)Ca^3b+48(dx+c)Aab^3+12(2dx+2c+\sin(2dx+2c))Cab^3-4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Cb^4-3Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] 1/12*(48*(d*x + c)*C*a^3*b + 48*(d*x + c)*A*a*b^3 + 12*(2*d*x + 2*c + sin(2
*d*x + 2*c))*C*a*b^3 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*b^4 - 3*A*a^4*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)) + 6*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*A
*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 72*C*a^2*b^2*sin
(d*x + c) + 12*A*b^4*sin(d*x + c) + 48*A*a^3*b*tan(d*x + c))/d
```

Fricas [A] time = 1.65472, size = 516, normalized size = 2.36

$$\frac{24\left(2\,Ca^3b+(2\,A+C)ab^3\right)dx\cos\left(dx+c\right)^2+3\left((A+2\,C)a^4+12\,Aa^2b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\left((A+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] 1/12*(24*(2*C*a^3*b + (2*A + C)*a*b^3)*d*x*cos(d*x + c)^2 + 3*((A + 2*C)*a^
4 + 12*A*a^2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*((A + 2*C)*a^4 +
 12*A*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*C*b^4*cos(d*x +
 c)^4 + 12*C*a*b^3*cos(d*x + c)^3 + 24*A*a^3*b*cos(d*x + c) + 3*A*a^4 + 2*(
18*C*a^2*b^2 + (3*A + 2*C)*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38456, size = 535, normalized size = 2.44

$$12\left(2\,Ca^3b+2\,Aab^3+Cab^3\right)(dx+c)+3\left(Aa^4+2\,Ca^4+12\,Aa^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Aa^4+2\,Ca^4+12\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] 1/6*(12*(2*C*a^3*b + 2*A*a*b^3 + C*a*b^3)*(d*x + c) + 3*(A*a^4 + 2*C*a^4 +
12*A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a^4 + 2*C*a^4 + 12*
A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 6*(A*a^4*tan(1/2*d*x + 1/2*
c)^3 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + A*a^4*tan(1/2*d*x + 1/2*c) + 8*A*
a^3*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 4*(18*C*a^2*b^
2*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^4*tan(1
/2*d*x + 1/2*c)^5 + 3*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b^2*tan(1/2*d
*x + 1/2*c)^3 + 6*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*C*b^4*tan(1/2*d*x + 1/2*
c)^3 + 18*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*C*a*b^3*tan(1/2*d*x + 1/2*c) +
 3*A*b^4*tan(1/2*d*x + 1/2*c) + 3*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

3.554 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=251

$$-\frac{2ab\left(a^2(2A+3C)+b^2(11A-6C)\right)\sin(c+dx)}{3d}+\frac{2ab\left(a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\left(a^2(4A+6C)+\right.}{}$$

```
[Out] (b^2*(2*A*b^2 + (12*a^2 + b^2)*C)*x)/2 + (2*a*b*(2*A*b^2 + a^2*(A + 2*C))*A
rcTanh[Sin[c + d*x]])/d - (2*a*b*(b^2*(11*A - 6*C) + a^2*(2*A + 3*C))*Sin[c
 + d*x])/(3*d) - (b^2*(3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Cos[c + d*x]*Sin[
c + d*x])/(6*d) + ((6*A*b^2 + a^2*(2*A + 3*C))*(a + b*Cos[c + d*x])^2*Tan[c
 + d*x])/(3*d) + (2*A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(
3*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.962218, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3047, 3033, 3023, 2735, 3770}

$$-\frac{2ab\left(a^2(2A+3C)+b^2(11A-6C)\right)\sin(c+dx)}{3d}+\frac{2ab\left(a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\left(a^2(4A+6C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] (b^2*(2*A*b^2 + (12*a^2 + b^2)*C)*x)/2 + (2*a*b*(2*A*b^2 + a^2*(A + 2*C))*A
rcTanh[Sin[c + d*x]])/d - (2*a*b*(b^2*(11*A - 6*C) + a^2*(2*A + 3*C))*Sin[c
 + d*x])/(3*d) - (b^2*(3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Cos[c + d*x]*Sin[
c + d*x])/(6*d) + ((6*A*b^2 + a^2*(2*A + 3*C))*(a + b*Cos[c + d*x])^2*Tan[c
 + d*x])/(3*d) + (2*A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(
3*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A(a+b\cos(c+dx))^4\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a$$

$$=\frac{2Ab(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{3d}+\frac{A(a+$$

$$=\frac{\left(6Ab^2+a^2(2A+3C)\right)(a+b\cos(c+dx))^2\tan(c+dx)}{3d}$$

$$=-\frac{b^2\left(3b^2(6A-C)+a^2(4A+6C)\right)\cos(c+dx)\sin(c+dx}{6d}$$

$$=-\frac{2ab\left(b^2(11A-6C)+a^2(2A+3C)\right)\sin(c+dx)}{3d}-\frac{b^2\left(3b\right.}{}$$

$$=\frac{1}{2}b^2\left(2Ab^2+\left(12a^2+b^2\right)C\right)x-\frac{2ab\left(b^2(11A-6C)+a^2(\right.}{3d}$$

$$=\frac{1}{2}b^2\left(2Ab^2+\left(12a^2+b^2\right)C\right)x+\frac{2ab\left(2Ab^2+a^2(A+2C\right.}{d}$$

Mathematica [A] time = 6.01673, size = 412, normalized size = 1.64

$$6b^2(c+dx)\left(C\left(12a^2+b^2\right)+2Ab^2\right)+\frac{4a^2\left(a^2(2A+3C)+18Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4a^2\left(a^2(2A+3C)+18Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-24ab\left(a^2(A+2C\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (6*b^2*(2*A*b^2 + (12*a^2 + b^2)*C)*(c + d*x) - 24*a*b*(2*A*b^2 + a^2*(A +
2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*A*b^2 + a^2*(A +
 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*A*(a + 12*b))/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^3 + (4*a^2*(18*A*b^2 + a^2*(2*A + 3*C))*Sin[(c +
d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*a^4*A*Sin[(c + d*x)/2])
/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (a^3*A*(a + 12*b))/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^2 + (4*a^2*(18*A*b^2 + a^2*(2*A + 3*C))*Sin[(c + d
```

```
*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 48*a*b^3*C*Sin[c + d*x] + 3
*b^4*C*Sin[2*(c + d*x)])/(12*d)
```

Maple [A] time = 0.069, size = 258, normalized size = 1.

$$Ab^4x + \frac{Ab^4c}{d} + \frac{Cb^4\cos(dx+c)\sin(dx+c)}{2d} + \frac{b^4Cx}{2} + \frac{Cb^4c}{2d} + 4\frac{aAb^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + 4\frac{Cab^3\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] A*b^4*x+1/d*A*b^4*c+1/2/d*C*b^4*cos(d*x+c)*sin(d*x+c)+1/2*b^4*C*x+1/2/d*C*b
^4*c+4/d*a*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*C*a*b^3*sin(d*x+c)+6/d*a^2*A
*b^2*tan(d*x+c)+6*a^2*b^2*C*x+6/d*a^2*b^2*C*c+2/d*A*a^3*b*sec(d*x+c)*tan(d*
x+c)+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b*C*ln(sec(d*x+c)+tan(d*
x+c))+2/3/d*A*a^4*tan(d*x+c)+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/d*a^4*C*
tan(d*x+c)
```

Maxima [A] time = 1.02472, size = 298, normalized size = 1.19

$$\frac{4\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Aa^4 + 72(dx+c)Ca^2b^2 + 12(dx+c)Ab^4 + 3(2dx+2c+\sin(2dx+2c))Cb^4 - 12Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 72*(d*x + c)*C*a^2*b^2 +
12*(d*x + c)*A*b^4 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*b^4 - 12*A*a^3*b*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)) + 24*C*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2
4*A*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*C*a*b^3*sin(
d*x + c) + 12*C*a^4*tan(d*x + c) + 72*A*a^2*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.67368, size = 509, normalized size = 2.03

$$\frac{3\left(12\,Ca^2b^2 + (2\,A + C)b^4\right)dx\cos\left(dx + c\right)^3 + 6\left((A + 2\,C)a^3b + 2\,Aab^3\right)\cos\left(dx + c\right)^3\log\left(\sin\left(dx + c\right) + 1\right) - 6\left((A + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(12*C*a^2*b^2 + (2*A + C)*b^4)*d*x*cos(d*x + c)^3 + 6*((A + 2*C)*a^3
*b + 2*A*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 6*((A + 2*C)*a^3*b +
 2*A*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (3*C*b^4*cos(d*x + c)^4
 + 24*C*a*b^3*cos(d*x + c)^3 + 12*A*a^3*b*cos(d*x + c) + 2*A*a^4 + 2*((2*A
+ 3*C)*a^4 + 18*A*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.71736, size = 536, normalized size = 2.14

$$\frac{3\left(12\,Ca^2b^2 + 2\,Ab^4 + Cb^4\right)(dx + c) + 12\left(Aa^3b + 2\,Ca^3b + 2\,Aab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 12\left(Aa^3b + 2\,Ca^3b + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] 1/6*(3*(12*C*a^2*b^2 + 2*A*b^4 + C*b^4)*(d*x + c) + 12*(A*a^3*b + 2*C*a^3*b
 + 2*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*(A*a^3*b + 2*C*a^3*b
+ 2*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 6*(8*C*a*b^3*tan(1/2*d*x
+ 1/2*c)^3 - C*b^4*tan(1/2*d*x + 1/2*c)^3 + 8*C*a*b^3*tan(1/2*d*x + 1/2*c)
+ C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 4*(3*A*a^4*t
an(1/2*d*x + 1/2*c)^5 + 3*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*b*tan(1/2*
d*x + 1/2*c)^5 + 18*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*A*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 6*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 36*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c) + 6*A*
a^3*b*tan(1/2*d*x + 1/2*c) + 18*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^3)/d
```

3.555 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=246

$$-\frac{b^2\left(3a^2(3A+4C)+2b^2(13A-12C)\right)\sin(c+dx)}{24d}+\frac{ab\left(a^2(23A+36C)+12Ab^2\right)\tan(c+dx)}{12d}+\frac{\left(24a^2b^2(A+2C)+a\right.}{}$$

```
[Out] 4*a*b^3*C*x + ((8*A*b^4 + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*ArcTanh[S
in[c + d*x]])/(8*d) - (b^2*(2*b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Sin[c
+ d*x])/(24*d) + (a*b*(12*A*b^2 + a^2*(23*A + 36*C))*Tan[c + d*x])/(12*d) +
 ((4*A*b^2 + a^2*(3*A + 4*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
 + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.934565, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^2\left(3a^2(3A+4C)+2b^2(13A-12C)\right)\sin(c+dx)}{24d}+\frac{ab\left(a^2(23A+36C)+12Ab^2\right)\tan(c+dx)}{12d}+\frac{\left(24a^2b^2(A+2C)+a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```


```
[Out] 4*a*b^3*C*x + ((8*A*b^4 + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*ArcTanh[S
in[c + d*x]])/(8*d) - (b^2*(2*b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Sin[c
+ d*x])/(24*d) + (a*b*(12*A*b^2 + a^2*(23*A + 36*C))*Tan[c + d*x])/(12*d) +
 ((4*A*b^2 + a^2*(3*A + 4*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
 + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (a$$

$$= \frac{Ab(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{A(a+$$

$$= \frac{\left(4Ab^2+a^2(3A+4C)\right)(a+b\cos(c+dx))^2\sec(c+dx)\,\mathrm{ta}}{8d}$$

$$= \frac{ab\left(12Ab^2+a^2(23A+36C)\right)\tan(c+dx)}{12d} + \frac{\left(4Ab^2+a^2(\right.}{}$$

$$= -\frac{b^2\left(2b^2(13A-12C)+3a^2(3A+4C)\right)\sin(c+dx)}{24d} + \frac{ab}{}$$

$$= 4ab^3Cx - \frac{b^2\left(2b^2(13A-12C)+3a^2(3A+4C)\right)\sin(c+d}{24d}$$

$$= 4ab^3Cx + \frac{\left(8Ab^4+24a^2b^2(A+2C)+a^4(3A+4C)\right)\tan}{8d}$$

Mathematica [B] time = 6.33435, size = 612, normalized size = 2.49

$$\frac{4\left(2a^3Ab\sin\left(\frac{1}{2}(c+dx)\right)+3a^3bC\sin\left(\frac{1}{2}(c+dx)\right)+3aAb^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4\left(2a^3Ab\sin\left(\frac{1}{2}(c+dx)\right)+3a^3bC\sin\left(\frac{1}{2}\right.\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (4*a*b^3*C*(c + d*x))/d + ((-3*a^4*A - 24*a^2*A*b^2 - 8*A*b^4 - 4*a^4*C - 4
8*a^2*b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(8*d) + ((3*a^4*A +
24*a^2*A*b^2 + 8*A*b^4 + 4*a^4*C + 48*a^2*b^2*C)*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]])/(8*d) + (a^4*A)/(16*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^4) + (9*a^4*A + 16*a^3*A*b + 72*a^2*A*b^2 + 12*a^4*C)/(48*d*(Cos[(c + d*x)
```

```
/2] - Sin[(c + d*x)/2])^2) + (2*a^3*A*b*Sin[(c + d*x)/2])/(3*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^3) - (a^4*A)/(16*d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^4) + (2*a^3*A*b*Sin[(c + d*x)/2])/(3*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^3) + (-9*a^4*A - 16*a^3*A*b - 72*a^2*A*b^2 - 12*a^4*C)/(48*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*a^3*A*b*Sin[(c + d*x)/2] + 3*a
*A*b^3*Sin[(c + d*x)/2] + 3*a^3*b*C*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + (4*(2*a^3*A*b*Sin[(c + d*x)/2] + 3*a*A*b^3*Sin[(c
 + d*x)/2] + 3*a^3*b*C*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (b^4*C*Sin[c + d*x])/d
```

Maple [A] time = 0.071, size = 316, normalized size = 1.3

$$\frac{Ab^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cb^4\sin(dx+c)}{d}+4\,\frac{aAb^3\tan(dx+c)}{d}+4\,ab^3Cx+4\,\frac{Cab^3c}{d}+3\,\frac{a^2Ab^2\sec(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*b^4*sin(d*x+c)+4/d*a*A*b^3*tan(d*
x+c)+4*a*b^3*C*x+4/d*C*a*b^3*c+3/d*a^2*A*b^2*sec(d*x+c)*tan(d*x+c)+3/d*a^2*
A*b^2*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*C*ln(sec(d*x+c)+tan(d*x+c))+8/3
/d*A*a^3*b*tan(d*x+c)+4/3/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^2+4/d*a^3*b*C*tan
(d*x+c)+1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^4*sec(d*x+c)*tan(d*x+
c)+3/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+
1/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.04228, size = 413, normalized size = 1.68

$$\frac{64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3b+192\,(dx+c)Cab^3-3\,Aa^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3*b + 192*(d*x + c)*C*a*b^3
- 3*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*
```

```
x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a
^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d
*x + c) - 1)) - 72*A*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 144*C*a^2*b^2*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 24*A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 48*C*b^4*sin(d*x + c) + 192*C*a^3*b*tan(d*x + c) + 192*A*a*b
^3*tan(d*x + c))/d
```

Fricas [A] time = 1.53185, size = 575, normalized size = 2.34

$$\frac{192\,Cab^3dx\cos\left(dx+c\right)^4+3\left(\left(3\,A+4\,C\right)a^4+24\,\left(A+2\,C\right)a^2b^2+8\,Ab^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(\left(3\,A+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/48*(192*C*a*b^3*d*x*cos(d*x + c)^4 + 3*((3*A + 4*C)*a^4 + 24*(A + 2*C)*a^
2*b^2 + 8*A*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*((3*A + 4*C)*a^4
+ 24*(A + 2*C)*a^2*b^2 + 8*A*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(24*C*b^4*cos(d*x + c)^4 + 32*A*a^3*b*cos(d*x + c) + 6*A*a^4 + 32*((2*A +
3*C)*a^3*b + 3*A*a*b^3)*cos(d*x + c)^3 + 3*((3*A + 4*C)*a^4 + 24*A*a^2*b^2)
*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53916, size = 797, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] 1/24*(96*(d*x + c)*C*a*b^3 + 48*C*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1) + 3*(3*A*a^4 + 4*C*a^4 + 24*A*a^2*b^2 + 48*C*a^2*b^2 + 8*A*b^4
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^4 + 4*C*a^4 + 24*A*a^2*b^2
+ 48*C*a^2*b^2 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^4*
tan(1/2*d*x + 1/2*c)^7 + 12*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 96*A*a^3*b*tan(1
/2*d*x + 1/2*c)^7 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b^2*tan(1/
2*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^4*tan(1/2*d*x
+ 1/2*c)^5 - 12*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*b*tan(1/2*d*x + 1/
2*c)^5 + 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^5 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^
3 - 12*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 -
288*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) + 12*C*a
^4*tan(1/2*d*x + 1/2*c) + 96*A*a^3*b*tan(1/2*d*x + 1/2*c) + 96*C*a^3*b*tan(
1/2*d*x + 1/2*c) + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*A*a*b^3*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.556 $\int (a+b\cos(c+dx))^4 \left(A + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=250

$$\frac{\left(a^2b^2(56A+85C)+2a^4(4A+5C)+6Ab^4\right)\tan(c+dx)}{15d}+\frac{ab\left(a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ab}{}$$

```
[Out] b^4*C*x + (a*b*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(
2*d) + ((6*A*b^4 + 2*a^4*(4*A + 5*C) + a^2*b^2*(56*A + 85*C))*Tan[c + d*x])
/(15*d) + (a*b*(6*A*b^2 + a^2*(29*A + 40*C))*Sec[c + d*x]*Tan[c + d*x])/(30
*d) + ((3*A*b^2 + a^2*(4*A + 5*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Ta
n[c + d*x])/(15*d) + (A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x
])/(5*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.902299, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3047, 3031, 3021, 2735, 3770}

$$\frac{\left(a^2b^2(56A+85C)+2a^4(4A+5C)+6Ab^4\right)\tan(c+dx)}{15d}+\frac{ab\left(a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```


```
[Out] b^4*C*x + (a*b*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(
2*d) + ((6*A*b^4 + 2*a^4*(4*A + 5*C) + a^2*b^2*(56*A + 85*C))*Tan[c + d*x])
/(15*d) + (a*b*(6*A*b^2 + a^2*(29*A + 40*C))*Sec[c + d*x]*Tan[c + d*x])/(30
*d) + ((3*A*b^2 + a^2*(4*A + 5*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Ta
n[c + d*x])/(15*d) + (A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x
])/(5*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
```

```
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a$$
$$= \frac{Ab(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{5d}+\frac{A(a+$$
$$= \frac{\left(3Ab^2+a^2(4A+5C)\right)(a+b\cos(c+dx))^2\sec^2(c+dx)\,t}{15d}$$
$$= \frac{ab\left(6Ab^2+a^2(29A+40C)\right)\sec(c+dx)\tan(c+dx)}{30d}+\frac{(3}{}$$
$$= \frac{\left(6Ab^4+2a^4(4A+5C)+a^2b^2(56A+85C)\right)\tan(c+dx)}{15d}$$
$$= b^4Cx+\frac{\left(6Ab^4+2a^4(4A+5C)+a^2b^2(56A+85C)\right)\tan}{15d}$$
$$= b^4Cx+\frac{ab\left(4b^2(A+2C)+a^2(3A+4C)\right)\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [A] time = 1.08064, size = 169, normalized size = 0.68

$$\frac{10a^2\left(a^2(2A+C)+6Ab^2\right)\tan^3(c+dx)+15ab\left(a^2(3A+4C)+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))+15\tan(c+dx)\left(ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (30*b^4*C*d*x + 15*a*b*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c +
d*x]] + 15*(2*(A*b^4 + a^4*(A + C) + 6*a^2*b^2*(A + C)) + a*b*(4*A*b^2 + a^
2*(3*A + 4*C))*Sec[c + d*x] + 2*a^3*A*b*Sec[c + d*x]^3)*Tan[c + d*x] + 10*a
^2*(6*A*b^2 + a^2*(2*A + C))*Tan[c + d*x]^3 + 6*a^4*A*Tan[c + d*x]^5)/(30*d
)
```

Maple [A] time = 0.07, size = 377, normalized size = 1.5

$$\frac{Ab^4\tan(dx+c)}{d}+b^4Cx+\frac{Cb^4c}{d}+2\,\frac{aAb^3\sec(dx+c)\tan(dx+c)}{d}+2\,\frac{aAb^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\,\frac{Cab^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/d*A*b^4*tan(d*x+c)+b^4*C*x+1/d*C*b^4*c+2/d*a*A*b^3*sec(d*x+c)*tan(d*x+c)+
2/d*a*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))
+4/d*a^2*A*b^2*tan(d*x+c)+2/d*a^2*A*b^2*tan(d*x+c)*sec(d*x+c)^2+6/d*a^2*b^2
*C*tan(d*x+c)+1/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^3+3/2/d*A*a^3*b*sec(d*x+c)*
tan(d*x+c)+3/2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^3*b*C*sec(d*x+c)*t
an(d*x+c)+2/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*A*a^4*tan(d*x+c)+1/5
/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+2/3/d
*a^4*C*tan(d*x+c)+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.02512, size = 439, normalized size = 1.76

$$\frac{4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+20\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^4+120\left(\tan(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 + 20
*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 120*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*A*a^2*b^2 + 60*(d*x + c)*C*b^4 - 15*A*a^3*b*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 60*C*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^
2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*A*a*b^3*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 120*C*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 360*C*
a^2*b^2*tan(d*x + c) + 60*A*b^4*tan(d*x + c))/d
```

Fricas [A] time = 1.62195, size = 610, normalized size = 2.44

$$\frac{60\,Cb^4dx\cos\left(dx+c\right)^5+15\left((3\,A+4\,C)a^3b+4\,(A+2\,C)ab^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left((3\,A+4\,C)a^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] 1/60*(60*C*b^4*d*x*cos(d*x + c)^5 + 15*((3*A + 4*C)*a^3*b + 4*(A + 2*C)*a*b
^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*((3*A + 4*C)*a^3*b + 4*(A + 2
*C)*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*A*a^3*b*cos(d*x +
c) + 6*A*a^4 + 2*(2*(4*A + 5*C)*a^4 + 30*(2*A + 3*C)*a^2*b^2 + 15*A*b^4)*co
s(d*x + c)^4 + 15*((3*A + 4*C)*a^3*b + 4*A*a*b^3)*cos(d*x + c)^3 + 2*((4*A
+ 5*C)*a^4 + 30*A*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32068, size = 1050, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] 1/30*(30*(d*x + c)*C*b^4 + 15*(3*A*a^3*b + 4*C*a^3*b + 4*A*a*b^3 + 8*C*a*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*a^3*b + 4*C*a^3*b + 4*A*a*b
^3 + 8*C*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*A*a^4*tan(1/2*d*
```

```
x + 1/2*c)^9 + 30*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 75*A*a^3*b*tan(1/2*d*x + 1
/2*c)^9 - 60*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^9 + 180*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*a*b^3*tan(1/2*d*x + 1
/2*c)^9 + 30*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 40*A*a^4*tan(1/2*d*x + 1/2*c)^7
 - 80*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 12
0*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 7
20*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 -
120*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 116*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 100*C
*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1080*C
*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 180*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 40*A*a
^4*tan(1/2*d*x + 1/2*c)^3 - 80*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 30*A*a^3*b*ta
n(1/2*d*x + 1/2*c)^3 - 120*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 - 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*a*b^3*
tan(1/2*d*x + 1/2*c)^3 - 120*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*A*a^4*tan(1/
2*d*x + 1/2*c) + 30*C*a^4*tan(1/2*d*x + 1/2*c) + 75*A*a^3*b*tan(1/2*d*x + 1
/2*c) + 60*C*a^3*b*tan(1/2*d*x + 1/2*c) + 180*A*a^2*b^2*tan(1/2*d*x + 1/2*c
) + 180*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a*b^3*tan(1/2*d*x + 1/2*c) +
30*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.557 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=307

$$\frac{4ab\left(2a^2(4A+5C)+5b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{\left(12a^2b^2(3A+4C)+a^4(5A+6C)+8b^4(A+2C)\right)\tanh^{-1}(\sin(c+$$

```
[Out] ((8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*ArcTanh[Sin[c
 + d*x]])/(16*d) + (4*a*b*(5*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Tan[c + d
*x])/(15*d) + ((24*A*b^4 + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(49*A + 66*C))*S
ec[c + d*x]*Tan[c + d*x])/(240*d) + (a*b*(4*A*b^2 + a^2*(39*A + 50*C))*Sec[
c + d*x]^2*Tan[c + d*x])/(60*d) + ((12*A*b^2 + 5*a^2*(5*A + 6*C))*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (2*A*b*(a + b*Cos[c +
d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (A*(a + b*Cos[c + d*x])^4*Sec
[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 1.12453, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3048, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{4ab\left(2a^2(4A+5C)+5b^2(2A+3C)\right)\tan(c+dx)}{15d}+\frac{\left(12a^2b^2(3A+4C)+a^4(5A+6C)+8b^4(A+2C)\right)\tanh^{-1}(\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```


```
[Out] ((8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*ArcTanh[Sin[c
 + d*x]])/(16*d) + (4*a*b*(5*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Tan[c + d
*x])/(15*d) + ((24*A*b^4 + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(49*A + 66*C))*S
ec[c + d*x]*Tan[c + d*x])/(240*d) + (a*b*(4*A*b^2 + a^2*(39*A + 50*C))*Sec[
c + d*x]^2*Tan[c + d*x])/(60*d) + ((12*A*b^2 + 5*a^2*(5*A + 6*C))*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (2*A*b*(a + b*Cos[c +
d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (A*(a + b*Cos[c + d*x])^4*Sec
[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}\int (a$$

$$= \frac{2Ab(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{15d}+\frac{A(a$$

$$= \frac{\left(12Ab^2+5a^2(5A+6C)\right)(a+b\cos(c+dx))^2\sec^3(c+dx}{120d}$$

$$= \frac{ab\left(4Ab^2+a^2(39A+50C)\right)\sec^2(c+dx)\tan(c+dx)}{60d}+$$

$$= \frac{\left(24Ab^4+15a^4(5A+6C)+10a^2b^2(49A+66C)\right)\sec(c+}{240d}$$

$$= \frac{\left(24Ab^4+15a^4(5A+6C)+10a^2b^2(49A+66C)\right)\sec(c+}{240d}$$

$$= \frac{\left(8b^4(A+2C)+12a^2b^2(3A+4C)+a^4(5A+6C)\right)\tanh^{-}}{16d}$$

$$= \frac{\left(8b^4(A+2C)+12a^2b^2(3A+4C)+a^4(5A+6C)\right)\tanh^{-}}{16d}$$

Mathematica [A] time = 4.90567, size = 204, normalized size = 0.66

$$\frac{15\left(12a^2b^2(3A+4C)+a^4(5A+6C)+8b^4(A+2C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(64ab\left(5\left(a^2(2A+C)+Ab^2\right)\tan\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] (15*(8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(15*(8*A*b^4 + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A
 + 6*C))*Sec[c + d*x] + 10*a^2*(36*A*b^2 + a^2*(5*A + 6*C))*Sec[c + d*x]^3
+ 40*a^4*A*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2)*(A + C) + 5*(A*b^2 + a^2
*(2*A + C))*Tan[c + d*x]^2 + 3*a^2*A*Tan[c + d*x]^4)))/(240*d)
```

Maple [A] time = 0.073, size = 511, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/2/d*A*b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/d
*C*b^4*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*a*A*b^3*tan(d*x+c)+4/3/d*a*A*b^3*tan
(d*x+c)*sec(d*x+c)^2+4/d*C*a*b^3*tan(d*x+c)+3/2/d*a^2*A*b^2*tan(d*x+c)*sec(
d*x+c)^3+9/4/d*a^2*A*b^2*sec(d*x+c)*tan(d*x+c)+9/4/d*a^2*A*b^2*ln(sec(d*x+c
)+tan(d*x+c))+3/d*a^2*b^2*C*sec(d*x+c)*tan(d*x+c)+3/d*a^2*b^2*C*ln(sec(d*x+
c)+tan(d*x+c))+32/15/d*A*a^3*b*tan(d*x+c)+4/5/d*A*a^3*b*tan(d*x+c)*sec(d*x+
c)^4+16/15/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^2+8/3/d*a^3*b*C*tan(d*x+c)+4/3/d
*a^3*b*C*tan(d*x+c)*sec(d*x+c)^2+1/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^5+5/24/d
*A*a^4*tan(d*x+c)*sec(d*x+c)^3+5/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)+5/16/d*A*
a^4*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^4
*C*sec(d*x+c)*tan(d*x+c)+3/8/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.0382, size = 629, normalized size = 2.05

$$\frac{128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3b+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^3b+640\left(\tan(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3*b
 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3*b + 640*(tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a*b^3 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 +
 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1
) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 30*C*a^4*(2*(3*s
in(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3
*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^2*b^2*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*l
og(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 720*C*a^2*b^2*(2*sin(d*x
+ c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
- 120*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) + 240*C*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 1920*C*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.72725, size = 716, normalized size = 2.33

$$\frac{15\left((5A+6C)a^4+12(3A+4C)a^2b^2+8(A+2C)b^4\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left((5A+6C)a^4+12(3 \ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
fricas")
```

```
[Out] 1/480*(15*((5*A + 6*C)*a^4 + 12*(3*A + 4*C)*a^2*b^2 + 8*(A + 2*C)*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*((5*A + 6*C)*a^4 + 12*(3*A + 4*C)*a^2
*b^2 + 8*(A + 2*C)*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(192*A*a^
3*b*cos(d*x + c) + 64*(2*(4*A + 5*C)*a^3*b + 5*(2*A + 3*C)*a*b^3)*cos(d*x +
 c)^5 + 40*A*a^4 + 15*((5*A + 6*C)*a^4 + 12*(3*A + 4*C)*a^2*b^2 + 8*A*b^4)*
cos(d*x + c)^4 + 64*((4*A + 5*C)*a^3*b + 5*A*a*b^3)*cos(d*x + c)^3 + 10*((5
*A + 6*C)*a^4 + 36*A*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)
^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.55314, size = 1485, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="
giac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 6*C*a^4 + 36*A*a^2*b^2 + 48*C*a^2*b^2 + 8*A*b^4 + 16*C
*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*A*a^4 + 6*C*a^4 + 36*A*a^2
*b^2 + 48*C*a^2*b^2 + 8*A*b^4 + 16*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + 2*(165*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 150*C*a^4*tan(1/2*d*x + 1/2*c)^1
1 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 - 960*C*a^3*b*tan(1/2*d*x + 1/2*c)^
11 + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 720*C*a^2*b^2*tan(1/2*d*x + 1/
2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 960*C*a*b^3*tan(1/2*d*x + 1
/2*c)^11 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*A*a^4*tan(1/2*d*x + 1/2*c
)^9 - 210*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^
9 + 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 2160*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*A*a*b^3*tan(1/2*d*x + 1
/2*c)^9 + 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*A*b^4*tan(1/2*d*x + 1/2
*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^4*tan(1/2*d*x + 1/2*c)^7
- 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^7
 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9600*C*a*b^3*tan(1/2*d*x + 1/2
*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^5
 + 60*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5
+ 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)
^5 + 9600*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^5
 + 25*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 224
0*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 12
```

```
60*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2160*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^
3 - 360*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*A*a^4*tan(1/2*d*x + 1/2*c) + 150
*C*a^4*tan(1/2*d*x + 1/2*c) + 960*A*a^3*b*tan(1/2*d*x + 1/2*c) + 960*C*a^3*
b*tan(1/2*d*x + 1/2*c) + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b^2
*tan(1/2*d*x + 1/2*c) + 960*A*a*b^3*tan(1/2*d*x + 1/2*c) + 960*C*a*b^3*tan(
1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^6)/d
```

3.558 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx$

Optimal. Leaf size=355

$$\frac{\left(84a^2b^2(4A+5C)+8a^4(6A+7C)+35b^4(2A+3C)\right)\tan(c+dx)}{105d}+\frac{ab\left(a^2(5A+6C)+2b^2(3A+4C)\right)\tanh^{-1}(\sin(c+d$$

```
[Out] (a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*ArcTanh[Sin[c + d*x]])/(4*d) + (
(35*b^4*(2*A + 3*C) + 84*a^2*b^2*(4*A + 5*C) + 8*a^4*(6*A + 7*C))*Tan[c + d
*x])/(105*d) + (a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*Sec[c + d*x]*Tan[
c + d*x])/(4*d) + ((4*A*b^4 + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A + 63*C))*
Sec[c + d*x]^2*Tan[c + d*x])/(105*d) + (a*b*(6*A*b^2 + a^2*(103*A + 126*C))
*Sec[c + d*x]^3*Tan[c + d*x])/(210*d) + ((2*A*b^2 + a^2*(6*A + 7*C))*(a + b
*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(35*d) + (2*A*b*(a + b*Cos[c
+ d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(21*d) + (A*(a + b*Cos[c + d*x])^4*S
ec[c + d*x]^6*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 1.23683, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3048, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(84a^2b^2(4A+5C)+8a^4(6A+7C)+35b^4(2A+3C)\right)\tan(c+dx)}{105d}+\frac{ab\left(a^2(5A+6C)+2b^2(3A+4C)\right)\tanh^{-1}(\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```


```
[Out] (a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*ArcTanh[Sin[c + d*x]])/(4*d) + (
(35*b^4*(2*A + 3*C) + 84*a^2*b^2*(4*A + 5*C) + 8*a^4*(6*A + 7*C))*Tan[c + d
*x])/(105*d) + (a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*Sec[c + d*x]*Tan[
c + d*x])/(4*d) + ((4*A*b^4 + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A + 63*C))*
Sec[c + d*x]^2*Tan[c + d*x])/(105*d) + (a*b*(6*A*b^2 + a^2*(103*A + 126*C))
*Sec[c + d*x]^3*Tan[c + d*x])/(210*d) + ((2*A*b^2 + a^2*(6*A + 7*C))*(a + b
*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(35*d) + (2*A*b*(a + b*Cos[c
+ d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(21*d) + (A*(a + b*Cos[c + d*x])^4*S
ec[c + d*x]^6*Tan[c + d*x])/(7*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
```

```
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^6(c+dx)\tan(c+dx)}{7d}+\frac{1}{7}\int (a$$
$$= \frac{2Ab(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{21d}+\frac{A(a}{}$$
$$= \frac{\left(2Ab^2+a^2(6A+7C)\right)(a+b\cos(c+dx))^2\sec^4(c+dx)\,t}{35d}$$
$$= \frac{ab\left(6Ab^2+a^2(103A+126C)\right)\sec^3(c+dx)\tan(c+dx)}{210d}$$
$$= \frac{\left(4Ab^4+4a^4(6A+7C)+3a^2b^2(50A+63C)\right)\sec^2(c+dx}{105d}$$
$$= \frac{\left(4Ab^4+4a^4(6A+7C)+3a^2b^2(50A+63C)\right)\sec^2(c+dx}{105d}$$
$$= \frac{ab\left(2b^2(3A+4C)+a^2(5A+6C)\right)\sec(c+dx)\tan(c+dx}{4d}$$
$$= \frac{ab\left(2b^2(3A+4C)+a^2(5A+6C)\right)\tanh^{-1}(\sin(c+dx))}{4d}+$$

Mathematica [A] time = 2.2006, size = 233, normalized size = 0.66

$$\frac{105ab\left(a^2(5A+6C)+2b^2(3A+4C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(84a^2\left(a^2(3A+C)+6Ab^2\right)\tan^4(c+dx)+14\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

```
[Out] (105*a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*ArcTanh[Sin[c + d*x]] + Tan[
c + d*x]*(420*(a^4 + 6*a^2*b^2 + b^4)*(A + C) + 105*a*b*(2*b^2*(3*A + 4*C)
+ a^2*(5*A + 6*C))*Sec[c + d*x] + 70*a*b*(6*A*b^2 + a^2*(5*A + 6*C))*Sec[c
+ d*x]^3 + 280*a^3*A*b*Sec[c + d*x]^5 + 140*(A*b^4 + 6*a^2*b^2*(2*A + C) +
a^4*(3*A + 2*C))*Tan[c + d*x]^2 + 84*a^2*(6*A*b^2 + a^2*(3*A + C))*Tan[c +
d*x]^4 + 60*a^4*A*Tan[c + d*x]^6))/(420*d)
```

Maple [A] time = 0.071, size = 591, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x)
```

```
[Out] 2/d*a^2*b^2*C*tan(d*x+c)*sec(d*x+c)^2+2/3/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^5
+1/d*a^3*b*C*tan(d*x+c)*sec(d*x+c)^3+1/d*a*A*b^3*tan(d*x+c)*sec(d*x+c)^3+2/
d*C*a*b^3*tan(d*x+c)*sec(d*x+c)+6/5/d*a^2*A*b^2*tan(d*x+c)*sec(d*x+c)^4+3/2
/d*a*A*b^3*sec(d*x+c)*tan(d*x+c)+8/5/d*a^2*A*b^2*tan(d*x+c)*sec(d*x+c)^2+5/
6/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^3+3/2/d*a^3*b*C*sec(d*x+c)*tan(d*x+c)+5/4
/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+1/d*C*b^4*tan(d*x+c)+2/3/d*A*b^4*tan(d*x+c
)+8/15/d*a^4*C*tan(d*x+c)+16/35/d*A*a^4*tan(d*x+c)+1/3/d*A*b^4*tan(d*x+c)*s
ec(d*x+c)^2+1/7/d*A*a^4*tan(d*x+c)*sec(d*x+c)^6+1/5/d*a^4*C*tan(d*x+c)*sec(
d*x+c)^4+2/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*b^2*C*tan(d*x+c)+3/2
/d*a*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+16/5/d*a^2*A*b^2*tan(d*x+c)+3/2/d*a^3*
b*C*ln(sec(d*x+c)+tan(d*x+c))+5/4/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/35/
d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+8/35/d
*A*a^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.04678, size = 637, normalized size = 1.79

$$\frac{24\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)Aa^{4}+56\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="
maxima")
```

```
[Out] 1/840*(24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*ta
n(d*x + c))*A*a^4 + 56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x +
 c))*C*a^4 + 336*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A
*a^2*b^2 + 1680*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b^2 + 280*(tan(d*x
+ c)^3 + 3*tan(d*x + c))*A*b^4 - 35*A*a^3*b*(2*(15*sin(d*x + c)^5 - 40*sin(
d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*
x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 210*
C*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 210*A*a*
b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)
^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*C*a*b^3*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)) + 840*C*b^4*tan(d*x + c))/d
```

Fricas [A] time = 1.66217, size = 783, normalized size = 2.21

$$\frac{105\left((5A+6C)a^3b+2(3A+4C)ab^3\right)\cos(dx+c)^7\log(\sin(dx+c)+1)-105\left((5A+6C)a^3b+2(3A+4C)ab^3\right)\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="
fricas")
```

```
[Out] 1/840*(105*((5*A + 6*C)*a^3*b + 2*(3*A + 4*C)*a*b^3)*cos(d*x + c)^7*log(sin
(d*x + c) + 1) - 105*((5*A + 6*C)*a^3*b + 2*(3*A + 4*C)*a*b^3)*cos(d*x + c)
^7*log(-sin(d*x + c) + 1) + 2*(4*(8*(6*A + 7*C)*a^4 + 84*(4*A + 5*C)*a^2*b^
2 + 35*(2*A + 3*C)*b^4)*cos(d*x + c)^6 + 280*A*a^3*b*cos(d*x + c) + 105*((5
*A + 6*C)*a^3*b + 2*(3*A + 4*C)*a*b^3)*cos(d*x + c)^5 + 60*A*a^4 + 4*(4*(6*
A + 7*C)*a^4 + 42*(4*A + 5*C)*a^2*b^2 + 35*A*b^4)*cos(d*x + c)^4 + 70*((5*A
 + 6*C)*a^3*b + 6*A*a*b^3)*cos(d*x + c)^3 + 12*((6*A + 7*C)*a^4 + 42*A*a^2*
b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46617, size = 1728, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="
giac")
```

```
[Out] 1/420*(105*(5*A*a^3*b + 6*C*a^3*b + 6*A*a*b^3 + 8*C*a*b^3)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - 105*(5*A*a^3*b + 6*C*a^3*b + 6*A*a*b^3 + 8*C*a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/2*d*x + 1/2*c)^13 + 4
20*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 1155*A*a^3*b*tan(1/2*d*x + 1/2*c)^13 - 1
050*C*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 2520*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^1
3 + 2520*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 - 1050*A*a*b^3*tan(1/2*d*x + 1/2
*c)^13 - 840*C*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 420*A*b^4*tan(1/2*d*x + 1/2*
c)^13 + 420*C*b^4*tan(1/2*d*x + 1/2*c)^13 - 840*A*a^4*tan(1/2*d*x + 1/2*c)^
11 - 1400*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 980*A*a^3*b*tan(1/2*d*x + 1/2*c)^
11 + 2520*C*a^3*b*tan(1/2*d*x + 1/2*c)^11 - 8400*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^11 - 11760*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 2520*A*a*b^3*tan(1/2*d*
x + 1/2*c)^11 + 3360*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 1960*A*b^4*tan(1/2*d
*x + 1/2*c)^11 - 2520*C*b^4*tan(1/2*d*x + 1/2*c)^11 + 3612*A*a^4*tan(1/2*d*
x + 1/2*c)^9 + 3164*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 2975*A*a^3*b*tan(1/2*d*x
 + 1/2*c)^9 - 1890*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 18984*A*a^2*b^2*tan(1/2
*d*x + 1/2*c)^9 + 24360*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 1890*A*a*b^3*tan
(1/2*d*x + 1/2*c)^9 - 4200*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 4060*A*b^4*tan(
1/2*d*x + 1/2*c)^9 + 6300*C*b^4*tan(1/2*d*x + 1/2*c)^9 - 2544*A*a^4*tan(1/2
*d*x + 1/2*c)^7 - 4368*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 26208*A*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 30240*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5040*A*b^4*tan
(1/2*d*x + 1/2*c)^7 - 8400*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 3612*A*a^4*tan(1/
2*d*x + 1/2*c)^5 + 3164*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 2975*A*a^3*b*tan(1/2
*d*x + 1/2*c)^5 + 1890*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18984*A*a^2*b^2*tan
(1/2*d*x + 1/2*c)^5 + 24360*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1890*A*a*b^3
*tan(1/2*d*x + 1/2*c)^5 + 4200*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 4060*A*b^4*
tan(1/2*d*x + 1/2*c)^5 + 6300*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 840*A*a^4*tan(
1/2*d*x + 1/2*c)^3 - 1400*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 980*A*a^3*b*tan(1/
2*d*x + 1/2*c)^3 - 2520*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 8400*A*a^2*b^2*tan
(1/2*d*x + 1/2*c)^3 - 11760*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2520*A*a*b^3
*tan(1/2*d*x + 1/2*c)^3 - 3360*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1960*A*b^4*
tan(1/2*d*x + 1/2*c)^3 - 2520*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 420*A*a^4*tan(
1/2*d*x + 1/2*c) + 420*C*a^4*tan(1/2*d*x + 1/2*c) + 1155*A*a^3*b*tan(1/2*d*
x + 1/2*c) + 1050*C*a^3*b*tan(1/2*d*x + 1/2*c) + 2520*A*a^2*b^2*tan(1/2*d*x
 + 1/2*c) + 2520*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 1050*A*a*b^3*tan(1/2*d*x
+ 1/2*c) + 840*C*a*b^3*tan(1/2*d*x + 1/2*c) + 420*A*b^4*tan(1/2*d*x + 1/2*c
) + 420*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.559 $\int (a+b\cos(c+dx))^3 \left(a^2-b^2\cos^2(c+dx)\right) dx$

Optimal. Leaf size=183

$$\frac{b\left(-32a^2b^2+83a^4-16b^4\right)\sin(c+dx)}{30d}+\frac{b\left(23a^2-16b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{60d}+\frac{ab^2\left(106a^2-71b^2\right)\sin(c+d}{120d}$$

```
[Out] (a*(8*a^4 + 8*a^2*b^2 - 9*b^4)*x)/8 + (b*(83*a^4 - 32*a^2*b^2 - 16*b^4)*Sin
[c + d*x])/(30*d) + (a*b^2*(106*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(1
20*d) + (b*(23*a^2 - 16*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) +
(a*b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*d) - (b*(a + b*Cos[c + d*x])^
4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.323211, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3016, 2753, 2734}

$$\frac{b\left(-32a^2b^2+83a^4-16b^4\right)\sin(c+dx)}{30d}+\frac{b\left(23a^2-16b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{60d}+\frac{ab^2\left(106a^2-71b^2\right)\sin(c+d}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(a^2 - b^2*Cos[c + d*x]^2),x]
```



```
[Out] (a*(8*a^4 + 8*a^2*b^2 - 9*b^4)*x)/8 + (b*(83*a^4 - 32*a^2*b^2 - 16*b^4)*Sin
[c + d*x])/(30*d) + (a*b^2*(106*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(1
20*d) + (b*(23*a^2 - 16*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) +
(a*b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*d) - (b*(a + b*Cos[c + d*x])^
4*Sin[c + d*x])/(5*d)
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(a^2-b^2\cos^2(c+dx)\right)dx = -\int (-a+b\cos(c+dx))(a+b\cos(c+dx))^4\,dx$$
$$= -\frac{b(a+b\cos(c+dx))^4\sin(c+dx)}{5d} - \frac{1}{5}\int (a+b\cos(c+dx))^3\left(-5a^2+\right.$$
$$= \frac{ab(a+b\cos(c+dx))^3\sin(c+dx)}{20d} - \frac{b(a+b\cos(c+dx))^4\sin(c+dx)}{5d}$$
$$= \frac{b\left(23a^2-16b^2\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60d} + \frac{ab(a+b\cos(c+dx)}{20d}$$
$$= \frac{1}{8}a\left(8a^4+8a^2b^2-9b^4\right)x + \frac{b\left(83a^4-32a^2b^2-16b^4\right)\sin(c+dx)}{30d} + \frac{ab}{}$$

Mathematica [A] time = 0.585719, size = 139, normalized size = 0.76

$$-\frac{-60a\left(8a^2b^2+8a^4-9b^4\right)(c+dx)+10b^3\left(8a^2+5b^2\right)\sin(3(c+dx))-120ab^2\left(2a^2-3b^2\right)\sin(2(c+dx))+60b\left(12a^2b^2\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(a^2 - b^2*Cos[c + d*x]^2),x]
```

```
[Out] -(-60*a*(8*a^4 + 8*a^2*b^2 - 9*b^4)*(c + d*x) + 60*b*(-24*a^4 + 12*a^2*b^2
+ 5*b^4)*Sin[c + d*x] - 120*a*b^2*(2*a^2 - 3*b^2)*Sin[2*(c + d*x)] + 10*b^3
*(8*a^2 + 5*b^2)*Sin[3*(c + d*x)] + 45*a*b^4*Sin[4*(c + d*x)] + 6*b^5*Sin[5
*(c + d*x)])/(480*d)
```

Maple [A] time = 0.026, size = 151, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{b^5\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)-3\,ab^4\left(1/4\,\left((\cos(dx+c))^3+3/2\,\cos(dx+c)\right)\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(a^2-b^2*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-3*a*b^4*(1/4*(
cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-2/3*a^2*b^3*(2+cos(d
*x+c)^2)*sin(d*x+c)+2*a^3*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a
^4*b*sin(d*x+c)+a^5*(d*x+c))
```

Maxima [A] time = 1.00753, size = 197, normalized size = 1.08

$$\frac{480\,(dx+c)a^5+240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3b^2+320\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^2b^3-45\,(12\,dx+12\,c+\mathrm{si}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(a^2-b^2*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(480*(d*x + c)*a^5 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3*b^2 + 3
20*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2*b^3 - 45*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a*b^4 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x +
c)^3 + 15*sin(d*x + c))*b^5 + 1440*a^4*b*sin(d*x + c))/d
```

Fricas [A] time = 1.49426, size = 308, normalized size = 1.68

$$\frac{15\left(8\,a^5+8\,a^3b^2-9\,ab^4\right)dx-\left(24\,b^5\cos(dx+c)^4+90\,ab^4\cos(dx+c)^3-360\,a^4b+160\,a^2b^3+64\,b^5+16\left(5\,a^2b^3+2\,b\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(a^2-b^2*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(8*a^5 + 8*a^3*b^2 - 9*a*b^4)*d*x - (24*b^5*cos(d*x + c)^4 + 90*a
*b^4*cos(d*x + c)^3 - 360*a^4*b + 160*a^2*b^3 + 64*b^5 + 16*(5*a^2*b^3 + 2*
```

```
b^5)*cos(d*x + c)^2 - 15*(8*a^3*b^2 - 9*a*b^4)*cos(d*x + c))*sin(d*x + c))/
d
```

Sympy [A] time = 2.75186, size = 321, normalized size = 1.75

$$\begin{cases} a^5x + \frac{3a^4b\sin(c+dx)}{d} + a^3b^2x\sin^2(c+dx) + a^3b^2x\cos^2(c+dx) + \frac{a^3b^2\sin(c+dx)\cos(c+dx)}{d} - \frac{4a^2b^3\sin^3(c+dx)}{3d} - \frac{2a^2b^3\sin(c+dx)\cos^2}{d} \\ x(a+b\cos(c))^3\left(a^2-b^2\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(a**2-b**2*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((a**5*x + 3*a**4*b*sin(c + d*x)/d + a**3*b**2*x*sin(c + d*x)**2 +
 a**3*b**2*x*cos(c + d*x)**2 + a**3*b**2*sin(c + d*x)*cos(c + d*x)/d - 4*a*
*2*b**3*sin(c + d*x)**3/(3*d) - 2*a**2*b**3*sin(c + d*x)*cos(c + d*x)**2/d
- 9*a*b**4*x*sin(c + d*x)**4/8 - 9*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 - 9*a*b**4*x*cos(c + d*x)**4/8 - 9*a*b**4*sin(c + d*x)**3*cos(c + d*x)/(
8*d) - 15*a*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 8*b**5*sin(c + d*x)**
5/(15*d) - 4*b**5*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - b**5*sin(c + d*x)
*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**3*(a**2 - b**2*cos(c)**2)
, True))
```

Giac [A] time = 1.42564, size = 198, normalized size = 1.08

$$-\frac{b^5\sin(5\,dx+5\,c)}{80\,d} - \frac{3\,ab^4\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(8\,a^5+8\,a^3b^2-9\,ab^4\right)x - \frac{\left(8\,a^2b^3+5\,b^5\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(2\,a^3b^2-3\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(a^2-b^2*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/80*b^5*sin(5*d*x + 5*c)/d - 3/32*a*b^4*sin(4*d*x + 4*c)/d + 1/8*(8*a^5 +
 8*a^3*b^2 - 9*a*b^4)*x - 1/48*(8*a^2*b^3 + 5*b^5)*sin(3*d*x + 3*c)/d + 1/4
*(2*a^3*b^2 - 3*a*b^4)*sin(2*d*x + 2*c)/d + 1/8*(24*a^4*b - 12*a^2*b^3 - 5*
b^5)*sin(d*x + c)/d
```

3.560 $\int (a+b\cos(c+dx))^2 \left(a^2-b^2\cos^2(c+dx)\right) dx$

Optimal. Leaf size=129

$$\frac{ab\left(13a^2-8b^2\right)\sin(c+dx)}{6d}+\frac{b^2\left(14a^2-9b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^4-3b^4\right)-\frac{b\sin(c+dx)(a+b\cos(c+dx}{4d}$$

```
[Out] ((8*a^4 - 3*b^4)*x)/8 + (a*b*(13*a^2 - 8*b^2)*Sin[c + d*x])/(6*d) + (b^2*(1
4*a^2 - 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (a*b*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(12*d) - (b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.201382, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3016, 2753, 2734}

$$\frac{ab\left(13a^2-8b^2\right)\sin(c+dx)}{6d}+\frac{b^2\left(14a^2-9b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^4-3b^4\right)-\frac{b\sin(c+dx)(a+b\cos(c+dx}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(a^2 - b^2*Cos[c + d*x]^2),x]
```


```
[Out] ((8*a^4 - 3*b^4)*x)/8 + (a*b*(13*a^2 - 8*b^2)*Sin[c + d*x])/(6*d) + (b^2*(1
4*a^2 - 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (a*b*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(12*d) - (b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(a^2-b^2\cos^2(c+dx)\right)\,dx = -\int(-a+b\cos(c+dx))(a+b\cos(c+dx))^3\,dx$$
$$= -\frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d} - \frac{1}{4}\int(a+b\cos(c+dx))^2\left(-4a^2+\right.$$
$$= \frac{ab(a+b\cos(c+dx))^2\sin(c+dx)}{12d} - \frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d}$$
$$= \frac{1}{8}\left(8a^4-3b^4\right)x + \frac{ab\left(13a^2-8b^2\right)\sin(c+dx)}{6d} + \frac{b^2\left(14a^2-9b^2\right)\cos(c}{24d}$$

Mathematica [A] time = 0.219931, size = 89, normalized size = 0.69

$$-\frac{-48ab\left(4a^2-3b^2\right)\sin(c+dx)-96a^4dx+16ab^3\sin(3(c+dx))+24b^4\sin(2(c+dx))+3b^4\sin(4(c+dx))+36b^4c+36}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(a^2 - b^2*Cos[c + d*x]^2),x]
```

```
[Out] -(36*b^4*c - 96*a^4*d*x + 36*b^4*d*x - 48*a*b*(4*a^2 - 3*b^2)*Sin[c + d*x]
+ 24*b^4*Sin[2*(c + d*x)] + 16*a*b^3*Sin[3*(c + d*x)] + 3*b^4*Sin[4*(c + d*
x)])/(96*d)
```

Maple [A] time = 0.021, size = 87, normalized size = 0.7

$$\frac{1}{d}\left(-b^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{2ab^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+2a^3b\,\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(a^2-b^2*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(-b^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-2/3*
a*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^3*b*sin(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 0.997582, size = 113, normalized size = 0.88

$$\frac{96\,(dx+c)a^4+64\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)ab^3-3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))b^4+192\,a^3}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(a^2-b^2*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*a^4 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3 - 3*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b^4 + 192*a^3*b*sin(d*
x + c))/d
```

Fricas [A] time = 1.40704, size = 188, normalized size = 1.46

$$\frac{3\left(8\,a^4-3\,b^4\right)dx-\left(6\,b^4\cos\left(dx+c\right)^3+16\,ab^3\cos\left(dx+c\right)^2+9\,b^4\cos\left(dx+c\right)-48\,a^3b+32\,ab^3\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(a^2-b^2*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^4 - 3*b^4)*d*x - (6*b^4*cos(d*x + c)^3 + 16*a*b^3*cos(d*x + c)
^2 + 9*b^4*cos(d*x + c) - 48*a^3*b + 32*a*b^3)*sin(d*x + c))/d
```

Sympy [A] time = 1.32987, size = 190, normalized size = 1.47

$$\begin{cases} a^4x+\frac{2a^3b\sin\left(c+dx\right)}{d}-\frac{4ab^3\sin^3\left(c+dx\right)}{3d}-\frac{2ab^3\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}-\frac{3b^4x\sin^4\left(c+dx\right)}{8}-\frac{3b^4x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}-\frac{3b^4x\cos^4\left(c+dx\right)}{8}-3b \\ x\left(a+b\cos\left(c\right)\right)^2\left(a^2-b^2\cos^2\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(a**2-b**2*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((a**4*x + 2*a**3*b*sin(c + d*x)/d - 4*a*b**3*sin(c + d*x)**3/(3*d
) - 2*a*b**3*sin(c + d*x)*cos(c + d*x)**2/d - 3*b**4*x*sin(c + d*x)**4/8 -
3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 - 3*b**4*x*cos(c + d*x)**4/8 - 3
*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 5*b**4*sin(c + d*x)*cos(c + d*x)
**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**2*(a**2 - b**2*cos(c)**2), True))
```

Giac [A] time = 1.53148, size = 123, normalized size = 0.95

$$-\frac{b^4 \sin(4\,dx + 4\,c)}{32\,d} - \frac{ab^3 \sin(3\,dx + 3\,c)}{6\,d} - \frac{b^4 \sin(2\,dx + 2\,c)}{4\,d} + \frac{1}{8}\left(8\,a^4 - 3\,b^4\right)x + \frac{\left(4\,a^3b - 3\,ab^3\right)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(a^2-b^2*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/32*b^4*sin(4*d*x + 4*c)/d - 1/6*a*b^3*sin(3*d*x + 3*c)/d - 1/4*b^4*sin(2
*d*x + 2*c)/d + 1/8*(8*a^4 - 3*b^4)*x + 1/2*(4*a^3*b - 3*a*b^3)*sin(d*x + c
)/d
```

3.561 $\int (a+b\cos(c+dx))\left(a^2-b^2\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=92

$$\frac{2b\left(2a^2-b^2\right)\sin(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2-b^2\right)+\frac{ab^2\sin(c+dx)\cos(c+dx)}{6d}-\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

```
[Out] (a*(2*a^2 - b^2)*x)/2 + (2*b*(2*a^2 - b^2)*Sin[c + d*x])/(3*d) + (a*b^2*Cos
[c + d*x]*Sin[c + d*x])/(6*d) - (b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*
d)
```

Rubi [A] time = 0.112424, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3016, 2753, 2734}

$$\frac{2b\left(2a^2-b^2\right)\sin(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2-b^2\right)+\frac{ab^2\sin(c+dx)\cos(c+dx)}{6d}-\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(a^2 - b^2*Cos[c + d*x]^2),x]
```

```
[Out] (a*(2*a^2 - b^2)*x)/2 + (2*b*(2*a^2 - b^2)*Sin[c + d*x])/(3*d) + (a*b^2*Cos
[c + d*x]*Sin[c + d*x])/(6*d) - (b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*
d)
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\left(a^2-b^2\cos^2(c+dx)\right)dx &= -\int (-a+b\cos(c+dx))(a+b\cos(c+dx))^2\,dx\\ &= -\frac{b(a+b\cos(c+dx))^2\sin(c+dx)}{3d}-\frac{1}{3}\int (a+b\cos(c+dx))\left(-3a^2+2\right.\\ &= \frac{1}{2}a\left(2a^2-b^2\right)x+\frac{2b\left(2a^2-b^2\right)\sin(c+dx)}{3d}+\frac{ab^2\cos(c+dx)\sin(c+d}{6d}\end{aligned}$$

Mathematica [A] time = 0.168031, size = 75, normalized size = 0.82

$$-\frac{\left(9b^3-12a^2b\right)\sin(c+dx)-12a^3dx+3ab^2\sin(2(c+dx))+6ab^2c+6ab^2dx+b^3\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(a^2 - b^2*Cos[c + d*x]^2),x]
```

```
[Out] -(6*a*b^2*c - 12*a^3*d*x + 6*a*b^2*d*x + (-12*a^2*b + 9*b^3)*Sin[c + d*x] +
 3*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.018, size = 75, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{b^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}-ab^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2b\sin(dx+c)+a^3(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(a^2-b^2*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/3*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)-a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+
1/2*d*x+1/2*c)+a^2*b*sin(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 0.994795, size = 99, normalized size = 1.08

$$\frac{12\,(dx+c)a^3-3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})ab^2+4\left(\sin{(dx+c)}^3-3\,\sin{(dx+c)}\right)b^3+12\,a^2b\sin{(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a^2-b^2*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*a^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^2 + 4*(sin(
d*x + c)^3 - 3*sin(d*x + c))*b^3 + 12*a^2*b*sin(d*x + c))/d
```

Fricas [A] time = 1.38727, size = 149, normalized size = 1.62

$$\frac{3\left(2\,a^3-ab^2\right)dx-\left(2\,b^3\cos{(dx+c)}^2+3\,ab^2\cos{(dx+c)}-6\,a^2b+4\,b^3\right)\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a^2-b^2*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^3 - a*b^2)*d*x - (2*b^3*cos(d*x + c)^2 + 3*a*b^2*cos(d*x + c) -
 6*a^2*b + 4*b^3)*sin(d*x + c))/d
```

Sympy [A] time = 0.962799, size = 131, normalized size = 1.42

$$\begin{cases} a^3x+\frac{a^2b\sin{(c+dx)}}{d}-\frac{ab^2x\sin^2{(c+dx)}}{2}-\frac{ab^2x\cos^2{(c+dx)}}{2}-\frac{ab^2\sin{(c+dx)}\cos{(c+dx)}}{2d}-\frac{2b^3\sin^3{(c+dx)}}{3d}-\frac{b^3\sin{(c+dx)}\cos^2{(c+dx)}}{d} & \text{for } d\neq \\ x\,(a+b\cos{(c)})\left(a^2-b^2\cos^2{(c)}\right) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a**2-b**2*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((a**3*x + a**2*b*sin(c + d*x)/d - a*b**2*x*sin(c + d*x)**2/2 - a*
b**2*x*cos(c + d*x)**2/2 - a*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*b**3*
sin(c + d*x)**3/(3*d) - b**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*
```

```
(a + b*cos(c))*(a**2 - b**2*cos(c)**2), True))
```

Giac [A] time = 1.38413, size = 100, normalized size = 1.09

$$-\frac{b^3\sin(3\,dx+3\,c)}{12\,d}-\frac{ab^2\sin(2\,dx+2\,c)}{4\,d}+\frac{1}{2}\left(2\,a^3-ab^2\right)x+\frac{\left(4\,a^2b-3\,b^3\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a^2-b^2*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/12*b^3*sin(3*d*x + 3*c)/d - 1/4*a*b^2*sin(2*d*x + 2*c)/d + 1/2*(2*a^3 -
a*b^2)*x + 1/4*(4*a^2*b - 3*b^3)*sin(d*x + c)/d
```

3.562 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=233

$$-\frac{a\left(3a^2C+3Ab^2+2b^2C\right)\sin(c+dx)}{3b^4d}-\frac{2a^3\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(4a^2C+b^2(4A+3C)\right)\sin(c+dx}{8b^3d}$$

```
[Out] ((8*a^4*C + 4*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*x)/(8*b^5) - (2*a^3*(A*b
^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*b^5*Sqrt[a + b]*d) - (a*(3*A*b^2 + 3*a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*b^
4*d) + ((4*a^2*C + b^2*(4*A + 3*C))*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) -
(a*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (C*Cos[c + d*x]^3*Sin[c + d*x
])/(4*b*d)
```

Rubi [A] time = 0.786423, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3050, 3049, 3023, 2735, 2659, 205}

$$-\frac{a\left(3a^2C+3Ab^2+2b^2C\right)\sin(c+dx)}{3b^4d}-\frac{2a^3\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(4a^2C+b^2(4A+3C)\right)\sin(c+dx}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*C + 4*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*x)/(8*b^5) - (2*a^3*(A*b
^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*b^5*Sqrt[a + b]*d) - (a*(3*A*b^2 + 3*a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*b^
4*d) + ((4*a^2*C + b^2*(4*A + 3*C))*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) -
(a*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (C*Cos[c + d*x]^3*Sin[c + d*x
])/(4*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
```

```
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{C\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos^2(c+dx)\left(3aC+b(4A+3C)\cos(c+dx)-4aC\cos^2(c+dx)\right)}{a+b\cos(c+dx)}}{4b}$$

$$= -\frac{aC\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{C\cos^3(c+dx)\sin(c+dx)}{4bd} + \int \frac{\cos(c+dx)\left(-8a\right.}{}$$

$$= \frac{\left(4a^2C+b^2(4A+3C)\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{aC\cos^2(c+dx)\sin(c+dx}{3b^2d}$$

$$= -\frac{a\left(3Ab^2+3a^2C+2b^2C\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2C+b^2(4A+3C)\right)\cos(c+dx)}{8b^3d}$$

$$= \frac{\left(8a^4C+4a^2b^2(2A+C)+b^4(4A+3C)\right)x}{8b^5} - \frac{a\left(3Ab^2+3a^2C+2b^2C\right)\sin(c}{3b^4d}$$

$$= \frac{\left(8a^4C+4a^2b^2(2A+C)+b^4(4A+3C)\right)x}{8b^5} - \frac{a\left(3Ab^2+3a^2C+2b^2C\right)\sin(c}{3b^4d}$$

$$= \frac{\left(8a^4C+4a^2b^2(2A+C)+b^4(4A+3C)\right)x}{8b^5} - \frac{2a^3\left(Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{ta}}{}\right.}{\sqrt{a-b}\,b^5\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.636615, size = 194, normalized size = 0.83

$$\frac{12(c+dx)\left(4a^2b^2(2A+C)+8a^4C+b^4(4A+3C)\right)+24b^2\left(C\left(a^2+b^2\right)+Ab^2\right)\sin(2(c+dx))-24ab\left(4a^2C+4Ab^2+3\right.}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a^4*C + 4*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*(c + d*x) + (192*a^3*
(A*b^2 + a^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2] - 24*a*b*(4*A*b^2 + 4*a^2*C + 3*b^2*C)*Sin[c + d*x] + 24*b^2*(A
*b^2 + (a^2 + b^2)*C)*Sin[2*(c + d*x)] - 8*a*b^3*C*Sin[3*(c + d*x)] + 3*b^4
*C*Sin[4*(c + d*x)])/(96*b^5*d)
```

Maple [B] time = 0.062, size = 1060, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -10/3/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*C*a-10/3/d/b^2/
(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*C*a-6/d/b^2/(tan(1/2*d*x+1/
2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a*A-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan
(1/2*d*x+1/2*c)^5*a^3*C-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*
c)^7*a^2*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*C*a-6/d/
b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*a^3*C-2/d/b^2/(tan(1/2*
d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*C*a-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d/b^2/(tan(1/2*d*x+1
/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*a*A-2/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*ta
n(1/2*d*x+1/2*c)^7*a^3*C+1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2
*c)*a^2*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*a*A-1/d/b^3
/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a^2*C+1/d/b^3/(tan(1/2*d*x
+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*a^2*C-2/d*a^5/b^5/((a+b)*(a-b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d/b^4/(tan(1/2*d*x+
1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*a^3*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*t
an(1/2*d*x+1/2*c)^7*a*A+1/d/b*arctan(tan(1/2*d*x+1/2*c))*A+3/4/d/b*arctan(t
an(1/2*d*x+1/2*c))*C-1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*
A+1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*A-3/4/d/b/(tan(1/2*
d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*C+1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*t
an(1/2*d*x+1/2*c)*A+5/4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*C
-1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*A-5/4/d/b/(tan(1/2*d
*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*C+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*
a^2*C+2/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4*C+3/4/d/b/(tan(1/2*d*x+1/2*c)^
2+1)^4*tan(1/2*d*x+1/2*c)^5*C+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.77419, size = 1312, normalized size = 5.63

$$\left[\frac{3\left(8\,Ca^6+4\,(2\,A-C)a^4b^2-(4\,A+C)a^2b^4-(4\,A+3\,C)b^6\right)dx-12\left(Ca^5+Aa^3b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [1/24*(3*(8*C*a^6 + 4*(2*A - C)*a^4*b^2 - (4*A + C)*a^2*b^4 - (4*A + 3*C)*b
^6)*d*x - 12*(C*a^5 + A*a^3*b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
 (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin
(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) -
 (24*C*a^5*b + 8*(3*A - C)*a^3*b^3 - 8*(3*A + 2*C)*a*b^5 - 6*(C*a^2*b^4 - C
*b^6)*cos(d*x + c)^3 + 8*(C*a^3*b^3 - C*a*b^5)*cos(d*x + c)^2 - 3*(4*C*a^4*
b^2 + (4*A - C)*a^2*b^4 - (4*A + 3*C)*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^5 - b^7)*d), 1/24*(3*(8*C*a^6 + 4*(2*A - C)*a^4*b^2 - (4*A + C)*a^2*b^4
 - (4*A + 3*C)*b^6)*d*x - 24*(C*a^5 + A*a^3*b^2)*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (24*C*a^5*b + 8*(3*A -
 C)*a^3*b^3 - 8*(3*A + 2*C)*a*b^5 - 6*(C*a^2*b^4 - C*b^6)*cos(d*x + c)^3 +
8*(C*a^3*b^3 - C*a*b^5)*cos(d*x + c)^2 - 3*(4*C*a^4*b^2 + (4*A - C)*a^2*b^4
 - (4*A + 3*C)*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33939, size = 775, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(8*C*a^4 + 8*A*a^2*b^2 + 4*C*a^2*b^2 + 4*A*b^4 + 3*C*b^4)*(d*x + c)
/b^5 + 48*(C*a^5 + A*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*b^5) - 2*(24*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*C*a
^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*C*a*b^
2*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 15*C*b^3*tan(1
/2*d*x + 1/2*c)^7 + 72*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*b*tan(1/2*d*
x + 1/2*c)^5 + 72*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 40*C*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 12*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*C*b^3*tan(1/2*d*x + 1/2*c)^
5 + 72*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 7
2*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 40*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 12*A
*b^3*tan(1/2*d*x + 1/2*c)^3 + 9*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 24*C*a^3*tan
(1/2*d*x + 1/2*c) - 12*C*a^2*b*tan(1/2*d*x + 1/2*c) + 24*A*a*b^2*tan(1/2*d*
x + 1/2*c) + 24*C*a*b^2*tan(1/2*d*x + 1/2*c) - 12*A*b^3*tan(1/2*d*x + 1/2*c
) - 15*C*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

3.563 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{\left(3a^2C+b^2(3A+2C)\right)\sin(c+dx)}{3b^3d}+\frac{2a^2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{ax\left(C\left(2a^2+b^2\right)+2Ab^2\right)}{2b^4}-\frac{aC\sin(c}{}$$

```
[Out] -(a*(2*A*b^2 + (2*a^2 + b^2)*C)*x)/(2*b^4) + (2*a^2*(A*b^2 + a^2*C)*ArcTan[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d
) + ((3*a^2*C + b^2*(3*A + 2*C))*Sin[c + d*x])/(3*b^3*d) - (a*C*Cos[c + d*x
]*Sin[c + d*x])/(2*b^2*d) + (C*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.47393, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3050, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(3a^2C+b^2(3A+2C)\right)\sin(c+dx)}{3b^3d}+\frac{2a^2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{ax\left(C\left(2a^2+b^2\right)+2Ab^2\right)}{2b^4}-\frac{aC\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```



```
[Out] -(a*(2*A*b^2 + (2*a^2 + b^2)*C)*x)/(2*b^4) + (2*a^2*(A*b^2 + a^2*C)*ArcTan[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d
) + ((3*a^2*C + b^2*(3*A + 2*C))*Sin[c + d*x])/(3*b^3*d) - (a*C*Cos[c + d*x
]*Sin[c + d*x])/(2*b^2*d) + (C*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{C\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2aC+b(3A+2C)\cos(c+dx)-3aC\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,d}{3b}$$

$$= -\frac{aC\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{C\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{-3a^2C+abC\cos(}{}}{}$$

$$= \frac{\left(3a^2C+b^2(3A+2C)\right)\sin(c+dx)}{3b^3d} - \frac{aC\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{C\cos^2(c}{}$$

$$= -\frac{a\left(2Ab^2+\left(2a^2+b^2\right)C\right)x}{2b^4} + \frac{\left(3a^2C+b^2(3A+2C)\right)\sin(c+dx)}{3b^3d} - \frac{aC\cos(}{}$$

$$= -\frac{a\left(2Ab^2+\left(2a^2+b^2\right)C\right)x}{2b^4} + \frac{\left(3a^2C+b^2(3A+2C)\right)\sin(c+dx)}{3b^3d} - \frac{aC\cos(}{}$$

$$= -\frac{a\left(2Ab^2+\left(2a^2+b^2\right)C\right)x}{2b^4} + \frac{2a^2\left(Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d} + \cdots$$

Mathematica [A] time = 0.435716, size = 152, normalized size = 0.86

$$\frac{-6a(c+dx)\left(C\left(2a^2+b^2\right)+2Ab^2\right)+3b\left(4a^2C+4Ab^2+3b^2C\right)\sin(c+dx)-\frac{24a^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-3ab^2(}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-6*a*(2*A*b^2 + (2*a^2 + b^2)*C)*(c + d*x) - (24*a^2*(A*b^2 + a^2*C)*ArcTa
nh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(4*
A*b^2 + 4*a^2*C + 3*b^2*C)*Sin[c + d*x] - 3*a*b^2*C*Sin[2*(c + d*x)] + b^3*
C*Sin[3*(c + d*x)])/(12*b^4*d)
```

Maple [B] time = 0.033, size = 551, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/b^3/(tan(1/2*d*
x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a^2*C+1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)
^3*tan(1/2*d*x+1/2*c)^5*C*a+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/
2*c)^5*C+4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*A+4/d/b^3/(t
an(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a^2*C+4/3/d/b/(tan(1/2*d*x+1/
2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2
*d*x+1/2*c)*A+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a^2*C+2
/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*C-1/d/b^2/(tan(1/2*d*x+1
/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*C*a-2/d/b^2*A*arctan(tan(1/2*d*x+1/2*c))*a-
2/d/b^4*C*arctan(tan(1/2*d*x+1/2*c))*a^3-1/d/b^2*C*arctan(tan(1/2*d*x+1/2*c
))*a+2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*A+2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68917, size = 1050, normalized size = 5.93

$$\left[-\frac{3\left(2\,Ca^5+(2\,A-C)a^3b^2-(2\,A+C)ab^4\right)dx+3\left(Ca^4+Aa^2b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(\ldots}{b^2\cos(dx+c)^2+2\,ab\cos(d\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [-1/6*(3*(2*C*a^5 + (2*A - C)*a^3*b^2 - (2*A + C)*a*b^4)*d*x + 3*(C*a^4 + A
*a^2*b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*C*a^4*b + 2*(3*A -
C)*a^2*b^3 - 2*(3*A + 2*C)*b^5 + 2*(C*a^2*b^3 - C*b^5)*cos(d*x + c)^2 - 3*(
C*a^3*b^2 - C*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d), -1/6*
(3*(2*C*a^5 + (2*A - C)*a^3*b^2 - (2*A + C)*a*b^4)*d*x - 6*(C*a^4 + A*a^2*b
^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x +
 c))) - (6*C*a^4*b + 2*(3*A - C)*a^2*b^3 - 2*(3*A + 2*C)*b^5 + 2*(C*a^2*b^3
 - C*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 - C*a*b^4)*cos(d*x + c))*sin(d*x +
c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.279, size = 440, normalized size = 2.49

$$-\frac{\frac{3\left(2\,Ca^3+2\,Aab^2+Cab^2\right)(dx+c)}{b^4}+\frac{12\left(Ca^4+Aa^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}-\frac{2\left(6\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(2*C*a^3 + 2*A*a*b^2 + C*a*b^2)*(d*x + c)/b^4 + 12*(C*a^4 + A*a^2*b
^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^4) - 2*(6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 +
6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*
```

```
tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^2*tan(1/2*
d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) - 3*C*a*b*tan(1/2*d*x + 1/2*c
) + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*
d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.564 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{2a\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2C+b^2(2A+C)\right)}{2b^3}-\frac{aC\sin(c+dx)}{b^2d}+\frac{C\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] ((2*a^2*C + b^2*(2*A + C))*x)/(2*b^3) - (2*a*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*
C*Sin[c + d*x])/(b^2*d) + (C*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.258365, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3050, 3023, 2735, 2659, 205}

$$-\frac{2a\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(\frac{2a^2C}{b^2}+2A+C\right)}{2b}-\frac{aC\sin(c+dx)}{b^2d}+\frac{C\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*A + C + (2*a^2*C)/b^2)*x)/(2*b) - (2*a*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*C*
Sin[c + d*x])/(b^2*d) + (C*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
```

```
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{C\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{aC+b(2A+C)\cos(c+dx)-2aC\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{aC\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{abC+\left(2a^2C+b^2(2A+C)\right)\cos(c+dx)}{a+b\cos(c+dx)}}{2b^2}$$

$$= \frac{\left(2a^2C+b^2(2A+C)\right)x}{2b^3} - \frac{aC\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(a\,(}{}$$

$$= \frac{\left(2a^2C+b^2(2A+C)\right)x}{2b^3} - \frac{aC\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a\,(}{}$$

$$= \frac{\left(2a^2C+b^2(2A+C)\right)x}{2b^3} - \frac{2a\left(Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{aC\sin(}{b^2}$$

Mathematica [A] time = 0.403958, size = 117, normalized size = 0.91

$$\frac{2(c+dx)\left(C\left(2a^2+b^2\right)+2Ab^2\right)+\frac{8a\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-4abC\sin(c+dx)+b^2C\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(2*A*b^2 + (2*a^2 + b^2)*C)*(c + d*x) + (8*a*(A*b^2 + a^2*C)*ArcTanh[((a
 - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*C*Sin[c
 + d*x] + b^2*C*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [B] time = 0.033, size = 296, normalized size = 2.3

$$-2\,\frac{(\tan(1/2\,dx+c/2))^3\,aC}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)^2} - \frac{C}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-2} - 2\,\frac{\tan(1/2\,dx+c/2)\,aC}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-1/d/b/(tan(1/2
*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^
2*tan(1/2*d*x+1/2*c)*a*C+1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c
)*C+2/d/b*arctan(tan(1/2*d*x+1/2*c))*A+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a
^2*C+1/d/b*arctan(tan(1/2*d*x+1/2*c))*C-2/d*a/b/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^3/b^3/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67143, size = 832, normalized size = 6.5

$$\left[\frac{\left(2\,Ca^4 + (2\,A - C)a^2b^2 - (2\,A + C)b^4\right)dx - \left(Ca^3 + Aab^2\right)\sqrt{-a^2 + b^2}\log\left(\frac{2\,ab\cos(dx+c) + \left(2\,a^2 - b^2\right)\cos(dx+c)^2 - 2\,\sqrt{-a^2+b^2}(a\cos(dx+c}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}\right.}{2\left(a^2b^3 - b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((2*C*a^4 + (2*A - C)*a^2*b^2 - (2*A + C)*b^4)*d*x - (C*a^3 + A*a*b^2)
*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 -
2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*C*a^3*b - 2*C*a*b^3 - (C*a^2
*b^2 - C*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d), 1/2*((2*C*a^
4 + (2*A - C)*a^2*b^2 - (2*A + C)*b^4)*d*x - 2*(C*a^3 + A*a*b^2)*sqrt(a^2 -
 b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*C*a
^3*b - 2*C*a*b^3 - (C*a^2*b^2 - C*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^
```

```
3 - b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22571, size = 269, normalized size = 2.1

$$\frac{\frac{\left(2\,Ca^2+2\,Ab^2+Cb^2\right)(dx+c)}{b^3}+\frac{4\left(Ca^3+Aab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^3}-\frac{2\left(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Cb\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*C*a^2 + 2*A*b^2 + C*b^2)*(d*x + c)/b^3 + 4*(C*a^3 + A*a*b^2)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2*(2
*C*a*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*tan(1/2*d*
x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2)
)/d
```

3.565 $\int \frac{A+C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aCx}{b^2}+\frac{C\sin(c+dx)}{bd}$$

```
[Out] -((a*C*x)/b^2) + (2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.125738, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3024, 2735, 2659, 205}

$$\frac{2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aCx}{b^2}+\frac{C\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((a*C*x)/b^2) + (2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*Sin[c + d*x])/(b*d)
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{C\sin(c+dx)}{bd}+\frac{\int \frac{Ab-aC\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= -\frac{aCx}{b^2}+\frac{C\sin(c+dx)}{bd}-\frac{\left(-Ab^2-a^2C\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\
&= -\frac{aCx}{b^2}+\frac{C\sin(c+dx)}{bd}+\frac{\left(2\left(Ab^2+a^2C\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{aCx}{b^2}+\frac{2\left(Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d}+\frac{C\sin(c+dx)}{bd}
\end{aligned}
$$

Mathematica [A] time = 0.210301, size = 82, normalized size = 0.95

$$
\frac{-\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-aC(c+dx)+bC\sin(c+dx)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-(a*C*(c + d*x)) - (2*(A*b^2 + a^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*C*Sin[c + d*x])/(b^2*d)
```

Maple [A] time = 0.028, size = 149, normalized size = 1.7

$$2\,\frac{C\tan\left(1/2\,dx+c/2\right)}{db\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}+1\right)}-2\,\frac{C\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)a}{db^{2}}+2\,\frac{A}{d\sqrt{\left(a+b\right)\left(a-b\right)}}\arctan\left(\frac{\left(a-b\right)\tan\left(1/2\,dx+c/2\right)}{\sqrt{\left(a+b\right)\left(a-b\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*C/b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)-2/d/b^2*C*arctan(tan(1/
2*d*x+1/2*c))*a+2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*A+2/d/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56617, size = 640, normalized size = 7.44

$$\left[-\frac{2\left(Ca^{3}-Cab^{2}\right)dx+\left(Ca^{2}+Ab^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^{2}-b^{2}\right)\cos(dx+c)^{2}+2\,\sqrt{-a^{2}+b^{2}}(a\cos(dx+c)+b)\sin(dx+c)-a^{2}+2\,b^{2}}{b^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+a^{2}}\right)-2}{2\left(a^{2}b^{2}-b^{4}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(C*a^3 - C*a*b^2)*d*x + (C*a^2 + A*b^2)*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(C*a^2*b - C*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), -((
```

```
C*a^3 - C*a*b^2)*d*x - (C*a^2 + A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
 c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (C*a^2*b - C*b^3)*sin(d*x + c))/
((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23958, size = 184, normalized size = 2.14

$$-\frac{\frac{(dx+c)Ca}{b^2}-\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}+\frac{2\left(Ca^2+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*C*a/b^2 - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*b) + 2*(C*a^2 + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*b^2))/d
```

3.566 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{b}$$

```
[Out] (C*x)/b - (2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rubi [A] time = 0.130941, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3058, 2659, 205, 3770}

$$-\frac{2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (C*x)/b - (2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 3058

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(C*x
)/(b*d), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{Cx}{b}+\frac{A\int \sec(c+dx)\,dx}{a}-\left(\frac{Ab}{a}+\frac{aC}{b}\right)\int \frac{1}{a+b\cos(c+dx)}\,dx$$

$$= \frac{Cx}{b}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\left(2\left(\frac{Ab}{a}+\frac{aC}{b}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\right.}{d}$$

$$= \frac{Cx}{b}-\frac{2\left(\frac{Ab}{a}+\frac{aC}{b}\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [C] time = 0.552859, size = 234, normalized size = 2.66

$$\frac{2\left(A+C\cos^2(c+dx)\right)\left(\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}\left(aCdx-Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+Ab\log\left(\right.\right.\right.}{abd\sqrt{\left(b^2-a^2\right)(\cos(2c)-i\sin(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(A + C*Cos[c + d*x]^2)*((a*C*d*x - A*b*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sqrt[-((a^2 - b^2)*
(Cos[c] - I*Sin[c])^2)] + 2*(A*b^2 + a^2*C)*ArcTan[((I*Cos[c] + Sin[c])*(b*
Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[
c])^2)]]*(I*Cos[c] + Sin[c])))/(a*b*d*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[(
-a^2 + b^2)*(Cos[2*c] - I*Sin[2*c])])
```

Maple [A] time = 0.057, size = 158, normalized size = 1.8

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{db}-2\,\frac{Ab}{da\sqrt{(a+b)\,(a-b)}}\arctan\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)\,(a-b)}}\right)-2\,\frac{aC}{db\sqrt{(a+b)\,(a-b)}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))*C-2/d/a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a/b/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/a/d*A*ln(tan(1/2*d*x+1
/2*c)-1)+1/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.24659, size = 786, normalized size = 8.93

$$\left[\frac{2\left(Ca^3-Cab^2\right)dx-\left(Ca^2+Ab^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(A\right.}{2\left(a^3b-ab^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(C*a^3 - C*a*b^2)*d*x - (C*a^2 + A*b^2)*sqrt(-a^2 + b^2)*log((2*a*b
*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*
x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
 + c) + a^2)) + (A*a^2*b - A*b^3)*log(sin(d*x + c) + 1) - (A*a^2*b - A*b^3)
*log(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d), 1/2*(2*(C*a^3 - C*a*b^2)*d*x
- 2*(C*a^2 + A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) + (A*a^2*b - A*b^3)*log(sin(d*x + c) + 1) - (A*a^2*b
- A*b^3)*log(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{a + b\cos\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.36475, size = 193, normalized size = 2.19

$$\frac{\frac{(dx+c)C}{b} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{2\left(Ca^2+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((d*x + c)*C/b + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a - 2*(C*a^2 + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b))/d
```

3.567 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{Ab\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

```
[Out] (2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*Sqrt[a - b]*Sqrt[a + b]*d) - (A*b*ArcTanh[Sin[c + d*x]])/(a^2*d) + (A*Tan[
c + d*x])/(a*d)
```

Rubi [A] time = 0.22504, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3056, 3001, 3770, 2659, 205}

$$\frac{2\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{Ab\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*Sqrt[a - b]*Sqrt[a + b]*d) - (A*b*ArcTanh[Sin[c + d*x]])/(a^2*d) + (A*Tan[
c + d*x])/(a*d)
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
```

```
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\tan(c+dx)}{ad}+\frac{\int \frac{(-Ab+aC\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{ad}-\frac{(Ab)\int \sec(c+dx)\,dx}{a^2}+\left(\frac{Ab^2}{a^2}+C\right)\int \frac{1}{a+b\cos(c+dx)}\,d$$

$$= -\frac{Ab\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}+\frac{\left(2\left(\frac{Ab^2}{a^2}+C\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(}\right.}{d}$$

$$= \frac{2\left(\frac{Ab^2}{a^2}+C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}-\frac{Ab\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c}{ad}$$

Mathematica [C] time = 2.18278, size = 306, normalized size = 3.22

$$\frac{2\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(-\frac{2i(\cos(c)-i\sin(c))\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(b\cos(c)-a)+b\sin(c)\right)}{\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\right)}{\sqrt{\left(b^2-a^2\right)(\cos(c)-i\sin(c))^2}}+\frac{aA\,\mathrm{si}}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right.\right.}\right.}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(A*b*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((2*I)*(A*b^2
+ a^2*C)*ArcTan[((I*Cos[c] + Sin[c])*(b*Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/
2]))/Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]]*(Cos[c] - I*Sin[c]))/Sqrt[
(-a^2 + b^2)*(Cos[c] - I*Sin[c])^2] + (a*A*Sin[(d*x)/2])/((Cos[c/2] - Sin[c
/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (a*A*Sin[(d*x)/2])/((Cos[c/2]
 + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a^2*d*(2*A + C + C*C
os[2*(c + d*x)]))
```

Maple [B] time = 0.064, size = 183, normalized size = 1.9

$$2\,\frac{Ab^2}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{C}{d\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{A}{ad}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*A*b^2+2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*C-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/d*A*b/a^2*ln(tan(1/2*d*x+1
/2*c)-1)-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1/d*A*b/a^2*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.59067, size = 952, normalized size = 10.02

$$\left[-\frac{\left(Ca^2+Ab^2\right)\sqrt{-a^2+b^2}\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Aa^2b\cdots\right.}{2\left(a^4-a^2b\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [-1/2*((C*a^2 + A*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c
) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*
sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)
) + (A*a^2*b - A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (A*a^2*b - A*b^3
)*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/(
(a^4 - a^2*b^2)*d*cos(d*x + c)), 1/2*(2*(C*a^2 + A*b^2)*sqrt(a^2 - b^2)*arc
tan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (A
*a^2*b - A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (A*a^2*b - A*b^3)*cos(
d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 -
 a^2*b^2)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{a + b\cos\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.4468, size = 221, normalized size = 2.33

$$-\frac{\frac{Ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{Ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}+\frac{2\left(Ca^2+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\sqrt{a^2\cdots}}\right.\right.}{\sqrt{a^2-b^2}a^2}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -(A*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - A*b*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a^2 + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a)
+ 2*(C*a^2 + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arc
tan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(s
qrt(a^2 - b^2)*a^2))/d
```

3.568 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{2b\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{Ab\tan(c+dx)}{a^2d}+\frac{A\tan(c+$$

```
[Out] (-2*b*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^3*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - (A*b*Tan[c + d*x])/(a^2*d) + (A*Sec[c + d*x]*Tan[c + d*
x])/(2*a*d)
```

Rubi [A] time = 0.473737, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2(A+2C)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{Ab\tan(c+dx)}{a^2d}+\frac{A\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```


```
[Out] (-2*b*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^3*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - (A*b*Tan[c + d*x])/(a^2*d) + (A*Sec[c + d*x]*Tan[c + d*
x])/(2*a*d)
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2Ab+a(A+2C)\cos(c+dx)+Ab\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,d}{2a}$$

$$= -\frac{Ab\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(2Ab^2+a^2(A+2C)+aAb\cos(c+d\right.}{a+b\cos(c+dx)}}{2a^2}$$

$$= -\frac{Ab\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(b\left(Ab^2+a^2C\right)\right)\int \frac{1}{a+b\cos(c}}{a^3}$$

$$= \frac{\left(2Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{Ab\tan(c+dx)}{a^2d} + \frac{A\sec(c+d}{2}$$

$$= -\frac{2b\left(Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(2Ab^2+a^2(A+2C)\right)\tanh^{-1}(s}{2a^3d}$$

Mathematica [C] time = 2.03703, size = 399, normalized size = 2.91

$$\frac{\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(-2\left(a^2(A+2C)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(a^2(A+2C)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(-2*(2*A*b^2 + a^2*(A + 2*C))*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 + a^2*(A + 2*C))*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (8*b*(A*b^2 + a^2*C)*ArcTan[((I*Cos[c] + S
in[c])*(b*Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos[c
] - I*Sin[c])^2)]]*(I*Cos[c] + Sin[c]))/Sqrt[(-a^2 + b^2)*(Cos[c] - I*Sin[c
])^2] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - (4*a*A*b*Sin[(d*x
)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a^2*
A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (4*a*A*b*Sin[(d*x)/2])/((Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(2*a^3*d*(2*A + C
+ C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.068, size = 362, normalized size = 2.6

$$-2\,{\frac {A{b}^{3}}{d{a}^{3}\sqrt {(a+b)(a-b)}}\arctan \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}} \right) }-2\,{\frac {bC}{da\sqrt {(a+b)(a-b)}}\arctan \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}} \right) }+ \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A-2/d*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*C+1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2-1/2/a/d*A*ln(tan(1/
2*d*x+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2-1/a/d*ln(tan(1/2*d*x
+1/2*c)-1)*C+1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/d*A/a^2/(tan(1/2*d*x+1/2*c)
-1)*b-1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)
+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C+1/
2/a/d*A/(tan(1/2*d*x+1/2*c)+1)+1/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.6404, size = 1239, normalized size = 9.04

$$\left[-\frac{2\left(Ca^{2}b+Ab^{3}\right)\sqrt{-a^{2}+b^{2}}\cos\left(dx+c\right)^{2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^{2}-b^{2}\right)\cos(dx+c)^{2}-2\,\sqrt{-a^{2}+b^{2}}(a\cos(dx+c)+b)\sin(dx+c)-a^{2}+2\,b^{2}}{b^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+a^{2}}\right)-\left((A+\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [-1/4*(2*(C*a^2*b + A*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)) - ((A + 2*C)*a^4 + (A - 2*C)*a^2*b^2 - 2*A*b^4)*cos(d*x + c)^2*log(
sin(d*x + c) + 1) + ((A + 2*C)*a^4 + (A - 2*C)*a^2*b^2 - 2*A*b^4)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) - 2*(A*a^4 - A*a^2*b^2 - 2*(A*a^3*b - A*a*b^3)
*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2), -1/4*(4*(C
*a^2*b + A*b^3)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^
2)*sin(d*x + c)))*cos(d*x + c)^2 - ((A + 2*C)*a^4 + (A - 2*C)*a^2*b^2 - 2*A
*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + ((A + 2*C)*a^4 + (A - 2*C)*a^2
*b^2 - 2*A*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(A*a^4 - A*a^2*b^
2 - 2*(A*a^3*b - A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*co
s(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7681, size = 327, normalized size = 2.39

$$\frac{\frac{(Aa^2+2Ca^2+2Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{(Aa^2+2Ca^2+2Ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{4(Ca^2b+Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan(\cdots}{\cdots}\right)\right)}{\sqrt{a^2-b^2}a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((A*a^2 + 2*C*a^2 + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (
A*a^2 + 2*C*a^2 + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 4*(C*a^
2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan(1/2*d*x + 1/2*c)^3
 + A*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 - 1)^2*a^2))/d
```

3.569 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{2b^2(a^2C+Ab^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{(a^2(2A+3C)+3Ab^2)\tan(c+dx)}{3a^3d}-\frac{b(a^2(A+2C)+2Ab^2)\tanh^{-1}(\sin(c}{2a^4d}$$

```
[Out] (2*b^2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^4*Sqrt[a - b]*Sqrt[a + b]*d) - (b*(2*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[
c + d*x]])/(2*a^4*d) + ((3*A*b^2 + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*a^3*d)
 - (A*b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c + d*
x])/(3*a*d)
```

Rubi [A] time = 0.740514, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^2(a^2C+Ab^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{(a^2(2A+3C)+3Ab^2)\tan(c+dx)}{3a^3d}-\frac{b(a^2(A+2C)+2Ab^2)\tanh^{-1}(\sin(c}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^2*(A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^4*Sqrt[a - b]*Sqrt[a + b]*d) - (b*(2*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[
c + d*x]])/(2*a^4*d) + ((3*A*b^2 + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*a^3*d)
 - (A*b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c + d*
x])/(3*a*d)
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3Ab+a(2A+3C)\cos(c+dx)+2Ab\cos^2(c+dx)\right)\sec^3(c+d}{a+b\cos(c+dx)}}{3a}$$

$$= -\frac{Ab\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \int \frac{2\left(3Ab^2+\frac{1}{2}a^2(\right.}{}$$

$$= \frac{\left(3Ab^2+a^2(2A+3C)\right)\tan(c+dx)}{3a^3d} - \frac{Ab\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(}{}$$

$$= \frac{\left(3Ab^2+a^2(2A+3C)\right)\tan(c+dx)}{3a^3d} - \frac{Ab\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(}{}$$

$$= -\frac{b\left(2Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(3Ab^2+a^2(2A+3C)\right)\tan}{3a^3d}$$

$$= \frac{2b^2\left(Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{b\left(2Ab^2+a^2(A+2C)\right)\tanh^{-1}(}{2a^4d}$$

Mathematica [B] time = 2.88109, size = 413, normalized size = 2.24

$$\frac{\frac{4a\left(a^2(2A+3C)+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a\left(a^2(2A+3C)+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} - \frac{24b^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + 6b\left(a^2(A+2C)+2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-24*b^2*(A*b^2 + a^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]])/Sqrt[-a^2 + b^2] + 6*b*(2*A*b^2 + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] - 6*b*(2*A*b^2 + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + (a^2*A*(a - 3*b))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^2 + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (
4*a*(3*A*b^2 + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(
```

```
c + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3 - (a^2*A*(a - 3*b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(
3*A*b^2 + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]))/(12*a^4*d)
```

Maple [B] time = 0.072, size = 554, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*C-1/3/a/d*A/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d*A/(tan(1/2
*d*x+1/2*c)-1)-1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)*b-1/d/a^3/(tan(1/2*d*x+1/
2*c)-1)*A*b^2-1/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/2/a/d*A/(tan(1/2*d*x+1/2*c)-
1)^2-1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)^2*b+1/2/d*A*b/a^2*ln(tan(1/2*d*x+1/
2*c)-1)+1/d*b^3/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A+1/d*b/a^2*ln(tan(1/2*d*x+1/2
*c)-1)*C-1/3/a/d*A/(tan(1/2*d*x+1/2*c)+1)^3-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)-
1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)*b-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b^2-1
/a/d/(tan(1/2*d*x+1/2*c)+1)*C+1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d*A/a^
2/(tan(1/2*d*x+1/2*c)+1)^2*b-1/2/d*A*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)-1/d*b^3
/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A-1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.4688, size = 1472, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [-1/12*(6*(C*a^2*b^2 + A*b^4)*sqrt(-a^2 + b^2)*cos(d*x + c)^3*log((2*a*b*co
s(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x +
 c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
c) + a^2)) + 3*((A + 2*C)*a^4*b + (A - 2*C)*a^2*b^3 - 2*A*b^5)*cos(d*x + c)
^3*log(sin(d*x + c) + 1) - 3*((A + 2*C)*a^4*b + (A - 2*C)*a^2*b^3 - 2*A*b^5
)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*((2*
A + 3*C)*a^5 + (A - 3*C)*a^3*b^2 - 3*A*a*b^4)*cos(d*x + c)^2 - 3*(A*a^4*b -
 A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3),
 1/12*(12*(C*a^2*b^2 + A*b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*((A + 2*C)*a^4*b + (A -
2*C)*a^2*b^3 - 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*((A + 2*C)
*a^4*b + (A - 2*C)*a^2*b^3 - 2*A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*((2*A + 3*C)*a^5 + (A - 3*C)*a^3*b^2 - 3*A*
a*b^4)*cos(d*x + c)^2 - 3*(A*a^4*b - A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))
/((a^6 - a^4*b^2)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41986, size = 502, normalized size = 2.73

$$-\frac{\frac{3\left(Aa^2b+2\,Ca^2b+2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{3\left(Aa^2b+2\,Ca^2b+2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{12\left(Ca^2b^2+Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\mathrm{arc}\right.}{\sqrt{a^2-b^2}a^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(A*a^2*b + 2*C*a^2*b + 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/
a^4 - 3*(A*a^2*b + 2*C*a^2*b + 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a^4 + 12*(C*a^2*b^2 + A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*a^4) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*
tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d
*x + 1/2*c)^5 - 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2
*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 6*
C*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.570 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=332

$$\frac{\left(a^2b^2(6A-7C)+12a^4C-b^4(3A+2C)\right)\sin(c+dx)}{3b^4d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab^2-5a^2b^2C+4a^4C-3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{b^5d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -((a*(2*A*b^2 + (4*a^2 + b^2)*C)*x)/b^5) + (2*a^2*(2*a^2*A*b^2 - 3*A*b^4 +
4*a^4*C - 5*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
((a - b)^(3/2)*b^5*(a + b)^(3/2)*d) + ((a^2*b^2*(6*A - 7*C) + 12*a^4*C - b^
4*(3*A + 2*C))*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) - (a*(A*b^2 + 2*a^2*C -
b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*A*b^2 + 4*a^2*C
 - b^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^
2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.11783, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(a^2b^2(6A-7C)+12a^4C-b^4(3A+2C)\right)\sin(c+dx)}{3b^4d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab^2-5a^2b^2C+4a^4C-3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{b^5d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*(2*A*b^2 + (4*a^2 + b^2)*C)*x)/b^5) + (2*a^2*(2*a^2*A*b^2 - 3*A*b^4 +
4*a^4*C - 5*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
((a - b)^(3/2)*b^5*(a + b)^(3/2)*d) + ((a^2*b^2*(6*A - 7*C) + 12*a^4*C - b^
4*(3*A + 2*C))*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) - (a*(A*b^2 + 2*a^2*C -
b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*A*b^2 + 4*a^2*C
 - b^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^
2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(3\left(Ab^2+a^2C\right)-ab(A+C)\cos(c\cdots\right.}{a+b\cos(\cdots}}{b\left(a^2-\cdots\right.}$$

$$= \frac{\left(3Ab^2+4a^2C-b^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\cdots}{b\left(a^2-b^2\right)d(a+b\cos(\cdots}$$

$$= -\frac{a\left(Ab^2+2a^2C-b^2C\right)\cos(c+dx)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3Ab^2+4a^2C-b^2C\right)\cos\cdots}{3b^2\left(a^2-\cdots\right.}$$

$$= \frac{\left(a^2b^2(6A-7C)+12a^4C-b^4(3A+2C)\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(Ab^2+2a^2C-b^2\cdots\right.}{b^3\left(\cdots\right.}$$

$$= -\frac{a\left(2Ab^2+\left(4a^2+b^2\right)C\right)x}{b^5} + \frac{\left(a^2b^2(6A-7C)+12a^4C-b^4(3A+2C)\right)\sin\cdots}{3b^4\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(2Ab^2+\left(4a^2+b^2\right)C\right)x}{b^5} + \frac{\left(a^2b^2(6A-7C)+12a^4C-b^4(3A+2C)\right)\sin\cdots}{3b^4\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(2Ab^2+\left(4a^2+b^2\right)C\right)x}{b^5} + \frac{2a^2\left(2a^2Ab^2-3Ab^4+4a^4C-5a^2b^2C\right)\tan^{-\cdots}}{(a-b)^{3/2}b^5(a+b)^{3/2}d}$$

Mathematica [A] time = 1.11899, size = 215, normalized size = 0.65

$$\frac{-12a(c+dx)\left(C\left(4a^2+b^2\right)+2Ab^2\right)+3b\left(3C\left(4a^2+b^2\right)+4Ab^2\right)\sin(c+dx)+\frac{12a^3b\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+\frac{24a^2\left(a^2b^2(2A-5C)+\cdots\right.}{\cdots}}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-12*a*(2*A*b^2 + (4*a^2 + b^2)*C)*(c + d*x) + (24*a^2*(-3*A*b^4 + a^2*b^2*
(2*A - 5*C) + 4*a^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(3/2) + 3*b*(4*A*b^2 + 3*(4*a^2 + b^2)*C)*Sin[c + d*x] + (12
*a^3*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]))
 - 6*a*b^2*C*Sin[2*(c + d*x)] + b^3*C*Sin[3*(c + d*x)])/(12*b^5*d)
```

Maple [B] time = 0.043, size = 828, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*A*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(tan(1/2*
d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)^5*a^2+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+
1)^3*C*tan(1/2*d*x+1/2*c)^5*a+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*
d*x+1/2*c)^5+4/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*A+12/d
/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a^2*C+4/3/d/b^2/(tan(1
/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1
)^3*A*tan(1/2*d*x+1/2*c)+6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1
/2*c)*a^2-2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)*a+2/d/b^2
/(tan(1/2*d*x+1/2*c)^2+1)^3*C*tan(1/2*d*x+1/2*c)-4/d/b^3*A*arctan(tan(1/2*d
*x+1/2*c))*a-8/d/b^5*C*arctan(tan(1/2*d*x+1/2*c))*a^3-2/d/b^3*C*arctan(tan(
1/2*d*x+1/2*c))*a+2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A+2/d*a^5/b^4/(a^2-b^2)*tan(1/2*d*x+1/2
*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C+4/d*a^4/b^3/(a+b)
/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*A-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*A+8/d*a^6/b^5/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-10/d*a^4/b^3/(a+b)/(a-
b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08631, size = 2171, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/6*(6*(4*C*a^7*b + (2*A - 7*C)*a^5*b^3 - 2*(2*A - C)*a^3*b^5 + (2*A + C)
*a*b^7)*d*x*cos(d*x + c) + 6*(4*C*a^8 + (2*A - 7*C)*a^6*b^2 - 2*(2*A - C)*a
^4*b^4 + (2*A + C)*a^2*b^6)*d*x + 3*(4*C*a^7 + (2*A - 5*C)*a^5*b^2 - 3*A*a^
3*b^4 + (4*C*a^6*b + (2*A - 5*C)*a^4*b^3 - 3*A*a^2*b^5)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*C*a^7*b + (6*A - 19*C)*a^5*b^3
- (9*A - 5*C)*a^3*b^5 + (3*A + 2*C)*a*b^7 + (C*a^4*b^4 - 2*C*a^2*b^6 + C*b^
8)*cos(d*x + c)^3 - 2*(C*a^5*b^3 - 2*C*a^3*b^5 + C*a*b^7)*cos(d*x + c)^2 +
(6*C*a^6*b^2 + (3*A - 10*C)*a^4*b^4 - 2*(3*A - C)*a^2*b^6 + (3*A + 2*C)*b^8
)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c)
+ (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d), -1/3*(3*(4*C*a^7*b + (2*A - 7*C)*a^5*b^
3 - 2*(2*A - C)*a^3*b^5 + (2*A + C)*a*b^7)*d*x*cos(d*x + c) + 3*(4*C*a^8 +
(2*A - 7*C)*a^6*b^2 - 2*(2*A - C)*a^4*b^4 + (2*A + C)*a^2*b^6)*d*x - 3*(4*C
*a^7 + (2*A - 5*C)*a^5*b^2 - 3*A*a^3*b^4 + (4*C*a^6*b + (2*A - 5*C)*a^4*b^3
 - 3*A*a^2*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*C*a^7*b + (6*A - 19*C)*a^5*b^3 - (9*A
 - 5*C)*a^3*b^5 + (3*A + 2*C)*a*b^7 + (C*a^4*b^4 - 2*C*a^2*b^6 + C*b^8)*cos
(d*x + c)^3 - 2*(C*a^5*b^3 - 2*C*a^3*b^5 + C*a*b^7)*cos(d*x + c)^2 + (6*C*a
^6*b^2 + (3*A - 10*C)*a^4*b^4 - 2*(3*A - C)*a^2*b^6 + (3*A + 2*C)*b^8)*cos(
d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^5
*b^5 - 2*a^3*b^7 + a*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46857, size = 593, normalized size = 1.79

$$-\frac{\frac{6\left(4\,Ca^{6}+2\,Aa^{4}b^{2}-5\,Ca^{4}b^{2}-3\,Aa^{2}b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{5}-b^{7}\right)\sqrt{a^{2}-b^{2}}}-\frac{6\left(Ca^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(a^{2}b^{4}-b^{6}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/3*(6*(4*C*a^6 + 2*A*a^4*b^2 - 5*C*a^4*b^2 - 3*A*a^2*b^4)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^5 - b^7)*sqrt(a^2 - b^2)) - 6
*(C*a^5*tan(1/2*d*x + 1/2*c) + A*a^3*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^4 -
b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 3*(4*
C*a^3 + 2*A*a*b^2 + C*a*b^2)*(d*x + c)/b^5 - 2*(9*C*a^2*tan(1/2*d*x + 1/2*c
)^5 + 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*C
*b^2*tan(1/2*d*x + 1/2*c)^5 + 18*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*b^2*tan
(1/2*d*x + 1/2*c)^3 + 2*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*C*a^2*tan(1/2*d*x
+ 1/2*c) - 3*C*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c) + 3*
C*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

3.571 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=262

$$-\frac{a\left(3a^2C+Ab^2-2b^2C\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{2a\left(a^2Ab^2-4a^2b^2C+3a^4C-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2C+Ab^2\right)\mathrm{si}}{bd\left(a^2-b^2\right)}$$

```
[Out] ((2*A*b^2 + (6*a^2 + b^2)*C)*x)/(2*b^4) - (2*a*(a^2*A*b^2 - 2*A*b^4 + 3*a^4
*C - 4*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
 b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(A*b^2 + 3*a^2*C - 2*b^2*C)*Sin[c + d*x
])/(b^3*(a^2 - b^2)*d) + ((2*A*b^2 + 3*a^2*C - b^2*C)*Cos[c + d*x]*Sin[c +
d*x])/(2*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.689676, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3049, 3023, 2735, 2659, 205}

$$-\frac{a\left(3a^2C+Ab^2-2b^2C\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{2a\left(a^2Ab^2-4a^2b^2C+3a^4C-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2C+Ab^2\right)\mathrm{si}}{bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] ((2*A*b^2 + (6*a^2 + b^2)*C)*x)/(2*b^4) - (2*a*(a^2*A*b^2 - 2*A*b^4 + 3*a^4
*C - 4*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
 b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(A*b^2 + 3*a^2*C - 2*b^2*C)*Sin[c + d*x
])/(b^3*(a^2 - b^2)*d) + ((2*A*b^2 + 3*a^2*C - b^2*C)*Cos[c + d*x]*Sin[c +
d*x])/(2*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2\left(Ab^2+a^2C\right)-ab(A+C)\cos(c+\ldots}{a+b\cos(c\ldots}}{b\left(a^2-\ldots\right.}$$

$$= \frac{\left(2Ab^2+3a^2C-b^2C\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\,\mathrm{s}\ldots}{b\left(a^2-b^2\right)d(a+b\cos(c\ldots}$$

$$= -\frac{a\left(Ab^2+3a^2C-2b^2C\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2+3a^2C-b^2C\right)\cos(c+dx)\,\mathrm{si}\ldots}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(2Ab^2+\left(6a^2+b^2\right)C\right)x}{2b^4} - \frac{a\left(Ab^2+3a^2C-2b^2C\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2+3\ldots\right.}{}$$

$$= \frac{\left(2Ab^2+\left(6a^2+b^2\right)C\right)x}{2b^4} - \frac{a\left(Ab^2+3a^2C-2b^2C\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2+3\ldots\right.}{}$$

$$= \frac{\left(2Ab^2+\left(6a^2+b^2\right)C\right)x}{2b^4} - \frac{2a\left(a^2Ab^2-2Ab^4+3a^4C-4a^2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-\ldots}}{\ldots}\right.}{(a-b)^{3/2}b^4(a+b)^{3/2}d}$$

Mathematica [A] time = 0.980046, size = 178, normalized size = 0.68

$$\frac{2(c+dx)\left(C\left(6a^2+b^2\right)+2Ab^2\right) - \frac{4a^2b\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - \frac{8a\left(a^2b^2(A-4C)+3a^4C-2Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - 8abC\sin(c+\ldots}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(2*A*b^2 + (6*a^2 + b^2)*C)*(c + d*x) - (8*a*(-2*A*b^4 + a^2*b^2*(A - 4*
C) + 3*a^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 +
 b^2)^(3/2) - 8*a*b*C*Sin[c + d*x] - (4*a^2*b*(A*b^2 + a^2*C)*Sin[c + d*x])
/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*C*Sin[2*(c + d*x)])/(4*b^4*d)
```

Maple [B] time = 0.039, size = 569, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-1/d/b^2/(tan(1
/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*C-4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1
)^2*tan(1/2*d*x+1/2*c)*a*C+1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1
/2*c)*C+2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*A+6/d/b^4*arctan(tan(1/2*d*x+1/2
*c))*a^2*C+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))*C-2/d*a^2/b/(a^2-b^2)*tan(1/2
*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A-2/d*a^4/b
^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)*C-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*a^5/b^4/(a+b)/(a-b
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
C+8/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91633, size = 1804, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/2*((6*C*a^6*b + (2*A - 11*C)*a^4*b^3 - 4*(A - C)*a^2*b^5 + (2*A + C)*b^7
)*d*x*cos(d*x + c) + (6*C*a^7 + (2*A - 11*C)*a^5*b^2 - 4*(A - C)*a^3*b^4 +
(2*A + C)*a*b^6)*d*x - (3*C*a^6 + (A - 4*C)*a^4*b^2 - 2*A*a^2*b^4 + (3*C*a^
5*b + (A - 4*C)*a^3*b^3 - 2*A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos
(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + a^2)) - (6*C*a^6*b + 2*(A - 5*C)*a^4*b^3 - 2*(A - 2*C)*a^2*b^5 -
 (C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7)*cos(d*x + c)^2 + 3*(C*a^5*b^2 - 2*C*a^3*
b^4 + C*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*c
os(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*C*a^6*b + (2*A - 11
*C)*a^4*b^3 - 4*(A - C)*a^2*b^5 + (2*A + C)*b^7)*d*x*cos(d*x + c) + (6*C*a^
7 + (2*A - 11*C)*a^5*b^2 - 4*(A - C)*a^3*b^4 + (2*A + C)*a*b^6)*d*x - 2*(3*
C*a^6 + (A - 4*C)*a^4*b^2 - 2*A*a^2*b^4 + (3*C*a^5*b + (A - 4*C)*a^3*b^3 -
2*A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt
(a^2 - b^2)*sin(d*x + c))) - (6*C*a^6*b + 2*(A - 5*C)*a^4*b^3 - 2*(A - 2*C)
*a^2*b^5 - (C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7)*cos(d*x + c)^2 + 3*(C*a^5*b^2
- 2*C*a^3*b^4 + C*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7
+ b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72435, size = 420, normalized size = 1.6

$$\frac{\frac{4\left(3\,Ca^{5}+Aa^{3}b^{2}-4\,Ca^{3}b^{2}-2\,Aab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}-\frac{4\left(Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(a^{2}b^{3}-b^{5}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/2*(4*(3*C*a^5 + A*a^3*b^2 - 4*C*a^3*b^2 - 2*A*a*b^4)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - 4*(C*a
^4*tan(1/2*d*x + 1/2*c) + A*a^2*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*C*a^2 +
 2*A*b^2 + C*b^2)*(d*x + c)/b^4 - 2*(4*C*a*tan(1/2*d*x + 1/2*c)^3 + C*b*tan
(1/2*d*x + 1/2*c)^3 + 4*C*a*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c)
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

3.572 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{2\left(3a^2b^2C-2a^4C+Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\left(a^2C+Ab^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2aCx}{b^3}+\frac{C\sin(c+dx)}{b^2d}$$

```
[Out] (-2*a*C*x)/b^3 - (2*(A*b^4 - 2*a^4*C + 3*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (C*Sin[c
 + d*x])/(b^2*d) + (a*(A*b^2 + a^2*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a +
 b*Cos[c + d*x]))
```

Rubi [A] time = 0.346346, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3032, 3023, 2735, 2659, 205}

$$-\frac{2\left(3a^2b^2C-2a^4C+Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\left(a^2C+Ab^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2aCx}{b^3}+\frac{C\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```



```
[Out] (-2*a*C*x)/b^3 - (2*(A*b^4 - 2*a^4*C + 3*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (C*Sin[c
 + d*x])/(b^2*d) + (a*(A*b^2 + a^2*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a +
 b*Cos[c + d*x]))
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b\left(Ab^2+a^2C\right)+a\left(a^2-b^2\right)C\cos(c+dx)-b\left(a^2-b^2\right)C\cos^{2}}{a+b\cos(c+dx)}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{C\sin(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b^2\left(Ab^2+a^2C\right)+2ab\left(a^2-b^2\right)C\,c}{a+b\cos(c+dx)}}{b^3\left(a^2-b^2\right)}$$

$$= -\frac{2aCx}{b^3} + \frac{C\sin(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(Ab^4-2a^4C+\right.}{}$$

$$= -\frac{2aCx}{b^3} + \frac{C\sin(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(Ab^4-2a^4C\right.\right.}{}$$

$$= -\frac{2aCx}{b^3} - \frac{2\left(Ab^4-2a^4C+3a^2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{C\sin(c+dx)}{b^2d}$$

Mathematica [A] time = 1.03547, size = 136, normalized size = 0.94

$$\frac{\frac{ab\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - \frac{2\left(3a^2b^2C-2a^4C+Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - 2aC(c+dx) + bC\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*C*(c + d*x) - (2*(A*b^4 - 2*a^4*C + 3*a^2*b^2*C)*ArcTanh[((a - b)*Tan
[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + b*C*Sin[c + d*x] + (
a*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(
b^3*d)
```

Maple [B] time = 0.037, size = 359, normalized size = 2.5

$$2\,\frac{C\tan(1/2\,dx+c/2)}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)} - 4\,\frac{C\arctan(\tan(1/2\,dx+c/2))\,a}{db^3} + 2\,\frac{a\tan(1/2\,dx+c/2)\,A}{d\left(a^2-b^2\right)\left(a(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*C/b^2*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)-4/d/b^3*C*arctan(tan(
1/2*d*x+1/2*c))*a+2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A+2/d/b^2*a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-2/d*b/(a+b)/(a-b)/((a+b
)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d*a
^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*C-6/d/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73507, size = 1388, normalized size = 9.64

$$\left[-\frac{4\left(Ca^5b-2Ca^3b^3+Cab^5\right)dx\cos(dx+c)+4\left(Ca^6-2Ca^4b^2+Ca^2b^4\right)dx+\left(2Ca^5-3Ca^3b^2-Aab^4+\left(2Ca^4b-3C\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(4*(C*a^5*b - 2*C*a^3*b^3 + C*a*b^5)*d*x*cos(d*x + c) + 4*(C*a^6 - 2*
C*a^4*b^2 + C*a^2*b^4)*d*x + (2*C*a^5 - 3*C*a^3*b^2 - A*a*b^4 + (2*C*a^4*b
- 3*C*a^2*b^3 - A*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)
*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2
```

```
)) - 2*(2*C*a^5*b + (A - 3*C)*a^3*b^3 - (A - C)*a*b^5 + (C*a^4*b^2 - 2*C*a^
2*b^4 + C*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*c
os(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d), -(2*(C*a^5*b - 2*C*a^3*b^3
+ C*a*b^5)*d*x*cos(d*x + c) + 2*(C*a^6 - 2*C*a^4*b^2 + C*a^2*b^4)*d*x - (2*
C*a^5 - 3*C*a^3*b^2 - A*a*b^4 + (2*C*a^4*b - 3*C*a^2*b^3 - A*b^5)*cos(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
+ c))) - (2*C*a^5*b + (A - 3*C)*a^3*b^3 - (A - C)*a*b^5 + (C*a^4*b^2 - 2*C*
a^2*b^4 + C*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d
*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43825, size = 478, normalized size = 3.32

$$-\frac{2\left(\frac{\left(2Ca^4-3Ca^2b^2-Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}+\frac{(dx+c)Ca}{b^3}-\frac{2Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ca^2b\tan\left(\frac{1}{2}dx+\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -2*((2*C*a^4 - 3*C*a^2*b^2 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + (d*x + c)*C*a/b^3 - (2*C*
a^3*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1
/2*d*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + C*b^3*tan(1/2*d*x + 1/
2*c)^3 + 2*C*a^3*tan(1/2*d*x + 1/2*c) + C*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*
b^2*tan(1/2*d*x + 1/2*c) - C*a*b^2*tan(1/2*d*x + 1/2*c) - C*b^3*tan(1/2*d*x
```

```
 + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(
1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)))/d
```

3.573 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{2a\left(a^2(-C)+Ab^2+2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Cx}{b^2}$$

```
[Out] (C*x)/b^2 + (2*a*(A*b^2 - a^2*C + 2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - ((A*b^2 + a^2*C)*
Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.196338, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3022, 2735, 2659, 205}

$$\frac{2a\left(a^2(-C)+Ab^2+2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Cx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (C*x)/b^2 + (2*a*(A*b^2 - a^2*C + 2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - ((A*b^2 + a^2*C)*
Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\int \frac{-ab(A+C)-\left(a^2-b^2\right)C\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}\\
&= \frac{Cx}{b^2}-\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\left(-a\left(a^2-b^2\right)C+ab^2(A+C)\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}\\
&= \frac{Cx}{b^2}-\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\left(2\left(-a\left(a^2-b^2\right)C+ab^2(A+C)\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-}\right.}{b^2\left(a^2-b^2\right)d}\\
&= \frac{Cx}{b^2}+\frac{2a\left(Ab^2-a^2C+2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}-\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.678512, size = 123, normalized size = 0.98

$$\frac{-\frac{b\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-\frac{2a\left(C\left(a^2-2b^2\right)-Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+C(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (C*(c + d*x) - (2*a*(-(A*b^2) + (a^2 - 2*b^2)*C)*ArcTanh[((a - b)*Tan[(c +
d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - (b*(A*b^2 + a^2*C)*Sin[c +
 d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^2*d)
```

Maple [B] time = 0.029, size = 320, normalized size = 2.5

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{db^{2}}-2\,\frac{b\tan\left(1/2\,dx+c/2\right)A}{d\left(a^{2}-b^{2}\right)\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b+a+b\right)}-2\,\frac{}{db\left(a^{2}-b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A-2/d/b/(a^2-b^2)*tan(1/2*
d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*a^2*C+2/d*a/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63014, size = 1172, normalized size = 9.3

$$\left[\frac{2\left(Ca^{4}b-2\,Ca^{2}b^{3}+Cb^{5}\right)dx\cos\left(dx+c\right)+2\left(Ca^{5}-2\,Ca^{3}b^{2}+Cab^{4}\right)dx-\left(Ca^{4}-(A+2\,C)a^{2}b^{2}+\left(Ca^{3}b-(A+2\,C)a\right.\right.}{2\left(\left(a^{4}b^{3}-2\,a^{2}b^{5}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(C*a^4*b - 2*C*a^2*b^3 + C*b^5)*d*x*cos(d*x + c) + 2*(C*a^5 - 2*C*a
^3*b^2 + C*a*b^4)*d*x - (C*a^4 - (A + 2*C)*a^2*b^2 + (C*a^3*b - (A + 2*C)*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(C*a^4*b +
 (A - C)*a^2*b^3 - A*b^5)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(
d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((C*a^4*b - 2*C*a^2*b^3 + C*b^
5)*d*x*cos(d*x + c) + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*d*x - (C*a^4 - (A + 2
*C)*a^2*b^2 + (C*a^3*b - (A + 2*C)*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arc
tan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (C*a^4*b + (A -
 C)*a^2*b^3 - A*b^5)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x +
 c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50064, size = 271, normalized size = 2.15

$$\frac{\frac{2\left(Ca^3-Aab^2-2\,Cab^2\right)\left(\pi\left\lfloor \frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{(dx+c)C}{b^2}-\frac{2\left(Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(C*a^3 - A*a*b^2 - 2*C*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*C/b^2 - 2*(C*a^2*t
an(1/2*d*x + 1/2*c) + A*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

3.574 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=134

$$-\frac{2b\left(2a^2A+a^2C-Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] (-2*b*(2*a^2*A - A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c + d*x]])/(a
^2*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
))
```

Rubi [A] time = 0.331843, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3056, 3001, 3770, 2659, 205}

$$-\frac{2b\left(2a^2A+a^2C-Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^2*A - A*b^2 + a^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c + d*x]])/(a
^2*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(A\left(a^2-b^2\right)-ab(A+C)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{A\int \sec(c+dx)\,dx}{a^2} + \frac{\left(b\left(Ab^2-a^2(2A+C)\right.\right.}{a^2\left(a^2-\right.}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(2b\left(Ab^2-a^2(2A\right.\right.}{}$$

$$= -\frac{2b\left(2a^2A-Ab^2+a^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} +$$

Mathematica [C] time = 1.82607, size = 306, normalized size = 2.28

$$\frac{2\cos(c+dx)(A\sec(c+dx)+C\cos(c+dx))\left(\frac{a\left(a^2C+Ab^2\right)(b\sin(dx)-a\sin(c))}{b(a-b)(a+b)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)(a+b\cos(c+dx))} + \frac{2b(\sin(c)+i\cos(c))\left(a^2(2A+C)-\right.}{\left(a^2-\right.}\right.}{a^2d(2A+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]*(C*Cos[c + d*x] + A*Sec[c + d*x])*(-(A*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]]) + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*b*(
-(A*b^2) + a^2*(2*A + C))*ArcTan[((I*Cos[c] + Sin[c])*(b*Sin[c] + (-a + b*C
os[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]]*(I*Cos[c]
 + Sin[c]))/((a^2 - b^2)*Sqrt[(-a^2 + b^2)*(Cos[c] - I*Sin[c])^2]) + (a*(A*
b^2 + a^2*C)*(-(a*Sin[c]) + b*Sin[d*x]))/((a - b)*b*(a + b)*(a + b*Cos[c +
d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))))/(a^2*d*(2*A + C + C*Co
s[2*(c + d*x)]))
```

Maple [B] time = 0.068, size = 342, normalized size = 2.6

$$2\,\frac{A\tan(1/2\,dx+c/2)\,b^2}{da\left(a^2-b^2\right)\left(a\left(\tan(1/2\,dx+c/2)\right)^2-\left(\tan(1/2\,dx+c/2)\right)^2b+a+b\right)} + 2\,\frac{a\tan(1/2\,dx+c/2)\,C}{d\left(a^2-b^2\right)\left(a\left(\tan(1/2\,dx+c/2)\right)^2-\left(\tan(1/2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)*A*b^2+2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-4/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d/a^2*b^3/(a+b)/(a-b)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2
/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*C-1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*A*ln(tan(1/2*d*
x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.0237, size = 1497, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((2*A + C)*a^3*b - A*a*b^3 + ((2*A + C)*a^2*b^2 - A*b^4)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 +
 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^5 - 2*A*a^3*b^2 + A*a*b^4
 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A
*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c)
```

```
)*log(-sin(d*x + c) + 1) + 2*(C*a^5 + (A - C)*a^3*b^2 - A*a*b^4)*sin(d*x +
c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*
b^4)*d), -1/2*(2*((2*A + C)*a^3*b - A*a*b^3 + ((2*A + C)*a^2*b^2 - A*b^4)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c))) - (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 +
A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + (A*a^5 - 2*A*a^3*b^2 + A*a*b^4
 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2
*(C*a^5 + (A - C)*a^3*b^2 - A*a*b^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^
2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\cos\left(c + dx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

Giac [A] time = 1.26806, size = 305, normalized size = 2.28

$$-\frac{\frac{2\left(2\,Aa^2b+Ca^2b-Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(2*A*a^2*b + C*a^2*b - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a
 - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) - A*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^2 + A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 2*(C*a^2*tan(1/2*d
*x + 1/2*c) + A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

3.575 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=180

$$\frac{2\left(3a^2Ab^2+a^4C-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(2Ab^2-a^2(A-C)\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{\left(a^2C+Ab^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (2*(3*a^2*A*b^2 - 2*A*b^4 + a^4*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - (2*A*b*ArcTanh[Sin[c + d*
x]])/(a^3*d) - ((2*A*b^2 - a^2*(A - C))*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) +
 ((A*b^2 + a^2*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.581978, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{2\left(3a^2Ab^2+a^4C-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(2Ab^2-a^2(A-C)\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{\left(a^2C+Ab^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (2*(3*a^2*A*b^2 - 2*A*b^4 + a^4*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - (2*A*b*ArcTanh[Sin[c + d*
x]])/(a^3*d) - ((2*A*b^2 - a^2*(A - C))*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) +
 ((A*b^2 + a^2*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-2Ab^2+a^2(A-C)-ab(A+C)\cos(c+dx)+\left(Ab^2+a^2C\right)\right.}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(2Ab^2-a^2(A-C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{(-}{}}{}$$

$$= -\frac{\left(2Ab^2-a^2(A-C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(2Ab}{}$$

$$= -\frac{2Ab\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\left(2Ab^2-a^2(A-C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+\right.}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(3a^2Ab^2-2Ab^4+a^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{2Ab\tanh^{-1}(\sin(c+d}{a^3d}$$

Mathematica [A] time = 1.74338, size = 219, normalized size = 1.22

$$\frac{2\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(-\frac{ab\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + \frac{2\left(3a^2Ab^2+a^4C-2Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + aA\tan(c+dx)\right.}{a^3d(2A+C\cos(2(c+dx))+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((2*(3*a^2*A*b^2 - 2*A*b^4 + a^4*C
)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2)
+ 2*A*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) - (a*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(a
 + b*Cos[c + d*x])) + a*A*Tan[c + d*x]))/(a^3*d*(2*A + C + C*Cos[2*(c + d*x
)]))
```

Maple [B] time = 0.076, size = 394, normalized size = 2.2

$$-2\frac{b^{3}\tan\left(1/2\,dx+c/2\right)A}{da^{2}\left(a^{2}-b^{2}\right)\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b+a+b\right)}-2\frac{b\tan\left(1/2\,dx+c/2\right)C}{d\left(a^{2}-b^{2}\right)\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}-\left(\tan\left(1\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/a^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)*A-2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C+6/d/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^2-4/d/a^3/(a+b)/(a-b
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
A*b^4+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*C+2/d*A*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*A/(tan
(1/2*d*x+1/2*c)-1)-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-2/d*A*b/a^3*ln(tan(1/2*
d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.9006, size = 1887, normalized size = 10.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/2*(((C*a^4*b + 3*A*a^2*b^3 - 2*A*b^5)*cos(d*x + c)^2 + (C*a^5 + 3*A*a^3*
b^2 - 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2
*((A*a^4*b^2 - 2*A*a^2*b^4 + A*b^6)*cos(d*x + c)^2 + (A*a^5*b - 2*A*a^3*b^3
 + A*a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*((A*a^4*b^2 - 2*A*a^2*b
^4 + A*b^6)*cos(d*x + c)^2 + (A*a^5*b - 2*A*a^3*b^3 + A*a*b^5)*cos(d*x + c)
)*log(-sin(d*x + c) + 1) + 2*(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4 + ((A - C)*a^
5*b - (3*A - C)*a^3*b^3 + 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b -
2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d
*x + c)), (((C*a^4*b + 3*A*a^2*b^3 - 2*A*b^5)*cos(d*x + c)^2 + (C*a^5 + 3*A
*a^3*b^2 - 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((A*a^4*b^2 - 2*A*a^2*b^4 + A*b^6)*
cos(d*x + c)^2 + (A*a^5*b - 2*A*a^3*b^3 + A*a*b^5)*cos(d*x + c))*log(sin(d*
x + c) + 1) + ((A*a^4*b^2 - 2*A*a^2*b^4 + A*b^6)*cos(d*x + c)^2 + (A*a^5*b
- 2*A*a^3*b^3 + A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (A*a^6 - 2*
A*a^4*b^2 + A*a^2*b^4 + ((A - C)*a^5*b - (3*A - C)*a^3*b^3 + 2*A*a*b^5)*cos
(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 +
(a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30631, size = 516, normalized size = 2.87

$$-\frac{2\left(\frac{\left(Ca^4+3\,Aa^2b^2-2\,Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{Ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{Ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{a^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -2*((C*a^4 + 3*A*a^2*b^2 - 2*A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + A*b*log(abs(tan(1/2*d*x +
 1/2*c) + 1))/a^3 - A*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + (A*a^3*tan
(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + C*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 - A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^
3 + A*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - C*a^2*b*tan
(1/2*d*x + 1/2*c) - A*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)))/d
```

3.576 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=265

$$-\frac{2b\left(4a^2Ab^2-a^2b^2C+2a^4C-3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b\left(3Ab^2-a^2(2A-C)\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2(A+2C)+\right.}{}$$

```
[Out] (-2*b*(4*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C - a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((6*A*b^
2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + (b*(3*A*b^2 - a^2*(2*
A - C))*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) - ((3*A*b^2 - a^2*(A - 2*C))*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sec[c + d*x
]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.06387, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b\left(4a^2Ab^2-a^2b^2C+2a^4C-3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b\left(3Ab^2-a^2(2A-C)\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2(A+2C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (-2*b*(4*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C - a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((6*A*b^
2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + (b*(3*A*b^2 - a^2*(2*
A - C))*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) - ((3*A*b^2 - a^2*(A - 2*C))*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sec[c + d*x
]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
```

```
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{\left(-3Ab^2+a^2(A-2C)-ab(A+C)\cos(c+dx)+2(\ldots\right.}{a+b\cos(c+\ldots}}{a\left(a^2-b\ldots\right.}$$

$$= -\frac{\left(3Ab^2-a^2(A-2C)\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d}+\frac{\left(Ab^2+a^2C\right)\sec(c+dx)\,\mathrm{t}\ldots}{a\left(a^2-b^2\right)d(a+b\cos(\ldots}$$

$$= \frac{b\left(3Ab^2-a^2(2A-C)\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d}-\frac{\left(3Ab^2-a^2(A-2C)\right)\sec(c+dx)\tan(\ldots}{2a^2\left(a^2-b^2\right)d}$$

$$= \frac{b\left(3Ab^2-a^2(2A-C)\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d}-\frac{\left(3Ab^2-a^2(A-2C)\right)\sec(c+dx)\tan(\ldots}{2a^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(6Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{b\left(3Ab^2-a^2(2A-C)\right)\tan(c+\ldots}{a^3\left(a^2-b^2\right)d}$$

$$= -\frac{2b\left(4a^2Ab^2-3Ab^4+2a^4C-a^2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}+\frac{\left(6Ab^2+\ldots\right.}{}$$

Mathematica [B] time = 6.32081, size = 712, normalized size = 2.69

$$\frac{2\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(a^2b^2C\sin(c+dx)+Ab^4\sin(c+dx)\right)}{a^3d(a-b)(a+b)(a+b\cos(c+dx))(2A+C\cos(2c+2dx)+C)}+\frac{\left(a^2(-A)-2a^2C-6Ab^2\right)\cos^2(c+dx)\left(A\ldots\right.}{a^4d(2A\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (4*b*(4*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C - a^2*b^2*C)*ArcTanh[((a - b)*Tan[(c
+ d*x)/2])/Sqrt[-a^2 + b^2]]*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2))/(a^4*(a
^2 - b^2)*Sqrt[-a^2 + b^2]*d*(2*A + C + C*Cos[2*c + 2*d*x])) + ((-(a^2*A) -
 6*A*b^2 - 2*a^2*C)*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*(C + A*Sec[c + d*x]^2))/(a^4*d*(2*A + C + C*Cos[2*c + 2*d*x])) + ((a^2*A +
 6*A*b^2 + 2*a^2*C)*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
*(C + A*Sec[c + d*x]^2))/(a^4*d*(2*A + C + C*Cos[2*c + 2*d*x])) + (A*Cos[c
+ d*x]^2*(C + A*Sec[c + d*x]^2))/(2*a^2*d*(2*A + C + C*Cos[2*c + 2*d*x])*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (4*A*b*Cos[c + d*x]^2*(C + A*Sec[c
 + d*x]^2)*Sin[(c + d*x)/2])/(a^3*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])) - (A*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2))/
(2*a^2*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^2) - (4*A*b*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(a^3
*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) +
(2*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(A*b^4*Sin[c + d*x] + a^2*b^2*C*Si
n[c + d*x]))/(a^3*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])*(2*A + C + C*Cos[2
*c + 2*d*x]))
```

Maple [B] time = 0.089, size = 638, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)*A+2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-8/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*b^5/a^4/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*A-4/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*C+2/d*b^3/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/2/d/a^2*A/(tan(1/2*d
*x+1/2*c)-1)^2-1/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)-3/d/a^4*ln(tan(1/2*d*x+
1/2*c)-1)*A*b^2-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^2*A/(tan(1/2*d*x
+1/2*c)-1)+2/d*A/a^3/(tan(1/2*d*x+1/2*c)-1)*b-1/2/d/a^2*A/(tan(1/2*d*x+1/2*
c)+1)^2+1/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)+3/d/a^4*ln(tan(1/2*d*x+1/2*c)+
1)*A*b^2+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)
+1)+2/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 79.7078, size = 2538, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/4*(2*((2*C*a^4*b^2 + (4*A - C)*a^2*b^4 - 3*A*b^6)*cos(d*x + c)^3 + (2*C
*a^5*b + (4*A - C)*a^3*b^3 - 3*A*a*b^5)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*lo
g((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(
a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b
*cos(d*x + c) + a^2)) - (((A + 2*C)*a^6*b + 4*(A - C)*a^4*b^3 - (11*A - 2*C
)*a^2*b^5 + 6*A*b^7)*cos(d*x + c)^3 + ((A + 2*C)*a^7 + 4*(A - C)*a^5*b^2 -
(11*A - 2*C)*a^3*b^4 + 6*A*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + (
((A + 2*C)*a^6*b + 4*(A - C)*a^4*b^3 - (11*A - 2*C)*a^2*b^5 + 6*A*b^7)*cos(
d*x + c)^3 + ((A + 2*C)*a^7 + 4*(A - C)*a^5*b^2 - (11*A - 2*C)*a^3*b^4 + 6*
A*a*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(A*a^7 - 2*A*a^5*b^2 +
A*a^3*b^4 - 2*((2*A - C)*a^5*b^2 - (5*A - C)*a^3*b^4 + 3*A*a*b^6)*cos(d*x +
 c)^2 - 3*(A*a^6*b - 2*A*a^4*b^3 + A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4
)*d*cos(d*x + c)^2), -1/4*(4*((2*C*a^4*b^2 + (4*A - C)*a^2*b^4 - 3*A*b^6)*c
os(d*x + c)^3 + (2*C*a^5*b + (4*A - C)*a^3*b^3 - 3*A*a*b^5)*cos(d*x + c)^2)
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - (((A + 2*C)*a^6*b + 4*(A - C)*a^4*b^3 - (11*A - 2*C)*a^2*b^5 + 6*A*b^7
)*cos(d*x + c)^3 + ((A + 2*C)*a^7 + 4*(A - C)*a^5*b^2 - (11*A - 2*C)*a^3*b^
4 + 6*A*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + (((A + 2*C)*a^6*b +
4*(A - C)*a^4*b^3 - (11*A - 2*C)*a^2*b^5 + 6*A*b^7)*cos(d*x + c)^3 + ((A +
2*C)*a^7 + 4*(A - C)*a^5*b^2 - (11*A - 2*C)*a^3*b^4 + 6*A*a*b^6)*cos(d*x +
c)^2)*log(-sin(d*x + c) + 1) - 2*(A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4 - 2*((2*A
```

```
 - C)*a^5*b^2 - (5*A - C)*a^3*b^4 + 3*A*a*b^6)*cos(d*x + c)^2 - 3*(A*a^6*b
- 2*A*a^4*b^3 + A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3
+ a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.57206, size = 477, normalized size = 1.8

$$\frac{\frac{4\left(2Ca^4b+4Aa^2b^3-Ca^2b^3-3Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{4\left(Ca^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/2*(4*(2*C*a^4*b + 4*A*a^2*b^3 - C*a^2*b^3 - 3*A*b^5)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) + 4*(C*a
^2*b^2*tan(1/2*d*x + 1/2*c) + A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (A*a^2 + 2
*C*a^2 + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (A*a^2 + 2*C*a^2
 + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(A*a*tan(1/2*d*x + 1
/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) - 4*A*b*t
an(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.577 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=335

$$\frac{2b^2\left(5a^2Ab^2-2a^2b^2C+3a^4C-4Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(-a^2b^2(7A-6C)+a^4(-(2A+3C))+12Ab^4\right)\tan(c}{3a^4d\left(a^2-b^2\right)}$$

```
[Out] (2*b^2*(5*a^2*A*b^2 - 4*A*b^4 + 3*a^4*C - 2*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(4
*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(a^5*d) - ((12*A*b^4 - a^2*b
^2*(7*A - 6*C) - a^4*(2*A + 3*C))*Tan[c + d*x])/(3*a^4*(a^2 - b^2)*d) + (b*
(2*A*b^2 - a^2*(A - C))*Sec[c + d*x]*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) - ((
4*A*b^2 - a^2*(A - 3*C))*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d)
 + ((A*b^2 + a^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Co
s[c + d*x]))
```

Rubi [A] time = 1.46962, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^2\left(5a^2Ab^2-2a^2b^2C+3a^4C-4Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(-a^2b^2(7A-6C)+a^4(-(2A+3C))+12Ab^4\right)\tan(c}{3a^4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (2*b^2*(5*a^2*A*b^2 - 4*A*b^4 + 3*a^4*C - 2*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(4
*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(a^5*d) - ((12*A*b^4 - a^2*b
^2*(7*A - 6*C) - a^4*(2*A + 3*C))*Tan[c + d*x])/(3*a^4*(a^2 - b^2)*d) + (b*
(2*A*b^2 - a^2*(A - C))*Sec[c + d*x]*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) - ((
4*A*b^2 - a^2*(A - 3*C))*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d)
 + ((A*b^2 + a^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Co
s[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
```

```
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-4Ab^2+a^2(A-3C)-ab(A+C)\cos(c+dx)+3\right.}{a+b\cos(c+}}{a\left(a^2-\right.}$$

$$= -\frac{\left(4Ab^2-a^2(A-3C)\right)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx}{a\left(a^2-b^2\right)d(a+b\cos}$$

$$= \frac{b\left(2Ab^2-a^2(A-C)\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(4Ab^2-a^2(A-3C)\right)\sec^2(}{3a^2\left(a^2-b^2\right.}$$

$$= -\frac{\left(12Ab^4-a^2b^2(7A-6C)-a^4(2A+3C)\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} + \frac{b\left(2Ab^2-a^2(A-\right.}{a^3}$$

$$= -\frac{\left(12Ab^4-a^2b^2(7A-6C)-a^4(2A+3C)\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} + \frac{b\left(2Ab^2-a^2(A-\right.}{a^3}$$

$$= -\frac{b\left(4Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\left(12Ab^4-a^2b^2(7A-6C)-\right.}{3a^4\left(a^2-\right.}$$

$$= \frac{2b^2\left(5a^2Ab^2-4Ab^4+3a^4C-2a^2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\left(4Ab^2\right.}{}$$

Mathematica [A] time = 6.2806, size = 593, normalized size = 1.77

$$\frac{2a^2A\sin\left(\frac{1}{2}(c+dx)\right)+3a^2C\sin\left(\frac{1}{2}(c+dx)\right)+9Ab^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2a^2A\sin\left(\frac{1}{2}(c+dx)\right)+3a^2C\sin\left(\frac{1}{2}(c+dx)\right)+9}{3a^4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(5*a^2*A*b^2 - 4*A*b^4 + 3*a^4*C - 2*a^2*b^2*C)*ArcTanh[((a - b)*Ta
n[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^5*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + (
(a^2*A*b + 4*A*b^3 + 2*a^2*b*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(
a^5*d) + ((-(a^2*A*b) - 4*A*b^3 - 2*a^2*b*C)*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]])/(a^5*d) + (A*(a - 6*b))/(12*a^3*d*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2) + (A*Sin[(c + d*x)/2])/(6*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3) + (A*Sin[(c + d*x)/2])/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3) - (A*(a - 6*b))/(12*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
+ (2*a^2*A*Sin[(c + d*x)/2] + 9*A*b^2*Sin[(c + d*x)/2] + 3*a^2*C*Sin[(c + d
*x)/2])/(3*a^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*a^2*A*Sin[(c +
 d*x)/2] + 9*A*b^2*Sin[(c + d*x)/2] + 3*a^2*C*Sin[(c + d*x)/2])/(3*a^4*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])) + (-(A*b^5*Sin[c + d*x]) - a^2*b^3*C*S
in[c + d*x])/(a^4*(a - b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

Maple [B] time = 0.089, size = 830, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^3-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-1/3/
d/a^2*A/(tan(1/2*d*x+1/2*c)-1)^3-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C-1/2/d/a^2
*A/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2-1/d/a^2*A/
(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)+2/d*b/a^3*ln(tan(1/
2*d*x+1/2*c)-1)*C+1/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2*b-4/d*b^3/a^5*ln(tan(1
/2*d*x+1/2*c)+1)*A-2/d*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-3/d/a^4/(tan(1/2*d*
x+1/2*c)-1)*A*b^2-1/d*A/a^3/(tan(1/2*d*x+1/2*c)-1)^2*b+4/d*b^3/a^5*ln(tan(1
/2*d*x+1/2*c)-1)*A-3/d/a^4/(tan(1/2*d*x+1/2*c)+1)*A*b^2-1/d*A/a^3/(tan(1/2*
d*x+1/2*c)-1)*b-1/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)*b-1/d*A*b/a^3*ln(tan(1/2*d
*x+1/2*c)+1)+1/d*A*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)+6/d*b^2/a/(a+b)/(a-b)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/d
*b^4/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*C-8/d*b^6/a^5/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+10/d/a^3/(a+b)/(a-b)/((a+b)*(a
-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^4-2/d*b
^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
```

```
*c)^2*b+a+b)*A-2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 70.9184, size = 2917, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/6*(3*((3*C*a^4*b^3 + (5*A - 2*C)*a^2*b^5 - 4*A*b^7)*cos(d*x + c)^4 + (3
*C*a^5*b^2 + (5*A - 2*C)*a^3*b^4 - 4*A*a*b^6)*cos(d*x + c)^3)*sqrt(-a^2 + b
^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 +
b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2)) + 3*(((A + 2*C)*a^6*b^2 + 2*(A - 2*C)*a^4*b^4 -
 (7*A - 2*C)*a^2*b^6 + 4*A*b^8)*cos(d*x + c)^4 + ((A + 2*C)*a^7*b + 2*(A -
2*C)*a^5*b^3 - (7*A - 2*C)*a^3*b^5 + 4*A*a*b^7)*cos(d*x + c)^3)*log(sin(d*x
 + c) + 1) - 3*(((A + 2*C)*a^6*b^2 + 2*(A - 2*C)*a^4*b^4 - (7*A - 2*C)*a^2*
b^6 + 4*A*b^8)*cos(d*x + c)^4 + ((A + 2*C)*a^7*b + 2*(A - 2*C)*a^5*b^3 - (7
*A - 2*C)*a^3*b^5 + 4*A*a*b^7)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(
A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4 + ((2*A + 3*C)*a^7*b + (5*A - 9*C)*a^5*b^3
- (19*A - 6*C)*a^3*b^5 + 12*A*a*b^7)*cos(d*x + c)^3 + ((2*A + 3*C)*a^8 + 2*
(A - 3*C)*a^6*b^2 - (10*A - 3*C)*a^4*b^4 + 6*A*a^2*b^6)*cos(d*x + c)^2 - 2*
(A*a^7*b - 2*A*a^5*b^3 + A*a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2
*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d
*x + c)^3), 1/6*(6*((3*C*a^4*b^3 + (5*A - 2*C)*a^2*b^5 - 4*A*b^7)*cos(d*x +
 c)^4 + (3*C*a^5*b^2 + (5*A - 2*C)*a^3*b^4 - 4*A*a*b^6)*cos(d*x + c)^3)*sqr
```

```
t(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) -
 3*(((A + 2*C)*a^6*b^2 + 2*(A - 2*C)*a^4*b^4 - (7*A - 2*C)*a^2*b^6 + 4*A*b^
8)*cos(d*x + c)^4 + ((A + 2*C)*a^7*b + 2*(A - 2*C)*a^5*b^3 - (7*A - 2*C)*a^
3*b^5 + 4*A*a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) + 3*(((A + 2*C)*a^
6*b^2 + 2*(A - 2*C)*a^4*b^4 - (7*A - 2*C)*a^2*b^6 + 4*A*b^8)*cos(d*x + c)^4
 + ((A + 2*C)*a^7*b + 2*(A - 2*C)*a^5*b^3 - (7*A - 2*C)*a^3*b^5 + 4*A*a*b^7
)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 2*(A*a^8 - 2*A*a^6*b^2 + A*a^4*b
^4 + ((2*A + 3*C)*a^7*b + (5*A - 9*C)*a^5*b^3 - (19*A - 6*C)*a^3*b^5 + 12*A
*a*b^7)*cos(d*x + c)^3 + ((2*A + 3*C)*a^8 + 2*(A - 3*C)*a^6*b^2 - (10*A - 3
*C)*a^4*b^4 + 6*A*a^2*b^6)*cos(d*x + c)^2 - 2*(A*a^7*b - 2*A*a^5*b^3 + A*a^
3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
 + c)^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69331, size = 652, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/3*(6*(3*C*a^4*b^2 + 5*A*a^2*b^4 - 2*C*a^2*b^4 - 4*A*b^6)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - a^5*b^2)*sqrt(a^2 - b^2)) + 6
*(C*a^2*b^3*tan(1/2*d*x + 1/2*c) + A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*
b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 3*(A*
a^2*b + 2*C*a^2*b + 4*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(A*
a^2*b + 2*C*a^2*b + 4*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 2*(3*
```

```
A*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan
(1/2*d*x + 1/2*c)^5 + 9*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 - 6*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 18*A*b^2*tan(1/2*d*x + 1/2*c)
^3 + 3*A*a^2*tan(1/2*d*x + 1/2*c) + 3*C*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*
tan(1/2*d*x + 1/2*c) + 9*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)
^2 - 1)^3*a^4))/d
```

3.578 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=372

$$-\frac{a\left(a^2b^2(2A-21C)+12a^4C-b^4(5A-6C)\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a\left(a^4b^2(2A-29C)-5a^2b^4(A-4C)+12a^6C+6Ab^6\right)\tan^{-1}}{b^5d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((2*A*b^2 + (12*a^2 + b^2)*C)*x)/(2*b^5) - (a*(6*A*b^6 + a^4*b^2*(2*A - 29*
C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) - (a*(a^2*b^2*(2*A - 21*C
) - b^4*(5*A - 6*C) + 12*a^4*C)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) + ((a
^2*b^2*(A - 10*C) - b^4*(4*A - C) + 6*a^4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*
b^3*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((3*A*b^4 - 4*a^4*C + 7*a^2*b^2*C)*C
os[c + d*x]^2*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.48927, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3047, 3049, 3023, 2735, 2659, 205}

$$-\frac{a\left(a^2b^2(2A-21C)+12a^4C-b^4(5A-6C)\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a\left(a^4b^2(2A-29C)-5a^2b^4(A-4C)+12a^6C+6Ab^6\right)\tan^{-1}}{b^5d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*A*b^2 + (12*a^2 + b^2)*C)*x)/(2*b^5) - (a*(6*A*b^6 + a^4*b^2*(2*A - 29*
C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) - (a*(a^2*b^2*(2*A - 21*C
) - b^4*(5*A - 6*C) + 12*a^4*C)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) + ((a
^2*b^2*(A - 10*C) - b^4*(4*A - C) + 6*a^4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*
b^3*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((3*A*b^4 - 4*a^4*C + 7*a^2*b^2*C)*C
os[c + d*x]^2*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(3\left(Ab^2+a^2C\right)-2ab(A+C)\cos(}{(a+b\cos(}}{2b\left(a^2\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(3Ab^4-4a^4C+7a^2b^2C\right)\cos^2(c+}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(}$$

$$= \frac{\left(a^2b^2(A-10C)-b^4(4A-C)+6a^4C\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2\right.}{2b\left(a^2-\right.}$$

$$= -\frac{a\left(a^2b^2(2A-21C)-b^4(5A-6C)+12a^4C\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^2b^2(A-10C)\right.}{}$$

$$= \frac{\left(2Ab^2+\left(12a^2+b^2\right)C\right)x}{2b^5} - \frac{a\left(a^2b^2(2A-21C)-b^4(5A-6C)+12a^4C\right)\sin}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(2Ab^2+\left(12a^2+b^2\right)C\right)x}{2b^5} - \frac{a\left(a^2b^2(2A-21C)-b^4(5A-6C)+12a^4C\right)\sin}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(2Ab^2+\left(12a^2+b^2\right)C\right)x}{2b^5} - \frac{a\left(2a^4Ab^2-5a^2Ab^4+6Ab^6+12a^6C-29a^4b^2\right.}{(a-b)^{5/2}b^5(a+}$$

Mathematica [A] time = 2.39855, size = 256, normalized size = 0.69

$$\frac{2(c+dx)\left(C\left(12a^2+b^2\right)+2Ab^2\right)+\frac{2a^3b\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+\frac{2a^2b\left(a^2b^2(10C-3A)-7a^4C+6Ab^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}+\frac{4a\left(a^4b^2(2A-29C)-5a^2b^4(A-\right.}{}}{4b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(2*A*b^2 + (12*a^2 + b^2)*C)*(c + d*x) + (4*a*(6*A*b^6 + a^4*b^2*(2*A -
29*C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 12*a*b*C*Sin[c + d*x] + (2*a^3*b*(A
*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (2*a
```

```
^2*b*(6*A*b^4 - 7*a^4*C + a^2*b^2*(-3*A + 10*C))*Sin[c + d*x])/((a - b)^2*(
a + b)^2*(a + b*Cos[c + d*x])) + b^2*C*Sin[2*(c + d*x)])/(4*b^5*d)
```

Maple [B] time = 0.046, size = 1428, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-6/d/b^4/(tan(1
/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C+2/d/b^3*arctan(tan(1/2*d*x+1/2*
c))*A+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*C-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*a^
3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan
(1/2*d*x+1/2*c)*A-1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+10/d
*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2
*d*x+1/2*c)*C+12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2*C-1/d/b^3/(tan(1/2*d*
x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*C-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-12/d*a^7
/b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*C+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/
2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-20/d*a^3/b/(a^4-2
*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*C+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*a^2/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-2/d*a^5
/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*A+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*a^6/b^4/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*C+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C
-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99712, size = 3391, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [1/4*(2*(12*C*a^8*b^2 + (2*A - 35*C)*a^6*b^4 - 3*(2*A - 11*C)*a^4*b^6 + 3*(
2*A - 3*C)*a^2*b^8 - (2*A + C)*b^10)*d*x*cos(d*x + c)^2 + 4*(12*C*a^9*b + (
2*A - 35*C)*a^7*b^3 - 3*(2*A - 11*C)*a^5*b^5 + 3*(2*A - 3*C)*a^3*b^7 - (2*A
 + C)*a*b^9)*d*x*cos(d*x + c) + 2*(12*C*a^10 + (2*A - 35*C)*a^8*b^2 - 3*(2*
A - 11*C)*a^6*b^4 + 3*(2*A - 3*C)*a^4*b^6 - (2*A + C)*a^2*b^8)*d*x - (12*C*
a^9 + (2*A - 29*C)*a^7*b^2 - 5*(A - 4*C)*a^5*b^4 + 6*A*a^3*b^6 + (12*C*a^7*
b^2 + (2*A - 29*C)*a^5*b^4 - 5*(A - 4*C)*a^3*b^6 + 6*A*a*b^8)*cos(d*x + c)^
2 + 2*(12*C*a^8*b + (2*A - 29*C)*a^6*b^3 - 5*(A - 4*C)*a^4*b^5 + 6*A*a^2*b^
7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
+ 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*C*a^9*b +
 (2*A - 33*C)*a^7*b^3 - (7*A - 27*C)*a^5*b^5 + (5*A - 6*C)*a^3*b^7 - (C*a^6
*b^4 - 3*C*a^4*b^6 + 3*C*a^2*b^8 - C*b^10)*cos(d*x + c)^3 + 4*(C*a^7*b^3 -
3*C*a^5*b^5 + 3*C*a^3*b^7 - C*a*b^9)*cos(d*x + c)^2 + (18*C*a^8*b^2 + (3*A
- 50*C)*a^6*b^4 - (9*A - 43*C)*a^4*b^6 + (6*A - 11*C)*a^2*b^8)*cos(d*x + c)
)*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2
 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5
- 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*C*a^8*b^2 + (2*A - 35*C)*a
^6*b^4 - 3*(2*A - 11*C)*a^4*b^6 + 3*(2*A - 3*C)*a^2*b^8 - (2*A + C)*b^10)*d
*x*cos(d*x + c)^2 + 2*(12*C*a^9*b + (2*A - 35*C)*a^7*b^3 - 3*(2*A - 11*C)*a
```

```
^5*b^5 + 3*(2*A - 3*C)*a^3*b^7 - (2*A + C)*a*b^9)*d*x*cos(d*x + c) + (12*C*
a^10 + (2*A - 35*C)*a^8*b^2 - 3*(2*A - 11*C)*a^6*b^4 + 3*(2*A - 3*C)*a^4*b^
6 - (2*A + C)*a^2*b^8)*d*x - (12*C*a^9 + (2*A - 29*C)*a^7*b^2 - 5*(A - 4*C)
*a^5*b^4 + 6*A*a^3*b^6 + (12*C*a^7*b^2 + (2*A - 29*C)*a^5*b^4 - 5*(A - 4*C)
*a^3*b^6 + 6*A*a*b^8)*cos(d*x + c)^2 + 2*(12*C*a^8*b + (2*A - 29*C)*a^6*b^3
 - 5*(A - 4*C)*a^4*b^5 + 6*A*a^2*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(
-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*C*a^9*b + (2*A
- 33*C)*a^7*b^3 - (7*A - 27*C)*a^5*b^5 + (5*A - 6*C)*a^3*b^7 - (C*a^6*b^4 -
 3*C*a^4*b^6 + 3*C*a^2*b^8 - C*b^10)*cos(d*x + c)^3 + 4*(C*a^7*b^3 - 3*C*a^
5*b^5 + 3*C*a^3*b^7 - C*a*b^9)*cos(d*x + c)^2 + (18*C*a^8*b^2 + (3*A - 50*C
)*a^6*b^4 - (9*A - 43*C)*a^4*b^6 + (6*A - 11*C)*a^2*b^8)*cos(d*x + c))*sin(
d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2*(
a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*a^
6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72114, size = 1544, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/2*(2*(12*C*a^7 + 2*A*a^5*b^2 - 29*C*a^5*b^2 - 5*A*a^3*b^4 + 20*C*a^3*b^4
+ 6*A*a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^5
 - 2*a^2*b^7 + b^9)*sqrt(a^2 - b^2)) - 2*(12*C*a^7*tan(1/2*d*x + 1/2*c)^7 -
 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 2*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 1
```

```
7*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 3
3*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2
*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13
*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^7 + C*b^
7*tan(1/2*d*x + 1/2*c)^7 + 36*C*a^7*tan(1/2*d*x + 1/2*c)^5 - 18*C*a^6*b*tan
(1/2*d*x + 1/2*c)^5 + 6*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 67*C*a^5*b^2*tan
(1/2*d*x + 1/2*c)^5 - 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*C*a^4*b^3*tan
(1/2*d*x + 1/2*c)^5 - 15*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 26*C*a^3*b^4*ta
n(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 5*C*a^2*b^5*tan
(1/2*d*x + 1/2*c)^5 - 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 3*C*b^7*tan(1/2*d*
x + 1/2*c)^5 + 36*C*a^7*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^6*b*tan(1/2*d*x + 1
/2*c)^3 + 6*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*C*a^5*b^2*tan(1/2*d*x + 1
/2*c)^3 + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*C*a^4*b^3*tan(1/2*d*x + 1
/2*c)^3 - 15*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 26*C*a^3*b^4*tan(1/2*d*x +
1/2*c)^3 - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*C*a^2*b^5*tan(1/2*d*x + 1
/2*c)^3 - 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 3*C*b^7*tan(1/2*d*x + 1/2*c)^3
 + 12*C*a^7*tan(1/2*d*x + 1/2*c) + 18*C*a^6*b*tan(1/2*d*x + 1/2*c) + 2*A*a^
5*b^2*tan(1/2*d*x + 1/2*c) - 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c) + 3*A*a^4*b^
3*tan(1/2*d*x + 1/2*c) - 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c) - 5*A*a^3*b^4*ta
n(1/2*d*x + 1/2*c) - 2*C*a^3*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^5*tan(1/2
*d*x + 1/2*c) + 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c) + 4*C*a*b^6*tan(1/2*d*x +
 1/2*c) - C*b^7*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 +
 a + b)^2) + (12*C*a^2 + 2*A*b^2 + C*b^2)*(d*x + c)/b^5)/d
```

3.579 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=262

$$\frac{\left(3a^2C+Ab^2-2b^2C\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)-15a^4b^2C+6a^6C+2Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2C+Ab^2\right)}{2bd\left(a^2-b\right.}$$

```
[Out] (-3*a*C*x)/b^4 + ((2*A*b^6 + 6*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*A
rcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a +
b)^(5/2)*d) + ((A*b^2 + 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)
*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a +
 b*Cos[c + d*x])^2) - (a*(2*A*b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Sin[c + d*
x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.8414, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3031, 3023, 2735, 2659, 205}

$$\frac{\left(3a^2C+Ab^2-2b^2C\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)-15a^4b^2C+6a^6C+2Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2C+Ab^2\right)}{2bd\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*a*C*x)/b^4 + ((2*A*b^6 + 6*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*A
rcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a +
b)^(5/2)*d) + ((A*b^2 + 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)
*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a +
 b*Cos[c + d*x])^2) - (a*(2*A*b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Sin[c + d*
x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(2\left(Ab^2+a^2C\right)-2ab(A+C)\cos(c+}{(a+b\cos(c+}}{2b\left(a^2-\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\text{si}}{2b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - $$

$$= -\frac{3aCx}{b^4} + \frac{\left(Ab^2+3a^2C-2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(}{2b\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= -\frac{3aCx}{b^4} + \frac{\left(Ab^2+3a^2C-2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(}{2b\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= -\frac{3aCx}{b^4} + \frac{\left(a^2Ab^4+2Ab^6+6a^6C-15a^4b^2C+12a^2b^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

Mathematica [A] time = 1.70445, size = 214, normalized size = 0.82

$$\frac{-\frac{a^2b\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{ab\left(a^2b^2(A-8C)+5a^4C-4Ab^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(a^2b^4(A+12C)-15a^4b^2C+6a^6C+2Ab^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} - 6aC(c+}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-6*a*C*(c + d*x) - (2*(2*A*b^6 + 6*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*
C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2
) + 2*b*C*Sin[c + d*x] - (a^2*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a +
```

```
 b)*(a + b*Cos[c + d*x])^2) + (a*b*(-4*A*b^4 + a^2*b^2*(A - 8*C) + 5*a^4*C)
*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^4*d)
```

Maple [B] time = 0.04, size = 1094, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*C/b^3*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)-6/d*C/b^4*a*arctan(ta
n(1/2*d*x+1/2*c))-1/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*b/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*A+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*C-8/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A-4/d*b/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2
)*tan(1/2*d*x+1/2*c)*A+4/d/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2*a^5/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*C+1/d/b^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^4/(a+b)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c)*C-8/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
*a^3/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*C+1/d/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*A
+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*A+6/d/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C-15/d/b^2/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*a^4*C+12/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53466, size = 2638, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [-1/4*(12*(C*a^7*b^2 - 3*C*a^5*b^4 + 3*C*a^3*b^6 - C*a*b^8)*d*x*cos(d*x + c
)^2 + 24*(C*a^8*b - 3*C*a^6*b^3 + 3*C*a^4*b^5 - C*a^2*b^7)*d*x*cos(d*x + c)
 + 12*(C*a^9 - 3*C*a^7*b^2 + 3*C*a^5*b^4 - C*a^3*b^6)*d*x + (6*C*a^8 - 15*C
*a^6*b^2 + (A + 12*C)*a^4*b^4 + 2*A*a^2*b^6 + (6*C*a^6*b^2 - 15*C*a^4*b^4 +
 (A + 12*C)*a^2*b^6 + 2*A*b^8)*cos(d*x + c)^2 + 2*(6*C*a^7*b - 15*C*a^5*b^3
 + (A + 12*C)*a^3*b^5 + 2*A*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(6*C*a^8*b - 17*C*a^6*b^3 - (3*A - 13*C)*a^4*b^5 + (3*A
- 2*C)*a^2*b^7 + 2*(C*a^6*b^3 - 3*C*a^4*b^5 + 3*C*a^2*b^7 - C*b^9)*cos(d*x
+ c)^2 + (9*C*a^7*b^2 + (A - 25*C)*a^5*b^4 - 5*(A - 4*C)*a^3*b^6 + 4*(A - C
)*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b
^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(
d*x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(6*(C*a^7*
b^2 - 3*C*a^5*b^4 + 3*C*a^3*b^6 - C*a*b^8)*d*x*cos(d*x + c)^2 + 12*(C*a^8*b
 - 3*C*a^6*b^3 + 3*C*a^4*b^5 - C*a^2*b^7)*d*x*cos(d*x + c) + 6*(C*a^9 - 3*C
*a^7*b^2 + 3*C*a^5*b^4 - C*a^3*b^6)*d*x - (6*C*a^8 - 15*C*a^6*b^2 + (A + 12
*C)*a^4*b^4 + 2*A*a^2*b^6 + (6*C*a^6*b^2 - 15*C*a^4*b^4 + (A + 12*C)*a^2*b^
6 + 2*A*b^8)*cos(d*x + c)^2 + 2*(6*C*a^7*b - 15*C*a^5*b^3 + (A + 12*C)*a^3*
b^5 + 2*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)
/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^8*b - 17*C*a^6*b^3 - (3*A - 13*C)
*a^4*b^5 + (3*A - 2*C)*a^2*b^7 + 2*(C*a^6*b^3 - 3*C*a^4*b^5 + 3*C*a^2*b^7 -
 C*b^9)*cos(d*x + c)^2 + (9*C*a^7*b^2 + (A - 25*C)*a^5*b^4 - 5*(A - 4*C)*a^
3*b^6 + 4*(A - C)*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8
+ 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9
- a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72454, size = 660, normalized size = 2.52

$$-\frac{\frac{\left(6\,Ca^6-15\,Ca^4b^2+Aa^2b^4+12\,Ca^2b^4+2\,Ab^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2\,a^2b^6+b^8\right)\sqrt{a^2-b^2}}+\frac{3\,(dx+c)Ca}{b^4}-\frac{4\,Ca^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -((6*C*a^6 - 15*C*a^4*b^2 + A*a^2*b^4 + 12*C*a^2*b^4 + 2*A*b^6)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt
(a^2 - b^2)) + 3*(d*x + c)*C*a/b^4 - (4*C*a^6*tan(1/2*d*x + 1/2*c)^3 - 5*C*
a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - A*a^3*b
^3*tan(1/2*d*x + 1/2*c)^3 + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^2*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*A*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*C*a^6*tan(1
/2*d*x + 1/2*c) + 5*C*a^5*b*tan(1/2*d*x + 1/2*c) - 7*C*a^4*b^2*tan(1/2*d*x
+ 1/2*c) + A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c
) - 3*A*a^2*b^4*tan(1/2*d*x + 1/2*c) - 4*A*a*b^5*tan(1/2*d*x + 1/2*c))/((a^
4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)
^2 + a + b)^2) - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3
))/d
```

3.580 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=203

$$-\frac{a\left(C\left(-5a^2b^2+2a^4+6b^4\right)+3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2b^2(A+6C)-3a^4C+2Ab^4\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a\left(a^2C\right.}{2b^2d\left(a^2-\right.}$$

```
[Out] (C*x)/b^3 - (a*(3*A*b^4 + (2*a^4 - 5*a^2*b^2 + 6*b^4)*C)*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) + (a*
(A*b^2 + a^2*C)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2)
+ ((2*A*b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Sin[c + d*x])/(2*b^2*(a^2 - b^2)
^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.464477, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3032, 3021, 2735, 2659, 205}

$$-\frac{a\left(C\left(-5a^2b^2+2a^4+6b^4\right)+3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2b^2(A+6C)-3a^4C+2Ab^4\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a\left(a^2C\right.}{2b^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (C*x)/b^3 - (a*(3*A*b^4 + (2*a^4 - 5*a^2*b^2 + 6*b^4)*C)*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) + (a*
(A*b^2 + a^2*C)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2)
+ ((2*A*b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Sin[c + d*x])/(2*b^2*(a^2 - b^2)
^2*d*(a + b*Cos[c + d*x]))
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
```

```
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b\left(Ab^2+a^2C\right)-a\left(Ab^2-\left(a^2-2b^2\right)C\right)\cos(c+dx)-2b(a}{(a+b\cos(c+dx))^2}}{2b^2\left(a^2-b^2\right)}$$

$$= \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\sin(c+d}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= \frac{Cx}{b^3} + \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\sin}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{Cx}{b^3} + \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\sin}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{Cx}{b^3} - \frac{a\left(3Ab^4+2a^4C-5a^2b^2C+6b^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} + \frac{a\left(Ab\right.}{2b^2\left(a^2-\right.}$$

Mathematica [A] time = 1.28578, size = 194, normalized size = 0.96

$$\frac{\frac{b\left(a^2b^2(A+6C)-3a^4C+2Ab^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{ab\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{2a\left(C\left(-5a^2b^2+2a^4+6b^4\right)+3Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + 2C(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*C*(c + d*x) + (2*a*(3*A*b^4 + (2*a^4 - 5*a^2*b^2 + 6*b^4)*C)*ArcTanh[((a
 - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*b*(A*b^2
 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (b*(2*A*
b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a +
b*Cos[c + d*x])))/(2*b^3*d)
```

Maple [B] time = 0.04, size = 1093, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*C+2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*A+2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^4/b^2/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*C+1/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2*a^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+6/d/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*a^2*C+2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a*A+2/d*b^2/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A
-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a
-b)^2*tan(1/2*d*x+1/2*c)*C-1/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^3*C+6/d/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^2*C-3/
d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*A-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/
2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/d*a^3/b/(a^4-2*
a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*C-6/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27732, size = 2279, normalized size = 11.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(4*(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8)*d*x*cos(d*x + c)^2
+ 8*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*d*x*cos(d*x + c) + 4*(C
*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*d*x - (2*C*a^7 - 5*C*a^5*b^2
+ 3*(A + 2*C)*a^3*b^4 + (2*C*a^5*b^2 - 5*C*a^3*b^4 + 3*(A + 2*C)*a*b^6)*cos
(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 + 3*(A + 2*C)*a^2*b^5)*cos(d*x + c
))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
- 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*C*a^7*b - (2*A + 7*C)*a^
5*b^3 + (A + 5*C)*a^3*b^5 + A*a*b^7 + (3*C*a^6*b^2 - (A + 9*C)*a^4*b^4 - (A
 - 6*C)*a^2*b^6 + 2*A*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^4*b^
7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8
 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d),
 1/2*(2*(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8)*d*x*cos(d*x + c)^2
+ 4*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*d*x*cos(d*x + c) + 2*(C
*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*d*x - (2*C*a^7 - 5*C*a^5*b^2
+ 3*(A + 2*C)*a^3*b^4 + (2*C*a^5*b^2 - 5*C*a^3*b^4 + 3*(A + 2*C)*a*b^6)*cos
(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 + 3*(A + 2*C)*a^2*b^5)*cos(d*x + c
))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x +
c))) - (2*C*a^7*b - (2*A + 7*C)*a^5*b^3 + (A + 5*C)*a^3*b^5 + A*a*b^7 + (3*
C*a^6*b^2 - (A + 9*C)*a^4*b^4 - (A - 6*C)*a^2*b^6 + 2*A*b^8)*cos(d*x + c))*
sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 +
2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*
a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

[Out] Timed out

Giac [B] time = 1.33529, size = 647, normalized size = 3.19

$$-\frac{\frac{\left(2\,Ca^5-5\,Ca^3b^2+3\,Aab^4+6\,Cab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{a^2-b^2}}-\frac{(dx+c)C}{b^3}+\frac{2\,Ca^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca^4b\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((2*C*a^5 - 5*C*a^3*b^2 + 3*A*a*b^4 + 6*C*a*b^4)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) - (
d*x + c)*C/b^3 + (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^4*b*tan(1/2*d*x +
1/2*c)^3 - 2*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^2*tan(1/2*d*x + 1
/2*c)^3 + A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^3*tan(1/2*d*x + 1/2*
c)^3 - A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*
C*a^5*tan(1/2*d*x + 1/2*c) + 3*C*a^4*b*tan(1/2*d*x + 1/2*c) - 2*A*a^3*b^2*t
an(1/2*d*x + 1/2*c) - 5*C*a^3*b^2*tan(1/2*d*x + 1/2*c) - A*a^2*b^3*tan(1/2*
d*x + 1/2*c) - 6*C*a^2*b^3*tan(1/2*d*x + 1/2*c) - A*a*b^4*tan(1/2*d*x + 1/2
*c) - 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.581 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=177

$$\frac{\left(a^2(2A+C)+b^2(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(a^2(-C)+3Ab^2+4b^2C\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{\left(a^2C+Ab^2\right)\sin(c\ \cdots}{2bd\left(a^2-b^2\right)(a+b\cos(\cdots}$$

```
[Out] ((a^2*(2*A + C) + b^2*(A + 2*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/
(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a*(3*A*b^2 - a^2*C + 4*b^2*C)
*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.262513, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3022, 2754, 12, 2659, 205}

$$\frac{\left(a^2(2A+C)+b^2(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(a^2(-C)+3Ab^2+4b^2C\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{\left(a^2C+Ab^2\right)\sin(c\ \cdots}{2bd\left(a^2-b^2\right)(a+b\cos(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((a^2*(2*A + C) + b^2*(A + 2*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/
(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a*(3*A*b^2 - a^2*C + 4*b^2*C)
*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2ab(A+C)+\left(Ab^2-a^2C+2b^2C\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{b\left(a^2(2A+C)+b^2(\cdots\right.}{a+b\cos(c+\cdots}}{2b\left(a^2-b\cdots\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(a^2(2A+C)+\cdots\right.}{\cdots}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(a^2(2A+C)+\cdots\right.}{\cdots}$$

$$= \frac{\left(2a^2A+Ab^2+a^2C+2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \cdots$$

Mathematica [A] time = 0.785877, size = 170, normalized size = 0.96

$$\frac{\frac{a\left(C\left(a^2-4b^2\right)-3Ab^2\right)\sin(c+dx)}{b(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{b(b-a)(a+b)(a+b\cos(c+dx))^2} - \frac{2\left(a^2(2A+C)+b^2(A+2C)\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(a^2*(2*A + C) + b^2*(A + 2*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(b*(-a
+ b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*(-3*A*b^2 + (a^2 - 4*b^2)*C)*Sin[
c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.033, size = 810, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*a^2*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*b*C-4/d*b/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)*A+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a
^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*b^2+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*C-4/d/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)*a*b*C+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*A+1/d*b^2/(a^4-2*a^2*b^2+
b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*A+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04314, size = 1562, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*A + C)*a^4 + (A + 2*C)*a^2*b^2 + ((2*A + C)*a^2*b^2 + (A + 2*C)*
b^4)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b + (A + 2*C)*a*b^3)*cos(d*x + c))*s
```

```
qrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*
sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((4*A + 3*C)*a^4*b - (5*A + 3*C
)*a^2*b^3 + A*b^5 - (C*a^5 - (3*A + 5*C)*a^3*b^2 + (3*A + 4*C)*a*b^4)*cos(d
*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x +
 c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3
*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*(((2*A + C)*a^4 + (A + 2*C)*a^2*b^2
 + ((2*A + C)*a^2*b^2 + (A + 2*C)*b^4)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b
+ (A + 2*C)*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((4*A + 3*C)*a^4*b - (5*A + 3*C)*a^2*b
^3 + A*b^5 - (C*a^5 - (3*A + 5*C)*a^3*b^2 + (3*A + 4*C)*a*b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 +
 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^
2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38806, size = 498, normalized size = 2.81

$$\frac{\left(2\,Aa^2+Ca^2+Ab^2+2\,Cb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Ca^2b\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*A*a^2 + C*a^2 + A*b^2 + 2*C*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) - (C*a^3*tan(1/2*d*x
```

```
 + 1/2*c)^3 + 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^2*b*tan(1/2*d*x + 1/
2*c)^3 - 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - A*b^3*tan(1/2*d*x + 1/2*c)^3 - C*a^3*tan(1/2*d*x + 1/2*c) + 4*A*a^2*b*t
an(1/2*d*x + 1/2*c) + 3*C*a^2*b*tan(1/2*d*x + 1/2*c) + 3*A*a*b^2*tan(1/2*d*
x + 1/2*c) + 4*C*a*b^2*tan(1/2*d*x + 1/2*c) - A*b^3*tan(1/2*d*x + 1/2*c))/(
(a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 + a + b)^2))/d
```

3.582 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$\frac{b\left(5a^2Ab^2-3a^4(2A+C)-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-a^2b^2(5A+2C)+a^4(-C)+2Ab^4\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{(a^2}{2ad\,(a^2}$$

```
[Out] (b*(5*a^2*A*b^2 - 2*A*b^4 - 3*a^4*(2*A + C))*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (A*ArcTanh[Sin[
c + d*x]])/(a^3*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a +
 b*Cos[c + d*x])^2) - ((2*A*b^4 - a^4*C - a^2*b^2*(5*A + 2*C))*Sin[c + d*x]
)/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.648892, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(5a^2Ab^2-3a^4(2A+C)-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-a^2b^2(5A+2C)+a^4(-C)+2Ab^4\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{(a^2}{2ad\,(a^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (b*(5*a^2*A*b^2 - 2*A*b^4 - 3*a^4*(2*A + C))*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (A*ArcTanh[Sin[
c + d*x]])/(a^3*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a +
 b*Cos[c + d*x])^2) - ((2*A*b^4 - a^4*C - a^2*b^2*(5*A + 2*C))*Sin[c + d*x]
)/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
```

```
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2A\left(a^2-b^2\right)-2ab(A+C)\cos(c+dx)+\left(Ab^2+a^2C\right)\cos^2}{(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a^2b^2(5A+2C)\right)\sin(c+dx}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a^2b^2(5A+2C)\right)\sin(c+dx}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a\right.}{2a^2\left(a^2-b^2\right.}$$

$$= -\frac{b\left(6a^4A-5a^2Ab^2+2Ab^4+3a^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{A\tanh^{-1}(\sin}{a^3d}$$

Mathematica [C] time = 3.58006, size = 409, normalized size = 1.94

$$\frac{\cos(c+dx)(A\sec(c+dx)+C\cos(c+dx))\left(\frac{4b(\sin(c)+i\cos(c))\left(-5a^2Ab^2+3a^4(2A+C)+2Ab^4\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(b\cos(c)-a)+b\sin(c)\right)}{\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\right)}{\left(a^2-b^2\right)^2\sqrt{\left(b^2-a^2\right)(\cos(c)-i\sin(c))^2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(C*Cos[c + d*x] + A*Sec[c + d*x])*(-4*A*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 4*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (4*b*(
-5*a^2*A*b^2 + 2*A*b^4 + 3*a^4*(2*A + C))*ArcTan[((I*Cos[c] + Sin[c])*(b*Si
n[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c]
```

```
)^2)]]*(I*Cos[c] + Sin[c]))/((a^2 - b^2)^2*Sqrt[(-a^2 + b^2)*(Cos[c] - I*Si
n[c])^2]) - (a*Sec[c]*((2*a^2 + b^2)*(-2*A*b^4 + a^4*C + a^2*b^2*(5*A + 2*C
))*Sin[c] + b*(-(a*(-7*A*b^4 + 4*a^4*C + a^2*b^2*(16*A + 5*C))*Sin[d*x]) +
b*(a*b*(-(A*b^2) + a^2*(4*A + 3*C))*Sin[2*c + d*x] - (-2*A*b^4 + a^4*C + a^
2*b^2*(5*A + 2*C))*Sin[c + 2*d*x]))))/(b*(a^2 - b^2)^2*(a + b*Cos[c + d*x])
^2)))/(2*a^3*d*(2*A + C + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.073, size = 1115, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^3-2/d/a^2/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*A*b^4+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2*C+1/d/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*a*b*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C+6/d*b^2/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d/a
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/
2*d*x+1/2*c)*A*b^3-2/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4+2/d/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^2*C-1/d*a/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*b*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b^2*C-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b
))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+5/d/a*b^3/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*A-2/d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-3/d*b*a/(a^4-2*a^2*b^2+b
^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*C-1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 46.9688, size = 2851, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*((3*(2*A + C)*a^6*b - 5*A*a^4*b^3 + 2*A*a^2*b^5 + (3*(2*A + C)*a^4*b^
3 - 5*A*a^2*b^5 + 2*A*b^7)*cos(d*x + c)^2 + 2*(3*(2*A + C)*a^5*b^2 - 5*A*a^
3*b^4 + 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
 (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin
(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) -
 2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^
4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^
3*b^5 - A*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(A*a^8 - 3*A*a^6*b
^2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b
^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(
d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*C*a^8 + (6*A - C)*a^6*b^2 - (9*A +
C)*a^4*b^4 + 3*A*a^2*b^6 + (C*a^7*b + (5*A + C)*a^5*b^3 - (7*A + 2*C)*a^3*b
^5 + 2*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b
^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^
7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), -1/2*((3*(
2*A + C)*a^6*b - 5*A*a^4*b^3 + 2*A*a^2*b^5 + (3*(2*A + C)*a^4*b^3 - 5*A*a^2
*b^5 + 2*A*b^7)*cos(d*x + c)^2 + 2*(3*(2*A + C)*a^5*b^2 - 5*A*a^3*b^4 + 2*A
*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^
2 - b^2)*sin(d*x + c))) - (A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6 +
(A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^7*b
 - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1)
 + (A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^
```

```
4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^
3*b^5 - A*a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (2*C*a^8 + (6*A - C
)*a^6*b^2 - (9*A + C)*a^4*b^4 + 3*A*a^2*b^6 + (C*a^7*b + (5*A + C)*a^5*b^3
- (7*A + 2*C)*a^3*b^5 + 2*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 -
3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 +
 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*
b^6)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\cos\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

Giac [B] time = 1.68566, size = 680, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((6*A*a^4*b + 3*C*a^4*b - 5*A*a^2*b^3 + 2*A*b^5)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) - A
*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a^3 - (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - C*a^4*b*tan(1/2*d*x + 1/2*c)^3
+ 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5
*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*
A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*C*a^5*t
an(1/2*d*x + 1/2*c) + C*a^4*b*tan(1/2*d*x + 1/2*c) + 6*A*a^3*b^2*tan(1/2*d*
x + 1/2*c) + C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*A*a^2*b^3*tan(1/2*d*x + 1/2
*c) + 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c) - 2
```

```
*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.583 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=275

$$-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-2a^6C-6Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(11a^2Ab^2+a^4(-(2A-3C))-6Ab^4\right)\tan(c+}{2a^3d\left(a^2-b^2\right)^2}$$

```
[Out] -(((15*a^2*A*b^4 - 6*A*b^6 - 2*a^6*C - a^4*b^2*(12*A + C))*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d)) -
(3*A*b*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((11*a^2*A*b^2 - 6*A*b^4 - a^4*(2*A
 - 3*C))*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Tan[c + d
*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b^4 - 2*a^4*C - a^2
*b^2*(6*A + C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.18937, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-2a^6C-6Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(11a^2Ab^2+a^4(-(2A-3C))-6Ab^4\right)\tan(c+}{2a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -(((15*a^2*A*b^4 - 6*A*b^6 - 2*a^6*C - a^4*b^2*(12*A + C))*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d)) -
(3*A*b*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((11*a^2*A*b^2 - 6*A*b^4 - a^4*(2*A
 - 3*C))*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Tan[c + d
*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b^4 - 2*a^4*C - a^2
*b^2*(6*A + C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
```

```
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(-3Ab^2+a^2(2A-C)-2ab(A+C)\cos(c+dx)+2\left(Ab^2\right.\right.}{(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3Ab^4-2a^4C-a^2b^2(6A+C)\right)\tan(c+}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

$$= -\frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\tan(c+}{2a\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= -\frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\tan(c+}{2a\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= -\frac{3Ab\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(12a^4Ab^2-15a^2Ab^4+6Ab^6+2a^6C+a^4b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3}{}$$

Mathematica [B] time = 6.33256, size = 649, normalized size = 2.36

$$\frac{\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(-a^2bC\sin(c+dx)-Ab^3\sin(c+dx)\right)}{a^2d(a-b)(a+b)(a+b\cos(c+dx))^2(2A+C\cos(2c+2dx)+C)} + \frac{\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(-7a^2Ab\right.}{a^3d(a-b)^2(a+b)^2(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-2*(12*a^4*A*b^2 - 15*a^2*A*b^4 + 6*A*b^6 + 2*a^6*C + a^4*b^2*C)*ArcTanh[(
(a - b)*Tan[(c + d*x)/2]]/Sqrt[-a^2 + b^2]]*Cos[c + d*x]^2*(C + A*Sec[c + d
*x]^2))/(a^4*(a^2 - b^2)^2*Sqrt[-a^2 + b^2]*d*(2*A + C + C*Cos[2*c + 2*d*x]
)) + (6*A*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(C + A*
Sec[c + d*x]^2))/(a^4*d*(2*A + C + C*Cos[2*c + 2*d*x])) - (6*A*b*Cos[c + d*
x]^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(C + A*Sec[c + d*x]^2))/(a^4*
d*(2*A + C + C*Cos[2*c + 2*d*x])) + (2*A*Cos[c + d*x]^2*(C + A*Sec[c + d*x]
^2)*Sin[(c + d*x)/2])/(a^3*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + (2*A*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*Sin[(c
 + d*x)/2])/(a^3*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) + (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(-(A*b^3*Sin[c + d*x
]) - a^2*b*C*Sin[c + d*x]))/(a^2*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])^2*(
2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(-7
*a^2*A*b^3*Sin[c + d*x] + 4*A*b^5*Sin[c + d*x] - 3*a^4*b*C*Sin[c + d*x]))/(
a^3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])*(2*A + C + C*Cos[2*c + 2*d*x
]))
```

Maple [B] time = 0.085, size = 1129, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -8/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^3-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+4/d/
a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^5/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*b*C-1/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*b^2*C-8/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2*b^3/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A+1/d/a^2/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)*A+4/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b
^5/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A-4/d/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*b*C
+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a^2-2
*a*b+b^2)*tan(1/2*d*x+1/2*c)*C+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-15/d/a^2/(a^4-2
*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
```

```
-b))^(1/2))*A*b^4+6/d/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6+2/d/(a^4-2*a^2*b^2+b^4)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a
^2+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*b^2*C-1/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)+3/d*A*b/a^4
*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-3/d*A*b/a^4*ln(t
an(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 82.808, size = 3401, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [-1/4*(((2*C*a^6*b^2 + (12*A + C)*a^4*b^4 - 15*A*a^2*b^6 + 6*A*b^8)*cos(d*x
 + c)^3 + 2*(2*C*a^7*b + (12*A + C)*a^5*b^3 - 15*A*a^3*b^5 + 6*A*a*b^7)*cos
(d*x + c)^2 + (2*C*a^8 + (12*A + C)*a^6*b^2 - 15*A*a^4*b^4 + 6*A*a^2*b^6)*c
os(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*((A*a^6*b^3 - 3*A
*a^4*b^5 + 3*A*a^2*b^7 - A*b^9)*cos(d*x + c)^3 + 2*(A*a^7*b^2 - 3*A*a^5*b^4
 + 3*A*a^3*b^6 - A*a*b^8)*cos(d*x + c)^2 + (A*a^8*b - 3*A*a^6*b^3 + 3*A*a^4
*b^5 - A*a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*((A*a^6*b^3 - 3*A
*a^4*b^5 + 3*A*a^2*b^7 - A*b^9)*cos(d*x + c)^3 + 2*(A*a^7*b^2 - 3*A*a^5*b^4
 + 3*A*a^3*b^6 - A*a*b^8)*cos(d*x + c)^2 + (A*a^8*b - 3*A*a^6*b^3 + 3*A*a^4
*b^5 - A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*A*a^9 - 6*A*a
```

```
^7*b^2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + ((2*A - 3*C)*a^7*b^2 - (13*A - 3*C)*a^
5*b^4 + 17*A*a^3*b^6 - 6*A*a*b^8)*cos(d*x + c)^2 + (4*(A - C)*a^8*b - 5*(4*
A - C)*a^6*b^3 + (25*A - C)*a^4*b^5 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x +
c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^1
1*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^
2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)), 1/2*(((2*C*a^6*b^2 + (12*A + C)*a
^4*b^4 - 15*A*a^2*b^6 + 6*A*b^8)*cos(d*x + c)^3 + 2*(2*C*a^7*b + (12*A + C)
*a^5*b^3 - 15*A*a^3*b^5 + 6*A*a*b^7)*cos(d*x + c)^2 + (2*C*a^8 + (12*A + C)
*a^6*b^2 - 15*A*a^4*b^4 + 6*A*a^2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*((A*a^6*b^3 - 3*
A*a^4*b^5 + 3*A*a^2*b^7 - A*b^9)*cos(d*x + c)^3 + 2*(A*a^7*b^2 - 3*A*a^5*b^
4 + 3*A*a^3*b^6 - A*a*b^8)*cos(d*x + c)^2 + (A*a^8*b - 3*A*a^6*b^3 + 3*A*a^
4*b^5 - A*a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*((A*a^6*b^3 - 3*
A*a^4*b^5 + 3*A*a^2*b^7 - A*b^9)*cos(d*x + c)^3 + 2*(A*a^7*b^2 - 3*A*a^5*b^
4 + 3*A*a^3*b^6 - A*a*b^8)*cos(d*x + c)^2 + (A*a^8*b - 3*A*a^6*b^3 + 3*A*a^
4*b^5 - A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*A*a^9 - 6*A*a^
7*b^2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + ((2*A - 3*C)*a^7*b^2 - (13*A - 3*C)*a^5
*b^4 + 17*A*a^3*b^6 - 6*A*a*b^8)*cos(d*x + c)^2 + (4*(A - C)*a^8*b - 5*(4*A
 - C)*a^6*b^3 + (25*A - C)*a^4*b^5 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c
))/((a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11
*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2
 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63396, size = 698, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -((2*C*a^6 + 12*A*a^4*b^2 + C*a^4*b^2 - 15*A*a^2*b^4 + 6*A*b^6)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt
(a^2 - b^2)) + 3*A*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3*A*b*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a^4 + (4*C*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 3*C*a
^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - C*a^3*
b^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*A*a*b^5
*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*C*a^5*b*tan(1/
2*d*x + 1/2*c) + 3*C*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b^3*tan(1/2*d*x
 + 1/2*c) - C*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*A*a^2*b^4*tan(1/2*d*x + 1/2*
c) - 5*A*a*b^5*tan(1/2*d*x + 1/2*c) - 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 -
 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
+ a + b)^2) + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/
d
```

3.584 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=378

$$-\frac{b\left(5a^4b^2(4A-C)-a^2b^4(29A-2C)+6a^6C+12Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{b\left(-a^2b^2(21A-2C)+a^4(6A-5C)+\right.}{2a^4d\left(a^2-b^2\right)^2}$$

```
[Out] -((b*(12*A*b^6 - a^2*b^4*(29*A - 2*C) + 5*a^4*b^2*(4*A - C) + 6*a^6*C)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^
(5/2)*d)) + ((12*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^5*d) -
(b*(12*A*b^4 + a^4*(6*A - 5*C) - a^2*b^2*(21*A - 2*C))*Tan[c + d*x])/(2*a^4
*(a^2 - b^2)^2*d) + ((6*A*b^4 + a^4*(A - 4*C) - a^2*b^2*(10*A - C))*Sec[c +
 d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sec[c + d*x]
*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((7*a^2*A*b^2 -
 4*A*b^4 + 3*a^4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a +
b*Cos[c + d*x]))
```

Rubi [A] time = 1.87584, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(5a^4b^2(4A-C)-a^2b^4(29A-2C)+6a^6C+12Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{b\left(-a^2b^2(21A-2C)+a^4(6A-5C)+\right.}{2a^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((b*(12*A*b^6 - a^2*b^4*(29*A - 2*C) + 5*a^4*b^2*(4*A - C) + 6*a^6*C)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^
(5/2)*d)) + ((12*A*b^2 + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^5*d) -
(b*(12*A*b^4 + a^4*(6*A - 5*C) - a^2*b^2*(21*A - 2*C))*Tan[c + d*x])/(2*a^4
*(a^2 - b^2)^2*d) + ((6*A*b^4 + a^4*(A - 4*C) - a^2*b^2*(10*A - C))*Sec[c +
 d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sec[c + d*x]
*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((7*a^2*A*b^2 -
 4*A*b^4 + 3*a^4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a +
```

```
b*Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(-2\left(2Ab^2-a^2(A-C)\right)-2ab(A+C)\cos(c+dx)+3\right.}{(a+b\cos(c+dx}}{2a\left(a^2-b\right.}$$

$$= \frac{\left(Ab^2+a^2C\right)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(7a^2Ab^2-4Ab^4+3a^4C\right)\sec(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c+}$$

$$= \frac{\left(6Ab^4+a^4(A-4C)-a^2b^2(10A-C)\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right.}{2a\left(a^2-b\right.}$$

$$= -\frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2(21A-2C)\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(6Ab^4+a^4(A-\right.}{}$$

$$= -\frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2(21A-2C)\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(6Ab^4+a^4(A-\right.}{}$$

$$= \frac{\left(12Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2\right.}{2a^4\left(a^2-b\right.}$$

$$= -\frac{b\left(20a^4Ab^2-29a^2Ab^4+12Ab^6+6a^6C-5a^4b^2C+2a^2b^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan}{\sqrt{}}\right.}{a^5(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [B] time = 6.3841, size = 856, normalized size = 2.26

$$\frac{\left(-Aa^2-2Ca^2-12Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(A\sec^2(c+dx)+C\right)\cos^2(c+dx)}{a^5d(2A+C+C\cos(2c+2dx))}+\frac{\left(Aa^2+2Ca^2+12\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*b*(20*a^4*A*b^2 - 29*a^2*A*b^4 + 12*A*b^6 + 6*a^6*C - 5*a^4*b^2*C + 2*a^
2*b^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]*Cos[c + d*x]^
2*(C + A*Sec[c + d*x]^2))/(a^5*(a^2 - b^2)^2*Sqrt[-a^2 + b^2]*d*(2*A + C +
C*Cos[2*c + 2*d*x])) + ((-(a^2*A) - 12*A*b^2 - 2*a^2*C)*Cos[c + d*x]^2*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(C + A*Sec[c + d*x]^2))/(a^5*d*(2*A +
C + C*Cos[2*c + 2*d*x])) + ((a^2*A + 12*A*b^2 + 2*a^2*C)*Cos[c + d*x]^2*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(C + A*Sec[c + d*x]^2))/(a^5*d*(2*A +
 C + C*Cos[2*c + 2*d*x])) + (A*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2))/(2*a^
3*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
 - (6*A*b*Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(a^4*d*(2
*A + C + C*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (A*Co
s[c + d*x]^2*(C + A*Sec[c + d*x]^2))/(2*a^3*d*(2*A + C + C*Cos[2*c + 2*d*x]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (6*A*b*Cos[c + d*x]^2*(C + A*S
ec[c + d*x]^2)*Sin[(c + d*x)/2])/(a^4*d*(2*A + C + C*Cos[2*c + 2*d*x])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])) + (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)
*(A*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(a^3*(a - b)*(a + b)*d*(a +
 b*Cos[c + d*x])^2*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*(C + A
*Sec[c + d*x]^2)*(9*a^2*A*b^4*Sin[c + d*x] - 6*A*b^6*Sin[c + d*x] + 5*a^4*b
^2*C*Sin[c + d*x] - 2*a^2*b^4*C*Sin[c + d*x]))/(a^4*(a - b)^2*(a + b)^2*d*(
a + b*Cos[c + d*x])*(2*A + C + C*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.099, size = 1497, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^5/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
```

```
+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b^2*C-20/d/a*b^3/(a^4-2
*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*A+29/d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-1/d/a^3*ln(tan(1/2*d*x+1/2*c
)-1)*C+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^2+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1
)*C-1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)
+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)+1/
2/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)+3/d*A/a^4/(tan(1/2*d*x+1/2*c)+1)*b-6/d/a
^5*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+3/d*A/a^4/(tan(1/2*d*x+1/2*c)-1)*b+6/d/a^
5*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-12/d*b^7/a^5
/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*A-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+6/d/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*b^2*C-6/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1
/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b
)^2*tan(1/2*d*x+1/2*c)*A-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*b^3/a/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-
2/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-
b)^2*tan(1/2*d*x+1/2*c)*C-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b^3/a/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*C+10/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+10/d/a^2/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*A*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.83124, size = 1608, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/2*(2*(6*C*a^6*b + 20*A*a^4*b^3 - 5*C*a^4*b^3 - 29*A*a^2*b^5 + 2*C*a^2*b^5
 + 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^9 - 2
*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) + 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*
A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 6*C*
a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a
^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*C*a
```

```
^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 2*C*a
^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^
7*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^6*b*tan(1
/2*d*x + 1/2*c)^5 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^5*b^2*tan(1/
2*d*x + 1/2*c)^5 - 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 15*C*a^4*b^3*tan(1
/2*d*x + 1/2*c)^5 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^3*b^4*tan(1
/2*d*x + 1/2*c)^5 + 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^2*b^5*tan(1
/2*d*x + 1/2*c)^5 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^7*tan(1/2*d*
x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x + 1/2
*c)^3 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*C*a^5*b^2*tan(1/2*d*x + 1/2*
c)^3 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*C*a^4*b^3*tan(1/2*d*x + 1/2
*c)^3 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^3*b^4*tan(1/2*d*x + 1/2
*c)^3 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^5*tan(1/2*d*x + 1/2
*c)^3 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^3
 + A*a^7*tan(1/2*d*x + 1/2*c) - 4*A*a^6*b*tan(1/2*d*x + 1/2*c) - 13*A*a^5*b
^2*tan(1/2*d*x + 1/2*c) + 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*A*a^4*b^3*ta
n(1/2*d*x + 1/2*c) + 5*C*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*A*a^3*b^4*tan(1/
2*d*x + 1/2*c) - 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*A*a^2*b^5*tan(1/2*d*
x + 1/2*c) - 2*C*a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*A*a*b^6*tan(1/2*d*x + 1/
2*c) - 12*A*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 -
 a - b)^2) + (A*a^2 + 2*C*a^2 + 12*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
)/a^5 - (A*a^2 + 2*C*a^2 + 12*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5
)/d
```

3.585 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=514

$$-\frac{a\left(a^4b^2(6A-167C)-a^2b^4(17A-146C)+60a^6C+2b^6(13A-12C)\right)\sin(c+dx)}{6b^5d\left(a^2-b^2\right)^3}+\frac{\left(-a^7b^2(2A-69C)+7a^5b^4(A-1\right.}{}$$

```
[Out] ((2*A*b^2 + (20*a^2 + b^2)*C)*x)/(2*b^6) + ((8*a*A*b^8 - a^7*b^2*(2*A - 69*
C) + 7*a^5*b^4*(A - 12*C) - 8*a^3*b^6*(A - 5*C) - 20*a^9*C)*ArcTan[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^6*Sqrt[a + b]*(a^2 - b^
2)^3*d) - (a*(a^4*b^2*(6*A - 167*C) - a^2*b^4*(17*A - 146*C) + 2*b^6*(13*A
- 12*C) + 60*a^6*C)*Sin[c + d*x])/(6*b^5*(a^2 - b^2)^3*d) + ((a^4*b^2*(A -
27*C) - a^2*b^4*(2*A - 23*C) + b^6*(6*A - C) + 10*a^6*C)*Cos[c + d*x]*Sin[c
 + d*x])/(2*b^4*(a^2 - b^2)^3*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((4*A*b^4 - 5*a^4*C + a^
2*b^2*(A + 10*C))*Cos[c + d*x]^3*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a +
b*Cos[c + d*x])^2) - ((12*A*b^6 + a^4*b^2*(2*A - 53*C) + 20*a^6*C + a^2*b^4
*(A + 48*C))*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Cos
[c + d*x]))
```

Rubi [A] time = 2.21043, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3047, 3049, 3023, 2735, 2659, 205}

$$-\frac{a\left(a^4b^2(6A-167C)-a^2b^4(17A-146C)+60a^6C+2b^6(13A-12C)\right)\sin(c+dx)}{6b^5d\left(a^2-b^2\right)^3}+\frac{\left(-a^7b^2(2A-69C)+7a^5b^4(A-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```



```
[Out] ((2*A*b^2 + (20*a^2 + b^2)*C)*x)/(2*b^6) + ((8*a*A*b^8 - a^7*b^2*(2*A - 69*
C) + 7*a^5*b^4*(A - 12*C) - 8*a^3*b^6*(A - 5*C) - 20*a^9*C)*ArcTan[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^6*Sqrt[a + b]*(a^2 - b^
2)^3*d) - (a*(a^4*b^2*(6*A - 167*C) - a^2*b^4*(17*A - 146*C) + 2*b^6*(13*A
- 12*C) + 60*a^6*C)*Sin[c + d*x])/(6*b^5*(a^2 - b^2)^3*d) + ((a^4*b^2*(A -
27*C) - a^2*b^4*(2*A - 23*C) + b^6*(6*A - C) + 10*a^6*C)*Cos[c + d*x]*Sin[c
```

```
 + d*x])/(2*b^4*(a^2 - b^2)^3*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((4*A*b^4 - 5*a^4*C + a^
2*b^2*(A + 10*C))*Cos[c + d*x]^3*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a +
b*Cos[c + d*x])^2) - ((12*A*b^6 + a^4*b^2*(2*A - 53*C) + 20*a^6*C + a^2*b^4
*(A + 48*C))*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Cos
[c + d*x]))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^4(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)\left(4\left(Ab^2+a^2C\right)-3ab(A+C)\cos(c+\right.}{(a+b\cos(c+}}{3b\left(a^2-\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^4(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(4Ab^4-5a^4C+a^2b^2(A+10C)\right)\mathrm{co}}{6b^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{co}}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^4(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(4Ab^4-5a^4C+a^2b^2(A+10C)\right)\mathrm{co}}{6b^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{co}}$$

$$= \frac{\left(a^4b^2(A-27C)-a^2b^4(2A-23C)+b^6(6A-C)+10a^6C\right)\cos(c+dx)\sin(c+}{2b^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{a\left(a^4b^2(6A-167C)-a^2b^4(17A-146C)+2b^6(13A-12C)+60a^6C\right)\sin(c}{6b^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(2Ab^2+\left(20a^2+b^2\right)C\right)x}{2b^6} - \frac{a\left(a^4b^2(6A-167C)-a^2b^4(17A-146C)+2b^6(1\right.}{6b^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(2Ab^2+\left(20a^2+b^2\right)C\right)x}{2b^6} - \frac{a\left(a^4b^2(6A-167C)-a^2b^4(17A-146C)+2b^6(1\right.}{6b^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(2Ab^2+\left(20a^2+b^2\right)C\right)x}{2b^6} + \frac{\left(8aAb^8-a^7b^2(2A-69C)+7a^5b^4(A-12C)-8\right.}{\sqrt{a-b}b^6\sqrt{a+}}$$

Mathematica [B] time = 6.6876, size = 1452, normalized size = 2.82

$$\frac{a\left(20Ca^8+2Ab^2a^6-69b^2Ca^6-7Ab^4a^4+84b^4Ca^4+8Ab^6a^2-40b^6Ca^2-8Ab^8\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{b^6\left(a^2-b^2\right)^3\sqrt{b^2-a^2}d} - \frac{960C(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*(2*a^6*A*b^2 - 7*a^4*A*b^4 + 8*a^2*A*b^6 - 8*A*b^8 + 20*a^8*C - 69*a^6*b
^2*C + 84*a^4*b^4*C - 40*a^2*b^6*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
```

```
[-a^2 + b^2]])/(b^6*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d) - (96*a^9*A*b^2*(c +
d*x) - 144*a^7*A*b^4*(c + d*x) - 144*a^5*A*b^6*(c + d*x) + 336*a^3*A*b^8*(c
 + d*x) - 144*a*A*b^10*(c + d*x) + 960*a^11*C*(c + d*x) - 1392*a^9*b^2*C*(c
 + d*x) - 1512*a^7*b^4*C*(c + d*x) + 3288*a^5*b^6*C*(c + d*x) - 1272*a^3*b^
8*C*(c + d*x) - 72*a*b^10*C*(c + d*x) + 288*a^8*A*b^3*(c + d*x)*Cos[c + d*x
] - 792*a^6*A*b^5*(c + d*x)*Cos[c + d*x] + 648*a^4*A*b^7*(c + d*x)*Cos[c +
d*x] - 72*a^2*A*b^9*(c + d*x)*Cos[c + d*x] - 72*A*b^11*(c + d*x)*Cos[c + d*
x] + 2880*a^10*b*C*(c + d*x)*Cos[c + d*x] - 7776*a^8*b^3*C*(c + d*x)*Cos[c
+ d*x] + 6084*a^6*b^5*C*(c + d*x)*Cos[c + d*x] - 396*a^4*b^7*C*(c + d*x)*Co
s[c + d*x] - 756*a^2*b^9*C*(c + d*x)*Cos[c + d*x] - 36*b^11*C*(c + d*x)*Cos
[c + d*x] + 144*a^7*A*b^4*(c + d*x)*Cos[2*(c + d*x)] - 432*a^5*A*b^6*(c + d
*x)*Cos[2*(c + d*x)] + 432*a^3*A*b^8*(c + d*x)*Cos[2*(c + d*x)] - 144*a*A*b
^10*(c + d*x)*Cos[2*(c + d*x)] + 1440*a^9*b^2*C*(c + d*x)*Cos[2*(c + d*x)]
- 4248*a^7*b^4*C*(c + d*x)*Cos[2*(c + d*x)] + 4104*a^5*b^6*C*(c + d*x)*Cos[
2*(c + d*x)] - 1224*a^3*b^8*C*(c + d*x)*Cos[2*(c + d*x)] - 72*a*b^10*C*(c +
 d*x)*Cos[2*(c + d*x)] + 24*a^6*A*b^5*(c + d*x)*Cos[3*(c + d*x)] - 72*a^4*A
*b^7*(c + d*x)*Cos[3*(c + d*x)] + 72*a^2*A*b^9*(c + d*x)*Cos[3*(c + d*x)] -
 24*A*b^11*(c + d*x)*Cos[3*(c + d*x)] + 240*a^8*b^3*C*(c + d*x)*Cos[3*(c +
d*x)] - 708*a^6*b^5*C*(c + d*x)*Cos[3*(c + d*x)] + 684*a^4*b^7*C*(c + d*x)*
Cos[3*(c + d*x)] - 204*a^2*b^9*C*(c + d*x)*Cos[3*(c + d*x)] - 12*b^11*C*(c
+ d*x)*Cos[3*(c + d*x)] - 96*a^8*A*b^3*Sin[c + d*x] + 228*a^6*A*b^5*Sin[c +
 d*x] - 288*a^4*A*b^7*Sin[c + d*x] - 144*a^2*A*b^9*Sin[c + d*x] - 960*a^10*
b*C*Sin[c + d*x] + 2232*a^8*b^3*C*Sin[c + d*x] - 1086*a^6*b^5*C*Sin[c + d*x
] - 750*a^4*b^7*C*Sin[c + d*x] + 270*a^2*b^9*C*Sin[c + d*x] - 6*b^11*C*Sin[
c + d*x] - 120*a^7*A*b^4*Sin[2*(c + d*x)] + 360*a^5*A*b^6*Sin[2*(c + d*x)]
- 480*a^3*A*b^8*Sin[2*(c + d*x)] - 1200*a^9*b^2*C*Sin[2*(c + d*x)] + 3300*a
^7*b^4*C*Sin[2*(c + d*x)] - 2772*a^5*b^6*C*Sin[2*(c + d*x)] + 372*a^3*b^8*C
*Sin[2*(c + d*x)] + 60*a*b^10*C*Sin[2*(c + d*x)] - 44*a^6*A*b^5*Sin[3*(c +
d*x)] + 128*a^4*A*b^7*Sin[3*(c + d*x)] - 144*a^2*A*b^9*Sin[3*(c + d*x)] - 4
40*a^8*b^3*C*Sin[3*(c + d*x)] + 1253*a^6*b^5*C*Sin[3*(c + d*x)] - 1143*a^4*
b^7*C*Sin[3*(c + d*x)] + 279*a^2*b^9*C*Sin[3*(c + d*x)] - 9*b^11*C*Sin[3*(c
 + d*x)] - 30*a^7*b^4*C*Sin[4*(c + d*x)] + 90*a^5*b^6*C*Sin[4*(c + d*x)] -
90*a^3*b^8*C*Sin[4*(c + d*x)] + 30*a*b^10*C*Sin[4*(c + d*x)] + 3*a^6*b^5*C*
Sin[5*(c + d*x)] - 9*a^4*b^7*C*Sin[5*(c + d*x)] + 9*a^2*b^9*C*Sin[5*(c + d*
x)] - 3*b^11*C*Sin[5*(c + d*x)])/(96*b^6*(-a^2 + b^2)^3*d*(a + b*Cos[c + d*
x])^3)
```

Maple [B] time = 0.05, size = 2919, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)

[Out] -8/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-8/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+40/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*A+1/d/b^4*arctan(tan(1/2*d*x+1/2*c))*C+1/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C-1/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*C+20/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^2*C-4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-20/d*a^9/b^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+8/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+7/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+69/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-84/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-30/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-4/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+44/3/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-24/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-12/d*a^8/b^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a

```
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-3/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1
/2*c)*C+34/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/
(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+6/d*a^5/b^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*C-24/d*a^8/b^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+212/3/
d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+
b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-12/d*a^8/b^5/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5*C+3/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+34/d*a^6/b^3/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-30/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
C-60/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a
*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.45533, size = 5520, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [1/12*(6*(20*C*a^10*b^3 + (2*A - 79*C)*a^8*b^5 - 4*(2*A - 29*C)*a^6*b^7 + 2
*(6*A - 37*C)*a^4*b^9 - 8*(A - 2*C)*a^2*b^11 + (2*A + C)*b^13)*d*x*cos(d*x
```

+ c)^3 + 18*(20*C*a^11*b^2 + (2*A - 79*C)*a^9*b^4 - 4*(2*A - 29*C)*a^7*b^6 + 2*(6*A - 37*C)*a^5*b^8 - 8*(A - 2*C)*a^3*b^10 + (2*A + C)*a*b^12)*d*x*cos (d*x + c)^2 + 18*(20*C*a^12*b + (2*A - 79*C)*a^10*b^3 - 4*(2*A - 29*C)*a^8* b^5 + 2*(6*A - 37*C)*a^6*b^7 - 8*(A - 2*C)*a^4*b^9 + (2*A + C)*a^2*b^11)*d* x*cos(d*x + c) + 6*(20*C*a^13 + (2*A - 79*C)*a^11*b^2 - 4*(2*A - 29*C)*a^9* b^4 + 2*(6*A - 37*C)*a^7*b^6 - 8*(A - 2*C)*a^5*b^8 + (2*A + C)*a^3*b^10)*d* x - 3*(20*C*a^12 + (2*A - 69*C)*a^10*b^2 - 7*(A - 12*C)*a^8*b^4 + 8*(A - 5* C)*a^6*b^6 - 8*A*a^4*b^8 + (20*C*a^9*b^3 + (2*A - 69*C)*a^7*b^5 - 7*(A - 12 *C)*a^5*b^7 + 8*(A - 5*C)*a^3*b^9 - 8*A*a*b^11)*cos(d*x + c)^3 + 3*(20*C*a^ 10*b^2 + (2*A - 69*C)*a^8*b^4 - 7*(A - 12*C)*a^6*b^6 + 8*(A - 5*C)*a^4*b^8 - 8*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20*C*a^11*b + (2*A - 69*C)*a^9*b^3 - 7* (A - 12*C)*a^7*b^5 + 8*(A - 5*C)*a^5*b^7 - 8*A*a^3*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt (-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(60*C*a^12*b + (6*A - 227*C)*a^10*b ^3 - (23*A - 313*C)*a^8*b^5 + (43*A - 170*C)*a^6*b^7 - 2*(13*A - 12*C)*a^4* b^9 - 3*(C*a^8*b^5 - 4*C*a^6*b^7 + 6*C*a^4*b^9 - 4*C*a^2*b^11 + C*b^13)*cos (d*x + c)^4 + 15*(C*a^9*b^4 - 4*C*a^7*b^6 + 6*C*a^5*b^8 - 4*C*a^3*b^10 + C* a*b^12)*cos(d*x + c)^3 + (110*C*a^10*b^3 + (11*A - 421*C)*a^8*b^5 - (43*A - 590*C)*a^6*b^7 + 2*(34*A - 171*C)*a^4*b^9 - 9*(4*A - 7*C)*a^2*b^11)*cos(d* x + c)^2 + 3*(50*C*a^11*b^2 + 5*(A - 38*C)*a^9*b^4 - (20*A - 263*C)*a^7*b^6 + (35*A - 146*C)*a^5*b^8 - (20*A - 23*C)*a^3*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^9 - 4*a^6*b^11 + 6*a^4*b^13 - 4*a^2*b^15 + b^17)*d*cos(d*x + c)^3 + 3*(a^9*b^8 - 4*a^7*b^10 + 6*a^5*b^12 - 4*a^3*b^14 + a*b^16)*d*cos(d*x + c)^2 + 3*(a^10*b^7 - 4*a^8*b^9 + 6*a^6*b^11 - 4*a^4*b^13 + a^2*b^15)*d*c os(d*x + c) + (a^11*b^6 - 4*a^9*b^8 + 6*a^7*b^10 - 4*a^5*b^12 + a^3*b^14)*d), 1/6*(3*(20*C*a^10*b^3 + (2*A - 79*C)*a^8*b^5 - 4*(2*A - 29*C)*a^6*b^7 + 2*(6*A - 37*C)*a^4*b^9 - 8*(A - 2*C)*a^2*b^11 + (2*A + C)*b^13)*d*x*cos(d*x + c)^3 + 9*(20*C*a^11*b^2 + (2*A - 79*C)*a^9*b^4 - 4*(2*A - 29*C)*a^7*b^6 + 2*(6*A - 37*C)*a^5*b^8 - 8*(A - 2*C)*a^3*b^10 + (2*A + C)*a*b^12)*d*x*cos (d*x + c)^2 + 9*(20*C*a^12*b + (2*A - 79*C)*a^10*b^3 - 4*(2*A - 29*C)*a^8*b ^5 + 2*(6*A - 37*C)*a^6*b^7 - 8*(A - 2*C)*a^4*b^9 + (2*A + C)*a^2*b^11)*d*x *cos(d*x + c) + 3*(20*C*a^13 + (2*A - 79*C)*a^11*b^2 - 4*(2*A - 29*C)*a^9*b ^4 + 2*(6*A - 37*C)*a^7*b^6 - 8*(A - 2*C)*a^5*b^8 + (2*A + C)*a^3*b^10)*d*x - 3*(20*C*a^12 + (2*A - 69*C)*a^10*b^2 - 7*(A - 12*C)*a^8*b^4 + 8*(A - 5*C)*a^6*b^6 - 8*A*a^4*b^8 + (20*C*a^9*b^3 + (2*A - 69*C)*a^7*b^5 - 7*(A - 12* C)*a^5*b^7 + 8*(A - 5*C)*a^3*b^9 - 8*A*a*b^11)*cos(d*x + c)^3 + 3*(20*C*a^1 0*b^2 + (2*A - 69*C)*a^8*b^4 - 7*(A - 12*C)*a^6*b^6 + 8*(A - 5*C)*a^4*b^8 - 8*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20*C*a^11*b + (2*A - 69*C)*a^9*b^3 - 7*(A - 12*C)*a^7*b^5 + 8*(A - 5*C)*a^5*b^7 - 8*A*a^3*b^9)*cos(d*x + c))*sqrt(a ^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6 0*C*a^12*b + (6*A - 227*C)*a^10*b^3 - (23*A - 313*C)*a^8*b^5 + (43*A - 170* C)*a^6*b^7 - 2*(13*A - 12*C)*a^4*b^9 - 3*(C*a^8*b^5 - 4*C*a^6*b^7 + 6*C*a^4 *b^9 - 4*C*a^2*b^11 + C*b^13)*cos(d*x + c)^4 + 15*(C*a^9*b^4 - 4*C*a^7*b^6 + 6*C*a^5*b^8 - 4*C*a^3*b^10 + C*a*b^12)*cos(d*x + c)^3 + (110*C*a^10*b^3 +

```
(11*A - 421*C)*a^8*b^5 - (43*A - 590*C)*a^6*b^7 + 2*(34*A - 171*C)*a^4*b^9
 - 9*(4*A - 7*C)*a^2*b^11)*cos(d*x + c)^2 + 3*(50*C*a^11*b^2 + 5*(A - 38*C)
*a^9*b^4 - (20*A - 263*C)*a^7*b^6 + (35*A - 146*C)*a^5*b^8 - (20*A - 23*C)*
a^3*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^9 - 4*a^6*b^11 + 6*a^4*b^13 -
 4*a^2*b^15 + b^17)*d*cos(d*x + c)^3 + 3*(a^9*b^8 - 4*a^7*b^10 + 6*a^5*b^12
 - 4*a^3*b^14 + a*b^16)*d*cos(d*x + c)^2 + 3*(a^10*b^7 - 4*a^8*b^9 + 6*a^6*
b^11 - 4*a^4*b^13 + a^2*b^15)*d*cos(d*x + c) + (a^11*b^6 - 4*a^9*b^8 + 6*a^
7*b^10 - 4*a^5*b^12 + a^3*b^14)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.95398, size = 1389, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/6*(6*(20*C*a^9 + 2*A*a^7*b^2 - 69*C*a^7*b^2 - 7*A*a^5*b^4 + 84*C*a^5*b^4
+ 8*A*a^3*b^6 - 40*C*a^3*b^6 - 8*A*a*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*sqrt(a^2 - b
^2)) - 2*(36*C*a^10*tan(1/2*d*x + 1/2*c)^5 - 81*C*a^9*b*tan(1/2*d*x + 1/2*c
)^5 + 6*A*a^8*b^2*tan(1/2*d*x + 1/2*c)^5 - 48*C*a^8*b^2*tan(1/2*d*x + 1/2*c
)^5 - 15*A*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 + 213*C*a^7*b^3*tan(1/2*d*x + 1/2
*c)^5 - 6*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 48*C*a^6*b^4*tan(1/2*d*x + 1/2
*c)^5 + 45*A*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 - 162*C*a^5*b^5*tan(1/2*d*x + 1
/2*c)^5 - 6*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 90*C*a^4*b^6*tan(1/2*d*x + 1
/2*c)^5 - 60*A*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^8*tan(1/2*d*x +
```

1/2*c)^5 + 72*C*a^10*tan(1/2*d*x + 1/2*c)^3 + 12*A*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 - 284*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 392*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 180*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^10*tan(1/2*d*x + 1/2*c) + 81*C*a^9*b*tan(1/2*d*x + 1/2*c) + 6*A*a^8*b^2*tan(1/2*d*x + 1/2*c) - 48*C*a^8*b^2*tan(1/2*d*x + 1/2*c) + 15*A*a^7*b^3*tan(1/2*d*x + 1/2*c) - 213*C*a^7*b^3*tan(1/2*d*x + 1/2*c) - 6*A*a^6*b^4*tan(1/2*d*x + 1/2*c) - 48*C*a^6*b^4*tan(1/2*d*x + 1/2*c) - 45*A*a^5*b^5*tan(1/2*d*x + 1/2*c) + 162*C*a^5*b^5*tan(1/2*d*x + 1/2*c) - 6*A*a^4*b^6*tan(1/2*d*x + 1/2*c) + 90*C*a^4*b^6*tan(1/2*d*x + 1/2*c) + 60*A*a^3*b^7*tan(1/2*d*x + 1/2*c) + 36*A*a^2*b^8*tan(1/2*d*x + 1/2*c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(20*C*a^2 + 2*A*b^2 + C*b^2)*(d*x + c)/b^6 - 6*(8*C*a*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 + 8*C*a*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^5))/d

3.586 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=369

$$-\frac{\left(5Ab^4-C\left(-23a^2b^2+12a^4+6b^4\right)\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)+28a^6b^2C-35a^4b^4C-8a^8C+2Ab^8\right)\tan^{-1}\left(\frac{\sqrt{a}\cdots}{\cdots}\right.}{b^5d(a-b)^{7/2}(a+b)^{7/2}}$$

```
[Out] (-4*a*C*x)/b^5 - ((2*A*b^8 - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C + a^2*b^
6*(3*A + 20*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(7/2)*b^5*(a + b)^(7/2)*d) - ((5*A*b^4 - (12*a^4 - 23*a^2*b^2 + 6*b^4)*C)
*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((3*A*b^4 - 4*a^4*
C + a^2*b^2*(2*A + 9*C))*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*
d*(a + b*Cos[c + d*x])^2) + (a*(2*A*b^6 + 4*a^6*C - 11*a^4*b^2*C + 3*a^2*b^
4*(A + 4*C))*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.57892, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3047, 3031, 3023, 2735, 2659, 205}

$$-\frac{\left(5Ab^4-C\left(-23a^2b^2+12a^4+6b^4\right)\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)+28a^6b^2C-35a^4b^4C-8a^8C+2Ab^8\right)\tan^{-1}\left(\frac{\sqrt{a}\cdots}{\cdots}\right.}{b^5d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```



```
[Out] (-4*a*C*x)/b^5 - ((2*A*b^8 - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C + a^2*b^
6*(3*A + 20*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(7/2)*b^5*(a + b)^(7/2)*d) - ((5*A*b^4 - (12*a^4 - 23*a^2*b^2 + 6*b^4)*C)
*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((3*A*b^4 - 4*a^4*
C + a^2*b^2*(2*A + 9*C))*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*
d*(a + b*Cos[c + d*x])^2) + (a*(2*A*b^6 + 4*a^6*C - 11*a^4*b^2*C + 3*a^2*b^
4*(A + 4*C))*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(3\left(Ab^2+a^2C\right)-3ab(A+C)\cos(c+}{(a+b\cos(c+}}{3b\left(a^2-\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\mathrm{co}}{6b^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{co}}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\mathrm{co}}{6b^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{co}}$$

$$= -\frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2C\right)\cos^3(c+dx)\,\mathrm{s}}{3b\left(a^2-b^2\right)d(a+b\cos(}$$

$$= -\frac{4aCx}{b^5} - \frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2C\right)\cos^3(}{3b\left(a^2-b^2\right)d(a}$$

$$= -\frac{4aCx}{b^5} - \frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2C\right)\cos^3(}{3b\left(a^2-b^2\right)d(a}$$

$$= -\frac{4aCx}{b^5} - \frac{\left(3a^2Ab^6+2Ab^8-8a^8C+28a^6b^2C-35a^4b^4C+20a^2b^6C\right)\tan^{-1}\left(\right.}{(a-b)^{7/2}b^5(a+b)^{7/2}d}$$

Mathematica [B] time = 4.08161, size = 849, normalized size = 2.3

$$\frac{\frac{24\left(8Ca^8-28b^2Ca^6+35b^4Ca^4-b^6(3A+20C)a^2-2Ab^8\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} + \frac{-96cCa^{10}-96Cdxa^{10}+96bC\sin(c+dx)a^9+144b^2cCa^8+144b^2Cdxa^8+120b^2C}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((24*(-2*A*b^8 + 8*a^8*C - 28*a^6*b^2*C + 35*a^4*b^4*C - a^2*b^6*(3*A + 20*
C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2
) + (-96*a^10*c*C + 144*a^8*b^2*c*C + 144*a^6*b^4*c*C - 336*a^4*b^6*c*C + 1
44*a^2*b^8*c*C - 96*a^10*C*d*x + 144*a^8*b^2*C*d*x + 144*a^6*b^4*C*d*x - 33
```

```
6*a^4*b^6*C*d*x + 144*a^2*b^8*C*d*x - 72*a*b*(a^2 - b^2)^3*(4*a^2 + b^2)*C*
(c + d*x)*Cos[c + d*x] - 144*a^2*b^2*(a^2 - b^2)^3*C*(c + d*x)*Cos[2*(c + d
*x)] - 24*a^7*b^3*c*C*Cos[3*(c + d*x)] + 72*a^5*b^5*c*C*Cos[3*(c + d*x)] -
72*a^3*b^7*c*C*Cos[3*(c + d*x)] + 24*a*b^9*c*C*Cos[3*(c + d*x)] - 24*a^7*b^
3*C*d*x*Cos[3*(c + d*x)] + 72*a^5*b^5*C*d*x*Cos[3*(c + d*x)] - 72*a^3*b^7*C
*d*x*Cos[3*(c + d*x)] + 24*a*b^9*C*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^5*Sin[
c + d*x] + 39*a^3*A*b^7*Sin[c + d*x] + 18*a*A*b^9*Sin[c + d*x] + 96*a^9*b*C
*Sin[c + d*x] - 228*a^7*b^3*C*Sin[c + d*x] + 135*a^5*b^5*C*Sin[c + d*x] + 9
0*a^3*b^7*C*Sin[c + d*x] - 18*a*b^9*C*Sin[c + d*x] + 6*a^4*A*b^6*Sin[2*(c +
 d*x)] + 54*a^2*A*b^8*Sin[2*(c + d*x)] + 120*a^8*b^2*C*Sin[2*(c + d*x)] - 3
36*a^6*b^4*C*Sin[2*(c + d*x)] + 300*a^4*b^6*C*Sin[2*(c + d*x)] - 18*a^2*b^8
*C*Sin[2*(c + d*x)] - 6*b^10*C*Sin[2*(c + d*x)] + 2*a^5*A*b^5*Sin[3*(c + d*
x)] - 5*a^3*A*b^7*Sin[3*(c + d*x)] + 18*a*A*b^9*Sin[3*(c + d*x)] + 44*a^7*b
^3*C*Sin[3*(c + d*x)] - 125*a^5*b^5*C*Sin[3*(c + d*x)] + 114*a^3*b^7*C*Sin[
3*(c + d*x)] - 18*a*b^9*C*Sin[3*(c + d*x)] + 3*a^6*b^4*C*Sin[4*(c + d*x)] -
 9*a^4*b^6*C*Sin[4*(c + d*x)] + 9*a^2*b^8*C*Sin[4*(c + d*x)] - 3*b^10*C*Sin
[4*(c + d*x)])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(24*b^5*d)
```

Maple [B] time = 0.049, size = 2199, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+12/d/b^4/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^7/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*C+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*b^2/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+
b^3)*tan(1/2*d*x+1/2*c)^5*A-116/3/d/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3*a^5/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
*C-2/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*C/b^4*tan(1/2*d*x+1/2*c)/(tan(
1/2*d*x+1/2*c)^2+1)-8/d*C/b^5*a*arctan(tan(1/2*d*x+1/2*c))+20/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*C-20/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2-3/d*b/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*a^2*A-28/d/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
```

```
)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C+8
/d/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^8*C+35/d/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*a^4*C+20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+4/3/d/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*A+40/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+2/d*a^3/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+5/d*a^
4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-18/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5*C+3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(
a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-3/d*a^2*b/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)*A+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+2/d*a^6/b^
3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+
3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-18/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*C+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-2/d*a^6/b^3/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5*C-5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.89175, size = 4302, normalized size = 11.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [-1/12*(48*(C*a^9*b^3 - 4*C*a^7*b^5 + 6*C*a^5*b^7 - 4*C*a^3*b^9 + C*a*b^11)
*d*x*cos(d*x + c)^3 + 144*(C*a^10*b^2 - 4*C*a^8*b^4 + 6*C*a^6*b^6 - 4*C*a^4
*b^8 + C*a^2*b^10)*d*x*cos(d*x + c)^2 + 144*(C*a^11*b - 4*C*a^9*b^3 + 6*C*a
^7*b^5 - 4*C*a^5*b^7 + C*a^3*b^9)*d*x*cos(d*x + c) + 48*(C*a^12 - 4*C*a^10*
b^2 + 6*C*a^8*b^4 - 4*C*a^6*b^6 + C*a^4*b^8)*d*x + 3*(8*C*a^11 - 28*C*a^9*b
^2 + 35*C*a^7*b^4 - (3*A + 20*C)*a^5*b^6 - 2*A*a^3*b^8 + (8*C*a^8*b^3 - 28*
C*a^6*b^5 + 35*C*a^4*b^7 - (3*A + 20*C)*a^2*b^9 - 2*A*b^11)*cos(d*x + c)^3
+ 3*(8*C*a^9*b^2 - 28*C*a^7*b^4 + 35*C*a^5*b^6 - (3*A + 20*C)*a^3*b^8 - 2*A
*a*b^10)*cos(d*x + c)^2 + 3*(8*C*a^10*b - 28*C*a^8*b^3 + 35*C*a^6*b^5 - (3*
A + 20*C)*a^4*b^7 - 2*A*a^2*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*
cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x
 + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + a^2)) - 2*(24*C*a^11*b - 92*C*a^9*b^3 + (4*A + 133*C)*a^7*b^5 + (7*A
 - 71*C)*a^5*b^7 - (11*A - 6*C)*a^3*b^9 + 6*(C*a^8*b^4 - 4*C*a^6*b^6 + 6*C*
a^4*b^8 - 4*C*a^2*b^10 + C*b^12)*cos(d*x + c)^3 + (44*C*a^9*b^3 + (2*A - 16
9*C)*a^7*b^5 - (7*A - 239*C)*a^5*b^7 + (23*A - 132*C)*a^3*b^9 - 18*(A - C)*
a*b^11)*cos(d*x + c)^2 + 3*(20*C*a^10*b^2 - 77*C*a^8*b^4 + (A + 110*C)*a^6*
b^6 + (8*A - 59*C)*a^4*b^8 - 3*(3*A - 2*C)*a^2*b^10)*cos(d*x + c))*sin(d*x
+ c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x +
c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*x
 + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*c
os(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d)
, -1/6*(24*(C*a^9*b^3 - 4*C*a^7*b^5 + 6*C*a^5*b^7 - 4*C*a^3*b^9 + C*a*b^11)
*d*x*cos(d*x + c)^3 + 72*(C*a^10*b^2 - 4*C*a^8*b^4 + 6*C*a^6*b^6 - 4*C*a^4*
b^8 + C*a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(C*a^11*b - 4*C*a^9*b^3 + 6*C*a^7
*b^5 - 4*C*a^5*b^7 + C*a^3*b^9)*d*x*cos(d*x + c) + 24*(C*a^12 - 4*C*a^10*b^
2 + 6*C*a^8*b^4 - 4*C*a^6*b^6 + C*a^4*b^8)*d*x - 3*(8*C*a^11 - 28*C*a^9*b^2
 + 35*C*a^7*b^4 - (3*A + 20*C)*a^5*b^6 - 2*A*a^3*b^8 + (8*C*a^8*b^3 - 28*C*
a^6*b^5 + 35*C*a^4*b^7 - (3*A + 20*C)*a^2*b^9 - 2*A*b^11)*cos(d*x + c)^3 +
3*(8*C*a^9*b^2 - 28*C*a^7*b^4 + 35*C*a^5*b^6 - (3*A + 20*C)*a^3*b^8 - 2*A*a
*b^10)*cos(d*x + c)^2 + 3*(8*C*a^10*b - 28*C*a^8*b^3 + 35*C*a^6*b^5 - (3*A
+ 20*C)*a^4*b^7 - 2*A*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos
(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (24*C*a^11*b - 92*C*a^9*b^
3 + (4*A + 133*C)*a^7*b^5 + (7*A - 71*C)*a^5*b^7 - (11*A - 6*C)*a^3*b^9 + 6
*(C*a^8*b^4 - 4*C*a^6*b^6 + 6*C*a^4*b^8 - 4*C*a^2*b^10 + C*b^12)*cos(d*x +
```

```
c)^3 + (44*C*a^9*b^3 + (2*A - 169*C)*a^7*b^5 - (7*A - 239*C)*a^5*b^7 + (23*
A - 132*C)*a^3*b^9 - 18*(A - C)*a*b^11)*cos(d*x + c)^2 + 3*(20*C*a^10*b^2 -
 77*C*a^8*b^4 + (A + 110*C)*a^6*b^6 + (8*A - 59*C)*a^4*b^8 - 3*(3*A - 2*C)*
a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 -
 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11
- 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b
^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7
*b^9 - 4*a^5*b^11 + a^3*b^13)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)

[Out] Timed out
```

Giac [B] time = 1.84877, size = 1142, normalized size = 3.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
giac")

[Out] -1/3*(3*(8*C*a^8 - 28*C*a^6*b^2 + 35*C*a^4*b^4 - 3*A*a^2*b^6 - 20*C*a^2*b^6
 - 2*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^5
- 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(a^2 - b^2)) + 12*(d*x + c)*C*a/b^5 - (
18*C*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 24*C*
a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A
*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*A
*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 105*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*
A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 27
*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 + 36*
C*a^9*tan(1/2*d*x + 1/2*c)^3 - 152*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*A*a
```

```
^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 32*A
*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 120*C*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 36
*A*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^9*tan(1/2*d*x + 1/2*c) + 42*C*a^8*
b*tan(1/2*d*x + 1/2*c) - 24*C*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*C*a^6*b^3*
tan(1/2*d*x + 1/2*c) + 6*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*C*a^5*b^4*tan(
1/2*d*x + 1/2*c) + 3*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*C*a^4*b^5*tan(1/2
*d*x + 1/2*c) + 6*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b^6*tan(1/2*d*x
 + 1/2*c) + 27*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 18*A*a*b^8*tan(1/2*d*x + 1/
2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) - 6*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*b^4))/d
```

3.587 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=304

$$\frac{a\left(a^2b^4(A-8C)+7a^4b^2C-2a^6C+4b^6(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{(-$$

```
[Out] (C*x)/b^4 + (a*(a^2*b^4*(A - 8*C) - 2*a^6*C + 7*a^4*b^2*C + 4*b^6*(A + 2*C)
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a
 + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 -
b^2)*d*(a + b*Cos[c + d*x])^3) - (a*(2*A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 8*C
))*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((4*A*b^6
 + 9*a^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A + 28*C))*Sin[c + d*x])/(
6*b^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.08444, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3031, 3021, 2735, 2659, 205}

$$\frac{a\left(a^2b^4(A-8C)+7a^4b^2C-2a^6C+4b^6(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{(-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```



```
[Out] (C*x)/b^4 + (a*(a^2*b^4*(A - 8*C) - 2*a^6*C + 7*a^4*b^2*C + 4*b^6*(A + 2*C)
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a
 + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 -
b^2)*d*(a + b*Cos[c + d*x])^3) - (a*(2*A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 8*C
))*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((4*A*b^6
 + 9*a^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A + 28*C))*Sin[c + d*x])/(
6*b^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(2\left(Ab^2+a^2C\right)-3ab(A+C)\cos(c+a}{(a+b\cos(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8C)\right)s}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8C)\right)s}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{Cx}{b^4} - \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos}$$

$$= \frac{Cx}{b^4} - \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos}$$

$$= \frac{Cx}{b^4} + \frac{a\left(a^2Ab^4+4Ab^6-2a^6C+7a^4b^2C-8a^2b^4C+8b^6C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+}}\right.}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

Mathematica [B] time = 6.23494, size = 723, normalized size = 2.38

$$-\frac{-6a^5Ab^4\sin(2(c+dx))+51a^4Ab^5\sin(c+dx)-a^4Ab^5\sin(3(c+dx))+54a^3Ab^6\sin(2(c+dx))+18a^2Ab^7\sin(c+dx)+10a^2Ab^7\sin(3(c+dx))+30a^7b^2C\sin(2(c+dx))-57}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((24*a*(-(a^2*b^4*(A - 8*C)) + 2*a^6*C - 7*a^4*b^2*C - 4*b^6*(A + 2*C))*Ar
cTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (-
```

```
24*a^9*c*C + 36*a^7*b^2*c*C + 36*a^5*b^4*c*C - 84*a^3*b^6*c*C + 36*a*b^8*c*
C - 24*a^9*C*d*x + 36*a^7*b^2*C*d*x + 36*a^5*b^4*C*d*x - 84*a^3*b^6*C*d*x +
 36*a*b^8*C*d*x + 18*b*(-a^2 + b^2)^3*(4*a^2 + b^2)*C*(c + d*x)*Cos[c + d*x
] - 36*a*b^2*(a^2 - b^2)^3*C*(c + d*x)*Cos[2*(c + d*x)] - 6*a^6*b^3*c*C*Cos
[3*(c + d*x)] + 18*a^4*b^5*c*C*Cos[3*(c + d*x)] - 18*a^2*b^7*c*C*Cos[3*(c +
 d*x)] + 6*b^9*c*C*Cos[3*(c + d*x)] - 6*a^6*b^3*C*d*x*Cos[3*(c + d*x)] + 18
*a^4*b^5*C*d*x*Cos[3*(c + d*x)] - 18*a^2*b^7*C*d*x*Cos[3*(c + d*x)] + 6*b^9
*C*d*x*Cos[3*(c + d*x)] + 51*a^4*A*b^5*Sin[c + d*x] + 18*a^2*A*b^7*Sin[c +
d*x] + 6*A*b^9*Sin[c + d*x] + 24*a^8*b*C*Sin[c + d*x] - 57*a^6*b^3*C*Sin[c
+ d*x] + 72*a^4*b^5*C*Sin[c + d*x] + 36*a^2*b^7*C*Sin[c + d*x] - 6*a^5*A*b^
4*Sin[2*(c + d*x)] + 54*a^3*A*b^6*Sin[2*(c + d*x)] + 12*a*A*b^8*Sin[2*(c +
d*x)] + 30*a^7*b^2*C*Sin[2*(c + d*x)] - 90*a^5*b^4*C*Sin[2*(c + d*x)] + 120
*a^3*b^6*C*Sin[2*(c + d*x)] - a^4*A*b^5*Sin[3*(c + d*x)] + 10*a^2*A*b^7*Sin
[3*(c + d*x)] + 6*A*b^9*Sin[3*(c + d*x)] + 11*a^6*b^3*C*Sin[3*(c + d*x)] -
32*a^4*b^5*C*Sin[3*(c + d*x)] + 36*a^2*b^7*C*Sin[3*(c + d*x)])/((a^2 - b^2)
^3*(a + b*Cos[c + d*x])^3))/(24*b^4*d)
```

Maple [B] time = 0.043, size = 2314, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-8/d*a^3/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*C+2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*C+4/d/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-2/d*b^3/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)*A-2/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+8/d*b^2*a/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))*C-4/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*a
^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b
```

```
+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+4/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
A-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+7/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^
2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*C+1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^4/b/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan
(1/2*d*x+1/2*c)^5*C-28/3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-24/d*b/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2-12/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C
*a^2-12/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+1/d*a^5/b^2/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5*C-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*a^2*b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C
-1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-4/d*a^6/b^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*C-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+6/d*a^4/b/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)*C+44/3/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.15287, size = 3852, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [1/12*(12*(C*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4*C*a^2*b^9 + C*b^11)*d*
x*cos(d*x + c)^3 + 36*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^6 - 4*C*a^3*b^8
+ C*a*b^10)*d*x*cos(d*x + c)^2 + 36*(C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 -
 4*C*a^4*b^7 + C*a^2*b^9)*d*x*cos(d*x + c) + 12*(C*a^11 - 4*C*a^9*b^2 + 6*C
*a^7*b^4 - 4*C*a^5*b^6 + C*a^3*b^8)*d*x - 3*(2*C*a^10 - 7*C*a^8*b^2 - (A -
8*C)*a^6*b^4 - 4*(A + 2*C)*a^4*b^6 + (2*C*a^7*b^3 - 7*C*a^5*b^5 - (A - 8*C)
*a^3*b^7 - 4*(A + 2*C)*a*b^9)*cos(d*x + c)^3 + 3*(2*C*a^8*b^2 - 7*C*a^6*b^4
 - (A - 8*C)*a^4*b^6 - 4*(A + 2*C)*a^2*b^8)*cos(d*x + c)^2 + 3*(2*C*a^9*b -
 7*C*a^7*b^3 - (A - 8*C)*a^5*b^5 - 4*(A + 2*C)*a^3*b^7)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*C*a^10*b - 23*C*a^8*b^3 + (13*A
+ 43*C)*a^6*b^5 - (11*A + 26*C)*a^4*b^7 - 2*A*a^2*b^9 + (11*C*a^8*b^3 - (A
+ 43*C)*a^6*b^5 + (11*A + 68*C)*a^4*b^7 - 4*(A + 9*C)*a^2*b^9 - 6*A*b^11)*c
os(d*x + c)^2 + 3*(5*C*a^9*b^2 - (A + 20*C)*a^7*b^4 + 5*(2*A + 7*C)*a^5*b^6
 - (7*A + 20*C)*a^3*b^8 - 2*A*a*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7
 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^
6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^1
0*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^
11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d), 1/6*(6*(C*a^8*b
^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4*C*a^2*b^9 + C*b^11)*d*x*cos(d*x + c)^3 +
 18*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^6 - 4*C*a^3*b^8 + C*a*b^10)*d*x*co
s(d*x + c)^2 + 18*(C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 - 4*C*a^4*b^7 + C*a
^2*b^9)*d*x*cos(d*x + c) + 6*(C*a^11 - 4*C*a^9*b^2 + 6*C*a^7*b^4 - 4*C*a^5*
b^6 + C*a^3*b^8)*d*x - 3*(2*C*a^10 - 7*C*a^8*b^2 - (A - 8*C)*a^6*b^4 - 4*(A
 + 2*C)*a^4*b^6 + (2*C*a^7*b^3 - 7*C*a^5*b^5 - (A - 8*C)*a^3*b^7 - 4*(A + 2
*C)*a*b^9)*cos(d*x + c)^3 + 3*(2*C*a^8*b^2 - 7*C*a^6*b^4 - (A - 8*C)*a^4*b^
6 - 4*(A + 2*C)*a^2*b^8)*cos(d*x + c)^2 + 3*(2*C*a^9*b - 7*C*a^7*b^3 - (A -
 8*C)*a^5*b^5 - 4*(A + 2*C)*a^3*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^10*b - 23*C*a
^8*b^3 + (13*A + 43*C)*a^6*b^5 - (11*A + 26*C)*a^4*b^7 - 2*A*a^2*b^9 + (11*
C*a^8*b^3 - (A + 43*C)*a^6*b^5 + (11*A + 68*C)*a^4*b^7 - 4*(A + 9*C)*a^2*b^
9 - 6*A*b^11)*cos(d*x + c)^2 + 3*(5*C*a^9*b^2 - (A + 20*C)*a^7*b^4 + 5*(2*A
 + 7*C)*a^5*b^6 - (7*A + 20*C)*a^3*b^8 - 2*A*a*b^10)*cos(d*x + c))*sin(d*x
+ c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c
```

```
)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x
+ c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos
(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.80198, size = 1141, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/3*(3*(2*C*a^7 - 7*C*a^5*b^2 - A*a^3*b^4 + 8*C*a^3*b^4 - 4*A*a*b^6 - 8*C*a
*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^
4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) + 3*(d*x + c)*C/b^4 - (6*C*a^8*t
an(1/2*d*x + 1/2*c)^5 - 15*C*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^6*b^2*tan
(1/2*d*x + 1/2*c)^5 + 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^3*tan
(1/2*d*x + 1/2*c)^5 + 12*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^4*b^4*tan
(1/2*d*x + 1/2*c)^5 - 27*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^3*b^5*ta
n(1/2*d*x + 1/2*c)^5 + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b^6*t
an(1/2*d*x + 1/2*c)^5 - 6*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*
d*x + 1/2*c)^5 + 12*C*a^8*tan(1/2*d*x + 1/2*c)^3 - 56*C*a^6*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 28*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 116*C*a^4*b^4*tan(1/2*d
*x + 1/2*c)^3 - 16*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 72*C*a^2*b^6*tan(1/2*
d*x + 1/2*c)^3 - 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^8*tan(1/2*d*x + 1/
2*c) + 15*C*a^7*b*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^2*tan(1/2*d*x + 1/2*c) -
 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c) - 45*C*a^5*b^3*tan(1/2*d*x + 1/2*c) + 12*
```

```
A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^4*tan(1/2*d*x + 1/2*c) + 27*A*a^
3*b^5*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b^5*tan(1/2*d*x + 1/2*c) + 12*A*a^2*b
^6*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b^6*tan(1/2*d*x + 1/2*c) + 6*A*a*b^7*tan
(1/2*d*x + 1/2*c) + 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3
*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a +
b)^3))/d
```

3.588 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=261

$$-\frac{b\left(a^2(4A+3C)+b^2(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(a^2b^2(2A-5C)+2a^4C+b^4(13A+18C)\right)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{(a\ldots}{\ldots}$$

```
[Out] -((b*(b^2*(A + 2*C) + a^2*(4*A + 3*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b^2 + a^2*C)*Sin[c
 + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((3*A*b^4 - 4*a^4*C
 + a^2*b^2*(2*A + 9*C))*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c +
 d*x])^2) + (a*(a^2*b^2*(2*A - 5*C) + 2*a^4*C + b^4*(13*A + 18*C))*Sin[c +
d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.566525, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3032, 3021, 2754, 12, 2659, 205}

$$-\frac{b\left(a^2(4A+3C)+b^2(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(a^2b^2(2A-5C)+2a^4C+b^4(13A+18C)\right)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{(a\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] -((b*(b^2*(A + 2*C) + a^2*(4*A + 3*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b^2 + a^2*C)*Sin[c
 + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((3*A*b^4 - 4*a^4*C
 + a^2*b^2*(2*A + 9*C))*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c +
 d*x])^2) + (a*(a^2*b^2*(2*A - 5*C) + 2*a^4*C + b^4*(13*A + 18*C))*Sin[c +
d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
```

```
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b\left(Ab^2+a^2C\right)-a\left(2Ab^2-\left(a^2-3b^2\right)C\right)\cos(c+dx)-3b(}{(a+b\cos(c+dx))^3}}{3b^2\left(a^2-b^2\right)}$$

$$= \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\sin(c+}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\sin(c+}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\sin(c+}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\sin(c+}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= -\frac{b\left(4a^2A+Ab^2+3a^2C+2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\left(Ab^2+a^2C\right.}{3b^2\left(a^2-b^2\right)d(a}$$

Mathematica [A] time = 1.15218, size = 224, normalized size = 0.86

$$\frac{\frac{2\sin(c+dx)\left(6b\left(9a^2b^2(A+C)+a^4(2A+C)-Ab^4\right)\cos(c+dx)+a\left(\left(a^2b^2(2A-5C)+2a^4C+b^4(13A+18C)\right)\cos(2(c+dx))+a^2b^2(22A+17C)+2a^4(6A+5C)+b^4(11A+18C)\right)}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((24*b*(b^2*(A + 2*C) + a^2*(4*A + 3*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(6*b*(-(A*b^4) + 9*a^2*b^2*(A + C
) + a^4*(2*A + C))*Cos[c + d*x] + a*(2*a^4*(6*A + 5*C) + a^2*b^2*(22*A + 17
*C) + b^4*(11*A + 18*C) + (a^2*b^2*(2*A - 5*C) + 2*a^4*C + b^4*(13*A + 18*C
))*Cos[2*(c + d*x)]))*Sin[c + d*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)
^3*d)
```

Maple [B] time = 0.039, size = 1727, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d*b^3/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5*A+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+3/d*b/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a*b^
2+4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+
b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+28/3/d*b^2/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*A+4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+12/d/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*b^2*C+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*a
^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*A-1/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*C-3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2+6/d/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d
*x+1/2*c)*C*a*b^2-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*A-1/d*b^3/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*A-3/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2-2/d*b^3/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42278, size = 2457, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*((4*A + 3*C)*a^5*b + (A + 2*C)*a^3*b^3 + ((4*A + 3*C)*a^2*b^4 + (A
 + 2*C)*b^6)*cos(d*x + c)^3 + 3*((4*A + 3*C)*a^3*b^3 + (A + 2*C)*a*b^5)*cos
(d*x + c)^2 + 3*((4*A + 3*C)*a^4*b^2 + (A + 2*C)*a^2*b^4)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sq
rt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(2*(3*A + 2*C)*a^7 + (4*A + 7*C)*
a^5*b^2 - 11*(A + C)*a^3*b^4 + A*a*b^6 + (2*C*a^7 + (2*A - 7*C)*a^5*b^2 + (
11*A + 23*C)*a^3*b^4 - (13*A + 18*C)*a*b^6)*cos(d*x + c)^2 + 3*((2*A + C)*a
^6*b + (7*A + 8*C)*a^4*b^3 - (10*A + 9*C)*a^2*b^5 + A*b^7)*cos(d*x + c))*si
n(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x
 + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*
x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d
*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*
((4*A + 3*C)*a^5*b + (A + 2*C)*a^3*b^3 + ((4*A + 3*C)*a^2*b^4 + (A + 2*C)*b
^6)*cos(d*x + c)^3 + 3*((4*A + 3*C)*a^3*b^3 + (A + 2*C)*a*b^5)*cos(d*x + c)
^2 + 3*((4*A + 3*C)*a^4*b^2 + (A + 2*C)*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*(3*A
+ 2*C)*a^7 + (4*A + 7*C)*a^5*b^2 - 11*(A + C)*a^3*b^4 + A*a*b^6 + (2*C*a^7
+ (2*A - 7*C)*a^5*b^2 + (11*A + 23*C)*a^3*b^4 - (13*A + 18*C)*a*b^6)*cos(d*
x + c)^2 + 3*((2*A + C)*a^6*b + (7*A + 8*C)*a^4*b^3 - (10*A + 9*C)*a^2*b^5
```

```
+ A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*
a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^
6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.7116, size = 930, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(4*A*a^2*b + 3*C*a^2*b + A*b^3 + 2*C*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
b^2)) + (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^5*tan(1/2*d*x + 1/2*c)^5 -
6*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*
a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A*
a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*A
*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 18*C*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5
*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^5*tan(1/2*d*x + 1/2*c)^3 + 4*C*a^5*tan(1/2
*d*x + 1/2*c)^3 + 16*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*C*a^3*b^2*tan(1/
2*d*x + 1/2*c)^3 - 28*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 36*C*a*b^4*tan(1/2*d
*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/2*c) + 6*C*a^5*tan(1/2*d*x + 1/2*c)
 + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) + 3*C*a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a
^3*b^2*tan(1/2*d*x + 1/2*c) + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b
```

```
^3*tan(1/2*d*x + 1/2*c) + 27*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*ta
n(1/2*d*x + 1/2*c) + 18*C*a*b^4*tan(1/2*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x
+ 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.589 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=252

$$\frac{a\left(a^2(2A+C)+b^2(3A+4C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-a^2b^2(11A+10C)+a^4C-2b^4(2A+3C)\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-a$$

```
[Out] (a*(a^2*(2*A + C) + b^2*(3*A + 4*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Sin[c + d*
x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (a*(5*A*b^2 - a^2*C + 6*b^
2*C)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((a^4*C -
 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Sin[c + d*x])/(6*b*(a^2 - b^2)^
3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.46314, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3022, 2754, 12, 2659, 205}

$$\frac{a\left(a^2(2A+C)+b^2(3A+4C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-a^2b^2(11A+10C)+a^4C-2b^4(2A+3C)\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-a$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*(a^2*(2*A + C) + b^2*(3*A + 4*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Sin[c + d*
x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (a*(5*A*b^2 - a^2*C + 6*b^
2*C)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((a^4*C -
 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Sin[c + d*x])/(6*b*(a^2 - b^2)^
3*d*(a + b*Cos[c + d*x]))
```

Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
```

```
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{-3ab(A+C)+\left(2Ab^2-a^2C+3b^2C\right)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{2\left(b^3(2A+3C)+\right.}{}}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4C-2b^4(2A\right.}{6b\left(\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4C-2b^4(2A\right.}{6b\left(\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4C-2b^4(2A\right.}{6b\left(\right.}$$

$$= \frac{a\left(2a^2A+3Ab^2+a^2C+4b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx)}$$

Mathematica [A] time = 1.08064, size = 224, normalized size = 0.89

$$\frac{\frac{2\sin(c+dx)\left(6a\left(-9a^2b^2(A+C)+a^4C-b^4(A+2C)\right)\cos(c+dx)-b\left(\left(a^2b^2(11A+10C)+a^4(-C)+2b^4(2A+3C)\right)\cos(2(c+dx))+a^2b^2(A+14C)+a^4(36A+25C)+2b^4(4A+3\right.\right.}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*a*(a^2*(2*A + C) + b^2*(3*A + 4*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2]
)/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(6*a*(a^4*C - 9*a^2*b^2*(A + C)
- b^4*(A + 2*C))*Cos[c + d*x] - b*(2*b^4*(4*A + 3*C) + a^2*b^2*(A + 14*C) +
 a^4*(36*A + 25*C) + (-(a^4*C) + 2*b^4*(2*A + 3*C) + a^2*b^2*(11*A + 10*C))
*Cos[2*(c + d*x)]))*Sin[c + d*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3
*d)
```

Maple [B] time = 0.036, size = 1726, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-3/d*b^2/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*A-2/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-1/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*C-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2-2/d/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^
2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a*b^2-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b
^3-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/3/d*b^3/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A-28/3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2-4/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+3/
d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*A+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1
/2*c)*C*a^2+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a*b^2-2/d/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)*C*b^3+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+3/d*a*b^2/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))*A+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+4/d*b^2*a/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
```

```
)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39121, size = 2456, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*((2*A + C)*a^6 + (3*A + 4*C)*a^4*b^2 + ((2*A + C)*a^3*b^3 + (3*A +
 4*C)*a*b^5)*cos(d*x + c)^3 + 3*((2*A + C)*a^4*b^2 + (3*A + 4*C)*a^2*b^4)*c
os(d*x + c)^2 + 3*((2*A + C)*a^5*b + (3*A + 4*C)*a^3*b^3)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sq
rt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*((18*A + 13*C)*a^6*b - (23*A + 11
*C)*a^4*b^3 + (7*A - 2*C)*a^2*b^5 - 2*A*b^7 - (C*a^6*b - 11*(A + C)*a^4*b^3
 + (7*A + 4*C)*a^2*b^5 + 2*(2*A + 3*C)*b^7)*cos(d*x + c)^2 - 3*(C*a^7 - (9*
A + 10*C)*a^5*b^2 + (8*A + 7*C)*a^3*b^4 + (A + 2*C)*a*b^6)*cos(d*x + c))*si
n(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x
 + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*
x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d
*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(
(2*A + C)*a^6 + (3*A + 4*C)*a^4*b^2 + ((2*A + C)*a^3*b^3 + (3*A + 4*C)*a*b^
5)*cos(d*x + c)^3 + 3*((2*A + C)*a^4*b^2 + (3*A + 4*C)*a^2*b^4)*cos(d*x + c
)^2 + 3*((2*A + C)*a^5*b + (3*A + 4*C)*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^
2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((18*A +
13*C)*a^6*b - (23*A + 11*C)*a^4*b^3 + (7*A - 2*C)*a^2*b^5 - 2*A*b^7 - (C*a^
6*b - 11*(A + C)*a^4*b^3 + (7*A + 4*C)*a^2*b^5 + 2*(2*A + 3*C)*b^7)*cos(d*x
 + c)^2 - 3*(C*a^7 - (9*A + 10*C)*a^5*b^2 + (8*A + 7*C)*a^3*b^4 + (A + 2*C)
```

```
*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a
^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^
3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^
4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6
 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39153, size = 930, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*A*a^3 + C*a^3 + 3*A*a*b^2 + 4*C*a*b^2)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
 b^2)) + (3*C*a^5*tan(1/2*d*x + 1/2*c)^5 + 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 + 12*C*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
 - 27*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
 + 12*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 -
 6*C*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^
5*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 28*C*a^4*b*t
an(1/2*d*x + 1/2*c)^3 - 32*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 16*C*a^2*b^3*
tan(1/2*d*x + 1/2*c)^3 - 4*A*b^5*tan(1/2*d*x + 1/2*c)^3 - 12*C*b^5*tan(1/2*
d*x + 1/2*c)^3 - 3*C*a^5*tan(1/2*d*x + 1/2*c) + 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 12*C*a^4*b*tan(1/2*d*x + 1/2*c) + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)
+ 27*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12
*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 6*C*a*b^
```

```
4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*x + 1/2*c) + 6*C*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.590 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$-\frac{b\left(-a^4b^2(8A-C)+7a^2Ab^4+4a^6(2A+C)-2Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(-13a^4b^2(2A+C)+17a^2Ab^4-2a^6C-\right.}{6a^3d\left(a^2-b^2\right)^3(a+b\cos(c}$$

```
[Out] -((b*(7*a^2*A*b^4 - 2*A*b^6 - a^4*b^2*(8*A - C) + 4*a^6*(2*A + C))*ArcTan[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2
)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/
(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((3*A*b^4 - 2*a^4*C - a^2*b^2*
(8*A + 3*C))*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) -
 ((17*a^2*A*b^4 - 6*A*b^6 - 2*a^6*C - 13*a^4*b^2*(2*A + C))*Sin[c + d*x])/(
6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.30167, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(-a^4b^2(8A-C)+7a^2Ab^4+4a^6(2A+C)-2Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(-13a^4b^2(2A+C)+17a^2Ab^4-2a^6C-\right.}{6a^3d\left(a^2-b^2\right)^3(a+b\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] -((b*(7*a^2*A*b^4 - 2*A*b^6 - a^4*b^2*(8*A - C) + 4*a^6*(2*A + C))*ArcTan[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2
)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/
(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((3*A*b^4 - 2*a^4*C - a^2*b^2*
(8*A + 3*C))*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) -
 ((17*a^2*A*b^4 - 6*A*b^6 - 2*a^6*C - 13*a^4*b^2*(2*A + C))*Sin[c + d*x])/(
6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3A\left(a^2-b^2\right)-3ab(A+C)\cos(c+dx)+2\left(Ab^2+a^2C\right)\cos}{(a+b\cos(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\sin(c+d}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\sin(c+d}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\sin(c+d}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-\right.}{6a^2\left(a^2-b^2\right.}$$

$$= -\frac{b\left(8a^6A-8a^4Ab^2+7a^2Ab^4-2Ab^6+4a^6C+a^4b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [C] time = 5.24793, size = 498, normalized size = 1.65

$$\frac{\cos(c+dx)(A\sec(c+dx)+C\cos(c+dx))\left(\frac{6b(\sin(c)+i\cos(c))\left(a^4b^2(C-8A)+7a^2Ab^4+4a^6(2A+C)-2Ab^6\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(b\cos(c)-\right.}{\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c)}}\right.}{a^4\left(a^2-b^2\right)^3\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(C*Cos[c + d*x] + A*Sec[c + d*x])*((-6*A*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/a^4 + (6*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/
a^4 + (6*b*(7*a^2*A*b^4 - 2*A*b^6 + a^4*b^2*(-8*A + C) + 4*a^6*(2*A + C))*A
rcTan[((I*Cos[c] + Sin[c])*(b*Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[
-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]]*(I*Cos[c] + Sin[c]))/(a^4*(a^2 - b^2
)^3*Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]) + (2*(A*b^2 + a^2*C)*Sec[c]
*(-(a*Sin[c]) + b*Sin[d*x]))/(a*b*(a^2 - b^2)*(a + b*Cos[c + d*x])^3) + ((-
17*a^2*A*b^4 + 6*A*b^6 + 2*a^6*C + 13*a^4*b^2*(2*A + C))*Sec[c]*Sin[d*x] -
3*a*b*(A*b^4 + a^2*b^2*(-2*A + C) + a^4*(6*A + 4*C))*Tan[c])/((a^3 - a*b^2)
^3*(a + b*Cos[c + d*x])) + ((-3*A*b^4 + 2*a^4*C + a^2*b^2*(8*A + 3*C))*Sec[
c]*Sin[d*x] + a*b*(A*b^2 - a^2*(6*A + 5*C))*Tan[c])/(a^2*(a^2 - b^2)^2*(a +
 b*Cos[c + d*x])^2)))/(3*d*(2*A + C + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.082, size = 2337, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*C*b^3+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+
b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+24/d*b^2/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*A+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+8/d*b^3/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))*A-1/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a
-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/a^4*A
*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)+6/d/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*C*a*b^2+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a*b^2+28/
3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2-2*a*b+b^2)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*C-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-
```

```
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2-8/d*b/(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*a^2*A+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+4/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*C-4/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+4/d*b^3/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^
2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2+2/d
*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b
+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2-7/d/a^2*b^5/(a^6-3*a^4*b^2+3*a^2*b
^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*A+2/d/a^4*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d/a^3/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5*A*b^6-44/3/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+4/d/a^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2
-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^6-6/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
A*b^4+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5+2/d/a^3/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)*A*b^6-6/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^4-1/d/a^2/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^
2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 159.947, size = 4764, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*(4*(2*A + C)*a^9*b - (8*A - C)*a^7*b^3 + 7*A*a^5*b^5 - 2*A*a^3*b^
7 + (4*(2*A + C)*a^6*b^4 - (8*A - C)*a^4*b^6 + 7*A*a^2*b^8 - 2*A*b^10)*cos(
d*x + c)^3 + 3*(4*(2*A + C)*a^7*b^3 - (8*A - C)*a^5*b^5 + 7*A*a^3*b^7 - 2*A
*a*b^9)*cos(d*x + c)^2 + 3*(4*(2*A + C)*a^8*b^2 - (8*A - C)*a^6*b^4 + 7*A*a
^4*b^6 - 2*A*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c
) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*
sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)
) - 6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^
8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 +
3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x
+ c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)
*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^
4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*
a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6
 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A
*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
2*(6*C*a^11 + 4*(9*A + C)*a^9*b^2 - (68*A + 11*C)*a^7*b^4 + (43*A + C)*a^5*
b^6 - 11*A*a^3*b^8 + (2*C*a^9*b^2 + (26*A + 11*C)*a^7*b^4 - (43*A + 13*C)*a
^5*b^6 + 23*A*a^3*b^8 - 6*A*a*b^10)*cos(d*x + c)^2 + 3*(2*C*a^10*b + (20*A
+ 7*C)*a^8*b^3 - 5*(7*A + 2*C)*a^6*b^5 + (20*A + C)*a^4*b^7 - 5*A*a^2*b^9)*
cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9
 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^
7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 -
4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a
^9*b^6 + a^7*b^8)*d), -1/6*(3*(4*(2*A + C)*a^9*b - (8*A - C)*a^7*b^3 + 7*A*
a^5*b^5 - 2*A*a^3*b^7 + (4*(2*A + C)*a^6*b^4 - (8*A - C)*a^4*b^6 + 7*A*a^2*
b^8 - 2*A*b^10)*cos(d*x + c)^3 + 3*(4*(2*A + C)*a^7*b^3 - (8*A - C)*a^5*b^5
 + 7*A*a^3*b^7 - 2*A*a*b^9)*cos(d*x + c)^2 + 3*(4*(2*A + C)*a^8*b^2 - (8*A
- C)*a^6*b^4 + 7*A*a^4*b^6 - 2*A*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arc
tan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*(A*a^11 - 4*A
*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5
 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*
a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*
b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(
sin(d*x + c) + 1) + 3*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A
```

```
*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*c
os(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*
a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*
b^7 + A*a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (6*C*a^11 + 4*(9*A
+ C)*a^9*b^2 - (68*A + 11*C)*a^7*b^4 + (43*A + C)*a^5*b^6 - 11*A*a^3*b^8 +
(2*C*a^9*b^2 + (26*A + 11*C)*a^7*b^4 - (43*A + 13*C)*a^5*b^6 + 23*A*a^3*b^8
 - 6*A*a*b^10)*cos(d*x + c)^2 + 3*(2*C*a^10*b + (20*A + 7*C)*a^8*b^3 - 5*(7
*A + 2*C)*a^6*b^5 + (20*A + C)*a^4*b^7 - 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x
 + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d*cos(d*
x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*c
os(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*
d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4977, size = 1172, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(8*A*a^6*b + 4*C*a^6*b - 8*A*a^4*b^3 + C*a^4*b^3 + 7*A*a^2*b^5 - 2*
A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^
2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) - 3*A*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^4 + 3*A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - (6*C*a^8*tan(1/2
*d*x + 1/2*c)^5 - 6*C*a^7*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^6*b^2*tan(1/2*d
*x + 1/2*c)^5 + 12*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^5*b^3*tan(1/2*
```

```
d*x + 1/2*c)^5 - 27*C*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b^4*tan(1/2*
d*x + 1/2*c)^5 + 12*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*A*a^3*b^5*tan(1/2
*d*x + 1/2*c)^5 + 3*C*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^6*tan(1/2*
d*x + 1/2*c)^5 - 15*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*d*x +
1/2*c)^5 + 12*C*a^8*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^6*b^2*tan(1/2*d*x + 1/2
*c)^3 + 16*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^4*tan(1/2*d*x + 1
/2*c)^3 - 28*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 56*A*a^2*b^6*tan(1/2*d*x +
1/2*c)^3 - 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^8*tan(1/2*d*x + 1/2*c) +
 6*C*a^7*b*tan(1/2*d*x + 1/2*c) + 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c) + 12*C*
a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c) + 27*C*a^5
*b^3*tan(1/2*d*x + 1/2*c) - 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) + 12*C*a^4*b^4
*tan(1/2*d*x + 1/2*c) - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c) - 3*C*a^3*b^5*tan
(1/2*d*x + 1/2*c) - 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*A*a*b^7*tan(1/2*d
*x + 1/2*c) + 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 -
 a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))
/d
```

3.591 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=376

$$-\frac{\left(-a^6b^2(20A+3C)+35a^4Ab^4-28a^2Ab^6-2a^8C+8Ab^8\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-a^4b^2(65A+4C)+68a^2Ab^4+\right.}{6a^4d\left(\right.}$$

```
[Out] -(((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 - 2*a^8*C - a^6*b^2*(20*A + 3*C))
*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a
+ b)^(7/2)*d)) - (4*A*b*ArcTanh[Sin[c + d*x]])/(a^5*d) + ((68*a^2*A*b^4 - 2
4*A*b^6 + a^6*(6*A - 11*C) - a^4*b^2*(65*A + 4*C))*Tan[c + d*x])/(6*a^4*(a^
2 - b^2)^3*d) + ((A*b^2 + a^2*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^3) - ((4*A*b^4 - 3*a^4*C - a^2*b^2*(9*A + 2*C))*Tan[c + d*x])/(
6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*b^4 - 4*A*b^6 -
2*a^6*C - 3*a^4*b^2*(4*A + C))*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 2.12741, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-a^6b^2(20A+3C)+35a^4Ab^4-28a^2Ab^6-2a^8C+8Ab^8\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-a^4b^2(65A+4C)+68a^2Ab^4+\right.}{6a^4d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] -(((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 - 2*a^8*C - a^6*b^2*(20*A + 3*C))
*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a
+ b)^(7/2)*d)) - (4*A*b*ArcTanh[Sin[c + d*x]])/(a^5*d) + ((68*a^2*A*b^4 - 2
4*A*b^6 + a^6*(6*A - 11*C) - a^4*b^2*(65*A + 4*C))*Tan[c + d*x])/(6*a^4*(a^
2 - b^2)^3*d) + ((A*b^2 + a^2*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^3) - ((4*A*b^4 - 3*a^4*C - a^2*b^2*(9*A + 2*C))*Tan[c + d*x])/(
6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*b^4 - 4*A*b^6 -
2*a^6*C - 3*a^4*b^2*(4*A + C))*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*
```

```
Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}+\frac{\int\frac{\left(-4Ab^2+a^2(3A-C)-3ab(A+C)\cos(c+dx)+3\left(Ab^2+\right.\right.}{(a+b\cos(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(4Ab^4-3a^4C-a^2b^2(9A+2C)\right)\tan(c+}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}$$

$$= \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}-\frac{\left(4Ab^4-3a^4C-a^2b^2(9A+2C)\right)\tan(c+}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}$$

$$= \frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A+4C)\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3d}+\frac{(A}{3a\left(a^2\right.}$$

$$= \frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A+4C)\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3d}+\frac{(A}{3a\left(a^2\right.}$$

$$= -\frac{4Ab\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A\right.}{6a^4\left(a^2-b^2\right)^3d}$$

$$= \frac{\left(20a^6Ab^2-35a^4Ab^4+28a^2Ab^6-8Ab^8+2a^8C+3a^6b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+b}}\right.}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 3.10924, size = 515, normalized size = 1.37

$$\frac{\cos(c+dx)\left(A\sec^2(c+dx)+C\right)\left(\frac{a\sin(c+dx)\left(6ab^2\left(-a^4b^2(53A+C)+57a^2Ab^4+a^6(6A-9C)-20Ab^6\right)\cos(2(c+dx))-b\left(a^6b^2(438A+13C)-5a^4b^4(61A-\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(C + A*Sec[c + d*x]^2)*((24*(-35*a^4*A*b^4 + 28*a^2*A*b^6 - 8
*A*b^8 + 2*a^8*C + a^6*b^2*(20*A + 3*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]]*Cos[c + d*x])/(-a^2 + b^2)^(7/2) + 96*A*b*Cos[c + d*x]*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 96*A*b*Cos[c + d*x]*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + (a*(24*a^9*A - 36*a^7*A*b^2 - 246*a^5*A*b^4 +
 318*a^3*A*b^6 - 120*a*A*b^8 - 54*a^7*b^2*C - 6*a^5*b^4*C - b*(-28*a^2*A*b^
6 + 72*A*b^8 - 5*a^4*b^4*(61*A - 4*C) - 72*a^8*(A - C) + a^6*b^2*(438*A + 1
3*C))*Cos[c + d*x] + 6*a*b^2*(57*a^2*A*b^4 - 20*A*b^6 + a^6*(6*A - 9*C) - a
^4*b^2*(53*A + C))*Cos[2*(c + d*x)] + 6*a^6*A*b^3*Cos[3*(c + d*x)] - 65*a^4
*A*b^5*Cos[3*(c + d*x)] + 68*a^2*A*b^7*Cos[3*(c + d*x)] - 24*A*b^9*Cos[3*(c
 + d*x)] - 11*a^6*b^3*C*Cos[3*(c + d*x)] - 4*a^4*b^5*C*Cos[3*(c + d*x)])*Si
n[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3)))/(12*a^5*d*(2*A + C + C
*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.097, size = 2234, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-4/3/d/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*C-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3+2/d*a^3/(a^6-3*a^4*b^2+
3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*C-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-1/d/a^4*A/(tan(1/2*d*x+1/2*
```

```
c)+1)-4/d*A*b/a^5*ln(tan(1/2*d*x+1/2*c)+1)+4/d*A*b/a^5*ln(tan(1/2*d*x+1/2*c
)-1)-3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a*b^2+3/d/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x
+1/2*c)*C*a*b^2-20/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-20/d*b^3/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5*A+3/d*b^2*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-
b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-40/d*b^3/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-1
2/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*C*a^2-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2-35/d/a/(a^6-3*a^4*b^
2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*A*b^4+28/d/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6-8/d/a^5/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*A*b^8+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^
6-6/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d/a^4/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5*A+116/3/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*b^5/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-12/d/a^4
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a^2+2*a*b+b^2)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
A*b^4+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5-2/d/a^3/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)*A*b^6-5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^4+18/d/a^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*
b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.7016, size = 1176, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/3*(3*(2*C*a^8 + 20*A*a^6*b^2 + 3*C*a^6*b^2 - 35*A*a^4*b^4 + 28*A*a^2*b^6
 - 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^11 - 3
*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2)) + 12*A*b*log(abs(tan(1/2*d
*x + 1/2*c) + 1))/a^5 - 12*A*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + (18
*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*
A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105
*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24
*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 11
7*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 -
42*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^9*tan(1/2*d*x + 1/2*c)^5 + 36*C*
a^8*b*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 - 32*C*
a^6*b^3*tan(1/2*d*x + 1/2*c)^3 - 236*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*C
*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 + 152*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^3 - 36
*A*b^9*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^8*b*tan(1/2*d*x + 1/2*c) + 27*C*a^7*
b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c) + 6*C*a^6*b^3*
tan(1/2*d*x + 1/2*c) + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) + 3*C*a^5*b^4*tan
(1/2*d*x + 1/2*c) - 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*C*a^4*b^5*tan(1/2
*d*x + 1/2*c) - 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 24*A*a^2*b^7*tan(1/2*d
*x + 1/2*c) + 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 18*A*b^9*tan(1/2*d*x + 1/2*
c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 6*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 - 1)*a^4))/d
```

3.592 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=522

$$\frac{\left(-a^2b^7(69A-2C)+7a^4b^5(12A-C)-8a^6b^3(5A-C)-8a^8bC+20Ab^9\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)^3}+\frac{b\left(a^4b^2(146A-17\right.}{}$$

```
[Out] ((20*A*b^9 - a^2*b^7*(69*A - 2*C) - 8*a^6*b^3*(5*A - C) + 7*a^4*b^5*(12*A -
 C) - 8*a^8*b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*S
qrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^3*d) + ((20*A*b^2 + a^2*(A + 2*C))*ArcTa
nh[Sin[c + d*x]])/(2*a^6*d) + (b*(60*A*b^6 - a^6*(24*A - 26*C) + a^4*b^2*(1
46*A - 17*C) - a^2*b^4*(167*A - 6*C))*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d)
 - ((10*A*b^6 - a^6*(A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(27*A - C))*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 + a^2*C)*Sec[c
 + d*x]*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((5*A*b^
4 - 4*a^4*C - a^2*b^2*(10*A + C))*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((20*A*b^6 - a^2*b^4*(53*A - 2*C) + 12*a
^6*C + a^4*b^2*(48*A + C))*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*
d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.65862, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{\left(-a^2b^7(69A-2C)+7a^4b^5(12A-C)-8a^6b^3(5A-C)-8a^8bC+20Ab^9\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)^3}+\frac{b\left(a^4b^2(146A-17\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] ((20*A*b^9 - a^2*b^7*(69*A - 2*C) - 8*a^6*b^3*(5*A - C) + 7*a^4*b^5*(12*A -
 C) - 8*a^8*b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*S
qrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^3*d) + ((20*A*b^2 + a^2*(A + 2*C))*ArcTa
nh[Sin[c + d*x]])/(2*a^6*d) + (b*(60*A*b^6 - a^6*(24*A - 26*C) + a^4*b^2*(1
46*A - 17*C) - a^2*b^4*(167*A - 6*C))*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d)
 - ((10*A*b^6 - a^6*(A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(27*A - C))*
```

```
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 + a^2*C)*Sec[c
 + d*x]*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((5*A*b^
4 - 4*a^4*C - a^2*b^2*(10*A + C))*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((20*A*b^6 - a^2*b^4*(53*A - 2*C) + 12*a
^6*C + a^4*b^2*(48*A + C))*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*
d*(a + b*Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\left(Ab^2+a^2C\right)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(-5Ab^2+a^2(3A-2C)-3ab(A+C)\cos(c+dx)+4(\right.}{(a+b\cos(c+dx}}{3a\left(a^2-b\right.}$$

$$= \frac{\left(Ab^2+a^2C\right)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5Ab^4-4a^4C-a^2b^2(10A+C)\right)\sec(\ldots}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(\ldots}$$

$$= \frac{\left(Ab^2+a^2C\right)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5Ab^4-4a^4C-a^2b^2(10A+C)\right)\sec(\ldots}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(\ldots}$$

$$= -\frac{\left(10Ab^6-a^6(A-6C)+a^4b^2(23A-2C)-a^2b^4(27A-C)\right)\sec(c+dx)\tan(\ldots}{2a^4\left(a^2-b^2\right)^3 d}$$

$$= \frac{b\left(60Ab^6-a^6(24A-26C)+a^4b^2(146A-17C)-a^2b^4(167A-6C)\right)\tan(c+\ldots}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{b\left(60Ab^6-a^6(24A-26C)+a^4b^2(146A-17C)-a^2b^4(167A-6C)\right)\tan(c+\ldots}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(20Ab^2+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^6d} + \frac{b\left(60Ab^6-a^6(24A-26C)+a\ldots\right.}{}$$

$$= \frac{\left(20Ab^9-a^2b^7(69A-2C)-8a^6b^3(5A-C)+7a^4b^5(12A-C)-8a^8bC\right)\tan^{-}\ldots}{a^6\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 5.91649, size = 740, normalized size = 1.42

$$\frac{\left(A\sec^2(c+dx)+C\right)\left(\frac{2a\sin(c+dx)\left(-6ab\left(3a^6b^2(3A-20C)+3a^4b^4(15C-103A)+5a^2b^6(80A-3C)+20a^8A-150Ab^8\right)\cos(c+dx)+12b^2\left(a^6b^2(85A-2C)-a^4b^4(\ldots\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((C + A*Sec[c + d*x]^2)*((96*b*(20*A*b^8 + 7*a^4*b^4*(12*A - C) - 8*a^8*C +
 8*a^6*b^2*(-5*A + C) + a^2*b^6*(-69*A + 2*C))*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]]*Cos[c + d*x]^2)/(-a^2 + b^2)^(7/2) - 48*(20*A*b^2
+ a^2*(A + 2*C))*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
48*(20*A*b^2 + a^2*(A + 2*C))*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + (2*a*(24*a^10*A - 324*a^8*A*b^2 + 1116*a^6*A*b^4 - 830*a^4*A*b
^6 - 61*a^2*A*b^8 + 180*A*b^10 + 144*a^8*b^2*C - 50*a^6*b^4*C - 7*a^4*b^6*C
 + 18*a^2*b^8*C - 6*a*b*(20*a^8*A - 150*A*b^8 + 3*a^6*b^2*(3*A - 20*C) + 5*
a^2*b^6*(80*A - 3*C) + 3*a^4*b^4*(-103*A + 15*C))*Cos[c + d*x] + 12*b^2*(20
*A*b^8 - 3*a^8*(7*A - 4*C) + a^6*b^2*(85*A - 2*C) + a^2*b^6*(-19*A + 2*C) -
 a^4*b^4*(55*A + 2*C))*Cos[2*(c + d*x)] - 138*a^7*A*b^3*Cos[3*(c + d*x)] +
738*a^5*A*b^5*Cos[3*(c + d*x)] - 840*a^3*A*b^7*Cos[3*(c + d*x)] + 300*a*A*b
^9*Cos[3*(c + d*x)] + 120*a^7*b^3*C*Cos[3*(c + d*x)] - 90*a^5*b^5*C*Cos[3*(
c + d*x)] + 30*a^3*b^7*C*Cos[3*(c + d*x)] - 24*a^6*A*b^4*Cos[4*(c + d*x)] +
 146*a^4*A*b^6*Cos[4*(c + d*x)] - 167*a^2*A*b^8*Cos[4*(c + d*x)] + 60*A*b^1
0*Cos[4*(c + d*x)] + 26*a^6*b^4*C*Cos[4*(c + d*x)] - 17*a^4*b^6*C*Cos[4*(c
+ d*x)] + 6*a^2*b^8*C*Cos[4*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^3*(a + b
*Cos[c + d*x])^3)))/(48*a^6*d*(2*A + C + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.119, size = 2988, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*C*b^3-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*C-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)
-1)^2+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)
^2+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)+1/
2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)+4/d
*A/a^5/(tan(1/2*d*x+1/2*c)-1)*b+10/d/a^6*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2+4/d
*A/a^5/(tan(1/2*d*x+1/2*c)+1)*b-10/d/a^6*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+12/
d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a*b^2+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
```

)*C*a*b^2+24/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2
-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C-6/d*b^4/a/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*C+1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+2/d*b^6/a
^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+12/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5*A-6/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-1/d*b^5/a^2/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5*C+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-8/d*b/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+24/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)
^3*A-44/3/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+4/d*b^6/a^3/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*C+12/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-7/d*b^5
/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+2/d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
C+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+84/d/a^2*b^5/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A-69/d/a^4*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)
*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-34/d/a^3/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*A*b^6+60/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-21
2/3/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+
b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^6+3/d/a^4/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*A-3/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+30/d/a/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)*A*b^4-6/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5-
34/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6+30/d/a/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A*b^4+6/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93487, size = 1445, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/6*(6*(8*C*a^8*b + 40*A*a^6*b^3 - 8*C*a^6*b^3 - 84*A*a^4*b^5 + 7*C*a^4*b^5
 + 69*A*a^2*b^7 - 2*C*a^2*b^7 - 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(a^2 - b
^2)) + 2*(36*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^7*b^3*tan(1/2*d*x +
1/2*c)^5 + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^6*b^4*tan(1/2*d*x +
1/2*c)^5 - 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^5*tan(1/2*d*x
+ 1/2*c)^5 - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^4*b^6*tan(1/2*d*x
+ 1/2*c)^5 + 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^3*b^7*tan(1/2*d*
x + 1/2*c)^5 - 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*b^8*tan(1/2*d*
x + 1/2*c)^5 - 81*A*a*b^9*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^10*tan(1/2*d*x +
1/2*c)^5 + 72*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 + 180*A*a^6*b^4*tan(1/2*d*x
+ 1/2*c)^3 - 116*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 - 392*A*a^4*b^6*tan(1/2*d
*x + 1/2*c)^3 + 56*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 + 284*A*a^2*b^8*tan(1/2
*d*x + 1/2*c)^3 - 12*C*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^10*tan(1/2*d
*x + 1/2*c)^3 + 36*C*a^8*b^2*tan(1/2*d*x + 1/2*c) + 60*C*a^7*b^3*tan(1/2*d*
x + 1/2*c) + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^4*tan(1/2*d*x +
1/2*c) + 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c) - 45*C*a^5*b^5*tan(1/2*d*x + 1/
2*c) - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^6*tan(1/2*d*x + 1/2*c)
 - 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c) + 15*C*a^3*b^7*tan(1/2*d*x + 1/2*c) -
 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c) + 6*C*a^2*b^8*tan(1/2*d*x + 1/2*c) + 81*
A*a*b^9*tan(1/2*d*x + 1/2*c) + 36*A*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a
^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
/2*c)^2 + a + b)^3) + 3*(A*a^2 + 2*C*a^2 + 20*A*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1))/a^6 - 3*(A*a^2 + 2*C*a^2 + 20*A*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^6 + 6*(A*a*tan(1/2*d*x + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^
3 + A*a*tan(1/2*d*x + 1/2*c) - 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 - 1)^2*a^5))/d
```

3.593 $\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{a\left(3a^2-b^2\right)\sin(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x\left(-4a^2b^2\right.}{8}$$

```
[Out] -((8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a - b]*Sqrt[a + b]*Arc
Tan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(b^5*d) + (a*(3*a^2 - b^2)
*Sin[c + d*x])/(3*b^4*d) - ((4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3
*d) + (a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) - (Cos[c + d*x]^3*Sin[c + d
*x])/(4*b*d)
```

Rubi [A] time = 0.614796, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3050, 3049, 3023, 2735, 2659, 205}

$$\frac{a\left(3a^2-b^2\right)\sin(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x\left(-4a^2b^2\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```



```
[Out] -((8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a - b]*Sqrt[a + b]*Arc
Tan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(b^5*d) + (a*(3*a^2 - b^2)
*Sin[c + d*x])/(3*b^4*d) - ((4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3
*d) + (a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) - (Cos[c + d*x]^3*Sin[c + d
*x])/(4*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
```

```
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{4bd}+\frac{\int \frac{\cos^2(c+dx)\left(-3a+b\cos(c+dx)+4a\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{4b}$$

$$= \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d}-\frac{\cos^3(c+dx)\sin(c+dx)}{4bd}+\frac{\int \frac{\cos(c+dx)\left(8a^2-ab\cos(c+\ldots\right.}{a+b\cos\ldots}}{12\ldots}$$

$$= -\frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d}+\frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d}-\frac{\cos^3(c+dx\ldots}{4\ldots}$$

$$= \frac{a\left(3a^2-b^2\right)\sin(c+dx)}{3b^4d}-\frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d}+\frac{a\cos^2(c+dx)\ldots}{3b^2d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5}+\frac{a\left(3a^2-b^2\right)\sin(c+dx)}{3b^4d}-\frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(\ldots}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5}+\frac{a\left(3a^2-b^2\right)\sin(c+dx)}{3b^4d}-\frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(\ldots}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5}+\frac{2a^3\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d}+\frac{a\left(3a^2-\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.966634, size = 168, normalized size = 0.87

$$\frac{-24a^2b^2\sin(2(c+dx))+24ab\left(4a^2-b^2\right)\sin(c+dx)+192a^3\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+48a^2b^2c+48a^2b^2dx\ldots}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-96*a^4*c + 48*a^2*b^2*c + 12*b^4*c - 96*a^4*d*x + 48*a^2*b^2*d*x + 12*b^4
*d*x + 192*a^3*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^
2 + b^2]] + 24*a*b*(4*a^2 - b^2)*Sin[c + d*x] - 24*a^2*b^2*Sin[2*(c + d*x)]
 + 8*a*b^3*Sin[3*(c + d*x)] - 3*b^4*Sin[4*(c + d*x)])/(96*b^5*d)
```

Maple [B] time = 0.035, size = 653, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^3+1/d/b^3/(tan(1/
2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^2+1/4/d/b/(tan(1/2*d*x+1/2*c)^2+
1)^4*tan(1/2*d*x+1/2*c)^7+6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/
2*c)^5*a^3+1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a^2-8/3/
d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a-7/4/d/b/(tan(1/2*d*
x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan
(1/2*d*x+1/2*c)^3*a^3-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)
^3*a^2+7/4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3-8/3/d/b^2/(t
an(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*a+2/d/b^4/(tan(1/2*d*x+1/2*c)
^2+1)^4*tan(1/2*d*x+1/2*c)*a^3-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d
*x+1/2*c)*a^2-1/4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)-2/d/b^5
*arctan(tan(1/2*d*x+1/2*c))*a^4+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/4/
d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a^5/b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88277, size = 842, normalized size = 4.36

$$\left[\frac{12\sqrt{-a^2+b^2}a^3\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-3\left(8\,a^4-4\,a^2b^2-b^4\right)dx-\left(6\,b^4\right.}{24\,b^5d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/24*(12*sqrt(-a^2 + b^2)*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(
d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2
*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 3*(8*a^4 - 4*a^2*b
^2 - b^4)*d*x - (6*b^4*cos(d*x + c)^3 - 8*a*b^3*cos(d*x + c)^2 - 24*a^3*b +
 8*a*b^3 + 3*(4*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(b^5*d), 1/24*(2
4*sqrt(a^2 - b^2)*a^3*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
 + c))) - 3*(8*a^4 - 4*a^2*b^2 - b^4)*d*x - (6*b^4*cos(d*x + c)^3 - 8*a*b^3
*cos(d*x + c)^2 - 24*a^3*b + 8*a*b^3 + 3*(4*a^2*b^2 - b^4)*cos(d*x + c))*si
n(d*x + c))/(b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48516, size = 500, normalized size = 2.59

$$-\frac{\frac{3\left(8a^4-4a^2b^2-b^4\right)(dx+c)}{b^5}+\frac{48\left(a^5-a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}-\frac{2\left(24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^2b\tan\left(\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/24*(3*(8*a^4 - 4*a^2*b^2 - b^4)*(d*x + c)/b^5 + 48*(a^5 - a^3*b^2)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2
*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) - 2*(
```

```
24*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 3*b^3*tan
(1/2*d*x + 1/2*c)^7 + 72*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 32*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 21*b^3*tan(1/2*d*x + 1/2*c)^
5 + 72*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 32*a*
b^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2
*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 3*b^3*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

3.594 $\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{\left(3a^2-b^2\right)\sin(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d}+\frac{ax\left(2a^2-b^2\right)}{2b^4}+\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}-\frac{\sin}{}$$

```
[Out] (a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(b^4*d) - ((3*a^2 - b^2)*Sin[c + d*x]
)/(3*b^3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (Cos[c + d*x]^2*Sin
[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.389628, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3050, 3049, 3023, 2735, 2659, 205}

$$-\frac{\left(3a^2-b^2\right)\sin(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d}+\frac{ax\left(2a^2-b^2\right)}{2b^4}+\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}-\frac{\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(b^4*d) - ((3*a^2 - b^2)*Sin[c + d*x]
)/(3*b^3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (Cos[c + d*x]^2*Sin
[c + d*x])/(3*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(-2a+b\cos(c+dx)+3a\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{3b}$$

$$= \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{3a^2-ab\cos(c+dx)-2\left(3a^2-b^2\right.}{a+b\cos(c+dx)}}{6b^2}$$

$$= -\frac{\left(3a^2-b^2\right)\sin(c+dx)}{3b^3d} + \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos^2(c+dx)\sin(c+dx)}{3bd}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{\left(3a^2-b^2\right)\sin(c+dx)}{3b^3d} + \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos^2(c+}{}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{\left(3a^2-b^2\right)\sin(c+dx)}{3b^3d} + \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos^2(c+}{}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{2a^2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d} - \frac{\left(3a^2-b^2\right)\sin(c}{3b^3d}$$

Mathematica [A] time = 0.422678, size = 125, normalized size = 0.83

$$-\frac{-6a\left(2a^2-b^2\right)(c+dx)+24a^2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)-3ab^2\sin(2(c+dx))+3b(2a-b)(2a+b)\sin(c+d}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(-6*a*(2*a^2 - b^2)*(c + d*x) + 24*a^2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*T
an[(c + d*x)/2])/Sqrt[-a^2 + b^2]] + 3*(2*a - b)*b*(2*a + b)*Sin[c + d*x] -
 3*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^4*d)
```

Maple [B] time = 0.031, size = 350, normalized size = 2.3

$$-2\,\frac{(\tan(1/2\,dx+c/2))^5\,a^2}{db^3\left((\tan(1/2\,dx+c/2))^2+1\right)^3} - \frac{a}{db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-3} - 4\,\frac{(\tan(1/2\,dx+c/2))^3\,a^2}{db^3\left((\tan(1/2\,dx+c/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a^2-1/d/b^2/(tan(1
/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a-4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1
)^3*tan(1/2*d*x+1/2*c)^3*a^2+8/3/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x
+1/2*c)^3-2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a^2+1/d/b^2
/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a+2/d/b^4*arctan(tan(1/2*d*x
+1/2*c))*a^3-1/d/b^2*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^4/b^4/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*a^2/b^2/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77885, size = 703, normalized size = 4.69

$$\left[\frac{3\sqrt{-a^2+b^2}a^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+3\left(2\,a^3-ab^2\right)dx-\left(2\,b^3\cos\left(dx\right.\right.}{6\,b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*sqrt(-a^2 + b^2)*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b
^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(2*a^3 - a*b^2)*d*
x - (2*b^3*cos(d*x + c)^2 - 3*a*b^2*cos(d*x + c) + 6*a^2*b - 2*b^3)*sin(d*x
 + c))/(b^4*d), -1/6*(6*sqrt(a^2 - b^2)*a^2*arctan(-(a*cos(d*x + c) + b)/(s
```

```
qrt(a^2 - b^2)*sin(d*x + c))) - 3*(2*a^3 - a*b^2)*d*x + (2*b^3*cos(d*x + c)
^2 - 3*a*b^2*cos(d*x + c) + 6*a^2*b - 2*b^3)*sin(d*x + c))/(b^4*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65114, size = 309, normalized size = 2.06

$$\frac{\frac{3\left(2a^3-ab^2\right)(dx+c)}{b^4}+\frac{12\left(a^4-a^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}-\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+\cdots\right.}{\cdots}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a^3 - a*b^2)*(d*x + c)/b^4 + 12*(a^4 - a^2*b^2)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^4) - 2*(6*a^2*tan(1/
2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*tan(1/2*d*x + 1/2*
c)^3 - 8*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c) - 3*a*b*ta
n(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.595 $\int \frac{\cos(c+dx)\left(1-\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{x\left(2a^2-b^2\right)}{2b^3}+\frac{a\sin(c+dx)}{b^2d}+\frac{2a\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] -((2*a^2 - b^2)*x)/(2*b^3) + (2*a*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(b^3*d) + (a*Sin[c + d*x])/(b^2*d) - (Co
s[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.200214, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3050, 3023, 2735, 2659, 205}

$$-\frac{x\left(2a^2-b^2\right)}{2b^3}+\frac{a\sin(c+dx)}{b^2d}+\frac{2a\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((2*a^2 - b^2)*x)/(2*b^3) + (2*a*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(b^3*d) + (a*Sin[c + d*x])/(b^2*d) - (Co
s[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(1-\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{-a+b\cos(c+dx)+2a\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= \frac{a\sin(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{-ab-\left(2a^2-b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{2b^2}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} + \frac{a\sin(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\left(a\left(a^2-b^2\right)\right)\int \frac{1}{a+b\cos(\cdots}}{b^3}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} + \frac{a\sin(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\left(2a\left(a^2-b^2\right)\right)\text{Subst}\left(\cdots\right.}{}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} + \frac{2a\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d} + \frac{a\sin(c+dx)}{b^2d} - \cos(\cdots$$

Mathematica [A] time = 0.303774, size = 98, normalized size = 0.9

$$\frac{-2\left(2a^2-b^2\right)(c+dx)+8a\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+4ab\sin(c+dx)+b^2(-\sin(2(c+dx)))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(2*a^2 - b^2)*(c + d*x) + 8*a*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c
+ d*x)/2])/Sqrt[-a^2 + b^2]] + 4*a*b*Sin[c + d*x] - b^2*Sin[2*(c + d*x)])/(
4*b^3*d)
```

Maple [B] time = 0.024, size = 269, normalized size = 2.5

$$2\,\frac{(\tan(1/2\,dx+c/2))^3\,a}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)^2} + \frac{1}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-2} + 2\,\frac{\tan(1/2\,dx+c/2)\,a}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a+1/d/b/(tan(1/2*d*
x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan
(1/2*d*x+1/2*c)*a-1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)-2/d/b
^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a^3/
b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)-2/d*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.467, size = 586, normalized size = 5.38

$$\left[-\frac{\left(2\,a^2-b^2\right)dx-\sqrt{-a^2+b^2}a\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(b^2\cos\left(dx+c\right)\right.}{2\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((2*a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
 + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (b^
2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(b^3*d), -1/2*((2*a^2 - b^2)*d*x - 2*
sqrt(a^2 - b^2)*a*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c
))) + (b^2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35877, size = 251, normalized size = 2.3

$$-\frac{\frac{\left(2\,a^{2}-b^{2}\right)(dx+c)}{b^{3}}+\frac{4\left(a^{3}-ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}b^{3}}-\frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,a\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*(d*x + c)/b^3 + 4*(a^3 - a*b^2)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2*(2*a*tan(1/2*d*x +
1/2*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.596 $\int \frac{1-\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{ax}{b^2}-\frac{\sin(c+dx)}{bd}$$

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(b^2*d) - Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.104991, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3024, 2735, 2659, 205}

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{ax}{b^2}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(b^2*d) - Sin[c + d*x]/(b*d)
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx &= -\frac{\sin(c+dx)}{bd}+\frac{\int \frac{b+a\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{ax}{b^2}-\frac{\sin(c+dx)}{bd}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\ &= \frac{ax}{b^2}-\frac{\sin(c+dx)}{bd}-\frac{\left(2\left(a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\ &= \frac{ax}{b^2}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{\sin(c+dx)}{bd}\end{aligned}$$

Mathematica [A] time = 0.13475, size = 69, normalized size = 0.95

$$\frac{-2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+a(c+dx)-b\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x) - 2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]] - b*Sin[c + d*x])/(b^2*d)
```

Maple [B] time = 0.022, size = 145, normalized size = 2.

$$-2\frac{\tan(1/2\,dx+c/2)}{db\left((\tan(1/2\,dx+c/2))^2+1\right)}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,a}{db^2}-2\,\frac{a^2}{db^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+}{\sqrt{(a+b)(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/b^2*arctan(tan(1/2*d
*x+1/2*c))*a-2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))+2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43336, size = 475, normalized size = 6.51

$$\left[\frac{2\,adx-2\,b\sin(dx+c)+\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,b^2d},\ \frac{adx-b\,\mathrm{s}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x - 2*b*sin(d*x + c) + sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c)
 + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
)/(b^2*d), (a*d*x - b*sin(d*x + c) - sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c
```

```
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56302, size = 165, normalized size = 2.26

$$\frac{\frac{(dx+c)a}{b^2} + \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + ar
ctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*s
qrt(a^2 - b^2)/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b
))/d
```

3.597 $\int \frac{(1-\cos^2(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{b}$$

```
[Out] -(x/b) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a*b*d) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.116554, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3058, 2659, 205, 3770}

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```


```
[Out] -(x/b) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a*b*d) + ArcTanh[Sin[c + d*x]]/(a*d)
```


Rule 3058

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(C*x
)/(b*d), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{x}{b}+\frac{\int \sec(c+dx)\,dx}{a}-\left(-\frac{a}{b}+\frac{b}{a}\right)\int \frac{1}{a+b\cos(c+dx)}\,dx$$

$$= -\frac{x}{b}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx\right.\right.}{d}$$

$$= -\frac{x}{b}+\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.135954, size = 115, normalized size = 1.51

$$-\frac{-2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+ac+adx+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((a*c + a*d*x - 2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]] + b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [B] time = 0.043, size = 153, normalized size = 2.

$$-2\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{db}+2\frac{a}{db\sqrt{\left(a+b\right)\left(a-b\right)}}\arctan\left(\frac{\left(a-b\right)\tan\left(1/2\,dx+c/2\right)}{\sqrt{\left(a+b\right)\left(a-b\right)}}\right)-2\frac{b}{da\sqrt{\left(a+b\right)\left(a-b\right)}}\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d/a*b/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/a*ln(tan(1/2*d*x+1/2*c)-1
)+1/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sec (c+dx)}{a+b\cos (c+dx)}\,dx-\int \frac{\cos ^2(c+dx)\sec (c+dx)}{a+b\cos (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -Integral(-sec(c + d*x)/(a + b*cos(c + d*x)), x) - Integral(cos(c + d*x)**2
*sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.34528, size = 176, normalized size = 2.32

$$-\frac{\frac{dx+c}{b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arc
tan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*sqr
t(a^2 - b^2)/(a*b))/d
```

3.598 $\int \frac{(1-\cos^2(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*d) - (b*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.206443, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3056, 3001, 3770, 2659, 205}

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*d) - (b*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a*d)
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\tan(c+dx)}{ad}+\frac{\int \frac{(-b-a\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{\tan(c+dx)}{ad}-\frac{b\int \sec(c+dx)\,dx}{a^2}+\frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2}$$

$$= -\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}-\frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x\right.}{a^2d}$$

$$= -\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx}{ad}$$

Mathematica [A] time = 0.270653, size = 112, normalized size = 1.37

$$\frac{-2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+a\tan(c+dx)+b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] +
 b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]) + a*Tan[c + d*x])/(a^2*d)
```

Maple [B] time = 0.049, size = 177, normalized size = 2.2

$$-2\,\frac{1}{d\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{b^2}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))+2/d/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*b^2-1/d/a/(tan(1/2*d*x+1/2*c)-1)+1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)-1
)-1/d/a/(tan(1/2*d*x+1/2*c)+1)-1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67416, size = 748, normalized size = 9.12

$$\left[-\frac{b\cos(dx+c)\log(\sin(dx+c)+1) - b\cos(dx+c)\log(-\sin(dx+c)+1) - \sqrt{-a^2+b^2}\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)+(2}{}\right.}{2\,a^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/2*(b*cos(d*x + c)*log(sin(d*x + c) + 1) - b*cos(d*x + c)*log(-sin(d*x +
 c) + 1) - sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*a*sin(
d*x + c))/(a^2*d*cos(d*x + c)), -1/2*(b*cos(d*x + c)*log(sin(d*x + c) + 1)
- b*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - 2*a*sin(d*x +
c))/(a^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx - \int \frac{\cos^2(c+dx)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] -Integral(-sec(c + d*x)**2/(a + b*cos(c + d*x)), x) - Integral(cos(c + d*x)
**2*sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [B] time = 1.29077, size = 203, normalized size = 2.48

$$-\frac{\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2} + \frac{2\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -(b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - b*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/a^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-
(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*sqrt(a^
2 - b^2)/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.599 $\int \frac{(1-\cos^2(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{(a^2-2b^2)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{2b\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}-\frac{b\tan(c+dx)}{a^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/(a^3*d) - ((a^2 - 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b*Tan[c
+ d*x])/(a^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.374339, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{(a^2-2b^2)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{2b\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}-\frac{b\tan(c+dx)}{a^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/(a^3*d) - ((a^2 - 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b*Tan[c
+ d*x])/(a^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
```

```
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2b-a\cos(c+dx)+b\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2a}$$

$$= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-a^2+2b^2+ab\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{2a^2}$$

$$= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(a^2-2b^2\right)\int \sec(c+dx)\,dx}{2a^3} + \frac{\left(b\left(a^2\right.\right.}{}$$

$$= -\frac{\left(a^2-2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{(2}{}$$

$$= \frac{2\sqrt{a-b}b\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d} - \frac{\left(a^2-2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{b\,\mathrm{t}}{}$$

Mathematica [B] time = 1.09951, size = 236, normalized size = 2.02

$$\frac{8b\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right) + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (8*b*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
+ 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4
*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*Tan[
c + d*x])/(4*a^3*d)
```

Maple [B] time = 0.052, size = 309, normalized size = 2.6

$$2\,\frac{b}{da\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right) - 2\,\frac{b^3}{da^3\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right) + \frac{1}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a/(tan(1/2*d*x+1/2*c
)-1)+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*b+1/2/d/a*ln(tan(1/2*d*x+1/2*c)-1)-1/d/
a^3*ln(tan(1/2*d*x+1/2*c)-1)*b^2-1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a/(
tan(1/2*d*x+1/2*c)+1)+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*b-1/2/d/a*ln(tan(1/2*d
*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8836, size = 907, normalized size = 7.75

$$\left[\frac{2\sqrt{-a^2+b^2}b\cos\left(dx+c\right)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(a^2-2\,b^2\right)\cos\left(dx\right.}{4\,a^3d\cos\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*sqrt(-a^2 + b^2)*b*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2*a^2
- b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c
) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^2 -
2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^2 - 2*b^2)*cos(d*x + c)^2*
```

```
log(-sin(d*x + c) + 1) - 2*(2*a*b*cos(d*x + c) - a^2)*sin(d*x + c))/(a^3*d*
cos(d*x + c)^2), 1/4*(4*sqrt(a^2 - b^2)*b*arctan(-(a*cos(d*x + c) + b)/(sqr
t(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (a^2 - 2*b^2)*cos(d*x + c)^2*l
og(sin(d*x + c) + 1) + (a^2 - 2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
- 2*(2*a*b*cos(d*x + c) - a^2)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39889, size = 296, normalized size = 2.53

$$-\frac{\frac{(a^2-2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{(a^2-2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{4\left(a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*((a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (a^2 - 2*b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 4*(a^2*b - b^3)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3) - 2*(a*tan(1/2*d*x
 + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 2*b*tan
(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.600 $\int \frac{\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{2b^2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d}-\frac{\left(a^2-3b^2\right)\tan(c+dx)}{3a^3d}+\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{b\tan(c+d}{2}$$

```
[Out] (-2*Sqrt[a - b]*b^2*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^4*d) + (b*(a^2 - 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - ((a^
2 - 3*b^2)*Tan[c + d*x])/(3*a^3*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d
) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.569702, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d}-\frac{\left(a^2-3b^2\right)\tan(c+dx)}{3a^3d}+\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{b\tan(c+d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a - b]*b^2*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^4*d) + (b*(a^2 - 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - ((a^
2 - 3*b^2)*Tan[c + d*x])/(3*a^3*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d
) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3b-a\cos(c+dx)+2b\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx}{3a}$$

$$= -\frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-2\left(a^2-3b^2\right)+ab\cos(c+a\right.}{a+b}}{}$$

$$= -\frac{\left(a^2-3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx}{3ad}$$

$$= -\frac{\left(a^2-3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx}{3ad}$$

$$= \frac{b\left(a^2-2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{\left(a^2-3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(}{2a^2d}$$

$$= -\frac{2\sqrt{a-b}b^2\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d} + \frac{b\left(a^2-2b^2\right)\tanh^{-1}(\sin(c+dx)}{2a^4d}$$

Mathematica [A] time = 2.50952, size = 256, normalized size = 1.65

$$-\frac{24b^2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right) + \frac{1}{2}\sec^3(c+dx)\left(4a\sin(c+dx)\left(\left(a^2-3b^2\right)\cos(2(c+dx)) - a^2 + 3ab\cos(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(24*b^2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]] + (Sec[c + d*x]^3*(9*b*(a^2 - 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*b*(a^2
 - 2*b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*a*(-a^2 - 3*b^2 + 3*a*b*Cos[c + d
*x] + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*a^4*d)
```

Maple [B] time = 0.057, size = 407, normalized size = 2.6

$$-2\frac{b^2}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\frac{b^4}{da^4\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*b^2+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))-1/3/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tan(1/2*d*x+
1/2*c)-1)^2-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2*b-1/2/d/a^2/(tan(1/2*d*x+1/2
*c)-1)*b-1/d*b^2/a^3/(tan(1/2*d*x+1/2*c)-1)-1/2/d*b/a^2*ln(tan(1/2*d*x+1/2*
c)-1)+1/d*b^3/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3
+1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2*b-1/2/
d/a^2/(tan(1/2*d*x+1/2*c)+1)*b-1/d*b^2/a^3/(tan(1/2*d*x+1/2*c)+1)+1/2/d*b/a
^2*ln(tan(1/2*d*x+1/2*c)+1)-1/d*b^3/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04623, size = 1045, normalized size = 6.74

$$\left[\frac{6\sqrt{-a^2+b^2}b^2\cos(dx+c)^3\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+3\left(a^2b-2\,b^3\right)\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*sqrt(-a^2 + b^2)*b^2*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(a
^2*b - 2*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(a^2*b - 2*b^3)*cos(
d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a^2*b*cos(d*x + c) - 2*a^3 + 2*(a^
3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a^4*d*cos(d*x + c)^3), -1/12*(1
2*sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
 + c)))*cos(d*x + c)^3 - 3*(a^2*b - 2*b^3)*cos(d*x + c)^3*log(sin(d*x + c)
+ 1) + 3*(a^2*b - 2*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(3*a^2*b
*cos(d*x + c) - 2*a^3 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a^
4*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24251, size = 359, normalized size = 2.32

$$\frac{\frac{3\left(a^{2}b-2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{3\left(a^{2}b-2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{4}}+\frac{12\left(a^{2}b^{2}-b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\right.}{\sqrt{a^{2}-b^{2}}}\right.\right.}{\sqrt{a^{2}-b^{2}}\,a^{4}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3*(a^2*b -
2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 12*(a^2*b^2 - b^4)*(pi*floo
```

```
r(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c)
 - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - 2*(3*a
*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*tan(1/2*d*
x + 1/2*c)^3 - 12*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c) +
 6*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.601 $\int \frac{\cos^4(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=237

$$\frac{a\left(15a^2-2b^2\right)\sin(c+dx)}{3b^5d}+\frac{2a^3\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^6d\sqrt{a-b}\sqrt{a+b}}-\frac{\left(20a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^4d}-\frac{x\left(-12a^2\right.}{}$$

```
[Out] -((40*a^4 - 12*a^2*b^2 - b^4)*x)/(8*b^6) + (2*a^3*(5*a^2 - 4*b^2)*ArcTan[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^6*Sqrt[a + b]*d)
+ (a*(15*a^2 - 2*b^2)*Sin[c + d*x])/(3*b^5*d) - ((20*a^2 - b^2)*Cos[c + d*x
]*Sin[c + d*x])/(8*b^4*d) + (5*a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^3*d) - (
5*Cos[c + d*x]^3*Sin[c + d*x])/(4*b^2*d) + (Cos[c + d*x]^4*Sin[c + d*x])/(b
*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.898475, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3050, 3049, 3023, 2735, 2659, 205}

$$\frac{a\left(15a^2-2b^2\right)\sin(c+dx)}{3b^5d}+\frac{2a^3\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^6d\sqrt{a-b}\sqrt{a+b}}-\frac{\left(20a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^4d}-\frac{x\left(-12a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -((40*a^4 - 12*a^2*b^2 - b^4)*x)/(8*b^6) + (2*a^3*(5*a^2 - 4*b^2)*ArcTan[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^6*Sqrt[a + b]*d)
+ (a*(15*a^2 - 2*b^2)*Sin[c + d*x])/(3*b^5*d) - ((20*a^2 - b^2)*Cos[c + d*x
]*Sin[c + d*x])/(8*b^4*d) + (5*a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^3*d) - (
5*Cos[c + d*x]^3*Sin[c + d*x])/(4*b^2*d) + (Cos[c + d*x]^4*Sin[c + d*x])/(b
*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
```

```
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
```

```
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \frac{\cos^4(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \frac{\int \frac{\cos^3(c+dx)\left(-4\left(a^2-b^2\right)+5\left(a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{5\cos^3(c+dx)\sin(c+dx)}{4b^2d} + \frac{\cos^4(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \int \frac{\cos^2(c+dx)\left(15a\left(a^2-b^2\right)-b\right.}{}$$

$$= \frac{5a\cos^2(c+dx)\sin(c+dx)}{3b^3d} - \frac{5\cos^3(c+dx)\sin(c+dx)}{4b^2d} + \frac{\cos^4(c+dx)\sin(c+}{bd(a+b\cos(c+dx}$$

$$= -\frac{\left(20a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{5a\cos^2(c+dx)\sin(c+dx)}{3b^3d} - \frac{5\cos^3(c+}{}$$

$$= \frac{a\left(15a^2-2b^2\right)\sin(c+dx)}{3b^5d} - \frac{\left(20a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{5a\cos^2(c+d}{3b}$$

$$= -\frac{\left(40a^4-12a^2b^2-b^4\right)x}{8b^6} + \frac{a\left(15a^2-2b^2\right)\sin(c+dx)}{3b^5d} - \frac{\left(20a^2-b^2\right)\cos(c+dx)}{8b^4d}$$

$$= -\frac{\left(40a^4-12a^2b^2-b^4\right)x}{8b^6} + \frac{a\left(15a^2-2b^2\right)\sin(c+dx)}{3b^5d} - \frac{\left(20a^2-b^2\right)\cos(c+dx)}{8b^4d}$$

$$= -\frac{\left(40a^4-12a^2b^2-b^4\right)x}{8b^6} + \frac{2a^3\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^6\sqrt{a+b}d} + \frac{a\left(15a^2-\right.}{}$$

Mathematica [A] time = 3.75048, size = 271, normalized size = 1.14

$$\frac{\frac{240a^3b^2\sin(2(c+dx))+24a^2b\left(40a^2-7b^2\right)\sin(c+dx)-40a^2b^3\sin(3(c+dx))+24b\left(12a^2b^2-40a^4+b^4\right)(c+dx)\cos(c+dx)+288a^3b^2c+288a^3b^2dx-960a^5c-960a^5dx-32ab}{a+b\cos(c+dx)}}{192b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-384*a^3*(5*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b
^2]])/Sqrt[-a^2 + b^2] + (-960*a^5*c + 288*a^3*b^2*c + 24*a*b^4*c - 960*a^5
*d*x + 288*a^3*b^2*d*x + 24*a*b^4*d*x + 24*b*(-40*a^4 + 12*a^2*b^2 + b^4)*(
c + d*x)*Cos[c + d*x] + 24*a^2*b*(40*a^2 - 7*b^2)*Sin[c + d*x] + 240*a^3*b^
```

```
2*Sin[2*(c + d*x)] - 32*a*b^4*Sin[2*(c + d*x)] - 40*a^2*b^3*Sin[3*(c + d*x)
] - 3*b^5*Sin[3*(c + d*x)] + 10*a*b^4*Sin[4*(c + d*x)] - 3*b^5*Sin[5*(c + d
*x)])/(a + b*Cos[c + d*x]))/(192*b^6*d)
```

Maple [B] time = 0.038, size = 708, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 8/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^3+3/d/b^4/(tan(1/
2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^2+1/4/d/b^2/(tan(1/2*d*x+1/2*c)^
2+1)^4*tan(1/2*d*x+1/2*c)^7+24/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x
+1/2*c)^5*a^3+3/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a^2-1
6/3/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a-7/4/d/b^2/(tan(
1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5+24/d/b^5/(tan(1/2*d*x+1/2*c)^2+1
)^4*tan(1/2*d*x+1/2*c)^3*a^3-3/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x
+1/2*c)^3*a^2+7/4/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3-16/
3/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*a+8/d/b^5/(tan(1/2*
d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*a^3-3/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4
*tan(1/2*d*x+1/2*c)*a^2-1/4/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/
2*c)-10/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^4+3/d/b^4*arctan(tan(1/2*d*x+1/2
*c))*a^2+1/4/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2/d*a^4/b^5*tan(1/2*d*x+1/2*c
)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)+10/d*a^5/b^6/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-8/d*a^3/b^
4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02855, size = 1634, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/24*(3*(40*a^6*b - 52*a^4*b^3 + 11*a^2*b^5 + b^7)*d*x*cos(d*x + c) + 3*(
40*a^7 - 52*a^5*b^2 + 11*a^3*b^4 + a*b^6)*d*x - 12*(5*a^6 - 4*a^4*b^2 + (5*
a^5*b - 4*a^3*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
 (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin
(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) -
 (120*a^6*b - 136*a^4*b^3 + 16*a^2*b^5 - 6*(a^2*b^5 - b^7)*cos(d*x + c)^4 +
 10*(a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (20*a^4*b^3 - 23*a^2*b^5 + 3*b^7)*co
s(d*x + c)^2 + (60*a^5*b^2 - 73*a^3*b^4 + 13*a*b^6)*cos(d*x + c))*sin(d*x +
 c))/((a^2*b^7 - b^9)*d*cos(d*x + c) + (a^3*b^6 - a*b^8)*d), -1/24*(3*(40*a
^6*b - 52*a^4*b^3 + 11*a^2*b^5 + b^7)*d*x*cos(d*x + c) + 3*(40*a^7 - 52*a^5
*b^2 + 11*a^3*b^4 + a*b^6)*d*x - 24*(5*a^6 - 4*a^4*b^2 + (5*a^5*b - 4*a^3*b
^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
b^2)*sin(d*x + c))) - (120*a^6*b - 136*a^4*b^3 + 16*a^2*b^5 - 6*(a^2*b^5 -
b^7)*cos(d*x + c)^4 + 10*(a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (20*a^4*b^3 - 2
3*a^2*b^5 + 3*b^7)*cos(d*x + c)^2 + (60*a^5*b^2 - 73*a^3*b^4 + 13*a*b^6)*co
s(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c) + (a^3*b^6 - a*b^
8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25701, size = 568, normalized size = 2.4

$$\frac{48\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)b^5}-\frac{3\left(40\,a^4-12\,a^2b^2-b^4\right)(dx+c)}{b^6}-\frac{48\left(5\,a^5-4\,a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\right.}{\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/24*(48*a^4*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*
x + 1/2*c)^2 + a + b)*b^5) - 3*(40*a^4 - 12*a^2*b^2 - b^4)*(d*x + c)/b^6 -
48*(5*a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))
/(sqrt(a^2 - b^2)*b^6) + 2*(96*a^3*tan(1/2*d*x + 1/2*c)^7 + 36*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 3*b^3*tan(1/2*d*x + 1/2*c)^7 + 288*a^3*tan(1/2*d*x + 1/2
*c)^5 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 64*a*b^2*tan(1/2*d*x + 1/2*c)^5 -
 21*b^3*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*tan(1/2*d*x + 1/2*c)^3 - 36*a^2*b*
tan(1/2*d*x + 1/2*c)^3 - 64*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^3*tan(1/2*d
*x + 1/2*c)^3 + 96*a^3*tan(1/2*d*x + 1/2*c) - 36*a^2*b*tan(1/2*d*x + 1/2*c)
 - 3*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^5))/d
```

3.602 $\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(12a^2-b^2\right)\sin(c+dx)}{3b^4d}-\frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{ax\left(4a^2-b^2\right)}{b^5}+\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}+\frac{\sin(\cdots}{bd(\cdots}$$

```
[Out] (a*(4*a^2 - b^2)*x)/b^5 - (2*a^2*(4*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) - ((12*a^2 - b^2)
*Sin[c + d*x])/(3*b^4*d) + (2*a*Cos[c + d*x]*Sin[c + d*x])/(b^3*d) - (4*Cos
[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(b*d*(a
 + b*Cos[c + d*x]))
```

Rubi [A] time = 0.616477, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3048, 3050, 3049, 3023, 2735, 2659, 205}

$$-\frac{\left(12a^2-b^2\right)\sin(c+dx)}{3b^4d}-\frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{ax\left(4a^2-b^2\right)}{b^5}+\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}+\frac{\sin(\cdots}{bd(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (a*(4*a^2 - b^2)*x)/b^5 - (2*a^2*(4*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) - ((12*a^2 - b^2)
*Sin[c + d*x])/(3*b^4*d) + (2*a*Cos[c + d*x]*Sin[c + d*x])/(b^3*d) - (4*Cos
[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(b*d*(a
 + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{4\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \int \frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(a^2\right.\right.}{}$$

$$= \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} - \frac{4\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+d}{bd(a+b\cos(c+dx))}$$

$$= -\frac{\left(12a^2-b^2\right)\sin(c+dx)}{3b^4d} + \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} - \frac{4\cos^2(c+dx)\sin(c+d}{3b^2d}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} - \frac{\left(12a^2-b^2\right)\sin(c+dx)}{3b^4d} + \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} - \frac{4\cos^2(c}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} - \frac{\left(12a^2-b^2\right)\sin(c+dx)}{3b^4d} + \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} - \frac{4\cos^2(c}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} - \frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^5\sqrt{a+b}\,d} - \frac{\left(12a^2-b^2\right)\sin(c+a}{3b^4d}$$

Mathematica [A] time = 2.25002, size = 217, normalized size = 1.15

$$\frac{\frac{-24a^2b^2\sin(2(c+dx))+12ab\left(b^2-8a^2\right)\sin(c+dx)+24ab\left(4a^2-b^2\right)(c+dx)\cos(c+dx)-24a^2b^2c-24a^2b^2dx+96a^4c+96a^4dx+4ab^3\sin(3(c+dx))+2b^4\sin(2(c+dx))-b^4 s}{a+b\cos(c+dx)}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((48*a^2*(4*a^2 - 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + (96*a^4*c - 24*a^2*b^2*c + 96*a^4*d*x - 24*a^2*b^2*d
*x + 24*a*b*(4*a^2 - b^2)*(c + d*x)*Cos[c + d*x] + 12*a*b*(-8*a^2 + b^2)*Si
n[c + d*x] - 24*a^2*b^2*Sin[2*(c + d*x)] + 2*b^4*Sin[2*(c + d*x)] + 4*a*b^3
*Sin[3*(c + d*x)] - b^4*Sin[4*(c + d*x)])/(a + b*Cos[c + d*x]))/(24*b^5*d)
```

Maple [B] time = 0.035, size = 403, normalized size = 2.1

$$-6\frac{(\tan(1/2\,dx+c/2))^5a^2}{db^4\left((\tan(1/2\,dx+c/2))^2+1\right)^3}-2\frac{(\tan(1/2\,dx+c/2))^5a}{db^3\left((\tan(1/2\,dx+c/2))^2+1\right)^3}-12\frac{(\tan(1/2\,dx+c/2))^3a^2}{db^4\left((\tan(1/2\,dx+c/2))^2+1\right)^3}+\frac{8}{3\,db^2}\Big(\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a^2-2/d/b^3/(tan(1
/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a-12/d/b^4/(tan(1/2*d*x+1/2*c)^2+
1)^3*tan(1/2*d*x+1/2*c)^3*a^2+8/3/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*
d*x+1/2*c)^3-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a^2+2/d/
b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a+8/d/b^5*arctan(tan(1/2*
d*x+1/2*c))*a^3-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^3/b^4*tan(1/2*d*
x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)-8/d*a^4/b^5/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+6/d*
a^2/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71853, size = 1374, normalized size = 7.27

$$\left[\frac{6\left(4a^5b-5a^3b^3+ab^5\right)dx\cos\left(dx+c\right)+6\left(4a^6-5a^4b^2+a^2b^4\right)dx+3\left(4a^5-3a^3b^2+\left(4a^4b-3a^2b^3\right)\cos\left(dx+c\right)\right)\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/6*(6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 6*(4*a^6 - 5*a^4*b
^2 + a^2*b^4)*d*x + 3*(4*a^5 - 3*a^3*b^2 + (4*a^4*b - 3*a^2*b^3)*cos(d*x +
c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*a^5*b - 13*a^3*b^3 + a
*b^5 + (a^2*b^4 - b^6)*cos(d*x + c)^3 - 2*(a^3*b^3 - a*b^5)*cos(d*x + c)^2
+ (6*a^4*b^2 - 7*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 - b^8
)*d*cos(d*x + c) + (a^3*b^5 - a*b^7)*d), 1/3*(3*(4*a^5*b - 5*a^3*b^3 + a*b^
5)*d*x*cos(d*x + c) + 3*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*d*x - 3*(4*a^5 - 3*a^
3*b^2 + (4*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*a^5*b - 13*a^3*b^3 + a*
b^5 + (a^2*b^4 - b^6)*cos(d*x + c)^3 - 2*(a^3*b^3 - a*b^5)*cos(d*x + c)^2 +
 (6*a^4*b^2 - 7*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 - b^8)
*d*cos(d*x + c) + (a^3*b^5 - a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24532, size = 378, normalized size = 2.

$$-\frac{\frac{6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b\right)b^{4}}-\frac{3\left(4\,a^{3}-ab^{2}\right)(dx+c)}{b^{5}}-\frac{6\left(4\,a^{4}-3\,a^{2}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\sqrt{a^{2}-b^{2}}}\right.\right.}{\sqrt{a^{2}-b^{2}}\,b^{5}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/3*(6*a^3*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
 + 1/2*c)^2 + a + b)*b^4) - 3*(4*a^3 - a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 3*
a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b
^2)*b^5) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 +
 18*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1
/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)
^3*b^4))/d
```

3.603 $\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(6a^2-b^2\right)}{2b^4}+\frac{3a\sin(c+dx)}{b^3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{bd(a+b\cos(c+dx))}-\frac{3\sin(c+dx)\cos(c}{2b^2d}$$

```
[Out] -((6*a^2 - b^2)*x)/(2*b^4) + (2*a*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + (3*a*Sin[c + d
*x])/(b^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(b*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.397235, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3050, 3023, 2735, 2659, 205}

$$\frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(6a^2-b^2\right)}{2b^4}+\frac{3a\sin(c+dx)}{b^3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{bd(a+b\cos(c+dx))}-\frac{3\sin(c+dx)\cos(c}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((6*a^2 - b^2)*x)/(2*b^4) + (2*a*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + (3*a*Sin[c + d
*x])/(b^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(b*d*(a + b*Cos[c + d*x]))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-2\left(a^2-b^2\right)+3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \frac{\int \frac{3a\left(a^2-b^2\right)-b\left(a^2-b^2\right)\cos(c+}{a+b\cos}}{2b^2\left(a^2\right.}$$

$$= \frac{3a\sin(c+dx)}{b^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{bd(a+b\cos(c+dx))} - \int \frac{3ab\left(a^2-\right.}{}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{3a\sin(c+dx)}{b^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c}{bd(a+b\cos(c+}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{3a\sin(c+dx)}{b^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c}{bd(a+b\cos(c+}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{3a\sin(c+dx)}{b^3d} - \frac{3\,\mathrm{co}}{}$$

Mathematica [A] time = 0.29238, size = 131, normalized size = 0.85

$$\frac{2\left(b^2-6a^2\right)(c+dx) - \frac{8a\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{4a^2b\sin(c+dx)}{a+b\cos(c+dx)} + 8ab\sin(c+dx) - b^2\sin(2(c+dx))}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(-6*a^2 + b^2)*(c + d*x) - (8*a*(3*a^2 - 2*b^2)*ArcTanh[((a - b)*Tan[(c
+ d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 8*a*b*Sin[c + d*x] + (4*a^
2*b*Sin[c + d*x])/(a + b*Cos[c + d*x]) - b^2*Sin[2*(c + d*x)])/(4*b^4*d)
```

Maple [B] time = 0.034, size = 321, normalized size = 2.1

$$4\,\frac{(\tan(1/2\,dx+c/2))^3\,a}{db^3\left((\tan(1/2\,dx+c/2))^2+1\right)^2} + \frac{1}{db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-2} + 4\,\frac{\tan(1/2\,dx+c/2)\,a}{db^3\left((\tan(1/2\,dx+c/2))^2+1\right)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a+1/d/b^2/(tan(1/2*
d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3+4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*t
an(1/2*d*x+1/2*c)*a-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)-6
/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2/
d*a^2/b^3*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)+6/d*a^3/b^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))-4/d*a/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62666, size = 1191, normalized size = 7.73

$$\left[-\frac{\left(6\,a^4b-7\,a^2b^3+b^5\right)dx\cos\left(dx+c\right)+\left(6\,a^5-7\,a^3b^2+ab^4\right)dx-\left(3\,a^4-2\,a^2b^2+\left(3\,a^3b-2\,ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*((6*a^4*b - 7*a^2*b^3 + b^5)*d*x*cos(d*x + c) + (6*a^5 - 7*a^3*b^2 +
a*b^4)*d*x - (3*a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(-a
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-
```

```
a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*a^4*b - 6*a^2*b^3 - (a^2*b^3 - b^5)*
cos(d*x + c)^2 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5
- b^7)*d*cos(d*x + c) + (a^3*b^4 - a*b^6)*d), -1/2*((6*a^4*b - 7*a^2*b^3 +
b^5)*d*x*cos(d*x + c) + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x - 2*(3*a^4 - 2*a^2*
b^2 + (3*a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^4*b - 6*a^2*b^3 - (a^2*b^3
 - b^5)*cos(d*x + c)^2 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((
a^2*b^5 - b^7)*d*cos(d*x + c) + (a^3*b^4 - a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.67666, size = 321, normalized size = 2.08

$$\frac{\frac{4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b\right)b^{3}}-\frac{\left(6\,a^{2}-b^{2}\right)(dx+c)}{b^{4}}-\frac{4\left(3\,a^{3}-2\,ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{4}}+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*a^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
+ 1/2*c)^2 + a + b)*b^3) - (6*a^2 - b^2)*(d*x + c)/b^4 - 4*(3*a^3 - 2*a*b^2
)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^4
) + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^3)
)/d
```

3.604 $\int \frac{\cos(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{a\sin(c+dx)}{b^2d(a+b\cos(c+dx))}+\frac{2ax}{b^3}-\frac{\sin(c+dx)}{b^2d}$$

```
[Out] (2*a*x)/b^3 - (2*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - Sin[c + d*x]/(b^2*d) - (a*Sin[c +
 d*x])/(b^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.258268, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3032, 3023, 2735, 2659, 205}

$$-\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{a\sin(c+dx)}{b^2d(a+b\cos(c+dx))}+\frac{2ax}{b^3}-\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 - (2*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - Sin[c + d*x]/(b^2*d) - (a*Sin[c +
 d*x])/(b^2*d*(a + b*Cos[c + d*x]))
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a\sin(c+dx)}{b^2d(a+b\cos(c+dx))} - \frac{\int \frac{-b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+dx)+b\left(a^2-b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{a\sin(c+dx)}{b^2d(a+b\cos(c+dx))} - \frac{\int \frac{-b^2\left(a^2-b^2\right)-2ab\left(a^2-b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b^3\left(a^2-b^2\right)}$$

$$= \frac{2ax}{b^3} - \frac{\sin(c+dx)}{b^2d} - \frac{a\sin(c+dx)}{b^2d(a+b\cos(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^3}$$

$$= \frac{2ax}{b^3} - \frac{\sin(c+dx)}{b^2d} - \frac{a\sin(c+dx)}{b^2d(a+b\cos(c+dx))} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x}\right.}{b^3d}$$

$$= \frac{2ax}{b^3} - \frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{\sin(c+dx)}{b^2d} - \frac{a\sin(c+dx)}{b^2d(a+b\cos(c+d}$$

Mathematica [A] time = 0.999303, size = 132, normalized size = 1.18

$$\frac{\frac{4a^2c+4a^2dx-4ab\sin(c+dx)+4ab(c+dx)\cos(c+dx)-b^2\sin(2(c+dx))}{a+b\cos(c+dx)} + \frac{4\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*(2*a^2 - b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqr
t[-a^2 + b^2] + (4*a^2*c + 4*a^2*d*x + 4*a*b*(c + d*x)*Cos[c + d*x] - 4*a*b
*Sin[c + d*x] - b^2*Sin[2*(c + d*x)])/(a + b*Cos[c + d*x]))/(2*b^3*d)
```

Maple [A] time = 0.032, size = 198, normalized size = 1.8

$$-2\,\frac{\tan(1/2\,dx+c/2)}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)} + 4\,\frac{\arctan(\tan(1/2\,dx+c/2))\,a}{db^3} - 2\,\frac{\tan(1/2\,dx+c/2)\,a}{db^2\left(a(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,dx+c/2)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/b^2*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+4/d/b^3*arctan(tan(1/2
*d*x+1/2*c))*a-2/d/b^2*tan(1/2*d*x+1/2*c)*a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)-4/d*a^2/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60559, size = 1002, normalized size = 8.95

$$\left[\frac{4\left(a^{3}b-ab^{3}\right)dx\cos\left(dx+c\right)+4\left(a^{4}-a^{2}b^{2}\right)dx+\left(2\,a^{3}-ab^{2}+\left(2\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^{2}-\cdots\right.}{\cdots}\right.}{2\left(\left(a^{2}b^{4}-b^{6}\right)d\cos\left(dx+c\right)+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*d*x*cos(d*x + c) + 4*(a^4 - a^2*b^2)*d*x + (2*a^3 -
 a*b^2 + (2*a^2*b - b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x
+ c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) +
b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a
^2)) - 2*(2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^4 - b^6)*d*cos(d*x + c) + (a^3*b^3 - a*b^5)*d), (2*(a^3*b - a*b^3)*d
*x*cos(d*x + c) + 2*(a^4 - a^2*b^2)*d*x - (2*a^3 - a*b^2 + (2*a^2*b - b^3)*
cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)
```

```
*sin(d*x + c))) - (2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*
x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c) + (a^3*b^3 - a*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.54517, size = 279, normalized size = 2.49

$$\frac{2\left(\frac{(dx+c)a}{b^3}+\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(2a^2-b^2)}{\sqrt{a^2-b^2}b^3}-\frac{2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\ldots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2a\tan\left(\frac{1}{2}dx\ldots\right.\right.}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 2*((d*x + c)*a/b^3 + (pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + ar
ctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(
2*a^2 - b^2)/(sqrt(a^2 - b^2)*b^3) - (2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/
2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((a*t
an(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)
^2 + a + b)*b^2))/d
```

3.605 $\int \frac{1-\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{\sin(c+dx)}{bd(a+b\cos(c+dx))}-\frac{x}{b^2}$$

```
[Out] -(x/b^2) + (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^2*Sqrt[a + b]*d) + Sin[c + d*x]/(b*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.118324, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3022, 12, 2735, 2659, 205}

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{\sin(c+dx)}{bd(a+b\cos(c+dx))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -(x/b^2) + (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^2*Sqrt[a + b]*d) + Sin[c + d*x]/(b*d*(a + b*Cos[c + d*x]))
```


Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= \frac{\sin(c+dx)}{bd(a+b\cos(c+dx))} - \frac{\int \frac{(a^2-b^2)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b(a^2-b^2)} \\
&= \frac{\sin(c+dx)}{bd(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b} \\
&= -\frac{x}{b^2} + \frac{\sin(c+dx)}{bd(a+b\cos(c+dx))} + \frac{a\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2} \\
&= -\frac{x}{b^2} + \frac{\sin(c+dx)}{bd(a+b\cos(c+dx))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d} \\
&= -\frac{x}{b^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d} + \frac{\sin(c+dx)}{bd(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.233445, size = 80, normalized size = 0.94

$$-\frac{\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - \frac{b\sin(c+dx)}{a+b\cos(c+dx)} + c + dx}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqr
t[-a^2 + b^2] - (b*Sin[c + d*x])/(a + b*Cos[c + d*x]))/(b^2*d))
```

Maple [A] time = 0.024, size = 116, normalized size = 1.4

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{db^2}+2\,\frac{\tan\left(1/2\,dx+c/2\right)}{db\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^2-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b+a+b\right)}+2\,\frac{a}{db^2\sqrt{(a+b)(a-b)}}\,\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2/d/b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)+2/d*a/b^2/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5555, size = 829, normalized size = 9.75

$$\left[-\frac{2\left(a^2b-b^3\right)dx\cos\left(dx+c\right)+2\left(a^3-ab^2\right)dx+\left(ab\cos\left(dx+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2}}{b^2\cos(dx+c)^2+2\,ab}\right.}{2\left(\left(a^2b^3-b^5\right)d\cos\left(dx+c\right)+\left(a^3b^2-ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d*x*cos(d*x + c) + 2*(a^3 - a*b^2)*d*x + (a*b*cos(d*
x + c) + a^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(
d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2
*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^2*b - b^3)*si
n(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c) + (a^3*b^2 - a*b^4)*d), -((a^2*
b - b^3)*d*x*cos(d*x + c) + (a^3 - a*b^2)*d*x - (a*b*cos(d*x + c) + a^2)*sq
rt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))
- (a^2*b - b^3)*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c) + (a^3*b^2 -
a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39042, size = 189, normalized size = 2.22

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b^2}+\frac{dx+c}{b^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)
*b^2) + (d*x + c)/b^2 - 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)*b))/d
```

3.606 $\int \frac{(1-\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\sin(c+dx)}{ad(a+b\cos(c+dx))}$$

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*
Sqrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a^2*d) - Sin[c + d*x]/(a*d*(a + b*C
os[c + d*x]))
```

Rubi [A] time = 0.170311, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3056, 12, 2747, 3770, 2659, 205}

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\sin(c+dx)}{ad(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*
Sqrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a^2*d) - Sin[c + d*x]/(a*d*(a + b*C
os[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
```

```
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-b^2\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\sin(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \sec(c+dx)\,dx}{a^2} - \frac{b\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{\sin(c+dx)}{ad(a+b\cos(c+dx))} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \text{ta}\right.}{a^2d}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{\sin(c+dx)}{ad(a+b\cos(c+dx))}$$

Mathematica [A] time = 0.18973, size = 123, normalized size = 1.31

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - \frac{a\sin(c+dx)}{a+b\cos(c+dx)} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - (a*Sin[c + d*x])/(a + b*Cos[c + d*x]))/(a^2*d)
```

Maple [A] time = 0.047, size = 137, normalized size = 1.5

$$-2\,\frac{\tan(1/2\,dx+c/2)}{da\left(a(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,dx+c/2))^2\,b+a+b\right)} - 2\,\frac{b}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)-2/d/a^2*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94589, size = 1056, normalized size = 11.23

$$\left[-\frac{\left(b^2\cos\left(dx+c\right)+ab\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(a^3-ab^2+\cdots\right.}{2\left(\left(a^4b-a^2b^3\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [-1/2*((b^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
(2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(
d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) -
(a^3 - a*b^2 + (a^2*b - b^3)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^3 - a
*b^2 + (a^2*b - b^3)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^3 - a*b^2)
*sin(d*x + c))/((a^4*b - a^2*b^3)*d*cos(d*x + c) + (a^5 - a^3*b^2)*d), -1/2
*(2*(b^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2 + (a^2*b - b^3)*cos(d*x + c))
*log(sin(d*x + c) + 1) + (a^3 - a*b^2 + (a^2*b - b^3)*cos(d*x + c))*log(-si
n(d*x + c) + 1) + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4*b - a^2*b^3)*d*cos(d*
x + c) + (a^5 - a^3*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sec(c+dx)}{a^2+2ab\cos(c+dx)+b^2\cos^2(c+dx)}\,dx-\int \frac{\cos^2(c+dx)\sec(c+dx)}{a^2+2ab\cos(c+dx)+b^2\cos^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] -Integral(-sec(c + d*x)/(a**2 + 2*a*b*cos(c + d*x) + b**2*cos(c + d*x)**2),
 x) - Integral(cos(c + d*x)**2*sec(c + d*x)/(a**2 + 2*a*b*cos(c + d*x) + b*
*2*cos(c + d*x)**2), x)
```

Giac [A] time = 1.6128, size = 223, normalized size = 2.37

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{2\tan\left(\frac{1}{2}d\cdots\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\,\mathrm{ta}\cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a
^2) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/a^2 + 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*
d*x + 1/2*c)^2 + a + b)*a))/d
```

3.607 $\int \frac{(1-\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}-\frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))}$$

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*
Sqrt[a - b]*Sqrt[a + b]*d) - (2*b*ArcTanh[Sin[c + d*x]])/(a^3*d) + (2*Tan[c
 + d*x])/(a^2*d) - Tan[c + d*x]/(a*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.399145, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3056, 3001, 3770, 2659, 205}

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}-\frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*
Sqrt[a - b]*Sqrt[a + b]*d) - (2*b*ArcTanh[Sin[c + d*x]])/(a^3*d) + (2*Tan[c
 + d*x])/(a^2*d) - Tan[c + d*x]/(a*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
```

```
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2\tan(c+dx)}{a^2d} - \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,c}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\tan(c+dx)}{a^2d} - \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))} - \frac{(2b)\int \sec(c+dx)\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \frac{}{a}}{a}$$

$$= -\frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2\tan(c+dx)}{a^2d} - \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))} - \frac{\left(2\left(a^2-2l\right.\right.}{}$$

$$= -\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} - \frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{2\tan(c+d}{a^2d}$$

Mathematica [A] time = 0.622995, size = 143, normalized size = 1.21

$$\frac{\frac{2\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{ab\sin(c+dx)}{a+b\cos(c+dx)} + a\tan(c+dx) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log\left(\sin\left(\right.\right.}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*(a^2 - 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqr
t[-a^2 + b^2] + 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b*Sin[c + d*x])/(a + b*Cos[c + d*x])
+ a*Tan[c + d*x])/(a^3*d)
```

Maple [B] time = 0.058, size = 231, normalized size = 2.

$$2\,\frac{\tan(1/2\,dx+c/2)\,b}{da^2\left(a\left(\tan(1/2\,dx+c/2)\right)^2-\left(\tan(1/2\,dx+c/2)\right)^2b+a+b\right)} - 2\,\frac{1}{da\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*tan(1/2*d*x+1/2*c)*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)-2/d/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))+4/d/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*b^2-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d*b/a^3*ln(tan(1/2*d*x
+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)-2/d*b/a^3*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17619, size = 1403, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*((a^2*b^2 - b^4)*cos(d*x + c)^2 + (
a^3*b - a*b^3)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*((a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4
 - a^2*b^2 + 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5*b - a^3*b^
3)*d*cos(d*x + c)^2 + (a^6 - a^4*b^2)*d*cos(d*x + c)), -(((a^2*b - 2*b^3)*c
os(d*x + c)^2 + (a^3 - 2*a*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + ((a^2*b^2 - b^4)*cos(d*x
```

```
+ c)^2 + (a^3*b - a*b^3)*cos(d*x + c))*log(sin(d*x + c) + 1) - ((a^2*b^2 -
b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*cos(d*x + c))*log(-sin(d*x + c) + 1)
- (a^4 - a^2*b^2 + 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5*b -
a^3*b^3)*d*cos(d*x + c)^2 + (a^6 - a^4*b^2)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.59419, size = 317, normalized size = 2.69

$$-\frac{2\left(\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\left(a^2-2\,b^2\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{a\tan\left(\cdots\right.}{\left(\cdots\right.}\cdots\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -2*(b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - b*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a^3 - (pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-
(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(a^2 -
2*b^2)/(sqrt(a^2 - b^2)*a^3) + (a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x
+ 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/2*c))/((a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a
 - b)*a^2))/d
```

3.608 $\int \frac{(1-\cos^2(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$\frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{\left(a^2-6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{3b\tan(c+dx)}{a^3d}+\frac{3\tan(c+dx)\sec(c+dx)}{2a^2d}-$$

```
[Out] (2*b*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a
^4*Sqrt[a - b]*Sqrt[a + b]*d) - ((a^2 - 6*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^
4*d) - (3*b*Tan[c + d*x])/(a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d)
 - (Sec[c + d*x]*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.672492, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{\left(a^2-6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{3b\tan(c+dx)}{a^3d}+\frac{3\tan(c+dx)\sec(c+dx)}{2a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a
^4*Sqrt[a - b]*Sqrt[a + b]*d) - ((a^2 - 6*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^
4*d) - (3*b*Tan[c + d*x])/(a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d)
 - (Sec[c + d*x]*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(-6b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+\ldots\right.}{a+\ldots}}{2a\ldots}$$

$$= -\frac{3b\tan(c+dx)}{a^3d} + \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(-a^4\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{3b\tan(c+dx)}{a^3d} + \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{ad(a+b\cos(c+dx))} - \frac{\left(a^2-\ldots\right.}{\ldots}$$

$$= -\frac{\left(a^2-6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{3b\tan(c+dx)}{a^3d} + \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \ldots$$

$$= \frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{\left(a^2-6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{3b\,\mathrm{ta}\ldots}{\ldots}$$

Mathematica [A] time = 3.42272, size = 271, normalized size = 1.69

$$\frac{\frac{8b\left(3b^2-2a^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((8*b*(-2*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/Sqrt[-a^2 + b^2] + 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 12*b^
2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*a^2*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + 12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2 - (4*a*b^2*Sin[c + d*x])/(a + b*Cos[c + d*x]) - 8*a*b*Tan[c + d*x]
```

```
)/(4*a^4*d)
```

Maple [B] time = 0.065, size = 364, normalized size = 2.3

$$-2\frac{b^2\tan(1/2\,dx+c/2)}{da^3\left(a(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,dx+c/2))^2b+a+b\right)}+4\frac{b}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c}{\sqrt{(a+b)(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b^2/a^3*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)+4/d/a^2*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))-6/d*b^3/a^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))+1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(ta
n(1/2*d*x+1/2*c)-1)+2/d/a^3/(tan(1/2*d*x+1/2*c)-1)*b+1/2/d/a^2*ln(tan(1/2*d
*x+1/2*c)-1)-3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*b^2-1/2/d/a^2/(tan(1/2*d*x+1/
2*c)+1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b
-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)+3/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.34888, size = 1688, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*((2*a^2*b^2 - 3*b^4)*cos(d*x + c)^3 + (2*a^3*b - 3*a*b^3)*cos(d*x +
 c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c
)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^4*b - 7*a^2*b^3 + 6*b
^5)*cos(d*x + c)^3 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*log(sin(d*
x + c) + 1) + ((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^3 + (a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(a^5 - a^3*b^2 - 6*
(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^6*b - a^4*b^3)*d*cos(d*x + c)^3 + (a^7 - a^5*b^2)*d*cos(d*x + c
)^2), 1/4*(4*((2*a^2*b^2 - 3*b^4)*cos(d*x + c)^3 + (2*a^3*b - 3*a*b^3)*cos(
d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*s
in(d*x + c))) - ((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^3 + (a^5 - 7*a^3*
b^2 + 6*a*b^4)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^4*b - 7*a^2*b^3
+ 6*b^5)*cos(d*x + c)^3 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) + 2*(a^5 - a^3*b^2 - 6*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 -
 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*b - a^4*b^3)*d*cos(d
*x + c)^3 + (a^7 - a^5*b^2)*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.81437, size = 363, normalized size = 2.27

$$-\frac{\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)a^3}+\frac{\left(a^2-6\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{\left(a^2-6\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{4\left(2\,a^2b-3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sg}\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
 + 1/2*c)^2 + a + b)*a^3) + (a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
)/a^4 - (a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 4*(2*a^2*b -
 3*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^
2)*a^4) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(
1/2*d*x + 1/2*c) - 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^
2*a^3))/d
```

3.609 $\int \frac{(1-\cos^2(c+dx))\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=195

$$-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}-\frac{\left(a^2-12b^2\right)\tan(c+dx)}{3a^4d}+\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{2b\tan(c+dx}{a^3}$$

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) + (b*(a^2 - 4*b^2)*ArcTanh[Sin[c + d*x]])/
(a^5*d) - ((a^2 - 12*b^2)*Tan[c + d*x])/(3*a^4*d) - (2*b*Sec[c + d*x]*Tan[c
 + d*x])/(a^3*d) + (4*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d) - (Sec[c + d*x
]^2*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.916285, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}-\frac{\left(a^2-12b^2\right)\tan(c+dx)}{3a^4d}+\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{2b\tan(c+dx}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) + (b*(a^2 - 4*b^2)*ArcTanh[Sin[c + d*x]])/
(a^5*d) - ((a^2 - 12*b^2)*Tan[c + d*x])/(3*a^4*d) - (2*b*Sec[c + d*x]*Tan[c
 + d*x])/(a^3*d) + (4*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d) - (Sec[c + d*x
]^2*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{4\sec^2(c+dx)\tan(c+dx)}{3a^2d} - \frac{\sec^2(c+dx)\tan(c+dx)}{ad(a+b\cos(c+dx))} + \frac{\int \frac{\left(-12b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos\right.}{}}{3}$$

$$= -\frac{2b\sec(c+dx)\tan(c+dx)}{a^3d} + \frac{4\sec^2(c+dx)\tan(c+dx)}{3a^2d} - \frac{\sec^2(c+dx)\tan(c+}{ad(a+b\cos(c+d}$$

$$= -\frac{\left(a^2-12b^2\right)\tan(c+dx)}{3a^4d} - \frac{2b\sec(c+dx)\tan(c+dx)}{a^3d} + \frac{4\sec^2(c+dx)\tan(c+}{3a^2d}$$

$$= -\frac{\left(a^2-12b^2\right)\tan(c+dx)}{3a^4d} - \frac{2b\sec(c+dx)\tan(c+dx)}{a^3d} + \frac{4\sec^2(c+dx)\tan(c+}{3a^2d}$$

$$= \frac{b\left(a^2-4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\left(a^2-12b^2\right)\tan(c+dx)}{3a^4d} - \frac{2b\sec(c+dx)\tan}{a^3d}$$

$$= -\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}d} + \frac{b\left(a^2-4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d} -$$

Mathematica [B] time = 6.21069, size = 475, normalized size = 2.44

$$\frac{b^3\sin(c+dx)}{a^4d(a+b\cos(c+dx))} + \frac{9b^2\sin\left(\frac{1}{2}(c+dx)\right)-a^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{9b^2\sin\left(\frac{1}{2}(c+dx)\right)-a^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2b^2\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^2*(3*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(a^5*Sqrt[-a^2 + b^2]*d) + ((-(a^2*b) + 4*b^3)*Log[Cos[(c + d*x)/2] - Sin
```

```
[(c + d*x)/2]])/(a^5*d) + ((a^2*b - 4*b^3)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]])/(a^5*d) + (a - 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^2) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
+ Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (-a
+ 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (-(a^2*Sin[(c +
 d*x)/2]) + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])) + (-(a^2*Sin[(c + d*x)/2]) + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^3*Sin[c + d*x])/(a^4*d*(a + b*Cos[c
 + d*x]))
```

Maple [B] time = 0.069, size = 458, normalized size = 2.4

$$2\,\frac{b^3\tan(1/2\,dx+c/2)}{da^4\left(a\,(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,dx+c/2))^2\,b+a+b\right)}-6\,\frac{b^2}{da^3\sqrt{(a+b)\,(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2}{\sqrt{(a+b)\,(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b^3/a^4*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)-6/d/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*b^2+8/d*b^4/a^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))-1/3/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(t
an(1/2*d*x+1/2*c)-1)^2-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2*b-1/d/a^3/(tan(1/2*
d*x+1/2*c)-1)*b-3/d*b^2/a^4/(tan(1/2*d*x+1/2*c)-1)-1/d*b/a^3*ln(tan(1/2*d*x
+1/2*c)-1)+4/d*b^3/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a^2/(tan(1/2*d*x+1/2*
c)+1)^3+1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2
*b-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b-3/d*b^2/a^4/(tan(1/2*d*x+1/2*c)+1)+1/d*
b/a^3*ln(tan(1/2*d*x+1/2*c)+1)-4/d*b^3/a^5*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78843, size = 1886, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/6*(3*((3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 + (3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^3)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*((a^4*b^2 - 5*a^2*b^4
 + 4*b^6)*cos(d*x + c)^4 + (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3)*lo
g(sin(d*x + c) + 1) - 3*((a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 + (a^
5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 2*(a^6
- a^4*b^2 - (a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - (a^6 - 7*a^4*b
^2 + 6*a^2*b^4)*cos(d*x + c)^2 - 2*(a^5*b - a^3*b^3)*cos(d*x + c))*sin(d*x
+ c))/((a^7*b - a^5*b^3)*d*cos(d*x + c)^4 + (a^8 - a^6*b^2)*d*cos(d*x + c)^
3), -1/6*(6*((3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 + (3*a^3*b^2 - 4*a*b^4)*cos
(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c))) - 3*((a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 + (a^5*b -
 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) + 3*((a^4*b^2 -
 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 + (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x
+ c)^3)*log(-sin(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - (a^5*b - 13*a^3*b^3 + 1
2*a*b^5)*cos(d*x + c)^3 - (a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^2 - 2*
(a^5*b - a^3*b^3)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x
+ c)^4 + (a^8 - a^6*b^2)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50607, size = 427, normalized size = 2.19

$$\frac{6 b^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+a+b\right)a^4}+\frac{3 \left(a^2b-4 b^3\right) \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^5}-\frac{3 \left(a^2b-4 b^3\right) \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a^5}+\frac{6 \left(3 a^2b^2-4 b^4\right)\left(\pi\left\lfloor \frac{dx+c}{2\pi}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(6*b^3*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
+ 1/2*c)^2 + a + b)*a^4) + 3*(a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/a^5 - 3*(a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 6*(3*
a^2*b^2 - 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan
(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt
(a^2 - b^2)*a^5) - 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/
2*c)^5 + 4*a^2*tan(1/2*d*x + 1/2*c)^3 - 18*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a
*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 - 1)^3*a^4))/d
```

3.610 $\int \frac{\cos^4(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=326

$$-\frac{\left(-59a^2b^2+60a^4+2b^4\right)\sin(c+dx)}{6b^5d\left(a^2-b^2\right)}-\frac{a^2\left(-33a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^6d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(5a^2-4b^2\right)\sin(c+dx)\,\mathrm{c}}{2b^2d\left(a^2-b^2\right)(a+b\,\mathrm{co}}$$

```
[Out] (a*(20*a^2 - 3*b^2)*x)/(2*b^6) - (a^2*(20*a^4 - 33*a^2*b^2 + 12*b^4)*ArcTan
[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^6*(a + b)^(3
/2)*d) - ((60*a^4 - 59*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(6*b^5*(a^2 - b^2)*d)
 + (a*(10*a^2 - 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b^4*(a^2 - b^2)*d) - (
(20*a^2 - 17*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^3*(a^2 - b^2)*d) + (Cos
[c + d*x]^4*Sin[c + d*x])/(2*b*d*(a + b*Cos[c + d*x])^2) + ((5*a^2 - 4*b^2)
*Cos[c + d*x]^3*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.04671, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3049, 3023, 2735, 2659, 205}

$$-\frac{\left(-59a^2b^2+60a^4+2b^4\right)\sin(c+dx)}{6b^5d\left(a^2-b^2\right)}-\frac{a^2\left(-33a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^6d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(5a^2-4b^2\right)\sin(c+dx)\,\mathrm{c}}{2b^2d\left(a^2-b^2\right)(a+b\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (a*(20*a^2 - 3*b^2)*x)/(2*b^6) - (a^2*(20*a^4 - 33*a^2*b^2 + 12*b^4)*ArcTan
[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^6*(a + b)^(3
/2)*d) - ((60*a^4 - 59*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(6*b^5*(a^2 - b^2)*d)
 + (a*(10*a^2 - 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b^4*(a^2 - b^2)*d) - (
(20*a^2 - 17*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^3*(a^2 - b^2)*d) + (Cos
[c + d*x]^4*Sin[c + d*x])/(2*b*d*(a + b*Cos[c + d*x])^2) + ((5*a^2 - 4*b^2)
*Cos[c + d*x]^3*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \frac{\cos^4(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)\left(-4\left(a^2-b^2\right)+5\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{\cos^4(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} + \frac{\left(5a^2-4b^2\right)\cos^3(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\cos^2(c+dx)(3(5}{}}{}$$

$$= -\frac{\left(20a^2-17b^2\right)\cos^2(c+dx)\sin(c+dx)}{6b^3\left(a^2-b^2\right)d} + \frac{\cos^4(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} + \frac{\left(5a^2-4b\right.}{2b^2\left(a^2-\right.}$$

$$= \frac{a\left(10a^2-9b^2\right)\cos(c+dx)\sin(c+dx)}{2b^4\left(a^2-b^2\right)d} - \frac{\left(20a^2-17b^2\right)\cos^2(c+dx)\sin(c+dx)}{6b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(60a^4-59a^2b^2+2b^4\right)\sin(c+dx)}{6b^5\left(a^2-b^2\right)d} + \frac{a\left(10a^2-9b^2\right)\cos(c+dx)\sin(c+dx)}{2b^4\left(a^2-b^2\right)d} - $$

$$= \frac{a\left(20a^2-3b^2\right)x}{2b^6} - \frac{\left(60a^4-59a^2b^2+2b^4\right)\sin(c+dx)}{6b^5\left(a^2-b^2\right)d} + \frac{a\left(10a^2-9b^2\right)\cos(c+d}{2b^4\left(a^2-b^2\right.}$$

$$= \frac{a\left(20a^2-3b^2\right)x}{2b^6} - \frac{\left(60a^4-59a^2b^2+2b^4\right)\sin(c+dx)}{6b^5\left(a^2-b^2\right)d} + \frac{a\left(10a^2-9b^2\right)\cos(c+d}{2b^4\left(a^2-b^2\right.}$$

$$= \frac{a\left(20a^2-3b^2\right)x}{2b^6} - \frac{a^2\left(20a^4-33a^2b^2+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^6(a+b)^{3/2}d} - \frac{\left(60a^4-\right.}{}$$

Mathematica [B] time = 7.20272, size = 979, normalized size = 3.

$$-\frac{12\left(-48a(c+dx)-\frac{6\left(16a^6-40b^2a^4+30b^4a^2-5b^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+16b\sin(c+dx)+\frac{ab\left(40a^4-72b^2a^2+29b^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}-\frac{b\left(8a^4-8b^2a^2+b^4\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}\right)}{b^4}+12\left(\frac{b(-\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-12*(-48*a*(c + d*x) - (6*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*ArcT
anh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 16*b
*Sin[c + d*x] - (b*(8*a^4 - 8*a^2*b^2 + b^4)*Sin[c + d*x])/((a - b)*(a + b)
*(a + b*Cos[c + d*x])^2) + (a*b*(40*a^4 - 72*a^2*b^2 + 29*b^4)*Sin[c + d*x]
)/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/b^4 + 12*((-2*(2*a^2 + b^2)*
ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) +
(b*(-4*a^2 + b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(
a + b*Cos[c + d*x])^2)) + (6*((-6*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sq
rt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + ((-(b*(2*a^2 + b^2)) + a*(2*a^2 - 5*b^2
)*Cos[c + d*x])*Sin[c + d*x])/(a + b*Cos[c + d*x])^2))/((a - b)^2*(a + b)^2
) + ((12*(640*a^8 - 1792*a^6*b^2 + 1680*a^4*b^4 - 560*a^2*b^6 + 35*b^8)*Arc
Tanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (38
40*a^9*c - 6912*a^7*b^2*c + 1728*a^5*b^4*c + 1920*a^3*b^6*c - 576*a*b^8*c +
 3840*a^9*d*x - 6912*a^7*b^2*d*x + 1728*a^5*b^4*d*x + 1920*a^3*b^6*d*x - 57
6*a*b^8*d*x + 768*b*(10*a^2 - 3*b^2)*(a^3 - a*b^2)^2*(c + d*x)*Cos[c + d*x]
 + 192*a*b^2*(10*a^2 - 3*b^2)*(a^2 - b^2)^2*(c + d*x)*Cos[2*(c + d*x)] - 38
40*a^8*b*Sin[c + d*x] + 7872*a^6*b^3*Sin[c + d*x] - 4256*a^4*b^5*Sin[c + d*
x] + 172*a^2*b^7*Sin[c + d*x] + 70*b^9*Sin[c + d*x] - 2880*a^7*b^2*Sin[2*(c
 + d*x)] + 6304*a^5*b^4*Sin[2*(c + d*x)] - 4022*a^3*b^6*Sin[2*(c + d*x)] +
607*a*b^8*Sin[2*(c + d*x)] - 320*a^6*b^3*Sin[3*(c + d*x)] + 696*a^4*b^5*Sin
[3*(c + d*x)] - 432*a^2*b^7*Sin[3*(c + d*x)] + 56*b^9*Sin[3*(c + d*x)] + 40
*a^5*b^4*Sin[4*(c + d*x)] - 80*a^3*b^6*Sin[4*(c + d*x)] + 40*a*b^8*Sin[4*(c
 + d*x)] - 8*a^4*b^5*Sin[5*(c + d*x)] + 16*a^2*b^7*Sin[5*(c + d*x)] - 8*b^9
*Sin[5*(c + d*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/b^6)/(384*d)
```

Maple [B] time = 0.039, size = 786, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -12/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a^2-3/d/b^4/(tan(
1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a-24/d/b^5/(tan(1/2*d*x+1/2*c)^2
+1)^3*tan(1/2*d*x+1/2*c)^3*a^2+8/3/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2
*d*x+1/2*c)^3-12/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a^2+3/
d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a+20/d/b^6*arctan(tan(1
/2*d*x+1/2*c))*a^3-3/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a-8/d*a^5/b^5/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+1
/d*a^4/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(
1/2*d*x+1/2*c)^3+8/d*a^3/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3-8/d*a^5/b^5/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)-1/d*a^4/b^4/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)+8/d*a^3/
b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x
+1/2*c)-20/d*a^6/b^6/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))+33/d*a^4/b^4/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arc
tan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-12/d*a^2/b^2/(a^2-b^2)/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16855, size = 2471, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/12*(6*(20*a^7*b^2 - 43*a^5*b^4 + 26*a^3*b^6 - 3*a*b^8)*d*x*cos(d*x + c)^
2 + 12*(20*a^8*b - 43*a^6*b^3 + 26*a^4*b^5 - 3*a^2*b^7)*d*x*cos(d*x + c) +
6*(20*a^9 - 43*a^7*b^2 + 26*a^5*b^4 - 3*a^3*b^6)*d*x + 3*(20*a^8 - 33*a^6*b
^2 + 12*a^4*b^4 + (20*a^6*b^2 - 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^2 + 2
*(20*a^7*b - 33*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2
*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*co
s(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + a^2)) - 2*(60*a^8*b - 119*a^6*b^3 + 61*a^4*b^5 - 2*a^2*b^7 + 2*
(a^4*b^5 - 2*a^2*b^7 + b^9)*cos(d*x + c)^4 - 5*(a^5*b^4 - 2*a^3*b^6 + a*b^8
)*cos(d*x + c)^3 + 2*(10*a^6*b^3 - 21*a^4*b^5 + 12*a^2*b^7 - b^9)*cos(d*x +
 c)^2 + (90*a^7*b^2 - 181*a^5*b^4 + 95*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin
(d*x + c))/((a^4*b^8 - 2*a^2*b^10 + b^12)*d*cos(d*x + c)^2 + 2*(a^5*b^7 - 2
*a^3*b^9 + a*b^11)*d*cos(d*x + c) + (a^6*b^6 - 2*a^4*b^8 + a^2*b^10)*d), 1/
6*(3*(20*a^7*b^2 - 43*a^5*b^4 + 26*a^3*b^6 - 3*a*b^8)*d*x*cos(d*x + c)^2 +
6*(20*a^8*b - 43*a^6*b^3 + 26*a^4*b^5 - 3*a^2*b^7)*d*x*cos(d*x + c) + 3*(20
*a^9 - 43*a^7*b^2 + 26*a^5*b^4 - 3*a^3*b^6)*d*x - 3*(20*a^8 - 33*a^6*b^2 +
12*a^4*b^4 + (20*a^6*b^2 - 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^2 + 2*(20*
a^7*b - 33*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*c
os(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (60*a^8*b - 119*a^6*b^3
+ 61*a^4*b^5 - 2*a^2*b^7 + 2*(a^4*b^5 - 2*a^2*b^7 + b^9)*cos(d*x + c)^4 - 5
*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*cos(d*x + c)^3 + 2*(10*a^6*b^3 - 21*a^4*b^5
+ 12*a^2*b^7 - b^9)*cos(d*x + c)^2 + (90*a^7*b^2 - 181*a^5*b^4 + 95*a^3*b^6
 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^8 - 2*a^2*b^10 + b^12)*d*co
s(d*x + c)^2 + 2*(a^5*b^7 - 2*a^3*b^9 + a*b^11)*d*cos(d*x + c) + (a^6*b^6 -
 2*a^4*b^8 + a^2*b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7102, size = 587, normalized size = 1.8

$$\frac{\frac{6\left(20\,a^{6}-33\,a^{4}b^{2}+12\,a^{2}b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{6}-b^{8}\right)\sqrt{a^{2}-b^{2}}}-\frac{6\left(8\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,a^{5}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,a^{4}b^{2}\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(20*a^6 - 33*a^4*b^2 + 12*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^2*b^6 - b^8)*sqrt(a^2 - b^2)) - 6*(8*a^6*tan(1/2*d*
x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d*x + 1/2
*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*c) + 9*a
^5*b*tan(1/2*d*x + 1/2*c) - 7*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^3*tan(
1/2*d*x + 1/2*c))/((a^2*b^5 - b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*
x + 1/2*c)^2 + a + b)^2) + 3*(20*a^3 - 3*a*b^2)*(d*x + c)/b^6 - 2*(36*a^2*t
an(1/2*d*x + 1/2*c)^5 + 9*a*b*tan(1/2*d*x + 1/2*c)^5 + 72*a^2*tan(1/2*d*x +
 1/2*c)^3 - 8*b^2*tan(1/2*d*x + 1/2*c)^3 + 36*a^2*tan(1/2*d*x + 1/2*c) - 9*
a*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^5))/d
```

3.611 $\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=268

$$\frac{a\left(12a^2-11b^2\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)}+\frac{a\left(-19a^2b^2+12a^4+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(4a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] -((12*a^2 - b^2)*x)/(2*b^5) + (a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/2)*d)
 + (a*(12*a^2 - 11*b^2)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)*d) - ((6*a^2 - 5*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*b*d*(a + b*Cos[c + d*x])^2) + ((4*a^2 - 3*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.750621, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3049, 3023, 2735, 2659, 205}

$$\frac{a\left(12a^2-11b^2\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)}+\frac{a\left(-19a^2b^2+12a^4+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(4a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((12*a^2 - b^2)*x)/(2*b^5) + (a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/2)*d)
 + (a*(12*a^2 - 11*b^2)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)*d) - ((6*a^2 - 5*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*b*d*(a + b*Cos[c + d*x])^2) + ((4*a^2 - 3*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} + \frac{\left(4a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\cos(c+dx)(2\ldots}{\ldots}}{\ldots}$$

$$= -\frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} + \frac{\left(4a^2-3b^2\right)\ldots}{2b^2\left(a^2-\ldots\right.}$$

$$= \frac{a\left(12a^2-11b^2\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)d} - \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+d\ldots}{2bd(a+b\ldots}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} + \frac{a\left(12a^2-11b^2\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)d} - \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+\ldots}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} + \frac{a\left(12a^2-11b^2\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)d} - \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+\ldots}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} + \frac{a\left(12a^4-19a^2b^2+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^5(a+b)^{3/2}d} + \frac{a\left(12a^2-\ldots\right.}{2b^4\ldots}$$

Mathematica [A] time = 5.00148, size = 374, normalized size = 1.4

$$\frac{72a^4b^2\sin(2(c+dx))-80a^3b^3\sin(c+dx)+8a^3b^3\sin(3(c+dx))-70a^2b^4\sin(2(c+dx))-a^2b^4\sin(4(c+dx))-16ab\left(-13a^2b^2+12a^4+b^4\right)(c+dx)\cos(c+dx)-4b^2\left(-13a^2b^2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-16*a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sq
rt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (-96*a^6*c + 56*a^4*b^2*c + 44*a^2*b^4*
c - 4*b^6*c - 96*a^6*d*x + 56*a^4*b^2*d*x + 44*a^2*b^4*d*x - 4*b^6*d*x - 16
*a*b*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*Cos[c + d*x] - 4*b^2*(12*a^4 - 1
3*a^2*b^2 + b^4)*(c + d*x)*Cos[2*(c + d*x)] + 96*a^5*b*Sin[c + d*x] - 80*a^
3*b^3*Sin[c + d*x] - 8*a*b^5*Sin[c + d*x] + 72*a^4*b^2*Sin[2*(c + d*x)] - 7
0*a^2*b^4*Sin[2*(c + d*x)] + 2*b^6*Sin[2*(c + d*x)] + 8*a^3*b^3*Sin[3*(c +
d*x)] - 8*a*b^5*Sin[3*(c + d*x)] - a^2*b^4*Sin[4*(c + d*x)] + b^6*Sin[4*(c
+ d*x)])/(a + b*Cos[c + d*x])^2)/(16*(a - b)*b^5*(a + b)*d)
```

Maple [B] time = 0.04, size = 704, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a+1/d/b^3/(tan(1/2*
d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3+6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*t
an(1/2*d*x+1/2*c)*a-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)-1
2/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))+6
/d*a^4/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(
1/2*d*x+1/2*c)^3-1/d*a^3/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3-6/d*a^2/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+6/d*a^4/b^4/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)+1/d*a^
3/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d
*x+1/2*c)-6/d*a^2/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a-b)*tan(1/2*d*x+1/2*c)+12/d*a^5/b^5/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-19/d*a^3/b^3/(a^2-b^2)/((a+b
)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+6/d*a/b
/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06507, size = 2171, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(2*(12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 +
4*(12*a^7*b - 25*a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 2*(12*a^8
 - 25*a^6*b^2 + 14*a^4*b^4 - a^2*b^6)*d*x - (12*a^7 - 19*a^5*b^2 + 6*a^3*b^
4 + (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(12*a^6*b - 19*a
^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c)
+ (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*si
n(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
- 2*(12*a^7*b - 23*a^5*b^3 + 11*a^3*b^5 - (a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d
*x + c)^3 + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + (18*a^6*b^2 -
35*a^4*b^4 + 17*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^7 - 2*a^2*b^9
+ b^11)*d*cos(d*x + c)^2 + 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*cos(d*x + c)
+ (a^6*b^5 - 2*a^4*b^7 + a^2*b^9)*d), -1/2*((12*a^6*b^2 - 25*a^4*b^4 + 14*a
^2*b^6 - b^8)*d*x*cos(d*x + c)^2 + 2*(12*a^7*b - 25*a^5*b^3 + 14*a^3*b^5 -
a*b^7)*d*x*cos(d*x + c) + (12*a^8 - 25*a^6*b^2 + 14*a^4*b^4 - a^2*b^6)*d*x
- (12*a^7 - 19*a^5*b^2 + 6*a^3*b^4 + (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*co
s(d*x + c)^2 + 2*(12*a^6*b - 19*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*
a^7*b - 23*a^5*b^3 + 11*a^3*b^5 - (a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^
3 + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + (18*a^6*b^2 - 35*a^4*b
^4 + 17*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^7 - 2*a^2*b^9 + b^11)*
d*cos(d*x + c)^2 + 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^6*b
^5 - 2*a^4*b^7 + a^2*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41852, size = 814, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(12*a^5 - 19*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(a^2 - b^2)))/((a^2*b^5 - b^7)*sqrt(a^2 - b^2)) - 2*(12*a^5*tan(1/2*d*x
 + 1/2*c)^7 - 18*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 7*a^3*b^2*tan(1/2*d*x + 1/2
*c)^7 + 18*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 - 4*a*b^4*tan(1/2*d*x + 1/2*c)^7
- b^5*tan(1/2*d*x + 1/2*c)^7 + 36*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*a^4*b*tan
(1/2*d*x + 1/2*c)^5 - 37*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 14*a^2*b^3*tan(1/
2*d*x + 1/2*c)^5 + 4*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*b^5*tan(1/2*d*x + 1/2
*c)^5 + 36*a^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 3
7*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 14*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a*
b^4*tan(1/2*d*x + 1/2*c)^3 - 3*b^5*tan(1/2*d*x + 1/2*c)^3 + 12*a^5*tan(1/2*
d*x + 1/2*c) + 18*a^4*b*tan(1/2*d*x + 1/2*c) - 7*a^3*b^2*tan(1/2*d*x + 1/2*
c) - 18*a^2*b^3*tan(1/2*d*x + 1/2*c) - 4*a*b^4*tan(1/2*d*x + 1/2*c) + b^5*t
an(1/2*d*x + 1/2*c))/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2
*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (12*a^2 - b^2)*(
d*x + c)/b^5)/d
```

3.612 $\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$-\frac{\left(-9a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{3ax}{b^4}+\frac{\sin(c+dx)\cos^2(c+dx)}{2bd(a+b\cos(c+dx))^2}$$

```
[Out] (3*a*x)/b^4 - ((6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (3*Sin[c + d*x])/
(2*b^3*d) + (Cos[c + d*x]^2*Sin[c + d*x])/(2*b*d*(a + b*Cos[c + d*x])^2) -
(a*(3*a^2 - 2*b^2)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.458377, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3048, 3032, 3023, 2735, 2659, 205}

$$-\frac{\left(-9a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{3ax}{b^4}+\frac{\sin(c+dx)\cos^2(c+dx)}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (3*a*x)/b^4 - ((6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (3*Sin[c + d*x])/
(2*b^3*d) + (Cos[c + d*x]^2*Sin[c + d*x])/(2*b*d*(a + b*Cos[c + d*x])^2) -
(a*(3*a^2 - 2*b^2)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
```

```
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2\left(a^2-b^2\right)+3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{\cos^2(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{b\left(3a^4-5a^2b^2+2b\right.}{}}{}$$

$$= -\frac{3\sin(c+dx)}{2b^3d} + \frac{\cos^2(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

$$= \frac{3ax}{b^4} - \frac{3\sin(c+dx)}{2b^3d} + \frac{\cos^2(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{3ax}{b^4} - \frac{3\sin(c+dx)}{2b^3d} + \frac{\cos^2(c+dx)\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx}{2b^3\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{3ax}{b^4} - \frac{\left(6a^4-9a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} - \frac{3\sin(c+dx)}{2b^3d} + \frac{\cos^2(c+}{2bd(a+}$$

Mathematica [A] time = 0.978476, size = 159, normalized size = 0.87

$$\frac{\frac{ab\left(4b^2-5a^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - \frac{2\left(-9a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{a^2b\sin(c+dx)}{(a+b\cos(c+dx))^2} + 6a(c+dx) - 2b\sin(c+dx)}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (6*a*(c + d*x) - (2*(6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 2*b*Sin[c + d*x] + (a^2*b*S
in[c + d*x])/(a + b*Cos[c + d*x])^2 + (a*b*(-5*a^2 + 4*b^2)*Sin[c + d*x])/(
(a - b)*(a + b)*(a + b*Cos[c + d*x])))/(2*b^4*d)
```

Maple [B] time = 0.037, size = 576, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d/b^3*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+6/d/b^4*arctan(tan(1/2
*d*x+1/2*c))*a-4/d*a^3/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+1/d*a^2/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+4/d/b/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)*tan(1/2*d*x+1/2*c)^3-4/d*a^3/b^
3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1
/2*c)-1/d*a^2/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-
b)*tan(1/2*d*x+1/2*c)+4/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2*a/(a-b)*tan(1/2*d*x+1/2*c)-6/d*a^4/b^4/(a^2-b^2)/((a+b)*(a-b))^(1/2)
*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+9/d*a^2/b^2/(a^2-b^2)
/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2
/d/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8494, size = 1879, normalized size = 10.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*x*cos(d*x + c)^2 + 24*(a^6*b - 2*a
^4*b^3 + a^2*b^5)*d*x*cos(d*x + c) + 12*(a^7 - 2*a^5*b^2 + a^3*b^4)*d*x + (
```

```
6*a^6 - 9*a^4*b^2 + 2*a^2*b^4 + (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*cos(d*x + c
)^2 + 2*(6*a^5*b - 9*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(
(2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*
cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + a^2)) - 2*(6*a^6*b - 11*a^4*b^3 + 5*a^2*b^5 + 2*(a^4*b^3 - 2*
a^2*b^5 + b^7)*cos(d*x + c)^2 + (9*a^5*b^2 - 17*a^3*b^4 + 8*a*b^6)*cos(d*x
+ c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c)^2 + 2*(a^5
*b^5 - 2*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^6*b^4 - 2*a^4*b^6 + a^2*b^8)*
d), 1/2*(6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*x*cos(d*x + c)^2 + 12*(a^6*b - 2
*a^4*b^3 + a^2*b^5)*d*x*cos(d*x + c) + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*d*x -
(6*a^6 - 9*a^4*b^2 + 2*a^2*b^4 + (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*cos(d*x +
c)^2 + 2*(6*a^5*b - 9*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^6*b - 11*a^
4*b^3 + 5*a^2*b^5 + 2*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + (9*a^5*b
^2 - 17*a^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^
8 + b^10)*d*cos(d*x + c)^2 + 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*cos(d*x + c)
 + (a^6*b^4 - 2*a^4*b^6 + a^2*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.73813, size = 450, normalized size = 2.47

$$\frac{\frac{\left(6\,a^4-9\,a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\right.}{\left(a\right.}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((6*a^4 - 9*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - (4*a^4*tan(1/2*d*x + 1/2*c)^3 -
5*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*a*b^3
*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*d*x + 1/2*c) + 5*a^3*b*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c))/(
(a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a +
b)^2) + 3*(d*x + c)*a/b^4 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 + 1)*b^3))/d
```

3.613 $\int \frac{\cos(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2}-\frac{x}{b^3}$$

```
[Out] -(x/b^3) + (a*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) - (a*Sin[c + d*x])/(2*b^2*d*(a +
 b*Cos[c + d*x])^2) + ((3*a^2 - 2*b^2)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.291669, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3032, 3021, 2735, 2659, 205}

$$\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2}-\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -(x/b^3) + (a*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) - (a*Sin[c + d*x])/(2*b^2*d*(a +
 b*Cos[c + d*x])^2) + ((3*a^2 - 2*b^2)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(1-\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+dx)+2b\left(a^2-b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-ab^2\left(a^2-b^2\right)-2\cdots}{a+b\cdots}}{2b^3\left(\cdots\right.}$$

$$= -\frac{x}{b^3} - \frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(a\left(2a^2-\cdots\right.\right.}{\cdots}$$

$$= -\frac{x}{b^3} - \frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(a\left(2a^2-\cdots\right.\right.}{\cdots}$$

$$= -\frac{x}{b^3} + \frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} - \frac{a\sin(c+dx)}{2b^2d(a+b\cos(c+dx))^2} + \frac{\cdots}{2b^2\left(\cdots\right.}$$

Mathematica [A] time = 1.59163, size = 291, normalized size = 1.95

$$\frac{\frac{\frac{\sin(c+dx)\left(b\left(a^2+2b^2\right)\cos(c+dx)+a\left(2a^2+b^2\right)\right)}{(a+b\cos(c+dx))^2}+\frac{6ab\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}}{(a-b)^2(a+b)^2} - \frac{-\frac{3b\left(-7a^2b^2+4a^4+2b^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}+\frac{ab\left(4a^2-3b^2\right)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+\frac{2a\left(-20a^2b^2+8a^4+15b^4\right)\tanh\cdots}{\left(b^2-a^2\right)\cdots}}{b^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(1 - Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-((8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*S
in[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2
*b^2 + 2*b^4)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/b^3
) + ((6*a*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
 + b^2] + ((a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
a + b*Cos[c + d*x])^2)/((a - b)^2*(a + b)^2))/(8*d)
```

Maple [B] time = 0.033, size = 475, normalized size = 3.2

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{db^{3}}+2\,\frac{a^{2}\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}}{db^{2}\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b+a+b\right)^{2}\left(a+b\right)}-\frac{a}{db\left(a+b\right)}\Bigg(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d/b^3*arctan(tan(1/2*d*x+1/2*c))+2/d*a^2/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3-1/d/b/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)*tan(1/2*d*x+1/2*c)^3-2/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)
^3+2/d*a^2/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*
tan(1/2*d*x+1/2*c)+1/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2*a/(a-b)*tan(1/2*d*x+1/2*c)-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)+2/d*a^3/b^3/(a^2-b^2)/((a+b)*(a-b))^
(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-3/d*a/b/(a^2-b^2
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66012, size = 1628, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*(4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 8*(a^5*b - 2*a^3*
b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*x + (2*a^5
- 3*a^3*b^2 + (2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3
)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*co
s(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5*b - 3*a
^3*b^3 + a*b^5 + (3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c))*sin(d*x + c)
)/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + 2*(a^5*b^4 - 2*a^3*b^6 +
a*b^8)*d*cos(d*x + c) + (a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*d), -1/2*(2*(a^4*b^
2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x
*cos(d*x + c) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*x - (2*a^5 - 3*a^3*b^2 + (2
*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (2*a^5*b - 3*a^3*b^3 + a*b^5 + (3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x
+ c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + 2*(a^5*
b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.68489, size = 392, normalized size = 2.63

$$-\frac{\frac{\left(2\,a^3-3\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2}{\left(a^2b^2-b^4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -((2*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arc
tan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((
a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b*t
an(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x +
1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + 3*a^2*b*tan(1/2*d*x + 1/2*c) - a*b^
2*tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^2*b^2 - b^4)*(a*ta
n(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (d*x + c)/b^3
)/d
```

3.614 $\int \frac{1-\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{a\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]/((a - b)^(3/2)*(a + b)^(
3/2)*d) + Sin[c + d*x]/(2*b*d*(a + b*Cos[c + d*x])^2) - (a*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.12988, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3022, 12, 2754, 2659, 205}

$$-\frac{a\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]/((a - b)^(3/2)*(a + b)^(
3/2)*d) + Sin[c + d*x]/(2*b*d*(a + b*Cos[c + d*x])^2) - (a*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1-\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{\int \frac{(a^2-b^2)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b}$$

$$= \frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{b}{a+b\cos(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{1}{a+b\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \text{t}\right.}{\left(a^2-b^2\right)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\sin(c+dx)}{2bd(a+b\cos(c+dx))^2} - \frac{a\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

Mathematica [A] time = 0.279818, size = 94, normalized size = 0.8

$$-\frac{\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{\sin(c+dx)(a\cos(c+dx)+b)}{(a+b\cos(c+dx))^2}}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
 + ((b + a*Cos[c + d*x])*Sin[c + d*x])/(a + b*Cos[c + d*x])^2)/(2*(a - b)*(
a + b)*d)
```

Maple [A] time = 0.021, size = 160, normalized size = 1.4

$$\frac{1}{d(a+b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2b+a+b\right)^{-2}-\frac{1}{d(a-b)}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(a\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x
+1/2*c)^3-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*t
an(1/2*d*x+1/2*c)+1/d/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24778, size = 1015, normalized size = 8.68

$$\left[\frac{\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{4\left(\left(a^4b^2-2\,a^2b^4+b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b-2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right)+\left(a^6-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(-a^2 + b^2)*log(
(2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*
cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + a^2)) - 2*(a^2*b - b^3 + (a^3 - a*b^2)*cos(d*x + c))*sin(d*x
+ c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3
+ a*b^5)*d*cos(d*x + c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d), 1/2*((b^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^2*b - b^3 + (a^3 - a*b^2)*cos(d*
x + c))*sin(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)^2 + 2*(a^
```

```
5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.08321, size = 239, normalized size = 2.04

$$-\frac{\frac{\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)^2\left(a^2-b^2\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(a^2 - b^2)^(3/2) -
(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
/2*c)^2 + a + b)^2*(a^2 - b^2)))/d
```

3.615 $\int \frac{(1-\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2-2b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{\sin(c+dx)}{2ad(a+b\cos(c+}$$

```
[Out] -((b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a
^3*(a - b)^(3/2)*(a + b)^(3/2)*d)) + ArcTanh[Sin[c + d*x]]/(a^3*d) - Sin[c
+ d*x]/(2*a*d*(a + b*Cos[c + d*x])^2) - ((a^2 - 2*b^2)*Sin[c + d*x])/(2*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.411733, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3056, 3001, 3770, 2659, 205}

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2-2b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{\sin(c+dx)}{2ad(a+b\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a
^3*(a - b)^(3/2)*(a + b)^(3/2)*d)) + ArcTanh[Sin[c + d*x]]/(a^3*d) - Sin[c
+ d*x]/(2*a*d*(a + b*Cos[c + d*x])^2) - ((a^2 - 2*b^2)*Sin[c + d*x])/(2*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{2ad(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{\sin(c+dx)}{2ad(a+b\cos(c+dx))^2} - \frac{\left(a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(2\left(a^2-b^2\right)^2-ab\left(a^2-\right.\right.}{a+b}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2ad(a+b\cos(c+dx))^2} - \frac{\left(a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \sec(c+dx)\,dx}{a^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sin(c+dx)}{2ad(a+b\cos(c+dx))^2} - \frac{\left(a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c+dx}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sin(c+d}{2ad(a+b\cos(c}$$

Mathematica [A] time = 0.949406, size = 180, normalized size = 1.16

$$\frac{-\frac{a\sin(c+dx)\left(b\left(a^2-2b^2\right)\cos(c+dx)+2a^3-3ab^2\right)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{2b\left(2b^2-3a^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\log}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*b*(-3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(3/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a*(2*a^3 - 3*a*b^2 + b*(a^2 - 2*b^2)
*Cos[c + d*x])*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2))/(2*a
^3*d)
```

Maple [B] time = 0.053, size = 496, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*
x+1/2*c)^3-1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)*tan(1/2*d*x+1/2*c)^3*b+2/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3*b^2-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)+1/d/a/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)*b+2/d/a^2/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)*
b^2-3/d/a*b/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))+2/d/a^3*b^3/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/
d/a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.27084, size = 2036, normalized size = 13.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*((3*a^4*b - 2*a^2*b^3 + (3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3
*b^2 - 2*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
```

```
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*
(a^6 - 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2
*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(a^6 -
 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*a^6 - 5*
a^4*b^2 + 3*a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sin(d*x +
 c))/((a^7*b^2 - 2*a^5*b^4 + a^3*b^6)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b
^3 + a^4*b^5)*d*cos(d*x + c) + (a^9 - 2*a^7*b^2 + a^5*b^4)*d), -1/2*((3*a^4
*b - 2*a^2*b^3 + (3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*a*b^
4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c))) - (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^
6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x
 + c) + 1) + (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d
*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) +
 1) + (2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*
x + c))*sin(d*x + c))/((a^7*b^2 - 2*a^5*b^4 + a^3*b^6)*d*cos(d*x + c)^2 + 2
*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c) + (a^9 - 2*a^7*b^2 + a^5*b^4)
*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61975, size = 420, normalized size = 2.71

$$-\frac{\frac{\left(3\,a^2b-2\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b^3\,\mathrm{ta}}{\left(a^4-a^2b^2\right)\left(a\,\mathrm{ta}\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -((3*a^2*b - 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arct
an((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^
5 - a^3*b^2)*sqrt(a^2 - b^2)) + (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1
/2*d*x + 1/2*c)^3 - 3*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x + 1/
2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - 3*a*b^2*
tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - a^2*b^2)*(a*tan(
1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) - log(abs(tan(1/2
*d*x + 1/2*c) + 1))/a^3 + log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d
```

3.616 $\int \frac{(1-\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=204

$$-\frac{(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{(5a^2-6b^2)\tan(c+dx)}{2a^3d(a^2-b^2)}-\frac{(2a^2-3b^2)\tan(c+dx)}{2a^2d(a^2-b^2)(a+b\cos(c+dx))}-\frac{3b\tan}{}$$

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d)) - (3*b*ArcTanh[Sin[c + d*x]])/
(a^4*d) + ((5*a^2 - 6*b^2)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)*d) - Tan[c + d*
x]/(2*a*d*(a + b*Cos[c + d*x])^2) - ((2*a^2 - 3*b^2)*Tan[c + d*x])/(2*a^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.735375, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{(5a^2-6b^2)\tan(c+dx)}{2a^3d(a^2-b^2)}-\frac{(2a^2-3b^2)\tan(c+dx)}{2a^2d(a^2-b^2)(a+b\cos(c+dx))}-\frac{3b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d)) - (3*b*ArcTanh[Sin[c + d*x]])/
(a^4*d) + ((5*a^2 - 6*b^2)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)*d) - Tan[c + d*
x]/(2*a*d*(a + b*Cos[c + d*x])^2) - ((2*a^2 - 3*b^2)*Tan[c + d*x])/(2*a^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
```

```
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}+\frac{\int \frac{\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(2a^2-3b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{\left(5a^4-11a^2b^2+6b^4\right.}{}}{}$$

$$= \frac{\left(5a^2-6b^2\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}-\frac{\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(2a^2-3b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= \frac{\left(5a^2-6b^2\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}-\frac{\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(2a^2-3b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= -\frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{\left(5a^2-6b^2\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}-\frac{\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-$$

$$= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}-\frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{\left(5a^2\right.}{}$$

Mathematica [A] time = 2.71054, size = 200, normalized size = 0.98

$$\frac{\frac{ab\sin(c+dx)\left(b\left(3a^2-4b^2\right)\cos(c+dx)+4a^3-5ab^2\right)}{(a-b)(a+b)(a+b\cos(c+dx))^2}-\frac{2\left(-9a^2b^2+2a^4+6b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+2a\tan(c+dx)+6b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(3/2) + 6*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
```

```
2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a*b*(4*a^3 - 5*a*b^2 + b
*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c
+ d*x])^2) + 2*a*Tan[c + d*x])/(2*a^4*d)
```

Maple [B] time = 0.067, size = 609, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 4/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d
*x+1/2*c)^3*b+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a+b)*tan(1/2*d*x+1/2*c)^3*b^2-4/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2*b^3/(a+b)*tan(1/2*d*x+1/2*c)^3+4/d/a/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)*b-1/d/a^2/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)*b
^2-4/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)*
tan(1/2*d*x+1/2*c)-2/d/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))+9/d/a^2/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2-6/d/a^4/(a^2-b^2)/((a+b)
*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^4-1/d/
a^3/(tan(1/2*d*x+1/2*c)-1)+3/d*b/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3/(tan(
1/2*d*x+1/2*c)+1)-3/d*b/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.45014, size = 2483, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^3 + 2*(2*a^5*b - 9*a^3*
b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 9*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c
))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
+ 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 6*((a^4*b^3 - 2*a^2*b^5 + b^7
)*cos(d*x + c)^3 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*((a^4*b^3 -
2*a^2*b^5 + b^7)*cos(d*x + c)^3 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x +
 c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1)
+ 2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + (5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos
(d*x + c)^2 + (8*a^6*b - 17*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c)
)/((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*cos(d*x + c)^3 + 2*(a^9*b - 2*a^7*b^3
+ a^5*b^5)*d*cos(d*x + c)^2 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)),
 -1/2*(((2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^3 + 2*(2*a^5*b - 9*a^3
*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 9*a^4*b^2 + 6*a^2*b^4)*cos(d*x +
c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x +
 c))) + 3*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^3 + 2*(a^5*b^2 - 2*a^3*
b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*l
og(sin(d*x + c) + 1) - 3*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^3 + 2*(a
^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*
cos(d*x + c))*log(-sin(d*x + c) + 1) - (2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + (5*
a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + (8*a^6*b - 17*a^4*b^3 + 9*
a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*cos
(d*x + c)^3 + 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2 + (a^10 - 2*
a^8*b^2 + a^6*b^4)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 1.69075, size = 482, normalized size = 2.36

$$\frac{\left(2\,a^4-9\,a^2b^2+6\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{4\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,ab^3\tan\left(\frac{1}{2}\,dx+\right.}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) + (4*a^3*b*tan(1/2*d*x + 1/2*c)^3
- 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*b^4
*tan(1/2*d*x + 1/2*c)^3 + 4*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^2*b^2*tan(1/2*
d*x + 1/2*c) - 5*a*b^3*tan(1/2*d*x + 1/2*c) - 4*b^4*tan(1/2*d*x + 1/2*c))/(
(a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a +
b)^2) - 3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 + 3*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^4 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)
*a^3))/d
```

3.617 $\int \frac{(1-\cos^2(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=271

$$\frac{b\left(-19a^2b^2+6a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\left(11a^2-12b^2\right)\tan(c+dx)}{2a^4d\left(a^2-b^2\right)}-\frac{\left(a^2-12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{(5}{}$$

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((a^2 - 12*b^2)*ArcTanh[Sin
[c + d*x]])/(2*a^5*d) - (b*(11*a^2 - 12*b^2)*Tan[c + d*x])/(2*a^4*(a^2 - b^
2)*d) + ((5*a^2 - 6*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)*d) -
 (Sec[c + d*x]*Tan[c + d*x])/(2*a*d*(a + b*Cos[c + d*x])^2) - ((3*a^2 - 4*b
^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.04766, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-19a^2b^2+6a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\left(11a^2-12b^2\right)\tan(c+dx)}{2a^4d\left(a^2-b^2\right)}-\frac{\left(a^2-12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{(5}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((a^2 - 12*b^2)*ArcTanh[Sin
[c + d*x]])/(2*a^5*d) - (b*(11*a^2 - 12*b^2)*Tan[c + d*x])/(2*a^4*(a^2 - b^
2)*d) + ((5*a^2 - 6*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)*d) -
 (Sec[c + d*x]*Tan[c + d*x])/(2*a*d*(a + b*Cos[c + d*x])^2) - ((3*a^2 - 4*b
^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
```

```
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}+\frac{\int \frac{\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(3a^2-4b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{\left(2\left(5a^4-11a^2b^2\right.\right.}{}}{}$$

$$= \frac{\left(5a^2-6b^2\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}-\frac{\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(3a^2-4b^2\right)\sec}{2a^2\left(a^2-b^2\right)a}$$

$$= -\frac{b\left(11a^2-12b^2\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)d}+\frac{\left(5a^2-6b^2\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}-\frac{\sec(c+dx}{2ad(a+b}$$

$$= -\frac{b\left(11a^2-12b^2\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)d}+\frac{\left(5a^2-6b^2\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}-\frac{\sec(c+dx}{2ad(a+b}$$

$$= -\frac{\left(a^2-12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d}-\frac{b\left(11a^2-12b^2\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)d}+\frac{\left(5a^2-6b^2\right)\mathrm{s}}{2a^3}$$

$$= \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}-\frac{\left(a^2-12b^2\right)\tanh^{-1}(\sin(c+d}{2a^5d}$$

Mathematica [A] time = 6.207, size = 414, normalized size = 1.53

$$-\frac{b^2\sin(c+dx)}{2a^3d(a+b\cos(c+dx))^2}+\frac{6b^4\sin(c+dx)-5a^2b^2\sin(c+dx)}{2a^4d(a-b)(a+b)(a+b\cos(c+dx))}-\frac{b\left(-19a^2b^2+6a^4+12b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{a^5d\left(a^2-b^2\right)\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[
-a^2 + b^2]])/(a^5*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d)) + ((a^2 - 12*b^2)*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^5*d) + ((-a^2 + 12*b^2)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^5*d) + 1/(4*a^3*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])) - 1/(4*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (
3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^2*
Sin[c + d*x])/(2*a^3*d*(a + b*Cos[c + d*x])^2) + (-5*a^2*b^2*Sin[c + d*x] +
 6*b^4*Sin[c + d*x])/(2*a^4*(a - b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

Maple [B] time = 0.076, size = 747, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -6/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/
2*d*x+1/2*c)^3*b^2-1/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2*b^3/(a+b)*tan(1/2*d*x+1/2*c)^3+6/d*b^4/a^4/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3-6/d/a^2/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*d*x+1/2*c)*b^2+1/d/
a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)*tan(1/2
*d*x+1/2*c)+6/d*b^4/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a-b)*tan(1/2*d*x+1/2*c)+6/d/a*b/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-19/d/a^3*b^3/(a^2-b^2)/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+12/d*b^5/a
^5/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tan(1/2*d*x+1/2*c)
-1)+3/d/a^4/(tan(1/2*d*x+1/2*c)-1)*b+1/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)-6/d
/a^5*ln(tan(1/2*d*x+1/2*c)-1)*b^2-1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/
a^3/(tan(1/2*d*x+1/2*c)+1)+3/d/a^4/(tan(1/2*d*x+1/2*c)+1)*b-1/2/d/a^3*ln(ta
n(1/2*d*x+1/2*c)+1)+6/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.81694, size = 2919, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((6*a^4*b^3 - 19*a^2*b^5 + 12*b^7)*cos(d*x + c)^4 + 2*(6*a^5*b^2 - 19
*a^3*b^4 + 12*a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*c
os(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos
(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^6*b^2 - 14*a^
4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 + 2*(a^7*b - 14*a^5*b^3 + 25*a^
3*b^5 - 12*a*b^7)*cos(d*x + c)^3 + (a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*
b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b^2 - 14*a^4*b^4 + 25*a^
2*b^6 - 12*b^8)*cos(d*x + c)^4 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*
b^7)*cos(d*x + c)^3 + (a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 2*(a^8 - 2*a^6*b^2 + a^4*b^4 - (11*a^5*b^3
 - 23*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 - (17*a^6*b^2 - 35*a^4*b^4 + 18*a^
2*b^6)*cos(d*x + c)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c))*sin(d
*x + c))/((a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 + 2*(a^10*b - 2*
a^8*b^3 + a^6*b^5)*d*cos(d*x + c)^3 + (a^11 - 2*a^9*b^2 + a^7*b^4)*d*cos(d*
x + c)^2), 1/4*(2*((6*a^4*b^3 - 19*a^2*b^5 + 12*b^7)*cos(d*x + c)^4 + 2*(6*
a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 19*a^4*b^3 + 1
2*a^2*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sq
rt(a^2 - b^2)*sin(d*x + c))) - ((a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8
)*cos(d*x + c)^4 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x +
 c)^3 + (a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*cos(d*x + c)^2)*log(si
n(d*x + c) + 1) + ((a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c
)^4 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^3 + (a^8
- 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) +
 1) + 2*(a^8 - 2*a^6*b^2 + a^4*b^4 - (11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*c
os(d*x + c)^3 - (17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^2 - 4*(
```

```
a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 2*a^7*
b^4 + a^5*b^6)*d*cos(d*x + c)^4 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*
x + c)^3 + (a^11 - 2*a^9*b^2 + a^7*b^4)*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45675, size = 860, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(6*a^4*b - 19*a^2*b^3 + 12*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(a^2 - b^2)))/((a^7 - a^5*b^2)*sqrt(a^2 - b^2)) - 2*(a^5*tan(1/2*d*x +
1/2*c)^7 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 18*a^3*b^2*tan(1/2*d*x + 1/2*c)
^7 + 7*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 1
2*b^5*tan(1/2*d*x + 1/2*c)^7 + 3*a^5*tan(1/2*d*x + 1/2*c)^5 + 4*a^4*b*tan(1
/2*d*x + 1/2*c)^5 + 14*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 37*a^2*b^3*tan(1/2*
d*x + 1/2*c)^5 - 18*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 36*b^5*tan(1/2*d*x + 1/2
*c)^5 + 3*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 14*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 37*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 18*a*b
^4*tan(1/2*d*x + 1/2*c)^3 - 36*b^5*tan(1/2*d*x + 1/2*c)^3 + a^5*tan(1/2*d*x
 + 1/2*c) - 4*a^4*b*tan(1/2*d*x + 1/2*c) - 18*a^3*b^2*tan(1/2*d*x + 1/2*c)
- 7*a^2*b^3*tan(1/2*d*x + 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c) + 12*b^5*t
an(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2
*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (a^2 - 12*b^2)*l
og(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - (a^2 - 12*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^5)/d
```

3.618 $\int \frac{(1-\cos^2(c+dx))\sec^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=335

$$-\frac{b^2\left(-33a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(-59a^2b^2+2a^4+60b^4\right)\tan(c+dx)}{6a^5d\left(a^2-b^2\right)}+\frac{b\left(3a^2-20b^2\right)\tanh^{-1}(\mathrm{s}}{2a^6d}$$

```
[Out] -((b^2*(12*a^4 - 33*a^2*b^2 + 20*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(a^6*(a - b)^(3/2)*(a + b)^(3/2)*d)) + (b*(3*a^2 - 20*b^2)*A
rcTanh[Sin[c + d*x]])/(2*a^6*d) - ((2*a^4 - 59*a^2*b^2 + 60*b^4)*Tan[c + d*
x])/(6*a^5*(a^2 - b^2)*d) - (b*(9*a^2 - 10*b^2)*Sec[c + d*x]*Tan[c + d*x])/
(2*a^4*(a^2 - b^2)*d) + ((17*a^2 - 20*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(6*
a^3*(a^2 - b^2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(2*a*d*(a + b*Cos[c + d*
x])^2) - ((4*a^2 - 5*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.39778, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^2\left(-33a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(-59a^2b^2+2a^4+60b^4\right)\tan(c+dx)}{6a^5d\left(a^2-b^2\right)}+\frac{b\left(3a^2-20b^2\right)\tanh^{-1}(\mathrm{s}}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x])^3,x]
```



```
[Out] -((b^2*(12*a^4 - 33*a^2*b^2 + 20*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(a^6*(a - b)^(3/2)*(a + b)^(3/2)*d)) + (b*(3*a^2 - 20*b^2)*A
rcTanh[Sin[c + d*x]])/(2*a^6*d) - ((2*a^4 - 59*a^2*b^2 + 60*b^4)*Tan[c + d*
x])/(6*a^5*(a^2 - b^2)*d) - (b*(9*a^2 - 10*b^2)*Sec[c + d*x]*Tan[c + d*x])/
(2*a^4*(a^2 - b^2)*d) + ((17*a^2 - 20*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(6*
a^3*(a^2 - b^2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(2*a*d*(a + b*Cos[c + d*
x])^2) - ((4*a^2 - 5*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
```

```
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}+\frac{\int \frac{\left(5\left(a^2-b^2\right)-4\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(4a^2-5b^2\right)\sec^2(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{\left(17a^4-37a^2\right.}{}}{}$$

$$= \frac{\left(17a^2-20b^2\right)\sec^2(c+dx)\tan(c+dx)}{6a^3\left(a^2-b^2\right)d}-\frac{\sec^2(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^2}-\frac{\left(4a^2-5b^2\right)}{2a^2\left(a^2-\right.}$$

$$= -\frac{b\left(9a^2-10b^2\right)\sec(c+dx)\tan(c+dx)}{2a^4\left(a^2-b^2\right)d}+\frac{\left(17a^2-20b^2\right)\sec^2(c+dx)\tan(c+dx}{6a^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(2a^4-59a^2b^2+60b^4\right)\tan(c+dx)}{6a^5\left(a^2-b^2\right)d}-\frac{b\left(9a^2-10b^2\right)\sec(c+dx)\tan(c+dx)}{2a^4\left(a^2-b^2\right)d}+$$

$$= -\frac{\left(2a^4-59a^2b^2+60b^4\right)\tan(c+dx)}{6a^5\left(a^2-b^2\right)d}-\frac{b\left(9a^2-10b^2\right)\sec(c+dx)\tan(c+dx)}{2a^4\left(a^2-b^2\right)d}+$$

$$= \frac{b\left(3a^2-20b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^6d}-\frac{\left(2a^4-59a^2b^2+60b^4\right)\tan(c+dx)}{6a^5\left(a^2-b^2\right)d}-\frac{b\left(9a\right.}{}$$

$$= -\frac{b^2\left(12a^4-33a^2b^2+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b\left(3a^2-20b^2\right)\tanh^{-1}(\mathrm{si}}{2a^6d}$$

Mathematica [A] time = 6.24713, size = 563, normalized size = 1.68

$$\frac{b^3\sin(c+dx)}{2a^4d(a+b\cos(c+dx))^2}+\frac{18b^2\sin\left(\frac{1}{2}(c+dx)\right)-a^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^5d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{18b^2\sin\left(\frac{1}{2}(c+dx)\right)-a^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^5d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{7a}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b^2*(12*a^4 - 33*a^2*b^2 + 20*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/(a^6*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((-3*a^2*b + 20*b^3)*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^6*d) + ((3*a^2*b - 20*b^3)*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^6*d) + (a - 9*b)/(12*a^4*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*a^3*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^3) + Sin[(c + d*x)/2]/(6*a^3*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^3) + (-a + 9*b)/(12*a^4*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2) + (-(a^2*Sin[(c + d*x)/2]) + 18*b^2*Sin[(c + d*x)/2])/(3*a^
5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (-(a^2*Sin[(c + d*x)/2]) + 18*
b^2*Sin[(c + d*x)/2])/(3*a^5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^
3*Sin[c + d*x])/(2*a^4*d*(a + b*Cos[c + d*x])^2) + (7*a^2*b^3*Sin[c + d*x]
- 8*b^5*Sin[c + d*x])/(2*a^5*(a - b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

Maple [B] time = 0.08, size = 843, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 8/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)*tan
(1/2*d*x+1/2*c)^3+1/d*b^4/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3-8/d*b^5/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+8/d/a^3/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)*tan(1/2*d*x+1/2*c)-1/d*b
^4/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)*tan(1/2*
d*x+1/2*c)-8/d*b^5/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
2/(a-b)*tan(1/2*d*x+1/2*c)-12/d/a^2/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2+33/d/a^4/(a^2-b^2)/((a+b)*(
```

```
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^4-20/d*b
^6/a^6/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))-1/3/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/a^3/(tan(1/2*d*x+1/
2*c)-1)^2-3/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)^2*b-3/2/d/a^4/(tan(1/2*d*x+1/2*c
)-1)*b-6/d*b^2/a^5/(tan(1/2*d*x+1/2*c)-1)-3/2/d*b/a^4*ln(tan(1/2*d*x+1/2*c)
-1)+10/d*b^3/a^6*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^
3+1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+3/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2*b-3
/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)*b-6/d*b^2/a^5/(tan(1/2*d*x+1/2*c)+1)+3/2/d*
b/a^4*ln(tan(1/2*d*x+1/2*c)+1)-10/d*b^3/a^6*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.26328, size = 3258, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*((12*a^4*b^4 - 33*a^2*b^6 + 20*b^8)*cos(d*x + c)^5 + 2*(12*a^5*b^3
 - 33*a^3*b^5 + 20*a*b^7)*cos(d*x + c)^4 + (12*a^6*b^2 - 33*a^4*b^4 + 20*a^
2*b^6)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*((3*a^6
*b^3 - 26*a^4*b^5 + 43*a^2*b^7 - 20*b^9)*cos(d*x + c)^5 + 2*(3*a^7*b^2 - 26
*a^5*b^4 + 43*a^3*b^6 - 20*a*b^8)*cos(d*x + c)^4 + (3*a^8*b - 26*a^6*b^3 +
43*a^4*b^5 - 20*a^2*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((3*a^6*
b^3 - 26*a^4*b^5 + 43*a^2*b^7 - 20*b^9)*cos(d*x + c)^5 + 2*(3*a^7*b^2 - 26*
a^5*b^4 + 43*a^3*b^6 - 20*a*b^8)*cos(d*x + c)^4 + (3*a^8*b - 26*a^6*b^3 + 4
```

```
3*a^4*b^5 - 20*a^2*b^7)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 2*(2*a^9 -
 4*a^7*b^2 + 2*a^5*b^4 - (2*a^7*b^2 - 61*a^5*b^4 + 119*a^3*b^6 - 60*a*b^8)*
cos(d*x + c)^4 - (4*a^8*b - 95*a^6*b^3 + 181*a^4*b^5 - 90*a^2*b^7)*cos(d*x
+ c)^3 - 2*(a^9 - 12*a^7*b^2 + 21*a^5*b^4 - 10*a^3*b^6)*cos(d*x + c)^2 - 5*
(a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 2*a^
8*b^4 + a^6*b^6)*d*cos(d*x + c)^5 + 2*(a^11*b - 2*a^9*b^3 + a^7*b^5)*d*cos(
d*x + c)^4 + (a^12 - 2*a^10*b^2 + a^8*b^4)*d*cos(d*x + c)^3), -1/12*(6*((12
*a^4*b^4 - 33*a^2*b^6 + 20*b^8)*cos(d*x + c)^5 + 2*(12*a^5*b^3 - 33*a^3*b^5
 + 20*a*b^7)*cos(d*x + c)^4 + (12*a^6*b^2 - 33*a^4*b^4 + 20*a^2*b^6)*cos(d*
x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c))) - 3*((3*a^6*b^3 - 26*a^4*b^5 + 43*a^2*b^7 - 20*b^9)*cos(d*x + c
)^5 + 2*(3*a^7*b^2 - 26*a^5*b^4 + 43*a^3*b^6 - 20*a*b^8)*cos(d*x + c)^4 + (
3*a^8*b - 26*a^6*b^3 + 43*a^4*b^5 - 20*a^2*b^7)*cos(d*x + c)^3)*log(sin(d*x
 + c) + 1) + 3*((3*a^6*b^3 - 26*a^4*b^5 + 43*a^2*b^7 - 20*b^9)*cos(d*x + c)
^5 + 2*(3*a^7*b^2 - 26*a^5*b^4 + 43*a^3*b^6 - 20*a*b^8)*cos(d*x + c)^4 + (3
*a^8*b - 26*a^6*b^3 + 43*a^4*b^5 - 20*a^2*b^7)*cos(d*x + c)^3)*log(-sin(d*x
 + c) + 1) - 2*(2*a^9 - 4*a^7*b^2 + 2*a^5*b^4 - (2*a^7*b^2 - 61*a^5*b^4 + 1
19*a^3*b^6 - 60*a*b^8)*cos(d*x + c)^4 - (4*a^8*b - 95*a^6*b^3 + 181*a^4*b^5
 - 90*a^2*b^7)*cos(d*x + c)^3 - 2*(a^9 - 12*a^7*b^2 + 21*a^5*b^4 - 10*a^3*b
^6)*cos(d*x + c)^2 - 5*(a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(d*x + c))*sin(d*x
+ c))/((a^10*b^2 - 2*a^8*b^4 + a^6*b^6)*d*cos(d*x + c)^5 + 2*(a^11*b - 2*a^
9*b^3 + a^7*b^5)*d*cos(d*x + c)^4 + (a^12 - 2*a^10*b^2 + a^8*b^4)*d*cos(d*x
 + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.54813, size = 636, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(12*a^4*b^2 - 33*a^2*b^4 + 20*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^8 - a^6*b^2)*sqrt(a^2 - b^2)) + 6*(8*a^3*b^3*tan(1/
2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*a*b^5*tan(1/2*d*x +
 1/2*c)^3 + 8*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c) +
 7*a^2*b^4*tan(1/2*d*x + 1/2*c) - 9*a*b^5*tan(1/2*d*x + 1/2*c) - 8*b^6*tan(
1/2*d*x + 1/2*c))/((a^7 - a^5*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*
x + 1/2*c)^2 + a + b)^2) + 3*(3*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^6 - 3*(3*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^6 -
 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*b^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 72*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*a*b*tan(1/2*d*x +
1/2*c) + 36*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^5))
/d
```

3.619 $\int \frac{a^2-b^2\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=16

$$ax-\frac{b\sin(c+dx)}{d}$$

```
[Out] a*x - (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.045377, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3016, 2637}

$$ax-\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] a*x - (b*Sin[c + d*x])/d
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a^2 - b^2 \cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = -\int (-a+b\cos(c+dx))\,dx$$
$$= ax - b\int \cos(c+dx)\,dx$$
$$= ax - \frac{b\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0096987, size = 28, normalized size = 1.75

$$ax - \frac{b\sin(c)\cos(dx)}{d} - \frac{b\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] a*x - (b*Cos[d*x]*Sin[c])/d - (b*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.025, size = 22, normalized size = 1.4

$$\frac{-\sin(dx+c)\,b + a\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(-sin(d*x+c)*b+a*(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31461, size = 38, normalized size = 2.38

$$\frac{adx - b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*x - b*sin(d*x + c))/d
```

Sympy [A] time = 0.800864, size = 32, normalized size = 2.

$$\begin{cases} ax - \frac{b\sin(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x\left(a^2-b^2\cos^2(c)\right)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x - b*sin(c + d*x)/d, Ne(d, 0)), (x*(a**2 - b**2*cos(c)**2)/(a
 + b*cos(c)), True))
```

Giac [B] time = 1.35752, size = 53, normalized size = 3.31

$$\frac{(dx + c)a - \frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a - 2*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.620 $\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$\frac{4a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}-x$$

```
[Out] -x + (4*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*d)
```

Rubi [A] time = 0.0996256, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3016, 2735, 2659, 205}

$$\frac{4a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -x + (4*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*d)
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\int \frac{-a+b\cos(c+dx)}{a+b\cos(c+dx)}\,dx\\ &= -x+(2a)\int \frac{1}{a+b\cos(c+dx)}\,dx\\ &= -x+\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\ &= -x+\frac{4a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.069373, size = 53, normalized size = 0.98

$$-\frac{4a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -x - (4*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2
+ b^2]*d)
```

Maple [A] time = 0.033, size = 61, normalized size = 1.1

$$-2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{d}+4\,\frac{a}{d\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*arctan(tan(1/2*d*x+1/2*c))+4/d*a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49614, size = 478, normalized size = 8.85

$$\left[-\frac{\left(a^2-b^2\right)dx+\sqrt{-a^2+b^2}a\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{\left(a^2-b^2\right)d},-\frac{\left(a^2-b^2\right)dx-2\,\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-((a^2 - b^2)*d*x + sqrt(-a^2 + b^2)*a*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/((a^2 - b^
2)*d), -((a^2 - b^2)*d*x - 2*sqrt(a^2 - b^2)*a*arctan(-(a*cos(d*x + c) + b)
/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2 - b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.68035, size = 115, normalized size = 2.13

$$-\frac{dx - \frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}} + c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(d*x - 4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/sqrt(a^2 - b
^2) + c)/d
```

3.621 $\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{2\left(a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{2ab\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (2*(a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - (2*a*b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c
+ d*x]))
```

Rubi [A] time = 0.116977, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3016, 2754, 12, 2659, 205}

$$\frac{2\left(a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{2ab\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - (2*a*b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c
+ d*x]))
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx &= -\int \frac{-a+b\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx\\
&= -\frac{2ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\int \frac{a^2+b^2}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{2ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\left(a^2+b^2\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{2ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\left(2\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= \frac{2\left(a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{2ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.243905, size = 91, normalized size = 0.98

$$\frac{2\left(\frac{(a^2+b^2)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{3/2}}-\frac{ab\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(((a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^
2 + b^2)^(3/2) - (a*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))
)/d
```

Maple [B] time = 0.037, size = 177, normalized size = 1.9

$$-4\,{\frac{ab\tan \left(1/2\,dx+c/2 \right) }{d \left({a}^{2}-{b}^{2} \right) \left(a \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}- \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}b+a+b \right) }}+2\,{\frac{{a}^{2}}{d \left(a+b \right) \left(a-b \right) \sqrt{ \left(a+b \right) \left(a-b \right) }}\arctan \left({\frac{ \left(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d*a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)+2/d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*a^2+2/d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47944, size = 783, normalized size = 8.42

$$\left[\frac{\left(a^3+ab^2+\left(a^2b+b^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(\left(a^4b-2\,a^2b^3+b^5\right)d\cos\left(dx+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3 + a*b^2 + (a^2*b + b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 4*(a^3*b - a*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)
*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), ((a^3 + a*b^2 + (a^2*b + b^
3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c))) - 2*(a^3*b - a*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 +
b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49066, size = 193, normalized size = 2.08

$$-\frac{2\left(\frac{2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)\left(a^2-b^2\right)}-\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\left(a^2+b^2\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(2*a*b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x +
 1/2*c)^2 + a + b)*(a^2 - b^2)) - (pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a
 - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
 - b^2)))*(a^2 + b^2)/(a^2 - b^2)^(3/2))/d
```

3.622 $\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=140

$$\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{b\left(2a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{ab\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

```
[Out] (2*a*(a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(5/2)*(a + b)^(5/2)*d) - (a*b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) - (b*(2*a^2 + b^2)*Sin[c + d*x])/((a^2 - b^2)^2*d*(a + b*Cos[c
 + d*x]))
```

Rubi [A] time = 0.232585, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3016, 2754, 12, 2659, 205}

$$\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{b\left(2a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{ab\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] (2*a*(a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(5/2)*(a + b)^(5/2)*d) - (a*b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) - (b*(2*a^2 + b^2)*Sin[c + d*x])/((a^2 - b^2)^2*d*(a + b*Cos[c
 + d*x]))
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a^2 - b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\int \frac{-a+b\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{2\left(a^2+b^2\right)-2ab\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(2a^2+b^2\right)\sin(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int -\frac{2a\left(a^2+2b^2\right)}{a+b\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(2a^2+b^2\right)\sin(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(a\left(a^2+2b^2\right)\right)\int \frac{}{a+b\,\mathrm{c}}}{\left(a^2-b^2\right)^2}$$

$$= -\frac{ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(2a^2+b^2\right)\sin(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(2a\left(a^2+2b^2\right)\right)\mathrm{Subs}}{}$$

$$= \frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{ab\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(2a^2+b^2\right)}{\left(a^2-b^2\right)^2 d(a+}$$

Mathematica [A] time = 0.623854, size = 116, normalized size = 0.83

$$-\frac{\frac{b\sin(c+dx)\left(b\left(2a^2+b^2\right)\cos(c+dx)+3a^3\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -(((2*a*(a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(5/2) + (b*(3*a^3 + b*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x
])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/d)
```

Maple [B] time = 0.038, size = 547, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a-2/d/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3-6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^2*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2+2/d/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)*a-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a
^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2/d*a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^
(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+4/d*a/(a^4-2*a^2
*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68785, size = 1287, normalized size = 9.19

$$\left[-\frac{\left(a^5+2a^3b^2+\left(a^3b^2+2ab^4\right)\cos(dx+c)^2+2\left(a^4b+2a^2b^3\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2}{b^2\cos(dx+}\right.}{2\left(\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)d\cos(dx+c)^2+2\left(a^7b-3a^5b^3+3a^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^5 + 2*a^3*b^2 + (a^3*b^2 + 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b + 2
*a^2*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
```

```
b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(3*a^5
*b - 3*a^3*b^3 + (2*a^4*b^2 - a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/((
a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*
b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^
2*b^6)*d), ((a^5 + 2*a^3*b^2 + (a^3*b^2 + 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*
b + 2*a^2*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c))) - (3*a^5*b - 3*a^3*b^3 + (2*a^4*b^2 - a^2*b^
4 - b^6)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x +
 c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.77027, size = 344, normalized size = 2.46

$$-\frac{2\left(\frac{\left(a^3+2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 \cdots}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2} \cdots\right.\right.}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arc
tan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((
a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 -
 2*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a^3*b*ta
n(1/2*d*x + 1/2*c) + 2*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/2
*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
```

```
/2*c)^2 + a + b)^2))/d
```

3.623 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=364

$$\frac{2\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}-\frac{4a\left(8a^2C+21Ab^2+18b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+d}}{315b^3d}$$

```
[Out] (-2*(16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*(21*A*b^2 + 8*a^2*C + 18*b^2*C)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315
*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(21*A*b^2 + 8*a^2*C + 18*b^2*C)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^3*d) + (2*(24*a^2*C + 7*b^2*(9*A
 + 7*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^3*d) - (4*a*C*Cos[
c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*C*Cos[c +
 d*x]^2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.805528, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3050, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}-\frac{4a\left(8a^2C+21Ab^2+18b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+d}}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (-2*(16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*(21*A*b^2 + 8*a^2*C + 18*b^2*C)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315
*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(21*A*b^2 + 8*a^2*C + 18*b^2*C)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^3*d) + (2*(24*a^2*C + 7*b^2*(9*A
 + 7*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^3*d) - (4*a*C*Cos[
c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*C*Cos[c +
 d*x]^2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
```

```
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9bd} + \frac{2\int \cos}{}$$

$$= -\frac{4aC\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2C\,c}{}$$

$$= \frac{2\left(24a^2C+7b^2(9A+7C)\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx}{315b^3d}$$

$$= -\frac{4a\left(21Ab^2+8a^2C+18b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{315b^3d}$$

$$= -\frac{4a\left(21Ab^2+8a^2C+18b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{315b^3d}$$

$$= -\frac{4a\left(21Ab^2+8a^2C+18b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{315b^3d}$$

$$= -\frac{2\left(16a^4C+6a^2b^2(7A+4C)-21b^4(9A+7C)\right)\sqrt{a+b\,\mathrm{co}}}{315b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.29654, size = 269, normalized size = 0.74

$$\frac{b(a+b\cos(c+dx))\left(2a\left(32a^2C+84Ab^2+57b^2C\right)\sin(c+dx)+b\left(\left(-24a^2C+252Ab^2+266b^2C\right)\sin(2(c+dx))+5bC(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(a*b^2*(147*A*b^2 - 4*a^2*C + 111*b^2
*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (16*a^4*C + 6*a^2*b^2*(7*A + 4*
C) - 21*b^4*(9*A + 7*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*(2*a*(84*
A*b^2 + 32*a^2*C + 57*b^2*C)*Sin[c + d*x] + b*((252*A*b^2 - 24*a^2*C + 266*
b^2*C)*Sin[2*(c + d*x)] + 5*b*C*(2*a*Sin[3*(c + d*x)] + 7*b*Sin[4*(c + d*x)
]))))/(1260*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.457, size = 1527, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(640*C*a*b^4+2240*C*b^5)*sin(
1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^5+8*C*a^2*b^3-960*C*a*b^4-207
2*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(336*A*a*b^4+504*A*b^5+8*C
*a^3*b^2-8*C*a^2*b^3+728*C*a*b^4+952*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+(-42*A*a^2*b^3-168*A*a*b^4-126*A*b^5-16*C*a^4*b-4*C*a^3*b^2-24*C*a
^2*b^3-204*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+42*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-42*a*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-42*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+42*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^4-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*b^5+16*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^5+20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*
b^2-36*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-16*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+16*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-24*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+24*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b^4-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*b^5)/b^4/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^4+A\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x
)
```

3.624 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=291

$$\frac{2\left(8a^2C+5b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(a^2-b^2\right)\left(8a^2C+35Ab^2+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx\right.}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*a*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) -
(2*(a^2 - b^2)*(35*A*b^2 + 8*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*(8*a^2*C + 5*b^2*(7*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(105*b^2*d) - (8*a*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b^
2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.47975, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3050, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2C+5b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(a^2-b^2\right)\left(8a^2C+35Ab^2+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx\right.}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] (2*a*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) -
(2*(a^2 - b^2)*(35*A*b^2 + 8*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*(8*a^2*C + 5*b^2*(7*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(105*b^2*d) - (8*a*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b^
2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd} + \frac{2\int\sqrt{a+}}{}$$

$$= -\frac{8aC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} + \frac{2C\cos(c+dx)(a}{}$$

$$= \frac{2\left(8a^2C+5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} -$$

$$= \frac{2\left(8a^2C+5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} -$$

$$= \frac{2\left(8a^2C+5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} -$$

$$= \frac{2a\left(35Ab^2+8a^2C+19b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.840625, size = 216, normalized size = 0.74

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))\left(-8a^2C+6abC\cos(c+dx)+70Ab^2+15b^2C\cos(2(c+dx))+65b^2C\right)+4\sqrt{\frac{a+b\cos(c+dx}{a+b}}}{210b^3d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(35*A*b^3 + 2*a^2*b*C + 25*b^3*C)*
EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*(
(a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(70*A*b^2 - 8*a^2*C + 65*b^2*C +
6*a*b*C*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(210*b^3*d*
Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.36, size = 1131, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-144*C*a*b^3-360*C*b^4)*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4-4*C*a^2*b^2+144*C*a*b^3+280*C*b
^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a*b^3-70*A*b^4+8*C*a^3*b
+2*C*a^2*b^2-86*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3
5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-35*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-35*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^4-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^3*b+19*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^2*b^2-19*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^
3-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
```

```
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^3/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.625 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=218

$$-\frac{2\left(2a^2C-3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{4aC\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sin}{}$$

```
[Out] (-2*(2*a^2*C - 3*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a
*(a^2 - b^2)*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*C*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*b*d) + (2*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d
*x])/(5*b*d)
```

Rubi [A] time = 0.325578, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3024, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(2a^2C-3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{4aC\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (-2*(2*a^2*C - 3*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a
*(a^2 - b^2)*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*C*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*b*d) + (2*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d
*x])/(5*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
```

```
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int \sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}b(5A\right.}{5b}$$

$$= -\frac{4aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+}{5bd}$$

$$= -\frac{4aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+}{5bd}$$

$$= -\frac{4aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+}{5bd}$$

$$= \frac{2\left(15A+\left(9-\frac{2a^2}{b^2}\right)C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{4a\left(a^2-\right.}{}$$

Mathematica [A] time = 0.92953, size = 181, normalized size = 0.83

$$\frac{-2(a+b)\left(2a^2C-15Ab^2-9b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+bC\sin(c+dx)\left(2a^2+8ab\cos(c+dx)+3b^2\cos(2(c\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*(a + b)*(-15*A*b^2 + 2*a^2*C - 9*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*a*(a^2 - b^2)*C*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*C*(2*a^2 +
3*b^2 + 8*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^2*
d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.355, size = 821, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*cos
(1/2*d*x+1/2*c)^7*b^3+16*C*cos(1/2*d*x+1/2*c)^5*a*b^2-48*C*cos(1/2*d*x+1/2*
c)^5*b^3+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-15*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+2*C*cos(1/2*d*x+1/2*c)^3*
a^2*b-24*C*cos(1/2*d*x+1/2*c)^3*a*b^2+30*C*cos(1/2*d*x+1/2*c)^3*b^3+2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-2*a*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^3+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*C*cos(1/2*d*x+1/2*c)*a^2*b+8*C*cos(1/2
*d*x+1/2*c)*a*b^2-6*C*cos(1/2*d*x+1/2*c)*b^3)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.626 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=231

$$-\frac{2\left(a^2C-b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{a+}}{3d}$$

```
[Out] (2*a*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2*C - b^2*(3*A + C))*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*a*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.647012, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3050, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(a^2C-b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{a+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] (2*a*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2*C - b^2*(3*A + C))*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*a*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int\frac{\left(\frac{3aA}{2}+\frac{1}{2}b(3A+C\right.}{}$$

$$=\frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}-\frac{2\int\frac{\left(-\frac{3}{2}aAb+\frac{1}{2}\left(a^2C-b^2(3A+\right.\right.}{\sqrt{a+b\,c}}}{3}$$

$$=\frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+(aA)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+}}$$

$$=\frac{2aC\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sqrt{a+b\cos(c}}{3}$$

$$=\frac{2aC\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2C-b^2(3A\right.}{3}$$

Mathematica [C] time = 2.35905, size = 371, normalized size = 1.61

$$\frac{\frac{4b(3A+C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2a(6A+C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2iC\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] ((4*b*(3*A + C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*(6*A + C)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] + ((2*I)*C*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Elli
pticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a -
 b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos
[c + d*x]]], (a + b)/(a - b)])))/(b^2*Sqrt[-(a + b)^(-1)]) + 4*C*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Maple [B] time = 0.383, size = 601, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b+2*C*cos(1/2*d*x
+1/2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3*b^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*C*cos(1/2*d*x+1/2*c)*a*b+2*C*cos(1/2*d*
x+1/2*c)*b^2)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

3.627 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=205

$$-\frac{(A-2C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -(((A - 2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (a*A*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]
) + (A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*Tan[
c + d*x])/d
```

Rubi [A] time = 0.61969, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3048, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(A-2C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] -(((A - 2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (a*A*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]
) + (A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*Tan[
c + d*x])/d
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{Ab}{2}+aC\cos(c+d\right.}{}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{Ab^2}{2}-\frac{1}{2}aAb\cos(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}}{b}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}(aA)\int \frac{1}{\sqrt{a+b\cos(c}}$$

$$= -\frac{(A-2C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b}}{}$$

$$= -\frac{(A-2C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{aA\sqrt{\frac{a+b}{}}}{d}$$

Mathematica [C] time = 2.25402, size = 374, normalized size = 1.82

$$\frac{\frac{2b(A+2C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i(A-2C)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)-2aF(i\right.\right.}{ab\sqrt{-\frac{1}{a+b}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] ((8*a*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(A + 2*C)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]
] - ((2*I)*(A - 2*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Elli
pticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a -
 b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos
[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b
*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 0.537, size = 833, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a-2*A*b)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+A*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*b-A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*b+2*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-2*C*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)*sin(1/2*d*x+1/2*c)^2+A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b-A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-2*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(2*cos(1/2*d*x+1/2*
c)^2-1)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x
)
```

3.628 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=277

$$-\frac{\left(Ab^2-4a^2(A+2C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{b(3A+8C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+Ab\tan(c+$$

```
[Out] -(A*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(3*A + 8*C)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c
 + d*x]]) - ((A*b^2 - 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]])
+ (A*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d) + (A*Sqrt[a + b*Cos[c
 + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.989618, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3048, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(Ab^2-4a^2(A+2C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{b(3A+8C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+Ab\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] -(A*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(3*A + 8*C)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c
 + d*x]]) - ((A*b^2 - 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]])
+ (A*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d) + (A*Sqrt[a + b*Cos[c
 + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{\left(\frac{Ab}{2} + \right.}{}$$

$$= \frac{Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{A\sqrt{a+b\cos(c+dx)}}{}$$

$$= \frac{Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{A\sqrt{a+b\cos(c+dx)}}{}$$

$$= \frac{Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{A\sqrt{a+b\cos(c+dx)}}{}$$

$$= -\frac{Ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{Ab\sqrt{a+b\cos}}{}$$

$$= -\frac{Ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b(3A+8C)\sqrt{}}{4d\sqrt{}}$$

Mathematica [C] time = 3.38998, size = 406, normalized size = 1.47

$$\frac{\frac{2\left(8a^2(A+2C)-3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}} - \frac{2iA\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)-\right.\right.}{a^2\sqrt{-}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(A + 4*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-3*A*b^2 + 8*a^2*(A + 2*C))*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]
)/(a*Sqrt[a + b*Cos[c + d*x]]) - ((2*I)*A*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
 d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*Sqrt[-(a + b)^(-
```

```
1)]) + (4*A*Sqrt[a + b*Cos[c + d*x]]*(2*a + b*Cos[c + d*x])*Sec[c + d*x]*Ta
n[c + d*x])/a)/(16*d)
```

Maple [B] time = 0.908, size = 1264, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*A*b^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*A*a*b+8*A*b^2)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-4*A*a^2-6*A*a*b-2*A*b^2)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a*b-A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*A*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+4*A*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))*a^2-A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*b^2-8*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+8*C*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2)*sin(1/2*d*x+1/2*c)^4+4
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-A*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*A*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b+4*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*a^2-A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-8*C*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+8*C*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a*b-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/
2))*a^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2+8*C
*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-8*a^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/a/(2*cos(1/2*d*x+1/2*c)
^2-1)^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
```

```
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x
)
```

3.629 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=365

$$-\frac{\left(3Ab^2-8a^2(2A+3C)\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}-\frac{\left(Ab^2-8a^2(2A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\cos(c+dx)}}+\frac{(3A}{}$$

```
[Out] ((3*A*b^2 - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b^2
 - 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(A*b^2 + 4*a^2*
(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(8*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b^2 - 8*a^2*(2*A +
 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^2*d) + (A*b*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a*d) + (A*Sqrt[a + b*Cos[c + d
*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.3604, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3048, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(3Ab^2-8a^2(2A+3C)\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}-\frac{\left(Ab^2-8a^2(2A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\cos(c+dx)}}+\frac{(3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] ((3*A*b^2 - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b^2
 - 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(A*b^2 + 4*a^2*
(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(8*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b^2 - 8*a^2*(2*A +
 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^2*d) + (A*b*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a*d) + (A*Sqrt[a + b*Cos[c + d
*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int$$

$$= \frac{Ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12ad}+\frac{A\sqrt{a+}}{}$$

$$= -\frac{\left(3Ab^2-8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d}$$

$$= -\frac{\left(3Ab^2-8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d}$$

$$= -\frac{\left(3Ab^2-8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d}$$

$$= -\frac{\left(A\left(16-\frac{3b^2}{a^2}\right)+24C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(A\left(16-\frac{3b^2}{a^2}\right)+24C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.49974, size = 601, normalized size = 1.65

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+24a^2C\sin(c+dx)-3Ab^2\sin(c+dx)\right)}{24a^2}+\frac{Ab\tan(c+dx)\sec(c+dx)}{12a}+\frac{1}{3}A\tan(c+dx)\sec^2(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -(b*((-8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-8*a^2*A - 9*A*b^2 - 24*a^2*C)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b
)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(16*a^2*A - 3*A*b^2 + 24*a^2*C)*Sqrt[
```

```
(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(
c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])
)*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2
 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 -
b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))))/(96*a^2*d) +
(Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(16*a^2*A*Sin[c + d*x] - 3*A*b^2*S
in[c + d*x] + 24*a^2*C*Sin[c + d*x]))/(24*a^2) + (A*b*Sec[c + d*x]*Tan[c +
d*x])/(12*a) + (A*Sec[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [B] time = 1.423, size = 2309, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*C*(-1/a
*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*b*(-1/2/a*cos(1/2*d*x+1/
2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*
b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
```

```
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2*C*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))+2*a*A*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5
/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/
2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x
)
```

3.630 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=443

$$\frac{2\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{231b^3d}-\frac{4a\left(8a^2C+33Ab^2+34b^2C\right)\sin(c+dx)(a+b\cos(c+dx))}{1155b^3d}$$

```
[Out] (-4*a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(1155*b^4*d*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(16*a^4*C + 6*a^2*b^2*(11*A + 8*C
) - 25*b^4*(11*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(1155*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(16*a^4*
C + 6*a^2*b^2*(11*A + 8*C) - 25*b^4*(11*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(1155*b^3*d) - (4*a*(33*A*b^2 + 8*a^2*C + 34*b^2*C)*(a + b*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(1155*b^3*d) + (2*(8*a^2*C + 3*b^2*(11*A +
9*C))*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(231*b^3*d) - (4*a*C*Cos[c +
 d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(33*b^2*d) + (2*C*Cos[c + d*
x]^2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 1.06326, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3050, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{231b^3d}-\frac{4a\left(8a^2C+33Ab^2+34b^2C\right)\sin(c+dx)(a+b\cos(c+dx))}{1155b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-4*a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(1155*b^4*d*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(16*a^4*C + 6*a^2*b^2*(11*A + 8*C
) - 25*b^4*(11*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(1155*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(16*a^4*
C + 6*a^2*b^2*(11*A + 8*C) - 25*b^4*(11*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(1155*b^3*d) - (4*a*(33*A*b^2 + 8*a^2*C + 34*b^2*C)*(a + b*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(1155*b^3*d) + (2*(8*a^2*C + 3*b^2*(11*A +
9*C))*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(231*b^3*d) - (4*a*C*Cos[c +
```

```
 d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(33*b^2*d) + (2*C*Cos[c + d*
x]^2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{11bd} + \frac{2\int}{}$$

$$= -\frac{4aC\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{33b^2d} + \frac{2}{}$$

$$= \frac{2\left(8a^2C+3b^2(11A+9C)\right)(a+b\cos(c+dx))^{5/2}\sin(c}{231b^3d}$$

$$= -\frac{4a\left(33Ab^2+8a^2C+34b^2C\right)(a+b\cos(c+dx))^{3/2}\sin}{1155b^3d}$$

$$= -\frac{2\left(16a^4C+6a^2b^2(11A+8C)-25b^4(11A+9C)\right)\sqrt{a}}{1155b^3d}$$

$$= -\frac{2\left(16a^4C+6a^2b^2(11A+8C)-25b^4(11A+9C)\right)\sqrt{a}}{1155b^3d}$$

$$= -\frac{2\left(16a^4C+6a^2b^2(11A+8C)-25b^4(11A+9C)\right)\sqrt{a}}{1155b^3d}$$

$$= -\frac{4a\left(8a^4C+3a^2b^2(11A+6C)-b^4(451A+348C)\right)\sqrt{a}}{1155b^4d\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

Mathematica [A] time = 1.74965, size = 331, normalized size = 0.75

$$\frac{b(a+b\cos(c+dx))\left(2\left(6a^2b^2(44A+27C)+64a^4C+5b^4(506A+435C)\right)\sin(c+dx)+b\left(16a\left(-3a^2C+132Ab^2+13\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(-4*a^4*b*C + 25*b^5*(11*A + 9*C)
 + 3*a^2*b^3*(187*A + 141*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(
8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b
*(a + b*Cos[c + d*x])*(2*(64*a^4*C + 6*a^2*b^2*(44*A + 27*C) + 5*b^4*(506*A
```

```
 + 435*C))*Sin[c + d*x] + b*(16*a*(132*A*b^2 - 3*a^2*C + 136*b^2*C)*Sin[2*(
c + d*x)] + 5*b*((132*A*b^2 + 4*a^2*C + 171*b^2*C)*Sin[3*(c + d*x)] + 7*b*C
*(8*a*Sin[4*(c + d*x)] + 3*b*Sin[5*(c + d*x)])))))/(9240*b^4*d*Sqrt[a + b*C
os[c + d*x]])
```

Maple [B] time = 0.428, size = 1791, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/1155*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6720*C
*b^6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-7840*C*a*b^5-16800*C*b^6)*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(2640*A*b^6+2320*C*a^2*b^4+15680*C*
a*b^5+18960*C*b^6)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-3432*A*a*b^5-3
960*A*b^6+8*C*a^3*b^3-3480*C*a^2*b^4-14456*C*a*b^5-11640*C*b^6)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1188*A*a^2*b^4+3432*A*a*b^5+3080*A*b^6+8*C*a^
4*b^2-8*C*a^3*b^3+2624*C*a^2*b^4+6616*C*a*b^5+4620*C*b^6)*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+(-66*A*a^3*b^3-594*A*a^2*b^4-1408*A*a*b^5-880*A*b^6-
16*C*a^5*b-4*C*a^4*b^2-36*C*a^3*b^3-732*C*a^2*b^4-1614*C*a*b^5-930*C*b^6)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-66*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^4*b^2+66*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3*b^3+902*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^2*b^4-902*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b^5+66*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*
b^2-341*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+27
5*A*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-16*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6+16*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b-36*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2+36*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
```

```
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a^3*b^3+696*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2*b^4-696*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a*b^5+16*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^6+32*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-273
*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+225*b^6*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^4/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + A\right)(b\cos{(dx+c)} + a)^{\frac{3}{2}}\cos{(dx+c)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos{(dx+c)}^5 + Ca\cos{(dx+c)}^4 + Ab\cos{(dx+c)}^3 + Aa\cos{(dx+c)}^2\right)\sqrt{b\cos{(dx+c)} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^5 + C*a*cos(d*x + c)^4 + A*b*cos(d*x + c)^3 + A*
a*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
 x)
```

3.631 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=356

$$\frac{2\left(8a^2C+7b^2(9A+7C)\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2C+63Ab^2+39b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{315b^2d}$$

```
[Out] (2*(8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*(63*A*b^2 + 8*a^2*C + 39*b^2*C)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315
*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(63*A*b^2 + 8*a^2*C + 39*b^2*C)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(8*a^2*C + 7*b^2*(9*A
+ 7*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d) - (8*a*C*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) + (2*C*Cos[c + d*x]*(a + b*C
os[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.652903, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3050, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2C+7b^2(9A+7C)\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2C+63Ab^2+39b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*(63*A*b^2 + 8*a^2*C + 39*b^2*C)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315
*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(63*A*b^2 + 8*a^2*C + 39*b^2*C)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(8*a^2*C + 7*b^2*(9*A
+ 7*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d) - (8*a*C*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) + (2*C*Cos[c + d*x]*(a + b*C
os[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd} + \frac{2\int(\ldots}{}$$

$$= -\frac{8aC(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d} + \frac{2C\cos(c+dx\ldots}{}$$

$$= \frac{2\left(8a^2C+7b^2(9A+7C)\right)(a+b\cos(c+dx))^{3/2}\sin(c+\ldots}{315b^2d}$$

$$= \frac{2a\left(63Ab^2+8a^2C+39b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+\ldots}{315b^2d}$$

$$= \frac{2a\left(63Ab^2+8a^2C+39b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+\ldots}{315b^2d}$$

$$= \frac{2a\left(63Ab^2+8a^2C+39b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+\ldots}{315b^2d}$$

$$= \frac{2\left(8a^4C+21b^4(9A+7C)+3a^2b^2(21A+11C)\right)\sqrt{a+b\ldots}}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.3193, size = 269, normalized size = 0.76

$$\frac{b(a+b\cos(c+dx))\left(b\left(2\left(6a^2C+126Ab^2+133b^2C\right)\sin(2(c+dx))+5bC(20a\sin(3(c+dx))+7b\sin(4(c+dx)))\right)-4a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(2*a*b^2*(126*A*b^2 + (a^2 + 93*b^2)*
C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (8*a^4*C + 21*b^4*(9*A + 7*C) +
3*a^2*b^2*(21*A + 11*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*(-4*a*(-2
52*A*b^2 + 8*a^2*C - 201*b^2*C)*Sin[c + d*x] + b*(2*(126*A*b^2 + 6*a^2*C +
133*b^2*C)*Sin[2*(c + d*x)] + 5*b*C*(20*a*Sin[3*(c + d*x)] + 7*b*Sin[4*(c +
 d*x)]))))/(1260*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.378, size = 1527, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1360*C*a*b^4+2240*C*b^5)*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^5-424*C*a^2*b^3-2040*C*a*b^4
-2072*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(756*A*a*b^4+504*A*b^5
-4*C*a^3*b^2+424*C*a^2*b^3+1568*C*a*b^4+952*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-252*A*a^2*b^3-378*A*a*b^4-126*A*b^5+8*C*a^4*b+2*C*a^3*b^2
-282*C*a^2*b^3-444*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-189*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5-63*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*
```

```
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+63*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^4+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^5-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^4*b+33*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2
-33*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5-8*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^4)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^4+Ca\cos\left(dx+c\right)^3+Ab\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + C*a*cos(d*x + c)^3 + A*b*cos(d*x + c)^2 + A*
a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x
)
```

3.632 $\int (a+b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=285

$$-\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2C+35Ab^2+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (4*a*(70*A*b^2 - 3*a^2*C + 41*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) -
(2*(a^2 - b^2)*(35*A*b^2 - 6*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[a + b*Cos[c +
 d*x]]) - (2*(6*a^2*C - 5*b^2*(7*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(105*b*d) - (4*a*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b*d)
 + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.469181, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3024, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2C+35Ab^2+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (4*a*(70*A*b^2 - 3*a^2*C + 41*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) -
(2*(a^2 - b^2)*(35*A*b^2 - 6*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[a + b*Cos[c +
 d*x]]) - (2*(6*a^2*C - 5*b^2*(7*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(105*b*d) - (4*a*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b*d)
 + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
```

```
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int(a+b\cos(c+dx))^{3/2}\left(\frac{1}{2}b\right.}{}$$

$$= -\frac{4aC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd} + \frac{2C(a+b\cos(c+dx))^{5/2}\sin}{7bd}$$

$$= -\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} - \frac{4aC(a}{}$$

$$= -\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} - \frac{4aC(a}{}$$

$$= -\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} - \frac{4aC(a}{}$$

$$= \frac{4a\left(70Ab^2-3a^2C+41b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} -$$

Mathematica [A] time = 0.836575, size = 224, normalized size = 0.79

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))\left(6a^2C+48abC\cos(c+dx)+70Ab^2+15b^2C\cos(2(c+dx))+65b^2C\right)+4\sqrt{\frac{a+b\cos(c+d}{a+b}}}{210}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*b^2*(7*A + 5*C) + 3*a^2*(35*A
 + 17*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(-70*A*b^2 + 3*a^2*C
- 41*b^2*C)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(70*A*b^2 + 6*a^2*C
+ 65*b^2*C + 48*a*b*C*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x
])/(210*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.403, size = 1131, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-312*C*a*b^3-360*C*b^4)*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4+108*C*a^2*b^2+312*C*a*b^3+280*C
*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a*b^3-70*A*b^4-6*C*a^3
*b-54*C*a^2*b^2-128*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+140*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-140
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-35*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-6*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a^3*b+82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^2*b^2-82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b^3+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^2/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.633 $\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=281

$$\frac{2a\left(5Ab^2-C\left(a^2-b^2\right)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+$$

```
[Out] (2*(a^2*C + b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*b)/(a + b)])/(5*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*(5*A*b^
2 - (a^2 - b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(5*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a
+ b*Cos[c + d*x]]) + (2*a*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) +
(2*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.92331, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3050, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2a\left(5Ab^2-C\left(a^2-b^2\right)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```



```
[Out] (2*(a^2*C + b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*b)/(a + b)])/(5*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*(5*A*b^
2 - (a^2 - b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(5*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a
+ b*Cos[c + d*x]]) + (2*a*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) +
(2*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
```

```
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int\sqrt{a+b\cos(c}$$

$$= \frac{2aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2C(a+b\cos(c+}{5}$$

$$= \frac{2aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2C(a+b\cos(c+}{5}$$

$$= \frac{2aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2C(a+b\cos(c+}{5}$$

$$= \frac{2\left(a^2C+b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(a^2C+b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.34794, size = 421, normalized size = 1.5

$$\frac{\frac{2\left(a^2(10A+C)+b^2(5A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(C\left(a^2+3b^2\right)+5Ab^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+}}\right.\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] ((8*a*b*(5*A + 2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(a^2*(10*A + C) + b^2*(5*A
 + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(5*A*b^2 + (a^2 + 3*b^2)*C)*S
qrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b
)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a
 + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)])))/(a*b^2*Sqrt[-(a + b)^(-1)]) + 4*C*Sqrt[a + b*Cos[c + d*x]]*(2*a
```

```
+ b*Cos[c + d*x])*Sin[c + d*x])/(10*d)
```

Maple [B] time = 0.415, size = 962, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*cos(1
/2*d*x+1/2*c)^7*b^3+12*C*cos(1/2*d*x+1/2*c)^5*a*b^2-16*C*cos(1/2*d*x+1/2*c)
^5*b^3+5*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-5*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*b^3-5*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(
a-b))^(1/2))*b+4*C*cos(1/2*d*x+1/2*c)^3*a^2*b-18*C*cos(1/2*d*x+1/2*c)^3*a*b
^2+10*C*cos(1/2*d*x+1/2*c)^3*b^3-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^3+a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2*b+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b^2-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*C*cos(
1/2*d*x+1/2*c)*a^2*b+6*C*cos(1/2*d*x+1/2*c)*a*b^2-2*C*cos(1/2*d*x+1/2*c)*b^
3)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x
)
```

3.634 $\int (a+b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\,dx$

Optimal. Leaf size=270

$$\frac{\left(a^2(3A-2C)+2b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{b(3A-2C)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}-\frac{a(3A-8C)}{}$$

```
[Out] -(a*(3*A - 8*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((a^2*(3*A - 2*C) + 2*b^2*(
3*A + C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (3*a*A*b*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) - (b*(3*A - 2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (A*
(a + b*Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.932268, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3048, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2(3A-2C)+2b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{b(3A-2C)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}-\frac{a(3A-8C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```


```
[Out] -(a*(3*A - 8*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((a^2*(3*A - 2*C) + 2*b^2*(
3*A + C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (3*a*A*b*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) - (b*(3*A - 2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (A*
(a + b*Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int \sqrt{a+b\cos(c+$$

$$= -\frac{b(3A-2C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{A(a+b\,\mathrm{co}}{}$$

$$= -\frac{b(3A-2C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{A(a+b\,\mathrm{co}}{}$$

$$= -\frac{b(3A-2C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{A(a+b\,\mathrm{co}}{}$$

$$= -\frac{a(3A-8C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b(3A}{}$$

$$= -\frac{a(3A-8C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(a^2(\right.}{}$$

<u>Mathematica</u> [C] time = 3.51614, size = 406, normalized size = 1.5

$$\frac{\frac{8\left(C\left(3a^2+b^2\right)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4\tan(c+dx)\sqrt{a+b\cos(c+dx)}(3aA+2bC\cos(c+dx)) + \frac{2ab(15A+8C)\sqrt{\frac{a+b\cos(}{a+b}}}{\sqrt{a+b\cos}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,
x]
```

```
[Out] ((8*(3*A*b^2 + (3*a^2 + b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*b*(15*A + 8*
C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(3*A - 8*C)*Sqrt[-((b*(-1 + Cos[c +
 d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*
(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
(a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(b*Sqrt[-(a
```

```
 + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(3*a*A + 2*b*C*Cos[c + d*x])*Tan[
c + d*x])/(12*d)
```

Maple [B] time = 0.44, size = 1221, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*A*a*b+8*C*a*b+16*C*b^2)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^2-6*A*a*b-4*C*a*b-4*C*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(3*A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a^2-3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a*b-3*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-6*A*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+9*A*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2))*a*b-8*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^2+8*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+2*C*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-2*C*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2+6*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-9*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+8
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-8*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*a^2*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2)))/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2,
 x)
```

3.635 $\int (a+b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=276

$$\frac{\left(4a^2(A+2C)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{ab(7A+8C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{b(5A-8C)\sqrt{}}{}$$

```
[Out] -(b*(5*A - 8*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (a*b*(7*A + 8*C)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a
 + b*Cos[c + d*x]]) + ((3*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c
 + d*x]]) + (3*A*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (A*(a + b
*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.951371, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3048, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2(A+2C)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{ab(7A+8C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{b(5A-8C)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] -(b*(5*A - 8*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (a*b*(7*A + 8*C)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a
 + b*Cos[c + d*x]]) + ((3*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c
 + d*x]]) + (3*A*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (A*(a + b
*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int$$

$$= \frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{A(a+b\cos(c+$$

$$= \frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{A(a+b\cos(c+$$

$$= \frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{A(a+b\cos(c+$$

$$= -\frac{b(5A-8C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{3A$$

$$= -\frac{b(5A-8C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{ab$$

Mathematica [C] time = 4.75934, size = 411, normalized size = 1.49

$$\frac{\frac{2\left(8a^2(A+2C)+b^2(A+8C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{8ab(A+8C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i(5A-8C)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(}{b}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,
x]
```

```
[Out] ((8*a*b*(A + 8*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*(A + 2*C) + b^2*(A +
8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*A - 8*C)*Sqrt[-((b*(-1 + Cos[
c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2
*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[
```

```
-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c + d*x]]*(2*a + 5*b*Cos[c + d*x])*Sec
[c + d*x]*Tan[c + d*x])/(16*d)
```

Maple [B] time = 0.46, size = 1526, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -1/4*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-40*A*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(28*A*a*b+40*A*b^2)*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+(-4*A*a^2-14*A*a*b-10*A*b^2)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*a^2+3*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2+5
*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-5*A*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-7*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a*b+8*C*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*a^2-8*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+8*C*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-8*C*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b)*sin(1/2*d*x+1/2*c)^4+4*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2+3*A*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))*b^2+5*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b-5*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-7*A*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+8*C*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-8*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a*b+8*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-8*C
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b)*sin(1/2*d*x+1/2*c)^2
-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-3*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-5*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+5*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a*b-8*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(
```

```
a-b))^(1/2))+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*C*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a)/(2*cos(1/2*d*x+1/2*c)^
2-1)^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3,
 x)
```

3.636 $\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=365

$$\frac{\left(8a^2(2A+3C)+3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(8a^2(2A+3C)+b^2(17A+48C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((3*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((8*a^2*
(2*A + 3*C) + b^2*(17*A + 48*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(A*b^2
 - 12*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 + 8*a^
2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) + (A*b*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*
x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.44125, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3048, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(8a^2(2A+3C)+3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(8a^2(2A+3C)+b^2(17A+48C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] -((3*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((8*a^2*
(2*A + 3*C) + b^2*(17*A + 48*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(A*b^2
 - 12*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 + 8*a^
2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) + (A*b*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*
x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int$$

$$=\frac{Ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{4d}+\frac{A(a+b}{}$$

$$=\frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad}$$

$$=\frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad}$$

$$=\frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad}$$

$$=-\frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.5723, size = 607, normalized size = 1.66

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+24a^2C\sin(c+dx)+3Ab^2\sin(c+dx)\right)}{24a}+\frac{1}{3}aA\tan(c+dx)\sec^2(c+dx)+\frac{7}{12}Ab\tan(c+dx)\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,
x]
```

```
[Out] -(b*((2*(-28*a*A*b - 96*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-56*a^2*A + 9*
A*b^2 - 120*a^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(16*a^2*A + 3*A*b^2
 + 24*a^2*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])
/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d
*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^
2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2
))))/(96*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(16*a^2*A*Sin[c +
d*x] + 3*A*b^2*Sin[c + d*x] + 24*a^2*C*Sin[c + d*x]))/(24*a) + (7*A*b*Sec[c
 + d*x]*Tan[c + d*x])/12 + (a*A*Sec[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [B] time = 1.127, size = 2424, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*(A*b^2+C*a^2)*(-1/a*cos(1/2*d*x+1/2*c)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
```

(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2)))+4*a*A*b*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3
/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-4*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))+2*A*a^2*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2
*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/

```
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16
/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(
a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4,
 x)
```

3.637 $\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=436

$$-\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{64a^2d}-\frac{b\left(Ab^2-4a^2(19A+28C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+\cdots}\right.}{64ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (b*(3*A*b^2 - 4*a^2*(13*A + 20*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(64*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (b*
(A*b^2 - 4*a^2*(19*A + 28*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^4
+ 24*a^2*b^2*(A + 2*C) + 16*a^4*(3*A + 4*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a^2*d*Sqrt[a + b*Cos[c
+ d*x]]) - (b*(3*A*b^2 - 4*a^2*(13*A + 20*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[
c + d*x])/(64*a^2*d) + ((A*b^2 + 4*a^2*(3*A + 4*C))*Sqrt[a + b*Cos[c + d*x]
]*Sec[c + d*x]*Tan[c + d*x])/(32*a*d) + (A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c
 + d*x]^2*Tan[c + d*x])/(8*d) + (A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^
3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 1.80648, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3048, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{64a^2d}-\frac{b\left(Ab^2-4a^2(19A+28C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+\cdots}\right.}{64ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (b*(3*A*b^2 - 4*a^2*(13*A + 20*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(64*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (b*
(A*b^2 - 4*a^2*(19*A + 28*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^4
+ 24*a^2*b^2*(A + 2*C) + 16*a^4*(3*A + 4*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a^2*d*Sqrt[a + b*Cos[c
+ d*x]]) - (b*(3*A*b^2 - 4*a^2*(13*A + 20*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[
```

```
c + d*x])/(64*a^2*d) + ((A*b^2 + 4*a^2*(3*A + 4*C))*Sqrt[a + b*Cos[c + d*x]
]*Sec[c + d*x]*Tan[c + d*x])/(32*a*d) + (A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c
 + d*x]^2*Tan[c + d*x])/(8*d) + (A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^
3*Tan[c + d*x])/(4*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int$$

$$= \frac{Ab\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{8d}+\frac{A(a}{}$$

$$= \frac{\left(Ab^2+4a^2(3A+4C)\right)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\,\mathrm{t}}{32ad}$$

$$= -\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\sqrt{a+b\cos(c+dx)}\tan(c}{64a^2d}$$

$$= -\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\sqrt{a+b\cos(c+dx)}\tan(c}{64a^2d}$$

$$= -\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\sqrt{a+b\cos(c+dx)}\tan(c}{64a^2d}$$

$$= -\frac{b\left(A\left(52-\frac{3b^2}{a^2}\right)+80C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{64d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{b\left(A\left(52-\frac{3b^2}{a^2}\right)+80C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{64d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.73891, size = 696, normalized size = 1.6

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec^2(c+dx)\left(12a^2A\sin(c+dx)+16a^2C\sin(c+dx)+Ab^2\sin(c+dx)\right)}{32a}+\frac{\sec(c+dx)\left(52a^2Ab\sin(c+dx)+80a^2bC\sin(c+dx)-3Ab^3\sin(c+\right.}{64a^2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,
x]
```

```
[Out] ((2*(48*a^3*A*b + 4*a*A*b^3 + 64*a^3*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(96*
```

```
a^4*A - 4*a^2*A*b^2 + 9*A*b^4 + 128*a^4*C + 16*a^2*b^2*C)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos
[c + d*x]] - ((2*I)*(-52*a^2*A*b^2 + 3*A*b^4 - 80*a^2*b^2*C)*Sqrt[(b - b*Co
s[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]
*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqr
t[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c +
 d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 -
2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a
*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(256*a^2*d) + (Sqrt[a +
 b*Cos[c + d*x]]*((Sec[c + d*x]^2*(12*a^2*A*Sin[c + d*x] + A*b^2*Sin[c + d*
x] + 16*a^2*C*Sin[c + d*x]))/(32*a) + (Sec[c + d*x]*(52*a^2*A*b*Sin[c + d*x
] - 3*A*b^3*Sin[c + d*x] + 80*a^2*b*C*Sin[c + d*x]))/(64*a^2) + (3*A*b*Sec[
c + d*x]^2*Tan[c + d*x])/8 + (a*A*Sec[c + d*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] time = 1.719, size = 3534, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*C*(-1
/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*(A*b^2+C*a^2)*(-1/2/a*co
s(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1
/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
```

F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,
(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*A*a
^2*(-1/4/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^4+7/24*b/a^2*cos(1/2*d*x+1/2*c)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)^3-1/96*(36*a^2+35*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5
/192*b*(20*a^2+21*b^2)/a^4*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-35/384*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+25/96/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-25/96*b^2/a^2*(sin(1/2*d*x+1/2
c)^2)^(1/2)((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+35/128/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b
^4/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))-3/16/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))*b^4)-2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))+4*a*A*b*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si

```
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d
*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^5,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^5,
 x)
```

3.638 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=523

$$\frac{2\left(24a^2C+11b^2(13A+11C)\right)\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{1287b^3d}-\frac{4a\left(24a^2C+143Ab^2+166b^2C\right)\sin(c+dx)(a+b\cos(c$$

```
[Out] (-2*(240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2
*b^4*(13299*A + 10223*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(45045*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2
 - b^2)*(120*a^4*C + 5*a^2*b^2*(143*A + 94*C) - 3*b^4*(2717*A + 2174*C))*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(45
045*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(120*a^4*C + 5*a^2*b^2*(143*A +
94*C) - 3*b^4*(2717*A + 2174*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(45
045*b^3*d) - (2*(240*a^4*C - 539*b^4*(13*A + 11*C) + 10*a^2*b^2*(143*A + 12
4*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45045*b^3*d) - (4*a*(143*A*
b^2 + 24*a^2*C + 166*b^2*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9009*
b^3*d) + (2*(24*a^2*C + 11*b^2*(13*A + 11*C))*(a + b*Cos[c + d*x])^(7/2)*Si
n[c + d*x])/(1287*b^3*d) - (12*a*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*
Sin[c + d*x])/(143*b^2*d) + (2*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(7/2)*
Sin[c + d*x])/(13*b*d)
```

Rubi [A] time = 1.27953, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3050, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(24a^2C+11b^2(13A+11C)\right)\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{1287b^3d}-\frac{4a\left(24a^2C+143Ab^2+166b^2C\right)\sin(c+dx)(a+b\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (-2*(240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2
*b^4*(13299*A + 10223*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(45045*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2
 - b^2)*(120*a^4*C + 5*a^2*b^2*(143*A + 94*C) - 3*b^4*(2717*A + 2174*C))*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(45
045*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(120*a^4*C + 5*a^2*b^2*(143*A +
```

```
94*C) - 3*b^4*(2717*A + 2174*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(45
045*b^3*d) - (2*(240*a^4*C - 539*b^4*(13*A + 11*C) + 10*a^2*b^2*(143*A + 12
4*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45045*b^3*d) - (4*a*(143*A*
b^2 + 24*a^2*C + 166*b^2*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9009*
b^3*d) + (2*(24*a^2*C + 11*b^2*(13*A + 11*C))*(a + b*Cos[c + d*x])^(7/2)*Si
n[c + d*x])/(1287*b^3*d) - (12*a*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*
Sin[c + d*x])/(143*b^2*d) + (2*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(7/2)*
Sin[c + d*x])/(13*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{13bd} + \frac{2\int}{}$$

$$= -\frac{12aC\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{143b^2d} +$$

$$= \frac{2\left(24a^2C+11b^2(13A+11C)\right)(a+b\cos(c+dx))^{7/2}\sin}{1287b^3d}$$

$$= -\frac{4a\left(143Ab^2+24a^2C+166b^2C\right)(a+b\cos(c+dx))^{5/2}}{9009b^3d}$$

$$= -\frac{2\left(240a^4C-539b^4(13A+11C)+10a^2b^2(143A+124\right.}{45045b^3d}$$

$$= -\frac{4a\left(120a^4C+5a^2b^2(143A+94C)-3b^4(2717A+21\right.}{45045b^3d}$$

$$= -\frac{4a\left(120a^4C+5a^2b^2(143A+94C)-3b^4(2717A+21\right.}{45045b^3d}$$

$$= -\frac{4a\left(120a^4C+5a^2b^2(143A+94C)-3b^4(2717A+21\right.}{45045b^3d}$$

$$= -\frac{2\left(240a^6C-1617b^6(13A+11C)+10a^4b^2(143A+76\right.}{450}$$

Mathematica [A] time = 2.58044, size = 395, normalized size = 0.76

$$\frac{b(a+b\cos(c+dx))\left(4a\left(10a^2b^2(572A+331C)+960a^4C+3b^4(71214A+60793C)\right)\sin(c+dx)+b\left(\left(120a^2b^2(1430A\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (32*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(a*b^2*(-60*a^4*C + 5*a^2*b^2*(4433*
A + 3337*C) + 3*b^4*(12441*A + 10277*C))*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] - (240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a
```

```
^2*b^4*(13299*A + 10223*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
- a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*(4*a*(
960*a^4*C + 10*a^2*b^2*(572*A + 331*C) + 3*b^4*(71214*A + 60793*C))*Sin[c +
 d*x] + b*((-1440*a^4*C + 120*a^2*b^2*(1430*A + 1457*C) + 77*b^4*(1976*A +
1897*C))*Sin[2*(c + d*x)] + 5*b*(2*a*(10868*A*b^2 + 60*a^2*C + 13939*b^2*C)
*Sin[3*(c + d*x)] + 7*b*((572*A*b^2 + 636*a^2*C + 880*b^2*C)*Sin[4*(c + d*x
)] + 9*b*C*(54*a*Sin[5*(c + d*x)] + 11*b*Sin[6*(c + d*x)])))))))/(720720*b^4
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.424, size = 2223, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1773
2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+1430*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b^2+17787*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^6-30669*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^5+30669*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^4+760*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^3-760*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b^2+240*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^6*b-13984*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^4+13044*C*a*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+700*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^5*b^2+21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a*b^6-39897*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
```

```
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^
5+39897*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^4+14
30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^3-1430*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b^2-443520*C*b^7
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+16302*a*A*b^6*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-240*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^7-21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*b^7-17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*b^7+240*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^7+
(766080*C*a*b^6+1330560*C*b^7)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-1
60160*A*b^7-450240*C*a^2*b^5-1915200*C*a*b^6-1798720*C*b^7)*sin(1/2*d*x+1/2
*c)^10*cos(1/2*d*x+1/2*c)+(297440*A*a*b^6+320320*A*b^7+90240*C*a^3*b^4+9004
80*C*a^2*b^5+2159680*C*a*b^6+1379840*C*b^7)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*
x+1/2*c)+(-194480*A*a^2*b^5-446160*A*a*b^6-296296*A*b^7+120*C*a^4*b^3-13536
0*C*a^3*b^4-828880*C*a^2*b^5-1324320*C*a*b^6-666512*C*b^7)*sin(1/2*d*x+1/2*
c)^6*cos(1/2*d*x+1/2*c)+(45760*A*a^3*b^4+194480*A*a^2*b^5+344344*A*a*b^6+13
6136*A*b^7+120*C*a^5*b^2-120*C*a^4*b^3+101840*C*a^3*b^4+378640*C*a^2*b^5+52
2368*C*a*b^6+198352*C*b^7)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1430*A
*a^4*b^3-22880*A*a^3*b^4-95238*A*a^2*b^5-97812*A*a*b^6-24024*A*b^7-240*C*a^
6*b-60*C*a^5*b^2-760*C*a^4*b^3-28360*C*a^3*b^4-104466*C*a^2*b^5-104304*C*a*
b^6-27258*C*b^7)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/b^4/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(
1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
```

```
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^6+2\,Cab\cos(dx+c)^5+2\,Aab\cos(dx+c)^3+Aa^2\cos(dx+c)^2+\left(Ca^2+Ab^2\right)\cos(dx+c)^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^6 + 2*C*a*b*cos(d*x + c)^5 + 2*A*a*b*cos(d*x +
 c)^3 + A*a^2*cos(d*x + c)^2 + (C*a^2 + A*b^2)*cos(d*x + c)^4)*sqrt(b*cos(d
*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
 x)
```

3.639 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=435

$$\frac{2\left(8a^2C+9b^2(11A+9C)\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2C+99Ab^2+67b^2C\right)\sin(c+dx)(a+b\cos(c+dx}{693b^2d}$$

```
[Out] (2*a*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*Sqrt[a + b
*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(8*a^4*C + 15*b^4*(11*A + 9*C) +
 3*a^2*b^2*(33*A + 19*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^4*C
 + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A + 19*C))*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(693*b^2*d) + (2*a*(99*A*b^2 + 8*a^2*C + 67*b^2*C)*(a + b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(693*b^2*d) + (2*(8*a^2*C + 9*b^2*(11*A + 9*
C))*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) - (8*a*C*(a + b*Co
s[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c
 + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 0.879467, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3050, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2C+9b^2(11A+9C)\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2C+99Ab^2+67b^2C\right)\sin(c+dx)(a+b\cos(c+dx}{693b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*Sqrt[a + b
*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(8*a^4*C + 15*b^4*(11*A + 9*C) +
 3*a^2*b^2*(33*A + 19*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^4*C
 + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A + 19*C))*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(693*b^2*d) + (2*a*(99*A*b^2 + 8*a^2*C + 67*b^2*C)*(a + b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(693*b^2*d) + (2*(8*a^2*C + 9*b^2*(11*A + 9*
C))*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) - (8*a*C*(a + b*Co
```

```
s[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c
 + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd} + \frac{2\int(a\cdots}{\cdots}$$

$$= -\frac{8aC(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d} + \frac{2C\cos(c+dx)(\cdots}{\cdots}$$

$$= \frac{2\left(8a^2C+9b^2(11A+9C)\right)(a+b\cos(c+dx))^{5/2}\sin(c+\cdots}{693b^2d}$$

$$= \frac{2a\left(99Ab^2+8a^2C+67b^2C\right)(a+b\cos(c+dx))^{3/2}\sin(c+\cdots}{693b^2d}$$

$$= \frac{2\left(8a^4C+15b^4(11A+9C)+3a^2b^2(33A+19C)\right)\sqrt{a+b\cdots}}{693b^2d}$$

$$= \frac{2\left(8a^4C+15b^4(11A+9C)+3a^2b^2(33A+19C)\right)\sqrt{a+b\cdots}}{693b^2d}$$

$$= \frac{2\left(8a^4C+15b^4(11A+9C)+3a^2b^2(33A+19C)\right)\sqrt{a+b\cdots}}{693b^2d}$$

$$= \frac{2a\left(8a^4C+3a^2b^2(33A+17C)+3b^4(319A+247C)\right)\sqrt{a\cdots}}{693b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.64698, size = 328, normalized size = 0.75

$$\frac{b(a+b\cos(c+dx))\left(\left(12a^2b^2(396A+311C)-64a^4C+6b^4(506A+435C)\right)\sin(c+dx)+b\left(4a\left(6a^2C+594Ab^2+619b^2\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(2*a^4*b*C + 15*b^5*(11*A + 9*C)
+ 3*a^2*b^3*(297*A + 221*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*(8*a
^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b
*(a + b*Cos[c + d*x])*((-64*a^4*C + 12*a^2*b^2*(396*A + 311*C) + 6*b^4*(506
*A + 435*C))*Sin[c + d*x] + b*(4*a*(594*A*b^2 + 6*a^2*C + 619*b^2*C)*Sin[2*
```

```
(c + d*x)] + b*((396*A*b^2 + 452*a^2*C + 513*b^2*C)*Sin[3*(c + d*x)] + 7*b*
C*(46*a*Sin[4*(c + d*x)] + 9*b*Sin[5*(c + d*x)])))))/(5544*b^3*d*Sqrt[a + b
*Cos[c + d*x]])
```

Maple [B] time = 0.434, size = 1791, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/693*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4032*C*
b^6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-7168*C*a*b^5-10080*C*b^6)*si
n(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(1584*A*b^6+4384*C*a^2*b^4+14336*C*a
*b^5+11376*C*b^6)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-3168*A*a*b^5-23
76*A*b^6-928*C*a^3*b^3-6576*C*a^2*b^4-13232*C*a*b^5-6984*C*b^6)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(2376*A*a^2*b^4+3168*A*a*b^5+1848*A*b^6-4*C*a^
4*b^2+928*C*a^3*b^3+5024*C*a^2*b^4+6064*C*a*b^5+2772*C*b^6)*sin(1/2*d*x+1/2
*c)^4*cos(1/2*d*x+1/2*c)+(-594*A*a^3*b^3-1188*A*a^2*b^4-1122*A*a*b^5-528*A*
b^6+8*C*a^5*b+2*C*a^4*b^2-642*C*a^3*b^3-1416*C*a^2*b^4-1338*C*a*b^5-558*C*b
^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-99*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-66*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^4+165*A*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))+99*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^4*b^2-99*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
3*b^3+957*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-
957*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-8*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-49*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-78*a^2*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+135*b^6*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*
```

```
d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^6-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^5*b+51*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^
2-51*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+741*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-741*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5)/b^3/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^2\cos\left(dx+c\right)^5+2\,Cab\cos\left(dx+c\right)^4+2\,Aab\cos\left(dx+c\right)^2+Aa^2\cos\left(dx+c\right)+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^3\right)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5 + 2*C*a*b*cos(d*x + c)^4 + 2*A*a*b*cos(d*x +
 c)^2 + A*a^2*cos(d*x + c) + (C*a^2 + A*b^2)*cos(d*x + c)^3)*sqrt(b*cos(d*x
```

```
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x
)
```

3.640 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=350

$$-\frac{2\left(10a^2C-7b^2(9A+7C)\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{4a\left(-5a^2C+84Ab^2+57b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+}}{315bd}$$

```
[Out] (-2*(10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*(84*A*b^2 - 5*a^2*C + 57*b^2*C)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(
315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (4*a*(84*A*b^2 - 5*a^2*C + 57*b^2*C)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) - (2*(10*a^2*C - 7*b^2*(9*
A + 7*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) - (4*a*C*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*C*(a + b*Cos[c + d*x])^(7
/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.648788, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3024, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(10a^2C-7b^2(9A+7C)\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{4a\left(-5a^2C+84Ab^2+57b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+}}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (-2*(10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*(84*A*b^2 - 5*a^2*C + 57*b^2*C)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(
315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (4*a*(84*A*b^2 - 5*a^2*C + 57*b^2*C)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) - (2*(10*a^2*C - 7*b^2*(9*
A + 7*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) - (4*a*C*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*C*(a + b*Cos[c + d*x])^(7
/2)*Sin[c + d*x])/(9*b*d)
```


Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int (a+b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9\right.}{9}$$

$$= -\frac{4aC(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd} + \frac{2C(a+b\cos(c+dx))^{7/2}\sin}{9bd}$$

$$= -\frac{2\left(10a^2C-7b^2(9A+7C)\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315bd} - \frac{4aC(}{}$$

$$= \frac{4a\left(84Ab^2-5a^2C+57b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd} - \frac{2\left(10a^2\right.}{}$$

$$= \frac{4a\left(84Ab^2-5a^2C+57b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd} - \frac{2\left(10a^2\right.}{}$$

$$= \frac{4a\left(84Ab^2-5a^2C+57b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd} - \frac{2\left(10a^2\right.}{}$$

$$= -\frac{2\left(10a^4C-21b^4(9A+7C)-3a^2b^2(161A+93C)\right)\sqrt{a+b\cos(c+d}}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.3186, size = 274, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(2a\left(20a^2C+924Ab^2+747b^2C\right)\sin(c+dx)+b\left(\left(300a^2C+252Ab^2+266b^2C\right)\sin(2(c+dx))+5b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(a*b^2*(5*a^2*(63*A + 31*C) + 3*b^2*(
119*A + 87*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-10*a^4*C + 21*b^4*
(9*A + 7*C) + 3*a^2*b^2*(161*A + 93*C))*((a + b)*EllipticE[(c + d*x)/2, (2*
```

```
b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c +
d*x])*(2*a*(924*A*b^2 + 20*a^2*C + 747*b^2*C)*Sin[c + d*x] + b*((252*A*b^2
+ 300*a^2*C + 266*b^2*C)*Sin[2*(c + d*x)] + 5*b*C*(38*a*Sin[3*(c + d*x)] +
7*b*Sin[4*(c + d*x)]))))/(1260*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.492, size = 1527, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2080*C*a*b^4+2240*C*b^5)*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^5-1360*C*a^2*b^3-3120*C*a*b^
4-2072*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1176*A*a*b^4+504*A*b
^5+320*C*a^3*b^2+1360*C*a^2*b^3+2408*C*a*b^4+952*C*b^5)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-462*A*a^2*b^3-588*A*a*b^4-126*A*b^5-10*C*a^4*b-160*C
*a^3*b^2-666*C*a^2*b^3-684*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)-168*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b^2+168*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+483
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-483*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-189*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5+10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3*b^2+114*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*b^4-10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^5+10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+279
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
```

```
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-279*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + A\right)(b\cos{(dx+c)} + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

3.641 $\int (a+b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx$

Optimal. Leaf size=342

$$\frac{2\left(3a^2C + b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d} + \frac{2\left(2a^2b^2(7A-C) - 3a^4C + b^4(7A+5C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{21bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(21*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(2*a^2*b^2*(7*A - C) - 3*a^4*C + b^4*(7*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(21*b*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*a^3*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*(3*a^2*C + b^2*(7*A
 + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*C*(a + b*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(7*d)
```

Rubi [A] time = 1.22306, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3050, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(3a^2C + b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d} + \frac{2\left(2a^2b^2(7A-C) - 3a^4C + b^4(7A+5C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{21bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```


```
[Out] (2*a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(21*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(2*a^2*b^2*(7*A - C) - 3*a^4*C + b^4*(7*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(21*b*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*a^3*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*(3*a^2*C + b^2*(7*A
 + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*C*(a + b*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(7*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int (a+b\cos(c$$

$$= \frac{2aC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2C(a+b\cos(c}{}$$

$$= \frac{2\left(3a^2C+b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}$$

$$= \frac{2\left(3a^2C+b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}$$

$$= \frac{2\left(3a^2C+b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}$$

$$= \frac{2a\left(49Ab^2+3a^2C+29b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2a\left(49Ab^2+3a^2C+29b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.08505, size = 468, normalized size = 1.37

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}\left(18a^2C+18abC\cos(c+dx)+14Ab^2+3b^2C\cos(2(c+dx))+13b^2C\right)+\frac{4b\left(9a^2(7A+3C)+b\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] ((4*b*(9*a^2*(7*A + 3*C) + b^2*(7*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*
(3*a^2*(14*A + C) + b^2*(49*A + 29*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)
*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
```

```
(a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sq
rt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(b^2*Sqrt[-(a + b)^(-1)]) + 2*
Sqrt[a + b*Cos[c + d*x]]*(14*A*b^2 + 18*a^2*C + 13*b^2*C + 18*a*b*C*Cos[c +
 d*x] + 3*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(42*d)
```

Maple [B] time = 0.456, size = 1209, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*C*b^4
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-96*C*a*b^3-72*C*b^4)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A*b^4+72*C*a^2*b^2+96*C*a*b^3+56*C*b^4)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A*a*b^3-14*A*b^4-18*C*a^3*b-36*C
*a^2*b^2-34*C*a*b^3-16*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+49*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-49*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-21*A*a^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b+14*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+7*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^4-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b+29*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-
29*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-3*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-2*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+5*C*b^4*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(
```

```
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos \left(dx + c\right)^{2} + A\right) \left(b \cos \left(dx + c\right) + a\right)^{\frac{5}{2}} \sec \left(dx + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x
)
```

3.642 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=327

$$\frac{a\left(a^2(15A-16C)+4b^2(15A+4C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2(15A-46C)-6b^2(5A+3C)\right)\sqrt{a+b\cos(c+}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((a^2*(15*A - 46*C) - 6*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(a*(a^2*(15*A - 16*C) + 4*b^2*(15*A + 4*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]])
+ (5*a^2*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (a*b*(15*A - 16*C)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (b*(5*A - 2*C)*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(5*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Tan[c + d*x])/d
```

Rubi [A] time = 1.27416, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3048, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{a\left(a^2(15A-16C)+4b^2(15A+4C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2(15A-46C)-6b^2(5A+3C)\right)\sqrt{a+b\cos(c+}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```



```
[Out] -((a^2*(15*A - 46*C) - 6*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(a*(a^2*(15*A - 16*C) + 4*b^2*(15*A + 4*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]])
+ (5*a^2*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (a*b*(15*A - 16*C)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (b*(5*A - 2*C)*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(5*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Tan[c + d*x])/d
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\tan(c+dx)}{d} + \int (a+b\cos(c+d$$

$$= -\frac{b(5A-2C)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{A(a+b}{}$$

$$= -\frac{ab(15A-16C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} - \frac{b(5A-}{}$$

$$= -\frac{ab(15A-16C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} - \frac{b(5A-}{}$$

$$= -\frac{ab(15A-16C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} - \frac{b(5A-}{}$$

$$= -\frac{\left(a^2(15A-46C)-6b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(a^2(15A-46C)-6b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.54963, size = 462, normalized size = 1.41

$$\frac{\frac{8a\left(15a^2C+45Ab^2+17b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2b\left(a^2(135A+46C)+6b^2(5A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4\sqrt{a+b\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,
x]
```

```
[Out] ((8*a*(45*A*b^2 + 15*a^2*C + 17*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(6*b^
2*(5*A + 3*C) + a^2*(135*A + 46*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(6
*b^2*(5*A + 3*C) + a^2*(-15*A + 46*C))*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a +
b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*Ell
ipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
```

```
- b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1
)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(22*a*b*C*Sin[c + d*x] + 3*b^2*C*Sin[2*(c
+ d*x)] + 15*a^2*A*Tan[c + d*x]))/(60*d)
```

Maple [B] time = 0.466, size = 1714, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-224*C*a*b^2-144*C*b^3)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(60*A*a^2*b+88*C*a^2*b+224*C*a*b^2+72*C*b^3)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-30*A*a^3-30*A*a^2*b-44*C*a^2*b-5
6*C*a*b^2-12*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+60*A*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-15*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^3+15*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b+30*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-30*A*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-75*A*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b-16*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^3+16*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*
b^2+46*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-46*C*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+18*C*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^2-18*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^2+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^3+60*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+30*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-30*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-75*A*a^2*b*(sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-16*a^3*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+16*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+46*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^3-46*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^2*b+18*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-18*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2,
 x)
```

3.643 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=329

$$\frac{b\left(a^2(33A+16C)+8b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12d\sqrt{a+b\cos(c+dx)}}+\frac{a\left(4a^2(A+2C)+15Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -(a*b*(27*A - 56*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(12*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(8*b^2*(3*A + C) +
a^2*(33*A + 16*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(12*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(15*A*b^2 + 4*a^2*(A
 + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) - (b^2*(21*A - 8*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(12*d) + (5*A*b*(a + b*Cos[c + d*x])^(3/2)*Tan[c
 + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(
2*d)
```

Rubi [A] time = 1.27282, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3048, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(a^2(33A+16C)+8b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12d\sqrt{a+b\cos(c+dx)}}+\frac{a\left(4a^2(A+2C)+15Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```


```
[Out] -(a*b*(27*A - 56*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(12*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(8*b^2*(3*A + C) +
a^2*(33*A + 16*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(12*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(15*A*b^2 + 4*a^2*(A
 + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) - (b^2*(21*A - 8*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(12*d) + (5*A*b*(a + b*Cos[c + d*x])^(3/2)*Tan[c
 + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(
2*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int($$

$$= \frac{5Ab(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{4d}+\frac{A(a+b\cos(c$$

$$= -\frac{b^2(21A-8C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}+\frac{5Ab(a$$

$$= -\frac{b^2(21A-8C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}+\frac{5Ab(a$$

$$= -\frac{b^2(21A-8C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}+\frac{5Ab(a$$

$$= -\frac{ab(27A-56C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-$$

$$= -\frac{ab(27A-56C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+$$

Mathematica [C] time = 4.14907, size = 445, normalized size = 1.35

$$\frac{\frac{8b\left(3a^2(A+12C)+4b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(24a^2(A+2C)+7b^2(9A+8C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec(c+dx)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,
x]
```

```
[Out] ((8*b*(4*b^2*(3*A + C) + 3*a^2*(A + 12*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*(
24*a^2*(A + 2*C) + 7*b^2*(9*A + 8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*
(27*A - 56*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c +
 d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a +
 b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)])))/Sqrt[-(a + b)^(-1)] + 4*Sqrt[a + b*Cos[c + d*x]]*
Sec[c + d*x]*(27*a*A*b*Sin[c + d*x] + 4*b^2*C*Sin[2*(c + d*x)] + 6*a^2*A*Ta
n[c + d*x]))/(48*d)
```

Maple [B] time = 0.517, size = 1897, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -1/12*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-216*A*a*b^2-64*C*a*b^2-192*C*b^
3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(132*A*a^2*b+216*A*a*b^2+64*C*a*
b^2+96*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-12*A*a^3-66*A*a^2*b
-54*A*a*b^2-16*C*a*b^2-16*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-4*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*(12*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3+45*A*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2-33*A*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-24*A*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^3+27*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^2*b-27*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+24*C*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-16*C*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-8*C*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*b^3-56*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^2*b+56*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2)*sin(1/2
*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*(12*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*a^3+45*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2-33*
```

```
A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-24*A*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+27*A*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b-27*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a*b^2+24*C*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-16*C
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-8*C*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-56*C*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+56*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b^2)*sin(1/2*d*x+1/2*c)^2-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))*a^3-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*a*b^2+33*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^2*b+24*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-27*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+27*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-24*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3+16*a^2*b*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))+56*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^2*b-56*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a*b^2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + 2\,Cab\cos(dx+c)^3 + 2\,Aab\cos(dx+c) + Aa^2 + \left(Ca^2 + Ab^2\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3,
 x)
```

3.644 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=363

$$\frac{\left(8a^2(2A+3C)+15Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{a\left(8a^2(2A+3C)+b^2(59A+96C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+d\right.}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ (a*(8*a^2*(2*A + 3*C) + b^2*(59*A + 96*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]])
 + (5*b*(A*b^2 + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*
A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d) +
(5*A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (A*(a
 + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.41634, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3048, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(8a^2(2A+3C)+15Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{a\left(8a^2(2A+3C)+b^2(59A+96C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+d\right.}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```


```
[Out] -((3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ (a*(8*a^2*(2*A + 3*C) + b^2*(59*A + 96*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]])
 + (5*b*(A*b^2 + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*
A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d) +
(5*A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (A*(a
 + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (\ldots$$

$$= \frac{5Ab(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{12d}+\frac{A(a\ldots}{\ldots}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)\ldots}{24d}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)\ldots}{24d}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)\ldots}{24d}$$

$$= -\frac{\left(3b^2(11A-16C)+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\ldots}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(3b^2(11A-16C)+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\ldots}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.9673, size = 477, normalized size = 1.31

$$\frac{\frac{2b\left(8a^2(13A+27C)-3b^2(A-16C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(\left(4a^2(2A+3C)+\frac{33Ab^2}{2}\right)\sin(2(c\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,
x]
```

```
[Out] ((8*a*b^2*(13*A + 72*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(-3*b^2*(A - 16*C) +
 8*a^2*(13*A + 27*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(3*b^2*(11*A - 1
6*C) + 8*a^2*(2*A + 3*C))*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((
b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
```

```
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(26*a*A*b*Sin[c + d*x] + ((33*A*b^2)/2 +
 4*a^2*(2*A + 3*C))*Sin[2*(c + d*x)] + 8*a^2*A*Tan[c + d*x]))/(96*d)
```

Maple [B] time = 1.378, size = 2673, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C*b^2*(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))+6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*C*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+2*a*(3*A*b^2+C*a^2)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2)))+6*A*a^2*b*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3
/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
```

```
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2*b*(A*b^2+3*C*a^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(
-2*b/(a-b))^(1/2))+2*A*a^3*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b
/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x
+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4,
 x)
```

3.645 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=437

$$\frac{b\left(4a^2(71A+108C)+15Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{b\left(4a^2(89A+132C)+b^2(133A+384C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{192d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -(b*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (
b*(4*a^2*(89*A + 132*C) + b^2*(133*A + 384*C))*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sqrt[a + b*Cos[c + d*x
]]) - ((5*A*b^4 - 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a*d*Sq
rt[a + b*Cos[c + d*x]]) + (b*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Sqrt[a + b*C
os[c + d*x]]*Tan[c + d*x])/(192*a*d) + ((5*A*b^2 + 4*a^2*(3*A + 4*C))*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(32*d) + (5*A*b*(a + b*Cos[c
 + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (A*(a + b*Cos[c + d*x]
)^(5/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 1.87455, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3048, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(4a^2(71A+108C)+15Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{b\left(4a^2(89A+132C)+b^2(133A+384C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{192d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] -(b*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (
b*(4*a^2*(89*A + 132*C) + b^2*(133*A + 384*C))*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sqrt[a + b*Cos[c + d*x
]]) - ((5*A*b^4 - 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a*d*Sq
rt[a + b*Cos[c + d*x]]) + (b*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Sqrt[a + b*C
```

```
os[c + d*x]]*Tan[c + d*x])/(192*a*d) + ((5*A*b^2 + 4*a^2*(3*A + 4*C))*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(32*d) + (5*A*b*(a + b*Cos[c
 + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d) + (A*(a + b*Cos[c + d*x]
)^(5/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int$$

$$=\frac{5Ab(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{24d}+\frac{A}{}$$

$$=\frac{\left(5Ab^2+4a^2(3A+4C)\right)\sqrt{a+b\cos(c+dx)}\sec(c+dx)}{32d}$$

$$=\frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\cos(c+dx)}\tan(c}{192ad}$$

$$=\frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\cos(c+dx)}\tan(c}{192ad}$$

$$=\frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\cos(c+dx)}\tan(c}{192ad}$$

$$=-\frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.82679, size = 704, normalized size = 1.61

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{96}\sec^2(c+dx)\left(36a^2A\sin(c+dx)+48a^2C\sin(c+dx)+59Ab^2\sin(c+dx)\right)+\frac{\sec(c+dx)\left(284a^2Ab\sin(\right.}{}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,
x]
```

```
[Out] ((2*(144*a^3*A*b + 236*a*A*b^3 + 192*a^3*b*C + 768*a*b^3*C)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] + (2*(288*a^4*A + 436*a^2*A*b^2 - 45*A*b^4 + 384*a^4*C + 1008*a^2*
```

```
b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-284*a^2*A*b^2 - 15*A*b^4 - 43
2*a^2*b^2*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])
/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d
*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^
2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2
)))/(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(36*a^2*A*Sin[c
+ d*x] + 59*A*b^2*Sin[c + d*x] + 48*a^2*C*Sin[c + d*x]))/96 + (Sec[c + d*x]
*(284*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 432*a^2*b*C*Sin[c + d*
x]))/(192*a) + (17*a*A*b*Sec[c + d*x]^2*Tan[c + d*x])/24 + (a^2*A*Sec[c + d
*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] time = 1.708, size = 3651, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*b*(A*b^2+3*C*a^2)*(-1/a*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2)))+2*A*a^3*(-1/4/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
^4+7/24*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
```

*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3-1/96*(36*a^2+35*b^2)/a^3*co
s(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b*(20*a^2+21*b^2)/a^4*cos(1/2*d*x+1/2
c)(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-35/384*b^3/a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-25/96*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35/128/a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3
/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/16/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^4)+2*a*(3*
A*b^2+C*a^2)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*
x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(
a-b))^(1/2))*b^2)-6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/

```
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+6*A*a^2*b*
(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b
^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/
16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*si
n(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5,
 x)
```

3.646 $\int (a+b\cos(c+dx))^{3/2}\left(a^2-b^2\cos^2(c+dx)\right)dx$

Optimal. Leaf size=246

$$\frac{2b\left(41a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-66a^2b^2+41a^4+25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{a+b\cos(c+dx)}}+\frac{4a\left(73a^2\right.}{}$$

```
[Out] (4*a*(73*a^2 - 41*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(41*a^4 - 66*a^
2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/(105*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(41*a^2 - 25*b^2)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (4*a*b*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(35*d) - (2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/
(7*d)
```

Rubi [A] time = 0.45753, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3016, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(41a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-66a^2b^2+41a^4+25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{a+b\cos(c+dx)}}+\frac{4a\left(73a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(a^2 - b^2*Cos[c + d*x]^2),x]
```



```
[Out] (4*a*(73*a^2 - 41*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(41*a^4 - 66*a^
2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/(105*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(41*a^2 - 25*b^2)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (4*a*b*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(35*d) - (2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/
(7*d)
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
```

```
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(a^2-b^2\cos^2(c+dx)\right)dx = -\int(-a+b\cos(c+dx))(a+b\cos(c+dx))^{5/2}dx$$
$$= -\frac{2b(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} - \frac{2}{7}\int(a+b\cos(c+dx))^{3/2}\Big(\frac{1}{2}$$
$$= \frac{4ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} - \frac{2b(a+b\cos(c+dx))^{5/2}\sin(c}{7d}$$
$$= \frac{2b\left(41a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{4ab(a+b\cos(c+}{3}$$
$$= \frac{2b\left(41a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{4ab(a+b\cos(c+}{3}$$
$$= \frac{2b\left(41a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{4ab(a+b\cos(c+}{3}$$
$$= \frac{4a\left(73a^2-41b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(41a^4-66\right.}{}$$

Mathematica [A] time = 1.16437, size = 212, normalized size = 0.86

$$\frac{-b\sin(c+dx)\left(\left(145b^3-32a^2b\right)\cos(c+dx)-128a^3+78ab^2\cos(2(c+dx))+178ab^2+15b^3\cos(3(c+dx))\right)-4\left(-66a^2b\right.}{210d\sqrt{a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(a^2 - b^2*Cos[c + d*x]^2),x]
```

```
[Out] (8*a*(73*a^3 + 73*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(41*a^4 - 66*a^2*b^2 + 25*b^
4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
 - b*(-128*a^3 + 178*a*b^2 + (-32*a^2*b + 145*b^3)*Cos[c + d*x] + 78*a*b^2*
Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*d*Sqrt[a + b
*Cos[c + d*x]])
```

Maple [B] time = 0.437, size = 824, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(a^2-b^2*cos(d*x+c)^2),x)
```

```
[Out] 2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(
1/2*d*x+1/2*c)^9*b^4+312*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)^
7*b^4-32*cos(1/2*d*x+1/2*c)^5*a^2*b^2-624*cos(1/2*d*x+1/2*c)^5*a*b^3+640*co
s(1/2*d*x+1/2*c)^5*b^4-64*cos(1/2*d*x+1/2*c)^3*a^3*b+48*cos(1/2*d*x+1/2*c)^
3*a^2*b^2+440*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4+41*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-66*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b^4-146*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+146*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+82*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-82*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a*b^3+64*cos(1/2*d*x+1/2*c)*a^3*b-16*cos(1/2*d*x+1/2*c)*a^2*b^
2-128*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(b^2 \cos\left(dx+c\right)^2 - a^2\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(a^2-b^2*cos(d*x+c)^2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*cos(d*x + c)^2 - a^2)*(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^3\cos\left(dx+c\right)^3+ab^2\cos\left(dx+c\right)^2-a^2b\cos\left(dx+c\right)-a^3\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(a^2-b^2*cos(d*x+c)^2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^3*cos(d*x + c)^3 + a*b^2*cos(d*x + c)^2 - a^2*b*cos(d*x + c) -
 a^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(a**2-b**2*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(a^2-b^2*cos(d*x+c)^2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.647 $\int \sqrt{a+b\cos(c+dx)}\left(a^2-b^2\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=197

$$-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(17a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)(a}{5}$$

```
[Out] (2*(17*a^2 - 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*
Sqrt[a + b*Cos[c + d*x]]) + (4*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
15*d) - (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.337622, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3016, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(17a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)(a}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(a^2 - b^2*Cos[c + d*x]^2),x]
```


```
[Out] (2*(17*a^2 - 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*
Sqrt[a + b*Cos[c + d*x]]) + (4*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
15*d) - (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(a^2-b^2\cos^2(c+dx)\right)dx = -\int(-a+b\cos(c+dx))(a+b\cos(c+dx))^{3/2}dx$$

$$= -\frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} - \frac{2}{5}\int\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}\right.$$

$$= \frac{4ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} - \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{4ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} - \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{4ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} - \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{2\left(17a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{4a\left(a^2-b^2\right)\sqrt{}}{15d}$$

Mathematica [A] time = 0.854269, size = 178, normalized size = 0.9

$$\frac{-b\sin(c+dx)\left(2a^2+8ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)-4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(17}{15d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(a^2 - b^2*Cos[c + d*x]^2),x]
```

```
[Out] (2*(17*a^3 + 17*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - b*(2*a^2 + 3*b^2 +
 8*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*d*Sqrt[a +
b*Cos[c + d*x]])
```

Maple [B] time = 0.395, size = 662, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/
2*d*x+1/2*c)^7*b^3+16*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b^
3+2*cos(1/2*d*x+1/2*c)^3*a^2*b-24*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d*x
+1/2*c)^3*b^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-17*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*cos(1/2*d*x+1/2*c)*a^2*b+8*
cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2
*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(b^2 \cos\left(dx+c\right)^2 - a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*cos(d*x + c)^2 - a^2)*sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2 \cos\left(dx+c\right)^2 - a^2\right)\sqrt{b\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - a^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(b^2 \cos\left(dx+c\right)^2 - a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*cos(d*x + c)^2 - a^2)*sqrt(b*cos(d*x + c) + a), x)
```

3.648 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=378

$$\frac{2\left(48a^2C+7b^2(9A+7C)\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^3d}-\frac{4a\left(32a^2C+42Ab^2+31b^2C\right)\sin(c+dx)\sqrt{a+b}}{315b^4d}$$

```
[Out] (2*(128*a^4*C + 21*b^4*(9*A + 7*C) + 12*a^2*b^2*(14*A + 9*C))*Sqrt[a + b*Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^5*d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) - (2*a*(128*a^4*C + 4*a^2*b^2*(42*A + 19*C) + 3*b^4*
(49*A + 37*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(315*b^5*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(42*A*b^2 + 32*a^
2*C + 31*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^4*d) + (2*(48
*a^2*C + 7*b^2*(9*A + 7*C))*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*b^3*d) - (16*a*C*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(63*b^2*d) + (2*C*Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(9*b*d)
```

Rubi [A] time = 0.926682, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3050, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(48a^2C+7b^2(9A+7C)\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^3d}-\frac{4a\left(32a^2C+42Ab^2+31b^2C\right)\sin(c+dx)\sqrt{a+b}}{315b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*(128*a^4*C + 21*b^4*(9*A + 7*C) + 12*a^2*b^2*(14*A + 9*C))*Sqrt[a + b*Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^5*d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) - (2*a*(128*a^4*C + 4*a^2*b^2*(42*A + 19*C) + 3*b^4*
(49*A + 37*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(315*b^5*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(42*A*b^2 + 32*a^
2*C + 31*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^4*d) + (2*(48
*a^2*C + 7*b^2*(9*A + 7*C))*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*b^3*d) - (16*a*C*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(63*b^2*d) + (2*C*Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
```

```
])/(9*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9bd} + \frac{2\int \frac{\cos^2(c+dx)\left(3aC+\frac{1}{2}b(9A+7C\right.}{\sqrt{a+b\,\mathrm{co}}}}{9}$$

$$= -\frac{16aC\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63b^2d} + \frac{2C\cos^3(c+dx)\sqrt{a+}}{9}$$

$$= \frac{2\left(48a^2C+7b^2(9A+7C)\right)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^3d} - \frac{16a}{}$$

$$= -\frac{4a\left(42Ab^2+32a^2C+31b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^4d} + \frac{2\left(48a^2C\right.}{}$$

$$= -\frac{4a\left(42Ab^2+32a^2C+31b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^4d} + \frac{2\left(48a^2C\right.}{}$$

$$= -\frac{4a\left(42Ab^2+32a^2C+31b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^4d} + \frac{2\left(48a^2C\right.}{}$$

$$= \frac{2\left(128a^4C+21b^4(9A+7C)+12a^2b^2(14A+9C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{315b^5d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.39748, size = 272, normalized size = 0.72

$$\frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b\left(32a^3bC+6ab^3(7A+6C)\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(12a^2b^2(14A+9C)+128a^4C+21b^4(9A+7C)\right)\left((a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],
x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(32*a^3*b*C + 6*a*b^3*(7*A + 6*C))
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (128*a^4*C + 21*b^4*(9*A + 7*C) +
12*a^2*b^2*(14*A + 9*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) - b*(a + b*Cos[c + d*x])*(32*a*(21
*A*b^2 + 2*(8*a^2 + 9*b^2)*C)*Sin[c + d*x] - b*(2*(126*A*b^2 + 96*a^2*C + 1
33*b^2*C)*Sin[2*(c + d*x)] + 5*b*C*(-16*a*Sin[3*(c + d*x)] + 7*b*Sin[4*(c +
```

```
d*x)])))))/(1260*b^5*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.422, size = 1527, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(-80*C*a*b^4+2240*C*b^5)*sin(
1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^5-64*C*a^2*b^3+120*C*a*b^4-20
72*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-84*A*a*b^4+504*A*b^5-64
*C*a^3*b^2+64*C*a^2*b^3-112*C*a*b^4+952*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(168*A*a^2*b^3+42*A*a*b^4-126*A*b^5+128*C*a^4*b+32*C*a^3*b^2+10
8*C*a^2*b^3+36*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1
68*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-168*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-189*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5-168*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-147*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^4+128*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^5-128*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4*b+108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
3*b^2-108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+
147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5-128*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-76*C*(sin(1/2*d*x+1/2*c)^2)^
```

```
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-111*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^4)/b^5/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{5}+A\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^5 + A*cos(d*x + c)^3)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x
)
```

3.649 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=305

$$\frac{2\left(24a^2C+5b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}+\frac{2\left(2a^2b^2(35A+16C)+48a^4C+5b^4(7A+5C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{105b^4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-4*a*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ (2*(48*a^4*C + 5*b^4*(7*A + 5*C) + 2*a^2*b^2*(35*A + 16*C))*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*(24*a^2*C + 5*b^2*(7*A + 5*C))*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^3*d) - (12*a*C*Cos[c + d*x]*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.589196, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3050, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(24a^2C+5b^2(7A+5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}+\frac{2\left(2a^2b^2(35A+16C)+48a^4C+5b^4(7A+5C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{105b^4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (-4*a*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ (2*(48*a^4*C + 5*b^4*(7*A + 5*C) + 2*a^2*b^2*(35*A + 16*C))*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*(24*a^2*C + 5*b^2*(7*A + 5*C))*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^3*d) - (12*a*C*Cos[c + d*x]*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(7*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos(c+dx)\left(2aC+\frac{1}{2}b(7A+5C)\operatorname{co}\right.}{\sqrt{a+b\cos(}}}{7b}$$

$$= -\frac{12aC\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35b^2d} + \frac{2C\cos^2(c+dx)\sqrt{a+b\,\mathrm{c}}}{7bd}$$

$$= \frac{2\left(24a^2C+5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} - \frac{12aC\cos(c+dx}{}$$

$$= \frac{2\left(24a^2C+5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} - \frac{12aC\cos(c+dx}{}$$

$$= \frac{2\left(24a^2C+5b^2(7A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} - \frac{12aC\cos(c+dx}{}$$

$$= -\frac{4a\left(35Ab^2+24a^2C+22b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(48a\right.}{}$$

Mathematica [A] time = 0.881804, size = 217, normalized size = 0.71

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))\left(48a^2C-36abC\cos(c+dx)+70Ab^2+15b^2C\cos(2(c+dx))+65b^2C\right)+4\sqrt{\frac{a+b\cos(c+d}{a+b}}}{210b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],
x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(35*A*b^3 - 12*a^2*b*C + 25*b^3*C)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(35*A*b^2 + 24*a^2*C + 22*b^2*
C)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(70*A*b^2 + 48*a^2*C + 65*b^2
*C - 36*a*b*C*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(210*
b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.393, size = 1131, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(24*C*a*b^3-360*C*b^4)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4+24*C*a^2*b^2-24*C*a*b^3+280*C*b^4
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a*b^3-70*A*b^4-48*C*a^3*b-
12*C*a^2*b^2-44*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-7
0*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+70*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3+70*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^4+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^3*b-44*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b^2+44*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*
b^3+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+32*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^4/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x
)
```

3.650 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=233

$$-\frac{2a\left(8a^2C+15Ab^2+7b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C+3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(8*a^2*C + 3*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*
(15*A*b^2 + 8*a^2*C + 7*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (8*a*C*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d) + (2*C*Cos[c + d*x]*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.348387, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3050, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(8a^2C+15Ab^2+7b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C+3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*(8*a^2*C + 3*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*
(15*A*b^2 + 8*a^2*C + 7*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (8*a*C*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d) + (2*C*Cos[c + d*x]*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
```

```
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{2\int \frac{aC+\frac{1}{2}b(5A+3C)\cos(c+dx)-2aC\,c}{\sqrt{a+b\cos(c+dx)}}}{5b}$$

$$= -\frac{8aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c}{5bd}$$

$$= -\frac{8aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c}{5bd}$$

$$= -\frac{8aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c}{5bd}$$

$$= \frac{2\left(8a^2C+3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(15Ab^2+\right.}{}$$

Mathematica [A] time = 0.929378, size = 190, normalized size = 0.82

$$\frac{-2a\left(8a^2C+15Ab^2+7b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(a+b)\left(8a^2C+15Ab^2+9b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(a + b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(15*A*b^2 + 8*a^2*C + 7*b^2*C
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
+ b*C*(-8*a^2 + 3*b^2 - 2*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c
+ d*x])/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.41, size = 892, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*cos
(1/2*d*x+1/2*c)^7*b^3+4*C*cos(1/2*d*x+1/2*c)^5*a*b^2+48*C*cos(1/2*d*x+1/2*c
)^5*b^3+15*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+15*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+8*C*cos(1/2*d*x+1/2*c)^3*a^2*b-6*C*cos(1/2
*d*x+1/2*c)^3*a*b^2-30*C*cos(1/2*d*x+1/2*c)^3*b^3+8*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^3+7*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b^2-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+8*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-9*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*b^3-8*C*cos(1/2*d*x+1/2*c)*a^2*b+2*C*cos(1/2*d*x+1/2*c)*a*b^2+
6*C*cos(1/2*d*x+1/2*c)*b^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.651 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{2\left(2a^2C+b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4aC\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

```
[Out] (-4*a*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*
b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*C + b^2*(3*A + C))*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*
b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(3*b*d)
```

Rubi [A] time = 0.205793, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3024, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(2a^2C+b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4aC\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*a*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*
b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*C + b^2*(3*A + C))*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*
b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(3*b*d)
```

Rule 3024

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{1}{2}b(3A+C)-aC\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{(2aC)\int\sqrt{a+b\cos(c+dx)}\,dx}{3b^2} + \frac{1}{3}\left(3A+C+\frac{2a^2C}{b^2}\right)$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{\left(2aC\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{((}{}$$

$$= -\frac{4aC\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(3A+C+\frac{2a^2C}{b^2}\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.683545, size = 148, normalized size = 0.85

$$\frac{2\left(C\left(2a^2+b^2\right)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bC\sin(c+dx)(a+b\cos(c+dx))-4aC(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*a*(a + b)*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)] + 2*(3*A*b^2 + (2*a^2 + b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*C*(a + b*Cos[c + d*x])*Si
n[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.378, size = 532, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
```

```
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+2*C*cos(1/2*d*x+1/2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3*b^2+2*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+
2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*C*cos(1/2*d*x+1/2
*c)*a*b+2*C*cos(1/2*d*x+1/2*c)*b^2)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c) + a), x)
```

3.652 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2aC\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*C*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.411264, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2aC\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*C*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
```

```
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{\int \frac{(-Ab+aC\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{C\int \sqrt{a+b\cos(c+dx)}\,dx}{b}$$

$$= A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx - \frac{(aC)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{\left(C\sqrt{a+b\cos(c+dx)}\right.}{b\sqrt{\frac{a+b}{}}}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,d}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2aC\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Mathematica [F] time = 8.42576, size = 0, normalized size = 0.

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]], x
]
```

Maple [A] time = 0.434, size = 249, normalized size = 1.4

$$2\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}\sqrt{\frac{2\,(\cdots}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b+C*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + C\cos^2{(c + dx)})\sec{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos{(dx + c)}^2 + A)\sec{(dx + c)}}{\sqrt{b\cos{(dx + c)} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.653 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{(A+2C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + ((A + 2*C)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]
) - (A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(a*d)
```

Rubi [A] time = 0.630307, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(A+2C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + ((A + 2*C)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]
) - (A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(a*d)
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{\int \frac{\left(-\frac{Ab}{2}+aC\cos(c+dx)-\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}}{a}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} - \frac{A\int\sqrt{a+b\cos(c+dx)}\,dx}{2a} - \int \frac{\left(\frac{Ab^2}{2}-\frac{1}{2}ab(\right.}{}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} - \frac{(Ab)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{2a} - \frac{1}{2}(-A-2C)\int$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(A+2C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 11.7702, size = 559, normalized size = 2.61

$$\frac{2A\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(A\sec^2(c+dx)+C\right)}{ad(2A+C\cos(2c+2dx)+C)} + \frac{\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(\frac{2iAb\sin(c+dx)\cos(}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],
x]
```

```
[Out] (2*A*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(C + A*Sec[c + d*x]^2)*Sin[c + d
*x])/(a*d*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*(C + A*Sec[c +
d*x]^2)*((8*a*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - (6*A*b*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]
] + ((2*I)*A*b*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x
])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*Ar
cSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*E
llipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
```

```
], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c +
 d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x]
)^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])
^2))))/(2*a*d*(2*A + C + C*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.672, size = 638, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))+2*A*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x
)
```

3.654 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=278

$$\frac{(4a^2(A+2C)+3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{3Ab\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{3Ab\sqrt{a+b\cos(c+\ldots}}{4a^2d\sqrt{\frac{a+\ldots}{}}}$$

```
[Out] (3*A*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a
^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (A*b*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*
x]]) + ((3*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) -
 (3*A*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a^2*d) + (A*Sqrt[a + b*Co
s[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.884024, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2(A+2C)+3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{3Ab\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{3Ab\sqrt{a+b\cos(c+\ldots}}{4a^2d\sqrt{\frac{a+\ldots}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (3*A*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a
^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (A*b*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*
x]]) + ((3*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) -
 (3*A*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a^2*d) + (A*Sqrt[a + b*Co
s[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-\frac{3Ab}{2}+a(A+2C)\cos(c+dx)+\frac{1}{2}Ab\,c}{\sqrt{a+b\cos(c+d}}}{2a}$$

$$= -\frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(}{2ad}$$

$$= -\frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(}{2ad}$$

$$= -\frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(}{2ad}$$

$$= \frac{3Ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{3Ab\sqrt{a+b\cos(c+dx)}\tan(c+d}{4a^2d}$$

$$= \frac{3Ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}$$

<u>Mathematica</u> [C] time = 6.73203, size = 603, normalized size = 2.17

$$\frac{\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A\sec^2(c+dx)+C\right)\left(\frac{A\tan(c+dx)\sec(c+dx)}{a}-\frac{3Ab\tan(c+dx)}{2a^2}\right)}{d(2A+C\cos(2c+2dx)+C)} + \frac{\cos^2(c+dx)\left(A\sec^2(c+dx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],
x]
```

```
[Out] (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] +
(2*(8*a^2*A + 9*A*b^2 + 16*a^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((6*I)*A*b^
2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*
Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(
```

```
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a
+ b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt
[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2
*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))))/(8*a^2
*d*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x
]]*(C + A*Sec[c + d*x]^2)*((-3*A*b*Tan[c + d*x])/(2*a^2) + (A*Sec[c + d*x]*
Tan[c + d*x])/a))/(d*(2*A + C + C*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.661, size = 814, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/2/a
*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x
)
```

3.655 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=370

$$\frac{\left(8a^2(2A+3C)+15Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^3d}+\frac{\left(8a^2(2A+3C)+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(8a}{}$$

```
[Out] -((15*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(24*a^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((5*A
*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(5*A*b^2
+ 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(8*a^3*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 + 8*a
^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^3*d) - (5*A*b*
Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a^2*d) + (A*Sqrt[a
+ b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 1.31926, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(8a^2(2A+3C)+15Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^3d}+\frac{\left(8a^2(2A+3C)+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(8a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] -((15*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(24*a^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((5*A
*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(5*A*b^2
+ 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(8*a^3*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 + 8*a
^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^3*d) - (5*A*b*
Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a^2*d) + (A*Sqrt[a
+ b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{A\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3ad}+\frac{\int \frac{\left(-\frac{5Ab}{2}+a(2A+3C)\cos(c+dx)+\frac{3}{2}\right.}{\sqrt{a+b\cos(}}}{3a}$$

$$=-\frac{5Ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12a^2d}+\frac{A\sqrt{a+b\cos(c+dx)}\sec}{3ad}$$

$$=\frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^3d}-\frac{5Ab\sqrt{a+b\cos(}}{}$$

$$=\frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^3d}-\frac{5Ab\sqrt{a+b\cos(}}{}$$

$$=\frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^3d}-\frac{5Ab\sqrt{a+b\cos(}}{}$$

$$=-\frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24a^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(15Ab^2+\right.}{}$$

$$=-\frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24a^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(A\left(16+\right.\right.}{}$$

Mathematica [C] time = 6.74647, size = 604, normalized size = 1.63

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+24a^2C\sin(c+dx)+15Ab^2\sin(c+dx)\right)}{24a^3}-\frac{5Ab\tan(c+dx)\sec(c+dx)}{12a^2}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3a}\right)}{d}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + b*Cos[c + d*x]],
x]
```

```
[Out] -(b*((40*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(40*a^2*A + 45*A*b^2 + 72*a^2*C
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(16*a^2*A + 15*A*b^2 + 24*a^2*C)*Sqr
t[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2
*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/
a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a
^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2
- b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))))/(96*a^3*d)
+ (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(16*a^2*A*Sin[c + d*x] + 15*A*b^
2*Sin[c + d*x] + 24*a^2*C*Sin[c + d*x]))/(24*a^3) - (5*A*b*Sec[c + d*x]*Tan
[c + d*x])/(12*a^2) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 1.063, size = 1562, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/a*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*(-1/3/a*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+1
5*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^
3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c) + a), x
)
```

3.656 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=473

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C+7Ab^2-b^2C\right)\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7b^2d\left(a^2-b^2\right)}-\frac{2a\left(48\right.}{}$$

```
[Out] (-2*a*(4*a^2*b^2*(70*A - 43*C) + 384*a^4*C - b^4*(175*A + 107*C))*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^5*(a^2 - b^2)
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(384*a^4*C + 5*b^4*(7*A + 5*C)
+ 4*a^2*b^2*(70*A + 29*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)])/(105*b^5*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2
+ a^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]) + (2*(2*a^2*b^2*(70*A - 31*C) + 192*a^4*C - 5*b^4*(7*A + 5*C))*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^4*(a^2 - b^2)*d) - (2*a*(35*A*b^2 +
 48*a^2*C - 13*b^2*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
35*b^3*(a^2 - b^2)*d) + (2*(7*A*b^2 + 8*a^2*C - b^2*C)*Cos[c + d*x]^2*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.12987, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C+7Ab^2-b^2C\right)\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7b^2d\left(a^2-b^2\right)}-\frac{2a\left(48\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*a*(4*a^2*b^2*(70*A - 43*C) + 384*a^4*C - b^4*(175*A + 107*C))*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^5*(a^2 - b^2)
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(384*a^4*C + 5*b^4*(7*A + 5*C)
+ 4*a^2*b^2*(70*A + 29*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)])/(105*b^5*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2
+ a^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]) + (2*(2*a^2*b^2*(70*A - 31*C) + 192*a^4*C - 5*b^4*(7*A + 5*C))*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^4*(a^2 - b^2)*d) - (2*a*(35*A*b^2 +
```

```
48*a^2*C - 13*b^2*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
35*b^3*(a^2 - b^2)*d) + (2*(7*A*b^2 + 8*a^2*C - b^2*C)*Cos[c + d*x]^2*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b^2*(a^2 - b^2)*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos^2(c+dx)\left(3\left(Ab^2+a^2C\right)-\frac{1}{2}ab(A+C)\cos\ldots}{\sqrt{a+b\ldots}}}{b\left(a^2\ldots\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(7Ab^2+8a^2C-b^2C\right)\cos^2(c+\ldots}{7b^2\left(a^2\ldots\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(35Ab^2+48a^2C-13b^2C\right)\cos\ldots}{35b^3\ldots}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2b^2(70A-31C)+192a^4C-\ldots\right.}{10\ldots}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2b^2(70A-31C)+192a^4C-\ldots\right.}{10\ldots}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2b^2(70A-31C)+192a^4C-\ldots\right.}{10\ldots}$$

$$= -\frac{2a\left(4a^2b^2(70A-43C)+384a^4C-b^4(175A+107C)\right)\sqrt{a+b\cos(c+dx)}E\left(\ldots\right.}{105b^5\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.6127, size = 358, normalized size = 0.76

$$\frac{b(a-b)(a+b)\left(420a^3\left(a^2C+Ab^2\right)\sin(c+dx)+\left(a^2-b^2\right)\left(348a^2C+140Ab^2+115b^2C\right)\sin(c+dx)(a+b\cos(c+dx))-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (-4*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(2*a^2*b^3*(35*A - 8*
C) + 96*a^4*b*C + 5*b^5*(7*A + 5*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
+ a*(4*a^2*b^2*(70*A - 43*C) + 384*a^4*C - b^4*(175*A + 107*C))*((a + b)*El
lipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b
```

```
)])) + (a - b)*b*(a + b)*(420*a^3*(A*b^2 + a^2*C)*Sin[c + d*x] + (a^2 - b^2
)*(140*A*b^2 + 348*a^2*C + 115*b^2*C)*(a + b*Cos[c + d*x])*Sin[c + d*x] - 7
8*a*b*(a^2 - b^2)*C*(a + b*Cos[c + d*x])*Sin[2*(c + d*x)] + 15*b^2*(a^2 - b
^2)*C*(a + b*Cos[c + d*x])*Sin[3*(c + d*x)]))/(210*(a - b)*b^5*(a + b)*(a^2
 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.828, size = 1788, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(32*C/b*(-1/
14/b*cos(1/2*d*x+1/2*c)^5*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/140/b^2*(-6*a+18*b)*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/420*(12*a^2-47*a*b+83*b^2)/b^3*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)+1/420*(12*a^2-47*a*b+83*b^2)/b^3*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/210*(-6*a^3+28*a^2*b-58*a*b^2+84*b^3)/b^4*(a-b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-16*C/b^2*(a+
4*b)*(-1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))))+8/b^3*(A*b^2+C*a^2+3*C*a*b+6*C*b^2)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
```

```
-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^5*(A*a*b
^2+2*A*b^3+C*a^3+2*C*a^2*b+3*C*a*b^2+4*C*b^3)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(A*a^2*b^2+
A*a*b^3+A*b^4+C*a^4+C*a^3*b+C*a^2*b^2+C*a*b^3+C*b^4)/b^5*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-2*a^3*(A*b^2+C*a^2)/b^5/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*
x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^5+A\cos\left(dx+c\right)^3\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^5 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2),
 x)
```

3.657 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2C+5Ab^2-b^2C\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2C\right.}{}$$

```
[Out] (2*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(5*b^4*(a^2 - b^2)*d*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]) - (4*a*(5*A*b^2 + 2*(4*a^2 + b^2)*C)*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(5*b^4*d*Sqr
t[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(b
*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a*(5*A*b^2 + 8*a^2*C - 3*b^2*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^3*(a^2 - b^2)*d) + (2*(5*A*b
^2 + 6*a^2*C - b^2*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
5*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.735158, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2C+5Ab^2-b^2C\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2),x]
```



```
[Out] (2*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(5*b^4*(a^2 - b^2)*d*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]) - (4*a*(5*A*b^2 + 2*(4*a^2 + b^2)*C)*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(5*b^4*d*Sqr
t[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(b
*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a*(5*A*b^2 + 8*a^2*C - 3*b^2*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^3*(a^2 - b^2)*d) + (2*(5*A*b
^2 + 6*a^2*C - b^2*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
5*b^2*(a^2 - b^2)*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
```

```
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(2\left(Ab^2+a^2C\right)-\frac{1}{2}ab(A+C)\right.}{\sqrt{a+}}}{b\left(\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(5Ab^2+6a^2C-b^2C\right)\cos(c+}{5b^2\left(a\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(5Ab^2+8a^2C-3b^2C\right)\sqrt{a+}}{5b^3\left(a^2-b\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(5Ab^2+8a^2C-3b^2C\right)\sqrt{a+}}{5b^3\left(a^2-b\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(5Ab^2+8a^2C-3b^2C\right)\sqrt{a+}}{5b^3\left(a^2-b\right.}$$

$$= \frac{2\left(2a^2b^2(5A-4C)+16a^4C-b^4(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{5b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.60632, size = 289, normalized size = 0.77

$$\frac{\frac{10a^2b\left(a^2C+Ab^2\right)\sin(c+dx)}{b^2-a^2} + \frac{2ab^2\left(C\left(4a^2+b^2\right)+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a-b)(a+b)} + \frac{2\left(2a^2b^2(5A-4C)+16a^4C-b^4(5A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c\right.\right.}{(a-b)(a+b)}}{5b^4d\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] ((2*a*b^2*(5*A*b^2 + (4*a^2 + b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a + b)) + (2*(2*a^2*b^2*(5*A
 - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((
a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*
b)/(a + b)]))/((a - b)*(a + b)) + (10*a^2*b*(A*b^2 + a^2*C)*Sin[c + d*x])/(
-a^2 + b^2) - 6*a*b*C*(a + b*Cos[c + d*x])*Sin[c + d*x] + b^2*C*(a + b*Cos[
c + d*x])*Sin[2*(c + d*x)])/(5*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.456, size = 1289, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C/b*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-8/b
^2*C*(a+3*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/1
2/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^4*(A*b^2+C*a^2+2*C*a*b+3*C*b^2)*(a-b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2)))-2*(A*a*b^2+A*b^3+C*a^3+C*a^2*b+C*a*b^2+C*b^3)/b^4*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))+2*a^2*(A*b^2+C*a^2)/b^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+
a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^4+A\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2),
 x)
```

3.658 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{2a\left(a^2C+Ab^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(C\left(8a^2+b^2\right)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^2C+3Ab^2-5b^2C\right.}{3b^3d\left(a^2\right.}$$

```
[Out] (-2*a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]) + (2*(3*A*b^2 + (8*a^2 + b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*(A*b^2 + a^2*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.423847, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3032, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a\left(a^2C+Ab^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(C\left(8a^2+b^2\right)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^2C+3Ab^2-5b^2C\right.}{3b^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]) + (2*(3*A*b^2 + (8*a^2 + b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*(A*b^2 + a^2*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
```

```
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}b\left(Ab^2+a^2C\right)+\frac{1}{2}a\left(Ab^2+2a^2C-b^2C\right)\cos(c+dx)-}{\sqrt{a+b\cos(c+dx)}}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d} - \frac{4\int}{}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d} - \frac{(a(3}{}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d} - \frac{(a(3}{}$$

$$= -\frac{2a\left(3Ab^2+8a^2C-5b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(3Ab^2\right.}{}$$

Mathematica [A] time = 1.39309, size = 209, normalized size = 0.82

$$-\frac{2\left(b\sin(c+dx)\left(bC\left(b^2-a^2\right)\cos(c+dx)-4a^3C+ab^2(C-3A)\right)-\left(a^2-b^2\right)\left(C\left(8a^2+b^2\right)+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.\right.}{3b^3d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-2*(a*(a + b)*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a^2 - b^2)*(3*A*b^2 + (8*a^2
+ b^2)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)] + b*(-4*a^3*C + a*b^2*(-3*A + C) + b*(-a^2 + b^2)*C*Cos[c + d*x])*S
in[c + d*x]))/(3*(a - b)*b^3*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.092, size = 885, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/b*C*(-1/6
/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6*b)*(
a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))))+2*C/b^3*(a+2*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(A*b^2+C*a^2+C*a*b+C*b^2)
/b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a*(A*b^2+C*a^2)/b^3/sin(1/2*d*x+1/
2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+
2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.659 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2C+Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4aC\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\right.}{b^2d\sqrt{a+b\cos(}}$$

```
[Out] (2*(A*b^2 + 2*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) -
(4*a*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.246881, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3022, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2C+Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4aC\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\right.}{b^2d\sqrt{a+b\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*(A*b^2 + 2*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) -
(4*a*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}ab(A+C)-\frac{1}{2}\left(Ab^2+2a^2C-b^2C\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(2aC)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2} + \frac{\left(Ab^2+2a^2C-b^2C\right)\int \sqrt{a}}{b^2\left(a^2-\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\left(Ab^2+2a^2C-b^2C\right)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+}}{b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(Ab^2+2a^2C-b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{4aC\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{b^2d\sqrt{a+b\cos(c+d}}$$

Mathematica [A] time = 0.669447, size = 166, normalized size = 0.82

$$\frac{-2b\left(a^2C+Ab^2\right)\sin(c+dx)+2(a+b)\left(2a^2C+Ab^2-b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-4aC\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+}{a+b}}}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b)*(A*b^2 + 2*a^2*C - b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*a*(a^2 - b^2)*C*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*b*(A*b^2 + a^2*C)*
Sin[c + d*x])/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.949, size = 490, normalized size = 2.4

$$-\frac{1}{d}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2b-a+b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-2\,\frac{C\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}}{b^2\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^4+(a+b)\left(\sin\left(1/2\,dx+c/\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C/b^2/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2*(
A*b^2+C*a^2)/b^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-
b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/(b^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.660 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{abd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\right.}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(A*b^2 + a^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(a*b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*C*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*
Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipti
cPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*
b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.702017, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{abd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\right.}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*(A*b^2 + a^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(a*b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*C*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*
Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipti
cPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*
b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}A\left(a^2-b^2\right)-\frac{1}{2}ab(A+C)\cos(c+dx)-\frac{1}{2}\left(Ab^2+a^2C\right)\cdots}{\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\left(-\frac{1}{2}Ab\left(a^2-b^2\right)-\frac{1}{2}a\left(a^2-b^2\right)C\cos(c+dx)\right)\sec(c+dx)\cdots}{\sqrt{a+b\cos(c+dx)}}}{ab\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a} + \frac{C\int \frac{1}{\sqrt{a+b\cos(c+dx)}}d\cdots}{b}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ab\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(Ab^2+a^2C\right)\sin\cdots}{a\left(a^2-b^2\right)d\sqrt{a+b\,c\cdots}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ab\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\cdots\right.}{bd\sqrt{a+b\cos(\cdots}}$$

Mathematica [F] time = 29.7452, size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),
 x]
```

Maple [A] time = 0.821, size = 539, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*(-A*b^2-C*a^2)/a/b/sin(1/2*d*x+1/2*c)^2/(-2*
sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*A/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.661 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{b\left(3Ab^2-a^2(A-2C)\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(3Ab^2-a^2(A-2C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi}{a^2d\sqrt{a+b\cos}}$$

```
[Out] ((3*A*b^2 - a^2*(A - 2*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2,
(2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (A
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/
(a*d*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]])
 - (b*(3*A*b^2 - a^2*(A - 2*C))*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.967668, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(3Ab^2-a^2(A-2C)\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(3Ab^2-a^2(A-2C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi}{a^2d\sqrt{a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] ((3*A*b^2 - a^2*(A - 2*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2,
(2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (A
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/
(a*d*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]])
 - (b*(3*A*b^2 - a^2*(A - 2*C))*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{3Ab}{2}+aC\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{a}$$

$$= -\frac{b(3Ab^2-a^2(A-2C))\sin(c+dx)}{a^2(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(-\frac{3}{4}Ab(a^2\right.}{}}{}$$

$$= -\frac{b(3Ab^2-a^2(A-2C))\sin(c+dx)}{a^2(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\left(\frac{3}{4}Ab^2(a^2\right.}{}}{}$$

$$= -\frac{b(3Ab^2-a^2(A-2C))\sin(c+dx)}{a^2(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{1}{\sqrt{a+b\cos}}}{2a}$$

$$= \frac{(3Ab^2-a^2(A-2C))\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b(3Ab^2-a^2(}{a^2(a^2-b^2)d}$$

$$= \frac{(3Ab^2-a^2(A-2C))\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{\frac{a+b\cos(c+dx}{a+b}}}{ad\sqrt{a+}}$$

Mathematica [C] time = 6.18306, size = 511, normalized size = 1.73

$$\frac{\cos^2(c+dx)\left(A\sec^2(c+dx)+C\right)\left(\frac{4\tan(c+dx)\left(b\left(a^2(A-2C)-3Ab^2\right)\cos(c+dx)+aA\left(a^2-b^2\right)\right)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} + \frac{\frac{8a\left(a^2C+Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2b(a^2(2}{}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*(((8*a*(A*b^2 + a^2*C)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos
[c + d*x]] + (2*b*(9*A*b^2 + a^2*(-7*A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
```

```
+ ((2*I)*(3*A*b^2 - a^2*(A - 2*C))*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]
*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*Ellipti
cE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))
/((a - b)*(a + b)) + (4*(a*A*(a^2 - b^2) + b*(-3*A*b^2 + a^2*(A - 2*C))*Cos
[c + d*x])*Tan[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2*a^2*d*
(2*A + C + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 1.043, size = 904, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b^2+C*
a^2)/a^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*A/a*(
-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A/a^2*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2),
 x)
```

3.662 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=370

$$\frac{b^2\left(15Ab^2-a^2(7A-8C)\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{b\left(15Ab^2-a^2(7A-8C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(4a^2(A+2C\right.}{}$$

```
[Out] -(b*(15*A*b^2 - a^2*(7*A - 8*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]) - (5*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 + 4*a^2*(A + 2*
C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) + (b^2*(15*A*b^2 - a^2*(7*A - 8*C
))*Sin[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b*Ta
n[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*
x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.34092, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b^2\left(15Ab^2-a^2(7A-8C)\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{b\left(15Ab^2-a^2(7A-8C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(4a^2(A+2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] -(b*(15*A*b^2 - a^2*(7*A - 8*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]) - (5*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 + 4*a^2*(A + 2*
C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) + (b^2*(15*A*b^2 - a^2*(7*A - 8*C
))*Sin[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b*Ta
n[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*
x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{5Ab}{2}+a(A+2C)\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,d}{2a}$$

$$= -\frac{5Ab\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}\left(15Ab^2+4a^2(A+2C)\right.\right.}{}}{}$$

$$= \frac{b^2\left(15Ab^2-a^2(7A-8C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5Ab\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+}{2ad\sqrt{a}}$$

$$= \frac{b^2\left(15Ab^2-a^2(7A-8C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5Ab\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+}{2ad\sqrt{a}}$$

$$= \frac{b^2\left(15Ab^2-a^2(7A-8C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5Ab\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+}{2ad\sqrt{a}}$$

$$= -\frac{b\left(15Ab^2-a^2(7A-8C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b^2\left(15Ab^2\right.}{4a^3\left(a^2\right.}$$

$$= -\frac{b\left(15Ab^2-a^2(7A-8C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{5Ab\sqrt{\frac{a+b\,c}{}}}{4a^2d}$$

Mathematica [C] time = 6.80069, size = 727, normalized size = 1.96

$$\frac{\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A\sec^2(c+dx)+C\right)\left(\frac{4\left(a^2b^2C\sin(c+dx)+Ab^4\sin(c+dx)\right)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{7Ab\tan(c+dx)}{2a^3}+\frac{A\tan(c+dx)\sec(c+dx)}{a^2}\right)}{d(2A+C\cos(2c+2dx)+C)} -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] -(Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((2*(4*a^3*A*b - 20*a*A*b^3 - 16*a^
3*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^4*A + 29*a^2*A*b^2 - 45*A*b^4 + 16
*a^4*C - 24*a^2*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
+ d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(7*a^2*A*b^2 -
15*A*b^4 - 8*a^2*b^2*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Co
s[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*Ellip
ticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a -
b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[
c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1
- Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos
[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[
c + d*x])^2))))/(8*a^3*(-a + b)*(a + b)*d*(2*A + C + C*Cos[2*c + 2*d*x])) +
 (Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(C + A*Sec[c + d*x]^2)*((4*(A*b^4
*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*
x])) - (7*A*b*Tan[c + d*x])/(2*a^3) + (A*Sec[c + d*x]*Tan[c + d*x])/a^2))/(
d*(2*A + C + C*Cos[2*c + 2*d*x]))
```

Maple [B] time = 1.335, size = 1561, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b^2+C
*a^2)*b/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/a^
2*b*A*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
```

```
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*(A*b^2+C*a^2)/a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2/a*A*(-1/2/a*cos(1/2*d*x+1/2*c)*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*
sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2),
 x)
```

3.663 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=521

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4\left(-a^2b^2(A-6C)-4a^4C+3Ab^4\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2b^2(15A-\right.}{}$$

```
[Out] (2*(4*a^4*b^2*(10*A - 53*C) - 5*a^2*b^4*(15*A - 11*C) + 128*a^6*C + 3*b^6*(
5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])
/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(4*a^2*
b^2*(10*A - 29*C) + 128*a^4*C - b^4*(45*A + 17*C))*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(3
*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*(3*A*b^4 - a^2*b^2*(A - 6
*C) - 4*a^4*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]]) - (4*a*(a^2*b^2*(10*A - 49*C) - b^4*(20*A - 7*C) + 32*a^4
*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) + (2*(a
^2*b^2*(15*A - 71*C) - b^4*(35*A - 3*C) + 48*a^4*C)*Cos[c + d*x]*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.33569, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3048, 3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4\left(-a^2b^2(A-6C)-4a^4C+3Ab^4\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2b^2(15A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(4*a^4*b^2*(10*A - 53*C) - 5*a^2*b^4*(15*A - 11*C) + 128*a^6*C + 3*b^6*(
5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])
/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(4*a^2*
b^2*(10*A - 29*C) + 128*a^4*C - b^4*(45*A + 17*C))*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(3
*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*(3*A*b^4 - a^2*b^2*(A - 6
```

```
*C) - 4*a^4*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]]) - (4*a*(a^2*b^2*(10*A - 49*C) - b^4*(20*A - 7*C) + 32*a^4
*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) + (2*(a
^2*b^2*(15*A - 71*C) - b^4*(35*A - 3*C) + 48*a^4*C)*Cos[c + d*x]*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\left(3\left(Ab^2+a^2C\right)-\frac{3}{2}ab(A+C)\right.}{(a+b}}{3b}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4\left(3Ab^4-a^2b^2(A-6C)-4a^4C\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4\left(3Ab^4-a^2b^2(A-6C)-4a^4C\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4\left(3Ab^4-a^2b^2(A-6C)-4a^4C\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4\left(3Ab^4-a^2b^2(A-6C)-4a^4C\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4\left(3Ab^4-a^2b^2(A-6C)-4a^4C\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= \frac{2\left(4a^4b^2(10A-53C)-5a^2b^4(15A-11C)+128a^6C+3b^6(5A+3C)\right)\sqrt{a+}}{15b^5\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 3.56358, size = 350, normalized size = 0.67

$$\frac{b\left(\frac{10a^3\left(a^2C+Ab^2\right)\sin(c+dx)}{a^2-b^2} - \frac{10a^2\left(5a^2b^2(A-3C)+11a^4C-9Ab^4\right)\sin(c+dx)(a+b\cos(c+dx))}{\left(a^2-b^2\right)^2} - 28aC\sin(c+dx)(a+b\cos(c+dx))^2 + 3bC\,s\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] ((2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*(-2*a*b^2*(-16*a^4*C + b^4*(15*A +
 4*C) + a^2*b^2*(-5*A + 22*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (4*a
^4*b^2*(10*A - 53*C) + 128*a^6*C + 3*b^6*(5*A + 3*C) + 5*a^2*b^4*(-15*A + 1
```

```
1*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x
)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + b*((10*a^3*(A*b^2 + a^2*C)*Sin
[c + d*x])/(a^2 - b^2) - (10*a^2*(-9*A*b^4 + 5*a^2*b^2*(A - 3*C) + 11*a^4*C
)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*a*C*(a + b*Cos[c +
d*x])^2*Sin[c + d*x] + 3*b*C*(a + b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*
b^5*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 2.425, size = 1735, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C/b^2*(-
1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-8
/b^3*C*(2*a+3*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2
*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^5*(A*b^2+3*C*a^2+4*C*a*b+3*C*b^2)
*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2)))-2*(2*A*a*b^2+A*b^3+4*C*a^3+3*C*a^2*b+2*C*a*b^2+C*b^3)/b^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^2/b^5*(3*A*b^2+5*C*a^2)/sin(1/2*d*x+1
/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
```

```
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b
+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a^3*(A*b^2+C*a^2)/b^5*(1/6/
b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1
/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/
2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{5}+A\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^5 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2),
 x)
```

3.664 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2C+Ab^2-b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}-\frac{4a\left(5a^2b^2C-3a^4C\right.}{3b^3d\left(a^2-b^2\right)^2\sqrt{}}$$

```
[Out] (-4*a*(a^2*b^2*(A - 14*C) - b^4*(3*A - 4*C) + 8*a^4*C)*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*b^2*(A - 8*C) + 16*a^4*C - b^4*(3*A
 + C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*C
os[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2))
- (4*a*(2*A*b^4 - 3*a^4*C + 5*a^2*b^2*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*b^2 + 2*a^2*C - b^2*C)*Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.829636, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2C+Ab^2-b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}-\frac{4a\left(5a^2b^2C-3a^4C\right.}{3b^3d\left(a^2-b^2\right)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-4*a*(a^2*b^2*(A - 14*C) - b^4*(3*A - 4*C) + 8*a^4*C)*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*b^2*(A - 8*C) + 16*a^4*C - b^4*(3*A
 + C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*C
os[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2))
- (4*a*(2*A*b^4 - 3*a^4*C + 5*a^2*b^2*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*b^2 + 2*a^2*C - b^2*C)*Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
```

```
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)\left(2\left(Ab^2+a^2C\right)-\frac{3}{2}ab(A+C)\cos\right.}{(a+b\cos}}{3b\left(a^2\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\sin(c}{3b^3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\sin(c}{3b^3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\sin(c}{3b^3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\sin(c}{3b^3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+}}$$

$$= -\frac{4a\left(a^2b^2(A-14C)-b^4(3A-4C)+8a^4C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{3b^4\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.71695, size = 306, normalized size = 0.78

$$\frac{2\left(\frac{b\sin(c+dx)\left(4ab\left(a^2b^2(A-8C)+5a^4C+b^4(C-3A)\right)\cos(c+dx)+2a^4Ab^2-10a^2Ab^4+C\left(b^3-a^2b\right)^2\cos(2(c+dx))-25a^4b^2C+16a^6C+b^6C\right)}{2\left(a^2-b^2\right)^2}+\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b\left(a^2\right.\right.}{}\right.}{3b^4d(a+b\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b*(-4*a^4*b*C + b^5*(3*A + C) +
a^2*b^3*(A + 7*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(a^2*b^2*(A
- 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)
/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b))
+ (b*(2*a^4*A*b^2 - 10*a^2*A*b^4 + 16*a^6*C - 25*a^4*b^2*C + b^6*C + 4*a*b*
(a^2*b^2*(A - 8*C) + 5*a^4*C + b^4*(-3*A + C))*Cos[c + d*x] + (-(a^2*b) + b
^3)^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b
```

```
*Cos[c + d*x])^(3/2))
```

Maple [B] time = 1.934, size = 1323, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/b^2*C*(-1
/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6*b)
*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))))+4*C/b^4*(a+b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(A*b^2+3*C*a^2+2*C*a*b+C*
b^2)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*a/b^4*(A*b^2+2*C*a^2)/sin(1/2*
d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*a^2*(A*b^2+C*a^2)/b^4*
(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*
d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^
2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
 x)
```

3.665 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=314

$$\frac{2\left(a^2b^2(A+9C)-5a^4C+3Ab^4\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(a^2C+Ab^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-8a^2C+Ab^2+9b^2C\right)\sqrt{\frac{a+b\cos}{a}}}{3b^3d\left(a^2-b^2\right)\sqrt{a+b}}$$

```
[Out] (-2*(3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]) + (2*a*(A*b^2 - 8*a^2*C + 9*b^2*C)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b^2 + a^2*C)*Sin[c + d*x])/(3*b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*A*b^4 - 5*a^4*C + a^2*b^2*
(A + 9*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.509611, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3032, 3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2b^2(A+9C)-5a^4C+3Ab^4\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(a^2C+Ab^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-8a^2C+Ab^2+9b^2C\right)\sqrt{\frac{a+b\cos}{a}}}{3b^3d\left(a^2-b^2\right)\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]) + (2*a*(A*b^2 - 8*a^2*C + 9*b^2*C)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b^2 + a^2*C)*Sin[c + d*x])/(3*b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*A*b^4 - 5*a^4*C + a^2*b^2*
(A + 9*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
```

```
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}b\left(Ab^2+a^2C\right)-\frac{1}{2}a\left(Ab^2-2a^2C+3b^2C\right)\cos(c+dx}{(a+b\cos(c+dx))^{3/2}}}{3b^2\left(a^2-b^2\right)}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)\sin(c}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)\sin(c}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)\sin(c}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= -\frac{2\left(3b^4(A-C)-8a^4C+a^2b^2(A+15C)\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.13299, size = 227, normalized size = 0.72

$$\frac{2\left(\frac{b\sin(c+dx)\left(\left(a^2b^3(A+9C)-5a^4bC+3Ab^5\right)\cos(c+dx)+2a^3b^2(A+4C)-4a^5C+2aAb^4\right)}{\left(a^2-b^2\right)^2}+\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(-a^2b^2(A+15C)+8a^4C+3b^4(C-A)\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right.}{(a-b)^2}\right.}{3b^3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((8*a^4*C + 3*b^4*(-A + C) - a^2*
b^2*(A + 15*C))*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*(a - b)*(-(A*b^2)
 + 8*a^2*C - 9*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (
```

```
b*(2*a*A*b^4 - 4*a^5*C + 2*a^3*b^2*(A + 4*C) + (3*A*b^5 - 5*a^4*b*C + a^2*b
^3*(A + 9*C))*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*d*(a + b*C
os[c + d*x])^(3/2))
```

Maple [B] time = 1.764, size = 926, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C/b^3/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2/b
^3*(A*b^2+3*C*a^2)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^
2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2)-2*a*(A*b^2+C*a^2)/b^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*
(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/
(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^
3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^3*
cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.666 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{4a\left(a^2(-C)+2Ab^2+3b^2C\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-2a^2C+Ab^2+3b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c}}$$

```
[Out] (4*a*(2*A*b^2 - (a^2 - 3*b^2)*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) - (2*(A*b^2 - 2*a^2*C + 3*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^(3/2)) - (4*a*(2*A*b^2 - a^2*C + 3*b^2*C)*Sin[c + d*x])/(3*b*(a^2
 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.390085, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3022, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a\left(a^2(-C)+2Ab^2+3b^2C\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-2a^2C+Ab^2+3b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*a*(2*A*b^2 - (a^2 - 3*b^2)*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) - (2*(A*b^2 - 2*a^2*C + 3*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^(3/2)) - (4*a*(2*A*b^2 - a^2*C + 3*b^2*C)*Sin[c + d*x])/(3*b*(a^2
 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
```

```
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}ab(A+C)+\frac{1}{2}\left(Ab^2-2a^2C+3b^2C\right)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^2-a^2C+3b^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}b\left(a^2(3A\right.}{}}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^2-a^2C+3b^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(Ab^2-2a^2\right.}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4a\left(2Ab^2-a^2C+3b^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2a\left(2Ab^2-\right.\right.}{}$$

$$= \frac{4a\left(2Ab^2-\left(a^2-3b^2\right)C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(Ab^2-2a^2C+3b^2C\right)\sqrt{}}{3b^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 1.76376, size = 205, normalized size = 0.69

$$\frac{2\left(\frac{b\sin(c+dx)\left(2ab\left(C\left(a^2-3b^2\right)-2Ab^2\right)\cos(c+dx)-5a^2b^2(A+C)+a^4C+Ab^4\right)}{\left(a^2-b^2\right)^2} + \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left((a-b)\left(2a^2C-Ab^2-3b^2C\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(2ab^2(2A+3C)-2a^3\right.\right.}{(a-b)^2}\right.}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-2*a^3*C + 2*a*b^2*(2*A + 3*C))
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (a - b)*(-(A*b^2) + 2*a^2*C - 3*b^
2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (b*(A*b^4 + a^4*C
- 5*a^2*b^2*(A + C) + 2*a*b*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x])*Sin[
c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 1.582, size = 856, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/b^2*a*C/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d
*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2)+2/b^2*(A*b^2+C*a^2)*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x
+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*c
os(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/(b^3*cos(d*x + c)^
3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.667 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{2\left(-a^2b^2(7A+3C)+a^4(-C)+3Ab^4\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(a^2C+Ab^2\right)\sqrt{\frac{a+b\cos(c+d}{a+b}}}{3abd\left(a^2-b^2\right)\sqrt{a+}}$$

```
[Out] (2*(3*A*b^4 - a^4*C - a^2*b^2*(7*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) + (2*(A*b^2 + a^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*b^2 + a^2*C)*S
in[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(3*A*b^4 -
 a^4*C - a^2*b^2*(7*A + 3*C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Rubi [A] time = 1.08117, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(-a^2b^2(7A+3C)+a^4(-C)+3Ab^4\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(a^2C+Ab^2\right)\sqrt{\frac{a+b\cos(c+d}{a+b}}}{3abd\left(a^2-b^2\right)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(3*A*b^4 - a^4*C - a^2*b^2*(7*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) + (2*(A*b^2 + a^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*b^2 + a^2*C)*S
in[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(3*A*b^4 -
 a^4*C - a^2*b^2*(7*A + 3*C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}A\left(a^2-b^2\right)-\frac{3}{2}ab(A+C)\cos(c+dx)+\frac{1}{2}\left(Ab^2+a^2C\right.}{(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\sin(c+}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\sin(c+}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\sin(c+}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2}{3a\left(a^{\cdot}\right.}$$

$$= \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(Ab\right.}{}$$

Mathematica [F] time = 36.6464, size = 0, normalized size = 0.

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),
 x]
```

Maple [A] time = 1.57, size = 875, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b^2+C
*a^2)/a^2/b/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*A/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(-A*b^2-C*a^2)/a/b*(1/6/b/(a-b)/
(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/
(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.668 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=416

$$-\frac{b\left(26a^2Ab^2+a^4(-(3A-8C))-15Ab^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\left(5Ab^2-a^2(3A-2C)\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\left(5Ab^2-a^2(3A-2C)\right)\sqrt{}}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] ((26*a^2*A*b^2 - 15*A*b^4 - a^4*(3*A - 8*C))*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - ((5*A*b^2 - a^2*(3*A - 2*C))*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a
 + b*Cos[c + d*x]]) - (5*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(5*A*
b^2 - a^2*(3*A - 2*C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*
x])^(3/2)) - (b*(26*a^2*A*b^2 - 15*A*b^4 - a^4*(3*A - 8*C))*Sin[c + d*x])/(
3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*(a
+ b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 1.40472, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(26a^2Ab^2+a^4(-(3A-8C))-15Ab^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\left(5Ab^2-a^2(3A-2C)\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\left(5Ab^2-a^2(3A-2C)\right)\sqrt{}}{3a^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((26*a^2*A*b^2 - 15*A*b^4 - a^4*(3*A - 8*C))*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - ((5*A*b^2 - a^2*(3*A - 2*C))*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a
 + b*Cos[c + d*x]]) - (5*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(5*A*
b^2 - a^2*(3*A - 2*C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*
x])^(3/2)) - (b*(26*a^2*A*b^2 - 15*A*b^4 - a^4*(3*A - 8*C))*Sin[c + d*x])/(
3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*(a
```

```
+ b*Cos[c + d*x])^(3/2))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(-\frac{5Ab}{2}+aC\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{a}$$

$$= -\frac{b\left(5Ab^2-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + 2\int \frac{\left(-\frac{15}{4}Ab\right.}{}$$

$$= -\frac{b\left(5Ab^2-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= -\frac{b\left(5Ab^2-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= -\frac{b\left(5Ab^2-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\left(\right.}{3a}$$

$$= \frac{\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(5}{}$$

Mathematica [C] time = 7.03227, size = 786, normalized size = 1.89

$$\frac{\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A\sec^2(c+dx)+C\right)\left(-\frac{4\left(a^2bC\sin(c+dx)+Ab^3\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} - \frac{8\left(5a^2Ab^3\sin(c+dx)+2a^4bC\sin(c+dx)-3Ab^5\sin\right.}{3a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right.}{d(2A+C\cos(2c+2dx)+C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Cos[c + d*x]^2*(C + A*Sec[c + d*x]^2)*((2*(36*a^3*A*b^2 - 20*a*A*b^4 + 12*
a^5*C + 4*a^3*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-33*a^4*A*b + 86*a^2*A*b
^3 - 45*A*b^5 + 8*a^4*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3*a^4*A*b
 + 26*a^2*A*b^3 - 15*A*b^5 + 8*a^4*b*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*
Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*Ellipti
cE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a +
b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*
x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x])
 + 2*(a + b*Cos[c + d*x])^2))))/(6*a^3*(-a + b)^2*(a + b)^2*d*(2*A + C + C*
Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(C + A*Sec[c
+ d*x]^2)*((-4*(A*b^3*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(3*a^2*(a^2 - b
^2)*(a + b*Cos[c + d*x])^2) - (8*(5*a^2*A*b^3*Sin[c + d*x] - 3*A*b^5*Sin[c
+ d*x] + 2*a^4*b*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])
) + (2*A*Tan[c + d*x])/a^3))/(d*(2*A + C + C*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.185, size = 1331, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3
/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*A/a^2*(-1/a*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+4*A/a^3*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))+2*(A*b^2+C*a^2)/a^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d
*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(
3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*
a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
 x)
```

3.669 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=389

$$\frac{2\left(-a^2b^2(23A+19C)+2a^4C-3b^4(3A+5C)\right)\sin(c+dx)}{15bd\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(a^2(-C)+4Ab^2+5b^2C\right)\sin(c+dx)}{15bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2C+Ab\right.}{5bd\left(a^2-b^2\right)(a}$$

```
[Out] (-2*(2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23*A + 19*C))*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*(a^2 - b^2)^3*d*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*(4*A*b^2 - (a^2 - 5*b^2)*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*(
a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])
/(5*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(5/2)) - (4*a*(4*A*b^2 - a^2*C + 5
*b^2*C)*Sin[c + d*x])/(15*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2)) + (
2*(2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23*A + 19*C))*Sin[c + d*x])/(15*b
*(a^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.605387, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3022, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-a^2b^2(23A+19C)+2a^4C-3b^4(3A+5C)\right)\sin(c+dx)}{15bd\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(a^2(-C)+4Ab^2+5b^2C\right)\sin(c+dx)}{15bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2C+Ab\right.}{5bd\left(a^2-b^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23*A + 19*C))*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*(a^2 - b^2)^3*d*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*(4*A*b^2 - (a^2 - 5*b^2)*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*(
a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])
/(5*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(5/2)) - (4*a*(4*A*b^2 - a^2*C + 5
*b^2*C)*Sin[c + d*x])/(15*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2)) + (
2*(2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23*A + 19*C))*Sin[c + d*x])/(15*b
*(a^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3022

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2
- b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2
 + a^2*C + b^2*(A + C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e,
 f, A, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}ab(A+C)+\frac{1}{2}\left(3Ab^2-2a^2C+5b^2C\right)\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{5b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{4a\left(4Ab^2-a^2C+5b^2C\right)\sin(c+dx)}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} + \frac{4\int \frac{\frac{3}{4}b\left(a^2(5A\right.}{}}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{4a\left(4Ab^2-a^2C+5b^2C\right)\sin(c+dx)}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(2a^4C-3\right.}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{4a\left(4Ab^2-a^2C+5b^2C\right)\sin(c+dx)}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(2a^4C-3\right.}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{4a\left(4Ab^2-a^2C+5b^2C\right)\sin(c+dx)}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(2a^4C-3\right.}{}$$

$$= -\frac{2\left(2a^4C-3b^4(3A+5C)-a^2b^2(23A+19C)\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2\left(a^2-b^2\right)^3 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{4a\left(\right.}{}$$

Mathematica [A] time = 2.56816, size = 314, normalized size = 0.81

$$\frac{2\left(\frac{b\sin(c+dx)\left(-4ab\left(-a^2b^2(27A+25C)+3a^4C-5b^4(A+2C)\right)\cos(c+dx)+\left(a^2b^4(23A+19C)-2a^4b^2C+3b^6(3A+5C)\right)\cos(2(c+dx))+68a^4Ab^2+13a^2Ab^4+48a^4b^2C+3\right.}{2\left(b^2-a^2\right)^3}\right.}{15b^2d(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(5/2)*((-2*a^4*C + 3*b^4*(3*A + 5*C) +
a^2*b^2*(23*A + 19*C))*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*a*(a - b)*
(-4*A*b^2 + (a^2 - 5*b^2)*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b
)^3 + (b*(68*a^4*A*b^2 + 13*a^2*A*b^4 + 15*A*b^6 - 2*a^6*C + 48*a^4*b^2*C +
 35*a^2*b^4*C + 15*b^6*C - 4*a*b*(3*a^4*C - 5*b^4*(A + 2*C) - a^2*b^2*(27*A
 + 25*C))*Cos[c + d*x] + (-2*a^4*b^2*C + 3*b^6*(3*A + 5*C) + a^2*b^4*(23*A
+ 19*C))*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(-a^2 + b^2)^3)))/(15*b^2*d*(a
+ b*Cos[c + d*x])^(5/2))
```

Maple [B] time = 2.747, size = 1305, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2/sin
(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(A*b^2+C*a^2)/b^2
*(1/20/b^2/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^3+4/15*a/b/(
a+b)^2/(a-b)^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+2/15*b*sin(1/2*d*x
+1/2*c)^2/(a-b)^3/(a+b)^3*cos(1/2*d*x+1/2*c)*(23*a^2+9*b^2)/(-(-2*cos(1/2*d
*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(15*a^2-8*a*b+9*b^2)/(15*a^5
+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4+15*b^5)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-1/15*(23*a^2+9*b^2)/(a+b)^3/(a-b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-4*a*C/b^2*(1/6/b/(a-
b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)
^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
```

```
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}}{b^4\cos(dx+c)^4 + 4\,ab^3\cos(dx+c)^3 + 6\,a^2b^2\cos(dx+c)^2 + 4\,a^3b\cos(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/(b^4*cos(d*x + c)^
4 + 4*a*b^3*cos(d*x + c)^3 + 6*a^2*b^2*cos(d*x + c)^2 + 4*a^3*b*cos(d*x + c
) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(7/2), x)
```

3.670 $\int \frac{a^2-b^2\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=157

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c +
d*x]]) - (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.233376, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3016, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c +
d*x]]) - (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a^2-b^2\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\int(-a+b\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,dx$$
$$= -\frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2}{3}\int \frac{\frac{1}{2}\left(-3a^2+b^2\right)-ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= -\frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(2a)\int\sqrt{a+b\cos(c+dx)}\,dx - \frac{1}{3}\left(-a^2+b^2\right)\int \frac{}{\sqrt{a}}$$
$$= -\frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(2a\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{((-}{}$$
$$= \frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} - \frac{2b\sqrt{}}{}$$

Mathematica [A] time = 0.497941, size = 134, normalized size = 0.85

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)(a+b\cos(c+dx))+4a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] - 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.718, size = 450, normalized size = 2.9

$$\frac{2}{3\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2b+a-b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,\left(\cos\left(1/2\,dx+c/2\right)\right)^5b^2+2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^3ab-6\,\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*
d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2
*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/2
*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^2 \cos{(dx+c)}^2 - a^2}{\sqrt{b\cos{(dx+c)} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*cos(d*x + c)^2 - a^2)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\sqrt{b\cos{(dx+c)}+a}(b\cos{(dx+c)}-a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*cos(d*x + c) + a)*(b*cos(d*x + c) - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2 \cos (dx+c)^2 - a^2}{\sqrt{b \cos (dx+c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*cos(d*x + c)^2 - a^2)/sqrt(b*cos(d*x + c) + a), x)
```

3.671 $\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (-2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.157303, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3016, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= -\int \frac{-a+b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx\\
&= (2a)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx-\int \sqrt{a+b\cos(c+dx)}\,dx\\
&= -\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}\\
&= -\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.165805, size = 83, normalized size = 0.72

$$-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[
c + d*x]])
```

Maple [A] time = 0.475, size = 218, normalized size = 1.9

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}\sqrt{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(2*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{b^{2}\cos\left(dx+c\right)^{2}-a^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*cos(d*x + c)^2 - a^2)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\cos\left(dx+c\right)-a}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*cos(d*x + c) - a)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2\cos\left(dx+c\right)^2-a^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*cos(d*x + c)^2 - a^2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.672 $\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{4ab\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2
- b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) - (4*a*b*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.239053, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3016, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4ab\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2
- b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) - (4*a*b*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\int \frac{-a+b\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{4ab\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2+b^2\right)+ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{4ab\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(2a)\int \sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2} - \int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{4ab\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\ldots}{\ldots}$$

$$= \frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} - \frac{4ab\sin\ldots}{\left(a^2-b^2\right)d\sqrt{\ldots}}$$

Mathematica [A] time = 0.355232, size = 134, normalized size = 0.81

$$\frac{-2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-4ab\sin(c+dx)+4a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] - 4*a*b*Sin[c + d*x])/((a - b)*(a + b)*d*Sqrt[a +
b*Cos[c + d*x]])
```

Maple [A] time = 0.533, size = 371, normalized size = 2.3

$$2\,\frac{1}{(a+b)\,(a-b)\sin(1/2\,dx+c/2)\,\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,b+a+b}\,d}\left(\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{-2\,\frac{b}{a-b}}\right)\sqrt{(\sin\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a^2-EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^2-2*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*a^2+2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*a*b-4*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(a-b)/(a+b
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^2 \cos(dx+c)^2 - a^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*cos(d*x + c)^2 - a^2)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\cos(dx+c)+a}\,(b\cos(dx+c)-a)}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*cos(d*x + c) + a)*(b*cos(d*x + c) - a)/(b^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2\cos\left(dx+c\right)^2-a^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*cos(d*x + c)^2 - a^2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.673 $\int \frac{a^2-b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{2b\left(5a^2+3b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{4ab\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(5a^2+\right.}{}$$

```
[Out] (2*(5*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*b*Sin[c + d*x])/(3*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*(5*a^2 + 3*b^2)*Sin[c + d*x])/(3*(a^2
- b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.34118, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3016, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b\left(5a^2+3b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{4ab\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(5a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(7/2),x]
```


```
[Out] (2*(5*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*b*Sin[c + d*x])/(3*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*(5*a^2 + 3*b^2)*Sin[c + d*x])/(3*(a^2
- b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a^2 - b^2\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx = -\int \frac{-a+b\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{4ab\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}\left(a^2+b^2\right)-ab\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{4ab\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(5a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}a\left(3a^2+5b^2\right)}{\sqrt{a}}}{3(}$$

$$= -\frac{4ab\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(5a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{(2a)\int \frac{1}{\sqrt{a+b\cos(}}}{3\left(a^2-b^2\right.}$$

$$= -\frac{4ab\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(5a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\left(5a^2+3b^2\right)\sqrt{\right.}{}$$

$$= \frac{2\left(5a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} -$$

Mathematica [A] time = 0.979563, size = 158, normalized size = 0.65

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2a(b-a)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{(a-b)^2} - \frac{b\sin(c+dx)\left(b\left(5a^2+3b^2\right)\cos(c+dx)+a\left(7a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2}\right)}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((5*a^2 + 3*b^2)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] + 2*a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
)/(a - b)^2 - (b*(a*(7*a^2 + b^2) + b*(5*a^2 + 3*b^2)*Cos[c + d*x])*Sin[c +
 d*x])/(a^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 1.384, size = 792, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b*sin(1/
2*d*x+1/2*c)^2/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2*b-
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+
2/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2)))+4*a*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-
b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(
-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3
*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^2 \cos(dx+c)^2 - a^2}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*cos(d*x + c)^2 - a^2)/(b*cos(d*x + c) + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\cos(dx+c)+a}\,(b\cos(dx+c)-a)}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*cos(d*x + c) + a)*(b*cos(d*x + c) - a)/(b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2\cos(dx+c)^2 - a^2}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*cos(d*x + c)^2 - a^2)/(b*cos(d*x + c) + a)^(7/2), x)
```

3.674 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=196

$$\frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(9A+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2aC\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{10b(11A+9C)}{23\ldots}$$

```
[Out] (2*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*b*(11*A + 9*C)*Ell
ipticF[(c + d*x)/2, 2])/(231*d) + (10*b*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(231*d) + (2*a*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*
d) + (2*b*(11*A + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*a*C*Cos
[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*b*C*Cos[c + d*x]^(9/2)*Sin[c + d*x
])/(11*d)
```

Rubi [A] time = 0.237284, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3034, 3023, 2748, 2635, 2639, 2641}

$$\frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(9A+7C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2aC\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{10b(11A+9C)}{23\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*b*(11*A + 9*C)*Ell
ipticF[(c + d*x)/2, 2])/(231*d) + (10*b*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(231*d) + (2*a*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*
d) + (2*b*(11*A + 9*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*a*C*Cos
[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*b*C*Cos[c + d*x]^(9/2)*Sin[c + d*x
])/(11*d)
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
```

```
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{2bC\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{2}{11}\int \cos^{\frac{5}{2}}(c+dx)\left(\frac{11aA}{2}\right.$$
$$= \frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2bC\cos^{\frac{9}{2}}(c+dx)\sin(c+}{11d}$$
$$= \frac{2aC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2bC\cos^{\frac{9}{2}}(c+dx)\sin(c+}{11d}$$
$$= \frac{2a(9A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2b(11A+9C)\cos}{}$$
$$= \frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10b(11A+9C)\sqrt{\cos(c+}}{231d}$$
$$= \frac{2a(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10b(11A+9C)F\left(\frac{1}{2}(c+d\right.}{231d}$$

Mathematica [A] time = 1.64802, size = 134, normalized size = 0.68

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(154a(36A+43C)\cos(c+dx)+770aC\cos(3(c+dx))+180b(11A+16C)\cos(2(c+dx))+858}{13860d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (1848*a*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 600*b*(11*A + 9*C)*Elliptic
F[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(8580*A*b + 7965*b*C + 154*a*(36*A +
 43*C)*Cos[c + d*x] + 180*b*(11*A + 16*C)*Cos[2*(c + d*x)] + 770*a*C*Cos[3*
(c + d*x)] + 315*b*C*Cos[4*(c + d*x)])*Sin[c + d*x])/(13860*d)
```

Maple [B] time = 0.355, size = 481, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*C*a-50400*C*b)*sin(1/2*d*x
+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b+24640*C*a+56880*C*b)*sin(1/2*d*x+1/
2*c)^8*cos(1/2*d*x+1/2*c)+(-5544*A*a-11880*A*b-22792*C*a-34920*C*b)*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(5544*A*a+9240*A*b+10472*C*a+13860*C*b)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1386*A*a-2640*A*b-1848*C*a-2790*C*b
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
+825*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+675*C*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^5+Ca\cos\left(dx+c\right)^4+Ab\cos\left(dx+c\right)^3+Aa\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^5 + C*a*cos(d*x + c)^4 + A*b*cos(d*x + c)^3 + A*
a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x
)
```

3.675 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=165

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(7A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aC\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2b(9A+7C)\dots}{1\dots}$$

```
[Out] (2*b*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*b*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (
2*a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*C*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```

Rubi [A] time = 0.207169, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3034, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(7A+5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aC\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2b(9A+7C)\dots}{1\dots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*b*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*b*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (
2*a*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*C*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{2bC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{9aA}{2}\right.$$
$$=\frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2bC\cos^{\frac{7}{2}}(c+dx)\sin(c+$$
$$=\frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2bC\cos^{\frac{7}{2}}(c+dx)\sin(c+$$
$$=\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2b(9A+7C)\,\mathrm{cc}}{}$$
$$=\frac{2b(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Mathematica [A] time = 0.91207, size = 119, normalized size = 0.72

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(5(84aA+18aC\cos(2(c+dx))+78aC+7bC\cos(3(c+dx)))+7b(36A+43C)\cos(c+dx))+}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (84*b*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 60*a*(7*A + 5*C)*EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(36*A + 43*C)*Cos[c + d*x] + 5*(84*
a*A + 78*a*C + 18*a*C*Cos[2*(c + d*x)] + 7*b*C*Cos[3*(c + d*x)]))*Sin[c + d
*x])/(630*d)
```

Maple [B] time = 0.431, size = 443, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*C*a+2240*C*b)*sin(1/2*d*x+1/2*
```

```
c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b-1080*C*a-2072*C*b)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(420*A*a+504*A*b+840*C*a+952*C*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-210*A*a-126*A*b-240*C*a-168*C*b)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+105*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*b+75*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^4+Ca\cos\left(dx+c\right)^3+Ab\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + C*a*cos(d*x + c)^3 + A*b*cos(d*x + c)^2 + A*
a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x
)
```

3.676 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=134

$$\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aC\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2b(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(7A+5C)\sin(c+dx)}{21d}$$

```
[Out] (2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(7*A + 5*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(21*d) + (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b*C*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.185441, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3034, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aC\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2b(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(7A+5C)\sin(c+dx)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(7*A + 5*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(21*d) + (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b*C*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx = \frac{2bC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{\cos(c+dx)}\left(\frac{7aA}{2}\right.$$
$$= \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2bC\cos^{\frac{5}{2}}(c+dx)\sin(c+$$
$$= \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2bC\cos^{\frac{5}{2}}(c+dx)\sin(c+$$
$$= \frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b(7A+5C)\sqrt{\cos(c+dx}}{21d}$$
$$= \frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b(7A+5C)F\left(\frac{1}{2}(c+dx)\right.}{21d}$$

Mathematica [A] time = 0.677965, size = 98, normalized size = 0.73

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(42aC\cos(c+dx)+70Ab+15bC\cos(2(c+dx))+65bC)+42a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (42*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 10*b*(7*A + 5*C)*EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A*b + 65*b*C + 42*a*C*Cos[c + d*x] +
 15*b*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 0.506, size = 401, normalized size = 3.

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-168\,aC-360\,C\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*C*a-360*C*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A*b+168*C*a+280*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-70*A*b-42*C*a-80*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+35*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+25*C*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.677 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bC\sin(c+dx)\cos^{\frac{3}{2}}(c$$

```
[Out] (2*b*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + C)*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*b*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.172979, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3034, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bC\sin(c+dx)\cos^{\frac{3}{2}}(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*b*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + C)*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*b*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{5aA}{2}+\frac{1}{2}b(5A+3C)\cos(c+dx)+}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{4}{15}\int$$

$$= \frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(a(3$$

$$= \frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + 2aC\sqrt{\text{co}}$$

Mathematica [A] time = 0.416156, size = 79, normalized size = 0.78

$$\frac{2\left(5a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(c+dx)\sqrt{\cos(c+dx)}(5a+3b\cos(c+dx))+3b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (2*(3*b*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*a*(3*A + C)*EllipticF[(c
+ d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*(5*a + 3*b*Cos[c + d*x])*Sin[c + d*x]))
/(15*d)
```

Maple [B] time = 0.387, size = 363, normalized size = 3.6

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(20\,aC+24\,Cb\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*C*a+24*C*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*C*a-6*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15
*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+5*a*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)(b\cos(dx+c) + a)}{\sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x
)
```

3.678 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (-2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(3*A + C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*b*C*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.173419, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3032, 3023, 2748, 2641, 2639}

$$-\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(3*A + C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*b*C*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{Ab}{2}-\frac{1}{2}a(A-C)\cos(c+dx)+\frac{1}{2}bC\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{4}{3}\int \frac{\frac{1}{4}b(3A+C)-}{\sqrt{c}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}-(a(A-C))\int \sqrt{\cos}$$

$$= -\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(}{d\sqrt{\cos(c}}$$

Mathematica [A] time = 0.468352, size = 78, normalized size = 0.82

$$\frac{\frac{2\sin(c+dx)(3aA+bC\cos(c+dx))}{\sqrt{\cos(c+dx)}}+6a(C-A)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2b(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (6*a*(-A + C)*EllipticE[(c + d*x)/2, 2] + 2*b*(3*A + C)*EllipticF[(c + d*x)
/2, 2] + (2*(3*a*A + b*C*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(3
*d)
```

Maple [B] time = 0.388, size = 294, normalized size = 3.1

$$-\frac{2}{3d}\left(4Cb\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+3\,Ab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*C*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-6*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+C*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*C*b
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x
)
```

3.679 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2Ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*b*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.185646, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3032, 3021, 2748, 2641, 2639}

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2Ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```



```
[Out] (-2*b*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*C)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*b*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3Ab}{2}+\frac{1}{2}a(A+3C)\cos(c+dx)+\frac{3}{2}bC\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2Ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4}{3}\int \frac{\frac{1}{4}a(A+3C)-\frac{3}{4}b(A-C)}{\sqrt{\cos(c+dx)}}$$

$$=\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2Ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-(b(A-C))\int \sqrt{\cos(c+dx)}\,dx$$

$$=-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin}{3d\cos^{\frac{3}{2}}}$$

Mathematica [A] time = 0.759188, size = 76, normalized size = 0.8

$$\frac{\frac{2A\sin(c+dx)(a+3b\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}+2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6b(C-A)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (6*b*(-A + C)*EllipticE[(c + d*x)/2, 2] + 2*a*(A + 3*C)*EllipticF[(c + d*x)
/2, 2] + (2*A*(a + 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d
)
```

Maple [B] time = 0.891, size = 614, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a*sin(1/2*d*x+1/2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b*sin(1/2*
d*x+1/2*c)^2-12*A*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*C*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a*sin(1/2*d*x+1/2*c)^2-6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b*sin(1/2
*d*x+1/2*c)^2-a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+2
*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*A*b*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2-3*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x
)
```

3.680 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$-\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2Ab\sin}{3d\cos^{\frac{3}{2}}}$$

```
[Out] (-2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(A + 3*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (2*A*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*C)*Sin[c + d
*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.195111, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3032, 3021, 2748, 2636, 2639, 2641}

$$-\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(3A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2Ab\sin}{3d\cos^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(A + 3*C)*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (2*A*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*C)*Sin[c + d
*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5Ab}{2}+\frac{1}{2}a(3A+5C)\cos(c+dx)+\frac{5}{2}bC\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2Ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4}{15}\int \frac{\frac{3}{4}a(3A+5C)+\frac{5}{4}b(A+3}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2Ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(b(A+3C))\int \frac{1}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2b(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2Ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA}{5d\,\mathrm{co}}$$

Mathematica [A] time = 0.733206, size = 122, normalized size = 0.92

$$\frac{-6a(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9aA\sin(2(c+dx))+6aA\tan(c+dx)+15aC\sin(2(c+dx))+10b(A+3}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*a*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(A +
3*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] + 9
*a*A*Sin[2*(c + d*x)] + 15*a*C*Sin[2*(c + d*x)] + 6*a*A*Tan[c + d*x])/(15*d
*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.153, size = 732, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*a
*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1
)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/
2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*a*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2
*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x
)
```

3.681 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}-\frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(3A+\ldots}{5d\sqrt{\ldots}}$$

```
[Out] (-2*b*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)
) + (2*A*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(5*A + 7*C)*Sin[c
+ d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*b*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqr
t[Cos[c + d*x]])
```

Rubi [A] time = 0.214679, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3032, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}-\frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(3A+\ldots}{5d\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-2*b*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)
) + (2*A*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(5*A + 7*C)*Sin[c
+ d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*b*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqr
t[Cos[c + d*x]])
```


Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
```

```
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{7Ab}{2}+\frac{1}{2}a(5A+7C)\cos(c+dx)+\frac{7}{2}bC\cos^2(c-}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2Ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{35}\int \frac{\frac{5}{4}a(5A+7C)+\frac{7}{4}b(3A+5}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2Ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(b(3A+5C))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}d$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2Ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(5A+7C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b(3}{5}$$

$$= -\frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2aA}{7d\cos}$$

Mathematica [A] time = 0.773908, size = 160, normalized size = 0.97

$$\frac{10a(5A+7C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+25aA\sin(2(c+dx))+30aA\tan(c+dx)+35aC\sin(2(c+dx))-42b(3A-}{105d\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] (-42*b*(3*A + 5*C)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(5*A
 + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 42*A*b*Sin[c + d*x]
+ 126*A*b*Cos[c + d*x]^2*Sin[c + d*x] + 210*b*C*Cos[c + d*x]^2*Sin[c + d*x]
 + 25*a*A*Sin[2*(c + d*x)] + 35*a*C*Sin[2*(c + d*x)] + 30*a*A*Tan[c + d*x])
/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 1.403, size = 841, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*A*b/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*A
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+2*C*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c
)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x
)
```

3.682 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=254

$$\frac{2\left(11a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{2\left(11a^2(7A\right.}{}$$

```
[Out] (4*a*b*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(11*a^2*(7*A + 5*
C) + 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (2*(11*a^2*(7
*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) +
(4*a*b*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(4*a^2*C +
b^2*(11*A + 9*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (8*a*b*C*Cos[c
+ d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*C*Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.479593, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(11a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{2\left(11a^2(7A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (4*a*b*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(11*a^2*(7*A + 5*
C) + 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (2*(11*a^2*(7
*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) +
(4*a*b*(9*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(4*a^2*C +
b^2*(11*A + 9*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (8*a*b*C*Cos[c
+ d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*C*Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(11*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
```

```
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2}{11}\int$$

$$= \frac{8abC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+b}{}$$

$$= \frac{2\left(4a^2C+b^2(11A+9C)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d} + \frac{8a}{}$$

$$= \frac{2\left(4a^2C+b^2(11A+9C)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d} + \frac{8a}{}$$

$$= \frac{2\left(11a^2(7A+5C)+5b^2(11A+9C)\right)\sqrt{\cos(c+dx)}\sin(c}{231d}$$

$$= \frac{4ab(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2\left(11a^2(7A+5C)+5b\right.}{}$$

Mathematica [A] time = 1.50415, size = 187, normalized size = 0.74

$$\frac{240\left(11a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\sqrt{\cos(c+dx)}\left(5\left(36\left(11a^2C+11Ab^2+16b^2C\right)c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (7392*a*b*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 240*(11*a^2*(7*A + 5*C) +
 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(308*
a*b*(36*A + 43*C)*Cos[c + d*x] + 5*(132*a^2*(14*A + 13*C) + 3*b^2*(572*A +
531*C) + 36*(11*A*b^2 + 11*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 308*a*b*C*C
os[3*(c + d*x)] + 63*b^2*C*Cos[4*(c + d*x)]))*Sin[c + d*x])/(27720*d)
```

Maple [B] time = 0.502, size = 649, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-24640*C*a*b-50400*C*b^2)*sin(1
/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^2+7920*C*a^2+49280*C*a*b+5688
0*C*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-11088*A*a*b-11880*A*b^2-
11880*C*a^2-45584*C*a*b-34920*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)+(4620*A*a^2+11088*A*a*b+9240*A*b^2+9240*C*a^2+20944*C*a*b+13860*C*b^2)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310*A*a^2-2772*A*a*b-2640*A*b^2-26
40*C*a^2-3696*C*a*b-2790*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-415
8*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+1155*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A*
b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-3234*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+825*a^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+675*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos(dx+c)^5+2\,Cab\cos(dx+c)^4+2\,Aab\cos(dx+c)^2+Aa^2\cos(dx+c)+\left(Ca^2+Ab^2\right)\cos(dx+c)^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5 + 2*C*a*b*cos(d*x + c)^4 + 2*A*a*b*cos(d*x +
 c)^2 + A*a^2*cos(d*x + c) + (C*a^2 + A*b^2)*cos(d*x + c)^3)*sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
 x)
```

3.683 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=205

$$\frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(4a^2C+b^2(9A+7C)\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{4ab(7A+5C)F}{21}$$

```
[Out] (2*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d)
+ (4*a*b*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a*b*(7*A + 5*C)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(4*a^2*C + b^2*(9*A + 7*C))*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a*b*C*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(63*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x
])/(9*d)
```

Rubi [A] time = 0.424298, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(4a^2C+b^2(9A+7C)\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{4ab(7A+5C)F}{21}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d)
+ (4*a*b*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a*b*(7*A + 5*C)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(4*a^2*C + b^2*(9*A + 7*C))*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a*b*C*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(63*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x
])/(9*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{9d}+\frac{2}{9}\int \sqrt{}$$

$$=\frac{8abC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\,c}{9}$$

$$=\frac{2\left(4a^2C+b^2(9A+7C)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{8abC}{}$$

$$=\frac{2\left(4a^2C+b^2(9A+7C)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{8abC}{}$$

$$=\frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4ab}{}$$

$$=\frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4ab}{}$$

Mathematica [A] time = 1.18616, size = 148, normalized size = 0.72

$$\frac{84\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7\left(36a^2C+36Ab^2+43b^2C\right)\cos(c+dx)\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (84*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 120*a
*b*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A*b^2
+ 36*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*b*(168*a*A + 156*a*C + 36*a*C*Cos[2
*(c + d*x)] + 7*b*C*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 0.401, size = 587, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1440*C*a*b+2240*C*b^2)*sin(1/2*d
*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^2-504*C*a^2-2160*C*a*b-2072*C*b^2)
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(840*A*a*b+504*A*b^2+504*C*a^2+168
0*C*a*b+952*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-420*A*a*b-126*
A*b^2-126*C*a^2-480*C*a*b-168*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+210*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*b^2+150*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
 x)
```

3.684 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{2\left(7a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(4a^2C+b^2(7A+5C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{4ab(5A+3C)E}{5}$$

```
[Out] (4*a*b*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2*(3*A + C) +
 b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(4*a^2*C + b^2*(7*
A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a*b*C*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(35*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*
Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.408697, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3050, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(7a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(4a^2C+b^2(7A+5C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{4ab(5A+3C)E}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (4*a*b*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2*(3*A + C) +
 b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(4*a^2*C + b^2*(7*
A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a*b*C*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(35*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*
Sin[c + d*x])/(7*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \frac{(a+b\cos(c$$

$$= \frac{8abC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{7d}$$

$$= \frac{2\left(4a^2C+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{8abC\cos^{\frac{3}{2}}(c+}{35}$$

$$= \frac{2\left(4a^2C+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{8abC\cos^{\frac{3}{2}}(c+}{35}$$

$$= \frac{4ab(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2(3A+C)+b^2(7A+5C)\right)F\left(\right.}{21d}$$

Mathematica [A] time = 0.985088, size = 126, normalized size = 0.74

$$\frac{10\left(7a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(70a^2C+84abC\cos(c+dx)+70Ab^2+15\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (84*a*b*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2*(3*A + C) + b^2*(
7*A + 5*C))*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A*b^2 + 70*a
^2*C + 65*b^2*C + 84*a*b*C*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c
+ d*x])/(105*d)
```

Maple [B] time = 0.4, size = 532, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-336*C*a*b-360*C*b^2)*sin(1/2*d*x+1
/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^2+140*C*a^2+336*C*a*b+280*C*b^2)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*b^2-70*C*a^2-84*C*a*b-80*C*b^2)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35
*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-210*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+35*a^2*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+25*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
 x)
```

3.685 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$-\frac{2\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4ab(3A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (-2*(5*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (4
*a*b*(3*A + C)*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a*b*(3*A - C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*d) - (2*b^2*(5*A - C)*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(5*d) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]])
```

Rubi [A] time = 0.406133, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3048, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4ab(3A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```



```
[Out] (-2*(5*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (4
*a*b*(3*A + C)*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a*b*(3*A - C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*d) - (2*b^2*(5*A - C)*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(5*d) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]])
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
```

```
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+dx))\left(2Ab-\right.}{}$$

$$= -\frac{2b^2(5A-C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2A(a+b\cos(c+dx))^2\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{4ab(3A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(5A-C)\cos^{\frac{3}{2}}(c+dx)\,\text{si}}{5d}$$

$$= -\frac{4ab(3A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(5A-C)\cos^{\frac{3}{2}}(c+dx)\,\text{si}}{5d}$$

$$= -\frac{2\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4ab(3A+C)F\left(\frac{1}{2}(\right.}{3d}$$

Mathematica [A] time = 1.03038, size = 119, normalized size = 0.72

$$\frac{-6\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(30a^2A+20abC\cos(c+dx)+3b^2C\cos(2(c+dx))+3b^2C\right)}{\sqrt{\cos(c+dx)}}+20ab(3A+C)F\left(\frac{1}{2}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (-6*(5*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 20*a*b*(3
*A + C)*EllipticF[(c + d*x)/2, 2] + ((30*a^2*A + 3*b^2*C + 20*a*b*C*Cos[c +
 d*x] + 3*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] time = 0.457, size = 694, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*b*(5*a+3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^2+10*C*a*b+3*C*b^2
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-15*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b
^2+10*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
 x)
```

3.686 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{2\left(a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4ab(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a}{3}$$

```
[Out] (-4*a*b*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*(b^2*(3*A + C) + a^2*(A +
 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (8*a*A*b*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]) - (2*b^2*(A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2
*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.40021, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3048, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\left(a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4ab(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-4*a*b*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*(b^2*(3*A + C) + a^2*(A +
 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (8*a*A*b*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]) - (2*b^2*(A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2
*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+dx))\left(2Ab \right.}{}$$

$$= \frac{8aAb\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{3}\int \frac{\frac{1}{4}\left(-4\right.}{}$$

$$= \frac{8aAb\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+b}{}$$

$$= \frac{8aAb\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+b}{}$$

$$= -\frac{4ab(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(b^2(3A+C)+a^2(A+3C)\right)F\left(\frac{1}{2}(c\right.}{3d}$$

Mathematica [A] time = 1.41676, size = 108, normalized size = 0.7

$$\frac{2\left(a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right) + \frac{2a^2A\tan(c+dx)+12aAb\sin(c+dx)+b^2C\sin(2(c+dx))}{\sqrt{\cos(c+dx)}} + 12ab(C-A)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (12*a*b*(-A + C)*EllipticE[(c + d*x)/2, 2] + 2*(b^2*(3*A + C) + a^2*(A + 3*
C))*EllipticF[(c + d*x)/2, 2] + (12*a*A*b*Sin[c + d*x] + b^2*C*Sin[2*(c + d
*x)] + 2*a^2*A*Tan[c + d*x])/Sqrt[Cos[c + d*x]])/(3*d)
```

Maple [B] time = 0.493, size = 871, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-8*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*b*(3*A*a+C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*a^2+6*A*a*b+C*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*b^2+6*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+3*C*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)*sin(1/2*d*x+1/2*c)^2+A*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+6*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a*b+3*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-6*C*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1
/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos(dx+c)^4+2\,Cab\cos(dx+c)^3+2\,Aab\cos(dx+c)+Aa^2+\left(Ca^2+Ab^2\right)\cos(dx+c)^2}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
 x)
```

3.687 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=169

$$-\frac{2\left(a^2(3A+5C)+5b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(a^2(3A+5C)+4Ab^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4ab(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] (-2*(5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (4
*a*b*(A + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (8*a*A*b*Sin[c + d*x])/(1
5*d*Cos[c + d*x]^(3/2)) + (2*(4*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c
 + d*x]^(5/2))
```

Rubi [A] time = 0.408209, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3048, 3031, 3021, 2748, 2641, 2639}

$$-\frac{2\left(a^2(3A+5C)+5b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(a^2(3A+5C)+4Ab^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4ab(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (4
*a*b*(A + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (8*a*A*b*Sin[c + d*x])/(1
5*d*Cos[c + d*x]^(3/2)) + (2*(4*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c
 + d*x]^(5/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(2Ab+\right.}{}$$

$$= \frac{8aAb\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}(4}{}$$

$$= \frac{8aAb\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(4Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2A(a+b\,c}{5}$$

$$= \frac{8aAb\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(4Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2A(a+b\,c}{5}$$

$$= -\frac{2\left(5b^2(A-C)+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4ab(A+3C)F\left(\frac{1}{2}(\right.}{3d}$$

Mathematica [A] time = 0.833093, size = 158, normalized size = 0.93

$$\frac{-6\left(a^2(3A+5C)+5b^2(A-C)\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9a^2A\sin(2(c+dx))+6a^2A\tan(c+dx)+15a^2C\sin(2(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] (-6*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x
)/2, 2] + 20*a*b*(A + 3*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2
0*a*A*b*Sin[c + d*x] + 9*a^2*A*Sin[2*(c + d*x)] + 15*A*b^2*Sin[2*(c + d*x)]
 + 15*a^2*C*Sin[2*(c + d*x)] + 6*a^2*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/
2))
```

Maple [B] time = 1.161, size = 913, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+4*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*A*a^2/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1
/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+4*a*A*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2
*(A*b^2+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*
x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos(dx+c)^4+2\,Cab\cos(dx+c)^3+2\,Aab\cos(dx+c)+Aa^2+\left(Ca^2+Ab^2\right)\cos(dx+c)^2}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
 x)
```

3.688 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=203

$$\frac{2\left(a^2(5A+7C)+7b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(a^2(5A+7C)+4Ab^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{4ab(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (-4*a*b*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*b^2*(A + 3*C)
+ a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a*A*b*Sin[c + d*x
])/(35*d*Cos[c + d*x]^(5/2)) + (2*(4*A*b^2 + a^2*(5*A + 7*C))*Sin[c + d*x])
/(21*d*Cos[c + d*x]^(3/2)) + (4*a*b*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos
[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(
7/2))
```

Rubi [A] time = 0.44869, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(a^2(5A+7C)+7b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(a^2(5A+7C)+4Ab^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{4ab(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-4*a*b*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*b^2*(A + 3*C)
+ a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a*A*b*Sin[c + d*x
])/(35*d*Cos[c + d*x]^(5/2)) + (2*(4*A*b^2 + a^2*(5*A + 7*C))*Sin[c + d*x])
/(21*d*Cos[c + d*x]^(3/2)) + (4*a*b*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos
[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(
7/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
```

```
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int\frac{(a+b\cos(c+dx))\left(2Ab\cdots\right.}{\cdots}$$

$$= \frac{8aAb\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}-\frac{4}{35}\int\frac{-\frac{5}{4}\left(\cdots\right.}{\cdots}$$

$$= \frac{8aAb\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(4Ab^2+a^2(5A+7C)\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\cdots}{\cdots}$$

$$= \frac{8aAb\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(4Ab^2+a^2(5A+7C)\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\cdots}{\cdots}$$

$$= \frac{2\left(7b^2(A+3C)+a^2(5A+7C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{8aAb\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}\cdots$$

$$= -\frac{4ab(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7b^2(A+3C)+a^2(5A+7C)\right)F\cdots}{21d}$$

Mathematica [A] time = 1.14537, size = 198, normalized size = 0.98

$$\frac{10\left(a^2(5A+7C)+7b^2(A+3C)\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+25a^2A\sin(2(c+dx))+30a^2A\tan(c+dx)+35a^2C\sin\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] (-84*a*b*(3*A + 5*C)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*b
^2*(A + 3*C) + a^2*(5*A + 7*C))*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2
] + 84*a*A*b*Sin[c + d*x] + 252*a*A*b*Cos[c + d*x]^2*Sin[c + d*x] + 420*a*b
*C*Cos[c + d*x]^2*Sin[c + d*x] + 25*a^2*A*Sin[2*(c + d*x)] + 35*A*b^2*Sin[2
*(c + d*x)] + 35*a^2*C*Sin[2*(c + d*x)] + 30*a^2*A*Tan[c + d*x])/(105*d*Cos
[c + d*x]^(5/2))
```

Maple [B] time = 1.512, size = 930, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5
*a*A*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(A*b^2+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2
*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+2*A*a^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))+4*a*b*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2),
 x)
```

3.689 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=295

$$\frac{2b\left(33a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2a\left(a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(8a^2C\right.}{}$$

```
[Out] (2*a*(a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(5*d) +
 (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2])/
(231*d) + (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(231*d) + (2*a*(99*A*b^2 + 8*a^2*C + 77*b^2*C)*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(165*d) + (2*b*(8*a^2*C + 3*b^2*(11*A + 9*C))*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(231*d) + (4*a*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(33*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^3*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.759514, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3050, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2b\left(33a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2a\left(a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(8a^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*a*(a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(5*d) +
 (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2])/
(231*d) + (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(231*d) + (2*a*(99*A*b^2 + 8*a^2*C + 77*b^2*C)*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(165*d) + (2*b*(8*a^2*C + 3*b^2*(11*A + 9*C))*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(231*d) + (4*a*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(33*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^3*Sin[c + d*x])/(11*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{11d}+\frac{2}{11}\int$$

$$= \frac{4aC\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{33d}+\frac{2C}{}$$

$$= \frac{2b\left(8a^2C+3b^2(11A+9C)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{231d}+$$

$$= \frac{2a\left(99Ab^2+8a^2C+77b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{165d}+$$

$$= \frac{2a\left(99Ab^2+8a^2C+77b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{165d}+$$

$$= \frac{2a\left(a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b}{}$$

$$= \frac{2a\left(a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b}{}$$

Mathematica [A] time = 1.5688, size = 215, normalized size = 0.73

$$\frac{80b\left(33a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(a^3(5A+3C)+ab^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (3696*(a^3*(5*A + 3*C) + a*b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 80*
b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2] + 2*S
qrt[Cos[c + d*x]]*(154*a*(36*A*b^2 + 12*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*
b*(1848*a^2*A + 572*A*b^2 + 1716*a^2*C + 531*b^2*C + 12*(11*A*b^2 + 33*a^2*
C + 16*b^2*C)*Cos[2*(c + d*x)] + 154*a*b*C*Cos[3*(c + d*x)] + 21*b^2*C*Cos[
4*(c + d*x)]))*Sin[c + d*x])/(9240*d)
```

Maple [B] time = 0.395, size = 793, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/1155*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6720*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*C*a*b^2-16800*C*b^3)*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(2640*A*b^3+7920*C*a^2*b+24640*C*a*b^2
+18960*C*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-5544*A*a*b^2-3960*A
*b^3-1848*C*a^3-11880*C*a^2*b-22792*C*a*b^2-11640*C*b^3)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(4620*A*a^2*b+5544*A*a*b^2+3080*A*b^3+1848*C*a^3+9240
*C*a^2*b+10472*C*a*b^2+4620*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-2310*A*a^2*b-1386*A*a*b^2-880*A*b^3-462*C*a^3-2640*C*a^2*b-1848*C*a*b^2-9
30*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*A*a^2*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+275*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1155*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a^3-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+825*a^2*b*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
```

```
2^(1/2))+225*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-693*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a^3-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+3\,Cab^2\cos\left(dx+c\right)^4+3\,Aa^2b\cos\left(dx+c\right)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.690 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=245

$$\frac{2a\left(7a^2(3A+C)+3b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\left(9a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(24a^2C\right.}{}$$

```
[Out] (2*b*(9*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d
) + (2*a*(7*a^2*(3*A + C) + 3*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(
21*d) + (2*a*(63*A*b^2 + 8*a^2*C + 45*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(63*d) + (2*b*(24*a^2*C + 7*b^2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(315*d) + (4*a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*
x])/(21*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(
9*d)
```

Rubi [A] time = 0.702488, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2a\left(7a^2(3A+C)+3b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\left(9a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(24a^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```



```
[Out] (2*b*(9*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d
) + (2*a*(7*a^2*(3*A + C) + 3*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(
21*d) + (2*a*(63*A*b^2 + 8*a^2*C + 45*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(63*d) + (2*b*(24*a^2*C + 7*b^2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(315*d) + (4*a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*
x])/(21*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(
9*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
```

```
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\sin(c+dx)}{9d}+\frac{2}{9}\int \frac{(a+b\cos(c}{}$$

$$= \frac{4aC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{21d}+\frac{2C\sqrt{\cos(c+dx)}}{}$$

$$= \frac{2b\left(24a^2C+7b^2(9A+7C)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d}+\frac{4aC\sqrt{\cos(c}}{}$$

$$= \frac{2a\left(63Ab^2+8a^2C+45b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{63d}+\frac{2b\left(24a^2C\right.}{}$$

$$= \frac{2a\left(63Ab^2+8a^2C+45b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{63d}+\frac{2b\left(24a^2C\right.}{}$$

$$= \frac{2b\left(9a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7a^2(3A+\right.}{}$$

Mathematica [A] time = 1.52486, size = 181, normalized size = 0.74

$$\frac{60a\left(7a^2(3A+C)+3b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(9a^2b(5A+3C)+b^3(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (84*(9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 60*
a*(7*a^2*(3*A + C) + 3*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(7*b*(36*A*b^2 + 108*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*(252*a*
A*b^2 + 84*a^3*C + 234*a*b^2*C + 54*a*b^2*C*Cos[2*(c + d*x)] + 7*b^3*C*Cos[
3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 0.516, size = 718, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2160*C*a*b^2+2240*C*b^3)*sin(1/2
*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^3-1512*C*a^2*b-3240*C*a*b^2-2072
*C*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1260*A*a*b^2+504*A*b^3+420
*C*a^3+1512*C*a^2*b+2520*C*a*b^2+952*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+(-630*A*a*b^2-126*A*b^3-210*C*a^3-378*C*a^2*b-720*C*a*b^2-168*C*b^
3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-945*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^2*b-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+315*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
315*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-1
47*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+105*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*C*
a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right) + a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos(dx+c)^5 + 3\,Cab^2\cos(dx+c)^4 + 3\,Aa^2b\cos(dx+c) + Aa^3 + \left(3\,Ca^2b + Ab^3\right)\cos(dx+c)^3 + \left(Ca^3\right.}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)\left(b\cos(dx+c) + a\right)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
 x)
```

3.691 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{2b\left(21a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a\left(5a^2(A-C)-3b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(6a^2(7A-\right.}{}$$

```
[Out] (-2*a*(5*a^2*(A - C) - 3*b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*b*(21*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*
d) - (2*b*(6*a^2*(7*A - 3*C) - b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) - (2*a*b^2*(35*A - 11*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) - (2*b*(7*A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(7*d) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.737913, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2b\left(21a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a\left(5a^2(A-C)-3b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(6a^2(7A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(5*a^2*(A - C) - 3*b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*b*(21*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*
d) - (2*b*(6*a^2*(7*A - 3*C) - b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) - (2*a*b^2*(35*A - 11*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) - (2*b*(7*A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(7*d) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
```

```
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+dx))^2\left(3Ab\right.}{}$$

$$= -\frac{2b(7A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2A(a+}{}$$

$$= -\frac{2ab^2(35A-11C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} - \frac{2b(7A-C)\sqrt{\cos(c+}}{}$$

$$= -\frac{2b\left(6a^2(7A-3C)-b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} - \frac{2ab^2}{}$$

$$= -\frac{2b\left(6a^2(7A-3C)-b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} - \frac{2ab^2}{}$$

$$= -\frac{2a\left(5a^2(A-C)-3b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2(3A+\right.}{}$$

Mathematica [A] time = 1.77046, size = 172, normalized size = 0.7

$$\frac{20b\left(21a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-84\left(5a^3(A-C)-3ab^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(5b(8\right.}{}}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (-84*(5*a^3*(A - C) - 3*a*b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 20*b
*(21*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2] + ((5*b*(28
```

```
*A*b^2 + 84*a^2*C + 29*b^2*C)*Cos[c + d*x] + 3*(140*a^3*A + 42*a*b^2*C + 42
*a*b^2*C*Cos[2*(c + d*x)] + 5*b^3*C*Cos[3*(c + d*x)]))*Sin[c + d*x])/Sqrt[C
os[c + d*x]])/(210*d)
```

Maple [B] time = 0.624, size = 943, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/105*(240*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*b^2*(7*a+5*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+28
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(5*A*b^2+15*C*a^2+1
8*C*a*b+10*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^3+35*A*b^3+105*C*a^2*b+63*C*
a*b^2+40*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-315*A*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^2+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+35*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*C*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+105*a^2*b*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)+25*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}\right.}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
 x)
```

3.692 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=218

$$\frac{2a\left(a^2(A+3C)+3b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(15a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2ab^2(5A-C)\,\mathrm{si}}{}$$

```
[Out] (-2*b*(15*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d) +
 (2*a*(3*b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) -
(2*a*b^2*(5*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d - (2*b^3*(35*A - 3*C)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (4*A*b*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.690981, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2a\left(a^2(A+3C)+3b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(15a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2ab^2(5A-C)\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-2*b*(15*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d) +
 (2*a*(3*b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) -
(2*a*b^2*(5*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d - (2*b^3*(35*A - 3*C)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (4*A*b*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
```

```
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int\frac{(a+b\cos(c+dx))^2\left(3Ab\right.}{}$$

$$=\frac{4Ab(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2A(a+b\cos(c+dx))^3\sin(c-}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$=-\frac{2b^3(35A-3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{4Ab(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx}}$$

$$=-\frac{2ab^2(5A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d}-\frac{2b^3(35A-3C)\cos^{\frac{3}{2}}(c+}{15d}$$

$$=-\frac{2ab^2(5A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d}-\frac{2b^3(35A-3C)\cos^{\frac{3}{2}}(c+}{15d}$$

$$=-\frac{2b\left(15a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(3b^2(3A+\right.}{}$$

Mathematica [A] time = 1.69354, size = 150, normalized size = 0.69

$$\frac{10\left(a^3(A+3C)+3ab^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\left(3b^3(5A+3C)-45a^2b(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{5a\left(2a^2A\tan(c+dx\right.}{}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (2*(-45*a^2*b*(A - C) + 3*b^3*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 10*(
3*a*b^2*(3*A + C) + a^3*(A + 3*C))*EllipticF[(c + d*x)/2, 2] + (6*(15*a^2*A
*b + b^3*C*Cos[c + d*x]^2)*Sin[c + d*x] + 5*a*(3*b^2*C*Sin[2*(c + d*x)] + 2
*a^2*A*Tan[c + d*x]))/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] time = 1.385, size = 1267, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(30*C*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-18*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*s
in(1/2*d*x+1/2*c)^2+10*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-
45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-45*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b-15*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*A*a
^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*C*b^3*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2+72*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*C*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-48*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8-30*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-180*A*a
^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-120*C*a*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+90*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+30
*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+120*C*a*b^2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-5*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-15*a^3*C*(sin(1/2*d*x+1/
```

```
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+30*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-90*C*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+90*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+90*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*
d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}\right.}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
 x)
```

3.693 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=229

$$\frac{2b\left(3a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2(3A+5C)+15b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(a^2(3A+5C\right.}{5d\sqrt{\ }}$$

```
[Out] (-2*a*(15*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) +
 (2*b*(b^2*(3*A + C) + 3*a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) +
(2*a*(8*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) - (
2*b^3*(9*A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (4*A*b*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c +
 d*x])^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.679485, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2b\left(3a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2(3A+5C)+15b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(a^2(3A+5C\right.}{5d\sqrt{\ }}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```



```
[Out] (-2*a*(15*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) +
 (2*b*(b^2*(3*A + C) + 3*a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) +
(2*a*(8*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) - (
2*b^3*(9*A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (4*A*b*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c +
 d*x])^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
```

```
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{(a+b\cos(c+dx))^2\left(3Ab\right.}{}$$

$$= \frac{4Ab(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^3\sin(c-}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(8Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4Ab(a+b\cos(c+dx))^2\sin}{5d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(8Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}-\frac{2b^3(9A-5C)\sqrt{\cos(c+dx)}\,s}{15d}$$

$$= \frac{2a\left(8Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}-\frac{2b^3(9A-5C)\sqrt{\cos(c+dx)}\,s}{15d}$$

$$= -\frac{2a\left(15b^2(A-C)+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(b^2(3A+C\right.}{}$$

Mathematica [A] time = 1.36079, size = 196, normalized size = 0.86

$$\frac{10b\left(3a^2(A+3C)+b^2(3A+C)\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6a\left(a^2(3A+5C)+15b^2(A-C)\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] (-6*a*(15*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(b^2*(3*A + C) + 3*a^2*(A + 3*C))*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 30*a^2*A*b*Sin[c + d*x] + 10*b^3*C*Cos[c + d*x]^2*Sin[c + d*x] + 9*a^3*A*Sin[2*(c + d*x)] + 45*a*A*b^2*Sin[2*(c + d*x)] + 15*a^3*C*Sin[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

Maple [B] time = 1.411, size = 1333, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/

```
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+6*A*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(3*A*b^2+C*a
^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos\left(dx+c\right)^5+3\,Cab^2\cos\left(dx+c\right)^4+3\,Aa^2b\cos\left(dx+c\right)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3\right.}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
```

```
cos(d*x + c)^2)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
 x)
```

3.694 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=243

$$\frac{2a\left(a^2(5A+7C)+21b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2b\left(3a^2(3A+5C)+5b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2(5A+\right.}{10}$$

```
[Out] (-2*b*(5*b^2*(A - C) + 3*a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*a*(21*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*
d) + (2*a*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sin[c + d*x])/(105*d*Cos[c + d*x]^
(3/2)) + (6*b*(8*A*b^2 + 7*a^2*(3*A + 5*C))*Sin[c + d*x])/(35*d*Sqrt[Cos[c
+ d*x]]) + (12*A*b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(35*d*Cos[c + d*x]^
(5/2)) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.7247, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2a\left(a^2(5A+7C)+21b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2b\left(3a^2(3A+5C)+5b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2(5A+\right.}{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-2*b*(5*b^2*(A - C) + 3*a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*a*(21*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*
d) + (2*a*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sin[c + d*x])/(105*d*Cos[c + d*x]^
(3/2)) + (6*b*(8*A*b^2 + 7*a^2*(3*A + 5*C))*Sin[c + d*x])/(35*d*Sqrt[Cos[c
+ d*x]]) + (12*A*b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(35*d*Cos[c + d*x]^
(5/2)) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
```

```
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int \frac{(a+b\cos(c+dx))^2\left(3Ab\right.}{}$$

$$= \frac{12Ab(a+b\cos(c+dx))^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^3\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a\left(24Ab^2+5a^2(5A+7C)\right)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{12Ab(a+b\cos(c+dx))^2}{35d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2a\left(24Ab^2+5a^2(5A+7C)\right)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{6b\left(8Ab^2+7a^2(3A+5C\right.}{35d\sqrt{\cos(c+d}}$$

$$= \frac{2a\left(24Ab^2+5a^2(5A+7C)\right)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{6b\left(8Ab^2+7a^2(3A+5C\right.}{35d\sqrt{\cos(c+d}}$$

$$= -\frac{2b\left(5b^2(A-C)+3a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(21b^2(A+3\right.}{}$$

Mathematica [A] time = 1.51516, size = 241, normalized size = 0.99

$$\frac{10a\left(a^2(5A+7C)+21b^2(A+3C)\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-42b\left(3a^2(3A+5C)+5b^2(A-C)\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] (-42*b*(5*b^2*(A - C) + 3*a^2*(3*A + 5*C))*Cos[c + d*x]^(5/2)*EllipticE[(c
+ d*x)/2, 2] + 10*a*(21*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Cos[c + d*x]^(5/2)
*EllipticF[(c + d*x)/2, 2] + 126*a^2*A*b*Sin[c + d*x] + 378*a^2*A*b*Cos[c +
 d*x]^2*Sin[c + d*x] + 210*A*b^3*Cos[c + d*x]^2*Sin[c + d*x] + 630*a^2*b*C*
Cos[c + d*x]^2*Sin[c + d*x] + 25*a^3*A*Sin[2*(c + d*x)] + 105*a*A*b^2*Sin[2
*(c + d*x)] + 35*a^3*C*Sin[2*(c + d*x)] + 30*a^3*A*Tan[c + d*x])/(105*d*Cos
[c + d*x]^(5/2))
```

Maple [B] time = 1.839, size = 1113, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6/5*A*a^2*b/
(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(3*A*b^2+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)
^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*A*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b^2+3*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}\right.}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2),
 x)
```

3.695 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=293

$$\frac{2b\left(3a^2(5A+7C)+7b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a\left(a^2(7A+9C)+9b^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(9a^2(5A\right.}{6}$$

```
[Out] (-2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*
d) + (2*b*(7*b^2*(A + 3*C) + 3*a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/
(21*d) + (2*a*(24*A*b^2 + 7*a^2*(7*A + 9*C))*Sin[c + d*x])/(315*d*Cos[c + d
*x]^(5/2)) + (2*b*(8*A*b^2 + 9*a^2*(5*A + 7*C))*Sin[c + d*x])/(63*d*Cos[c +
 d*x]^(3/2)) + (2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sin[c + d*x])/(15
*d*Sqrt[Cos[c + d*x]]) + (4*A*b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*
Cos[c + d*x]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c
+ d*x]^(9/2))
```

Rubi [A] time = 0.788261, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2b\left(3a^2(5A+7C)+7b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a\left(a^2(7A+9C)+9b^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(9a^2(5A\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```


```
[Out] (-2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*
d) + (2*b*(7*b^2*(A + 3*C) + 3*a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/
(21*d) + (2*a*(24*A*b^2 + 7*a^2*(7*A + 9*C))*Sin[c + d*x])/(315*d*Cos[c + d
*x]^(5/2)) + (2*b*(8*A*b^2 + 9*a^2*(5*A + 7*C))*Sin[c + d*x])/(63*d*Cos[c +
 d*x]^(3/2)) + (2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sin[c + d*x])/(15
*d*Sqrt[Cos[c + d*x]]) + (4*A*b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*
Cos[c + d*x]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c
+ d*x]^(9/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
```

```
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2}{9}\int \frac{(a+b\cos(c+dx))^2\left(3Ab+\right.}{}$$

$$= \frac{4Ab(a+b\cos(c+dx))^2\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^3\sin(c+}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a\left(24Ab^2+7a^2(7A+9C)\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{4Ab(a+b\cos(c+dx))^2\sin}{21d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a\left(24Ab^2+7a^2(7A+9C)\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b\left(8Ab^2+9a^2(5A+7C)\right)}{63d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a\left(24Ab^2+7a^2(7A+9C)\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b\left(8Ab^2+9a^2(5A+7C)\right)}{63d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2b\left(7b^2(A+3C)+3a^2(5A+7C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(24Ab^2+7a^2\right.}{315d}$$

$$= -\frac{2a\left(9b^2(3A+5C)+a^2(7A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(7b^2(A+3C\right.}{}$$

Mathematica [A] time = 5.2872, size = 250, normalized size = 0.85

$$\frac{10\left(3a^2b(5A+7C)+7b^3(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-14\left(a^3(7A+9C)+9ab^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{14a\left(a^2(7A+9C\right.}{3\cos}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
```

```
[Out] (-14*(9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2] + 10
*(7*b^3*(A + 3*C) + 3*a^2*b*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2] + (70*a^
3*A*Sin[c + d*x])/(3*Cos[c + d*x]^(9/2)) + (90*a^2*A*b*Sin[c + d*x])/Cos[c
+ d*x]^(7/2) + (14*a*(27*A*b^2 + a^2*(7*A + 9*C))*Sin[c + d*x])/(3*Cos[c +
d*x]^(5/2)) + (10*b*(7*A*b^2 + 3*a^2*(5*A + 7*C))*Sin[c + d*x])/Cos[c + d*x
]^(3/2) + (14*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sin[c + d*x])/Sqrt[Co
s[c + d*x]])/(105*d)
```

Maple [B] time = 2.273, size = 1270, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5
*a*(3*A*b^2+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/
2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b^2+3*C*a^2)*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*A*a^2*b*(-1/56*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3*(-1/144*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x
+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+
7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+6*C*a*b^2*(-(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/
```

```
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+\right.}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2)
, x)
```

3.696 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=382

$$\frac{8ab\left(11a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(78a^2b^2(9A+7C)+39a^4(5A+3C)+7b^4(13A+11C)\right)E\left(\frac{1}{2}\right.}{195d}$$

```
[Out] (2*(39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C) + 7*b^4*(13*A + 11*C))*Elli
pticE[(c + d*x)/2, 2])/(195*d) + (8*a*b*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A +
 9*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a*b*(11*a^2*(7*A + 5*C) + 5*
b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(192*a^4*C
+ 77*b^4*(13*A + 11*C) + 11*a^2*b^2*(637*A + 491*C))*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(6435*d) + (4*a*b*(1573*A*b^2 + 96*a^2*C + 1259*b^2*C)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(9009*d) + (2*(48*a^2*C + 11*b^2*(13*A + 11*C))*Cos
[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(1287*d) + (16*a*C*Cos
[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(143*d) + (2*C*Cos[c +
 d*x]^(3/2)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(13*d)
```

Rubi [A] time = 1.1538, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3050, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{8ab\left(11a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(78a^2b^2(9A+7C)+39a^4(5A+3C)+7b^4(13A+11C)\right)E\left(\frac{1}{2}\right.}{195d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C) + 7*b^4*(13*A + 11*C))*Elli
pticE[(c + d*x)/2, 2])/(195*d) + (8*a*b*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A +
 9*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a*b*(11*a^2*(7*A + 5*C) + 5*
b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(192*a^4*C
+ 77*b^4*(13*A + 11*C) + 11*a^2*b^2*(637*A + 491*C))*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(6435*d) + (4*a*b*(1573*A*b^2 + 96*a^2*C + 1259*b^2*C)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(9009*d) + (2*(48*a^2*C + 11*b^2*(13*A + 11*C))*Cos
[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(1287*d) + (16*a*C*Cos
[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(143*d) + (2*C*Cos[c +
 d*x]^(3/2)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(13*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^4\sin(c+dx)}{13d} + \frac{2}{13}\int$$

$$= \frac{16aC\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{143d} + \frac{2C}{}$$

$$= \frac{2\left(48a^2C+11b^2(13A+11C)\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c}{1287d}$$

$$= \frac{4ab\left(1573Ab^2+96a^2C+1259b^2C\right)\cos^{\frac{5}{2}}(c+dx)\sin(c}{9009d}$$

$$= \frac{2\left(192a^4C+77b^4(13A+11C)+11a^2b^2(637A+491C)\right.}{6435d}$$

$$= \frac{2\left(192a^4C+77b^4(13A+11C)+11a^2b^2(637A+491C)\right.}{6435d}$$

$$= \frac{2\left(39a^4(5A+3C)+78a^2b^2(9A+7C)+7b^4(13A+11C\right.}{195d}$$

$$= \frac{2\left(39a^4(5A+3C)+78a^2b^2(9A+7C)+7b^4(13A+11C\right.}{195d}$$

Mathematica [A] time = 3.01578, size = 281, normalized size = 0.74

$$\frac{24960ab\left(11a^2(7A+5C)+5b^2(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+7392\left(78a^2b^2(9A+7C)+39a^4(5A+3C)+7b^4(13A+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] (7392*(39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C) + 7*b^4*(13*A + 11*C))*E
llipticE[(c + d*x)/2, 2] + 24960*a*b*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*
C))*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(154*(936*a^4*C + 156*
a^2*b^2*(36*A + 43*C) + b^4*(1118*A + 1171*C))*Cos[c + d*x] + 5*b*(312*a*(4
4*a^2*(14*A + 13*C) + b^2*(572*A + 531*C)) + 3744*a*(11*A*b^2 + 11*a^2*C +
16*b^2*C)*Cos[2*(c + d*x)] + 77*(52*A*b^3 + 312*a^2*b*C + 89*b^3*C)*Cos[3*(
c + d*x)] + 6552*a*b^2*C*Cos[4*(c + d*x)] + 693*b^3*C*Cos[5*(c + d*x)]))*Si
n[c + d*x])/(720720*d)
```

Maple [B] time = 0.531, size = 1017, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*C
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(1048320*C*a*b^3+1330560*C*b^
4)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^4-960960*C*a^2*b^2
-2620800*C*a*b^3-1798720*C*b^4)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(4
11840*A*a*b^3+320320*A*b^4+411840*C*a^3*b+1921920*C*a^2*b^2+2957760*C*a*b^3
+1379840*C*b^4)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-432432*A*a^2*b^2-
617760*A*a*b^3-296296*A*b^4-72072*C*a^4-617760*C*a^3*b-1777776*C*a^2*b^2-18
15840*C*a*b^3-666512*C*b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(240240
*A*a^3*b+432432*A*a^2*b^2+480480*A*a*b^3+136136*A*b^4+72072*C*a^4+480480*C*
a^3*b+816816*C*a^2*b^2+720720*C*a*b^3+198352*C*b^4)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-120120*A*a^3*b-108108*A*a^2*b^2-137280*A*a*b^3-24024*A*b
^4-18018*C*a^4-137280*C*a^3*b-144144*C*a^2*b^2-145080*C*a*b^3-27258*C*b^4)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+60060*A*a^3*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+42900*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45045*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^4-162162*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-21021*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*b^4+42900*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35100*C*a*b^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-27027*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-126126*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*b^2-17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\left(C\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sqrt(cos(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.697 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=329

$$\frac{2\left(66a^2b^2(7A+5C)+77a^4(3A+C)+5b^4(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{8ab\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+\right.}{15d}$$

```
[Out] (8*a*b*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15
*d) + (2*(77*a^4*(3*A + C) + 66*a^2*b^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*E
llipticF[(c + d*x)/2, 2])/(231*d) + (2*(64*a^4*C + 15*b^4*(11*A + 9*C) + 9*
a^2*b^2*(143*A + 101*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(693*d) + (4*a*b*
(891*A*b^2 + 96*a^2*C + 673*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d
) + (2*(16*a^2*C + 3*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(231*d) + (16*a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(99*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*
Sin[c + d*x])/(11*d)
```

Rubi [A] time = 1.08621, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(66a^2b^2(7A+5C)+77a^4(3A+C)+5b^4(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{8ab\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```



```
[Out] (8*a*b*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15
*d) + (2*(77*a^4*(3*A + C) + 66*a^2*b^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*E
llipticF[(c + d*x)/2, 2])/(231*d) + (2*(64*a^4*C + 15*b^4*(11*A + 9*C) + 9*
a^2*b^2*(143*A + 101*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(693*d) + (4*a*b*
(891*A*b^2 + 96*a^2*C + 673*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d
) + (2*(16*a^2*C + 3*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(231*d) + (16*a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(99*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*
Sin[c + d*x])/(11*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4\sin(c+dx)}{11d}+\frac{2}{11}\int\frac{(a+b\cos(c}{}$$

$$= \frac{16aC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\sin(c+dx)}{99d}+\frac{2C\sqrt{\cos(c+dx}}{}$$

$$= \frac{2\left(16a^2C+3b^2(11A+9C)\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+}{231d}$$

$$= \frac{4ab\left(891Ab^2+96a^2C+673b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d}+\frac{2\left(16a^2\right.}{}$$

$$= \frac{2\left(64a^4C+15b^4(11A+9C)+9a^2b^2(143A+101C)\right)\sqrt{\cos(c+dx)}\,\mathrm{si}}{693d}$$

$$= \frac{2\left(64a^4C+15b^4(11A+9C)+9a^2b^2(143A+101C)\right)\sqrt{\cos(c+dx)}\,\mathrm{si}}{693d}$$

$$= \frac{8ab\left(3a^2(5A+3C)+b^2(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(77a^4(3A+\right.}{}$$

Mathematica [A] time = 2.19815, size = 243, normalized size = 0.74

$$\frac{240\left(66a^2b^2(7A+5C)+77a^4(3A+C)+5b^4(11A+9C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+14784\left(3a^3b(5A+3C)+ab^3(9A+7C)\right)E}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (14784*(3*a^3*b*(5*A + 3*C) + a*b^3*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2]
+ 240*(77*a^4*(3*A + C) + 66*a^2*b^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*Elli
pticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(616*a*b*(36*A*b^2 + 36*a^2*C
+ 43*b^2*C)*Cos[c + d*x] + 5*(1848*a^4*C + 792*a^2*b^2*(14*A + 13*C) + 3*b^
4*(572*A + 531*C) + 36*(11*A*b^4 + 66*a^2*b^2*C + 16*b^4*C)*Cos[2*(c + d*x)
] + 616*a*b^3*C*Cos[3*(c + d*x)] + 63*b^4*C*Cos[4*(c + d*x)]))*Sin[c + d*x]
)/(27720*d)
```

Maple [B] time = 0.465, size = 924, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-49280*C*a*b^3-50400*C*b^4)*sin
(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^4+47520*C*a^2*b^2+98560*C*a
*b^3+56880*C*b^4)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-22176*A*a*b^3-1
1880*A*b^4-22176*C*a^3*b-71280*C*a^2*b^2-91168*C*a*b^3-34920*C*b^4)*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(27720*A*a^2*b^2+22176*A*a*b^3+9240*A*b^4+
4620*C*a^4+22176*C*a^3*b+55440*C*a^2*b^2+41888*C*a*b^3+13860*C*b^4)*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-13860*A*a^2*b^2-5544*A*a*b^3-2640*A*b^4-
2310*C*a^4-5544*C*a^3*b-15840*C*a^2*b^2-7392*C*a*b^3-2790*C*b^4)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-13860*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b^3+3465*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6930*a^
2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8316*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^3*b-6468*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+1155*a
^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
```

```
F(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*C
*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\left(C\right.}{\sqrt{\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/sqrt(cos(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)),
 x)
```

3.698 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=320

$$\frac{8ab\left(7a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(-18a^2b^2(5A+3C)+15a^4(A-C)-b^4(9A+7C)\right)E\left(\frac{1}{2}(c+dx\right.}{15d}$$

```
[Out] (-2*(15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) - b^4*(9*A + 7*C))*EllipticE[(
c + d*x)/2, 2])/(15*d) + (8*a*b*(7*a^2*(3*A + C) + b^2*(7*A + 5*C))*Ellipti
cF[(c + d*x)/2, 2])/(21*d) - (4*a*b*(a^2*(63*A - 31*C) - 6*b^2*(7*A + 5*C))
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63*d) - (2*b^2*(3*a^2*(105*A - 41*C) - 7
*b^2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) - (2*a*b*(21*A -
 5*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d) - (2*b
*(9*A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) +
(2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.17898, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{8ab\left(7a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(-18a^2b^2(5A+3C)+15a^4(A-C)-b^4(9A+7C)\right)E\left(\frac{1}{2}(c+dx\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) - b^4*(9*A + 7*C))*EllipticE[(
c + d*x)/2, 2])/(15*d) + (8*a*b*(7*a^2*(3*A + C) + b^2*(7*A + 5*C))*Ellipti
cF[(c + d*x)/2, 2])/(21*d) - (4*a*b*(a^2*(63*A - 31*C) - 6*b^2*(7*A + 5*C))
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63*d) - (2*b^2*(3*a^2*(105*A - 41*C) - 7
*b^2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) - (2*a*b*(21*A -
 5*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d) - (2*b
*(9*A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) +
(2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+dx))^3\left(4Ab\right.}{}$$

$$= -\frac{2b(9A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\sin(c+dx)}{9d} + \frac{2A(a+}{}$$

$$= -\frac{2ab(21A-5C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{21d} - \frac{2b(9}{}$$

$$= -\frac{2b^2\left(3a^2(105A-41C)-7b^2(9A+7C)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d} -$$

$$= -\frac{4ab\left(a^2(63A-31C)-6b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{63d} - \frac{2}{}$$

$$= -\frac{4ab\left(a^2(63A-31C)-6b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{63d} - \frac{2}{}$$

$$= -\frac{2\left(15a^4(A-C)-18a^2b^2(5A+3C)-b^4(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

Mathematica [A] time = 3.87668, size = 216, normalized size = 0.68

$$\frac{40\left(7a^3b(3A+C)+ab^3(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-14\left(-18a^2b^2(5A+3C)+15a^4(A-C)-b^4(9A+7C)\right)E\left(\frac{1}{2}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (-14*(15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) - b^4*(9*A + 7*C))*EllipticE[
(c + d*x)/2, 2] + 40*(7*a^3*b*(3*A + C) + a*b^3*(7*A + 5*C))*EllipticF[(c +
 d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(120*a*b*(28*A*b^2 + 28*a^2*C + 23*b^2*C)
*Sin[c + d*x] + 14*(18*A*b^4 + 108*a^2*b^2*C + 19*b^4*C)*Sin[2*(c + d*x)] +
 5*(72*a*b^3*C*Sin[3*(c + d*x)] + 7*b^4*C*Sin[4*(c + d*x)] + 504*a^4*A*Tan[
c + d*x])))/12)/(105*d)
```

Maple [B] time = 0.626, size = 1209, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/315*(-1120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*b^3*(9*a+7*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c
)-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(63*A*b^2+378*
C*a^2+540*C*a*b+259*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+56*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(30*A*a*b^2+9*A*b^3+30*C*
a^3+54*C*a^2*b+60*C*a*b^2+17*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^4+140*A*a*
b^3+21*A*b^4+140*C*a^3*b+126*C*a^2*b^2+160*C*a*b^3+28*C*b^4)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-1890*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-189*A*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1260*
```

```
A*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)+420*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)-315*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-1134*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-147*C*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+420*a
^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+300*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos\left(dx+c\right)^6+4\,Cab^3\cos\left(dx+c\right)^5+4\,Aa^3b\cos\left(dx+c\right)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos\left(dx+c\right)^4+4\left(C\right.}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(3/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2),
 x)
```

3.699 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=300

$$\frac{2\left(42a^2b^2(3A+C)+7a^4(A+3C)+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{8ab\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (-8*a*b*(5*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*(42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(
c + d*x)/2, 2])/(21*d) - (2*b^2*(3*a^2*(49*A - 13*C) - b^2*(7*A + 5*C))*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (4*a*b^3*(175*A - 27*C)*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(105*d) - (2*b^2*(21*A - C)*Sqrt[Cos[c + d*x]]*(a + b*
Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + (16*A*b*(a + b*Cos[c + d*x])^3*Sin[c
+ d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x]
)/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.14876, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(42a^2b^2(3A+C)+7a^4(A+3C)+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{8ab\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-8*a*b*(5*a^2*(A - C) - b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*(42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(
c + d*x)/2, 2])/(21*d) - (2*b^2*(3*a^2*(49*A - 13*C) - b^2*(7*A + 5*C))*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (4*a*b^3*(175*A - 27*C)*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(105*d) - (2*b^2*(21*A - C)*Sqrt[Cos[c + d*x]]*(a + b*
Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + (16*A*b*(a + b*Cos[c + d*x])^3*Sin[c
+ d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x]
)/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
```

```
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+dx))^3\left(4Ab+\right.}{}$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A(a+b\cos(c+dx))^4\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2(21A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{16Ab(a}{}$$

$$= -\frac{4ab^3(175A-27C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} - \frac{2b^2(21A-C)\sqrt{\cos(c}}{}$$

$$= -\frac{2b^2\left(3a^2(49A-13C)-b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} - \frac{4a}{}$$

$$= -\frac{2b^2\left(3a^2(49A-13C)-b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} - \frac{4a}{}$$

$$= -\frac{8ab\left(5a^2(A-C)-b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(42a^2b^2(3A+\right.}{}$$

Mathematica [A] time = 2.73897, size = 206, normalized size = 0.69

$$\frac{10\left(42a^2b^2(3A+C)+7a^4(A+3C)+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-168\left(5a^3b(A-C)-ab^3(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (-168*(5*a^3*b*(A - C) - a*b^3*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 10*
(42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(c + d
*x)/2, 2] + 105*Sqrt[Cos[c + d*x]]*(((2*A*b^4)/3 + 4*a^2*b^2*C + (23*b^4*C)
/42)*Sin[c + d*x] + (4*a*b^3*C*Sin[2*(c + d*x)])/5 + (b^4*C*Sin[3*(c + d*x)
])/14 + 8*a^3*A*b*Tan[c + d*x] + (2*a^4*A*Sec[c + d*x]*Tan[c + d*x])/3))/(1
05*d)
```

Maple [B] time = 1.575, size = 1715, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(210*C*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+50*C*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4
*sin(1/2*d*x+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^
2-420*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+420*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3
-630*a^2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+420*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*
b+252*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-210*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+280*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+920*C*b^4*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-280*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4-440*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+70*A*a^4*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2+70*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+80*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+480*C*b^4*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^10-960*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8
+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2-35*A*a^4*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-35*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*a^4*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+420*C*a^2*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+168*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1
344*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+1680*C*a^2*b^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2016*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^6-1680*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1680*C*a^2*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1008*C*a*b^3*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^4+840*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+12
```

```
60*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2+840*A*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-840*A*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*
b^3*sin(1/2*d*x+1/2*c)^2+420*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x
+1/2*c)^2-840*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-504*C*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\left(Ca^{3}\right.}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(5/
```

```
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2),
 x)
```

3.700 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=321

$$\frac{8ab\left(a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(30a^2b^2(A-C)+a^4(3A+5C)-b^4(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2}{}$$

```
[Out] (-2*(30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3*A + 5*C))*EllipticE[(c +
 d*x)/2, 2])/(5*d) + (8*a*b*(b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c +
d*x)/2, 2])/(3*d) - (4*a*b*(2*b^2*(33*A - 5*C) + 3*a^2*(3*A + 5*C))*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(b^2*(59*A - 3*C) + 3*a^2*(3*A +
5*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(16*A*b^2 + a^2*(3*A + 5
*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (16*A*
b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a
+ b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 1.22486, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{8ab\left(a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(30a^2b^2(A-C)+a^4(3A+5C)-b^4(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-2*(30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3*A + 5*C))*EllipticE[(c +
 d*x)/2, 2])/(5*d) + (8*a*b*(b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c +
d*x)/2, 2])/(3*d) - (4*a*b*(2*b^2*(33*A - 5*C) + 3*a^2*(3*A + 5*C))*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(b^2*(59*A - 3*C) + 3*a^2*(3*A +
5*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(16*A*b^2 + a^2*(3*A + 5
*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (16*A*
b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a
+ b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+dx))^3\left(4Ab+\right.}{}$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^4\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(16Ab^2+a^2(3A+5C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{16Ab(a+}{}$$

$$= -\frac{2b^2\left(b^2(59A-3C)+3a^2(3A+5C)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2\left(1\right.}{}$$

$$= -\frac{4ab\left(2b^2(33A-5C)+3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} - \frac{2b}{}$$

$$= -\frac{4ab\left(2b^2(33A-5C)+3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} - \frac{2b}{}$$

$$= -\frac{2\left(30a^2b^2(A-C)-b^4(5A+3C)+a^4(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} +$$

Mathematica [A] time = 1.57635, size = 233, normalized size = 0.73

$$\frac{40ab\left(a^2(A+3C)+b^2(3A+C)\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(30a^2b^2(A-C)+a^4(3A+5C)-b^4(5A+3C)\right)\cos^{\frac{3}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] (-6*(30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3*A + 5*C))*Cos[c + d*x]^(
3/2)*EllipticE[(c + d*x)/2, 2] + 40*a*b*(b^2*(3*A + C) + a^2*(A + 3*C))*Cos
[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 40*a^3*A*b*Sin[c + d*x] + 40*a*
b^3*C*Cos[c + d*x]^2*Sin[c + d*x] + 6*b^4*C*Cos[c + d*x]^3*Sin[c + d*x] + 9
*a^4*A*Sin[2*(c + d*x)] + 90*a^2*A*b^2*Sin[2*(c + d*x)] + 15*a^4*C*Sin[2*(c
 + d*x)] + 6*a^4*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.729, size = 1622, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C*b^4*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+1/3*(16*C*a*b^3-12*C*b^4)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(
2*A*b^4+12*C*a^2*b^2-16*C*a*b^3+6*C*b^4)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))+8*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
```

```
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-2*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*a^2*b^2*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*C*a*b^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-2/5*A*a^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4
+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*A*a^3*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(6*A*b^2+C*a^2)*(-(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2),
```

```
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos(dx+c)^6 + 4\,Cab^3\cos(dx+c)^5 + 4\,Aa^3b\cos(dx+c) + Aa^4 + \left(6\,Ca^2b^2 + Ab^4\right)\cos(dx+c)^4 + 4\left(C\right.}{\cos(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(7/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2),
 x)
```

3.701 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=316

$$\frac{2\left(42a^2b^2(A+3C)+a^4(5A+7C)+7b^4(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{8ab\left(a^2(3A+5C)+5b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (-8*a*b*(5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*(7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(
c + d*x)/2, 2])/(21*d) + (4*a*b*(96*A*b^2 + a^2*(101*A + 175*C))*Sin[c + d*
x])/(105*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(b^2*(87*A - 35*C) + 5*a^2*(5*A + 7
*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(48*A*b^2 + 5*a^2*(5*A +
 7*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (1
6*A*b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A
*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.14316, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\left(42a^2b^2(A+3C)+a^4(5A+7C)+7b^4(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{8ab\left(a^2(3A+5C)+5b^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-8*a*b*(5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d)
+ (2*(7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(
c + d*x)/2, 2])/(21*d) + (4*a*b*(96*A*b^2 + a^2*(101*A + 175*C))*Sin[c + d*
x])/(105*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(b^2*(87*A - 35*C) + 5*a^2*(5*A + 7
*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(48*A*b^2 + 5*a^2*(5*A +
 7*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (1
6*A*b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A
*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int\frac{(a+b\cos(c+dx))^3\left(4Ab\right.}{}$$

$$=\frac{16Ab(a+b\cos(c+dx))^3\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^4\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2\left(48Ab^2+5a^2(5A+7C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{16Ab(}{}$$

$$=\frac{4ab\left(96Ab^2+a^2(101A+175C)\right)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}}+\frac{2\left(48Ab^2+5a^2(5A+\right.}{1}$$

$$=\frac{4ab\left(96Ab^2+a^2(101A+175C)\right)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}}-\frac{2b^2\left(b^2(87A-35C)\right.}{}$$

$$=\frac{4ab\left(96Ab^2+a^2(101A+175C)\right)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}}-\frac{2b^2\left(b^2(87A-35C)\right.}{}$$

$$=-\frac{8ab\left(5b^2(A-C)+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7b^4(3A+C\right.}{}$$

Mathematica [A] time = 2.53575, size = 276, normalized size = 0.87

$$10\left(42a^2b^2(A+3C)+a^4(5A+7C)+7b^4(3A+C)\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-168ab\left(a^2(3A+5C)+5b^2(A-C)\right)\text{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] (-168*a*b*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*x]^(5/2)*EllipticE[(c
 + d*x)/2, 2] + 10*(7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C
))*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 168*a^3*A*b*Sin[c + d*x]
+ 504*a^3*A*b*Cos[c + d*x]^2*Sin[c + d*x] + 840*a*A*b^3*Cos[c + d*x]^2*Sin[
c + d*x] + 840*a^3*b*C*Cos[c + d*x]^2*Sin[c + d*x] + 70*b^4*C*Cos[c + d*x]^
3*Sin[c + d*x] + 25*a^4*A*Sin[2*(c + d*x)] + 210*a^2*A*b^2*Sin[2*(c + d*x)]
 + 35*a^4*C*Sin[2*(c + d*x)] + 30*a^4*A*Tan[c + d*x])/(105*d*Cos[c + d*x]^(
5/2))
```

Maple [B] time = 1.888, size = 1531, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^4*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))-4*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^4*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
```

```
2))+12*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8/5*A*a^3*b/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*a^2*(6*A*b^2+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
+2*A*a^4*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+
5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+8*a*b*(A*b^2+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2),
```

```
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos(dx+c)^6 + 4\,Cab^3\cos(dx+c)^5 + 4\,Aa^3b\cos(dx+c) + Aa^4 + \left(6\,Ca^2b^2 + Ab^4\right)\cos(dx+c)^4 + 4\left(Ca^3\right.}{\cos(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(9/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2),
 x)
```

3.702 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{8ab\left(a^2(5A+7C)+7b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(18a^2b^2(3A+5C)+a^4(7A+9C)+15b^4(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

```
[Out] (-2*(15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*EllipticE[(
c + d*x)/2, 2])/(15*d) + (8*a*b*(7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Ellipti
cF[(c + d*x)/2, 2])/(21*d) + (4*a*b*(32*A*b^2 + a^2*(101*A + 147*C))*Sin[c
+ d*x])/(315*d*Cos[c + d*x]^(3/2)) + (2*(192*A*b^4 + 21*a^4*(7*A + 9*C) + 7
*a^2*b^2*(155*A + 261*C))*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]) + (2*(48
*A*b^2 + 7*a^2*(7*A + 9*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos
[c + d*x]^(5/2)) + (16*A*b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c
 + d*x]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2))
```

Rubi [A] time = 1.15572, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{8ab\left(a^2(5A+7C)+7b^2(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(18a^2b^2(3A+5C)+a^4(7A+9C)+15b^4(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```


```
[Out] (-2*(15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*EllipticE[(
c + d*x)/2, 2])/(15*d) + (8*a*b*(7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Ellipti
cF[(c + d*x)/2, 2])/(21*d) + (4*a*b*(32*A*b^2 + a^2*(101*A + 147*C))*Sin[c
+ d*x])/(315*d*Cos[c + d*x]^(3/2)) + (2*(192*A*b^4 + 21*a^4*(7*A + 9*C) + 7
*a^2*b^2*(155*A + 261*C))*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]) + (2*(48
*A*b^2 + 7*a^2*(7*A + 9*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos
[c + d*x]^(5/2)) + (16*A*b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c
 + d*x]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\cos(c+dx))^3\left(4Ab\right.}{}$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^4\sin(c}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2\left(48Ab^2+7a^2(7A+9C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{16Ab(}{}$$

$$= \frac{4ab\left(32Ab^2+a^2(101A+147C)\right)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(48Ab^2+7a^2(7A+\right.}{3}$$

$$= \frac{4ab\left(32Ab^2+a^2(101A+147C)\right)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(192Ab^4+21a^4(7A\right.}{}$$

$$= \frac{4ab\left(32Ab^2+a^2(101A+147C)\right)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(192Ab^4+21a^4(7A\right.}{}$$

$$= -\frac{2\left(15b^4(A-C)+18a^2b^2(3A+5C)+a^4(7A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

Mathematica [A] time = 5.16363, size = 268, normalized size = 0.82

$$\frac{2\left(60\left(a^3b(5A+7C)+7ab^3(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-21\left(18a^2b^2(3A+5C)+a^4(7A+9C)+15b^4(A-C)\right)E\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
```

```
[Out] (2*(-21*(15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*Ellipti
cE[(c + d*x)/2, 2] + 60*(7*a*b^3*(A + 3*C) + a^3*b*(5*A + 7*C))*EllipticF[(
c + d*x)/2, 2] + (35*a^4*A*Sin[c + d*x])/Cos[c + d*x]^(9/2) + (180*a^3*A*b*
Sin[c + d*x])/Cos[c + d*x]^(7/2) + (7*a^2*(54*A*b^2 + a^2*(7*A + 9*C))*Sin[
c + d*x])/Cos[c + d*x]^(5/2) + (60*a*b*(7*A*b^2 + a^2*(5*A + 7*C))*Sin[c +
d*x])/Cos[c + d*x]^(3/2) + (21*(15*A*b^4 + 18*a^2*b^2*(3*A + 5*C) + a^4*(7*
A + 9*C))*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(315*d)
```

Maple [B] time = 2.263, size = 1451, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^4*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*a^2*(6*A
*b^2+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*a*b*(A*b^2+C*a^2)*(-1/6*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/
2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))+8*A*a^3*b*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-
1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^4*(-1/144*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)
^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*b^2*(A*b^2+6*C*a^2)*(
```

```
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\left(C\right.}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(11
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(11/2)
, x)
```

3.703 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=377

$$\frac{2\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+77b^4(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{8ab\left(a^2(7A+9C)+3b^2(3A+5C)\right)E\left(\frac{1}{2}(c+\right.}{15d}$$

```
[Out] (-8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(1
5*d) + (2*(77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*
EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a*b*(96*A*b^2 + a^2*(673*A + 891*C)
)*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(5/2)) + (2*(64*A*b^4 + 15*a^4*(9*A +
11*C) + 9*a^2*b^2*(101*A + 143*C))*Sin[c + d*x])/(693*d*Cos[c + d*x]^(3/2))
 + (8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sin[c + d*x])/(15*d*Sqrt[Co
s[c + d*x]]) + (2*(16*A*b^2 + 3*a^2*(9*A + 11*C))*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) + (16*A*b*(a + b*Cos[c + d*x])^3*Sin
[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c +
d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 1.23302, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+77b^4(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{8ab\left(a^2(7A+9C)+3b^2(3A+5C)\right)E\left(\frac{1}{2}(c+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```


```
[Out] (-8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(1
5*d) + (2*(77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*
EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a*b*(96*A*b^2 + a^2*(673*A + 891*C)
)*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(5/2)) + (2*(64*A*b^4 + 15*a^4*(9*A +
11*C) + 9*a^2*b^2*(101*A + 143*C))*Sin[c + d*x])/(693*d*Cos[c + d*x]^(3/2))
 + (8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sin[c + d*x])/(15*d*Sqrt[Co
s[c + d*x]]) + (2*(16*A*b^2 + 3*a^2*(9*A + 11*C))*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) + (16*A*b*(a + b*Cos[c + d*x])^3*Sin
[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c +
```

```
d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
```

```
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\cos(c+dx))^3\left(4Ab\right.}{}$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^4\sin(c+}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2\left(16Ab^2+3a^2(9A+11C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)} + \frac{16Ab(a}{}$$

$$= \frac{4ab\left(96Ab^2+a^2(673A+891C)\right)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(16Ab^2+3a^2(9A+\right.}{23}$$

$$= \frac{4ab\left(96Ab^2+a^2(673A+891C)\right)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(64Ab^4+15a^4(9A+\right.}{}$$

$$= \frac{4ab\left(96Ab^2+a^2(673A+891C)\right)\sin(c+dx)}{3465d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(64Ab^4+15a^4(9A+\right.}{}$$

$$= \frac{2\left(77b^4(A+3C)+66a^2b^2(5A+7C)+5a^4(9A+11C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{231d}$$

$$= -\frac{8ab\left(3b^2(3A+5C)+a^2(7A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2\left(77b^4(A+3\right.}{}$$

Mathematica [A] time = 4.45073, size = 284, normalized size = 0.75

$$\frac{10\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+77b^4(A+3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-616\left(a^3b(7A+9C)+3ab^3(3A+5C)\right)E\left(\frac{1}{2}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/
2),x]
```

```
[Out] (-616*(3*a*b^3*(3*A + 5*C) + a^3*b*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2] +
 10*(77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*Ellipt
```

```
icF[(c + d*x)/2, 2] + (2*(1540*a^3*A*b + 308*a*b*(9*A*b^2 + a^2*(7*A + 9*C)
)*Cos[c + d*x]^2 + 15*(77*A*b^4 + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*
C))*Cos[c + d*x]^3 + 924*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Cos[c +
d*x]^4)*Sin[c + d*x] + 45*((66*a^2*A*b^2 + a^4*(9*A + 11*C))*Sin[2*(c + d*x
)] + 14*a^4*A*Tan[c + d*x]))/(3*Cos[c + d*x]^(9/2)))/(1155*d)
```

Maple [B] time = 2.655, size = 1521, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8/5
*a*b*(A*b^2+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/
2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^4*(-1/352*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/154*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+2*b^2*(A*b^2+6*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+2*a^2*(6*A*b^2+C*a^2)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+8*A*a^3*b*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(
```

```
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-
7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+8*C*a*b^3*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(13/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\left(Ca^{3}\right.}{\cos\left(dx+c\right)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/cos(d*x + c)^(13
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/cos(d*x + c)^(13/2)
, x)
```

3.704 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=299

$$-\frac{2a\left(7a^2b^2(3A+C)+21a^4C+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d}+\frac{2\left(3a^2b^2(5A+3C)+15a^4C+b^4(9A+7C)\right)E\left(\frac{1}{2}(c+d\right.}{15b^5d}$$

```
[Out] (2*(15*a^4*C + 3*a^2*b^2*(5*A + 3*C) + b^4*(9*A + 7*C))*EllipticE[(c + d*x)
/2, 2])/(15*b^5*d) - (2*a*(21*a^4*C + 7*a^2*b^2*(3*A + C) + b^4*(7*A + 5*C)
)*EllipticF[(c + d*x)/2, 2])/(21*b^6*d) + (2*a^4*(A*b^2 + a^2*C)*EllipticPi
[(2*b)/(a + b), (c + d*x)/2, 2])/(b^6*(a + b)*d) - (2*a*(7*A*b^2 + 7*a^2*C
+ 5*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*b^4*d) + (2*(9*a^2*C + b^2*
(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*b^3*d) - (2*a*C*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(7*b^2*d) + (2*C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(
9*b*d)
```

Rubi [A] time = 1.50171, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2a\left(7a^2b^2(3A+C)+21a^4C+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d}+\frac{2\left(3a^2b^2(5A+3C)+15a^4C+b^4(9A+7C)\right)E\left(\frac{1}{2}(c+d\right.}{15b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```



```
[Out] (2*(15*a^4*C + 3*a^2*b^2*(5*A + 3*C) + b^4*(9*A + 7*C))*EllipticE[(c + d*x)
/2, 2])/(15*b^5*d) - (2*a*(21*a^4*C + 7*a^2*b^2*(3*A + C) + b^4*(7*A + 5*C)
)*EllipticF[(c + d*x)/2, 2])/(21*b^6*d) + (2*a^4*(A*b^2 + a^2*C)*EllipticPi
[(2*b)/(a + b), (c + d*x)/2, 2])/(b^6*(a + b)*d) - (2*a*(7*A*b^2 + 7*a^2*C
+ 5*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*b^4*d) + (2*(9*a^2*C + b^2*
(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*b^3*d) - (2*a*C*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(7*b^2*d) + (2*C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(
9*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
```

```
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{2C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9bd} + \frac{2\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7aC}{2}+\frac{1}{2}b(9A+7C)\cos(c+dx)-\frac{9}{2}aC\cos^2(c+\right.}{a+b\cos(c+dx)}}{9b}$$

$$= -\frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7b^2d} + \frac{2C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9bd} + \frac{4\int \frac{\cos^{\frac{3}{2}}(c+d}{}}{}$$

$$= \frac{2\left(9a^2C+b^2(9A+7C)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45b^3d} - \frac{2aC\cos^{\frac{5}{2}}(c+dx)\sin(c+}{7b^2d}$$

$$= -\frac{2a\left(7Ab^2+7a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^4d} + \frac{2\left(9a^2C+b^2(9A+7\right.}{}$$

$$= -\frac{2a\left(7Ab^2+7a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^4d} + \frac{2\left(9a^2C+b^2(9A+7\right.}{}$$

$$= \frac{2\left(15a^4C+3a^2b^2(5A+3C)+b^4(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^5d} - \frac{2a\left(7Ab^2+7\right.}{}$$

$$= \frac{2\left(15a^4C+3a^2b^2(5A+3C)+b^4(9A+7C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^5d} - \frac{2a\left(21a^4C+\right.}{}$$

Mathematica [A] time = 2.58339, size = 364, normalized size = 1.22

$$\sin(c+dx)\sqrt{\cos(c+dx)}\left(7b\left(36a^2C+36Ab^2+43b^2C\right)\cos(c+dx)-5\left(84a^3C+84aAb^2+18ab^2C\cos(2(c+dx))+78a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(7*b*(36*A*b^2 + 36*a^2*C + 43*b^2*C)*Cos[c + d*x] - 5*
(84*a*A*b^2 + 84*a^3*C + 78*a*b^2*C + 18*a*b^2*C*Cos[2*(c + d*x)] - 7*b^3*C
*Cos[3*(c + d*x)]))*Sin[c + d*x] + 6*(((35*a^4*C + 7*b^4*(9*A + 7*C) + a^2*
b^2*(35*A + 13*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*
a*(7*A*b^2 + 7*a^2*C + 6*b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (7*(15*a^4*C + 3*a^2*b^2*
(5*A + 3*C) + b^4*(9*A + 7*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]]
, -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^
2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^
2*Sqrt[Sin[c + d*x]^2])))/(630*b^4*d)
```

Maple [B] time = 0.716, size = 1554, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-75*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a^2*b^4+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b^3-105*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*a^4*b^2+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^5*b+315*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^4*b^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^5+315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^4-315*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^3*b^3+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^5+315*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b^3-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^4-315*A*(sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^4*b^2-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^5+189*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a^2*b^4+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*b^2-315*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a^5*b+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^5-189*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^3*b^3-315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^6+189*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*b^6+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^6+147*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*b^6+(-1120*C*a*b^5+1120*C*b^6)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^10+(-720*C*a^2*b^4+2960*C*a*b^5-2240*C*b^6)*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-504*A*a*b^5+504*A*b^6-504*C*a^3*b^3+1584*C*a^2*b^4-3152*C
*a*b^5+2072*C*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-420*A*a^2*b^4+
924*A*a*b^5-504*A*b^6-420*C*a^4*b^2+924*C*a^3*b^3-1344*C*a^2*b^4+1792*C*a*b
^5-952*C*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(210*A*a^2*b^4-336*A*
a*b^5+126*A*b^6+210*C*a^4*b^2-336*C*a^3*b^3+366*C*a^2*b^4-408*C*a*b^5+168*C
*b^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/b^6/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a), x
)
```

3.705 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{2\left(7a^2b^2(3A+C)+21a^4C+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^5d}-\frac{2a\left(5a^2C+5Ab^2+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d}-\frac{2a^3\left(a^2C+\right.}{}$$

```
[Out] (-2*a*(5*A*b^2 + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b^4*d) +
(2*(21*a^4*C + 7*a^2*b^2*(3*A + C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2
, 2])/(21*b^5*d) - (2*a^3*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(b^5*(a + b)*d) + (2*(7*a^2*C + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(21*b^3*d) - (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b^
2*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 1.12037, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(7a^2b^2(3A+C)+21a^4C+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^5d}-\frac{2a\left(5a^2C+5Ab^2+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d}-\frac{2a^3\left(a^2C+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```


```
[Out] (-2*a*(5*A*b^2 + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b^4*d) +
(2*(21*a^4*C + 7*a^2*b^2*(3*A + C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2
, 2])/(21*b^5*d) - (2*a^3*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(b^5*(a + b)*d) + (2*(7*a^2*C + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(21*b^3*d) - (2*a*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b^
2*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
```

```
1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5aC}{2}+\frac{1}{2}b(7A+5C)\cos(c+dx)-\frac{7}{2}aC\cos^2(c+$$

$$= -\frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5b^2d} + \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7bd} + \frac{4\int \frac{\sqrt{\cos(c+d}}{}}{}$$

$$= \frac{2\left(7a^2C+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^3d} - \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c}{5b^2d}$$

$$= \frac{2\left(7a^2C+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^3d} - \frac{2aC\cos^{\frac{3}{2}}(c+dx)\sin(c}{5b^2d}$$

$$= -\frac{2a\left(5Ab^2+5a^2C+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d} + \frac{2\left(7a^2C+b^2(7A+5C)\right)\sqrt{\cos}}{21b^3d}$$

$$= -\frac{2a\left(5Ab^2+5a^2C+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d} + \frac{2\left(21a^4C+7a^2b^2(3A+C)+\right.}{21}$$

Mathematica [A] time = 2.26058, size = 293, normalized size = 1.23

$$-\frac{2a\left(35a^2C+35Ab^2+13b^2C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 2\sin(c+dx)\sqrt{\cos(c+dx)}\left(70a^2C-42abC\cos(c+dx)+70Ab^2+15b^2C\cos(2(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] ((-2*a*(35*A*b^2 + 35*a^2*C + 13*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b) + (4*(35*A*b^2 - 28*a^2*C + 25*b^2*C)*((a + b)*EllipticF[(c
 + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + 2*S
qrt[Cos[c + d*x]]*(70*A*b^2 + 70*a^2*C + 65*b^2*C - 42*a*b*C*Cos[c + d*x] +
 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x] + (42*(5*A*b^2 + 5*a^2*C + 3*b^2*C
)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[
ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin
[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(210*b
^3*d)
```

Maple [B] time = 0.577, size = 1244, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((240*C*a*b^
4-240*C*b^5)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(168*C*a^2*b^3-528*C*a
*b^4+360*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a*b^4-140*A*
b^5+140*C*a^3*b^2-308*C*a^2*b^3+448*C*a*b^4-280*C*b^5)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-70*A*a*b^4+70*A*b^5-70*C*a^3*b^2+112*C*a^2*b^3-122*C*
a*b^4+80*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^3*b^2-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3+35*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^4-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^5+105*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2*b^3-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^4-105*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))*a^3*b^2+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^5-105*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a^4*b+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b^2-35*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^3+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^4-25*C*(sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^5+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*b-105*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a^3*b^2+63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3-63*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a*b^4-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^5)/b^5
/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x
)
```

3.706 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=181

$$-\frac{2a\left(C\left(3a^2+b^2\right)+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}+\frac{2\left(5a^2C+b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2a^2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+$$

```
[Out] (2*(5*a^2*C + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) - (2*a*
(3*A*b^2 + (3*a^2 + b^2)*C)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) + (2*a^2*(
A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) -
 (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*C*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.78887, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3050, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2a\left(C\left(3a^2+b^2\right)+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}+\frac{2\left(5a^2C+b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2a^2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*(5*a^2*C + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) - (2*a*
(3*A*b^2 + (3*a^2 + b^2)*C)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) + (2*a^2*(
A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) -
 (2*a*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*C*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(5*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+\frac{1}{2}b(5A+3C)\cos(c+dx)-\frac{5}{2}aC\cos^2(c+\cdots\right.}{a+b\cos(c+dx)}}{5b}$$

$$= -\frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{4\int \frac{-\frac{5a^2C}{4}+ab\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} - \frac{4\int \frac{\frac{5}{4}a^2bC+\frac{5}{4}\cdots}{\sqrt{\cos\cdots}}}{\cdots}$$

$$= \frac{2\left(5a^2C+b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} - \frac{2aC\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \cdots$$

$$= \frac{2\left(5a^2C+b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} - \frac{2a\left(3Ab^2+\left(3a^2+b^2\right)C\right)F\left(\frac{1}{2}(c\cdots\right.}{3b^4d}\cdots$$

Mathematica [A] time = 2.28379, size = 248, normalized size = 1.37

$$\frac{\frac{2\left(5a^2C+15Ab^2+9b^2C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\left(5a^2C+5Ab^2+3b^2C\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2\cdots\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{30b\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] ((2*(15*A*b^2 + 5*a^2*C + 9*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b) + 8*a*C*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(a + b)) + 4*C*Sqrt[Cos[c + d*x]]*(-5*a + 3*b*Cos[c
+ d*x])*Sin[c + d*x] + (6*(5*A*b^2 + 5*a^2*C + 3*b^2*C)*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
```

```
d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(30*b^2*d)
```

Maple [B] time = 0.558, size = 948, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((24*C*a*b^3-2
4*C*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*C*a^2*b^2-44*C*a*b^3+2
4*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*C*a^2*b^2+16*C*a*b^3-
6*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-15*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^2*b^2+15*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*a^3*b*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+5*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*a*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2*b^2+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^
4-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^4)/b^4/(a-b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x
)
```

3.707 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{2\left(3a^2C+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aCE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2C\sin(c+dx)}{3b}$$

```
[Out] (-2*a*C*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2*C + b^2*(3*A + C))*E
llipticF[(c + d*x)/2, 2])/(3*b^3*d) - (2*a*(A*b^2 + a^2*C)*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*C*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*b*d)
```

Rubi [A] time = 0.53404, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3050, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2C+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aCE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2C\sin(c+dx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```



```
[Out] (-2*a*C*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2*C + b^2*(3*A + C))*E
llipticF[(c + d*x)/2, 2])/(3*b^3*d) - (2*a*(A*b^2 + a^2*C)*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*C*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*b*d)
```


Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
```

```
))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx = \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{aC}{2}+\frac{1}{2}b(3A+C)\cos(c+dx)-\frac{3}{2}aC\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}$$

$$= \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{1}{2}abC-\frac{1}{2}\left(3a^2C+b^2(3A+C)\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b^2} - \frac{(aC)}{}$$

$$= -\frac{2aCE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{\left(a\left(Ab^2+a^2C\right)\right)\int}{}$$

$$= -\frac{2aCE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2\left(3a^2C+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d} - \frac{2a\left(Ab^2\right.}{}$$

Mathematica [A] time = 1.72793, size = 200, normalized size = 1.54

$$\frac{\frac{6C\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}} + \frac{4(3A+C)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\right.}{a+b}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] ((-2*a*C*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*(3*A + C)*
((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]))/(a + b) + 4*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (6*C*(2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(6*b*d)
```

Maple [B] time = 0.454, size = 686, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*C*a*b^2-4*
C*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b^2+2*C*b^3)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b^2+3*
a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*a*b^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))*a^3+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^2)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x
)
```

3.708 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=85

$$\frac{2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aCF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*C*EllipticF[(c + d*x)/2, 2])/(
b^2*d) + (2*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2
*(a + b)*d)
```

Rubi [A] time = 0.27191, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3060, 2639, 3002, 2641, 2805}

$$\frac{2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aCF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*C*EllipticF[(c + d*x)/2, 2])/(
b^2*d) + (2*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2
*(a + b)*d)
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx &= -\frac{\int \frac{-Ab+aC\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b}+\frac{C\int \sqrt{\cos(c+dx)}\,dx}{b}\\
&= \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{(aC)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}+\left(A+\frac{a^2C}{b^2}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b
\end{aligned}$$

$$= \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aCF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\left(A+\frac{a^2C}{b^2}\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A] time = 0.893085, size = 131, normalized size = 1.54

$$\frac{\frac{C\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}+\frac{(2A+C)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),
x]
```

```
[Out] (((2*A + C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (C*(-2*a*b
*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[S
qrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos
[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/d
```

Maple [A] time = 0.442, size = 259, normalized size = 3.1

$$-2\,{\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{{b}^{2}\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}}\left(A\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b^2-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a*b+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+C*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c) + a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
 x)
```

3.709 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(b
*d) - (2*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*b*(a
 + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.501498, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3056, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```


```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(b
*d) - (2*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*b*(a
 + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
```

```
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{-\frac{Ab}{2}-\frac{1}{2}a(A-C)\cos(c+dx)-\frac{1}{2}Ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{A\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{2\int \frac{\frac{Ab^2}{2}-\frac{1}{2}abC\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab}$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{C\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \left(\frac{Ab}{a}+\frac{aC}{b}\right)\int$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2\left(\frac{Ab}{a}+\frac{aC}{b}\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{(a+b)d}$$

Mathematica [A] time = 3.74686, size = 209, normalized size = 1.87

$$-\frac{\frac{2A\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}} + \frac{4a(A-C)\left((a+b)F\left(\frac{1}{2}(c+dx)\right.\right.}{b(a-}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),
x]
```

```
[Out] -((6*A*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*a*(A - C)*
((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]))/(b*(a + b)) - (4*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*A*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

Maple [B] time = 1.113, size = 407, normalized size = 3.6

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,\frac{C\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\mathrm{Elliptic}}{b\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin(1/2\,dx+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*(-A
*b^2-C*a^2)/a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + A}{(b\cos{(dx+c)} + a)\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
 x)
```

3.710 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=140

$$\frac{2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2Ab\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*A*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x)/2, 2])
/(3*a*d) + (2*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*A*b*Sin[
c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.729576, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2Ab\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```


```
[Out] (2*A*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x)/2, 2])
/(3*a*d) + (2*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*A*b*Sin[
c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3Ab}{2}+\frac{1}{2}a(A+3C)\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(3Ab^2+a^2(A+3C)\right)+aAb\cos(c+dx)+\frac{3}{4}Ab^2\cos}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3a^2}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b\left(3Ab^2+a^2(A+3C)\right)-\frac{1}{4}aAb^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{3a^2b}$$

$$= \frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{A\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{3a}$$

$$= \frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2\left(\frac{Ab^2}{a^2}+C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A] time = 5.72485, size = 223, normalized size = 1.59

$$\frac{\frac{2\left(2a^2(A+3C)+9Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6A\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+\right.\right.}{a\sqrt{\sin^2(c+dx)}}}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),
x]
```

```
[Out] ((2*(9*A*b^2 + 2*a^2*(A + 3*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b) + 8*a*A*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2])/(a + b)) + (4*A*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Co
s[c + d*x]^(3/2) + (6*A*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[
c + d*x]^2]))/(6*a^2*d)
```

Maple [B] time = 1.135, size = 463, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b^2+C*a^2
)/a^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))-2*A/a^2*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin
(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right) + a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
 x)
```

3.711 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=206

$$-\frac{2\left(a^2(3A+5C)+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}-\frac{2b\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}+\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)}{5a^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(5*A*b^2 + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*A
*b*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (2*b*(A*b^2 + a^2*C)*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(5*a*d*
Cos[c + d*x]^(5/2)) - (2*A*b*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) + (
2*(5*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*a^3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.07326, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(a^2(3A+5C)+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}-\frac{2b\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}+\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)}{5a^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])),x]
```


```
[Out] (-2*(5*A*b^2 + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*A
*b*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (2*b*(A*b^2 + a^2*C)*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(5*a*d*
Cos[c + d*x]^(5/2)) - (2*A*b*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) + (
2*(5*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*a^3*d*Sqrt[Cos[c + d*x]])
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{-\frac{5Ab}{2}+\frac{1}{2}a(3A+5C)\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{3}{4}\left(5Ab^2+a^2(3A+5C)\right)+aAb\cos(c+dx)-\frac{5}{4}Ab^2}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}}{15a^2}$$

$$= \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5a^3d\sqrt{\cos(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sin(c+dx)}{5a^3d\sqrt{\cos(c+dx)}} -$$

$$= -\frac{2\left(5Ab^2+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} + \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2Ab\sin(c+}{3a^2d\cos^{\frac{3}{2}}(c+}$$

$$= -\frac{2\left(5Ab^2+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2AbF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{2b\left(Ab^2+\right.}{}$$

Mathematica [A] time = 4.19485, size = 299, normalized size = 1.45

$$-\frac{\frac{2\left(a^2b(19A+45C)+45Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{\left(6a^3(3A+5C)+40aAb^2\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}-\frac{2\left(3\left(a^2(3A+5C)+5Ab^2\right)\sin(2(c+dx))+6a}{\cos^{\frac{3}{2}}(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])),
x]
```

```
[Out] -((2*(45*A*b^3 + a^2*b*(19*A + 45*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b) + ((40*a*A*b^2 + 6*a^3*(3*A + 5*C))*(2*EllipticF[(c + d*x)/2,
 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (6*(5*A
*b^2 + a^2*(3*A + 5*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Si
n[c + d*x]^2]) - (2*(-10*a*A*b*Sin[c + d*x] + 3*(5*A*b^2 + a^2*(3*A + 5*C))
*Sin[2*(c + d*x)] + 6*a^2*A*Tan[c + d*x]))/Cos[c + d*x]^(3/2))/(30*a^3*d)
```

Maple [B] time = 1.759, size = 786, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(A*b^2+C*a^2)
*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/5*A/a/(8*sin(1/2*d*x+1/2*c)^6-12*
sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*
d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
-2*A/a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2+C*a^2)/a^3*(-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(7/2)),
 x)
```

3.712 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=270

$$\frac{2\left(a^2(5A+7C)+7Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d}+\frac{2b\left(a^2(3A+5C)+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d}+\frac{2b^2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{a^4d(a+b)}$$

```
[Out] (2*b*(5*A*b^2 + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^4*d) + (2*
(7*A*b^2 + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*a^3*d) + (2*b^2*
(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^4*(a + b)*d)
+ (2*A*Sin[c + d*x])/(7*a*d*Cos[c + d*x]^(7/2)) - (2*A*b*Sin[c + d*x])/(5*a
^2*d*Cos[c + d*x]^(5/2)) + (2*(7*A*b^2 + a^2*(5*A + 7*C))*Sin[c + d*x])/(21
*a^3*d*Cos[c + d*x]^(3/2)) - (2*b*(5*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])
/(5*a^4*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.49436, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(a^2(5A+7C)+7Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d}+\frac{2b\left(a^2(3A+5C)+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d}+\frac{2b^2\left(a^2C+Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{a^4d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*b*(5*A*b^2 + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^4*d) + (2*
(7*A*b^2 + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*a^3*d) + (2*b^2*
(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^4*(a + b)*d)
+ (2*A*Sin[c + d*x])/(7*a*d*Cos[c + d*x]^(7/2)) - (2*A*b*Sin[c + d*x])/(5*a
^2*d*Cos[c + d*x]^(5/2)) + (2*(7*A*b^2 + a^2*(5*A + 7*C))*Sin[c + d*x])/(21
*a^3*d*Cos[c + d*x]^(3/2)) - (2*b*(5*A*b^2 + a^2*(3*A + 5*C))*Sin[c + d*x])
/(5*a^4*d*Sqrt[Cos[c + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{-\frac{7Ab}{2}+\frac{1}{2}a(5A+7C)\cos(c+dx)+\frac{5}{2}Ab\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{5}{4}\left(7Ab^2+a^2(5A+7C)\right)+aAb\cos(c+dx)-\frac{21}{4}A}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}}{35a^2}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2+a^2(5A+7C)\right)\sin(c+dx)}{21a^3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2+a^2(5A+7C)\right)\sin(c+dx)}{21a^3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2+a^2(5A+7C)\right)\sin(c+dx)}{21a^3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(5Ab^2+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d} + \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2Ab\sin(c+}{5a^2d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2b\left(5Ab^2+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d} + \frac{2\left(7Ab^2+a^2(5A+7C)\right)F\left(\frac{1}{2}(c+\right.}{21a^3d}$$

Mathematica [A] time = 3.92258, size = 342, normalized size = 1.27

$$\frac{\frac{\left(7a^2b^2(19A+45C)+10a^4(5A+7C)+315Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{8\left(a^3(22A+35C)+35aAb^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} + 5\left(\left(a^3(5A+7C)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*(a + b*Cos[c + d*x])),
x]
```

```
[Out] (((315*A*b^4 + 10*a^4*(5*A + 7*C) + 7*a^2*b^2*(19*A + 45*C))*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(35*a*A*b^2 + a^3*(22*A + 35*C))*
((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/
```

```
2, 2]))/(a + b) + (21*(5*A*b^2 + a^2*(3*A + 5*C))*(-2*a*b*EllipticE[ArcSin[
Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]]
, -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*
Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]) + (-42*b*(a^2*A + (5*A*b^2 + a^2*(3*
A + 5*C))*Cos[c + d*x]^2)*Sin[c + d*x] + 5*((7*a*A*b^2 + a^3*(5*A + 7*C))*S
in[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x]))/Cos[c + d*x]^(5/2))/(105*a^4*d)
```

Maple [B] time = 2.205, size = 982, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b^2+C*a^2
)*b^3/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/5*A/a^2*b/(8*sin(1/2*d*x+1/2*c)^
6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*(A*b^2+C*a^2)/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A/a*(
-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-
2*(A*b^2+C*a^2)/a^4*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin
(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(9/2)),
 x)
```

3.713 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{11}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=344

$$-\frac{2b\left(a^2(5A+7C)+7Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d}-\frac{2\left(3a^2b^2(3A+5C)+a^4(7A+9C)+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^5d}-\frac{2b^3\left(a^2C\right.}{}$$

```
[Out] (-2*(15*A*b^4 + 3*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*EllipticE[(c + d*x
)/2, 2])/(15*a^5*d) - (2*b*(7*A*b^2 + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/
2, 2])/(21*a^4*d) - (2*b^3*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(a^5*(a + b)*d) + (2*A*Sin[c + d*x])/(9*a*d*Cos[c + d*x]^(9/2))
- (2*A*b*Sin[c + d*x])/(7*a^2*d*Cos[c + d*x]^(7/2)) + (2*(9*A*b^2 + a^2*(7*
A + 9*C))*Sin[c + d*x])/(45*a^3*d*Cos[c + d*x]^(5/2)) - (2*b*(7*A*b^2 + a^2
*(5*A + 7*C))*Sin[c + d*x])/(21*a^4*d*Cos[c + d*x]^(3/2)) + (2*(15*A*b^4 +
3*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*Sin[c + d*x])/(15*a^5*d*Sqrt[Cos[c
 + d*x]])
```

Rubi [A] time = 1.9357, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2b\left(a^2(5A+7C)+7Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d}-\frac{2\left(3a^2b^2(3A+5C)+a^4(7A+9C)+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^5d}-\frac{2b^3\left(a^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(11/2)*(a + b*Cos[c + d*x])),x]
```


```
[Out] (-2*(15*A*b^4 + 3*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*EllipticE[(c + d*x
)/2, 2])/(15*a^5*d) - (2*b*(7*A*b^2 + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/
2, 2])/(21*a^4*d) - (2*b^3*(A*b^2 + a^2*C)*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(a^5*(a + b)*d) + (2*A*Sin[c + d*x])/(9*a*d*Cos[c + d*x]^(9/2))
- (2*A*b*Sin[c + d*x])/(7*a^2*d*Cos[c + d*x]^(7/2)) + (2*(9*A*b^2 + a^2*(7*
A + 9*C))*Sin[c + d*x])/(45*a^3*d*Cos[c + d*x]^(5/2)) - (2*b*(7*A*b^2 + a^2
*(5*A + 7*C))*Sin[c + d*x])/(21*a^4*d*Cos[c + d*x]^(3/2)) + (2*(15*A*b^4 +
3*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*Sin[c + d*x])/(15*a^5*d*Sqrt[Cos[c
 + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{11}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{9ad\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{-\frac{9Ab}{2}+\frac{1}{2}a(7A+9C)\cos(c+dx)+\frac{7}{2}Ab\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}dx}{9a}$$

$$= \frac{2A\sin(c+dx)}{9ad\cos^{\frac{9}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{4\int \frac{\frac{7}{4}\left(9Ab^2+a^2(7A+9C)\right)+aAb\cos(c+dx)-\frac{45}{4}Ab}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}}{63a^2}$$

$$= \frac{2A\sin(c+dx)}{9ad\cos^{\frac{9}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(9Ab^2+a^2(7A+9C)\right)\sin(c+dx)}{45a^3d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2A\sin(c+dx)}{9ad\cos^{\frac{9}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(9Ab^2+a^2(7A+9C)\right)\sin(c+dx)}{45a^3d\cos^{\frac{5}{2}}(c+dx)} -$$

$$= \frac{2A\sin(c+dx)}{9ad\cos^{\frac{9}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(9Ab^2+a^2(7A+9C)\right)\sin(c+dx)}{45a^3d\cos^{\frac{5}{2}}(c+dx)} -$$

$$= \frac{2A\sin(c+dx)}{9ad\cos^{\frac{9}{2}}(c+dx)} - \frac{2Ab\sin(c+dx)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(9Ab^2+a^2(7A+9C)\right)\sin(c+dx)}{45a^3d\cos^{\frac{5}{2}}(c+dx)} -$$

$$= -\frac{2\left(15Ab^4+3a^2b^2(3A+5C)+a^4(7A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^5d} + \frac{2A\sin(c+a}{9ad\cos^{\frac{9}{2}}(c+}$$

$$= -\frac{2\left(15Ab^4+3a^2b^2(3A+5C)+a^4(7A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^5d} - \frac{2b\left(7Ab^2+a^2\right.}{}$$

Mathematica [A] time = 4.11208, size = 431, normalized size = 1.25

$$\frac{\frac{2\left(7\left(9a^2Ab^2+a^4(7A+9C)\right)\sin(2(c+dx))+6\sin(c+dx)\left(-5ab\left(a^2(5A+7C)+7Ab^2\right)\cos^2(c+dx)+7\left(3a^2b^2(3A+5C)+a^4(7A+9C)+15Ab^4\right)\cos^3(c+dx)-15a^3Ab\right)+70a^4A\right.}{\cos^{\frac{7}{2}}(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(11/2)*(a + b*Cos[c + d*x]))
,x]
```

```
[Out] (-3*((2*(315*A*b^5 + 7*a^2*b^3*(19*A + 45*C) + a^4*b*(99*A + 133*C))*Ellipt
icPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*(140*a*A*b^4 + 7*a^5*(7*A
 + 9*C) + 4*a^3*b^2*(22*A + 35*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + (14*(15*A*b^4 + 3*
a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
+ d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*
a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a*b*Sqrt[Sin[c + d*x]^2])) + (2*(6*(-15*a^3*A*b - 5*a*b*(7*A*b^2 + a^2*
(5*A + 7*C))*Cos[c + d*x]^2 + 7*(15*A*b^4 + 3*a^2*b^2*(3*A + 5*C) + a^4*(7*
A + 9*C))*Cos[c + d*x]^3)*Sin[c + d*x] + 7*(9*a^2*A*b^2 + a^4*(7*A + 9*C))*
Sin[2*(c + d*x)] + 70*a^4*A*Tan[c + d*x]))/Cos[c + d*x]^(7/2))/(630*a^5*d)
```

Maple [B] time = 2.906, size = 1320, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(A*b^2+C*a^2)
*b^4/a^5/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a*A*(-1/144*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)
^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-2/5*(A*b^2+C*a^2)/a^3/(
8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/si
n(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)-2*b*(A*b^2+C*a^2)/a^4*(-1/6*cos(1/2*d*x+1/2*c)*(
```

```
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))-2/a^2*b*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(A*b^2+C*a^2)/a^5*(-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)/(a+b*cos(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)/(a+b*cos(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c) + a)\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)/(a+b*cos(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(11/2))
, x)
```

3.714 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=370

$$-\frac{a\left(a^2b^2(9A-20C)+21a^4C-4b^4(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^5d\left(a^2-b^2\right)}+\frac{\left(3a^2b^2(5A-8C)+35a^4C-2b^4(5A+3C)\right)E\left(\frac{1}{2}(c+dx\right.}{5b^4d\left(a^2-b^2\right)}$$

```
[Out] ((3*a^2*b^2*(5*A - 8*C) + 35*a^4*C - 2*b^4*(5*A + 3*C))*EllipticE[(c + d*x)
/2, 2])/(5*b^4*(a^2 - b^2)*d) - (a*(a^2*b^2*(9*A - 20*C) + 21*a^4*C - 4*b^4
*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b^
4 - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/((a - b)*b^5*(a + b)^2*d) - (a*(3*A*b^2 + 7*a^2*C - 4*b^2*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d) + ((5*A*b^2 + 7*a^2*C - 2*b^2*C
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)
*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.37583, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{a\left(a^2b^2(9A-20C)+21a^4C-4b^4(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^5d\left(a^2-b^2\right)}+\frac{\left(3a^2b^2(5A-8C)+35a^4C-2b^4(5A+3C)\right)E\left(\frac{1}{2}(c+dx\right.}{5b^4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```



```
[Out] ((3*a^2*b^2*(5*A - 8*C) + 35*a^4*C - 2*b^4*(5*A + 3*C))*EllipticE[(c + d*x)
/2, 2])/(5*b^4*(a^2 - b^2)*d) - (a*(a^2*b^2*(9*A - 20*C) + 21*a^4*C - 4*b^4
*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b^
4 - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/((a - b)*b^5*(a + b)^2*d) - (a*(3*A*b^2 + 7*a^2*C - 4*b^2*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d) + ((5*A*b^2 + 7*a^2*C - 2*b^2*C
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)
*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}\left(Ab^2+a^2C\right)-ab(A+C)\cos(c}{a+b\cos}}{b\left(a^2\right.}$$

$$= \frac{\left(5Ab^2+7a^2C-2b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^{\frac{5}{2}}(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos}$$

$$= -\frac{a\left(3Ab^2+7a^2C-4b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^3\left(a^2-b^2\right)d} + \frac{\left(5Ab^2+7a^2C-2b^2C\right.}{5b^2\left(a\right.}$$

$$= -\frac{a\left(3Ab^2+7a^2C-4b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^3\left(a^2-b^2\right)d} + \frac{\left(5Ab^2+7a^2C-2b^2C\right.}{5b^2\left(a\right.}$$

$$= \frac{\left(3a^2b^2(5A-8C)+35a^4C-2b^4(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4\left(a^2-b^2\right)d} - \frac{a\left(3Ab^2+7a^2\right.}{}$$

$$= \frac{\left(3a^2b^2(5A-8C)+35a^4C-2b^4(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4\left(a^2-b^2\right)d} - \frac{a\left(a^2b^2(9A-\right.}{}$$

Mathematica [A] time = 4.17658, size = 358, normalized size = 0.97

$$\frac{4\sqrt{\cos(c+dx)}\left(-\frac{15a^2(a^2C+Ab^2)\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}-20aC\sin(c+dx)+3bC\sin(2(c+dx))\right)+\frac{2(a^2b^2(15A-32C)+35a^4C-6b^4(5A+3C))\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+d\right.}{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
2,x]
```

```
[Out] (((2*(a^2*b^2*(15*A - 32*C) + 35*a^4*C - 6*b^4*(5*A + 3*C))*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(15*A*b^2 + (14*a^2 + b^2)*C)*((
a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2]))/(a + b) + (6*(3*a^2*b^2*(5*A - 8*C) + 35*a^4*C - 2*b^4*(5*A + 3*C))*(
-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b
)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*(-20*a*C*Sin[c + d*x] - (15*a^2*(A*b^2 +
a^2*C)*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) + 3*b*C*Sin[2*(c +
d*x)]))/(60*b^3*d)
```

Maple [B] time = 1.872, size = 1337, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^2*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)-4/3/b^3*C*(2*a+3*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/b^4*(
```

```
A*b^2+3*C*a^2+4*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-
2*(2*A*a*b^2+A*b^3+4*C*a^3+3*C*a^2*b+2*C*a*b^2+C*b^3)/b^5*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*a^2/b^4*(3*
A*b^2+5*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2+C*a^2)/b^5*(-1/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+
1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2,
 x)
```

3.715 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=292

$$\frac{\left(a^2b^2(3A-16C)+15a^4C-2b^4(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}-\frac{a\left(5a^2C+Ab^2-4b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{a\left(-a^2b^2(A-7C\right.}{}$$

```
[Out] -((a*(A*b^2 + 5*a^2*C - 4*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 - b^2
)*d)) + ((a^2*b^2*(3*A - 16*C) + 15*a^4*C - 2*b^4*(3*A + C))*EllipticF[(c +
 d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a*(3*A*b^4 - a^2*b^2*(A - 7*C) - 5*a^
4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^4*(a + b)^2*d) +
 ((3*A*b^2 + 5*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^
2 - b^2)*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b
^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.96948, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2b^2(3A-16C)+15a^4C-2b^4(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}-\frac{a\left(5a^2C+Ab^2-4b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{a\left(-a^2b^2(A-7C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -((a*(A*b^2 + 5*a^2*C - 4*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 - b^2
)*d)) + ((a^2*b^2*(3*A - 16*C) + 15*a^4*C - 2*b^4*(3*A + C))*EllipticF[(c +
 d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a*(3*A*b^4 - a^2*b^2*(A - 7*C) - 5*a^
4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^4*(a + b)^2*d) +
 ((3*A*b^2 + 5*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^
2 - b^2)*d) - ((A*b^2 + a^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b
^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}\left(Ab^2+a^2C\right)-ab(A+C)\cos(c\right.}{a+b\cos}}{b\left(a^2\right.}$$

$$= \frac{\left(3Ab^2+5a^2C-2b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx}{b\left(a^2-b^2\right)d(a+b\cos}$$

$$= \frac{\left(3Ab^2+5a^2C-2b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx}{b\left(a^2-b^2\right)d(a+b\cos}$$

$$= -\frac{a\left(Ab^2+5a^2C-4b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3Ab^2+5a^2C-2b^2C\right)\sqrt{\cos(c+}}{3b^2\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(Ab^2+5a^2C-4b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(a^2b^2(3A-16C)+15a^4C-2b^4\right.}{3b^4\left(a^2-b\right.}$$

Mathematica [A] time = 2.87611, size = 303, normalized size = 1.04

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a\left(a^2C+Ab^2\right)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2C\right)-\frac{\frac{2a\left(5a^2C-3Ab^2-8b^2C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(C\left(2a^2+b^2\right)+3Ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.\right.}{a+b}}{}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*C + (3*a*(A*b^2 + a^2*C))/((a^2 - b^2)*(a + b*Cos[
c + d*x])))*Sin[c + d*x] - ((2*a*(-3*A*b^2 + 5*a^2*C - 8*b^2*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(3*A*b^2 + (2*a^2 + b^2)*C)*((
a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2]))/(a + b) - (6*(A*b^2 + 5*a^2*C - 4*b^2*C)*(2*a*b*EllipticE[ArcSin[Sqrt
[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1
] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin
[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(12*b^2*d)
```

Maple [B] time = 1.661, size = 1102, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^2*C*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4*C/b^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(A*b^2+3*C*a^2+2*C*a*b+C*b^2)/b^4*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a/b^3*(A*b^2+2
*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b^2+C*a^2)/b^4*(-1/a*b^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
```

```
*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^
2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2,
 x)
```

3.716 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{a\left(-3a^2C+Ab^2+4b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(3a^2C+Ab^2-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2b^2(A+5C)-3a^4C+Ab^4\right)\Pi\left(\right.}{b^3d(a-b)(a+b)^2}$$

```
[Out] ((A*b^2 + 3*a^2*C - 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)
 + (a*(A*b^2 - 3*a^2*C + 4*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^3*(a^2 - b^
2)*d) - ((A*b^4 - 3*a^4*C + a^2*b^2*(A + 5*C))*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) - ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.662286, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3048, 3059, 2639, 3002, 2641, 2805}

$$\frac{a\left(-3a^2C+Ab^2+4b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(3a^2C+Ab^2-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2b^2(A+5C)-3a^4C+Ab^4\right)\Pi\left(\right.}{b^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] ((A*b^2 + 3*a^2*C - 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)
 + (a*(A*b^2 - 3*a^2*C + 4*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^3*(a^2 - b^
2)*d) - ((A*b^4 - 3*a^4*C + a^2*b^2*(A + 5*C))*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) - ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
```

```
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(Ab^2+a^2C\right)-ab(A+C)\cos(c+dx)-\frac{1}{2}\left(Ab\right.}{\sqrt{\cos(c+dx)}(a+b\cos(}}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}b\left(Ab^2+a^2C\right)+\frac{1}{2}a\left(Ab^2-3a^2C+4b^2C\right)c}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{a\left(Ab^2-3a^2C+4b^2C\right)F\left(\frac{1}{2}(c+dx\right.}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 3.24116, size = 284, normalized size = 1.31

$$-\frac{\frac{4\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(C\left(a^2-2b^2\right)-Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(3a^2C+Ab^2-2b^2C\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos\right.\right.\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(b-a)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
2,x]
```

```
[Out] -((4*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*C
os[c + d*x])) + ((2*(-(A*b^2) + (a^2 - 2*b^2)*C)*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/(a + b) + (8*a*(A + C)*((a + b)*EllipticF[(c + d*x)/2, 2]
- a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(A*b^2 + 3*a^2
*C - 2*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b
/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*
x]^2]))/((-a + b)*(a + b)))/(4*b*d)
```

Maple [B] time = 1.713, size = 834, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C/b^3/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-4/b^2*(A*b^2+3*C*a^2)/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-2*a*(A*b^2+C*a^2)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2,
 x)
```

3.717 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(a^2(-C)+Ab^2+2b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2C+Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{abd\left(a^2-b^2\right)}-\frac{\left(-3a^2b^2(A+C)+a^4C+Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{ab^2d(a-b)(a+b)^2}$$

```
[Out] -(((A*b^2 + a^2*C)*EllipticE[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d)) - ((A*b^
2 - a^2*C + 2*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((A*b
^4 + a^4*C - 3*a^2*b^2*(A + C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.705348, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3056, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(a^2(-C)+Ab^2+2b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2C+Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{abd\left(a^2-b^2\right)}-\frac{\left(-3a^2b^2(A+C)+a^4C+Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{ab^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```


```
[Out] -(((A*b^2 + a^2*C)*EllipticE[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d)) - ((A*b^
2 - a^2*C + 2*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((A*b
^4 + a^4*C - 3*a^2*b^2*(A + C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-Ab^2+a^2(2A+C)\right)-ab(A+C)\cos(c+dx)-}{\sqrt{\cos(c+dx)}(a+b\cos(c+}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}b\left(Ab^2-a^2(2A+C)\right)+\frac{1}{2}a\left(Ab^2-\left(a^2-2b^2\right)C\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{ab\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a^2C+2b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{(A}{}$$

Mathematica [A] time = 2.41571, size = 273, normalized size = 1.28

$$\frac{\frac{4\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(a^2(4A+C)-3Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx}\right.\right.\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2
),x]
```

```
[Out] ((4*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Co
s[c + d*x])) + ((2*(-3*A*b^2 + a^2*(4*A + C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(a + b) - (8*a*(A + C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(A*b^2 + a^2*C)*(
2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b
)*(a + b)))/(4*a*d)
```

Maple [B] time = 1.197, size = 804, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/b
*a*C/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^2*(A*b^2+C*a^2)*(-1/a*b^2/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/
(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))
), x)
```

3.718 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=270

$$\frac{\left(a^2C+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{abd\left(a^2-b^2\right)}+\frac{\left(3Ab^2-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\left(-a^2b^2(5A+C)+a^4(-C)+3Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a^2bd(a-b)(a+b)^2}$$

```
[Out] ((3*A*b^2 - a^2*(2*A - C))*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) +
 ((A*b^2 + a^2*C)*EllipticF[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d) + ((3*A*b^
4 - a^4*C - a^2*b^2*(5*A + C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b^2 - a^2*(2*A - C))*Sin[c + d*x])/(a^2*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.06579, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2C+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{abd\left(a^2-b^2\right)}+\frac{\left(3Ab^2-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\left(-a^2b^2(5A+C)+a^4(-C)+3Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a^2bd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```


```
[Out] ((3*A*b^2 - a^2*(2*A - C))*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) +
 ((A*b^2 + a^2*C)*EllipticF[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d) + ((3*A*b^
4 - a^4*C - a^2*b^2*(5*A + C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b^2 - a^2*(2*A - C))*Sin[c + d*x])/(a^2*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-3Ab^2+2a^2\left(A-\frac{C}{2}\right)\right)-ab(A+C)\cos(}{\cos^{\frac{3}{2}}(c+dx)(a+b}}{a\left(a^2-\right.}$$

$$= -\frac{\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+d}$$

$$= -\frac{\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+d}$$

$$= \frac{\left(3Ab^2-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{}{a}$$

$$= \frac{\left(3Ab^2-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} + \frac{(3Ab}{}$$

Mathematica [A] time = 4.04672, size = 310, normalized size = 1.15

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{\left(a^2bC+Ab^3\right)\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}+2A\tan(c+dx)\right)-\frac{-\frac{2\left(a^2b(10A+C)-9Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{\left(8aAb^2-4a^3(A-C)\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(\right.}{a+b}\right.}{b}}{}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2
),x]
```

```
[Out] (-(((-2*(-9*A*b^3 + a^2*b*(10*A + C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b) + ((8*a*A*b^2 - 4*a^3*(A - C))*(2*EllipticF[(c + d*x)/2, 2] -
 (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(-3*A*b^2
 + a^2*(2*A - C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(
a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi
[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c +
d*x]^2]))/((-a + b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]*(((A*b^3 + a^2*b*C)*Si
n[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])) + 2*A*Tan[c + d*x]))/(4*a^2
*d)
```

Maple [B] time = 1.803, size = 899, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b^2+C*a^
2)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin
(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b^2-C*a^2)/a/b*(-1/a*b^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)
), x)
```

3.719 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=336

$$-\frac{\left(5Ab^2-a^2(2A-3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(5Ab^2-a^2(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}-\frac{\left(-a^2b^2(7A-C)-3a^4C+5Ab^4\right)\Pi}{a^3d(a-b)(a+b)}$$

```
[Out] -((b*(5*A*b^2 - a^2*(4*A - C))*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*
d)) - ((5*A*b^2 - a^2*(2*A - 3*C))*EllipticF[(c + d*x)/2, 2])/(3*a^2*(a^2 -
 b^2)*d) - ((5*A*b^4 - a^2*b^2*(7*A - C) - 3*a^4*C)*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) - ((5*A*b^2 - a^2*(2*A - 3*C)
)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) + (b*(5*A*b^2 - a^
2*(4*A - C))*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((A*b^2
 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x]))
```

Rubi [A] time = 1.45327, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(5Ab^2-a^2(2A-3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(5Ab^2-a^2(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}-\frac{\left(-a^2b^2(7A-C)-3a^4C+5Ab^4\right)\Pi}{a^3d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```


```
[Out] -((b*(5*A*b^2 - a^2*(4*A - C))*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*
d)) - ((5*A*b^2 - a^2*(2*A - 3*C))*EllipticF[(c + d*x)/2, 2])/(3*a^2*(a^2 -
 b^2)*d) - ((5*A*b^4 - a^2*b^2*(7*A - C) - 3*a^4*C)*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) - ((5*A*b^2 - a^2*(2*A - 3*C)
)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) + (b*(5*A*b^2 - a^
2*(4*A - C))*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((A*b^2
 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-5Ab^2+2a^2\left(A-\frac{3C}{2}\right)\right)-ab(A+C)\,c}{\cos^{\frac{5}{2}}(c+dx)(a}}{a\left(a^2\right.}$$

$$= -\frac{\left(5Ab^2-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c}$$

$$= -\frac{\left(5Ab^2-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b\left(5Ab^2-a^2(4A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + $$

$$= -\frac{\left(5Ab^2-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b\left(5Ab^2-a^2(4A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + $$

$$= -\frac{b\left(5Ab^2-a^2(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} - \frac{\left(5Ab^2-a^2(2A-3C)\right)\sin(c+dx}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{b\left(5Ab^2-a^2(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} - \frac{\left(5Ab^2-a^2(2A-3C)\right)F\left(\frac{1}{2}(c+d\right.}{3a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 5.23104, size = 336, normalized size = 1.

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^2\left(a^2C+Ab^2\right)\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2A\tan(c+dx)(a\sec(c+dx)-6b)\right)+\frac{\frac{2\left(a^2b^2(44A-9C)+4a^4(A+3C)-45Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8(a^3}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2
),x]
```

```
[Out] (((2*(-45*A*b^4 + a^2*b^2*(44*A - 9*C) + 4*a^4*(A + 3*C))*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-10*a*A*b^2 + a^3*(7*A - 3*C))*((a
+ b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
]))/(a + b) + (6*(-5*A*b^2 + a^2*(4*A - C))*(-2*a*b*EllipticE[ArcSin[Sqrt[C
os[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
```

```
+ (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*
((3*b^2*(A*b^2 + a^2*C)*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) +
2*A*(-6*b + a*Sec[c + d*x])*Tan[c + d*x]))/(12*a^3*d)
```

Maple [B] time = 2.176, size = 1019, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*A*b^3/a^3/(-
2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-4*A/a^3*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+2*(A*b^2+C*a^2)/a^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b
+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)
), x)
```

3.720 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=427

$$\frac{b\left(7Ab^2-a^2(4A-3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\left(a^2-b^2\right)}+\frac{\left(-3a^2b^2(8A-5C)-2a^4(3A+5C)+35Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\left(a^2-b^2\right)}+\frac{b\left(-3a^2b^2(\right.}{}$$

```
[Out] ((35*A*b^4 - 3*a^2*b^2*(8*A - 5*C) - 2*a^4*(3*A + 5*C))*EllipticE[(c + d*x)
/2, 2])/(5*a^4*(a^2 - b^2)*d) + (b*(7*A*b^2 - a^2*(4*A - 3*C))*EllipticF[(c
 + d*x)/2, 2])/(3*a^3*(a^2 - b^2)*d) + (b*(7*A*b^4 - 3*a^2*b^2*(3*A - C) -
5*a^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^4*(a - b)*(a + b)^2*
d) - ((7*A*b^2 - a^2*(2*A - 5*C))*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c
+ d*x]^(5/2)) + (b*(7*A*b^2 - a^2*(4*A - 3*C))*Sin[c + d*x])/(3*a^3*(a^2 -
b^2)*d*Cos[c + d*x]^(3/2)) - ((35*A*b^4 - 3*a^2*b^2*(8*A - 5*C) - 2*a^4*(3*
A + 5*C))*Sin[c + d*x])/(5*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((A*b^2
+ a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d
*x]))
```

Rubi [A] time = 1.80726, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{b\left(7Ab^2-a^2(4A-3C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\left(a^2-b^2\right)}+\frac{\left(-3a^2b^2(8A-5C)-2a^4(3A+5C)+35Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\left(a^2-b^2\right)}+\frac{b\left(-3a^2b^2(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] ((35*A*b^4 - 3*a^2*b^2*(8*A - 5*C) - 2*a^4*(3*A + 5*C))*EllipticE[(c + d*x)
/2, 2])/(5*a^4*(a^2 - b^2)*d) + (b*(7*A*b^2 - a^2*(4*A - 3*C))*EllipticF[(c
 + d*x)/2, 2])/(3*a^3*(a^2 - b^2)*d) + (b*(7*A*b^4 - 3*a^2*b^2*(3*A - C) -
5*a^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^4*(a - b)*(a + b)^2*
d) - ((7*A*b^2 - a^2*(2*A - 5*C))*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c
+ d*x]^(5/2)) + (b*(7*A*b^2 - a^2*(4*A - 3*C))*Sin[c + d*x])/(3*a^3*(a^2 -
b^2)*d*Cos[c + d*x]^(3/2)) - ((35*A*b^4 - 3*a^2*b^2*(8*A - 5*C) - 2*a^4*(3*
A + 5*C))*Sin[c + d*x])/(5*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((A*b^2
+ a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d
```

```
*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-7Ab^2+2a^2\left(A-\frac{5C}{2}\right)\right)-ab(A+C)\cos}{\cos^{\frac{7}{2}}(c+dx)(a+b}}{a\left(a^2-\right.}$$

$$= -\frac{\left(7Ab^2 - a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+}$$

$$= -\frac{\left(7Ab^2 - a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)} + \frac{b\left(7Ab^2-a^2(4A-3C)\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + -$$

$$= -\frac{\left(7Ab^2 - a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)} + \frac{b\left(7Ab^2-a^2(4A-3C)\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} -$$

$$= -\frac{\left(7Ab^2 - a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)} + \frac{b\left(7Ab^2-a^2(4A-3C)\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} -$$

$$= \frac{\left(35Ab^4 - 3a^2b^2(8A-5C) - 2a^4(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4\left(a^2-b^2\right)d} - \frac{\left(7Ab^2-a^2(2A\right.}{5a^2\left(a^2-b^2\right.}$$

$$= \frac{\left(35Ab^4 - 3a^2b^2(8A-5C) - 2a^4(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4\left(a^2-b^2\right)d} + \frac{b\left(7Ab^2-a^2(4\right.}{3a}$$

Mathematica [A] time = 7.01756, size = 496, normalized size = 1.16

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{-a^2b^3C\sin(c+dx)-Ab^5\sin(c+dx)}{a^4\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{2\sec(c+dx)\left(3a^2A\sin(c+dx)+5a^2C\sin(c+dx)+15Ab^2\sin(c+dx)\right)}{5a^4} - \frac{4Ab\tan(c+dx)\sec(c+dx)}{3a^3} + \frac{2A}{}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^2
),x]
```

```
[Out] -((2*(-58*a^4*A*b - 272*a^2*A*b^3 + 315*A*b^5 - 150*a^4*b*C + 135*a^2*b^3*C
)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + ((-36*a^5*A - 184*a^
```

```
3*A*b^2 + 280*a*A*b^4 - 60*a^5*C + 120*a^3*b^2*C)*(2*EllipticF[(c + d*x)/2,
 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(-18
*a^4*A*b - 72*a^2*A*b^3 + 105*A*b^5 - 30*a^4*b*C + 45*a^2*b^3*C)*Cos[2*(c +
 d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Elli
pticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -A
rcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[1 - Cos[c + d*x]^
2]*(-1 + 2*Cos[c + d*x]^2)))/(60*a^4*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*
x]]*((2*Sec[c + d*x]*(3*a^2*A*Sin[c + d*x] + 15*A*b^2*Sin[c + d*x] + 5*a^2*
C*Sin[c + d*x]))/(5*a^4) + (-(A*b^5*Sin[c + d*x]) - a^2*b^3*C*Sin[c + d*x])
/(a^4*(a^2 - b^2)*(a + b*Cos[c + d*x])) - (4*A*b*Sec[c + d*x]*Tan[c + d*x])
/(3*a^3) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5*a^2)))/d
```

Maple [B] time = 2.956, size = 1353, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*(3*A*b^2+
C*a^2)/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/5*A/a^2/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*(3*A*b^2+C*a^2)/a^4*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1)-4*A/a^3*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))-2*(A*b^2+C*a^2)*b/a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/
```

```
2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(7/2)
), x)
```

3.721 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=433

$$\frac{\left(a^4b^2(9A-223C)-a^2b^4(15A-128C)+105a^6C+8b^6(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}-\frac{a\left(a^2b^2(3A-65C)+35a^4C-3b^4(3\right.}{4b^4d\left(a^2-b\right.}$$

```
[Out] -(a*(a^2*b^2*(3*A - 65*C) - 3*b^4*(3*A - 8*C) + 35*a^4*C)*EllipticE[(c + d*
x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + ((a^4*b^2*(9*A - 223*C) - a^2*b^4*(15*A
 - 128*C) + 105*a^6*C + 8*b^6*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(12*b^5
*(a^2 - b^2)^2*d) - (a*(15*A*b^6 + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A -
21*C) + 35*a^6*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b
^5*(a + b)^3*d) + ((a^2*b^2*(3*A - 61*C) - b^4*(21*A - 8*C) + 35*a^4*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2)
+ ((5*A*b^4 - 7*a^4*C + a^2*b^2*(A + 13*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.60115, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3048, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^4b^2(9A-223C)-a^2b^4(15A-128C)+105a^6C+8b^6(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}-\frac{a\left(a^2b^2(3A-65C)+35a^4C-3b^4(3\right.}{4b^4d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```



```
[Out] -(a*(a^2*b^2*(3*A - 65*C) - 3*b^4*(3*A - 8*C) + 35*a^4*C)*EllipticE[(c + d*
x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + ((a^4*b^2*(9*A - 223*C) - a^2*b^4*(15*A
 - 128*C) + 105*a^6*C + 8*b^6*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(12*b^5
*(a^2 - b^2)^2*d) - (a*(15*A*b^6 + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A -
21*C) + 35*a^6*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b
^5*(a + b)^3*d) + ((a^2*b^2*(3*A - 61*C) - b^4*(21*A - 8*C) + 35*a^4*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2)
+ ((5*A*b^4 - 7*a^4*C + a^2*b^2*(A + 13*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x]
```

```
)/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}\left(Ab^2+a^2C\right)-2ab(A+C)\cos\cdots\right.}{(a+b\cos\cdots}}{2b\left(\cdots\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(5Ab^4-7a^4C+a^2b^2(A+13C)\right)\cdots}{4b^2\left(a^2-b^2\right)^2 d(a+b\cdots}$$

$$= \frac{\left(a^2b^2(3A-61C)-b^4(21A-8C)+35a^4C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} - \frac{(Ab\cdots}{2\cdots}$$

$$= \frac{\left(a^2b^2(3A-61C)-b^4(21A-8C)+35a^4C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} - \frac{(Ab\cdots}{2\cdots}$$

$$= -\frac{a\left(a^2b^2(3A-65C)-3b^4(3A-8C)+35a^4C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^2b^2(3A\cdots\right.}{\cdots}$$

$$= -\frac{a\left(a^2b^2(3A-65C)-3b^4(3A-8C)+35a^4C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4b^2(9A\cdots\right.}{\cdots}$$

Mathematica [A] time = 4.40414, size = 432, normalized size = 1.

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(ab\left(a^2b^2(9A-83C)+49a^4C+b^4(16C-27A)\right)\cos(c+dx)+3a^4Ab^2-21a^2Ab^4+4C\left(b^3-a^2b\right)^2\cos(2(c+dx))-57a^4b^2C+35a^6C+4b^6C\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{2(a^{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*a^4*A*b^2 - 21*a^2*A*b^4 + 35*a^6*C - 57*a^4*b^2*
C + 4*b^6*C + a*b*(a^2*b^2*(9*A - 83*C) + 49*a^4*C + b^4*(-27*A + 16*C))*Co
s[c + d*x] + 4*(-(a^2*b) + b^3)^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/((a^2 -
 b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(a^3*b^2*(3*A - 73*C) + 35*a^5*C + a*
b^4*(15*A + 56*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (16
*(-7*a^4*C + 2*b^4*(3*A + C) + a^2*b^2*(3*A + 14*C))*((a + b)*EllipticF[(c
```

```
+ d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(
a^2*b^2*(3*A - 65*C) + 35*a^4*C + 3*b^4*(-3*A + 8*C))*(-2*a*b*EllipticE[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*
x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -
1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(48*b^
3*d)
```

Maple [B] time = 2.78, size = 2240, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^3*C*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2*C/b^4*(3*a+2*b)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))+2*(A*b^2+6*C*a^2+3*C*a*b+C*b^2)/b^5*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a/b^4*(3*A
*b^2+10*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2+C*a^2)/b^5*(-1/2
/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2
-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/
```

```
a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*
b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2)))+2*a^2/b^5*(3*A*b^2+5*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*
b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2
-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3,
 x)
```

3.722 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=345

$$-\frac{a\left(-a^2b^2(A+33C)+15a^4C+b^4(7A+24C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^2(A+29C)-15a^4C+b^4(5A-8C)\right)E\left(\frac{1}{2}(c+d$$

```
[Out] -((b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*EllipticE[(c + d*x)/2,
2])/(4*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*C + b^4*(7*A + 24*C) - a^2*b^2*(A
+ 33*C))*EllipticF[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 + 1
5*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d) - ((A*b^2 + a^2*C)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (
(3*A*b^4 - 5*a^4*C + a^2*b^2*(3*A + 11*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.14075, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{a\left(-a^2b^2(A+33C)+15a^4C+b^4(7A+24C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^2(A+29C)-15a^4C+b^4(5A-8C)\right)E\left(\frac{1}{2}(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*EllipticE[(c + d*x)/2,
2])/(4*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*C + b^4*(7*A + 24*C) - a^2*b^2*(A
+ 33*C))*EllipticF[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 + 1
5*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d) - ((A*b^2 + a^2*C)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (
(3*A*b^4 - 5*a^4*C + a^2*b^2*(3*A + 11*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}\left(Ab^2+a^2C\right)-2ab(A+C)\cos\cdots\right.}{(a+b\cos\cdots}}{2b\left(a\cdots\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(3Ab^4-5a^4C+a^2b^2(3A+11C)\cdots\right.}{4b^2\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= -\frac{\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(3Ab^4-5a^4C+a^2b^2(3A+11C)\cdots\right.}{4b^2\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= -\frac{\left(b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2C\right)\cdots}{2b\left(a^2-b^2\cdots\right.}$$

$$= -\frac{\left(b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a\left(15a^4C+b\cdots\right.}{\cdots}$$

Mathematica [A] time = 4.51761, size = 372, normalized size = 1.08

$$\frac{\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(\left(a^2b^3(A+13C)-7a^4bC+5Ab^5\right)\cos(c+dx)+a^3b^2(3A+11C)-5a^5C+3aAb^4\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} + \frac{\frac{\left(a^2b^2(5A-7C)+5a^4C+b^4(A+8C)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{8a\left(C\left(a^2-\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
3,x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(3*a*A*b^4 - 5*a^5*C + a^3*b^2*(3*A + 11*C) + (5*A*b
^5 - 7*a^4*b*C + a^2*b^3*(A + 13*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^
2)^2*(a + b*Cos[c + d*x])^2) + (((a^2*b^2*(5*A - 7*C) + 5*a^4*C + b^4*(A +
8*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(-3*A*b^2 +
 (a^2 - 4*b^2)*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(
a + b), (c + d*x)/2, 2]))/(a + b) + ((15*a^4*C + b^4*(-5*A + 8*C) - a^2*b^2
*(A + 29*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b
)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/
a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x
]^2]))/((a - b)^2*(a + b)^2))/(8*b^2*d)
```

Maple [B] time = 2.484, size = 1966, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-4/b^3*(A*b^2+6*C*a^2)/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*a^2*(A*b^2+C*a^2)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a
^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
```

```
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-
3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-4*a/b^4*(A*b^2+2*C*
a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

[Out] Timed out

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3,
 x)
```

3.723 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=348

$$\frac{\left(a^2b^2(3A-5C)+3a^4C+b^4(3A+8C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{\left(a^2b^2(5A+9C)-3a^4C+Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{\left(-3a^4b^2\right.}{}$$

```
[Out] ((A*b^4 - 3*a^4*C + a^2*b^2*(5*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(4*a*b^
2*(a^2 - b^2)^2*d) + ((a^2*b^2*(3*A - 5*C) + 3*a^4*C + b^4*(3*A + 8*C))*Ell
ipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((A*b^6 - 3*a^4*b^2*(A -
2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(4*a*(a - b)^2*b^3*(a + b)^3*d) - ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((A*b^4 - 3*a
^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*b*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.13, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3048, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2b^2(3A-5C)+3a^4C+b^4(3A+8C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{\left(a^2b^2(5A+9C)-3a^4C+Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{\left(-3a^4b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] ((A*b^4 - 3*a^4*C + a^2*b^2*(5*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(4*a*b^
2*(a^2 - b^2)^2*d) + ((a^2*b^2*(3*A - 5*C) + 3*a^4*C + b^4*(3*A + 8*C))*Ell
ipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((A*b^6 - 3*a^4*b^2*(A -
2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(4*a*(a - b)^2*b^3*(a + b)^3*d) - ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((A*b^4 - 3*a
^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*b*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(Ab^2+a^2C\right)-2ab(A+C)\cos(c+dx)+\frac{1}{2}(\cdots}{\sqrt{\cos(c+dx)}(a+b\cos\cdots}}{2b\left(a^2-\cdots\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(Ab^4-3a^4C+a^2b^2(5A+9C)\right)\cdots}{4ab\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(Ab^4-3a^4C+a^2b^2(5A+9C)\right)\cdots}{4ab\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= \frac{\left(Ab^4-3a^4C+a^2b^2(5A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+\cdots}}{2b\left(a^2-b^2\right)d(a+b\,\mathrm{c}\cdots}$$

$$= \frac{\left(Ab^4-3a^4C+a^2b^2(5A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^2b^2(3A-5C)+3a^4C\cdots\right.}{4b^3\cdots}$$

Mathematica [A] time = 3.6215, size = 368, normalized size = 1.06

$$\frac{\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(-a^2b^2(5A+9C)+3a^4C-Ab^4\right)\cos(c+dx)-7a^3b^2(A+C)+a^5C+aAb^4\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{\frac{\left(a^2b^2(9A+5C)+a^4C-3Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{8\left(a^3(2A+C)+ab^2(A+2C\right.}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
3,x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*A*b^4 + a^5*C - 7*a^3*b^2*(A + C) + b*(-(A*b^4) +
 3*a^4*C - a^2*b^2*(5*A + 9*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*
(a + b*Cos[c + d*x])^2) - (((-3*A*b^4 + a^4*C + a^2*b^2*(9*A + 5*C))*Ellipt
icPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(a^3*(2*A + C) + a*b^2*(A
 + 2*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2]))/(a + b) + ((-(A*b^4) + 3*a^4*C - a^2*b^2*(5*A + 9*C))*(-2*
a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[
Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2
*(a + b)^2))/(8*a*b*d)
```

Maple [B] time = 2.326, size = 1934, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12/
b^2*C*a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b^2+C*a^2)/b^3*(-1/2/a*b^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-
b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b
/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/b
^3*(A*b^2+3*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3,
 x)
```

3.724 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=345

$$\frac{\left(-7a^2b^2(A+C)+a^4C+Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{\left(-a^2b^2(9A+5C)+a^4(-C)+3Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd\left(a^2-b^2\right)^2}+\frac{\left(5a^4b^2(3A+2\right.}{}$$

```
[Out] ((3*A*b^4 - a^4*C - a^2*b^2*(9*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(4*a^2*
b*(a^2 - b^2)^2*d) + ((A*b^4 + a^4*C - 7*a^2*b^2*(A + C))*EllipticF[(c + d*
x)/2, 2])/(4*a*b^2*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^2*b^4*(2*A - C) - a^6
*C + 5*a^4*b^2*(3*A + 2*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a
^2*(a - b)^2*b^2*(a + b)^3*d) + ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b^4 - a^4*C - a^2
*b^2*(9*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(
a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.07334, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-7a^2b^2(A+C)+a^4C+Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{\left(-a^2b^2(9A+5C)+a^4(-C)+3Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd\left(a^2-b^2\right)^2}+\frac{\left(5a^4b^2(3A+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```



```
[Out] ((3*A*b^4 - a^4*C - a^2*b^2*(9*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(4*a^2*
b*(a^2 - b^2)^2*d) + ((A*b^4 + a^4*C - 7*a^2*b^2*(A + C))*EllipticF[(c + d*
x)/2, 2])/(4*a*b^2*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^2*b^4*(2*A - C) - a^6
*C + 5*a^4*b^2*(3*A + 2*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a
^2*(a - b)^2*b^2*(a + b)^3*d) + ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b^4 - a^4*C - a^2
*b^2*(9*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(
a + b*Cos[c + d*x]))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-3Ab^2+a^2(4A+C)\right)-2ab(A+C)\cos(c+dx)+}{\sqrt{\cos(c+dx)}(a+b\cos(c+a}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3Ab^4-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos}}{4a^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3Ab^4-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos}}{4a^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= \frac{\left(3Ab^4-a^4C-a^2b^2(9A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2b\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\,s}{2a\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= \frac{\left(3Ab^4-a^4C-a^2b^2(9A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2b\left(a^2-b^2\right)^2d} + \frac{\left(Ab^4+a^4C-7a^2b^2(A+C\right.}{4ab^2\left(a^2-b^2\right.}$$

Mathematica [A] time = 4.90274, size = 370, normalized size = 1.07

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(a^2b^2(9A+5C)+a^4C-3Ab^4\right)\cos(c+dx)+a^3b^2(11A+3C)+3a^5C-5aAb^4\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\frac{2\left(a^2b^2(C-19A)+a^4(16A+5C)+9Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16\left(aAb\right.}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3
),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(-5*a*A*b^4 + 3*a^5*C + a^3*b^2*(11*A + 3*C) + b*(-3
*A*b^4 + a^4*C + a^2*b^2*(9*A + 5*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b
^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(9*A*b^4 + a^2*b^2*(-19*A + C) + a^4*(1
6*A + 5*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(a*A*b
^2 - a^3*(4*A + 3*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*(-3*A*b^4 + a^4*C + a^2*b^2*(9*A
 + 5*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*El
lipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a),
-ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]
))/((a - b)^2*(a + b)^2))/(16*a^2*d)
```

Maple [B] time = 2.148, size = 1846, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b^2+C*a^2)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2
-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b
^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/
4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-4*a*C/b^2*(-1/a*b^2/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))
), x)
```

3.725 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=417

$$-\frac{\left(-a^2b^2(11A+3C)-3a^4C+5Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{\left(-a^2b^2(29A+C)+a^4(8A-5C)+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{(5}{}$$

```
[Out] -((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*EllipticE[(c + d*x)/2,
2])/(4*a^3*(a^2 - b^2)^2*d) - ((5*A*b^4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*E
llipticF[(c + d*x)/2, 2])/(4*a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 + 3*a^6*C
- a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(4*a^3*(a - b)^2*b*(a + b)^3*d) + ((15*A*b^4 + a^4*(8*A - 5*C
) - a^2*b^2*(29*A + C))*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d
*x]]) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]
]*(a + b*Cos[c + d*x])^2) - ((5*A*b^4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sin
[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.54741, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(-a^2b^2(11A+3C)-3a^4C+5Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{\left(-a^2b^2(29A+C)+a^4(8A-5C)+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{(5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```


```
[Out] -((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*EllipticE[(c + d*x)/2,
2])/(4*a^3*(a^2 - b^2)^2*d) - ((5*A*b^4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*E
llipticF[(c + d*x)/2, 2])/(4*a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 + 3*a^6*C
- a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(4*a^3*(a - b)^2*b*(a + b)^3*d) + ((15*A*b^4 + a^4*(8*A - 5*C
) - a^2*b^2*(29*A + C))*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d
*x]]) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]
]*(a + b*Cos[c + d*x])^2) - ((5*A*b^4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sin
[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-5Ab^2+a^2(4A-C)\right)-2ab(A+C)}{\cos^{\frac{3}{2}}(c+dx)(}}{2a\,(}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} - \frac{\left(5Ab^4-3a^4C-a^2b^2(11A\right.}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= \frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2+a^2\right.}{2a\left(a^2-b^2\right)d\sqrt{\cos(c}}$$

$$= \frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2+a^2\right.}{2a\left(a^2-b^2\right)d\sqrt{\cos(c}}$$

$$= -\frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(15Ab^4+a^4(8\right.}{4a}$$

$$= -\frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} - \frac{\left(5Ab^4-3a^4C\right.}{}$$

Mathematica [A] time = 5.37121, size = 429, normalized size = 1.03

$$\frac{\frac{\sqrt{\cos(c+dx)}\left(b^2\left(-a^2b^2(29A+C)+a^4(8A-5C)+15Ab^4\right)\sin(2(c+dx))+2ab\left(a^2b^2(C-47A)+a^4(16A-7C)+25Ab^4\right)\sin(c+dx)+16A\left(a^3-ab^2\right)^2\tan(c+dx)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\left(-a^2b^3\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3
),x]
```

```
[Out] (-((((45*A*b^5 - a^2*b^3*(95*A + 3*C) + a^4*b*(56*A + 9*C))*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(5*a*A*b^4 + 2*a^5*(A - C) - a^3*b
^2*(10*A + C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]))/(b*(a + b)) + ((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2
*(29*A + C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b
)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/
```

```
a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^
2]))/((a - b)^2*(a + b)^2)) + (Sqrt[Cos[c + d*x]]*(2*a*b*(25*A*b^4 + a^4*(1
6*A - 7*C) + a^2*b^2*(-47*A + C))*Sin[c + d*x] + b^2*(15*A*b^4 + a^4*(8*A -
 5*C) - a^2*b^2*(29*A + C))*Sin[2*(c + d*x)] + 16*A*(a^3 - a*b^2)^2*Tan[c +
 d*x]))/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(8*a^3*d)
```

Maple [B] time = 2.79, size = 2023, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b^2-C*a^2)/a/b*(-1/2/a*b^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b
)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9
/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
```

```
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^3*(-(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b^
2+C*a^2)/a^2/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2
)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)
), x)
```

3.726 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=494

$$\frac{\left(-a^2b^2(61A-3C)+a^4(8A-21C)+35Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-a^2b^2(65A-3C)+3a^4(8A-3C)+35Ab^4\right)E\left(\frac{1}{2}(c\right.}{4a^4d\left(a^2-b^2\right)^2}$$

```
[Out] (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*A - 3*C))*EllipticE[(c + d*x
)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A*b^4 + a^4*(8*A - 21*C) - a^2*b^2*
(61*A - 3*C))*EllipticF[(c + d*x)/2, 2])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A*
b^6 - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((35*A*b^4
+ a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2
)^2*d*Cos[c + d*x]^(3/2)) - (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*
A - 3*C))*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + ((A*b^
2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c
 + d*x])^2) - ((7*A*b^4 - 5*a^4*C - a^2*b^2*(13*A + C))*Sin[c + d*x])/(4*a^
2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.09596, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-a^2b^2(61A-3C)+a^4(8A-21C)+35Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-a^2b^2(65A-3C)+3a^4(8A-3C)+35Ab^4\right)E\left(\frac{1}{2}(c\right.}{4a^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```


```
[Out] (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*A - 3*C))*EllipticE[(c + d*x
)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A*b^4 + a^4*(8*A - 21*C) - a^2*b^2*
(61*A - 3*C))*EllipticF[(c + d*x)/2, 2])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A*
b^6 - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((35*A*b^4
+ a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2
)^2*d*Cos[c + d*x]^(3/2)) - (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*
A - 3*C))*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + ((A*b^
```

```
2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c
 + d*x])^2) - ((7*A*b^4 - 5*a^4*C - a^2*b^2*(13*A + C))*Sin[c + d*x])/(4*a^
2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-7Ab^2+a^2(4A-3C)\right)-2ab(A+C)}{\cos^{\frac{5}{2}}(c+dx)(a}}{2a\left(a\right.}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} - \frac{\left(7Ab^4-5a^4C-a^2b^2(13A\right.}{4a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)(}$$

$$= \frac{\left(35Ab^4+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(Ab^2+a^2\right.}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c}$$

$$= \frac{\left(35Ab^4+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(35Ab^4+3a^4(8A\right.}{4a^4\left(a^2\right.}$$

$$= \frac{\left(35Ab^4+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(35Ab^4+3a^4(8A\right.}{4a^4\left(a^2\right.}$$

$$= \frac{b\left(35Ab^4+3a^4(8A-3C)-a^2b^2(65A-3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d} + \frac{\left(35Ab^4+a^4(8\right.}{1}$$

$$= \frac{b\left(35Ab^4+3a^4(8A-3C)-a^2b^2(65A-3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d} + \frac{\left(35Ab^4+a^4(8\right.}{}$$

Mathematica [A] time = 7.23324, size = 547, normalized size = 1.11

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{a^2b^2C\sin(c+dx)+Ab^4\sin(c+dx)}{2a^3\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{17a^2Ab^4\sin(c+dx)-3a^2b^4C\sin(c+dx)+9a^4b^2C\sin(c+dx)-11Ab^6\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2(a+b\cos(c+dx))} - \frac{6Ab\tan(c+dx)}{a^4} + \frac{2A\,\mathrm{t}}{}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3
),x]
```

```
[Out] ((2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 + 48*a^6*C - 57*a
^4*b^2*C + 27*a^2*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)
```

```
 + ((160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 96*a^5*b*C + 24*a^3*b^3*C)
*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a + b)))/b + (2*(72*a^4*A*b^2 - 195*a^2*A*b^4 + 105*A*b^6 - 27*a^4*b^
2*C + 9*a^2*b^4*C)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2
 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/
(a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(48*a^4*(a - b)^2
*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((A*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c
+ d*x])/(2*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (17*a^2*A*b^4*Sin[c +
d*x] - 11*A*b^6*Sin[c + d*x] + 9*a^4*b^2*C*Sin[c + d*x] - 3*a^2*b^4*C*Sin[c
 + d*x])/(4*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) - (6*A*b*Tan[c + d*x])/
a^4 + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 3.81, size = 2140, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-12*A*b^3/a^4/(
-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b^2+C*a^2)/a^2*(-1/2/a*b^2/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+
b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
```

```
2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-6/a^4*b*A*(-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^3
*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*A*b^2/a^3*(-1/a*b^2/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)
/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)
), x)
```

3.727 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=553

$$-\frac{\left(3a^2C-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(3a^2C-2abC-8b^2(3A+2C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{24b^2d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*C - 8*b^2*(3*A + 2*C))*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(24*a*b^2*d) - (Sqrt[a + b]*(3*a^2*C - 2*a*b*C - 8*b^2*(3*A + 2*
C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) - (a*Sqrt[a + b]*(8*A*b^2
 + (a^2 + 4*b^2)*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^3*d
) - ((3*a^2*C - 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
24*b^2*d*Sqrt[Cos[c + d*x]]) - (a*C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(4*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 1.53172, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(3a^2C-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(3a^2C-2abC-8b^2(3A+2C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{24b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*C - 8*b^2*(3*A + 2*C))*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(24*a*b^2*d) - (Sqrt[a + b]*(3*a^2*C - 2*a*b*C - 8*b^2*(3*A + 2*
C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) - (a*Sqrt[a + b]*(8*A*b^2
```

```
 + (a^2 + 4*b^2)*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^3*d
) - ((3*a^2*C - 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
24*b^2*d*Sqrt[Cos[c + d*x]]) - (a*C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(4*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(3*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{\int \frac{\sqrt{a}}{}}{}$$

$$= -\frac{aC\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd} + \frac{C\sqrt{c}}{}$$

$$= -\frac{\left(3a^2C-8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(3a^2C-8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a\sqrt{a+b}\left(8Ab^2+\left(a^2+4b^2\right)C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin\right.}{8}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24A+\left(16-\frac{3a^2}{b^2}\right)C\right)\cot(c+dx)E\left(\sin\right.}{24}$$

Mathematica [C] time = 6.30593, size = 1220, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2
),x]
```

```
[Out] ((-4*a*(24*A*b^2 - a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(48*
a*A*b + 28*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
```

```
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(24*A*b^2 - 3*a^2*C + 16*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b
*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-
2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[
((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*b*d) + (Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]*((a*C*Sin[c + d*x])/(12*b) + (C*Sin[2*(c + d*x)])/6)
)/d
```

Maple [B] time = 0.291, size = 2526, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b
^2+10*C*cos(d*x+c)^4*a*b^2-C*cos(d*x+c)^3*a^2*b+3*C*cos(d*x+c)^2*a^2*b+6*C*
cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+8*C*cos(d*x+c
)^5*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
```

^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*a^2*b-28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*
b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b
^3+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-
28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+6*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*a^3-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*a^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*C*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/

```
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
3+8*C*cos(d*x+c)^3*b^3-3*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3+3*C*cos(d
*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+c)^2*b^3)/sin(d*x+c)/b^2/cos(d
*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

3.728 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=455

$$\frac{\sqrt{a+b}\left(a^2C-4b^2(2A+C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{\sqrt{a+b}(C(}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[
a + b]*(8*A*b + (a + 2*b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (
Sqrt[a + b]*(a^2*C - 4*b^2*(2*A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(4*b^2*d) + (a*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sq
rt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(2*d)
```

Rubi [A] time = 1.08326, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3050, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(a^2C-4b^2(2A+C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{\sqrt{a+b}(C(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[
a + b]*(8*A*b + (a + 2*b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (
Sqrt[a + b]*(a^2*C - 4*b^2*(2*A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
```

```
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(4*b^2*d) + (a*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sq
rt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(2*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\frac{1}{2}a(4A+C)+b}{\sqrt{c}}$$

$$= \frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}{2d}$$

$$= \frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}{2d}$$

$$= \frac{\sqrt{a+b}\left(a^2C-4b^2(2A+C)\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{4b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{4bd}$$

Mathematica [C] time = 8.74247, size = 1169, normalized size = 2.57

$$-\frac{4a(8aA+3aC)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c\right.}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\quad}{\quad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(8*a*A + 3*a*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
```

```
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*A*b + 4*b*C)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*a*C*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d)
```

Maple [B] time = 0.191, size = 2165, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-C*cos(d*x+c)^3*a*b-2*C*cos(d*x+c)^2*a*b+3*C
*cos(d*x+c)^4*a*b+C*cos(d*x+c)^3*a^2-16*A*cos(d*x+c)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+32*A*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
```

```
(d*x+c)))^(1/2)*a*b-4*C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-2*C*cos(d*x+c)^2*sin(d*x+c)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*c
os(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*b^2+C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-4*C*cos(d*x+c)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-2*C*cos(d*x
+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*cos(d*x+c)^2*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*b^2+16*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-C*cos(d*x+c)^2*a^2
+2*C*cos(d*x+c)^5*b^2-2*C*cos(d*x+c)^3*b^2+16*A*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+2*C*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a*b+C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8
*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*b^2+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*b^2)/b/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)
), x)
```

3.729 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=439

$$-\frac{(2A-C)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}(2aA-aC-2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (
Sqrt[a + b]*(2*a*A - 2*A*b - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
- (a*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (
2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*A -
 C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.06127, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3048, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2A-C)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}(2aA-aC-2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (
Sqrt[a + b]*(2*a*A - 2*A*b - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
- (a*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
```

```
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (
2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*A -
 C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
```

```
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{Ab}{2}-\frac{1}{2}a(A-C)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2A-C)\sqrt{a+b\cos(c+dx)}\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2A-C)\sqrt{a+b\cos(c+dx)}\sin(}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}(2A-C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{ad}$$

Mathematica [C] time = 20.1035, size = 1166, normalized size = 2.66

$$-\frac{4abC\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)F\left(\sin^{-1}\right.}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \overline{\qquad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((-4*a
*b*C*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
```

```
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-2*a*A + 2*a*C)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*A*b + b*C)*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*d)
```

Maple [B] time = 0.212, size = 1585, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d/(a+b*cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
```

```
)^(1/2))*a-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-2*C*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-
2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b+C*cos(d*x+c)^3*b+2*A*cos(d*x+c)^2*b+C*cos(d*x+c)^2*a-C*
cos(d*x+c)^2*b+2*A*cos(d*x+c)*a-2*A*cos(d*x+c)*b-C*cos(d*x+c)*a-2*a*A)/sin(
d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\cos^{2}\left(c+dx\right)\right)\sqrt{a+b\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

3.730 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=394

$$\frac{2Ab(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2\sqrt{a+b}(Ab-a(A+3C))}{}$$

```
[Out] (2*A*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) - (
2*Sqrt[a + b]*(A*b - a*(A + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d
) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*
A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.768762, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3048, 3053, 2809, 2998, 2816, 2994}

$$\frac{2Ab(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2\sqrt{a+b}(Ab-a(A+3C))}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (2*A*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) - (
2*Sqrt[a + b]*(A*b - a*(A + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d
) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*
```

```
A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{Ab}{2}+\frac{1}{2}a(A+3C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{Ab}{2}+\frac{1}{2}a(A+3C)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+}}}{d}$$

$$= \frac{2A(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-se}{a}}}{3a^2d}$$

Mathematica [A] time = 8.7452, size = 318, normalized size = 0.81

$$-\frac{2\left(\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(-a(a(A+3C)+b(A-3C))\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+b\left(A\,\mathrm{ta}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (-2*(-((A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/Cos[c + d*x]^(3/2)) + Sqrt[C
os[(c + d*x)/2]^2]*(A*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(b*(
A - 3*C) + a*(A + 3*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(6*a*C*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + A*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d
*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))))/(3*a*d*Sqrt[a + b*Cos[c + d*x]
])
```

Maple [B] time = 0.247, size = 1753, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(1/2)*(6*C*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+3*C*cos(d*x+c)^2*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b+12*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+6*C*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
-6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
```

```
1/2))*a*b-A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*C*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+3*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*cos(d*x
+c)^3*a*b+A*cos(d*x+c)^3*b^2+A*cos(d*x+c)^2*a^2+A*cos(d*x+c)^2*a*b-A*cos(d*
x+c)^2*b^2-2*A*cos(d*x+c)*a*b-A*a^2)/a/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

3.731 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=345

$$-\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-3a^2(3A+5C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 3*a^2*(3*A + 5*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b + 15*a*C)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.840176, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3048, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-3a^2(3A+5C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 3*a^2*(3*A + 5*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b + 15*a*C)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
```

```
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{Ab}{2}+\frac{1}{2}a(3A+5C)\cos(c+d}{\cos^{\frac{5}{2}}(c+dx)\sqrt{}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx}{15ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx}{15ad\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-3a^2(3A+5C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{15a^3d}$$

Mathematica [C] time = 6.33669, size = 1288, normalized size = 3.73

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] -((-4*a*(2*a^2*A*b - 2*A*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
```

```
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(9*a^3*A -
2*a*A*b^2 + 15*a^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi
[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]])) + 2*(9*a^2*A*b - 2*A*b^3 + 15*a^2*b*C)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(15*a^2*d) + (Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(9*a^2*A*Sin[c + d*x] - 2*
A*b^2*Sin[c + d*x] + 15*a^2*C*Sin[c + d*x]))/(15*a^2) + (2*A*b*Sec[c + d*x]
*Tan[c + d*x])/(15*a) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.203, size = 2434, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos(d*x
+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b+A*cos(d*x+c)^4*a*b^2-5*A*cos(d*x+c)^3*a^
2*b-2*A*cos(d*x+c)^3*a*b^2-4*A*cos(d*x+c)*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
```

)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*
A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15*C*cos(d*x+c
)^3*a^3-15*C*cos(d*x+c)^2*a^3+2*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9
*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2+15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x

```
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*A*sin(d*x
+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+
c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2
), x)
```

3.732 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=415

$$-\frac{2\left(4Ab^2-5a^2(5A+7C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(5a^2(5A+7C)+6aAb+8Ab^2\right)\cot(c+dx}{1\dots}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^2 + a^2*(19*A + 35*C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(6*a*A*b + 8*A*b^2 + 5*
a^2*(5*A + 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*A*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*A*b*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b^2
- 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[
c + d*x]^(3/2))
```

Rubi [A] time = 1.18859, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3048, 3055, 2998, 2816, 2994}

$$-\frac{2\left(4Ab^2-5a^2(5A+7C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(5a^2(5A+7C)+6aAb+8Ab^2\right)\cot(c+dx}{1\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^2 + a^2*(19*A + 35*C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(6*a*A*b + 8*A*b^2 + 5*
a^2*(5*A + 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*A*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*A*b*Sqrt[a
```

```
 + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b^2
- 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[
c + d*x]^(3/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{Ab}{2}+\frac{1}{2}a(5A+7C)\cos(c+}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(A\left(19+\frac{8b^2}{a^2}\right)+35C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{105a^2d}$$

Mathematica [C] time = 6.41011, size = 1373, normalized size = 3.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 17*a^2*A*b^2 - 8*A*b^4 + 35*a^4*C - 35*a^2*b^2*C)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*A*b - 8*a*A*b^3 - 35*a^3*b*C)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-19*a^2*A*b^2
 - 8*A*b^4 - 35*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*El
lipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Se
c[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d
*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] - 4*A*b^2*Sin[c + d*x] +
 35*a^2*C*Sin[c + d*x]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^2*A*b*Sin[c + d*
x] + 8*A*b^3*Sin[c + d*x] + 35*a^2*b*C*Sin[c + d*x]))/(105*a^3) + (2*A*b*Se
c[c + d*x]^2*Tan[c + d*x])/(35*a) + (2*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.26, size = 2767, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] 2/105/d*(15*A*a^4+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-2*A*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+35*
C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b+19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b-2*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^4*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^4-25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4
-25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
```

```
-(a-b)/(a+b))^(1/2))*a^4-8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b+35*C*cos(d*x+c)^4*a^2*b^2+70*C*cos(d*x+c)^3*a^3*b-35*C*cos(d*x+c)^5*a
^3*b-35*C*cos(d*x+c)^5*a^2*b^2-35*C*cos(d*x+c)^4*a^3*b+4*A*cos(d*x+c)^3*a*b
^3-A*cos(d*x+c)^2*a^2*b^2+18*A*cos(d*x+c)*a^3*b-25*A*cos(d*x+c)^5*a^3*b-19*
A*cos(d*x+c)^5*a^2*b^2+4*A*cos(d*x+c)^5*a*b^3-19*A*cos(d*x+c)^4*a^3*b+20*A*
cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^4*a*b^3+26*A*cos(d*x+c)^3*a^3*b-25*A*co
s(d*x+c)^4*a^4-35*C*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+35*C*cos(d*x+c)^
2*a^4-8*A*cos(d*x+c)^5*b^4+8*A*cos(d*x+c)^4*b^4+19*A*cos(d*x+c)^4*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

3.733 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=638

$$-\frac{\left(3a^2C-4b^2(4A+3C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{a\left(-3a^2C+80Ab^2+52b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{64b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(80*A*b^2 - 3*a^2*C + 52*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(64*b^2*d) - (Sqrt[a + b]*(3*a^3*C - 2*a^2*b*C - 8*b^3*(4*A +
 3*C) - 4*a*b^2*(20*A + 13*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d
) - (Sqrt[a + b]*(3*a^4*C + 24*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3*d) + (a*(80*A*b^2 - 3*a^
2*C + 52*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(64*b^2*d*Sqrt[Cos[c
 + d*x]]) - ((3*a^2*C - 4*b^2*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(32*b*d) - (a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(8*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 1.99476, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(3a^2C-4b^2(4A+3C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{a\left(-3a^2C+80Ab^2+52b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{64b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(80*A*b^2 - 3*a^2*C + 52*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(64*b^2*d) - (Sqrt[a + b]*(3*a^3*C - 2*a^2*b*C - 8*b^3*(4*A +
 3*C) - 4*a*b^2*(20*A + 13*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
```

```
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d
) - (Sqrt[a + b]*(3*a^4*C + 24*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3*d) + (a*(80*A*b^2 - 3*a^
2*C + 52*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(64*b^2*d*Sqrt[Cos[c
 + d*x]]) - ((3*a^2*C - 4*b^2*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(32*b*d) - (a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(8*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(4*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd} + \frac{\int}{}$$

$$= -\frac{aC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{8bd} +$$

$$= -\frac{\left(3a^2C-4b^2(4A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}{32bd}$$

$$= \frac{a\left(80Ab^2-3a^2C+52b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+}{64b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{a\left(80Ab^2-3a^2C+52b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+}{64b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(3a^4C+24a^2b^2(2A+C)+16b^4(4A+3C)\right)}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(80Ab^2-3a^2C+52b^2C\right)\cot(c+dx)}{}$$

Mathematica [C] time = 6.37685, size = 1270, normalized size = 1.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]
^2),x]
```

```
[Out] -((-4*a*(-112*a*A*b^2 + a^3*C - 76*a*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4
*a*(-128*a^2*A*b - 64*A*b^3 - 76*a^2*b*C - 48*b^3*C)*((Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-80*a*A*b^2 + 3*a^3*C - 52*
a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[
Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqr
t[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])
/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi
[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*
x]])))/(128*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((16*A*b^2
 + a^2*C + 14*b^2*C)*Sin[c + d*x])/(32*b) + (3*a*C*Sin[2*(c + d*x)])/16 + (
b*C*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.354, size = 3800, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 1/64/d/(a+b*cos(d*x+c))^(1/2)*(128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
```

)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-32*A*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
x+c),(-(a-b)/(a+b))^(1/2))(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a*b^3-96*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-80*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-80*A*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*a*b^3-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+76*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-24*C*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3-48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-52*C*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b^2-52*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+128*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-
32*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*a*b^3-96*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^2*b^2-80*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-80*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-2*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+76*
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2-24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b

```
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^3-48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-52*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-52*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^3-26*C*cos(d*x+c)^4*a^2*b^2+C*cos(d*x+c)^3*a^3*b+64*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4-128*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*b^4+48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^4-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-96*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+3*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-16*C*cos(d*x+c)^6
*b^4-8*C*cos(d*x+c)^4*b^4+32*A*cos(d*x+c)^2*b^4+24*C*cos(d*x+c)^2*b^4-3*C*c
os(d*x+c)*a^4-112*A*cos(d*x+c)^3*a*b^3-80*A*cos(d*x+c)^2*a^2*b^2+3*C*cos(d*
x+c)^2*a^4-32*A*cos(d*x+c)^4*b^4+52*C*cos(d*x+c)^2*a*b^3+2*C*cos(d*x+c)*a^3
*b+52*C*cos(d*x+c)*a^2*b^2+24*C*cos(d*x+c)*a*b^3-40*C*cos(d*x+c)^5*a*b^3-36
*C*cos(d*x+c)^3*a*b^3+80*A*cos(d*x+c)^2*a*b^3+80*A*cos(d*x+c)*a^2*b^2+32*A*
cos(d*x+c)*a*b^3+64*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+48*C*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^4-6*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-96*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-3*C*cos(d*x
+c)^2*a^3*b-26*C*cos(d*x+c)^2*a^2*b^2)/sin(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.734 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=553

$$\frac{\left(3a^2C+8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(3a^2C+48aAb+14abC+24Ab^2+16b^2C\right)\cot(c+d$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2*C + 8*b^2*(3*A + 2*C))*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(48*a*A*b + 24*A*b^2 + 3*a^2*C + 14*a
*b*C + 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (a*Sqrt[a +
b]*(24*A*b^2 - a^2*C + 12*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(8*b^2*d) + ((3*a^2*C + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (a*C*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.5808, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2C+8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(3a^2C+48aAb+14abC+24Ab^2+16b^2C\right)\cot(c+d$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2*C + 8*b^2*(3*A + 2*C))*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(48*a*A*b + 24*A*b^2 + 3*a^2*C + 14*a
*b*C + 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
```

```
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (a*Sqrt[a +
b]*(24*A*b^2 - a^2*C + 12*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(8*b^2*d) + ((3*a^2*C + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (a*C*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\sqrt{a+b\cos(c}}{}$$

$$= \frac{aC\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{C\sqrt{\cos(c+dx)}(a+}{}$$

$$= \frac{\left(3a^2C+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} + \frac{aC\sqrt{\cos(c}}{}$$

$$= \frac{\left(3a^2C+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} + \frac{aC\sqrt{\cos(c}}{}$$

$$= -\frac{a\sqrt{a+b}\left(24Ab^2-a^2C+12b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{8b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(3a^2C+8b^2(3A+2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{24abd}$$

Mathematica [C] time = 6.41524, size = 1221, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

```
[Out] ((-4*a*(48*a^2*A + 24*A*b^2 + 17*a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
```

```
]]) - 4*a*(96*a*A*b + 52*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(24*A*b^2 + 3*a^2*C + 16*b^2*C)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((7*a*C*Sin[c + d*x])/12 + (b*C*Sin[2*(c
 + d*x)])/6))/d
```

Maple [B] time = 0.329, size = 2716, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b
^2+22*C*cos(d*x+c)^4*a*b^2+17*C*cos(d*x+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-6
*C*cos(d*x+c)^2*a*b^2-14*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+48*A*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+8*C*cos(d*x+c)^5*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c
```

)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^
2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a^2*b-52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16
*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*a*b^2+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+
16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*b^3+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+72*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b-52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+48*A*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b

```
+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3+3*C*cos
(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^
3-24*A*cos(d*x+c)^2*b^3)/b/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x +
c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x +
c)), x)
```

3.735 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=509

$$-\frac{\sqrt{a+b}\left(3a^2C+8Ab^2+4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}-\frac{b(4A}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*A - 5*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) -
 (Sqrt[a + b]*(8*a*A - 16*A*b - 5*a*C - 2*b*C)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(4*d) - (Sqrt[a + b]*(8*A*b^2 + 3*a^2*C + 4*b^2*C)*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) - (a*(8*A - 5*C)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*A - C)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*A*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.55323, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3048, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2C+8Ab^2+4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}-\frac{b(4A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(8*A - 5*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) -
 (Sqrt[a + b]*(8*a*A - 16*A*b - 5*a*C - 2*b*C)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
```

```
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(4*d) - (Sqrt[a + b]*(8*A*b^2 + 3*a^2*C + 4*b^2*C)*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) - (a*(8*A - 5*C)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*A - C)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*A*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3Ab}{2}\right.}{}$$

$$= -\frac{b(4A-C)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{2A(a+b}{}$$

$$= -\frac{a(8A-5C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{b(4A-C)\sqrt{\cos(c+d}}{}$$

$$= -\frac{a(8A-5C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{b(4A-C)\sqrt{\cos(c+d}}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+3a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{4bd}$$

$$= \frac{(a-b)\sqrt{a+b}(8A-5C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}$$

Mathematica [C] time = 6.42767, size = 1209, normalized size = 2.38

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

```
[Out] ((4*a*(-8*a*A*b - 7*a*b*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(8*a^2*A - 8*A
*b^2 - 8*a^2*C - 4*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
```

```
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])) - 2*(8*a*A*b - 5*a*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
+ b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((b*C*Sin[c + d*x])/2 + 2*a*A*Tan[c + d*x]))/d
```

Maple [B] time = 0.245, size = 2610, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(8*A*cos(d*x+c)^2*a*b-8*A*cos(d*x+c)*a*b-2*C*cos(d*x+c)*a*b+7*C*cos(
d*x+c)^3*a*b-5*C*cos(d*x+c)^2*a*b-8*A*a^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+16*A*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b-2*b^2*C*cos(d*x+c)^2+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
```

(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+5*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^

```
2+5*C*cos(d*x+c)^2*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+5*C*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*cos(d*
x+c)*a^2+2*C*cos(d*x+c)^4*b^2-5*C*cos(d*x+c)*a^2)/(a+b*cos(d*x+c))^(1/2)/si
n(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3
/2), x)
```

3.736 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=500

$$\frac{\sqrt{a+b}\left(2a^2(A+3C)-a(8Ab-3bC)+6Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3ad}$$

```
[Out] ((a - b)*b*Sqrt[a + b]*(8*A - 3*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
d) + (Sqrt[a + b]*(6*A*b^2 + 2*a^2*(A + 3*C) - a*(8*A*b - 3*b*C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(3*a*d) - (3*a*Sqrt[a + b]*C*Cot[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/d + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]) - (b*(8*A - 3*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.52525, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3048, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(2a^2(A+3C)-a(8Ab-3bC)+6Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```


```
[Out] ((a - b)*b*Sqrt[a + b]*(8*A - 3*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
d) + (Sqrt[a + b]*(6*A*b^2 + 2*a^2*(A + 3*C) - a*(8*A*b - 3*b*C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
```

```
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(3*a*d) - (3*a*Sqrt[a + b]*C*Cot[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/d + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]) - (b*(8*A - 3*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3Ab}{2}\right.}{}$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2A(a+b\cos(c+dx))^{3/2}\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b(8A-3C)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b(8A-3C)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

$$=-\frac{3a\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{d}$$

$$=\frac{(a-b)b\sqrt{a+b}(8A-3C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3ad}$$

Mathematica [C] time = 6.39757, size = 1219, normalized size = 2.44

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

```
[Out] ((-4*a*(2*a^2*A - 2*A*b^2 + 6*a^2*C + 3*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
```

```
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(-8*a*A*b + 12*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])) + 2*(-8*A*b^2 + 3*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*
Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2
*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[(
(a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(6*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]*((8*A*b*Tan[c + d*x])/3 + (2*a*A*Sec[c + d*x]*Tan[c + d*
x])/3))/d
```

Maple [B] time = 0.172, size = 2126, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d/(a+b*cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)^3*a*b+8*A*cos(d*x+c)^2*a*b-10
*A*cos(d*x+c)*a*b+3*C*cos(d*x+c)^3*a*b-3*C*cos(d*x+c)^2*a*b-2*A*a^2+8*A*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b-8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
```

)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b+6*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a^2+3*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*b^2+
3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^2+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*A*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2-8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*cos(d*x+c)*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2
-3*C*cos(d*x+c)^3*b^2-12*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+3*C*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-12*C*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a*b+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*cos(d*x+c)^2*a^2-8*A*cos(d*x+c)^2*
b^2+18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*a*b+18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+8*A*cos(d*x+c)^3*
b^2+3*C*cos(d*x+c)^4*b^2)/sin(d*x+c)/cos(d*x+c)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5
/2), x)
```

3.737 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=465

$$-\frac{2\sqrt{a+b}\left(a^2(3A+5C)-2ab(2A+5C)+Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a-}{a-}\right.}{5ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b^2 + a^2*(3*A + 5*C))*Cot[c + d*x]*EllipticE[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(5*a^2*d) - (2*Sqrt[a + b]*(A*b^2 - 2*a*b*(2*A + 5*C) + a^2*(3*A +
 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a*d) - (2*b*Sqrt[a + b]*C*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*A*b*Sqrt[a + b*Cos[c + d*
x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 1.17591, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3048, 3047, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(3A+5C)-2ab(2A+5C)+Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a-}{a-}\right.}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b^2 + a^2*(3*A + 5*C))*Cot[c + d*x]*EllipticE[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(5*a^2*d) - (2*Sqrt[a + b]*(A*b^2 - 2*a*b*(2*A + 5*C) + a^2*(3*A +
 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
```

```
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a*d) - (2*b*Sqrt[a + b]*C*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*A*b*Sqrt[a + b*Cos[c + d*
x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3Ab}{2}\right.}{}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^{3/2}\sin(c}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^{3/2}\sin(c}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(Ab^2+a^2(3A+5C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c-}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{5a^2d}$$

Mathematica [C] time = 6.49543, size = 1296, normalized size = 2.79

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(7/2),x]
```

```
[Out] -((-4*a*(-(a^2*A*b) + A*b^3 - 5*a^2*b*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*
(3*a^3*A + a*A*b^2 + 5*a^3*C - 5*a*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*A*b + A*b^3 + 5*a^2*b*C)*((I*Cos
```

```
[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*
a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
 (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(5*a*d) +
 (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(3*a^2*A*Sin
[c + d*x] + A*b^2*Sin[c + d*x] + 5*a^2*C*Sin[c + d*x]))/(5*a) + (4*A*b*Sec[
c + d*x]*Tan[c + d*x])/5 + (2*a*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.19, size = 2819, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*(-A*a^3-3*A*cos(d*x+c)^2*a*b^2-5*C*cos(d*x+c)^3*a^2*b+5*C*cos(d*x+c)
^4*a^2*b+3*A*cos(d*x+c)^4*a^2*b+2*A*cos(d*x+c)^4*a*b^2+A*cos(d*x+c)^3*a*b^2
-3*A*cos(d*x+c)*a^2*b-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*C*cos(d*x+c)^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^
2-5*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a*b^2+10*C*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-5*C*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b^2+A*cos(d*x+c)^4*b^3+3*A*cos(d*x+c)^3*a^3-2*A*cos(d*x+c)^2*a^3+5*C*cos
(d*x+c)^3*a^3-5*C*cos(d*x+c)^2*a^3-A*cos(d*x+c)^3*b^3+3*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
```

*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+5*C*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*C*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^3+5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^2+10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+
10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b

```
+A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(5/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7
/2), x)
```

3.738 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=418

$$\frac{2\left(5a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2A+35a^2C-57aAb-105abC-6Ab^2\right.}{}$$

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(3*A*b^2 - a^2*(41*A + 70*C))*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(105*a^3*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2*A - 57*a*A*b -
 6*A*b^2 + 35*a^2*C - 105*a*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^
2*d) + (6*A*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/
2)) + (2*(3*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(105*a*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.23199, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3048, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2A+35a^2C-57aAb-105abC-6Ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```


```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(3*A*b^2 - a^2*(41*A + 70*C))*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(105*a^3*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2*A - 57*a*A*b -
 6*A*b^2 + 35*a^2*C - 105*a*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^
```

```
2*d) + (6*A*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/
2)) + (2*(3*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(105*a*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3Ab}{2}\right.}{}$$

$$= \frac{6Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{6Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab^2+5a^2(5A+7C)\right)\sqrt{a}}{105ad\cos^{\frac{3}{2}}(}$$

$$= \frac{6Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab^2+5a^2(5A+7C)\right)\sqrt{a}}{105ad\cos^{\frac{3}{2}}(}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\left(3Ab^2-a^2(41A+70C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+}}\right.\right.}{105a^3d}$$

Mathematica [C] time = 6.48618, size = 1371, normalized size = 3.28

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(9/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 31*a^2*A*b^2 + 6*A*b^4 + 35*a^4*C - 35*a^2*b^2*C)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*A*b + 6*a*A*b^3 - 140*a^3*b*C)*((Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
```

```
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-82*a^2*A*b^
2 + 6*A*b^4 - 140*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*
Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x]
 + 35*a^2*C*Sin[c + d*x]))/(105*a) + (4*Sec[c + d*x]*(41*a^2*A*b*Sin[c + d*
x] - 3*A*b^3*Sin[c + d*x] + 70*a^2*b*C*Sin[c + d*x]))/(105*a^2) + (16*A*b*S
ec[c + d*x]^2*Tan[c + d*x])/35 + (2*a*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.248, size = 2971, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(-15*A*a^4+6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*A
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-140*C*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
```

(-(a-b)/(a+b))^(1/2))*a^2*b^2+140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-140*C*cos(d*x+c)^4*a^2*b^2-175*C*cos(d*x+c)^3*a^3*b+35*C*cos(d*x+c)^5*a^3*b+140*C*cos(d*x+c)^5*a^2*b^2+140*C*cos(d*x+c)^4*a^3*b+105*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+105*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+

```
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+3*A*cos(d*x+c)^3*a*b^3-
27*A*cos(d*x+c)^2*a^2*b^2-39*A*cos(d*x+c)*a^3*b+25*A*cos(d*x+c)^5*a^3*b+82*
A*cos(d*x+c)^5*a^2*b^2+3*A*cos(d*x+c)^5*a*b^3+82*A*cos(d*x+c)^4*a^3*b-55*A*
cos(d*x+c)^4*a^2*b^2-6*A*cos(d*x+c)^4*a*b^3-68*A*cos(d*x+c)^3*a^3*b+25*A*co
s(d*x+c)^4*a^4+35*C*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4-35*C*cos(d*x+c)^
2*a^4-6*A*cos(d*x+c)^5*b^4+6*A*cos(d*x+c)^4*b^4-82*A*cos(d*x+c)^4*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9
/2), x)
```

3.739 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=502

$$-\frac{4b\left(2Ab^2-a^2(44A+63C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(7a^2(7A+9C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21
*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d) + (2*(a - b)*Sqrt[a + b
]*(6*a*A*b^2 + 8*A*b^3 - 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*(3*A*b^2 + 7*a^2*(7*A + 9*C))*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) - (4*b*
(2*A*b^2 - a^2*(44*A + 63*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a
^2*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9
*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 1.73621, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3048, 3047, 3055, 2998, 2816, 2994}

$$-\frac{4b\left(2Ab^2-a^2(44A+63C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(7a^2(7A+9C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2)
,x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21
*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d) + (2*(a - b)*Sqrt[a + b
]*(6*a*A*b^2 + 8*A*b^3 - 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Cot[c
```

```
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*(3*A*b^2 + 7*a^2*(7*A + 9*C))*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) - (4*b*
(2*A*b^2 - a^2*(44*A + 63*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a
^2*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9
*d*Cos[c + d*x]^(9/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{3Ab}{2}\right.}{}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(3Ab^2+7a^2(7A+9C)\right)\sqrt{a}}{315ad\cos^{\frac{5}{2}}(}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(3Ab^2+7a^2(7A+9C)\right)\sqrt{a}}{315ad\cos^{\frac{5}{2}}(}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(3Ab^2+7a^2(7A+9C)\right)\sqrt{a}}{315ad\cos^{\frac{5}{2}}(}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^4+21a^4(7A+9C)+3a^2b^2(11A+21C)\right)\cot(c}{315a}$$

Mathematica [C] time = 6.62042, size = 1485, normalized size = 2.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(11/2),x]
```

```
[Out] -((-4*a*(-39*a^4*A*b + 31*a^2*A*b^3 + 8*A*b^5 - 63*a^4*b*C + 63*a^2*b^3*C)*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 33*a^3*A*b^2 + 8*a*A*b^4 + 1
89*a^5*C + 63*a^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
```

```
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])) + 2*(147*a^4*A*b + 33*a^2*A*b^3 + 8*A*b^5 + 189*a^4*b*C + 63
*a^2*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSi
nh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*
x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
 d*x]])))/(315*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Se
c[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 63*a^2*C*Sin[c
 + d*x]))/(315*a) + (4*Sec[c + d*x]^2*(44*a^2*A*b*Sin[c + d*x] - 2*A*b^3*Si
n[c + d*x] + 63*a^2*b*C*Sin[c + d*x]))/(315*a^2) + (2*Sec[c + d*x]*(147*a^4
*A*Sin[c + d*x] + 33*a^2*A*b^2*Sin[c + d*x] + 8*A*b^4*Sin[c + d*x] + 189*a^
4*C*Sin[c + d*x] + 63*a^2*b^2*C*Sin[c + d*x]))/(315*a^3) + (20*A*b*Sec[c +
d*x]^3*Tan[c + d*x])/63 + (2*a*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.395, size = 4110, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(-35*A*a^5-189*C*cos(d*x+c)^3*a^4*b+189*C*cos(d*x+c)^6*a^4*b+126*C
*cos(d*x+c)^6*a^3*b^2+63*C*cos(d*x+c)^6*a^2*b^3+63*C*cos(d*x+c)^5*a^3*b^2-6
3*C*cos(d*x+c)^5*a^2*b^3-189*C*cos(d*x+c)^4*a^3*b^2-52*A*cos(d*x+c)^3*a^4*b
+A*cos(d*x+c)^3*a^2*b^3-53*A*cos(d*x+c)^2*a^3*b^2-85*A*cos(d*x+c)*a^4*b+147
*A*cos(d*x+c)^6*a^4*b+88*A*cos(d*x+c)^6*a^3*b^2+33*A*cos(d*x+c)^6*a^2*b^3-4
*A*cos(d*x+c)^6*a*b^4-10*A*cos(d*x+c)^5*a^4*b+33*A*cos(d*x+c)^5*a^3*b^2-34*
A*cos(d*x+c)^5*a^2*b^3+8*A*cos(d*x+c)^5*a*b^4+8*A*cos(d*x+c)^6*b^5-189*C*co
```

s(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^5+147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^5+189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-189*C*cos(d*x+c)^4*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+186*A*cos(
d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^4*b-68*A*cos(d*x+c)^4*a^3*b^2-4*A*cos(d*x+c)^4*a*b^4+147*A*cos
(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^5-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+189*C*cos(d
*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a^5+147*A*cos(d*x+c)^5*a^5-8*A*cos(d*x+c)^5*b^5-98*A*cos(d*x+c)^4
*a^5-14*A*cos(d*x+c)^2*a^5+189*C*cos(d*x+c)^5*a^5-126*C*cos(d*x+c)^4*a^5-63
*C*cos(d*x+c)^2*a^5-8*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+252*C*cos(d*x+c)^5*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+63*
C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a^3*b^2-189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b-63*C*cos(d*x+c)^5*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3
*b^2-63*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a^2*b^3+186*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+33*A*cos(d*x+c)

^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b^2+2*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^4*b-33*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*A*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*
A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^4+252*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+63*C*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
^2-189*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^4*b-63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-63*C*cos(d*x+c)^4
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b^3+33*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*cos(d
*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^4-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*cos(d*x+c)^5*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*A
*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^3)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^(
9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(1
1/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(1
1/2), x)
```

3.740 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=746

$$-\frac{\left(15a^2C-16b^2(5A+4C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{a\left(-15a^2C+240Ab^2+172b^2C\right)\sin(c+dx)}{320bd}$$

```
[Out] ((a - b)*Sqrt[a + b]*(45*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A +
141*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[a + b]*(4
5*a^4*C - 30*a^3*b*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C) - 8
*a*b^3*(260*A + 193*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d) - (
a*Sqrt[a + b]*(3*a^4*C + 40*a^2*b^2*(2*A + C) + 80*b^4*(4*A + 3*C))*Cot[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(128*b^3*d) - ((45*a^4*C - 256*b^4*
(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(1920*b^2*d*Sqrt[Cos[c + d*x]]) + (a*(240*A*b^2 - 15*a^2*C + 172*b^2*
C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) - ((
15*a^2*C - 16*b^2*(5*A + 4*C))*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(240*b*d) - (3*a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(40*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(
7/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 2.75778, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(15a^2C-16b^2(5A+4C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{a\left(-15a^2C+240Ab^2+172b^2C\right)\sin(c+dx)}{320bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(45*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A +
141*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
```

```
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[a + b]*(4
5*a^4*C - 30*a^3*b*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C) - 8
*a*b^3*(260*A + 193*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d) - (
a*Sqrt[a + b]*(3*a^4*C + 40*a^2*b^2*(2*A + C) + 80*b^4*(4*A + 3*C))*Cot[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(128*b^3*d) - ((45*a^4*C - 256*b^4*
(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(1920*b^2*d*Sqrt[Cos[c + d*x]]) + (a*(240*A*b^2 - 15*a^2*C + 172*b^2*
C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) - ((
15*a^2*C - 16*b^2*(5*A + 4*C))*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(240*b*d) - (3*a*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(40*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(
7/2)*Sin[c + d*x])/(5*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd} + \frac{\int}{}$$

$$= -\frac{3aC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd} +$$

$$= -\frac{\left(15a^2C-16b^2(5A+4C)\right)\sqrt{\cos(c+dx)}(a+b\cos(c}{240bd}$$

$$= \frac{a\left(240Ab^2-15a^2C+172b^2C\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}{320bd}$$

$$= -\frac{\left(45a^4C-256b^4(5A+4C)-12a^2b^2(220A+141C)\right)}{1920b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(45a^4C-256b^4(5A+4C)-12a^2b^2(220A+141C)\right)}{1920b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a\sqrt{a+b}\left(3a^4C+40a^2b^2(2A+C)+80b^4(4A+3C)\right)}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(45a^4C-256b^4(5A+4C)-12a^2b^2(220\right.}{}$$

Mathematica [C] time = 6.48163, size = 1341, normalized size = 1.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]
^2),x]
```

```
[Out] -((-4*a*(-4720*a^2*A*b^2 - 1280*A*b^4 + 15*a^4*C - 3236*a^2*b^2*C - 1024*b^
4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3840*a^3*A*b - 6080*a*A*b^3 - 2292
*a^3*b*C - 4624*a*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])) + 2*(-2640*a^2*A*b^2 - 1280*A*b^4 + 45*a^4*C - 1692*a^2*b^2*
C - 1024*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*A
rcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])
/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
+ d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos
[c + d*x]])))/(3840*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((a
*(1040*A*b^2 + 15*a^2*C + 898*b^2*C)*Sin[c + d*x])/(960*b) + ((80*A*b^2 + 9
3*a^2*C + 88*b^2*C)*Sin[2*(c + d*x)])/480 + (21*a*b*C*Sin[3*(c + d*x)])/160
 + (b^2*C*Sin[4*(c + d*x)])/40))/d
```

Maple [B] time = 0.546, size = 4724, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/1920/d/(a+b*cos(d*x+c))^(1/2)*(-15*C*cos(d*x+c)^3*a^4*b+1752*C*cos(d*x+c
)^5*a^2*b^3+774*C*cos(d*x+c)^4*a^3*b^2+4720*A*cos(d*x+c)^3*a^2*b^3+2640*A*c
os(d*x+c)^2*a^3*b^2-3840*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2720*A*cos(d*x+c)^4*a*b^4+4
5*C*cos(d*x+c)*a^5+384*C*cos(d*x+c)^7*b^5+128*C*cos(d*x+c)^5*b^5+640*A*cos(
d*x+c)^5*b^5-45*C*cos(d*x+c)^2*a^5+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-2292*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1544*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^3-4624*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^4-45*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+1692*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1692*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+1024*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^4+1200*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^3*b^2+7200*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4+2080*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^
3-6080*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^4+2640*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2640*A*cos(d*x+c)*s
```

in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^3+1280*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+2400*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2+9600*A*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b^4+30*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-2292*C*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1
544*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^3-4624*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-45*C*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*
b+1692*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b^2+1692*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+1024*C*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^4+1200*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2+7200*C*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4+640*
A*cos(d*x+c)^3*b^5-1280*A*cos(d*x+c)^2*b^5+512*C*cos(d*x+c)^3*b^5-1024*C*co
s(d*x+c)^2*b^5-45*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^5+1024*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+90*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5+1280*A*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^5-45*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(

```
-(a-b)/(a+b))^(1/2))*a^5+1024*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+90*C*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5-
3840*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b^2+2080*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-6080*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+2640*A*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2640*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^3+1280*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^4+2400*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2+9600*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4+1280*A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5
-30*C*cos(d*x+c)*a^4*b-1692*C*cos(d*x+c)*a^3*b^2-1544*C*cos(d*x+c)*a^2*b^3-
1024*C*cos(d*x+c)*a*b^4+45*C*cos(d*x+c)^2*a^4*b+918*C*cos(d*x+c)^2*a^3*b^2-
1692*C*cos(d*x+c)^2*a^2*b^3-1032*C*cos(d*x+c)^2*a*b^4+664*C*cos(d*x+c)^4*a*
b^4+1484*C*cos(d*x+c)^3*a^2*b^3+1392*C*cos(d*x+c)^6*a*b^4-2640*A*cos(d*x+c)
^2*a^2*b^3-1440*A*cos(d*x+c)^2*a*b^4-2640*A*cos(d*x+c)*a^3*b^2-2080*A*cos(d
*x+c)*a^2*b^3-1280*A*cos(d*x+c)*a*b^4)/sin(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
```

```
c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + 2\,Cab\cos(dx+c)^3 + 2\,Aab\cos(dx+c) + Aa^2 + \left(Ca^2 + Ab^2\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.741 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=635

$$\frac{\left(5a^2C+4b^2(4A+3C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}+\frac{a\left(15a^2C+432Ab^2+284b^2C\right)\sin(c+dx)\sqrt{a+}}{192bd\sqrt{\cos(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(192*b*d) + (Sqrt[a + b]*(15*a^3*C + 24*b^3*(4*A + 3*C) +
2*a^2*b*(192*A + 59*C) + 4*a*b^2*(108*A + 71*C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(192*b*d) + (Sqrt[a + b]*(5*a^4*C - 120*a^2*b^2*(2*A + C) - 16*b^4
*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d) + (
a*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(192*b*d*Sqrt[Cos[c + d*x]]) + ((5*a^2*C + 4*b^2*(4*A + 3*C))*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d) + (5*a*C*Sqrt[Cos[c +
d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sqrt[Cos[c + d*x
]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 2.05793, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(5a^2C+4b^2(4A+3C)\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}+\frac{a\left(15a^2C+432Ab^2+284b^2C\right)\sin(c+dx)\sqrt{a+}}{192bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
```

```
d*x]))/(a - b)])/(192*b*d) + (Sqrt[a + b]*(15*a^3*C + 24*b^3*(4*A + 3*C) +
2*a^2*b*(192*A + 59*C) + 4*a*b^2*(108*A + 71*C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(192*b*d) + (Sqrt[a + b]*(5*a^4*C - 120*a^2*b^2*(2*A + C) - 16*b^4
*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d) + (
a*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(192*b*d*Sqrt[Cos[c + d*x]]) + ((5*a^2*C + 4*b^2*(4*A + 3*C))*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d) + (5*a*C*Sqrt[Cos[c +
d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sqrt[Cos[c + d*x
]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
```

```
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4d}+\frac{1}{4}\int \frac{(a+b\cos(c}{}$$

$$= \frac{5aC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24d}+\frac{C\sqrt{\cos(c+dx)}}{}$$

$$= \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32d}$$

$$= \frac{a\left(432Ab^2+15a^2C+284b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{192bd\sqrt{\cos(c+dx)}}+\frac{(5a}{}$$

$$= \frac{a\left(432Ab^2+15a^2C+284b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{192bd\sqrt{\cos(c+dx)}}+\frac{(5a}{}$$

$$= \frac{\sqrt{a+b}\left(5a^4C-120a^2b^2(2A+C)-16b^4(4A+3C)\right)\cot(c+dx)\Pi\left(\right.}{64b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(432Ab^2+15a^2C+284b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{192bd}$$

Mathematica [C] time = 6.57282, size = 1275, normalized size = 2.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

```
[Out] ((-4*a*(384*a^3*A + 528*a*A*b^2 + 133*a^3*C + 356*a*b^2*C)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(1152*a^2*A*b + 192*A*b^3 + 644*a^2*b*C + 144*b^3*C)*((Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(432*a*A*b
^2 + 15*a^3*C + 284*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*
Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(b*Sqrt[Cos[c + d*x]])))/(384*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]*(((48*A*b^2 + 59*a^2*C + 42*b^2*C)*Sin[c + d*x])/96 + (17*a*b*C*Sin[
2*(c + d*x)])/48 + (b^2*C*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.555, size = 3991, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

[Out] -1/192/d/(a+b*cos(d*x+c))^(1/2)*(-1152*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+96*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+1440*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+432*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+432*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+118*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+720*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+284*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+284*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-1152*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+96*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+1440*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+432*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+432*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+118*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(

a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+720*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+284*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+284*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+254*C*cos(d*x+c)^4*a^2*b^2+133*C*cos(d*x+c)^3*a^3*b-192*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4+384*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4-144*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*C*cos(d*x+c)^6*b^4+24*C*cos(d*x+c)^4*b^4-96*A*cos(d*x+c)^2*b^4-72*C*cos(d*x+c)^2*b^4-15*C*cos(d*x+c)*a^4+384*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+528*A*cos(d*x+c)^3*a*b^3+432*A*cos(d*x+c)^2*a^2*b^2+15*C*cos(d*x+c)^2*a^4+96*A*cos(d*x+c)^4*b^4-284*C*cos(d*x+c)^2*a*b^3-118*C*cos(d*x+c)*a^3*b-284*C*cos(d*x+c)*a^2*b^2-72*C*cos(d*x+c)*a*b^3+184*C*cos(d*x+c)^5*a*b^3+172*C*cos(d*x+c)^3*a*b^3-432*A*cos(d*x+c)^2*a*b^3-432*A*cos(d*x+c)*a^2*b^2-96*A*cos(d*x+c)*a*b^3+384*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-192*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+384*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-144*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((

```
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*C*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*b^4-15*C*cos(d*x+c)^2*a^3*b+30*C*cos(d*x+c)^2*a^2*b^2)/b/cos(d*x+c)^(1/2
)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x +
c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x +
c)), x)
```

3.742 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=609

$$-\frac{\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(a^2(48A-33C)-2ab(72A+13C)-8b^2(3A\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(24*a*d) - (Sqrt[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*
A + 2*C) - 2*a*b*(72*A + 13*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) -
 (5*a*Sqrt[a + b]*(8*A*b^2 + (a^2 + 4*b^2)*C)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(8*b*d) - ((a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) - (a*b*(8*A - 3*
C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*(6*
A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) +
(2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 2.11365, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3048, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(a^2(48A-33C)-2ab(72A+13C)-8b^2(3A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
```

```
c + d*x]))/(a - b)])/(24*a*d) - (Sqrt[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*
A + 2*C) - 2*a*b*(72*A + 13*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) -
 (5*a*Sqrt[a + b]*(8*A*b^2 + (a^2 + 4*b^2)*C)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(8*b*d) - ((a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) - (a*b*(8*A - 3*
C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*(6*
A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) +
(2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
```

```
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+dx))^{3/2}\left(\right.}{}$$

$$= -\frac{b(6A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2A(a}{}$$

$$= -\frac{ab(8A-3C)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{b(6A}{}$$

$$= -\frac{\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$

$$= -\frac{5a\sqrt{a+b}\left(8Ab^2+\left(a^2+4b^2\right)C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\sqrt{}}\right.\right.}{8bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(a^2(48A-33C)-8b^2(3A+2C)\right)\cot(c+dx)E\left(\sin^{-}\right.}{24ad}$$

Mathematica [C] time = 6.5892, size = 1262, normalized size = 2.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

```
[Out] ((4*a*(-96*a^2*A*b - 24*A*b^3 - 59*a^2*b*C - 16*b^3*C)*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) + 4*a*(48*a^3*A - 144*a*A*b^2 - 48*a^3*C - 76*a*b^2*C)*((Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(48*a^2*A*b - 24*A
*b^3 - 33*a^2*b*C - 16*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]
*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
+ d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a
*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(b*Sqrt[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]*((13*a*b*C*Sin[c + d*x])/12 + (b^2*C*Sin[2*(c + d*x)])/6 + 2*a^2*A*T
an[c + d*x]))/d
```

Maple [B] time = 0.35, size = 3513, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*(-48*A*a^3+24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b^2+34*C*cos(d
*x+c)^4*a*b^2+59*C*cos(d*x+c)^3*a^2*b-33*C*cos(d*x+c)^2*a^2*b-18*C*cos(d*x+
```

```
c)^2*a*b^2-26*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2-48*A*cos(d*x+c)*a^2*
b+144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+8*C*cos(d*x+c)^5*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+120*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*a*b^2+26*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-76*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+33*C*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-144*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^2+48*A*cos(d*x+c)*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+240*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-
144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a*b^2+120*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+26*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+33*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+30*C*sin(d*x+c)*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+33*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*a^3+48*A*cos(d*x+c)^2*a^2*b+144*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^
3+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*a^3+33*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3+33*C*cos(d*x+c)
^2*a^3-16*C*cos(d*x+c)^2*b^3-33*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A
*cos(d*x+c)^2*b^3-48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b-48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx\right.}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3
/2), x)
```

3.743 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=567

$$\frac{\sqrt{a+b}\left(8a^2(A+3C)-a(56Ab-27bC)+6b^2(12A+C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{12d}$$

```
[Out] ((a - b)*b*Sqrt[a + b]*(56*A - 27*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12
*d) + (Sqrt[a + b]*(6*b^2*(12*A + C) + 8*a^2*(A + 3*C) - a*(56*A*b - 27*b*C
))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12*d) - (Sqrt[a + b]*(8*A*b^2 + 15*a
^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (a*b
*(56*A - 27*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*
x]]) - (b^2*(8*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(2*d) + (10*A*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(3/2))
```

Rubi [A] time = 1.98146, antiderivative size = 567, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {3048, 3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(8a^2(A+3C)-a(56Ab-27bC)+6b^2(12A+C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```


```
[Out] ((a - b)*b*Sqrt[a + b]*(56*A - 27*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12
```

```
*d) + (Sqrt[a + b]*(6*b^2*(12*A + C) + 8*a^2*(A + 3*C) - a*(56*A*b - 27*b*C
))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12*d) - (Sqrt[a + b]*(8*A*b^2 + 15*a
^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (a*b
*(56*A - 27*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*
x]]) - (b^2*(8*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(2*d) + (10*A*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(3/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+dx))^{3/2}\left(\right.}{}$$

$$= \frac{10Ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A(a+b\cos(c+dx))^{5/2}\,\mathrm{s}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{b^2(8A-C)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{10Ab(a}{}$$

$$= -\frac{ab(56A-27C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\cos(c+dx)}} - \frac{b^2(8A-C)\sqrt{\cos}}{}$$

$$= -\frac{ab(56A-27C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\cos(c+dx)}} - \frac{b^2(8A-C)\sqrt{\cos}}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+15a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{cc}}}{\sqrt{a+b}\sqrt{\mathrm{c}}}\right.\right.}{4d}$$

$$= \frac{(a-b)b\sqrt{a+b}(56A-27C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|\,-\right.}{12d}$$

Mathematica [C] time = 6.54511, size = 1256, normalized size = 2.22

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

```
[Out] ((-4*a*(8*a^3*A + 16*a*A*b^2 + 24*a^3*C + 33*a*b^2*C)*Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]) - 4*a*(-56*a^2*A*b + 24*A*b^3 + 72*a^2*b*C + 12*b^3*C)*((Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
```

```
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-56*a*A*b^2 + 27*a
*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[S
in[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/
(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x
]])))/(24*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b^2*C*Sin[c +
 d*x])/2 + (14*a*A*b*Tan[c + d*x])/3 + (2*a^2*A*Sec[c + d*x]*Tan[c + d*x])/
3))/d
```

Maple [B] time = 0.194, size = 3195, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/12/d*(-8*A*a^3-56*A*cos(d*x+c)^2*a*b^2+33*C*cos(d*x+c)^4*a*b^2+27*C*cos(
d*x+c)^3*a^2*b-27*C*cos(d*x+c)^2*a^2*b-6*C*cos(d*x+c)^2*a*b^2+8*A*cos(d*x+c
)^3*a^2*b+56*A*cos(d*x+c)^3*a*b^2-64*A*cos(d*x+c)*a^2*b-27*C*cos(d*x+c)^3*a
*b^2+6*C*cos(d*x+c)^5*b^3-56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-72*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+6
*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
```

1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*a*b^2+27*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+27*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+72*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^2+6*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+56*A*cos(d*x+c)^2*a^2*b+56*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b+8*A*cos(d*x+c)^2*a^3-6*C*cos(d*x+c)^3*b^3+8*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3+24*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3+56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+72*A*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2-56*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-56*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-72*C*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+27*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*A*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3-56*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b+90*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+90*C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a^2*b+27*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*

```
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*A*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^3+48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-12*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+24*C*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*b^3-24*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+48*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-12*C*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+\right.}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5
/2), x)
```

3.744 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=606

$$\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{\left(6a^2(3A+5C)+b^2(46A-15C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(15*a*d) + (Sqrt[a + b]*(30*A*b^3 - a*b^2*(46*A - 15*C
) - 6*a^3*(3*A + 5*C) + a^2*(34*A*b + 90*b*C))*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(15*a*d) - (5*a*b*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/d + (2*(5*A*b^2 + a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) - ((b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C
))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*
b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 2.03055, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3048, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{\left(6a^2(3A+5C)+b^2(46A-15C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
```

```
c + d*x]))/(a - b)])/(15*a*d) + (Sqrt[a + b]*(30*A*b^3 - a*b^2*(46*A - 15*C
) - 6*a^3*(3*A + 5*C) + a^2*(34*A*b + 90*b*C))*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(15*a*d) - (5*a*b*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/d + (2*(5*A*b^2 + a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) - ((b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C
))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*
b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
```

```
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+dx))^{3/2}\left(\right.}{}$$

$$= \frac{2Ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^{5/2}\sin}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab(a+}{}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{\left(b^2(46A\right.}{}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{\left(b^2(46A\right.}{}$$

$$= -\frac{5ab\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(b^2(46A-15C)+6a^2(3A+5C)\right)\cot(c+dx)E\left(\sin^{-}\right.}{15ad}$$

Mathematica [C] time = 6.57563, size = 1309, normalized size = 2.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(7/2),x]
```

```
[Out] ((4*a*(-16*a^2*A*b + 16*A*b^3 - 60*a^2*b*C - 15*b^3*C)*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) + 4*a*(18*a^3*A + 46*a*A*b^2 + 30*a^3*C - 90*a*b^2*C)*((Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(18*a^2*A*b + 46*A*
b^3 + 30*a^2*b*C - 15*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*
Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(b*Sqrt[Cos[c + d*x]])))/(30*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]*((2*Sec[c + d*x]*(9*a^2*A*Sin[c + d*x] + 23*A*b^2*Sin[c + d*x] + 15*a
^2*C*Sin[c + d*x]))/15 + (22*a*A*b*Sec[c + d*x]*Tan[c + d*x])/15 + (2*a^2*A
*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.22, size = 3489, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*(-6*A*a^3-68*A*cos(d*x+c)^2*a*b^2+15*C*cos(d*x+c)^4*a*b^2-30*C*cos(
d*x+c)^3*a^2*b+30*C*cos(d*x+c)^4*a^2*b+18*A*cos(d*x+c)^4*a^2*b+22*A*cos(d*x
+c)^4*a*b^2+10*A*cos(d*x+c)^3*a^2*b+46*A*cos(d*x+c)^3*a*b^2-28*A*cos(d*x+c)
*a^2*b-15*C*cos(d*x+c)^3*a*b^2+15*C*cos(d*x+c)^5*b^3-18*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+150*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-90*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+150*C*cos(d*x+
c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a*b^2-90*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+15*C*cos(d*x+c)^3*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+46*A*c
os(d*x+c)^4*b^3+18*A*cos(d*x+c)^3*a^3-12*A*cos(d*x+c)^2*a^3+30*C*cos(d*x+c)
^3*a^3-15*C*cos(d*x+c)^4*b^3-30*C*cos(d*x+c)^2*a^3-46*A*cos(d*x+c)^3*b^3+18
*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3-18*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-46*A*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*C*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3-30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c
)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-18*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3-46*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-30*C*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
```

```
*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3-46*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+90*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-30*C*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+34*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+46*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-18*A*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b-46*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+90*C*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-30*C
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b+34*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+46*A*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3
0*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^3+15*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+30*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx\right.}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7
/2), x)
```

3.745 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=540

$$\frac{2\left(a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b}\left(a^2b(29A+49C)+a^3(-(5A+7C))-9ab^2(3A+7C\right.}{}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(3*A*b^2 + a^2*(29*A + 49*C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(21*a^2*d) - (2*Sqrt[a + b]*(3*A*b^3 - 9*a*b^2*(3*A + 7*C) -
 a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(21*a*d) - (2*b^2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d + (2*(3*A*b^2 + a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(21*d*Cos[c + d*x]^(3/2)) + (2*A*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(7*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x]
)/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.56169, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3048, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\left(a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b}\left(a^2b(29A+49C)+a^3(-(5A+7C))-9ab^2(3A+7C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(3*A*b^2 + a^2*(29*A + 49*C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(21*a^2*d) - (2*Sqrt[a + b]*(3*A*b^3 - 9*a*b^2*(3*A + 7*C) -
```

```
a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(21*a*d) - (2*b^2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d + (2*(3*A*b^2 + a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(21*d*Cos[c + d*x]^(3/2)) + (2*A*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(7*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x]
)/(7*d*Cos[c + d*x]^(7/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\cos(c+dx))^{3/2}\left(\frac{5A}{2}\right.}{}$$

$$= \frac{2Ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^{5/2}\sin(}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2\left(3Ab^2+a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2Ab(a+b}{}$$

$$= \frac{2\left(3Ab^2+a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2Ab(a+b}{}$$

$$= -\frac{2b^2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{d}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(3Ab^2+a^2(29A+49C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{21a^2d}$$

Mathematica [C] time = 6.63517, size = 1378, normalized size = 2.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(9/2),x]
```

```
[Out] ((-4*a*(5*a^4*A - 2*a^2*A*b^2 - 3*A*b^4 + 7*a^4*C + 14*a^2*b^2*C)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(-29*a^3*A*b - 3*a*A*b^3 - 49*a^3*b*C + 21*a*b^3*C
)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
```

```
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-29
*a^2*A*b^2 - 3*A*b^4 - 49*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(21*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(5*a^2*A*Sin[c + d*x] + 9*A*b^2*Sin[c +
d*x] + 7*a^2*C*Sin[c + d*x]))/21 + (2*Sec[c + d*x]*(29*a^2*A*b*Sin[c + d*x]
 + 3*A*b^3*Sin[c + d*x] + 49*a^2*b*C*Sin[c + d*x]))/(21*a) + (6*a*A*b*Sec[c
 + d*x]^2*Tan[c + d*x])/7 + (2*a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.264, size = 3373, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*(-3*A*a^4-3*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+29*A*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+27*A*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2+3*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-49*C*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-49
```

*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2+49*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-29*A*cos(d*x+c)^3*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b-29*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+29*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b+27*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+7*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*A*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^4+5*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4+7*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*A*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+
5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4+3*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-49*C*cos(d*x+c)^3*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4
9*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2+49*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-29*A*cos(d*x+c)^4*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b-49*C*cos(d*x+c)^4*a^2*b^2-56*C*cos(d*x+c)^3*a^3*b+7*C*cos(d*x+c)^5*a^3*
b+49*C*cos(d*x+c)^5*a^2*b^2+49*C*cos(d*x+c)^4*a^3*b+63*C*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*

```
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2+63*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-12*A*cos(d*x+c)^3*a*b^3-18*A*cos(d*x+c)^2*
a^2*b^2-12*A*cos(d*x+c)*a^3*b+5*A*cos(d*x+c)^5*a^3*b+29*A*cos(d*x+c)^5*a^2*
b^2+9*A*cos(d*x+c)^5*a*b^3+29*A*cos(d*x+c)^4*a^3*b-11*A*cos(d*x+c)^4*a^2*b^
2+3*A*cos(d*x+c)^4*a*b^3-22*A*cos(d*x+c)^3*a^3*b+5*A*cos(d*x+c)^4*a^4+7*C*c
os(d*x+c)^4*a^4-2*A*cos(d*x+c)^2*a^4-7*C*cos(d*x+c)^2*a^4+3*A*cos(d*x+c)^5*
b^4-3*A*cos(d*x+c)^4*b^4-29*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*C*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*
b^3+42*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-21*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+42*C*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*b^3)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx\right.}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9
/2), x)
```

3.746 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=504

$$\frac{2b\left(a^2(163A+231C)+5Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(7a^2(7A+9C)+15Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+d}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A +
161*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*Sqrt[a
+ b]*(10*A*b^3 + 21*a^3*(7*A + 9*C) + 15*a*b^2*(11*A + 21*C) - 6*a^2*b*(19*
A + 28*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*(15*A*b^2 +
7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*
x]^(5/2)) + (2*b*(5*A*b^2 + a^2*(163*A + 231*C))*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(315*a*d*Cos[c + d*x]^(3/2)) + (10*A*b*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 1.75809, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3048, 3047, 3055, 2998, 2816, 2994}

$$\frac{2b\left(a^2(163A+231C)+5Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(7a^2(7A+9C)+15Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+d}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2)
,x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A +
161*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*Sqrt[a
+ b]*(10*A*b^3 + 21*a^3*(7*A + 9*C) + 15*a*b^2*(11*A + 21*C) - 6*a^2*b*(19*
```

```
A + 28*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*(15*A*b^2 +
7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*
x]^(5/2)) + (2*b*(5*A*b^2 + a^2*(163*A + 231*C))*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(315*a*d*Cos[c + d*x]^(3/2)) + (10*A*b*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\cos(c+dx))^{3/2}\left(\frac{5A}{2}\right.}{}$$

$$= \frac{10Ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^{5/2}\sin}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{10Ab(a}{}$$

$$= \frac{2\left(15Ab^2+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(5Ab\right.}{}$$

$$= \frac{2\left(15Ab^2+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(5Ab\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(10Ab^4-21a^4(7A+9C)-3a^2b^2(93A+161C)\right)\cot}{31}$$

Mathematica [C] time = 6.69406, size = 1485, normalized size = 2.95

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(11/2),x]
```

```
[Out] -((-4*a*(-114*a^4*A*b + 124*a^2*A*b^3 - 10*A*b^5 - 168*a^4*b*C + 168*a^2*b^
3*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 279*a^3*A*b^2 - 10*a*A*
b^4 + 189*a^5*C + 483*a^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a
```

```
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 279*a^2*A*b^3 - 10*A*b^5 + 189*a^
4*b*C + 483*a^2*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
+ d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])
*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*S
qrt[Cos[c + d*x]])))/(315*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]*((2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x] + 63
*a^2*C*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*A*b*Sin[c + d*x] + 5
*A*b^3*Sin[c + d*x] + 231*a^2*b*C*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]*
(147*a^4*A*Sin[c + d*x] + 279*a^2*A*b^2*Sin[c + d*x] - 10*A*b^4*Sin[c + d*x
] + 189*a^4*C*Sin[c + d*x] + 483*a^2*b^2*C*Sin[c + d*x]))/(315*a^2) + (38*a
*A*b*Sec[c + d*x]^3*Tan[c + d*x])/63 + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x]
)/9))/d
```

Maple [B] time = 0.376, size = 4330, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d*(35*A*a^5+294*C*cos(d*x+c)^3*a^4*b-189*C*cos(d*x+c)^6*a^4*b-231*C*c
os(d*x+c)^6*a^3*b^2-483*C*cos(d*x+c)^6*a^2*b^3-483*C*cos(d*x+c)^5*a^3*b^2+4
83*C*cos(d*x+c)^5*a^2*b^3+714*C*cos(d*x+c)^4*a^3*b^2+82*A*cos(d*x+c)^3*a^4*
b+80*A*cos(d*x+c)^3*a^2*b^3+170*A*cos(d*x+c)^2*a^3*b^2+130*A*cos(d*x+c)*a^4
*b-147*A*cos(d*x+c)^6*a^4*b-163*A*cos(d*x+c)^6*a^3*b^2-279*A*cos(d*x+c)^6*a
^2*b^3-5*A*cos(d*x+c)^6*a*b^4-65*A*cos(d*x+c)^5*a^4*b-279*A*cos(d*x+c)^5*a^
3*b^2+199*A*cos(d*x+c)^5*a^2*b^3+10*A*cos(d*x+c)^5*a*b^4+10*A*cos(d*x+c)^6*
```

b^5+189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-10*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-261*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+272*A*cos(d*x+c)^4*a^3*b^2-5*A*cos(d*x+c)^4*a*b^4-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-10*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-147*A*cos(d*x+c)^5*a^5-10*A*cos(d*x+c)^5*b^5+98*A*cos(d*x+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-189*C*cos(d*x+c)^5*a^5+126*C*cos(d*x+c)^4*a^5+63*C*cos(d*x+c)^2*a^5-10*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-357*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b-483*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2+189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+483*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2+483*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3-261*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a

^4*b-279*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-155*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+279*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-357*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b-483*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+189*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+483*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+483*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-279*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-155*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+279*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-105*C*cos(d*x+c)^5*a^4*b-315*C*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos

```
(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3-315*C*sin(d*x+c)*cos(d*x+c)^4*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3)/(a+b*c
os(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
1/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+\right.}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
1/2), x)
```

3.747 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=587

$$-\frac{2\left(-a^2b^2(205A+297C)-15a^4(9A+11C)+4Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(a^2(229A+297C)+3Ab^2\right)\sin}{693ad\cos^{\frac{5}{2}}(}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A +
957*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^4*d) + (2*(a - b)*Sqrt[a
+ b]*(6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) -
 6*a^3*b*(101*A + 132*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*d) +
(2*(5*A*b^2 + 3*a^2*(9*A + 11*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2
31*d*Cos[c + d*x]^(7/2)) + (2*b*(3*A*b^2 + a^2*(229*A + 297*C))*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(693*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b^4 - 15
*a^4*(9*A + 11*C) - a^2*b^2*(205*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(693*a^2*d*Cos[c + d*x]^(3/2)) + (10*A*b*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + b*Cos[c + d*x])^(5/2)
*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 2.5454, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3048, 3047, 3055, 2998, 2816, 2994}

$$-\frac{2\left(-a^2b^2(205A+297C)-15a^4(9A+11C)+4Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(a^2(229A+297C)+3Ab^2\right)\sin}{693ad\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2)
,x]
```


```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A +
957*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
```

```
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^4*d) + (2*(a - b)*Sqrt[a
+ b]*(6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) -
 6*a^3*b*(101*A + 132*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*d) +
(2*(5*A*b^2 + 3*a^2*(9*A + 11*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2
31*d*Cos[c + d*x]^(7/2)) + (2*b*(3*A*b^2 + a^2*(229*A + 297*C))*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(693*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b^4 - 15
*a^4*(9*A + 11*C) - a^2*b^2*(205*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(693*a^2*d*Cos[c + d*x]^(3/2)) + (10*A*b*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*A*(a + b*Cos[c + d*x])^(5/2)
*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\cos(c+dx))^{3/2}}{}$$

$$= \frac{10Ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^{5/2}\,\mathrm{s}}{11d\cos^{\frac{11}{2}}(c+dx}$$

$$= \frac{2\left(5Ab^2+3a^2(9A+11C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)} + \frac{10Ab(}{}$$

$$= \frac{2\left(5Ab^2+3a^2(9A+11C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\left(3A\right.}{}$$

$$= \frac{2\left(5Ab^2+3a^2(9A+11C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\left(3A\right.}{}$$

$$= \frac{2\left(5Ab^2+3a^2(9A+11C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{231d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\left(3A\right.}{}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(8Ab^4+3a^2b^2(17A+33C)+a^4(741A+957C)\right)\,\mathrm{c}}{6}$$

Mathematica [C] time = 6.84887, size = 1591, normalized size = 2.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(13/2),x]
```

```
[Out] ((-4*a*(135*a^6*A - 78*a^4*A*b^2 - 49*a^2*A*b^4 - 8*A*b^6 + 165*a^6*C - 66*
a^4*b^2*C - 99*a^2*b^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-741*a^5*A*b -
 51*a^3*A*b^3 - 8*a*A*b^5 - 957*a^5*b*C - 99*a^3*b^3*C)*((Sqrt[((a + b)*Cot
```

```
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-741*a^4*A*b^2 - 51*a^2*
A*b^4 - 8*A*b^6 - 957*a^4*b^2*C - 99*a^2*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]
, (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*S
qrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(693*a^3*d) + (Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^4*(81*a^2*A*Sin[c + d*x] + 1
13*A*b^2*Sin[c + d*x] + 99*a^2*C*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(22
9*a^2*A*b*Sin[c + d*x] + 3*A*b^3*Sin[c + d*x] + 297*a^2*b*C*Sin[c + d*x]))/
(693*a) + (2*Sec[c + d*x]^2*(135*a^4*A*Sin[c + d*x] + 205*a^2*A*b^2*Sin[c +
 d*x] - 4*A*b^4*Sin[c + d*x] + 165*a^4*C*Sin[c + d*x] + 297*a^2*b^2*C*Sin[c
 + d*x]))/(693*a^2) + (2*Sec[c + d*x]*(741*a^4*A*b*Sin[c + d*x] + 51*a^2*A*
b^3*Sin[c + d*x] + 8*A*b^5*Sin[c + d*x] + 957*a^4*b*C*Sin[c + d*x] + 99*a^2
*b^3*C*Sin[c + d*x]))/(693*a^3) + (46*a*A*b*Sec[c + d*x]^4*Tan[c + d*x])/99
 + (2*a^2*A*Sec[c + d*x]^5*Tan[c + d*x])/11))/d
```

Maple [B] time = 0.575, size = 4694, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/693/d*(-63*A*a^6-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
```

/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*b^6+135*A*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^6+165*C*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^6*a^6-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*b^6+135*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^6+165*C*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^5*a^6+891*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^4*b^2+2*A*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^4+8*A
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a*b^5-957*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^6*a^5*b-957*C*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^6*a^4*b
^2-99*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^6*a^3*b^3-99*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^6*a^2*b^4+957*C*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^6*a^5*b+891*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^6*a^4*b^2+99*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^6*a^3*b^3-741*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^5*sin(d*x+c)*a^5*b-741*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^4*b^2-51*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^3*b^
3-51*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co

s(d*x+c)^5*sin(d*x+c)*a^2*b^4-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a*b^5+741*A*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a^5*b+663*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b^2+51*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^3*b^3+2*A*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a^2*b^4+8*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^5-957*C*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5*b-957*C*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^5*a^4*b^2-99*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^3-99*C*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b
^4+957*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^5*a^5*b-741*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^5*b-741*A*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*a^4*b^2+99*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^3-51*A*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^3-51*A*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^2*b^4-8*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^5+741*A*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5*b+663
*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c

```
os(d*x+c)))^(1/2)*a^4*b^2+51*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^3+8*A*cos(d*x+c)^7*b^6-
8*A*cos(d*x+c)^6*b^6+135*A*cos(d*x+c)^6*a^6+165*C*cos(d*x+c)^6*a^6-54*A*cos
(d*x+c)^4*a^6-66*C*cos(d*x+c)^4*a^6-18*A*cos(d*x+c)^2*a^6-99*C*cos(d*x+c)^2
*a^6-160*A*cos(d*x+c)^4*a^4*b^2+A*cos(d*x+c)^4*a^2*b^4-86*A*cos(d*x+c)^3*a^
5*b-116*A*cos(d*x+c)^3*a^3*b^3-274*A*cos(d*x+c)^2*a^4*b^2-224*A*cos(d*x+c)*
a^5*b+135*A*cos(d*x+c)^7*a^5*b+741*A*cos(d*x+c)^7*a^4*b^2+205*A*cos(d*x+c)^
7*a^3*b^3+51*A*cos(d*x+c)^7*a^2*b^4-4*A*cos(d*x+c)^7*a*b^5+741*A*cos(d*x+c)
^6*a^5*b-307*A*cos(d*x+c)^6*a^4*b^2+51*A*cos(d*x+c)^6*a^3*b^3-52*A*cos(d*x+
c)^6*a^2*b^4+8*A*cos(d*x+c)^6*a*b^5-566*A*cos(d*x+c)^5*a^5*b-140*A*cos(d*x+
c)^5*a^3*b^3-594*C*cos(d*x+c)^4*a^4*b^2-396*C*cos(d*x+c)^3*a^5*b+957*C*cos(
d*x+c)^6*a^5*b-363*C*cos(d*x+c)^6*a^4*b^2+99*C*cos(d*x+c)^6*a^3*b^3-99*C*co
s(d*x+c)^6*a^2*b^4-726*C*cos(d*x+c)^5*a^5*b-396*C*cos(d*x+c)^5*a^3*b^3+165*
C*cos(d*x+c)^7*a^5*b+957*C*cos(d*x+c)^7*a^4*b^2+297*C*cos(d*x+c)^7*a^3*b^3+
99*C*cos(d*x+c)^7*a^2*b^4-4*A*cos(d*x+c)^5*a*b^5)/(a+b*cos(d*x+c))^(1/2)/a^
3/sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx\right.}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(1
3/2), x)
```

3.748 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=554

$$\frac{\left(15a^2C+8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^3d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(15a^2C-10abC+8b^2(3A+2C)\right)\cot(c+dx)\sqrt{\frac{a(1-\mathrm{se}}{}}}{24b^3d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(15*a^2*C + 8*b^2*(3*A + 2*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^3*d) + (Sqrt[a + b]*(15*a^2*C - 10*a*b*C + 8*b^2*(3*A
+ 2*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^3*d) + (a*Sqrt[a + b]*(8*A
*b^2 + 5*a^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8
*b^4*d) + ((15*a^2*C + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(24*b^3*d*Sqrt[Cos[c + d*x]]) - (5*a*C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(12*b^2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 1.52341, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(15a^2C+8b^2(3A+2C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^3d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(15a^2C-10abC+8b^2(3A+2C)\right)\cot(c+dx)\sqrt{\frac{a(1-\mathrm{se}}{}}}{24b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(15*a^2*C + 8*b^2*(3*A + 2*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^3*d) + (Sqrt[a + b]*(15*a^2*C - 10*a*b*C + 8*b^2*(3*A
+ 2*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
```

```
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^3*d) + (a*Sqrt[a + b]*(8*A
*b^2 + 5*a^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8
*b^4*d) + ((15*a^2*C + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(24*b^3*d*Sqrt[Cos[c + d*x]]) - (5*a*C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(12*b^2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd}+\frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+b(3A+2C)\cos(\right.}{\sqrt{a+b\cos(c+}}}{3b}$$

$$=-\frac{5aC\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12b^2d}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,c}}{3bc}$$

$$=\frac{\left(15a^2C+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^3d\sqrt{\cos(c+dx)}}-\frac{5aC\sqrt{\cos(c+dx)}}{}$$

$$=\frac{\left(15a^2C+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^3d\sqrt{\cos(c+dx)}}-\frac{5aC\sqrt{\cos(c+dx)}}{}$$

$$=\frac{a\sqrt{a+b}\left(8Ab^2+5a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{8b^4d}$$

$$=-\frac{(a-b)\sqrt{a+b}\left(15a^2C+8b^2(3A+2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{24ab^3d}$$

Mathematica [C] time = 13.5434, size = 1216, normalized size = 2.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] ((-4*a*(24*A*b^2 + 5*a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
```

```
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a^2
*b*C*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
24*A*b^2 + 15*a^2*C + 16*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(48*b^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]*((-5*a*C*Sin[c + d*x])/(12*b^2) + (C*Sin[2*(c + d*x)])/(6*b)))
/d
```

Maple [B] time = 0.233, size = 2336, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/24/d/(a+b*cos(d*x+c))^(1/2)*(-24*A*cos(d*x+c)^2*a*b^2+24*A*cos(d*x+c)*a*b
^2+2*C*cos(d*x+c)^4*a*b^2-5*C*cos(d*x+c)^3*a^2*b+15*C*cos(d*x+c)^2*a^2*b-18
*C*cos(d*x+c)^2*a*b^2-10*C*cos(d*x+c)*a^2*b+16*C*cos(d*x+c)*a*b^2-8*C*cos(d
*x+c)^5*b^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
```

os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1

```
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-8*C*cos(d*x+c)^3*b^3-15*C
*cos(d*x+c)^2*a^3+16*C*cos(d*x+c)^2*b^3+15*C*cos(d*x+c)*a^3-24*A*cos(d*x+c)
^3*b^3+24*A*cos(d*x+c)^2*b^3)/sin(d*x+c)/b^3/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a
), x)
```

3.749 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=455

$$-\frac{\sqrt{a+b}\left(3a^2C+4b^2(2A+C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}-\frac{3aC\,\mathrm{s}}{}$$

```
[Out] (3*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) - ((3
*a - 2*b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) - (Sqr
t[a + b]*(3*a^2*C + 4*b^2*(2*A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^3*d) - (3*a*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d
*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(2*b*d)
```

Rubi [A] time = 1.05464, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3050, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2C+4b^2(2A+C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}-\frac{3aC\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```



```
[Out] (3*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) - ((3
*a - 2*b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) - (Sqr
t[a + b]*(3*a^2*C + 4*b^2*(2*A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
```

```
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^3*d) - (3*a*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d
*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(2*b*d)
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \frac{\int \frac{\frac{aC}{2}+b(2A+C)\cos(c+dx)-\frac{3}{2}aC\cos}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}}{2b}$$

$$= -\frac{3aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin}{2bd}$$

$$= -\frac{3aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin}{2bd}$$

$$= -\frac{\sqrt{a+b}\left(8A+\left(4+\frac{3a^2}{b^2}\right)C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{4bd}$$

$$= \frac{3(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4b^2d}$$

Mathematica [C] time = 12.006, size = 1169, normalized size = 2.57

$$-\frac{4a^2C\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}$$

$$\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} - $$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) - ((-4
*a^2*C*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
```

```
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-8*A*b - 4*b*C)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 6*a*C*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*b*d)
```

Maple [B] time = 0.161, size = 1635, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^2-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
```

```
(a-b)/(a+b))^(1/2))*a^2-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*C*cos(d*x+c)*sin(d*x+c)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*b^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-4*C*cos(d*x+c)*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+16*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^
2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^2+2*C*cos(d*x+c)^4*b^2-C*cos(d*x+c)^3*a*b-3*C*cos(d*x+c)^2*a^2+3*C*cos(d
*x+c)^2*a*b-2*b^2*C*cos(d*x+c)^2+3*C*cos(d*x+c)*a^2-2*C*cos(d*x+c)*a*b)/sin
(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a
), x)
```

3.750 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=393

$$\frac{\sqrt{a+b}(aC+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd}+\frac{aC\sqrt{a+b}\cot(c+dx)\sqrt{\ldots}}{\ldots}$$

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*(2*A*b + a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d) + (a*Sqrt
[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (C*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.727906, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3062, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(aC+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd}+\frac{aC\sqrt{a+b}\cot(c+dx)\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*(2*A*b + a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d) + (a*Sqrt
[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (C*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 3062

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -
Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*f*Sqrt[a + b*Sin[e + f*x]
]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - A*b
*d)*Sin[e + f*x] - C*(b*c + a*d)*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^
(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
```

```
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\int \frac{-aC+2Ab\cos(c+dx)-aC\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\int \frac{-aC+2Ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{(aC)\int \frac{\sqrt{\cos(}}{\sqrt{a+b\,\mathrm{co}}}}{2b}$$

$$= \frac{a\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{abd}$$

Mathematica [A] time = 15.1079, size = 342, normalized size = 0.87

$$\frac{8\cos\left(\frac{1}{2}(c+dx)\right)\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\sqrt{\cos(c+dx)}\left(2Ab\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]),x]
```

```
[Out] (8*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[c + d*x]]*((a + b)*
C*Cos[(c + d*x)/2]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*A*b*Cos[(c + d*x)
/2]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*C*Cos[(c + d*x)/2]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)] + C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(a
+ b*Cos[c + d*x])*Sin[(c + d*x)/2]))/(b*d*(1 + Cos[c + d*x])^(5/2)*Sqrt[a +
 b*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])
```

Maple [B] time = 0.123, size = 939, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)*cos(d*x+c)^2*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b+C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a+C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)^4*b+C*cos(d*x+c)^3*a-C*cos(d*x+c)^3*b-
C*cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(1/2)/b/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2 + a\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x
))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c
))), x)
```

3.751 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=343

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*A*
Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*C*Cot
[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rubi [A] time = 0.444629, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3054, 2809, 12, 2801, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*A*
Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*C*Cot
[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```


Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = C\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{A}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a\cdots}}{bd}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a\cdots}}{bd}$$

$$= \frac{2A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{a^2d}$$

Mathematica [B] time = 16.9422, size = 2642, normalized size = 7.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x
]]),x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((1
+ Cos[c + d*x])^(3/2)*(-((A*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]) +
 (C*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]] - (2*A*b*Cos[c + d*x]^(3/2
))/(a*Sqrt[a + b*Cos[c + d*x]]))*Sec[(c + d*x)/2]^2*(2*A*(a + b)*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] - 2*a*(A - C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
 4*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*b*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*A*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - A*b*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Tan[(c + d*x)/2]))/(2*a*d*Sqrt[a + b*Cos[c + d*x]]*(-(b*(1 + Co
s[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*(2*A*(a + b)*Sqrt[(a + b*
```

Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] - 2*a*(A - C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4
*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*b*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*A*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Tan[(c + d*x)/2]))/(4*a*(a + b*Cos[c + d*x])^(3/2)) + (3*Sqrt[1 +
Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*(2*A*(a + b)*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] - 2*a*(A - C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*
C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*b*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*A*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Tan[(c + d*x)/2]))/(4*a*Sqrt[a + b*Cos[c + d*x]]) - ((1 + Cos[c + d*
x])^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(2*A*(a + b)*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] - 2*a*(A - C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*C
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*b*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*A*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Tan[(c + d*x)/2]))/(2*a*Sqrt[a + b*Cos[c + d*x]]) - ((1 + Cos[c + d*x
])^(3/2)*Sec[(c + d*x)/2]^2*((3*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*C
os[(3*(c + d*x))/2]*Sec[(c + d*x)/2])/2 + a*A*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sec[(c + d*x)/2]^2 - (A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Se
c[(c + d*x)/2]^2)/2 + (A*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos
[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))] - (a*(A - C)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))
)/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (2*a*C*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(
1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))] + (A*b*Sec[(c + d*x)/2]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Sin[(3*(c + d*x))/2])/(2*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]) + (a*A*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c +
d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/2])/Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])] - (A*b*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d
*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(2*Sqrt[Cos[c +

```
d*x]/(1 + Cos[c + d*x])]) + (A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec
[(c + d*x)/2]*Sin[(3*(c + d*x))/2]*Tan[(c + d*x)/2])/2 - (a*(A - C)*Sqrt[(a
 + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) -
(2*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x
)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-
a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (A*(a + b)*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c
 + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(2*a*Sqrt[a + b*Cos[
c + d*x]])))
```

Maple [B] time = 0.191, size = 992, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)/a/sin(d*x+c)/cos(
```

```
d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^3 + a\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3
/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/
2)), x)
```

3.752 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt{a+b}(a(A+3C)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{4Ab(a-b)\sqrt{a}}{}$$

```
[Out] (-4*A*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*d) +
(2*Sqrt[a + b]*(2*A*b + a*(A + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3
/2))
```

Rubi [A] time = 0.508221, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3056, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a(A+3C)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{4Ab(a-b)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (-4*A*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*d) +
(2*Sqrt[a + b]*(2*A*b + a*(A + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3
/2))
```


Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-Ab+\frac{1}{2}a(A+3C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(2Ab)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a} + \frac{(2Ab}{}$$

$$= -\frac{4A(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{3a^3d}$$

Mathematica [A] time = 11.9357, size = 349, normalized size = 1.23

$$\frac{2\left(\frac{8\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(a(a(A+3C)-2Ab)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+Ab\cos(c}{(\cos(c+dx)+1)^{3/2}}\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x
]]),x]
```

```
[Out] (2*(A*(a - 2*b*Cos[c + d*x])*(a + b*Cos[c + d*x])*Sin[c + d*x] + (8*(Cos[(c
 + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]
*Sec[(c + d*x)/2]^2]*(2*A*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] + a*(-2*A*b + a*(A + 3*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*b*Cos[c + d*
x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1 + Cos[c +
d*x])^(3/2)))/(3*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.172, size = 1185, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(1/2)*(3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+6*C*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+
2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*A*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*cos(d*x+c)^3*a*b-2*A*cos(d*x+c)^3*b^2+A*c
os(d*x+c)^2*a^2-2*A*cos(d*x+c)^2*a*b+2*A*cos(d*x+c)^2*b^2+A*cos(d*x+c)*a*b-
A*a^2)/a^2/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c) + a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/
2)), x)
```

3.753 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=354

$$\frac{2\sqrt{a+b}\left(-3a^2(3A+5C)+2aAb-8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}+$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^4*d) + (2*Sqrt[a + b]*(2*a*A*b - 8*A*b^2 - 3*a^2*(3*A +
5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*A*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (8*A*b*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.810106, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 3055, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(-3a^2(3A+5C)+2aAb-8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^4*d) + (2*Sqrt[a + b]*(2*a*A*b - 8*A*b^2 - 3*a^2*(3*A +
5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*A*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (8*A*b*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
- Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{-2Ab+\frac{1}{2}a(3A+5C)\cos(c+dx)+Ab\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,d\ldots}{5a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{8Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int\ldots}{\ldots}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{8Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{15}\ldots$$

$$= \frac{2(a-b)\sqrt{a+b}\left(A\left(9+\frac{8b^2}{a^2}\right)+15C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+\ldots}{a-\ldots}\right.\ldots}{15a^2d}$$

Mathematica [C] time = 6.41438, size = 1298, normalized size = 3.67

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x
]]),x]
```

```
[Out] -((-4*a*(7*a^2*A*b + 8*A*b^3 + 15*a^2*b*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*
a*(9*a^3*A + 8*a*A*b^2 + 15*a^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 8*A*b^3 + 15*a^2*b*C)*((I*Cos[(c
+ d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqr
t[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((
a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(15*a^3*d) +
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(9*a^2*A*Sin[
c + d*x] + 8*A*b^2*Sin[c + d*x] + 15*a^2*C*Sin[c + d*x]))/(15*a^3) - (8*A*b
*Sec[c + d*x]*Tan[c + d*x])/(15*a^2) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5
*a)))/d
```

Maple [B] time = 0.18, size = 2235, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3-4*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos(d
*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2-10*A*cos(d*x+c)
```

```
^3*a^2*b+8*A*cos(d*x+c)^3*a*b^2+A*cos(d*x+c)*a^2*b-9*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
+8*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15*C*cos(d*
x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3-8*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*
C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c
)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
```

```
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(
d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^5 + a\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2+A}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/
2)), x)
```

3.754 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=429

$$\frac{2\left(5a^2(5A+7C)+24Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b}\left(-a^2(44Ab+70bC)-5a^3(5A+7C)+12aAb^2-48\right.}{}$$

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(24*A*b^2 + a^2*(22*A + 35*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(105*a^5*d) - (2*Sqrt[a + b]*(12*a*A*b^2 - 48*A*b^3 - 5*a^
3*(5*A + 7*C) - a^2*(44*A*b + 70*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
105*a^4*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*a*d*Cos[c + d*x
]^(7/2)) - (12*A*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a^2*d*Cos[c +
 d*x]^(5/2)) + (2*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(105*a^3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.19778, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2(5A+7C)+24Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b}\left(-a^2(44Ab+70bC)-5a^3(5A+7C)+12aAb^2-48\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(24*A*b^2 + a^2*(22*A + 35*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(105*a^5*d) - (2*Sqrt[a + b]*(12*a*A*b^2 - 48*A*b^3 - 5*a^
3*(5*A + 7*C) - a^2*(44*A*b + 70*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
105*a^4*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*a*d*Cos[c + d*x
```

```
]^(7/2)) - (12*A*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a^2*d*Cos[c +
 d*x]^(5/2)) + (2*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(105*a^3*d*Cos[c + d*x]^(3/2))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{-3Ab+\frac{1}{2}a(5A+7C)\cos(c+dx)+2Ab\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{7a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{12Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{12Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(2\right.}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{12Ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(2\right.}{}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\left(24Ab^2+a^2(22A+35C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{105a^5d}$$

Mathematica [C] time = 6.46398, size = 1376, normalized size = 3.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + b*Cos[c + d*x
]]),x]
```

```
[Out] ((-4*a*(25*a^4*A + 32*a^2*A*b^2 + 48*A*b^4 + 35*a^4*C + 70*a^2*b^2*C)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - 4*a*(44*a^3*A*b + 48*a*A*b^3 + 70*a^3*b*C)*((Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(44*a^2*A*b^2
 + 48*A*b^4 + 70*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*E
llipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*S
ec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(b*Sqrt[Cos[c + d*x]])))/(105*a^4*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 24*A*b^2*Sin[c + d*x]
 + 35*a^2*C*Sin[c + d*x]))/(105*a^3) - (4*Sec[c + d*x]*(22*a^2*A*b*Sin[c +
d*x] + 24*A*b^3*Sin[c + d*x] + 35*a^2*b*C*Sin[c + d*x]))/(105*a^4) - (12*A*
b*Sec[c + d*x]^2*Tan[c + d*x])/(35*a^2) + (2*A*Sec[c + d*x]^3*Tan[c + d*x])
/(7*a)))/d
```

Maple [B] time = 0.24, size = 2767, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-15*A*a^4+48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-12*
A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^4*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b+70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^2-70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b+44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3*b-12*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*
x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
```

```
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)
^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b+70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3*b+70*C*cos(d*x+c)^4*a^2*b^2+35*C*cos(d*x+c)^3*a^3*b+35*C*cos(
d*x+c)^5*a^3*b-70*C*cos(d*x+c)^5*a^2*b^2-70*C*cos(d*x+c)^4*a^3*b+24*A*cos(d
*x+c)^3*a*b^3-6*A*cos(d*x+c)^2*a^2*b^2+3*A*cos(d*x+c)*a^3*b+25*A*cos(d*x+c)
^5*a^3*b-44*A*cos(d*x+c)^5*a^2*b^2+24*A*cos(d*x+c)^5*a*b^3-44*A*cos(d*x+c)^
4*a^3*b+50*A*cos(d*x+c)^4*a^2*b^2-48*A*cos(d*x+c)^4*a*b^3+16*A*cos(d*x+c)^3
*a^3*b+25*A*cos(d*x+c)^4*a^4+35*C*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4-35
*C*cos(d*x+c)^2*a^4-48*A*cos(d*x+c)^5*b^4+48*A*cos(d*x+c)^4*b^4+44*A*cos(d*
x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b^2)/(a+b*cos(d*x+c))^(1/2)/a^4/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(9/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)^{6}+a\cos\left(dx+c\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b*cos(d*x + c)^6 + a*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(9/
2)), x)
```

3.755 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=604

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(5a^2C+4Ab^2-b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2b^2d\left(a^2-b^2\right)}-\frac{a\left(15a\right.}{}$$

```
[Out] ((8*A*b^2 + 15*a^2*C - 7*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*S
qrt[a + b]*d) - ((8*A*b^2 + (15*a^2 + 5*a*b - 2*b^2)*C)*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(4*b^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(8*A*b^2 + 15*a^2*C +
4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^4*d) - (2*(A*b
^2 + a^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Co
s[c + d*x]]) - (a*(8*A*b^2 + 15*a^2*C - 7*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((4*A*b^2 + 5*a^2*C
 - b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*
(a^2 - b^2)*d)
```

Rubi [A] time = 1.71422, antiderivative size = 604, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3048, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(5a^2C+4Ab^2-b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2b^2d\left(a^2-b^2\right)}-\frac{a\left(15a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2),
x]
```


```
[Out] ((8*A*b^2 + 15*a^2*C - 7*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*S
```

```
qrt[a + b]*d) - ((8*A*b^2 + (15*a^2 + 5*a*b - 2*b^2)*C)*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(4*b^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(8*A*b^2 + 15*a^2*C +
4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^4*d) - (2*(A*b
^2 + a^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Co
s[c + d*x]]) - (a*(8*A*b^2 + 15*a^2*C - 7*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((4*A*b^2 + 5*a^2*C
 - b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*
(a^2 - b^2)*d)
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
```

```
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}\left(Ab^2+a^2C\right)-\frac{1}{2}ab(A+C)\right.}{\sqrt{a+}}}{b\left(\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4Ab^2+5a^2C-b^2C\right)\sqrt{\cos(c+}}{2b^2\left(a^2\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{a\left(8Ab^2+15a^2C-7b^2C\right)\sqrt{a+b}}{4b^3\left(a^2-b^2\right)d\sqrt{\cos}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{a\left(8Ab^2+15a^2C-7b^2C\right)\sqrt{a+b}}{4b^3\left(a^2-b^2\right)d\sqrt{\cos}}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+15a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid\right.}{4b^4d}$$

$$= \frac{\left(8Ab^2+15a^2C-7b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{4b^3\sqrt{a+b}d}$$

Mathematica [C] time = 6.50712, size = 1276, normalized size = 2.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((C*Sin[c + d*x])/(2*b^2) - (2
*(a*A*b^2*Sin[c + d*x] + a^3*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(a + b*Cos[
c + d*x]))))/d - ((-4*a*(5*a^3*C - 5*a*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 4*a*(8*A*b^3 + 4*a^2*b*C + 4*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a*A*b^2 + 15*a^3*C - 7*a*b^2*C)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(8*(a - b)*
b^2*(a + b)*d)
```

Maple [B] time = 0.249, size = 3546, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/(a+b*cos(d*x+c))^(1/2)*(-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
```

```
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-16*A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-10*C*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b-4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b^2+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+22*C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^2*b^2+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-7*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-7*C*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*
A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3-10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+22*C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
```

```
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b-7*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^2-7*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^3-2*C*cos(d*x+c)^4*a^2*b^2+5*C*cos(d*x+c)^3*a
^3*b-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*b^4+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*b^4-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+8*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+15*C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^4+2*C*cos(d*x+c)^4*b^4-2*C*cos(d*x+c)^2*b^4-15*C*cos(d*x+c)*a^4+8*A*cos(
d*x+c)^2*a^2*b^2+15*C*cos(d*x+c)^2*a^4+7*C*cos(d*x+c)^2*a*b^3+10*C*cos(d*x+
c)*a^3*b+7*C*cos(d*x+c)*a^2*b^2-2*C*cos(d*x+c)*a*b^3-5*C*cos(d*x+c)^3*a*b^3
-8*A*cos(d*x+c)^2*a*b^3-8*A*cos(d*x+c)*a^2*b^2+8*A*cos(d*x+c)*a*b^3-8*A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^4+16*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*a^4+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-15*C*cos(d*x+c)^2*a^3*b-5*C*cos(d*x+c)
^2*a^2*b^2)/sin(d*x+c)/b^3/(a^2-b^2)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

3.756 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=503

$$\frac{\left(3a^2C+2Ab^2-b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(3a^2C+2Ab^2-b^2C\right)\mathrm{c}}{}$$

```
[Out] -(((2*A*b^2 + 3*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sq
rt[a + b]*d)) + ((2*A*b^2 + a*(3*a + b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*b^2*Sqrt[a + b]*d) + (3*a*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(b^3*d) - (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((2*A*b^2 + 3*a^2*C - b^2
*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c +
d*x]])
```

Rubi [A] time = 1.23815, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3048, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2C+2Ab^2-b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(3a^2C+2Ab^2-b^2C\right)\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] -(((2*A*b^2 + 3*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sq
rt[a + b]*d)) + ((2*A*b^2 + a*(3*a + b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
```

```
])/(a*b^2*Sqrt[a + b]*d) + (3*a*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(b^3*d) - (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((2*A*b^2 + 3*a^2*C - b^2
*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c +
d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
```

```
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(Ab^2+a^2C\right)-\frac{1}{2}ab(A+C)\cos(c+dx}{\sqrt{\cos(c+dx)}\sqrt{}}}{b\left(a^2\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2Ab^2+3a^2C-b^2C\right)\sqrt{a+b\,c}}{b^2\left(a^2-b^2\right)d\sqrt{co}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2Ab^2+3a^2C-b^2C\right)\sqrt{a+b\,c}}{b^2\left(a^2-b^2\right)d\sqrt{co}}$$

$$= \frac{3a\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{b^3d}$$

$$= -\frac{\left(2Ab^2+3a^2C-b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{ab^2\sqrt{a+b}d}$$

Mathematica [C] time = 6.36833, size = 1234, normalized size = 2.45

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(b*(-a^2 +
 b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*C - b^2*C)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(2*a*A*b + 2*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
```

```
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])) + 2*(2*A*b^2 + 3*a^2*C - b^2*C)*((I*Cos[(c + d*
x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Co
s[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*(a - b)*b*(a +
 b)*d)
```

Maple [B] time = 0.183, size = 2499, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(2*A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a*b^2+C*cos(d*x+c)^3*a^2*b-3*C*
cos(d*x+c)^2*a^2*b-C*cos(d*x+c)^2*a*b^2+2*C*cos(d*x+c)*a^2*b+C*cos(d*x+c)*a
*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-2*C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-C*sin(d*x+c)*(co
```

s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a
*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*a*b^2+2*A*cos(d*x+c)*b^3-C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-2*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b
^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+2*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-C*cos(d*x+c)^3*b^3+3*C*cos(d*x+c)^2*a^3
+C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3-2*A*cos(d*x+c)^2*b^3-2*A*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b^2/(a^2-b^2)/cos(d*x+c)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3
/2), x)
```

3.757 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{a^2bd\sqrt{a+b}}$$

```
[Out] (2*(A*b^2 + a^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*b*Sqrt[a + b]*d)
 + (2*(A*b - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) -
(2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) - (
2*(A*b^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])
```

Rubi [A] time = 0.80226, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3052, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (2*(A*b^2 + a^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*b*Sqrt[a + b]*d)
 + (2*(A*b - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) -
(2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
```

```
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) - (
2*(A*b^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])
```

Rule 3052

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Dist[C/
(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[1/b
, Int[(A*b - a*C*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{Ab-aC\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{b} + \frac{C\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= \frac{2\left(Ab^2+a^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+b}d}$$

Mathematica [C] time = 6.38617, size = 1225, normalized size = 2.91

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(a*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*A - A*b^2)*Sqrt[((a + b)*Cot
```

```
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - 4*a*(-(a*A*b) - a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])) + 2*(-(A*b^2) - a^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]
], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a*(a - b)*(a + b)*d)
```

Maple [B] time = 0.219, size = 2046, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/d*(-A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a*b^2+C*cos(d*x+c)^2*a^2*b-C*cos(d
*x+c)*a^2*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+co
```

```
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*
b^2-A*cos(d*x+c)*b^3+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*b^3+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+
A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^3+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*cos(d*x+c)^2*a^3+C*cos
(d*x+c)*a^3+A*cos(d*x+c)^2*b^3)/(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/a/b/sin(d*
x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
 c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^3+2\,ab\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
 c))), x)
```

3.758 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=308

$$\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2Ab^2-a^2(A-C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\right.}{a^3d\sqrt{a+b}}$$

```
[Out] (-2*(2*A*b^2 - a^2*(A - C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
+ b]*d) - (2*(2*A*b + a*(A - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sq
rt[a + b]*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.624556, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3056, 2998, 2816, 2994}

$$\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2Ab^2-a^2(A-C)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\right.}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (-2*(2*A*b^2 - a^2*(A - C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
+ b]*d) - (2*(2*A*b + a*(A - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sq
rt[a + b]*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(-2Ab^2+a^2(A-C)\right)-\frac{1}{2}ab(A+C}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c-}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{((a-b)(2Ab+a(A-C)))\int}{a\left(a^2-\right.}$$

$$= -\frac{2\left(2Ab^2-a^2(A-C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{a^3\sqrt{a+b}d}$$

Mathematica [C] time = 6.46727, size = 1269, normalized size = 4.12

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] ((-4*a*(2*a^2*A*b - 2*A*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(a^3*A - 2*a
*A*b^2 - a^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])) + 2*(a^2*A*b - 2*A*b^3 - a^2*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
+ b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
```

```
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*(-a + b)*(a + b)*d) + (Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(A*b^3*Sin[c + d*x] + a^2*b*C*Sin[c
 + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/a^2))
/d
```

Maple [B] time = 0.29, size = 2278, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(a*A*b^2-A*a^3+A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x
+c)*a*b^2-C*cos(d*x+c)^2*a^2*b+C*cos(d*x+c)*a^2*b-A*cos(d*x+c)*a^2*b-A*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-2*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a*b^2+A*cos(d*x+c)*a^3+2*A*cos(d*x+c)*b^3-2*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+2*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^
2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
```

```
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+A*cos(d*x+c)^
2*a^2*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+C*cos(d*x+c)^2*a^3-C*cos(d*x
+c)*a^3-2*A*cos(d*x+c)^2*b^3-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b)/a^2/(a^2
-b^2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
```

```
3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{4}+2\,ab\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
3/2)), x)
```

3.759 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{2\left(4Ab^2-a^2(A-3C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2(A+3C\right.}{}$$

```
[Out] (2*b*(8*A*b^2 - a^2*(5*A - 3*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*
Sqrt[a + b]*d) + (2*(6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(a
*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2 -
 a^2*(A - 3*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d
*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.979306, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 3055, 2998, 2816, 2994}

$$-\frac{2\left(4Ab^2-a^2(A-3C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2(A+3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),
x]
```


```
[Out] (2*b*(8*A*b^2 - a^2*(5*A - 3*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*
Sqrt[a + b]*d) + (2*(6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(a
*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2 -
```

```
 a^2*(A - 3*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d
*Cos[c + d*x]^(3/2))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(-4Ab^2+a^2(A-3C)\right)-\frac{1}{2}ab(A+}{\cos^{\frac{5}{2}}(c+dx}}{a\left(a\right.}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(4Ab^2-a^2(A-3C)\right)\sqrt{a+}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(4Ab^2-a^2(A-3C)\right)\sqrt{a+}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b\left(8Ab^2-a^2(5A-3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3a^4\sqrt{a+b}d}$$

Mathematica [C] time = 6.61165, size = 1327, normalized size = 3.39

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] ((-4*a*(a^4*A + 7*a^2*A*b^2 - 8*A*b^4 + 3*a^4*C - 3*a^2*b^2*C)*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(5*a^3*A*b - 8*a*A*b^3 - 3*a^3*b*C)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(5*a^2*A*b^2 - 8*A*b^4
- 3*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Ar
cSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/
(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c +
 d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[
c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]*((2*(A*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(a^3*(a^2 - b
^2)*(a + b*Cos[c + d*x])) - (10*A*b*Tan[c + d*x])/(3*a^3) + (2*A*Sec[c + d*
x]*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] time = 0.185, size = 2667, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(1/2)*(a^2*A*b^2-A*a^4+3*C*cos(d*x+c)^3*a^2*b^2-5*A
*cos(d*x+c)^3*a^2*b^2-5*A*cos(d*x+c)^2*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+8*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a*b^3+5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+3*C*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+
A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*C*cos(d*x+c)^3*a^3*b+8*A*cos(d*x+
c)^3*b^4-8*A*cos(d*x+c)^2*b^4-5*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-4*A*cos(d*x+c)^3*a*b^3
+4*A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b+A*cos(d*x+c)^3*a^3*b+A*cos(d
*x+c)^2*a^4+8*A*cos(d*x+c)^2*a*b^3-4*A*cos(d*x+c)*a*b^3+3*C*cos(d*x+c)^2*a^
3*b-3*C*cos(d*x+c)^2*a^2*b^2+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
```

```
2*b^2-8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*A*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^2+8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*cos(d*x+c)^2*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b^2+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b)/a^3/(a^2-b^2)/si
n(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^5 + 2\,ab\cos(dx+c)^4 + a^2\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
5/2)), x)
```

3.760 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=494

$$\frac{2b\left(8Ab^2-a^2(3A-5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2\left(6Ab^2-a^2(A-5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)}+\frac{}{ad\left(a\right.}$$

```
[Out] (-2*(16*A*b^4 - 2*a^2*b^2*(4*A - 5*C) - a^4*(3*A + 5*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(5*a^5*Sqrt[a + b]*d) - (2*(12*a*A*b^2 + 16*A*b^3 + 2*a^2*
b*(2*A + 5*C) + a^3*(3*A + 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^4*
Sqrt[a + b]*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c +
d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]) - (2*(6*A*b^2 - a^2*(A - 5*C))*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) +
(2*b*(8*A*b^2 - a^2*(3*A - 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*
a^3*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.44608, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 3055, 2998, 2816, 2994}

$$\frac{2b\left(8Ab^2-a^2(3A-5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2\left(6Ab^2-a^2(A-5C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)}+\frac{}{ad\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),
x]
```


```
[Out] (-2*(16*A*b^4 - 2*a^2*b^2*(4*A - 5*C) - a^4*(3*A + 5*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(5*a^5*Sqrt[a + b]*d) - (2*(12*a*A*b^2 + 16*A*b^3 + 2*a^2*
b*(2*A + 5*C) + a^3*(3*A + 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
```

```
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^4*
Sqrt[a + b]*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c +
d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]) - (2*(6*A*b^2 - a^2*(A - 5*C))*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) +
(2*b*(8*A*b^2 - a^2*(3*A - 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*
a^3*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(-6Ab^2+a^2(A-5C)\right)-\frac{1}{2}ab(A+}{\cos^{\frac{7}{2}}(c+dx}}{a\,(a}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-a^2(A-5C)\right)\sqrt{a+}}{5a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-a^2(A-5C)\right)\sqrt{a+}}{5a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-a^2(A-5C)\right)\sqrt{a+}}{5a^2\left(a^2-b^2\right)d}$$

$$= -\frac{2\left(16Ab^4-2a^2b^2(4A-5C)-a^4(3A+5C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{5a^5\sqrt{a+b}d}$$

Mathematica [C] time = 6.72412, size = 1418, normalized size = 2.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(
3/2)),x]
```

```
[Out] ((-4*a*(4*a^4*A*b + 12*a^2*A*b^3 - 16*A*b^5 + 10*a^4*b*C - 10*a^2*b^3*C)*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*A + 8*a^3*A*b^2 - 16*a*A*b^4 + 5*a^5
*C - 10*a^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
```

```
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])) + 2*(3*a^4*A*b + 8*a^2*A*b^3 - 16*A*b^5 + 5*a^4*b*C - 10*a^2*b^3*C
)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
+ d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b
)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b)
, ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/
(5*a^4*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(
(2*Sec[c + d*x]*(3*a^2*A*Sin[c + d*x] + 11*A*b^2*Sin[c + d*x] + 5*a^2*C*Sin
[c + d*x]))/(5*a^4) - (2*(A*b^5*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x]))/(a^
4*(a^2 - b^2)*(a + b*Cos[c + d*x])) - (6*A*b*Sec[c + d*x]*Tan[c + d*x])/(5*
a^3) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5*a^2)))/d
```

Maple [B] time = 0.243, size = 4066, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d*(8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-A*a^3*b^2+A*a^5+5*C*cos(d*x+c)^3*a^4*b-5*
C*cos(d*x+c)^4*a^3*b^2+5*A*cos(d*x+c)^3*a^4*b+6*A*cos(d*x+c)^3*a^2*b^3+6*A*
cos(d*x+c)^2*a^3*b^2-2*A*cos(d*x+c)*a^4*b+16*A*cos(d*x+c)^4*b^5-3*A*cos(d*x
+c)^3*a^5-5*C*cos(d*x+c)^3*a^5+3*A*cos(d*x+c)^4*a^3*b^2-8*A*cos(d*x+c)^4*a*
b^4+2*A*cos(d*x+c)^2*a^5+5*C*cos(d*x+c)^2*a^5+10*C*cos(d*x+c)^4*a^2*b^3+10*
C*cos(d*x+c)^3*a^3*b^2-5*C*cos(d*x+c)^4*a^4*b-3*A*cos(d*x+c)^4*a^4*b-8*A*co
s(d*x+c)^4*a^2*b^3-8*A*cos(d*x+c)^3*a^3*b^2+16*A*cos(d*x+c)^3*a*b^4-16*A*co
s(d*x+c)^3*b^5-5*C*cos(d*x+c)^2*a^3*b^2-10*C*cos(d*x+c)^3*a^2*b^3-8*A*cos(d
*x+c)^2*a*b^4+2*A*cos(d*x+c)*a^2*b^3+16*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
```

c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-16*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+16*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(

```
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-16*A*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^4+A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*A*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^3+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+3*A*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^5-16*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+5*C*sin(d*x+c)*
cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^5-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^5+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-16*A*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^5-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^5)/(a+b*cos(d*x+c))^(1/2)/a^4/(a^2-b^2)/sin(d*x+c)/
cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^6 + 2\,ab\cos(dx+c)^5 + a^2\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^2*cos(d*x + c)^6 + 2*a*b*cos(d*x + c)^5 + a^2*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(
7/2)), x)
```

3.761 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=650

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(a^2b^2(A+9C)-5a^4C+3Ab^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(8Ab^4-C\left(-\right.\right.}{}$$

```
[Out] ((8*A*b^4 - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Cot[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) + ((2*a*A*b^3 - 6*A*b^4 + 15*a^4*C
 + 5*a^3*b*C - 21*a^2*b^2*C - 3*a*b^3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) + (5*a*Sqrt[a + b]*C*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b^4*d) - (2*(A*b^2 + a^2*C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*A*b^4
 - 5*a^4*C + a^2*b^2*(A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^
2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((8*A*b^4 - (15*a^4 - 26*a^2*b^2 +
 3*b^4)*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sq
rt[Cos[c + d*x]])
```

Rubi [A] time = 2.47235, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3048, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(a^2b^2(A+9C)-5a^4C+3Ab^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(8Ab^4-C\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2),
x]
```


```
[Out] ((8*A*b^4 - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Cot[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
```

```
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) + ((2*a*A*b^3 - 6*A*b^4 + 15*a^4*C
 + 5*a^3*b*C - 21*a^2*b^2*C - 3*a*b^3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) + (5*a*Sqrt[a + b]*C*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b^4*d) - (2*(A*b^2 + a^2*C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*A*b^4
 - 5*a^4*C + a^2*b^2*(A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^
2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((8*A*b^4 - (15*a^4 - 26*a^2*b^2 +
 3*b^4)*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sq
rt[Cos[c + d*x]])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}\left(Ab^2+a^2C\right)-\frac{3}{2}ab(A+C)}{(a+b}}{3b}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= \frac{5a\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^4d}$$

$$= \frac{\left(8Ab^4-\left(15a^4-26a^2b^2+3b^4\right)C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{3a(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [C] time = 6.64003, size = 1366, normalized size = 2.1

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(a*A*b^2*Sin[c + d*x] + a
^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (4*(2*A*b
^4*Sin[c + d*x] - 3*a^4*C*Sin[c + d*x] + 5*a^2*b^2*C*Sin[c + d*x]))/(3*b^2*
(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(2*a^2*A*b^2 - 2*A*b^4 +
5*a^4*C - 8*a^2*b^2*C + 3*b^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-8*a*A*
b^3 + 4*a^3*b*C - 12*a*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])) + 2*(-8*A*b^4 + 15*a^4*C - 26*a^2*b^2*C + 3*b^4*C)*((I*
Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)
/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) +
(2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/
b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(6*(a
- b)^2*b^2*(a + b)^2*d)
```

Maple [B] time = 0.378, size = 6463, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*
x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5
/2), x)
```

3.762 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=563

$$\frac{2\left(a^2b^2(3A+7C)-3a^4C+Ab^4\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bC-3a^3C+3aAb^2\right.}{}$$

```
[Out] (-2*(A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a^2*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(3*a*A*b^2 - A*b^3 - 3*a^3*C
 - a^2*b*C + 6*a*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2
*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b^3*d) - (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(A*b^4 - 3*a^4*C + a^2*b^2*
(3*A + 7*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])
```

Rubi [A] time = 1.61337, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3048, 3051, 2809, 2993, 2998, 2816, 2994}

$$\frac{2\left(a^2b^2(3A+7C)-3a^4C+Ab^4\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bC-3a^3C+3aAb^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^(5/2),
x]
```


```
[Out] (-2*(A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a^2*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(3*a*A*b^2 - A*b^3 - 3*a^3*C
```

```
 - a^2*b*C + 6*a*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2
*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b^3*d) - (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(A*b^4 - 3*a^4*C + a^2*b^2*
(3*A + 7*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])
```

<u>Rule 3048</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

<u>Rule 3051</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

<u>Rule 2809</u>

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

<u>Rule 2993</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
```

```
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(Ab^2+a^2C\right)-\frac{3}{2}ab(A+C)\cos(c+dx}{\sqrt{\cos(c+dx)}(a+b\cos}}{3b\left(a^2-b\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}b\left(Ab^2+a^2C\right)+\left(\frac{3}{2}a\left(a^2-b^2\right)C-\frac{3}{2}ab}{\sqrt{\cos(c+dx)}(a+b\cos(c}}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{b^3d}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{b^3d}$$

$$= -\frac{2\left(Ab^4-3a^4C+a^2b^2(3A+7C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{3a^2(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [C] time = 6.53945, size = 1388, normalized size = 2.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(A*b^2*Sin[c + d*x] + a^2*
C*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(-3*a^2*A*b
^2*Sin[c + d*x] - A*b^4*Sin[c + d*x] + 3*a^4*C*Sin[c + d*x] - 7*a^2*b^2*C*S
in[c + d*x]))/(3*a*b*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(a^2*
A*b^2 - A*b^4 + a^4*C - a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3*a^
3*A*b - a*A*b^3 - a^3*b*C - 3*a*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
```

```
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-3*a^2*A*b^2 - A*b^4 + 3*a^4*C - 7*a^2*b^
2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[
(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Co
s[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
+ b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])
))/(3*a*(a - b)^2*b*(a + b)^2*d)
```

Maple [B] time = 0.326, size = 6425, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5
/2), x)
```

3.763 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{4b\left(Ab^2-a^2(3A+2C)\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2(-(3A+C))+3ab(A\right.}{}$$

```
[Out] (4*b*(3*a^2*A - A*b^2 + 2*a^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*
(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 + 3*a*b*(A + C) - a^2*(3*A + C))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b
^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^(3/2)) + (4*b*(A*b^2 - a^2*(3*A + 2*C))*Sin[c + d*x])/(3*a*(a^2 -
 b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.02992, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 2993, 2998, 2816, 2994}

$$\frac{4b\left(Ab^2-a^2(3A+2C)\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2(-(3A+C))+3ab(A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),
x]
```

```
[Out] (4*b*(3*a^2*A - A*b^2 + 2*a^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*
(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 + 3*a*b*(A + C) - a^2*(3*A + C))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b
^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[
```

```
c + d*x])^(3/2)) + (4*b*(A*b^2 - a^2*(3*A + 2*C))*Sin[c + d*x])/(3*a*(a^2 -
 b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(-2Ab^2+a^2(3A+C)\right)-\frac{3}{2}ab(A+C)\cos}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b\left(Ab^2-a^2(3A+2C)\right)\sin}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b\left(Ab^2-a^2(3A+2C)\right)\sin}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= -\frac{4b\left(Ab^2-a^2(3A+2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{a(}}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.50088, size = 1364, normalized size = 3.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(
5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(A*b^2*Sin[c + d*x] + a^2*
C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(3*a^2*A*b^2
```

```
*Sin[c + d*x] - A*b^4*Sin[c + d*x] + 2*a^2*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3*a^4*A - 5*a^2*A*b^2 + 2*A*b
^4 + a^4*C - a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6*a^3*A*b + 2*a
*A*b^3 - 4*a^3*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) + 2*(-6*a^2*A*b^2 + 2*A*b^4 - 4*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d
*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(3*a^2*(a - b)^2*(a + b)
^2*d)
```

Maple [B] time = 0.598, size = 4582, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3/d*(6*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
```

+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-3*C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3-3*A*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-6*A*
cos(d*x+c)^3*a^2*b^3-12*A*cos(d*x+c)^2*a^3*b^2-6*A*cos(d*x+c)*a^4*b-3*A*cos
(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a^5-C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-6*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+6*A*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-9*A*cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
^4*b+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^4*b-7*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-C*cos(d*x+c)^3*a^5+C*c
os(d*x+c)*a^5-C*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a^5-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+4*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^3+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+12*A*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*A*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^3-4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-5*C*cos(d*x+c)*sin(d*x+c)*(cos(

d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-7*C*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b^2-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+4*C*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+8*C*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3*b^2+4*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+5*C*cos(d*x+c)^3*a^3*b^2+5*A*cos(d*
x+c)^3*a^3*b^2-A*cos(d*x+c)^3*a*b^4+2*A*cos(d*x+c)^3*b^5-2*A*cos(d*x+c)^2*b
^5-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^5-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+6*A*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-2*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^3-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^4-4*C*cos(d*x+c)*a^4*b+3*C*cos(d*x+c)*a^3*b^2+4*C*cos(d*x+c)^2
*a^4*b-8*C*cos(d*x+c)^2*a^3*b^2+4*C*cos(d*x+c)^2*a^2*b^3-4*C*cos(d*x+c)^3*a
^2*b^3+4*A*cos(d*x+c)^2*a^2*b^3+4*A*cos(d*x+c)^2*a*b^4+7*A*cos(d*x+c)*a^3*b
^2+2*A*cos(d*x+c)*a^2*b^3-3*A*cos(d*x+c)*a*b^4+6*A*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-2
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^4-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-6*A*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2
-A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^3+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+

```
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+4*C*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
^2+4*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^3-C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-4*C*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b^2+6*A*cos(d*x+c)^2*a^4*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5)/(a+b*cos(d*x+c))^(3/2)/
a^2/(a+b)^2/(a-b)^2/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
 c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^4 + 3\,ab^2\cos(dx+c)^3 + 3\,a^2b\cos(dx+c)^2 + a^3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^
```

```
3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
 c))), x)
```

3.764 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=449

$$-\frac{4\left(-a^2b^2(4A+C)+a^4(-C)+2Ab^4\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2b(9A+\right.}{}$$

```
[Out] (2*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(6*a*A*b^2 + 8*A*b^3 - 3*a
^3*(A - C) - a^2*b*(9*A + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqr
t[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(3*a*(a^2 - b^2)
*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) - (4*(2*A*b^4 - a^4*C - a
^2*b^2*(4*A + C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.15759, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 3055, 2998, 2816, 2994}

$$-\frac{4\left(-a^2b^2(4A+C)+a^4(-C)+2Ab^4\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2b(9A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),
x]
```



```
[Out] (2*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(6*a*A*b^2 + 8*A*b^3 - 3*a
^3*(A - C) - a^2*b*(9*A + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqr
```

```
t[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(3*a*(a^2 - b^2)
*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) - (4*(2*A*b^4 - a^4*C - a
^2*b^2*(4*A + C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(-4Ab^2+a^2(3A-C)\right)-\frac{3}{2}ab}{\cos^{\frac{3}{2}}(c}}{3}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} - \frac{4\left(2Ab^4-a^4C-a^2b^2(\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} - \frac{4\left(2Ab^4-a^4C-a^2b^2(\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+}}$$

$$= \frac{2\left(8Ab^4+3a^4(A-C)-a^2b^2(15A+C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{3a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.72854, size = 1421, normalized size = 3.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(
5/2)),x]
```

```
[Out] -((-4*a*(9*a^4*A*b - 17*a^2*A*b^3 + 8*A*b^5 + a^4*b*C - a^2*b^3*C)*Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - 4*a*(3*a^5*A - 15*a^3*A*b^2 + 8*a*A*b^4 - 3*a^5*C - a
^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
+ 2*(3*a^4*A*b - 15*a^2*A*b^3 + 8*A*b^5 - 3*a^4*b*C - a^2*b^3*C)*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a -
b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(A*b^
3*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d
*x])^2) - (2*(9*a^2*A*b^3*Sin[c + d*x] - 5*A*b^5*Sin[c + d*x] + 3*a^4*b*C*S
in[c + d*x] + a^2*b^3*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c +
d*x])) + (2*A*Tan[c + d*x])/a^3))/d
```

Maple [B] time = 0.841, size = 6176, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(
3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^5 + 3\,ab^2\cos(dx+c)^4 + 3\,a^2b\cos(dx+c)^3 + a^3\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^
3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(
3/2)), x)
```

3.765 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=549

$$\frac{2\left(-a^2b^2(13A-C)+a^4(A-5C)+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{4\left(5a^2Ab^2+2a^4C-3Ab^4\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}$$

```
[Out] (-4*b*(8*A*b^4 + a^4*(4*A - 3*C) - a^2*b^2*(14*A - C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(3*a^5*Sqrt[a + b]*(a^2 - b^2)*d) - (2*(12*a*A*b^3 + 16*A*b^
4 - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*C) - a^3*(9*A*b - 3*b*C))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 + a^2*
C)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)) + (4*(5*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C)*Sin[c + d*x])/(3*a^2*(a^2 -
b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*A*b^4 + a^4*(
A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^
3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.67229, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3056, 3055, 2998, 2816, 2994}

$$\frac{2\left(-a^2b^2(13A-C)+a^4(A-5C)+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{4\left(5a^2Ab^2+2a^4C-3Ab^4\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),
x]
```



```
[Out] (-4*b*(8*A*b^4 + a^4*(4*A - 3*C) - a^2*b^2*(14*A - C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(3*a^5*Sqrt[a + b]*(a^2 - b^2)*d) - (2*(12*a*A*b^3 + 16*A*b^
4 - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*C) - a^3*(9*A*b - 3*b*C))*Cot[c + d*x]
```

```
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 + a^2*
C)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)) + (4*(5*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C)*Sin[c + d*x])/(3*a^2*(a^2 -
b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*A*b^4 + a^4*(
A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^
3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}\left(2Ab^2-a^2(A-C)\right)-\frac{3}{2}ab(A-}{\cos^{\frac{5}{2}}(c+d}}{3a}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4\left(5a^2Ab^2-3Ab^4+2\right.}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4\left(5a^2Ab^2-3Ab^4+2\right.}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4\left(5a^2Ab^2-3Ab^4+2\right.}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+}$$

$$= -\frac{4b\left(8Ab^4+a^4(4A-3C)-a^2b^2(14A-C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{3a^5(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.81543, size = 1471, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(
5/2)),x]
```

```
[Out] ((-4*a*(a^6*A + 15*a^4*A*b^2 - 32*a^2*A*b^4 + 16*A*b^6 + 3*a^6*C - 5*a^4*b^
2*C + 2*a^2*b^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*A*b - 28*a^3*A*
b^3 + 16*a*A*b^5 - 6*a^5*b*C + 2*a^3*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
```

```
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*A*b^2 - 28*a^2*A*b^4 + 16*A*b^
6 - 6*a^4*b^2*C + 2*a^2*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)
]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(b*Sqrt[Cos[c + d*x]]))))/(3*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(A*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c +
d*x]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(6*a^2*A*b^4*Sin[c +
 d*x] - 4*A*b^6*Sin[c + d*x] + 3*a^4*b^2*C*Sin[c + d*x] - a^2*b^4*C*Sin[c +
 d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) - (16*A*b*Tan[c + d*x])/
(3*a^4) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 0.347, size = 7087, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(
5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{6}+3\,ab^{2}\cos\left(dx+c\right)^{5}+3\,a^{2}b\cos\left(dx+c\right)^{4}+a^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^
3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(
5/2)), x)
```

3.766 $\int \cos^m(c+dx)(a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=318

$$-\frac{\sin(c+dx)\left(a^2(m+4)(A(m+2)+C(m+1))+b^2(m+1)(A(m+4)+C(m+3))\right)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] ((2*a^2*C + b^2*(C*(3 + m) + A*(4 + m)))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])
/(d*(2 + m)*(4 + m)) + (2*a*b*C*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3 +
m)*(4 + m)) + (C*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/
(d*(4 + m)) - ((a^2*(4 + m)*(C*(1 + m) + A*(2 + m)) + b^2*(1 + m)*(C*(3 + m
) + A*(4 + m)))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c
+ d*x]^2]) - (2*a*b*(C*(2 + m) + A*(3 + m))*Cos[c + d*x]^(2 + m)*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)
*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.858477, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3050, 3033, 3023, 2748, 2643}

$$-\frac{\sin(c+dx)\left(a^2(m+4)(A(m+2)+C(m+1))+b^2(m+1)(A(m+4)+C(m+3))\right)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((2*a^2*C + b^2*(C*(3 + m) + A*(4 + m)))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])
/(d*(2 + m)*(4 + m)) + (2*a*b*C*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3 +
m)*(4 + m)) + (C*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/
(d*(4 + m)) - ((a^2*(4 + m)*(C*(1 + m) + A*(2 + m)) + b^2*(1 + m)*(C*(3 + m
) + A*(4 + m)))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c
+ d*x]^2]) - (2*a*b*(C*(2 + m) + A*(3 + m))*Cos[c + d*x]^(2 + m)*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)
*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^{1+m}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{d(4+m)} + \frac{\int \cos}{}$$

$$= \frac{2abC\cos^{2+m}(c+dx)\sin(c+dx)}{d(3+m)(4+m)} + \frac{C\cos^{1+m}(c+dx)(a+}{d}$$

$$= \frac{\left(2a^2C+b^2C(3+m)+Ab^2(4+m)\right)\cos^{1+m}(c+dx)\sin(c}{d(2+m)(4+m)}$$

$$= \frac{\left(2a^2C+b^2C(3+m)+Ab^2(4+m)\right)\cos^{1+m}(c+dx)\sin(c}{d(2+m)(4+m)}$$

$$= \frac{\left(2a^2C+b^2C(3+m)+Ab^2(4+m)\right)\cos^{1+m}(c+dx)\sin(c}{d(2+m)(4+m)}$$

Mathematica [A] time = 2.3779, size = 250, normalized size = 0.79

$$\frac{\sin(c+dx)\cos^{m+1}(c+dx)\left(\cos(c+dx)\left(\cos(c+dx)\left(bC\cos(c+dx)\left(-\frac{2a\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(c+dx)\right)}{m+4}-\frac{b\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+5}{2};\frac{m+7}{2}\right.}{m+5}\right.\right.\right.\right.}{d\sqrt{\sin^2(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*(-((a^2*A*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2
, Cos[c + d*x]^2])/(1 + m)) + Cos[c + d*x]*((-2*a*A*b*Hypergeometric2F1[1/2
, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + Cos[c + d*x]*(-(((A*b^2
+ a^2*C)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3 +
 m)) + b*C*Cos[c + d*x]*((-2*a*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2,
 Cos[c + d*x]^2])/(4 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + m)/
2, (7 + m)/2, Cos[c + d*x]^2])/(5 + m)))))*Sin[c + d*x])/(d*Sqrt[Sin[c + d*
x]^2])
```

Maple [F] time = 1.61, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a+b\cos(dx+c))^2 \left(A+C(\cos(dx+c))^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\cos\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.767 $\int \cos^m(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=217

$$-\frac{a(A(m+2)+C(m+1))\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}+\frac{aC\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

```
[Out] (a*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (b*C*Cos[c + d*x]^(2
+ m)*Sin[c + d*x])/(d*(3 + m)) - (a*(C*(1 + m) + A*(2 + m))*Cos[c + d*x]^(1
 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (b*(C*(2 + m) + A*(3 + m))
*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c +
d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.371077, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3034, 3023, 2748, 2643}

$$-\frac{a(A(m+2)+C(m+1))\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}+\frac{aC\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```


```
[Out] (a*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (b*C*Cos[c + d*x]^(2
+ m)*Sin[c + d*x])/(d*(3 + m)) - (a*(C*(1 + m) + A*(2 + m))*Cos[c + d*x]^(1
 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (b*(C*(2 + m) + A*(3 + m))
*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c +
d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```


Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
```

```
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)dx=\frac{bC\cos^{2+m}(c+dx)\sin(c+dx)}{d(3+m)}+\frac{\int\cos^m(c+dx)\left(aA(3+\right.}{}$$

$$=\frac{aC\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)}+\frac{bC\cos^{2+m}(c+dx)\sin(c}{d(3+m)}$$

$$=\frac{aC\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)}+\frac{bC\cos^{2+m}(c+dx)\sin(c}{d(3+m)}$$

$$=\frac{aC\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)}+\frac{bC\cos^{2+m}(c+dx)\sin(c}{d(3+m)}$$

Mathematica [A] time = 1.00144, size = 194, normalized size = 0.89

$$\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(\cos(c+dx)\left(C\cos(c+dx)\left(-\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(c+dx)\right)}{m+3}-\frac{b\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m}{}\right.}{m+4}\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + Cos[c + d*x]*(-((A*b*Hypergeometri
c2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m)) + C*Cos[c + d*x]*
(-((a*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3 + m)
) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c + d*
x]^2])/(4 + m))))*Sqrt[Sin[c + d*x]^2])/d
```

Maple [F] time = 0.85, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a+b\cos(dx+c))\left(A+C(\cos(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)\cos(dx+c)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\cos\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.768 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{a\left(a^2C+Ab^2\right)\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2C+Ab^2\right)\sin}{}$$

```
[Out] (a*(A*b^2 + a^2*C)*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*
Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 -
m)/2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*AppellF1[1/2, -m
/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*
x]^m*Sin[c + d*x])/(b*(a^2 - b^2)*d*(Cos[c + d*x]^2)^(m/2)) + (a*C*Cos[c +
d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*S
in[c + d*x])/(b^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (C*Cos[c + d*x]^(2 + m)
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.456262, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3064, 2643, 2823, 3189, 429}

$$\frac{a\left(a^2C+Ab^2\right)\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2C+Ab^2\right)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*(A*b^2 + a^2*C)*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*
Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 -
m)/2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*AppellF1[1/2, -m
/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*
x]^m*Sin[c + d*x])/(b*(a^2 - b^2)*d*(Cos[c + d*x]^2)^(m/2)) + (a*C*Cos[c +
d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*S
in[c + d*x])/(b^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (C*Cos[c + d*x]^(2 + m)
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3064

```
Int[(((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)
*(x_)]^2))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Dist[(a*C)
/b^2, Int[(d*Sin[e + f*x])^n, x], x] + (Dist[(A*b^2 + a^2*C)/b^2, Int[(d*Si
n[e + f*x])^n/(a + b*Sin[e + f*x]), x], x] + Dist[C/(b*d), Int[(d*Sin[e + f
*x])^(n + 1), x], x]) /; FreeQ[{a, b, d, e, f, A, C, n}, x] && NeQ[a^2 - b^
2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = -\frac{(aC)\int \cos^m(c+dx)\,dx}{b^2}+\frac{C\int \cos^{1+m}(c+dx)\,dx}{b}+\left(A+\frac{a^2C}{b^2}\right)\int \frac{\cos^m}{a+b\,c}$$

$$=\frac{aC\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(1+m)\sqrt{\sin^2(c+dx)}}-\frac{C\cos^{2+m}(}{}$$

$$=\frac{aC\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(1+m)\sqrt{\sin^2(c+dx)}}-\frac{C\cos^{2+m}(}{}$$

$$=\frac{a\left(A+\frac{a^2C}{b^2}\right)F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{-1+m}(c+dx)\cos}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 28.8291, size = 10459, normalized size = 29.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.612, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m\left(A+C(\cos(dx+c))^2\right)}{a+b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

3.769 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=514

$$\frac{\sin(c+dx)\left(a^2b^2(A(-m)+A+C(m+2))+a^4(-C)(m+1)+Ab^4m\right)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2\right.}{b^2d\left(a^2-b^2\right)^2}$$

```
[Out] ((A*b^4*m - a^4*C*(1 + m) + a^2*b^2*(A - A*m + C*(2 + m)))*AppellF1[1/2, (1
 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c
 + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*x])/(b^2*(a^2 - b^2
)^2*d) - ((A*b^4*m - a^4*C*(1 + m) + a^2*b^2*(A - A*m + C*(2 + m)))*AppellF
1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*C
os[c + d*x]^m*Sin[c + d*x])/(a*b*(a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2)) +
((A*b^2 + a^2*C)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x])) - ((a^2*C*(1 + m) - b^2*(C - A*m))*Cos[c + d*x]^(1 + m)*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^
2*(a^2 - b^2)*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) + ((A*b^2 + a^2*C)*(1 + m)*Co
s[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x
]^2]*Sin[c + d*x])/(a*b*(a^2 - b^2)*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.880797, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3056, 3063, 2643, 2823, 3189, 429}

$$\frac{\sin(c+dx)\left(a^2b^2(A(-m)+A+C(m+2))+a^4(-C)(m+1)+Ab^4m\right)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2\right.}{b^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] ((A*b^4*m - a^4*C*(1 + m) + a^2*b^2*(A - A*m + C*(2 + m)))*AppellF1[1/2, (1
 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c
 + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*x])/(b^2*(a^2 - b^2
)^2*d) - ((A*b^4*m - a^4*C*(1 + m) + a^2*b^2*(A - A*m + C*(2 + m)))*AppellF
1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*C
os[c + d*x]^m*Sin[c + d*x])/(a*b*(a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2)) +
((A*b^2 + a^2*C)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b
```

```
*Cos[c + d*x])) - ((a^2*C*(1 + m) - b^2*(C - A*m))*Cos[c + d*x]^(1 + m)*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^
2*(a^2 - b^2)*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) + ((A*b^2 + a^2*C)*(1 + m)*Co
s[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x
]^2]*Sin[c + d*x])/(a*b*(a^2 - b^2)*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3063

```
Int[(((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)
*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2))/((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Dist[(b*B - a*C)/b^2, Int[(d*Sin[e + f*x])^n, x], x] +
 (Dist[(A*b^2 - a*b*B + a^2*C)/b^2, Int[(d*Sin[e + f*x])^n/(a + b*Sin[e + f
*x]), x], x] + Dist[C/(b*d), Int[(d*Sin[e + f*x])^(n + 1), x], x]) /; FreeQ
[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\cos^m(c+dx)\left(Ab^2m+a^2(A+C+Cm)-ab(A\cdots\right.}{a+b\cdots}}{a\left(\cdots\right.}$$

$$= \frac{\left(Ab^2+a^2C\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(\left(Ab^2+a^2C\right)(1+m)\right)\int\cos^{1+m}(\cdots}{ab\left(a^2-b^2\right)\cdots}$$

$$= \frac{\left(Ab^2+a^2C\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(Am-C\left(1-\frac{a^2(1+m)}{b^2}\right)\right)\cos^{1+m}(c\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{\left(Ab^2+a^2C\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(Am-C\left(1-\frac{a^2(1+m)}{b^2}\right)\right)\cos^{1+m}(c\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{\left(Ab^4m-a^4C(1+m)+a^2b^2(A-Am+C(2+m))\right)F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+\cdots\right.}{b^2\left(a^2-b^2\right)^2 d\cdots}$$

Mathematica [B] time = 34.2314, size = 17999, normalized size = 35.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m \left(A + C (\cos(dx+c))^2\right)}{(a + b\cos(dx+c))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)\cos(dx+c)^m}{(b\cos(dx+c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\cos(dx+c)^m}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{m}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

3.770 $\int \cos(c+dx)(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=105

$$-\frac{(aC+bB)\sin^3(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)}{d}+\frac{(4aB+3bC)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aB+3bC)+\frac{bC\sin(c\right.}{}$$

```
[Out] ((4*a*B + 3*b*C)*x)/8 + ((b*B + a*C)*Sin[c + d*x])/d + ((4*a*B + 3*b*C)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(b*B + a*C)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.207602, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3029, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{(aC+bB)\sin^3(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)}{d}+\frac{(4aB+3bC)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aB+3bC)+\frac{bC\sin(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

```
[Out] ((4*a*B + 3*b*C)*x)/8 + ((b*B + a*C)*Sin[c + d*x])/d + ((4*a*B + 3*b*C)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(b*B + a*C)*Sin[c + d*x]^3)/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos^2(c+dx)(a+b\cos(c+dx))(B+C\cos(c$$

$$= \int \cos^2(c+dx)\left(aB+(bB+aC)\cos(c+dx)+\right.$$

$$= \frac{bC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx$$

$$= \frac{bC\cos^3(c+dx)\sin(c+dx)}{4d}+(bB+aC)\int \cos$$

$$= \frac{(4aB+3bC)\cos(c+dx)\sin(c+dx)}{8d}+\frac{bC\cos^3}{}$$

$$= \frac{1}{8}(4aB+3bC)x+\frac{(bB+aC)\sin(c+dx)}{d}+\frac{(4a}{}$$

Mathematica [A] time = 0.220838, size = 91, normalized size = 0.87

$$\frac{-32(aC+bB)\sin^3(c+dx)+96(aC+bB)\sin(c+dx)+24(aB+bC)\sin(2(c+dx))+48aBc+48aBdx+3bC\sin(4(c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (48*a*B*c + 36*b*c*C + 48*a*B*d*x + 36*b*C*d*x + 96*(b*B + a*C)*Sin[c + d*x
] - 32*(b*B + a*C)*Sin[c + d*x]^3 + 24*(a*B + b*C)*Sin[2*(c + d*x)] + 3*b*C
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.016, size = 107, normalized size = 1.

$$\frac{1}{d}\left(Cb\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{bB\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aC\left(2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b
*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2
```

```
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.12894, size = 136, normalized size = 1.3

$$\frac{24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba-32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ca-32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Bb+3\,(12}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*C*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b + 3*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*b)/d
```

Fricas [A] time = 1.69143, size = 205, normalized size = 1.95

$$\frac{3\,(4\,Ba+3\,Cb)dx+\left(6\,Cb\cos\left(dx+c\right)^{3}+8\,(Ca+Bb)\cos\left(dx+c\right)^{2}+16\,Ca+16\,Bb+3\,(4\,Ba+3\,Cb)\cos\left(dx+c\right)\right)\sin\left(d\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/24*(3*(4*B*a + 3*C*b)*d*x + (6*C*b*cos(d*x + c)^3 + 8*(C*a + B*b)*cos(d*x
 + c)^2 + 16*C*a + 16*B*b + 3*(4*B*a + 3*C*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.78366, size = 255, normalized size = 2.43

$$\begin{cases}\frac{Bax\sin^{2}\left(c+dx\right)}{2}+\frac{Bax\cos^{2}\left(c+dx\right)}{2}+\frac{Ba\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2Bb\sin^{3}\left(c+dx\right)}{3d}+\frac{Bb\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}+\frac{2Ca\sin^{3}\left(c+dx\right)}{3d}+\frac{Ca\sin\left(c+dx\right)\cos^{2}\left(\right.}{d}\\ x\left(a+b\cos\left(c\right)\right)\left(B\cos\left(c\right)+C\cos^{2}\left(c\right)\right)\cos\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 + B*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + 2*B*b*sin(c + d*x)**3/(3*d) + B*b*sin(c + d*x)*co
s(c + d*x)**2/d + 2*C*a*sin(c + d*x)**3/(3*d) + C*a*sin(c + d*x)*cos(c + d*
x)**2/d + 3*C*b*x*sin(c + d*x)**4/8 + 3*C*b*x*sin(c + d*x)**2*cos(c + d*x)*
*2/4 + 3*C*b*x*cos(c + d*x)**4/8 + 3*C*b*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 5*C*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))*(
B*cos(c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.57557, size = 120, normalized size = 1.14

$$\frac{1}{8}(4Ba + 3Cb)x + \frac{Cb\sin(4dx + 4c)}{32d} + \frac{(Ca + Bb)\sin(3dx + 3c)}{12d} + \frac{(Ba + Cb)\sin(2dx + 2c)}{4d} + \frac{3(Ca + Bb)\sin(dx +}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] 1/8*(4*B*a + 3*C*b)*x + 1/32*C*b*sin(4*d*x + 4*c)/d + 1/12*(C*a + B*b)*sin(
3*d*x + 3*c)/d + 1/4*(B*a + C*b)*sin(2*d*x + 2*c)/d + 3/4*(C*a + B*b)*sin(d
*x + c)/d
```

3.771 $\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=84

$$\frac{(3aB+2bC)\sin(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aC+bB)+\frac{bC\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] ((b*B + a*C)*x)/2 + ((3*a*B + 2*b*C)*Sin[c + d*x])/(3*d) + ((b*B + a*C)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (b*C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0806476, antiderivative size = 104, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3023, 2734}

$$\frac{\left(a^2(-C)+3abB+2b^2C\right)\sin(c+dx)}{3bd}+\frac{(3bB-aC)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}x(aC+bB)+\frac{C\sin(c+dx)(a+b\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*B + a*C)*x)/2 + ((3*a*b*B - a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*b*d) + ((
3*b*B - a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b*Cos[c + d*x])^2*S
in[c + d*x])/(3*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C(a+b\cos(c+dx))^2\sin(c+dx)}{3bd}+\frac{\int (a+b\cos(c+dx))(2$$

$$=\frac{1}{2}(bB+aC)x+\frac{\left(3abB-a^2C+2b^2C\right)\sin(c+dx)}{3bd}+\frac{(3bB-$$

Mathematica [A] time = 0.153093, size = 75, normalized size = 0.89

$$\frac{3(4aB+3bC)\sin(c+dx)+3(aC+bB)\sin(2(c+dx))+6acC+6aCdx+6bBc+6bBdx+bC\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*b*B*c + 6*a*c*C + 6*b*B*d*x + 6*a*C*d*x + 3*(4*a*B + 3*b*C)*Sin[c + d*x]
 + 3*(b*B + a*C)*Sin[2*(c + d*x)] + b*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.018, size = 85, normalized size = 1.

$$\frac{1}{d}\left(\frac{Cb\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+bB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*C*b*(2+cos(d*x+c)^2)*sin(d*x+c)+b*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c))
```

Maxima [A] time = 1.02136, size = 107, normalized size = 1.27

$$\frac{3(2dx+2c+\sin(2dx+2c))Ca+3(2dx+2c+\sin(2dx+2c))Bb-4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Cb+12Ba\sin}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a + 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*B*b - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*b + 12*B*a*sin(d*x + c))
/d
```

Fricas [A] time = 1.68881, size = 149, normalized size = 1.77

$$\frac{3\,(Ca+Bb)dx+\left(2\,Cb\cos\left(dx+c\right)^{2}+6\,Ba+4\,Cb+3\,(Ca+Bb)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(C*a + B*b)*d*x + (2*C*b*cos(d*x + c)^2 + 6*B*a + 4*C*b + 3*(C*a + B
*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.45925, size = 170, normalized size = 2.02

$$\begin{cases}\frac{Ba\sin\left(c+dx\right)}{d}+\frac{Bbx\sin^{2}\left(c+dx\right)}{2}+\frac{Bbx\cos^{2}\left(c+dx\right)}{2}+\frac{Bb\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Cax\sin^{2}\left(c+dx\right)}{2}+\frac{Cax\cos^{2}\left(c+dx\right)}{2}+\frac{Ca\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+ 2C\\x\left(a+b\cos\left(c\right)\right)\left(B\cos\left(c\right)+C\cos^{2}\left(c\right)\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*sin(c + d*x)/d + B*b*x*sin(c + d*x)**2/2 + B*b*x*cos(c + d*x
)**2/2 + B*b*sin(c + d*x)*cos(c + d*x)/(2*d) + C*a*x*sin(c + d*x)**2/2 + C*
a*x*cos(c + d*x)**2/2 + C*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*b*sin(c +
 d*x)**3/(3*d) + C*b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*c
os(c))*(B*cos(c) + C*cos(c)**2), True))
```

Giac [A] time = 1.50569, size = 92, normalized size = 1.1

$$\frac{1}{2}(Ca+Bb)x+\frac{Cb\sin(3dx+3c)}{12d}+\frac{(Ca+Bb)\sin(2dx+2c)}{4d}+\frac{(4Ba+3Cb)\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/2*(C*a + B*b)*x + 1/12*C*b*sin(3*d*x + 3*c)/d + 1/4*(C*a + B*b)*sin(2*d*x
 + 2*c)/d + 1/4*(4*B*a + 3*C*b)*sin(d*x + c)/d
```

3.772 $\int (a+b\cos(c+dx)) \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=52

$$\frac{(aC+bB)\sin(c+dx)}{d}+\frac{1}{2}x(2aB+bC)+\frac{bC\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] ((2*a*B + b*C)*x)/2 + ((b*B + a*C)*Sin[c + d*x])/d + (b*C*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rubi [A] time = 0.065021, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3029, 2734}

$$\frac{(aC+bB)\sin(c+dx)}{d}+\frac{1}{2}x(2aB+bC)+\frac{bC\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x
]
```

```
[Out] ((2*a*B + b*C)*x)/2 + ((b*B + a*C)*Sin[c + d*x])/d + (b*C*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int (a+b\cos(c+dx))(B+C\cos(c+dx))\,dx$$
$$=\frac{1}{2}(2aB+bC)x+\frac{(bB+aC)\sin(c+dx)}{d}+\frac{bC\,\mathrm{co}}{}$$

Mathematica [A] time = 0.0811117, size = 51, normalized size = 0.98

$$\frac{4(aC+bB)\sin(c+dx)+4aBdx+bC\sin(2(c+dx))+2bcC+2bCdx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + 
d*x],x]
```

```
[Out] (2*b*c*C + 4*a*B*d*x + 2*b*C*d*x + 4*(b*B + a*C)*Sin[c + d*x] + b*C*Sin[2*(
c + d*x)])/(4*d)
```

Maple [A] time = 0.035, size = 57, normalized size = 1.1

$$\frac{1}{d}\left(Cb\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+bB\sin(dx+c)+aC\sin(dx+c)+Ba(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(C*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b*B*sin(d*x+c)+a*C*sin(d
*x+c)+B*a*(d*x+c))
```

Maxima [A] time = 1.02688, size = 74, normalized size = 1.42

$$\frac{4(dx+c)Ba+(2dx+2c+\sin(2dx+2c))Cb+4Ca\sin(dx+c)+4Bb\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*C*b + 4*C*a*sin(d*x
 + c) + 4*B*b*sin(d*x + c))/d
```

Fricas [A] time = 1.67619, size = 104, normalized size = 2.

$$\frac{(2\,Ba+Cb)dx+(Cb\cos(dx+c)+2\,Ca+2\,Bb)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="fricas")
```

```
[Out] 1/2*((2*B*a + C*b)*d*x + (C*b*cos(d*x + c) + 2*C*a + 2*B*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B+C\cos(c+dx))\,(a+b\cos(c+dx))\cos(c+dx)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*(a + b*cos(c + d*x))*cos(c + d*x)*sec(c + d*x
), x)
```

Giac [B] time = 1.32393, size = 163, normalized size = 3.13

$$\frac{(2\,Ba+Cb)(dx+c)+\frac{2\bigl(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="giac")
```

```
[Out] 1/2*((2*B*a + C*b)*(d*x + c) + 2*(2*C*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*b*tan(
1/2*d*x + 1/2*c)^3 - C*b*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*tan(1/2*d*x + 1/2*c
) + 2*B*b*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^2)/d
```

3.773 $\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=35

$$x(aC+bB)+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin(c+dx)}{d}$$

```
[Out] (b*B + a*C)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*C*Sin[c + d*x])/d
```

Rubi [A] time = 0.15909, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3029, 2968, 3023, 2735, 3770}

$$x(aC+bB)+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2
,x]
```

```
[Out] (b*B + a*C)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*C*Sin[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx &= \int (a+b\cos(c+dx))(B+C\cos(c+dx))\sec(c\\ &= \int \left(aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+\right.\\ &= \frac{bC\sin(c+dx)}{d}+\int (aB+(bB+aC)\cos(c+d\\ &= (bB+aC)x+\frac{bC\sin(c+dx)}{d}+(aB)\int \sec(c+\\ &= (bB+aC)x+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\sin}{}\end{aligned}$$

Mathematica [A] time = 0.02509, size = 46, normalized size = 1.31

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+aCx+bBx+\frac{bC\sin(c)\cos(dx)}{d}+\frac{bC\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] b*B*x + a*C*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*C*Cos[d*x]*Sin[c])/d + (
b*C*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.046, size = 56, normalized size = 1.6

$$bBx + aCx + \frac{Ba\ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{Bbc}{d} + \frac{Cb\sin(dx+c)}{d} + \frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] b*B*x+a*C*x+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b*c+b*C*sin(d*x+c)/d+1/
d*a*C*c
```

Maxima [A] time = 1.05372, size = 78, normalized size = 2.23

$$\frac{2(dx+c)Ca + 2(dx+c)Bb + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Cb\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + 2*(d*x + c)*B*b + B*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*C*b*sin(d*x + c))/d
```

Fricas [A] time = 1.46613, size = 142, normalized size = 4.06

$$\frac{2(Ca+Bb)dx + Ba\log(\sin(dx+c)+1) - Ba\log(-\sin(dx+c)+1) + 2Cb\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a + B*b)*d*x + B*a*log(sin(d*x + c) + 1) - B*a*log(-sin(d*x + c)
+ 1) + 2*C*b*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.90246, size = 107, normalized size = 3.06

$$\frac{Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ca+Bb)(dx+c)+\frac{2\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="giac")
```

```
[Out] (B*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (C*a + B*b)*(d*x + c) + 2*C*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.774 $\int (a+b\cos(c+dx)) \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=35

$$\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}+bCx$$

```
[Out] b*C*x + ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rubi [A] time = 0.171184, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3029, 2968, 3021, 2735, 3770}

$$\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}+bCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3
,x]
```

```
[Out] b*C*x + ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int (a+b\cos(c+dx))(B+C\cos(c+dx))\sec^2($$
$$= \int \left(aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+\right.$$
$$= \frac{aB\tan(c+dx)}{d} + \int (bB+aC+bC\cos(c+dx$$
$$= bCx + \frac{aB\tan(c+dx)}{d} - (-bB-aC)\int \sec(c+$$
$$= bCx + \frac{(bB+aC)\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\,\text{ta}}{}$$

Mathematica [A] time = 0.0127727, size = 43, normalized size = 1.23

$$\frac{aB\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+bCx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] b*C*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*
B*Tan[c + d*x])/d
```

Maple [A] time = 0.039, size = 65, normalized size = 1.9

$$bCx + \frac{bB\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Ba\tan(dx+c)}{d} + \frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Cbc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] b*C*x+1/d*b*B*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/d+1/d*a*C*ln(sec(d*x
+c)+tan(d*x+c))+1/d*C*b*c
```

Maxima [B] time = 1.01172, size = 99, normalized size = 2.83

$$\frac{2(dx+c)Cb + Ca(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Bb(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2B}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C*b + C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*B*a*tan(d*x + c))
/d
```

Fricas [B] time = 1.39602, size = 225, normalized size = 6.43

$$\frac{2Cbdx\cos(dx+c) + (Ca+Bb)\cos(dx+c)\log(\sin(dx+c)+1) - (Ca+Bb)\cos(dx+c)\log(-\sin(dx+c)+1) + 2Ba}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*b*d*x*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (C*a + B*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*a*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51948, size = 113, normalized size = 3.23

$$\frac{(dx+c)Cb+(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="giac")
```

```
[Out] ((d*x + c)*C*b + (C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (C*a + B*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.775 $\int (a+b\cos(c+dx)) \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=61

$$\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(aB+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((a*B + 2*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((b*B + a*C)*Tan[c + d*x])/d
+ (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.196138, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(aB+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4
,x]
```

```
[Out] ((a*B + 2*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((b*B + a*C)*Tan[c + d*x])/d
+ (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int (a+b\cos(c+dx))(B+C\cos(c+dx))\sec^3(c$$
$$= \int \left(aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+d\right.$$
$$= \frac{aB\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (2(bB+aC)$$
$$= \frac{aB\sec(c+dx)\tan(c+dx)}{2d}+(bB+aC)\int \sec^2($$
$$= \frac{(aB+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\sec(c+d}{2}$$
$$= \frac{(aB+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(bB+aC)\,\mathrm{ta}}{d}$$

Mathematica [A] time = 0.0232534, size = 75, normalized size = 1.23

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tan(c+dx)}{d}+\frac{bB\tan(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (a*B*ArcTanh[Sin[c + d*x]])/(2*d) + (b*C*ArcTanh[Sin[c + d*x]])/d + (b*B*Ta
n[c + d*x])/d + (a*C*Tan[c + d*x])/d + (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.042, size = 86, normalized size = 1.4

$$\frac{Cb\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{bB\tan(dx+c)}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Ba\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba\ln(\sec(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*C*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b*B*tan(d*x+c)+1/d*a*C*tan(d*x+c)+1/2
*a*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02818, size = 128, normalized size = 2.1

$$-\frac{Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,Cb(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))-}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="maxima")
```

```
[Out] -1/4*(B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*C*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*C*a*tan(d*x + c) - 4*B*b*tan(d*x + c))/d
```

Fricas [A] time = 1.3892, size = 247, normalized size = 4.05

$$\frac{(Ba+2\,Cb)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(Ba+2\,Cb)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(Ba+2\left(Ca+Bb\right)\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="fricas")
```

```
[Out] 1/4*((B*a + 2*C*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (B*a + 2*C*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*a + 2*(C*a + B*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52171, size = 204, normalized size = 3.34

$$\frac{(Ba+2Cb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(Ba+2Cb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-2Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-2}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="giac")
```

```
[Out] 1/2*((B*a + 2*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + 2*C*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(B*a*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1
/2*d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1/2*d*x + 1/2*c)
 + 2*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

3.776 $\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=93

$$\frac{(2aB+3bC)\tan(c+dx)}{3d}+\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aC+bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*B + 3*b*C)*Tan[c + d*x])/
(3*d) + ((b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.236785, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3029, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2aB+3bC)\tan(c+dx)}{3d}+\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aC+bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5
,x]
```

```
[Out] ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*B + 3*b*C)*Tan[c + d*x])/
(3*d) + ((b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int (a+b\cos(c+dx))(B+C\cos(c+dx))\sec^4($$
$$= \int \left(aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+\right.$$
$$= \frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (3(bB+aC$$
$$= \frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}+(bB+aC)\int \mathrm{se}$$
$$= \frac{(bB+aC)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aB\sec^2(c}{}$$
$$= \frac{(bB+aC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2aB+3bC)}{3}$$

Mathematica [A] time = 0.261972, size = 67, normalized size = 0.72

$$\frac{3(aC+bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aC+bB)\sec(c+dx)+2aB\tan^2(c+dx)+6aB+6bC\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] (3*(b*B + a*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*B + 6*b*C + 3*(b*B
 + a*C)*Sec[c + d*x] + 2*a*B*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.047, size = 128, normalized size = 1.4

$$\frac{Cb\tan(dx+c)}{d}+\frac{bB\sec(dx+c)\tan(dx+c)}{2d}+\frac{bB\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aC\tan(dx+c)\sec(dx+c)}{2d}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/d*C*b*tan(d*x+c)+1/2/d*b*B*sec(d*x+c)*tan(d*x+c)+1/2/d*b*B*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x
+c))+2/3*a*B*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.1175, size = 171, normalized size = 1.84

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba-3\,Ca\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Bb\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-\cdots}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, al
gorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a - 3*C*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*C*b*tan(d*x + c))/d
```

Fricas [A] time = 1.52108, size = 298, normalized size = 3.2

$$\frac{3\,(Ca+Bb)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(Ca+Bb)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(2\,Ba+3\,Cb)\cos\cdots\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, al
gorithm="fricas")
```

```
[Out] 1/12*(3*(C*a + B*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(C*a + B*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*B*a + 3*C*b)*cos(d*x + c)^2 +
 2*B*a + 3*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50747, size = 284, normalized size = 3.05

$$3\left(Ca+Bb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ca+Bb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(C*a + B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a*tan(1/2*d*x + 1/2*c)^5 - 3*C*a*tan
(1/2*d*x + 1/2*c)^5 - 3*B*b*tan(1/2*d*x + 1/2*c)^5 + 6*C*b*tan(1/2*d*x + 1/
2*c)^5 - 4*B*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*b*tan(1/2*d*x + 1/2*c)^3 + 6*B
*a*tan(1/2*d*x + 1/2*c) + 3*C*a*tan(1/2*d*x + 1/2*c) + 3*B*b*tan(1/2*d*x +
1/2*c) + 6*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.777 $\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{(aC+bB)\tan^3(c+dx)}{3d}+\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(3aB+4bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aB+4bC)\tan(c+dx)\sec(c+$$

```
[Out] ((3*a*B + 4*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((b*B + a*C)*Tan[c + d*x])/
d + ((3*a*B + 4*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((b*B + a*C)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.227083, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{(aC+bB)\tan^3(c+dx)}{3d}+\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(3aB+4bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aB+4bC)\tan(c+dx)\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6
,x]
```

```
[Out] ((3*a*B + 4*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((b*B + a*C)*Tan[c + d*x])/
d + ((3*a*B + 4*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((b*B + a*C)*Tan[c + d*x]^3)/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a+b\cos(c+dx))(B+C\cos(c+dx))\sec^5(c$$

$$= \int \left(aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+d\right.$$

$$= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4(bB+aC)$$

$$= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d}+(bB+aC)\int \sec$$

$$= \frac{(3aB+4bC)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aB\sec^3(}{}$$

$$= \frac{(3aB+4bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(bB+aC)\,\mathrm{t}}{d}$$

Mathematica [A] time = 0.5755, size = 85, normalized size = 0.75

$$\frac{3(3aB+4bC)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(aC+bB)(\cos(2(c+dx))+2)\sec(c+dx)+6aB\sec^2(c+\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```

```
[Out] (3*(3*a*B + 4*b*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*a*B + 12*b*C + 8
*(b*B + a*C)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*a*B*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

Maple [A] time = 0.047, size = 171, normalized size = 1.5

$$\frac{Cb\tan(dx+c)\sec(dx+c)}{2d}+\frac{Cb\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2bB\tan(dx+c)}{3d}+\frac{bB\tan(dx+c)(\sec(dx+c))^2}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/2/d*C*b*tan(d*x+c)*sec(d*x+c)+1/2/d*C*b*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b
*B*tan(d*x+c)+1/3/d*b*B*tan(d*x+c)*sec(d*x+c)^2+2/3/d*a*C*tan(d*x+c)+1/3/d*
a*C*tan(d*x+c)*sec(d*x+c)^2+1/4*a*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*B*sec(d
```

```
*x+c)*tan(d*x+c)/d+3/8/d*B*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.07738, size = 220, normalized size = 1.93

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Bb-3\,Ba\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(s\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, al
gorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*b - 3*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*C*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.57776, size = 352, normalized size = 3.09

$$\frac{3\left(3\,Ba+4\,Cb\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Ba+4\,Cb\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(Ca+B\right.\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, al
gorithm="fricas")
```

```
[Out] 1/48*(3*(3*B*a + 4*C*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*B*a + 4
*C*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(C*a + B*b)*cos(d*x + c
)^3 + 3*(3*B*a + 4*C*b)*cos(d*x + c)^2 + 6*B*a + 8*(C*a + B*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5065, size = 410, normalized size = 3.6

$$3\,(3\,Ba + 4\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(3\,Ba + 4\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,Ba\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{7} - 24\,Ca\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a + 4*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*B*a + 4*C
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*B*a*tan(1/2*d*x + 1/2*c)^7 -
 24*C*a*tan(1/2*d*x + 1/2*c)^7 - 24*B*b*tan(1/2*d*x + 1/2*c)^7 + 12*C*b*tan
(1/2*d*x + 1/2*c)^7 + 9*B*a*tan(1/2*d*x + 1/2*c)^5 + 40*C*a*tan(1/2*d*x + 1
/2*c)^5 + 40*B*b*tan(1/2*d*x + 1/2*c)^5 - 12*C*b*tan(1/2*d*x + 1/2*c)^5 + 9
*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*C*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*b*tan(1/
2*d*x + 1/2*c)^3 - 12*C*b*tan(1/2*d*x + 1/2*c)^3 + 15*B*a*tan(1/2*d*x + 1/2
*c) + 24*C*a*tan(1/2*d*x + 1/2*c) + 24*B*b*tan(1/2*d*x + 1/2*c) + 12*C*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.778 $\int \cos(c+dx)(a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=189

$$\frac{\left(4a^2B+6abC+3b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2B+6abC+3b^2B\right)-\frac{\left(5a(aC+2bB)+4b^2C\right)\sin^3(c+dx)}{15d}+\frac{(}{}$$

```
[Out] ((4*a^2*B + 3*b^2*B + 6*a*b*C)*x)/8 + ((4*b^2*C + 5*a*(2*b*B + a*C))*Sin[c
+ d*x])/(5*d) + ((4*a^2*B + 3*b^2*B + 6*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (b*(5*b*B + 6*a*C)*Cos[c + d*x]^3*Sin[c + d*x])/(20*d) + (b*C*Cos[c
+ d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) - ((4*b^2*C + 5*a*(2*b*B
+ a*C))*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.356517, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3029, 2990, 3023, 2748, 2635, 8, 2633}

$$\frac{\left(4a^2B+6abC+3b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2B+6abC+3b^2B\right)-\frac{\left(5a(aC+2bB)+4b^2C\right)\sin^3(c+dx)}{15d}+\frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)
,x]
```


```
[Out] ((4*a^2*B + 3*b^2*B + 6*a*b*C)*x)/8 + ((4*b^2*C + 5*a*(2*b*B + a*C))*Sin[c
+ d*x])/(5*d) + ((4*a^2*B + 3*b^2*B + 6*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (b*(5*b*B + 6*a*C)*Cos[c + d*x]^3*Sin[c + d*x])/(20*d) + (b*C*Cos[c
+ d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) - ((4*b^2*C + 5*a*(2*b*B
+ a*C))*Sin[c + d*x]^3)/(15*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int \cos^2(c+dx)(a+b\cos(c+dx))^2(B+C\cos$$

$$= \frac{bC\cos^3(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{5d}$$

$$= \frac{b(5bB+6aC)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bC\,c}{}$$

$$= \frac{b(5bB+6aC)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bC\,c}{}$$

$$= \frac{\left(4a^2B+3b^2B+6abC\right)\cos(c+dx)\sin(c+dx}{8d}$$

$$= \frac{1}{8}\left(4a^2B+3b^2B+6abC\right)x + \frac{\left(4b^2C+5a(2bB\right.}{}$$

Mathematica [A] time = 0.455521, size = 146, normalized size = 0.77

$$\frac{60(c+dx)\left(4a^2B+6abC+3b^2B\right)+60\left(6a^2C+12abB+5b^2C\right)\sin(c+dx)+120\left(a^2B+2abC+b^2B\right)\sin(2(c+dx))+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (60*(4*a^2*B + 3*b^2*B + 6*a*b*C)*(c + d*x) + 60*(12*a*b*B + 6*a^2*C + 5*b^
2*C)*Sin[c + d*x] + 120*(a^2*B + b^2*B + 2*a*b*C)*Sin[2*(c + d*x)] + 10*(8*
a*b*B + 4*a^2*C + 5*b^2*C)*Sin[3*(c + d*x)] + 15*b*(b*B + 2*a*C)*Sin[4*(c +
 d*x)] + 6*b^2*C*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.019, size = 184, normalized size = 1.

$$\frac{1}{d}\left(\frac{b^2C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+b^2B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*b^2*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^2*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a*b*C*(1/4*(cos(d*x
```

```
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a*b*B*(2+cos(d*x+c)^2)*
sin(d*x+c)+1/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00866, size = 238, normalized size = 1.26

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2-160\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^2-320\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Bab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 160*(sin(d*x + c)^3 - 3
*sin(d*x + c))*C*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b + 30*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a*b + 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^2 + 32*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^2)/d
```

Fricas [A] time = 1.55587, size = 350, normalized size = 1.85

$$\frac{15\left(4\,Ba^2+6\,Cab+3\,Bb^2\right)dx+\left(24\,Cb^2\cos(dx+c)^4+30\left(2\,Cab+Bb^2\right)\cos(dx+c)^3+80\,Ca^2+160\,Bab+64\,Cb^2+8}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/120*(15*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*d*x + (24*C*b^2*cos(d*x + c)^4 + 30
*(2*C*a*b + B*b^2)*cos(d*x + c)^3 + 80*C*a^2 + 160*B*a*b + 64*C*b^2 + 8*(5*
C*a^2 + 10*B*a*b + 4*C*b^2)*cos(d*x + c)^2 + 15*(4*B*a^2 + 6*C*a*b + 3*B*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 8.6023, size = 462, normalized size = 2.44

$$\begin{cases} \frac{Ba^2x\sin^2(c+dx)}{2}+\frac{Ba^2x\cos^2(c+dx)}{2}+\frac{Ba^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{4Bab\sin^3(c+dx)}{3d}+\frac{2Bab\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3Bb^2x\sin^4(c+dx)}{8}+\frac{3Bb^2x\sin^2}{} \\ x\,(a+b\cos(c))^2\left(B\cos(c)+C\cos^2(c)\right)\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*x*sin(c + d*x)**2/2 + B*a**2*x*cos(c + d*x)**2/2 + B*a**2
*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*B*a*b*sin(c + d*x)**3/(3*d) + 2*B*a*b*
sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b**2*x*sin(c + d*x)**4/8 + 3*B*b**2*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*b**2*x*cos(c + d*x)**4/8 + 3*B*b**2
*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*B*b**2*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + 2*C*a**2*sin(c + d*x)**3/(3*d) + C*a**2*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*C*a*b*x*sin(c + d*x)**4/4 + 3*C*a*b*x*sin(c + d*x)**2*cos(c + d*x)
**2/2 + 3*C*a*b*x*cos(c + d*x)**4/4 + 3*C*a*b*sin(c + d*x)**3*cos(c + d*x)/
(4*d) + 5*C*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*C*b**2*sin(c + d*x)*
*5/(15*d) + 4*C*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*b**2*sin(c +
 d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**2*(B*cos(c) + C*cos(
c)**2)*cos(c), True))
```

Giac [A] time = 1.37336, size = 211, normalized size = 1.12

$$\frac{Cb^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(4\,Ba^2+6\,Cab+3\,Bb^2\right)x+\frac{\left(2\,Cab+Bb^2\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(4\,Ca^2+8\,Bab+5\,Cb^2\right)\sin(3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/80*C*b^2*sin(5*d*x + 5*c)/d + 1/8*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*x + 1/32*
(2*C*a*b + B*b^2)*sin(4*d*x + 4*c)/d + 1/48*(4*C*a^2 + 8*B*a*b + 5*C*b^2)*s
in(3*d*x + 3*c)/d + 1/4*(B*a^2 + 2*C*a*b + B*b^2)*sin(2*d*x + 2*c)/d + 1/8*
(6*C*a^2 + 12*B*a*b + 5*C*b^2)*sin(d*x + c)/d
```

3.779 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=170

$$\frac{\left(4a^2bB+a^3(-C)+8ab^2C+4b^3B\right)\sin(c+dx)}{6bd}+\frac{\left(-2a^2C+8abB+9b^2C\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2C+8abB+\right.$$

```
[Out] ((8*a*b*B + 4*a^2*C + 3*b^2*C)*x)/8 + ((4*a^2*b*B + 4*b^3*B - a^3*C + 8*a*b
^2*C)*Sin[c + d*x])/(6*b*d) + ((8*a*b*B - 2*a^2*C + 9*b^2*C)*Cos[c + d*x]*S
in[c + d*x])/(24*d) + ((4*b*B - a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
12*b*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.193054, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3023, 2753, 2734}

$$\frac{\left(4a^2bB+a^3(-C)+8ab^2C+4b^3B\right)\sin(c+dx)}{6bd}+\frac{\left(-2a^2C+8abB+9b^2C\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2C+8abB+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] ((8*a*b*B + 4*a^2*C + 3*b^2*C)*x)/8 + ((4*a^2*b*B + 4*b^3*B - a^3*C + 8*a*b
^2*C)*Sin[c + d*x])/(6*b*d) + ((8*a*b*B - 2*a^2*C + 9*b^2*C)*Cos[c + d*x]*S
in[c + d*x])/(24*d) + ((4*b*B - a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
12*b*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{C(a+b\cos(c+dx))^3\sin(c+dx)}{4bd} + \frac{\int (a+b\cos(c+dx))^2}{}$$

$$= \frac{(4bB-aC)(a+b\cos(c+dx))^2\sin(c+dx)}{12bd} + \frac{C(a+b\cos}{}$$

$$= \frac{1}{8}\left(8abB+4a^2C+3b^2C\right)x + \frac{\left(4a^2bB+4b^3B-a^3C+8ab\right.}{6bd}$$

Mathematica [A] time = 0.433045, size = 118, normalized size = 0.69

$$\frac{12(c+dx)\left(4a^2C+8abB+3b^2C\right)+24\left(4a^2B+6abC+3b^2B\right)\sin(c+dx)+24\left(a^2C+2abB+b^2C\right)\sin(2(c+dx))+8b}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (12*(8*a*b*B + 4*a^2*C + 3*b^2*C)*(c + d*x) + 24*(4*a^2*B + 3*b^2*B + 6*a*b
*C)*Sin[c + d*x] + 24*(2*a*b*B + a^2*C + b^2*C)*Sin[2*(c + d*x)] + 8*b*(b*B
 + 2*a*C)*Sin[3*(c + d*x)] + 3*b^2*C*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.016, size = 152, normalized size = 0.9

$$\frac{1}{d}\left(b^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2abC(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(b^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*a*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+2*
a*b*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+a^2*B*sin(d*x+c))
```

Maxima [A] time = 1.03815, size = 192, normalized size = 1.13

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^2+48\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab-64\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Cab-32\,(}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 48*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a*b - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a*b - 32*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*B*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*C*b^2 + 96*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.50271, size = 274, normalized size = 1.61

$$\frac{3\left(4\,Ca^2+8\,Bab+3\,Cb^2\right)dx+\left(6\,Cb^2\cos(dx+c)^3+24\,Ba^2+32\,Cab+16\,Bb^2+8\left(2\,Cab+Bb^2\right)\cos(dx+c)^2+3\left(4\,Ca\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(4*C*a^2 + 8*B*a*b + 3*C*b^2)*d*x + (6*C*b^2*cos(d*x + c)^3 + 24*B*
a^2 + 32*C*a*b + 16*B*b^2 + 8*(2*C*a*b + B*b^2)*cos(d*x + c)^2 + 3*(4*C*a^2
 + 8*B*a*b + 3*C*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.94598, size = 340, normalized size = 2.

$$\begin{cases} \frac{Ba^2\sin(c+dx)}{d} + Babx\sin^2(c+dx) + Babx\cos^2(c+dx) + \frac{Bab\sin(c+dx)\cos(c+dx)}{d} + \frac{2Bb^2\sin^3(c+dx)}{3d} + \frac{Bb^2\sin(c+dx)\cos^2(c+dx)}{d} + \\ x(a+b\cos(c))^2\left(B\cos(c) + C\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*sin(c + d*x)/d + B*a*b*x*sin(c + d*x)**2 + B*a*b*x*cos(c
+ d*x)**2 + B*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*B*b**2*sin(c + d*x)**3/(3
*d) + B*b**2*sin(c + d*x)*cos(c + d*x)**2/d + C*a**2*x*sin(c + d*x)**2/2 +
C*a**2*x*cos(c + d*x)**2/2 + C*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*C*a
*b*sin(c + d*x)**3/(3*d) + 2*C*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*b**
2*x*sin(c + d*x)**4/8 + 3*C*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*
b**2*x*cos(c + d*x)**4/8 + 3*C*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
C*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**2*
(B*cos(c) + C*cos(c)**2), True))
```

Giac [A] time = 1.52493, size = 167, normalized size = 0.98

$$\frac{Cb^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(4\,Ca^2 + 8\,Bab + 3\,Cb^2\right)x + \frac{\left(2\,Cab + Bb^2\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(Ca^2 + 2\,Bab + Cb^2\right)\sin(2\,dx+2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/32*C*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*C*a^2 + 8*B*a*b + 3*C*b^2)*x + 1/12*
(2*C*a*b + B*b^2)*sin(3*d*x + 3*c)/d + 1/4*(C*a^2 + 2*B*a*b + C*b^2)*sin(2*
d*x + 2*c)/d + 1/4*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*sin(d*x + c)/d
```

3.780 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=107

$$\frac{2\left(a^2C+3abB+b^2C\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2B+2abC+b^2B\right)+\frac{b(2aC+3bB)\sin(c+dx)\cos(c+dx)}{6d}+\frac{C\sin(c+dx)(a$$

```
[Out] ((2*a^2*B + b^2*B + 2*a*b*C)*x)/2 + (2*(3*a*b*B + a^2*C + b^2*C)*Sin[c + d*
x])/(3*d) + (b*(3*b*B + 2*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.159257, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3029, 2753, 2734}

$$\frac{2\left(a^2C+3abB+b^2C\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2B+2abC+b^2B\right)+\frac{b(2aC+3bB)\sin(c+dx)\cos(c+dx)}{6d}+\frac{C\sin(c+dx)(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
,x]
```


```
[Out] ((2*a^2*B + b^2*B + 2*a*b*C)*x)/2 + (2*(3*a*b*B + a^2*C + b^2*C)*Sin[c + d*
x])/(3*d) + (b*(3*b*B + 2*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int (a+b\cos(c+dx))^2(B+C\cos(c+dx))\,dx$$

$$=\frac{C(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+b$$

$$=\frac{1}{2}\left(2a^2B+b^2B+2abC\right)x+\frac{2\left(3abB+a^2C+b\right.}{3d}$$

Mathematica [A] time = 0.216213, size = 90, normalized size = 0.84

$$\frac{6(c+dx)\left(2a^2B+2abC+b^2B\right)+3\left(4a^2C+8abB+3b^2C\right)\sin(c+dx)+3b(2aC+bB)\sin(2(c+dx))+b^2C\sin(3(c+d}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (6*(2*a^2*B + b^2*B + 2*a*b*C)*(c + d*x) + 3*(8*a*b*B + 4*a^2*C + 3*b^2*C)*
Sin[c + d*x] + 3*b*(b*B + 2*a*C)*Sin[2*(c + d*x)] + b^2*C*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A] time = 0.039, size = 114, normalized size = 1.1

$$\frac{1}{d}\left(\frac{b^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+b^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,abC\,(1/2\,\cos(dx+c)\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(1/3*b^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*a*b*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*B*s
in(d*x+c)+a^2*C*sin(d*x+c)+a^2*B*(d*x+c))
```

Maxima [A] time = 1.035, size = 146, normalized size = 1.36

$$\frac{12\,(dx+c)Ba^2+6\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Cab+3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bb^2-4\left(\sin{(dx+c)}^3-3\,\sin{(dx+}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b + 3*(2*
d*x + 2*c + sin(2*d*x + 2*c))*B*b^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C
*b^2 + 12*C*a^2*sin(d*x + c) + 24*B*a*b*sin(d*x + c))/d
```

Fricas [A] time = 1.4496, size = 201, normalized size = 1.88

$$\frac{3\left(2\,Ba^2+2\,Cab+Bb^2\right)dx+\left(2\,Cb^2\cos{(dx+c)}^2+6\,Ca^2+12\,Bab+4\,Cb^2+3\left(2\,Cab+Bb^2\right)\cos{(dx+c)}\right)\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] 1/6*(3*(2*B*a^2 + 2*C*a*b + B*b^2)*d*x + (2*C*b^2*cos(d*x + c)^2 + 6*C*a^2
+ 12*B*a*b + 4*C*b^2 + 3*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56496, size = 343, normalized size = 3.21

$$3\left(2\,Ba^2+2\,Cab+Bb^2\right)(dx+c)+\frac{2\left(6\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Cb^2\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(2*B*a^2 + 2*C*a*b + B*b^2)*(d*x + c) + 2*(6*C*a^2*tan(1/2*d*x + 1/2
*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*b*tan(1/2*d*x + 1/2*c)^5 -
3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*
tan(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^2*tan(1/2*
d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*
c) + 6*C*a*b*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.781 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}x\left(2a^2C+4abB+b^2C\right)+\frac{a^2B\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aC+2bB)\sin(c+dx)}{2d}+\frac{bC\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

```
[Out] ((4*a*b*B + 2*a^2*C + b^2*C)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (b*(2
*b*B + 3*a*C)*Sin[c + d*x])/(2*d) + (b*C*(a + b*Cos[c + d*x])*Sin[c + d*x])
/(2*d)
```

Rubi [A] time = 0.244916, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3029, 2990, 3023, 2735, 3770}

$$\frac{1}{2}x\left(2a^2C+4abB+b^2C\right)+\frac{a^2B\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aC+2bB)\sin(c+dx)}{2d}+\frac{bC\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^2,x]
```


```
[Out] ((4*a*b*B + 2*a^2*C + b^2*C)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (b*(2
*b*B + 3*a*C)*Sin[c + d*x])/(2*d) + (b*C*(a + b*Cos[c + d*x])*Sin[c + d*x])
/(2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int (a+b\cos(c+dx))^2(B+C\cos(c+dx))\,\mathrm{sec}$$

$$=\frac{bC(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int\left(2a\right.$$

$$=\frac{b(2bB+3aC)\sin(c+dx)}{2d}+\frac{bC(a+b\cos(c+}{2}$$

$$=\frac{1}{2}\left(4abB+2a^2C+b^2C\right)x+\frac{b(2bB+3aC)\sin}{2d}$$

$$=\frac{1}{2}\left(4abB+2a^2C+b^2C\right)x+\frac{a^2B\tanh^{-1}(\sin(}{d}$$

Mathematica [A] time = 0.219161, size = 120, normalized size = 1.4

$$\frac{2(c+dx)\left(2a^2C+4abB+b^2C\right)-4a^2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2B\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] (2*(4*a*b*B + 2*a^2*C + b^2*C)*(c + d*x) - 4*a^2*B*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*a^2*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*b*(
b*B + 2*a*C)*Sin[c + d*x] + b^2*C*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.05, size = 120, normalized size = 1.4

$$\frac{b^2C\cos(dx+c)\sin(dx+c)}{2d}+\frac{b^2Cx}{2}+\frac{b^2Cc}{2d}+\frac{b^2B\sin(dx+c)}{d}+2\,\frac{abC\sin(dx+c)}{d}+2\,abBx+2\,\frac{Babc}{d}+a^2Cx+\frac{Ca^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/2/d*b^2*C*cos(d*x+c)*sin(d*x+c)+1/2*b^2*C*x+1/2/d*b^2*C*c+1/d*b^2*B*sin(d
*x+c)+2/d*a*b*C*sin(d*x+c)+2*a*b*B*x+2/d*B*a*b*c+a^2*C*x+1/d*a^2*C*c+1/d*a^
2*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.052, size = 134, normalized size = 1.56

$$\frac{4\,(dx+c)Ca^2+8\,(dx+c)Bab+(2\,dx+2\,c+\sin(2\,dx+2\,c))Cb^2+2\,Ba^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a^2 + 8*(d*x + c)*B*a*b + (2*d*x + 2*c + sin(2*d*x + 2*c
))*C*b^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*C*a*
```

```
b*sin(d*x + c) + 4*B*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.47149, size = 213, normalized size = 2.48

$$\frac{Ba^2\log(\sin(dx+c)+1)-Ba^2\log(-\sin(dx+c)+1)+\left(2\,Ca^2+4\,Bab+Cb^2\right)dx+\left(Cb^2\cos(dx+c)+4\,Cab+2\,Bb^2\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*log(sin(d*x + c) + 1) - B*a^2*log(-sin(d*x + c) + 1) + (2*C*a^2
+ 4*B*a*b + C*b^2)*d*x + (C*b^2*cos(d*x + c) + 4*C*a*b + 2*B*b^2)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61288, size = 240, normalized size = 2.79

$$\frac{2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(2\,Ca^2+4\,Bab+Cb^2\right)(dx+c)+\frac{2\left(4\,Cab\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] 1/2*(2*B*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (2*C*a^2 + 4*B*a*b + C*b^2)*(d*x + c) + 2*(4*C*a*b*tan(
1/2*d*x + 1/2*c)^3 + 2*B*b^2*tan(1/2*d*x + 1/2*c)^3 - C*b^2*tan(1/2*d*x + 1
/2*c)^3 + 4*C*a*b*tan(1/2*d*x + 1/2*c) + 2*B*b^2*tan(1/2*d*x + 1/2*c) + C*b
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.782 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=60

$$\frac{a^2B\tan(c+dx)}{d}+\frac{a(aC+2bB)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aC+bB)+\frac{b^2C\sin(c+dx)}{d}$$

```
[Out] b*(b*B + 2*a*C)*x + (a*(2*b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (b^2*C*Sin[
c + d*x])/d + (a^2*B*Tan[c + d*x])/d
```

Rubi [A] time = 0.242722, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3029, 2988, 3023, 2735, 3770}

$$\frac{a^2B\tan(c+dx)}{d}+\frac{a(aC+2bB)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aC+bB)+\frac{b^2C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^3,x]
```

```
[Out] b*(b*B + 2*a*C)*x + (a*(2*b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (b^2*C*Sin[
c + d*x])/d + (a^2*B*Tan[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
```

```
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int (a+b\cos(c+dx))^2(B+C\cos(c+dx))\sec^2($$

$$=\frac{a^2B\tan(c+dx)}{d}-\int\left(-a(2bB+aC)-b(bB+\right.$$

$$=\frac{b^2C\sin(c+dx)}{d}+\frac{a^2B\tan(c+dx)}{d}-\int(-a(2b$$

$$=b(bB+2aC)x+\frac{b^2C\sin(c+dx)}{d}+\frac{a^2B\tan(c+}{d}$$

$$=b(bB+2aC)x+\frac{a(2bB+aC)\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A] time = 0.473368, size = 109, normalized size = 1.82

$$\frac{a^2B\tan(c+dx)+b(c+dx)(2aC+bB)-a(aC+2bB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(aC+2bB)\log\left(\sin\left(\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] (b*(b*B + 2*a*C)*(c + d*x) - a*(2*b*B + a*C)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + a*(2*b*B + a*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2
*C*Sin[c + d*x] + a^2*B*Tan[c + d*x])/d
```

Maple [A] time = 0.046, size = 104, normalized size = 1.7

$$b^2Bx+2\,abCx+2\,\frac{abB\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2B\tan(dx+c)}{d}+\frac{Bb^2c}{d}+\frac{a^2C\ln(\sec(dx+c)+\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] b^2*B*x+2*a*b*C*x+2/d*a*b*B*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*tan(d*x+c)/d+1/
d*B*b^2*c+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+b^2*C*sin(d*x+c)/d+2/d*a*b*C*
c
```

Maxima [A] time = 1.10479, size = 139, normalized size = 2.32

$$\frac{4\,(dx+c)Cab+2\,(dx+c)Bb^2+Ca^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,Bab(\log(\sin(dx+c)+1)-\log(}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*C*a*b + 2*(d*x + c)*B*b^2 + C*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*B*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
```

```
 - 1)) + 2*C*b^2*sin(d*x + c) + 2*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.59998, size = 294, normalized size = 4.9

$$\frac{2\left(2\,Cab+Bb^{2}\right)dx\cos\left(dx+c\right)+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\log\left(-\sin\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] 1/2*(2*(2*C*a*b + B*b^2)*d*x*cos(d*x + c) + (C*a^2 + 2*B*a*b)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (C*a^2 + 2*B*a*b)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(C*b^2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.69251, size = 205, normalized size = 3.42

$$\frac{\left(2\,Cab+Bb^{2}\right)\left(dx+c\right)+\left(Ca^{2}+2\,Bab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(Ca^{2}+2\,Bab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Ba^{2}\tan\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

[Out] ((2*C*a*b + B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (C*a^2 + 2*B*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(B*a^2*tan(1/2*d*x + 1/2*c)^3 - C*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a^2*tan(1/2*d*x + 1/2*c) + C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d

3.783 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{\left(a^2B+4abC+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2B\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aC+2bB)\tan(c+dx)}{d}+b^2Cx$$

```
[Out] b^2*C*x + ((a^2*B + 2*b^2*B + 4*a*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2
*b*B + a*C)*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.279569, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3029, 2988, 3021, 2735, 3770}

$$\frac{\left(a^2B+4abC+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2B\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aC+2bB)\tan(c+dx)}{d}+b^2Cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^4,x]
```

```
[Out] b^2*C*x + ((a^2*B + 2*b^2*B + 4*a*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2
*b*B + a*C)*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
```

```
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int (a+b\cos(c+dx))^2(B+C\cos(c+dx))\sec$$

$$= \frac{a^2B\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int \Big(-2a(2bB$$

$$= \frac{a(2bB+aC)\tan(c+dx)}{d} + \frac{a^2B\sec(c+dx)\,\mathrm{t}}{2d}$$

$$= b^2Cx + \frac{a(2bB+aC)\tan(c+dx)}{d} + \frac{a^2B\sec(c}{}$$

$$= b^2Cx + \frac{\left(a^2B+2b^2B+4abC\right)\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [A] time = 0.257165, size = 67, normalized size = 0.84

$$\frac{\left(a^2B+4abC+2b^2B\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)(aB\sec(c+dx)+2aC+4bB)+2b^2Cdx}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```

```
[Out] (2*b^2*C*d*x + (a^2*B + 2*b^2*B + 4*a*b*C)*ArcTanh[Sin[c + d*x]] + a*(4*b*B
 + 2*a*C + a*B*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.051, size = 133, normalized size = 1.7

$$b^2Cx+\frac{Cb^2c}{d}+\frac{b^2B\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{abC\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{abB\tan(dx+c)}{d}+\frac{a^2C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] b^2*C*x+1/d*b^2*C*c+1/d*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*C*ln(sec(d*
x+c)+tan(d*x+c))+2/d*a*b*B*tan(d*x+c)+1/d*a^2*C*tan(d*x+c)+1/2*a^2*B*sec(d*
x+c)*tan(d*x+c)/d+1/2/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.14292, size = 189, normalized size = 2.36

$$\frac{4(dx+c)Cb^2-Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Cab(\log(\sin(dx+c)+1)-\log(\sin}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*b^2 - B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*C*a*b*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
```

```
) - 1)) + 4*C*a^2*tan(d*x + c) + 8*B*a*b*tan(d*x + c))/d
```

Fricas [A] time = 1.50077, size = 335, normalized size = 4.19

$$\frac{4 Cb^2dx\cos(dx+c)^2 + (Ba^2+4Cab+2Bb^2)\cos(dx+c)^2\log(\sin(dx+c)+1) - (Ba^2+4Cab+2Bb^2)\cos(dx+c)^2}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] 1/4*(4*C*b^2*d*x*cos(d*x + c)^2 + (B*a^2 + 4*C*a*b + 2*B*b^2)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (B*a^2 + 4*C*a*b + 2*B*b^2)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) + 2*(B*a^2 + 2*(C*a^2 + 2*B*a*b)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47228, size = 257, normalized size = 3.21

$$\frac{2(dx+c)Cb^2 + (Ba^2+4Cab+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - (Ba^2+4Cab+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*C*b^2 + (B*a^2 + 4*C*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (B*a^2 + 4*C*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + 2*(B*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*B
*a*b*tan(1/2*d*x + 1/2*c)^3 + B*a^2*tan(1/2*d*x + 1/2*c) + 2*C*a^2*tan(1/2*
d*x + 1/2*c) + 4*B*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

3.784 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{\left(2a^2B+6abC+3b^2B\right)\tan(c+dx)}{3d}+\frac{\left(a^2C+2abB+2b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2B\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a(a$$

```
[Out] ((2*a*b*B + a^2*C + 2*b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a^2*B + 3*b
^2*B + 6*a*b*C)*Tan[c + d*x])/(3*d) + (a*(2*b*B + a*C)*Sec[c + d*x]*Tan[c +
 d*x])/(2*d) + (a^2*B*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.360039, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2988, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(2a^2B+6abC+3b^2B\right)\tan(c+dx)}{3d}+\frac{\left(a^2C+2abB+2b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2B\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^5,x]
```


```
[Out] ((2*a*b*B + a^2*C + 2*b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a^2*B + 3*b
^2*B + 6*a*b*C)*Tan[c + d*x])/(3*d) + (a*(2*b*B + a*C)*Sec[c + d*x]*Tan[c +
 d*x])/(2*d) + (a^2*B*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
```

```
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int (a+b\cos(c+dx))^2(B+C\cos(c+dx))\sec$$
$$= \frac{a^2B\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}\int \left(-3a(2b\right.$$
$$= \frac{a(2bB+aC)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2B\,\mathrm{s}}{}$$
$$= \frac{a(2bB+aC)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2B\,\mathrm{s}}{}$$
$$= \frac{\left(2abB+a^2C+2b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d} +$$
$$= \frac{\left(2abB+a^2C+2b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d} +$$

Mathematica [A] time = 0.447025, size = 92, normalized size = 0.79

$$\frac{3\left(a^2C+2abB+2b^2C\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2\left(a^2B\tan^2(c+dx)+3a^2B+6abC+3b^2B\right)+3a(aC+2bB\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^5,x]
```

```
[Out] (3*(2*a*b*B + a^2*C + 2*b^2*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(2
*b*B + a*C)*Sec[c + d*x] + 2*(3*a^2*B + 3*b^2*B + 6*a*b*C + a^2*B*Tan[c + d
*x]^2)))/(6*d)
```

Maple [A] time = 0.052, size = 174, normalized size = 1.5

$$\frac{b^2C\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^2B\tan(dx+c)}{d} + 2\,\frac{abC\tan(dx+c)}{d} + \frac{abB\sec(dx+c)\tan(dx+c)}{d} + \frac{abB\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*B*tan(d*x+c)+2/d*a*b*C*tan(d*x+
c)+1/d*a*b*B*sec(d*x+c)*tan(d*x+c)+1/d*a*b*B*ln(sec(d*x+c)+tan(d*x+c))+1/2/
```

```
d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/3*a^2
*B*tan(d*x+c)/d+1/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.00288, size = 232, normalized size = 2.

$$\frac{4\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ba^2 - 3Ca^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 6Bab\left(\frac{2\sin(\ldots}{\sin(dx\ldots}\right.\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 - 3*C*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*B*
a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*C*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
24*C*a*b*tan(d*x + c) + 12*B*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.3828, size = 371, normalized size = 3.2

$$\frac{3\left(Ca^2 + 2Bab + 2Cb^2\right)\cos(dx+c)^3\log(\sin(dx+c)+1) - 3\left(Ca^2 + 2Bab + 2Cb^2\right)\cos(dx+c)^3\log(-\sin(dx+c)+1\ldots}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="fricas")
```

```
[Out] 1/12*(3*(C*a^2 + 2*B*a*b + 2*C*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(C*a^2 + 2*B*a*b + 2*C*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
B*a^2 + 2*(2*B*a^2 + 6*C*a*b + 3*B*b^2)*cos(d*x + c)^2 + 3*(C*a^2 + 2*B*a*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.79041, size = 397, normalized size = 3.42

$$3\left(Ca^2 + 2\,Bab + 2\,Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(Ca^2 + 2\,Bab + 2\,Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(6\,Ba^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(C*a^2 + 2*B*a*b + 2*C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
C*a^2 + 2*B*a*b + 2*C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a^2*
tan(1/2*d*x + 1/2*c)^5 - 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d
*x + 1/2*c)^5 + 12*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2
*c)^5 - 4*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*C*a*b*tan(1/2*d*x + 1/2*c)^3 -
12*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) + 3*C*a^2*ta
n(1/2*d*x + 1/2*c) + 6*B*a*b*tan(1/2*d*x + 1/2*c) + 12*C*a*b*tan(1/2*d*x +
1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.785 $\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{(2a^2C+4abB+3b^2C)\tan(c+dx)}{3d}+\frac{(3a^2B+8abC+4b^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2B+8abC+4b^2B)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] ((3*a^2*B + 4*b^2*B + 8*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*b*B + 2
*a^2*C + 3*b^2*C)*Tan[c + d*x])/(3*d) + ((3*a^2*B + 4*b^2*B + 8*a*b*C)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*(2*b*B + a*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d) + (a^2*B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.376542, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2988, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2a^2C+4abB+3b^2C)\tan(c+dx)}{3d}+\frac{(3a^2B+8abC+4b^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2B+8abC+4b^2B)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^6,x]
```


```
[Out] ((3*a^2*B + 4*b^2*B + 8*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*b*B + 2
*a^2*C + 3*b^2*C)*Tan[c + d*x])/(3*d) + ((3*a^2*B + 4*b^2*B + 8*a*b*C)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*(2*b*B + a*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d) + (a^2*B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a+b\cos(c+dx))^2(B+C\cos(c+dx))\sec^5($$

$$= \frac{a^2B\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}\int \left(-4a(2bB\right.$$

$$= \frac{a(2bB+aC)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2B\,\text{se}}{}$$

$$= \frac{a(2bB+aC)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2B\,\text{se}}{}$$

$$= \frac{\left(3a^2B+4b^2B+8abC\right)\sec(c+dx)\tan(c+dx)}{8d}$$

$$= \frac{\left(3a^2B+4b^2B+8abC\right)\tanh^{-1}(\sin(c+dx))}{8d} +$$

Mathematica [A] time = 0.719009, size = 120, normalized size = 0.77

$$\frac{3\left(3a^2B+8abC+4b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(3a^2B+8abC+4b^2B\right)\sec(c+dx)+24\left(a^2C+2abB+b^2\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^6,x]
```

```
[Out] (3*(3*a^2*B + 4*b^2*B + 8*a*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
2*a*b*B + a^2*C + b^2*C) + 3*(3*a^2*B + 4*b^2*B + 8*a*b*C)*Sec[c + d*x] + 6
*a^2*B*Sec[c + d*x]^3 + 8*a*(2*b*B + a*C)*Tan[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.054, size = 241, normalized size = 1.5

$$\frac{b^2C\tan(dx+c)}{d} + \frac{b^2B\sec(dx+c)\tan(dx+c)}{2d} + \frac{b^2B\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{abC\tan(dx+c)\sec(dx+c)}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/d*b^2*C*tan(d*x+c)+1/2/d*b^2*B*sec(d*x+c)*tan(d*x+c)+1/2/d*b^2*B*ln(sec(d
*x+c)+tan(d*x+c))+1/d*a*b*C*tan(d*x+c)*sec(d*x+c)+1/d*a*b*C*ln(sec(d*x+c)+t
an(d*x+c))+4/3/d*a*b*B*tan(d*x+c)+2/3/d*a*b*B*tan(d*x+c)*sec(d*x+c)^2+2/3/d
*a^2*C*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/4*a^2*B*sec(d*x+c)^
3*tan(d*x+c)/d+3/8*a^2*B*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^2*B*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 1.04523, size = 308, normalized size = 1.97

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca^{2}+32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Bab-3\,Ba^{2}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\text{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a*b - 3*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*C*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*B*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*C*b^2*tan(d*x +
 c))/d
```

Fricas [A] time = 1.45799, size = 443, normalized size = 2.84

$$\frac{3\left(3\,Ba^{2}+8\,Cab+4\,Bb^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Ba^{2}+8\,Cab+4\,Bb^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(3*B*a^2 + 8*C*a*b + 4*B*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(3*B*a^2 + 8*C*a*b + 4*B*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
```

```
*(8*(2*C*a^2 + 4*B*a*b + 3*C*b^2)*cos(d*x + c)^3 + 6*B*a^2 + 3*(3*B*a^2 + 8
*C*a*b + 4*B*b^2)*cos(d*x + c)^2 + 8*(C*a^2 + 2*B*a*b)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.75789, size = 645, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a^2 + 8*C*a*b + 4*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(3*B*a^2 + 8*C*a*b + 4*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*
B*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*B*a*b*t
an(1/2*d*x + 1/2*c)^7 + 24*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b^2*tan(1/2*
d*x + 1/2*c)^7 - 24*C*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*tan(1/2*d*x + 1/
2*c)^5 + 40*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 80*B*a*b*tan(1/2*d*x + 1/2*c)^5
- 24*C*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*C*
b^2*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*C*a^2*tan(
1/2*d*x + 1/2*c)^3 - 80*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*C*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*C*b^2*tan(1/2*d*x + 1/2*
c)^3 + 15*B*a^2*tan(1/2*d*x + 1/2*c) + 24*C*a^2*tan(1/2*d*x + 1/2*c) + 48*B
*a*b*tan(1/2*d*x + 1/2*c) + 24*C*a*b*tan(1/2*d*x + 1/2*c) + 12*B*b^2*tan(1/
2*d*x + 1/2*c) + 24*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

3.786 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=243

$$\frac{\left(52a^2b^2C+15a^3bB-3a^4C+60ab^3B+16b^4C\right)\sin(c+dx)}{30bd}+\frac{\left(-3a^2C+15abB+16b^2C\right)\sin(c+dx)(a+b\cos(c+dx))}{60bd}$$

```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*x)/8 + ((15*a^3*b*B + 60*a*b^
3*B - 3*a^4*C + 52*a^2*b^2*C + 16*b^4*C)*Sin[c + d*x])/(30*b*d) + ((30*a^2*
b*B + 45*b^3*B - 6*a^3*C + 71*a*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(120*d) +
 ((15*a*b*B - 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*
b*d) + ((5*b*B - a*C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (C*(a
 + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.293494, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3023, 2753, 2734}

$$\frac{\left(52a^2b^2C+15a^3bB-3a^4C+60ab^3B+16b^4C\right)\sin(c+dx)}{30bd}+\frac{\left(-3a^2C+15abB+16b^2C\right)\sin(c+dx)(a+b\cos(c+dx))}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*x)/8 + ((15*a^3*b*B + 60*a*b^
3*B - 3*a^4*C + 52*a^2*b^2*C + 16*b^4*C)*Sin[c + d*x])/(30*b*d) + ((30*a^2*
b*B + 45*b^3*B - 6*a^3*C + 71*a*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(120*d) +
 ((15*a*b*B - 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*
b*d) + ((5*b*B - a*C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (C*(a
 + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} + \frac{\int (a+b\cos(c+dx))^3(4\cdots}{\cdots}$$

$$= \frac{(5bB-aC)(a+b\cos(c+dx))^3\sin(c+dx)}{20bd} + \frac{C(a+b\cos(c\cdots}{\cdots}$$

$$= \frac{\left(15abB-3a^2C+16b^2C\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60bd} + \cdots$$

$$= \frac{1}{8}\left(12a^2bB+3b^3B+4a^3C+9ab^2C\right)x + \frac{\left(15a^3bB+60ab^3B\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.662376, size = 176, normalized size = 0.72

$$\frac{60(c+dx)\left(12a^2bB+4a^3C+9ab^2C+3b^3B\right)+10b\left(12a^2C+12abB+5b^2C\right)\sin(3(c+dx))+60\left(18a^2bC+8a^3B+18ab\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (60*(12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*(c + d*x) + 60*(8*a^3*B +
18*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*Sin[c + d*x] + 120*(3*a^2*b*B + b^3*B +
a^3*C + 3*a*b^2*C)*Sin[2*(c + d*x)] + 10*b*(12*a*b*B + 12*a^2*C + 5*b^2*C)*
Sin[3*(c + d*x)] + 15*b^2*(b*B + 3*a*C)*Sin[4*(c + d*x)] + 6*b^3*C*Sin[5*(c
```

```
 + d*x)])/(480*d)
```

Maple [A] time = 0.016, size = 227, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Cb^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+b^3B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^3*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*C*a*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*b^2*B*(2+cos(d*x+c)^2)*
sin(d*x+c)+a^2*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*B*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3
*B*sin(d*x+c))
```

Maxima [A] time = 1.02662, size = 293, normalized size = 1.21

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^3+360\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b-480\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 360*(2*d*x + 2*c + sin(
2*d*x + 2*c))*B*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b - 480
*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^2 + 45*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*C*a*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c)
 + 8*sin(2*d*x + 2*c))*B*b^3 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 1
5*sin(d*x + c))*C*b^3 + 480*B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.52349, size = 423, normalized size = 1.74

$$\frac{15\left(4\,Ca^3+12\,Ba^2b+9\,Cab^2+3\,Bb^3\right)dx+\left(24\,Cb^3\cos(dx+c)^4+120\,Ba^3+240\,Ca^2b+240\,Bab^2+64\,Cb^3+30\left(3\,C\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B*b^3)*d*x + (24*C*b^3*cos(
d*x + c)^4 + 120*B*a^3 + 240*C*a^2*b + 240*B*a*b^2 + 64*C*b^3 + 30*(3*C*a*b
^2 + B*b^3)*cos(d*x + c)^3 + 8*(15*C*a^2*b + 15*B*a*b^2 + 4*C*b^3)*cos(d*x
+ c)^2 + 15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [A] time = 10.0765, size = 552, normalized size = 2.27

$$\begin{cases} \frac{Ba^3\sin(c+dx)}{d} + \frac{3Ba^2bx\sin^2(c+dx)}{2} + \frac{3Ba^2bx\cos^2(c+dx)}{2} + \frac{3Ba^2b\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Bab^2\sin^3(c+dx)}{d} + \frac{3Bab^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Bb^3 \\ x(a+b\cos(c))^3\left(B\cos(c)+C\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**3*sin(c + d*x)/d + 3*B*a**2*b*x*sin(c + d*x)**2/2 + 3*B*a**
2*b*x*cos(c + d*x)**2/2 + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*
a*b**2*sin(c + d*x)**3/d + 3*B*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*
b**3*x*sin(c + d*x)**4/8 + 3*B*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3
*B*b**3*x*cos(c + d*x)**4/8 + 3*B*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) +
 5*B*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a**3*x*sin(c + d*x)**2/2 +
 C*a**3*x*cos(c + d*x)**2/2 + C*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*
a**2*b*sin(c + d*x)**3/d + 3*C*a**2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*C*
a*b**2*x*sin(c + d*x)**4/8 + 9*C*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4
 + 9*C*a*b**2*x*cos(c + d*x)**4/8 + 9*C*a*b**2*sin(c + d*x)**3*cos(c + d*x)
/(8*d) + 15*C*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*C*b**3*sin(c +
d*x)**5/(15*d) + 4*C*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*b**3*si
n(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**3*(B*cos(c) + C
*cos(c)**2), True))
```

Giac [A] time = 1.62097, size = 254, normalized size = 1.05

$$\frac{Cb^3\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(4\,Ca^3 + 12\,Ba^2b + 9\,Cab^2 + 3\,Bb^3\right)x + \frac{\left(3\,Cab^2 + Bb^3\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{\left(12\,Ca^2b + 12\,Bab^2 + 5 \right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/80*C*b^3*sin(5*d*x + 5*c)/d + 1/8*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B
*b^3)*x + 1/32*(3*C*a*b^2 + B*b^3)*sin(4*d*x + 4*c)/d + 1/48*(12*C*a^2*b +
12*B*a*b^2 + 5*C*b^3)*sin(3*d*x + 3*c)/d + 1/4*(C*a^3 + 3*B*a^2*b + 3*C*a*b
^2 + B*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*B*a^3 + 18*C*a^2*b + 18*B*a*b^2 + 5
*C*b^3)*sin(d*x + c)/d
```

3.787 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=171

$$\frac{\left(16a^2bB+3a^3C+12ab^2C+4b^3B\right)\sin(c+dx)}{6d}+\frac{b\left(6a^2C+20abB+9b^2C\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(12a^2bC+8a^3\right.$$

```
[Out] ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*x)/8 + ((16*a^2*b*B + 4*b^3*
B + 3*a^3*C + 12*a*b^2*C)*Sin[c + d*x])/(6*d) + (b*(20*a*b*B + 6*a^2*C + 9*
b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*b*B + 3*a*C)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(12*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d
)
```

Rubi [A] time = 0.261676, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3029, 2753, 2734}

$$\frac{\left(16a^2bB+3a^3C+12ab^2C+4b^3B\right)\sin(c+dx)}{6d}+\frac{b\left(6a^2C+20abB+9b^2C\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(12a^2bC+8a^3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
,x]
```


```
[Out] ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*x)/8 + ((16*a^2*b*B + 4*b^3*
B + 3*a^3*C + 12*a*b^2*C)*Sin[c + d*x])/(6*d) + (b*(20*a*b*B + 6*a^2*C + 9*
b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*b*B + 3*a*C)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(12*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d
)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\,dx$$
$$=\frac{C(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a+b$$
$$=\frac{(4bB+3aC)(a+b\cos(c+dx))^2\sin(c+dx)}{12d}+$$
$$=\frac{1}{8}\left(8a^3B+12ab^2B+12a^2bC+3b^3C\right)x+\frac{(16a}{}$$

Mathematica [A] time = 0.400583, size = 140, normalized size = 0.82

$$\frac{12(c+dx)\left(12a^2bC+8a^3B+12ab^2B+3b^3C\right)+24b\left(3a^2C+3abB+b^2C\right)\sin(2(c+dx))+24\left(12a^2bB+4a^3C+9ab^2C\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (12*(8*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*(c + d*x) + 24*(12*a^2*b*
B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*Sin[c + d*x] + 24*b*(3*a*b*B + 3*a^2*C +
 b^2*C)*Sin[2*(c + d*x)] + 8*b^2*(b*B + 3*a*C)*Sin[3*(c + d*x)] + 3*b^3*C*S
in[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.045, size = 180, normalized size = 1.1

$$\frac{1}{d}\left(Cb^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^3B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Cab^2\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*(C*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*b^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+C*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a*
b^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*C*(1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*B*sin(d*x+c)+a^3*C*sin(d*x+c)+a^3*B*(d*x+c
))
```

Maxima [A] time = 1.04951, size = 231, normalized size = 1.35

$$\frac{96(dx+c)Ba^3+72(2dx+2c+\sin(2dx+2c))Ca^2b+72(2dx+2c+\sin(2dx+2c))Bab^2-96\left(\sin(dx+c)^3-3\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*B*a^3 + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2*b + 72
*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^2 - 96*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*C*a*b^2 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b^3 + 3*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*b^3 + 96*C*a^3*sin(d*x + c) +
 288*B*a^2*b*sin(d*x + c))/d
```

Fricas [A] time = 1.48211, size = 321, normalized size = 1.88

$$\frac{3\left(8Ba^3+12Ca^2b+12Bab^2+3Cb^3\right)dx+\left(6Cb^3\cos(dx+c)^3+24Ca^3+72Ba^2b+48Cab^2+16Bb^3+8\left(3Cab^2+Bb\right.\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] 1/24*(3*(8*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 3*C*b^3)*d*x + (6*C*b^3*cos(d*
x + c)^3 + 24*C*a^3 + 72*B*a^2*b + 48*C*a*b^2 + 16*B*b^3 + 8*(3*C*a*b^2 + B
*b^3)*cos(d*x + c)^2 + 9*(4*C*a^2*b + 4*B*a*b^2 + C*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57755, size = 724, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(8*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 3*C*b^3)*(d*x + c) + 2*(24*C*a
^3*tan(1/2*d*x + 1/2*c)^7 + 72*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*C*a^2*b*
tan(1/2*d*x + 1/2*c)^7 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 72*C*a*b^2*tan
(1/2*d*x + 1/2*c)^7 + 24*B*b^3*tan(1/2*d*x + 1/2*c)^7 - 15*C*b^3*tan(1/2*d*
x + 1/2*c)^7 + 72*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 216*B*a^2*b*tan(1/2*d*x +
1/2*c)^5 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*B*a*b^2*tan(1/2*d*x + 1/2
*c)^5 + 120*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 40*B*b^3*tan(1/2*d*x + 1/2*c)^
5 + 9*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 216*
B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*B*a
*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 40*B*b^3
*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 24*C*a^3*tan(1/2
*d*x + 1/2*c) + 72*B*a^2*b*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b*tan(1/2*d*x +
```

```
1/2*c) + 36*B*a*b^2*tan(1/2*d*x + 1/2*c) + 72*C*a*b^2*tan(1/2*d*x + 1/2*c)
+ 24*B*b^3*tan(1/2*d*x + 1/2*c) + 15*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

3.788 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{b\left(8a^2C+9abB+2b^2C\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(6a^2bB+2a^3C+3ab^2C+b^3B\right)+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(5aC+3bB)}{}$$

```
[Out] ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*
x]])/d + (b*(9*a*b*B + 8*a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*d) + (b^2*(3*b*B
 + 5*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*C*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(3*d)
```

Rubi [A] time = 0.479173, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2990, 3033, 3023, 2735, 3770}

$$\frac{b\left(8a^2C+9abB+2b^2C\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(6a^2bB+2a^3C+3ab^2C+b^3B\right)+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(5aC+3bB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^2,x]
```


```
[Out] ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*
x]])/d + (b*(9*a*b*B + 8*a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*d) + (b^2*(3*b*B
 + 5*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*C*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\sec$$
$$= \frac{bC(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (a$$
$$= \frac{b^2(3bB+5aC)\cos(c+dx)\sin(c+dx)}{6d} + \frac{bC(}{}$$
$$= \frac{b\left(9abB+8a^2C+2b^2C\right)\sin(c+dx)}{3d} + \frac{b^2(3b}{}$$
$$= \frac{1}{2}\left(6a^2bB+b^3B+2a^3C+3ab^2C\right)x + \frac{b\left(9ab\right.}{}$$
$$= \frac{1}{2}\left(6a^2bB+b^3B+2a^3C+3ab^2C\right)x + \frac{a^3B\tan}{}$$

Mathematica [A] time = 0.379683, size = 159, normalized size = 1.16

$$\frac{6(c+dx)\left(6a^2bB+2a^3C+3ab^2C+b^3B\right)+9b\left(4a^2C+4abB+b^2C\right)\sin(c+dx)-12a^3B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] (6*(6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*(c + d*x) - 12*a^3*B*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*B*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*b*(4*a*b*B + 4*a^2*C + b^2*C)*Sin[c + d*x] + 3*b^2*(b*B + 3*a*
C)*Sin[2*(c + d*x)] + b^3*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.057, size = 207, normalized size = 1.5

$$\frac{C\sin(dx+c)(\cos(dx+c))^2 b^3}{3d} + \frac{2Cb^3\sin(dx+c)}{3d} + \frac{b^3B\cos(dx+c)\sin(dx+c)}{2d} + \frac{b^3Bx}{2} + \frac{b^3Bc}{2d} + \frac{3C\cos(dx+c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*b^3+2/3/d*C*b^3*sin(d*x+c)+1/2/d*b^3*B*cos(
d*x+c)*sin(d*x+c)+1/2*b^3*B*x+1/2/d*b^3*B*c+3/2/d*C*a*b^2*cos(d*x+c)*sin(d*
x+c)+3/2*a*b^2*C*x+3/2/d*C*a*b^2*c+3/d*a*b^2*B*sin(d*x+c)+3/d*a^2*b*C*sin(d
*x+c)+3*a^2*b*B*x+3/d*B*a^2*b*c+a^3*C*x+1/d*a^3*C*c+1/d*a^3*B*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 1.07524, size = 205, normalized size = 1.5

$$\frac{12\,(dx+c)Ca^3+36\,(dx+c)Ba^2b+9\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Cab^2+3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bb^3-4\left(\sin{(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*C*a^3 + 36*(d*x + c)*B*a^2*b + 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*C*a*b^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^3 - 4*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*C*b^3 + 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 36*C*a^2*b*sin(d*x + c) + 36*B*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.56102, size = 317, normalized size = 2.31

$$\frac{3\,Ba^3\log{(\sin{(dx+c)}+1)}-3\,Ba^3\log{(-\sin{(dx+c)}+1)}+3\left(2\,Ca^3+6\,Ba^2b+3\,Cab^2+Bb^3\right)dx+\left(2\,Cb^3\cos{(dx+c)}^2\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*B*a^3*log(sin(d*x + c) + 1) - 3*B*a^3*log(-sin(d*x + c) + 1) + 3*(2*
C*a^3 + 6*B*a^2*b + 3*C*a*b^2 + B*b^3)*d*x + (2*C*b^3*cos(d*x + c)^2 + 18*C
*a^2*b + 18*B*a*b^2 + 4*C*b^3 + 3*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.78, size = 424, normalized size = 3.09

$$6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(2\,Ca^3+6\,Ba^2b+3\,Cab^2+Bb^3\right)(dx+c)+\frac{2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*B*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*C*a^3 + 6*B*a^2*b + 3*C*a*b^2 + B*b^3)*(d*x + c) +
 2*(18*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 -
 9*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^
3*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^2*t
an(1/2*d*x + 1/2*c)^3 + 4*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^2*b*tan(1/2
*d*x + 1/2*c) + 18*B*a*b^2*tan(1/2*d*x + 1/2*c) + 9*C*a*b^2*tan(1/2*d*x + 1
/2*c) + 3*B*b^3*tan(1/2*d*x + 1/2*c) + 6*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.789 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=131

$$-\frac{b\left(2a^2B-3abC-b^2B\right)\sin(c+dx)}{d}+\frac{1}{2}bx\left(6a^2C+6abB+b^2C\right)+\frac{a^2(aC+3bB)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aB-bC)\sin}{}$$

```
[Out] (b*(6*a*b*B + 6*a^2*C + b^2*C)*x)/2 + (a^2*(3*b*B + a*C)*ArcTanh[Sin[c + d*
x]])/d - (b*(2*a^2*B - b^2*B - 3*a*b*C)*Sin[c + d*x])/d - (b^2*(2*a*B - b*C
)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*B*(a + b*Cos[c + d*x])^2*Tan[c + d*
x])/d
```

Rubi [A] time = 0.464639, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2989, 3033, 3023, 2735, 3770}

$$-\frac{b\left(2a^2B-3abC-b^2B\right)\sin(c+dx)}{d}+\frac{1}{2}bx\left(6a^2C+6abB+b^2C\right)+\frac{a^2(aC+3bB)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aB-bC)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^3,x]
```


```
[Out] (b*(6*a*b*B + 6*a^2*C + b^2*C)*x)/2 + (a^2*(3*b*B + a*C)*ArcTanh[Sin[c + d*
x]])/d - (b*(2*a^2*B - b^2*B - 3*a*b*C)*Sin[c + d*x])/d - (b^2*(2*a*B - b*C
)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*B*(a + b*Cos[c + d*x])^2*Tan[c + d*
x])/d
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\sec^2($$

$$= \frac{aB(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+b$$

$$= -\frac{b^2(2aB-bC)\cos(c+dx)\sin(c+dx)}{2d} + \frac{aB(a}{}$$

$$= -\frac{b\left(2a^2B-b^2B-3abC\right)\sin(c+dx)}{d} - \frac{b^2(2aB}{}$$

$$= \frac{1}{2}b\left(6abB+6a^2C+b^2C\right)x - \frac{b\left(2a^2B-b^2B-3\right.}{d}$$

$$= \frac{1}{2}b\left(6abB+6a^2C+b^2C\right)x + \frac{a^2(3bB+aC)\tan}{d}$$

Mathematica [A] time = 0.632534, size = 217, normalized size = 1.66

$$\frac{2b(c+dx)\left(6a^2C+6abB+b^2C\right) - 4a^2(aC+3bB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2(aC+3bB)\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] (2*b*(6*a*b*B + 6*a^2*C + b^2*C)*(c + d*x) - 4*a^2*(3*b*B + a*C)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*(3*b*B + a*C)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*a^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*a^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*b^2*(b*B + 3*a*C)*Sin[c + d*x] + b^3*C*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.053, size = 168, normalized size = 1.3

$$\frac{Cb^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{b^3Cx}{2} + \frac{Cb^3c}{2d} + \frac{b^3B\sin(dx+c)}{d} + 3\frac{Cab^2\sin(dx+c)}{d} + 3ab^2Bx + 3\frac{Bab^2c}{d} + 3a^2bCx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*C*b^3*cos(d*x+c)*sin(d*x+c)+1/2*b^3*C*x+1/2/d*C*b^3*c+1/d*b^3*B*sin(d
*x+c)+3/d*C*a*b^2*sin(d*x+c)+3*a*b^2*B*x+3/d*B*a*b^2*c+3*a^2*b*C*x+3/d*a^2*
b*C*c+3/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x
+c))+1/d*a^3*B*tan(d*x+c)
```

Maxima [A] time = 1.01296, size = 194, normalized size = 1.48

$$\frac{12\,(dx+c)Ca^2b+12\,(dx+c)Bab^2+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Cb^3+2\,Ca^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] 1/4*(12*(d*x + c)*C*a^2*b + 12*(d*x + c)*B*a*b^2 + (2*d*x + 2*c + sin(2*d*x
 + 2*c))*C*b^3 + 2*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
6*B*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*C*a*b^2*sin(
d*x + c) + 4*B*b^3*sin(d*x + c) + 4*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.45581, size = 369, normalized size = 2.82

$$\frac{\left(6\,Ca^2b+6\,Bab^2+Cb^3\right)dx\cos\left(dx+c\right)+\left(Ca^3+3\,Ba^2b\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(Ca^3+3\,Ba^2b\right)\cos\left(dx+\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] 1/2*((6*C*a^2*b + 6*B*a*b^2 + C*b^3)*d*x*cos(d*x + c) + (C*a^3 + 3*B*a^2*b)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (C*a^3 + 3*B*a^2*b)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (C*b^3*cos(d*x + c)^2 + 2*B*a^3 + 2*(3*C*a*b^2 + B*b^3
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43093, size = 316, normalized size = 2.41

$$-\frac{\frac{4\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\left(6\,Ca^{2}b+6\,Bab^{2}+Cb^{3}\right)(dx+c)-2\left(Ca^{3}+3\,Ba^{2}b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(Ca^{3}+3\,Ba^{2}b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] -1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*C*a^2*
b + 6*B*a*b^2 + C*b^3)*(d*x + c) - 2*(C*a^3 + 3*B*a^2*b)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(C*a^3 + 3*B*a^2*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(6*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 - C
*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a*b^2*tan(1/2*d*x + 1/2*c) + 2*B*b^3*tan(
1/2*d*x + 1/2*c) + C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

3.790 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{a\left(a^2B+6abC+6b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aC+2bB)\tan(c+dx)}{d}-\frac{b^2(aB-2bC)\sin(c+dx)}{2d}+b^2x(3aC+bB)$$

```
[Out] b^2*(b*B + 3*a*C)*x + (a*(a^2*B + 6*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]])
/(2*d) - (b^2*(a*B - 2*b*C)*Sin[c + d*x])/(2*d) + (a^2*(2*b*B + a*C)*Tan[c
+ d*x])/d + (a*B*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.413213, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2989, 3031, 3023, 2735, 3770}

$$\frac{a\left(a^2B+6abC+6b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aC+2bB)\tan(c+dx)}{d}-\frac{b^2(aB-2bC)\sin(c+dx)}{2d}+b^2x(3aC+bB)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^4,x]
```



```
[Out] b^2*(b*B + 3*a*C)*x + (a*(a^2*B + 6*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]])
/(2*d) - (b^2*(a*B - 2*b*C)*Sin[c + d*x])/(2*d) + (a^2*(2*b*B + a*C)*Tan[c
+ d*x])/d + (a*B*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\sec$$
$$= \frac{aB(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx}{2d}$$
$$= \frac{a^2(2bB+aC)\tan(c+dx)}{d} + \frac{aB(a+b\cos(c+}{}$$
$$= -\frac{b^2(aB-2bC)\sin(c+dx)}{2d} + \frac{a^2(2bB+aC)\tan}{d}$$
$$= b^2(bB+3aC)x - \frac{b^2(aB-2bC)\sin(c+dx)}{2d} +$$
$$= b^2(bB+3aC)x + \frac{a\left(a^2B+6b^2B+6abC\right)\tan}{2d}$$

Mathematica [B] time = 2.16565, size = 277, normalized size = 2.23

$$\frac{-2a\left(a^2B+6abC+6b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2a\left(a^2B+6abC+6b^2B\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```

```
[Out] (4*b^2*(b*B + 3*a*C)*(c + d*x) - 2*a*(a^2*B + 6*b^2*B + 6*a*b*C)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(a^2*B + 6*b^2*B + 6*a*b*C)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2 + (4*a^2*(3*b*B + a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]) - (a^3*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a^2*(3*b*B
 + a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*b^3*C*S
in[c + d*x])/(4*d)
```

Maple [A] time = 0.056, size = 172, normalized size = 1.4

$$\frac{Cb^3\sin(dx+c)}{d} + b^3Bx + \frac{Bb^3c}{d} + 3\,ab^2Cx + 3\,\frac{Cab^2c}{d} + 3\,\frac{ab^2B\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\,\frac{a^2bC\ln(\sec(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*C*b^3*sin(d*x+c)+b^3*B*x+1/d*b^3*B*c+3*a*b^2*C*x+3/d*C*a*b^2*c+3/d*a*b^
2*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2
*b*B*tan(d*x+c)+1/d*a^3*C*tan(d*x+c)+1/2/d*a^3*B*sec(d*x+c)*tan(d*x+c)+1/2/
d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01802, size = 228, normalized size = 1.84

$$\frac{12\,(dx+c)Cab^2+4\,(dx+c)Bb^3-Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Ca^2b(\log\left(\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] 1/4*(12*(d*x + c)*C*a*b^2 + 4*(d*x + c)*B*b^3 - B*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*C*b^3*sin(d*x + c) + 4*C*a^3*tan(d*x
 + c) + 12*B*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 1.5192, size = 401, normalized size = 3.23

$$\frac{4\left(3\,Cab^2+Bb^3\right)dx\cos\left(dx+c\right)^2+\left(Ba^3+6\,Ca^2b+6\,Bab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(Ba^3+6\,Ca^2b+6\,Bab^2\right.}{4\,d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] 1/4*(4*(3*C*a*b^2 + B*b^3)*d*x*cos(d*x + c)^2 + (B*a^3 + 6*C*a^2*b + 6*B*a*
b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (B*a^3 + 6*C*a^2*b + 6*B*a*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*C*b^3*cos(d*x + c)^2 + B*a^3
+ 2*(C*a^3 + 3*B*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33928, size = 323, normalized size = 2.6

$$\frac{4\,Cb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 2\left(3\,Cab^2 + Bb^3\right)(dx + c) + \left(Ba^3 + 6\,Ca^2b + 6\,Bab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Ba^3 + 6\,Ca^2b +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] 1/2*(4*C*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*C*a*b
^2 + B*b^3)*(d*x + c) + (B*a^3 + 6*C*a^2*b + 6*B*a*b^2)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (B*a^3 + 6*C*a^2*b + 6*B*a*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^3*tan(1/2*d*x + 1/2*c)^
3 - 6*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + B*a^3*tan(1/2*d*x + 1/2*c) + 2*C*a^3
*tan(1/2*d*x + 1/2*c) + 6*B*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

3.791 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{a\left(2a^2B+9abC+8b^2B\right)\tan(c+dx)}{3d}+\frac{\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3aC+5bB)\tan(c+dx)}{6d}$$

```
[Out] b^3*C*x + ((3*a^2*b*B + 2*b^3*B + a^3*C + 6*a*b^2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*(2*a^2*B + 8*b^2*B + 9*a*b*C)*Tan[c + d*x])/(3*d) + (a^2*(5*b*B
 + 3*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a*B*(a + b*Cos[c + d*x])^2*Se
c[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.429343, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2989, 3031, 3021, 2735, 3770}

$$\frac{a\left(2a^2B+9abC+8b^2B\right)\tan(c+dx)}{3d}+\frac{\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3aC+5bB)\tan(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^5,x]
```


```
[Out] b^3*C*x + ((3*a^2*b*B + 2*b^3*B + a^3*C + 6*a*b^2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*(2*a^2*B + 8*b^2*B + 9*a*b*C)*Tan[c + d*x])/(3*d) + (a^2*(5*b*B
 + 3*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a*B*(a + b*Cos[c + d*x])^2*Se
c[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\sec^4($$

$$= \frac{aB(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx}{3d}$$

$$= \frac{a^2(5bB+3aC)\sec(c+dx)\tan(c+dx)}{6d} + \frac{aB(a}{}$$

$$= \frac{a\left(2a^2B+8b^2B+9abC\right)\tan(c+dx)}{3d} + \frac{a^2(5bB}{}$$

$$= b^3Cx + \frac{a\left(2a^2B+8b^2B+9abC\right)\tan(c+dx)}{3d} +$$

$$= b^3Cx + \frac{\left(3a^2bB+2b^3B+a^3C+6ab^2C\right)\tanh^{-1}}{2d}$$

Mathematica [A] time = 0.56333, size = 108, normalized size = 0.74

$$\frac{3\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)\tanh^{-1}(\sin(c+dx))+3a\tan(c+dx)\left(2a^2B+a(aC+3bB)\sec(c+dx)+6abC+6b^2B\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^5,x]
```

```
[Out] (6*b^3*C*d*x + 3*(3*a^2*b*B + 2*b^3*B + a^3*C + 6*a*b^2*C)*ArcTanh[Sin[c +
d*x]] + 3*a*(2*a^2*B + 6*b^2*B + 6*a*b*C + a*(3*b*B + a*C)*Sec[c + d*x])*Ta
n[c + d*x] + 2*a^3*B*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.06, size = 223, normalized size = 1.5

$$b^3Cx + \frac{Cb^3c}{d} + \frac{b^3B\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\frac{Cab^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\frac{ab^2B\tan(dx+c)}{d} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] b^3*C*x+1/d*C*b^3*c+1/d*b^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*C*a*b^2*ln(sec(
d*x+c)+tan(d*x+c))+3/d*a*b^2*B*tan(d*x+c)+3/d*a^2*b*C*tan(d*x+c)+3/2/d*a^2*
b*B*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^3
*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^3*B*
tan(d*x+c)+1/3/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.03589, size = 292, normalized size = 2.01

$$4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3+12(dx+c)Cb^3-3Ca^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 12*(d*x + c)*C*b^3 - 3*C*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*C*a*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 6*B*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 36*C*a^2*b*tan(d*x + c) + 36*B*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.54638, size = 458, normalized size = 3.16

$$12Cb^3dx\cos(dx+c)^3+3\left(Ca^3+3Ba^2b+6Cab^2+2Bb^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(Ca^3+3Ba^2b+6C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="fricas")
```

```
[Out] 1/12*(12*C*b^3*d*x*cos(d*x + c)^3 + 3*(C*a^3 + 3*B*a^2*b + 6*C*a*b^2 + 2*B*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(C*a^3 + 3*B*a^2*b + 6*C*a*b^
2 + 2*B*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*B*a^3 + 2*(2*B*a^
3 + 9*C*a^2*b + 9*B*a*b^2)*cos(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.54403, size = 454, normalized size = 3.13

$$\frac{6\,(dx+c)Cb^3+3\left(Ca^3+3\,Ba^2b+6\,Cab^2+2\,Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ca^3+3\,Ba^2b+6\,Cab^2+2\,Bb^3\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*C*b^3 + 3*(C*a^3 + 3*B*a^2*b + 6*C*a*b^2 + 2*B*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(C*a^3 + 3*B*a^2*b + 6*C*a*b^2 + 2*B*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*
C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*C*a^2*
b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^3*tan(
1/2*d*x + 1/2*c)^3 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^2*tan(1/2
*d*x + 1/2*c)^3 + 6*B*a^3*tan(1/2*d*x + 1/2*c) + 3*C*a^3*tan(1/2*d*x + 1/2*
c) + 9*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 18*
B*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.792 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=188

$$\frac{\left(6a^2bB+2a^3C+9ab^2C+3b^3B\right)\tan(c+dx)}{3d}+\frac{\left(12a^2bC+3a^3B+12ab^2B+8b^3C\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(3a^2B+1\right.}{}$$

```
[Out] ((3*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 8*b^3*C)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((6*a^2*b*B + 3*b^3*B + 2*a^3*C + 9*a*b^2*C)*Tan[c + d*x])/(3*d) + (a*(3
*a^2*B + 10*b^2*B + 12*a*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(3*b*
B + 2*a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*B*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.548636, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2989, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(6a^2bB+2a^3C+9ab^2C+3b^3B\right)\tan(c+dx)}{3d}+\frac{\left(12a^2bC+3a^3B+12ab^2B+8b^3C\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(3a^2B+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^6,x]
```


```
[Out] ((3*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 8*b^3*C)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((6*a^2*b*B + 3*b^3*B + 2*a^3*C + 9*a*b^2*C)*Tan[c + d*x])/(3*d) + (a*(3
*a^2*B + 10*b^2*B + 12*a*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(3*b*
B + 2*a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*B*(a + b*Cos[c + d*x])^2
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\,\mathrm{sec}$$

$$= \frac{aB(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+d}{4d}$$

$$= \frac{a^2(3bB+2aC)\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{aI}{}$$

$$= \frac{a\left(3a^2B+10b^2B+12abC\right)\sec(c+dx)\tan(c}{8d}$$

$$= \frac{a\left(3a^2B+10b^2B+12abC\right)\sec(c+dx)\tan(c}{8d}$$

$$= \frac{\left(3a^3B+12ab^2B+12a^2bC+8b^3C\right)\tanh^{-1}(\mathrm{si}}{8d}$$

$$= \frac{\left(3a^3B+12ab^2B+12a^2bC+8b^3C\right)\tanh^{-1}(\mathrm{si}}{8d}$$

Mathematica [A] time = 0.799638, size = 140, normalized size = 0.74

$$\frac{3\left(12a^2bC+3a^3B+12ab^2B+8b^3C\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(9a\left(a^2B+4abC+4b^2B\right)\sec(c+dx)+24\left(3a^2\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^6,x]
```

```
[Out] (3*(3*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 8*b^3*C)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*b*B + b^3*B + a^3*C + 3*a*b^2*C) + 9*a*(a^2*B + 4*b^2
*B + 4*a*b*C)*Sec[c + d*x] + 6*a^3*B*Sec[c + d*x]^3 + 8*a^2*(3*b*B + a*C)*T
an[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.059, size = 290, normalized size = 1.5

$$\frac{Cb^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^3B\tan(dx+c)}{d}+3\,\frac{Cab^2\tan(dx+c)}{d}+\frac{3\,ab^2B\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{3\,ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/d*C*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^3*B*tan(d*x+c)+3/d*C*a*b^2*tan(d*
x+c)+3/2/d*a*b^2*B*sec(d*x+c)*tan(d*x+c)+3/2/d*a*b^2*B*ln(sec(d*x+c)+tan(d*
x+c))+3/2/d*a^2*b*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*C*ln(sec(d*x+c)+tan(d
*x+c))+2/d*a^2*b*B*tan(d*x+c)+1/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)^2+2/3/d*a^3
*C*tan(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan(d*x+c)*se
c(d*x+c)^3+3/8/d*a^3*B*sec(d*x+c)*tan(d*x+c)+3/8/d*a^3*B*ln(sec(d*x+c)+tan(
d*x+c))
```

Maxima [A] time = 1.07162, size = 369, normalized size = 1.96

$$\frac{16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^3+48\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^2b-3\,Ba^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2*b - 3*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*C*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*b^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 144*C*a*b^2*tan(d*x + c) + 4
```

```
8*B*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.6031, size = 510, normalized size = 2.71

$$\frac{3\left(3\,Ba^3+12\,Ca^2b+12\,Bab^2+8\,Cb^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Ba^3+12\,Ca^2b+12\,Bab^2+8\,Cb^3\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(3*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 8*C*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(3*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 8*C*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*B*a^3 + 8*(2*C*a^3 + 6*B*a^2*b + 9*C*a*
b^2 + 3*B*b^3)*cos(d*x + c)^3 + 9*(B*a^3 + 4*C*a^2*b + 4*B*a*b^2)*cos(d*x +
 c)^2 + 8*(C*a^3 + 3*B*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.95453, size = 791, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 8*C*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(3*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 8*C*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(15*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*C*a^3*tan
(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*C*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*C*a*b^2*tan(1/2*d
*x + 1/2*c)^7 - 24*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*tan(1/2*d*x + 1/2
*c)^5 + 40*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 120*B*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
 216*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*B
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 216*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^3*tan(1/
2*d*x + 1/2*c)^3 + 15*B*a^3*tan(1/2*d*x + 1/2*c) + 24*C*a^3*tan(1/2*d*x + 1
/2*c) + 72*B*a^2*b*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b*tan(1/2*d*x + 1/2*c) +
 36*B*a*b^2*tan(1/2*d*x + 1/2*c) + 72*C*a*b^2*tan(1/2*d*x + 1/2*c) + 24*B*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.793 $\int (a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{\left(30a^2bC+8a^3B+30ab^2B+15b^3C\right)\tan(c+dx)}{15d}+\frac{\left(9a^2bB+3a^3C+12ab^2C+4b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(4a^2B+\right.}{}$$

```
[Out] ((9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((8*a^3*B + 30*a*b^2*B + 30*a^2*b*C + 15*b^3*C)*Tan[c + d*x])/(15*d) + ((
9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (a*(4*a^2*B + 12*b^2*B + 15*a*b*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a^2*(7*b*B + 5*a*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*B*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.561617, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3029, 2989, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(30a^2bC+8a^3B+30ab^2B+15b^3C\right)\tan(c+dx)}{15d}+\frac{\left(9a^2bB+3a^3C+12ab^2C+4b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(4a^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^7,x]
```

```
[Out] ((9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((8*a^3*B + 30*a*b^2*B + 30*a^2*b*C + 15*b^3*C)*Tan[c + d*x])/(15*d) + ((
9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (a*(4*a^2*B + 12*b^2*B + 15*a*b*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a^2*(7*b*B + 5*a*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*B*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int (a+b\cos(c+dx))^3(B+C\cos(c+dx))\sec$$

$$= \frac{aB(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+d}{5d}$$

$$= \frac{a^2(7bB+5aC)\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{aB}{}$$

$$= \frac{a\left(4a^2B+12b^2B+15abC\right)\sec^2(c+dx)\tan(c}{15d}$$

$$= \frac{a\left(4a^2B+12b^2B+15abC\right)\sec^2(c+dx)\tan(c}{15d}$$

$$= \frac{\left(9a^2bB+4b^3B+3a^3C+12ab^2C\right)\sec(c+dx}{8d}$$

$$= \frac{\left(9a^2bB+4b^3B+3a^3C+12ab^2C\right)\tanh^{-1}(\sin}{8d}$$

Mathematica [A] time = 3.41093, size = 181, normalized size = 0.77

$$\frac{15\left(9a^2bB+3a^3C+12ab^2C+4b^3B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5a\left(2a^2B+3abC+3b^2B\right)\tan^2(c+dx)+15\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^7,x]
```

```
[Out] (15*(9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*Sec[c + d*x] +
30*a^2*(3*b*B + a*C)*Sec[c + d*x]^3 + 8*(15*(a^3*B + 3*a*b^2*B + 3*a^2*b*C
+ b^3*C) + 5*a*(2*a^2*B + 3*b^2*B + 3*a*b*C)*Tan[c + d*x]^2 + 3*a^3*B*Tan[c
 + d*x]^4)))/(120*d)
```

Maple [A] time = 0.059, size = 382, normalized size = 1.6

$$\frac{Cb^3\tan(dx+c)}{d}+\frac{b^3B\sec(dx+c)\tan(dx+c)}{2d}+\frac{b^3B\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{3\,Cab^2\tan(dx+c)\sec(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/d*C*b^3*tan(d*x+c)+1/2/d*b^3*B*sec(d*x+c)*tan(d*x+c)+1/2/d*b^3*B*ln(sec(d
*x+c)+tan(d*x+c))+3/2/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)+3/2/d*C*a*b^2*ln(sec(
d*x+c)+tan(d*x+c))+2/d*a*b^2*B*tan(d*x+c)+1/d*a*b^2*B*tan(d*x+c)*sec(d*x+c)
^2+2/d*a^2*b*C*tan(d*x+c)+1/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^2*b*B
*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^2*b*B*sec(d*x+c)*tan(d*x+c)+9/8/d*a^2*b*B*
ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*C*s
ec(d*x+c)*tan(d*x+c)+3/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*a^3*B*tan
(d*x+c)+1/5/d*a^3*B*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^3*B*tan(d*x+c)*sec(d*x
+c)^2
```

Maxima [A] time = 1.06425, size = 460, normalized size = 1.95

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^2b+240\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b + 240*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a*b^2 - 15*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 45*B*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 180*C*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) - 60*B*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*C*b^3*tan(d*x +
 c))/d
```

Fricas [A] time = 1.58003, size = 612, normalized size = 2.59

$$\frac{15\left(3\,Ca^3+9\,Ba^2b+12\,Cab^2+4\,Bb^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ca^3+9\,Ba^2b+12\,Cab^2+4\,Bb^3\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*cos(d*x
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(8*B*a^3 + 30*C*a^2*b + 30*B*a*b^2 + 1
5*C*b^3)*cos(d*x + c)^4 + 24*B*a^3 + 15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 +
 4*B*b^3)*cos(d*x + c)^3 + 8*(4*B*a^3 + 15*C*a^2*b + 15*B*a*b^2)*cos(d*x +
c)^2 + 30*(C*a^3 + 3*B*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**7,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86497, size = 975, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,
algorithm="giac")
```

```
[Out] 1/120*(15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*C*a^3*t
an(1/2*d*x + 1/2*c)^9 - 225*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*C*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*C*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 60*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*C*b^3*tan(1/2*d
*x + 1/2*c)^9 - 160*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 30*C*a^3*tan(1/2*d*x + 1
/2*c)^7 + 90*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*C*a^2*b*tan(1/2*d*x + 1/2
*c)^7 - 960*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 360*C*a*b^2*tan(1/2*d*x + 1/2*
c)^7 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*C*b^3*tan(1/2*d*x + 1/2*c)^7
+ 464*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*C*b^3*tan(1/2*d*x + 1/2*c)^5 - 16
0*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*B*a^2
*b*tan(1/2*d*x + 1/2*c)^3 - 960*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a*b^
2*tan(1/2*d*x + 1/2*c)^3 - 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*B*b^3*t
an(1/2*d*x + 1/2*c)^3 - 480*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^3*tan(1/
2*d*x + 1/2*c) + 75*C*a^3*tan(1/2*d*x + 1/2*c) + 225*B*a^2*b*tan(1/2*d*x +
1/2*c) + 360*C*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a*b^2*tan(1/2*d*x + 1/2*c
) + 180*C*a*b^2*tan(1/2*d*x + 1/2*c) + 60*B*b^3*tan(1/2*d*x + 1/2*c) + 120*
C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.794 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{\left(-3a^2C+3abB-2b^2C\right)\sin(c+dx)}{3b^3d}-\frac{2a^3(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)(bB-aC)}{2b^4}+\frac{(bB-aC)\text{si}}{}$$

```
[Out] ((2*a^2 + b^2)*(b*B - a*C)*x)/(2*b^4) - (2*a^3*(b*B - a*C)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a
*b*B - 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(3*b^3*d) + ((b*B - a*C)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^2*d) + (C*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.569581, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2990, 3049, 3023, 2735, 2659, 205}

$$-\frac{\left(-3a^2C+3abB-2b^2C\right)\sin(c+dx)}{3b^3d}-\frac{2a^3(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)(bB-aC)}{2b^4}+\frac{(bB-aC)\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x
]),x]
```


```
[Out] ((2*a^2 + b^2)*(b*B - a*C)*x)/(2*b^4) - (2*a^3*(b*B - a*C)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a
*b*B - 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(3*b^3*d) + ((b*B - a*C)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^2*d) + (C*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$= \frac{C\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2aC+2bC\cos(c+dx)+3(bB-aC)\right.}{a+b\cos(c+dx)}}{3b}$$

$$= \frac{(bB-aC)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{C\cos^2(c+dx)\sin(c+dx)}{3bd}$$

$$= -\frac{\left(3abB-3a^2C-2b^2C\right)\sin(c+dx)}{3b^3d} + \frac{(bB-aC)\cos(c+dx)\sin}{2b^2d}$$

$$= \frac{\left(2a^2+b^2\right)(bB-aC)x}{2b^4} - \frac{\left(3abB-3a^2C-2b^2C\right)\sin(c+dx)}{3b^3d} + \frac{(b}{}$$

$$= \frac{\left(2a^2+b^2\right)(bB-aC)x}{2b^4} - \frac{\left(3abB-3a^2C-2b^2C\right)\sin(c+dx)}{3b^3d} + \frac{(b}{}$$

$$= \frac{\left(2a^2+b^2\right)(bB-aC)x}{2b^4} - \frac{2a^3(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d}$$

Mathematica [A] time = 0.464254, size = 152, normalized size = 0.85

$$\frac{6\left(2a^2+b^2\right)(c+dx)(bB-aC)+3b\left(4a^2C-4abB+3b^2C\right)\sin(c+dx)-\frac{24a^3(aC-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+3b^2(bB-aC}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c
 + d*x]),x]
```

```
[Out] (6*(2*a^2 + b^2)*(b*B - a*C)*(c + d*x) - (24*a^3*(-(b*B) + a*C)*ArcTanh[((a
 - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(-4*a*b*B
 + 4*a^2*C + 3*b^2*C)*Sin[c + d*x] + 3*b^2*(b*B - a*C)*Sin[2*(c + d*x)] + b
^3*C*Sin[3*(c + d*x)])/(12*b^4*d)
```

Maple [B] time = 0.037, size = 641, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a*B-1/d/b/(tan(1/2
*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*B+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^
3*tan(1/2*d*x+1/2*c)^5*a^2*C+1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x
+1/2*c)^5*C*a+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*C-4/d/b
^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a*B+4/d/b^3/(tan(1/2*d*x
+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a^2*C+4/3/d/b/(tan(1/2*d*x+1/2*c)^2+1)^
3*tan(1/2*d*x+1/2*c)^3*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2
*c)*a*B+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*a^2*C+2/d/b/(
tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*C+1/d/b/(tan(1/2*d*x+1/2*c)^2+
1)^3*tan(1/2*d*x+1/2*c)*B-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/
2*c)*C*a+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*B+1/d/b*arctan(tan(1/2*d*x+
1/2*c))*B-2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3*C-1/d/b^2*C*arctan(tan(1/2
*d*x+1/2*c))*a-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*B+2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64978, size = 1164, normalized size = 6.54

$$\left[-\frac{3\left(2\,Ca^5-2\,Ba^4b-Ca^3b^2+Ba^2b^3-Cab^4+Bb^5\right)dx-3\left(Ca^4-Ba^3b\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2}{b^2\cos(dx+c)^2-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [-1/6*(3*(2*C*a^5 - 2*B*a^4*b - C*a^3*b^2 + B*a^2*b^3 - C*a*b^4 + B*b^5)*d*
x - 3*(C*a^4 - B*a^3*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*C*a^4
*b - 6*B*a^3*b^2 - 2*C*a^2*b^3 + 6*B*a*b^4 - 4*C*b^5 + 2*(C*a^2*b^3 - C*b^5
)*cos(d*x + c)^2 - 3*(C*a^3*b^2 - B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d), -1/6*(3*(2*C*a^5 - 2*B*a^4*b - C*a^3*b
^2 + B*a^2*b^3 - C*a*b^4 + B*b^5)*d*x - 6*(C*a^4 - B*a^3*b)*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^4*b
- 6*B*a^3*b^2 - 2*C*a^2*b^3 + 6*B*a*b^4 - 4*C*b^5 + 2*(C*a^2*b^3 - C*b^5)*c
os(d*x + c)^2 - 3*(C*a^3*b^2 - B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.8073, size = 486, normalized size = 2.73

$$-\frac{\frac{3\left(2\,Ca^{3}-2\,Ba^{2}b+Cab^{2}-Bb^{3}\right)(dx+c)}{b^{4}}+\frac{12\left(Ca^{4}-Ba^{3}b\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}b^{4}}-\frac{2\left(6\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/6*(3*(2*C*a^3 - 2*B*a^2*b + C*a*b^2 - B*b^3)*(d*x + c)/b^4 + 12*(C*a^4 -
 B*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 -
b^2)*b^4) - 2*(6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C
*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x
 + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) - 3*C*a*b*tan(1/2*d*x + 1/2*c) + 3
*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^3*b^3))/d
```

3.795 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(-2a^2C+2abB-b^2C\right)}{2b^3}+\frac{(bB-aC)\sin(c+dx)}{b^2d}+\frac{C\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] -((2*a*b*B - 2*a^2*C - b^2*C)*x)/(2*b^3) + (2*a^2*(b*B - a*C)*ArcTan[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((
b*B - a*C)*Sin[c + d*x])/(b^2*d) + (C*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.355128, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3029, 2990, 3023, 2735, 2659, 205}

$$\frac{2a^2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(-2a^2C+2abB-b^2C\right)}{2b^3}+\frac{(bB-aC)\sin(c+dx)}{b^2d}+\frac{C\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])
,x]
```

```
[Out] -((2*a*b*B - 2*a^2*C - b^2*C)*x)/(2*b^3) + (2*a^2*(b*B - a*C)*ArcTan[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((
b*B - a*C)*Sin[c + d*x])/(b^2*d) + (C*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$= \frac{C\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{aC+bC\cos(c+dx)+2(bB-aC)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{2b}$$

$$= \frac{(bB-aC)\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{abC-(2abB-}{a+}}{}$$

$$= -\frac{\left(2abB-2a^2C-b^2C\right)x}{2b^3} + \frac{(bB-aC)\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx}{2b}$$

$$= -\frac{\left(2abB-2a^2C-b^2C\right)x}{2b^3} + \frac{(bB-aC)\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx}{2b}$$

$$= -\frac{\left(2abB-2a^2C-b^2C\right)x}{2b^3} + \frac{2a^2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d}$$

Mathematica [A] time = 0.31783, size = 121, normalized size = 0.9

$$\frac{2(c+dx)\left(2a^2C-2abB+b^2C\right)+\frac{8a^2(aC-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+4b(bB-aC)\sin(c+dx)+b^2C\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```

```
[Out] (2*(-2*a*b*B + 2*a^2*C + b^2*C)*(c + d*x) + (8*a^2*(-(b*B) + a*C)*ArcTanh[(
(a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*b*(b*B -
a*C)*Sin[c + d*x] + b^2*C*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [B] time = 0.034, size = 367, normalized size = 2.7

$$2\,\frac{(\tan(1/2\,dx+c/2))^3\,B}{db\left((\tan(1/2\,dx+c/2))^2+1\right)^2} - 2\,\frac{(\tan(1/2\,dx+c/2))^3\,aC}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)^2} - \frac{C}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-2/d/b^2/(tan(1/2*d*
x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*t
an(1/2*d*x+1/2*c)^3*C+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*B
-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C+1/d/b/(tan(1/2*d
*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*a*
B+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*C+1/d/b*arctan(tan(1/2*d*x+1/2*c))
*C+2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*B-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5619, size = 918, normalized size = 6.85

$$\left[\frac{\left(2\,Ca^4 - 2\,Ba^3b - Ca^2b^2 + 2\,Bab^3 - Cb^4\right)dx + \left(Ca^3 - Ba^2b\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c) + \left(2\,a^2 - b^2\right)\cos(dx+c)^2 + 2\,\sqrt{-a^2+b^2}(a\cos(d}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a}}{2\left(a^2b^3 - b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] [1/2*((2*C*a^4 - 2*B*a^3*b - C*a^2*b^2 + 2*B*a*b^3 - C*b^4)*d*x + (C*a^3 -
B*a^2*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
```

```
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*C*a^3*b - 2*B*a^2*b^
2 - 2*C*a*b^3 + 2*B*b^4 - (C*a^2*b^2 - C*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^3 - b^5)*d), 1/2*((2*C*a^4 - 2*B*a^3*b - C*a^2*b^2 + 2*B*a*b^3 - C*b
^4)*d*x - 2*(C*a^3 - B*a^2*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*C*a^3*b - 2*B*a^2*b^2 - 2*C*a*b^3 + 2*
B*b^4 - (C*a^2*b^2 - C*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.59666, size = 306, normalized size = 2.28

$$\frac{\frac{(2\,Ca^2-2\,Bab+Cb^2)(dx+c)}{b^3}+\frac{4\left(Ca^3-Ba^2b\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{2\left(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Bb\tan\left(\frac{1}{2}\,c\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algo
rithm="giac")
```

```
[Out] 1/2*((2*C*a^2 - 2*B*a*b + C*b^2)*(d*x + c)/b^3 + 4*(C*a^3 - B*a^2*b)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2*(2
*C*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*
x + 1/2*c)^3 + 2*C*a*tan(1/2*d*x + 1/2*c) - 2*B*b*tan(1/2*d*x + 1/2*c) - C*
b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.796 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{2a(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{x(bB-aC)}{b^2}+\frac{C\sin(c+dx)}{bd}$$

```
[Out] ((b*B - a*C)*x)/b^2 - (2*a*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.131223, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3023, 12, 2735, 2659, 205}

$$-\frac{2a(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{x(bB-aC)}{b^2}+\frac{C\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```


```
[Out] ((b*B - a*C)*x)/b^2 - (2*a*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*Sin[c + d*x])/(b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{C\sin(c+dx)}{bd}+\frac{\int \frac{(bB-aC)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{C\sin(c+dx)}{bd}+\frac{(bB-aC)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{(bB-aC)x}{b^2}+\frac{C\sin(c+dx)}{bd}-\frac{(a(bB-aC))\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\
&= \frac{(bB-aC)x}{b^2}+\frac{C\sin(c+dx)}{bd}-\frac{(2a(bB-aC))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}\right.\right.}{b^2d}\\
&= \frac{(bB-aC)x}{b^2}-\frac{2a(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d}+\frac{C\sin(c+dx)}{bd}
\end{aligned}
$$

Mathematica [A] time = 0.213006, size = 85, normalized size = 0.96

$$
\frac{-\frac{2a(aC-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+(c+dx)(bB-aC)+bC\sin(c+dx)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((b*B - a*C)*(c + d*x) - (2*a*(-(b*B) + a*C)*ArcTanh[((a - b)*Tan[(c + d*x)
/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*C*Sin[c + d*x])/(b^2*d)
```

Maple [B] time = 0.033, size = 172, normalized size = 1.9

$$2\frac{C\tan(1/2\,dx+c/2)}{db\left((\tan(1/2\,dx+c/2))^2+1\right)}+2\frac{\arctan(\tan(1/2\,dx+c/2))\,B}{db}-2\frac{C\arctan(\tan(1/2\,dx+c/2))\,a}{db^2}-2\frac{Ba}{db\sqrt{(a+b)\,(a-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*C/b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/b*arctan(tan(1/2*d*
x+1/2*c))*B-2/d/b^2*C*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a/b/((a+b)*(a-b))^(1
/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d/b^2/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55324, size = 689, normalized size = 7.74

$$\left[-\frac{2\left(Ca^3-Ba^2b-Cab^2+Bb^3\right)dx-\left(Ca^2-Bab\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\left(a^2b^2-b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/2*(2*(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x - (C*a^2 - B*a*b)*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(C*a^2*b - C*b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d), -((C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x - (C*a^2 - B*a*b)*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (C*a^2*b - C*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.66405, size = 192, normalized size = 2.16

$$-\frac{\frac{(Ca-Bb)(dx+c)}{b^2}-\frac{2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}+\frac{2\left(Ca^2-Bab\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((C*a - B*b)*(d*x + c)/b^2 - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*b) + 2*(C*a^2 - B*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/(sqrt(a^2 - b^2)*b^2))/d
```

3.797 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{Cx}{b}$$

```
[Out] (C*x)/b + (2*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.144358, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3029, 2735, 2659, 205}

$$\frac{2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{Cx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])
,x]
```

```
[Out] (C*x)/b + (2*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx &= \int \frac{B+C\cos(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{Cx}{b}-\frac{(-bB+aC)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{Cx}{b}+\frac{(2(bB-aC))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\
&= \frac{Cx}{b}+\frac{2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.120246, size = 68, normalized size = 1.01

$$
\frac{\frac{2(aC-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+C(c+dx)}{bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c +
 d*x]),x]
```

```
[Out] (C*(c + d*x) + (2*(-(b*B) + a*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2])/(b*d)
```

Maple [A] time = 0.049, size = 113, normalized size = 1.7

$$2\,{\frac {\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) C}{db}}+2\,{\frac {B}{d\sqrt { \left(a+b \right) \left(a-b \right) }}\arctan \left({\frac { \left(a-b \right) \tan \left(1/2\,dx+c/2 \right) }{\sqrt { \left(a+b \right) \left(a-b \right) }}} \right) }-2\,{\frac {aC}{db\sqrt { \left(a+b \right) \left(a-b \right) }}\arctan \left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))*C+2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*a/b/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45848, size = 524, normalized size = 7.82

$$\left[\frac{2\left(Ca^{2}-Cb^{2}\right)dx+\left(Ca-Bb\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2\,ab\cos\left(dx+c\right)+\left(2\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+2\,\sqrt{-a^{2}+b^{2}}\left(a\cos\left(dx+c\right)+b\right)\sin\left(dx+c\right)-a^{2}+2\,b^{2}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}}\right)}{2\left(a^{2}b-b^{3}\right)d},\ \frac{\left(Ca^{2}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] [1/2*(2*(C*a^2 - C*b^2)*d*x + (C*a - B*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)))/((a^2*b - b^3)*d), ((C*a^2 - C*b^2)*d*x - (C*a - B*b)*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b -
 b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\cos(c + dx))\cos(c + dx)\sec(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a + b*cos(c + d*x)
), x)
```

Giac [A] time = 1.67828, size = 135, normalized size = 2.01

$$\frac{\frac{(dx+c)C}{b} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ca-Bb)}{\sqrt{a^2-b^2}\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, algo
rithm="giac")
```

```
[Out] ((d*x + c)*C/b - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arcta
n((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(C*a
- B*b)/(sqrt(a^2 - b^2)*b))/d
```

3.798 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (-2*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + (B*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rubi [A] time = 0.208097, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3029, 3001, 3770, 2659, 205}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x
]),x]
```



```
[Out] (-2*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + (B*ArcTanh[Sin[c + d*x]])/(a*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
```

```
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{(B+C\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$=\frac{B\int \sec(c+dx)\,dx}{a}+\frac{(-bB+aC)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}$$

$$=\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(2(bB-aC))\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x\right.}{ad}$$

$$=-\frac{2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.159663, size = 112, normalized size = 1.47

$$\frac{\frac{2(bB-aC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+B\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c
 + d*x]),x]
```

```
[Out] ((2*(b*B - a*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2] + B*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]))/(a*d)
```

Maple [A] time = 0.058, size = 135, normalized size = 1.8

$$-\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\,\frac{bB}{ad\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{C}{d\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/a/d*B*ln(tan(1/2*d*x+1/2*c)-1)-2/d/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b*B+2/d/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/a/d*B*ln(tan(1/2*d*x+1/2*c)
+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.18883, size = 689, normalized size = 9.07

$$\left[\frac{(Ca-Bb)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Ba^2-Bb^2\right)\log(\sin(dx}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*((C*a - B*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (B*a^2 - B*b^2
)*log(sin(d*x + c) + 1) - (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a
*b^2)*d), 1/2*(2*(C*a - B*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) - (B
*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\cos(c+dx))\cos(c+dx)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2/(a + b*cos(c + d
*x)), x)
```

Giac [A] time = 1.61138, size = 173, normalized size = 2.28

$$\frac{\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ca-Bb)}{\sqrt{a^2-b^2}a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - B*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(C*a - B*b)/
(sqrt(a^2 - b^2)*a))/d
```

3.799 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{2b(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

```
[Out] (2*b*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) - ((b*B - a*C)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (
B*Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.272975, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 3000, 12, 2747, 3770, 2659, 205}

$$\frac{2b(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x
]),x]
```

```
[Out] (2*b*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) - ((b*B - a*C)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (
B*Tan[c + d*x])/(a*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{B\tan(c+dx)}{ad} + \frac{\int \frac{(-bB+aC)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{B\tan(c+dx)}{ad} + \frac{(-bB+aC)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB-aC)\int \sec(c+dx)\,dx}{a^2} + \frac{(b(bB-aC))\int \frac{}{a+}}{a^2}$$

$$= -\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{B\tan(c+dx)}{ad} + \frac{(2b(bB-aC}{}$$

$$= \frac{2b(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} - \frac{(bB-aC)\tanh^{-1}(\sin(c}{a^2d}$$

Mathematica [A] time = 0.52719, size = 129, normalized size = 1.3

$$\frac{-\frac{2b(bB-aC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + (bB-aC)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c
 + d*x]),x]
```

```
[Out] ((-2*b*(b*B - a*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2] + (b*B - a*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a*B*Tan[c + d*x])/(a^2*d)
```

Maple [B] time = 0.061, size = 228, normalized size = 2.3

$$-\frac{B}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{bB}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - \frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + 2\,\frac{b^2B}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)

[Out] -1/a/d*B/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*b*B-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/a/d*B/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*b*B+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.39427, size = 1049, normalized size = 10.6

$$\left[\frac{\left(Cab - Bb^2\right)\sqrt{-a^2+b^2}\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(Ca^3-Ba\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] [1/2*((C*a*b - B*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c)), -1/2*(2*(C*a*b - B*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d

```
*x + c)*log(sin(d*x + c) + 1) + (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x
 + c)*log(-sin(d*x + c) + 1) - 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^
2*b^2)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72007, size = 236, normalized size = 2.38

$$\frac{\frac{(Ca-Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{(Ca-Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}+\frac{2\left(Cab-Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan}{}\right.\right.}{\sqrt{a^2-b^2}a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] ((C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - (C*a - B*b)*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/a^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a) + 2*(C*a*b - B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2))/d
```

3.800 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{2b^2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2B-2abC+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(bB-aC)\tan(c+dx)}{a^2d}+\frac{B\tan(c$$

```
[Out] (-2*b^2*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
3*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2*B + 2*b^2*B - 2*a*b*C)*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - ((b*B - a*C)*Tan[c + d*x])/(a^2*d) + (B*Sec[c + d*x]*Ta
n[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.584607, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2B-2abC+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(bB-aC)\tan(c+dx)}{a^2d}+\frac{B\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c + d*x
]),x]
```



```
[Out] (-2*b^2*(b*B - a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
3*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2*B + 2*b^2*B - 2*a*b*C)*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - ((b*B - a*C)*Tan[c + d*x])/(a^2*d) + (B*Sec[c + d*x]*Ta
n[c + d*x])/(2*a*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(B+C\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2(bB-aC)+aB\cos(c+dx)+bB\cos^2(c+dx)\right)\text{se}}{a+b\cos(c+dx)}}{2a}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \int \frac{\left(a^2B+2b^2I\right.}{}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(b^2(bB-a\right.}{}$$

$$= \frac{\left(a^2B+2b^2B-2abC\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(bB-aC)\tan(c+dx}{a^2d}$$

$$= -\frac{2b^2(bB-aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(a^2B+2b^2B-2abC\right)\text{t}}{2a^3d}$$

Mathematica [B] time = 1.64749, size = 300, normalized size = 2.1

$$\frac{\frac{8b^2(bB-aC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 2\left(a^2B-2abC+2b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\left(a^2B-2abC+2b^2B\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c
 + d*x]),x]
```

```
[Out] ((8*b^2*(b*B - a*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 2*(a^2*B + 2*b^2*B - 2*a*b*C)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 2*(a^2*B + 2*b^2*B - 2*a*b*C)*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (a^2*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a*(-(b*B
) + a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^2*B)/
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(-(b*B) + a*C)*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*a^3*d)
```

Maple [B] time = 0.07, size = 410, normalized size = 2.9

$$\frac{B}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{B}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{bB}{da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{B}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2
/(tan(1/2*d*x+1/2*c)-1)*b*B-1/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/2/a/d*B*ln(tan
(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*b^2*B+1/d*b/a^2*ln(tan(
1/2*d*x+1/2*c)-1)*C-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/2/a/d*B/(tan(1/2*d*x+1/2*c)+1
)^2+1/2/a/d*B/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*b*B-1/a
/d/(tan(1/2*d*x+1/2*c)+1)*C+1/2/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(t
an(1/2*d*x+1/2*c)+1)*b^2*B-1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 27.519, size = 1334, normalized size = 9.33

$$\left[\frac{2\left(Cab^2 - Bb^3\right)\sqrt{-a^2 + b^2}\cos\left(dx + c\right)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right) + \left(Ba^4 - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/4*(2*(C*a*b^2 - B*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
+ b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 a^2)) + (B*a^4 - 2*C*a^3*b + B*a^2*b^2 + 2*C*a*b^3 - 2*B*b^4)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - (B*a^4 - 2*C*a^3*b + B*a^2*b^2 + 2*C*a*b^3 - 2*B
*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*a^4 - B*a^2*b^2 + 2*(C*a
^4 - B*a^3*b - C*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3
*b^2)*d*cos(d*x + c)^2), 1/4*(4*(C*a*b^2 - B*b^3)*sqrt(a^2 - b^2)*arctan(-(
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (B*a^4
 - 2*C*a^3*b + B*a^2*b^2 + 2*C*a*b^3 - 2*B*b^4)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (B*a^4 - 2*C*a^3*b + B*a^2*b^2 + 2*C*a*b^3 - 2*B*b^4)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) + 2*(B*a^4 - B*a^2*b^2 + 2*(C*a^4 - B*a^3*b -
C*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x
 + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.71797, size = 363, normalized size = 2.54

$$\frac{\frac{(Ba^2-2\,Cab+2\,Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{(Ba^2-2\,Cab+2\,Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{4\,(Cab^2-Bb^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\right.}{}\right.\right.}{\sqrt{a^2-b^2}\,a^3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((B*a^2 - 2*C*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (
B*a^2 - 2*C*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 4*(C*a*
b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3) + 2*(B*a*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1/2*d*x + 1/2*c)^3
 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1/2*d*x + 1/2*c) + 2*C*a*tan(1/2*
d*x + 1/2*c) - 2*B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*
a^2))/d
```

3.801 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=263

$$\frac{\left(2a^2bB-3a^3C+2ab^2C-b^3B\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2bB-3a^3C+4ab^2C-3b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(bB-aC}{bd\left(a^2-\right.}$$

```
[Out] -((4*a*b*B - 6*a^2*C - b^2*C)*x)/(2*b^4) + (2*a^2*(2*a^2*b*B - 3*b^3*B - 3*
a^3*C + 4*a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*b*B - b^3*B - 3*a^3*C + 2*a*b^2*C
)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*b*B - 3*a^2*C + b^2*C)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.749419, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2989, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(2a^2bB-3a^3C+2ab^2C-b^3B\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2bB-3a^3C+4ab^2C-3b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(bB-aC}{bd\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x
])^2,x]
```


```
[Out] -((4*a*b*B - 6*a^2*C - b^2*C)*x)/(2*b^4) + (2*a^2*(2*a^2*b*B - 3*b^3*B - 3*
a^3*C + 4*a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*b*B - b^3*B - 3*a^3*C + 2*a*b^2*C
)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*b*B - 3*a^2*C + b^2*C)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \int \frac{\cos(c+dx)\left(-2a(bB-aC)+b(bB-a\ldots\right.}{\ldots}$$

$$= -\frac{\left(2abB-3a^2C+b^2C\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} + \frac{a(bB-aC)\cos^2\ldots}{b\left(a^2-b^2\right)d(\ldots}$$

$$= \frac{\left(2a^2bB-b^3B-3a^3C+2ab^2C\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2abB-3a^2C+b^2\ldots\right.}{2b^2\ldots}$$

$$= -\frac{\left(4abB-6a^2C-b^2C\right)x}{2b^4} + \frac{\left(2a^2bB-b^3B-3a^3C+2ab^2C\right)\sin(c\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4abB-6a^2C-b^2C\right)x}{2b^4} + \frac{\left(2a^2bB-b^3B-3a^3C+2ab^2C\right)\sin(c\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4abB-6a^2C-b^2C\right)x}{2b^4} + \frac{2a^2\left(2a^2bB-3b^3B-3a^3C+4ab^2C\right)\mathrm{t}\ldots}{(a-b)^{3/2}b^4(a+b)\ldots}$$

Mathematica [A] time = 1.02155, size = 184, normalized size = 0.7

$$\frac{2(c+dx)\left(6a^2C-4abB+b^2C\right)-\frac{8a^2\left(-2a^2bB+3a^3C-4ab^2C+3b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{4a^3b(bB-aC)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+4b(bB-2aC}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c
 + d*x])^2,x]
```

```
[Out] (2*(-4*a*b*B + 6*a^2*C + b^2*C)*(c + d*x) - (8*a^2*(-2*a^2*b*B + 3*b^3*B +
3*a^3*C - 4*a*b^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(3/2) + 4*b*(b*B - 2*a*C)*Sin[c + d*x] + (4*a^3*b*(b*B - a*C)*
Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*C*Sin[2*(c + d*x
)])/(4*b^4*d)
```

Maple [B] time = 0.043, size = 643, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-4/d/b^3/(tan(1/2*
d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)
^2*tan(1/2*d*x+1/2*c)^3*C+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/
2*c)*B-4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C+1/d/b^2/(t
an(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C-4/d/b^3*arctan(tan(1/2*d*x+1/
2*c))*a*B+6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2*C+1/d/b^2*arctan(tan(1/2*d
*x+1/2*c))*C+2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C+4/d*a^4/b^3/(a+b)/(a-b
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
B-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B-6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+8/d*a^3/b^2/(a+b)/(a-b)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04238, size = 2120, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*((6*C*a^6*b - 4*B*a^5*b^2 - 11*C*a^4*b^3 + 8*B*a^3*b^4 + 4*C*a^2*b^5 -
 4*B*a*b^6 + C*b^7)*d*x*cos(d*x + c) + (6*C*a^7 - 4*B*a^6*b - 11*C*a^5*b^2
+ 8*B*a^4*b^3 + 4*C*a^3*b^4 - 4*B*a^2*b^5 + C*a*b^6)*d*x - (3*C*a^6 - 2*B*a
^5*b - 4*C*a^4*b^2 + 3*B*a^3*b^3 + (3*C*a^5*b - 2*B*a^4*b^2 - 4*C*a^3*b^3 +
 3*B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*C
*a^6*b - 4*B*a^5*b^2 - 10*C*a^4*b^3 + 6*B*a^3*b^4 + 4*C*a^2*b^5 - 2*B*a*b^6
 - (C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7)*cos(d*x + c)^2 + (3*C*a^5*b^2 - 2*B*a^
4*b^3 - 6*C*a^3*b^4 + 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^
6 + a*b^8)*d), 1/2*((6*C*a^6*b - 4*B*a^5*b^2 - 11*C*a^4*b^3 + 8*B*a^3*b^4 +
 4*C*a^2*b^5 - 4*B*a*b^6 + C*b^7)*d*x*cos(d*x + c) + (6*C*a^7 - 4*B*a^6*b -
 11*C*a^5*b^2 + 8*B*a^4*b^3 + 4*C*a^3*b^4 - 4*B*a^2*b^5 + C*a*b^6)*d*x - 2*
(3*C*a^6 - 2*B*a^5*b - 4*C*a^4*b^2 + 3*B*a^3*b^3 + (3*C*a^5*b - 2*B*a^4*b^2
 - 4*C*a^3*b^3 + 3*B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^6*b - 4*B*a^5*b^2 -
10*C*a^4*b^3 + 6*B*a^3*b^4 + 4*C*a^2*b^5 - 2*B*a*b^6 - (C*a^4*b^3 - 2*C*a^2
*b^5 + C*b^7)*cos(d*x + c)^2 + (3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*C*a^3*b^4 + 4
*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2
```

```
*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40385, size = 456, normalized size = 1.73

$$\frac{\frac{4\left(3\,Ca^5-2\,Ba^4b-4\,Ca^3b^2+3\,Ba^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{4\left(Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/2*(4*(3*C*a^5 - 2*B*a^4*b - 4*C*a^3*b^2 + 3*B*a^2*b^3)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - 4*(C
*a^4*tan(1/2*d*x + 1/2*c) - B*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*C*a^2 -
 4*B*a*b + C*b^2)*(d*x + c)/b^4 - 2*(4*C*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*b*t
an(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 + 4*C*a*tan(1/2*d*x + 1/
2*c) - 2*B*b*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^2*b^3))/d
```

3.802 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{2a\left(a^2bB-2a^3C+3ab^2C-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(bB-aC)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(bB-2aC)}{b^3}+\frac{C\sin(}{b^2}$$

```
[Out] ((b*B - 2*a*C)*x)/b^3 - (2*a*(a^2*b*B - 2*b^3*B - 2*a^3*C + 3*a*b^2*C)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^
(3/2)*d) + (C*Sin[c + d*x])/(b^2*d) - (a^2*(b*B - a*C)*Sin[c + d*x])/(b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.468126, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3029, 2988, 3023, 2735, 2659, 205}

$$-\frac{2a\left(a^2bB-2a^3C+3ab^2C-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(bB-aC)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(bB-2aC)}{b^3}+\frac{C\sin(}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])
^2,x]
```

```
[Out] ((b*B - 2*a*C)*x)/b^3 - (2*a*(a^2*b*B - 2*b^3*B - 2*a^3*C + 3*a*b^2*C)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^
(3/2)*d) + (C*Sin[c + d*x])/(b^2*d) - (a^2*(b*B - a*C)*Sin[c + d*x])/(b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= -\frac{a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{ab(bB-aC)+\left(a^2-b^2\right)(bB-aC)\cos(c+}{a+b\cos(c+}}{b^2\left(a^2-\right.}$$

$$= \frac{C\sin(c+dx)}{b^2d} - \frac{a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{ab^2(bB-aC)+b(a}{a+}}{b^3}$$

$$= \frac{(bB-2aC)x}{b^3} + \frac{C\sin(c+dx)}{b^2d} - \frac{a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} -$$

$$= \frac{(bB-2aC)x}{b^3} + \frac{C\sin(c+dx)}{b^2d} - \frac{a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} -$$

$$= \frac{(bB-2aC)x}{b^3} - \frac{2a\left(a^2bB-2b^3B-2a^3C+3ab^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+}}\right.}{(a-b)^{3/2}b^3(a+b)^{3/2}d}$$

Mathematica [A] time = 0.788902, size = 147, normalized size = 0.95

$$\frac{\frac{2a\left(-a^2bB+2a^3C-3ab^2C+2b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{a^2b(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + (c+dx)(bB-2aC) + bC\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```

```
[Out] ((b*B - 2*a*C)*(c + d*x) + (2*a*(-(a^2*b*B) + 2*b^3*B + 2*a^3*C - 3*a*b^2*C
)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2)
+ b*C*Sin[c + d*x] + (a^2*b*(-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(
a + b*Cos[c + d*x])))/(b^3*d)
```

Maple [B] time = 0.041, size = 445, normalized size = 2.9

$$2\,\frac{C\tan(1/2\,dx+c/2)}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)} + 2\,\frac{B\arctan(\tan(1/2\,dx+c/2))}{db^2} - 4\,\frac{C\arctan(\tan(1/2\,dx+c/2))\,a}{db^3} - 2\,\frac{}{db\left(a^2-b^2\right)\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*C/b^2*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/b^2*B*arctan(tan(
1/2*d*x+1/2*c))-4/d/b^3*C*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^2/b/(a^2-b^2)*
tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B+2/
d/b^2*a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)*C-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4/d*a^4/b^3/(a+
b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*C-6/d/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97066, size = 1701, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] [-1/2*(2*(2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B
*b^6)*d*x*cos(d*x + c) + 2*(2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 +
 2*C*a^2*b^4 - B*a*b^5)*d*x + (2*C*a^5 - B*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^
3 + (2*C*a^4*b - B*a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt(-a
```

```
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-
a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*C*a^5*b - B*a^4*b^2 - 3*C*a^3*b^3
+ B*a^2*b^4 + C*a*b^5 + (C*a^4*b^2 - 2*C*a^2*b^4 + C*b^6)*cos(d*x + c))*sin
(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b
^5 + a*b^7)*d), -((2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*
a*b^5 - B*b^6)*d*x*cos(d*x + c) + (2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^
3*b^3 + 2*C*a^2*b^4 - B*a*b^5)*d*x - (2*C*a^5 - B*a^4*b - 3*C*a^3*b^2 + 2*B
*a^2*b^3 + (2*C*a^4*b - B*a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (2*C*a^5*b - B*a^4*b^2 - 3*C*a^3*b^3 + B*a^2*b^4 + C*a*b^5 + (C*a^4*b^2
 - 2*C*a^2*b^4 + C*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 +
 b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49013, size = 502, normalized size = 3.24

$$-\frac{\frac{2\left(2\,Ca^4-Ba^3b-3\,Ca^2b^2+2\,Bab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\left(2\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] -(2*(2*C*a^4 - B*a^3*b - 3*C*a^2*b^2 + 2*B*a*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(2*C*a^3*
```

```
tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*tan(1/2*d
*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + C*b^3*tan(1/2*d*x + 1/2*c)
^3 + 2*C*a^3*tan(1/2*d*x + 1/2*c) - B*a^2*b*tan(1/2*d*x + 1/2*c) + C*a^2*b*
tan(1/2*d*x + 1/2*c) - C*a*b^2*tan(1/2*d*x + 1/2*c) - C*b^3*tan(1/2*d*x + 1
/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) + (2*C*a - B*b)*(d*x + c)/b^3)/d
```

3.803 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=122

$$-\frac{2\left(a^3C-2ab^2C+b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(bB-aC)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Cx}{b^2}$$

```
[Out] (C*x)/b^2 - (2*(b^3*B + a^3*C - 2*a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(b*B - a*C)*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.1736, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3021, 2735, 2659, 205}

$$-\frac{2\left(a^3C-2ab^2C+b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(bB-aC)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Cx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (C*x)/b^2 - (2*(b^3*B + a^3*C - 2*a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(b*B - a*C)*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{a(bB-aC)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b(bB-aC)-\left(a^2-b^2\right)C\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{Cx}{b^2} + \frac{a(bB-aC)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(b^3B+a^3C-2ab^2C\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= \frac{Cx}{b^2} + \frac{a(bB-aC)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(b^3B+a^3C-2ab^2C\right)\right)\text{Subst}\left(\int \frac{}{a+b}\right.}{b^2\left(a^2-b^2\right.}$$

$$= \frac{Cx}{b^2} - \frac{2\left(b^3B+a^3C-2ab^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} + \frac{a(bB-aC)\sin(c}{b\left(a^2-b^2\right)d(a+b\cos}$$

Mathematica [A] time = 0.543648, size = 119, normalized size = 0.98

$$\frac{-\frac{2\left(a^3C-2ab^2C+b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{ab(bB-aC)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + C(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (C*(c + d*x) - (2*(b^3*B + a^3*C - 2*a*b^2*C)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (a*b*(b*B - a*C)*Sin[c + d*x]
)/((a - b)*(a + b)*(a + b*Cos[c + d*x]))))/(b^2*d)
```

Maple [B] time = 0.036, size = 320, normalized size = 2.6

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{db^2}+2\,\frac{a\tan\left(1/2\,dx+c/2\right)B}{d\left(a^2-b^2\right)\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^2-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b+a+b\right)}-2\,\frac{}{db\left(a^2-b^2\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*C+2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d/b*a^2/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-2/d*b/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*B-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79058, size = 1210, normalized size = 9.92

$$\left[\frac{2\left(Ca^4b-2\,Ca^2b^3+Cb^5\right)dx\cos\left(dx+c\right)+2\left(Ca^5-2\,Ca^3b^2+Cab^4\right)dx-\left(Ca^4-2\,Ca^2b^2+Bab^3+\left(Ca^3b-2\,Cab^3+\right.\right.}{2\left(\left(a^4b^3-2\,a^2b^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(2*(C*a^4*b - 2*C*a^2*b^3 + C*b^5)*d*x*cos(d*x + c) + 2*(C*a^5 - 2*C*a
^3*b^2 + C*a*b^4)*d*x - (C*a^4 - 2*C*a^2*b^2 + B*a*b^3 + (C*a^3*b - 2*C*a*b
^3 + B*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(C*a
^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5
 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((C*a^4*b - 2*C*
a^2*b^3 + C*b^5)*d*x*cos(d*x + c) + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*d*x - (
C*a^4 - 2*C*a^2*b^2 + B*a*b^3 + (C*a^3*b - 2*C*a*b^3 + B*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - (C*a^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*sin(d*x + c))/((a^4*b^3 -
2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33685, size = 269, normalized size = 2.2

$$\frac{\frac{2\left(Ca^3-2\,Cab^2+Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{(dx+c)C}{b^2}-\frac{2\left(Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(C*a^3 - 2*C*a*b^2 + B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*C/b^2 - 2*(C*a^2*tan
(1/2*d*x + 1/2*c) - B*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d
*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

3.804 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aB-bC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bB-aC)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (2*(a*B - b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - ((b*B - a*C)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 0.153697, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3029, 2754, 12, 2659, 205}

$$\frac{2(aB-bC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bB-aC)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])
^2,x]
```

```
[Out] (2*(a*B - b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - ((b*B - a*C)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*
Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{B+C\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$=-\frac{(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{-aB+bC}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}$$

$$=-\frac{(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(aB-bC)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$

$$=-\frac{(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(2(aB-bC))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x}\right.}{\left(a^2-b^2\right)}$$

$$=\frac{2(aB-bC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx)}$$

Mathematica [A] time = 0.321673, size = 97, normalized size = 0.97

$$\frac{\frac{2(aB-bC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c +
 d*x])^2,x]
```

```
[Out] ((2*(a*B - b*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c
 + d*x])))/d
```

Maple [B] time = 0.055, size = 234, normalized size = 2.3

$$-2\,\frac{\tan(1/2\,dx+c/2)\,bB}{d\left(a^2-b^2\right)\left(a\,(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,dx+c/2))^2\,b+a+b\right)}+2\,\frac{\tan(1/2\,dx+c/2)\,aC}{d\left(a^2-b^2\right)\left(a\,(\tan(1/2\,dx+c/2))^2-(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)*b*B+2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)*a*C+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d/(a+b)/(a-b)/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67439, size = 846, normalized size = 8.46

$$\left[-\frac{\left(Ba^2 - Cab + \left(Bab - Cb^2\right)\cos\left(dx + c\right)\right)\sqrt{-a^2 + b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\left(\left(a^4b - 2\,a^2b^3 + b^5\right)d\cos\left(dx + c\right) + \left(a^5 - 2\,a^3b^2 + ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] [-1/2*((B*a^2 - C*a*b + (B*a*b - C*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(
(2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*
cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + a^2)) - 2*(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*sin(d*x + c))/(
(a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), ((
B*a^2 - C*a*b + (B*a*b - C*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (C*a^3 - B*a^2*b - C*a*b^
2 + B*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 -
 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\cos(c + dx))\cos(c + dx)\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a + b*cos(c + d*x)
)**2, x)
```

Giac [A] time = 1.38674, size = 212, normalized size = 2.12

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ba-Cb)}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(B*a - C*b)/(a^2 - b^
2)^(3/2) + (C*a*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/((a*tan(1/
2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

3.805 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{2\left(2a^2bB+a^3(-C)-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] (-2*(2*a^2*b*B - b^3*B - a^3*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (B*ArcTanh[Sin[c + d*x]])/(a
^2*d) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.385672, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 3000, 3001, 3770, 2659, 205}

$$-\frac{2\left(2a^2bB+a^3(-C)-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x
])^2,x]
```

```
[Out] (-2*(2*a^2*b*B - b^3*B - a^3*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (B*ArcTanh[Sin[c + d*x]])/(a
^2*d) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(\left(a^2-b^2\right)B-a(bB-aC)\cos(c+dx)\right)\sec(c}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{B\int \sec(c+dx)\,dx}{a^2} - \frac{\left(2a^2bB-\right.}{}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(2a\right.\right.}{}$$

$$= -\frac{2\left(2a^2bB-b^3B-a^3C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{B\tanh^{-1}(\sin}{a^2d}$$

Mathematica [A] time = 0.588287, size = 191, normalized size = 1.44

$$\frac{\cos(c+dx)(B\sec(c+dx)+C)\left(\frac{2\left(-2a^2bB+a^3C+b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{ab(bB-aC)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.}{a^2d(B+C\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c
 + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(C + B*Sec[c + d*x])*((2*(-2*a^2*b*B + b^3*B + a^3*C)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - B*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (a*b*(b*B - a*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]
))))/(a^2*d*(B + C*Cos[c + d*x]))
```

Maple [B] time = 0.074, size = 342, normalized size = 2.6

$$-\frac{B}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\,\frac{b^2\tan(1/2\,dx+c/2)\,B}{da\left(a^2-b^2\right)\left(a\left(\tan(1/2\,dx+c/2)\right)^2-\left(\tan(1/2\,dx+c/2)\right)^2b+a+b\right)}-2\,\frac{}{d\left(a^2-b^2\right)\left(a\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*B*ln(tan(1/2*d*x+1/2*c)-1)+2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d*b/(a^2-b^2)*tan(1
/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-4/d*b/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*B+2/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*b^3*B+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/a^2*B*ln(tan(1/2*d
*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 29.9143, size = 1534, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*((C*a^4 - 2*B*a^3*b + B*a*b^3 + (C*a^3*b - 2*B*a^2*b^2 + B*b^4)*cos(d*
x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (B*a^5 - 2*B*a^3*b^2 + B
*a*b^4 + (B*a^4*b - 2*B*a^2*b^3 + B*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1
) - (B*a^5 - 2*B*a^3*b^2 + B*a*b^4 + (B*a^4*b - 2*B*a^2*b^3 + B*b^5)*cos(d*
```

```
x + c))*log(-sin(d*x + c) + 1) - 2*(C*a^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b
^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d), 1/2*(2*(C*a^4 - 2*B*a^3*b + B*a*b^3 + (C*a^3*b - 2*B
*a^2*b^2 + B*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4 + (B*a^4
*b - 2*B*a^2*b^3 + B*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) - (B*a^5 - 2*
B*a^3*b^2 + B*a*b^4 + (B*a^4*b - 2*B*a^2*b^3 + B*b^5)*cos(d*x + c))*log(-si
n(d*x + c) + 1) - 2*(C*a^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*sin(d*x + c
))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34965, size = 304, normalized size = 2.29

$$-\frac{\frac{2\left(Ca^3-2\,Ba^2b+Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -(2*(C*a^3 - 2*B*a^2*b + B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) - B*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^2 + B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(C*a*b*tan(1/2*d
*x + 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1
```

```
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

3.806 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{2b\left(3a^2bB-2a^3C+ab^2C-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2B+abC-2b^2B\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b(bB-aC)\tan(c+}{ad\left(a^2-b^2\right)(a+b\cos(}$$

```
[Out] (2*b*(3*a^2*b*B - 2*b^3*B - 2*a^3*C + a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*b*B - a*C
)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((a^2*B - 2*b^2*B + a*b*C)*Tan[c + d*x])
/(a^2*(a^2 - b^2)*d) + (b*(b*B - a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x]))
```

Rubi [A] time = 0.788006, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 3000, 3055, 3001, 3770, 2659, 205}

$$\frac{2b\left(3a^2bB-2a^3C+ab^2C-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2B+abC-2b^2B\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b(bB-aC)\tan(c+}{ad\left(a^2-b^2\right)(a+b\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x
])^2,x]
```



```
[Out] (2*b*(3*a^2*b*B - 2*b^3*B - 2*a^3*C + a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*b*B - a*C
)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((a^2*B - 2*b^2*B + a*b*C)*Tan[c + d*x])
/(a^2*(a^2 - b^2)*d) + (b*(b*B - a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2B-2b^2B+abC-a(bB-aC)\cos(c+dx)\cdots\right.}{a+b\cos(\cdots}}{a\left(a^2-\cdots\right.}$$

$$= \frac{\left(a^2B-2b^2B+abC\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx)}\cdots$$

$$= \frac{\left(a^2B-2b^2B+abC\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx)}\cdots$$

$$= -\frac{(2bB-aC)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2B-2b^2B+abC\right)\tan(c+\cdots}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b\left(3a^2bB-2b^3B-2a^3C+ab^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(2b\cdots}{}$$

Mathematica [A] time = 1.79234, size = 240, normalized size = 1.27

$$\frac{-\frac{2b\left(-3a^2bB+2a^3C-ab^2C+2b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{ab^2(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + aB\tan(c+dx) - aC\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c
 + d*x])^2,x]
```

```
[Out] ((-2*b*(-3*a^2*b*B + 2*b^3*B + 2*a^3*C - a*b^2*C)*ArcTanh[((a - b)*Tan[(c +
 d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 2*b*B*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - a*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b*
B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a*C*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (a*b^2*(-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])) + a*B*Tan[c + d*x])/(a^3*d)
```

Maple [B] time = 0.075, size = 502, normalized size = 2.7

$$-\frac{B}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)-1\right)bB}{da^3}-\frac{C}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\,\frac{}{da^2\left(a^2-b^2\right)\left(a\left(\tan(1/\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*B/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*b*B-1/d/
a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B+2/d*b^2/a/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C+6/d*b^
2/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*B-4/d*b^4/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*b+2/d*b^3/a^2/(a+b)
/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*C-1/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b*
B+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 76.5396, size = 2419, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [-1/2*(((2*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4 + 2*B*b^5)*cos(d*x + c)^2 + (2
*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b
^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 +
b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2)) - ((C*a^5*b - 2*B*a^4*b^2 - 2*C*a^3*b^3 + 4*B*a
^2*b^4 + C*a*b^5 - 2*B*b^6)*cos(d*x + c)^2 + (C*a^6 - 2*B*a^5*b - 2*C*a^4*b
^2 + 4*B*a^3*b^3 + C*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*log(sin(d*x + c) +
1) + ((C*a^5*b - 2*B*a^4*b^2 - 2*C*a^3*b^3 + 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^
6)*cos(d*x + c)^2 + (C*a^6 - 2*B*a^5*b - 2*C*a^4*b^2 + 4*B*a^3*b^3 + C*a^2*
b^4 - 2*B*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(B*a^6 - 2*B*a^4*
b^2 + B*a^2*b^4 + (B*a^5*b + C*a^4*b^2 - 3*B*a^3*b^3 - C*a^2*b^4 + 2*B*a*b^
5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c
)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)), -1/2*(2*((2*C*a^3*b^2 -
3*B*a^2*b^3 - C*a*b^4 + 2*B*b^5)*cos(d*x + c)^2 + (2*C*a^4*b - 3*B*a^3*b^2
- C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
 c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((C*a^5*b - 2*B*a^4*b^2 - 2*C*a^
3*b^3 + 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*cos(d*x + c)^2 + (C*a^6 - 2*B*a^5*
b - 2*C*a^4*b^2 + 4*B*a^3*b^3 + C*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*log(si
n(d*x + c) + 1) + ((C*a^5*b - 2*B*a^4*b^2 - 2*C*a^3*b^3 + 4*B*a^2*b^4 + C*a
*b^5 - 2*B*b^6)*cos(d*x + c)^2 + (C*a^6 - 2*B*a^5*b - 2*C*a^4*b^2 + 4*B*a^3
*b^3 + C*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(B*a
^6 - 2*B*a^4*b^2 + B*a^2*b^4 + (B*a^5*b + C*a^4*b^2 - 3*B*a^3*b^3 - C*a^2*b
^4 + 2*B*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*
d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4668, size = 545, normalized size = 2.88

$$\frac{\frac{2\left(2Ca^3b-3Ba^2b^2-Cab^3+2Bb^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}-\frac{2\left(Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-B\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] (2*(2*C*a^3*b - 3*B*a^2*b^2 - C*a*b^3 + 2*B*b^4)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) - 2*(B*a^3*tan
(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x
+ 1/2*c)^3 - C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^3*tan(1/2*d*x + 1/2*c)^
3 + B*a^3*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c) - B*a*b^2*tan
(1/2*d*x + 1/2*c) + C*a*b^2*tan(1/2*d*x + 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + (C*a - 2*B*b)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1))/a^3 - (C*a - 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)
/d
```

3.807 $\int \frac{\cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=398

$$\frac{\left(-11a^2b^3B+21a^3b^2C+6a^4bB-12a^5C-6ab^4C+2b^5B\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-15a^2b^3B+29a^3b^2C+6a^4bB-12a^5C-20a\right.}{b^5d(a-b)^{5/2}(a-}$$

```
[Out] -((6*a*b*B - 12*a^2*C - b^2*C)*x)/(2*b^5) + (a^2*(6*a^4*b*B - 15*a^2*b^3*B
+ 12*b^5*B - 12*a^5*C + 29*a^3*b^2*C - 20*a*b^4*C)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*b
*B - 11*a^2*b^3*B + 2*b^5*B - 12*a^5*C + 21*a^3*b^2*C - 6*a*b^4*C)*Sin[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*b*B - 6*a*b^3*B - 6*a^4*C + 10*a^2*
b^2*C - b^4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(b*B
 - a*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + (a*(2*a^2*b*B - 5*b^3*B - 4*a^3*C + 7*a*b^2*C)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.7654, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2989, 3047, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(-11a^2b^3B+21a^3b^2C+6a^4bB-12a^5C-6ab^4C+2b^5B\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-15a^2b^3B+29a^3b^2C+6a^4bB-12a^5C-20a\right.}{b^5d(a-b)^{5/2}(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x
])^3,x]
```


```
[Out] -((6*a*b*B - 12*a^2*C - b^2*C)*x)/(2*b^5) + (a^2*(6*a^4*b*B - 15*a^2*b^3*B
+ 12*b^5*B - 12*a^5*C + 29*a^3*b^2*C - 20*a*b^4*C)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*b
*B - 11*a^2*b^3*B + 2*b^5*B - 12*a^5*C + 21*a^3*b^2*C - 6*a*b^4*C)*Sin[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*b*B - 6*a*b^3*B - 6*a^4*C + 10*a^2*
b^2*C - b^4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(b*B
 - a*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + (a*(2*a^2*b*B - 5*b^3*B - 4*a^3*C + 7*a*b^2*C)*Cos[c + d*x]^2*Sin[c
```

```
+ d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos^4(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-3a(bB-aC)+2b(b\right.}{}}{}$$

$$= \frac{a(bB-aC)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2bB-5b^3B-4a^3C\right.}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(3a^3bB-6ab^3B-6a^4C+10a^2b^2C-b^4C\right)\cos(c+dx)\sin(c+}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(6a^4bB-11a^2b^3B+2b^5B-12a^5C+21a^3b^2C-6ab^4C\right)\sin(c+}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(6abB-12a^2C-b^2C\right)x}{2b^5} + \frac{\left(6a^4bB-11a^2b^3B+2b^5B-12a^5C\right.}{2b^4\left(a^2-\right.}$$

$$= -\frac{\left(6abB-12a^2C-b^2C\right)x}{2b^5} + \frac{\left(6a^4bB-11a^2b^3B+2b^5B-12a^5C\right.}{2b^4\left(a^2-\right.}$$

$$= -\frac{\left(6abB-12a^2C-b^2C\right)x}{2b^5} + \frac{a^2\left(6a^4bB-15a^2b^3B+12b^5B-12a\right.}{(a-}$$

Mathematica [A] time = 3.48729, size = 734, normalized size = 1.84

$$\frac{16ab\left(a^2-b^2\right)^2(c+dx)\left(12a^2C-6abB+b^2C\right)\cos(c+dx)+4\left(b^3-a^2b\right)^2(c+dx)\left(12a^2C-6abB+b^2C\right)\cos(2(c+dx))+48a^6b^2B\sin(c+dx)+36a^5b^3B\sin(2(c+dx))-84a^4b^4B}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c
 + d*x])^3,x]
```

```
[Out] ((16*a^2*(-6*a^4*b*B + 15*a^2*b^3*B - 12*b^5*B + 12*a^5*C - 29*a^3*b^2*C +
20*a*b^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b
```

```
^2)^(5/2) + (-48*a^7*b*B*c + 72*a^5*b^3*B*c - 24*a*b^7*B*c + 96*a^8*c*C - 1
36*a^6*b^2*c*C - 12*a^4*b^4*c*C + 48*a^2*b^6*c*C + 4*b^8*c*C - 48*a^7*b*B*d
*x + 72*a^5*b^3*B*d*x - 24*a*b^7*B*d*x + 96*a^8*C*d*x - 136*a^6*b^2*C*d*x -
 12*a^4*b^4*C*d*x + 48*a^2*b^6*C*d*x + 4*b^8*C*d*x + 16*a*b*(a^2 - b^2)^2*(
-6*a*b*B + 12*a^2*C + b^2*C)*(c + d*x)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*
(-6*a*b*B + 12*a^2*C + b^2*C)*(c + d*x)*Cos[2*(c + d*x)] + 48*a^6*b^2*B*Sin
[c + d*x] - 84*a^4*b^4*B*Sin[c + d*x] + 8*a^2*b^6*B*Sin[c + d*x] + 4*b^8*B*
Sin[c + d*x] - 96*a^7*b*C*Sin[c + d*x] + 160*a^5*b^3*C*Sin[c + d*x] - 32*a^
3*b^5*C*Sin[c + d*x] - 8*a*b^7*C*Sin[c + d*x] + 36*a^5*b^3*B*Sin[2*(c + d*x
)] - 64*a^3*b^5*B*Sin[2*(c + d*x)] + 16*a*b^7*B*Sin[2*(c + d*x)] - 72*a^6*b
^2*C*Sin[2*(c + d*x)] + 130*a^4*b^4*C*Sin[2*(c + d*x)] - 48*a^2*b^6*C*Sin[2
*(c + d*x)] + 2*b^8*C*Sin[2*(c + d*x)] + 4*a^4*b^4*B*Sin[3*(c + d*x)] - 8*a
^2*b^6*B*Sin[3*(c + d*x)] + 4*b^8*B*Sin[3*(c + d*x)] - 8*a^5*b^3*C*Sin[3*(c
 + d*x)] + 16*a^3*b^5*C*Sin[3*(c + d*x)] - 8*a*b^7*C*Sin[3*(c + d*x)] + a^4
*b^4*C*Sin[4*(c + d*x)] - 2*a^2*b^6*C*Sin[4*(c + d*x)] + b^8*C*Sin[4*(c + d
*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*b^5*d)
```

Maple [B] time = 0.05, size = 1504, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-
b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B
-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a
-b)^2*tan(1/2*d*x+1/2*c)*C-1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+4/d*a^5/b^3/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*B-1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d*a^3/b/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3*B-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+
1)^2*tan(1/2*d*x+1/2*c)*a*C-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+
1/2*c)^3*a*C+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d/b^3*arctan(tan(1/2*d*x+1/2*c)
)*C+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
```

```
)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d/b^3
/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C-6/d/b^4*arctan(tan(1/2*d*x
+1/2*c))*a*B+12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2*C+2/d/b^3/(tan(1/2*d*x
+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*ta
n(1/2*d*x+1/2*c)^3*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*
B+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-12/d*a^7/
b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*C+1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-15/d*a^4/b^2/(a
^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*B+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.92849, size = 4001, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [1/4*(2*(12*C*a^8*b^2 - 6*B*a^7*b^3 - 35*C*a^6*b^4 + 18*B*a^5*b^5 + 33*C*a^
4*b^6 - 18*B*a^3*b^7 - 9*C*a^2*b^8 + 6*B*a*b^9 - C*b^10)*d*x*cos(d*x + c)^2
 + 4*(12*C*a^9*b - 6*B*a^8*b^2 - 35*C*a^7*b^3 + 18*B*a^6*b^4 + 33*C*a^5*b^5
 - 18*B*a^4*b^6 - 9*C*a^3*b^7 + 6*B*a^2*b^8 - C*a*b^9)*d*x*cos(d*x + c) + 2
```

```
*(12*C*a^10 - 6*B*a^9*b - 35*C*a^8*b^2 + 18*B*a^7*b^3 + 33*C*a^6*b^4 - 18*B
*a^5*b^5 - 9*C*a^4*b^6 + 6*B*a^3*b^7 - C*a^2*b^8)*d*x + (12*C*a^9 - 6*B*a^8
*b - 29*C*a^7*b^2 + 15*B*a^6*b^3 + 20*C*a^5*b^4 - 12*B*a^4*b^5 + (12*C*a^7*
b^2 - 6*B*a^6*b^3 - 29*C*a^5*b^4 + 15*B*a^4*b^5 + 20*C*a^3*b^6 - 12*B*a^2*b
^7)*cos(d*x + c)^2 + 2*(12*C*a^8*b - 6*B*a^7*b^2 - 29*C*a^6*b^3 + 15*B*a^5*
b^4 + 20*C*a^4*b^5 - 12*B*a^3*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(12*C*a^9*b - 6*B*a^8*b^2 - 33*C*a^7*b^3 + 17*B*a^6*b^4
+ 27*C*a^5*b^5 - 13*B*a^4*b^6 - 6*C*a^3*b^7 + 2*B*a^2*b^8 - (C*a^6*b^4 - 3*
C*a^4*b^6 + 3*C*a^2*b^8 - C*b^10)*cos(d*x + c)^3 + 2*(2*C*a^7*b^3 - B*a^6*b
^4 - 6*C*a^5*b^5 + 3*B*a^4*b^6 + 6*C*a^3*b^7 - 3*B*a^2*b^8 - 2*C*a*b^9 + B*
b^10)*cos(d*x + c)^2 + (18*C*a^8*b^2 - 9*B*a^7*b^3 - 50*C*a^6*b^4 + 25*B*a^
5*b^5 + 43*C*a^4*b^6 - 20*B*a^3*b^7 - 11*C*a^2*b^8 + 4*B*a*b^9)*cos(d*x + c
))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^
2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5
 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*C*a^8*b^2 - 6*B*a^7*b^3 -
 35*C*a^6*b^4 + 18*B*a^5*b^5 + 33*C*a^4*b^6 - 18*B*a^3*b^7 - 9*C*a^2*b^8 +
6*B*a*b^9 - C*b^10)*d*x*cos(d*x + c)^2 + 2*(12*C*a^9*b - 6*B*a^8*b^2 - 35*C
*a^7*b^3 + 18*B*a^6*b^4 + 33*C*a^5*b^5 - 18*B*a^4*b^6 - 9*C*a^3*b^7 + 6*B*a
^2*b^8 - C*a*b^9)*d*x*cos(d*x + c) + (12*C*a^10 - 6*B*a^9*b - 35*C*a^8*b^2
+ 18*B*a^7*b^3 + 33*C*a^6*b^4 - 18*B*a^5*b^5 - 9*C*a^4*b^6 + 6*B*a^3*b^7 -
C*a^2*b^8)*d*x - (12*C*a^9 - 6*B*a^8*b - 29*C*a^7*b^2 + 15*B*a^6*b^3 + 20*C
*a^5*b^4 - 12*B*a^4*b^5 + (12*C*a^7*b^2 - 6*B*a^6*b^3 - 29*C*a^5*b^4 + 15*B
*a^4*b^5 + 20*C*a^3*b^6 - 12*B*a^2*b^7)*cos(d*x + c)^2 + 2*(12*C*a^8*b - 6*
B*a^7*b^2 - 29*C*a^6*b^3 + 15*B*a^5*b^4 + 20*C*a^4*b^5 - 12*B*a^3*b^6)*cos(
d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c))) - (12*C*a^9*b - 6*B*a^8*b^2 - 33*C*a^7*b^3 + 17*B*a^6*b^4 + 27*
C*a^5*b^5 - 13*B*a^4*b^6 - 6*C*a^3*b^7 + 2*B*a^2*b^8 - (C*a^6*b^4 - 3*C*a^4
*b^6 + 3*C*a^2*b^8 - C*b^10)*cos(d*x + c)^3 + 2*(2*C*a^7*b^3 - B*a^6*b^4 -
6*C*a^5*b^5 + 3*B*a^4*b^6 + 6*C*a^3*b^7 - 3*B*a^2*b^8 - 2*C*a*b^9 + B*b^10)
*cos(d*x + c)^2 + (18*C*a^8*b^2 - 9*B*a^7*b^3 - 50*C*a^6*b^4 + 25*B*a^5*b^5
 + 43*C*a^4*b^6 - 20*B*a^3*b^7 - 11*C*a^2*b^8 + 4*B*a*b^9)*cos(d*x + c))*si
n(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*
a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.62264, size = 1821, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(12*C*a^7 - 6*B*a^6*b - 29*C*a^5*b^2 + 15*B*a^4*b^3 + 20*C*a^3*b^4 -
 12*B*a^2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-
(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b
^5 - 2*a^2*b^7 + b^9)*sqrt(a^2 - b^2)) - 2*(12*C*a^7*tan(1/2*d*x + 1/2*c)^7
 - 6*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 9
*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 9
*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 1
6*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 2
*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4
*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*B*b^
7*tan(1/2*d*x + 1/2*c)^7 + C*b^7*tan(1/2*d*x + 1/2*c)^7 + 36*C*a^7*tan(1/2*
d*x + 1/2*c)^5 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 18*C*a^6*b*tan(1/2*d*x
 + 1/2*c)^5 + 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 67*C*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 35*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*C*a^4*b^3*tan(1/2*d*
x + 1/2*c)^5 - 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 26*C*a^3*b^4*tan(1/2*d
*x + 1/2*c)^5 - 10*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 5*C*a^2*b^5*tan(1/2*d
*x + 1/2*c)^5 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 4*C*a*b^6*tan(1/2*d*x +
1/2*c)^5 + 2*B*b^7*tan(1/2*d*x + 1/2*c)^5 - 3*C*b^7*tan(1/2*d*x + 1/2*c)^5
+ 36*C*a^7*tan(1/2*d*x + 1/2*c)^3 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 18*
C*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*C*
a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*C
*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 26*
C*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*
C*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*C*a
*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^7*tan(1/2*d*x + 1/2*c)^3 + 3*C*b^7*tan(
1/2*d*x + 1/2*c)^3 + 12*C*a^7*tan(1/2*d*x + 1/2*c) - 6*B*a^6*b*tan(1/2*d*x
+ 1/2*c) + 18*C*a^6*b*tan(1/2*d*x + 1/2*c) - 9*B*a^5*b^2*tan(1/2*d*x + 1/2*
c) - 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c) + 9*B*a^4*b^3*tan(1/2*d*x + 1/2*c) -
```

```
 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*
C*a^3*b^4*tan(1/2*d*x + 1/2*c) + 2*B*a^2*b^5*tan(1/2*d*x + 1/2*c) + 13*C*a^
2*b^5*tan(1/2*d*x + 1/2*c) - 4*B*a*b^6*tan(1/2*d*x + 1/2*c) + 4*C*a*b^6*tan
(1/2*d*x + 1/2*c) - 2*B*b^7*tan(1/2*d*x + 1/2*c) - C*b^7*tan(1/2*d*x + 1/2*
c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x
+ 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (12*C*a^2 - 6*B*a*b +
 C*b^2)*(d*x + c)/b^5)/d
```

3.808 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(-3a^2C+abB+2b^2C\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2b^3B+15a^3b^2C+2a^4bB-6a^5C-12ab^4C+6b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a+b}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((b*B - 3*a*C)*x)/b^4 - (a*(2*a^4*b*B - 5*a^2*b^3*B + 6*b^5*B - 6*a^5*C + 1
5*a^3*b^2*C - 12*a*b^4*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*b*B - 3*a^2*C + 2*b^2*C)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^2*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(a^2*b*B - 4*b^3*B - 3*
a^3*C + 6*a*b^2*C)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]
))
```

Rubi [A] time = 1.28441, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2989, 3031, 3023, 2735, 2659, 205}

$$-\frac{\left(-3a^2C+abB+2b^2C\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2b^3B+15a^3b^2C+2a^4bB-6a^5C-12ab^4C+6b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a+b}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x
])^3,x]
```


```
[Out] ((b*B - 3*a*C)*x)/b^4 - (a*(2*a^4*b*B - 5*a^2*b^3*B + 6*b^5*B - 6*a^5*C + 1
5*a^3*b^2*C - 12*a*b^4*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*b*B - 3*a^2*C + 2*b^2*C)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^2*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(a^2*b*B - 4*b^3*B - 3*
a^3*C + 6*a*b^2*C)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]
))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a(bB-aC)+2b(bB\right.}{}}{}$$

$$= \frac{a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(a^2bB-4b^3B-3a^3C+\right.}{2b^3\left(a^2-b^2\right)^2d(a+}$$

$$= -\frac{\left(abB-3a^2C+2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(bB-aC)\cos^2(c+dx)\,\mathrm{si}}{2b\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= \frac{(bB-3aC)x}{b^4} - \frac{\left(abB-3a^2C+2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(bB-aC)\,\mathrm{c}}{2b\left(a^2-b^2\right.}$$

$$= \frac{(bB-3aC)x}{b^4} - \frac{\left(abB-3a^2C+2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(bB-aC)\,\mathrm{c}}{2b\left(a^2-b^2\right.}$$

$$= \frac{(bB-3aC)x}{b^4} - \frac{a\left(2a^4bB-5a^2b^3B+6b^5B-6a^5C+15a^3b^2C-1\right.}{(a-b)^{5/2}b^4(a+b)^{5}}$$

Mathematica [A] time = 2.17692, size = 232, normalized size = 0.83

$$\frac{\frac{a^2b\left(-3a^2bB+5a^3C-8ab^2C+6b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}-\frac{2a\left(5a^2b^3B-15a^3b^2C-2a^4bB+6a^5C+12ab^4C-6b^5B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{a^3b(bB-aC)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c
 + d*x])^3,x]
```

```
[Out] (2*(b*B - 3*a*C)*(c + d*x) - (2*a*(-2*a^4*b*B + 5*a^2*b^3*B - 6*b^5*B + 6*a
^5*C - 15*a^3*b^2*C + 12*a*b^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*C*Sin[c + d*x] + (a^3*b*(b*B - a*C)*S
in[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^2*b*(-3*a^2*b*B
+ 6*b^3*B + 5*a^3*C - 8*a*b^2*C)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*
Cos[c + d*x])))/(2*b^4*d)
```

Maple [B] time = 0.045, size = 1301, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*C/b^3*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/b^3*B*arctan(tan(
1/2*d*x+1/2*c))-6/d*C/b^4*a*arctan(tan(1/2*d*x+1/2*c))-2/d*a^4/b^2/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*B+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*a^2/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*B+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*C-8/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*a^3/b/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*
d*x+1/2*c)*B+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/
```

```
(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+1/d*a^4/b^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*C-8/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^3*C-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*
(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+5/d*a^3
/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*B-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d/b^4/(a^4-2*a^2*b^2+b
^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*a^6*C-15/d/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^4*C+12/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.57718, size = 3380, normalized size = 12.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*C*a^7*b^2 - B*a^6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6
 - 3*B*a^2*b^7 - 3*C*a*b^8 + B*b^9)*d*x*cos(d*x + c)^2 + 8*(3*C*a^8*b - B*a
^7*b^2 - 9*C*a^6*b^3 + 3*B*a^5*b^4 + 9*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^
7 + B*a*b^8)*d*x*cos(d*x + c) + 4*(3*C*a^9 - B*a^8*b - 9*C*a^7*b^2 + 3*B*a^
6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^3*b^6 + B*a^2*b^7)*d*x - (6*C*a^8
 - 2*B*a^7*b - 15*C*a^6*b^2 + 5*B*a^5*b^3 + 12*C*a^4*b^4 - 6*B*a^3*b^5 + (6
```

```
*C*a^6*b^2 - 2*B*a^5*b^3 - 15*C*a^4*b^4 + 5*B*a^3*b^5 + 12*C*a^2*b^6 - 6*B*
a*b^7)*cos(d*x + c)^2 + 2*(6*C*a^7*b - 2*B*a^6*b^2 - 15*C*a^5*b^3 + 5*B*a^4
*b^4 + 12*C*a^3*b^5 - 6*B*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(6*C*a^8*b - 2*B*a^7*b^2 - 17*C*a^6*b^3 + 7*B*a^5*b^4 +
13*C*a^4*b^5 - 5*B*a^3*b^6 - 2*C*a^2*b^7 + 2*(C*a^6*b^3 - 3*C*a^4*b^5 + 3*C
*a^2*b^7 - C*b^9)*cos(d*x + c)^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 - 25*C*a^5*b^
4 + 9*B*a^4*b^5 + 20*C*a^3*b^6 - 6*B*a^2*b^7 - 4*C*a*b^8)*cos(d*x + c))*sin
(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*
(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^
6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(2*(3*C*a^7*b^2 - B*a^6*b^3 - 9*C*a^
5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 - 3*C*a*b^8 + B*b^9)*d*x*co
s(d*x + c)^2 + 4*(3*C*a^8*b - B*a^7*b^2 - 9*C*a^6*b^3 + 3*B*a^5*b^4 + 9*C*a
^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^7 + B*a*b^8)*d*x*cos(d*x + c) + 2*(3*C*a^9
 - B*a^8*b - 9*C*a^7*b^2 + 3*B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^
3*b^6 + B*a^2*b^7)*d*x - (6*C*a^8 - 2*B*a^7*b - 15*C*a^6*b^2 + 5*B*a^5*b^3
+ 12*C*a^4*b^4 - 6*B*a^3*b^5 + (6*C*a^6*b^2 - 2*B*a^5*b^3 - 15*C*a^4*b^4 +
5*B*a^3*b^5 + 12*C*a^2*b^6 - 6*B*a*b^7)*cos(d*x + c)^2 + 2*(6*C*a^7*b - 2*B
*a^6*b^2 - 15*C*a^5*b^3 + 5*B*a^4*b^4 + 12*C*a^3*b^5 - 6*B*a^2*b^6)*cos(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (6*C*a^8*b - 2*B*a^7*b^2 - 17*C*a^6*b^3 + 7*B*a^5*b^4 + 13*C*a^4
*b^5 - 5*B*a^3*b^6 - 2*C*a^2*b^7 + 2*(C*a^6*b^3 - 3*C*a^4*b^5 + 3*C*a^2*b^7
 - C*b^9)*cos(d*x + c)^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 - 25*C*a^5*b^4 + 9*B*
a^4*b^5 + 20*C*a^3*b^6 - 6*B*a^2*b^7 - 4*C*a*b^8)*cos(d*x + c))*sin(d*x + c
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 +
3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44441, size = 733, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -((6*C*a^6 - 2*B*a^5*b - 15*C*a^4*b^2 + 5*B*a^3*b^3 + 12*C*a^2*b^4 - 6*B*a*
b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*C*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*C*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*B*a^4*b^2*tan
(1/2*d*x + 1/2*c)^3 - 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*b^3*tan(
1/2*d*x + 1/2*c)^3 + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*B*a^2*b^4*tan(1
/2*d*x + 1/2*c)^3 + 4*C*a^6*tan(1/2*d*x + 1/2*c) - 2*B*a^5*b*tan(1/2*d*x +
1/2*c) + 5*C*a^5*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b^2*tan(1/2*d*x + 1/2*c)
- 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c) + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*C
*a^3*b^3*tan(1/2*d*x + 1/2*c) + 6*B*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^3
 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 +
a + b)^2) + (3*C*a - B*b)*(d*x + c)/b^4 - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

3.809 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-6ab^4C+2b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(bB-aC)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-3a^3\right.}{2b^2d\left(a^2\right.}$$

```
[Out] (C*x)/b^3 + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3*b^2*C - 6*a*b^4*C)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^
(5/2)*d) - (a^2*(b*B - a*C)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (a*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*a*b^2*C)*Sin[c + d*x])/(2
*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.615256, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3029, 2988, 3021, 2735, 2659, 205}

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-6ab^4C+2b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(bB-aC)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-3a^3\right.}{2b^2d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])
^3,x]
```

```
[Out] (C*x)/b^3 + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3*b^2*C - 6*a*b^4*C)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^
(5/2)*d) - (a^2*(b*B - a*C)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (a*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*a*b^2*C)*Sin[c + d*x])/(2
*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$=-\frac{a^2(bB-aC)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{\int \frac{2ab(bB-aC)+\left(a^2-2b^2\right)(bB-aC)\,\mathrm{co}}{(a+b\cos}}{2b^2\left(a\right.}$$

$$=-\frac{a^2(bB-aC)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-4b^3B-3a^3C+6a\right.}{2b^2\left(a^2-b^2\right)^2d(a+b\,\mathrm{c}}$$

$$=\frac{Cx}{b^3}-\frac{a^2(bB-aC)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-4b^3B-3a^3C\right.}{2b^2\left(a^2-b^2\right)^2d(a}$$

$$=\frac{Cx}{b^3}-\frac{a^2(bB-aC)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-4b^3B-3a^3C\right.}{2b^2\left(a^2-b^2\right)^2d(a}$$

$$=\frac{Cx}{b^3}+\frac{\left(a^2b^3B+2b^5B-2a^5C+5a^3b^2C-6ab^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^3(a+b)^{5/2}d}$$

Mathematica [A] time = 1.38075, size = 204, normalized size = 0.97

$$\frac{\frac{ab\left(a^2bB-3a^3C+6ab^2C-4b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}+\frac{2\left(-a^2b^3B-5a^3b^2C+2a^5C+6ab^4C-2b^5B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{a^2b(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+2C(c+a}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^3,x]
```

```
[Out] (2*C*(c + d*x) + (2*(-(a^2*b^3*B) - 2*b^5*B + 2*a^5*C - 5*a^3*b^2*C + 6*a*b
^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + (a^2*b*(-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d
*x])^2) + (a*b*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*a*b^2*C)*Sin[c + d*x])/((a
- b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^3*d)
```

Maple [B] time = 0.042, size = 1023, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*C-1/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d*b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d/b/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*C+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2*C+1/d*a^2/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-
4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a-b)^2
*tan(1/2*d*x+1/2*c)*B-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d/b/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^3*C+6/d
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/
2*d*x+1/2*c)*a^2*C+1/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*b^2/(a^4-2*a^2*b^2+b^4)/
((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-
2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-6/d*b*a/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0715, size = 2462, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] [1/4*(4*(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8)*d*x*cos(d*x + c)^2
+ 8*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*d*x*cos(d*x + c) + 4*(C
*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*d*x + (2*C*a^7 - 5*C*a^5*b^2
- B*a^4*b^3 + 6*C*a^3*b^4 - 2*B*a^2*b^5 + (2*C*a^5*b^2 - 5*C*a^3*b^4 - B*a^
2*b^5 + 6*C*a*b^6 - 2*B*b^7)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 -
B*a^3*b^4 + 6*C*a^2*b^5 - 2*B*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + a^2)) - 2*(2*C*a^7*b - 7*C*a^5*b^3 + 3*B*a^4*b^4 + 5*C*a^3*b^5 -
 3*B*a^2*b^6 + (3*C*a^6*b^2 - B*a^5*b^3 - 9*C*a^4*b^4 + 5*B*a^3*b^5 + 6*C*a
^2*b^6 - 4*B*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^
10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2
*(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8)*d*x*cos(d*x + c)^2 + 4*(C*
a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*d*x*cos(d*x + c) + 2*(C*a^8 -
3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*d*x - (2*C*a^7 - 5*C*a^5*b^2 - B*a^4
*b^3 + 6*C*a^3*b^4 - 2*B*a^2*b^5 + (2*C*a^5*b^2 - 5*C*a^3*b^4 - B*a^2*b^5 +
 6*C*a*b^6 - 2*B*b^7)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 - B*a^3*b
^4 + 6*C*a^2*b^5 - 2*B*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*C*a^7*b - 7*C*a^5*b^3 +
3*B*a^4*b^4 + 5*C*a^3*b^5 - 3*B*a^2*b^6 + (3*C*a^6*b^2 - B*a^5*b^3 - 9*C*a^
4*b^4 + 5*B*a^3*b^5 + 6*C*a^2*b^6 - 4*B*a*b^7)*cos(d*x + c))*sin(d*x + c))/
((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3
*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^
4*b^7 - a^2*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32382, size = 614, normalized size = 2.91

$$-\frac{\frac{\left(2\,Ca^5-5\,Ca^3b^2-Ba^2b^3+6\,Cab^4-2\,Bb^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{a^2-b^2}}-\frac{(dx+c)C}{b^3}+\frac{2\,Ca^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca^4b\,\mathrm{t}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] -((2*C*a^5 - 5*C*a^3*b^2 - B*a^2*b^3 + 6*C*a*b^4 - 2*B*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
 b^2)) - (d*x + c)*C/b^3 + (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^4*b*tan(
1/2*d*x + 1/2*c)^3 + B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 + 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 - 4*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*C*a^5*tan(1/2*d*x + 1
/2*c) + 3*C*a^4*b*tan(1/2*d*x + 1/2*c) - B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5
*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 6*C*a^
2*b^3*tan(1/2*d*x + 1/2*c) + 4*B*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*
a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^2))/d
```

3.810 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(a^2(-C)+3abB-2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2bB+a^3C-4ab^2C+2b^3B\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a(bB-aC)\sin(c}{2bd\left(a^2-b^2\right)(a+b\,\mathrm{co}}$$

```
[Out] -(((3*a*b*B - a^2*C - 2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(b*B - a*C)*Sin[c + d*x])/(2*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*b*B + 2*b^3*B + a^3*C - 4*a
*b^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.228082, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3021, 2754, 12, 2659, 205}

$$-\frac{\left(a^2(-C)+3abB-2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2bB+a^3C-4ab^2C+2b^3B\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a(bB-aC)\sin(c}{2bd\left(a^2-b^2\right)(a+b\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((3*a*b*B - a^2*C - 2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(b*B - a*C)*Sin[c + d*x])/(2*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*b*B + 2*b^3*B + a^3*C - 4*a
*b^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{a(bB-aC)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b(bB-aC)-\left(abB+a^2C-2b^2C\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(bB-aC)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} -$$

$$= \frac{a(bB-aC)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} -$$

$$= \frac{a(bB-aC)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} -$$

$$= -\frac{\left(3abB-a^2C-2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(bB-aC)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+d$$

Mathematica [A] time = 0.820671, size = 172, normalized size = 0.96

$$\frac{\frac{\left(a^2bB+a^3C-4ab^2C+2b^3B\right)\sin(c+dx)}{b(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(a^2C-3abB+2b^2C\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a(bB-aC)\sin(c+dx)}{b(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*b*B + a^2*C + 2*b^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*(b*B - a*C)*Sin[c + d*x])/((a - b)*b*(
a + b)*(a + b*Cos[c + d*x])^2) + ((a^2*b*B + 2*b^3*B + a^3*C - 4*a*b^2*C)*S
in[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.036, size = 886, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*b^2*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2*C-4/d/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*a*b*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(
a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*b*B+2/d/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c)*b^2*B+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*C-4/d/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)*a*b*C-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+2/d/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87706, size = 1616, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*((C*a^4 - 3*B*a^3*b + 2*C*a^2*b^2 + (C*a^2*b^2 - 3*B*a*b^3 + 2*C*b^4)
*cos(d*x + c)^2 + 2*(C*a^3*b - 3*B*a^2*b^2 + 2*C*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5 - 3*C*a^4*b - B*a^3*b^2 +
3*C*a^2*b^3 - B*a*b^4 + (C*a^5 + B*a^4*b - 5*C*a^3*b^2 + B*a^2*b^3 + 4*C*a*
b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((C*a^4 - 3*B*a
^3*b + 2*C*a^2*b^2 + (C*a^2*b^2 - 3*B*a*b^3 + 2*C*b^4)*cos(d*x + c)^2 + 2*(
C*a^3*b - 3*B*a^2*b^2 + 2*C*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*B*a^5 - 3*C*a^4*b -
 B*a^3*b^2 + 3*C*a^2*b^3 - B*a*b^4 + (C*a^5 + B*a^4*b - 5*C*a^3*b^2 + B*a^2
*b^3 + 4*C*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42876, size = 528, normalized size = 2.93

$$\frac{\frac{\left(Ca^2-3\,Bab+2\,Cb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((C*a^2 - 3*B*a*b + 2*C*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - C*a^3*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*C
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d
*x + 1/2*c) + C*a^3*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c) - 3
*C*a^2*b*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) - 4*C*a*b^2*ta
n(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.811 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{\left(2a^2B-3abC+b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-C)+3abB-2b^2C\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(bB-aC)\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

```
[Out] ((2*a^2*B + b^2*B - 3*a*b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((b*B - a*C)*Sin[c + d*x])/(2*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*a*b*B - a^2*C - 2*b^2*C)*Sin[c + d*x
])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.253487, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3029, 2754, 12, 2659, 205}

$$\frac{\left(2a^2B-3abC+b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-C)+3abB-2b^2C\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(bB-aC)\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])
^3,x]
```

```
[Out] ((2*a^2*B + b^2*B - 3*a*b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((b*B - a*C)*Sin[c + d*x])/(2*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*a*b*B - a^2*C - 2*b^2*C)*Sin[c + d*x
])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{B+C\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{(bB-aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2(aB-bC)+(bB-aC)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= -\frac{(bB-aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3abB-a^2C-2b^2C)\sin(c+a}{2(a^2-b^2)^2\,d(a+b\cos(c+dx}$$

$$= -\frac{(bB-aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3abB-a^2C-2b^2C)\sin(c+a}{2(a^2-b^2)^2\,d(a+b\cos(c+dx}$$

$$= -\frac{(bB-aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3abB-a^2C-2b^2C)\sin(c+a}{2(a^2-b^2)^2\,d(a+b\cos(c+dx}$$

$$= \frac{(2a^2B+b^2B-3abC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(bB-aC)\sin}{2(a^2-b^2)\,d(a+b}$$

Mathematica [A] time = 0.653494, size = 157, normalized size = 0.96

$$\frac{\frac{(a^2C-3abB+2b^2C)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2(2a^2B-3abC+b^2B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{5/2}} + \frac{(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c +
 d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*B + b^2*B - 3*a*b*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a
+ b)*(a + b*Cos[c + d*x])^2) + ((-3*a*b*B + a^2*C + 2*b^2*C)*Sin[c + d*x])/
((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.059, size = 886, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B+2/d/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*a^2*C+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*b*C+2/d/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*b^2*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*b*B+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b^2*B+2/d/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)*a^2*C-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*b*C+2/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)*C+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-3/d*b*a/
(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00056, size = 1616, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] [-1/4*((2*B*a^4 - 3*C*a^3*b + B*a^2*b^2 + (2*B*a^2*b^2 - 3*C*a*b^3 + B*b^4)
*cos(d*x + c)^2 + 2*(2*B*a^3*b - 3*C*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*C*a^5 - 4*B*a^4*b - C*a^3*b^2 +
5*B*a^2*b^3 - C*a*b^4 - B*b^5 + (C*a^4*b - 3*B*a^3*b^2 + C*a^2*b^3 + 3*B*a*
b^4 - 2*C*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((2*B*a^4 - 3*C
*a^3*b + B*a^2*b^2 + (2*B*a^2*b^2 - 3*C*a*b^3 + B*b^4)*cos(d*x + c)^2 + 2*(
2*B*a^3*b - 3*C*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*C*a^5 - 4*B*a^4*b -
 C*a^3*b^2 + 5*B*a^2*b^3 - C*a*b^4 - B*b^5 + (C*a^4*b - 3*B*a^3*b^2 + C*a^2
*b^3 + 3*B*a*b^4 - 2*C*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35438, size = 527, normalized size = 3.21

$$\frac{\frac{\left(2\,Ba^2-3\,Cab+Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, al
gorithm="giac")

[Out] ((2*B*a^2 - 3*C*a*b + B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*C*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 3*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*b^3*
tan(1/2*d*x + 1/2*c)^3 - 2*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*C*a^3*tan(1/2*d
*x + 1/2*c) - 4*B*a^2*b*tan(1/2*d*x + 1/2*c) + C*a^2*b*tan(1/2*d*x + 1/2*c)
 - 3*B*a*b^2*tan(1/2*d*x + 1/2*c) + C*a*b^2*tan(1/2*d*x + 1/2*c) + B*b^3*ta
n(1/2*d*x + 1/2*c) + 2*C*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.812 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(-5a^2b^3B-a^3b^2C+6a^4bB-2a^5C+2b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2bB-3a^3C-2b^3B\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b(}{2ad\left(a^2\right.}$$

```
[Out] -(((6*a^4*b*B - 5*a^2*b^3*B + 2*b^5*B - 2*a^5*C - a^3*b^2*C)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d))
+ (B*ArcTanh[Sin[c + d*x]])/(a^3*d) + (b*(b*B - a*C)*Sin[c + d*x])/(2*a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(5*a^2*b*B - 2*b^3*B - 3*a^3*C)*Sin
[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.801641, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-5a^2b^3B-a^3b^2C+6a^4bB-2a^5C+2b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2bB-3a^3C-2b^3B\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b(}{2ad\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x
])^3,x]
```


```
[Out] -(((6*a^4*b*B - 5*a^2*b^3*B + 2*b^5*B - 2*a^5*C - a^3*b^2*C)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d))
+ (B*ArcTanh[Sin[c + d*x]])/(a^3*d) + (b*(b*B - a*C)*Sin[c + d*x])/(2*a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(5*a^2*b*B - 2*b^3*B - 3*a^3*C)*Sin
[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2-b^2\right)B-2a(bB-aC)\cos(c+dx)+\ldots}{(a+b\cos(\ldots}}{2a\left(a^2\ldots}$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2bB-2b^3B-3a^3C\right)\sin(\ldots}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c\ldots}$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2bB-2b^3B-3a^3C\right)\sin(\ldots}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c\ldots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5\ldots}{2a\ldots}$$

$$= -\frac{\left(6a^4bB-5a^2b^3B+2b^5B-2a^5C-a^3b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx\ldots}{\sqrt{a+b}}\ldots}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 1.27408, size = 269, normalized size = 1.26

$$\frac{\cos(c+dx)(B\sec(c+dx)+C)\left(\frac{ab\left(5a^2bB-3a^3C-2b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(5a^2b^3B+a^3b^2C-6a^4bB+2a^5C-2b^5B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a^2b(b\ldots}{(a-b)(a+\ldots}\ldots}{2a^3d(B+C\cos(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c
 + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(C + B*Sec[c + d*x])*((-2*(-6*a^4*b*B + 5*a^2*b^3*B - 2*b^5*B
 + 2*a^5*C + a^3*b^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(-a^2 + b^2)^(5/2) - 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*B*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*(b*B - a*C)*Sin[c + d*x])
/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b*(5*a^2*b*B - 2*b^3*B - 3*a
^3*C)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/(2*a^3*d*(
B + C*Cos[c + d*x]))
```

Maple [B] time = 0.084, size = 1045, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*B*ln(tan(1/2*d*x+1/2*c)-1)+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B+1/d/a/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*B-2/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*a*b*C-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C+6/d/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1
/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a-b)^
2*tan(1/2*d*x+1/2*c)*B-2/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-4/d*a/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b*C+1/d/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2
*d*x+1/2*c)*b^2*C-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+5/d/a/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^3*B
-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*b^5*B+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+1/d/(a^4-2*a^2
*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
```

```
^(1/2))*b^2*C+1/d/a^3*B*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 140.4, size = 3032, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [1/4*((2*C*a^7 - 6*B*a^6*b + C*a^5*b^2 + 5*B*a^4*b^3 - 2*B*a^2*b^5 + (2*C*a
^5*b^2 - 6*B*a^4*b^3 + C*a^3*b^4 + 5*B*a^2*b^5 - 2*B*b^7)*cos(d*x + c)^2 +
2*(2*C*a^6*b - 6*B*a^5*b^2 + C*a^4*b^3 + 5*B*a^3*b^4 - 2*B*a*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^
2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(B*a^8 - 3*B*a^6*b^2 + 3*
B*a^4*b^4 - B*a^2*b^6 + (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*cos
(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c
))*log(sin(d*x + c) + 1) - 2*(B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6
 + (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*cos(d*x + c)^2 + 2*(B*a^
7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c))*log(-sin(d*x + c)
+ 1) - 2*(4*C*a^7*b - 6*B*a^6*b^2 - 5*C*a^5*b^3 + 9*B*a^4*b^4 + C*a^3*b^5 -
 3*B*a^2*b^6 + (3*C*a^6*b^2 - 5*B*a^5*b^3 - 3*C*a^4*b^4 + 7*B*a^3*b^5 - 2*B
*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3
*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos
(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), 1/2*((2*C*a^7 - 6*
B*a^6*b + C*a^5*b^2 + 5*B*a^4*b^3 - 2*B*a^2*b^5 + (2*C*a^5*b^2 - 6*B*a^4*b^
3 + C*a^3*b^4 + 5*B*a^2*b^5 - 2*B*b^7)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 6*B*
```

```
a^5*b^2 + C*a^4*b^3 + 5*B*a^3*b^4 - 2*B*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2
)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^8 - 3
*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6 + (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b
^6 - B*b^8)*cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b
^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^
4 - B*a^2*b^6 + (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*cos(d*x + c
)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c))*log(-
sin(d*x + c) + 1) - (4*C*a^7*b - 6*B*a^6*b^2 - 5*C*a^5*b^3 + 9*B*a^4*b^4 +
C*a^3*b^5 - 3*B*a^2*b^6 + (3*C*a^6*b^2 - 5*B*a^5*b^3 - 3*C*a^4*b^4 + 7*B*a^
3*b^5 - 2*B*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^
5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42158, size = 649, normalized size = 3.03

$$\frac{\frac{\left(2\,Ca^5-6\,Ba^4b+Ca^3b^2+5\,Ba^2b^3-2\,Bb^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,d\right.\right.\right.}{a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] ((2*C*a^5 - 6*B*a^4*b + C*a^3*b^2 + 5*B*a^2*b^3 - 2*B*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
```

```
b^2)) + B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - B*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (4*C*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*B*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 - 3*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^2*b^3*tan(1/2*d
*x + 1/2*c)^3 - C*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*B*a*b^4*tan(1/2*d*x +
1/2*c)^3 - 2*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*C*a^4*b*tan(1/2*d*x + 1/2*c)
- 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*C*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*B
*a^2*b^3*tan(1/2*d*x + 1/2*c) - C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*B*a*b^4*
tan(1/2*d*x + 1/2*c) + 2*B*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.813 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=299

$$\frac{b\left(-15a^2b^3B+5a^3b^2C+12a^4bB-6a^5C-2ab^4C+6b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2b^2B+5a^3bC+2a^4B-2ab\cdots\right.}{2a^3d\left(a^2-b\cdots\right.}$$

```
[Out] (b*(12*a^4*b*B - 15*a^2*b^3*B + 6*b^5*B - 6*a^5*C + 5*a^3*b^2*C - 2*a*b^4*C
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a
 + b)^(5/2)*d) - ((3*b*B - a*C)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((2*a^4*B
- 11*a^2*b^2*B + 6*b^4*B + 5*a^3*b*C - 2*a*b^3*C)*Tan[c + d*x])/(2*a^3*(a^2
 - b^2)^2*d) + (b*(b*B - a*C)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (b*(6*a^2*b*B - 3*b^3*B - 4*a^3*C + a*b^2*C)*Tan[c + d*x])/(2
*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.80346, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 3000, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-15a^2b^3B+5a^3b^2C+12a^4bB-6a^5C-2ab^4C+6b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2b^2B+5a^3bC+2a^4B-2ab\cdots\right.}{2a^3d\left(a^2-b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x
])^3,x]
```

```
[Out] (b*(12*a^4*b*B - 15*a^2*b^3*B + 6*b^5*B - 6*a^5*C + 5*a^3*b^2*C - 2*a*b^4*C
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a
 + b)^(5/2)*d) - ((3*b*B - a*C)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((2*a^4*B
- 11*a^2*b^2*B + 6*b^4*B + 5*a^3*b*C - 2*a*b^3*C)*Tan[c + d*x])/(2*a^3*(a^2
 - b^2)^2*d) + (b*(b*B - a*C)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (b*(6*a^2*b*B - 3*b^3*B - 4*a^3*C + a*b^2*C)*Tan[c + d*x])/(2
*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$=\frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{\int \frac{\left(2a^2B-3b^2B+abC-2a(bB-aC)\cos\right.}{(a+}}{2a}$$

$$=\frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{b\left(6a^2bB-3b^3B-4a^3C+a\right.}{2a^2\left(a^2-b^2\right)^2d(a+b}$$

$$=\frac{\left(2a^4B-11a^2b^2B+6b^4B+5a^3bC-2ab^3C\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d}+\frac{}{2a\left(a\right.}$$

$$=\frac{\left(2a^4B-11a^2b^2B+6b^4B+5a^3bC-2ab^3C\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d}+\frac{}{2a\left(a\right.}$$

$$=-\frac{(3bB-aC)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{\left(2a^4B-11a^2b^2B+6b^4B+5\right.}{2a^3\left(a^2\right.}$$

$$=\frac{b\left(12a^4bB-15a^2b^3B+6b^5B-6a^5C+5a^3b^2C-2ab^4C\right)\tan^{-1}\left(\right.}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 5.85741, size = 352, normalized size = 1.18

$$\frac{\frac{ab^2\left(-7a^2bB+5a^3C-2ab^2C+4b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}+\frac{a^2b^2(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}-\frac{2b\left(-15a^2b^3B+5a^3b^2C+12a^4bB-6a^5C-2ab^4C+6b^5B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c
 + d*x])^3,x]
```

```
[Out] ((-2*b*(12*a^4*b*B - 15*a^2*b^3*B + 6*b^5*B - 6*a^5*C + 5*a^3*b^2*C - 2*a*b
^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + 2*(3*b*B - a*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-3*b*B
+ a*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*B*Sin[(c + d*x)/2])/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*a*B*Sin[(c + d*x)/2])/(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + (a^2*b^2*(-(b*B) + a*C)*Sin[c + d*x])/((a - b
)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b^2*(-7*a^2*b*B + 4*b^3*B + 5*a^3*C
- 2*a*b^2*C)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*a
^4*d)
```

Maple [B] time = 0.091, size = 1358, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*B/(tan(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*b*B-1/d/
a^3*ln(tan(1/2*d*x+1/2*c)-1)*C-8/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/a^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*
d*x+1/2*c)^3*b^2*C+1/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d*b^4/a^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*C-8/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b
```

```
^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+4/d*b^5/a^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/
2*d*x+1/2*c)*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b^2*C-1/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-2/d*b^4/a^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*C+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6
/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/d*b^3/a/(a^4-2
*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*C-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/d/a^3*B/(tan(1/2*d*x+1/2*c)
+1)-3/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*b*B+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44892, size = 775, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] ((6*C*a^5*b - 12*B*a^4*b^2 - 5*C*a^3*b^3 + 15*B*a^2*b^4 + 2*C*a*b^5 - 6*B*b
^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2
+ a^4*b^4)*sqrt(a^2 - b^2)) + (6*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*B*a^3
*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*B*a^2*
b^4*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*B*a*b^5
*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^6*tan(1/
2*d*x + 1/2*c)^3 + 6*C*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*B*a^3*b^3*tan(1/2*d
*x + 1/2*c) + 5*C*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*B*a^2*b^4*tan(1/2*d*x +
1/2*c) - 3*C*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*B*a*b^5*tan(1/2*d*x + 1/2*c)
- 2*C*a*b^5*tan(1/2*d*x + 1/2*c) + 4*B*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*
a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^2) + (C*a - 3*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (C*a - 3*
B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*B*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

3.814 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c\right.$

Optimal. Leaf size=303

$$-\frac{2\left(-8a^2C+14abB-25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2\left(a^2-b^2\right)\left(-8a^2C+14abB-25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - 19*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + (2*(a^2 - b^2)*(14*a*b*B - 8*a^2*C - 25*b^2*C)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt
[a + b*Cos[c + d*x]]) - (2*(14*a*b*B - 8*a^2*C - 25*b^2*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*b*B - 4*a*C)*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.604872, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3029, 2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2C+14abB-25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2\left(a^2-b^2\right)\left(-8a^2C+14abB-25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```


```
[Out] (-2*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - 19*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + (2*(a^2 - b^2)*(14*a*b*B - 8*a^2*C - 25*b^2*C)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt
[a + b*Cos[c + d*x]]) - (2*(14*a*b*B - 8*a^2*C - 25*b^2*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*b*B - 4*a*C)*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(B+C\cos(c$$

$$= \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd}$$

$$= \frac{2(7bB-4aC)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d}$$

$$= -\frac{2\left(14abB-8a^2C-25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^2d}$$

$$= -\frac{2\left(14abB-8a^2C-25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^2d}$$

$$= -\frac{2\left(14abB-8a^2C-25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^2d}$$

$$= -\frac{2\left(14a^2bB-63b^3B-8a^3C-19ab^2C\right)\sqrt{a+b\,c}}{105b^3d\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

Mathematica [A] time = 0.981228, size = 232, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(-16a^2C+28abB+115b^2C\right)\sin(c+dx)+3b(2(aC+7bB)\sin(2(c+dx))+5bC\sin(3(c+dx)))\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(49*a*b*B + 2*a^2*C + 25*b^2*C)*
EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C +
 19*a*b^2*C)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((28*a*b*B - 16*a^2*C
 + 115*b^2*C)*Sin[c + d*x] + 3*b*(2*(7*b*B + a*C)*Sin[2*(c + d*x)] + 5*b*C*
Sin[3*(c + d*x)])))/(210*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.746, size = 1305, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-144*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(112*B*a*b^3+168*B*b^4-4*C*a^2*b
^2+144*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*B*a^
2*b^2-56*B*a*b^3-42*B*b^4+8*C*a^3*b+2*C*a^2*b^2-86*C*a*b^3-80*C*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b-14*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3-14*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*a^3*b+14*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a^2*b^2+
63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a*b^3-63*B*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^4-8*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^4-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b+19*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-19*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*cos(
d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.815 $\int \sqrt{a+b\cos(c+dx)} \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=231

$$-\frac{2\left(a^2-b^2\right)(5bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-2a^2C+5abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(5*a*b*B - 2*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(5*b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*b*B -
2*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*C*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.346391, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(5bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-2a^2C+5abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*a*b*B - 2*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(5*b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*b*B -
2*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*C*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{2\int\sqrt{a+b\cos(c+a}}{}$$

$$=\frac{2(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd}+\frac{2C(a+b\cos}{}$$

$$=\frac{2(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd}+\frac{2C(a+b\cos}{}$$

$$=\frac{2(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd}+\frac{2C(a+b\cos}{}$$

$$=\frac{2\left(5abB-2a^2C+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.857163, size = 179, normalized size = 0.77

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(-2a^2C+5abB+9b^2C\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(7aC+5bB)F\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*b*B + 7*a*C)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (5*a*b*B - 2*a^2*C + 9*b^2*C)*((a + b)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2*b
*(a + b*Cos[c + d*x])*(5*b*B + a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x])/(15*
b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.74, size = 993, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b^3+16*C*a*b^2+24*C*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*B*a*b^2-10*B*b^3-2*C*a^2*b-8*C*a
*b^2-6*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2*b-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^2+2*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
2*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^3)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.816 $\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=171

$$-\frac{2C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aC+3bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sin(c+dx)\sqrt{a}}{3d}$$

```
[Out] (2*(3*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*C*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```

Rubi [A] time = 0.302879, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aC+3bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sin(c+dx)\sqrt{a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```


```
[Out] (2*(3*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*C*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))\,dx$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3a}{}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{\left((a^2-b^2)\right.}{}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left((3bB+a\right.}{}$$

$$= \frac{2(3bB+aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.582878, size = 146, normalized size = 0.85

$$\frac{-2C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(a+b)(aC+3bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bC\sin(c+dx)(a}{3bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```

```
[Out] (2*(a + b)*(3*b*B + a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*C*(a + b*Cos[c + d*x])*Sin[c +
 d*x])/(3*b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.715, size = 600, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*C*cos(1/2*d*x+1/
2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3*b^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*C*cos(1/2*d*x+1/2*c)*a*b+2*C*cos(1/2*d*x+1
/2*c)*b^2)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d
*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c), x)
```

3.817 $\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{2bB\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aB\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.461494, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3029, 3002, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2bB\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aB\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```


```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))\sec(c$$

$$= B\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx + C\int$$

$$= (aB)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + (bB)\int \frac{}{\sqrt{a+b}}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(aB}{}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2bB}{}$$

Mathematica [A] time = 2.38467, size = 107, normalized size = 0.6

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(B\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+C(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*C*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + B*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.791, size = 247, normalized size = 1.4

$$-2\,{\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}}\sqrt{\frac{2\,(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-B*a*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d
*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^2, x)
```

3.818 $\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=213

$$\frac{(aB+2bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aC+bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{B\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

```
[Out] -((B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a*B + 2*b*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + ((b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.706322, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3029, 2999, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(aB+2bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aC+bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{B\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```

```
[Out] -((B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a*B + 2*b*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + ((b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))\sec^2(\ldots$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}(bB+\ldots\right.}{\ldots}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{1}{2}b(bB+2\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}(-bB-2a\ldots$$

$$= -\frac{B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{B\sqrt{a\ldots}}{\ldots}$$

$$= -\frac{B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(aB\ldots}{\ldots}$$

Mathematica [C] time = 10.6428, size = 372, normalized size = 1.75

$$\frac{\frac{2(4aC+bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4B\tan(c+dx)\sqrt{a+b\cos(c+dx)} - \frac{2iB\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sin\ldots\right.\right.\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```

```
[Out] ((8*b*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(b*B + 4*a*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] - ((2*I)*B*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*B*Sqrt[a + b*Cos[c
```

```
 + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 1.405, size = 746, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*a*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2)))-2*(B*b+C*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1
/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^3, x)
```

3.819 $\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=292

$$\frac{\left(4a^2B+4abC-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aC+bB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aC+3bB)\sqrt{}}{4d}$$

```
[Out] -((b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*b*B + 4*a*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt
[a + b*Cos[c + d*x]]) + ((4*a^2*B - b^2*B + 4*a*b*C)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*C
os[c + d*x]]) + ((b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*
d) + (B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 1.0475, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 2999, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2B+4abC-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aC+bB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aC+3bB)\sqrt{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```



```
[Out] -((b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*b*B + 4*a*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt
[a + b*Cos[c + d*x]]) + ((4*a^2*B - b^2*B + 4*a*b*C)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*C
os[c + d*x]]) + ((b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*
d) + (B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))\sec^3($$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} +$$

$$= \frac{(bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{B}{}$$

$$= \frac{(bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{B}{}$$

$$= \frac{(bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{B}{}$$

$$= -\frac{(bB+4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+}\right.}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(bB+4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+}\right.}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 4.28936, size = 420, normalized size = 1.44

$$\frac{\frac{2\left(8a^2B+4abC-3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}}-\frac{2i(4aC+bB)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right.\right.\right.\right.}{a^2b}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```

```
[Out] ((8*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*B - 3*b^2*B + 4*a*b*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*Sq
rt[a + b*Cos[c + d*x]]) - ((2*I)*(b*B + 4*a*C)*Sqrt[-((b*(-1 + Cos[c + d*x]
))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*b*Sqrt[-(a
+ b)^(-1)]) + (4*Sqrt[a + b*Cos[c + d*x]]*(2*a*B + (b*B + 4*a*C)*Cos[c + d*
x])*Sec[c + d*x]*Tan[c + d*x])/a)/(16*d)
```

Maple [B] time = 1.334, size = 1290, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(B*b+C*a)
*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*C*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2))+2*B*a*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/
a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
```

```
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^4, x)
```

3.820 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=378

$$-\frac{2\left(-8a^2C+18abB-49b^2C\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2bB-8a^3C-39ab^2C-75b^3B\right)\sin(c+dx)\sqrt{a+}}{315b^2d}$$

```
[Out] (-2*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 33*a^2*b^2*C - 147*b^4*C)*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*b*B - 75*b^3*B - 8*a^
3*C - 39*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(18*a^2*b*B - 75
*b^3*B - 8*a^3*C - 39*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*
b^2*d) - (2*(18*a*b*B - 8*a^2*C - 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(315*b^2*d) + (2*(9*b*B - 4*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(63*b^2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(9*b*d)
```

Rubi [A] time = 0.788744, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3029, 2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2C+18abB-49b^2C\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2bB-8a^3C-39ab^2C-75b^3B\right)\sin(c+dx)\sqrt{a+}}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] (-2*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 33*a^2*b^2*C - 147*b^4*C)*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*b*B - 75*b^3*B - 8*a^
3*C - 39*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(18*a^2*b*B - 75
*b^3*B - 8*a^3*C - 39*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*
b^2*d) - (2*(18*a*b*B - 8*a^2*C - 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(315*b^2*d) + (2*(9*b*B - 4*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(63*b^2*d) + (2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c +
```

```
d*x])/(9*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
```

```
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(B+C\,c$$

$$= \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+}{9bd}$$

$$= \frac{2(9bB-4aC)(a+b\cos(c+dx))^{5/2}\sin(c+d}{63b^2d}$$

$$= -\frac{2\left(18abB-8a^2C-49b^2C\right)(a+b\cos(c+a}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-39ab^2C\right)\sqrt{a+}}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-39ab^2C\right)\sqrt{a+}}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-39ab^2C\right)\sqrt{a+}}{315b^2d}$$

$$= -\frac{2\left(18a^3bB-246ab^3B-8a^4C-33a^2b^2C-\right.}{315b^3d\sqrt{}}$$

Mathematica [A] time = 1.56573, size = 291, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(72a^2bB-32a^3C+804ab^2C+690b^3B\right)\sin(c+dx)+b\left(2\left(6a^2C+144abB+133b^2C\right)\sin(2(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(153*a^2*b*B + 75*b^3*B + 2*a^3*
C + 186*a*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-18*a^3*b*B + 246
*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*((a + b)*EllipticE[(c + d*x)
/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*C
os[c + d*x])*((72*a^2*b*B + 690*b^3*B - 32*a^3*C + 804*a*b^2*C)*Sin[c + d*x
] + b*(2*(144*a*b*B + 6*a^2*C + 133*b^2*C)*Sin[2*(c + d*x)] + 5*b*(2*(9*b*B
```

```
 + 10*a*C)*Sin[3*(c + d*x)] + 7*b*C*Sin[4*(c + d*x)]))))/(1260*b^3*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [B] time = 0.79, size = 1635, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^5+1360*C*a*b^4+2240*
C*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-936*B*a*b^4-1080*B*b^5-424
*C*a^2*b^3-2040*C*a*b^4-2072*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(324*B*a^2*b^3+936*B*a*b^4+840*B*b^5-4*C*a^3*b^2+424*C*a^2*b^3+1568*C*a*b^
4+952*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*B*a^3*b^2-162*B*a
^2*b^3-384*B*a*b^4-240*B*b^5+8*C*a^4*b+2*C*a^3*b^2-282*C*a^2*b^3-444*C*a*b^
4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-18*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+246*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^2*b^3-246*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a*b^4+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^4*b-93*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^3+75*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-8*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+33*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-33*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b^4-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
```

```
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*b^5-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^5-31*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39
*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4)/b^3/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*co
s(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{4}+Ba\cos\left(dx+c\right)^{2}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + B*a*cos(d*x + c)^2 + (C*a + B*b)*cos(d*x + c
)^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*co
s(d*x + c), x)
```

3.821 $\int (a+b\cos(c+dx))^{3/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=297

$$\frac{2\left(-6a^2C+21abB+25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2C+21abB+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 82*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(21*a*b*B - 6*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(21*a*b*B - 6*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(35*b*d) + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(7*b*d)
```

Rubi [A] time = 0.454838, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-6a^2C+21abB+25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2C+21abB+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 82*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(21*a*b*B - 6*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(21*a*b*B - 6*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(35*b*d) + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(7*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{2C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{2\int(a+b\cos(c+$$

$$=\frac{2(7bB-2aC)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd}+\frac{2C(a$$

$$=\frac{2\left(21abB-6a^2C+25b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{105bd}$$

$$=\frac{2\left(21abB-6a^2C+25b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{105bd}$$

$$=\frac{2\left(21abB-6a^2C+25b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+d}{105bd}$$

$$=\frac{2\left(21a^2bB+63b^3B-6a^3C+82ab^2C\right)\sqrt{a+b\cos(c+dx}}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.09261, size = 233, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(\left(12a^2C+168abB+115b^2C\right)\sin(c+dx)+3b(2(8aC+7bB)\sin(2(c+dx))+5bC\sin(3(c+dx)))\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(84*a*b*B + 51*a^2*C + 25*b^2*C)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (21*a^2*b*B + 63*b^3*B - 6*a^3*C +
 82*a*b^2*C)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((168*a*b*B + 12*a^2*
C + 115*b^2*C)*Sin[c + d*x] + 3*b*(2*(7*b*B + 8*a*C)*Sin[2*(c + d*x)] + 5*b
*C*Sin[3*(c + d*x)])))/(210*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.85, size = 1305, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-312*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(252*B*a*b^3+168*B*b^4+108*C*a^2
*b^2+312*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-84*B*
a^2*b^2-126*B*a*b^3-42*B*b^4-6*C*a^3*b-54*C*a^2*b^2-128*C*a*b^3-80*C*b^4)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+21*B*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*a^3*b-21*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*a^2*b^2+63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*a*b^3-63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b
^4-21*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+21*B*a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-6*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+6*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^2*b^2-82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a*b^3+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4-31*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b
^2+25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^2/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3 + Ba\cos(dx+c) + (Ca+Bb)\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*a*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.822 $\int (a+b\cos(c+dx))^{3/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=225

$$-\frac{2\left(a^2-b^2\right)(3aC+5bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2C+20abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^
2 - b^2)*(5*b*B + 3*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*b*B + 3*a
*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.448775, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(3aC+5bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2C+20abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```


```
[Out] (2*(20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^
2 - b^2)*(5*b*B + 3*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*b*B + 3*a
*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int (a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))\,d$$

$$= \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2\left(20abB+3a^2C+9b^2C\right)\sqrt{a+b\cos(c+dx}}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.768727, size = 203, normalized size = 0.9

$$\frac{2\left(b\left(15a^2B+12abC+5b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(3a^2C+20abB+9b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+\right.\right.\right.}{15bd\sqrt{a+b\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (2*(b*(15*a^2*B + 5*b^2*B + 12*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)] + (20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
 - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]) + b*(a + b*Cos[c + d*x])*(5*b*B
 + 6*a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[a + b*Cos[c + d*
x]])
```

Maple [B] time = 0.703, size = 993, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b^3+36*C*a*b^2+24*C*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*B*a*b^2-10*B*b^3-12*C*a^2*b-18*C
*a*b^2-6*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*b^3*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^2-3*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))+3*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-3*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3 + Ba\cos(dx+c) + (Ca+Bb)\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*a*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c), x)
```

3.823 $\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{2\left(a^2(-C)+3abB+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aC+3bB)\sqrt{a+}}{3a}$$

```
[Out] (2*(3*b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a*b*B - a^2*C + b^
2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*B*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*b*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.833, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3029, 2990, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(a^2(-C)+3abB+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aC+3bB)\sqrt{a+}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```


```
[Out] (2*(3*b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a*b*B - a^2*C + b^
2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*B*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*b*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))\,s$$

$$= \frac{2bC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int$$

$$= \frac{2bC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2\int}{}$$

$$= \frac{2bC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \left(a^2B\right)$$

$$= \frac{2(3bB+4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(3bB+4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.38276, size = 406, normalized size = 1.72

$$\frac{\frac{4\left(3a^2C+6abB+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(6a^2B+4abC+3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i(4aC+3bB)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] ((4*(6*a*b*B + 3*a^2*C + b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^2*B + 3*b
^2*B + 4*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*b*B + 4*a*C)*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Cs
c[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*b*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
```

```
x])/(6*d)
```

Maple [B] time = 0.67, size = 738, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+2*C*cos(1/2*d*x+1/2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3
*b^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2-4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-
2*C*cos(1/2*d*x+1/2*c)*a*b+2*C*cos(1/2*d*x+1/2*c)*b^2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ba\cos\left(dx+c\right)+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*a*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^2, x)
```

3.824 $\int (a+b\cos(c+dx))^{3/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=232

$$\frac{\left(a^2B+2abC+2b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aB-2bC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aC+3bB)}{}$$

```
[Out] -(((a*B - 2*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2*B + 2*b^2*B + 2*a*b*C
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (a*B*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.805116, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3029, 2989, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2B+2abC+2b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aB-2bC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aC+3bB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```



```
[Out] -(((a*B - 2*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2*B + 2*b^2*B + 2*a*b*C
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (a*B*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int (a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))\,\mathrm{s}$$

$$= \frac{aB\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}a\right.}{}$$

$$= \frac{aB\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{1}{2}a\right.}{}}{}$$

$$= \frac{aB\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}(a(3b$$

$$= -\frac{(aB-2bC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(aB-2bC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.45338, size = 398, normalized size = 1.72

$$\frac{\frac{2\left(4a^2C+5abB+2b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{8b(2aC+bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i(2bC-aB)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+d}{b-a}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] ((8*b*(b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(5*a*b*B + 4*a^2*C + 2*b^2
*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-(a*B) + 2*b*C)*Sqrt[-((b*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*
(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sq
rt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*
```

```
Sqrt[-(a + b)^(-1)]) + 4*a*B*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 0.705, size = 1167, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*B*a^2-2*B*a*b)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2+2*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-B*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+B*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b-3*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*a*b+2*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+2*C*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*C*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*C*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))*a^2)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2+2*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b+2*C*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b^2-2*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^3+Ba\cos\left(dx+c\right)+\left(Ca+Bb\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*a*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + B\cos{(dx+c)}\right)(b\cos{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^3, x)
```

3.825 $\int (a+b\cos(c+dx))^{3/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=295

$$\frac{\left(4a^2C+7abB+8b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{\left(4a^2B+12abC+3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+$$

```
[Out] -((5*b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((7*a*b*B + 4*a^2*C + 8*b
^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*B + 3*b^2*B + 12*a*b*C)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4
*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d) + (a*B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rubi [A] time = 1.18185, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 2989, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2C+7abB+8b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{\left(4a^2B+12abC+3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```


```
[Out] -((5*b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((7*a*b*B + 4*a^2*C + 8*b
^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*B + 3*b^2*B + 12*a*b*C)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4
*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*b*B + 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d) + (a*B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int (a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))\,\mathrm{se}$$

$$=\frac{aB\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}$$

$$=\frac{(5bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+$$

$$=\frac{(5bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+$$

$$=\frac{(5bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+$$

$$=-\frac{(5bB+4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{(5bB+4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 4.86122, size = 422, normalized size = 1.43

$$\frac{2\left(8a^2B+20abC+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{8b(aB+4bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] ((8*b*(a*B + 4*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*B + b^2*B + 20*a*b*
C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*b*B + 4*a*C)*Sqrt[-((b*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-
2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sq
rt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*B + (5*b*B + 4*a*C)*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Maple [B] time = 1.904, size = 1403, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b*(B*b+2*C*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+2*a*(2*B*b+C*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
)+2*a^2*B*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1
/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2
*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^4, x)
```

3.826 $\int (a+b\cos(c+dx))^{3/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=375

$$\frac{\left(16a^2B+30abC+3b^2B\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(16a^2B+42abC+17b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-$$

```
[Out] -((16*a^2*B + 3*b^2*B + 30*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a
^2*B + 17*b^2*B + 42*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*b*B -
 b^3*B + 8*a^3*C + 6*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a^
2*B + 3*b^2*B + 30*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) +
 ((7*b*B + 6*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d
) + (a*B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.55105, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 2989, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2B+30abC+3b^2B\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(16a^2B+42abC+17b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```



```
[Out] -((16*a^2*B + 3*b^2*B + 30*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a
^2*B + 17*b^2*B + 42*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*b*B -
 b^3*B + 8*a^3*C + 6*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a^
2*B + 3*b^2*B + 30*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) +
 ((7*b*B + 6*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d
```

```
) + (a*B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int (a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))\,\mathrm{s}$$

$$=\frac{aB\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+}{3d}$$

$$=\frac{(7bB+6aC)\sqrt{a+b\cos(c+dx)}\sec(c+dx)}{12d}$$

$$=\frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\cos(c+dx}}{24ad}$$

$$=\frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\cos(c+dx}}{24ad}$$

$$=\frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\cos(c+dx}}{24ad}$$

$$=-\frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\cos(c+d}}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\cos(c+d}}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.55706, size = 634, normalized size = 1.69

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2B\sin(c+dx)+30abC\sin(c+dx)+3b^2B\sin(c+dx)\right)}{24a}+\frac{1}{12}\sec^2(c+dx)(6aC\sin(c+dx)+7bB\sin(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] ((2*(28*a*b^2*B + 24*a^2*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(56*a^2*b*B - 9*
b^3*B + 48*a^3*C + 6*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2
*b*B - 3*b^3*B - 30*a*b^2*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
 b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*
EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sq
rt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a +
b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b
*Cos[c + d*x])^2)))/(96*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
7*b*B*Sin[c + d*x] + 6*a*C*Sin[c + d*x]))/12 + (Sec[c + d*x]*(16*a^2*B*Sin[
c + d*x] + 3*b^2*B*Sin[c + d*x] + 30*a*b*C*Sin[c + d*x]))/(24*a) + (a*B*Sec
[c + d*x]^2*Tan[c + d*x])/3))/d
```

Maple [B] time = 2.221, size = 2327, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*B*(-1
/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*
```

c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*
b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*b*(B*b+2*C*a)*(-1/a*cos(1/2
*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+2*a*(2*B*b+C*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^
2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^

```
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^5, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*se
c(d*x + c)^5, x)
```

3.827 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2} \left(B\cos(c+dx) + C\cos^2(c+\right.$

Optimal. Leaf size=462

$$-\frac{2\left(-8a^2C+22abB-81b^2C\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2bB-40a^3C-335ab^2C-539b^3B\right)\sin(c+dx)(}{3465b^2d}$$

```
[Out] (-2*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*C - 255*a^3*b^2*C -
 3705*a*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a
^3*b*B - 1254*a*b^3*B - 40*a^4*C - 285*a^2*b^2*C - 675*b^4*C)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) - (2*(110*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 285*a
^2*b^2*C - 675*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) -
 (2*(110*a^2*b*B - 539*b^3*B - 40*a^3*C - 335*a*b^2*C)*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) - (2*(22*a*b*B - 8*a^2*C - 81*b^2*C)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*b*B - 4*a*C)*(a +
 b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c + d*x]*(a + b*
Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 0.979887, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3029, 2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2C+22abB-81b^2C\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2bB-40a^3C-335ab^2C-539b^3B\right)\sin(c+dx)(}{3465b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (-2*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*C - 255*a^3*b^2*C -
 3705*a*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a
^3*b*B - 1254*a*b^3*B - 40*a^4*C - 285*a^2*b^2*C - 675*b^4*C)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) - (2*(110*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 285*a
^2*b^2*C - 675*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) -
```

```
(2*(110*a^2*b*B - 539*b^3*B - 40*a^3*C - 335*a*b^2*C)*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) - (2*(22*a*b*B - 8*a^2*C - 81*b^2*C)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*b*B - 4*a*C)*(a +
 b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c + d*x]*(a + b*
Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(B+C$$

$$= \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+}{11bd}$$

$$= \frac{2(11bB-4aC)(a+b\cos(c+dx))^{7/2}\sin(c+}{99b^2d}$$

$$= -\frac{2\left(22abB-8a^2C-81b^2C\right)(a+b\cos(c+}{693b^2d}$$

$$= -\frac{2\left(110a^2bB-539b^3B-40a^3C-335ab^2C\right)}{3465b^2d}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-285a^2b^2\right.}{346}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-285a^2b^2\right.}{346}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-285a^2b^2\right.}{346}$$

$$= -\frac{2\left(110a^4bB-3069a^2b^3B-1617b^5B-40a^5\right.}{}$$

Mathematica [A] time = 2.03145, size = 357, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(18660a^2b^2C+880a^3bB-320a^4C+32868ab^3B+13050b^4C\right)\sin(c+dx)+b\left(4\left(1650a^2bB+30a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(1705*a^3*b*B + 2871*a*b^3*B +
10*a^4*C + 3315*a^2*b^2*C + 675*b^4*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + (-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C
+ 3705*a*b^4*C)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
```

```
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((880*a^3*b*B + 32
868*a*b^3*B - 320*a^4*C + 18660*a^2*b^2*C + 13050*b^4*C)*Sin[c + d*x] + b*(
4*(1650*a^2*b*B + 1463*b^3*B + 30*a^3*C + 3095*a*b^2*C)*Sin[2*(c + d*x)] +
5*b*((836*a*b*B + 452*a^2*C + 513*b^2*C)*Sin[3*(c + d*x)] + 7*b*((22*b*B +
46*a*C)*Sin[4*(c + d*x)] + 9*b*C*Sin[5*(c + d*x)])))))/(27720*b^3*d*Sqrt[a
+ b*Cos[c + d*x]])
```

Maple [B] time = 1.177, size = 1983, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-255*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+3705*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-3705*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-245*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-390*a^2*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-40*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^5*b+255*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^4*b^2+20160*C*b^6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
12-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^6+675*b^
6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+40*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-40*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-3069*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^2*b^4+(-12320*B*b^6-35840*C*a*b^5-50400*C*b^6)*si
n(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(22880*B*a*b^5+24640*B*b^6+21920*C*a
^2*b^4+71680*C*a*b^5+56880*C*b^6)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
```

```
-14960*B*a^2*b^4-34320*B*a*b^5-22792*B*b^6-4640*C*a^3*b^3-32880*C*a^2*b^4-6
6160*C*a*b^5-34920*C*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(3520*B*a
^3*b^3+14960*B*a^2*b^4+26488*B*a*b^5+10472*B*b^6-20*C*a^4*b^2+4640*C*a^3*b^
3+25120*C*a^2*b^4+30320*C*a*b^5+13860*C*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+(-110*B*a^4*b^2-1760*B*a^3*b^3-7326*B*a^2*b^4-7524*B*a*b^5-1848*B
*b^6+40*C*a^5*b+10*C*a^4*b^2-3210*C*a^3*b^3-7080*C*a^2*b^4-6690*C*a*b^5-279
0*C*b^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+110*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-110*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b^5+110*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^5*b-1364*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^3+1254*a*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+3069*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b
^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*co
s(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^5 + Ba^2\cos\left(dx+c\right)^2 + \left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^4 + \left(Ca^2+2\,Bab\right)\cos\left(dx+c\right)^3\right)\sqrt{b\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5 + B*a^2*cos(d*x + c)^2 + (2*C*a*b + B*b^2)*c
os(d*x + c)^4 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*co
s(d*x + c), x)
```

3.828 $\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=372

$$\frac{2\left(-10a^2C+45abB+49b^2C\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2bB-10a^3C+114ab^2C+75b^3B\right)\sin(c+dx)\sqrt{}}{315bd}$$

[Out] (2*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 279*a^2*b^2*C + 147*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 114*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 114*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) + (2*(45*a*b*B - 10*a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) + (2*(9*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)

Rubi [A] time = 0.629096, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-10a^2C+45abB+49b^2C\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2bB-10a^3C+114ab^2C+75b^3B\right)\sin(c+dx)\sqrt{}}{315bd}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] (2*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 279*a^2*b^2*C + 147*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 114*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 114*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) + (2*(45*a*b*B - 10*a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) + (2*(9*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int(a+b\cos(c+$$

$$= \frac{2(9bB-2aC)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd} + \frac{2C(a+$$

$$= \frac{2\left(45abB-10a^2C+49b^2C\right)(a+b\cos(c+dx))^{3/2}\sin(c+$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+114ab^2C\right)\sqrt{a+b\cos(c+}}{315bd}$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+114ab^2C\right)\sqrt{a+b\cos(c+}}{315bd}$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+114ab^2C\right)\sqrt{a+b\cos(c+}}{315bd}$$

$$= \frac{2\left(45a^3bB+435ab^3B-10a^4C+279a^2b^2C+147b^4C\right)\sqrt{}}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.53207, size = 291, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(2\left(540a^2bB+20a^3C+747ab^2C+345b^3B\right)\sin(c+dx)+b\left(\left(300a^2C+540abB+266b^2C\right)\sin(2(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(405*a^2*b*B + 75*b^3*B + 155*a^
3*C + 261*a*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (45*a^3*b*B + 43
5*a*b^3*B - 10*a^4*C + 279*a^2*b^2*C + 147*b^4*C)*((a + b)*EllipticE[(c + d
```

```
*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a +
b*Cos[c + d*x])*(2*(540*a^2*b*B + 345*b^3*B + 20*a^3*C + 747*a*b^2*C)*Sin[c
 + d*x] + b*((540*a*b*B + 300*a^2*C + 266*b^2*C)*Sin[2*(c + d*x)] + 5*b*(2*
(9*b*B + 19*a*C)*Sin[3*(c + d*x)] + 7*b*C*Sin[4*(c + d*x)]))))/(1260*b^2*d*
Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.863, size = 1635, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^5+2080*C*a*b^4+2240*
C*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1440*B*a*b^4-1080*B*b^5-13
60*C*a^2*b^3-3120*C*a*b^4-2072*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1080*B*a^2*b^3+1440*B*a*b^4+840*B*b^5+320*C*a^3*b^2+1360*C*a^2*b^3+2408
*C*a*b^4+952*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-270*B*a^3*b^2
-540*B*a^2*b^3-510*B*a*b^4-240*B*b^5-10*C*a^4*b-160*C*a^3*b^2-666*C*a^2*b^3
-684*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-45*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+435*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-435*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^4*b-30*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*b^3+75*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+10*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+279*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-279*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
```

```
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^5+10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^5-124*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^3*b^2+114*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^4)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+Ba^{2}\cos\left(dx+c\right)+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2), x
)
```

3.829 $\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=288

$$\frac{2\left(15a^2C+56abB+25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(15a^2C+56abB+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105bd\sqrt{a+b\cos(c+dx)}}$$

[Out] (2*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(a^2 - b^2)*(56*a*b*B + 15*a^2*C + 25*b^2*C)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(56*a*b*B + 15*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*b*B + 5*a*C)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d)

Rubi [A] time = 0.577827, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(15a^2C+56abB+25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(15a^2C+56abB+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]

[Out] (2*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(a^2 - b^2)*(56*a*b*B + 15*a^2*C + 25*b^2*C)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(56*a*b*B + 15*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*b*B + 5*a*C)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d)

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int (a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))\,d$$

$$= \frac{2C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{2}{7}\int (a$$

$$= \frac{2(7bB+5aC)(a+b\cos(c+dx))^{3/2}\sin(c+dx}{35d}$$

$$= \frac{2\left(56abB+15a^2C+25b^2C\right)\sqrt{a+b\cos(c+}}{105d}$$

$$= \frac{2\left(56abB+15a^2C+25b^2C\right)\sqrt{a+b\cos(c+}}{105d}$$

$$= \frac{2\left(56abB+15a^2C+25b^2C\right)\sqrt{a+b\cos(c+}}{105d}$$

$$= \frac{2\left(161a^2bB+63b^3B+15a^3C+145ab^2C\right)\sqrt{a}}{105bd\sqrt{\frac{a+b\cos(}{a+b}}}$$

Mathematica [A] time = 1.03859, size = 254, normalized size = 0.88

$$b\sin(c+dx)(a+b\cos(c+dx))\left(90a^2C+6b(15aC+7bB)\cos(c+dx)+154abB+15b^2C\cos(2(c+dx))+65b^2C\right)+2b$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (2*b*(105*a^3*B + 119*a*b^2*B + 135*a^2*b*C + 25*b^3*C)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*(161*a^2*b*B + 63
*b^3*B + 15*a^3*C + 145*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b
```

```
)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)]) + b*(a + b*Cos[c + d*x])*(154*a*b*B + 90*a^2*C + 65*b^2*C + 6*b*(7*
b*B + 15*a*C)*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*
b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.827, size = 1305, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-480*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(392*B*a*b^3+168*B*b^4+360*C*a^2
*b^2+480*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-154*B
*a^2*b^2-196*B*a*b^3-42*B*b^4-90*C*a^3*b-180*C*a^2*b^2-170*C*a*b^3-80*C*b^4
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+161*B*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*a^3*b-161*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*a^2*b^2+63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*a*b^3-63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*b^4-56*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+5
6*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+15*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-15*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+145*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-145*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a*b^3-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^4-10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b^2+25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
```

```
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
))/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+Ba^{2}\cos\left(dx+c\right)+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*se
c(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c), x)
```

3.830 $\int (a+b\cos(c+dx))^{5/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{2\left(10a^2bB-8a^3C+8ab^2C+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2C+35abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(35*a*b*B + 23*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*
a^2*b*B + 5*b^3*B - 8*a^3*C + 8*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*a^3*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*b*B + 8*a*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rubi [A] time = 1.11715, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 2990, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(10a^2bB-8a^3C+8ab^2C+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2C+35abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```


```
[Out] (2*(35*a*b*B + 23*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*
a^2*b*B + 5*b^3*B - 8*a^3*C + 8*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*a^3*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*b*B + 8*a*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*C*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int (a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))\,\mathrm{se}$$

$$= \frac{2bC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2b(5bB+8aC)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d}$$

$$= \frac{2b(5bB+8aC)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d}$$

$$= \frac{2b(5bB+8aC)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d}$$

$$= \frac{2\left(35abB+23a^2C+9b^2C\right)\sqrt{a+b\cos(c+dx}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(35abB+23a^2C+9b^2C\right)\sqrt{a+b\cos(c+dx}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.69307, size = 453, normalized size = 1.55

$$\frac{4\left(45a^2bB+15a^3C+17ab^2C+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2bC+30a^3B+35ab^2B+9b^3C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i\left(23a^2C+35abB+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] ((4*(45*a^2*b*B + 5*b^3*B + 15*a^3*C + 17*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
+ (2*(30*a^3*B + 35*a*b^2*B + 23*a^2*b*C + 9*b^3*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d
*x]] + ((2*I)*(35*a*b*B + 23*a^2*C + 9*b^2*C)*Sqrt[-((b*(-1 + Cos[c + d*x])
)/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
 b)^(-1)]) + 4*b*Sqrt[a + b*Cos[c + d*x]]*(5*b*B + 11*a*C + 3*b*C*Cos[c + d
*x])*Sin[c + d*x])/(30*d)
```

Maple [B] time = 0.731, size = 1067, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b^3+56*C*a*b^2+24*C*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*B*a*b^2-10*B*b^3-22*C*a^2*b-28*C
*a*b^2-6*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-35*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^2*b+5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+23*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-23
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-8*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+8*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2)))/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+Ba^2\cos\left(dx+c\right)+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+\left(Ca^2+2\,Bab\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*se
c(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^2, x)
```

3.831 $\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(\; dx)\,dx$

Optimal. Leaf size=296

$$\frac{\left(4a^2bC+3a^3B+12ab^2B+2b^3C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{\left(3a^2B-14abC-6b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((3*a^2*B - 6*b^2*B - 14*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^3*B
+ 12*a*b^2*B + 4*a^2*b*C + 2*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5
*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*a*B - 2*b*C)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*B*(a + b*Cos[c + d*x])^(3/2)*Tan[
c + d*x])/d
```

Rubi [A] time = 1.1187, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 2989, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2bC+3a^3B+12ab^2B+2b^3C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{\left(3a^2B-14abC-6b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```


```
[Out] -((3*a^2*B - 6*b^2*B - 14*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^3*B
+ 12*a*b^2*B + 4*a^2*b*C + 2*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5
*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*a*B - 2*b*C)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*B*(a + b*Cos[c + d*x])^(3/2)*Tan[
c + d*x])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int (a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))$$

$$= \frac{aB(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int \sqrt{}$$

$$= -\frac{b(3aB-2bC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{3d}$$

$$= -\frac{b(3aB-2bC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{3d}$$

$$= -\frac{b(3aB-2bC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{3d}$$

$$= -\frac{\left(3a^2B-6b^2B-14abC\right)\sqrt{a+b\cos(c+dx}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(3a^2B-6b^2B-14abC\right)\sqrt{a+b\cos(c+dx}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.74055, size = 442, normalized size = 1.49

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)}\left(3a^2B+2b^2C\cos(c+dx)\right)+\frac{8b\left(9a^2C+9abB+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(27a^2bB+12a^3C\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] ((8*b*(9*a*b*B + 9*a^2*C + b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(27*a^2*b*B
+ 6*b^3*B + 12*a^3*C + 14*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-3
*a^2*B + 6*b^2*B + 14*a*b*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[
(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-
2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
 + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt
[a + b*Cos[c + d*x]]*(3*a^2*B + 2*b^2*C*Cos[c + d*x])*Tan[c + d*x])/(12*d)
```

Maple [B] time = 0.813, size = 1563, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*B*a^2*b+8*C*a*b^2+16*C*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*B*a^3-6*B*a^2*b-4*C*a*b^2-4*C*b^3
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(3*B*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-3*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^2*b-6*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2
+6*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-3*B*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-12*B*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a*b^2+15*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))*a^2*b-14*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+14
*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-4*C*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-2*C*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^3+6*C*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*a^3)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^2*b+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+3*B*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+12*a*b^2*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+14*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b-14*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a*b^2+4*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-6*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^2\cos\left(dx+c\right)^4 + Ba^2\cos\left(dx+c\right) + \left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3 + \left(Ca^2+2\,Bab\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*se
c(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^3, x)
```

3.832 $\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=315

$$\frac{\left(11a^2bB+4a^3C+16ab^2C+8b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{\left(4a^2C+9abB-8b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((9*a*b*B + 4*a^2*C - 8*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((11*a^2*b*
B + 8*b^3*B + 4*a^3*C + 16*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4*a
^2*B + 15*b^2*B + 20*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(7*b*B +
 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*B*(a + b*Cos[c +
d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 1.14735, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 2989, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(11a^2bB+4a^3C+16ab^2C+8b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{\left(4a^2C+9abB-8b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```


```
[Out] -((9*a*b*B + 4*a^2*C - 8*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((11*a^2*b*
B + 8*b^3*B + 4*a^3*C + 16*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4*a
^2*B + 15*b^2*B + 20*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(7*b*B +
 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*B*(a + b*Cos[c +
d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int (a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))\,\mathrm{se}$$

$$=\frac{aB(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+}{2d}$$

$$=\frac{a(7bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}$$

$$=\frac{a(7bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}$$

$$=\frac{a(7bB+4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}$$

$$=-\frac{\left(9abB+4a^2C-8b^2C\right)\sqrt{a+b\cos(c+dx)}E}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{\left(9abB+4a^2C-8b^2C\right)\sqrt{a+b\cos(c+dx)}E}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.59062, size = 451, normalized size = 1.43

$$\frac{8b\left(a^2B+12abC+4b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(36a^2bC+8a^3B+21ab^2B+8b^3C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i\left(-4a^2C-9abB+8b^2C\right)\csc(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] ((8*b*(a^2*B + 4*b^2*B + 12*a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^3*B +
21*a*b^2*B + 36*a^2*b*C + 8*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-9
*a*b*B - 4*a^2*C + 8*b^2*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*a*Sqr
t[a + b*Cos[c + d*x]]*(2*a*B + (9*b*B + 4*a*C)*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x])/(16*d)
```

Maple [B] time = 2.168, size = 1742, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C*b^2*(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*C*a*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-6*a*b*(B
*b+C*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^2*(3*B*b+C*a)*(-1/a*cos(1/
2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
```

```
1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a^3*B*(-1/2/a*cos(1/2*d*x+1/2*c)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+Ba^{2}\cos\left(dx+c\right)+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*se
c(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^4, x)
```

3.833 $\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=376

$$\frac{\left(16a^2B+54abC+33b^2B\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(66a^2bC+16a^3B+59ab^2B+48b^3C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((16*a^2*B + 33*b^2*B + 54*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a^
3*B + 59*a*b^2*B + 66*a^2*b*C + 48*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) +
((20*a^2*b*B + 5*b^3*B + 8*a^3*C + 30*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d
*x]]) + ((16*a^2*B + 33*b^2*B + 54*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
d*x])/(24*d) + (a*(3*b*B + 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan
[c + d*x])/(4*d) + (a*B*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 1.56455, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3029, 2989, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2B+54abC+33b^2B\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(66a^2bC+16a^3B+59ab^2B+48b^3C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```


```
[Out] -((16*a^2*B + 33*b^2*B + 54*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a^
3*B + 59*a*b^2*B + 66*a^2*b*C + 48*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) +
((20*a^2*b*B + 5*b^3*B + 8*a^3*C + 30*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d
*x]]) + ((16*a^2*B + 33*b^2*B + 54*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
```

d*x])/(24*d) + (a*(3*b*B + 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan [c + d*x])/(4*d) + (a*B*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d *x])/(3*d)

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int (a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))\,\mathrm{se}$$

$$= \frac{aB(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{a(3bB+2aC)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\,\mathrm{t}}{4d}$$

$$= \frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\cos(c+dx}}{24d}$$

$$= \frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\cos(c+dx}}{24d}$$

$$= \frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\cos(c+dx}}{24d}$$

$$= -\frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\cos(c+d}}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\cos(c+d}}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.96234, size = 486, normalized size = 1.29

$$\frac{\frac{8b\left(6a^2C+13abB+24b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(104a^2bB+48a^3C+126ab^2C-3b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec^2(c+dx)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] ((8*b*(13*a*b*B + 6*a^2*C + 24*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(104*a^2
*b*B - 3*b^3*B + 48*a^3*C + 126*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
```

```
I)*(16*a^2*B + 33*b^2*B + 54*a*b*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))
]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*Elliptic
E[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
 + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) +
 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*a*(13*b*B + 6*a*C)*Sin[c + d*
x] + (8*a^2*B + (33*b^2*B)/2 + 27*a*b*C)*Sin[2*(c + d*x)] + 8*a^2*B*Tan[c +
 d*x]))/(96*d)
```

Maple [B] time = 2.523, size = 2438, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^2*(3*B*b+C*a)*(-1/2/a*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/
2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*a^3*B*(-1/3/a*cos(1
/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1
/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
```

```
*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16
*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*b^2*(B*b+3*C*a)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))+6*a*b*(B*b+C*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^5, x)
```

3.834 $\int (a+b\cos(c+dx))^{5/2} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=465

$$\frac{\left(284a^2bB+128a^3C+264ab^2C+15b^3B\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{\left(356a^2bB+128a^3C+472ab^2C+133b^3B\right)\cdots}{192d\sqrt{a+b\cos(c\cdots}}$$

```
[Out] -((284*a^2*b*B + 15*b^3*B + 128*a^3*C + 264*a*b^2*C)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + ((356*a^2*b*B + 133*b^3*B + 128*a^3*C + 472*a*b^2*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sq
rt[a + b*Cos[c + d*x]]) + ((48*a^4*B + 120*a^2*b^2*B - 5*b^4*B + 160*a^3*b*
C + 40*a*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((284*a^2*b*B + 15*b
^3*B + 128*a^3*C + 264*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192
*a*d) + ((36*a^2*B + 59*b^2*B + 104*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]*Tan[c + d*x])/(96*d) + (a*(11*b*B + 8*a*C)*Sqrt[a + b*Cos[c + d*x]]*S
ec[c + d*x]^2*Tan[c + d*x])/(24*d) + (a*B*(a + b*Cos[c + d*x])^(3/2)*Sec[c
+ d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 1.99725, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3029, 2989, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(284a^2bB+128a^3C+264ab^2C+15b^3B\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{\left(356a^2bB+128a^3C+472ab^2C+133b^3B\right)\cdots}{192d\sqrt{a+b\cos(c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```


```
[Out] -((284*a^2*b*B + 15*b^3*B + 128*a^3*C + 264*a*b^2*C)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + ((356*a^2*b*B + 133*b^3*B + 128*a^3*C + 472*a*b^2*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sq
rt[a + b*Cos[c + d*x]]) + ((48*a^4*B + 120*a^2*b^2*B - 5*b^4*B + 160*a^3*b*
```

```
C + 40*a*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((284*a^2*b*B + 15*b
^3*B + 128*a^3*C + 264*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192
*a*d) + ((36*a^2*B + 59*b^2*B + 104*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]*Tan[c + d*x])/(96*d) + (a*(11*b*B + 8*a*C)*Sqrt[a + b*Cos[c + d*x]]*S
ec[c + d*x]^2*Tan[c + d*x])/(24*d) + (a*B*(a + b*Cos[c + d*x])^(3/2)*Sec[c
+ d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int (a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))\,\mathrm{s}$$

$$= \frac{aB(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c}{4d}$$

$$= \frac{a(11bB+8aC)\sqrt{a+b\cos(c+dx)}\sec^2(c+}{24d}$$

$$= \frac{\left(36a^2B+59b^2B+104abC\right)\sqrt{a+b\cos(c+}}{96d}$$

$$= \frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)\sqrt{}}{192ad}$$

$$= \frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)\sqrt{}}{192ad}$$

$$= \frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)\sqrt{}}{192ad}$$

$$= -\frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)}{192ad\sqrt{\frac{a+b\cos}{}}}$$

$$= -\frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)}{192ad\sqrt{\frac{a+b\cos}{}}}$$

Mathematica [C] time = 6.71027, size = 729, normalized size = 1.57

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{96}\sec^2(c+dx)\left(36a^2B\sin(c+dx)+104abC\sin(c+dx)+59b^2B\sin(c+dx)\right)+\frac{\sec(c+dx)\left(284a^2bB\sin\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] ((2*(144*a^3*b*B + 236*a*b^3*B + 416*a^2*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (
2*(288*a^4*B + 436*a^2*b^2*B - 45*b^4*B + 832*a^3*b*C - 24*a*b^3*C)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt
[a + b*Cos[c + d*x]] - ((2*I)*(-284*a^2*b^2*B - 15*b^4*B - 128*a^3*b*C - 26
4*a*b^3*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(
a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (
a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x
]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)
/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))
)/(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(17*a*b*B*Sin[c +
d*x] + 8*a^2*C*Sin[c + d*x]))/24 + (Sec[c + d*x]^2*(36*a^2*B*Sin[c + d*x] +
 59*b^2*B*Sin[c + d*x] + 104*a*b*C*Sin[c + d*x]))/96 + (Sec[c + d*x]*(284*a
^2*b*B*Sin[c + d*x] + 15*b^3*B*Sin[c + d*x] + 128*a^3*C*Sin[c + d*x] + 264*
a*b^2*C*Sin[c + d*x]))/(192*a) + (a^2*B*Sec[c + d*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] time = 3.414, size = 3548, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6*a*b*(B*b+
C*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
```

*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*b^2)+2*a^2*(3*B*b+C*a)*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*
b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*
x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
)+2*b^2*(B*b+3*C*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*C
b^3(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^3*B*(-1/4/a*cos(1/2*d*x+1/2*c
)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^4+7/24*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(

```
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3-1/96*(36*a^2+
35*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b*(20*a^2+21*b^2)/a^4*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3
5/384*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96/a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-25/96*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35/128/a^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/16/a^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
)*b^4))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*se
c(d*x + c)^6, x)
```

3.835 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(10a^2bB-8a^3C-7ab^2C+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2C+10abB-9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (-2*(10*a*b*B - 8*a^2*C - 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(10*a^2*b*B + 5*b^3*B - 8*a^3*C - 7*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d)
 + (2*C*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.486591, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2990, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(10a^2bB-8a^3C-7ab^2C+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2C+10abB-9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c + d
*x]],x]
```


```
[Out] (-2*(10*a*b*B - 8*a^2*C - 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(10*a^2*b*B + 5*b^3*B - 8*a^3*C - 7*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d)
 + (2*C*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2C\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{2\int \frac{aC+\frac{3}{2}bC\cos(c+dx}{\sqrt{a+}}}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+dx)\sqrt{a-}}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+dx)\sqrt{a-}}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+dx)\sqrt{a-}}{}$$

$$= -\frac{2\left(10abB-8a^2C-9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

Mathematica [A] time = 0.889603, size = 180, normalized size = 0.73

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(8a^2C-10abB+9b^2C\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(2aC+5bB)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos
[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*b*B + 2*a*C)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (-10*a*b*B + 8*a^2*C + 9*b^2*C)*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2
*b*(a + b*Cos[c + d*x])*(5*b*B - 4*a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x])/
(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.779, size = 993, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b^3-4*C*a*b^2+24*C*b^3)*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*B*a*b^2-10*B*b^3+8*C*a^2*b+2*C*a*
b^2-6*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a*b^2-8*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-7*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-8*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
```

```
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)/sqrt(b*cos(d*x +
 c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos{(dx+c)}^2 + B\cos{(dx+c)}\right)\cos{(dx+c)}}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)/sqrt(b*cos(d*x +
 c) + a), x)
```

3.836 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{2\left(-2a^2C+3abB-b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sin}{}$$

```
[Out] (2*(3*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*b*B - 2*a^2*
C - b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.222296, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-2a^2C+3abB-b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*b*B - 2*a^2*
C - b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{bC}{2}+\frac{1}{2}(3bB-2aC)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{(3bB-2aC)\int\sqrt{a+b\cos(c+dx)}\,dx}{3b^2} - \frac{(3al}{}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left((3bB-2aC)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+}}{3b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(3abB-2a^2C-b^2C\right)\sqrt{\frac{a+}{}}}{3b^2d\sqrt{a+b}}$$

Mathematica [A] time = 0.774383, size = 154, normalized size = 0.84

$$\frac{2\left(2a^2C-3abB+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a+b)(2aC-3bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bC\sin(c}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a + b)*(-3*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] + 2*(-3*a*b*B + 2*a^2*C + b^2*C)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*C*(a + b*Co
s[c + d*x])*Sin[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.747, size = 671, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
```

```
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*C*cos(1/2*d*x+1/2*c)^3*a*b+6*C*cos(
1/2*d*x+1/2*c)^3*b^2-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2-C*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b+2*C*cos(1/2*d*x+1/2*c)*a*b-2*C*cos(1/2*d*x+1/2*c)*b^2)/b^
2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.837 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=130

$$\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]])
```

Rubi [A] time = 0.237164, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b*Cos[c + d
*x]],x]
```

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B+C\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=\frac{C\int\sqrt{a+b\cos(c+dx)}\,dx}{b}+\frac{(bB-aC)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$=\frac{\left(C\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left((bB-aC)\sqrt{\frac{a+b}{\cdots}}\right.}{b\sqrt{\cdots}}$$

$$=\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+\cdots}{a+b}}}{bd\sqrt{a+b\cos\cdots}}$$

Mathematica [A] time = 3.1837, size = 93, normalized size = 0.72

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((bB-aC)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+C(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b*Cos
[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*C*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + (b*B - a*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.65, size = 249, normalized size = 1.9

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}\sqrt{\frac{2\cdots}{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-C*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/sqrt(b*cos(d*x +
 c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/sqrt(b*cos(d*x +
c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\sec\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)/sqrt(a + b*cos(c +
d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/sqrt(b*cos(d*x +
 c) + a), x)
```

3.838 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.401341, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3029, 3002, 2663, 2661, 2807, 2805}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c +
 d*x]],x]
```

```
[Out] (2*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + C\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\left(C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{}{\sqrt{\frac{a}{a+b}}}}{\sqrt{a+b\cos(c+}}$$
$$= \frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.18558, size = 81, normalized size = 0.69

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(B\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+CF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + b*C
os[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(C*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + B*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d
*x]])
```

Maple [A] time = 0.682, size = 194, normalized size = 1.6

$$2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2b+a+b}d}\sqrt{\frac{2\,(\cos}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(B*EllipticPi(
```

```
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-C*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2)))/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^2/sqrt(b*cos(d*x
 + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^2/sqrt(b*cos(d*x
 + c) + a), x)
```

3.839 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=216

$$-\frac{(bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{B\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + (B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - ((b*
B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.719831, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3029, 3000, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{B\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c +
 d*x]],x]
```

```
[Out] -((B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + (B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - ((b*
B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(a*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=\frac{B\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad}+\frac{\int \frac{\left(\frac{1}{2}(-bB+2aC)-\frac{1}{2}bB\cos^2(c+dx)\right)\sec}{\sqrt{a+b\cos(c+dx)}}}{a}$$

$$=\frac{B\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad}-\frac{\int \frac{\left(\frac{1}{2}b(bB-2aC)-\frac{1}{2}abB\cos(c+dx)\right)\sec(}{\sqrt{a+b\cos(c+dx)}}}{ab}$$

$$=\frac{B\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad}+\frac{1}{2}B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=-\frac{B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{B\sqrt{a+b\cos(c+dx)}\,\mathrm{ta}}{ad}$$

$$=-\frac{B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{d\sqrt{a+b\cos(c+}}$$

Mathematica [C] time = 6.42467, size = 320, normalized size = 1.48

$$\frac{\frac{2(4aC-3bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4B\tan(c+dx)\sqrt{a+b\cos(c+dx)}-\frac{2iB\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\,\mathrm{s}\right.\right.\right.}{}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + b*C
os[c + d*x]],x]
```

```
[Out] ((2*(-3*b*B + 4*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*B*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x
]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*
b*Sqrt[-(a + b)^(-1)]) + 4*B*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

Maple [B] time = 1.052, size = 639, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/a*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^3/sqrt(b*cos(d*x
 + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^3/sqrt(b*cos(d*x
 + c) + a), x)
```

3.840 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{\left(4a^2B-4abC+3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3bB-4aC)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3bB-4aC)}{}$$

```
[Out] ((3*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((b*B - 4*a*C)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((4*a^2*B + 3*b^2*B - 4*a*b*C)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - ((3*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*
x])/(4*a^2*d) + (B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d)
```

Rubi [A] time = 1.12592, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2B-4abC+3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3bB-4aC)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3bB-4aC)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + b*Cos[c +
 d*x]],x]
```


```
[Out] ((3*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((b*B - 4*a*C)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((4*a^2*B + 3*b^2*B - 4*a*b*C)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - ((3*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*
x])/(4*a^2*d) + (B*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

<u>Rule 3000</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

<u>Rule 3055</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

<u>Rule 3059</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B+C\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(\frac{1}{2}(-3bB+4aC)+aB\,c\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{(3bB-4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{B\sqrt{a+b\cos(c+\cdots}}{\cdots}$$

$$= -\frac{(3bB-4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{B\sqrt{a+b\cos(c+\cdots}}{\cdots}$$

$$= -\frac{(3bB-4aC)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{B\sqrt{a+b\cos(c+\cdots}}{\cdots}$$

$$= \frac{(3bB-4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(3bB-4aC)\sqrt{\cdots}}{\cdots}$$

$$= \frac{(3bB-4aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(bB-4aC)\sqrt{\cdots}}{4ad\cdots}$$

Mathematica [C] time = 5.71005, size = 420, normalized size = 1.4

$$\frac{\frac{2\left(8a^2B-12abC+9b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}((4aC-3bB)\cos(c+dx)+2aB\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + b*C
os[c + d*x]],x]
```

```
[Out] ((8*a*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*B + 9*b^2*B - 12*a*b*C)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sq
rt[a + b*Cos[c + d*x]] + ((2*I)*(3*b*B - 4*a*C)*Sqrt[-((b*(-1 + Cos[c + d*x
]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
 b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*B + (-3*b*B + 4*a*C)*Cos[c + d
*x])*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d)
```

Maple [B] time = 1.417, size = 1182, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/a*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*B*(-1/2/a*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
```

```
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/
2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^4/sqrt(b*cos(d*x
 + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{4}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^4/sqrt(b*cos(d*x
 + c) + a), x)
```

3.841 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$\frac{2a(bB-aC)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-6a^2C+5abB+b^2C\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2\left(20a^2bB\right.}{}$$

```
[Out] (-2*(40*a^3*b*B - 25*a*b^3*B - 48*a^4*C + 24*a^2*b^2*C + 9*b^4*C)*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*(a^2 - b^2)*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*b*B + 5*b^3*B - 48*a^3*C
 - 12*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(b*B - a*C)*Cos[c
+ d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*
a^2*b*B - 5*b^3*B - 24*a^3*C + 9*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*b*B - 6*a^2*C + b^2*C)*Cos[c + d*x]*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.874876, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3029, 2989, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(bB-aC)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-6a^2C+5abB+b^2C\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2\left(20a^2bB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x
])^(3/2),x]
```


```
[Out] (-2*(40*a^3*b*B - 25*a*b^3*B - 48*a^4*C + 24*a^2*b^2*C + 9*b^4*C)*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*(a^2 - b^2)*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*b*B + 5*b^3*B - 48*a^3*C
 - 12*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(b*B - a*C)*Cos[c
+ d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*
a^2*b*B - 5*b^3*B - 24*a^3*C + 9*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*b*B - 6*a^2*C + b^2*C)*Cos[c + d*x]*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(-2a(bB-aC)+\frac{1}{2}\right.}{}}{}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(5abB-6a^2C+b^2C\right)}{}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2bB-5b^3B-24a\right.}{}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2bB-5b^3B-24a\right.}{}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2bB-5b^3B-24a\right.}{}$$

$$= -\frac{2\left(40a^3bB-25ab^3B-48a^4C+24a^2b^2C+9b^4C\right)\sqrt{a+b\cos(c}}{15b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.73833, size = 304, normalized size = 0.79

$$\frac{\frac{30a^3b(aC-bB)\sin(c+dx)}{b^2-a^2} + \frac{2b^2\left(-10a^2bB+12a^3C+3ab^2C-5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a-b)(a+b)} + \frac{2\left(-24a^2b^2C-40a^3bB+48a^4C+25ab^3B-9b^4C\right)\sqrt{\frac{a+b\cos(c+}{a+b}}}{(a-b)(a+b)}}{15b^4d\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] ((2*b^2*(-10*a^2*b*B - 5*b^3*B + 12*a^3*C + 3*a*b^2*C)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a + b)) + (
2*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/((a - b)*(a + b)) + (30*a^3*b*(-(b
```

```
*B) + a*C)*Sin[c + d*x]])/(-a^2 + b^2) + 2*b*(5*b*B - 9*a*C)*(a + b*Cos[c +
d*x]])*Sin[c + d*x] + 3*b^2*C*(a + b*Cos[c + d*x])*Sin[2*(c + d*x)]])/(15*b^4
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 2.812, size = 1308, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C/b*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8/b
^2*(B*b-C*a-3*C*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^4*(B*a*b+2*B*b^2-C*a^2-2*C*a*b-
3*C*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2)))+2*(B*a^2*b+B*a*b^2+B*b^3-C*a^3-C*a^2*b-C*a*b^2-C*b
^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a^3*(B*b-C*a)/b^4/sin(1/2*d*x+1
/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b
```

```
+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^2/(b*cos(d*x + c
) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{4}+B\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos \left(dx+c\right)^{2}+B\cos \left(dx+c\right)\right)\cos \left(dx+c\right)^{2}}{\left(b\cos \left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^2/(b*cos(d*x + c
) + a)^(3/2), x)
```

3.842 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2a^2(bB-aC)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2C+6abB-b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2bB-8a^3C+5ab\right.}{3}$$

```
[Out] (2*(6*a^2*b*B - 3*b^3*B - 8*a^3*C + 5*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - (2*(6*a*b*B - 8*a^2*C - b^2*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*a^2*(b*B - a*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.567283, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2988, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2(bB-aC)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2C+6abB-b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2bB-8a^3C+5ab\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])
^(3/2),x]
```


```
[Out] (2*(6*a^2*b*B - 3*b^3*B - 8*a^3*C + 5*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - (2*(6*a*b*B - 8*a^2*C - b^2*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*a^2*(b*B - a*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}ab(bB-aC)+\frac{1}{2}\left(2a^2-b^2\right)(bB-aC}{\sqrt{a+b}}}{b^2\left(a\right.}$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c}{3b^2d}$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c}{3b^2d}$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c}{3b^2d}$$

$$= \frac{2\left(6a^2bB-3b^3B-8a^3C+5ab^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.38697, size = 189, normalized size = 0.72

$$\frac{2\left(b\sin(c+dx)\left(\frac{a\left(-4a^2C+3abB+b^2C\right)}{b^2-a^2}+bC\cos(c+dx)\right)+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a-b)\left(8a^2C-6abB+b^2C\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(6a^2bB-8a^3C+5ab^2C-3b^3B\right)\cdots\right)}{a-b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (2*((Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((6*a^2*b*B - 3*b^3*B - 8*a^3*C + 5
*a*b^2*C)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (a - b)*(-6*a*b*B + 8*a^2
*C + b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b) + b*((a*(3*a*b*
B - 4*a^2*C + b^2*C))/(-a^2 + b^2) + b*C*Cos[c + d*x])*Sin[c + d*x]))/(3*b^
3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 2.411, size = 954, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^3*(4*
b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b-2*C*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-6*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*a^2*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
```

```
(1/2)+2*a^2*(B*b-C*a)/b^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)/(b*cos(d*x + c)
+ a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)/(b*cos(d*x + c)
+ a)^(3/2), x)
```

3.843 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=204

$$\frac{2a(bB-aC)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2C+abB+b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(bB-2aC)\sqrt{\frac{a+b\cos(}{a+}}}{b^2d\sqrt{a+b}}$$

```
[Out] (-2*(a*b*B - 2*a^2*C + b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(b*B - a*C)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.282259, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(bB-aC)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2C+abB+b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(bB-2aC)\sqrt{\frac{a+b\cos(}{a+}}}{b^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a*b*B - 2*a^2*C + b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(b*B - a*C)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(bB-aC)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}b(bB-aC)+\frac{1}{2}\left(abB-2a^2C+b^2C\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(bB-aC)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(bB-2aC)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b^2} - \frac{\left(abB-2a^2C\right.}{}$$

$$= \frac{2a(bB-aC)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\left(abB-2a^2C+b^2C\right)\sqrt{a+b\cos(c+dx)}\right)\int}{b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{2\left(abB-2a^2C+b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(bB-2aC)\sqrt{}}{b^2d\sqrt{}}$$

Mathematica [A] time = 0.798803, size = 170, normalized size = 0.83

$$-\frac{2\left(\left(a^2-b^2\right)(2aC-bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a+b)\left(2a^2C-abB-b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\right.}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-((a + b)*(-(a*b*B) + 2*a^2*C - b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*(-(b*B) + 2*a*C)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a
*b*(-(b*B) + a*C)*Sin[c + d*x]))/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c +
d*x]])
```

Maple [B] time = 1.668, size = 515, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*C*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*a*(B*b-C*a)/b^2/sin(
1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x
)
```

3.844 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2(bB-aC)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(b*B - a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*C*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a
+ b*Cos[c + d*x]]) - (2*(b*B - a*C)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rubi [A] time = 0.336096, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2(bB-aC)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])
^(3/2),x]
```


```
[Out] (2*(b*B - a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*C*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a
+ b*Cos[c + d*x]]) - (2*(b*B - a*C)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B+C\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$=-\frac{2(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{2\int \frac{\frac{1}{2}(-aB+bC)-\frac{1}{2}(bB-aC)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$=-\frac{2(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{C\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}+\frac{(bB-aC}{}$$

$$=-\frac{2(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{\left((bB-aC)\sqrt{a+b\cos(c+dx)}\right)\int}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cos}{}}}$$

$$=\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{bd\sqrt{a+b}}$$

Mathematica [A] time = 0.546509, size = 151, normalized size = 0.82

$$\frac{2\left(C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b(aC-bB)\sin(c+dx)-(a+b)(aC-bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right.}{bd(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (2*(-((a + b)*(-(b*B) + a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*C*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-(b*B) + a*C)*Sin[c + d*x]))
/((a - b)*b*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 1.431, size = 428, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2b-a+b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\frac{C\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}}{b\sqrt{-2\,b\left(\sin(1/2\,dx+c/2)\right)^4+(a+b)\left(\sin(1/2\,dx+c/2)\right)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*(B*b-C*a)/b/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x + c)
+ a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d
*x + c)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\cos(c + dx))\cos(c + dx)\sec(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a + b*cos(c + d*x)
)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c)\right)\sec(dx + c)}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x + c)
+ a)^(3/2), x)
```

3.845 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{2b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(b*B - a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.64334, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (-2*(b*B - a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B+C\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$=\frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{2\int\frac{\left(\frac{1}{2}\left(a^2-b^2\right)B-\frac{1}{2}a(bB-aC)\cos(c+dx\cdots}{\sqrt{a+b\cdots}}}{a\left(a^2\cdots\right.}$$

$$=\frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{2\int-\frac{b\left(a^2-b^2\right)B\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)}-\frac{(b\cdots}{}$$

$$=\frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{B\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a}-\frac{((bB-\cdots}{}$$

$$=-\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b(bB-\cdots}{a\left(a^2-b^2\right)\cdots}$$

$$=-\frac{2(bB-aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(\cdots}{a+b}}}{ad\sqrt{\cdots}}$$

Mathematica [C] time = 3.82337, size = 460, normalized size = 2.42

$$\frac{\cos(c+dx)(B\sec(c+dx)+C)\left(\frac{4b(bB-aC)\sin(c+dx)}{(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(2a^2B+abC-3b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{4a(aC-bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}\cdots}{\cdots}\right.}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]*(C + B*Sec[c + d*x])*(-(((4*a*(-(b*B) + a*C)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
+ d*x]] + (2*(2*a^2*B - 3*b^2*B + a*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*(b*B - a*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b))) +
 (4*b*(b*B - a*C)*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2
*a*d*(B + C*Cos[c + d*x]))
```

Maple [A] time = 1.636, size = 429, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2b-a+b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\frac{(-bB+aC)\sqrt{-2\,b\left(\sin(1/2\,dx+c/2)\right)^4+(a+b)\left(\sin(1/2\cdots\right.}}{a\left(\sin(1/2\,dx+c/2)\right)^2\left(-2\left(\sin(1/2\,dx+c/2)\right)^2b+a+b\right)\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-B*b+C*a
)/a/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
```

```
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*B/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*x + c
) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*x + c
) + a)^(3/2), x)
```

3.846 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{b\left(a^2B+2abC-3b^2B\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2B+2abC-3b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{a^2d\sqrt{a+}}$$

```
[Out] -(((a^2*B - 3*b^2*B + 2*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]))
 + (B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*b*B - 2*a*C)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b
*Cos[c + d*x]]) + (b*(a^2*B - 3*b^2*B + 2*a*b*C)*Sin[c + d*x])/(a^2*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (B*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c +
 d*x]])
```

Rubi [A] time = 1.16418, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 3000, 3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(a^2B+2abC-3b^2B\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2B+2abC-3b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{a^2d\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x
])^(3/2),x]
```



```
[Out] -(((a^2*B - 3*b^2*B + 2*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]))
 + (B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*b*B - 2*a*C)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b
*Cos[c + d*x]]) + (b*(a^2*B - 3*b^2*B + 2*a*b*C)*Sin[c + d*x])/(a^2*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (B*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c +
 d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{B\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-3bB+2aC)+\frac{1}{2}bB\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{b\left(a^2B-3b^2B+2abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2\int}{}$$

$$= \frac{b\left(a^2B-3b^2B+2abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2\int}{}$$

$$= \frac{b\left(a^2B-3b^2B+2abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{B\int}{}$$

$$= -\frac{\left(a^2B-3b^2B+2abC\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\left(\right.}{a^2}$$

$$= -\frac{\left(a^2B-3b^2B+2abC\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{B\sqrt{}}{}$$

Mathematica [C] time = 5.52263, size = 482, normalized size = 1.59

$$\frac{\frac{4\tan(c+dx)\left(b\left(a^2B+2abC-3b^2B\right)\cos(c+dx)+aB\left(a^2-b^2\right)\right)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} + \frac{\frac{2\left(-7a^2bB+4a^3C-6ab^2C+9b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(a^2B+2abC-3b^2B\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+d}{a+b}}}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (((-8*a*b*(-(b*B) + a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-7*a^2*b*B + 9*b^3*B
 + 4*a^3*C - 6*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(a^2*B - 3*b^2
*B + 2*a*b*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c
 + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((a - b)*(a + b)) + (4
*(a*(a^2 - b^2)*B + b*(a^2*B - 3*b^2*B + 2*a*b*C)*Cos[c + d*x])*Tan[c + d*x
])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

Maple [B] time = 1.92, size = 908, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(B*b-C*a)
*b/a^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*B/a*(-1
/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
```

```
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*(-B*b+C*a)/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+
a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*x + c
) + a)^(3/2), x)
```

3.847 $\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=413

$$\frac{2a(bB-aC)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a^2\left(3a^2bB-6a^3C+10ab^2C-7b^3B\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2C+abB+b^2C\right)\sin(c}{3b^3d\left(a^2\right.}$$

```
[Out] (2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 16*a^5*C + 28*a^3*b^2*C - 8*a*b^4*
C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(
a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^3*b*B - 9*a*b^
3*B - 16*a^4*C + 16*a^2*b^2*C + b^4*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*a*(b*B - a*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2*b*B - 7*b^3*B - 6*a^3*C + 10*
a*b^2*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
2*(a*b*B - 2*a^2*C + b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(
a^2 - b^2)*d)
```

Rubi [A] time = 0.950623, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3029, 2989, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(bB-aC)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a^2\left(3a^2bB-6a^3C+10ab^2C-7b^3B\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2C+abB+b^2C\right)\sin(c}{3b^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x
])^(5/2),x]
```


```
[Out] (2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 16*a^5*C + 28*a^3*b^2*C - 8*a*b^4*
C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(
a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^3*b*B - 9*a*b^
3*B - 16*a^4*C + 16*a^2*b^2*C + b^4*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*a*(b*B - a*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2*b*B - 7*b^3*B - 6*a^3*C + 10*
a*b^2*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
```

```
2*(a*b*B - 2*a^2*C + b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(
a^2 - b^2)*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos^3(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)\left(-2a(bB-aC)+\frac{3}{2}\right.}{}}{}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(bB-aC)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2\left(8a^4bB-15a^2b^3B+3b^5B-16a^5C+28a^3b^2C-8ab^4C\right)\sqrt{a+}}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

<u>Mathematica</u> [A] time = 2.83381, size = 334, normalized size = 0.81

$$\frac{2\left(\frac{b\sin(c+dx)\left(2ab\left(-16a^2b^2C-5a^3bB+10a^4C+9ab^3B+2b^4C\right)\cos(c+dx)+16a^3b^3B+C\left(b^3-a^2b\right)^2\cos(2(c+dx))-25a^4b^2C-8a^5bB+16a^6C+b^6C\right)}{2\left(a^2-b^2\right)^2}+\frac{\left(\frac{a+b\cos(c+dx}{a+b}\right.}{}\right.}{3b^4d(a+b\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(2*a^3*b*B - 6*a*b^3*B - 4*a
^4*C + 7*a^2*b^2*C + b^4*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-8*a^4
*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*((a +
b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(
```

```
a + b)])))/((a - b)^2*(a + b)) + (b*(-8*a^5*b*B + 16*a^3*b^3*B + 16*a^6*C -
 25*a^4*b^2*C + b^6*C + 2*a*b*(-5*a^3*b*B + 9*a*b^3*B + 10*a^4*C - 16*a^2*b
^2*C + 2*b^4*C)*Cos[c + d*x] + (-(a^2*b) + b^3)^2*C*Cos[2*(c + d*x)])*Sin[c
 + d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 3.297, size = 1389, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^4*(4*
b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b-2*C*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+17*a^2
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a^2/b^4*(3*B*b-4*C*a)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2)-2*a^3*(B*b-C*a)/b^4*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x
+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*
a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/
```

```
(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^2/(b*cos(d*x + c
) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^4+B\cos\left(dx+c\right)^3\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^2/(b*cos(d*x + c
) + a)^(5/2), x)
```

3.848 $\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2a^2(bB-aC)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2bB-5a^3C+9ab^2C-6b^3B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2bB-8a^3C+9ab^2C-\right.}{3b^3d\left(a^2-b\right.}$$

```
[Out] (-2*(2*a^3*b*B - 6*a*b^3*B - 8*a^4*C + 15*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*b*B - 3*b^3*B - 8*a^3*C + 9
*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*(b*B - a*C
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*
a^2*b*B - 6*b^3*B - 5*a^3*C + 9*a*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.638588, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3029, 2988, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2(bB-aC)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2bB-5a^3C+9ab^2C-6b^3B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2bB-8a^3C+9ab^2C-\right.}{3b^3d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d*x])
^(5/2),x]
```


```
[Out] (-2*(2*a^3*b*B - 6*a*b^3*B - 8*a^4*C + 15*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*b*B - 3*b^3*B - 8*a^3*C + 9
*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*(b*B - a*C
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*
a^2*b*B - 6*b^3*B - 5*a^3*C + 9*a*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}ab(bB-aC)+\frac{1}{2}\left(2a^2-3b^2\right)(\ldots}{(\ldots}}{3\ldots}$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3C\ldots\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a\ldots}}$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3C\ldots\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a\ldots}}$$

$$= -\frac{2a^2(bB-aC)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3C\ldots\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a\ldots}}$$

$$= -\frac{2\left(2a^3bB-6ab^3B-8a^4C+15a^2b^2C-3b^4C\right)\sqrt{a+b\cos(c+dx\ldots}}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.21774, size = 274, normalized size = 0.83

$$2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(a^2bB+2a^3C-6ab^2C+3b^3B\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(-15a^2b^2C-2a^3bB+8a^4C+6ab^3B+3b^4C\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{(a-b)^2(a+b)} - \cdots \right) \over 3b^3d(a+b\cos(c+dx))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(a^2*b*B + 3*b^3*B + 2*a^3*C
 - 6*a*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-2*a^3*b*B + 6*a*b^3
*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*((a + b)*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b))
 - (a*b*(a*(-(a^2*b*B) + 5*b^3*B + 4*a^3*C - 8*a*b^2*C) + b*(-2*a^2*b*B + 6
*b^3*B + 5*a^3*C - 9*a*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(
3*b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 2.745, size = 950, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*C*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*a/b^3*(2*B*b-3*C*a)/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*a^2*(B*b-C*a)/
b^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(
```

```
1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2
*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b
^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)/(b*cos(d*x + c)
+ a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)/(b*cos(d*x + c)
+ a)^(5/2), x)
```

3.849 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=307

$$\frac{2\left(a^2bB+2a^3C-6ab^2C+3b^3B\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(bB-aC)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2C+abB-3b^2C\right)\sqrt{\frac{a+b\cos(c}{a+b}}}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\,c}}$$

```
[Out] (-2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) + (2*(a*b*B + 2*a^2*C - 3*b^2*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*a*(b*B - a*C)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*
Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.415313, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2bB+2a^3C-6ab^2C+3b^3B\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(bB-aC)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2C+abB-3b^2C\right)\sqrt{\frac{a+b\cos(c}{a+b}}}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\,c}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) + (2*(a*b*B + 2*a^2*C - 3*b^2*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*a*(b*B - a*C)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*
Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(bB-aC)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}b(bB-aC)-\frac{1}{2}\left(abB+2a^2C-3b^2C\right)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(bB-aC)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-6ab^2C\right)\sin(c+}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a(bB-aC)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-6ab^2C\right)\sin(c+}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a(bB-aC)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-6ab^2C\right)\sin(c+}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= -\frac{2\left(a^2bB+3b^3B+2a^3C-6ab^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(a\right.}{}$$

Mathematica [A] time = 1.90922, size = 224, normalized size = 0.73

$$\frac{2\left(\frac{b\sin(c+dx)\left(b\left(a^2bB+2a^3C-6ab^2C+3b^3B\right)\cos(c+dx)+a\left(2a^2bB+a^3C-5ab^2C+2b^3B\right)\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2bB+2a^3C-6ab^2C+3b^3B\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-\right.}{(a-b)^2}\right.}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2*b*B + 3*b^3*B + 2*a^3*C -
 6*a*b^2*C)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(a*b*B + 2*a^2*
C - 3*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (b*(a*(2*
a^2*b*B + 2*b^3*B + a^3*C - 5*a*b^2*C) + b*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6
*a*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c
```

```
+ d*x])^(3/2))
```

Maple [B] time = 2.59, size = 860, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2/b^2*(B*b-2*C*a)/sin(1/2*d*x+1/2*c)^2/(-2*s
in(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a*(B*b-C*a)/b^2*(1/6/b/(a-b)/(a+b)*cos(1
/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+
b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*
x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^3*
cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x
)
```

3.850 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{2\left(a^2(-C)+4abB-3b^2C\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(4*a*b*B - a^2*C - 3*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(b*B -
a*C)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(4*a*b
*B - a^2*C - 3*b^2*C)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d
*x]])
```

Rubi [A] time = 0.483174, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2(-C)+4abB-3b^2C\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d*x])
^(5/2),x]
```


```
[Out] (2*(4*a*b*B - a^2*C - 3*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(b*B -
a*C)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(4*a*b
*B - a^2*C - 3*b^2*C)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d
*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B+C\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2(bB-aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aB-bC)+\frac{1}{2}(bB-aC)\cos(c+}{(a+b\cos(c+dx))^{3/2}}}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(bB-aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4abB-a^2C-3b^2C\right)\sin(}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c}}$$

$$= -\frac{2(bB-aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4abB-a^2C-3b^2C\right)\sin(}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c}}$$

$$= -\frac{2(bB-aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4abB-a^2C-3b^2C\right)\sin(}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c}}$$

$$= \frac{2\left(4abB-a^2C-3b^2C\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(}{}$$

Mathematica [A] time = 1.54199, size = 193, normalized size = 0.7

$$\frac{2\left(\frac{\sin(c+dx)\left(b\left(a^2C-4abB+3b^2C\right)\cos(c+dx)-5a^2bB+2a^3C+2ab^2C+b^3B\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2C-4abB+3b^2C\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-b)(aC-bB)F\left(\frac{1}{2}(c+dx)\right.\right.}{b(a-b)^2}\right.}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-4*a*b*B + a^2*C + 3*b^2*C)*E
llipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(-(b*B) + a*C)*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)]))/((a - b)^2*b)) + ((-5*a^2*b*B + b^3*B + 2*a^3*C
+ 2*a*b^2*C + b*(-4*a*b*B + a^2*C + 3*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/(a
```

```
^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 2.373, size = 750, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b/sin(1
/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(B*b-C*a)/b*(1/6/b/
(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2
*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x + c)
+ a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)}{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d
*x + c)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c)
 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x + c)
+ a)^(5/2), x)
```

3.851 $\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{2b\left(7a^2bB-4a^3C-3b^3B\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(bB-aC)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(7*a^2*b*B - 3*b^3*B - 4*a^3*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]) + (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(b*B - a*C)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(7*a^2*b*B - 3*b^3*B - 4*
a^3*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.24977, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {3029, 3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b\left(7a^2bB-4a^3C-3b^3B\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(bB-aC)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c + d*x
])^(5/2),x]
```


```
[Out] (-2*(7*a^2*b*B - 3*b^3*B - 4*a^3*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]) + (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(b*B - a*C)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(7*a^2*b*B - 3*b^3*B - 4*
a^3*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B+C\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}\left(a^2-b^2\right)B-\frac{3}{2}a(bB-aC)\cos(c+\cdots}{(a+b\cdots}}{3a\,(\cdots}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2bB-3b^3B-4a^3C\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos\cdots}}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2bB-3b^3B-4a^3C\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos\cdots}}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2bB-3b^3B-4a^3C\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos\cdots}}$$

$$= -\frac{2\left(7a^2bB-3b^3B-4a^3C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \cdots$$

$$= -\frac{2\left(7a^2bB-3b^3B-4a^3C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \cdots$$

Mathematica [C] time = 6.77455, size = 743, normalized size = 2.13

$$\frac{\cos(c+dx)\sqrt{a+b\cos(c+dx)}(B\sec(c+dx)+C)\left(-\frac{2\left(abC\sin(c+dx)-b^2B\sin(c+dx)\right)}{3a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{2\left(-7a^2b^2B\sin(c+dx)+4a^3bC\sin(c+dx)+3b^4B\sin(c+\cdots}{3a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\cdots\right.}{d(B+C\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]*(C + B*Sec[c + d*x])*((2*(-12*a^3*b*B + 4*a*b^3*B + 6*a^4*C +
 2*a^2*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4*B - 19*a^2*b^2*B + 9*b^4*
B + 4*a^3*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-7*a^2*b^2*B + 3*b^4*B
 + 4*a^3*b*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x]
)/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]]
, (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])
^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^
2))))/(6*a^2*(a - b)^2*(a + b)^2*d*(B + C*Cos[c + d*x])) + (Cos[c + d*x]*Sq
rt[a + b*Cos[c + d*x]]*(C + B*Sec[c + d*x])*((-2*(-(b^2*B*Sin[c + d*x]) + a
*b*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-7*a^2*b
^2*B*Sin[c + d*x] + 3*b^4*B*Sin[c + d*x] + 4*a^3*b*C*Sin[c + d*x]))/(3*a^2*
(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/(d*(B + C*Cos[c + d*x]))
```

Maple [B] time = 2.623, size = 854, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*B/a^2/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*B/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))+2*(-B*b+C*a)/a*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+
```

```
1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*co
s(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*x + c
) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*x + c
) + a)^(5/2), x)
```

3.852 $\int \frac{(B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=437

$$\frac{b\left(-26a^2b^2B+14a^3bC+3a^4B-6ab^3C+15b^4B\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2B+2abC-5b^2B\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2B+2abC-5\right.}{3a^2d\left(a\right.}$$

```
[Out] -((3*a^4*B - 26*a^2*b^2*B + 15*b^4*B + 14*a^3*b*C - 6*a*b^3*C)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2*B - 5*b^2*B + 2*a*b*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*b*B - 2*a*C)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a
 + b*Cos[c + d*x]]) + (b*(3*a^2*B - 5*b^2*B + 2*a*b*C)*Sin[c + d*x])/(3*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4*B - 26*a^2*b^2*B + 1
5*b^4*B + 14*a^3*b*C - 6*a*b^3*C)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt
[a + b*Cos[c + d*x]]) + (B*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 1.63666, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3029, 3000, 3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(-26a^2b^2B+14a^3bC+3a^4B-6ab^3C+15b^4B\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2B+2abC-5b^2B\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2B+2abC-5\right.}{3a^2d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] -((3*a^4*B - 26*a^2*b^2*B + 15*b^4*B + 14*a^3*b*C - 6*a*b^3*C)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2*B - 5*b^2*B + 2*a*b*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*b*B - 2*a*C)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a
 + b*Cos[c + d*x]]) + (b*(3*a^2*B - 5*b^2*B + 2*a*b*C)*Sin[c + d*x])/(3*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4*B - 26*a^2*b^2*B + 1
```

```
5*b^4*B + 14*a^3*b*C - 6*a*b^3*C)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt
[a + b*Cos[c + d*x]]) + (B*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B+C\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$=\frac{B\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}+\frac{\int \frac{\left(\frac{1}{2}(-5bB+2aC)+\frac{3}{2}bB\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{a}$$

$$=\frac{b\left(3a^2B-5b^2B+2abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{B\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}+$$

$$=\frac{b\left(3a^2B-5b^2B+2abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{b\left(3a^4B-26a^2b^2B+15b^4B\right.}{3a^3\left(a^2-b^2\right)^2}$$

$$=\frac{b\left(3a^2B-5b^2B+2abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{b\left(3a^4B-26a^2b^2B+15b^4B\right.}{3a^3\left(a^2-b^2\right)^2}$$

$$=\frac{b\left(3a^2B-5b^2B+2abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{b\left(3a^4B-26a^2b^2B+15b^4B\right.}{3a^3\left(a^2-b^2\right)^2}$$

$$=-\frac{\left(3a^4B-26a^2b^2B+15b^4B+14a^3bC-6ab^3C\right)\sqrt{a+b\cos(c+dx}}{3a^3\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{\left(3a^4B-26a^2b^2B+15b^4B+14a^3bC-6ab^3C\right)\sqrt{a+b\cos(c+dx}}{3a^3\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.17229, size = 750, normalized size = 1.72

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(ab^2C\sin(c+dx)-b^3B\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{2\left(-10a^2b^3B\sin(c+dx)+7a^3b^2C\sin(c+dx)-3ab^4C\sin(c+dx)+6b^5B\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{B\tan(c+dx)}{a^3}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] ((2*(36*a^3*b^2*B - 20*a*b^4*B - 24*a^4*b*C + 8*a^2*b^3*C)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
+ d*x]] + (2*(-33*a^4*b*B + 86*a^2*b^3*B - 45*b^5*B + 12*a^5*C - 38*a^3*b^2
*C + 18*a*b^4*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3*a^4*b*B + 26*a^2*
b^3*B - 15*b^5*B - 14*a^3*b^2*C + 6*a*b^4*C)*Sqrt[(b - b*Cos[c + d*x])/(a +
 b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-
(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c
 + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c +
d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(12*a^3*(-a + b)^2*(a + b)^2*d) + (Sqrt
[a + b*Cos[c + d*x]]*((2*(-(b^3*B*Sin[c + d*x]) + a*b^2*C*Sin[c + d*x]))/(3
*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-10*a^2*b^3*B*Sin[c + d*x] +
 6*b^5*B*Sin[c + d*x] + 7*a^3*b^2*C*Sin[c + d*x] - 3*a*b^4*C*Sin[c + d*x]))
/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (B*Tan[c + d*x])/a^3))/d
```

Maple [B] time = 3.721, size = 1341, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*(2*B*b-
C*a)/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*B/a^2
*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
```

```
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*(-2*B*b+C*a)/a^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(B*b-C*a)*b/a^2*(1/6/b/(a-b)/(a+b)*cos(
1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a
+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*x + c
) + a)^(5/2), x)
```

3.853 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+\right.$

Optimal. Leaf size=170

$$\frac{10(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aB+7bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aC+bB)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aB+7bC)\sin}{}$$

```
[Out] (2*(9*a*B + 7*b*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(b*B + a*C)*Elli
pticF[(c + d*x)/2, 2])/(21*d) + (10*(b*B + a*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*(9*a*B + 7*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) +
 (2*(b*B + a*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*C*Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.267145, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2968, 3023, 2748, 2635, 2639, 2641}

$$\frac{10(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aB+7bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aC+bB)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aB+7bC)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] (2*(9*a*B + 7*b*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(b*B + a*C)*Elli
pticF[(c + d*x)/2, 2])/(21*d) + (10*(b*B + a*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*(9*a*B + 7*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) +
 (2*(b*B + a*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*C*Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(B+C\cos(c$$
$$= \int \cos^{\frac{5}{2}}(c+dx)\left(aB+(bB+aC)\cos(c+dx)+\right.$$
$$= \frac{2bC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \cos^{\frac{5}{2}}(c+$$
$$= \frac{2bC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}+(bB+aC)\int c$$
$$= \frac{2(9aB+7bC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2(bB}{}$$
$$= \frac{2(9aB+7bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10(bB+aC)\sqrt{}}{}$$
$$= \frac{2(9aB+7bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10(bB+aC)F}{2}$$

Mathematica [A] time = 1.34683, size = 125, normalized size = 0.74

$$\frac{300(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84(9aB+7bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(7(36aB+43bC)\cos(c+dx)+}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (84*(9*a*B + 7*b*C)*EllipticE[(c + d*x)/2, 2] + 300*(b*B + a*C)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a*B + 43*b*C)*Cos[c + d*x] + 5*(
78*b*B + 78*a*C + 18*(b*B + a*C)*Cos[2*(c + d*x)] + 7*b*C*Cos[3*(c + d*x)])
)*Sin[c + d*x])/(630*d)
```

Maple [B] time = 0.59, size = 451, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b+720*C*a+2240*C*b)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*B*a-1080*B*b-1080*C*a-2072*C*b)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*B*a+840*B*b+840*C*a+952*C*b)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*B*a-240*B*b-240*C*a-168*C*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a+75*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)*cos(d*x
+ c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^4 + Ba\cos\left(dx+c\right)^2 + \left(Ca+Bb\right)\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + B*a*cos(d*x + c)^2 + (C*a + B*b)*cos(d*x + c
)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)*cos(d*x
+ c)^(3/2), x)
```

3.854 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c\right.$

Optimal. Leaf size=140

$$\frac{2(7aB+5bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aC+bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aB+5bC)\text{si}}{}$$

```
[Out] (6*(b*B + a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a*B + 5*b*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(b*B + a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
b*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.239993, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(7aB+5bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aC+bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aB+5bC)\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] (6*(b*B + a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a*B + 5*b*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(b*B + a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
b*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(B+C\cos$$

$$= \int \cos^{\frac{3}{2}}(c+dx)\left(aB+(bB+aC)\cos(c+dx)\right.$$

$$= \frac{2bC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c$$

$$= \frac{2bC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (bB+aC)\int$$

$$= \frac{2(7aB+5bC)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b}{}$$

$$= \frac{6(bB+aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7aB+5bC)}{2}$$

Mathematica [A] time = 0.913246, size = 103, normalized size = 0.74

$$\frac{10(7aB+5bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)+126(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(42(aC+bB)\cos(c+dx)+7}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (126*(b*B + a*C)*EllipticE[(c + d*x)/2, 2] + 10*(7*a*B + 5*b*C)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a*B + 65*b*C + 42*(b*B + a*C)*Cos[c
 + d*x] + 15*b*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 0.752, size = 413, normalized size = 3.

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-168\,bB-168\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b-168*C*a-360*C*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*B*a+168*B*b+168*C*a+280*C*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*B*a-42*B*b-42*C*a-80*C*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+35*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+25*C*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)*sqrt(cos
(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ba\cos\left(dx+c\right)+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*a*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)^
2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.855 $\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{2(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aC+bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bC\sin(c+dx)\,\mathrm{c}}{5d}$$

```
[Out] (2*(5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*C)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(b*B + a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*b*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.219386, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aC+bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bC\sin(c+dx)\,\mathrm{c}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c +
 d*x]],x]
```

```
[Out] (2*(5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*C)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(b*B + a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*b*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(B+C\cos(c+dx))\,dx$$

$$= \int \sqrt{\cos(c+dx)}\left(aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+\right.$$

$$= \frac{2bC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}(5aB\right.$$

$$= \frac{2bC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (bB+aC)\int \cos^{\frac{3}{2}}(c+dx)\,d$$

$$= \frac{2(5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(bB+aC)\sqrt{\cos(c+dx)}\,\mathrm{si}}{3d}$$

$$= \frac{2(5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(bB+aC)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

Mathematica [A] time = 0.421684, size = 86, normalized size = 0.8

$$\frac{2\left(5(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(5aC+5bB+3bC\cos(c+dx))\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```

```
[Out] (2*(3*(5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2] + 5*(b*B + a*C)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*b*B + 5*a*C + 3*b*C*Cos[c + d*x])*Si
n[c + d*x]))/(15*d)
```

Maple [B] time = 0.622, size = 371, normalized size = 3.4

$$-\frac{2}{15\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,Cb\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6+(20\,bB+20\,aC+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b+20*C*a+24*C*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*B*b-10*C*a-6*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+
5*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/sqrt(cos
(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))*sqrt(cos(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/sqrt(cos
(d*x + c)), x)
```

3.856 $\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2(3aB+bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*(b*B + a*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*B + b*C)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*b*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.20494, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2968, 3023, 2748, 2641, 2639}

$$\frac{2(3aB+bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bC\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]
^(3/2),x]
```


```
[Out] (2*(b*B + a*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*B + b*C)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*b*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\cos(c+dx))(B+C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$=\int \frac{aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$=\frac{2bC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}(3aB+bC)+\frac{3}{2}(bB+}{\sqrt{\cos(c+}}$$

$$=\frac{2bC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+(bB+aC)\int \sqrt{\cos(c+dx)}$$

$$=\frac{2(bB+aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3aB+bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [A] time = 0.231759, size = 67, normalized size = 0.89

$$\frac{2\left((3aB+bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+bC\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```

```
[Out] (2*(3*(b*B + a*C)*EllipticE[(c + d*x)/2, 2] + (3*a*B + b*C)*EllipticF[(c +
d*x)/2, 2] + b*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 0.75, size = 326, normalized size = 4.4

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+3\,Ba\sqrt{\left(\sin\left(1/2\,dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*b+C*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a-2*C*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))/sqrt(cos(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(3/2), x)
```

3.857 $\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{2(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aB-bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(a*B - b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(b*B + a*C)*EllipticF[(c
+ d*x)/2, 2])/d + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.210657, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3029, 2968, 3021, 2748, 2641, 2639}

$$\frac{2(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aB-bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]
^(5/2),x]
```

```
[Out] (-2*(a*B - b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(b*B + a*C)*EllipticF[(c
+ d*x)/2, 2])/d + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{(a+b\cos(c+dx))(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \int \frac{aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}(bB+aC)-\frac{1}{2}(aB-bC)\cos(c+dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (bB+aC)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + (-aB+ \\
&= -\frac{2(aB-bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(bB+aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.346534, size = 64, normalized size = 0.9

$$
\frac{2\left((aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(bC-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{aB\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```

```
[Out] (2*((-(a*B) + b*C)*EllipticE[(c + d*x)/2, 2] + (b*B + a*C)*EllipticF[(c + d
*x)/2, 2] + (a*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [B] time = 0.575, size = 244, normalized size = 3.4

$$
-2\,\frac{bB\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(}}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2*(b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*B*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))/cos(d*x + c)
^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(5/2), x)
```

3.858 $\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{2(aB+3bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aC+bB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(b*B + a*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*B + 3*b*C)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(b*B + a*C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.227216, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(aB+3bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aC+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aC+bB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]
^(7/2),x]
```



```
[Out] (-2*(b*B + a*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*B + 3*b*C)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(b*B + a*C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \int \frac{(a+b\cos(c+dx))(B+C\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$
$$= \int \frac{aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx$$
$$= \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}(bB+aC)+\frac{1}{2}(aB+3bC)\cos(c+$$
$$= \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (bB+aC)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx + \frac{1}{3}(aB$$
$$= \frac{2(aB+3bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(bB+}{d\sqrt{}}$$
$$= -\frac{2(bB+aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(aB+3bC)F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A] time = 0.46622, size = 107, normalized size = 1.04

$$\frac{2\left((aB+3bC)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-3(aC+bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+aB\tan(c+dx)+3aC\sin(c+\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```

```
[Out] (2*(-3*(b*B + a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (a*B + 3*
b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*b*B*Sin[c + d*x] + 3*
a*C*Sin[c + d*x] + a*B*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.444, size = 428, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b+C*a)*(-(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))/cos(d*x + c)
^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(7/2), x)
```

3.859 $\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{2(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aB+5bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aC+bB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aB+5bC)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a}{5d}$$

```
[Out] (-2*(3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*C)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(b*B + a*C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a*B + 5*b*
C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.247685, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3029, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(aC+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aB+5bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aC+bB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aB+5bC)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]
^(9/2),x]
```


```
[Out] (-2*(3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*C)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(b*B + a*C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a*B + 5*b*
C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx = \int \frac{(a+b\cos(c+dx))(B+C\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$= \int \frac{aB+(bB+aC)\cos(c+dx)+bC\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}(bB+aC)+\frac{1}{2}(3aB+5bC)\cos(c}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (bB+aC)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx + \frac{1}{5}(3aB$$

$$= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3aB+5bC)}{5d\sqrt{\cos(c}}$$

$$= -\frac{2(3aB+5bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(bB+aC)F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

Mathematica [A] time = 0.798197, size = 134, normalized size = 0.96

$$\frac{10(aC+bB)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6(3aB+5bC)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9aB\sin(2(c+dx))+6aB\tan(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (-6*(3*a*B + 5*b*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(b*B
+ a*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*b*B*Sin[c + d*x] +
 10*a*C*Sin[c + d*x] + 9*a*B*Sin[2*(c + d*x)] + 15*b*C*Sin[2*(c + d*x)] + 6
*a*B*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.744, size = 663, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(B*b+C*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(
1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*B*a/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^2+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))/cos(d*x + c)
^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)/cos(d*x
+ c)^(9/2), x)
```

3.860 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c\right.$

Optimal. Leaf size=264

$$\frac{2\left(9a^2B+14abC+7b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2B+14abC+7b^2B\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10\left(11a(aC+2bB)+\right.}{2}$$

```
[Out] (2*(9*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(
9*b^2*C + 11*a*(2*b*B + a*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*(9*b
^2*C + 11*a*(2*b*B + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(9
*a^2*B + 7*b^2*B + 14*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(
9*b^2*C + 11*a*(2*b*B + a*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*
b*(11*b*B + 13*a*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*C*Cos[c
+ d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.461829, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2990, 3023, 2748, 2635, 2639, 2641}

$$\frac{2\left(9a^2B+14abC+7b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2B+14abC+7b^2B\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10\left(11a(aC+2bB)+\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```


```
[Out] (2*(9*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(
9*b^2*C + 11*a*(2*b*B + a*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*(9*b
^2*C + 11*a*(2*b*B + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(9
*a^2*B + 7*b^2*B + 14*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(
9*b^2*C + 11*a*(2*b*B + a*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*
b*(11*b*B + 13*a*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*C*Cos[c
+ d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(B+C\,\mathrm{co}$$

$$= \frac{2bC\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+}{11d}$$

$$= \frac{2b(11bB+13aC)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d}+$$

$$= \frac{2b(11bB+13aC)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d}+$$

$$= \frac{2\left(9a^2B+7b^2B+14abC\right)\cos^{\frac{3}{2}}(c+dx)\sin(c}{45d}$$

$$= \frac{2\left(9a^2B+7b^2B+14abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

$$= \frac{2\left(9a^2B+7b^2B+14abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

Mathematica [A] time = 1.67936, size = 196, normalized size = 0.74

$$\frac{1200\left(11a^2C+22abB+9b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(9a^2B+14abC+7b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (3696*(9*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticE[(c + d*x)/2, 2] + 1200*(22*a
*b*B + 11*a^2*C + 9*b^2*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]
*(154*(36*a^2*B + 43*b^2*B + 86*a*b*C)*Cos[c + d*x] + 180*(22*a*b*B + 11*a^
2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 770*b*(b*B + 2*a*C)*Cos[3*(c + d*x)] + 1
5*(1144*a*b*B + 572*a^2*C + 531*b^2*C + 21*b^2*C*Cos[4*(c + d*x)]))*Sin[c +
```

```
d*x])/(27720*d)
```

Maple [B] time = 0.647, size = 666, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^2-24640*C*a*b-50400*
C*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(15840*B*a*b+24640*B*b^2+79
20*C*a^2+49280*C*a*b+56880*C*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-5544*B*a^2-23760*B*a*b-22792*B*b^2-11880*C*a^2-45584*C*a*b-34920*C*b^2)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(5544*B*a^2+18480*B*a*b+10472*B*b^2+9
240*C*a^2+20944*C*a*b+13860*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-1386*B*a^2-5280*B*a*b-1848*B*b^2-2640*C*a^2-3696*C*a*b-2790*C*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1650*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+825*a^2*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+675*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3234*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2*cos(d*
x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^5 + Ba^2\cos(dx+c)^2 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^4 + \left(Ca^2 + 2\,Bab\right)\cos(dx+c)^3\right)\sqrt{\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5 + B*a^2*cos(d*x + c)^2 + (2*C*a*b + B*b^2)*c
os(d*x + c)^4 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.861 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2 \left(B\cos(c+dx)+C\cos^2(c\right.$

Optimal. Leaf size=223

$$\frac{2\left(7a^2B+10abC+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2B+10abC+5b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(9a(aC+2bB)+7b\right.}{15}$$

```
[Out] (2*(7*b^2*C + 9*a*(2*b*B + a*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*
a^2*B + 5*b^2*B + 10*a*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2*B
 + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*b^2*
C + 9*a*(2*b*B + a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*b*
B + 11*a*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*C*Cos[c + d*x]^(
5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.423099, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2990, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(7a^2B+10abC+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2B+10abC+5b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(9a(aC+2bB)+7b\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```


```
[Out] (2*(7*b^2*C + 9*a*(2*b*B + a*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*
a^2*B + 5*b^2*B + 10*a*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2*B
 + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*b^2*
C + 9*a*(2*b*B + a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*b*
B + 11*a*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*C*Cos[c + d*x]^(
5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(B+C\cos$$

$$=\frac{2bC\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+d}{9d}$$

$$=\frac{2b(9bB+11aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2}{}$$

$$=\frac{2b(9bB+11aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2}{}$$

$$=\frac{2\left(7a^2B+5b^2B+10abC\right)\sqrt{\cos(c+dx)}\sin(c}{21d}$$

$$=\frac{2\left(7b^2C+9a(2bB+aC)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

Mathematica [A] time = 1.38698, size = 167, normalized size = 0.75

$$\frac{60\left(7a^2B+10abC+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(9a^2C+18abB+7b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (84*(18*a*b*B + 9*a^2*C + 7*b^2*C)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^2*B
+ 5*b^2*B + 10*a*b*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(72
*a*b*B + 36*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*(84*a^2*B + 78*b^2*B + 156*a
*b*C + 18*b*(b*B + 2*a*C)*Cos[2*(c + d*x)] + 7*b^2*C*Cos[3*(c + d*x)]))*Sin
[c + d*x])/(630*d)
```

Maple [B] time = 0.641, size = 610, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^2+1440*C*a*b+2240*C*b^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1008*B*a*b-1080*B*b^2-504*C*a^2-
2160*C*a*b-2072*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*B*a^2+1
008*B*a*b+840*B*b^2+504*C*a^2+1680*C*a*b+952*C*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-210*B*a^2-252*B*a*b-240*B*b^2-126*C*a^2-480*C*a*b-168*C*
b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+75*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+1
50*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2*sqrt(c
os(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+Ba^2\cos\left(dx+c\right)+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+\left(Ca^2+2\,Bab\right)\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2*sqrt(c
os(d*x + c)), x)
```

3.862 $\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=182

$$\frac{2\left(5a^2B+6abC+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a(aC+2bB)+5b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a(aC+2bB)+5b^2C\right)\sin(\cdots}{21d}$$

```
[Out] (2*(5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*b
^2*C + 7*a*(2*b*B + a*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*b^2*C +
 7*a*(2*b*B + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(7*b*B +
 9*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b*C*Cos[c + d*x]^(3/2)
*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.393585, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2990, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(5a^2B+6abC+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a(aC+2bB)+5b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a(aC+2bB)+5b^2C\right)\sin(\cdots}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c
 + d*x]],x]
```



```
[Out] (2*(5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*b
^2*C + 7*a*(2*b*B + a*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*b^2*C +
 7*a*(2*b*B + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(7*b*B +
 9*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b*C*Cos[c + d*x]^(3/2)
*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(B+C\cos(c+dx))\,d$$

$$= \frac{2bC\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{}$$

$$= \frac{2b(7bB+9aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bC\cos^{\frac{3}{2}}(c+}{}$$

$$= \frac{2b(7bB+9aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bC\cos^{\frac{3}{2}}(c+}{}$$

$$= \frac{2\left(5a^2B+3b^2B+6abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5b^2C+7a\right.}{}$$

$$= \frac{2\left(5a^2B+3b^2B+6abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5b^2C+7a\right.}{}$$

Mathematica [A] time = 1.0325, size = 139, normalized size = 0.76

$$\frac{10\left(7a^2C+14abB+5b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(5a^2B+6abC+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(5\left(\right.\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```

```
[Out] (42*(5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2] + 10*(14*a*b*B
+ 7*a^2*C + 5*b^2*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*b*(
b*B + 2*a*C)*Cos[c + d*x] + 5*(28*a*b*B + 14*a^2*C + 13*b^2*C + 3*b^2*C*Cos
[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] time = 0.722, size = 548, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^2-336*C*a*b-360*C*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*B*a*b+168*B*b^2+140*C*a^2+336*C*a
*b+280*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*B*a*b-42*B*b^2-
70*C*a^2-84*C*a*b-80*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*a*b*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/sqrt(c
os(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/sqrt(c
os(d*x + c)), x)
```

3.863 $\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{2\left(3a^2B+2abC+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aC+2bB)+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aC+5bB)\sin(c+dx)\sqrt{\cos(}}{15d}$$

```
[Out] (2*(3*b^2*C + 5*a*(2*b*B + a*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3*a
^2*B + b^2*B + 2*a*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(5*b*B + 7*
a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*C*Sqrt[Cos[c + d*x]]*(a
 + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.356083, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3029, 2990, 3023, 2748, 2641, 2639}

$$\frac{2\left(3a^2B+2abC+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aC+2bB)+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aC+5bB)\sin(c+dx)\sqrt{\cos(}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(3/2),x]
```


```
[Out] (2*(3*b^2*C + 5*a*(2*b*B + a*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3*a
^2*B + b^2*B + 2*a*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(5*b*B + 7*
a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*C*Sqrt[Cos[c + d*x]]*(a
 + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^2(B+C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}a}{}$$

$$= \frac{2b(5bB+7aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bC\sqrt{\cos(c+d}}{}$$

$$= \frac{2b(5bB+7aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bC\sqrt{\cos(c+d}}{}$$

$$= \frac{2\left(3b^2C+5a(2bB+aC)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(3a^2B+b^2B\right.}{}$$

Mathematica [A] time = 0.569829, size = 106, normalized size = 0.76

$$\frac{2\left(5\left(3a^2B+2abC+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3\left(5a^2C+10abB+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\sqrt{\cos(c+dx)}(10a}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```

```
[Out] (2*(3*(10*a*b*B + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*B
 + b^2*B + 2*a*b*C)*EllipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(5*b*B
 + 10*a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 0.742, size = 487, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b^2+40*C*a*b+24*C*b^2)*sin(1/2*
```

```
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*B*b^2-20*C*a*b-6*C*b^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*a*b*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(3/2), x)
```

3.864 $\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{2\left(3a^2C+6abB+b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B-2abC-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2C\sin(c+d}{3}$$

```
[Out] (-2*(a^2*B - b^2*B - 2*a*b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*b*B +
3*a^2*C + b^2*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*B*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (2*b^2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.335469, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3029, 2988, 3023, 2748, 2641, 2639}

$$\frac{2\left(3a^2C+6abB+b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B-2abC-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2C\sin(c+d}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(5/2),x]
```

```
[Out] (-2*(a^2*B - b^2*B - 2*a*b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*b*B +
3*a^2*C + b^2*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*B*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (2*b^2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^2(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{-\frac{1}{2}a(2bB+aC)+\frac{1}{2}\left(a^2B-b^2B-\right.}{\sqrt{c}}$$

$$= \frac{2a^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{4}{3}\int$$

$$= \frac{2a^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \left(a^2B\right.$$

$$= -\frac{2\left(a^2B-b^2B-2abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(6abB+3a^2\right.}{}$$

Mathematica [A] time = 0.624118, size = 102, normalized size = 0.84

$$\frac{2\left(\left(3a^2C+6abB+b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(-3a^2B+6abC+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(3a^2B+b^2C\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```

```
[Out] (2*((-3*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2] + (6*a*b*B + 3
*a^2*C + b^2*C)*EllipticF[(c + d*x)/2, 2] + ((3*a^2*B + b^2*C*Cos[c + d*x])
*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)
```

Maple [B] time = 0.66, size = 404, normalized size = 3.3

$$-\frac{2}{3\,d}\left(4\,b^2C\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+6\,abB\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(4*b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*a*b*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*B*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*a^2*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*
C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(5/2), x)
```

3.865 $\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{2\left(a^2B+6abC+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2C+2abB-b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aC+2bB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(2*a*b*B + a^2*C - b^2*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*B + 3*
b^2*B + 6*a*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*B*Sin[c + d*x])/
(3*d*Cos[c + d*x]^(3/2)) + (2*a*(2*b*B + a*C)*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

Rubi [A] time = 0.359727, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3029, 2988, 3021, 2748, 2641, 2639}

$$\frac{2\left(a^2B+6abC+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2C+2abB-b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aC+2bB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(7/2),x]
```


```
[Out] (-2*(2*a*b*B + a^2*C - b^2*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*B + 3*
b^2*B + 6*a*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*B*Sin[c + d*x])/
(3*d*Cos[c + d*x]^(3/2)) + (2*a*(2*b*B + a*C)*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^2(B+C\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}a(2bB+aC)-\frac{1}{2}\left(a^2B+3b^2B\right.}{\cos}$$

$$= \frac{2a^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{4}{3}\int \frac{\frac{1}{4}\left(-\right.}{}$$

$$= \frac{2a^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{3}\left(-a^2B-\right.$$

$$= -\frac{2\left(2abB+a^2C-b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2B+3b^2B\right.}{}$$

Mathematica [A] time = 1.0967, size = 105, normalized size = 0.83

$$\frac{2\left(\left(a^2B+6abC+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(a^2C+2abB-b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(3(aC+2bB)\cos(c+dx)+aB)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(7/2),x]
```

```
[Out] (2*(-3*(2*a*b*B + a^2*C - b^2*C)*EllipticE[(c + d*x)/2, 2] + (a^2*B + 3*b^2
*B + 6*a*b*C)*EllipticF[(c + d*x)/2, 2] + (a*(a*B + 3*(2*b*B + a*C)*Cos[c +
 d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 1.567, size = 677, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a*b*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a^2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(2*B*b+C*a)*(-(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(7/2), x)
```

3.866 $\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(a^2C+2abB+3b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2B+10abC+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2B+10abC+5b^2B\right)\sin(c}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(3*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2
*a*b*B + a^2*C + 3*b^2*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*B*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(2*b*B + a*C)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(3*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rubi [A] time = 0.391498, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2988, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(a^2C+2abB+3b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2B+10abC+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2B+10abC+5b^2B\right)\sin(c}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(9/2),x]
```



```
[Out] (-2*(3*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2
*a*b*B + a^2*C + 3*b^2*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*B*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(2*b*B + a*C)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(3*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^2(B+C\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2bB+aC)-\frac{1}{2}\left(3a^2B+5b^2\right.}{}$$

$$= \frac{2a^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int$$

$$= \frac{2a^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}\left(-3a^2\right.$$

$$= \frac{2\left(2abB+a^2C+3b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2B\sin(c+}{5d\cos^{\frac{5}{2}}(c+}$$

$$= -\frac{2\left(3a^2B+5b^2B+10abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(2abB+\right.}{}$$

Mathematica [A] time = 1.05368, size = 175, normalized size = 1.02

$$\frac{10\left(a^2C+2abB+3b^2C\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(3a^2B+10abC+5b^2B\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9a^2B}{15d\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(9/2),x]
```

```
[Out] (-6*(3*a^2*B + 5*b^2*B + 10*a*b*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2
, 2] + 10*(2*a*b*B + a^2*C + 3*b^2*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x
)/2, 2] + 20*a*b*B*Sin[c + d*x] + 10*a^2*C*Sin[c + d*x] + 9*a^2*B*Sin[2*(c
+ d*x)] + 15*b^2*B*Sin[2*(c + d*x)] + 30*a*b*C*Sin[2*(c + d*x)] + 6*a^2*B*T
an[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 2.03, size = 750, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
*(2*B*b+C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(B*b+2*C*a)*(-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a
^2*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos(dx+c)^3 + Ba^2 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^2 + \left(Ca^2 + 2\,Bab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c) + a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(9/2), x)
```

3.867 $\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{2\left(5a^2B+14abC+7b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(3a^2C+6abB+5b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2B+14abC+7b^2B\right)\sin(}{21d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(6*a*b*B + 3*a^2*C + 5*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*
a^2*B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*B*Si
n[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*a*(2*b*B + a*C)*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)) + (2*(5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])/(21
*d*Cos[c + d*x]^(3/2)) + (2*(6*a*b*B + 3*a^2*C + 5*b^2*C)*Sin[c + d*x])/(5*
d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.437819, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2988, 3021, 2748, 2636, 2641, 2639}

$$\frac{2\left(5a^2B+14abC+7b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(3a^2C+6abB+5b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2B+14abC+7b^2B\right)\sin(}{21d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(11/2),x]
```



```
[Out] (-2*(6*a*b*B + 3*a^2*C + 5*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*
a^2*B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*B*Si
n[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*a*(2*b*B + a*C)*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)) + (2*(5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])/(21
*d*Cos[c + d*x]^(3/2)) + (2*(6*a*b*B + 3*a^2*C + 5*b^2*C)*Sin[c + d*x])/(5*
d*Sqrt[Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^2(B+C\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(2bB+aC)-\frac{1}{2}\left(5a^2B+7b^2\right.}{}$$

$$= \frac{2a^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int$$

$$= \frac{2a^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{7}\left(-5a^2\right.$$

$$= \frac{2a^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(2bB+aC)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(5a^2B\right.}{}$$

$$= -\frac{2\left(6abB+3a^2C+5b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5a^2B+7\right.}{}$$

Mathematica [A] time = 4.19834, size = 191, normalized size = 0.89

$$\frac{2\left(5\left(5a^2B+14abC+7b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-21\left(3a^2C+6abB+5b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{5\left(5a^2B+14abC+7b^2B\right)\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}+\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(11/2),x]
```

```
[Out] (2*(-21*(6*a*b*B + 3*a^2*C + 5*b^2*C)*EllipticE[(c + d*x)/2, 2] + 5*(5*a^2*
B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d*x)/2, 2] + (15*a^2*B*Sin[c + d*x])
/Cos[c + d*x]^(7/2) + (21*a*(2*b*B + a*C)*Sin[c + d*x])/Cos[c + d*x]^(5/2)
+ (5*(5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21*
(6*a*b*B + 3*a^2*C + 5*b^2*C)*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)
```

Maple [B] time = 2.485, size = 859, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*(B*b+2*C*a)
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*C*(-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*a^2*B*(-1/56*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a*(2*B*b+C*a
)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*
sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos(dx+c)^3 + Ba^2 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^2 + \left(Ca^2 + 2\,Bab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\left(b\cos(dx+c) + a\right)^2}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^2/cos(d*
x + c)^(11/2), x)
```

3.868 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(\right.$

Optimal. Leaf size=305

$$\frac{2\left(165a^2bC+77a^3B+165ab^2B+45b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(27a^2bB+9a^3C+21ab^2C+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2}{\ }$$

```
[Out] (2*(27*a^2*b*B + 7*b^3*B + 9*a^3*C + 21*a*b^2*C)*EllipticE[(c + d*x)/2, 2])
/(15*d) + (2*(77*a^3*B + 165*a*b^2*B + 165*a^2*b*C + 45*b^3*C)*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(77*a^3*B + 165*a*b^2*B + 165*a^2*b*C + 45*b^3*
C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(27*a^2*b*B + 7*b^3*B + 9*
a^3*C + 21*a*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(33*a*b*
B + 26*a^2*C + 9*b^2*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*b^2*(1
1*b*B + 15*a*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*C*Cos[c + d*
x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.638124, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2990, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(165a^2bC+77a^3B+165ab^2B+45b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(27a^2bB+9a^3C+21ab^2C+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2}{\ }$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```



```
[Out] (2*(27*a^2*b*B + 7*b^3*B + 9*a^3*C + 21*a*b^2*C)*EllipticE[(c + d*x)/2, 2])
/(15*d) + (2*(77*a^3*B + 165*a*b^2*B + 165*a^2*b*C + 45*b^3*C)*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(77*a^3*B + 165*a*b^2*B + 165*a^2*b*C + 45*b^3*
C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(27*a^2*b*B + 7*b^3*B + 9*
a^3*C + 21*a*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(33*a*b*
B + 26*a^2*C + 9*b^2*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*b^2*(1
1*b*B + 15*a*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*C*Cos[c + d*
x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(B+C\cos$$

$$= \frac{2bC\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c}{11d}$$

$$= \frac{2b^2(11bB+15aC)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d}$$

$$= \frac{2b\left(33abB+26a^2C+9b^2C\right)\cos^{\frac{5}{2}}(c+dx)\sin}{77d}$$

$$= \frac{2b\left(33abB+26a^2C+9b^2C\right)\cos^{\frac{5}{2}}(c+dx)\sin}{77d}$$

$$= \frac{2\left(77a^3B+165ab^2B+165a^2bC+45b^3C\right)\sqrt{}}{231d}$$

$$= \frac{2\left(27a^2bB+7b^3B+9a^3C+21ab^2C\right)E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [A] time = 1.91069, size = 235, normalized size = 0.77

$$\frac{240\left(165a^2bC+77a^3B+165ab^2B+45b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(27a^2bB+9a^3C+21ab^2C+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (3696*(27*a^2*b*B + 7*b^3*B + 9*a^3*C + 21*a*b^2*C)*EllipticE[(c + d*x)/2,
2] + 240*(77*a^3*B + 165*a*b^2*B + 165*a^2*b*C + 45*b^3*C)*EllipticF[(c + d
*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(154*(108*a^2*b*B + 43*b^3*B + 36*a^3*C +
129*a*b^2*C)*Cos[c + d*x] + 180*b*(33*a*b*B + 33*a^2*C + 16*b^2*C)*Cos[2*(c
 + d*x)] + 770*b^2*(b*B + 3*a*C)*Cos[3*(c + d*x)] + 15*(616*a^3*B + 1716*a*
b^2*B + 1716*a^2*b*C + 531*b^3*C + 21*b^3*C*Cos[4*(c + d*x)]))*Sin[c + d*x]
)/(27720*d)
```

Maple [B] time = 0.684, size = 825, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^3-36960*C*a*b^2-5040
0*C*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(23760*B*a*b^2+24640*B*b^
3+23760*C*a^2*b+73920*C*a*b^2+56880*C*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x
+1/2*c)+(-16632*B*a^2*b-35640*B*a*b^2-22792*B*b^3-5544*C*a^3-35640*C*a^2*b-
68376*C*a*b^2-34920*C*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4620*B*
a^3+16632*B*a^2*b+27720*B*a*b^2+10472*B*b^3+5544*C*a^3+27720*C*a^2*b+31416*
C*a*b^2+13860*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310*B*a^3-4
158*B*a^2*b-7920*B*a*b^2-1848*B*b^3-1386*C*a^3-7920*C*a^2*b-5544*C*a*b^2-27
90*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-6237*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2*b-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+1155*a^3*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+2475*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a^3-4851*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+2475*a^2*b*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+675*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
```

```
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3*sqrt(c
os(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^{3}\cos\left(dx+c\right)^{5}+Ba^{3}\cos\left(dx+c\right)+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+3\left(Ca^{2}b+Bab^{2}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + B*a^3*cos(d*x + c) + (3*C*a*b^2 + B*b^3)*c
os(d*x + c)^4 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b)*
cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.869 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=255

$$\frac{2\left(21a^2bB+7a^3C+15ab^2C+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(27a^2bC+15a^3B+27ab^2B+7b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(2\right.}{}$$

```
[Out] (2*(15*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 7*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(15*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*C + 15*a*b^2*C)*EllipticF[(c +
d*x)/2, 2])/(21*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*C + 15*a*b^2*C)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a*b*B + 22*a^2*C + 7*b^2*C)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*b*B + 13*a*C)*Cos[c + d*
x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.581075, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2990, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(21a^2bB+7a^3C+15ab^2C+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(27a^2bC+15a^3B+27ab^2B+7b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c
 + d*x]],x]
```



```
[Out] (2*(15*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 7*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(15*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*C + 15*a*b^2*C)*EllipticF[(c +
d*x)/2, 2])/(21*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*C + 15*a*b^2*C)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a*b*B + 22*a^2*C + 7*b^2*C)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*b*B + 13*a*C)*Cos[c + d*
x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(9*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(B+C\cos(c+dx))\,d$$

$$= \frac{2bC\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{9d} + \frac{2}{9}\int$$

$$= \frac{2b^2(9bB+13aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bC\cos^{\frac{3}{2}}(c}{}$$

$$= \frac{2b\left(27abB+22a^2C+7b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + $$

$$= \frac{2b\left(27abB+22a^2C+7b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + $$

$$= \frac{2\left(15a^3B+27ab^2B+27a^2bC+7b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + $$

$$= \frac{2\left(15a^3B+27ab^2B+27a^2bC+7b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + $$

Mathematica [A] time = 1.15429, size = 197, normalized size = 0.77

$$\frac{60\left(21a^2bB+7a^3C+15ab^2C+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(27a^2bC+15a^3B+27ab^2B+7b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```

```
[Out] (84*(15*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 7*b^3*C)*EllipticE[(c + d*x)/2, 2
] + 60*(21*a^2*b*B + 5*b^3*B + 7*a^3*C + 15*a*b^2*C)*EllipticF[(c + d*x)/2,
 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a*b*B + 108*a^2*C + 43*b^2*C)*Cos[c + d*
x] + 5*(252*a^2*b*B + 78*b^3*B + 84*a^3*C + 234*a*b^2*C + 18*b^2*(b*B + 3*a
*C)*Cos[2*(c + d*x)] + 7*b^3*C*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 0.661, size = 745, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^3+2160*C*a*b^2+2240*C*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*B*a*b^2-1080*B*b^3-1512*C
*a^2*b-3240*C*a*b^2-2072*C*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*B*a^2*b+1512*B*a*b^2+840*B*b^3+420*C*a^3+1512*C*a^2*b+2520*C*a*b^2+952*C
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*B*a^2*b-378*B*a*b^2-240
*B*b^3-210*C*a^3-378*C*a^2*b-720*C*a*b^2-168*C*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+315*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*b^3*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+105*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*C*a*b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-567*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/sqrt(c
os(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^4+Ba^3+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^3+3\left(Ca^2b+Bab^2\right)\cos\left(dx+c\right)^2+\left(Ca^3+3\,Ba^2b\right)\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/sqrt(c
os(d*x + c)), x)
```

3.870 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{2\left(21a^2bC+21a^3B+21ab^2B+5b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^2bB+5a^3C+9ab^2C+3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(18}{}$$

```
[Out] (2*(15*a^2*b*B + 3*b^3*B + 5*a^3*C + 9*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*C + 5*b^3*C)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(21*a*b*B + 18*a^2*C + 5*b^2*C)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(21*d) + (2*b^2*(7*b*B + 11*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*b*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d)
```

Rubi [A] time = 0.554093, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2990, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(21a^2bC+21a^3B+21ab^2B+5b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^2bB+5a^3C+9ab^2C+3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(18}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(3/2),x]
```



```
[Out] (2*(15*a^2*b*B + 3*b^3*B + 5*a^3*C + 9*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*C + 5*b^3*C)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(21*a*b*B + 18*a^2*C + 5*b^2*C)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(21*d) + (2*b^2*(7*b*B + 11*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*b*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^3(B+C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int$$

$$= \frac{2b^2(7bB+11aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bC\sqrt{\cos(c}}{}$$

$$= \frac{2b\left(21abB+18a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

$$= \frac{2b\left(21abB+18a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

$$= \frac{2\left(15a^2bB+3b^3B+5a^3C+9ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2}{}$$

Mathematica [A] time = 1.25879, size = 158, normalized size = 0.77

$$\frac{10\left(21a^2bC+21a^3B+21ab^2B+5b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(15a^2bB+5a^3C+9ab^2C+3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```

```
[Out] (42*(15*a^2*b*B + 3*b^3*B + 5*a^3*C + 9*a*b^2*C)*EllipticE[(c + d*x)/2, 2]
+ 10*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*C + 5*b^3*C)*EllipticF[(c + d*x)/2,
2] + b*Sqrt[Cos[c + d*x]]*(42*b*(b*B + 3*a*C)*Cos[c + d*x] + 5*(42*a*b*B +
42*a^2*C + 13*b^2*C + 3*b^2*C*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] time = 0.707, size = 664, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^3-504*C*a*b^2-360*C*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*B*a*b^2+168*B*b^3+420*C*a^2*b+5
04*C*a*b^2+280*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*B*a*b^2
-42*B*b^3-210*C*a^2*b-126*C*a*b^2-80*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b^3+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+1
05*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos(dx+c)^4 + Ba^3 + (3\,Cab^2 + Bb^3)\cos(dx+c)^3 + 3\,(Ca^2b + Bab^2)\cos(dx+c)^2 + (Ca^3 + 3\,Ba^2b)\cos}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(3/2), x)
```

3.871 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{2\left(9a^2bB+3a^3C+3ab^2C+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(-15a^2bC+5a^3B-15ab^2B-3b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(6a^2B-\right.}{}$$

```
[Out] (-2*(5*a^3*B - 15*a*b^2*B - 15*a^2*b*C - 3*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(9*a^2*b*B + b^3*B + 3*a^3*C + 3*a*b^2*C)*EllipticF[(c + d*x)/
2, 2])/(3*d) - (2*b*(6*a^2*B - b^2*B - 3*a*b*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d) - (2*b^2*(5*a*B - b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) +
 (2*a*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.558461, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2989, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(9a^2bB+3a^3C+3ab^2C+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(-15a^2bC+5a^3B-15ab^2B-3b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(6a^2B-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(5/2),x]
```


```
[Out] (-2*(5*a^3*B - 15*a*b^2*B - 15*a^2*b*C - 3*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(9*a^2*b*B + b^3*B + 3*a^3*C + 3*a*b^2*C)*EllipticF[(c + d*x)/
2, 2])/(3*d) - (2*b*(6*a^2*B - b^2*B - 3*a*b*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d) - (2*b^2*(5*a*B - b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) +
 (2*a*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3 \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^3(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+d$$

$$= -\frac{2b^2(5aB-bC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aB(a+b\cos(}{d\sqrt{c}}$$

$$= -\frac{2b\left(6a^2B-b^2B-3abC\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(}{}$$

$$= -\frac{2b\left(6a^2B-b^2B-3abC\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(}{}$$

$$= -\frac{2\left(5a^3B-15ab^2B-15a^2bC-3b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2}{}$$

Mathematica [A] time = 1.11544, size = 150, normalized size = 0.74

$$\frac{10\left(9a^2bB+3a^3C+3ab^2C+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(90a^2bC-30a^3B+90ab^2B+18b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)(3(}{}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```

```
[Out] ((-30*a^3*B + 90*a*b^2*B + 90*a^2*b*C + 18*b^3*C)*EllipticE[(c + d*x)/2, 2]
 + 10*(9*a^2*b*B + b^3*B + 3*a^3*C + 3*a*b^2*C)*EllipticF[(c + d*x)/2, 2] +
 ((10*b^2*(b*B + 3*a*C)*Cos[c + d*x] + 3*(10*a^3*B + b^3*C + b^3*C*Cos[2*(c
```

```
+ d*x)]))*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] time = 0.718, size = 867, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b^2*(5*B*b+15*C*a+6*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*B*a^3+5*B*b
^3+15*C*a*b^2+3*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*a^2*b*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+15*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3-45*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+15*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*C*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-45*C*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2*b-9*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos(dx+c)^4 + Ba^3 + \left(3\,Cab^2 + Bb^3\right)\cos(dx+c)^3 + 3\left(Ca^2b + Bab^2\right)\cos(dx+c)^2 + \left(Ca^3 + 3\,Ba^2b\right)\cos(\right.}{\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\left(b\cos(dx+c) + a\right)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(5/2), x)
```

3.872 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2\left(9a^2bC+a^3B+9ab^2B+b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2bB+a^3C-3ab^2C-b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(3aC+7bB)\,\mathrm{s}}{3d\sqrt{\cos(c+}}$$

```
[Out] (-2*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/d +
(2*(a^3*B + 9*a*b^2*B + 9*a^2*b*C + b^3*C)*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*a^2*(7*b*B + 3*a*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(
a*B - b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*B*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.532273, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2989, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\left(9a^2bC+a^3B+9ab^2B+b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2bB+a^3C-3ab^2C-b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(3aC+7bB)\,\mathrm{s}}{3d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(7/2),x]
```


```
[Out] (-2*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/d +
(2*(a^3*B + 9*a*b^2*B + 9*a^2*b*C + b^3*C)*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*a^2*(7*b*B + 3*a*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(
a*B - b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*B*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^3(B+C\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+\ldots}{\ldots}$$

$$= \frac{2a^2(7bB+3aC)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2aB(a+b\cos(c+dx))^2\sin\ldots}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(7bB+3aC)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aB-bC)\sqrt{\cos(c+dx)}\,\mathrm{s}\ldots}{3d}$$

$$= \frac{2a^2(7bB+3aC)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aB-bC)\sqrt{\cos(c+dx)}\,\mathrm{s}\ldots}{3d}$$

$$= -\frac{2\left(3a^2bB-b^3B+a^3C-3ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^3\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.01865, size = 165, normalized size = 0.86

$$\frac{2\left(9a^2bC+a^3B+9ab^2B+b^3C\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+d\ldots\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 2*(a^3*B + 9*a*b^2*B + 9*a^2*b*C + b^3*C)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 18*a^2*b*B*Sin[c + d*x] + 6*a^3*C*Sin[c + d*x
] + b^3*C*Sin[2*(c + d*x)] + 2*a^3*B*Tan[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.844, size = 1212, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*C*b^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+18*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2
*d*x+1/2*c)^2-6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+2*B*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+18*B*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*a*b^2*sin(1/2*d*x+1/2*c)^2-36*B*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-18*C*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+18*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*
b*sin(1/2*d*x+1/2*c)^2+2*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^
2-12*C*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-8*C*b^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*b^3-a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a*b^2*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+2*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*B*a^2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+9*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b^2-9*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
```

```
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+6*C*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*C*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{4}+Ba^{3}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{3}+3\left(Ca^{2}b+Bab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{3}+3\,Ba^{2}b\right)\cos\left(\right.}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(7/2), x)
```

3.873 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{2\left(3a^2bB+a^3C+9ab^2C+3b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(15a^2bC+3a^3B+15ab^2B-5b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(3a^2B+\right.}{}$$

```
[Out] (-2*(3*a^3*B + 15*a*b^2*B + 15*a^2*b*C - 5*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(3*a^2*b*B + 3*b^3*B + a^3*C + 9*a*b^2*C)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*(9*b*B + 5*a*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2
)) + (2*a*(3*a^2*B + 14*b^2*B + 15*a*b*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*a*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
)
```

Rubi [A] time = 0.546613, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2989, 3031, 3021, 2748, 2641, 2639}

$$\frac{2\left(3a^2bB+a^3C+9ab^2C+3b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(15a^2bC+3a^3B+15ab^2B-5b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(3a^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(9/2),x]
```


```
[Out] (-2*(3*a^3*B + 15*a*b^2*B + 15*a^2*b*C - 5*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(3*a^2*b*B + 3*b^3*B + a^3*C + 9*a*b^2*C)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*(9*b*B + 5*a*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2
)) + (2*a*(3*a^2*B + 14*b^2*B + 15*a*b*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*a*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^3(B+C\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+ \cdots}{\cdots}$$

$$= \frac{2a^2(9bB+5aC)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aB(a+b\cos(c+dx))^2\sin \cdots}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(9bB+5aC)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(3a^2B+14b^2B+15abC\right) \cdots}{5d\sqrt{\cos(c+dx)} \cdots}$$

$$= \frac{2a^2(9bB+5aC)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(3a^2B+14b^2B+15abC\right) \cdots}{5d\sqrt{\cos(c+dx)} \cdots}$$

$$= -\frac{2\left(3a^3B+15ab^2B+15a^2bC-5b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \cdots$$

Mathematica [A] time = 2.02626, size = 176, normalized size = 0.86

$$\frac{9a\left(a^2B+5abC+5b^2B\right)\sin(2(c+dx))+10\left(3a^2bB+a^3C+9ab^2C+3b^3B\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(15a^2bC+ \cdots\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(9/2),x]
```

```
[Out] (-6*(3*a^3*B + 15*a*b^2*B + 15*a^2*b*C - 5*b^3*C)*Cos[c + d*x]^(3/2)*Ellipt
icE[(c + d*x)/2, 2] + 10*(3*a^2*b*B + 3*b^3*B + a^3*C + 9*a*b^2*C)*Cos[c +
d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*(3*b*B + a*C)*Sin[c + d*x] +
9*a*(a^2*B + 5*b^2*B + 5*a*b*C)*Sin[2*(c + d*x)] + 6*a^3*B*Tan[c + d*x])/(1
5*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 2.143, size = 997, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a^2*(3*B*b+C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b*(B*b+C*a)
*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-
2/5*a^3*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
```

```
/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{4}+Ba^{3}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{3}+3\left(Ca^{2}b+Bab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{3}+3\,Ba^{2}b\right)\cos(}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(9/2), x)
```

3.874 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=255

$$\frac{2\left(21a^2bC+5a^3B+21ab^2B+21b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(9a^2bB+3a^3C+15ab^2C+5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2\right.}{}$$

```
[Out] (-2*(9*a^2*b*B + 5*b^3*B + 3*a^3*C + 15*a*b^2*C)*EllipticE[(c + d*x)/2, 2])
/(5*d) + (2*(5*a^3*B + 21*a*b^2*B + 21*a^2*b*C + 21*b^3*C)*EllipticF[(c + d
*x)/2, 2])/(21*d) + (2*a^2*(11*b*B + 7*a*C)*Sin[c + d*x])/(35*d*Cos[c + d*x
]^(5/2)) + (2*a*(5*a^2*B + 18*b^2*B + 21*a*b*C)*Sin[c + d*x])/(21*d*Cos[c +
 d*x]^(3/2)) + (2*(9*a^2*b*B + 5*b^3*B + 3*a^3*C + 15*a*b^2*C)*Sin[c + d*x]
)/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7
*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.59507, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2989, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(21a^2bC+5a^3B+21ab^2B+21b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(9a^2bB+3a^3C+15ab^2C+5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(11/2),x]
```


```
[Out] (-2*(9*a^2*b*B + 5*b^3*B + 3*a^3*C + 15*a*b^2*C)*EllipticE[(c + d*x)/2, 2])
/(5*d) + (2*(5*a^3*B + 21*a*b^2*B + 21*a^2*b*C + 21*b^3*C)*EllipticF[(c + d
*x)/2, 2])/(21*d) + (2*a^2*(11*b*B + 7*a*C)*Sin[c + d*x])/(35*d*Cos[c + d*x
]^(5/2)) + (2*a*(5*a^2*B + 18*b^2*B + 21*a*b*C)*Sin[c + d*x])/(21*d*Cos[c +
 d*x]^(3/2)) + (2*(9*a^2*b*B + 5*b^3*B + 3*a^3*C + 15*a*b^2*C)*Sin[c + d*x]
)/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7
*d*Cos[c + d*x]^(7/2))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^3(B+C\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\cos(c+$$

$$= \frac{2a^2(11bB+7aC)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aB(a+b\cos(c+dx))^2 s}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(11bB+7aC)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\left(5a^2B+18b^2B+21ab\right.}{21d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2a^2(11bB+7aC)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\left(5a^2B+18b^2B+21ab\right.}{21d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2\left(5a^3B+21ab^2B+21a^2bC+21b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} +$$

$$= -\frac{2\left(9a^2bB+5b^3B+3a^3C+15ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} +$$

Mathematica [A] time = 3.42871, size = 221, normalized size = 0.87

$$\frac{2\left(5\left(21a^2bC+5a^3B+21ab^2B+21b^3C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-21\left(9a^2bB+3a^3C+15ab^2C+5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{5a(5}{}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(11/2),x]
```

```
[Out] (2*(-21*(9*a^2*b*B + 5*b^3*B + 3*a^3*C + 15*a*b^2*C)*EllipticE[(c + d*x)/2,
 2] + 5*(5*a^3*B + 21*a*b^2*B + 21*a^2*b*C + 21*b^3*C)*EllipticF[(c + d*x)/
2, 2] + (15*a^3*B*Sin[c + d*x])/Cos[c + d*x]^(7/2) + (21*a^2*(3*b*B + a*C)*
Sin[c + d*x])/Cos[c + d*x]^(5/2) + (5*a*(5*a^2*B + 21*b^2*B + 21*a*b*C)*Sin
[c + d*x])/Cos[c + d*x]^(3/2) + (21*(9*a^2*b*B + 5*b^3*B + 3*a^3*C + 15*a*b
^2*C)*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)
```

Maple [B] time = 2.678, size = 944, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a
*b*(B*b+C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(B*b+3*C*a)*(-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*a
^3*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))-2/5*a^2*(3*B*b+C*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos{(dx+c)}^4 + Ba^3 + \left(3\,Cab^2 + Bb^3\right)\cos{(dx+c)}^3 + 3\left(Ca^2b + Bab^2\right)\cos{(dx+c)}^2 + \left(Ca^3 + 3\,Ba^2b\right)\cos}{\cos{(dx+c)}^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos{(dx+c)}^2 + B\cos{(dx+c)}\right)(b\cos{(dx+c)} + a)^3}{\cos{(dx+c)}^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(11/2), x)
```

3.875 $\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=305

$$\frac{2\left(15a^2bB+5a^3C+21ab^2C+7b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(27a^2bC+7a^3B+27ab^2B+15b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7\right.}{}$$

```
[Out] (-2*(7*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 15*b^3*C)*EllipticE[(c + d*x)/2, 2
])/(15*d) + (2*(15*a^2*b*B + 7*b^3*B + 5*a^3*C + 21*a*b^2*C)*EllipticF[(c +
 d*x)/2, 2])/(21*d) + (2*a^2*(13*b*B + 9*a*C)*Sin[c + d*x])/(63*d*Cos[c + d
*x]^(7/2)) + (2*a*(7*a^2*B + 22*b^2*B + 27*a*b*C)*Sin[c + d*x])/(45*d*Cos[c
 + d*x]^(5/2)) + (2*(15*a^2*b*B + 7*b^3*B + 5*a^3*C + 21*a*b^2*C)*Sin[c + d
*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*(7*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 15
*b^3*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a*B*(a + b*Cos[c + d*x
])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.626176, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2989, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2\left(15a^2bB+5a^3C+21ab^2C+7b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(27a^2bC+7a^3B+27ab^2B+15b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(13/2),x]
```


```
[Out] (-2*(7*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 15*b^3*C)*EllipticE[(c + d*x)/2, 2
])/(15*d) + (2*(15*a^2*b*B + 7*b^3*B + 5*a^3*C + 21*a*b^2*C)*EllipticF[(c +
 d*x)/2, 2])/(21*d) + (2*a^2*(13*b*B + 9*a*C)*Sin[c + d*x])/(63*d*Cos[c + d
*x]^(7/2)) + (2*a*(7*a^2*B + 22*b^2*B + 27*a*b*C)*Sin[c + d*x])/(45*d*Cos[c
 + d*x]^(5/2)) + (2*(15*a^2*b*B + 7*b^3*B + 5*a^3*C + 21*a*b^2*C)*Sin[c + d
*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*(7*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 15
*b^3*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a*B*(a + b*Cos[c + d*x
])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^3(B+C\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\cos(c+$$

$$= \frac{2a^2(13bB+9aC)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aB(a+b\cos(c+dx))^2\sin}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(13bB+9aC)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\left(7a^2B+22b^2B+27abC\right.}{45d\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2a^2(13bB+9aC)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\left(7a^2B+22b^2B+27abC\right.}{45d\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2a^2(13bB+9aC)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\left(7a^2B+22b^2B+27abC\right.}{45d\cos^{\frac{5}{2}}(c+d}$$

$$= -\frac{2\left(7a^3B+27ab^2B+27a^2bC+15b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} +$$

Mathematica [A] time = 4.69778, size = 266, normalized size = 0.87

$$\frac{2\left(15\left(15a^2bB+5a^3C+21ab^2C+7b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-21\left(27a^2bC+7a^3B+27ab^2B+15b^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{7a(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(13/2),x]
```

```
[Out] (2*(-21*(7*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 15*b^3*C)*EllipticE[(c + d*x)/
2, 2] + 15*(15*a^2*b*B + 7*b^3*B + 5*a^3*C + 21*a*b^2*C)*EllipticF[(c + d*x
)/2, 2] + (35*a^3*B*Sin[c + d*x])/Cos[c + d*x]^(9/2) + (45*a^2*(3*b*B + a*C
)*Sin[c + d*x])/Cos[c + d*x]^(7/2) + (7*a*(7*a^2*B + 27*b^2*B + 27*a*b*C)*S
in[c + d*x])/Cos[c + d*x]^(5/2) + (15*(15*a^2*b*B + 7*b^3*B + 5*a^3*C + 21*
a*b^2*C)*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21*(7*a^3*B + 27*a*b^2*B + 27*
a^2*b*C + 15*b^3*C)*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(315*d)
```

Maple [B] time = 3.329, size = 1193, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*(B*b+3*C*
a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*b^3*(-(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*a^2*(3*B*b+C*a)*(-1/5
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*
a*b*(B*b+C*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^3*B*(-1/144*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/1
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2
),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{4}+Ba^{3}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{3}+3\left(Ca^{2}b+Bab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{3}+3\,Ba^{2}b\right)\cos\left(\right.}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2
),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4 + B*a^3 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^3
 + 3*(C*a^2*b + B*a*b^2)*cos(d*x + c)^2 + (C*a^3 + 3*B*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
3/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2
),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^3/cos(d*
x + c)^(13/2), x)
```

3.876 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=246

$$-\frac{2\left(-7a^2b^2C+21a^3bB-21a^4C+7ab^3B-5b^4C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^5d}+\frac{2\left(5a^2+3b^2\right)(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d}+\frac{2a^4(bB-}{}$$

```
[Out] (2*(5*a^2 + 3*b^2)*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(5*b^4*d) - (2*(2
1*a^3*b*B + 7*a*b^3*B - 21*a^4*C - 7*a^2*b^2*C - 5*b^4*C)*EllipticF[(c + d*
x)/2, 2])/(21*b^5*d) + (2*a^4*(b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(b^5*(a + b)*d) - (2*(7*a*b*B - 7*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(5*b^2*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 1.20395, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2990, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(-7a^2b^2C+21a^3bB-21a^4C+7ab^3B-5b^4C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^5d}+\frac{2\left(5a^2+3b^2\right)(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d}+\frac{2a^4(bB-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```

```
[Out] (2*(5*a^2 + 3*b^2)*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(5*b^4*d) - (2*(2
1*a^3*b*B + 7*a*b^3*B - 21*a^4*C - 7*a^2*b^2*C - 5*b^4*C)*EllipticF[(c + d*
x)/2, 2])/(21*b^5*d) + (2*a^4*(b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(b^5*(a + b)*d) - (2*(7*a*b*B - 7*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(5*b^2*d) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+C\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5aC}{2}+\frac{5}{2}bC\cos(c+dx)+\frac{7}{2}(\right.}{a+b\cos(c+dx)}}{7b}$$

$$= \frac{2(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5b^2d} + \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+}{7bd}$$

$$= -\frac{2\left(7abB-7a^2C-5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(bB-a}{}$$

$$= -\frac{2\left(7abB-7a^2C-5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(bB-a}{}$$

$$= \frac{2\left(5a^2+3b^2\right)(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d} - \frac{2\left(7abB-7a^2C-5b^2\right.}{}$$

$$= \frac{2\left(5a^2+3b^2\right)(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d} - \frac{2\left(21a^3bB+7ab^3B-\right.}{}$$

Mathematica [A] time = 2.60045, size = 309, normalized size = 1.26

$$\frac{\frac{2\left(35a^2bB-35a^3C-13ab^2C+63b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 2\sin(c+dx)\sqrt{\cos(c+dx)}\left(70a^2C+42b(bB-aC)\cos(c+dx)-70abB+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x]),x]
```

```
[Out] ((2*(35*a^2*b*B + 63*b^3*B - 35*a^3*C - 13*a*b^2*C)*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2])/(a + b) + (4*(28*a*b*B - 28*a^2*C + 25*b^2*C)*((a + b)*
EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(
a + b) + 2*Sqrt[Cos[c + d*x]]*(-70*a*b*B + 70*a^2*C + 65*b^2*C + 42*b*(b*B
- a*C)*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x] - (42*(5*a^2
+ 3*b^2)*(-(b*B) + a*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[
```

```
Sin[c + d*x]^2]))/(210*b^3*d)
```

Maple [B] time = 0.807, size = 1375, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((240*C*a*b^
4-240*C*b^5)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*a*b^4+168*B*b^
5+168*C*a^2*b^3-528*C*a*b^4+360*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+(-140*B*a^2*b^3+308*B*a*b^4-168*B*b^5+140*C*a^3*b^2-308*C*a^2*b^3+448*C
*a*b^4-280*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(70*B*a^2*b^3-112
*B*a*b^4+42*B*b^5-70*C*a^3*b^2+112*C*a^2*b^3-122*C*a*b^4+80*C*b^5)*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*
a^4*b-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b^2+105*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2*b^3-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^4+63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^5-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*b+105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3
*b^2-35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*
b^4-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^5+105*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^4*b-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b^2+63*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2*b^3-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^4+105*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^5-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*b+35*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
```

```
))*a^3*b^2-35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3+25*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^4-25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^5)/b^5/(a-b)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(5/2)/(b*cos(d*x
 + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(5/2)/(b*cos(d*x
 + c) + a), x)
```

3.877 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{2\left(3a^2+b^2\right)(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2\left(-5a^2C+5abB-3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}-\frac{2a^3(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\right.}{b^4d(a+b)}$$

```
[Out] (-2*(5*a*b*B - 5*a^2*C - 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2
*(3*a^2 + b^2)*(b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) - (2*a^3*(b
*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(
b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*C*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.898939, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2990, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2+b^2\right)(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2\left(-5a^2C+5abB-3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}-\frac{2a^3(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|\right.}{b^4d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```



```
[Out] (-2*(5*a*b*B - 5*a^2*C - 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2
*(3*a^2 + b^2)*(b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) - (2*a^3*(b
*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(
b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*C*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(5*b*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+\frac{3}{2}bC\cos(c+dx)+\frac{5}{2}(\right.}{a+b\cos(c+dx)}}{5b}$$

$$= \frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5bd}$$

$$= \frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5bd}$$

$$= -\frac{2\left(5abB-5a^2C-3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2(bB-aC)\sqrt{\cos(c}}{3b}$$

$$= -\frac{2\left(5abB-5a^2C-3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2\left(3a^2+b^2\right)(bB-}{3}$$

Mathematica [A] time = 2.27272, size = 264, normalized size = 1.45

$$\frac{\frac{2b^2(5a^2C-5abB+9b^2C)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{6(5a^2C-5abB+3b^2C)\sin(c+dx)\left((2a^2-b^2)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2}{a\sqrt{\sin^2(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x]),x]
```

```
[Out] ((2*b^2*(-5*a*b*B + 5*a^2*C + 9*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 2*b^2*(5*b*B + 4*a*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*b^2*Sqrt[Cos[c + d*
x]]*(5*b*B - 5*a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x] + (6*(-5*a*b*B + 5*a^
2*C + 3*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(
b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^
2]))/(30*b^4*d)
```

Maple [B] time = 0.71, size = 1074, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*C*a*b^3
+24*C*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*a*b^3-20*B*b^4-20*
C*a^2*b^2+44*C*a*b^3-24*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10
*B*a*b^3+10*B*b^4+10*C*a^2*b^2-16*C*a*b^3+6*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-15*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b^3+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*a^2*b^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
```

```
))*b^4-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3*b-15*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^3*b+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-9*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a*b^3+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-15*a^4*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+15*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+5*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))*a^4)/b^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a), x)
```

3.878 $\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{2\left(-3a^2C+3abB-b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2C\sin}{}$$

```
[Out] (2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b^2*d) - (2*(3*a*b*B - 3*a^2*C -
 b^2*C)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) + (2*a^2*(b*B - a*C)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*C*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.604984, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2990, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(-3a^2C+3abB-b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2C\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] (2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b^2*d) - (2*(3*a*b*B - 3*a^2*C -
 b^2*C)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) + (2*a^2*(b*B - a*C)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*C*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{aC}{2}+\frac{1}{2}bC\cos(c+dx)+\frac{3}{2}(bB-aC)\cos^2(}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3b}$$

$$= \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{1}{2}abC+\frac{1}{2}\left(3abB-3a^2C-b^2C\right)\cos(c+}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3b^2}$$

$$= \frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{(}{}$$

$$= \frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2\left(3abB-3a^2C-b^2C\right)F\left(\frac{1}{2}(c+\right.}{3b^3d}$$

Mathematica [A] time = 1.35343, size = 209, normalized size = 1.53

$$\frac{-\frac{3(bB-aC)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}} + \frac{(3bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(\right.}{a+b}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x]),x]
```

```
[Out] (((3*b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + C*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)) + 2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (3*(b*B - a*C)*(2*a*b*Elli
pticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[C
os[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(3*b*d)
```

Maple [B] time = 0.819, size = 786, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-4*C*a*b^2+4*
C*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(2*C*a*b^2-2*C*b^3)*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a*b^2*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b^3-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*a^3*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*a*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2*b+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a), x)
```

3.879 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=89

$$\frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(b*B - a*C)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) - (2*a*(b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(b^2*(a + b)*d)
```

Rubi [A] time = 0.29845, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3029, 3002, 2639, 2803, 2641, 2805}

$$\frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])),x]
```


```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(b*B - a*C)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) - (2*a*(b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(b^2*(a + b)*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx=\int \frac{\sqrt{\cos(c+dx)}(B+C\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$=\frac{C\int\sqrt{\cos(c+dx)}\,dx}{b}-\frac{(-bB+aC)\int\frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}$$

$$=\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{(bB-aC)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}-\frac{(a(bB-aC))\int\frac{1}{\sqrt{\cos(c+dx)}(a+}}{b^2}$$

$$=\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c-\right.}{b^2(a+b)d}$$

Mathematica [A] time = 0.855516, size = 131, normalized size = 1.47

$$\frac{bB\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)-\frac{2C\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-a\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{\sqrt{\sin^2(c+dx)}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Co
s[c + d*x])),x]
```

```
[Out] (b*B*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*C*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - a*EllipticPi[-(b/a), -ArcS
in[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(b^2*d)
```

Maple [A] time = 0.706, size = 295, normalized size = 3.3

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{(a-b)\,b^2\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\Big(B\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-B*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-C*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2+C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-C*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a*b+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+C*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*sqrt(co
s(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*sqrt(co
s(d*x + c))), x)
```

3.880 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=61

$$\frac{2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*C*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(b*B - a*C)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Rubi [A] time = 0.242194, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3029, 3002, 2641, 2805}

$$\frac{2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])),x]
```

```
[Out] (2*C*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(b*B - a*C)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx &= \int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx \\
&= \frac{C\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{(-bB+aC)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b} \\
&= \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}
\end{aligned}
$$

Mathematica [A] time = 0.197041, size = 58, normalized size = 0.95

$$\frac{2\left((bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)+C(a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Co
s[c + d*x])),x]
```

```
[Out] (2*((a + b)*C*EllipticF[(c + d*x)/2, 2] + (b*B - a*C)*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]))/(b*(a + b)*d)
```

Maple [A] time = 0.67, size = 217, normalized size = 3.6

$$-2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{b\left(a-b\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(B\mathrm{EllipticI}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(B*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b+C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-C*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))*a)/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*cos(d*x
 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*cos(d*x
 + c)^(3/2)), x)
```

3.881 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(b*B - a*C)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*B*Sin[c + d*x])/(a*d*Sqrt[Cos[c
+ d*x]])
```

Rubi [A] time = 0.408887, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3029, 3000, 3059, 2639, 12, 2805}

$$-\frac{2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(b*B - a*C)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*B*Sin[c + d*x])/(a*d*Sqrt[Cos[c
+ d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(-bB+aC)-\frac{1}{2}aB\cos(c+dx)-\frac{1}{2}bB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{2\int \frac{b(bB-aC)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab} - \frac{B\int \sqrt{\cos(c+dx)}\,dx}{a}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(bB-aC)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,d}{a}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 2.39108, size = 210, normalized size = 2.44

$$\frac{-\frac{2B\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{2(2aC-3bB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a+b}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Co
s[c + d*x])),x]
```

```
[Out] ((2*(-3*b*B + 2*a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (
2*a*B*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(a + b)))/b + (4*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*B*(-2*a*
b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[
Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

Maple [B] time = 1.307, size = 327, normalized size = 3.8

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\frac{(-bB+aC)\,b\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2}}{a\left(-2\,ab+2\,b^2\right)\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin(1/2\,dx+c/\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-B*b+C*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*B/a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*cos(d*x
 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*cos(d*x
 + c)^(5/2)), x)
```

3.882 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(bB-aC)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\,\mathrm{si}}{3ad\,\mathrm{co}}$$

```
[Out] (2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*B*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*b*(b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a^2*(a + b)*d) + (2*B*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*(b
*B - a*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.871981, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(bB-aC)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\,\mathrm{s}}{3ad\,\mathrm{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c +
d*x])),x]
```

```
[Out] (2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*B*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*b*(b*B - a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a^2*(a + b)*d) + (2*B*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*(b
*B - a*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx=\int \frac{B+C\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$=\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\int \frac{-\frac{3}{2}(bB-aC)+\frac{1}{2}aB\cos(c+dx)+\frac{1}{2}bB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$=\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(bB-aC)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{4\int \frac{\frac{1}{4}\left(a^2B+3b^2B-3abC\right)+\frac{1}{4}a(4bB-3aC)\cos}{\sqrt{\cos(c+dx)}(a+b\,c}}{3a^2}$$

$$=\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(bB-aC)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{4\int \frac{-\frac{1}{4}b\left(a^2B+3b^2B-3abC\right)-\frac{1}{4}ab^2B\cos(c+d}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3a^2b}$$

$$=\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(bB-aC)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{B\int}{}$$

$$=\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2b(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{a^2(a+b)d}$$

Mathematica [A] time = 2.1674, size = 262, normalized size = 1.75

$$\frac{\frac{2a\left(2a^2B-9abC+9b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{6(bB-aC)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\right.\right.}{b\sqrt{\sin^2(c+dx)}}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Co
s[c + d*x])),x]
```

```
[Out] ((2*a*(2*a^2*B + 9*b^2*B - 9*a*b*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + (a*(8*a*b*B - 6*a^2*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (4*a^2*B*Sin[c + d*x
])/Cos[c + d*x]^(3/2) + (12*a*(-(b*B) + a*C)*Sin[c + d*x])/Sqrt[Cos[c + d*x
]] - (6*(b*B - a*C)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*
(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c +
d*x]^2]))/(6*a^3*d)
```

Maple [B] time = 1.961, size = 468, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(B*b-C*a
)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*B/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2
+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+2*(-B*b+C*a)/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
```

```
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*cos(d*x
 + c)^(7/2)), x)
```

3.883 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=217

$$-\frac{2\left(3a^2B-5abC+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}-\frac{2b^2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}+\frac{2\left(3a^2B-5abC+5b^2B\right)\sin(c+dx)}{5a^3d\sqrt{\cos(c+dx)}}-$$

```
[Out] (-2*(3*a^2*B + 5*b^2*B - 5*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2
*(b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (2*b^2*(b*B - a*C)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*B*Sin[c + d*x])
/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(b*B - a*C)*Sin[c + d*x])/(3*a^2*d*Cos[c +
 d*x]^(3/2)) + (2*(3*a^2*B + 5*b^2*B - 5*a*b*C)*Sin[c + d*x])/(5*a^3*d*Sqrt
[Cos[c + d*x]])
```

Rubi [A] time = 1.23606, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(3a^2B-5abC+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}-\frac{2b^2(bB-aC)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}+\frac{2\left(3a^2B-5abC+5b^2B\right)\sin(c+dx)}{5a^3d\sqrt{\cos(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*(a + b*Cos[c +
d*x])),x]
```


```
[Out] (-2*(3*a^2*B + 5*b^2*B - 5*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2
*(b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (2*b^2*(b*B - a*C)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*B*Sin[c + d*x])
/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(b*B - a*C)*Sin[c + d*x])/(3*a^2*d*Cos[c +
 d*x]^(3/2)) + (2*(3*a^2*B + 5*b^2*B - 5*a*b*C)*Sin[c + d*x])/(5*a^3*d*Sqrt
[Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{-\frac{5}{2}(bB-aC)+\frac{3}{2}aB\cos(c+dx)+\frac{3}{2}bB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{5a}$$

$$= \frac{2B\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(bB-aC)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{3}{4}(3a^2B+5b^2B-5abC)+\frac{1}{4}a(4bB+5aC)}{\cos^{\frac{3}{2}}(c+dx)(a+}}{15a}$$

$$= \frac{2B\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(bB-aC)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3a^2B+5b^2B-5abC)\sin(c+}{5a^3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2B\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(bB-aC)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3a^2B+5b^2B-5abC)\sin(c+}{5a^3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(3a^2B+5b^2B-5abC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} + \frac{2B\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(bB-aC)\,\mathrm{s}}{3a^2d\cos^{\frac{3}{2}}}$$

$$= -\frac{2(3a^2B+5b^2B-5abC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{2b}{}$$

Mathematica [A] time = 3.86291, size = 313, normalized size = 1.44

$$\frac{\frac{2(-19a^2bB+10a^3C+45ab^2C-45b^3B)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{2a(9a^2B-20abC+20b^2B)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b} + \frac{2(3(3a^2B-5abC+5b^2B)\sin(2(c}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*(a + b*Co
s[c + d*x])),x]
```

```
[Out] ((2*(-19*a^2*b*B - 45*b^3*B + 10*a^3*C + 45*a*b^2*C)*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(a + b) - (2*a*(9*a^2*B + 20*b^2*B - 20*a*b*C)*(2*Elli
pticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
+ b)))/b - (6*(3*a^2*B + 5*b^2*B - 5*a*b*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[C
```

```
os[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a*b*Sqrt[Sin[c + d*x]^2]) + (2*(10*a*(-(b*B) + a*C)*Sin[c + d*x] +
 3*(3*a^2*B + 5*b^2*B - 5*a*b*C)*Sin[2*(c + d*x)] + 6*a^2*B*Tan[c + d*x]))/
Cos[c + d*x]^(3/2))/(30*a^3*d)
```

Maple [B] time = 2.697, size = 787, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^3*(B*b-C*a)
/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/5*B/a/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(
-B*b+C*a)/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b-C*a)/a^3*b*(
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2} + B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right) + a\right)\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)*cos(d*x
 + c)^(9/2)), x)
```

3.884 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=389

$$\frac{\left(-16a^2b^3B+20a^3b^2C+15a^4bB-21a^5C+4ab^4C-2b^5B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^5d\left(a^2-b^2\right)}-\frac{\left(24a^2b^2C+25a^3bB-35a^4C-20ab^3B+\right.}{5b^4d\left(a^2-b^2\right)}$$

```
[Out] -((25*a^3*b*B - 20*a*b^3*B - 35*a^4*C + 24*a^2*b^2*C + 6*b^4*C)*EllipticE[(
c + d*x)/2, 2])/(5*b^4*(a^2 - b^2)*d) + ((15*a^4*b*B - 16*a^2*b^3*B - 2*b^5
*B - 21*a^5*C + 20*a^3*b^2*C + 4*a*b^4*C)*EllipticF[(c + d*x)/2, 2])/(3*b^5
*(a^2 - b^2)*d) - (a^3*(5*a^2*b*B - 7*b^3*B - 7*a^3*C + 9*a*b^2*C)*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^5*(a + b)^2*d) + ((5*a^2*b*B
- 2*b^3*B - 7*a^3*C + 4*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a
^2 - b^2)*d) - ((5*a*b*B - 7*a^2*C + 2*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(5*b^2*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.38222, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2989, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-16a^2b^3B+20a^3b^2C+15a^4bB-21a^5C+4ab^4C-2b^5B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^5d\left(a^2-b^2\right)}-\frac{\left(24a^2b^2C+25a^3bB-35a^4C-20ab^3B+\right.}{5b^4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] -((25*a^3*b*B - 20*a*b^3*B - 35*a^4*C + 24*a^2*b^2*C + 6*b^4*C)*EllipticE[(
c + d*x)/2, 2])/(5*b^4*(a^2 - b^2)*d) + ((15*a^4*b*B - 16*a^2*b^3*B - 2*b^5
*B - 21*a^5*C + 20*a^3*b^2*C + 4*a*b^4*C)*EllipticF[(c + d*x)/2, 2])/(3*b^5
*(a^2 - b^2)*d) - (a^3*(5*a^2*b*B - 7*b^3*B - 7*a^3*C + 9*a*b^2*C)*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^5*(a + b)^2*d) + ((5*a^2*b*B
- 2*b^3*B - 7*a^3*C + 4*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a
^2 - b^2)*d) - ((5*a*b*B - 7*a^2*C + 2*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(5*b^2*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\cos^{\frac{7}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$=\frac{a(bB-aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(bB-aC)+b(bB\right.}{}$$

$$=-\frac{\left(5abB-7a^2C+2b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5b^2\left(a^2-b^2\right)d}+\frac{a(bB-aC)\,c}{b\left(a^2-b^2\right)}$$

$$=\frac{\left(5a^2bB-2b^3B-7a^3C+4ab^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^3\left(a^2-b^2\right)d}-\frac{\left(5ab\right.}{}$$

$$=\frac{\left(5a^2bB-2b^3B-7a^3C+4ab^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^3\left(a^2-b^2\right)d}-\frac{\left(5ab\right.}{}$$

$$=-\frac{\left(25a^3bB-20ab^3B-35a^4C+24a^2b^2C+6b^4C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4\left(a^2-b^2\right)d}$$

$$=-\frac{\left(25a^3bB-20ab^3B-35a^4C+24a^2b^2C+6b^4C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4\left(a^2-b^2\right)d}$$

Mathematica [A] time = 4.56678, size = 373, normalized size = 0.96

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{15a^3(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+10(bB-2aC)\sin(c+dx)+3bC\sin(2(c+dx))\right)+\frac{\frac{2\left(-32a^2b^2C-25a^3bB+35a^4C+40ab^3B-18b^4C\right)\Pi\left(\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] (((2*(-25*a^3*b*B + 40*a*b^3*B + 35*a^4*C - 32*a^2*b^2*C - 18*b^4*C)*Ellipt
icPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-10*a^2*b*B - 5*b^3*B +
14*a^3*C + a*b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(-25*a^3*b*B + 20*a*b^3*B + 35*a^4
*C - 24*a^2*b^2*C - 6*b^4*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]],
```

```
-1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)
*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*
Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*(10*(b*B -
2*a*C)*Sin[c + d*x] + (15*a^3*(b*B - a*C)*Sin[c + d*x])/((a^2 - b^2)*(a + b
*Cos[c + d*x])) + 3*b*C*Sin[2*(c + d*x)]))/(60*b^3*d)
```

Maple [B] time = 3.045, size = 1348, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^2*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+4/3/b^3*(B*b-2*C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2
/b^4*(2*B*a*b+2*B*b^2-3*C*a^2-4*C*a*b-3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))+2*(3*B*a^2*b+2*B*a*b^2+B*b^3-4*C*a^3-3*C*a^2*b-2*C*a*b^2-C*
b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+4*a^3/b^4*(4*B*b-5*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(
B*b-C*a)/b^5*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
```

```
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(5/2)/(b*cos(d*x
 + c) + a)^2, x)
```

3.885 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$-\frac{\left(16a^2b^2C+9a^3bB-15a^4C-12ab^3B+2b^4C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(3a^2bB-5a^3C+4ab^2C-2b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\ldots$$

```
[Out] ((3*a^2*b*B - 2*b^3*B - 5*a^3*C + 4*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^
3*(a^2 - b^2)*d) - ((9*a^3*b*B - 12*a*b^3*B - 15*a^4*C + 16*a^2*b^2*C + 2*b
^4*C)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*b*B -
5*b^3*B - 5*a^3*C + 7*a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^4*(a + b)^2*d) - ((3*a*b*B - 5*a^2*C + 2*b^2*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.02452, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2989, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(16a^2b^2C+9a^3bB-15a^4C-12ab^3B+2b^4C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(3a^2bB-5a^3C+4ab^2C-2b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] ((3*a^2*b*B - 2*b^3*B - 5*a^3*C + 4*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^
3*(a^2 - b^2)*d) - ((9*a^3*b*B - 12*a*b^3*B - 15*a^4*C + 16*a^2*b^2*C + 2*b
^4*C)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*b*B -
5*b^3*B - 5*a^3*C + 7*a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^4*(a + b)^2*d) - ((3*a*b*B - 5*a^2*C + 2*b^2*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(b*B - a*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(bB-aC)+b(\right.}{}$$

$$= -\frac{\left(3abB-5a^2C+2b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(bB-aC)}{b\left(a^2-b\right.}$$

$$= -\frac{\left(3abB-5a^2C+2b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(bB-aC)}{b\left(a^2-b\right.}$$

$$= \frac{\left(3a^2bB-2b^3B-5a^3C+4ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{\left(3abB-5a^2\right.}{}$$

$$= \frac{\left(3a^2bB-2b^3B-5a^3C+4ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{\left(9a^3bB-12\right.}{}$$

Mathematica [A] time = 3.01854, size = 322, normalized size = 1.06

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a^2(aC-bB)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2C\right) - \frac{\frac{2\left(-3a^2bB+5a^3C-8ab^2C+6b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(2a^2C-3abB+b^2C\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\right.}{a+b}}{}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*C + (3*a^2*(-(b*B) + a*C))/((a^2 - b^2)*(a + b*Cos
[c + d*x])))*Sin[c + d*x] - ((2*(-3*a^2*b*B + 6*b^3*B + 5*a^3*C - 8*a*b^2*C
)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-3*a*b*B + 2*a^2
*C + b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + (6*(-3*a^2*b*B + 2*b^3*B + 5*a^3*C - 4*a*b^2*
C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a
```

```
 - b)*(a + b)))/(12*b^2*d)
```

Maple [B] time = 2.637, size = 1066, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*C*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+6*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-9*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)-4*a^2/b^3*(3*B*b-4*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(B*b-
C*a)/b^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^2, x)
```

3.886 $\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=224

$$\frac{\left(a^2bB-3a^3C+4ab^2C-2b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-3a^2C+abB+2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(a^2bB-3a^3C+5ab^2C\right.}{b^3d(a-}$$

```
[Out] -(((a*b*B - 3*a^2*C + 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*
d)) + ((a^2*b*B - 2*b^3*B - 3*a^3*C + 4*a*b^2*C)*EllipticF[(c + d*x)/2, 2])
/(b^3*(a^2 - b^2)*d) - (a*(a^2*b*B - 3*b^3*B - 3*a^3*C + 5*a*b^2*C)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) + (a*(b*B - a
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.704721, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2989, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2bB-3a^3C+4ab^2C-2b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-3a^2C+abB+2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(a^2bB-3a^3C+5ab^2C\right.}{b^3d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] -(((a*b*B - 3*a^2*C + 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*
d)) + ((a^2*b*B - 2*b^3*B - 3*a^3*C + 4*a*b^2*C)*EllipticF[(c + d*x)/2, 2])
/(b^3*(a^2 - b^2)*d) - (a*(a^2*b*B - 3*b^3*B - 3*a^3*C + 5*a*b^2*C)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) + (a*(b*B - a
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(bB-aC)+b(bB-aC)\cos(\ldots}{\sqrt{\cos(c+\ldots}}}{b\ldots}$$

$$= \frac{a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}ab(bB-aC)+\frac{1}{2}\left(a^2bB-2b^3B-\ldots\right.}{\sqrt{\cos(c+dx)}(a+\ldots}}{b^2\left(a^2\ldots\right.}$$

$$= -\frac{\left(abB-3a^2C+2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{a(bB-aC)\sqrt{\cos(c+\ldots}}{b\left(a^2-b^2\right)d(a+b\ldots}$$

$$= -\frac{\left(abB-3a^2C+2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{\left(a^2bB-2b^3B-3a^3\ldots\right.}{b^3\ldots}$$

Mathematica [A] time = 2.53201, size = 284, normalized size = 1.27

$$\frac{\frac{\frac{2\left(a^2C+abB-2b^2C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{2\left(3a^2C-abB-2b^2C\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}\ldots}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] ((-4*a*(-(b*B) + a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*
Cos[c + d*x])) + ((2*(a*b*B + a^2*C - 2*b^2*C)*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(a + b) + (8*(-(b*B) + a*C)*((a + b)*EllipticF[(c + d*x)/2,
```

```
2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(-(a*b*B) +
 3*a^2*C - 2*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[
c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Maple [B] time = 2.114, size = 849, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+4*a/b^2*(2*B*b-3*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(B*b-
C*a)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^2, x)
```

3.887 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{\left(a^2C+abB-2b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(a^2bB+a^3C-3ab^2C+b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

```
[Out] ((b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a*b*B + a^2*C
 - 2*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((a^2*b*B + b^
3*B + a^3*C - 3*a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b
)*b^2*(a + b)^2*d) - ((b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.63593, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 2999, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2C+abB-2b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(a^2bB+a^3C-3ab^2C+b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^2),x]
```

```
[Out] ((b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a*b*B + a^2*C
 - 2*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((a^2*b*B + b^
3*B + a^3*C - 3*a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b
)*b^2*(a + b)^2*d) - ((b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+C\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= -\frac{(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(bB-aC)-(aB-bC)\cos(c+dx)-\frac{1}{2}(bB-aC)\cos}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{-a^2+b^2}$$

$$= -\frac{(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}b(bB-aC)+\frac{1}{2}\left(abB+a^2C-2b^2C\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b\left(a^2-b^2\right)}$$

$$= \frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(abB+a^2\right.}{}$$

$$= \frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(abB+a^2C-2b^2C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{\left(a^2bB+\right.}{}$$

Mathematica [A] time = 2.31026, size = 262, normalized size = 1.32

$$\frac{\frac{4(aC-bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{-\frac{2(bB-aC)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}+\frac{2(aC-}{}}{(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Co
s[c + d*x])^2),x]
```

```
[Out] ((4*(-(b*B) + a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos
[c + d*x])) - ((2*(-(b*B) + a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b) + ((4*a*B - 4*b*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(b*B - a*C)*(2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/
```

```
(4*d)
```

Maple [B] time = 1.829, size = 808, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(B*b-2*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(B*b-C*a)/b^2*(-1/a*b^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2
-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^2*sqrt(
cos(d*x + c))), x)
```

3.888 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2bB+a^3(-C)-ab^2C-b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}+\frac{b(bB}{ac}$$

```
[Out] -(((b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - ((b*B - a*C)
*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((3*a^2*b*B - b^3*B - a^3*C
 - a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (b*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x]))
```

Rubi [A] time = 0.713893, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3029, 3000, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2bB+a^3(-C)-ab^2C-b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}+\frac{b(bB}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^2),x]
```


```
[Out] -(((b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - ((b*B - a*C)
*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((3*a^2*b*B - b^3*B - a^3*C
 - a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (b*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2B-b^2B-abC\right)-a(bB-aC)\cos(c+dx)-\frac{1}{2}b(bB-}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b\left(2a^2B-b^2B-abC\right)+\frac{1}{2}ab(bB-aC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab\left(a^2-b^2\right)}$$

$$= -\frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(bB-aC}{2b}$$

$$= -\frac{(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(3a^2bB-b^3B-a^3C}{a(a}$$

Mathematica [A] time = 2.55043, size = 276, normalized size = 1.38

$$\frac{\frac{4b(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(4a^2B-abC-3b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2(bB-aC)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Co
s[c + d*x])^2),x]
```

```
[Out] ((4*b*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[
c + d*x])) + ((2*(4*a^2*B - 3*b^2*B - a*b*C)*EllipticPi[(2*b)/(a + b), (c +
```

```
 d*x)/2, 2])/(a + b) + (4*a*(-(b*B) + a*C)*(2*EllipticF[(c + d*x)/2, 2] - (
2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(b*B - a*C)
*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[
Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b
)*(a + b)))/(4*a*d)
```

Maple [B] time = 1.646, size = 721, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*(B*b-C*a)/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^
2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^2*cos(d
*x + c)^(3/2)), x)
```

3.889 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=256

$$\frac{(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2B+abC-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\left(5a^2bB-3a^3C+ab^2C-3b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a^2d(a-b)(a+b)^2}$$

```
[Out] -(((2*a^2*B - 3*b^2*B + a*b*C)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*
d)) + ((b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - ((5*a^2*b
*B - 3*b^3*B - 3*a^3*C + a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2*B - 3*b^2*B + a*b*C)*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.01263, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2B+abC-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\left(5a^2bB-3a^3C+ab^2C-3b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])^2),x]
```

```
[Out] -(((2*a^2*B - 3*b^2*B + a*b*C)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*
d)) + ((b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - ((5*a^2*b
*B - 3*b^3*B - 3*a^3*C + a*b^2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2*B - 3*b^2*B + a*b*C)*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
```

```
1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2B-3b^2B+abC\right)-a(bB-aC)\cos(}{\cos^{\frac{3}{2}}(c+dx)(a+b}}{a\left(a^2-\right.}$$

$$= \frac{\left(2a^2B-3b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+a}$$

$$= \frac{\left(2a^2B-3b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+a}$$

$$= -\frac{\left(2a^2B-3b^2B+abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(2a^2B-3b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} -$$

$$= -\frac{\left(2a^2B-3b^2B+abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{\left(5a^2b\right.}{}$$

Mathematica [A] time = 3.9354, size = 320, normalized size = 1.25

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^2(bB-aC)\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}+2B\tan(c+dx)\right) - \frac{\frac{2\left(-10a^2bB+4a^3C-3ab^2C+9b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{8a\left(a^2B+abC-2b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\right.}{b(a+b)}}{}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Co
s[c + d*x])^2),x]
```

```
[Out] (-((((2*(-10*a^2*b*B + 9*b^3*B + 4*a^3*C - 3*a*b^2*C)*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(a + b) - (8*a*(a^2*B - 2*b^2*B + a*b*C)*((a + b)*Elli
pticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a
 + b)) - (2*(2*a^2*B - 3*b^2*B + a*b*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2
*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*
x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]*
((b^2*(b*B - a*C)*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])) + 2*B*T
```

```
an[c + d*x]))/(4*a^2*d)
```

Maple [B] time = 2.086, size = 883, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*B/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*B/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-B*b+C*a)/a*(-1/a*b^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^2*cos(d
*x + c)^(5/2)), x)
```

3.890 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=345

$$\frac{\left(2a^2B+3abC-5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(4a^2bB-2a^3C+3ab^2C-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b\left(7a^2bB-5a^3C+3ab^2C\right.}{a^3d(a}$$

```
[Out] ((4*a^2*b*B - 5*b^3*B - 2*a^3*C + 3*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(a^
3*(a^2 - b^2)*d) + ((2*a^2*B - 5*b^2*B + 3*a*b*C)*EllipticF[(c + d*x)/2, 2]
)/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*b*B - 5*b^3*B - 5*a^3*C + 3*a*b^2*C)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^
2*B - 5*b^2*B + 3*a*b*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3
/2)) - ((4*a^2*b*B - 5*b^3*B - 2*a^3*C + 3*a*b^2*C)*Sin[c + d*x])/(a^3*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.3846, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(2a^2B+3abC-5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(4a^2bB-2a^3C+3ab^2C-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b\left(7a^2bB-5a^3C+3ab^2C\right.}{a^3d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c +
d*x])^2),x]
```


```
[Out] ((4*a^2*b*B - 5*b^3*B - 2*a^3*C + 3*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(a^
3*(a^2 - b^2)*d) + ((2*a^2*B - 5*b^2*B + 3*a*b*C)*EllipticF[(c + d*x)/2, 2]
)/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*b*B - 5*b^3*B - 5*a^3*C + 3*a*b^2*C)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^
2*B - 5*b^2*B + 3*a*b*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3
/2)) - ((4*a^2*b*B - 5*b^3*B - 2*a^3*C + 3*a*b^2*C)*Sin[c + d*x])/(a^3*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2B-5b^2B+3abC\right)-a(bB-aC)\cos(c}{\cos^{\frac{5}{2}}(c+dx)(a+b\,c}}{a\left(a^2-b\right.}$$

$$= \frac{\left(2a^2B-5b^2B+3abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+d}$$

$$= \frac{\left(2a^2B-5b^2B+3abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2bB-5b^3B-2a^3C+3ab^2C\right)\sin(c+a}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(2a^2B-5b^2B+3abC\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2bB-5b^3B-2a^3C+3ab^2C\right)\sin(c+a}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(4a^2bB-5b^3B-2a^3C+3ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2B-5b^2B+3abC\right)\sin}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{\left(4a^2bB-5b^3B-2a^3C+3ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2B-5b^2B+3abC\right)F\left(\right.}{3a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 6.37077, size = 367, normalized size = 1.06

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^3(bB-aC)\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2\tan(c+dx)(aB\sec(c+dx)+3aC-6bB)\right)+\frac{\frac{2\left(44a^2b^2B-30a^3bC+4a^4B+27ab^3C-45b^4B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Co
s[c + d*x])^2),x]
```

```
[Out] (((2*(4*a^4*B + 44*a^2*b^2*B - 45*b^4*B - 30*a^3*b*C + 27*a*b^3*C)*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*a*(-7*a^2*b*B + 10*b^3*B +
3*a^3*C - 6*a*b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b
```

```
)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) - (6*(-4*a^2*b*B + 5*b^3*B + 2*a^3
*C - 3*a*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-
(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*
x]^2]))/((a - b)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*((3*b^3*(b*B - a*C)*Sin[c
+ d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) + 2*(-6*b*B + 3*a*C + a*B*Sec[c
+ d*x])*Tan[c + d*x]))/(12*a^3*d)
```

Maple [B] time = 3.378, size = 1031, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*B*b-C
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*B/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*(-2*B*b+C*a)/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*
c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(B*b-C*a)*b/a^2*(-1/a*b^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
```

```
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^2*cos(d
*x + c)^(7/2)), x)
```

3.891 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=461

$$-\frac{\left(-99a^3b^3B+223a^4b^2C-128a^2b^4C+45a^5bB-105a^6C+72ab^5B-8b^6C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^3B+65a^3b^2C+\right.}{}$$

```
[Out] ((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - 35*a^5*C + 65*a^3*b^2*C - 24*a*b^4*
C)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) - ((45*a^5*b*B - 99*a
^3*b^3*B + 72*a*b^5*B - 105*a^6*C + 223*a^4*b^2*C - 128*a^2*b^4*C - 8*b^6*C
)*EllipticF[(c + d*x)/2, 2])/(12*b^5*(a^2 - b^2)^2*d) + (a^2*(15*a^4*b*B -
38*a^2*b^3*B + 35*b^5*B - 35*a^5*C + 86*a^3*b^2*C - 63*a*b^4*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*b*
B - 33*a*b^3*B - 35*a^4*C + 61*a^2*b^2*C - 8*b^4*C)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(12*b^3*(a^2 - b^2)^2*d) + (a*(b*B - a*C)*Cos[c + d*x]^(5/2)*Sin[
c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(3*a^2*b*B - 9*b^
3*B - 7*a^3*C + 13*a*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.5447, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3029, 2989, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(-99a^3b^3B+223a^4b^2C-128a^2b^4C+45a^5bB-105a^6C+72ab^5B-8b^6C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^3B+65a^3b^2C+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^3,x]
```

```
[Out] ((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - 35*a^5*C + 65*a^3*b^2*C - 24*a*b^4*
C)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) - ((45*a^5*b*B - 99*a
^3*b^3*B + 72*a*b^5*B - 105*a^6*C + 223*a^4*b^2*C - 128*a^2*b^4*C - 8*b^6*C
)*EllipticF[(c + d*x)/2, 2])/(12*b^5*(a^2 - b^2)^2*d) + (a^2*(15*a^4*b*B -
38*a^2*b^3*B + 35*b^5*B - 35*a^5*C + 86*a^3*b^2*C - 63*a*b^4*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*b*
B - 33*a*b^3*B - 35*a^4*C + 61*a^2*b^2*C - 8*b^4*C)*Sqrt[Cos[c + d*x]]*Sin[
```

```
c + d*x])/(12*b^3*(a^2 - b^2)^2*d) + (a*(b*B - a*C)*Cos[c + d*x]^(5/2)*Sin[
c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(3*a^2*b*B - 9*b^
3*B - 7*a^3*C + 13*a*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
```

```
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(bB-aC)+2b\right.}{}$$

$$= \frac{a(bB-aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(3a^2bB-9b^3B-7a^3C\right.}{4b^2\left(a^2-b^2\right.}$$

$$= -\frac{\left(15a^3bB-33ab^3B-35a^4C+61a^2b^2C-8b^4C\right)\sqrt{\cos(c+dx)}\,\mathrm{s}}{12b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(15a^3bB-33ab^3B-35a^4C+61a^2b^2C-8b^4C\right)\sqrt{\cos(c+dx)}\,\mathrm{s}}{12b^3\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-35a^5C+65a^3b^2C-24ab^4C\right)E\left(\frac{1}{2}\right.}{4b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-35a^5C+65a^3b^2C-24ab^4C\right)E\left(\frac{1}{2}\right.}{4b^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 4.6727, size = 466, normalized size = 1.01

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(ab\left(-83a^2b^2C-21a^3bB+49a^4C+39ab^3B+16b^4C\right)\cos(c+dx)+33a^3b^3B+4C\left(b^3-a^2b\right)^2\cos(2(c+dx))-57a^4b^2C-15a^5bB+35a^6C+4b^6C\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(-15*a^5*b*B + 33*a^3*b^3*B + 35*a^6*C - 57*a^4*b^2*
C + 4*b^6*C + a*b*(-21*a^3*b*B + 39*a*b^3*B + 49*a^4*C - 83*a^2*b^2*C + 16*
b^4*C)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*C*Cos[2*(c + d*x)])*Sin[c + d*x]
)/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(-15*a^4*b*B + 21*a^2*b^3*B
- 24*b^5*B + 35*a^5*C - 73*a^3*b^2*C + 56*a*b^4*C)*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2])/(a + b) + (16*(-3*a^3*b*B + 12*a*b^3*B + 7*a^4*C - 14*a^
2*b^2*C - 2*b^4*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(-15*a^4*b*B + 29*a^2*b^3*B - 8*b^5
*B + 35*a^5*C - 65*a^3*b^2*C + 24*a*b^4*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Co
s[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c +
 d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(48*b^3*d)
```

Maple [B] time = 3.932, size = 2195, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^5/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*C*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+9*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-18*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2)-8*a^2/b^4*(3*B*b-5*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(B*b
-C*a)/b^5*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^
2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
```

```
s(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^
2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*
b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))-2*a^3/b^5*(4*B*b-5*C*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(5/2)/(b*cos(d*x
 + c) + a)^3, x)
```

3.892 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=367

$$\frac{\left(-5a^2b^3B+33a^3b^2C+3a^4bB-15a^5C-24ab^4C+8b^5B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(29a^2b^2C+3a^3bB-15a^4C-9ab^3B-8b^4C\right.}{4b^3d\left(a^2-b^2\right)^2}$$

```
[Out] -((3*a^3*b*B - 9*a*b^3*B - 15*a^4*C + 29*a^2*b^2*C - 8*b^4*C)*EllipticE[(c
+ d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*b*B - 5*a^2*b^3*B + 8*b^5*B
 - 15*a^5*C + 33*a^3*b^2*C - 24*a*b^4*C)*EllipticF[(c + d*x)/2, 2])/(4*b^4*
(a^2 - b^2)^2*d) - (a*(3*a^4*b*B - 6*a^2*b^3*B + 15*b^5*B - 15*a^5*C + 38*a
^3*b^2*C - 35*a*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b
)^2*b^4*(a + b)^3*d) + (a*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(a^2*b*B - 7*b^3*B - 5*a^3*C +
11*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 1.10009, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2989, 3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-5a^2b^3B+33a^3b^2C+3a^4bB-15a^5C-24ab^4C+8b^5B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(29a^2b^2C+3a^3bB-15a^4C-9ab^3B-8b^4C\right.}{4b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^3,x]
```


```
[Out] -((3*a^3*b*B - 9*a*b^3*B - 15*a^4*C + 29*a^2*b^2*C - 8*b^4*C)*EllipticE[(c
+ d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*b*B - 5*a^2*b^3*B + 8*b^5*B
 - 15*a^5*C + 33*a^3*b^2*C - 24*a*b^4*C)*EllipticF[(c + d*x)/2, 2])/(4*b^4*
(a^2 - b^2)^2*d) - (a*(3*a^4*b*B - 6*a^2*b^3*B + 15*b^5*B - 15*a^5*C + 38*a
^3*b^2*C - 35*a*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b
)^2*b^4*(a + b)^3*d) + (a*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(a^2*b*B - 7*b^3*B - 5*a^3*C +
11*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
```

```
Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$=\frac{a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(bB-aC)+2b\right.}{}}{}$$

$$=\frac{a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-7b^3B-5a^3C+\right.}{4b^2\left(a^2-b^2\right.}$$

$$=\frac{a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2bB-7b^3B-5a^3C+\right.}{4b^2\left(a^2-b^2\right.}$$

$$=-\frac{\left(3a^3bB-9ab^3B-15a^4C+29a^2b^2C-8b^4C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d}$$

$$=-\frac{\left(3a^3bB-9ab^3B-15a^4C+29a^2b^2C-8b^4C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d}$$

<u>Mathematica</u> [A] time = 4.67253, size = 394, normalized size = 1.07

$$\frac{\frac{\left(-7a^2b^2C-a^3bB+5a^4C-5ab^3B+8b^4C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(a^2bB+a^3C-4ab^2C+2b^3B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(-29a^2b^2C-3a^3bB+15a^4C+9ab^3B+8b^4C\right)\sin(c+dx}{(a-b)^2(a+b)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] ((-2*a*Sqrt[Cos[c + d*x]]*(a*(-(a^2*b*B) + 7*b^3*B + 5*a^3*C - 11*a*b^2*C)
+ b*(-3*a^2*b*B + 9*b^3*B + 7*a^3*C - 13*a*b^2*C)*Cos[c + d*x])*Sin[c + d*x
])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((-(a^3*b*B) - 5*a*b^3*B + 5*a
^4*C - 7*a^2*b^2*C + 8*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b) + (8*(a^2*b*B + 2*b^3*B + a^3*C - 4*a*b^2*C)*((a + b)*EllipticF[(c +
d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((-3*a
^3*b*B + 9*a*b^3*B + 15*a^4*C - 29*a^2*b^2*C + 8*b^4*C)*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
```

```
d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8
*b^2*d)
```

Maple [B] time = 3.355, size = 1977, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+12*a/b^3*(B*b-2*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(B*b-C
*a)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-
b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)
```

```
^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2)))+2*a^2/b^4*(3*B*b-4*C*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^3, x)
```

3.893 $\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=344

$$\frac{\left(-5a^2b^2C+a^3bB+3a^4C-7ab^3B+8b^4C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{\left(a^2bB+3a^3C-9ab^2C+5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{(-10a}{}$$

```
[Out] -((a^2*b*B + 5*b^3*B + 3*a^3*C - 9*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(4*b
^2*(a^2 - b^2)^2*d) + ((a^3*b*B - 7*a*b^3*B + 3*a^4*C - 5*a^2*b^2*C + 8*b^4
*C)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*b*B - 10*a^2
*b^3*B - 3*b^5*B + 3*a^5*C - 6*a^3*b^2*C + 15*a*b^4*C)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*(b*B - a*C)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((
a^2*b*B + 5*b^3*B + 3*a^3*C - 9*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.09723, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2989, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-5a^2b^2C+a^3bB+3a^4C-7ab^3B+8b^4C\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{\left(a^2bB+3a^3C-9ab^2C+5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{(-10a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^3,x]
```


```
[Out] -((a^2*b*B + 5*b^3*B + 3*a^3*C - 9*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(4*b
^2*(a^2 - b^2)^2*d) + ((a^3*b*B - 7*a*b^3*B + 3*a^4*C - 5*a^2*b^2*C + 8*b^4
*C)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*b*B - 10*a^2
*b^3*B - 3*b^5*B + 3*a^5*C - 6*a^3*b^2*C + 15*a*b^4*C)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*(b*B - a*C)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((
a^2*b*B + 5*b^3*B + 3*a^3*C - 9*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a(bB-aC)+2b(bB-aC)\cos(c}{\sqrt{\cos(c+dx}}}{2b}$$

$$= \frac{a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+5b^3B+3a^3C-9\right.}{4b\left(a^2-b^2\right)^2}$$

$$= \frac{a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+5b^3B+3a^3C-9\right.}{4b\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(a^2bB+5b^3B+3a^3C-9ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{a(bB-aC)}{2b\left(a^2-b^2\right.}$$

$$= -\frac{\left(a^2bB+5b^3B+3a^3C-9ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^3bB-7ab\right.}{}$$

Mathematica [A] time = 3.35566, size = 364, normalized size = 1.06

$$\frac{\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(a^2bB+3a^3C-9ab^2C+5b^3B\right)\cos(c+dx)+a\left(3a^2bB+a^3C-7ab^2C+3b^3B\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{\left(-5a^2bB+a^3C+5ab^2C-b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{8\left(a^2C-3abB+2b^2C\right)(}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*(3*a^2*b*B + 3*b^3*B + a^3*C - 7*a*b^2*C) + b*(a^
2*b*B + 5*b^3*B + 3*a^3*C - 9*a*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) - (((-5*a^2*b*B - b^3*B + a^3*C + 5*a*b^2*C)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(-3*a*b*B + a^2*C
+ 2*b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2]))/(a + b) + ((a^2*b*B + 5*b^3*B + 3*a^3*C - 9*a*b^2*C)*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt
```

```
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^
2*(a + b)^2))/(8*b*d)
```

Maple [B] time = 3.077, size = 1937, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(B*b-3*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(B*b-C*a)/b^3*(-1/2/a*b
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
```

```
,-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)-2*a/b^3*(2*B*b-3*C*a)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2
/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2
))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^3, x)
```

3.894 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=337

$$\frac{\left(3a^2bB+a^3C-7ab^2C+3b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(5a^2bB+a^3(-C)-5ab^2C+b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(10a^2b^3B-10a^3\ldots\right.}{}$$

```
[Out] ((5*a^2*b*B + b^3*B - a^3*C - 5*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(4*a*b*
(a^2 - b^2)^2*d) + ((3*a^2*b*B + 3*b^3*B + a^3*C - 7*a*b^2*C)*EllipticF[(c
+ d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*b*B + 10*a^2*b^3*B - b^5*B
+ a^5*C - 10*a^3*b^2*C - 3*a*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) - ((b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*a^2*b*B + b^3*B - a
^3*C - 5*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a
+ b*Cos[c + d*x]))
```

Rubi [A] time = 1.02569, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 2999, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(3a^2bB+a^3C-7ab^2C+3b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(5a^2bB+a^3(-C)-5ab^2C+b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(10a^2b^3B-10a^3\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^3),x]
```


```
[Out] ((5*a^2*b*B + b^3*B - a^3*C - 5*a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(4*a*b*
(a^2 - b^2)^2*d) + ((3*a^2*b*B + 3*b^3*B + a^3*C - 7*a*b^2*C)*EllipticF[(c
+ d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*b*B + 10*a^2*b^3*B - b^5*B
+ a^5*C - 10*a^3*b^2*C - 3*a*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) - ((b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*a^2*b*B + b^3*B - a
^3*C - 5*a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a
+ b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+C\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(bB-aC)-2(aB-bC)\cos(c+dx)+\frac{1}{2}(bB-aC)\cos}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}}{2\left(a^2-b^2\right)}$$

$$= -\frac{(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2bB+b^3B-a^3C-5ab^2C\right)\sqrt{\cos(}}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2bB+b^3B-a^3C-5ab^2C\right)\sqrt{\cos(}}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(5a^2bB+b^3B-a^3C-5ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} - \frac{(bB-aC)\sqrt{\cos(c+dx)}\sin}{2\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{\left(5a^2bB+b^3B-a^3C-5ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} + \frac{\left(3a^2bB+3b^3B+a^3C-7a\right.}{4b^2\left(a^2-b\right.}$$

Mathematica [A] time = 4.23426, size = 369, normalized size = 1.09

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(-5a^2bB+a^3C+5ab^2C-b^3B\right)\cos(c+dx)+a\left(-7a^2bB+3a^3C+3ab^2C+b^3B\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} + \frac{\frac{2\left(-9a^2bB+5a^3C+ab^2C+3b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16a\left(2a^2B-3a\right.}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Co
s[c + d*x])^3),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(a*(-7*a^2*b*B + b^3*B + 3*a^3*C + 3*a*b^2*C) + b*(-
5*a^2*b*B - b^3*B + a^3*C + 5*a*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*b*B + 3*b^3*B + 5*a^3*C + a*b^
2*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*a*(2*a^2*B +
b^2*B - 3*a*b*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(b*(a + b)) - (2*(-5*a^2*b*B - b^3*B + a^3*C + 5*a
*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ell
ipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -
```

```
ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2])
)/((a - b)^2*(a + b)^2))/(16*a*d)
```

Maple [B] time = 3.082, size = 1850, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))-2*a*(B*b-C*a)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*
b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b
^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^
2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(B*b-2*C*a)/b^2*(-1/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+
1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^3*sqrt(
cos(d*x + c))), x)
```

3.895 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(7a^2bB-3a^3C-3ab^2C-b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(9a^2bB-5a^3C-ab^2C-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{\left(-6a^2b^3B-10a\right.}{}$$

```
[Out] -((9*a^2*b*B - 3*b^3*B - 5*a^3*C - a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(4*a
^2*(a^2 - b^2)^2*d) - ((7*a^2*b*B - b^3*B - 3*a^3*C - 3*a*b^2*C)*EllipticF[
(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*b*B - 6*a^2*b^3*B + 3*b
^5*B - 3*a^5*C - 10*a^3*b^2*C + a*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) + (b*(b*B - a*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(9*a^2*b*B
- 3*b^3*B - 5*a^3*C - a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2
 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.17187, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(7a^2bB-3a^3C-3ab^2C-b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(9a^2bB-5a^3C-ab^2C-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{\left(-6a^2b^3B-10a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^3),x]
```


```
[Out] -((9*a^2*b*B - 3*b^3*B - 5*a^3*C - a*b^2*C)*EllipticE[(c + d*x)/2, 2])/(4*a
^2*(a^2 - b^2)^2*d) - ((7*a^2*b*B - b^3*B - 3*a^3*C - 3*a*b^2*C)*EllipticF[
(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*b*B - 6*a^2*b^3*B + 3*b
^5*B - 3*a^5*C - 10*a^3*b^2*C + a*b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) + (b*(b*B - a*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(9*a^2*b*B
- 3*b^3*B - 5*a^3*C - a*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2
 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2B-3b^2B-abC\right)-2a(bB-aC)\cos(c+dx)+\frac{1}{2}b(b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(9a^2bB-3b^3B-5a^3C-ab^2C\right)\sqrt{\cos(}}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(9a^2bB-3b^3B-5a^3C-ab^2C\right)\sqrt{\cos(}}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{\left(9a^2bB-3b^3B-5a^3C-ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{b(bB-aC)\sqrt{\cos(c+dx)}\,\mathrm{si}}{2a\left(a^2-b^2\right)d(a+b\cos(c}$$

$$= -\frac{\left(9a^2bB-3b^3B-5a^3C-ab^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} - \frac{\left(7a^2bB-b^3B-3a^3C-3a\right.}{4ab\left(a^2-b\right.}$$

Mathematica [A] time = 4.73436, size = 387, normalized size = 1.12

$$\frac{\frac{\left(-19a^2b^2B-9a^3bC+16a^4B+3ab^3C+9b^4B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(-4a^2bB+2a^3C+ab^2C+b^3B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}+\frac{\left(-9a^2bB+5a^3C+ab^2C+3b^3B\right)\sin(c+dx)\left(\left(2a^2-b\right.\right.}{}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Co
s[c + d*x])^3),x]
```

```
[Out] ((-2*b*Sqrt[Cos[c + d*x]]*(a*(-11*a^2*b*B + 5*b^3*B + 7*a^3*C - a*b^2*C) +
b*(-9*a^2*b*B + 3*b^3*B + 5*a^3*C + a*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/((
a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((16*a^4*B - 19*a^2*b^2*B + 9*b^4*B
 - 9*a^3*b*C + 3*a*b^3*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b
) + (8*a*(-4*a^2*b*B + b^3*B + 2*a^3*C + a*b^2*C)*((a + b)*EllipticF[(c + d
*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((-
9*a^2*b*B + 3*b^3*B + 5*a^3*C + a*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (
```

```
2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d
*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a^2*d)
```

Maple [B] time = 3.027, size = 1744, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(B*b-C*a)/b*(
-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(
a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*
b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
5/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2)))+2*C/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a
```

```
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^
2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3
,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^3*cos(d
*x + c)^(3/2)), x)
```

3.896 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=420

$$\frac{\left(11a^2bB-7a^3C+ab^2C-5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2B+9a^3bC+8a^4B-3ab^3C+15b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-$$

```
[Out] -((8*a^4*B - 29*a^2*b^2*B + 15*b^4*B + 9*a^3*b*C - 3*a*b^3*C)*EllipticE[(c
+ d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) + ((11*a^2*b*B - 5*b^3*B - 7*a^3*C +
a*b^2*C)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*b*B
- 38*a^2*b^3*B + 15*b^5*B - 15*a^5*C + 6*a^3*b^2*C - 3*a*b^4*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*B -
 29*a^2*b^2*B + 15*b^4*B + 9*a^3*b*C - 3*a*b^3*C)*Sin[c + d*x])/(4*a^3*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(b*B - a*C)*Sin[c + d*x])/(2*a*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b*(11*a^2*b*B - 5*b^3*
B - 7*a^3*C + a*b^2*C)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.57548, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3029, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(11a^2bB-7a^3C+ab^2C-5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2B+9a^3bC+8a^4B-3ab^3C+15b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])^3),x]
```


```
[Out] -((8*a^4*B - 29*a^2*b^2*B + 15*b^4*B + 9*a^3*b*C - 3*a*b^3*C)*EllipticE[(c
+ d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) + ((11*a^2*b*B - 5*b^3*B - 7*a^3*C +
a*b^2*C)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*b*B
- 38*a^2*b^3*B + 15*b^5*B - 15*a^5*C + 6*a^3*b^2*C - 3*a*b^4*C)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*B -
 29*a^2*b^2*B + 15*b^4*B + 9*a^3*b*C - 3*a*b^3*C)*Sin[c + d*x])/(4*a^3*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(b*B - a*C)*Sin[c + d*x])/(2*a*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b*(11*a^2*b*B - 5*b^3*
```

```
B - 7*a^3*C + a*b^2*C)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
x]]*(a + b*Cos[c + d*x]))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2B-5b^2B+abC\right)-2a(bB-aC)}{\cos^{\frac{3}{2}}(c+dx)(a}}{2a\left(a\right.}$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{b\left(11a^2bB-5b^3B-7a^3C-\right.}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= \frac{\left(8a^4B-29a^2b^2B+15b^4B+9a^3bC-3ab^3C\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b(bB}{2a\left(a^2-b^2\right)d\sqrt{\text{co}}}$$

$$= \frac{\left(8a^4B-29a^2b^2B+15b^4B+9a^3bC-3ab^3C\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b(bB}{2a\left(a^2-b^2\right)d\sqrt{\text{co}}}$$

$$= -\frac{\left(8a^4B-29a^2b^2B+15b^4B+9a^3bC-3ab^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4B-29\right.}{}$$

$$= -\frac{\left(8a^4B-29a^2b^2B+15b^4B+9a^3bC-3ab^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(11a^2bB-\right.}{}$$

Mathematica [A] time = 5.16708, size = 462, normalized size = 1.1

$$\frac{\frac{\sqrt{\cos(c+dx)}\left(b^2\left(-29a^2b^2B+9a^3bC+8a^4B-3ab^3C+15b^4B\right)\sin(2(c+dx))+2ab\left(-47a^2b^2B+11a^3bC+16a^4B-5ab^3C+25b^4B\right)\sin(c+dx)+16B\left(a^3-ab^2\right)^2\tan(c+dx)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Co
s[c + d*x])^3),x]
```

```
[Out] (-((((56*a^4*b*B - 95*a^2*b^3*B + 45*b^5*B - 16*a^5*C + 19*a^3*b^2*C - 9*a*
b^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^4*B -
 10*a^2*b^2*B + 5*b^4*B + 4*a^3*b*C - a*b^3*C)*((a + b)*EllipticF[(c + d*x)
```

```
/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((8*a^
4*B - 29*a^2*b^2*B + 15*b^4*B + 9*a^3*b*C - 3*a*b^3*C)*(-2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]],
-1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + (Sq
rt[Cos[c + d*x]]*(2*a*b*(16*a^4*B - 47*a^2*b^2*B + 25*b^4*B + 11*a^3*b*C -
5*a*b^3*C)*Sin[c + d*x] + b^2*(8*a^4*B - 29*a^2*b^2*B + 15*b^4*B + 9*a^3*b*
C - 3*a*b^3*C)*Sin[2*(c + d*x)] + 16*(a^3 - a*b^2)^2*B*Tan[c + d*x]))/((a^2
 - b^2)^2*(a + b*Cos[c + d*x])^2))/(8*a^3*d)
```

Maple [B] time = 3.873, size = 2002, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*B/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-B*b+C*a)/a*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2
-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b
```

```
^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/
4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*B/a^3*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2*b*B/a^2*(-1/
a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))
**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^3*cos(d
*x + c)^(5/2)), x)
```

3.897 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2\right.$

Optimal. Leaf size=560

$$\frac{\left(-3a^2C+6abB+16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\left(-3a^2C+6abB+16b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec}{a+}}}{24ab^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*b*B - 3*a^2*C + 16*b^2*C)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(a + 2*b)*(6*b*B - 3*a*C + 8*b*C)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt[a + b]*(2*a^2*b*B - 8
*b^3*B - a^3*C - 4*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(8*b^3*d) + ((6*a*b*B - 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*b*B - a*C)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d) + (C*Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 1.65908, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2C+6abB+16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\left(-3a^2C+6abB+16b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec}{a+}}}{24ab^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*b*B - 3*a^2*C + 16*b^2*C)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(a + 2*b)*(6*b*B - 3*a*C + 8*b*C)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
```

```
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt[a + b]*(2*a^2*b*B - 8
*b^3*B - a^3*C - 4*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(8*b^3*d) + ((6*a*b*B - 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*b*B - a*C)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d) + (C*Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(B+C\cos($$

$$= \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+d}{3bd}$$

$$= \frac{(2bB-aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin}{4bd}$$

$$= \frac{\left(6abB-3a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(6abB-3a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(2a^2bB-8b^3B-a^3C-4ab^2C\right)\cot(c+}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(6abB-3a^2C+16b^2C\right)\cot(c}{}$$

Mathematica [C] time = 6.32859, size = 1224, normalized size = 2.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] -((-4*a*(-18*a*b*B + a^2*C - 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-
24*b^2*B - 28*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])) + 2*(-6*a*b*B + 3*a^2*C - 16*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]],
 (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sq
rt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*b*d) + (Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]*(((6*b*B + a*C)*Sin[c + d*x])/(12*b) + (C*Sin[2*(
c + d*x)])/6))/d
```

Maple [B] time = 0.227, size = 2949, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(10*C*cos(d*x+c)^4*a*b^2-C*cos(d*x+c)^3*a^2*
b+3*C*cos(d*x+c)^2*a^2*b+6*C*cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)*a^2*b-16*C*c
os(d*x+c)*a*b^2+6*B*cos(d*x+c)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2-6*B*cos(d*x+c
)*a^2*b-12*B*cos(d*x+c)*a*b^2+18*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^3
+8*C*cos(d*x+c)^5*b^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*a^2*b-28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-3*C*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^
3-12*B*cos(d*x+c)^2*b^3+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)
```

```
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3
*b^3-3*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3+3*C*cos(d*x+c)*a^3+6*B*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^2-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*B*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*B*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
a^2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b^2/cos(d*x+c)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.898 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=473

$$-\frac{\sqrt{a+b}\left(a^2(-C)+4abB+4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{(aC}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(a*C + 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*b*B - a^2*C + 4*b^2*C)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*b*B + a*C)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.17413, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 3003, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(a^2(-C)+4abB+4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{(aC}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(a*C + 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*b*B - a^2*C + 4*b^2*C)*Cot[c + d*x]*Elliptic
```

```
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*b*B + a*C)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))\,dx$$

$$= \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{4}\int \frac{aC+\ldots}{\ldots}$$

$$= \frac{(4bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx\ldots}}{\ldots}$$

$$= \frac{(4bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx\ldots}}{\ldots}$$

$$= -\frac{\sqrt{a+b}\left(4abB-a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\ldots\right.\right.}{4b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}(4bB+aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+\ldots}}{\sqrt{a+b}\sqrt{\cos(c+\ldots}}\ldots\right.\right.}{4abd}$$

<u>Mathematica</u> [C] time = 21.0447, size = 1175, normalized size = 2.48

$$-\frac{4a(4bB+3aC)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+d\ldots\right.}{a}}\ldots}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos\ldots}}$$

$$\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(4*b*B + 3*a*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
```

```
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a*B + 4*b*C)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*b*B + a*C)*
((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]
) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8
*d)
```

Maple [B] time = 0.142, size = 2052, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x
)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-2*C*cos(d*x+c)*a*b+4*B*cos(d*x+c)^2*a*b-4*B
*cos(d*x+c)*a*b+3*C*cos(d*x+c)^3*a*b-C*cos(d*x+c)^2*a*b+4*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*b^2*C*c
os(d*x+c)^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
```

```
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-4*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^2-4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*b^2-2*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+
C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^3*b^2-4*B*cos(d*x+c)^2*
b^2+C*cos(d*x+c)^2*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*b+2*C*cos(d*x+c)^4*b^2-C*cos(d*x+c)*a^2-8*B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b)/sin(d*x+c)/b/cos(d*x+c
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)+B\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

3.899 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{a+b}(2B+C)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(aC+2bB)\cot(c+}{}$$

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*B + C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (Sqrt[a + b]*(2*b*B
 + a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (C*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.842322, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {3029, 3003, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(2B+C)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(aC+2bB)\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*B + C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (Sqrt[a + b]*(2*b*B
 + a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
```

```
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (C*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\int \frac{-aC+2aB\cos(c+}{\cos^{\frac{3}{2}}(c+}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\int \frac{-aC+2aB\cos}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}$$

$$= -\frac{\sqrt{a+b}(2bB+aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{bd}$$

$$= -\frac{(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{ad}$$

Mathematica [B] time = 18.2087, size = 3054, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] ((1 + Cos[c + d*x])^(3/2)*((B*Sqrt[a + b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
+ C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])*Sec[(c + d*x)/2]^2*(2*(a +
 b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(b*B + a*(-B + C))*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - 8*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
4*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*C*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*C*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*C*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Tan[(c + d*x)/2]))/(4*d*Sqrt[a + b*Cos[c + d*x]]*((b*(1 + Cos[c
+ d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*(2*(a + b)*C*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] - 4*(b*B + a*(-B + C))*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
] - 8*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticP
```

i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*(a + b*Cos[c + d*x])^(3/2)) - (3*Sqrt[1 + Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*(2*(a + b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(b*B + a*(-B + C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 8*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(2*(a + b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(b*B + a*(-B + C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 8*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*((3*b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Cos[(3*(c + d*x))/2]*Sec[(c + d*x)/2])/2 + a*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2 - (b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2)/2 + ((a + b)*C*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (2*(b*B + a*(-B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (4*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*C*Sec[(c + d*x)/2]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c

```
+ d*x]))*Sin[(3*(c + d*x))/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) +
(a*C*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
Cos[c + d*x]))*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b
*C*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Co
s[c + d*x]))*Tan[(c + d*x)/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) +
(b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x
))/2]*Tan[(c + d*x)/2])/2 - (2*(b*B + a*(-B + C))*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (4*b*B*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*
x)/2]^2)/(a + b)]) + (2*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*C*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt
[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))
/(4*Sqrt[a + b*Cos[c + d*x]])))
```

Maple [B] time = 0.215, size = 1693, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x
)
```

```
[Out] -1/d*(2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a-2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+4*B*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+4*
B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a-4*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+8*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+2*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
```

```
1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a-2*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b+
4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)^2*a+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a+C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*b+2*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*a-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2
*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)^4*b+C*cos(d*x+c)^3*a-C*cos(d*x+c)^3*b-C*
cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\cos(c + dx))\sqrt{a + b\cos(c + dx)}}{\sqrt{\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c)\right)\sqrt{b\cos(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

3.900 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{2\sqrt{a+b}(bB-a(B-C))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2B(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt
[a + b]*(b*B - a*(B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqr
t[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rubi [A] time = 0.640728, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 2991, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(bB-a(B-C))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2B(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt
[a + b]*(b*B - a*(B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqr
t[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= (bC)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{aB+(bB+aC)\cos}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,c}}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad}$$

Mathematica [A] time = 13.14, size = 275, normalized size = 0.78

$$2(a(B+C)+b(B-C))\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+\frac{2B\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] (-2*(a + b)*B*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2
*(b*(B - C) + a*(B + C))*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] - 4*b*C*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
```

```
 b)] + (2*B*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]])/(d*S
qrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.158, size = 1687, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x
)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a-C*cos(d*x+c)^2*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+2*C*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*b+2*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a-2*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+4*C*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*b+B*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a+B*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b-B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)^2*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b+C*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a-C*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c
```

```
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+B*cos(d*x+c)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b-B*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*c
os(d*x+c)*a)/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)+B\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

3.901 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2(a-b)\sqrt{a+b}(3aC+bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)\sqrt{a+b}(}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(b*B + 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*(a - b)*Sqrt[a + b]*(B - 3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(3*a*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))
```

Rubi [A] time = 0.627171, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3029, 2999, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}(3aC+bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)\sqrt{a+b}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(b*B + 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*(a - b)*Sqrt[a + b]*(B - 3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(3*a*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}(bB+3aC)+}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}((a-b)(B-3C))\int$$

$$= \frac{2(a-b)\sqrt{a+b}(bB+3aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{3a^2d}$$

Mathematica [A] time = 14.2008, size = 407, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(3aC\sin(c+dx)+bB\sin(c+dx))}{3a}+\frac{2}{3}B\tan(c+dx)\sec(c+dx)\right)}{d} + \frac{4\left(\frac{\cos(c+dx)}{\cos(c+dx)+1}\right)^{3/2}\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```

```
[Out] (4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
1 + Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(b*B +
3*a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*a*(a + b)*(B + 3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] - (b*B + 3*a*C)*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Cos[c + d*x]^(5/2)*Sqrt[a +
b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(b*B*Sin[c + d*x] + 3*a*C*Sin[c + d*x]))/(3*a) + (2*B*Sec[c + d*x]*Ta
```

```
n[c + d*x])/3))/d
```

Maple [B] time = 0.116, size = 1727, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x
)
```

```
[Out] -2/3/d*(-a^2*B+B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+3*C*cos(d*x+c)^3*a*b-3
*C*cos(d*x+c)^2*a*b+B*cos(d*x+c)^3*a*b+B*cos(d*x+c)^2*a^2-B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^2*sin(d*x+c)*a*b-3*C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-3*C*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*a^2+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*cos(d*x+c)^3*b^2-B*cos(d*x+c)^2*b^2+B
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*C*cos(d*x+c)^
2*a^2+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a*b-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-3*
C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
```

```
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b-3*C*cos(d*x+c)*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^2)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(C\cos\left(dx+c\right)+B\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

3.902 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=350

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2B+5abC-2b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a\ \cdots}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B - 2*b^2*B + 5*a*b*C)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*B + 2*b*B - 5*a*C)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(b*B + 5*a*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.95211, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 2999, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2B+5abC-2b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a\ \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B - 2*b^2*B + 5*a*b*C)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*B + 2*b*B - 5*a*C)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(b*B + 5*a*C)*Sqrt[a + b*Cos[c
```

```
+ d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(bB+5aC)+$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB+5aC)\sqrt{a+b\,\mathrm{c}}}{15ad\cos}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB+5aC)\sqrt{a+b\,\mathrm{c}}}{15ad\cos}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2B-2b^2B+5abC\right)\cot(c+dx)E\left(\sin^{-1}\right.}{15a^3}$$

Mathematica [C] time = 6.35049, size = 1315, normalized size = 3.76

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```

```
[Out] -((-4*a*(2*a^2*b*B - 2*b^3*B - 5*a^3*C + 5*a*b^2*C)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(9*a^3*B - 2*a*b^2*B + 5*a^2*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*b*B - 2*b^3*B + 5*a*b^2*C)*((
I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)])
+ (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
)/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(15*
a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(b
*B*Sin[c + d*x] + 5*a*C*Sin[c + d*x]))/(15*a) + (2*Sec[c + d*x]*(9*a^2*B*Si
n[c + d*x] - 2*b^2*B*Sin[c + d*x] + 5*a*b*C*Sin[c + d*x]))/(15*a^2) + (2*B*
Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.156, size = 2481, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x
)
```

```
[Out] 2/15/d*(3*a^3*B-5*C*cos(d*x+c)^4*a*b^2-5*C*cos(d*x+c)^3*a^2*b+10*C*cos(d*x+
c)^2*a^2*b-5*C*cos(d*x+c)^4*a^2*b-B*cos(d*x+c)^2*a*b^2+4*B*cos(d*x+c)*a^2*b
+2*B*cos(d*x+c)^3*a*b^2+5*C*cos(d*x+c)^3*a*b^2+5*B*cos(d*x+c)^3*a^2*b-9*B*c
os(d*x+c)^4*a^2*b-B*cos(d*x+c)^4*a*b^2-2*B*cos(d*x+c)^3*b^3-9*B*cos(d*x+c)^
3*a^3+2*B*cos(d*x+c)^4*b^3+6*B*cos(d*x+c)^2*a^3-2*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-7*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^3*a^2*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+9*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+9*
B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a^2*b-2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-7*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+2*
B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*a*b^2+5*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*C*cos(d*x+c)^3*a^3+5*C*co
s(d*x+c)*a^3-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*C*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
```

```
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+5*C*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+9*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^3-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3-9*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a
^3+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^2*a^3-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3-9*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3+5*C
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(5/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

3.903 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(25a^2B+7abC-4b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25B-63C)+2ab(3B-7C)+8b^2B\right)\cot(c$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*b*B + 8*b^3*B + 63*a^3*C - 14*a*b^2*C)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(8*b^2*B
 + a^2*(25*B - 63*C) + 2*a*b*(3*B - 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*
x]^(7/2)) + (2*(b*B + 7*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d
*Cos[c + d*x]^(5/2)) + (2*(25*a^2*B - 4*b^2*B + 7*a*b*C)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.29121, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 2999, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2B+7abC-4b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25B-63C)+2ab(3B-7C)+8b^2B\right)\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*b*B + 8*b^3*B + 63*a^3*C - 14*a*b^2*C)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(8*b^2*B
 + a^2*(25*B - 63*C) + 2*a*b*(3*B - 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
```

```
)/(105*a^3*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*
x]^(7/2)) + (2*(b*B + 7*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d
*Cos[c + d*x]^(5/2)) + (2*(25*a^2*B - 4*b^2*B + 7*a*b*C)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos(c+dx)}(B+C\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(bB+7aC)+ }{c}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(bB+7aC)\sqrt{a+b\,c}}{35ad\cos}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(bB+7aC)\sqrt{a+b\,c}}{35ad\cos}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(bB+7aC)\sqrt{a+b\,c}}{35ad\cos}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(19a^2bB+8b^3B+63a^3C-14ab^2C\right)\cot(c}{}$$

Mathematica [C] time = 6.42372, size = 1408, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(11/2),x]
```

```
[Out] ((-4*a*(25*a^4*B - 17*a^2*b^2*B - 8*b^4*B - 14*a^3*b*C + 14*a*b^3*C)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*b*B - 8*a*b^3*B - 63*a^4*C + 14*a^2*b^
2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
```

```
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-19*a^2*b^2*B - 8*b^4*B - 63*a^3*b*C + 14*a*b^3*C)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(b*B*Sin[c + d*x] + 7*a
*C*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]^2*(25*a^2*B*Sin[c + d*x] - 4*b^2
*B*Sin[c + d*x] + 7*a*b*C*Sin[c + d*x]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^
2*b*B*Sin[c + d*x] + 8*b^3*B*Sin[c + d*x] + 63*a^3*C*Sin[c + d*x] - 14*a*b^
2*C*Sin[c + d*x]))/(105*a^3) + (2*B*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.239, size = 3427, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),
x)
```

```
[Out] -2/105/d*(-63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+14*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+49*C*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3*b-15*a^4*B+7*C*cos(d*x+c)^3*a^2*b^2+63*C*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
```

c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6
3*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4-19*B*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-63*C*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+
14*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2+49*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*C*cos(d*x+c)^4*a^2
*b^2+63*C*cos(d*x+c)^5*a^3*b+7*C*cos(d*x+c)^5*a^2*b^2-35*C*cos(d*x+c)^4*a^3
*b-14*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b^2-14*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-21*C*cos(d*x+c)*
a^4+63*C*cos(d*x+c)^4*a^4-14*C*cos(d*x+c)^5*a*b^3-42*C*cos(d*x+c)^3*a^4+8*B
*cos(d*x+c)^5*b^4-8*B*cos(d*x+c)^4*b^4+25*B*cos(d*x+c)^4*a^4-10*B*cos(d*x+c
)^2*a^4+14*C*cos(d*x+c)^4*a*b^3+19*B*cos(d*x+c)^4*a^3*b-20*B*cos(d*x+c)^4*a
^2*b^2+8*B*cos(d*x+c)^4*a*b^3-26*B*cos(d*x+c)^3*a^3*b-4*B*cos(d*x+c)^3*a*b^
3+B*cos(d*x+c)^2*a^2*b^2-18*B*cos(d*x+c)*a^3*b+25*B*cos(d*x+c)^5*a^3*b+19*B
*cos(d*x+c)^5*a^2*b^2-4*B*cos(d*x+c)^5*a*b^3-28*C*cos(d*x+c)^2*a^3*b-8*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^
4*sin(d*x+c)*b^4+25*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-63*C*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4-8*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*si
n(d*x+c)*b^4+25*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4-63*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-19*B*cos(d*x+c
)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*a^2*b^2-8*B*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)

```
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+2*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*si
n(d*x+c)*a^2*b^2+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+14*C*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3-19*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^
3*sin(d*x+c)*a^3*b-19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2-8*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+19*
B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3*b+2*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*B*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+
14*C*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a*b^3)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)
^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(
d*x + c)^(11/2), x)
```

3.904 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(\right.$

Optimal. Leaf size=670

$$\frac{\left(-3a^2C+8abB+12b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{\left(24a^2bB-9a^3C+156ab^2C+128b^3B\right)\sin(c+d}{192b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 156*a*b^2*C)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a^3*C - 6*a^
2*b*(4*B + C) - 8*b^3*(16*B + 9*C) - 4*a*b^2*(28*B + 39*C))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*b*B - 96*a*b^3*B - 3*
a^4*C - 24*a^2*b^2*C - 48*b^4*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(64*b^3*d) + ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 156*a*b^2*C)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((8*a*b*B
- 3*a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(32*b*d) + ((8*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(24*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 2.27303, antiderivative size = 670, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2C+8abB+12b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{\left(24a^2bB-9a^3C+156ab^2C+128b^3B\right)\sin(c+d}{192b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 156*a*b^2*C)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
```

```
*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a^3*C - 6*a^
2*b*(4*B + C) - 8*b^3*(16*B + 9*C) - 4*a*b^2*(28*B + 39*C))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*b*B - 96*a*b^3*B - 3*
a^4*C - 24*a^2*b^2*C - 48*b^4*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(64*b^3*d) + ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 156*a*b^2*C)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((8*a*b*B
- 3*a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(32*b*d) + ((8*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(24*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(4*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
```

```
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(B+C$$

$$= \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+}{4bd}$$

$$= \frac{(8bB-3aC)\sqrt{\cos(c+dx)}(a+b\cos(c+dx)}{24bd}$$

$$= \frac{\left(8abB-3a^2C+12b^2C\right)\sqrt{\cos(c+dx)}\sqrt{a+}{32bd}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+156ab^2C\right)\sqrt{a+}{192b^2d\sqrt{\cos(c+dx}}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+156ab^2C\right)\sqrt{a+}{192b^2d\sqrt{\cos(c+dx}}$$

$$= \frac{\sqrt{a+b}\left(8a^3bB-96ab^3B-3a^4C-24a^2b^2C\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24a^2bB+128b^3B-9a^3C+\right.}{}$$

Mathematica [C] time = 6.42683, size = 1284, normalized size = 1.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] -((-4*a*(-136*a^2*b*B - 128*b^3*B + 3*a^3*C - 228*a*b^2*C)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(-416*a*b^2*B - 228*a^2*b*C - 144*b^3*C)*((Sqrt[((a + b)*
```

```
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-24*a^2*b*B - 128*b^3
*B + 9*a^3*C - 156*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*E
llipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*S
ec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(b*Sqrt[Cos[c + d*x]]))))/(384*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]*(((56*a*b*B + 3*a^2*C + 42*b^2*C)*Sin[c + d*x])/(96*b) + ((8*b*B +
9*a*C)*Sin[2*(c + d*x)])/48 + (b*C*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.347, size = 4048, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
)
```

```
[Out] 1/192/d/(a+b*cos(d*x+c))^(1/2)*(-6*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+228*C*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^2-72*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
```

```
+c),(-(a-b)/(a+b))^(1/2))*a*b^3-144*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+9*C*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b-156*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-128*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+48*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3
*b-576*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*a*b^3-112*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+416*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-24*B*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-24*B*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b^2-128*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^3-64*B*cos(d*x+c)^3*b^4-156*C*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+128*B*cos(
d*x+c)^2*b^4+9*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+228*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-72*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-
144*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*a^2*b^2+9*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b-156*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-156*C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
```

ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-78*C*cos(d*x+
c)^4*a^2*b^2+3*C*cos(d*x+c)^3*a^3*b-136*B*cos(d*x+c)^3*a^2*b^2-24*B*cos(d*x
+c)^2*a^3*b+48*B*cos(d*x+c)^2*a*b^3+112*B*cos(d*x+c)*a^2*b^2+128*B*cos(d*x+
c)*a*b^3+144*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^4-18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-288*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+9*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-48*C*cos(d*x+
c)^6*b^4-24*C*cos(d*x+c)^4*b^4+72*C*cos(d*x+c)^2*b^4-9*C*cos(d*x+c)*a^4+9*C
*cos(d*x+c)^2*a^4+156*C*cos(d*x+c)^2*a*b^3+6*C*cos(d*x+c)*a^3*b+156*C*cos(d
*x+c)*a^2*b^2+72*C*cos(d*x+c)*a*b^3-120*C*cos(d*x+c)^5*a*b^3-108*C*cos(d*x+
c)^3*a*b^3-64*B*cos(d*x+c)^5*b^4+144*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-18*C*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^4-288*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-176*B*cos(d*x+c)^4*a*b^3+24*B*cos(d*x+c)^
2*a^2*b^2+24*B*cos(d*x+c)*a^3*b-9*C*cos(d*x+c)^2*a^3*b-78*C*cos(d*x+c)^2*a^
2*b^2-128*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^4+48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b-576*B*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-
112*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2+416*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-24*B*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2-128*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)/sin(d*x+c)/b^2/cos(d*x
+c)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)*sq
rt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.905 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=566

$$\frac{\left(3a^2C+30abB+16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(3a^2C+30abB+14abC+12b^2B+16b^2C\right)\cot(c+dx}{24}$$

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*b*B + 3*a^2*C + 16*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(30*a*b*B + 12*b^2*B + 3*a^2*C + 14
*a*b*C + 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (Sqrt[a +
b]*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(8*b^2*d) + ((30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + ((6*b*B + 7*a*C)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b*C*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.85762, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2C+30abB+16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(3a^2C+30abB+14abC+12b^2B+16b^2C\right)\cot(c+dx}{24}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(30*a*b*B + 3*a^2*C + 16*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(30*a*b*B + 12*b^2*B + 3*a^2*C + 14
*a*b*C + 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
```

```
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (Sqrt[a +
b]*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(8*b^2*d) + ((30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + ((6*b*B + 7*a*C)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b*C*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx)$$

$$= \frac{bC\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int$$

$$= \frac{(6bB+7aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx}{12d}$$

$$= \frac{\left(30abB+3a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx}{24bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(30abB+3a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx}{24bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(6a^2bB+8b^3B-a^3C+12ab^2C\right)\cot(c+dx)\Pi}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(30abB+3a^2C+16b^2C\right)\cot(c+dx)E\Big(}{}$$

Mathematica [C] time = 6.31525, size = 1227, normalized size = 2.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((-4*a*(42*a*b*B + 17*a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
48*a^2*B + 24*b^2*B + 52*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(30*a*b*B + 3*a^2*C + 16*b^2*C)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(48*d) + (Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((6*b*B + 7*a*C)*Sin[c + d*x])/12 + (b*
C*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.223, size = 3139, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
)

[Out] 1/24/d/(a+b*cos(d*x+c))^(1/2)*(-22*C*cos(d*x+c)^4*a*b^2-17*C*cos(d*x+c)^3*a
^2*b+3*C*cos(d*x+c)^2*a^2*b+6*C*cos(d*x+c)^2*a*b^2+14*C*cos(d*x+c)*a^2*b+16
*C*cos(d*x+c)*a*b^2-30*B*cos(d*x+c)^2*a^2*b+30*B*cos(d*x+c)^2*a*b^2+30*B*co
s(d*x+c)*a^2*b+12*B*cos(d*x+c)*a*b^2-42*B*cos(d*x+c)^3*a*b^2-12*B*cos(d*x+c
)^4*b^3-8*C*cos(d*x+c)^5*b^3-30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-72*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a*b^2-14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*a^2*b+52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*
b-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a*b^2-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-72*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-14*C*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b+52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-3*C*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*a^3+12*B*cos(d*x+c)^2*b^3+48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+6*C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*C

```
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3-16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^3-8*C*cos(d*x+c)^3*b^3-3*C*cos(d*x+c)^2*a^3+16*C*cos(d*x+c)
^2*b^3+3*C*cos(d*x+c)*a^3-30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-36*B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2
*b-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2+48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+24*B*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^3-30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b-30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-36*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-12*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-48*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*b^3+24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
```

```
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/sq
rt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/sq
rt(cos(d*x + c)), x)
```

3.906 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=472

$$-\frac{\sqrt{a+b}\left(3a^2C+12abB+4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{(5aC}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + 5*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*d) + (Sqrt[a + b]*(8*a*B + 4*b*B + 5*a*C + 2*b*C)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*d) - (Sqrt[a + b]*(12*a*b*B + 3*a^2*C + 4*b^2*C)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + ((4*b*B + 5*a*C)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.31672, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2C+12abB+4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{(5aC}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + 5*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*d) + (Sqrt[a + b]*(8*a*B + 4*b*B + 5*a*C + 2*b*C)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
```

```
))/(a - b)])/(4*d) - (Sqrt[a + b]*(12*a*b*B + 3*a^2*C + 4*b^2*C)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + ((4*b*B + 5*a*C)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{bC\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\frac{1}{2}\cdots}{\cdots}$$

$$= \frac{(4bB+5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c\cdots}}{\cdots}$$

$$= \frac{(4bB+5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c\cdots}}{\cdots}$$

$$= -\frac{\sqrt{a+b}\left(12abB+3a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-\cdots}\right.}{4b\cdots}$$

$$= -\frac{(a-b)\sqrt{a+b}(4bB+5aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos\cdots}}{\sqrt{a+b}\sqrt{\cos\cdots}}\right.\right.}{4ad}$$

<u>Mathematica</u> [C] time = 6.33437, size = 1198, normalized size = 2.54

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4
*a*(8*a^2*B + 4*b^2*B + 7*a*b*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(16*a*b*
B + 8*a^2*C + 4*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
```

```
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(4*b^2*B + 5*a*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d)
```

Maple [B] time = 0.249, size = 2430, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
)
```

```
[Out] -1/4/d*(-2*C*cos(d*x+c)*a*b+4*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b+7*C*cos
(d*x+c)^3*a*b-5*C*cos(d*x+c)^2*a*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*b^2*C*cos(d*x+c)^2+4*B*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*C*sin(d*x+c)*
```

```
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*C*c
os(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+5*C*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2-8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^3*b^2-4
*B*cos(d*x+c)^2*b^2+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+5*C*cos(d*x+c)^2*a^2+2*C*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*a*b+5*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)^4*b^2
-5*C*cos(d*x+c)*a^2-16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*b-16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/
sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^2+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(3/2), x)
```

3.907 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=449

$$-\frac{(2aB-bC)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}(2a(B-C)-b(4B+C))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F}{d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*B - b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a*(B - C) - b*(4*B + C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d - (Sqrt[a + b]*(2*b*B + 3*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/d + (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*a*B - b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]])
```

Rubi [A] time = 1.30499, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2aB-bC)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}(2a(B-C)-b(4B+C))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(5/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(2*a*B - b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a*(B - C) - b*(4*B + C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
```

```
)])/d - (Sqrt[a + b]*(2*b*B + 3*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/d + (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*a*B - b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a(2bB+aC)\ldots}{\ldots}$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aB-bC)\sqrt{a+b\ldots}}{d\sqrt{\cos\ldots}}$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aB-bC)\sqrt{a+b\ldots}}{d\sqrt{\cos\ldots}}$$

$$= -\frac{\sqrt{a+b}(2bB+3aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(\ldots}}{\sqrt{a+b}\sqrt{\cos(\ldots}}\right.\right.}{d}$$

$$= \frac{(a-b)\sqrt{a+b}(2aB-bC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+\ldots}}{\sqrt{a+b}\sqrt{\cos(c\ldots}}\right.\right.}{ad}$$

Mathematica [C] time = 6.35598, size = 1196, normalized size = 2.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*
a*(-2*a*b*B - 2*a^2*C - b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a^2*B -
 2*b^2*B - 4*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
```

```
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) - 2*(2*a*b*B - b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(2*d)
```

Maple [B] time = 0.138, size = 2185, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
)
```

```
[Out] -1/d*(-2*a^2*B-C*cos(d*x+c)*a*b+2*B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+C*c
os(d*x+c)^2*a*b+4*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*b^2-2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-b^2*C*cos(d*x+c)^2-2*B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*
B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2+4*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-2*B*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+6*C*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
a*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2+2*B*cos(d*x+c)*a^2+C*cos(d*x+c)^3*b^2-4*C*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a*b+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+4*B*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*b^2+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/(a+b*cos(d*x+c))^(1/2)/sin(
d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(5/2), x)
```

3.908 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=418

$$\frac{2\sqrt{a+b}\left(a^2(B-3C)-ab(4B-6C)+3b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*b*B + 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*d) + (2*Sqrt[a + b]*(3*b^2*B - a*b*(4*B - 6*C) + a^2*(B - 3*C))*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (2*b*Sqrt[a + b]*C*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.994344, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {3029, 2989, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2(B-3C)-ab(4B-6C)+3b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(4*b*B + 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*d) + (2*Sqrt[a + b]*(3*b^2*B - a*b*(4*B - 6*C) + a^2*(B - 3*C))*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (2*b*Sqrt[a + b]*C*Cot[c + d*x]*Ellip
```

```
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
```

```
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a(4bB+3aC$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a(4bB+3aC}{\cos$$

$$= -\frac{2b\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}(4bB+3aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{3ad}$$

Mathematica **[C]** time = 6.35058, size = 1236, normalized size = 2.96

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] ((-4*a*(a^2*B - b^2*B + 3*a*b*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-4*a*b*
B - 3*a^2*C + 3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(-4*b^2*B - 3*a*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
```

```
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]*(4*b*B*Sin[c + d*x] + 3*a*C*Sin[c + d*x]))/3
 + (2*a*B*Sec[c + d*x]*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.135, size = 2318, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
)
```

```
[Out] -2/3/d*(-a^2*B+4*B*cos(d*x+c)^2*a*b-5*B*cos(d*x+c)*a*b+3*C*cos(d*x+c)^3*a*b
-3*C*cos(d*x+c)^2*a*b+B*cos(d*x+c)^3*a*b+B*cos(d*x+c)^2*a^2-4*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)*a*b-3*C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+6*C*cos(d*x+c)^2*sin(d*
x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
b^2-3*C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*a^2-3*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-3*
```

```
C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a^2+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*B*cos(d*x+
c)^3*b^2-4*B*cos(d*x+c)^2*b^2+3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+B*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*C*cos(d*x+c)^2*a^2+6*C*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*a*b-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-3*C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*
C*cos(d*x+c)*a^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^2+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(7/2), x)
```

3.909 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2B+20abC+3b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}+\frac{2(5}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B + 3*b^2*B + 20*a*b*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*B - 3*b*B - 5*a*C +
15*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (2*a*B*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*b*B + 5*a*C)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.0507, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 2989, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2B+20abC+3b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}+\frac{2(5}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B + 3*b^2*B + 20*a*b*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*B - 3*b*B - 5*a*C +
15*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (2*a*B*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*b*B + 5*a*C)*Sqrt
```

```
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a(6bB+5aC$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6bB+5aC)\sqrt{a+}{15d\,\mathrm{co}}$$

$$= \frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(6bB+5aC)\sqrt{a+}{15d\,\mathrm{co}}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2B+3b^2B+20abC\right)\cot(c+dx)E\left(\sin\right.}{15a}$$

Mathematica [C] time = 6.42739, size = 1314, normalized size = 3.72

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] -((-4*a*(-3*a^2*b*B + 3*b^3*B - 5*a^3*C + 5*a*b^2*C)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - 4*a*(9*a^3*B + 3*a*b^2*B + 20*a^2*b*C)*((Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*b*B + 3*b^3*B + 20*a*b^2*C)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
```

```
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
15*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(
6*b*B*Sin[c + d*x] + 5*a*C*Sin[c + d*x]))/15 + (2*Sec[c + d*x]*(9*a^2*B*Sin
[c + d*x] + 3*b^2*B*Sin[c + d*x] + 20*a*b*C*Sin[c + d*x]))/(15*a) + (2*a*B*
Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.176, size = 2666, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
)
```

```
[Out] -2/15/d*(-3*a^3*B+20*C*cos(d*x+c)^4*a*b^2+20*C*cos(d*x+c)^3*a^2*b-25*C*cos(
d*x+c)^2*a^2*b+5*C*cos(d*x+c)^4*a^2*b-9*B*cos(d*x+c)^2*a*b^2-9*B*cos(d*x+c)
*a^2*b+3*B*cos(d*x+c)^3*a*b^2-20*C*cos(d*x+c)^3*a*b^2+9*B*cos(d*x+c)^4*a^2*
b+6*B*cos(d*x+c)^4*a*b^2-3*B*cos(d*x+c)^3*b^3+9*B*cos(d*x+c)^3*a^3+3*B*cos(
d*x+c)^4*b^3-6*B*cos(d*x+c)^2*a^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+15*C*cos(d*x+c)^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b^2+12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
```

```
c)^3*a^2*b-9*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+12*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*a^2*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+15*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-20*C*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2+5*C*cos(d*x+c)^3*a^3-5*C*cos(d*x+c)*a^3+5*C*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3+20*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-20*C*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b+20*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b-20*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-9*B*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^
3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^3*b^3+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-9*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-3*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*b^3+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-20*C*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*c
```

```
os(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(9/2), x)
```

3.910 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(25a^2B+42abC+3b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(a-b)\sqrt{a+b}\left(a^2(-(25B-63C))+3ab(19B-7C)+6b^2\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*b*B - 6*b^3*B + 63*a^3*C + 21*a*b^2*C)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a + b]*(6*b^2*B
 - a^2*(25*B - 63*C) + 3*a*b*(19*B - 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(105*a^2*d) + (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + (2*(8*b*B + 7*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*B + 3*b^2*B + 42*a*b*C)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.4174, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2B+42abC+3b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(a-b)\sqrt{a+b}\left(a^2(-(25B-63C))+3ab(19B-7C)+6b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*b*B - 6*b^3*B + 63*a^3*C + 21*a*b^2*C)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a + b]*(6*b^2*B
 - a^2*(25*B - 63*C) + 3*a*b*(19*B - 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
```

```
])/(105*a^2*d) + (2*a*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + (2*(8*b*B + 7*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*B + 3*b^2*B + 42*a*b*C)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(a+b\cos(c+dx))^{3/2}(B+C\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$=\frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int \frac{\frac{1}{2}a(8bB+7aC$$

$$=\frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(8bB+7aC)\sqrt{a+}}{35d\,c}$$

$$=\frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(8bB+7aC)\sqrt{a+}}{35d\,c}$$

$$=\frac{2aB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(8bB+7aC)\sqrt{a+}}{35d\,c}$$

$$=\frac{2(a-b)\sqrt{a+b}\left(82a^2bB-6b^3B+63a^3C+21ab^2C\right)\cot(c}{}$$

Mathematica [C] time = 6.52025, size = 1407, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] ((-4*a*(25*a^4*B - 31*a^2*b^2*B + 6*b^4*B + 21*a^3*b*C - 21*a*b^3*C)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*b*B + 6*a*b^3*B - 63*a^4*C - 21*a^2*b^
2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
```

```
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-82*a^2*b^2*B + 6*b^4*B - 63*a^3*b*C - 21*a*b^3*C)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(8*b*B*Sin[c + d*x] + 7
*a*C*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*B*Sin[c + d*x] + 3*b^2*B
*Sin[c + d*x] + 42*a*b*C*Sin[c + d*x]))/(105*a) + (2*Sec[c + d*x]*(82*a^2*b
*B*Sin[c + d*x] - 6*b^3*B*Sin[c + d*x] + 63*a^3*C*Sin[c + d*x] + 21*a*b^2*C
*Sin[c + d*x]))/(105*a^2) + (2*a*B*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.261, size = 3413, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),
x)
```

```
[Out] -2/105/d*(-63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-21*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+84*C*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3*b-15*a^4*B-63*C*cos(d*x+c)^3*a^2*b^2+63*C*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
```

+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+
63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^4-82*B*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-63*C*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-21*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2+84*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+21*C*cos(d*x+c)^4*a^
2*b^2+63*C*cos(d*x+c)^5*a^3*b+42*C*cos(d*x+c)^5*a^2*b^2+21*C*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^2+21*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-21*C*cos(d*x+c)*a^4+63*C*cos(d*x+c)^4*
a^4+21*C*cos(d*x+c)^5*a*b^3-42*C*cos(d*x+c)^3*a^4-6*B*cos(d*x+c)^5*b^4+6*B*
cos(d*x+c)^4*b^4+25*B*cos(d*x+c)^4*a^4-10*B*cos(d*x+c)^2*a^4-21*C*cos(d*x+c
)^4*a*b^3+82*B*cos(d*x+c)^4*a^3*b-55*B*cos(d*x+c)^4*a^2*b^2-6*B*cos(d*x+c)^
4*a*b^3-68*B*cos(d*x+c)^3*a^3*b+3*B*cos(d*x+c)^3*a*b^3-27*B*cos(d*x+c)^2*a^
2*b^2-39*B*cos(d*x+c)*a^3*b+25*B*cos(d*x+c)^5*a^3*b+82*B*cos(d*x+c)^5*a^2*b
^2+3*B*cos(d*x+c)^5*a*b^3-63*C*cos(d*x+c)^2*a^3*b+6*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+25
B(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^4*sin(d*x+c)*a^4-63*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4+6*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^4+25*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^3*sin(d*x+c)*a^4-63*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4-82*B*cos(d*x+c)^4*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+6*B*c
os(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a*b^3+82*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b

```
)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+51*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2-6
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^4*sin(d*x+c)*a*b^3-21*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3-82*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b
-82*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*a^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+82*B*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b+51*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-21*C*cos(d*x+c)^3
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*a*b^3)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{2}+Ba+\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2 + B*a + (C*a + B*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(3/2)/co
s(d*x + c)^(11/2), x)
```

3.911 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2\right.$

Optimal. Leaf size=779

$$\frac{\left(-15a^2C+50abB+64b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{\left(50a^2bB-15a^3C+172ab^2C+120b^3B\right)\mathrm{si}}{320}$$

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 1692*a^2*b^2
*C + 1024*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[
a + b]*(45*a^4*C - 30*a^3*b*(5*B + C) - 16*b^4*(45*B + 64*C) - 8*a*b^3*(355
*B + 193*C) - 4*a^2*b^2*(295*B + 423*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(1920*b^2*d) + (Sqrt[a + b]*(10*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5
*C - 40*a^3*b^2*C - 240*a*b^4*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(128*b^3*d) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 1692*a^2*b^2*C
 + 1024*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sqrt[Cos[
c + d*x]]) + ((50*a^2*b*B + 120*b^3*B - 15*a^3*C + 172*a*b^2*C)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) + ((50*a*b*B - 15*
a^2*C + 64*b^2*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(240*b*d) + ((10*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(40*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(7/
2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 3.29026, antiderivative size = 779, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-15a^2C+50abB+64b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{\left(50a^2bB-15a^3C+172ab^2C+120b^3B\right)\mathrm{si}}{320}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 1692*a^2*b^2
*C + 1024*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[
a + b]*(45*a^4*C - 30*a^3*b*(5*B + C) - 16*b^4*(45*B + 64*C) - 8*a*b^3*(355
*B + 193*C) - 4*a^2*b^2*(295*B + 423*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(1920*b^2*d) + (Sqrt[a + b]*(10*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5
*C - 40*a^3*b^2*C - 240*a*b^4*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(128*b^3*d) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 1692*a^2*b^2*C
 + 1024*b^4*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sqrt[Cos[
c + d*x]]) + ((50*a^2*b*B + 120*b^3*B - 15*a^3*C + 172*a*b^2*C)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) + ((50*a*b*B - 15*
a^2*C + 64*b^2*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(240*b*d) + ((10*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(40*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(7/
2)*Sin[c + d*x])/(5*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(B+$$

$$= \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}\sin(c}{5bd}$$

$$= \frac{(10bB-3aC)\sqrt{\cos(c+dx)}(a+b\cos(c+}{40bd}$$

$$= \frac{\left(50abB-15a^2C+64b^2C\right)\sqrt{\cos(c+dx)}(a}{240bd}$$

$$= \frac{\left(50a^2bB+120b^3B-15a^3C+172ab^2C\right)\sqrt{}}{320}$$

$$= \frac{\left(150a^3bB+2840ab^3B-45a^4C+1692a^2b\right.}{1920b^2d}$$

$$= \frac{\left(150a^3bB+2840ab^3B-45a^4C+1692a^2b\right.}{1920b^2d}$$

$$= \frac{\sqrt{a+b}\left(10a^4bB-240a^2b^3B-96b^5B-3a\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(150a^3bB+2840ab^3B-4\right.}{}$$

Mathematica [C] time = 6.5218, size = 1353, normalized size = 1.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] -((-4*a*(-1330*a^3*b*B - 3560*a*b^3*B + 15*a^4*C - 3236*a^2*b^2*C - 1024*b^
4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
```

```
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6440*a^2*b^2*B - 1440*b^4*B - 2292
*a^3*b*C - 4624*a*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])) + 2*(-150*a^3*b*B - 2840*a*b^3*B + 45*a^4*C - 1692*a^2*b^2*C
 - 1024*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Ar
cSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/
(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c +
 d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[
c + d*x]])))/(3840*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((5
90*a^2*b*B + 420*b^3*B + 15*a^3*C + 898*a*b^2*C)*Sin[c + d*x])/(960*b) + ((
170*a*b*B + 93*a^2*C + 88*b^2*C)*Sin[2*(c + d*x)])/480 + (b*(10*b*B + 21*a*
C)*Sin[3*(c + d*x)])/160 + (b^2*C*Sin[4*(c + d*x)])/40))/d
```

Maple [B] time = 0.647, size = 5164, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)*sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+Ba^2\cos\left(dx+c\right)+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+\left(Ca^2+2\,Bab\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c) + (2*C*a*b + B*b^2)*cos
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sq
rt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.912 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=664

$$\frac{\left(5a^2C+24abB+12b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\left(264a^2bB+15a^3C+284ab^2C+128b^3B\right)\sin(c}{192bd\sqrt{\cos(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*b*B + 128*b^3*B + 15*a^3*C + 284*a*b^2*C)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*a^3*C + 8*b
^3*(16*B + 9*C) + 2*a^2*b*(132*B + 59*C) + 4*a*b^2*(52*B + 71*C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sqrt[a + b]*(40*a^3*b*B + 160*a*b^3*
B - 5*a^4*C + 120*a^2*b^2*C + 48*b^4*C)*Cot[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(64*b^2*d) + ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 284*a*b^2*C
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((2
4*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(32*d) + ((8*b*B + 11*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(24*d) + (b*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 2.36107, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(5a^2C+24abB+12b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\left(264a^2bB+15a^3C+284ab^2C+128b^3B\right)\sin(c}{192bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*b*B + 128*b^3*B + 15*a^3*C + 284*a*b^2*C)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
```

```
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*a^3*C + 8*b
^3*(16*B + 9*C) + 2*a^2*b*(132*B + 59*C) + 4*a*b^2*(52*B + 71*C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sqrt[a + b]*(40*a^3*b*B + 160*a*b^3*
B - 5*a^4*C + 120*a^2*b^2*C + 48*b^4*C)*Cot[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(64*b^2*d) + ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 284*a*b^2*C
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((2
4*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(32*d) + ((8*b*B + 11*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(24*d) + (b*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx$$

$$= \frac{bC\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{1}{4}\int$$

$$= \frac{(8bB+11aC)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+}{24d}$$

$$= \frac{\left(24abB+5a^2C+12b^2C\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}{32d}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+284ab^2C\right)\sqrt{a+b\cos(c}}{192bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+284ab^2C\right)\sqrt{a+b\cos(c}}{192bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(40a^3bB+160ab^3B-5a^4C+120a^2b^2C+48b}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(264a^2bB+128b^3B+15a^3C+284ab^2C}{}$$

<u>Mathematica</u> [C] time = 6.40537, size = 1287, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((-4*a*(472*a^2*b*B + 128*b^3*B + 133*a^3*C + 356*a*b^2*C)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(384*a^3*B + 608*a*b^2*B + 644*a^2*b*C + 144*b^3*C)*((Sqr
```

```
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(264*a^2*b*
B + 128*b^3*B + 15*a^3*C + 284*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
+ b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(384*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*(((104*a*b*B + 59*a^2*C + 42*b^2*C)*Sin[c + d*x])/96 + (b
*(8*b*B + 17*a*C)*Sin[2*(c + d*x)])/48 + (b^2*C*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.357, size = 4238, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
)
```

```
[Out] 1/192/d/(a+b*cos(d*x+c))^(1/2)*(-118*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+644*C*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b^2-72*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
```

*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-720*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*C*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b-284*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-128*B*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-240*B*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^3*b-960*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a*b^3-208*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+608*B*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-264*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-264*B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b^2-128*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3-64*B*cos(d*x+c)^3*b^4-284*C*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+128*
B*cos(d*x+c)^2*b^4-15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-118*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+644*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-72*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^3-720*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a^2*b^2-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-284*C*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-284*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))

^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-254
*C*cos(d*x+c)^4*a^2*b^2-133*C*cos(d*x+c)^3*a^3*b+384*B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-472*B*cos(d*
x+c)^3*a^2*b^2-264*B*cos(d*x+c)^2*a^3*b+144*B*cos(d*x+c)^2*a*b^3+208*B*cos(
d*x+c)*a^2*b^2+128*B*cos(d*x+c)*a*b^3+144*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+30*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-288*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^4-15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^4-48*C*cos(d*x+c)^6*b^4-24*C*cos(d*x+c)^4*b^4+72*C*cos(d*x+c
)^2*b^4+15*C*cos(d*x+c)*a^4-15*C*cos(d*x+c)^2*a^4+284*C*cos(d*x+c)^2*a*b^3+
118*C*cos(d*x+c)*a^3*b+284*C*cos(d*x+c)*a^2*b^2+72*C*cos(d*x+c)*a*b^3-184*C
*cos(d*x+c)^5*a*b^3-172*C*cos(d*x+c)^3*a*b^3-64*B*cos(d*x+c)^5*b^4+144*C*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^4+30*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-288*C*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-272*B*c
os(d*x+c)^4*a*b^3+264*B*cos(d*x+c)^2*a^2*b^2+264*B*cos(d*x+c)*a^3*b+15*C*co
s(d*x+c)^2*a^3*b-30*C*cos(d*x+c)^2*a^2*b^2+384*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-128*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^4-240*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b-960*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-208*B*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b^2+608*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-264*B*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-264*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)

```
)^(1/2))*a^2*b^2-128*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/sq
rt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/sq
rt(cos(d*x + c)), x)
```

3.913 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=563

$$\frac{\left(33a^2C+54abB+16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(a^2(48B+33C)+ab(54B+26C)+4b^2(3B+4C)\right)}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*b*B + 33*a^2*C + 16*b^2*C)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(24*a*d) + (Sqrt[a + b]*(4*b^2*(3*B + 4*C) + a*b*(54*B + 26*
C) + a^2*(48*B + 33*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) - (Sqrt[a
 + b]*(30*a^2*b*B + 8*b^3*B + 5*a^3*C + 20*a*b^2*C)*Cot[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(8*b*d) + ((54*a*b*B + 33*a^2*C + 16*b^2*C)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (b*(2*b*B + 3*a
*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*C*
Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.82428, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(33a^2C+54abB+16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\left(a^2(48B+33C)+ab(54B+26C)+4b^2(3B+4C)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*b*B + 33*a^2*C + 16*b^2*C)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(24*a*d) + (Sqrt[a + b]*(4*b^2*(3*B + 4*C) + a*b*(54*B + 26*
```

```
C) + a^2*(48*B + 33*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) - (Sqrt[a
 + b]*(30*a^2*b*B + 8*b^3*B + 5*a^3*C + 20*a*b^2*C)*Cot[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(8*b*d) + ((54*a*b*B + 33*a^2*C + 16*b^2*C)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (b*(2*b*B + 3*a
*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*C*
Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{bC\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int$$

$$= \frac{b(2bB+3aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d}$$

$$= \frac{\left(54abB+33a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(54abB+33a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(30a^2bB+8b^3B+5a^3C+20ab^2C\right)\cot(c+dx)\Pi}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(54abB+33a^2C+16b^2C\right)\cot(c+dx)E\Big(}{2}$$

Mathematica [C] time = 6.51835, size = 1251, normalized size = 2.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] ((-4*a*(48*a^3*B + 66*a*b^2*B + 59*a^2*b*C + 16*b^3*C)*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - 4*a*(144*a^2*b*B + 24*b^3*B + 48*a^3*C + 76*a*b^2*C)*((Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(54*a*b^2*B + 33*a
^2*b*C + 16*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[
I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*
x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec
[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[
Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b*
(6*b*B + 13*a*C)*Sin[c + d*x])/12 + (b^2*C*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.36, size = 3512, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
)
```

```
[Out] -1/24/d*(34*C*cos(d*x+c)^4*a*b^2+59*C*cos(d*x+c)^3*a^2*b-33*C*cos(d*x+c)^2*
a^2*b-18*C*cos(d*x+c)^2*a*b^2-26*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+5
4*B*cos(d*x+c)^2*a^2*b-54*B*cos(d*x+c)^2*a*b^2-54*B*cos(d*x+c)*a^2*b-12*B*c
os(d*x+c)*a*b^2+66*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^3+8*C*cos(d*x+c
)^5*b^3+54*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b+120*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+26*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*a^2*b-76*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+33*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*b^3+120*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+26*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+33
*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+33*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-12*
B*cos(d*x+c)^2*b^3-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+30*C*sin(d*x+c)*(cos(d*x+c)/(1+
```

cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+33*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^3+8*C*cos(d*x+c)^3*b^3+33*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-33*C
*cos(d*x+c)*a^3+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3+54*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+180*B*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a
^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2-48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+48*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^2*b+48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+54*B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+54*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+180*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^3)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(3/2), x)
```

3.914 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=547

$$-\frac{\left(8a^2B-9abC-4b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(8a^2(B-C)-3ab(8B+3C)-2b^2(2B+C)\right)\cot(c+dx}{4a}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*B - 4*b^2*B - 9*a*b*C)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*a*d) - (Sqrt[a + b]*(8*a^2*(B - C) - 2*b^2*(2*B + C) - 3*a*b*
(8*B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(20*a*
b*B + 15*a^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*d) - ((8*a^2*B - 4*b^2*B - 9*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*a*B - b*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.78503, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2989, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(8a^2B-9abC-4b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(8a^2(B-C)-3ab(8B+3C)-2b^2(2B+C)\right)\cot(c+dx}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*B - 4*b^2*B - 9*a*b*C)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*a*d) - (Sqrt[a + b]*(8*a^2*(B - C) - 2*b^2*(2*B + C) - 3*a*b*
```

```
(8*B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(20*a*
b*B + 15*a^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*d) - ((8*a^2*B - 4*b^2*B - 9*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*a*B - b*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3061

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3053

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2809

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2998

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 2816

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$=\frac{2aB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\sqrt{a+b\cos}}{}$$

$$=-\frac{b(4aB-bC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+d}{2d}$$

$$=-\frac{\left(8a^2B-4b^2B-9abC\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$=-\frac{\left(8a^2B-4b^2B-9abC\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$=-\frac{\sqrt{a+b}\left(20abB+15a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\mathrm{s}\right.}{}$$

$$=\frac{(a-b)\sqrt{a+b}\left(8a^2B-4b^2B-9abC\right)\cot(c+dx)E\left(\sin^{-}\right.}{4a}$$

Mathematica [C] time = 6.50593, size = 1241, normalized size = 2.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] ((4*a*(-16*a^2*b*B - 4*b^3*B - 8*a^3*C - 11*a*b^2*C)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) + 4*a*(8*a^3*B - 24*a*b^2*B - 24*a^2*b*C - 4*b^3*C)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(8*a^2*b*B - 4*b^3*B -
9*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSin
h[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x
])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
d*x]])))/(8*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b^2*C*Sin[c
 + d*x])/2 + 2*a^2*B*Tan[c + d*x]))/d
```

Maple [B] time = 0.178, size = 3270, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
)
```

```
[Out] -1/4/d*(-8*a^3*B+9*C*cos(d*x+c)^2*a^2*b-9*C*cos(d*x+c)^2*a*b^2-9*C*cos(d*x+
c)*a^2*b-2*C*cos(d*x+c)*a*b^2+8*B*cos(d*x+c)^2*a^2*b+4*B*cos(d*x+c)^2*a*b^2
-8*B*cos(d*x+c)*a^2*b-4*B*cos(d*x+c)*a*b^2+11*C*cos(d*x+c)^3*a*b^2-8*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
3+4*B*cos(d*x+c)^3*b^3+8*B*cos(d*x+c)*a^3-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-24*C*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*a^2*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+9*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+9*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*a*b^2-24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B*cos(d*x+c)^2*b
^3+24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a^2*b+2*C*cos(d*x+c)^4*b^3-2*C*cos(d*x+c)^2*b^3+8*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3+4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*b^3-4*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*b^3+8*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
```

pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+30*C*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^2*b-4*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+24*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-8
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+30*C*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-8*B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+4
*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*b^3+40*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+40*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2)/(a+b*cos(
d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(5/2), x)
```

3.915 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=536

$$-\frac{\left(6a^2C+14abB-3b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(-2a^2(B-3C)+2ab(7B-9C)-3b^2(6B+C)\right)\cot(c$$

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*b*B + 6*a^2*C - 3*b^2*C)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*a*b*(7*B - 9*C) - 2*a^2*(B - 3*C) - 3
*b^2*(6*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (b*Sqrt[a + b]*(
2*b*B + 5*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(2*b
*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (
(14*a*b*B + 6*a^2*C - 3*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.76022, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(6a^2C+14abB-3b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(-2a^2(B-3C)+2ab(7B-9C)-3b^2(6B+C)\right)\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(14*a*b*B + 6*a^2*C - 3*b^2*C)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*a*b*(7*B - 9*C) - 2*a^2*(B - 3*C) - 3
```

```
*b^2*(6*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (b*Sqrt[a + b]*(
2*b*B + 5*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(2*b
*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (
(14*a*b*B + 6*a^2*C - 3*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+b\cos}}{}$$

$$= \frac{2a(2bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aB(a+}{}$$

$$= \frac{2a(2bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(14abB}{}$$

$$= \frac{2a(2bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(14abB}{}$$

$$= -\frac{b\sqrt{a+b}(2bB+5aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(14abB+6a^2C-3b^2C\right)\cot(c+dx)E\left(\sin\right.}{3}$$

Mathematica [C] time = 6.50404, size = 1269, normalized size = 2.37

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] ((-4*a*(2*a^3*B + 4*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-14*a^2*b*B + 6*b^3*B - 6*a^3*C + 18*a*b^2*C)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
```

```
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-14*a*b^2*B - 6*a^2*b*
C + 3*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(6*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(7*a*b*B*Sin[c + d*x] + 3*a^2*C*Sin[c + d*x]))/3 + (2*a^2*B*Sec[c + d
*x]*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.155, size = 3204, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
)
```

```
[Out] -1/3/d*(-2*a^3*B+6*C*cos(d*x+c)^3*a^2*b-6*C*cos(d*x+c)^2*a^2*b-3*C*cos(d*x+
c)^2*a*b^2+14*B*cos(d*x+c)^2*a^2*b-14*B*cos(d*x+c)^2*a*b^2-16*B*cos(d*x+c)*
a^2*b+14*B*cos(d*x+c)^3*a*b^2+3*C*cos(d*x+c)^3*a*b^2+2*B*cos(d*x+c)^3*a^2*b
+2*B*cos(d*x+c)^2*a^3-14*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+1
```

8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-18*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+30*C*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-14*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-14*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+14*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+3*C*cos(d*x+c)^4*b^3-3*C*cos(d*x+c)^3*b^3+6*C*cos(d*x+c)^2*a^3-6*C*cos(d*x+c)*a^3+6*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-6*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-14*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)

```
/(a+b))^(1/2))*a*b^2+6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3+2*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+12*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*b^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*b^3-6*B*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+12*B*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(7/2), x)
```

3.916 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=493

$$\frac{2\sqrt{a+b}\left(a^2b(17B-35C)+a^3(-(9B-5C))-ab^2(23B-45C)+15b^3B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{15ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B + 23*b^2*B + 35*a*b*C)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a*d) + (2*Sqrt[a + b]*(15*b^3*B - a*b^2*(23*B - 45*C) +
a^2*b*(17*B - 35*C) - a^3*(9*B - 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a*d) - (2*b^2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d + (2*a*(8*b*B + 5*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Co
s[c + d*x]^(3/2)) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```

Rubi [A] time = 1.36184, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 2989, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2b(17B-35C)+a^3(-(9B-5C))-ab^2(23B-45C)+15b^3B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B + 23*b^2*B + 35*a*b*C)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a*d) + (2*Sqrt[a + b]*(15*b^3*B - a*b^2*(23*B - 45*C) +
a^2*b*(17*B - 35*C) - a^3*(9*B - 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
```

```
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a*d) - (2*b^2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d + (2*a*(8*b*B + 5*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Co
s[c + d*x]^(3/2)) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\sqrt{a+b\cos}}{}$$

$$= \frac{2a(8bB+5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aB(a}{}$$

$$= \frac{2a(8bB+5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aB(a}{}$$

$$= -\frac{2b^2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2B+23b^2B+35abC\right)\cot(c+dx)E\left(\right.}{1}$$

Mathematica [C] time = 6.53903, size = 1319, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] ((4*a*(-8*a^2*b*B + 8*b^3*B - 5*a^3*C - 10*a*b^2*C)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) + 4*a*(9*a^3*B + 23*a*b^2*B + 35*a^2*b*C - 15*b^3*C)*((Sqrt[((a + b)*C
```

```
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(9*a^2*b*B + 23*b^3*B +
 35*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(15*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c
+ d*x]^2*(11*a*b*B*Sin[c + d*x] + 5*a^2*C*Sin[c + d*x]))/15 + (2*Sec[c + d*
x]*(9*a^2*B*Sin[c + d*x] + 23*b^2*B*Sin[c + d*x] + 35*a*b*C*Sin[c + d*x]))/
15 + (2*a^2*B*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.18, size = 3274, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
)
```

```
[Out] -2/15/d*(-3*a^3*B+35*C*cos(d*x+c)^4*a*b^2+35*C*cos(d*x+c)^3*a^2*b-40*C*cos(
d*x+c)^2*a^2*b+5*C*cos(d*x+c)^4*a^2*b-34*B*cos(d*x+c)^2*a*b^2-14*B*cos(d*x+
c)*a^2*b+23*B*cos(d*x+c)^3*a*b^2-35*C*cos(d*x+c)^3*a*b^2+5*B*cos(d*x+c)^3*a
^2*b+9*B*cos(d*x+c)^4*a^2*b+11*B*cos(d*x+c)^4*a*b^2-23*B*cos(d*x+c)^3*b^3+9
*B*cos(d*x+c)^3*a^3+23*B*cos(d*x+c)^4*b^3-6*B*cos(d*x+c)^2*a^3-23*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
```

EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+45*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+17*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+23*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-9*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-23*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+17*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+23*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+45*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*C*cos(d*x+c)^3*a^3-5*C*cos(d*x+c)*a^3+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+35*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-35*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+35*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-35*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-9*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-23*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip

```
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^3*a^3-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^2*a^3-23*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+9*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^
3-35*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2+30*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-15*C*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^3+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^3*b^3-15*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+30*C*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*b^3+15*B*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(9/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(9/2), x)
```

3.917 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=434

$$\frac{2\left(25a^2B+77abC+45b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25B-63C)-8ab(15B-7C)+15b^2(B\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*b*B + 15*b^3*B + 63*a^3*C + 161*a*b^2*C)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(a - b)*Sqrt[a + b]*(a^2*
(25*B - 63*C) + 15*b^2*(B - 7*C) - 8*a*b*(15*B - 7*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a*d) + (2*a*(10*b*B + 7*a*C)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*B + 45*b^2*B + 77*a*b*C
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*
B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.45233, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {3029, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2B+77abC+45b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25B-63C)-8ab(15B-7C)+15b^2(B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*b*B + 15*b^3*B + 63*a^3*C + 161*a*b^2*C)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(a - b)*Sqrt[a + b]*(a^2*
(25*B - 63*C) + 15*b^2*(B - 7*C) - 8*a*b*(15*B - 7*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
```

```
x]))/(a - b)])/(105*a*d) + (2*a*(10*b*B + 7*a*C)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*B + 45*b^2*B + 77*a*b*C
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*
B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+b\cos}}{}$$

$$= \frac{2a(10bB+7aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aB(a}{}$$

$$= \frac{2a(10bB+7aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a\right.}{}$$

$$= \frac{2a(10bB+7aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(145a^2bB+15b^3B+63a^3C+161ab^2C\right)}{}$$

Mathematica [C] time = 6.59775, size = 1409, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] ((-4*a*(25*a^4*B - 10*a^2*b^2*B - 15*b^4*B + 56*a^3*b*C - 56*a*b^3*C)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - 4*a*(-145*a^3*b*B - 15*a*b^3*B - 63*a^4*C - 161*a^
2*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
```

```
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
 2*(-145*a^2*b^2*B - 15*b^4*B - 63*a^3*b*C - 161*a*b^3*C)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a*d) + (Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(15*a*b*B*Sin[c +
d*x] + 7*a^2*C*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*B*Sin[c + d*x]
 + 45*b^2*B*Sin[c + d*x] + 77*a*b*C*Sin[c + d*x]))/105 + (2*Sec[c + d*x]*(1
45*a^2*b*B*Sin[c + d*x] + 15*b^3*B*Sin[c + d*x] + 63*a^3*C*Sin[c + d*x] + 1
61*a*b^2*C*Sin[c + d*x]))/(105*a) + (2*a^2*B*Sec[c + d*x]^3*Tan[c + d*x])/7
))/d
```

Maple [B] time = 0.26, size = 3628, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),
x)
```

```
[Out] 2/105/d*(63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-119*C*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3*b+15*a^4*B+238*C*cos(d*x+c)^3*a^2*b^2-63*C*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
```

x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
-63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^4+145*B*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-105*C*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*
b^3+63*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b+161*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-119*C*cos(d*x+c)
^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b-161*C*cos(d*x+c)^4*a^2*b^2-63*C*cos(d*x+c)^5*a^3*b-77*C*cos(d*x+
c)^5*a^2*b^2-35*C*cos(d*x+c)^4*a^3*b-161*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-161*C*sin
(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^2*b^2+21*C*cos(d*x+c)*a^4-63*C*cos(d*x+c)^4*a^4-161*C*cos(d*x
+c)^5*a*b^3+42*C*cos(d*x+c)^3*a^4-15*B*cos(d*x+c)^5*b^4+15*B*cos(d*x+c)^4*b
^4-25*B*cos(d*x+c)^4*a^4+10*B*cos(d*x+c)^2*a^4+161*C*cos(d*x+c)^4*a*b^3-145
*B*cos(d*x+c)^4*a^3*b+55*B*cos(d*x+c)^4*a^2*b^2-15*B*cos(d*x+c)^4*a*b^3+110
*B*cos(d*x+c)^3*a^3*b+60*B*cos(d*x+c)^3*a*b^3+90*B*cos(d*x+c)^2*a^2*b^2+60*
B*cos(d*x+c)*a^3*b-25*B*cos(d*x+c)^5*a^3*b-145*B*cos(d*x+c)^5*a^2*b^2-45*B*
cos(d*x+c)^5*a*b^3+98*C*cos(d*x+c)^2*a^3*b+15*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4-25*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4
*sin(d*x+c)*a^4+63*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^4-25*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*si
n(d*x+c)*a^4+63*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4+145*B*cos(d*x+c)^4*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+15*B*cos(
d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/

```
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a*b^3-145*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-135*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2-15
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^4*sin(d*x+c)*a*b^3+161*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3+145*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3
*b+145*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*a^2*b^2+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-145*B*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b-135*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+161*C*cos(
d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a*b^3-105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3)/(a+b*cos(d*x+c))^(1/2)/a/sin(d
*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\cos{(dx+c)}^3+Ba^2+\left(2\,Cab+Bb^2\right)\cos{(dx+c)}^2+\left(Ca^2+2\,Bab\right)\cos{(dx+c)}\right)\sqrt{b\cos{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}\right)(b\cos{(dx+c)}+a)^{\frac{5}{2}}}{\cos{(dx+c)}^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(11/2), x)
```

3.918 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(163a^2bB+75a^3C+135ab^2C+5b^3B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2B+135abC+75b^2B\right)\sin(c+dx)\sqrt{a+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*B + 279*a^2*b^2*B - 10*b^4*B + 435*a^3*b*C
+ 45*a*b^3*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*S
qrt[a + b]*(10*b^3*B - 6*a^2*b*(19*B - 60*C) + 3*a^3*(49*B - 25*C) + 15*a*b
^2*(11*B - 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a*(4*b
*B + 3*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)
) + (2*(49*a^2*B + 75*b^2*B + 135*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(163*a^2*b*B + 5*b^3*B + 75*a^3*C + 13
5*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/
2)) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/
2))
```

Rubi [A] time = 1.95532, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {3029, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(163a^2bB+75a^3C+135ab^2C+5b^3B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2B+135abC+75b^2B\right)\sin(c+dx)\sqrt{a+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(13/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*B + 279*a^2*b^2*B - 10*b^4*B + 435*a^3*b*C
+ 45*a*b^3*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*S
```

```
qrt[a + b]*(10*b^3*B - 6*a^2*b*(19*B - 60*C) + 3*a^3*(49*B - 25*C) + 15*a*b
^2*(11*B - 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a*(4*b
*B + 3*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)
) + (2*(49*a^2*B + 75*b^2*B + 135*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(163*a^2*b*B + 5*b^3*B + 75*a^3*C + 13
5*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/
2)) + (2*a*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/
2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\cos(}}{}$$

$$= \frac{2a(4bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aB(a+}{}$$

$$= \frac{2a(4bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2B\right.}{}$$

$$= \frac{2a(4bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2B\right.}{}$$

$$= \frac{2a(4bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2B\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4B+279a^2b^2B-10b^4B+435a^3bC\right.}{}$$

Mathematica [C] time = 6.71798, size = 1517, normalized size = 2.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] -((-4*a*(-114*a^4*b*B + 124*a^2*b^3*B - 10*b^5*B - 75*a^5*C + 30*a^3*b^2*C
+ 45*a*b^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
```

```
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*B + 279*a^3*b^2*B
- 10*a*b^4*B + 435*a^4*b*C + 45*a^2*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b*B + 279*a^2*b^3*B - 10*b^5*
B + 435*a^3*b^2*C + 45*a*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]]))))/(315*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]^4*(19*a*b*B*Sin[c + d*x] + 9*a^2*C*Sin[c + d
*x]))/63 + (2*Sec[c + d*x]^3*(49*a^2*B*Sin[c + d*x] + 75*b^2*B*Sin[c + d*x]
 + 135*a*b*C*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*b*B*Sin[c + d*
x] + 5*b^3*B*Sin[c + d*x] + 75*a^3*C*Sin[c + d*x] + 135*a*b^2*C*Sin[c + d*x
]))/(315*a) + (2*Sec[c + d*x]*(147*a^4*B*Sin[c + d*x] + 279*a^2*b^2*B*Sin[c
 + d*x] - 10*b^4*B*Sin[c + d*x] + 435*a^3*b*C*Sin[c + d*x] + 45*a*b^3*C*Sin
[c + d*x]))/(315*a^2) + (2*a^2*B*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.37, size = 4392, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
```

x)

[Out] -2/315/d*(-35*a^5*B-80*B*cos(d*x+c)^3*a^2*b^3-45*C*cos(d*x+c)^5*a*b^4+147*B
*cos(d*x+c)^6*a^4*b+163*B*cos(d*x+c)^6*a^3*b^2+279*B*cos(d*x+c)^6*a^2*b^3+5
*B*cos(d*x+c)^6*a*b^4+65*B*cos(d*x+c)^5*a^4*b+279*B*cos(d*x+c)^5*a^3*b^2-19
9*B*cos(d*x+c)^5*a^2*b^3-272*B*cos(d*x+c)^4*a^3*b^2+5*B*cos(d*x+c)^4*a*b^4-
82*B*cos(d*x+c)^3*a^4*b+75*C*cos(d*x+c)^6*a^4*b+435*C*cos(d*x+c)^6*a^3*b^2+
135*C*cos(d*x+c)^6*a^2*b^3-165*C*cos(d*x+c)^5*a^3*b^2+45*C*cos(d*x+c)^5*a^2
*b^3-14*B*cos(d*x+c)^2*a^5-147*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+75*C*cos(d*x+c)^4*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
^5-30*C*cos(d*x+c)^3*a^5-10*B*cos(d*x+c)^5*a*b^4-170*B*cos(d*x+c)^2*a^3*b^2
-130*B*cos(d*x+c)*a^4*b-279*B*cos(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2-279*B*cos(d*x+c)^5*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^2*b^3+10*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+261*B*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+279*B*cos(d*x+c
)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*a^3*b^2+155*B*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3-10*B*cos(d*x+c)^5*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^4-45*C*c
os(d*x+c)^5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a*b^4-147*B*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b-279*B*cos(d*x+c)^4*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2
-279*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^2*b^3+10*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+261*B*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))

^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^4*b+279*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2+155*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-10*B*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*a*b^4-45*C*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^4-45*C*cos(d*x+c)*a^5+75*C*cos(d*x
+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a^5+10*B*cos(d*x+c)^5*b^5-98*B*cos(d*x+c)^4*a^5+147*B*cos(d*x+c)^5*
a^5+75*C*cos(d*x+c)^5*a^5+435*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+405*C*cos(d*x+c)^5*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
^3*b^2-435*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b-435*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2-45*C*cos(d*x
+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a^2*b^3+435*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+405*C*cos(d*x+c)^4*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-435
*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^4*b-435*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-45*C*cos(d*x+c)^4*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^3-180*C*cos(d*x+c)^4*a^2*b^3-270*C*cos(d*x+c)^3*a^3*b^2-330*C*cos(d*x+c
)^4*a^4*b-180*C*cos(d*x+c)^2*a^4*b+45*C*cos(d*x+c)^6*a*b^4-10*B*cos(d*x+c)^
6*b^5+435*C*cos(d*x+c)^5*a^4*b+45*C*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3+45*C*sin(d*x+c)*
cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1

```
/2)*a^2*b^3-147*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5+10*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+147*B*cos(d*x+
c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^5-147*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5+10*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+147*B*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^5)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(13/2), x)
```

3.919 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=622

$$\frac{2\left(1025a^2b^2B+1793a^3bC+675a^4B+55ab^3C-20b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3465a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(1145a^2bB+539a^3C+825a\right.}{346}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*b*B + 255*a^2*b^3*B + 40*b^5*B + 1617*a^5*
C + 3069*a^3*b^2*C - 110*a*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*
a^4*d) + (2*(a - b)*Sqrt[a + b]*(40*b^4*B + 3*a^4*(225*B - 539*C) - 6*a^3*b
*(505*B - 209*C) + 15*a^2*b^2*(19*B - 121*C) + 10*a*b^3*(3*B - 11*C))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d) + (2*a*(14*b*B + 11*a*C)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2*B +
113*b^2*B + 209*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c
+ d*x]^(7/2)) + (2*(1145*a^2*b*B + 15*b^3*B + 539*a^3*C + 825*a*b^2*C)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*a*d*Cos[c + d*x]^(5/2)) + (2*(675*
a^4*B + 1025*a^2*b^2*B - 20*b^4*B + 1793*a^3*b*C + 55*a*b^3*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Cos[c + d*x]^(3/2)) + (2*a*B*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 2.75315, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {3029, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(1025a^2b^2B+1793a^3bC+675a^4B+55ab^3C-20b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3465a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(1145a^2bB+539a^3C+825a\right.}{346}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(15/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*b*B + 255*a^2*b^3*B + 40*b^5*B + 1617*a^5*
C + 3069*a^3*b^2*C - 110*a*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
```

```
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*
a^4*d) + (2*(a - b)*Sqrt[a + b]*(40*b^4*B + 3*a^4*(225*B - 539*C) - 6*a^3*b
*(505*B - 209*C) + 15*a^2*b^2*(19*B - 121*C) + 10*a*b^3*(3*B - 11*C))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d) + (2*a*(14*b*B + 11*a*C)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2*B +
113*b^2*B + 209*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c
+ d*x]^(7/2)) + (2*(1145*a^2*b*B + 15*b^3*B + 539*a^3*C + 825*a*b^2*C)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*a*d*Cos[c + d*x]^(5/2)) + (2*(675*
a^4*B + 1025*a^2*b^2*B - 20*b^4*B + 1793*a^3*b*C + 55*a*b^3*C)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Cos[c + d*x]^(3/2)) + (2*a*B*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{5/2}(B+C\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{2a(14bB+11aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2aB(}{}$$

$$= \frac{2a(14bB+11aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81\right.}{}$$

$$= \frac{2a(14bB+11aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81\right.}{}$$

$$= \frac{2a(14bB+11aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81\right.}{}$$

$$= \frac{2a(14bB+11aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3705a^4bB+255a^2b^3B+40b^5B+1617a\right.}{}$$

Mathematica [C] time = 6.84064, size = 1640, normalized size = 2.64

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2),x]

[Out] ((-4*a*(675*a^6*B - 390*a^4*b^2*B - 245*a^2*b^4*B - 40*b^6*B + 1254*a^5*b*C - 1364*a^3*b^3*C + 110*a*b^5*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3705*a^5*b*B - 255*a^3*b^3*B - 40*a*b^5*B - 1617*a^6*C - 3069*a^4*b^2*C + 110*a^2*b^4*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-3705*a^4*b^2*B - 255*a^2*b^4*B - 40*b^6*B - 1617*a^5*b*C - 3069*a^3*b^3*C + 110*a*b^5*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(3465*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^5*(23*a*b*B*Sin[c + d*x] + 11*a^2*C*Sin[c + d*x]))/99 + (2*Sec[c + d*x]^4*(81*a^2*B*Sin[c + d*x] + 113*b^2*B*Sin[c + d*x] + 209*a*b*C*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(1145*a^2*b*B*Sin[c + d*x] + 15*b^3*B*Sin[c + d*x] + 539*a^3*C*Sin[c + d*x] + 825*a*b^2*C*Sin[c + d*x]))/(3465*a) + (2*Sec[c + d*x]^2*(675*a^4*B*Sin[c + d*x] + 1025*a^2*b^2*B*Sin[c + d*x] - 20*b^4*B*Sin[c + d*x] + 1793*a^3*b*C*Sin[c + d*x] + 55*a*b^3*C*Sin[c + d*x]))/(3465*a^2) + (2*Sec[c + d*x]*(3705*a^4*b*B*Sin[c + d*x] + 255*a^2*b^3*B*Sin[c + d*x] + 40*b^5*B*Sin[c + d*x] + 1617*a^5*C*Sin[c + d*x] + 3069*a^3*b^2*C*Sin[c + d*x] - 110*a*b^4*C*Sin[c + d*x]))/(3465*a^3) + (2*a^2*B*Sec[c + d*x]^5*Tan[c + d*x])/11))/d

Maple [B] time = 0.592, size = 5373, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),
x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{3}+Ba^{2}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3 + B*a^2 + (2*C*a*b + B*b^2)*cos(d*x + c)^2 +
 (C*a^2 + 2*B*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(13/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(5/2)/co
s(d*x + c)^(15/2), x)
```

3.920 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=571

$$-\frac{\left(-15a^2C+18abB-16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(-15a^2C+18abB+10abC-12b^2B-16b^2C\right)\cot$$

```
[Out] ((a - b)*Sqrt[a + b]*(18*a*b*B - 15*a^2*C - 16*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b^3*d) - (Sqrt[a + b]*(18*a*b*B - 12*b^2*B - 15*a^2*C +
 10*a*b*C - 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^3*d) - (Sqrt
[a + b]*(6*a^2*b*B + 8*b^3*B - 5*a^3*C - 4*a*b^2*C)*Cot[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(8*b^4*d) - ((18*a*b*B - 15*a^2*C - 16*b^2*C)*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*b^3*d*Sqrt[Cos[c + d*x]]) + ((6*b*B -
 5*a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*b^2*d
) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 1.72722, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(-15a^2C+18abB-16b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(-15a^2C+18abB+10abC-12b^2B-16b^2C\right)\cot$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos
[c + d*x]],x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(18*a*b*B - 15*a^2*C - 16*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b^3*d) - (Sqrt[a + b]*(18*a*b*B - 12*b^2*B - 15*a^2*C +
```

10*a*b*C - 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^3*d) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - 5*a^3*C - 4*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^4*d) - ((18*a*b*B - 15*a^2*C - 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*b^3*d*Sqrt[Cos[c + d*x]]) + ((6*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*b^2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rule 3029

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a*b*B + a^2*C, 0]

Rule 2990

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3049

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{(6bB-5aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12b^2d} + \frac{C\cos\cdots}{\cdots}$$

$$= -\frac{\left(18abB-15a^2C-16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^3d\sqrt{\cos(c+dx)}} + \frac{(6b\cdots}{\cdots}$$

$$= -\frac{\left(18abB-15a^2C-16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^3d\sqrt{\cos(c+dx)}} + \frac{(6b\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+b}\left(6a^2bB+8b^3B-5a^3C-4ab^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{\cdots}\right.}{8b^4d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(18abB-15a^2C-16b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a+\cdots}}\right.\right.}{24ab^3d}$$

Mathematica [C] time = 6.40931, size = 1229, normalized size = 2.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a +
 b*Cos[c + d*x]],x]
```

```
[Out] ((-4*a*(-6*a*b*B + 5*a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(2
4*b^2*B + 4*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])) + 2*(-18*a*b*B + 15*a^2*C + 16*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]],
 (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sq
rt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*b^2*d) + (Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]*(((6*b*B - 5*a*C)*Sin[c + d*x])/(12*b^2) + (C*S
in[2*(c + d*x)])/(6*b)))/d
```

Maple [B] time = 0.238, size = 2949, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(-2*C*cos(d*x+c)^4*a*b^2+5*C*cos(d*x+c)^3*a^
2*b-15*C*cos(d*x+c)^2*a^2*b+18*C*cos(d*x+c)^2*a*b^2+10*C*cos(d*x+c)*a^2*b-1
6*C*cos(d*x+c)*a*b^2-18*B*cos(d*x+c)^2*a^2*b+18*B*cos(d*x+c)^2*a*b^2+18*B*c
os(d*x+c)*a^2*b-12*B*cos(d*x+c)*a*b^2-6*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c
)^4*b^3+8*C*cos(d*x+c)^5*b^3-18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-24*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a*b^2-10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*a^2*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+15*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*
b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-24*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-10*C*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+15*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*a^3-12*B*cos(d*x+c)^2*b^3-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+15*C*sin(d*x+c)*(cos(d*x
```

```
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*
C*cos(d*x+c)^3*b^3+15*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-15*C*cos(d*x
+c)*a^3-18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2+48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-18*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-18*B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2+36*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)
/b^3/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/sqrt(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/sqrt(b*cos
(d*x + c) + a), x)
```

3.921 $\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=479

$$\frac{\sqrt{a+b}\left(-3a^2C+4abB-4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+\frac{(4bB}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B - 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) - (Sqrt[a + b]*(3*a*C - 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) + (Sqrt[a + b]*(4*a*b*B - 3*a^2*C - 4*b^2*C)*Cot[c + d*x
]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*b*B - 3*a*C)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 1.1956, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(-3a^2C+4abB-4b^2C\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+\frac{(4bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos
[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B - 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) - (Sqrt[a + b]*(3*a*C - 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) + (Sqrt[a + b]*(4*a*b*B - 3*a^2*C - 4*b^2*C)*Cot[c + d*x
```

```
]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*b*B - 3*a*C)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
```

```
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd}+\frac{\int \frac{\frac{aC}{2}+bC\cos(c+dx)}{\sqrt{\cos(c+d}}}{}$$

$$=\frac{(4bB-3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

$$=\frac{(4bB-3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

$$=\frac{\sqrt{a+b}\left(4abB-3a^2C-4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4b^3d}$$

$$=-\frac{(a-b)\sqrt{a+b}(4bB-3aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{4ab^2d}$$

Mathematica [C] time = 12.6034, size = 1175, normalized size = 2.45

$$-\frac{4a(4bB-aC)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right.}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd}+\frac{}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a +
 b*Cos[c + d*x]],x]
```

```
[Out] (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + ((-4
*a*(4*b*B - a*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*C*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*b*B - 3*a*C)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(8*b*d)
```

Maple [B] time = 0.148, size = 1871, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-2*C*cos(d*x+c)*a*b+4*B*cos(d*x+c)^2*a*b-4*B
*cos(d*x+c)*a*b-C*cos(d*x+c)^3*a*b+3*C*cos(d*x+c)^2*a*b+4*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*b^2*C*c
```

```
os(d*x+c)^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-4*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^2-4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^
2-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^3*b^2-4*B*cos(d*x
+c)^2*b^2-3*C*cos(d*x+c)^2*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-3*C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*
B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)^4*b^2+3*C*cos(d*x+c)*a^2+4*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a*b)/sin(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos
(d*x + c) + a), x)
```

3.922 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=391

$$-\frac{\sqrt{a+b}(2bB-aC)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{C\sin(c+dx)\sqrt{a\ \cdots}}{bd\sqrt{\cos(\cdots}}$$

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) - (Sqrt[a + b]*(2*b*B -
 a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (C*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.21342, antiderivative size = 427, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 3003, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}(2bB-aC)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{C\sin(c+dx)\sqrt{\mathrm{co}\cdots}}{d\sqrt{a+b\cos(c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) - (Sqrt[a + b]*(2*b*B -
 a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*C*Sin[c +
```

```
d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (C*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
```

```
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+C\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{1}{2}\int \frac{aC+2aB\cos(c+dx)+(2bB-aC)\cos^2(c+}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{abC+(2abB-a(2bB-aC))\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{2b} + \frac{(2bB-aC)\int}{}$$

$$= -\frac{\sqrt{a+b}(2bB-aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+b}}}{b^2d}$$

$$= -\frac{\sqrt{a+b}(2bB-aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+b}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{abd}$$

Mathematica [C] time = 18.0589, size = 4017, normalized size = 10.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]),x]
```

```
[Out] ((1 + Cos[c + d*x])^(3/2)*((B*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]
+ (C*Cos[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]])*Sec[(c + d*x)/2]^2*((2*I
)*(a - b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] +
 (4*I)*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))] - (8*I)*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]
], -((a + b)/(a - b))] + (4*I)*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*
Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)
/2] - b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(
```

c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[a + b*Cos[c + d*x]]*(((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*Sqrt[(a - b)/(a + b)]*(a + b*Cos[c + d*x])^(3/2)) - (3*Sqrt[1 + Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*((2*I)*(a - b)*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*b*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*((3*b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Cos[(3*(c + d*x))/2]*Sec[(c + d*x)/2])/2 + a*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + C

```
os[c + d*x])]*Sec[(c + d*x)/2]^2 - (b*Sqrt[(a - b)/(a + b)]*C*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2)/2 + (I*(a - b)*C*EllipticE[I*A
rcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + ((2*I)*(b*B - a*C)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
 ((4*I)*b*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
)^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((2*I)*a*C
*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/
2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
+ ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*Sqrt[(a - b)/(a +
b)]*C*Sec[(c + d*x)/2]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
Sin[c + d*x]/(1 + Cos[c + d*x]))*Sin[(3*(c + d*x))/2])/(2*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]) + (a*Sqrt[(a - b)/(a + b)]*C*((Cos[c + d*x]*Sin[c + d
*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/
2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*Sqrt[(a - b)/(a + b)]*C*((Co
s[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d
*x]))*Tan[(c + d*x)/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (b*Sqrt
[(a - b)/(a + b)]*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*
Sin[(3*(c + d*x))/2]*Tan[(c + d*x)/2])/2 - (2*Sqrt[(a - b)/(a + b)]*(b*B -
a*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]) - ((a - b)*Sqrt[(a - b)/(a + b)]*C*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2])/S
qrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (4*b*Sqrt[(a - b)/(a + b)]*
B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^
2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]) - (2*a*Sqrt[(a - b)/(a + b)]*C*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)*Tan[(c + d*x)/
2]^2)/(a + b)])))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]]))
```

Maple [B] time = 0.183, size = 1002, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -1/d/(a+b*cos(d*x+c))^(1/2)*(4*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*b-2*B*cos(d*x+c)*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+C*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+4*B*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b-2*B*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b+C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)^3*b+C*cos(d*x+c)^2*a-C*cos(d*x+c)^2*b-C*
cos(d*x+c)*a)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B+C\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos(dx+c)^2+B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

3.923 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=228

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}$$

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*
C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rubi [A] time = 0.392587, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3029, 3006, 2809, 2816}

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[
c + d*x]]),x]
```


```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*
C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3006

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= B\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + C\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}B\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c\cdots}{a-b}}}{ad}$$

Mathematica [A] time = 1.46398, size = 146, normalized size = 0.64

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left((B-C)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2C\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right.\right.}{d\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a +
b*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*((B - C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
 2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])
```

Maple [A] time = 0.123, size = 197, normalized size = 0.9

$$2\,\frac{(\sin(dx+c))^2}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))\sqrt{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\left(B\mathrm{EllipticF}\left(\frac{-1}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(B*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))+2*C*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2)))/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B+C\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos(dx+c)^2+B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

3.924 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=230

$$\frac{2B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(B-C)\cot(c+dx)}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(B - C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.438111, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3029, 2998, 2816, 2994}

$$\frac{2B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(B-C)\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[
c + d*x]]),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(B - C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=B\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx+(-B+C)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$=\frac{2(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{a^2d}$$

Mathematica [A] time = 13.2316, size = 299, normalized size = 1.3

$$\frac{2\left(B\sin(c+dx)(a+b\cos(c+dx))-\frac{2\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\left(-2a(B+C)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-}{a+}\right.\right.}{}\right.}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{cc}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a +
b*Cos[c + d*x]]),x]
```

```
[Out] (2*(B*(a + b*Cos[c + d*x])*Sin[c + d*x] - (2*Sqrt[2]*(Cos[(c + d*x)/2]^2)^(
3/2)*(2*(a + b)*B*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(B + C)*Cos[(c + d*x)/2]^2*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + B*Cos[c
 + d*x]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2)))/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.149, size = 935, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+2*C*cos(d*x+c)*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+B*cos(d*
x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a-B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
```

```
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+C*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+B*cos(d*x+c)*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a-B*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*
x+c)^2-B*cos(d*x+c)*a)/a/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)}}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos{(dx+c)}+B)\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}{b\cos{(dx+c)}^3 + a\cos{(dx+c)}^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

3.925 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=290

$$\frac{2\sqrt{a+b}(a(B-3C)+2bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d} - \frac{2(a-b)\sqrt{a+b}(\cdots}{\cdots}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*b*B - 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*b*B + a*(B - 3*C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^2*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos
[c + d*x]^(3/2))
```

Rubi [A] time = 0.629157, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3029, 3000, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a(B-3C)+2bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d} - \frac{2(a-b)\sqrt{a+b}(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[
c + d*x]]),x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*b*B - 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*b*B + a*(B - 3*C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^2*d) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos
[c + d*x]^(3/2))
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B+C\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=\frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\int \frac{\frac{1}{2}(-2bB+3aC)+\frac{1}{2}aB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{3a}$$

$$=\frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{(2bB+a(B-3C))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}}{3a}$$

$$=-\frac{2(a-b)\sqrt{a+b}(2bB-3aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{3a^3d}$$

Mathematica [A] time = 16.4475, size = 416, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(3aC\sin(c+dx)-2bB\sin(c+dx))}{3a^2}+\frac{2B\tan(c+dx)\sec(c+dx)}{3a}\right)}{d}+\frac{8\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a +
b*Cos[c + d*x]]),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(-2*b*B + 3*a*C)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*b*B + a
*(B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + (2*b*B - 3*a*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^(3/
```

```
2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]*(-2*b*B*Sin[c + d*x] + 3*a*C*Sin[c + d*x]))/(3*a^2) + (2*
B*Sec[c + d*x]*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 0.12, size = 1536, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -2/3/d*(-a^2*B-2*B*cos(d*x+c)^2*a*b+B*cos(d*x+c)*a*b+3*C*cos(d*x+c)^3*a*b-3
*C*cos(d*x+c)^2*a*b+B*cos(d*x+c)^3*a*b+B*cos(d*x+c)^2*a^2+2*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a*b-3*C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-3*C*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*a^2+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*B*cos(d*x+c)^3*b^2+2*B*cos(d*x+c)
^2*b^2+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*C*c
os(d*x+c)^2*a^2-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*C*cos(d*x+c)*
a^2-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
```

```
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b-2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2)/(a+b*
cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^4 + a\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(7/2)), x)
```

3.926 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=363

$$-\frac{2\sqrt{a+b}\left(a^2(9B-5C)-2ab(B+5C)+8b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B + 8*b^2*B - 10*a*b*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*b^2*B + a^2*(9*B - 5*C) - 2*a*
b*(B + 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*B*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(4*b*B - 5*a*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.964515, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 3000, 3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(9B-5C)-2ab(B+5C)+8b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + b*Cos[
c + d*x]]),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*B + 8*b^2*B - 10*a*b*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*b^2*B + a^2*(9*B - 5*C) - 2*a*
b*(B + 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*B*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(4*b*B - 5*a*
```

```
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B+C\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=\frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2\int \frac{\frac{1}{2}(-4bB+5aC)+\frac{3}{2}aB\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}$$

$$=\frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{2(4bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{2(4bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2(a-b)\sqrt{a+b}\left(9a^2B+8b^2B-10abC\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|-}{15a^4d}$$

Mathematica [C] time = 6.39929, size = 1319, normalized size = 3.63

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a +
b*Cos[c + d*x]]),x]
```

```
[Out] -((-4*a*(7*a^2*b*B + 8*b^3*B - 5*a^3*C - 10*a*b^2*C)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - 4*a*(9*a^3*B + 8*a*b^2*B - 10*a^2*b*C)*((Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
```

```
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*b*B + 8*b^3*B - 10*a*b^2*C)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
15*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2
*(-4*b*B*Sin[c + d*x] + 5*a*C*Sin[c + d*x]))/(15*a^2) + (2*Sec[c + d*x]*(9*
a^2*B*Sin[c + d*x] + 8*b^2*B*Sin[c + d*x] - 10*a*b*C*Sin[c + d*x]))/(15*a^3
) + (2*B*Sec[c + d*x]^2*Tan[c + d*x])/(5*a)))/d
```

Maple [B] time = 0.163, size = 2480, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x
)
```

```
[Out] -2/15/d*(-3*a^3*B-10*C*cos(d*x+c)^4*a*b^2-10*C*cos(d*x+c)^3*a^2*b+5*C*cos(d
*x+c)^2*a^2*b+5*C*cos(d*x+c)^4*a^2*b-4*B*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)*a^
2*b+8*B*cos(d*x+c)^3*a*b^2+10*C*cos(d*x+c)^3*a*b^2-10*B*cos(d*x+c)^3*a^2*b+
9*B*cos(d*x+c)^4*a^2*b-4*B*cos(d*x+c)^4*a*b^2-8*B*cos(d*x+c)^3*b^3+9*B*cos(
d*x+c)^3*a^3+8*B*cos(d*x+c)^4*b^3-6*B*cos(d*x+c)^2*a^3-8*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a
*b^2+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^3*a^2*b+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a
```

^2*b-9*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-8*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+10*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*C*cos(d*x+c)^3*a^3-5*C*cos(d*x+c)*a^3+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-9*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3+9*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3+10*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^(5/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^5 + a\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(sqrt(b*cos(d*x + c) + a)*cos
(d*x + c)^(9/2)), x)
```

3.927 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=620

$$\frac{2a(bB-aC)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(-5a^2C+4abB+b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2b^2d\left(a^2-b^2\right)}+\frac{\left(12a^2bB\right.}{}$$

```
[Out] -((12*a^2*b*B - 4*b^3*B - 15*a^3*C + 7*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(4*a*b^3*Sqrt[a + b]*d) + ((a*b*(12*B - 5*C) - 15*a^2*C + 2*b^2*(2*
B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*Sqrt[a + b]*d) + (Sqrt[a
+ b]*(12*a*b*B - 15*a^2*C - 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^4*d) + (2*a*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(
a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*b*B - 4*b^3*B - 15*a^3*C
+ 7*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sq
rt[Cos[c + d*x]]) - ((4*a*b*B - 5*a^2*C + b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.89073, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2989, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2a(bB-aC)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(-5a^2C+4abB+b^2C\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2b^2d\left(a^2-b^2\right)}+\frac{\left(12a^2bB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(3/2),x]
```



```
[Out] -((12*a^2*b*B - 4*b^3*B - 15*a^3*C + 7*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
```

```
- b)])/(4*a*b^3*Sqrt[a + b]*d) + ((a*b*(12*B - 5*C) - 15*a^2*C + 2*b^2*(2*
B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*Sqrt[a + b]*d) + (Sqrt[a
+ b]*(12*a*b*B - 15*a^2*C - 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^4*d) + (2*a*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(
a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*b*B - 4*b^3*B - 15*a^3*C
+ 7*a*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sq
rt[Cos[c + d*x]]) - ((4*a*b*B - 5*a^2*C + b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(bB-aC)\right.}{}}{}$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(4abB-5a^2C+b^2C\right)\sqrt{}}{}$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(12a^2bB-4b^3B-15a^3\right.}{4b^3}$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(12a^2bB-4b^3B-15a^3\right.}{4b^3}$$

$$= \frac{\sqrt{a+b}\left(12abB-15a^2C-4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{4b^4d}$$

$$= -\frac{\left(12a^2bB-4b^3B-15a^3C+7ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\sqrt{}}\right.\right.}{4ab^3\sqrt{a+b}d}$$

Mathematica [C] time = 6.5628, size = 1297, normalized size = 2.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((C*Sin[c + d*x])/(2*b^2) - (2
*(-(a^2*b*B*Sin[c + d*x]) + a^3*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(a + b*C
os[c + d*x]))))/d - ((-4*a*(-4*a^2*b*B + 4*b^3*B + 5*a^3*C - 5*a*b^2*C)*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
```

```
qrt[a + b*Cos[c + d*x]]) - 4*a*(-8*a*b^2*B + 4*a^2*b*C + 4*b^3*C)*((Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-12*a^2*b*B +
4*b^3*B + 15*a^3*C - 7*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]))))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(8*(a - b)*b^2*(a + b)*d)
```

Maple [B] time = 0.224, size = 4001, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x
)
```

```
[Out] -1/4/d*(10*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-2*C*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^3-22*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
```

(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+7*C*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^2-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^4-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b+24*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-8*B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^3+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-4*B*cos(d*x+c)^3*b^4
+7*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3+4*B*cos(d*x+c)^2*b^4-15*C*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+10*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^3-22*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+7*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^
2+7*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3+2*C*cos(d*x+c)^4*a^2*b^2-5*C*cos(d*x+c)^3*a^3*b+4*B*cos(d*x+c

)^3*a^2*b^2+12*B*cos(d*x+c)^2*a^3*b-4*B*cos(d*x+c)^2*a*b^3+8*B*cos(d*x+c)*a
^2*b^2+4*B*cos(d*x+c)*a*b^3+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-8*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-15*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^4-2*C*cos(d*x+c)^4*b^4+2*C*cos(d*x+c)^2*b^4+15*C*cos(d*x+c)*a^4-15*C*cos(d
*x+c)^2*a^4-7*C*cos(d*x+c)^2*a*b^3-10*C*cos(d*x+c)*a^3*b-7*C*cos(d*x+c)*a^2
*b^2+2*C*cos(d*x+c)*a*b^3+5*C*cos(d*x+c)^3*a*b^3+4*C*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+30*C*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a^4-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-12*B*cos(d*x+c)^2*a^2*b^2-12*
B*cos(d*x+c)*a^3*b+15*C*cos(d*x+c)^2*a^3*b+5*C*cos(d*x+c)^2*a^2*b^2-4*B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^3*b+24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-8*B*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^2-8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^3+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+12*B*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^2-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b^3/(a^2-b
^2)/cos(d*x+c)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^(3/2), x)
```

3.928 $\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=500

$$-\frac{\left(-3a^2C+2abB+b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{2a(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(-3a^2C+2abB+b^2\right.}{}$$

```
[Out] ((2*a*b*B - 3*a^2*C + b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt
[a + b]*d) - ((2*b*B - 3*a*C - b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*b*B - 3*a*C)*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) + (2*a*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((2*a*b*B - 3*a^2*C + b^2*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]])
```

Rubi [A] time = 1.4119, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(-3a^2C+2abB+b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{2a(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(-3a^2C+2abB+b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] ((2*a*b*B - 3*a^2*C + b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt
[a + b]*d) - ((2*b*B - 3*a*C - b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
```

```
*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*b*B - 3*a*C)*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) + (2*a*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((2*a*b*B - 3*a^2*C + b^2*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$=\frac{2a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{2\int \frac{-\frac{1}{2}a(bB-aC)+\frac{1}{2}b(bB-aC)\,c}{\sqrt{\cos(}}}{}$$

$$=\frac{2a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{\left(2abB-3a^2C+b^2C\right)\sqrt{a}}{b^2\left(a^2-b^2\right)}$$

$$=\frac{2a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{\left(2abB-3a^2C+b^2C\right)\sqrt{a}}{b^2\left(a^2-b^2\right)}$$

$$=-\frac{\sqrt{a+b}(2bB-3aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{b^3d}$$

$$=\frac{\left(2abB-3a^2C+b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{ab^2\sqrt{a+b}d}$$

Mathematica [C] time = 6.37936, size = 1234, normalized size = 2.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(a*b*B*Sin[c + d*x]) + a^2*C*Sin[c + d*x]))/(b*(-a^
2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*C - b^2*C)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-2*b^2*B + 2*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
```

```
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*a*b*B + 3*a^2*C - b^2*C)*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*(a - b)*b
*(a + b)*d)
```

Maple [B] time = 0.142, size = 2885, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x
)
```

```
[Out] -1/d*(C*cos(d*x+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)^2*a*b^2+2*C*
cos(d*x+c)*a^2*b+C*cos(d*x+c)*a*b^2-2*B*cos(d*x+c)^2*a^2*b+2*B*cos(d*x+c)^2
*a*b^2+2*B*cos(d*x+c)*a^2*b-2*B*cos(d*x+c)*a*b^2-2*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
```

icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+6*C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-2
*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-6*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-C*cos(d*x+c)^3*b^3+3*C*cos(
d*x+c)^2*a^3+C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3-2*B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+2
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^

```
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*b^3+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b^2/(a^2-b^2)/cos(d
*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^(3/2), x)
```

3.929 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{2a(bB-aC)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{}}\right.\right.}{b^2d}$$

```
[Out] (-2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) + (2
*(b*B - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) - (2*Sq
rt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (2*a*(
b*B - a*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])
```

Rubi [A] time = 0.725629, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {3029, 2992, 2809, 2794, 2795, 2816, 2994}

$$\frac{2a(bB-aC)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{}}\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^(3/2)),x]
```


```
[Out] (-2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) + (2
*(b*B - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) - (2*Sq
rt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
```

```
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (2*a*(
b*B - a*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{C\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(bB-aC)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{b^2d}$$

$$= -\frac{2(bB-aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{ab\sqrt{a+b}d}$$

Mathematica [C] time = 18.2077, size = 1012, normalized size = 2.43

$$\frac{2\sqrt{\cos(c+dx)}(aC\sin(c+dx)-bB\sin(c+dx))}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\left(\frac{i\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}E\left(i\sinh^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\middle|-\frac{2a}{-a-b}\right)\sec(c+dx)}{b\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{\frac{(a+b\cos(c+dx))\sec(c+dx)}{a+b}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(b*B*Sin[c + d*x]) + a*C*Sin[c + d*x]))/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) - (-4*a*(a*B - b*C)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(b*B - a*C)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
```

```
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/((-a + b)*(a + b)*d)
```

Maple [B] time = 0.154, size = 2013, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x
)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*a*b-B*cos(d*x+c)^2*a*b+B*cos(d*x+c
)*a*b-C*cos(d*x+c)^2*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-b^2*B*cos(d*x+c)-B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*a^2+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-2*C*cos(d*x+c)*sin
(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*b^2+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*cos(d*
x+c)^2*b^2+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*b^2+C*cos(d*x+c)^2*a^2-C*cos(d*x+c)*sin(d*x+c)*(c
```

```
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b-C*cos(d*x+c)*a^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-B*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b)/sin(d*x+c)/b/(a^2-b^2)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*s
qrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)+B\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\cos(c + dx))\sqrt{\cos(c + dx)}}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx + c)^2 + B\cos(dx + c)}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*s
qrt(cos(d*x + c))), x)
```

3.930 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{2(bB-aC)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-aC)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{a^2d\sqrt{a+b}}$$

```
[Out] (2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*
(B + C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*(b*B - a*
C)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rubi [A] time = 0.64027, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3029, 2993, 2998, 2816, 2994}

$$-\frac{2(bB-aC)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-aC)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{a^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] (2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*
(B + C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*(b*B - a*
C)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$$

$$=-\frac{2(bB-aC)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{\int \frac{bB-aC+(aB-bC)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$=-\frac{2(bB-aC)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{(B+C)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{a+b}$$

$$=\frac{2(bB-aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{a^2\sqrt{a+b}d}$$

Mathematica [C] time = 6.35877, size = 1223, normalized size = 4.31

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(-(b^2*B*Sin[c + d*x]) + a*b*C*Sin[c + d*x]))/(a*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*B - b^2*B)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-(a*b*B) + a^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-(b^2*B) + a*b*C)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d
```

```
*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a*(a - b)*(a + b)*d)
```

Maple [B] time = 0.162, size = 1633, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x
)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^2-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-C*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
```

```
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^2*a*b+B*cos(d*x+c)^2*b^2+C*cos(d*x+c)^2*a^
2-C*cos(d*x+c)^2*a*b+B*cos(d*x+c)*a*b-b^2*B*cos(d*x+c)-C*cos(d*x+c)*a^2+C*c
os(d*x+c)*a*b)/(a^2-b^2)/a/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*c
os(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^3 + 2\,ab\cos(dx+c)^2 + a^2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*c
os(d*x + c)^(3/2)), x)
```

3.931 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{2b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2B+abC-2b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\right.\right.}{a^3d\sqrt{a+b}}$$

```
[Out] (2*(a^2*B - 2*b^2*B + a*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
 + b]*d) - (2*(2*b*B + a*(B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d) + (2*b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.744563, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3029, 3000, 2998, 2816, 2994}

$$\frac{2b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2B+abC-2b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\right.\right.}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])^(3/2)),x]
```


```
[Out] (2*(a^2*B - 2*b^2*B + a*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
 + b]*d) - (2*(2*b*B + a*(B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d) + (2*b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$=\frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\int\frac{\frac{1}{2}\left(a^2B-2b^2B+abC\right)-\frac{1}{2}a(bB-aC)\,\mathrm{c}}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+d}}}{a\left(a^2-b^2\right)}$$

$$=\frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{((a-b)(2bB+a(B-C)))\int\frac{}{\sqrt{}}}{a\left(a^2-b\right.}$$

$$=\frac{2\left(a^2B-2b^2B+abC\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{a^3\sqrt{a+b}d}$$

Mathematica [C] time = 6.48916, size = 1281, normalized size = 4.2

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(2*a^2*b*B - 2*b^3*B - a^3*C + a*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(a^3*B - 2*a*b^2*B + a^2*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
```

```
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])) + 2*(a^2*b*B - 2*b^3*B + a*b^2*C)*((I*Cos[(c + d*
x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Co
s[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*(-a + b)*(a
+ b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(b^3*B*Sin[c +
 d*x]) + a*b^2*C*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2
*B*Tan[c + d*x])/a^2))/d
```

Maple [B] time = 0.162, size = 2282, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x
)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(a*b^2*B-a^3*B-C*cos(d*x+c)^2*a^2*b+C*cos(d*x+c
)^2*a*b^2+C*cos(d*x+c)*a^2*b-C*cos(d*x+c)*a*b^2+B*cos(d*x+c)^2*a^2*b+B*cos(
d*x+c)^2*a*b^2-B*cos(d*x+c)*a^2*b-2*B*cos(d*x+c)*a*b^2-B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+B*cos(d*x+c)
*a^3-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-C*sin(d
```

x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*B*cos(d*x+c)*b^3-2*B*cos(d*x+
c)^2*b^3-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*B*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+2*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+2*B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3)/
a^2/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*c
os(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^4 + 2\,ab\cos(dx+c)^3 + a^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*c
os(d*x + c)^(5/2)), x)
```

3.932 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=393

$$\frac{2\left(a^2B+3abC-4b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2bB-3a^3\right.}{}$$

```
[Out] (-2*(5*a^2*b*B - 8*b^3*B - 3*a^3*C + 6*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(4*b*B + a*(B - 3*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*b*(b*B - a*C)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(
a^2*B - 4*b^2*B + 3*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.08041, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(a^2B+3abC-4b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(bB-aC)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2bB-3a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[c +
d*x])^(3/2)),x]
```


```
[Out] (-2*(5*a^2*b*B - 8*b^3*B - 3*a^3*C + 6*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(4*b*B + a*(B - 3*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*b*(b*B - a*C)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(
```

```
a^2*B - 4*b^2*B + 3*a*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2B-4b^2B+3abC\right)-\frac{1}{2}a(bB-aC)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}}{a\left(a^2-\right.}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2B-4b^2B+3abC\right)\sqrt{a+}}{3a^2\left(a^2-b^2\right)d\cos}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2B-4b^2B+3abC\right)\sqrt{a+}}{3a^2\left(a^2-b^2\right)d\cos}$$

$$= -\frac{2\left(5a^2bB-8b^3B-3a^3C+6ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle| -\frac{a}{a}\right.}{3a^4\sqrt{a+b}d}$$

<u>Mathematica</u> [C] time = 6.68048, size = 1357, normalized size = 3.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(a^4*B + 7*a^2*b^2*B - 8*b^4*B - 6*a^3*b*C + 6*a*b^3*C)*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(5*a^3*b*B - 8*a*b^3*B - 3*a^4*C + 6*a^2*b^2*C)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
```

```
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(5*a^2*b^2*
B - 8*b^4*B - 3*a^3*b*C + 6*a*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-5*b*B*Sin[c + d*x] + 3*
a*C*Sin[c + d*x]))/(3*a^3) - (2*(-(b^4*B*Sin[c + d*x]) + a*b^3*C*Sin[c + d*
x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*B*Sec[c + d*x]*Tan[c + d*x
])/(3*a^2)))/d
```

Maple [B] time = 0.15, size = 3334, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x
)
```

```
[Out] -2/3/d*(a^2*b^2*B-a^4*B+3*C*cos(d*x+c)^3*a^2*b^2-3*C*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*B*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
```

+b))^(1/2))*b^4+8*B*cos(d*x+c)^3*b^4+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^4+6*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^3-8*B*cos(d*x+c)^2*b^4-3*C*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*
cos(d*x+c)^3*a^3*b-5*B*cos(d*x+c)^3*a^2*b^2-5*B*cos(d*x+c)^2*a^3*b+8*B*cos(
d*x+c)^2*a*b^3-4*B*cos(d*x+c)*a*b^3-3*C*cos(d*x+c)*a^4+3*C*cos(d*x+c)^2*a^4
+6*C*cos(d*x+c)^2*a*b^3+3*C*cos(d*x+c)*a^2*b^2-6*C*cos(d*x+c)^3*a*b^3+B*cos
(d*x+c)^2*a^4+B*cos(d*x+c)^3*a^3*b-4*B*cos(d*x+c)^3*a*b^3+4*B*cos(d*x+c)^2*
a^2*b^2+4*B*cos(d*x+c)*a^3*b-3*C*cos(d*x+c)^2*a^3*b-6*C*cos(d*x+c)^2*a^2*b^
2-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b+6*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*C*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b-8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^4+B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*C*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^4-5*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*B*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^3+6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+5*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(

```
a+b))^(1/2))*a^3*b+5*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*B*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*a^3*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+5*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b+5*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)/(a+b*cos(d*
x+c))^(1/2)/a^3/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*c
os(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos{(dx+c)}+B)\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}{b^2\cos{(dx+c)}^5 + 2\,ab\cos{(dx+c)}^4 + a^2\cos{(dx+c)}^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(3/2)*c
os(d*x + c)^(7/2)), x)
```

3.933 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=674

$$\frac{2a(bB-aC)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2bB-5a^3C+9ab^2C-6b^3B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(26a^2b^2C+6a^3\right.}{}$$

```
[Out] ((6*a^3*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((a^2*b*(6*B - 5*
C) - 3*b^3*(4*B - C) - 15*a^3*C + a*b^2*(2*B + 21*C))*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a + b]*(2*b*B - 5*a
*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d) + (2*a*(b*B - a*
C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^(3/2)) + (2*a*(2*a^2*b*B - 6*b^3*B - 5*a^3*C + 9*a*b^2*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((6*a^3
*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 2.12424, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3029, 2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2a(bB-aC)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2bB-5a^3C+9ab^2C-6b^3B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(26a^2b^2C+6a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] ((6*a^3*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
```

```
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((a^2*b*(6*B - 5*
C) - 3*b^3*(4*B - C) - 15*a^3*C + a*b^2*(2*B + 21*C))*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a + b]*(2*b*B - 5*a
*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d) + (2*a*(b*B - a*
C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^(3/2)) + (2*a*(2*a^2*b*B - 6*b^3*B - 5*a^3*C + 9*a*b^2*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((6*a^3
*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(bB-aC)\right.}{}}{}$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3\right.}{3b^2\left(a^2-\right.}$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3\right.}{3b^2\left(a^2-\right.}$$

$$= \frac{2a(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3\right.}{3b^2\left(a^2-\right.}$$

$$= -\frac{\sqrt{a+b}(2bB-5aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{b^4d}$$

$$= \frac{\left(6a^3bB-14ab^3B-15a^4C+26a^2b^2C-3b^4C\right)\cot(c+dx)E\left(\sin\right.}{3a(a-b)b^3(a}$$

Mathematica [C] time = 6.67922, size = 1396, normalized size = 2.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(a^2*b*B*Sin[c + d*x])
+ a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-3
*a^3*b*B*Sin[c + d*x] + 7*a*b^3*B*Sin[c + d*x] + 6*a^4*C*Sin[c + d*x] - 10*
a^2*b^2*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d +
((-4*a*(-2*a^3*b*B + 2*a*b^3*B + 5*a^4*C - 8*a^2*b^2*C + 3*b^4*C)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
```

```
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(2*a^2*b^2*B + 6*b^4*B + 4*a^3*b*C - 12*a*b^3*C)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^3
*b*B + 14*a*b^3*B + 15*a^4*C - 26*a^2*b^2*C + 3*b^4*C)*((I*Cos[(c + d*x)/2]
*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
 d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(6*(a - b)^2*b^2*(a +
b)^2*d)
```

Maple [B] time = 0.282, size = 8611, normalized size = 12.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x
)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2} + B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3} + B\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3} + 3\,ab^{2}\cos\left(dx+c\right)^{2} + 3\,a^{2}b\cos\left(dx+c\right) + a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(
d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^(3/2)/(b*cos(d*x
 + c) + a)^(5/2), x)
```

3.934 $\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=545

$$-\frac{2a\left(3a^3C-7ab^2C+4b^3B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bC-3a^3C+ab^2B\right.}{}$$

```
[Out] (2*(4*b^3*B + 3*a^3*C - 7*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*b^2*B - 3*b^3*B - 3*a^3*C - a^2*b*C +
6*a*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2
)*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3
*d) + (2*a*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) - (2*a*(4*b^3*B + 3*a^3*C - 7*a*b^2*C)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.49626, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3029, 2989, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a\left(3a^3C-7ab^2C+4b^3B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bC-3a^3C+ab^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(5/2),x]
```



```
[Out] (2*(4*b^3*B + 3*a^3*C - 7*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*b^2*B - 3*b^3*B - 3*a^3*C - a^2*b*C +
6*a*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
```

```
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2
)*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3
*d) + (2*a*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) - (2*a*(4*b^3*B + 3*a^3*C - 7*a*b^2*C)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$=\frac{2a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}-\frac{2\int \frac{-\frac{1}{2}a(bB-aC)+\frac{3}{2}b(bB-aC)\ldots}{\sqrt{\cos(c+dx)}\ldots}}{3b\,(\ldots}$$

$$=\frac{2a(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}-\frac{2\int \frac{-\frac{1}{2}ab(bB-aC)+\left(\frac{3}{2}a\left(a^2-b^2\right)\ldots\right.}{\sqrt{\cos(c+dx)}(a+\ldots}}{3b^2\left(a^2\ldots\right.}$$

$$=-\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\ldots}}{b^3d}$$

$$=-\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\ldots}}{b^3d}$$

$$=\frac{2\left(4b^3B+3a^3C-7ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\ldots\right.}{3a(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [C] time = 6.51322, size = 1342, normalized size = 2.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(a*b*B*Sin[c + d*x]) + a
^2*C*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(4*b^3*B
*Sin[c + d*x] + 3*a^3*C*Sin[c + d*x] - 7*a*b^2*C*Sin[c + d*x]))/(3*b*(-a^2
+ b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(-(a^2*b*B) + b^3*B + a^3*C - a
*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(4*a*b^2*B - a^2*b*C - 3*b^3*C)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
```

```
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*b^3*
B + 3*a^3*C - 7*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[
c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x
])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b
*Sqrt[Cos[c + d*x]])))/(3*(a - b)^2*b*(a + b)^2*d)
```

Maple [B] time = 0.233, size = 5749, normalized size = 10.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x
)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*
x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a)^(5/2), x)
```

3.935 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2\left(3a^2B-4abC+b^2B\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2B-4abC+b^2B\right)\cot(c+d}{}$$

```
[Out] (-2*(3*a^2*B + b^2*B - 4*a*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(
a - b)*(a + b)^(3/2)*d) + (2*(3*a*B - b*B + a*C - 3*b*C)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(b*B - a*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*
a^2*B + b^2*B - 4*a*b*C)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.989335, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 2999, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2B-4abC+b^2B\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2B-4abC+b^2B\right)\cot(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] (-2*(3*a^2*B + b^2*B - 4*a*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(
a - b)*(a + b)^(3/2)*d) + (2*(3*a*B - b*B + a*C - 3*b*C)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(b*B - a*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*
```

```
a^2*B + b^2*B - 4*a*b*C)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\cos(c+dx)}(B+C\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$=-\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}-\frac{2\int\frac{\frac{1}{2}(bB-aC)-\frac{3}{2}(aB-bC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$=-\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2\left(3a^2B+b^2B-4abC\right)\sin(c+}{3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$=-\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2\left(3a^2B+b^2B-4abC\right)\sin(c+}{3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$=-\frac{2\left(3a^2B+b^2B-4abC\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{3a^2(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.44566, size = 1335, normalized size = 3.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(b*B*Sin[c + d*x]) + a*C
*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*b*B*Sin[
c + d*x] + b^3*B*Sin[c + d*x] - 4*a*b^2*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2
*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*b*B) + b^3*B + a^3*C - a*b^2*C)*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3*B + a*b^2*B - 4*a^2*b*C)*((Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*b*B + b^
3*B - 4*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
 + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Co
s[c + d*x]])))/(3*a*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 0.181, size = 4237, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x
)

[Out] -2/3/d/(a+b*cos(d*x+c))^(3/2)*(-5*C*cos(d*x+c)^3*a^2*b^2+3*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4-3*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
4+5*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b+7*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*C*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-4*
C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3*b-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^
2+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-B*cos(d*x+c)^3*b^4+C*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-4
*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^3+B*cos(d*x+c)^2*b^4+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+3*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b

))^(1/2))*a^2*b^2-3*B*cos(d*x+c)*a^4-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-3*B*cos(d*x+c)^3*a^2*b^2-6
*B*cos(d*x+c)^2*a^3*b-2*B*cos(d*x+c)^2*a*b^3-B*cos(d*x+c)*a^2*b^2-C*cos(d*x
+c)*a^4-4*C*cos(d*x+c)^2*a*b^3+4*C*cos(d*x+c)*a^3*b-3*C*cos(d*x+c)*a^2*b^2+
4*C*cos(d*x+c)^3*a*b^3+C*cos(d*x+c)^3*a^4+3*B*cos(d*x+c)^2*a^4+2*B*cos(d*x+
c)^3*a^3*b+2*B*cos(d*x+c)^3*a*b^3+4*B*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)*a
^3*b-4*C*cos(d*x+c)^2*a^3*b+8*C*cos(d*x+c)^2*a^2*b^2-4*C*cos(d*x+c)^2*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2+C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b^3+B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-3
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^4-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*B*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-B*s
in(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3-4*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+4*C*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*B
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3*b+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+B*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-7*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a^3*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-5*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-B*cos(d*x

```
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+2*B*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^4)/sin(d*x+c)/a/(a+b)^2/(a-b)^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(5/2)*s
qrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)+B\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(5/2)*s
qrt(cos(d*x + c))), x)
```

3.936 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=429

$$-\frac{2\left(6a^2bB-3a^3C-ab^2C-2b^3B\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(B+C)+ab(3B+C\right.}{}$$

```
[Out] (2*(6*a^2*b*B - 2*b^3*B - 3*a^3*C - a*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*b^2*B - 3*a^2*(B + C) + a*b*(3
*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*
d) + (2*b*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])^(3/2)) - (2*(6*a^2*b*B - 2*b^3*B - 3*a^3*C - a*b^2*C)*S
in[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])
```

Rubi [A] time = 1.10738, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 3000, 2993, 2998, 2816, 2994}

$$-\frac{2\left(6a^2bB-3a^3C-ab^2C-2b^3B\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(bB-aC)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(B+C)+ab(3B+C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^(5/2)),x]
```


```
[Out] (2*(6*a^2*b*B - 2*b^3*B - 3*a^3*C - a*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*b^2*B - 3*a^2*(B + C) + a*b*(3
*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*
```

```
d) + (2*b*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])^(3/2)) - (2*(6*a^2*b*B - 2*b^3*B - 3*a^3*C - a*b^2*C)*S
in[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2B-2b^2B-abC\right)-\frac{3}{2}a(bB-aC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(6a^2bB-2b^3B-3a^3C-ab^2C\right)\sin}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$= \frac{2b(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(6a^2bB-2b^3B-3a^3C-ab^2C\right)\sin}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}$$

$$= \frac{2\left(6a^2bB-2b^3B-3a^3C-ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.55093, size = 1384, normalized size = 3.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(b^2*B*Sin[c + d*x]) +
a*b*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-6*a^2*
b^2*B*Sin[c + d*x] + 2*b^4*B*Sin[c + d*x] + 3*a^3*b*C*Sin[c + d*x] + a*b^3*
C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3
*a^4*B - 5*a^2*b^2*B + 2*b^4*B - a^3*b*C + a*b^3*C)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-6*a^3*b*B + 2*a*b^3*B + 3*a^4*C + a^2*b^2*C)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*b^2*B + 2*b^4*B
 + 3*a^3*b*C + a*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
 + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x]
)*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*
Sqrt[Cos[c + d*x]])))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 0.512, size = 5200, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x
)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(5/2)*c
os(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)+B\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^3\cos\left(dx+c\right)^4+3\,ab^2\cos\left(dx+c\right)^3+3\,a^2b\cos\left(dx+c\right)^2+a^3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(5/2)*c
os(d*x + c)^(3/2)), x)
```

3.937 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=456

$$\frac{2b\left(8a^2bB-5a^3C+ab^2C-4b^3B\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(bB-aC)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3B+C)\right.}{}$$

```
[Out] (2*(3*a^4*B - 15*a^2*b^2*B + 8*b^4*B + 6*a^3*b*C - 2*a*b^3*C)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*(8*b^3*B - 3*a^3
*(B - C) + 2*a*b^2*(3*B - C) - 3*a^2*b*(3*B + C))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*b*(b*B - a*C)*Sin[c + d*x]
)/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (2*b*
(8*a^2*b*B - 4*b^3*B - 5*a^3*C + a*b^2*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.29535, antiderivative size = 456, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3029, 3000, 3055, 2998, 2816, 2994}

$$\frac{2b\left(8a^2bB-5a^3C+ab^2C-4b^3B\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(bB-aC)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3B+C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c +
d*x])^(5/2)),x]
```



```
[Out] (2*(3*a^4*B - 15*a^2*b^2*B + 8*b^4*B + 6*a^3*b*C - 2*a*b^3*C)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*(8*b^3*B - 3*a^3
*(B - C) + 2*a*b^2*(3*B - C) - 3*a^2*b*(3*B + C))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
```

```
(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*b*(b*B - a*C)*Sin[c + d*x]
)/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (2*b*
(8*a^2*b*B - 4*b^3*B - 5*a^3*C + a*b^2*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2B-4b^2B+abC\right)-\frac{3}{2}a(b\cdots}{\cos^{\frac{3}{2}}(c+\cdots}}{3\cdots}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2bB-4b^3B-5a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+\cdots}}$$

$$= \frac{2b(bB-aC)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2bB-4b^3B-5a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+\cdots}}$$

$$= \frac{2\left(3a^4B-15a^2b^2B+8b^4B+6a^3bC-2ab^3C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(\cdots}}{\sqrt{a+b}\sqrt{\cos\cdots}}\right.\right.}{3a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.71643, size = 1431, normalized size = 3.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] -((-4*a*(9*a^4*b*B - 17*a^2*b^3*B + 8*b^5*B - 3*a^5*C + 5*a^3*b^2*C - 2*a*b
^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B
 + 6*a^4*b*C - 2*a^2*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
```

```
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])) + 2*(3*a^4*b*B - 15*a^2*b^3*B + 8*b^5*B + 6*a^3*b^2*C - 2
*a*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x]
)/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d
*x]])))/(3*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*((2*(-(b^3*B*Sin[c + d*x]) + a*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2 -
 b^2)*(a + b*Cos[c + d*x])^2) + (2*(-9*a^2*b^3*B*Sin[c + d*x] + 5*b^5*B*Sin
[c + d*x] + 6*a^3*b^2*C*Sin[c + d*x] - 2*a*b^4*C*Sin[c + d*x]))/(3*a^3*(a^2
 - b^2)^2*(a + b*Cos[c + d*x])) + (2*B*Tan[c + d*x])/a^3))/d
```

Maple [B] time = 0.751, size = 6498, normalized size = 14.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x
)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(5/2)*c
os(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)+B)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^5+3\,ab^2\cos(dx+c)^4+3\,a^2b\cos(dx+c)^3+a^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/((b*cos(d*x + c) + a)^(5/2)*c
os(d*x + c)^(5/2)), x)
```

3.938 $\int \cos^2(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2\right.$

Optimal. Leaf size=156

$$-\frac{\sin^3(c+dx)(5aB+5Ab+4bC)}{15d}+\frac{\sin(c+dx)(5aB+5Ab+4bC)}{5d}+\frac{\sin(c+dx)\cos(c+dx)(4aA+3aC+3bB)}{8d}+\frac{1}{8}x$$

```
[Out] ((4*a*A + 3*b*B + 3*a*C)*x)/8 + ((5*A*b + 5*a*B + 4*b*C)*Sin[c + d*x])/(5*d
) + ((4*a*A + 3*b*B + 3*a*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((b*B + a*C
)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (b*C*Cos[c + d*x]^4*Sin[c + d*x])/(5
*d) - ((5*A*b + 5*a*B + 4*b*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.229286, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{\sin^3(c+dx)(5aB+5Ab+4bC)}{15d}+\frac{\sin(c+dx)(5aB+5Ab+4bC)}{5d}+\frac{\sin(c+dx)\cos(c+dx)(4aA+3aC+3bB)}{8d}+\frac{1}{8}x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```



```
[Out] ((4*a*A + 3*b*B + 3*a*C)*x)/8 + ((5*A*b + 5*a*B + 4*b*C)*Sin[c + d*x])/(5*d
) + ((4*a*A + 3*b*B + 3*a*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((b*B + a*C
)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (b*C*Cos[c + d*x]^4*Sin[c + d*x])/(5
*d) - ((5*A*b + 5*a*B + 4*b*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{bC\cos^4(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^2(c$$

$$= \frac{(bB+aC)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{bC\,c}{}$$

$$= \frac{(bB+aC)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{bC\,c}{}$$

$$= \frac{(4aA+3bB+3aC)\cos(c+dx)\sin(c+dx)}{8d}$$

$$= \frac{1}{8}(4aA+3bB+3aC)x+\frac{(5Ab+5aB+4b}{5d}$$

Mathematica [A] time = 0.506928, size = 117, normalized size = 0.75

$$\frac{-160\sin^3(c+dx)(aB+Ab+2bC)+480\sin(c+dx)(aB+Ab+bC)+15(4(c+dx)(4aA+3aC+3bB)+8\sin(2(c+d$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (480*(A*b + a*B + b*C)*Sin[c + d*x] - 160*(A*b + a*B + 2*b*C)*Sin[c + d*x]^
3 + 96*b*C*Sin[c + d*x]^5 + 15*(4*(4*a*A + 3*b*B + 3*a*C)*(c + d*x) + 8*(b*
B + a*(A + C))*Sin[2*(c + d*x)] + (b*B + a*C)*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.02, size = 173, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Cb\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+bB\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3d}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b*B*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*C*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*b*(2+cos(d*x+c)^2)*sin(d*x+c)
+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 0.965829, size = 224, normalized size = 1.44

$$\frac{120(2dx+2c+\sin(2dx+2c))Aa-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba+15(12dx+12c+\sin(4dx+4c)+8\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 160*(sin(d*x + c)^3 - 3*s
in(d*x + c))*B*a + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c
))*C*a - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b + 15*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b + 32*(3*sin(d*x + c)^5 - 10*sin(d*
x + c)^3 + 15*sin(d*x + c))*C*b)/d
```

Fricas [A] time = 1.77518, size = 305, normalized size = 1.96

$$\frac{15\,((4\,A+3\,C)a+3\,Bb)dx+\left(24\,Cb\cos\left(dx+c\right)^{4}+30\,(Ca+Bb)\cos\left(dx+c\right)^{3}+8\,(5\,Ba+(5\,A+4\,C)b)\cos\left(dx+c\right)^{2}+8\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*((4*A + 3*C)*a + 3*B*b)*d*x + (24*C*b*cos(d*x + c)^4 + 30*(C*a +
B*b)*cos(d*x + c)^3 + 8*(5*B*a + (5*A + 4*C)*b)*cos(d*x + c)^2 + 80*B*a + 1
6*(5*A + 4*C)*b + 15*((4*A + 3*C)*a + 3*B*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.0238, size = 428, normalized size = 2.74

$$\begin{cases}\frac{Aax\sin^{2}(c+dx)}{2}+\frac{Aax\cos^{2}(c+dx)}{2}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Ab\sin^{3}(c+dx)}{3d}+\frac{Ab\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{2Ba\sin^{3}(c+dx)}{3d}+\frac{Ba\sin(c+dx)\cos^{2}}{d} \\ x\,(a+b\cos(c))\left(A+B\cos(c)+C\cos^{2}(c)\right)\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + 2*A*b*sin(c + d*x)**3/(3*d) + A*b*sin(c + d*x)*co
s(c + d*x)**2/d + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(c + d*
x)**2/d + 3*B*b*x*sin(c + d*x)**4/8 + 3*B*b*x*sin(c + d*x)**2*cos(c + d*x)*
*2/4 + 3*B*b*x*cos(c + d*x)**4/8 + 3*B*b*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 5*B*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*C*a*x*sin(c + d*x)**4/8 + 3
*C*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*a*x*cos(c + d*x)**4/8 + 3*C*
a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*a*sin(c + d*x)*cos(c + d*x)**3/(
8*d) + 8*C*b*sin(c + d*x)**5/(15*d) + 4*C*b*sin(c + d*x)**3*cos(c + d*x)**2
```

```
/(3*d) + C*b*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))*(
A + B*cos(c) + C*cos(c)**2)*cos(c)**2, True))
```

Giac [A] time = 1.17417, size = 174, normalized size = 1.12

$$\frac{1}{8}(4Aa + 3Ca + 3Bb)x + \frac{Cb\sin(5dx + 5c)}{80d} + \frac{(Ca + Bb)\sin(4dx + 4c)}{32d} + \frac{(4Ba + 4Ab + 5Cb)\sin(3dx + 3c)}{48d} + \frac{(A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/8*(4*A*a + 3*C*a + 3*B*b)*x + 1/80*C*b*sin(5*d*x + 5*c)/d + 1/32*(C*a + B
*b)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a + 4*A*b + 5*C*b)*sin(3*d*x + 3*c)/d +
1/4*(A*a + C*a + B*b)*sin(2*d*x + 2*c)/d + 1/8*(6*B*a + 6*A*b + 5*C*b)*sin(
d*x + c)/d
```

3.939 $\int \cos(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c$

Optimal. Leaf size=128

$$\frac{\sin(c+dx)(3aA+2aC+2bB)}{3d}+\frac{\sin(c+dx)\cos(c+dx)(4aB+4Ab+3bC)}{8d}+\frac{1}{8}x(4aB+4Ab+3bC)+\frac{(aC+bB)\sin(c$$

```
[Out] ((4*A*b + 4*a*B + 3*b*C)*x)/8 + ((3*a*A + 2*b*B + 2*a*C)*Sin[c + d*x])/(3*d
) + ((4*A*b + 4*a*B + 3*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((b*B + a*C
)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + (b*C*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d)
```

Rubi [A] time = 0.138539, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3033, 3023, 2734}

$$\frac{\sin(c+dx)(3aA+2aC+2bB)}{3d}+\frac{\sin(c+dx)\cos(c+dx)(4aB+4Ab+3bC)}{8d}+\frac{1}{8}x(4aB+4Ab+3bC)+\frac{(aC+bB)\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```



```
[Out] ((4*A*b + 4*a*B + 3*b*C)*x)/8 + ((3*a*A + 2*b*B + 2*a*C)*Sin[c + d*x])/(3*d
) + ((4*A*b + 4*a*B + 3*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((b*B + a*C
)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + (b*C*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{bC\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos(c$$

$$= \frac{(bB+aC)\cos^2(c+dx)\sin(c+dx)}{3d}+\frac{bC}{}$$

$$= \frac{1}{8}(4Ab+4aB+3bC)x+\frac{(3aA+2bB+2a}{3d}$$

Mathematica [A] time = 0.323008, size = 118, normalized size = 0.92

$$\frac{24\sin(c+dx)(4aA+3aC+3bB)+24\sin(2(c+dx))(aB+Ab+bC)+48aBc+48aBdx+8aC\sin(3(c+dx))+48Abc}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (48*A*b*c + 48*a*B*c + 36*b*c*C + 48*A*b*d*x + 48*a*B*d*x + 36*b*C*d*x + 24
*(4*a*A + 3*b*B + 3*a*C)*Sin[c + d*x] + 24*(A*b + a*B + b*C)*Sin[2*(c + d*x
)] + 8*b*B*Sin[3*(c + d*x)] + 8*a*C*Sin[3*(c + d*x)] + 3*b*C*Sin[4*(c + d*x
)])/(96*d)
```

Maple [A] time = 0.018, size = 141, normalized size = 1.1

$$\frac{1}{d}\left(Cb\left(\frac{\sin\left(dx+c\right)}{4}\left(\left(\cos\left(dx+c\right)\right)^{3}+\frac{3\cos\left(dx+c\right)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{bB\left(2+\left(\cos\left(dx+c\right)\right)^{2}\right)\sin\left(dx+c\right)}{3}+\frac{aC\left(2+(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b
*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*
x+1/2*c)+a*A*sin(d*x+c))
```

Maxima [A] time = 0.987773, size = 178, normalized size = 1.39

$$\frac{24\left(2dx+2c+\sin\left(2dx+2c\right)\right)Ba-32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ca+24\left(2dx+2c+\sin\left(2dx+2c\right)\right)Ab-32\left(\sin\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*C*a + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b - 32*(sin(d*x + c)^
3 - 3*sin(d*x + c))*B*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*C*b + 96*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.70532, size = 239, normalized size = 1.87

$$\frac{3\left(4Ba+\left(4A+3C\right)b\right)dx+\left(6Cb\cos\left(dx+c\right)^{3}+8\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+8\left(3A+2C\right)a+16Bb+3\left(4Ba+\left(4A+3\right.\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/24*(3*(4*B*a + (4*A + 3*C)*b)*d*x + (6*C*b*cos(d*x + c)^3 + 8*(C*a + B*b)
*cos(d*x + c)^2 + 8*(3*A + 2*C)*a + 16*B*b + 3*(4*B*a + (4*A + 3*C)*b)*cos(
d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.55172, size = 320, normalized size = 2.5

$$\begin{cases} \frac{Aa\sin(c+dx)}{d} + \frac{Abx\sin^2(c+dx)}{2} + \frac{Abx\cos^2(c+dx)}{2} + \frac{Ab\sin(c+dx)\cos(c+dx)}{2d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{Ba\sin(c+dx)\cos(c+dx)}{2d} + \\ x(a+b\cos(c))\left(A+B\cos(c)+C\cos^2(c)\right)\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d + A*b*x*sin(c + d*x)**2/2 + A*b*x*cos(c + d*x
)**2/2 + A*b*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*b*sin(c +
 d*x)**3/(3*d) + B*b*sin(c + d*x)*cos(c + d*x)**2/d + 2*C*a*sin(c + d*x)**3
/(3*d) + C*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*b*x*sin(c + d*x)**4/8 + 3
*C*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*b*x*cos(c + d*x)**4/8 + 3*C*
b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*b*sin(c + d*x)*cos(c + d*x)**3/(
8*d), Ne(d, 0)), (x*(a + b*cos(c))*(A + B*cos(c) + C*cos(c)**2)*cos(c), Tru
e))
```

Giac [A] time = 1.21832, size = 138, normalized size = 1.08

$$\frac{1}{8}(4Ba + 4Ab + 3Cb)x + \frac{Cb\sin(4dx+4c)}{32d} + \frac{(Ca+Bb)\sin(3dx+3c)}{12d} + \frac{(Ba+Ab+Cb)\sin(2dx+2c)}{4d} + \frac{(4Aa +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/8*(4*B*a + 4*A*b + 3*C*b)*x + 1/32*C*b*sin(4*d*x + 4*c)/d + 1/12*(C*a + B
*b)*sin(3*d*x + 3*c)/d + 1/4*(B*a + A*b + C*b)*sin(2*d*x + 2*c)/d + 1/4*(4*
A*a + 3*C*a + 3*B*b)*sin(d*x + c)/d
```

3.940 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=80

$$\frac{\sin(c+dx)(aB+Ab+bC)}{d}+\frac{1}{2}x(a(2A+C)+bB)+\frac{(aC+bB)\sin(c+dx)\cos(c+dx)}{2d}-\frac{bC\sin^3(c+dx)}{3d}$$

```
[Out] ((b*B + a*(2*A + C))*x)/2 + ((A*b + a*B + b*C)*Sin[c + d*x])/d + ((b*B + a*
C)*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (b*C*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0985468, antiderivative size = 113, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3023, 2734}

$$\frac{\sin(c+dx)\left(a(3bB-aC)+b^2(3A+2C)\right)}{3bd}+\frac{1}{2}x(a(2A+C)+bB)+\frac{(3bB-aC)\sin(c+dx)\cos(c+dx)}{6d}+\frac{C\sin(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*B + a*(2*A + C))*x)/2 + ((b^2*(3*A + 2*C) + a*(3*b*B - a*C))*Sin[c + d*
x])/(3*b*d) + ((3*b*B - a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(3*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{C(a+b\cos(c+dx))^2\sin(c+dx)}{3bd}+\frac{\int (a+b\cos(c+d$$

$$=\frac{1}{2}(bB+a(2A+C))x+\frac{\left(b^2(3A+2C)+a(3bB-aC)\right)}{3bd}$$

Mathematica [A] time = 0.181079, size = 85, normalized size = 1.06

$$\frac{3\sin(c+dx)(4aB+4Ab+3bC)+12aAdx+3(aC+bB)\sin(2(c+dx))+6acC+6aCdx+6bBc+6bBdx+bC\sin(3(c}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*b*B*c + 6*a*c*C + 12*a*A*d*x + 6*b*B*d*x + 6*a*C*d*x + 3*(4*A*b + 4*a*B
+ 3*b*C)*Sin[c + d*x] + 3*(b*B + a*C)*Sin[2*(c + d*x)] + b*C*Sin[3*(c + d*x
)])/(12*d)
```

Maple [A] time = 0.018, size = 102, normalized size = 1.3

$$\frac{1}{d}\left(\frac{Cb\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+bB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*C*b*(2+cos(d*x+c)^2)*sin(d*x+c)+b*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b*sin(d*x+c)+B*
a*sin(d*x+c)+a*A*(d*x+c))
```

Maxima [A] time = 0.991426, size = 132, normalized size = 1.65

$$\frac{12(dx+c)Aa+3(2dx+2c+\sin(2dx+2c))Ca+3(2dx+2c+\sin(2dx+2c))Bb-4\left(\sin(dx+c)^3-3\sin(dx+c\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a + 3*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*B*b - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*b + 1
2*B*a*sin(d*x + c) + 12*A*b*sin(d*x + c))/d
```

Fricas [A] time = 1.65776, size = 173, normalized size = 2.16

$$\frac{3\left((2A+C)a+Bb\right)dx+\left(2Cb\cos\left(dx+c\right)^2+6Ba+2\left(3A+2C\right)b+3\left(Ca+Bb\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((2*A + C)*a + B*b)*d*x + (2*C*b*cos(d*x + c)^2 + 6*B*a + 2*(3*A + 2
*C)*b + 3*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 0.75824, size = 189, normalized size = 2.36

$$\begin{cases} Aax+\frac{Ab\sin\left(c+dx\right)}{d}+\frac{Ba\sin\left(c+dx\right)}{d}+\frac{Bbx\sin^2\left(c+dx\right)}{2}+\frac{Bbx\cos^2\left(c+dx\right)}{2}+\frac{Bb\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Cax\sin^2\left(c+dx\right)}{2}+\frac{Cax\cos^2\left(c+dx\right)}{2}+\frac{Ca \\ x\left(a+b\cos\left(c\right)\right)\left(A+B\cos\left(c\right)+C\cos^2\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a*x + A*b*sin(c + d*x)/d + B*a*sin(c + d*x)/d + B*b*x*sin(c +
d*x)**2/2 + B*b*x*cos(c + d*x)**2/2 + B*b*sin(c + d*x)*cos(c + d*x)/(2*d) +
 C*a*x*sin(c + d*x)**2/2 + C*a*x*cos(c + d*x)**2/2 + C*a*sin(c + d*x)*cos(c
 + d*x)/(2*d) + 2*C*b*sin(c + d*x)**3/(3*d) + C*b*sin(c + d*x)*cos(c + d*x)
**2/d, Ne(d, 0)), (x*(a + b*cos(c))*(A + B*cos(c) + C*cos(c)**2), True))
```

Giac [A] time = 1.14909, size = 103, normalized size = 1.29

$$\frac{1}{2}(2Aa+Ca+Bb)x+\frac{Cb\sin(3dx+3c)}{12d}+\frac{(Ca+Bb)\sin(2dx+2c)}{4d}+\frac{(4Ba+4Ab+3Cb)\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*A*a + C*a + B*b)*x + 1/12*C*b*sin(3*d*x + 3*c)/d + 1/4*(C*a + B*b)*s
in(2*d*x + 2*c)/d + 1/4*(4*B*a + 4*A*b + 3*C*b)*sin(d*x + c)/d
```

3.941 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}x(2aB+2Ab+bC)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{(aC+bB)\sin(c+dx)}{d}+\frac{bC\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] ((2*A*b + 2*a*B + b*C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + ((b*B + a*C)*
Sin[c + d*x])/d + (b*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.13973, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3033, 3023, 2735, 3770}

$$\frac{1}{2}x(2aB+2Ab+bC)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{(aC+bB)\sin(c+dx)}{d}+\frac{bC\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```

```
[Out] ((2*A*b + 2*a*B + b*C)*x)/2 + (a*A*ArcTanh[Sin[c + d*x]])/d + ((b*B + a*C)*
Sin[c + d*x])/d + (b*C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx &= \frac{bC\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2aA+\right.\\ &= \frac{(bB+aC)\sin(c+dx)}{d}+\frac{bC\cos(c+dx)\sin}{2d}\\ &= \frac{1}{2}(2Ab+2aB+bC)x+\frac{(bB+aC)\sin(c+}{d}\\ &= \frac{1}{2}(2Ab+2aB+bC)x+\frac{aA\tanh^{-1}(\sin(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.115718, size = 68, normalized size = 0.99

$$\frac{4aA\tanh^{-1}(\sin(c+dx))+4(aC+bB)\sin(c+dx)+4aBdx+4Abdx+bC\sin(2(c+dx))+2bcC+2bCdx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```

```
[Out] (2*b*c*C + 4*A*b*d*x + 4*a*B*d*x + 2*b*C*d*x + 4*a*A*ArcTanh[Sin[c + d*x]]
+ 4*(b*B + a*C)*Sin[c + d*x] + b*C*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.042, size = 100, normalized size = 1.5

$$Abx + \frac{Abc}{d} + \frac{bB\sin(dx+c)}{d} + \frac{Cb\cos(dx+c)\sin(dx+c)}{2d} + \frac{bCx}{2} + \frac{Cbc}{2d} + \frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d} + Bax +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] A*b*x+1/d*A*b*c+1/d*b*B*sin(d*x+c)+1/2*b*C*cos(d*x+c)*sin(d*x+c)/d+1/2*b*C*
x+1/2/d*C*b*c+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+B*a*x+1/d*B*a*c+a*C*sin(d*x
+c)/d
```

Maxima [A] time = 1.22437, size = 111, normalized size = 1.61

$$\frac{4(dx+c)Ba + 4(dx+c)Ab + (2dx+2c+\sin(2dx+2c))Cb + 4Aa\log(\sec(dx+c)+\tan(dx+c)) + 4Ca\sin(dx+c}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a + 4*(d*x + c)*A*b + (2*d*x + 2*c + sin(2*d*x + 2*c))*C
*b + 4*A*a*log(sec(d*x + c) + tan(d*x + c)) + 4*C*a*sin(d*x + c) + 4*B*b*si
n(d*x + c))/d
```

Fricas [A] time = 1.74916, size = 192, normalized size = 2.78

$$\frac{(2Ba+(2A+C)b)dx + Aa\log(\sin(dx+c)+1) - Aa\log(-\sin(dx+c)+1) + (Cb\cos(dx+c)+2Ca+2Bb)\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] 1/2*((2*B*a + (2*A + C)*b)*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d
*x + c) + 1) + (C*b*cos(d*x + c) + 2*C*a + 2*B*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\cos(c + dx))\left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(
c + d*x), x)
```

Giac [B] time = 1.20825, size = 215, normalized size = 3.12

$$\frac{2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + (2\,Ba + 2\,Ab + Cb)(dx + c) + \frac{2\left(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 \cdots\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (2*B*a + 2*A*b + C*b)*(d*x + c) + 2*(2*C*a*tan(1/2*d*x + 1/
2*c)^3 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 - C*b*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*
tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.942 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=52

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+x(aC+bB)+\frac{bC\sin(c+dx)}{d}$$

```
[Out] (b*B + a*C)*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*C*Sin[c + d*x])/
d + (a*A*Tan[c + d*x])/d
```

Rubi [A] time = 0.13825, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3031, 3023, 2735, 3770}

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+x(aC+bB)+\frac{bC\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```

```
[Out] (b*B + a*C)*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*C*Sin[c + d*x])/
d + (a*A*Tan[c + d*x])/d
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{aA\tan(c+dx)}{d}-\int\left(-Ab-aB-(bB+a\right.$$

$$=\frac{bC\sin(c+dx)}{d}+\frac{aA\tan(c+dx)}{d}-\int(-A$$

$$=(bB+aC)x+\frac{bC\sin(c+dx)}{d}+\frac{aA\tan(c}{d}$$

$$=(bB+aC)x+\frac{(Ab+aB)\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A] time = 0.0202367, size = 71, normalized size = 1.37

$$\frac{aA\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+aCx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+bBx+\frac{bC\sin(c)\cos(dx)}{d}+\frac{bC\cos(c)\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```

```
[Out] b*B*x + a*C*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])
/d + (b*C*Cos[d*x]*Sin[c])/d + (b*C*Cos[c]*Sin[d*x])/d + (a*A*Tan[c + d*x])
```

```
/d
```

Maple [A] time = 0.049, size = 88, normalized size = 1.7

$$bBx + aCx + \frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{A\tan(dx+c)\,a}{d} + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bbc}{d} + \frac{Cb\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] b*B*x+a*C*x+1/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+a*A*tan(d*x+c)/d+1/d*B*a*ln(s
ec(d*x+c)+tan(d*x+c))+1/d*B*b*c+b*C*sin(d*x+c)/d+1/d*a*C*c
```

Maxima [A] time = 0.999385, size = 124, normalized size = 2.38

$$\frac{2(dx+c)Ca + 2(dx+c)Bb + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ab(\log(\sin(dx+c)+1) - \log(\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + 2*(d*x + c)*B*b + B*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + A*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2
*C*b*sin(d*x + c) + 2*A*a*tan(d*x + c))/d
```

Fricas [A] time = 1.7503, size = 265, normalized size = 5.1

$$\frac{2(Ca+Bb)dx\cos(dx+c) + (Ba+Ab)\cos(dx+c)\log(\sin(dx+c)+1) - (Ba+Ab)\cos(dx+c)\log(-\sin(dx+c)+1}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a + B*b)*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x
+ c) + 1) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*b*cos(d*
x + c) + A*a)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.19432, size = 178, normalized size = 3.42

$$\frac{(Ca+Bb)(dx+c)+(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] ((C*a + B*b)*(d*x + c) + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)
^3 - C*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.943 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=69

$$\frac{(a(A+2C)+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+bCx$$

```
[Out] b*C*x + ((2*b*B + a*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*
Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.168074, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3031, 3021, 2735, 3770}

$$\frac{(a(A+2C)+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+bCx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```

```
[Out] b*C*x + ((2*b*B + a*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*
Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\int \Big(-2(A\\ &= \frac{(Ab+aB)\tan(c+dx)}{d}+\frac{aA\sec(c+dx)\tan\\ &= bCx+\frac{(Ab+aB)\tan(c+dx)}{d}+\frac{aA\sec(c\\ &= bCx+\frac{(2bB+a(A+2C))\tanh^{-1}(\sin(c+\end{aligned}$$

Mathematica [A] time = 0.0275851, size = 92, normalized size = 1.33

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{Ab\tan(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```

```
[Out] b*C*x + (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (b*B*ArcTanh[Sin[c + d*x]])/d +
 (a*C*ArcTanh[Sin[c + d*x]])/d + (A*b*Tan[c + d*x])/d + (a*B*Tan[c + d*x])/
```

```
d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.053, size = 117, normalized size = 1.7

$$\frac{Ab\tan(dx+c)}{d}+\frac{bB\ln(\sec(dx+c)+\tan(dx+c))}{d}+bCx+\frac{Cbc}{d}+\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] A*b*tan(d*x+c)/d+1/d*b*B*ln(sec(d*x+c)+tan(d*x+c))+b*C*x+1/d*C*b*c+1/2*a*A*
sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/
d+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01096, size = 176, normalized size = 2.55

$$\frac{4(dx+c)Cb-Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+2Ca(\log(\sin(dx+c)+1)-\log(\sin(dx}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*b - A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 2*B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 4*B*a*tan(d*x + c) + 4*A*b*tan(d*x + c))/d
```

Fricas [A] time = 1.8409, size = 305, normalized size = 4.42

$$\frac{4Cbdx\cos(dx+c)^2+((A+2C)a+2Bb)\cos(dx+c)^2\log(\sin(dx+c)+1)-((A+2C)a+2Bb)\cos(dx+c)^2\log(-s}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] 1/4*(4*C*b*d*x*cos(d*x + c)^2 + ((A + 2*C)*a + 2*B*b)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - ((A + 2*C)*a + 2*B*b)*cos(d*x + c)^2*log(-sin(d*x + c) +
1) + 2*(A*a + 2*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.24343, size = 227, normalized size = 3.29

$$\frac{2\,(dx+c)Cb+(Aa+2\,Ca+2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Ca+2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa\cdots\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*C*b + (A*a + 2*C*a + 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (A*a + 2*C*a + 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*ta
n(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1
/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.944 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=101

$$\frac{\tan(c+dx)(2aA+3aC+3bB)}{3d}+\frac{(aB+Ab+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(}{}$$

```
[Out] ((A*b + a*B + 2*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*A + 3*b*B + 3*a*C
)*Tan[c + d*x])/(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.220181, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\tan(c+dx)(2aA+3aC+3bB)}{3d}+\frac{(aB+Ab+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```



```
[Out] ((A*b + a*B + 2*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*A + 3*b*B + 3*a*C
)*Tan[c + d*x])/(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}-\frac{1}{3}\int\left(-3(Ab\right.$$
$$= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\,\mathrm{se}}{}$$
$$= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\,\mathrm{se}}{}$$
$$= \frac{(Ab+aB+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A}{}$$
$$= \frac{(Ab+aB+2bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2}{}$$

Mathematica [A] time = 0.419357, size = 73, normalized size = 0.72

$$\frac{3(aB+Ab+2bC)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aB+Ab)\sec(c+dx)+2aA\tan^2(c+dx)+6a(A+C)+6bB\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```

```
[Out] (3*(A*b + a*B + 2*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*b*B + 6*a*(A
 + C) + 3*(A*b + a*B)*Sec[c + d*x] + 2*a*A*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.058, size = 160, normalized size = 1.6

$$\frac{Ab\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{bB\tan(dx+c)}{d}+\frac{Cb\ln(\sec(dx+c)+\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/2*A*b*sec(d*x+c)*tan(d*x+c)/d+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b*B
*tan(d*x+c)+1/d*C*b*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*A*tan(d*x+c)/d+1/3*a*A*
sec(d*x+c)^2*tan(d*x+c)/d+1/2*a*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(
d*x+c)+tan(d*x+c))+1/d*a*C*tan(d*x+c)
```

Maxima [A] time = 0.976989, size = 219, normalized size = 2.17

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa-3\,Ba\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ab\left(\frac{2\,\sin(dx}{\sin(dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 6*C*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*C*a*t
an(d*x + c) + 12*B*b*tan(d*x + c))/d
```

Fricas [A] time = 1.72559, size = 331, normalized size = 3.28

$$\frac{3\left(Ba+(A+2\,C)b\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba+(A+2\,C)b\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] 1/12*(3*(B*a + (A + 2*C)*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a +
 (A + 2*C)*b)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*((2*A + 3*C)*a +
 3*B*b)*cos(d*x + c)^2 + 2*A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.23133, size = 352, normalized size = 3.49

$$\frac{3\,(Ba+Ab+2\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba+Ab+2\,Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(B*a + A*b + 2*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*
b + 2*C*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*
c)^5 - 3*B*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*tan(1/2*d*x + 1/2*c)^5 - 3*A*b*
tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x +
 1/2*c)^3 - 12*C*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 +
 6*A*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 6*C*a*tan(1/2*d*
x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.945 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{\tan(c+dx)(2aB+2Ab+3bC)}{3d}+\frac{(3aA+4aC+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec(c+dx)(3aA+4aC+4b\ldots}{8d}$$

```
[Out] ((3*a*A + 4*b*B + 4*a*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((2*A*b + 2*a*B + 3
*b*C)*Tan[c + d*x])/(3*d) + ((3*a*A + 4*b*B + 4*a*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((A*b + a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c +
 d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.239113, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\tan(c+dx)(2aB+2Ab+3bC)}{3d}+\frac{(3aA+4aC+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec(c+dx)(3aA+4aC+4b\ldots}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^5,x]
```


```
[Out] ((3*a*A + 4*b*B + 4*a*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((2*A*b + 2*a*B + 3
*b*C)*Tan[c + d*x])/(3*d) + ((3*a*A + 4*b*B + 4*a*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((A*b + a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*A*Sec[c +
 d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}\int \Big(-4(A$$

$$= \frac{(Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{aA}{}$$

$$= \frac{(Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{aA}{}$$

$$= \frac{(3aA+4bB+4aC)\sec(c+dx)\tan(c+d}{8d}$$

$$= \frac{(3aA+4bB+4aC)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [A] time = 0.653965, size = 100, normalized size = 0.73

$$\frac{3(3aA+4aC+4bB)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\Big(3\sec(c+dx)(3aA+4aC+4bB) + 8(aB+Ab)\tan^2(c+dx) +}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```

```
[Out] (3*(3*a*A + 4*b*B + 4*a*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(A*b +
a*B + b*C) + 3*(3*a*A + 4*b*B + 4*a*C)*Sec[c + d*x] + 6*a*A*Sec[c + d*x]^3
+ 8*(A*b + a*B)*Tan[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.058, size = 223, normalized size = 1.6

$$\frac{2\,Ab\tan(dx+c)}{3\,d} + \frac{Ab(\sec(dx+c))^2\tan(dx+c)}{3\,d} + \frac{bB\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{bB\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 2/3*A*b*tan(d*x+c)/d+1/3*A*b*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*b*B*sec(d*x+c)
*tan(d*x+c)+1/2/d*b*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*b*tan(d*x+c)+1/4*a*A*
sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A*ln(sec(
d*x+c)+tan(d*x+c))+2/3*a*B*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d+1
```

```
/2/d*a*C*tan(d*x+c)*sec(d*x+c)+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02229, size = 294, normalized size = 2.15

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab-3\,Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*b - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*C*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)) - 12*B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*C*b*tan(d*x + c))/d
```

Fricas [A] time = 1.81406, size = 400, normalized size = 2.92

$$\frac{3\left((3A+4C)a+4Bb\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left((3A+4C)a+4Bb\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)}{48\,d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="fricas")
```

```
[Out] 1/48*(3*((3*A + 4*C)*a + 4*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*((
3*A + 4*C)*a + 4*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(2*B*a +
 (2*A + 3*C)*b)*cos(d*x + c)^3 + 3*((3*A + 4*C)*a + 4*B*b)*cos(d*x + c)^2 +
 6*A*a + 8*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.26706, size = 578, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a + 4*C*a + 4*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A
*a + 4*C*a + 4*B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a*tan(1/2*
d*x + 1/2*c)^7 - 24*B*a*tan(1/2*d*x + 1/2*c)^7 + 12*C*a*tan(1/2*d*x + 1/2*c
)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan(1/2*d*x + 1/2*c)^7 - 24*C*
b*tan(1/2*d*x + 1/2*c)^7 + 9*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*
x + 1/2*c)^5 - 12*C*a*tan(1/2*d*x + 1/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^
5 - 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 72*C*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a*t
an(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x
+ 1/2*c)^3 - 40*A*b*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3
- 72*C*b*tan(1/2*d*x + 1/2*c)^3 + 15*A*a*tan(1/2*d*x + 1/2*c) + 24*B*a*tan(
1/2*d*x + 1/2*c) + 12*C*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c
) + 12*B*b*tan(1/2*d*x + 1/2*c) + 24*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^4)/d
```

3.946 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=165

$$\frac{\tan^3(c+dx)(4aA+5aC+5bB)}{15d}+\frac{\tan(c+dx)(4aA+5aC+5bB)}{5d}+\frac{(3aB+3Ab+4bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c}{}$$

```
[Out] ((3*A*b + 3*a*B + 4*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*A + 5*b*B + 5
*a*C)*Tan[c + d*x])/(5*d) + ((3*A*b + 3*a*B + 4*b*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((A*b + a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*A*Sec[c +
 d*x]^4*Tan[c + d*x])/(5*d) + ((4*a*A + 5*b*B + 5*a*C)*Tan[c + d*x]^3)/(15*
d)
```

Rubi [A] time = 0.255276, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3031, 3021, 2748, 3767, 3768, 3770}

$$\frac{\tan^3(c+dx)(4aA+5aC+5bB)}{15d}+\frac{\tan(c+dx)(4aA+5aC+5bB)}{5d}+\frac{(3aB+3Ab+4bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^6,x]
```

```
[Out] ((3*A*b + 3*a*B + 4*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*A + 5*b*B + 5
*a*C)*Tan[c + d*x])/(5*d) + ((3*A*b + 3*a*B + 4*b*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((A*b + a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*A*Sec[c +
 d*x]^4*Tan[c + d*x])/(5*d) + ((4*a*A + 5*b*B + 5*a*C)*Tan[c + d*x]^3)/(15*
d)
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
```

```
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{aA\sec^4(c+dx)\tan(c+dx)}{5d}-\frac{1}{5}\int\left(-5(Ab\right.$$
$$= \frac{(Ab+aB)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{aA\,\mathrm{s}}{}$$
$$= \frac{(Ab+aB)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{aA\,\mathrm{s}}{}$$
$$= \frac{(3Ab+3aB+4bC)\sec(c+dx)\tan(c+dx)}{8d}$$
$$= \frac{(3Ab+3aB+4bC)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Mathematica [A] time = 1.28905, size = 123, normalized size = 0.75

$$\frac{15(3aB+3Ab+4bC)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5\tan^2(c+dx)(a(2A+C)+bB)+15(a(A+C)+bB)+3aA\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^6,x]
```

```
[Out] (15*(3*A*b + 3*a*B + 4*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(3*A*b
 + 3*a*B + 4*b*C)*Sec[c + d*x] + 30*(A*b + a*B)*Sec[c + d*x]^3 + 8*(15*(b*B
 + a*(A + C)) + 5*(b*B + a*(2*A + C))*Tan[c + d*x]^2 + 3*a*A*Tan[c + d*x]^4
)))/(120*d)
```

Maple [A] time = 0.062, size = 287, normalized size = 1.7

$$\frac{Ab(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3Ab\sec(dx+c)\tan(dx+c)}{8d}+\frac{3Ab\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2bB\tan(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/4*A*b*sec(d*x+c)^3*tan(d*x+c)/d+3/8*A*b*sec(d*x+c)*tan(d*x+c)/d+3/8/d*A*b
*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b*B*tan(d*x+c)+1/3/d*b*B*tan(d*x+c)*sec(d*
x+c)^2+1/2/d*C*b*tan(d*x+c)*sec(d*x+c)+1/2/d*C*b*ln(sec(d*x+c)+tan(d*x+c))+
```

```
8/15*a*A*tan(d*x+c)/d+1/5*a*A*sec(d*x+c)^4*tan(d*x+c)/d+4/15*a*A*sec(d*x+c)
^2*tan(d*x+c)/d+1/4*a*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*B*sec(d*x+c)*tan(d*
x+c)/d+3/8/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a*C*tan(d*x+c)+1/3/d*a*C*t
an(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.03263, size = 359, normalized size = 2.18

$$\frac{16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Aa+80\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca+80\left(\tan\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a + 80
*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a + 80*(tan(d*x + c)^3 + 3*tan(d*x + c
))*B*b - 15*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
15*A*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
 c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*C*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)))/d
```

Fricas [A] time = 1.77455, size = 467, normalized size = 2.83

$$\frac{15\,(3\,Ba+(3\,A+4\,C)b)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,(3\,Ba+(3\,A+4\,C)b)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(3*B*a + (3*A + 4*C)*b)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15
*(3*B*a + (3*A + 4*C)*b)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*((4*
A + 5*C)*a + 5*B*b)*cos(d*x + c)^4 + 15*(3*B*a + (3*A + 4*C)*b)*cos(d*x + c
)^3 + 8*((4*A + 5*C)*a + 5*B*b)*cos(d*x + c)^2 + 24*A*a + 30*(B*a + A*b)*co
```

```
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.20227, size = 639, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,
algorithm="giac")
```

```
[Out] 1/120*(15*(3*B*a + 3*A*b + 4*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(
3*B*a + 3*A*b + 4*C*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a*tan(
1/2*d*x + 1/2*c)^9 - 75*B*a*tan(1/2*d*x + 1/2*c)^9 + 120*C*a*tan(1/2*d*x +
1/2*c)^9 - 75*A*b*tan(1/2*d*x + 1/2*c)^9 + 120*B*b*tan(1/2*d*x + 1/2*c)^9 -
 60*C*b*tan(1/2*d*x + 1/2*c)^9 - 160*A*a*tan(1/2*d*x + 1/2*c)^7 + 30*B*a*ta
n(1/2*d*x + 1/2*c)^7 - 320*C*a*tan(1/2*d*x + 1/2*c)^7 + 30*A*b*tan(1/2*d*x
+ 1/2*c)^7 - 320*B*b*tan(1/2*d*x + 1/2*c)^7 + 120*C*b*tan(1/2*d*x + 1/2*c)^
7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*C*a*tan(1/2*d*x + 1/2*c)^5 + 400*B
*b*tan(1/2*d*x + 1/2*c)^5 - 160*A*a*tan(1/2*d*x + 1/2*c)^3 - 30*B*a*tan(1/2
*d*x + 1/2*c)^3 - 320*C*a*tan(1/2*d*x + 1/2*c)^3 - 30*A*b*tan(1/2*d*x + 1/2
*c)^3 - 320*B*b*tan(1/2*d*x + 1/2*c)^3 - 120*C*b*tan(1/2*d*x + 1/2*c)^3 + 1
20*A*a*tan(1/2*d*x + 1/2*c) + 75*B*a*tan(1/2*d*x + 1/2*c) + 120*C*a*tan(1/2
*d*x + 1/2*c) + 75*A*b*tan(1/2*d*x + 1/2*c) + 120*B*b*tan(1/2*d*x + 1/2*c)
+ 60*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.947 $\int \cos(c+dx)(a+b\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2\right.$

Optimal. Leaf size=224

$$\frac{\sin(c+dx)\left(5a^2(3A+2C)+20abB+2b^2(5A+4C)\right)}{15d}+\frac{\sin(c+dx)\cos^2(c+dx)\left(2a^2C+10abB+5Ab^2+4b^2C\right)}{15d}+\frac{\sin}{}$$

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B + 6*a*b*C)*x)/8 + ((20*a*b*B + 5*a^2*(3*A + 2
*C) + 2*b^2*(5*A + 4*C))*Sin[c + d*x])/(15*d) + ((8*a*A*b + 4*a^2*B + 3*b^2
*B + 6*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((5*A*b^2 + 10*a*b*B + 2*a
^2*C + 4*b^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (b*(5*b*B + 2*a*C)*Co
s[c + d*x]^3*Sin[c + d*x])/(20*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^
2*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.325013, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3049, 3033, 3023, 2734}

$$\frac{\sin(c+dx)\left(5a^2(3A+2C)+20abB+2b^2(5A+4C)\right)}{15d}+\frac{\sin(c+dx)\cos^2(c+dx)\left(2a^2C+10abB+5Ab^2+4b^2C\right)}{15d}+\frac{\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B + 6*a*b*C)*x)/8 + ((20*a*b*B + 5*a^2*(3*A + 2
*C) + 2*b^2*(5*A + 4*C))*Sin[c + d*x])/(15*d) + ((8*a*A*b + 4*a^2*B + 3*b^2
*B + 6*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((5*A*b^2 + 10*a*b*B + 2*a
^2*C + 4*b^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (b*(5*b*B + 2*a*C)*Co
s[c + d*x]^3*Sin[c + d*x])/(20*d) + (C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^
2*Sin[c + d*x])/(5*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^2(c+dx)(a+b\cos(c+dx))^2\sin(c+}{5d}$$

$$= \frac{b(5bB+2aC)\cos^3(c+dx)\sin(c+dx)}{20d} + $$

$$= \frac{\left(5Ab^2+10abB+2a^2C+4b^2C\right)\cos^2(c+}{15d}$$

$$= \frac{1}{8}\left(8aAb+4a^2B+3b^2B+6abC\right)x + \frac{(20a}{}$$

Mathematica [A] time = 0.813885, size = 169, normalized size = 0.75

$$\frac{60(c+dx)\left(4a^2B+8aAb+6abC+3b^2B\right)+60\sin(c+dx)\left(a^2(8A+6C)+12abB+b^2(6A+5C)\right)+120\sin(2(c+dx))\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (60*(8*a*A*b + 4*a^2*B + 3*b^2*B + 6*a*b*C)*(c + d*x) + 60*(12*a*b*B + b^2*
(6*A + 5*C) + a^2*(8*A + 6*C))*Sin[c + d*x] + 120*(a^2*B + b^2*B + 2*a*b*(A
 + C))*Sin[2*(c + d*x)] + 10*(4*A*b^2 + 8*a*b*B + 4*a^2*C + 5*b^2*C)*Sin[3*
(c + d*x)] + 15*b*(b*B + 2*a*C)*Sin[4*(c + d*x)] + 6*b^2*C*Sin[5*(c + d*x)]
)/(480*d)
```

Maple [A] time = 0.02, size = 244, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Ab^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+b^2B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2C\sin\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*B*(1/4*(cos(d*x+c)^3+3/2*cos
(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^2*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c)+2*a*A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2/3*a*b*
B*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*si
n(d*x+c)+3/8*d*x+3/8*c)+A*a^2*sin(d*x+c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+1/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.00349, size = 315, normalized size = 1.41

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2-160\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^2+240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 160*(sin(d*x + c)^3 - 3
*sin(d*x + c))*C*a^2 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b - 320*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a*b + 30*(12*d*x + 12*c + sin(4*d*x + 4*c)
 + 8*sin(2*d*x + 2*c))*C*a*b - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^2
+ 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^2 + 32*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^2 + 480*A*a^2*sin
(d*x + c))/d
```

Fricas [A] time = 1.7603, size = 414, normalized size = 1.85

$$\frac{15\left(4\,Ba^2+2\,(4\,A+3\,C)ab+3\,Bb^2\right)dx+\left(24\,Cb^2\cos\left(dx+c\right)^4+30\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+40\,(3\,A+2\,C)a^2+160\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(4*B*a^2 + 2*(4*A + 3*C)*a*b + 3*B*b^2)*d*x + (24*C*b^2*cos(d*x +
 c)^4 + 30*(2*C*a*b + B*b^2)*cos(d*x + c)^3 + 40*(3*A + 2*C)*a^2 + 160*B*a*
b + 16*(5*A + 4*C)*b^2 + 8*(5*C*a^2 + 10*B*a*b + (5*A + 4*C)*b^2)*cos(d*x +
 c)^2 + 15*(4*B*a^2 + 2*(4*A + 3*C)*a*b + 3*B*b^2)*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [A] time = 3.50372, size = 570, normalized size = 2.54

$$\begin{cases}\frac{Aa^2\sin\left(c+dx\right)}{d}+Aabx\sin^2\left(c+dx\right)+Aabx\cos^2\left(c+dx\right)+\frac{Aab\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{2Ab^2\sin^3\left(c+dx\right)}{3d}+\frac{Ab^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\ldots\\ x\left(a+b\cos\left(c\right)\right)^2\left(A+B\cos\left(c\right)+C\cos^2\left(c\right)\right)\cos\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*x*sin(c + d*x)**2 + A*a*b*x*cos(c
+ d*x)**2 + A*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*A*b**2*sin(c + d*x)**3/(3
```

```
*d) + A*b**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*x*sin(c + d*x)**2/2 +
B*a**2*x*cos(c + d*x)**2/2 + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*B*a
*b*sin(c + d*x)**3/(3*d) + 2*B*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b**
2*x*sin(c + d*x)**4/8 + 3*B*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*
b**2*x*cos(c + d*x)**4/8 + 3*B*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
B*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*C*a**2*sin(c + d*x)**3/(3*d)
+ C*a**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*a*b*x*sin(c + d*x)**4/4 + 3*C
*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*C*a*b*x*cos(c + d*x)**4/4 + 3*
C*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 5*C*a*b*sin(c + d*x)*cos(c + d*x
)**3/(4*d) + 8*C*b**2*sin(c + d*x)**5/(15*d) + 4*C*b**2*sin(c + d*x)**3*cos
(c + d*x)**2/(3*d) + C*b**2*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(
a + b*cos(c))**2*(A + B*cos(c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.21233, size = 248, normalized size = 1.11

$$\frac{Cb^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(4\,Ba^2+8\,Aab+6\,Cab+3\,Bb^2\right)x+\frac{\left(2\,Cab+Bb^2\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(4\,Ca^2+8\,Bab+4\,Ab^2+\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/80*C*b^2*sin(5*d*x + 5*c)/d + 1/8*(4*B*a^2 + 8*A*a*b + 6*C*a*b + 3*B*b^2)
*x + 1/32*(2*C*a*b + B*b^2)*sin(4*d*x + 4*c)/d + 1/48*(4*C*a^2 + 8*B*a*b +
4*A*b^2 + 5*C*b^2)*sin(3*d*x + 3*c)/d + 1/4*(B*a^2 + 2*A*a*b + 2*C*a*b + B*
b^2)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^2 + 6*C*a^2 + 12*B*a*b + 6*A*b^2 + 5*C
*b^2)*sin(d*x + c)/d
```

3.948 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) d$

Optimal. Leaf size=191

$$\frac{\sin(c+dx)\left(4a^2bB+a^3(-C)+4ab^2(3A+2C)+4b^3B\right)}{6bd}+\frac{\sin(c+dx)\cos(c+dx)\left(-2a^2C+8abB+12Ab^2+9b^2C\right)}{24d}+\frac{1}{8}$$

```
[Out] ((8*a*b*B + 4*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/8 + ((4*a^2*b*B + 4*b^3*B
 - a^3*C + 4*a*b^2*(3*A + 2*C))*Sin[c + d*x])/(6*b*d) + ((12*A*b^2 + 8*a*b*
B - 2*a^2*C + 9*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*b*B - a*C)*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(12*b*d) + (C*(a + b*Cos[c + d*x])^3*Si
n[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.226021, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3023, 2753, 2734}

$$\frac{\sin(c+dx)\left(4a^2bB+a^3(-C)+4ab^2(3A+2C)+4b^3B\right)}{6bd}+\frac{\sin(c+dx)\cos(c+dx)\left(-2a^2C+8abB+12Ab^2+9b^2C\right)}{24d}+\frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] ((8*a*b*B + 4*a^2*(2*A + C) + b^2*(4*A + 3*C))*x)/8 + ((4*a^2*b*B + 4*b^3*B
 - a^3*C + 4*a*b^2*(3*A + 2*C))*Sin[c + d*x])/(6*b*d) + ((12*A*b^2 + 8*a*b*
B - 2*a^2*C + 9*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*b*B - a*C)*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(12*b*d) + (C*(a + b*Cos[c + d*x])^3*Si
n[c + d*x])/(4*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{C(a+b\cos(c+dx))^3\sin(c+dx)}{4bd}+\frac{\int(a+b\cos(c+\ldots}{}$$

$$=\frac{(4bB-aC)(a+b\cos(c+dx))^2\sin(c+dx)}{12bd}+\frac{C(a+b\ldots}{}$$

$$=\frac{1}{8}\left(8abB+4a^2(2A+C)+b^2(4A+3C)\right)x+\frac{\left(4a^2bB\ldots\right.}{}$$

Mathematica [A] time = 0.592331, size = 137, normalized size = 0.72

$$\frac{12(c+dx)\left(4a^2(2A+C)+8abB+b^2(4A+3C)\right)+24\sin(c+dx)\left(4a^2B+8aAb+6abC+3b^2B\right)+24\sin(2(c+dx))\left(a\ldots\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (12*(8*a*b*B + 4*a^2*(2*A + C) + b^2*(4*A + 3*C))*(c + d*x) + 24*(8*a*A*b +
 4*a^2*B + 3*b^2*B + 6*a*b*C)*Sin[c + d*x] + 24*(A*b^2 + 2*a*b*B + a^2*C +
b^2*C)*Sin[2*(c + d*x)] + 8*b*(b*B + 2*a*C)*Sin[3*(c + d*x)] + 3*b^2*C*Sin[
4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.019, size = 200, normalized size = 1.1

$$\frac{1}{d}\left(b^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2abC\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(b^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*a*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*
b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*B*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*A*
b*sin(d*x+c)+a^2*B*sin(d*x+c)+A*a^2*(d*x+c))
```

Maxima [A] time = 0.982046, size = 252, normalized size = 1.32

$$\frac{96(dx+c)Aa^2+24(2dx+2c+\sin(2dx+2c))Ca^2+48(2dx+2c+\sin(2dx+2c))Bab-64\left(\sin(dx+c)^3-3\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/96*(96*(d*x + c)*A*a^2 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 48*(
2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b - 64*(sin(d*x + c)^3 - 3*sin(d*x + c)
)*C*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 - 32*(sin(d*x + c)^3 -
3*sin(d*x + c))*B*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
 2*c))*C*b^2 + 96*B*a^2*sin(d*x + c) + 192*A*a*b*sin(d*x + c))/d
```

Fricas [A] time = 1.68043, size = 320, normalized size = 1.68

$$\frac{3\left(4(2A+C)a^2+8Bab+(4A+3C)b^2\right)dx+\left(6Cb^2\cos(dx+c)^3+24Ba^2+16(3A+2C)ab+16Bb^2+8\left(2Cab+Bb\right.\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/24*(3*(4*(2*A + C)*a^2 + 8*B*a*b + (4*A + 3*C)*b^2)*d*x + (6*C*b^2*cos(d*
x + c)^3 + 24*B*a^2 + 16*(3*A + 2*C)*a*b + 16*B*b^2 + 8*(2*C*a*b + B*b^2)*c
os(d*x + c)^2 + 3*(4*C*a^2 + 8*B*a*b + (4*A + 3*C)*b^2)*cos(d*x + c))*sin(d
*x + c))/d
```

Sympy [A] time = 1.72981, size = 420, normalized size = 2.2

$$\begin{cases} Aa^2x + \frac{2Aab\sin(c+dx)}{d} + \frac{Ab^2x\sin^2(c+dx)}{2} + \frac{Ab^2x\cos^2(c+dx)}{2} + \frac{Ab^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{Ba^2\sin(c+dx)}{d} + Babx\sin^2(c+dx) + Bab \\ x(a+b\cos(c))^2\left(A + B\cos(c) + C\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**2*x + 2*A*a*b*sin(c + d*x)/d + A*b**2*x*sin(c + d*x)**2/2 +
 A*b**2*x*cos(c + d*x)**2/2 + A*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*
*2*sin(c + d*x)/d + B*a*b*x*sin(c + d*x)**2 + B*a*b*x*cos(c + d*x)**2 + B*a
*b*sin(c + d*x)*cos(c + d*x)/d + 2*B*b**2*sin(c + d*x)**3/(3*d) + B*b**2*si
n(c + d*x)*cos(c + d*x)**2/d + C*a**2*x*sin(c + d*x)**2/2 + C*a**2*x*cos(c
+ d*x)**2/2 + C*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*C*a*b*sin(c + d*x)
**3/(3*d) + 2*C*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*b**2*x*sin(c + d*x
)**4/8 + 3*C*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*b**2*x*cos(c +
d*x)**4/8 + 3*C*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*b**2*sin(c +
d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**2*(A + B*cos(c) +
 C*cos(c)**2), True))
```

Giac [A] time = 1.18103, size = 197, normalized size = 1.03

$$\frac{Cb^2\sin(4\,dx + 4\,c)}{32\,d} + \frac{1}{8}\left(8\,Aa^2 + 4\,Ca^2 + 8\,Bab + 4\,Ab^2 + 3\,Cb^2\right)x + \frac{\left(2\,Cab + Bb^2\right)\sin(3\,dx + 3\,c)}{12\,d} + \frac{\left(Ca^2 + 2\,Bab + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/32*C*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^2 + 4*C*a^2 + 8*B*a*b + 4*A*b^2
+ 3*C*b^2)*x + 1/12*(2*C*a*b + B*b^2)*sin(3*d*x + 3*c)/d + 1/4*(C*a^2 + 2*B
*a*b + A*b^2 + C*b^2)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^2 + 8*A*a*b + 6*C*a*b
```

```
+ 3*B*b^2)*sin(d*x + c)/d
```

3.949 $\int (a+b\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ s dx) dx

Optimal. Leaf size=134

$$\frac{\sin(c+dx)\left(2a^2C+6abB+3Ab^2+2b^2C\right)}{3d}+\frac{1}{2}x\left(2a^2B+2ab(2A+C)+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(2aC+3}{}$$

```
[Out] ((2*a^2*B + b^2*B + 2*a*b*(2*A + C))*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d
 + ((3*A*b^2 + 6*a*b*B + 2*a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*d) + (b*(3*b*B
 + 2*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b*Cos[c + d*x])^2*Sin[
c + d*x])/(3*d)
```

Rubi [A] time = 0.331725, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3049, 3033, 3023, 2735, 3770}

$$\frac{\sin(c+dx)\left(2a^2C+6abB+3Ab^2+2b^2C\right)}{3d}+\frac{1}{2}x\left(2a^2B+2ab(2A+C)+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(2aC+3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```


```
[Out] ((2*a^2*B + b^2*B + 2*a*b*(2*A + C))*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d
 + ((3*A*b^2 + 6*a*b*B + 2*a^2*C + 2*b^2*C)*Sin[c + d*x])/(3*d) + (b*(3*b*B
 + 2*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (C*(a + b*Cos[c + d*x])^2*Sin[
c + d*x])/(3*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + b\cos(c + dx))^2 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec(c + dx)\,dx = \frac{C(a + b\cos(c + dx))^2\sin(c + dx)}{3d} + \frac{1}{3}\int($$
$$= \frac{b(3bB + 2aC)\cos(c + dx)\sin(c + dx)}{6d} + \frac{C}{}$$
$$= \frac{\left(3Ab^2 + 6abB + 2a^2C + 2b^2C\right)\sin(c + dx}{3d}$$
$$= \frac{1}{2}\left(2a^2B + b^2B + 2ab(2A + C)\right)x + \frac{\left(3Ab^2\right.}{}$$
$$= \frac{1}{2}\left(2a^2B + b^2B + 2ab(2A + C)\right)x + \frac{a^2A\,\mathrm{t}}{}$$

Mathematica [A] time = 0.499848, size = 158, normalized size = 1.18

$$\frac{6(c + dx)\left(2a^2B + 2ab(2A + C) + b^2B\right) + 3\sin(c + dx)\left(4a^2C + 8abB + 4Ab^2 + 3b^2C\right) - 12a^2A\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \mathrm{s}\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (6*(2*a^2*B + b^2*B + 2*a*b*(2*A + C))*(c + d*x) - 12*a^2*A*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] + 12*a^2*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 3*(4*A*b^2 + 8*a*b*B + 4*a^2*C + 3*b^2*C)*Sin[c + d*x] + 3*b*(b*B + 2*
a*C)*Sin[2*(c + d*x)] + b^2*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.053, size = 204, normalized size = 1.5

$$\frac{Ab^2\sin(dx + c)}{d} + \frac{b^2B\cos(dx + c)\sin(dx + c)}{2d} + \frac{b^2Bx}{2} + \frac{b^2Bc}{2d} + \frac{C\sin(dx + c)(\cos(dx + c))^2b^2}{3d} + \frac{2b^2C\sin(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 1/d*A*b^2*sin(d*x+c)+1/2/d*b^2*B*cos(d*x+c)*sin(d*x+c)+1/2*b^2*B*x+1/2/d*B*
b^2*c+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*b^2+2/3*b^2*C*sin(d*x+c)/d+2*a*A*b*x+
```

```
2/d*A*a*b*c+2/d*a*b*B*sin(d*x+c)+a*b*C*cos(d*x+c)*sin(d*x+c)/d+a*b*C*x+1/d*
a*b*C*c+1/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*x+1/d*B*a^2*c+1/d*a^2*C*s
in(d*x+c)
```

Maxima [A] time = 0.988509, size = 203, normalized size = 1.51

$$\frac{12\,(dx+c)Ba^2+24\,(dx+c)Aab+6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Cab+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^2-4\left(\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^2 + 24*(d*x + c)*A*a*b + 6*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*C*a*b + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^2 - 4*(sin(d*x + c)^
3 - 3*sin(d*x + c))*C*b^2 + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 12*
C*a^2*sin(d*x + c) + 24*B*a*b*sin(d*x + c) + 12*A*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.83762, size = 313, normalized size = 2.34

$$\frac{3\,Aa^2\log(\sin(dx+c)+1)-3\,Aa^2\log(-\sin(dx+c)+1)+3\left(2\,Ba^2+2\,(2\,A+C)ab+Bb^2\right)dx+\left(2\,Cb^2\cos(dx+c)^2+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*A*a^2*log(sin(d*x + c) + 1) - 3*A*a^2*log(-sin(d*x + c) + 1) + 3*(2*
B*a^2 + 2*(2*A + C)*a*b + B*b^2)*d*x + (2*C*b^2*cos(d*x + c)^2 + 6*C*a^2 +
12*B*a*b + 2*(3*A + 2*C)*b^2 + 3*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.2862, size = 467, normalized size = 3.49

$$6\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(2\,Ba^2+4\,Aab+2\,Cab+Bb^2\right)(dx+c)+\frac{2(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="giac")
```

```
[Out] 1/6*(6*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*B*a^2 + 4*A*a*b + 2*C*a*b + B*b^2)*(d*x + c) + 2*(
6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*b*
tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2
*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 +
 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) + 12*B*a*b*t
an(1/2*d*x + 1/2*c) + 6*C*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x +
1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.950 $\int (a+b\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \mathrm{se}$ $dx)\,dx$

Optimal. Leaf size=126

$$\frac{1}{2}x\left(2a^2C+4abB+2Ab^2+b^2C\right)-\frac{b\sin(c+dx)(2aA-2aC-bB)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)}{}$$

```
[Out] ((2*A*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*x)/2 + (a*(2*A*b + a*B)*ArcTanh[Sin[
c + d*x]])/d - (b*(2*a*A - b*B - 2*a*C)*Sin[c + d*x])/d - (b^2*(2*A - C)*Co
s[c + d*x]*Sin[c + d*x])/(2*d) + (A*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.322606, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3033, 3023, 2735, 3770}

$$\frac{1}{2}x\left(2a^2C+4abB+2Ab^2+b^2C\right)-\frac{b\sin(c+dx)(2aA-2aC-bB)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```


```
[Out] ((2*A*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*x)/2 + (a*(2*A*b + a*B)*ArcTanh[Sin[
c + d*x]])/d - (b*(2*a*A - b*B - 2*a*C)*Sin[c + d*x])/d - (b^2*(2*A - C)*Co
s[c + d*x]*Sin[c + d*x])/(2*d) + (A*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \int (a$$
$$= -\frac{b^2(2A-C)\cos(c+dx)\sin(c+dx)}{2d} + \frac{A(}{}$$
$$= -\frac{b(2aA-bB-2aC)\sin(c+dx)}{d} - \frac{b^2(2A}{}$$
$$= \frac{1}{2}\left(2Ab^2+4abB+2a^2C+b^2C\right)x - \frac{b(2aA}{}$$
$$= \frac{1}{2}\left(2Ab^2+4abB+2a^2C+b^2C\right)x + \frac{a(2Ab}{}$$

Mathematica [A] time = 1.02564, size = 155, normalized size = 1.23

$$\frac{2(c+dx)\left(2a^2C+4abB+2Ab^2+b^2C\right)+\tan(c+dx)\left(4a^2A+4b(2aC+bB)\cos(c+dx)+b^2C\cos(2(c+dx))+b^2C\right)-}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (2*(2*A*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*(c + d*x) - 4*a*(2*A*b + a*B)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a*(2*A*b + a*B)*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + (4*a^2*A + b^2*C + 4*b*(b*B + 2*a*C)*Cos[c + d*x]
+ b^2*C*Cos[2*(c + d*x)])*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.063, size = 171, normalized size = 1.4

$$Ab^2x+\frac{Ab^2c}{d}+\frac{b^2B\sin(dx+c)}{d}+\frac{b^2C\cos(dx+c)\sin(dx+c)}{2d}+\frac{b^2Cx}{2}+\frac{b^2Cc}{2d}+2\frac{aAb\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] A*b^2*x+1/d*A*b^2*c+1/d*b^2*B*sin(d*x+c)+1/2/d*b^2*C*cos(d*x+c)*sin(d*x+c)+
1/2*b^2*C*x+1/2/d*b^2*C*c+2/d*a*A*b*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*B*x+2/d
*B*a*b*c+2/d*a*b*C*sin(d*x+c)+1/d*A*a^2*tan(d*x+c)+1/d*a^2*B*ln(sec(d*x+c)+
```

```
tan(d*x+c))+a^2*C*x+1/d*a^2*C*c
```

Maxima [A] time = 1.02231, size = 200, normalized size = 1.59

$$\frac{4\,(dx+c)Ca^2+8\,(dx+c)Bab+4\,(dx+c)Ab^2+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Cb^2+2\,Ba^2(\log{(\sin{(dx+c)}+1)}-\log{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a^2 + 8*(d*x + c)*B*a*b + 4*(d*x + c)*A*b^2 + (2*d*x + 2
*c + sin(2*d*x + 2*c))*C*b^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 4*A*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*C
*a*b*sin(d*x + c) + 4*B*b^2*sin(d*x + c) + 4*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.81429, size = 366, normalized size = 2.9

$$\frac{\left(2\,Ca^2+4\,Bab+(2\,A+C)b^2\right)dx\cos{(dx+c)}+\left(Ba^2+2\,Aab\right)\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-\left(Ba^2+2\,Aab\right)\cos{(}}{2\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="fricas")
```

```
[Out] 1/2*((2*C*a^2 + 4*B*a*b + (2*A + C)*b^2)*d*x*cos(d*x + c) + (B*a^2 + 2*A*a*
b)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^2 + 2*A*a*b)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (C*b^2*cos(d*x + c)^2 + 2*A*a^2 + 2*(2*C*a*b + B*b^2)*
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24385, size = 309, normalized size = 2.45

$$-\frac{\frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \left(2\,Ca^2 + 4\,Bab + 2\,Ab^2 + Cb^2\right)(dx+c) - 2\left(Ba^2 + 2\,Aab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 2\left(Ba^2 + 2\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="giac")
```

```
[Out] -1/2*(4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (2*C*a^2
+ 4*B*a*b + 2*A*b^2 + C*b^2)*(d*x + c) - 2*(B*a^2 + 2*A*a*b)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) + 2*(B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*(4*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^2*tan(1/2*d*x + 1/2*c)^3 - C
*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*a*b*tan(1/2*d*x + 1/2*c) + 2*B*b^2*tan(1/
2*d*x + 1/2*c) + C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2
)/d
```

3.951 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx$

Optimal. Leaf size=118

$$\frac{\left(a^2(A+2C)+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(aB+Ab)\tan(c+dx)}{d}+\frac{A\tan(c+dx)\sec(c+dx)(a+b\cos(c+$$

```
[Out] b*(b*B + 2*a*C)*x + ((2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c + d*
x]])/(2*d) - (b^2*(A - 2*C)*Sin[c + d*x])/(2*d) + (a*(A*b + a*B)*Tan[c + d*
x])/d + (A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.359696, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3031, 3023, 2735, 3770}

$$\frac{\left(a^2(A+2C)+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(aB+Ab)\tan(c+dx)}{d}+\frac{A\tan(c+dx)\sec(c+dx)(a+b\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```


```
[Out] b*(b*B + 2*a*C)*x + ((2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c + d*
x]])/(2*d) - (b^2*(A - 2*C)*Sin[c + d*x])/(2*d) + (a*(A*b + a*B)*Tan[c + d*
x])/d + (A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec(c+dx)\tan(c}{2d}$$
$$= \frac{a(Ab+aB)\tan(c+dx)}{d} + \frac{A(a+b\cos(c}{}$$
$$= -\frac{b^2(A-2C)\sin(c+dx)}{2d} + \frac{a(Ab+aB)\,t}{d}$$
$$= b(bB+2aC)x - \frac{b^2(A-2C)\sin(c+dx)}{2d}$$
$$= b(bB+2aC)x + \frac{\left(2Ab^2+4abB+a^2(A+\right.}{}$$

Mathematica [B] time = 1.48669, size = 277, normalized size = 2.35

$$-2\left(a^2(A+2C)+4abB+2Ab^2\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+2\left(a^2(A+2C)+4abB+2Ab^2\right)\log\left(\sin\left(\tfrac{1}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (4*b*(b*B + 2*a*C)*(c + d*x) - 2*(2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2 + (4*a*(2*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]) - (a^2*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*
(2*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*b
^2*C*Sin[c + d*x])/(4*d)
```

Maple [A] time = 0.068, size = 184, normalized size = 1.6

$$\frac{Ab^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + b^2Bx + \frac{Bb^2c}{d} + \frac{b^2C\sin(dx+c)}{d} + 2\,\frac{aAb\tan(dx+c)}{d} + 2\,\frac{abB\ln(\sec(dx+c)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^2*B*x+1/d*B*b^2*c+b^2*C*sin(d*x+c)/d+
2*a*A*b*tan(d*x+c)/d+2/d*a*b*B*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*C*x+2/d*a*b*
C*c+1/2/d*A*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^2*ln(sec(d*x+c)+tan(d*x+c))
+a^2*B*tan(d*x+c)/d+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02194, size = 255, normalized size = 2.16

$$\frac{8\,(dx+c)Cab+4\,(dx+c)Bb^2-Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+2\,Ca^2(\log\left(\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="maxima")
```

```
[Out] 1/4*(8*(d*x + c)*C*a*b + 4*(d*x + c)*B*b^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a^2*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 4*C*b^2*sin(d*x + c) + 4*B*a^2*tan(d*x + c) + 8*A*a*b*tan(d*x
+ c))/d
```

Fricas [A] time = 1.80475, size = 406, normalized size = 3.44

$$\frac{4\left(2\,Cab+Bb^2\right)dx\cos\left(dx+c\right)^2+\left((A+2\,C)a^2+4\,Bab+2\,Ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left((A+2\,C)a^2+4\,B\right.}{4\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="fricas")
```

```
[Out] 1/4*(4*(2*C*a*b + B*b^2)*d*x*cos(d*x + c)^2 + ((A + 2*C)*a^2 + 4*B*a*b + 2*
A*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - ((A + 2*C)*a^2 + 4*B*a*b + 2*
A*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*C*b^2*cos(d*x + c)^2 +
A*a^2 + 2*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28627, size = 323, normalized size = 2.74

$$\frac{\frac{4\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 2\left(2\,Cab + Bb^2\right)(dx+c) + \left(Aa^2 + 2\,Ca^2 + 4\,Bab + 2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Aa^2 + 2\,Ca^2 \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="giac")
```

```
[Out] 1/2*(4*C*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(2*C*a*b
 + B*b^2)*(d*x + c) + (A*a^2 + 2*C*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (A*a^2 + 2*C*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 4*A*a*b*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x + 1/2*c) +
 2*B*a^2*tan(1/2*d*x + 1/2*c) + 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^2)/d
```

3.952 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^4(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{\tan(c+dx)\left(a^2(2A+3C)+6abB+2Ab^2\right)}{3d}+\frac{\left(a^2B+2ab(A+2C)+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(3aB+2Ab)\tan(c+$$

```
[Out] b^2*C*x + ((a^2*B + 2*b^2*B + 2*a*b*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d)
 + ((2*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*d) + (a*(2*A*b +
 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a + b*Cos[c + d*x])^2*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.366121, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3031, 3021, 2735, 3770}

$$\frac{\tan(c+dx)\left(a^2(2A+3C)+6abB+2Ab^2\right)}{3d}+\frac{\left(a^2B+2ab(A+2C)+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(3aB+2Ab)\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```



```
[Out] b^2*C*x + ((a^2*B + 2*b^2*B + 2*a*b*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d)
 + ((2*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*d) + (a*(2*A*b +
 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a + b*Cos[c + d*x])^2*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{a(2Ab+3aB)\sec(c+dx)\tan(c+dx)}{6d} + $$

$$= \frac{\left(2Ab^2+6abB+a^2(2A+3C)\right)\tan(c+dx}{3d}$$

$$= b^2Cx + \frac{\left(2Ab^2+6abB+a^2(2A+3C)\right)\tan}{3d}$$

$$= b^2Cx + \frac{\left(a^2B+2b^2B+2ab(A+2C)\right)\tanh}{2d}$$

Mathematica [A] time = 0.615442, size = 104, normalized size = 0.74

$$\frac{3\left(a^2B+2ab(A+2C)+2b^2B\right)\tanh^{-1}(\sin(c+dx))+3\tan(c+dx)\left(2a^2(A+C)+a(aB+2Ab)\sec(c+dx)+4abB+2A\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (6*b^2*C*d*x + 3*(a^2*B + 2*b^2*B + 2*a*b*(A + 2*C))*ArcTanh[Sin[c + d*x]]
+ 3*(2*A*b^2 + 4*a*b*B + 2*a^2*(A + C) + a*(2*A*b + a*B)*Sec[c + d*x])*Tan[
c + d*x] + 2*a^2*A*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.066, size = 225, normalized size = 1.6

$$\frac{Ab^2\tan(dx+c)}{d} + \frac{b^2B\ln(\sec(dx+c)+\tan(dx+c))}{d} + b^2Cx + \frac{Cb^2c}{d} + \frac{aAb\sec(dx+c)\tan(dx+c)}{d} + \frac{aAb\ln(\sec(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*A*b^2*tan(d*x+c)+1/d*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+b^2*C*x+1/d*b^2*C*
c+a*A*b*sec(d*x+c)*tan(d*x+c)/d+1/d*a*A*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b
*B*tan(d*x+c)+2/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*A*a^2*tan(d*x+c)+1/
3/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+1/2*a^2*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a
```

```
^2*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*tan(d*x+c)
```

Maxima [A] time = 1.00142, size = 298, normalized size = 2.11

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{2}+12\left(dx+c\right)Cb^{2}-3\,Ba^{2}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 12*(d*x + c)*C*b^2 - 3*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 6*A*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 12*C*a*b*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 12*C*a^2*tan(d*x + c) + 24*B*a*b*tan(d*x + c) + 12*A*b^2*tan(d*x + c))
/d
```

Fricas [A] time = 1.81486, size = 444, normalized size = 3.15

$$\frac{12\,Cb^{2}dx\cos\left(dx+c\right)^{3}+3\left(Ba^{2}+2\left(A+2\,C\right)ab+2\,Bb^{2}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^{2}+2\left(A+2\,C\right)ab+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="fricas")
```

```
[Out] 1/12*(12*C*b^2*d*x*cos(d*x + c)^3 + 3*(B*a^2 + 2*(A + 2*C)*a*b + 2*B*b^2)*c
os(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^2 + 2*(A + 2*C)*a*b + 2*B*b^2)
*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*A*a^2 + 2*((2*A + 3*C)*a^2 +
6*B*a*b + 3*A*b^2)*cos(d*x + c)^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23938, size = 491, normalized size = 3.48

$$\frac{6\,(dx+c)Cb^2+3\left(Ba^2+2\,Aab+4\,Cab+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ba^2+2\,Aab+4\,Cab+2\,Bb^2\right)\log\left(\left|\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*C*b^2 + 3*(B*a^2 + 2*A*a*b + 4*C*a*b + 2*B*b^2)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^2 + 2*A*a*b + 4*C*a*b + 2*B*b^2)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x
 + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c
)^5 - 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 24
*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*t
an(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x +
1/2*c) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A
*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.953 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^5(c+dx)\,dx$

Optimal. Leaf size=184

$$\frac{\tan(c+dx)\left(2a^2B+4aAb+6abC+3b^2B\right)}{3d}+\frac{\left(a^2(3A+4C)+8abB+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c+dx\ldots}{}$$

```
[Out] ((8*a*b*B + 4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((4*a*A*b + 2*a^2*B + 3*b^2*B + 6*a*b*C)*Tan[c + d*x])/(3*d) + ((2*A*b^2
 + 8*a*b*B + a^2*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A*b +
2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (A*(a + b*Cos[c + d*x])^2*Sec[c
 + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.465376, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\tan(c+dx)\left(2a^2B+4aAb+6abC+3b^2B\right)}{3d}+\frac{\left(a^2(3A+4C)+8abB+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c+dx\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```



```
[Out] ((8*a*b*B + 4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((4*a*A*b + 2*a^2*B + 3*b^2*B + 6*a*b*C)*Tan[c + d*x])/(3*d) + ((2*A*b^2
 + 8*a*b*B + a^2*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A*b +
2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (A*(a + b*Cos[c + d*x])^2*Sec[c
 + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{a(Ab+2aB)\sec^2(c+dx)\tan(c+dx)}{6d} +$$
$$= \frac{\left(2Ab^2+8abB+a^2(3A+4C)\right)\sec(c+d}{8d}$$
$$= \frac{\left(2Ab^2+8abB+a^2(3A+4C)\right)\sec(c+d}{8d}$$
$$= \frac{\left(8abB+4b^2(A+2C)+a^2(3A+4C)\right)\text{ta}}{8d}$$
$$= \frac{\left(8abB+4b^2(A+2C)+a^2(3A+4C)\right)\text{ta}}{8d}$$

Mathematica [A] time = 1.07744, size = 137, normalized size = 0.74

$$\frac{3\left(a^2(3A+4C)+8abB+4b^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(3a^2B+a(aB+2Ab)\tan^2(c+dx)+6ab\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (3*(8*a*b*B + 4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(3*(4*A*b^2 + 8*a*b*B + a^2*(3*A + 4*C))*Sec[c + d*x] + 6*a^2*A*
Sec[c + d*x]^3 + 8*(3*a^2*B + 3*b^2*B + 6*a*b*(A + C) + a*(2*A*b + a*B)*Tan
[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.069, size = 321, normalized size = 1.7

$$\frac{Ab^2\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ab^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{b^2B\tan(dx+c)}{d}+\frac{b^2C\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/2/d*A*b^2*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d
*b^2*B*tan(d*x+c)+1/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))+4/3*a*A*b*tan(d*x+c)/
d+2/3*a*A*b*sec(d*x+c)^2*tan(d*x+c)/d+1/d*a*b*B*sec(d*x+c)*tan(d*x+c)+1/d*a
*b*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*C*tan(d*x+c)+1/4/d*A*a^2*tan(d*x+c)*
sec(d*x+c)^3+3/8/d*A*a^2*sec(d*x+c)*tan(d*x+c)+3/8/d*A*a^2*ln(sec(d*x+c)+ta
n(d*x+c))+2/3*a^2*B*tan(d*x+c)/d+1/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*
a^2*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.98514, size = 423, normalized size = 2.3

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{2}+32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aab-3\,Aa^{2}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a*b - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(2*sin(d*
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)
) + 24*C*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*C*a*b*tan
(d*x + c) + 48*B*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.91735, size = 510, normalized size = 2.77

$$\frac{3\left((3\,A+4\,C)a^{2}+8\,Bab+4\,(A+2\,C)b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left((3\,A+4\,C)a^{2}+8\,Bab+4\,(A+2\,C)b^{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="fricas")
```

```
[Out] 1/48*(3*((3*A + 4*C)*a^2 + 8*B*a*b + 4*(A + 2*C)*b^2)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*((3*A + 4*C)*a^2 + 8*B*a*b + 4*(A + 2*C)*b^2)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(8*(2*B*a^2 + 2*(2*A + 3*C)*a*b + 3*B*b^2)
*cos(d*x + c)^3 + 6*A*a^2 + 3*((3*A + 4*C)*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x
 + c)^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28139, size = 851, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 4*C*a^2 + 8*B*a*b + 4*A*b^2 + 8*C*b^2)*log(abs(tan(1/2*d
*x + 1/2*c) + 1)) - 3*(3*A*a^2 + 4*C*a^2 + 8*B*a*b + 4*A*b^2 + 8*C*b^2)*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B
*a^2*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*ta
n(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 - 48*C*a*b*tan(1/2*d
*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/
2*c)^7 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 -
 12*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 80*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 24*B*a
*b*tan(1/2*d*x + 1/2*c)^5 + 144*C*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^2*tan
```

```
(1/2*d*x + 1/2*c)^5 + 72*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*
c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 -
144*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b
^2*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 24*B*a^2*tan(1/
2*d*x + 1/2*c) + 12*C*a^2*tan(1/2*d*x + 1/2*c) + 48*A*a*b*tan(1/2*d*x + 1/2
*c) + 24*B*a*b*tan(1/2*d*x + 1/2*c) + 48*C*a*b*tan(1/2*d*x + 1/2*c) + 12*A*
b^2*tan(1/2*d*x + 1/2*c) + 24*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

3.954 $\int (a+b\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ s dx) dx

Optimal. Leaf size=232

$$\frac{\tan(c+dx)\left(2a^2(4A+5C)+20abB+5b^2(2A+3C)\right)}{15d}+\frac{\left(3a^2B+6aAb+8abC+4b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c}{}$$

```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((2
0*a*b*B + 5*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Tan[c + d*x])/(15*d) + ((6
*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((
2*A*b^2 + 10*a*b*B + a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) +
 (a*(2*A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.507059, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(2a^2(4A+5C)+20abB+5b^2(2A+3C)\right)}{15d}+\frac{\left(3a^2B+6aAb+8abC+4b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```


```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((2
0*a*b*B + 5*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Tan[c + d*x])/(15*d) + ((6
*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((
2*A*b^2 + 10*a*b*B + a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) +
 (a*(2*A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c}{5d}$$

$$= \frac{a(2Ab+5aB)\sec^3(c+dx)\tan(c+dx)}{20d}$$

$$= \frac{\left(2Ab^2+10abB+a^2(4A+5C)\right)\sec^2(c+}{15d}$$

$$= \frac{\left(2Ab^2+10abB+a^2(4A+5C)\right)\sec^2(c+}{15d}$$

$$= \frac{\left(6aAb+3a^2B+4b^2B+8abC\right)\sec(c+d}{8d}$$

$$= \frac{\left(6aAb+3a^2B+4b^2B+8abC\right)\tanh^{-1}(s}{8d}$$

Mathematica [A] time = 2.47562, size = 167, normalized size = 0.72

$$15\left(3a^2B+6aAb+8abC+4b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5\tan^2(c+dx)\left(a^2(2A+C)+2abB+Ab^2\right)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (15*(6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c +
 d*x]*(15*(6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*Sec[c + d*x] + 30*a*(2*A*
```

```
b + a*B)*Sec[c + d*x]^3 + 8*(15*(2*a*b*B + a^2*(A + C) + b^2*(A + C)) + 5*(
A*b^2 + 2*a*b*B + a^2*(2*A + C))*Tan[c + d*x]^2 + 3*a^2*A*Tan[c + d*x]^4)))
/(120*d)
```

Maple [A] time = 0.069, size = 404, normalized size = 1.7

$$\frac{2\,Ab^2\tan(dx+c)}{3d}+\frac{Ab^2\tan(dx+c)(\sec(dx+c))^2}{3d}+\frac{b^2B\sec(dx+c)\tan(dx+c)}{2d}+\frac{b^2B\ln(\sec(dx+c)+\tan(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 2/3/d*A*b^2*tan(d*x+c)+1/3/d*A*b^2*tan(d*x+c)*sec(d*x+c)^2+1/2/d*b^2*B*sec(
d*x+c)*tan(d*x+c)+1/2/d*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*C*tan(d*x+c
)+1/2*a*A*b*sec(d*x+c)^3*tan(d*x+c)/d+3/4*a*A*b*sec(d*x+c)*tan(d*x+c)/d+3/4
/d*a*A*b*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*a*b*B*tan(d*x+c)+2/3/d*a*b*B*tan(d
*x+c)*sec(d*x+c)^2+1/d*a*b*C*tan(d*x+c)*sec(d*x+c)+1/d*a*b*C*ln(sec(d*x+c)+
tan(d*x+c))+8/15/d*A*a^2*tan(d*x+c)+1/5/d*A*a^2*tan(d*x+c)*sec(d*x+c)^4+4/1
5/d*A*a^2*tan(d*x+c)*sec(d*x+c)^2+1/4*a^2*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a
^2*B*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^2*B*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^
2*C*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.033, size = 482, normalized size = 2.08

$$\frac{16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^2+80\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^2+160\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 +
80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 + 160*(tan(d*x + c)^3 + 3*tan(d*
x + c))*B*a*b + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^2 - 15*B*a^2*(2*(3
*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*A*a*b*(2*(3*sin(d*
x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(
```

```
sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*C*a*b*(2*sin(d*x + c)/(s
in(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*B*
b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 240*C*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.87832, size = 598, normalized size = 2.58

$$\frac{15\left(3\,Ba^2+2\,(3\,A+4\,C)ab+4\,Bb^2\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ba^2+2\,(3\,A+4\,C)ab+4\,Bb^2\right)\cos\left(dx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(3*B*a^2 + 2*(3*A + 4*C)*a*b + 4*B*b^2)*cos(d*x + c)^5*log(sin(d*
x + c) + 1) - 15*(3*B*a^2 + 2*(3*A + 4*C)*a*b + 4*B*b^2)*cos(d*x + c)^5*log
(-sin(d*x + c) + 1) + 2*(8*(2*(4*A + 5*C)*a^2 + 20*B*a*b + 5*(2*A + 3*C)*b^
2)*cos(d*x + c)^4 + 15*(3*B*a^2 + 2*(3*A + 4*C)*a*b + 4*B*b^2)*cos(d*x + c)
^3 + 24*A*a^2 + 8*((4*A + 5*C)*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x + c)^2 + 3
0*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20769, size = 1034, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="giac")
```

```
[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 8*C*a*b + 4*B*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 15*(3*B*a^2 + 6*A*a*b + 8*C*a*b + 4*B*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(120*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^2*tan(1/2*d*x
 + 1/2*c)^9 + 120*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 150*A*a*b*tan(1/2*d*x + 1/
2*c)^9 + 240*B*a*b*tan(1/2*d*x + 1/2*c)^9 - 120*C*a*b*tan(1/2*d*x + 1/2*c)^
9 + 120*A*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*B*b^2*tan(1/2*d*x + 1/2*c)^9 + 12
0*C*b^2*tan(1/2*d*x + 1/2*c)^9 - 160*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^
2*tan(1/2*d*x + 1/2*c)^7 - 320*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 60*A*a*b*tan(
1/2*d*x + 1/2*c)^7 - 640*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 240*C*a*b*tan(1/2*d
*x + 1/2*c)^7 - 320*A*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*B*b^2*tan(1/2*d*x +
1/2*c)^7 - 480*C*b^2*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^2*tan(1/2*d*x + 1/2*c
)^5 + 400*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*a*b*tan(1/2*d*x + 1/2*c)^5 +
 400*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*
A*a^2*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 320*C*a^2*
tan(1/2*d*x + 1/2*c)^3 - 60*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 640*B*a*b*tan(1/
2*d*x + 1/2*c)^3 - 240*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 320*A*b^2*tan(1/2*d*x
 + 1/2*c)^3 - 120*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*C*b^2*tan(1/2*d*x + 1/
2*c)^3 + 120*A*a^2*tan(1/2*d*x + 1/2*c) + 75*B*a^2*tan(1/2*d*x + 1/2*c) + 1
20*C*a^2*tan(1/2*d*x + 1/2*c) + 150*A*a*b*tan(1/2*d*x + 1/2*c) + 240*B*a*b*
tan(1/2*d*x + 1/2*c) + 120*C*a*b*tan(1/2*d*x + 1/2*c) + 120*A*b^2*tan(1/2*d
*x + 1/2*c) + 60*B*b^2*tan(1/2*d*x + 1/2*c) + 120*C*b^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.955 $\int \cos(c+dx)(a+b\cos(c+dx))^3 \left(A + B\cos(c + dx) + C\cos^2\right.$

Optimal. Leaf size=327

$$\frac{\sin(c+dx)\left(5a^3(3A+2C)+30a^2bB+6ab^2(5A+4C)+8b^3B\right)}{15d}+\frac{b\sin(c+dx)\cos^3(c+dx)\left(6a^2C+42abB+30Ab^2+\right.}{120d}$$

```
[Out] ((8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*x)/16 + ((3
0*a^2*b*B + 8*b^3*B + 5*a^3*(3*A + 2*C) + 6*a*b^2*(5*A + 4*C))*Sin[c + d*x]
)/(15*d) + ((8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*
Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((12*a^2*b*B + 4*b^3*B + a^3*C + 3*a*b^
2*(5*A + 4*C))*Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (b*(30*A*b^2 + 42*a*b*
B + 6*a^2*C + 25*b^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(120*d) + ((2*b*B + a*
C)*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(10*d) + (C*Cos[c +
d*x]^2*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.606592, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3049, 3033, 3023, 2734}

$$\frac{\sin(c+dx)\left(5a^3(3A+2C)+30a^2bB+6ab^2(5A+4C)+8b^3B\right)}{15d}+\frac{b\sin(c+dx)\cos^3(c+dx)\left(6a^2C+42abB+30Ab^2+\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] ((8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*x)/16 + ((3
0*a^2*b*B + 8*b^3*B + 5*a^3*(3*A + 2*C) + 6*a*b^2*(5*A + 4*C))*Sin[c + d*x]
)/(15*d) + ((8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*
Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((12*a^2*b*B + 4*b^3*B + a^3*C + 3*a*b^
2*(5*A + 4*C))*Cos[c + d*x]^2*Sin[c + d*x])/(15*d) + (b*(30*A*b^2 + 42*a*b*
B + 6*a^2*C + 25*b^2*C)*Cos[c + d*x]^3*Sin[c + d*x])/(120*d) + ((2*b*B + a*
C)*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(10*d) + (C*Cos[c +
d*x]^2*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(6*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{C\cos^2(c+dx)(a+b\cos(c+dx))^3\sin(c\ }{6d}$$
$$= \frac{(2bB+aC)\cos^2(c+dx)(a+b\cos(c+dx}{10d}$$
$$= \frac{b\left(30Ab^2+42abB+6a^2C+25b^2C\right)\cos^3}{120d}$$
$$= \frac{\left(12a^2bB+4b^3B+a^3C+3ab^2(5A+4C)\right.}{15d}$$
$$= \frac{1}{16}\left(8a^3B+18ab^2B+6a^2b(4A+3C)+b\right.$$

Mathematica [A] time = 1.10767, size = 368, normalized size = 1.13

$$\frac{120\sin(c+dx)\left(a^3(8A+6C)+18a^2bB+3ab^2(6A+5C)+5b^3B\right)+15\sin(2(c+dx))\left(48a^2b(A+C)+16a^3B+48ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (1440*a^2*A*b*c + 360*A*b^3*c + 480*a^3*B*c + 1080*a*b^2*B*c + 1080*a^2*b*c
*C + 300*b^3*c*C + 1440*a^2*A*b*d*x + 360*A*b^3*d*x + 480*a^3*B*d*x + 1080*
a*b^2*B*d*x + 1080*a^2*b*C*d*x + 300*b^3*C*d*x + 120*(18*a^2*b*B + 5*b^3*B
+ 3*a*b^2*(6*A + 5*C) + a^3*(8*A + 6*C))*Sin[c + d*x] + 15*(16*a^3*B + 48*a
*b^2*B + 48*a^2*b*(A + C) + b^3*(16*A + 15*C))*Sin[2*(c + d*x)] + 240*a*A*b
^2*Sin[3*(c + d*x)] + 240*a^2*b*B*Sin[3*(c + d*x)] + 100*b^3*B*Sin[3*(c + d
*x)] + 80*a^3*C*Sin[3*(c + d*x)] + 300*a*b^2*C*Sin[3*(c + d*x)] + 30*A*b^3*
Sin[4*(c + d*x)] + 90*a*b^2*B*Sin[4*(c + d*x)] + 90*a^2*b*C*Sin[4*(c + d*x)
] + 45*b^3*C*Sin[4*(c + d*x)] + 12*b^3*B*Sin[5*(c + d*x)] + 36*a*b^2*C*Sin[
5*(c + d*x)] + 5*b^3*C*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.023, size = 370, normalized size = 1.1

$$\frac{1}{d}\left(Ab^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^3B\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(A*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*b^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+C*b^3*(1/6*(cos(d*x+c
)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+a*A*b^2*(
2+cos(d*x+c)^2)*sin(d*x+c)+3*a*b^2*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+3/8*d*x+3/8*c)+3/5*C*a*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(
d*x+c)+3*A*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*b*B*(2+cos(d
*x+c)^2)*sin(d*x+c)+3*a^2*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c)+A*a^3*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.02471, size = 486, normalized size = 1.49

$$\frac{240\,(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right))Ba^3 - 320\left(\sin\left(dx + c\right)^3 - 3\,\sin\left(dx + c\right)\right)Ca^3 + 720\,(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right))Aa^2b - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3
*sin(d*x + c))*C*a^3 + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b - 960*(
sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b + 90*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*C*a^2*b - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*
A*a*b^2 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^
2 + 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a*b^2 +
30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^3 + 64*(3*si
n(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^3 - 5*(4*sin(2*d*x
+ 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*C*b^3
+ 960*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.92528, size = 610, normalized size = 1.87

$$\frac{15\left(8\,Ba^3 + 6\,(4\,A + 3\,C)a^2b + 18\,Bab^2 + (6\,A + 5\,C)b^3\right)dx + \left(40\,Cb^3\cos\left(dx + c\right)^5 + 48\left(3\,Cab^2 + Bb^3\right)\cos\left(dx + c\right)^4 + 8\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(8*B*a^3 + 6*(4*A + 3*C)*a^2*b + 18*B*a*b^2 + (6*A + 5*C)*b^3)*d*
x + (40*C*b^3*cos(d*x + c)^5 + 48*(3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + 80*(
3*A + 2*C)*a^3 + 480*B*a^2*b + 96*(5*A + 4*C)*a*b^2 + 128*B*b^3 + 10*(18*C*
a^2*b + 18*B*a*b^2 + (6*A + 5*C)*b^3)*cos(d*x + c)^3 + 16*(5*C*a^3 + 15*B*a
^2*b + 3*(5*A + 4*C)*a*b^2 + 4*B*b^3)*cos(d*x + c)^2 + 15*(8*B*a^3 + 6*(4*A
 + 3*C)*a^2*b + 18*B*a*b^2 + (6*A + 5*C)*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 6.98618, size = 966, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)/d + 3*A*a**2*b*x*sin(c + d*x)**2/2 + 3*A*a**
2*b*x*cos(c + d*x)**2/2 + 3*A*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*
a*b**2*sin(c + d*x)**3/d + 3*A*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*
b**3*x*sin(c + d*x)**4/8 + 3*A*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3
*A*b**3*x*cos(c + d*x)**4/8 + 3*A*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) +
 5*A*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*x*sin(c + d*x)**2/2 +
 B*a**3*x*cos(c + d*x)**2/2 + B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*
a**2*b*sin(c + d*x)**3/d + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*
a*b**2*x*sin(c + d*x)**4/8 + 9*B*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4
 + 9*B*a*b**2*x*cos(c + d*x)**4/8 + 9*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)
/(8*d) + 15*B*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*b**3*sin(c +
d*x)**5/(15*d) + 4*B*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**3*si
n(c + d*x)*cos(c + d*x)**4/d + 2*C*a**3*sin(c + d*x)**3/(3*d) + C*a**3*sin(
c + d*x)*cos(c + d*x)**2/d + 9*C*a**2*b*x*sin(c + d*x)**4/8 + 9*C*a**2*b*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*C*a**2*b*x*cos(c + d*x)**4/8 + 9*C*a*
*2*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*C*a**2*b*sin(c + d*x)*cos(c +
d*x)**3/(8*d) + 8*C*a*b**2*sin(c + d*x)**5/(5*d) + 4*C*a*b**2*sin(c + d*x)*
*3*cos(c + d*x)**2/d + 3*C*a*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*C*b**3
*x*sin(c + d*x)**6/16 + 15*C*b**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15
*C*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*C*b**3*x*cos(c + d*x)**6/1
6 + 5*C*b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*C*b**3*sin(c + d*x)**3
*cos(c + d*x)**3/(6*d) + 11*C*b**3*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(
```

```
d, 0)), (x*(a + b*cos(c))**3*(A + B*cos(c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.18892, size = 382, normalized size = 1.17

$$\frac{Cb^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(8\,Ba^3+24\,Aa^2b+18\,Ca^2b+18\,Bab^2+6\,Ab^3+5\,Cb^3\right)x+\frac{\left(3\,Cab^2+Bb^3\right)\sin(5\,dx+5\,c)}{80\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/192*C*b^3*sin(6*d*x + 6*c)/d + 1/16*(8*B*a^3 + 24*A*a^2*b + 18*C*a^2*b +
18*B*a*b^2 + 6*A*b^3 + 5*C*b^3)*x + 1/80*(3*C*a*b^2 + B*b^3)*sin(5*d*x + 5*
c)/d + 1/64*(6*C*a^2*b + 6*B*a*b^2 + 2*A*b^3 + 3*C*b^3)*sin(4*d*x + 4*c)/d
+ 1/48*(4*C*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 15*C*a*b^2 + 5*B*b^3)*sin(3*d*x
 + 3*c)/d + 1/64*(16*B*a^3 + 48*A*a^2*b + 48*C*a^2*b + 48*B*a*b^2 + 16*A*b^
3 + 15*C*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^3 + 6*C*a^3 + 18*B*a^2*b + 18
*A*a*b^2 + 15*C*a*b^2 + 5*B*b^3)*sin(d*x + c)/d
```

3.956 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$

Optimal. Leaf size=277

$$\frac{\sin(c+dx)\left(4a^2b^2(20A+13C)+15a^3bB-3a^4C+60ab^3B+4b^4(5A+4C)\right)}{30bd}+\frac{\sin(c+dx)\cos(c+dx)\left(30a^2bB-6a^3C\right.}{120d}$$

```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*(2*A + C) + 3*a*b^2*(4*A + 3*C))*x)/8 + ((15
*a^3*b*B + 60*a*b^3*B - 3*a^4*C + 4*b^4*(5*A + 4*C) + 4*a^2*b^2*(20*A + 13*
C))*Sin[c + d*x])/(30*b*d) + ((30*a^2*b*B + 45*b^3*B - 6*a^3*C + a*b^2*(100
*A + 71*C))*Cos[c + d*x]*Sin[c + d*x])/(120*d) + ((4*b^2*(5*A + 4*C) + 3*a*
(5*b*B - a*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) + ((5*b*B - a*
C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (C*(a + b*Cos[c + d*x])^
4*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.419204, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3023, 2753, 2734}

$$\frac{\sin(c+dx)\left(4a^2b^2(20A+13C)+15a^3bB-3a^4C+60ab^3B+4b^4(5A+4C)\right)}{30bd}+\frac{\sin(c+dx)\cos(c+dx)\left(30a^2bB-6a^3C\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*(2*A + C) + 3*a*b^2*(4*A + 3*C))*x)/8 + ((15
*a^3*b*B + 60*a*b^3*B - 3*a^4*C + 4*b^4*(5*A + 4*C) + 4*a^2*b^2*(20*A + 13*
C))*Sin[c + d*x])/(30*b*d) + ((30*a^2*b*B + 45*b^3*B - 6*a^3*C + a*b^2*(100
*A + 71*C))*Cos[c + d*x]*Sin[c + d*x])/(120*d) + ((4*b^2*(5*A + 4*C) + 3*a*
(5*b*B - a*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) + ((5*b*B - a*
C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (C*(a + b*Cos[c + d*x])^
4*Sin[c + d*x])/(5*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{C(a+b\cos(c+dx))^4\sin(c+dx)}{5bd}+\frac{\int(a+b\cos(c+dx}{}$$

$$=\frac{(5bB-aC)(a+b\cos(c+dx))^3\sin(c+dx)}{20bd}+\frac{C(a+b}{}$$

$$=\frac{\left(4b^2(5A+4C)+3a(5bB-aC)\right)(a+b\cos(c+dx))^2\sin}{60bd}$$

$$=\frac{1}{8}\left(12a^2bB+3b^3B+4a^3(2A+C)+3ab^2(4A+3C)\right)x$$

Mathematica [A] time = 0.903856, size = 288, normalized size = 1.04

$$\frac{60\sin(c+dx)\left(6a^2b(4A+3C)+8a^3B+18ab^2B+b^3(6A+5C)\right)+120\sin(2(c+dx))\left(3a^2bB+a^3C+3ab^2(A+C)+b^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (480*a^3*A*c + 720*a*A*b^2*c + 720*a^2*b*B*c + 180*b^3*B*c + 240*a^3*c*C +
540*a*b^2*c*C + 480*a^3*A*d*x + 720*a*A*b^2*d*x + 720*a^2*b*B*d*x + 180*b^3
```

```
*B*d*x + 240*a^3*C*d*x + 540*a*b^2*C*d*x + 60*(8*a^3*B + 18*a*b^2*B + 6*a^2
*b*(4*A + 3*C) + b^3*(6*A + 5*C))*Sin[c + d*x] + 120*(3*a^2*b*B + b^3*B + a
^3*C + 3*a*b^2*(A + C))*Sin[2*(c + d*x)] + 40*A*b^3*Sin[3*(c + d*x)] + 120*
a*b^2*B*Sin[3*(c + d*x)] + 120*a^2*b*C*Sin[3*(c + d*x)] + 50*b^3*C*Sin[3*(c
 + d*x)] + 15*b^3*B*Sin[4*(c + d*x)] + 45*a*b^2*C*Sin[4*(c + d*x)] + 6*b^3*
C*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.022, size = 301, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Cb^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+b^3B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^3*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*C*a*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*b^3*(2+cos(d*x+c)^2
)*sin(d*x+c)+a*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*b*C*(2+cos(d*x+c)^2)*s
in(d*x+c)+3*a*A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*B*(1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+3*A*a^2*b*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c))
```

Maxima [A] time = 0.978894, size = 389, normalized size = 1.4

$$\frac{480(dx+c)Aa^3+120(2dx+2c+\sin(2dx+2c))Ca^3+360(2dx+2c+\sin(2dx+2c))Ba^2b-480\left(\sin(dx+c)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/480*(480*(d*x + c)*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 3
60*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d
*x + c))*C*a^2*b + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 - 480*(sin(
d*x + c)^3 - 3*sin(d*x + c))*B*a*b^2 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c)
 + 8*sin(2*d*x + 2*c))*C*a*b^2 - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^
```

```
3 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^3 + 32*(
3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^3 + 480*B*a^3*s
in(d*x + c) + 1440*A*a^2*b*sin(d*x + c))/d
```

Fricas [A] time = 1.73846, size = 498, normalized size = 1.8

$$\frac{15\left(4\left(2A+C\right)a^{3}+12\,Ba^{2}b+3\left(4A+3C\right)ab^{2}+3\,Bb^{3}\right)dx+\left(24\,Cb^{3}\cos\left(dx+c\right)^{4}+120\,Ba^{3}+120\left(3A+2C\right)a^{2}b+240\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(4*(2*A + C)*a^3 + 12*B*a^2*b + 3*(4*A + 3*C)*a*b^2 + 3*B*b^3)*d*
x + (24*C*b^3*cos(d*x + c)^4 + 120*B*a^3 + 120*(3*A + 2*C)*a^2*b + 240*B*a*
b^2 + 16*(5*A + 4*C)*b^3 + 30*(3*C*a*b^2 + B*b^3)*cos(d*x + c)^3 + 8*(15*C*
a^2*b + 15*B*a*b^2 + (5*A + 4*C)*b^3)*cos(d*x + c)^2 + 15*(4*C*a^3 + 12*B*a
^2*b + 3*(4*A + 3*C)*a*b^2 + 3*B*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.88104, size = 685, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**3*x + 3*A*a**2*b*sin(c + d*x)/d + 3*A*a*b**2*x*sin(c + d*x)
**2/2 + 3*A*a*b**2*x*cos(c + d*x)**2/2 + 3*A*a*b**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 2*A*b**3*sin(c + d*x)**3/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x
)**2/d + B*a**3*sin(c + d*x)/d + 3*B*a**2*b*x*sin(c + d*x)**2/2 + 3*B*a**2*
b*x*cos(c + d*x)**2/2 + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*a*
b**2*sin(c + d*x)**3/d + 3*B*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b*
*3*x*sin(c + d*x)**4/8 + 3*B*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B
*b**3*x*cos(c + d*x)**4/8 + 3*B*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5
*B*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + C*a**3*x*sin(c + d*x)**2/2 + C
*a**3*x*cos(c + d*x)**2/2 + C*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*a*
*2*b*sin(c + d*x)**3/d + 3*C*a**2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*C*a*
```

```
b**2*x*sin(c + d*x)**4/8 + 9*C*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 9*C*a*b**2*x*cos(c + d*x)**4/8 + 9*C*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 15*C*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*C*b**3*sin(c + d*
x)**5/(15*d) + 4*C*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*b**3*sin(
c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**3*(A + B*cos(c) +
 C*cos(c)**2), True))
```

Giac [A] time = 1.20787, size = 306, normalized size = 1.1

$$\frac{Cb^3 \sin(5\,dx + 5\,c)}{80\,d} + \frac{1}{8}\left(8\,Aa^3 + 4\,Ca^3 + 12\,Ba^2b + 12\,Aab^2 + 9\,Cab^2 + 3\,Bb^3\right)x + \frac{\left(3\,Cab^2 + Bb^3\right)\sin(4\,dx + 4\,c)}{32\,d} + \frac{(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/80*C*b^3*sin(5*d*x + 5*c)/d + 1/8*(8*A*a^3 + 4*C*a^3 + 12*B*a^2*b + 12*A*
a*b^2 + 9*C*a*b^2 + 3*B*b^3)*x + 1/32*(3*C*a*b^2 + B*b^3)*sin(4*d*x + 4*c)/
d + 1/48*(12*C*a^2*b + 12*B*a*b^2 + 4*A*b^3 + 5*C*b^3)*sin(3*d*x + 3*c)/d +
 1/4*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2 + 3*C*a*b^2 + B*b^3)*sin(2*d*x + 2*c)/d
 + 1/8*(8*B*a^3 + 24*A*a^2*b + 18*C*a^2*b + 18*B*a*b^2 + 6*A*b^3 + 5*C*b^3)
*sin(d*x + c)/d
```

3.957 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \text{se}$ $dx)\,dx$

Optimal. Leaf size=207

$$\frac{\sin(c+dx)\left(16a^2bB + 3a^3C + 6ab^2(3A+2C) + 4b^3B\right)}{6d} + \frac{b\sin(c+dx)\cos(c+dx)\left(6a^2C + 20abB + 12Ab^2 + 9b^2C\right)}{24d} + \frac{1}{8}$$

```
[Out] ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^3*(4*A + 3*C))*x)/8 + (a^3*
A*ArcTanh[Sin[c + d*x]])/d + ((16*a^2*b*B + 4*b^3*B + 3*a^3*C + 6*a*b^2*(3*
A + 2*C))*Sin[c + d*x])/(6*d) + (b*(12*A*b^2 + 20*a*b*B + 6*a^2*C + 9*b^2*C
)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*b*B + 3*a*C)*(a + b*Cos[c + d*x])
^2*Sin[c + d*x])/(12*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.563557, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3049, 3033, 3023, 2735, 3770}

$$\frac{\sin(c+dx)\left(16a^2bB + 3a^3C + 6ab^2(3A+2C) + 4b^3B\right)}{6d} + \frac{b\sin(c+dx)\cos(c+dx)\left(6a^2C + 20abB + 12Ab^2 + 9b^2C\right)}{24d} + \frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```



```
[Out] ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^3*(4*A + 3*C))*x)/8 + (a^3*
A*ArcTanh[Sin[c + d*x]])/d + ((16*a^2*b*B + 4*b^3*B + 3*a^3*C + 6*a*b^2*(3*
A + 2*C))*Sin[c + d*x])/(6*d) + (b*(12*A*b^2 + 20*a*b*B + 6*a^2*C + 9*b^2*C
)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*b*B + 3*a*C)*(a + b*Cos[c + d*x])
^2*Sin[c + d*x])/(12*d) + (C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a$$

$$= \frac{(4bB+3aC)(a+b\cos(c+dx))^2\sin(c+dx}{12d}$$

$$= \frac{b\left(12Ab^2+20abB+6a^2C+9b^2C\right)\cos(c+}{24d}$$

$$= \frac{\left(16a^2bB+4b^3B+3a^3C+6ab^2(3A+2C)\right)}{6d}$$

$$= \frac{1}{8}\left(8a^3B+12ab^2B+12a^2b(2A+C)+b^3(4.\right.$$

$$= \frac{1}{8}\left(8a^3B+12ab^2B+12a^2b(2A+C)+b^3(4.\right.$$

Mathematica [A] time = 0.934392, size = 218, normalized size = 1.05

$$\frac{12(c+dx)\left(12a^2b(2A+C)+8a^3B+12ab^2B+b^3(4A+3C)\right)+24\sin(c+dx)\left(12a^2bB+4a^3C+3ab^2(4A+3C)+3b^3B\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (12*(8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^3*(4*A + 3*C))*(c + d*x)
 - 96*a^3*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*a^3*A*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 24*(12*a^2*b*B + 3*b^3*B + 4*a^3*C + 3*a*b^
2*(4*A + 3*C))*Sin[c + d*x] + 24*b*(A*b^2 + 3*a*b*B + 3*a^2*C + b^2*C)*Sin[
2*(c + d*x)] + 8*b^2*(b*B + 3*a*C)*Sin[3*(c + d*x)] + 3*b^3*C*Sin[4*(c + d*
x)])/(96*d)
```

Maple [A] time = 0.061, size = 362, normalized size = 1.8

$$\frac{a^3C\sin(dx+c)}{d}+3\,Aa^2bx+\frac{Ab^3c}{2\,d}+\frac{3\,Cb^3c}{8\,d}+\frac{3\,a^2bCx}{2}+\frac{2\,b^3B\sin(dx+c)}{3\,d}+\frac{3\,ab^2Bx}{2}+\frac{Ba^3c}{d}+\frac{Aa^3\ln(\sec(dx+c)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] a^3*C*sin(d*x+c)/d+3*A*a^2*b*x+1/2/d*A*b^3*c+3/8/d*C*b^3*c+3/2*a^2*b*C*x+2/
3/d*b^3*B*sin(d*x+c)+3/2*a*b^2*B*x+1/d*a^3*B*c+1/d*A*a^3*ln(sec(d*x+c)+tan(
d*x+c))+3/8*b^3*C*x+3/2/d*B*a*b^2*c+3/8/d*C*b^3*cos(d*x+c)*sin(d*x+c)+2/d*C
*a*b^2*sin(d*x+c)+3/2/d*a^2*b*C*c+1/4/d*C*b^3*sin(d*x+c)*cos(d*x+c)^3+3/d*a
*A*b^2*sin(d*x+c)+3/d*a^2*b*B*sin(d*x+c)+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*b^
3+1/2/d*A*b^3*cos(d*x+c)*sin(d*x+c)+3/d*A*a^2*b*c+a^3*B*x+3/2/d*a*b^2*B*cos
(d*x+c)*sin(d*x+c)+1/d*C*sin(d*x+c)*cos(d*x+c)^2*a*b^2+3/2/d*a^2*b*C*cos(d*
x+c)*sin(d*x+c)+1/2*A*b^3*x
```

Maxima [A] time = 0.994961, size = 323, normalized size = 1.56

$$\frac{96\,(dx+c)Ba^3 + 288\,(dx+c)Aa^2b + 72\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ca^2b + 72\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Bab^2 - 96}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*B*a^3 + 288*(d*x + c)*A*a^2*b + 72*(2*d*x + 2*c + sin(2*
d*x + 2*c))*C*a^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^2 - 96*(sin
(d*x + c)^3 - 3*sin(d*x + c))*C*a*b^2 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))
*A*b^3 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b^3 + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*b^3 + 96*A*a^3*log(sec(d*x + c) + ta
n(d*x + c)) + 96*C*a^3*sin(d*x + c) + 288*B*a^2*b*sin(d*x + c) + 288*A*a*b^
2*sin(d*x + c))/d
```

Fricas [A] time = 1.9642, size = 463, normalized size = 2.24

$$\frac{12\,Aa^3\log\left(\sin\left(dx+c\right)+1\right) - 12\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right) + 3\left(8\,Ba^3 + 12\,(2\,A+C)a^2b + 12\,Bab^2 + (4\,A+3\,C)b^3\right)d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="fricas")
```

```
[Out] 1/24*(12*A*a^3*log(sin(d*x + c) + 1) - 12*A*a^3*log(-sin(d*x + c) + 1) + 3*
(8*B*a^3 + 12*(2*A + C)*a^2*b + 12*B*a*b^2 + (4*A + 3*C)*b^3)*d*x + (6*C*b^
3*cos(d*x + c)^3 + 24*C*a^3 + 72*B*a^2*b + 24*(3*A + 2*C)*a*b^2 + 16*B*b^3
```

```
+ 8*(3*C*a*b^2 + B*b^3)*cos(d*x + c)^2 + 3*(12*C*a^2*b + 12*B*a*b^2 + (4*A
+ 3*C)*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.22459, size = 976, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="giac")
```

```
[Out] 1/24*(24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(8*B*a^3 + 24*A*a^2*b + 12*C*a^2*b + 12*B*a*b^2 +
4*A*b^3 + 3*C*b^3)*(d*x + c) + 2*(24*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 72*B*a^
2*b*tan(1/2*d*x + 1/2*c)^7 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*a*b^2
*tan(1/2*d*x + 1/2*c)^7 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 72*C*a*b^2*ta
n(1/2*d*x + 1/2*c)^7 - 12*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*B*b^3*tan(1/2*d
*x + 1/2*c)^7 - 15*C*b^3*tan(1/2*d*x + 1/2*c)^7 + 72*C*a^3*tan(1/2*d*x + 1/
2*c)^5 + 216*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*C*a^2*b*tan(1/2*d*x + 1/2*
c)^5 + 216*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)
^5 + 120*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^3*tan(1/2*d*x + 1/2*c)^5 +
 40*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^
3*tan(1/2*d*x + 1/2*c)^3 + 216*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^2*b*
tan(1/2*d*x + 1/2*c)^3 + 216*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 + 120*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^3*tan(1/
2*d*x + 1/2*c)^3 + 40*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^3*tan(1/2*d*x +
1/2*c)^3 + 24*C*a^3*tan(1/2*d*x + 1/2*c) + 72*B*a^2*b*tan(1/2*d*x + 1/2*c)
```

```
+ 36*C*a^2*b*tan(1/2*d*x + 1/2*c) + 72*A*a*b^2*tan(1/2*d*x + 1/2*c) + 36*B*
a*b^2*tan(1/2*d*x + 1/2*c) + 72*C*a*b^2*tan(1/2*d*x + 1/2*c) + 12*A*b^3*tan
(1/2*d*x + 1/2*c) + 24*B*b^3*tan(1/2*d*x + 1/2*c) + 15*C*b^3*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.958 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^2(c+dx)\,dx$

Optimal. Leaf size=192

$$\frac{b\sin(c+dx)\left(a^2(-(6A-8C))+9abB+b^2(3A+2C)\right)}{3d}+\frac{1}{2}x\left(6a^2bB+2a^3C+3ab^2(2A+C)+b^3B\right)+\frac{a^2(aB+3Ab)\tan}{d}$$

```
[Out] ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*(2*A + C))*x)/2 + (a^2*(3*A*b + a*B
)*ArcTanh[Sin[c + d*x]])/d + (b*(9*a*b*B - a^2*(6*A - 8*C) + b^2*(3*A + 2*C
))*Sin[c + d*x])/(3*d) - (b^2*(6*a*A - 3*b*B - 5*a*C)*Cos[c + d*x]*Sin[c +
d*x])/(6*d) - (b*(3*A - C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (A*
(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.586682, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3033, 3023, 2735, 3770}

$$\frac{b\sin(c+dx)\left(a^2(-(6A-8C))+9abB+b^2(3A+2C)\right)}{3d}+\frac{1}{2}x\left(6a^2bB+2a^3C+3ab^2(2A+C)+b^3B\right)+\frac{a^2(aB+3Ab)\tan}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```


```
[Out] ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*(2*A + C))*x)/2 + (a^2*(3*A*b + a*B
)*ArcTanh[Sin[c + d*x]])/d + (b*(9*a*b*B - a^2*(6*A - 8*C) + b^2*(3*A + 2*C
))*Sin[c + d*x])/(3*d) - (b^2*(6*a*A - 3*b*B - 5*a*C)*Cos[c + d*x]*Sin[c +
d*x])/(6*d) - (b*(3*A - C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (A*
(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\tan(c+dx)}{d} + \int (a$$

$$= -\frac{b(3A-C)(a+b\cos(c+dx))^2\sin(c+dx$$

$$= -\frac{b^2(6aA-3bB-5aC)\cos(c+dx)\sin(c+$$

$$= \frac{b\left(9abB-a^2(6A-8C)+b^2(3A+2C)\right)\sin$$

$$= \frac{1}{2}\left(6a^2bB+b^3B+2a^3C+3ab^2(2A+C)\right)x$$

$$= \frac{1}{2}\left(6a^2bB+b^3B+2a^3C+3ab^2(2A+C)\right)x$$

Mathematica [A] time = 1.23384, size = 266, normalized size = 1.39

$$\frac{6(c+dx)\left(6a^2bB+2a^3C+3ab^2(2A+C)+b^3B\right)+3b\sin(c+dx)\left(3\left(4a^2C+4abB+b^2C\right)+4Ab^2\right)-12a^2(aB+3Ab)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (6*(6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*(2*A + C))*(c + d*x) - 12*a^2*(3*
A*b + a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2*(3*A*b + a*B)*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a^3*A*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]) + (12*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 3*b*(4*A*b^2 + 3*(4*a*b*B + 4*a^2*C + b^2*C))*
Sin[c + d*x] + 3*b^2*(b*B + 3*a*C)*Sin[2*(c + d*x)] + b^3*C*Sin[3*(c + d*x)
])/(12*d)
```

Maple [A] time = 0.064, size = 278, normalized size = 1.5

$$\frac{Ab^3\sin(dx+c)}{d}+\frac{B\cos(dx+c)\,b^3\sin(dx+c)}{2d}+\frac{b^3Bx}{2}+\frac{b^3Bc}{2d}+\frac{C\sin(dx+c)\,(\cos(dx+c))^2\,b^3}{3d}+\frac{2\,Cb^3\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*b^3*sin(d*x+c)+1/2/d*b^3*B*cos(d*x+c)*sin(d*x+c)+1/2*b^3*B*x+1/2/d*b^
3*B*c+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*b^3+2/3/d*C*b^3*sin(d*x+c)+3*a*A*b^2*
x+3/d*A*a*b^2*c+3/d*a*b^2*B*sin(d*x+c)+3/2/d*C*a*b^2*cos(d*x+c)*sin(d*x+c)+
3/2*a*b^2*C*x+3/2/d*C*a*b^2*c+3/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b
*B*x+3/d*B*a^2*b*c+3/d*a^2*b*C*sin(d*x+c)+1/d*A*a^3*tan(d*x+c)+1/d*a^3*B*ln
(sec(d*x+c)+tan(d*x+c))+a^3*C*x+1/d*a^3*C*c
```

Maxima [A] time = 1.00017, size = 292, normalized size = 1.52

$$\frac{12\,(dx+c)Ca^3+36\,(dx+c)Ba^2b+36\,(dx+c)Aab^2+9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Cab^2+3\,(2\,dx+2\,c+\sin(2\,dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*C*a^3 + 36*(d*x + c)*B*a^2*b + 36*(d*x + c)*A*a*b^2 + 9*
(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c
))*B*b^3 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*b^3 + 6*B*a^3*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 18*A*a^2*b*(log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) + 36*C*a^2*b*sin(d*x + c) + 36*B*a*b^2*sin(d*x + c) +
 12*A*b^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.84635, size = 486, normalized size = 2.53

$$\frac{3\left(2\,Ca^3+6\,Ba^2b+3\,(2\,A+C)ab^2+Bb^3\right)dx\cos(dx+c)+3\left(Ba^3+3\,Aa^2b\right)\cos(dx+c)\log(\sin(dx+c)+1)-3\left(Ba^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*C*a^3 + 6*B*a^2*b + 3*(2*A + C)*a*b^2 + B*b^3)*d*x*cos(d*x + c) +
 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(B*a^3 + 3*A*
a^2*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*C*b^3*cos(d*x + c)^3 + 6*A*
a^3 + 3*(3*C*a*b^2 + B*b^3)*cos(d*x + c)^2 + 2*(9*C*a^2*b + 9*B*a*b^2 + (3*
A + 2*C)*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25768, size = 564, normalized size = 2.94

$$-\frac{\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\left(2\,Ca^3 + 6\,Ba^2b + 6\,Aab^2 + 3\,Cab^2 + Bb^3\right)(dx+c) - 6\left(Ba^3 + 3\,Aa^2b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="giac")
```

```
[Out] -1/6*(12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(2*C*a
^3 + 6*B*a^2*b + 6*A*a*b^2 + 3*C*a*b^2 + B*b^3)*(d*x + c) - 6*(B*a^3 + 3*A*
a^2*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(B*a^3 + 3*A*a^2*b)*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*(18*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b
^2*tan(1/2*d*x + 1/2*c)^5 - 9*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^3*tan(
1/2*d*x + 1/2*c)^5 - 3*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^3*tan(1/2*d*x +
 1/2*c)^5 + 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^2*tan(1/2*d*x + 1/
```

```
2*c)^3 + 12*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^3*tan(1/2*d*x + 1/2*c)^3 +
 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a*b^2*tan(1/2*d*x + 1/2*c) + 9*C*a*
b^2*tan(1/2*d*x + 1/2*c) + 6*A*b^3*tan(1/2*d*x + 1/2*c) + 3*B*b^3*tan(1/2*d
*x + 1/2*c) + 6*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

3.959 $\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{se}$ $dx)\,dx$

Optimal. Leaf size=204

$$-\frac{b\sin(c+dx)\left(4a^2B+9aAb-6abC-2b^2B\right)}{2d}+\frac{a\left(a^2(A+2C)+6abB+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}bx\left(6a^2C+6ab\right.$$

```
[Out] (b*(2*A*b^2 + 6*a*b*B + 6*a^2*C + b^2*C)*x)/2 + (a*(6*A*b^2 + 6*a*b*B + a^2
*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b*(9*a*A*b + 4*a^2*B - 2*b^2*B
- 6*a*b*C)*Sin[c + d*x])/(2*d) - (b^2*(4*A*b + 2*a*B - b*C)*Cos[c + d*x]*Si
n[c + d*x])/(2*d) + ((3*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/(
2*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.645678, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3033, 3023, 2735, 3770}

$$-\frac{b\sin(c+dx)\left(4a^2B+9aAb-6abC-2b^2B\right)}{2d}+\frac{a\left(a^2(A+2C)+6abB+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}bx\left(6a^2C+6ab\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```


```
[Out] (b*(2*A*b^2 + 6*a*b*B + 6*a^2*C + b^2*C)*x)/2 + (a*(6*A*b^2 + 6*a*b*B + a^2
*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b*(9*a*A*b + 4*a^2*B - 2*b^2*B
- 6*a*b*C)*Sin[c + d*x])/(2*d) - (b^2*(4*A*b + 2*a*B - b*C)*Cos[c + d*x]*Si
n[c + d*x])/(2*d) + ((3*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/(
2*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+}{2d}$$
$$= \frac{(3Ab+2aB)(a+b\cos(c+dx))^2\tan(c+}{2d}$$
$$= -\frac{b^2(4Ab+2aB-bC)\cos(c+dx)\sin(c+}{2d}$$
$$= -\frac{b\left(9aAb+4a^2B-2b^2B-6abC\right)\sin(c+}{2d}$$
$$= \frac{1}{2}b\left(2Ab^2+6abB+6a^2C+b^2C\right)x - \frac{b\left(9a\right.}{}$$
$$= \frac{1}{2}b\left(2Ab^2+6abB+6a^2C+b^2C\right)x + \frac{a\left(6\right.}{}$$

Mathematica [A] time = 3.20042, size = 318, normalized size = 1.56

$$2b(c+dx)\left(6a^2C+6abB+2Ab^2+b^2C\right)-2a\left(a^2(A+2C)+6abB+6Ab^2\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+2a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (2*b*(2*A*b^2 + 6*a*b*B + 6*a^2*C + b^2*C)*(c + d*x) - 2*a*(6*A*b^2 + 6*a*b
*B + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(6*A*b^2
 + 6*a*b*B + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3
*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a^2*(3*A*b + a*B)*Sin[(c +
 d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^3*A)/(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2 + (4*a^2*(3*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 4*b^2*(b*B + 3*a*C)*Sin[c + d*x] + b^3*C*Sin[2
*(c + d*x)])/(4*d)
```

Maple [A] time = 0.076, size = 267, normalized size = 1.3

$$Ab^3x + \frac{Ab^3c}{d} + \frac{b^3B\sin(dx+c)}{d} + \frac{Cb^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{b^3Cx}{2} + \frac{Cb^3c}{2d} + 3\frac{aAb^2\ln(\sec(dx+c)+\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] A*b^3*x+1/d*A*b^3*c+1/d*b^3*B*sin(d*x+c)+1/2/d*C*b^3*cos(d*x+c)*sin(d*x+c)+
1/2*b^3*C*x+1/2/d*C*b^3*c+3/d*a*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*B*x
+3/d*B*a*b^2*c+3/d*C*a*b^2*sin(d*x+c)+3/d*A*a^2*b*tan(d*x+c)+3/d*a^2*b*B*ln
(sec(d*x+c)+tan(d*x+c))+3*a^2*b*C*x+3/d*a^2*b*C*c+1/2/d*A*a^3*sec(d*x+c)*ta
n(d*x+c)+1/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*B*tan(d*x+c)+1/d*a^3
*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00006, size = 328, normalized size = 1.61

$$\frac{12\,(dx+c)Ca^2b+12\,(dx+c)Bab^2+4\,(dx+c)Ab^3+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Cb^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="maxima")
```

```
[Out] 1/4*(12*(d*x + c)*C*a^2*b + 12*(d*x + c)*B*a*b^2 + 4*(d*x + c)*A*b^3 + (2*d
*x + 2*c + sin(2*d*x + 2*c))*C*b^3 - A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a^3*(log(sin(d*
x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^2*b*(log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 12*C*a*b^2*sin(d*x + c) + 4*B*b^3*sin(d*x + c) + 4*B*a^3*tan(d*x +
 c) + 12*A*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 1.92889, size = 500, normalized size = 2.45

$$\frac{2\left(6\,Ca^2b+6\,Bab^2+(2\,A+C)b^3\right)dx\cos\left(dx+c\right)^2+\left((A+2\,C)a^3+6\,Ba^2b+6\,Aab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="fricas")
```

```
[Out] 1/4*(2*(6*C*a^2*b + 6*B*a*b^2 + (2*A + C)*b^3)*d*x*cos(d*x + c)^2 + ((A + 2
*C)*a^3 + 6*B*a^2*b + 6*A*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - ((A
```

```
 + 2*C)*a^3 + 6*B*a^2*b + 6*A*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(C*b^3*cos(d*x + c)^3 + A*a^3 + 2*(3*C*a*b^2 + B*b^3)*cos(d*x + c)^2 +
2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30229, size = 726, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="giac")
```

```
[Out] 1/2*((6*C*a^2*b + 6*B*a*b^2 + 2*A*b^3 + C*b^3)*(d*x + c) + (A*a^3 + 2*C*a^3
 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a^3 + 2*C
*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^3
*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 6*A*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 6*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*b^3*tan(1/2*d*x +
 1/2*c)^7 - C*b^3*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 -
 2*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*
b^2*tan(1/2*d*x + 1/2*c)^5 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*C*b^3*tan(1
/2*d*x + 1/2*c)^5 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x +
1/2*c)^3 + 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 6*C*a*b^2*tan(1/2*d*x + 1/2*c
)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*b^3*tan(1/2*d*x + 1/2*c)^3 + A*a
^3*tan(1/2*d*x + 1/2*c) + 2*B*a^3*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b*tan(1/2*
d*x + 1/2*c) + 6*C*a*b^2*tan(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c
) + C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.960 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \mathrm{s}$ $dx)\,dx$

Optimal. Leaf size=196

$$\frac{a\tan(c+dx)\left(a^2(2A+3C)+6abB+3Ab^2\right)}{3d}+\frac{\left(3a^2b(A+2C)+a^3B+6ab^2B+2Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\sin(c}{}$$

```
[Out] b^2*(b*B + 3*a*C)*x + ((2*A*b^3 + a^3*B + 6*a*b^2*B + 3*a^2*b*(A + 2*C))*Ar
cTanh[Sin[c + d*x]])/(2*d) - (b^2*(5*A*b + 3*a*B - 6*b*C)*Sin[c + d*x])/(6*
d) + (a*(3*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*d) + ((A*b +
 a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (A*(a + b*C
os[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.641592, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3031, 3023, 2735, 3770}

$$\frac{a\tan(c+dx)\left(a^2(2A+3C)+6abB+3Ab^2\right)}{3d}+\frac{\left(3a^2b(A+2C)+a^3B+6ab^2B+2Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```


```
[Out] b^2*(b*B + 3*a*C)*x + ((2*A*b^3 + a^3*B + 6*a*b^2*B + 3*a^2*b*(A + 2*C))*Ar
cTanh[Sin[c + d*x]])/(2*d) - (b^2*(5*A*b + 3*a*B - 6*b*C)*Sin[c + d*x])/(6*
d) + (a*(3*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*d) + ((A*b +
 a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (A*(a + b*C
os[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c}{3d}$$
$$= \frac{(Ab+aB)(a+b\cos(c+dx))^2\sec(c+dx}{2d}$$
$$= \frac{a\left(3Ab^2+6abB+a^2(2A+3C)\right)\tan(c+}{3d}$$
$$= -\frac{b^2(5Ab+3aB-6bC)\sin(c+dx)}{6d} + \frac{a\,(}{}$$
$$= b^2(bB+3aC)x - \frac{b^2(5Ab+3aB-6bC)\,\mathrm{s}}{6d}$$
$$= b^2(bB+3aC)x + \frac{\left(2Ab^3+a^3B+6ab^2B\right.}{}$$

Mathematica [B] time = 5.74256, size = 429, normalized size = 2.19

$$\frac{\frac{4a\sin\left(\frac{1}{2}(c+dx)\right)\left(a^2(2A+3C)+9abB+9Ab^2\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a\sin\left(\frac{1}{2}(c+dx)\right)\left(a^2(2A+3C)+9abB+9Ab^2\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} - 6\left(3a^2b(A+2C)+a^3B+6ab^2B+2Ab^3\right)\log\left(\mathrm{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (12*b^2*(b*B + 3*a*C)*(c + d*x) - 6*(2*A*b^3 + a^3*B + 6*a*b^2*B + 3*a^2*b*
(A + 2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(2*A*b^3 + a^3*B +
6*a*b^2*B + 3*a^2*b*(A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (
a^2*(9*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*a^3
*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (4*a*(9*A*b^
2 + 9*a*b*B + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3 - (a^2*(9*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
 + (4*a*(9*A*b^2 + 9*a*b*B + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 12*b^3*C*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.075, size = 294, normalized size = 1.5

$$\frac{Ab^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + b^3Bx + \frac{Bb^3c}{d} + \frac{Cb^3\sin(dx+c)}{d} + 3\frac{aAb^2\tan(dx+c)}{d} + 3\frac{ab^2B\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] 1/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+b^3*B*x+1/d*b^3*B*c+1/d*C*b^3*sin(d*x+c
)+3/d*a*A*b^2*tan(d*x+c)+3/d*a*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*C*x+
3/d*C*a*b^2*c+3/2/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(d*x+
c)+tan(d*x+c))+3/d*a^2*b*B*tan(d*x+c)+3/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))
+2/3/d*A*a^3*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^3*B*sec
(d*x+c)*tan(d*x+c)+1/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*C*tan(d*x+
c)
```

Maxima [A] time = 1.02746, size = 378, normalized size = 1.93

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}+36\left(dx+c\right)Cab^{2}+12\left(dx+c\right)Bb^{3}-3\,Ba^{3}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 36*(d*x + c)*C*a*b^2 + 12
*(d*x + c)*B*b^3 - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) - 9*A*a^2*b*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*C*a^2*b*(
log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*B*a*b^2*(log(sin(d*x +
c) + 1) - log(sin(d*x + c) - 1)) + 6*A*b^3*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) + 12*C*b^3*sin(d*x + c) + 12*C*a^3*tan(d*x + c) + 36*B*a^2*
b*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 2.06222, size = 543, normalized size = 2.77

$$\frac{12\left(3\,Cab^{2}+Bb^{3}\right)dx\cos\left(dx+c\right)^{3}+3\left(Ba^{3}+3\left(A+2\,C\right)a^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*C*a*b^2 + B*b^3)*d*x*cos(d*x + c)^3 + 3*(B*a^3 + 3*(A + 2*C)*a^
2*b + 6*B*a*b^2 + 2*A*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^3
+ 3*(A + 2*C)*a^2*b + 6*B*a*b^2 + 2*A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c)
 + 1) + 2*(6*C*b^3*cos(d*x + c)^3 + 2*A*a^3 + 2*((2*A + 3*C)*a^3 + 9*B*a^2*
b + 9*A*a*b^2)*cos(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25153, size = 591, normalized size = 3.02

$$\frac{\frac{12\,Cb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(3\,Cab^2+Bb^3\right)(dx+c)+3\left(Ba^3+3\,Aa^2b+6\,Ca^2b+6\,Bab^2+2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="giac")
```

```
[Out] 1/6*(12*C*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(3*C*a*
b^2 + B*b^3)*(d*x + c) + 3*(B*a^3 + 3*A*a^2*b + 6*C*a^2*b + 6*B*a*b^2 + 2*A
*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^3 + 3*A*a^2*b + 6*C*a^2*b
 + 6*B*a*b^2 + 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan
(1/2*d*x + 1/2*c)^5 - 3*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^3*tan(1/2*d*x
+ 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*b*tan(1/2*d*x + 1/
```

```
2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^3
 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36
*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3*B*a^3*ta
n(1/2*d*x + 1/2*c) + 6*C*a^3*tan(1/2*d*x + 1/2*c) + 9*A*a^2*b*tan(1/2*d*x +
 1/2*c) + 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.961 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \mathrm{s}$ $dx)\, dx$

Optimal. Leaf size=223

$$\frac{\tan(c+dx)\left(6a^2b(2A+3C)+4a^3B+16ab^2B+3Ab^3\right)}{6d}+\frac{\left(a^3(3A+4C)+12a^2bB+12ab^2(A+2C)+8b^3B\right)\tanh^{-1}(\mathrm{si}}{8d}$$

```
[Out] b^3*C*x + ((12*a^2*b*B + 8*b^3*B + 12*a*b^2*(A + 2*C) + a^3*(3*A + 4*C))*Ar
cTanh[Sin[c + d*x]])/(8*d) + ((3*A*b^3 + 4*a^3*B + 16*a*b^2*B + 6*a^2*b*(2*
A + 3*C))*Tan[c + d*x])/(6*d) + (a*(6*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 4*C))
*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A*b + 4*a*B)*(a + b*Cos[c + d*x])^
2*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d
*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.670789, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3031, 3021, 2735, 3770}

$$\frac{\tan(c+dx)\left(6a^2b(2A+3C)+4a^3B+16ab^2B+3Ab^3\right)}{6d}+\frac{\left(a^3(3A+4C)+12a^2bB+12ab^2(A+2C)+8b^3B\right)\tanh^{-1}(\mathrm{si}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```


```
[Out] b^3*C*x + ((12*a^2*b*B + 8*b^3*B + 12*a*b^2*(A + 2*C) + a^3*(3*A + 4*C))*Ar
cTanh[Sin[c + d*x]])/(8*d) + ((3*A*b^3 + 4*a^3*B + 16*a*b^2*B + 6*a^2*b*(2*
A + 3*C))*Tan[c + d*x])/(6*d) + (a*(6*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 4*C))
*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A*b + 4*a*B)*(a + b*Cos[c + d*x])^
2*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d
*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c}{4d}$$

$$= \frac{(3Ab+4aB)(a+b\cos(c+dx))^2\sec^2(c+}{12d}$$

$$= \frac{a\left(6Ab^2+20abB+3a^2(3A+4C)\right)\sec(c}{24d}$$

$$= \frac{\left(3Ab^3+4a^3B+16ab^2B+6a^2b(2A+3C\right.}{6d}$$

$$= b^3Cx + \frac{\left(3Ab^3+4a^3B+16ab^2B+6a^2b(\right.}{6d}$$

$$= b^3Cx + \frac{\left(12a^2bB+8b^3B+12ab^2(A+2C\right.}{}$$

Mathematica [A] time = 1.25263, size = 165, normalized size = 0.74

$$\frac{3\left(a^3(3A+4C)+12a^2bB+12ab^2(A+2C)+8b^3B\right)\tanh^{-1}(\sin(c+dx))+3\tan(c+dx)\left(a\sec(c+dx)\left(a^2(3A+4C)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (24*b^3*C*d*x + 3*(12*a^2*b*B + 8*b^3*B + 12*a*b^2*(A + 2*C) + a^3*(3*A + 4
*C))*ArcTanh[Sin[c + d*x]] + 3*(8*(A*b^3 + a^3*B + 3*a*b^2*B + 3*a^2*b*(A +
 C)) + a*(12*A*b^2 + 12*a*b*B + a^2*(3*A + 4*C))*Sec[c + d*x] + 2*a^3*A*Sec
[c + d*x]^3)*Tan[c + d*x] + 8*a^2*(3*A*b + a*B)*Tan[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.085, size = 389, normalized size = 1.7

$$\frac{Ab^3\tan(dx+c)}{d} + \frac{b^3B\ln(\sec(dx+c)+\tan(dx+c))}{d} + b^3Cx + \frac{Cb^3c}{d} + \frac{3\,aAb^2\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{3\,aAb^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/d*A*b^3*tan(d*x+c)+1/d*b^3*B*ln(sec(d*x+c)+tan(d*x+c))+b^3*C*x+1/d*C*b^3*
c+3/2/d*a*A*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*a*A*b^2*ln(sec(d*x+c)+tan(d*x+c
))+3/d*a*b^2*B*tan(d*x+c)+3/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a^2*b
*tan(d*x+c)+1/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^2*b*B*sec(d*x+c)*ta
n(d*x+c)+3/2/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*C*tan(d*x+c)+1/4
/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+3/8/d*A*
a^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^3*B*tan(d*x+c)+1/3/d*a^3*B*tan(d*x+c)
*sec(d*x+c)^2+1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*C*ln(sec(d*x+c)+t
an(d*x+c))
```

Maxima [A] time = 1.00369, size = 502, normalized size = 2.25

$$\frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^3+48\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^2b+48\left(dx+c\right)Cb^3-3\,Aa^3\left(\frac{2\left(3\,\sin(dx+c)^3 \cdots}{\sin(dx+c)^4-2 \cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^2*b + 48*(d*x + c)*C*b^3 - 3*A*a^3*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^
2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^2*b*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) + 72*C*a*b^2*(log(sin(d*x + c) + 1) - log(sin(
d*x + c) - 1)) + 24*B*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 144*C*a^2*b*tan(d*x + c) + 144*B*a*b^2*tan(d*x + c) + 48*A*b^3*tan(d*x + c
))/d
```

Fricas [A] time = 2.19594, size = 624, normalized size = 2.8

$$\frac{48\,Cb^3dx\cos\left(dx+c\right)^4+3\left(\left(3\,A+4\,C\right)a^3+12\,Ba^2b+12\,\left(A+2\,C\right)ab^2+8\,Bb^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="fricas")
```

```
[Out] 1/48*(48*C*b^3*d*x*cos(d*x + c)^4 + 3*((3*A + 4*C)*a^3 + 12*B*a^2*b + 12*(A
 + 2*C)*a*b^2 + 8*B*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*((3*A + 4
*C)*a^3 + 12*B*a^2*b + 12*(A + 2*C)*a*b^2 + 8*B*b^3)*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(6*A*a^3 + 8*(2*B*a^3 + 3*(2*A + 3*C)*a^2*b + 9*B*a*b^2
 + 3*A*b^3)*cos(d*x + c)^3 + 3*((3*A + 4*C)*a^3 + 12*B*a^2*b + 12*A*a*b^2)*
cos(d*x + c)^2 + 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29304, size = 1025, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*C*b^3 + 3*(3*A*a^3 + 4*C*a^3 + 12*B*a^2*b + 12*A*a*b^2 +
 24*C*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^3 + 4*
C*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 24*C*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x
 + 1/2*c)^7 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan(1/2*d*x + 1/
2*c)^7 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^2*b*tan(1/2*d*x + 1/2*c
)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^7
 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*B*
```

```
a^3*tan(1/2*d*x + 1/2*c)^5 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 120*A*a^2*b*
tan(1/2*d*x + 1/2*c)^5 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 216*C*a^2*b*ta
n(1/2*d*x + 1/2*c)^5 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 216*B*a*b^2*tan(
1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c
)^3 - 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^
3 - 216*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3
- 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 15
*A*a^3*tan(1/2*d*x + 1/2*c) + 24*B*a^3*tan(1/2*d*x + 1/2*c) + 12*C*a^3*tan(
1/2*d*x + 1/2*c) + 72*A*a^2*b*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b*tan(1/2*d*x
 + 1/2*c) + 72*C*a^2*b*tan(1/2*d*x + 1/2*c) + 36*A*a*b^2*tan(1/2*d*x + 1/2*
c) + 72*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*A*b^3*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.962 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ s dx) dx

Optimal. Leaf size=278

$$\frac{\tan(c+dx)\left(2a^3(4A+5C)+30a^2bB+15ab^2(2A+3C)+15b^3B\right)}{15d}+\frac{\left(3a^2b(3A+4C)+3a^3B+12ab^2B+4b^3(A+2C)\right.}{8d}$$

```
[Out] ((3*a^3*B + 12*a*b^2*B + 4*b^3*(A + 2*C) + 3*a^2*b*(3*A + 4*C))*ArcTanh[Sin
[c + d*x]])/(8*d) + ((30*a^2*b*B + 15*b^3*B + 15*a*b^2*(2*A + 3*C) + 2*a^3*
(4*A + 5*C))*Tan[c + d*x])/(15*d) + ((6*A*b^3 + 15*a^3*B + 50*a*b^2*B + 15*
a^2*b*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a*(3*A*b^2 + 15*a*b
*B + 2*a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + ((3*A*b + 5*a
*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*
Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.917961, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\tan(c+dx)\left(2a^3(4A+5C)+30a^2bB+15ab^2(2A+3C)+15b^3B\right)}{15d}+\frac{\left(3a^2b(3A+4C)+3a^3B+12ab^2B+4b^3(A+2C)\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```



```
[Out] ((3*a^3*B + 12*a*b^2*B + 4*b^3*(A + 2*C) + 3*a^2*b*(3*A + 4*C))*ArcTanh[Sin
[c + d*x]])/(8*d) + ((30*a^2*b*B + 15*b^3*B + 15*a*b^2*(2*A + 3*C) + 2*a^3*
(4*A + 5*C))*Tan[c + d*x])/(15*d) + ((6*A*b^3 + 15*a^3*B + 50*a*b^2*B + 15*
a^2*b*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a*(3*A*b^2 + 15*a*b
*B + 2*a^2*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + ((3*A*b + 5*a
*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*
Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c}{5d}$$
$$= \frac{(3Ab+5aB)(a+b\cos(c+dx))^2\sec^3(c+}{20d}$$
$$= \frac{a\left(3Ab^2+15abB+2a^2(4A+5C)\right)\sec^2(}{30d}$$
$$= \frac{\left(6Ab^3+15a^3B+50ab^2B+15a^2b(3A+\right.}{40d}$$
$$= \frac{\left(6Ab^3+15a^3B+50ab^2B+15a^2b(3A+\right.}{40d}$$
$$= \frac{\left(3a^3B+12ab^2B+4b^3(A+2C)+3a^2b(3\right.}{8d}$$
$$= \frac{\left(3a^3B+12ab^2B+4b^3(A+2C)+3a^2b(3\right.}{8d}$$

Mathematica [A] time = 4.6066, size = 204, normalized size = 0.73

$$15\left(3a^2b(3A+4C)+3a^3B+12ab^2B+4b^3(A+2C)\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5a\tan^2(c+dx)\left(a^2(2A+C\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (15*(3*a^3*B + 12*a*b^2*B + 4*b^3*(A + 2*C) + 3*a^2*b*(3*A + 4*C))*ArcTanh[
Sin[c + d*x]] + Tan[c + d*x]*(15*(4*A*b^3 + 3*a^3*B + 12*a*b^2*B + 3*a^2*b*
(3*A + 4*C))*Sec[c + d*x] + 30*a^2*(3*A*b + a*B)*Sec[c + d*x]^3 + 8*(15*(3*
a^2*b*B + b^3*B + a^3*(A + C) + 3*a*b^2*(A + C)) + 5*a*(3*A*b^2 + 3*a*b*B +
```

```
a^2*(2*A + C))*Tan[c + d*x]^2 + 3*a^3*A*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.079, size = 504, normalized size = 1.8

$$\frac{Ab^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ab^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{b^3B\tan(dx+c)}{d}+\frac{Cb^3\ln(\sec(dx+c)+\tan(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/2/d*A*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d
*b^3*B*tan(d*x+c)+1/d*C*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*A*b^2*tan(d*x+c
)+1/d*a*A*b^2*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a*b^2*B*sec(d*x+c)*tan(d*x+c)+3
/2/d*a*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*C*a*b^2*tan(d*x+c)+3/4/d*A*a^2*b
*tan(d*x+c)*sec(d*x+c)^3+9/8/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+9/8/d*A*a^2*b*
ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*b*B*tan(d*x+c)+1/d*a^2*b*B*tan(d*x+c)*sec
(d*x+c)^2+3/2/d*a^2*b*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*C*ln(sec(d*x+c)+t
an(d*x+c))+8/15/d*A*a^3*tan(d*x+c)+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+4/15
/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3+3/8/d*
a^3*B*sec(d*x+c)*tan(d*x+c)+3/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^3
*C*tan(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.01116, size = 610, normalized size = 2.19

$$16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3+80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^3+240\left(\tan(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d*
x + c))*B*a^2*b + 240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a*b^2 - 15*B*a^3*
(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 45*A*a^2*b*(2*(3
*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
```

```
3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*C*a^2*b*(2*sin(d*
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)
) - 180*B*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)) - 60*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*C*b^3*(log(sin(d*x +
 c) + 1) - log(sin(d*x + c) - 1)) + 720*C*a*b^2*tan(d*x + c) + 240*B*b^3*ta
n(d*x + c))/d
```

Fricas [A] time = 2.30307, size = 711, normalized size = 2.56

$$\frac{15\left(3\,Ba^3+3\,(3\,A+4\,C)a^2b+12\,Bab^2+4\,(A+2\,C)b^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ba^3+3\,(3\,A+4\,C)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(3*B*a^3 + 3*(3*A + 4*C)*a^2*b + 12*B*a*b^2 + 4*(A + 2*C)*b^3)*co
s(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*B*a^3 + 3*(3*A + 4*C)*a^2*b + 12
*B*a*b^2 + 4*(A + 2*C)*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(2
*(4*A + 5*C)*a^3 + 30*B*a^2*b + 15*(2*A + 3*C)*a*b^2 + 15*B*b^3)*cos(d*x +
c)^4 + 24*A*a^3 + 15*(3*B*a^3 + 3*(3*A + 4*C)*a^2*b + 12*B*a*b^2 + 4*A*b^3)
*cos(d*x + c)^3 + 8*((4*A + 5*C)*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x + c
)^2 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27234, size = 1335, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="giac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*C*a^2*b + 12*B*a*b^2 + 4*A*b^3 + 8*C*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*B*a^3 + 9*A*a^2*b + 12*C*a^2*
b + 12*B*a*b^2 + 4*A*b^3 + 8*C*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*
(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 120*C
*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^
2*b*tan(1/2*d*x + 1/2*c)^9 - 180*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b
^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 360*C*a*b^
2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(
1/2*d*x + 1/2*c)^9 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^3*tan(1/2*d*
x + 1/2*c)^7 - 320*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 90*A*a^2*b*tan(1/2*d*x +
1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 360*C*a^2*b*tan(1/2*d*x + 1
/2*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 360*B*a*b^2*tan(1/2*d*x + 1/
2*c)^7 - 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*A*b^3*tan(1/2*d*x + 1/2*
c)^7 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1/2*c)^5
+ 400*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 2160*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*B
*a^3*tan(1/2*d*x + 1/2*c)^3 - 320*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*A*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 360*C*a^2*b*
tan(1/2*d*x + 1/2*c)^3 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 360*B*a*b^2*t
an(1/2*d*x + 1/2*c)^3 - 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*b^3*tan
(1/2*d*x + 1/2*c)^3 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/2*
d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 120*C*a^3*tan(1/2*d*x + 1/2*
c) + 225*A*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) +
180*C*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c) + 180*B
*a*b^2*tan(1/2*d*x + 1/2*c) + 360*C*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*t
an(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 - 1)^5)/d
```

3.963 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^7(c+dx)\,dx$

Optimal. Leaf size=336

$$\frac{\tan(c+dx)\left(6a^2b(4A+5C)+8a^3B+30ab^2B+5b^3(2A+3C)\right)}{15d}+\frac{\left(a^3(5A+6C)+18a^2bB+6ab^2(3A+4C)+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{16d}$$

```
[Out] ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))*ArcTanh[Sin
[c + d*x]])/(16*d) + ((8*a^3*B + 30*a*b^2*B + 5*b^3*(2*A + 3*C) + 6*a^2*b*(
4*A + 5*C))*Tan[c + d*x])/(15*d) + ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A +
4*C) + a^3*(5*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((A*b^3 + 4*a^3
*B + 12*a*b^2*B + 3*a^2*b*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a*(6*A*b^2 + 42*a*b*B + 5*a^2*(5*A + 6*C))*Sec[c + d*x]^3*Tan[c + d*x])/
(120*d) + ((A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x]
)/(10*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.913314, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(6a^2b(4A+5C)+8a^3B+30ab^2B+5b^3(2A+3C)\right)}{15d}+\frac{\left(a^3(5A+6C)+18a^2bB+6ab^2(3A+4C)+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^7,x]
```


```
[Out] ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))*ArcTanh[Sin
[c + d*x]])/(16*d) + ((8*a^3*B + 30*a*b^2*B + 5*b^3*(2*A + 3*C) + 6*a^2*b*(
4*A + 5*C))*Tan[c + d*x])/(15*d) + ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A +
4*C) + a^3*(5*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((A*b^3 + 4*a^3
*B + 12*a*b^2*B + 3*a^2*b*(4*A + 5*C))*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a*(6*A*b^2 + 42*a*b*B + 5*a^2*(5*A + 6*C))*Sec[c + d*x]^3*Tan[c + d*x])/
(120*d) + ((A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x]
)/(10*d) + (A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c}{6d}$$

$$= \frac{(Ab+2aB)(a+b\cos(c+dx))^2\sec^4(c+}{10d}$$

$$= \frac{a\left(6Ab^2+42abB+5a^2(5A+6C)\right)\sec^3(}{120d}$$

$$= \frac{\left(Ab^3+4a^3B+12ab^2B+3a^2b(4A+5C)\right.}{15d}$$

$$= \frac{\left(Ab^3+4a^3B+12ab^2B+3a^2b(4A+5C)\right.}{15d}$$

$$= \frac{\left(18a^2bB+8b^3B+6ab^2(3A+4C)+a^3(5\right.}{16d}$$

$$= \frac{\left(18a^2bB+8b^3B+6ab^2(3A+4C)+a^3(5\right.}{16d}$$

Mathematica [A] time = 2.91056, size = 252, normalized size = 0.75

$$\frac{15\left(a^3(5A+6C)+18a^2bB+6ab^2(3A+4C)+8b^3B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(16\left(5\tan^2(c+dx)\left(3a^2b(2A\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^7,x]
```

```
[Out] (15*(18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))*ArcTanh[
Sin[c + d*x]] + Tan[c + d*x]*(15*(18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C
) + a^3*(5*A + 6*C))*Sec[c + d*x] + 10*a*(18*A*b^2 + 18*a*b*B + a^2*(5*A +
6*C))*Sec[c + d*x]^3 + 40*a^3*A*Sec[c + d*x]^5 + 16*(15*(a^3*B + 3*a*b^2*B
+ 3*a^2*b*(A + C) + b^3*(A + C)) + 5*(A*b^3 + 2*a^3*B + 3*a*b^2*B + 3*a^2*b
*(2*A + C))*Tan[c + d*x]^2 + 3*a^2*(3*A*b + a*B)*Tan[c + d*x]^4)))/(240*d)
```

Maple [A] time = 0.076, size = 644, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/d*a*b^2*B*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)^3+9
/8/d*a^2*b*B*sec(d*x+c)*tan(d*x+c)+3/4/d*a*A*b^2*tan(d*x+c)*sec(d*x+c)^3+3/
2/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)+3/5/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)^4+1/d
*a^2*b*C*tan(d*x+c)*sec(d*x+c)^2+9/8/d*a*A*b^2*sec(d*x+c)*tan(d*x+c)+4/5/d*
A*a^2*b*tan(d*x+c)*sec(d*x+c)^2+1/d*C*b^3*tan(d*x+c)+2/3/d*A*b^3*tan(d*x+c)
+1/2/d*b^3*B*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*a^3*B*tan(d*x+c)+3/8/d*a^3*C*
ln(sec(d*x+c)+tan(d*x+c))+5/16/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+9/8/d*a^2*
b*B*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*A*b^3*tan(d*x+c)*sec(d*x+c)^2+3/2/d*C*a
*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*b*C*tan(d*x+c)+9/8/d*a*A*b^2*ln(sec(
d*x+c)+tan(d*x+c))+8/5/d*A*a^2*b*tan(d*x+c)+1/5/d*a^3*B*tan(d*x+c)*sec(d*x+
c)^4+4/15/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2+1/6/d*A*a^3*tan(d*x+c)*sec(d*x+c)
^5+1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+5/24/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3
+3/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+5/16/d*A*a^3*sec(d*x+c)*tan(d*x+c)+2/d*a
*b^2*B*tan(d*x+c)+1/2/d*b^3*B*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.06592, size = 763, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 +
96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2*b + 480*(
tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b + 480*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*B*a*b^2 + 160*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^3 - 5*A*a^3*(2*(1
5*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3
*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log
(sin(d*x + c) - 1)) - 30*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 90*B*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 90*A*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4
 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1
)) - 360*C*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) - 120*B*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*C*b^3*tan(d*x + c)
)/d
```

Fricas [A] time = 2.01562, size = 824, normalized size = 2.45

$$\frac{15\left((5A+6C)a^3+18Ba^2b+6(3A+4C)ab^2+8Bb^3\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left((5A+6C)a^3+18Ba^2 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="fricas")
```

```
[Out] 1/480*(15*((5*A + 6*C)*a^3 + 18*B*a^2*b + 6*(3*A + 4*C)*a*b^2 + 8*B*b^3)*co
s(d*x + c)^6*log(sin(d*x + c) + 1) - 15*((5*A + 6*C)*a^3 + 18*B*a^2*b + 6*(
3*A + 4*C)*a*b^2 + 8*B*b^3)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(
8*B*a^3 + 6*(4*A + 5*C)*a^2*b + 30*B*a*b^2 + 5*(2*A + 3*C)*b^3)*cos(d*x + c
)^5 + 15*((5*A + 6*C)*a^3 + 18*B*a^2*b + 6*(3*A + 4*C)*a*b^2 + 8*B*b^3)*cos
(d*x + c)^4 + 40*A*a^3 + 16*(4*B*a^3 + 3*(4*A + 5*C)*a^2*b + 15*B*a*b^2 + 5
*A*b^3)*cos(d*x + c)^3 + 10*((5*A + 6*C)*a^3 + 18*B*a^2*b + 18*A*a*b^2)*cos
(d*x + c)^2 + 48*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36199, size = 1850, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="giac")
```

```
[Out] 1/240*(15*(5*A*a^3 + 6*C*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 24*C*a*b^2 + 8*B*b
^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*A*a^3 + 6*C*a^3 + 18*B*a^2*b
 + 18*A*a*b^2 + 24*C*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
2*(165*A*a^3*tan(1/2*d*x + 1/2*c)^11 - 240*B*a^3*tan(1/2*d*x + 1/2*c)^11 +
150*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*A*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 4
50*B*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 720*C*a^2*b*tan(1/2*d*x + 1/2*c)^11 +
450*A*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 720*B*a*b^2*tan(1/2*d*x + 1/2*c)^11 +
 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 240*A*b^3*tan(1/2*d*x + 1/2*c)^11 +
120*B*b^3*tan(1/2*d*x + 1/2*c)^11 - 240*C*b^3*tan(1/2*d*x + 1/2*c)^11 + 25*
A*a^3*tan(1/2*d*x + 1/2*c)^9 + 560*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 210*C*a^3
*tan(1/2*d*x + 1/2*c)^9 + 1680*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 630*B*a^2*b
*tan(1/2*d*x + 1/2*c)^9 + 2640*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 630*A*a*b^2
*tan(1/2*d*x + 1/2*c)^9 + 2640*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 1080*C*a*b^
2*tan(1/2*d*x + 1/2*c)^9 + 880*A*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*B*b^3*tan
(1/2*d*x + 1/2*c)^9 + 1200*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^3*tan(1/2
*d*x + 1/2*c)^7 - 1248*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^3*tan(1/2*d*x
+ 1/2*c)^7 - 3744*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*B*a^2*b*tan(1/2*d*x
+ 1/2*c)^7 - 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*A*a*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 4320*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*C*a*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 1440*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*B*b^3*tan(1/2*d*x + 1/
2*c)^7 - 2400*C*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^3*tan(1/2*d*x + 1/2*c)
^5 + 1248*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^5 +
```

```
3744*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 180*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 180*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
4320*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
1440*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 2400
*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 560*B*a^3
*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*A*a^2*b*t
an(1/2*d*x + 1/2*c)^3 - 630*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2640*C*a^2*b*t
an(1/2*d*x + 1/2*c)^3 - 630*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2640*B*a*b^2*t
an(1/2*d*x + 1/2*c)^3 - 1080*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 880*A*b^3*tan
(1/2*d*x + 1/2*c)^3 - 360*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 1200*C*b^3*tan(1/2
*d*x + 1/2*c)^3 + 165*A*a^3*tan(1/2*d*x + 1/2*c) + 240*B*a^3*tan(1/2*d*x +
1/2*c) + 150*C*a^3*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b*tan(1/2*d*x + 1/2*c)
+ 450*B*a^2*b*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b*tan(1/2*d*x + 1/2*c) + 450
*A*a*b^2*tan(1/2*d*x + 1/2*c) + 720*B*a*b^2*tan(1/2*d*x + 1/2*c) + 360*C*a*
b^2*tan(1/2*d*x + 1/2*c) + 240*A*b^3*tan(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1
/2*d*x + 1/2*c) + 240*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^6)/d
```

3.964 $\int \cos(c+dx)(a+b\cos(c+dx))^4 \left(A + B\cos(c + dx) + C\cos^2\right.$

Optimal. Leaf size=445

$$\frac{\sin(c+dx)\left(84a^2b^2(5A+4C)+35a^4(3A+2C)+280a^3bB+224ab^3B+8b^4(7A+6C)\right)}{105d}+\frac{\sin(c+dx)\cos^2(c+dx)\left(4a^2\right.}{}$$

```
[Out] ((8*a^4*B + 36*a^2*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5
*C))*x)/16 + ((280*a^3*b*B + 224*a*b^3*B + 35*a^4*(3*A + 2*C) + 84*a^2*b^2*
(5*A + 4*C) + 8*b^4*(7*A + 6*C))*Sin[c + d*x])/(105*d) + ((8*a^4*B + 36*a^2
*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5*C))*Cos[c + d*x]*
Sin[c + d*x])/(16*d) + ((91*a^3*b*B + 112*a*b^3*B + 4*a^4*C + 4*b^4*(7*A +
6*C) + 3*a^2*b^2*(63*A + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])/(105*d) + (b*(
336*a^2*b*B + 175*b^3*B + 24*a^3*C + 4*a*b^2*(126*A + 103*C))*Cos[c + d*x]^
3*Sin[c + d*x])/(840*d) + ((14*A*b^2 + 21*a*b*B + 4*a^2*C + 12*b^2*C)*Cos[c
 + d*x]^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(70*d) + ((7*b*B + 4*a*C)*Co
s[c + d*x]^2*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(42*d) + (C*Cos[c + d*x]^
2*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.04241, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3049, 3033, 3023, 2734}

$$\frac{\sin(c+dx)\left(84a^2b^2(5A+4C)+35a^4(3A+2C)+280a^3bB+224ab^3B+8b^4(7A+6C)\right)}{105d}+\frac{\sin(c+dx)\cos^2(c+dx)\left(4a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```



```
[Out] ((8*a^4*B + 36*a^2*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5
*C))*x)/16 + ((280*a^3*b*B + 224*a*b^3*B + 35*a^4*(3*A + 2*C) + 84*a^2*b^2*
(5*A + 4*C) + 8*b^4*(7*A + 6*C))*Sin[c + d*x])/(105*d) + ((8*a^4*B + 36*a^2
*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5*C))*Cos[c + d*x]*
Sin[c + d*x])/(16*d) + ((91*a^3*b*B + 112*a*b^3*B + 4*a^4*C + 4*b^4*(7*A +
6*C) + 3*a^2*b^2*(63*A + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])/(105*d) + (b*(
336*a^2*b*B + 175*b^3*B + 24*a^3*C + 4*a*b^2*(126*A + 103*C))*Cos[c + d*x]^
3*Sin[c + d*x])/(840*d) + ((14*A*b^2 + 21*a*b*B + 4*a^2*C + 12*b^2*C)*Cos[c
 + d*x]^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(70*d) + ((7*b*B + 4*a*C)*Co
s[c + d*x]^2*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(42*d) + (C*Cos[c + d*x]^
```

```
2*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(7*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{C\cos^2(c+dx)(a+b\cos(c+dx))^4\sin(c+}{7d}$$
$$= \frac{(7bB+4aC)\cos^2(c+dx)(a+b\cos(c+dx}{42d}$$
$$= \frac{\left(14Ab^2+21abB+4a^2C+12b^2C\right)\cos^2(c}{70d}$$
$$= \frac{b\left(336a^2bB+175b^3B+24a^3C+4ab^2(126\right.}{840d}$$
$$= \frac{\left(91a^3bB+112ab^3B+4a^4C+4b^4(7A+6C\right.}{}$$
$$= \frac{1}{16}\left(8a^4B+36a^2b^2B+5b^4B+8a^3b(4A+\right.$$

Mathematica [A] time = 1.42587, size = 528, normalized size = 1.19

$$\frac{105\sin(c+dx)\left(48a^2b^2(6A+5C)+16a^4(4A+3C)+192a^3bB+160ab^3B+5b^4(8A+7C)\right)+105\sin(2(c+dx))\left(64a^3b(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (13440*a^3*A*b*c + 10080*a*A*b^3*c + 3360*a^4*B*c + 15120*a^2*b^2*B*c + 210
0*b^4*B*c + 10080*a^3*b*c*C + 8400*a*b^3*c*C + 13440*a^3*A*b*d*x + 10080*a*
A*b^3*d*x + 3360*a^4*B*d*x + 15120*a^2*b^2*B*d*x + 2100*b^4*B*d*x + 10080*a
^3*b*C*d*x + 8400*a*b^3*C*d*x + 105*(192*a^3*b*B + 160*a*b^3*B + 16*a^4*(4*
A + 3*C) + 48*a^2*b^2*(6*A + 5*C) + 5*b^4*(8*A + 7*C))*Sin[c + d*x] + 105*(
16*a^4*B + 96*a^2*b^2*B + 15*b^4*B + 64*a^3*b*(A + C) + 4*a*b^3*(16*A + 15*
C))*Sin[2*(c + d*x)] + 3360*a^2*A*b^2*Sin[3*(c + d*x)] + 700*A*b^4*Sin[3*(c
 + d*x)] + 2240*a^3*b*B*Sin[3*(c + d*x)] + 2800*a*b^3*B*Sin[3*(c + d*x)] +
560*a^4*C*Sin[3*(c + d*x)] + 4200*a^2*b^2*C*Sin[3*(c + d*x)] + 735*b^4*C*Si
n[3*(c + d*x)] + 840*a*A*b^3*Sin[4*(c + d*x)] + 1260*a^2*b^2*B*Sin[4*(c + d
*x)] + 315*b^4*B*Sin[4*(c + d*x)] + 840*a^3*b*C*Sin[4*(c + d*x)] + 1260*a*b
^3*C*Sin[4*(c + d*x)] + 84*A*b^4*Sin[5*(c + d*x)] + 336*a*b^3*B*Sin[5*(c +
d*x)] + 504*a^2*b^2*C*Sin[5*(c + d*x)] + 147*b^4*C*Sin[5*(c + d*x)] + 35*b^
4*B*Sin[6*(c + d*x)] + 140*a*b^3*C*Sin[6*(c + d*x)] + 15*b^4*C*Sin[7*(c + d
*x)])/(6720*d)
```

Maple [A] time = 0.024, size = 505, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^4*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1
/7*C*b^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+4
*a*A*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*a
*b^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*C*a*b^3*(1/6*(cos(d
*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+2*a^2
*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^2*b^2*B*(1/4*(cos(d*x+c)^3+3/2*cos(d
*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*a^2*b^2*C*(8/3+cos(d*x+c)^4+4/3*cos(d*
x+c)^2)*sin(d*x+c)+4*A*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4/3*
a^3*b*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^3*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*
x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^4*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.04286, size = 672, normalized size = 1.51

$$\frac{1680\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^4-2240\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ca^4+6720\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/6720*(1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 2240*(sin(d*x + c)^3
- 3*sin(d*x + c))*C*a^4 + 6720*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3*b - 8
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3*b + 840*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3*b - 13440*(sin(d*x + c)^3 - 3*sin(d*
x + c))*A*a^2*b^2 + 1260*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
2*c))*B*a^2*b^2 + 2688*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
 c))*C*a^2*b^2 + 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*A*a*b^3 + 1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B
*a*b^3 - 140*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*C*a*b^3 + 448*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 1
5*sin(d*x + c))*A*b^4 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*
```

```
d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*
x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*C*b^4 + 6720*A*a^4*sin(d*x
+ c))/d
```

Fricas [A] time = 2.08945, size = 852, normalized size = 1.91

$$\frac{105\left(8\,Ba^4+8\,(4\,A+3\,C)a^3b+36\,Ba^2b^2+4\,(6\,A+5\,C)ab^3+5\,Bb^4\right)dx+\left(240\,Cb^4\cos\left(dx+c\right)^6+280\left(4\,Cab^3+Bb^4\right)c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/1680*(105*(8*B*a^4 + 8*(4*A + 3*C)*a^3*b + 36*B*a^2*b^2 + 4*(6*A + 5*C)*a
*b^3 + 5*B*b^4)*d*x + (240*C*b^4*cos(d*x + c)^6 + 280*(4*C*a*b^3 + B*b^4)*c
os(d*x + c)^5 + 560*(3*A + 2*C)*a^4 + 4480*B*a^3*b + 1344*(5*A + 4*C)*a^2*b
^2 + 3584*B*a*b^3 + 128*(7*A + 6*C)*b^4 + 48*(42*C*a^2*b^2 + 28*B*a*b^3 + (
7*A + 6*C)*b^4)*cos(d*x + c)^4 + 70*(24*C*a^3*b + 36*B*a^2*b^2 + 4*(6*A + 5
*C)*a*b^3 + 5*B*b^4)*cos(d*x + c)^3 + 16*(35*C*a^4 + 140*B*a^3*b + 42*(5*A
+ 4*C)*a^2*b^2 + 112*B*a*b^3 + 4*(7*A + 6*C)*b^4)*cos(d*x + c)^2 + 105*(8*B
*a^4 + 8*(4*A + 3*C)*a^3*b + 36*B*a^2*b^2 + 4*(6*A + 5*C)*a*b^3 + 5*B*b^4)*
cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 13.1221, size = 1334, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Piecewise((A*a**4*sin(c + d*x)/d + 2*A*a**3*b*x*sin(c + d*x)**2 + 2*A*a**3*
b*x*cos(c + d*x)**2 + 2*A*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*A*a**2*b**
2*sin(c + d*x)**3/d + 6*A*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a*
b**3*x*sin(c + d*x)**4/2 + 3*A*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3
*A*a*b**3*x*cos(c + d*x)**4/2 + 3*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 5*A*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*A*b**4*sin(c + d*x)*
```

```
*5/(15*d) + 4*A*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**4*sin(c +
 d*x)*cos(c + d*x)**4/d + B*a**4*x*sin(c + d*x)**2/2 + B*a**4*x*cos(c + d*x
)**2/2 + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*B*a**3*b*sin(c + d*x)**
3/(3*d) + 4*B*a**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**2*b**2*x*sin(c
 + d*x)**4/4 + 9*B*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*B*a**2
*b**2*x*cos(c + d*x)**4/4 + 9*B*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d
) + 15*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*B*a*b**3*sin(c +
 d*x)**5/(15*d) + 16*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*B*a
*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*b**4*x*sin(c + d*x)**6/16 + 15*B
*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**4*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*B*b**4*x*cos(c + d*x)**6/16 + 5*B*b**4*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + 5*B*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*
B*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*C*a**4*sin(c + d*x)**3/(3*d)
 + C*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*a**3*b*x*sin(c + d*x)**4/2 +
 3*C*a**3*b*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*C*a**3*b*x*cos(c + d*x)**
4/2 + 3*C*a**3*b*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*C*a**3*b*sin(c + d*
x)*cos(c + d*x)**3/(2*d) + 16*C*a**2*b**2*sin(c + d*x)**5/(5*d) + 8*C*a**2*
b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 6*C*a**2*b**2*sin(c + d*x)*cos(c +
 d*x)**4/d + 5*C*a*b**3*x*sin(c + d*x)**6/4 + 15*C*a*b**3*x*sin(c + d*x)**4
*cos(c + d*x)**2/4 + 15*C*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/4 + 5*C*
a*b**3*x*cos(c + d*x)**6/4 + 5*C*a*b**3*sin(c + d*x)**5*cos(c + d*x)/(4*d)
+ 10*C*a*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*C*a*b**3*sin(c + d
*x)*cos(c + d*x)**5/(4*d) + 16*C*b**4*sin(c + d*x)**7/(35*d) + 8*C*b**4*sin
(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*C*b**4*sin(c + d*x)**3*cos(c + d*x)*
*4/d + C*b**4*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a + b*cos(c))*
*4*(A + B*cos(c) + C*cos(c)**2)*cos(c), True))
```

Giac [A] time = 1.21344, size = 527, normalized size = 1.18

$$\frac{Cb^4 \sin(7\,dx + 7\,c)}{448\,d} + \frac{1}{16}\left(8\,Ba^4 + 32\,Aa^3b + 24\,Ca^3b + 36\,Ba^2b^2 + 24\,Aab^3 + 20\,Cab^3 + 5\,Bb^4\right)x + \frac{\left(4\,Cab^3 + Bb^4\right)\sin}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/448*C*b^4*sin(7*d*x + 7*c)/d + 1/16*(8*B*a^4 + 32*A*a^3*b + 24*C*a^3*b +
36*B*a^2*b^2 + 24*A*a*b^3 + 20*C*a*b^3 + 5*B*b^4)*x + 1/192*(4*C*a*b^3 + B*
b^4)*sin(6*d*x + 6*c)/d + 1/320*(24*C*a^2*b^2 + 16*B*a*b^3 + 4*A*b^4 + 7*C*
b^4)*sin(5*d*x + 5*c)/d + 1/64*(8*C*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3 + 12*C
*a*b^3 + 3*B*b^4)*sin(4*d*x + 4*c)/d + 1/192*(16*C*a^4 + 64*B*a^3*b + 96*A*
```

```
a^2*b^2 + 120*C*a^2*b^2 + 80*B*a*b^3 + 20*A*b^4 + 21*C*b^4)*sin(3*d*x + 3*c
)/d + 1/64*(16*B*a^4 + 64*A*a^3*b + 64*C*a^3*b + 96*B*a^2*b^2 + 64*A*a*b^3
+ 60*C*a*b^3 + 15*B*b^4)*sin(2*d*x + 2*c)/d + 1/64*(64*A*a^4 + 48*C*a^4 + 1
92*B*a^3*b + 288*A*a^2*b^2 + 240*C*a^2*b^2 + 160*B*a*b^3 + 40*A*b^4 + 35*C*
b^4)*sin(d*x + c)/d
```

3.965 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=375

$$\frac{\sin(c+dx)\left(a^3b^2(190A+121C)+224a^2b^3B+24a^4bB-4a^5C+32ab^4(5A+4C)+32b^5B\right)}{60bd}+\frac{\sin(c+dx)\left(24a^2bB-4a\right.}{}$$

```
[Out] ((32*a^3*b*B + 24*a*b^3*B + 8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*
(6*A + 5*C))*x)/16 + ((24*a^4*b*B + 224*a^2*b^3*B + 32*b^5*B - 4*a^5*C + 32
*a*b^4*(5*A + 4*C) + a^3*b^2*(190*A + 121*C))*Sin[c + d*x])/(60*b*d) + ((48
*a^3*b*B + 232*a*b^3*B - 8*a^4*C + 15*b^4*(6*A + 5*C) + 2*a^2*b^2*(130*A +
89*C))*Cos[c + d*x]*Sin[c + d*x])/(240*d) + ((24*a^2*b*B + 32*b^3*B - 4*a^3
*C + a*b^2*(70*A + 53*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) +
((5*b^2*(6*A + 5*C) + 4*a*(6*b*B - a*C))*(a + b*Cos[c + d*x])^3*Sin[c + d*x
])/(120*b*d) + ((6*b*B - a*C)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d)
 + (C*(a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.682631, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3023, 2753, 2734}

$$\frac{\sin(c+dx)\left(a^3b^2(190A+121C)+224a^2b^3B+24a^4bB-4a^5C+32ab^4(5A+4C)+32b^5B\right)}{60bd}+\frac{\sin(c+dx)\left(24a^2bB-4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] ((32*a^3*b*B + 24*a*b^3*B + 8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*
(6*A + 5*C))*x)/16 + ((24*a^4*b*B + 224*a^2*b^3*B + 32*b^5*B - 4*a^5*C + 32
*a*b^4*(5*A + 4*C) + a^3*b^2*(190*A + 121*C))*Sin[c + d*x])/(60*b*d) + ((48
*a^3*b*B + 232*a*b^3*B - 8*a^4*C + 15*b^4*(6*A + 5*C) + 2*a^2*b^2*(130*A +
89*C))*Cos[c + d*x]*Sin[c + d*x])/(240*d) + ((24*a^2*b*B + 32*b^3*B - 4*a^3
*C + a*b^2*(70*A + 53*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) +
((5*b^2*(6*A + 5*C) + 4*a*(6*b*B - a*C))*(a + b*Cos[c + d*x])^3*Sin[c + d*x
])/(120*b*d) + ((6*b*B - a*C)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d)
 + (C*(a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C(a+b\cos(c+dx))^5\sin(c+dx)}{6bd}+\frac{\int (a+b\cos(c+dx}{}$$

$$=\frac{(6bB-aC)(a+b\cos(c+dx))^4\sin(c+dx)}{30bd}+\frac{C(a+b\,c}{}$$

$$=\frac{\left(5b^2(6A+5C)+4a(6bB-aC)\right)(a+b\cos(c+dx))^3\,\mathrm{si}}{120bd}$$

$$=\frac{\left(24a^2bB+32b^3B-4a^3C+ab^2(70A+53C)\right)(a+b\,\mathrm{co}}{120bd}$$

$$=\frac{1}{16}\left(32a^3bB+24ab^3B+8a^4(2A+C)+12a^2b^2(4A+3\right.$$

Mathematica [A] time = 1.35124, size = 432, normalized size = 1.15

$$\frac{120\sin(c+dx)\left(8a^3b(4A+3C)+36a^2b^2B+8a^4B+4ab^3(6A+5C)+5b^4B\right)+15\sin(2(c+dx))\left(96a^2b^2(A+C)+64a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (960*a^4*A*c + 2880*a^2*A*b^2*c + 360*A*b^4*c + 1920*a^3*b*B*c + 1440*a*b^3
*B*c + 480*a^4*c*C + 2160*a^2*b^2*c*C + 300*b^4*c*C + 960*a^4*A*d*x + 2880*
a^2*A*b^2*d*x + 360*A*b^4*d*x + 1920*a^3*b*B*d*x + 1440*a*b^3*B*d*x + 480*a
^4*C*d*x + 2160*a^2*b^2*C*d*x + 300*b^4*C*d*x + 120*(8*a^4*B + 36*a^2*b^2*B
 + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5*C))*Sin[c + d*x] + 15*(
64*a^3*b*B + 64*a*b^3*B + 16*a^4*C + 96*a^2*b^2*(A + C) + b^4*(16*A + 15*C)
)*Sin[2*(c + d*x)] + 320*a*A*b^3*Sin[3*(c + d*x)] + 480*a^2*b^2*B*Sin[3*(c
+ d*x)] + 100*b^4*B*Sin[3*(c + d*x)] + 320*a^3*b*C*Sin[3*(c + d*x)] + 400*a
*b^3*C*Sin[3*(c + d*x)] + 30*A*b^4*Sin[4*(c + d*x)] + 120*a*b^3*B*Sin[4*(c
+ d*x)] + 180*a^2*b^2*C*Sin[4*(c + d*x)] + 45*b^4*C*Sin[4*(c + d*x)] + 12*b
^4*B*Sin[5*(c + d*x)] + 48*a*b^3*C*Sin[5*(c + d*x)] + 5*b^4*C*Sin[6*(c + d*
x)])/(960*d)
```

Maple [A] time = 0.022, size = 431, normalized size = 1.2

$$\frac{1}{d}\left(Cb^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{b^4B\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*b^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
/5*C*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*b^4*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a*b^3*B*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*a^2*b^2*C*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a*A*b^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+2*a^2*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a^3*b*C*(2+cos(d*x+c)^
2)*sin(d*x+c)+6*a^2*A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^3*b
*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*C*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+4*A*a^3*b*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

Maxima [A] time = 0.994822, size = 560, normalized size = 1.49

$$\frac{960\,(dx+c)Aa^4+240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^4+960\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3b-1280\left(\sin(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/960*(960*(d*x + c)*A*a^4 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 9
60*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b - 1280*(sin(d*x + c)^3 - 3*sin(
d*x + c))*C*a^3*b + 1440*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^2 - 1920*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b^2 + 180*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2*b^2 - 1280*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*A*a*b^3 + 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*B*a*b^3 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*
a*b^3 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^4 +
64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^4 - 5*(4*si
n(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c)
)*C*b^4 + 960*B*a^4*sin(d*x + c) + 3840*A*a^3*b*sin(d*x + c))/d
```

Fricas [A] time = 1.92701, size = 698, normalized size = 1.86

$$15\left(8\,(2\,A+C)a^4+32\,Ba^3b+12\,(4\,A+3\,C)a^2b^2+24\,Bab^3+(6\,A+5\,C)b^4\right)dx+\left(40\,Cb^4\cos\left(dx+c\right)^5+240\,Ba^4+320\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(8*(2*A + C)*a^4 + 32*B*a^3*b + 12*(4*A + 3*C)*a^2*b^2 + 24*B*a*b
^3 + (6*A + 5*C)*b^4)*d*x + (40*C*b^4*cos(d*x + c)^5 + 240*B*a^4 + 320*(3*A
 + 2*C)*a^3*b + 960*B*a^2*b^2 + 128*(5*A + 4*C)*a*b^3 + 128*B*b^4 + 48*(4*C
*a*b^3 + B*b^4)*cos(d*x + c)^4 + 10*(36*C*a^2*b^2 + 24*B*a*b^3 + (6*A + 5*C
)*b^4)*cos(d*x + c)^3 + 32*(10*C*a^3*b + 15*B*a^2*b^2 + 2*(5*A + 4*C)*a*b^3
 + 2*B*b^4)*cos(d*x + c)^2 + 15*(8*C*a^4 + 32*B*a^3*b + 12*(4*A + 3*C)*a^2*
b^2 + 24*B*a*b^3 + (6*A + 5*C)*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 7.48967, size = 1066, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*a**4*x + 4*A*a**3*b*sin(c + d*x)/d + 3*A*a**2*b**2*x*sin(c + d
*x)**2 + 3*A*a**2*b**2*x*cos(c + d*x)**2 + 3*A*a**2*b**2*sin(c + d*x)*cos(c
 + d*x)/d + 8*A*a*b**3*sin(c + d*x)**3/(3*d) + 4*A*a*b**3*sin(c + d*x)*cos(
c + d*x)**2/d + 3*A*b**4*x*sin(c + d*x)**4/8 + 3*A*b**4*x*sin(c + d*x)**2*c
os(c + d*x)**2/4 + 3*A*b**4*x*cos(c + d*x)**4/8 + 3*A*b**4*sin(c + d*x)**3*
cos(c + d*x)/(8*d) + 5*A*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**4*s
in(c + d*x)/d + 2*B*a**3*b*x*sin(c + d*x)**2 + 2*B*a**3*b*x*cos(c + d*x)**2
 + 2*B*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*B*a**2*b**2*sin(c + d*x)**3/d
 + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*b**3*x*sin(c + d*x)
**4/2 + 3*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*B*a*b**3*x*cos(c +
 d*x)**4/2 + 3*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a*b**3*sin
(c + d*x)*cos(c + d*x)**3/(2*d) + 8*B*b**4*sin(c + d*x)**5/(15*d) + 4*B*b**
4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**4*sin(c + d*x)*cos(c + d*x)*
*4/d + C*a**4*x*sin(c + d*x)**2/2 + C*a**4*x*cos(c + d*x)**2/2 + C*a**4*sin
(c + d*x)*cos(c + d*x)/(2*d) + 8*C*a**3*b*sin(c + d*x)**3/(3*d) + 4*C*a**3*
b*sin(c + d*x)*cos(c + d*x)**2/d + 9*C*a**2*b**2*x*sin(c + d*x)**4/4 + 9*C*
a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*C*a**2*b**2*x*cos(c + d*x
)**4/4 + 9*C*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*C*a**2*b**2*
sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*C*a*b**3*sin(c + d*x)**5/(15*d) + 1
6*C*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*C*a*b**3*sin(c + d*x)*
cos(c + d*x)**4/d + 5*C*b**4*x*sin(c + d*x)**6/16 + 15*C*b**4*x*sin(c + d*x
)**4*cos(c + d*x)**2/16 + 15*C*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
5*C*b**4*x*cos(c + d*x)**6/16 + 5*C*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d
) + 5*C*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*b**4*sin(c + d*x)
*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**4*(A + B*cos(c) + C*
cos(c)**2), True))
```

Giac [A] time = 1.1937, size = 440, normalized size = 1.17

$$\frac{Cb^4\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(16\,Aa^4+8\,Ca^4+32\,Ba^3b+48\,Aa^2b^2+36\,Ca^2b^2+24\,Bab^3+6\,Ab^4+5\,Cb^4\right)x+\frac{\left(4\,Cab^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/192*C*b^4*sin(6*d*x + 6*c)/d + 1/16*(16*A*a^4 + 8*C*a^4 + 32*B*a^3*b + 48
*A*a^2*b^2 + 36*C*a^2*b^2 + 24*B*a*b^3 + 6*A*b^4 + 5*C*b^4)*x + 1/80*(4*C*a
*b^3 + B*b^4)*sin(5*d*x + 5*c)/d + 1/64*(12*C*a^2*b^2 + 8*B*a*b^3 + 2*A*b^4
```

```
 + 3*C*b^4)*sin(4*d*x + 4*c)/d + 1/48*(16*C*a^3*b + 24*B*a^2*b^2 + 16*A*a*b
^3 + 20*C*a*b^3 + 5*B*b^4)*sin(3*d*x + 3*c)/d + 1/64*(16*C*a^4 + 64*B*a^3*b
 + 96*A*a^2*b^2 + 96*C*a^2*b^2 + 64*B*a*b^3 + 16*A*b^4 + 15*C*b^4)*sin(2*d*
x + 2*c)/d + 1/8*(8*B*a^4 + 32*A*a^3*b + 24*C*a^3*b + 36*B*a^2*b^2 + 24*A*a
*b^3 + 20*C*a*b^3 + 5*B*b^4)*sin(d*x + c)/d
```

3.966 $\int(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=290

$$\frac{\sin(c+dx)\left(2a^2b^2(85A+56C)+95a^3bB+12a^4C+80ab^3B+4b^4(5A+4C)\right)}{30d}+\frac{\sin(c+dx)\left(12a^2C+35abB+20Ab^2\right.}{60d}$$

```
[Out] ((8*a^4*B + 24*a^2*b^2*B + 3*b^4*B + 16*a^3*b*(2*A + C) + 4*a*b^3*(4*A + 3*
C))*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + ((95*a^3*b*B + 80*a*b^3*B + 12
*a^4*C + 4*b^4*(5*A + 4*C) + 2*a^2*b^2*(85*A + 56*C))*Sin[c + d*x])/(30*d)
+ (b*(130*a^2*b*B + 45*b^3*B + 24*a^3*C + 4*a*b^2*(40*A + 29*C))*Cos[c + d*
x]*Sin[c + d*x])/(120*d) + ((20*A*b^2 + 35*a*b*B + 12*a^2*C + 16*b^2*C)*(a
+ b*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + ((5*b*B + 4*a*C)*(a + b*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d
)
```

Rubi [A] time = 0.905955, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3049, 3033, 3023, 2735, 3770}

$$\frac{\sin(c+dx)\left(2a^2b^2(85A+56C)+95a^3bB+12a^4C+80ab^3B+4b^4(5A+4C)\right)}{30d}+\frac{\sin(c+dx)\left(12a^2C+35abB+20Ab^2\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```


```
[Out] ((8*a^4*B + 24*a^2*b^2*B + 3*b^4*B + 16*a^3*b*(2*A + C) + 4*a*b^3*(4*A + 3*
C))*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + ((95*a^3*b*B + 80*a*b^3*B + 12
*a^4*C + 4*b^4*(5*A + 4*C) + 2*a^2*b^2*(85*A + 56*C))*Sin[c + d*x])/(30*d)
+ (b*(130*a^2*b*B + 45*b^3*B + 24*a^3*C + 4*a*b^2*(40*A + 29*C))*Cos[c + d*
x]*Sin[c + d*x])/(120*d) + ((20*A*b^2 + 35*a*b*B + 12*a^2*C + 16*b^2*C)*(a
+ b*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + ((5*b*B + 4*a*C)*(a + b*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d
)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(a+b\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}\int($$

$$= \frac{(5bB+4aC)(a+b\cos(c+dx))^3\sin(c+d}{20d}$$

$$= \frac{\left(20Ab^2+35abB+12a^2C+16b^2C\right)(a+b}{60d}$$

$$= \frac{b\left(130a^2bB+45b^3B+24a^3C+4ab^2(40A\right.}{120d}$$

$$= \frac{\left(95a^3bB+80ab^3B+12a^4C+4b^4(5A+4\right.}{30d}$$

$$= \frac{1}{8}\left(8a^4B+24a^2b^2B+3b^4B+16a^3b(2A+\right.$$

$$= \frac{1}{8}\left(8a^4B+24a^2b^2B+3b^4B+16a^3b(2A+\right.$$

Mathematica [A] time = 1.22174, size = 382, normalized size = 1.32

$$\frac{60\sin(c+dx)\left(12a^2b^2(4A+3C)+32a^3bB+8a^4C+24ab^3B+b^4(6A+5C)\right)+120b\sin(2(c+dx))\left(6a^2bB+4a^3C+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (1920*a^3*A*b*c + 960*a*A*b^3*c + 480*a^4*B*c + 1440*a^2*b^2*B*c + 180*b^4*
B*c + 960*a^3*b*c*C + 720*a*b^3*c*C + 1920*a^3*A*b*d*x + 960*a*A*b^3*d*x +
480*a^4*B*d*x + 1440*a^2*b^2*B*d*x + 180*b^4*B*d*x + 960*a^3*b*C*d*x + 720*
a*b^3*C*d*x - 480*a^4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 480*a^4*
A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*(32*a^3*b*B + 24*a*b^3*B +
8*a^4*C + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*Sin[c + d*x] + 120*b*(6
*a^2*b*B + b^3*B + 4*a^3*C + 4*a*b^2*(A + C))*Sin[2*(c + d*x)] + 40*A*b^4*S
in[3*(c + d*x)] + 160*a*b^3*B*Sin[3*(c + d*x)] + 240*a^2*b^2*C*Sin[3*(c + d
*x)] + 50*b^4*C*Sin[3*(c + d*x)] + 15*b^4*B*Sin[4*(c + d*x)] + 60*a*b^3*C*S
in[4*(c + d*x)] + 6*b^4*C*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.063, size = 543, normalized size = 1.9

$$\frac{3\,ab^3Cx}{2}+2\,\frac{A\cos\left(dx+c\right)ab^3\sin\left(dx+c\right)}{d}+\frac{Cab^3\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^3}{d}+\frac{3\,C\cos\left(dx+c\right)ab^3\sin\left(dx+c\right)}{2\,d}+2\,\frac{C\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] 3/2*a*b^3*C*x+2/d*a*A*b^3*cos(d*x+c)*sin(d*x+c)+1/d*C*a*b^3*sin(d*x+c)*cos(
d*x+c)^3+3/2/d*C*a*b^3*cos(d*x+c)*sin(d*x+c)+2/d*C*cos(d*x+c)^2*sin(d*x+c)*
a^2*b^2+2/d*a^3*b*C*cos(d*x+c)*sin(d*x+c)+3/8/d*b^4*B*c+3*a^2*b^2*B*x+2/3/d
*A*b^4*sin(d*x+c)+8/15/d*C*b^4*sin(d*x+c)+2*a*A*b^3*x+4*A*a^3*b*x+2*a^3*b*C
*x+1/d*a^4*B*c+1/d*a^4*C*sin(d*x+c)+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+2/d
*a*A*b^3*c+3/2/d*C*a*b^3*c+4/d*A*a^3*b*c+2/d*a^3*b*C*c+1/5/d*C*b^4*sin(d*x+
c)*cos(d*x+c)^4+4/15/d*C*b^4*sin(d*x+c)*cos(d*x+c)^2+6/d*a^2*A*b^2*sin(d*x+
c)+4/d*a^2*b^2*C*sin(d*x+c)+1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*b^4+a^4*B*x+3/8
*b^4*B*x+4/d*a^3*b*B*sin(d*x+c)+3/d*a^2*b^2*B*c+1/4/d*b^4*B*sin(d*x+c)*cos(
d*x+c)^3+3/8/d*b^4*B*cos(d*x+c)*sin(d*x+c)+8/3/d*a*b^3*B*sin(d*x+c)+4/3/d*B
*sin(d*x+c)*cos(d*x+c)^2*a*b^3+3/d*a^2*b^2*B*cos(d*x+c)*sin(d*x+c)
```

Maxima [A] time = 1.01442, size = 459, normalized size = 1.58

$$\frac{480\,(dx+c)Ba^4+1920\,(dx+c)Aa^3b+480\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ca^3b+720\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^2b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/480*(480*(d*x + c)*B*a^4 + 1920*(d*x + c)*A*a^3*b + 480*(2*d*x + 2*c + si
n(2*d*x + 2*c))*C*a^3*b + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b^2 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b^2 + 480*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a*b^3 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^3 + 60*(1
2*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a*b^3 - 160*(sin(d*
x + c)^3 - 3*sin(d*x + c))*A*b^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*B*b^4 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*C*b^4 + 480*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 480*C*a^4*
sin(d*x + c) + 1920*B*a^3*b*sin(d*x + c) + 2880*A*a^2*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.96246, size = 640, normalized size = 2.21

$$\frac{60\,Aa^4\log\left(\sin\left(dx+c\right)+1\right)-60\,Aa^4\log\left(-\sin\left(dx+c\right)+1\right)+15\left(8\,Ba^4+16\,(2\,A+C)a^3b+24\,Ba^2b^2+4\,(4\,A+3\,C)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="fricas")
```

```
[Out] 1/120*(60*A*a^4*log(sin(d*x + c) + 1) - 60*A*a^4*log(-sin(d*x + c) + 1) + 1
5*(8*B*a^4 + 16*(2*A + C)*a^3*b + 24*B*a^2*b^2 + 4*(4*A + 3*C)*a*b^3 + 3*B*
b^4)*d*x + (24*C*b^4*cos(d*x + c)^4 + 120*C*a^4 + 480*B*a^3*b + 240*(3*A +
2*C)*a^2*b^2 + 320*B*a*b^3 + 16*(5*A + 4*C)*b^4 + 30*(4*C*a*b^3 + B*b^4)*co
s(d*x + c)^3 + 8*(30*C*a^2*b^2 + 20*B*a*b^3 + (5*A + 4*C)*b^4)*cos(d*x + c)
^2 + 15*(16*C*a^3*b + 24*B*a^2*b^2 + 4*(4*A + 3*C)*a*b^3 + 3*B*b^4)*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.33528, size = 1477, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,
algorithm="giac")
```

[Out] 1/120*(120*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 120*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 15*(8*B*a^4 + 32*A*a^3*b + 16*C*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 12*C*a*b^3 + 3*B*b^4)*(d*x + c) + 2*(120*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 240*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 300*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 75*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 120*C*b^4*tan(1/2*d*x + 1/2*c)^9 + 480*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 1920*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 2880*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 1920*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1280*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 120*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 320*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 30*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 160*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 720*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 2880*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 4320*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 2400*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1600*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 400*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 464*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 480*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 1920*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 2880*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 1920*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 1280*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 320*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 160*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*C*a^4*tan(1/2*d*x + 1/2*c) + 480*B*a^3*b*tan(1/2*d*x + 1/2*c) + 240*C*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x + 1/2*c) + 300*C*a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c) + 75*B*b^4*tan(1/2*d*x + 1/2*c) + 120*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d

3.967 $\int (a+b\cos(c+dx))^4 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \text{s}$ $dx)\, dx$

Optimal. Leaf size=273

$$\frac{b\sin(c+dx)\left(a^3(-(12A-19C))+34a^2bB+8ab^2(3A+2C)+4b^3B\right)}{6d}+\frac{b^2\sin(c+dx)\cos(c+dx)\left(a^2(-(24A-26C))+\right.}{24d}$$

```
[Out] ((32*a^3*b*B + 16*a*b^3*B + 8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C
))*x)/8 + (a^3*(4*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*(34*a^2*b*B + 4*
b^3*B - a^3*(12*A - 19*C) + 8*a*b^2*(3*A + 2*C))*Sin[c + d*x])/(6*d) + (b^2
*(32*a*b*B - a^2*(24*A - 26*C) + 3*b^2*(4*A + 3*C))*Cos[c + d*x]*Sin[c + d*
x])/(24*d) - (b*(12*a*A - 4*b*B - 7*a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x
])/(12*d) - (b*(4*A - C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (A*(a
 + b*Cos[c + d*x])^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.906428, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3033, 3023, 2735, 3770}

$$\frac{b\sin(c+dx)\left(a^3(-(12A-19C))+34a^2bB+8ab^2(3A+2C)+4b^3B\right)}{6d}+\frac{b^2\sin(c+dx)\cos(c+dx)\left(a^2(-(24A-26C))+\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```



```
[Out] ((32*a^3*b*B + 16*a*b^3*B + 8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C
))*x)/8 + (a^3*(4*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*(34*a^2*b*B + 4*
b^3*B - a^3*(12*A - 19*C) + 8*a*b^2*(3*A + 2*C))*Sin[c + d*x])/(6*d) + (b^2
*(32*a*b*B - a^2*(24*A - 26*C) + 3*b^2*(4*A + 3*C))*Cos[c + d*x]*Sin[c + d*
x])/(24*d) - (b*(12*a*A - 4*b*B - 7*a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x
])/(12*d) - (b*(4*A - C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (A*(a
 + b*Cos[c + d*x])^4*Tan[c + d*x])/d
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
```

```
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\tan(c+dx)}{d} + \int (\ldots$$

$$= -\frac{b(4A-C)(a+b\cos(c+dx))^3\sin(c+\ldots}{4d}$$

$$= -\frac{b(12aA-4bB-7aC)(a+b\cos(c+dx)\ldots}{12d}$$

$$= \frac{b^2\left(32abB-a^2(24A-26C)+3b^2(4A+\ldots\right.}{24d}$$

$$= \frac{b\left(34a^2bB+4b^3B-a^3(12A-19C)+8a\ldots\right.}{6d}$$

$$= \frac{1}{8}\left(32a^3bB+16ab^3B+8a^4C+24a^2b^2(2\ldots\right.$$

$$= \frac{1}{8}\left(32a^3bB+16ab^3B+8a^4C+24a^2b^2(2\ldots\right.$$

Mathematica [A] time = 2.9465, size = 383, normalized size = 1.4

$$\frac{32b\sin(c+dx)\left(36a^2bB+24a^3C+4ab^2(6A+5C)+5b^3B\right)+b^2\sec(c+dx)\left(3\sin(3(c+dx))\left(48a^2C+32abB+8Ab^2\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (32*b*(36*a^2*b*B + 5*b^3*B + 24*a^3*C + 4*a*b^2*(6*A + 5*C))*Sin[c + d*x]
+ b^2*Sec[c + d*x]*(3*(8*A*b^2 + 32*a*b*B + 48*a^2*C + 9*b^2*C)*Sin[3*(c +
d*x)] + b*(8*(b*B + 4*a*C)*Sin[4*(c + d*x)] + 3*b*C*Sin[5*(c + d*x)])) + 24
*(48*a^2*A*b^2*c + 4*A*b^4*c + 32*a^3*b*B*c + 16*a*b^3*B*c + 8*a^4*c*C + 24
*a^2*b^2*c*C + 3*b^4*c*C + 48*a^2*A*b^2*d*x + 4*A*b^4*d*x + 32*a^3*b*B*d*x
```

```
+ 16*a*b^3*B*d*x + 8*a^4*C*d*x + 24*a^2*b^2*C*d*x + 3*b^4*C*d*x - 8*a^3*(4*
A*b + a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 32*a^3*A*b*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 8*a^4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + (8*a^4*A + 4*a*b^3*B + 6*a^2*b^2*C + b^4*(A + C))*Tan[c + d*x]))/(
192*d)
```

Maple [A] time = 0.072, size = 434, normalized size = 1.6

$$a^4Cx + \frac{4\,C\sin(dx+c)(\cos(dx+c))^2\,ab^3}{3\,d} + 3\,\frac{C\cos(dx+c)\,a^2b^2\sin(dx+c)}{d} + \frac{2\,b^4B\sin(dx+c)}{3\,d} + 4\,a^3bBx + 2\,ab^3Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] a^4*C*x+4/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a*b^3+3/d*a^2*b^2*C*cos(d*x+c)*sin(
d*x+c)+2/3/d*b^4*B*sin(d*x+c)+4*a^3*b*B*x+2*a*b^3*B*x+1/2/d*A*b^4*c+3/8/d*C
*b^4*c+6*a^2*A*b^2*x+3*a^2*b^2*C*x+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*
A*a^4*tan(d*x+c)+1/d*a^4*C*c+3/8*b^4*C*x+1/4/d*C*b^4*sin(d*x+c)*cos(d*x+c)^
3+3/8/d*C*b^4*cos(d*x+c)*sin(d*x+c)+4/d*a*A*b^3*sin(d*x+c)+8/3/d*C*a*b^3*si
n(d*x+c)+4/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b*C*sin(d*x+c)+1/2/d
*A*b^4*cos(d*x+c)*sin(d*x+c)+6/d*A*a^2*b^2*c+3/d*a^2*b^2*C*c+1/2*A*b^4*x+2/
d*a*b^3*B*cos(d*x+c)*sin(d*x+c)+2/d*a*b^3*B*c+4/d*B*a^3*b*c+6/d*a^2*b^2*B*s
in(d*x+c)+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*b^4
```

Maxima [A] time = 1.00064, size = 412, normalized size = 1.51

$$\frac{96\,(dx+c)Ca^4 + 384\,(dx+c)Ba^3b + 576\,(dx+c)Aa^2b^2 + 144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^2b^2 + 96\,(2\,dx+2\,c+\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*C*a^4 + 384*(d*x + c)*B*a^3*b + 576*(d*x + c)*A*a^2*b^2
+ 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2*b^2 + 96*(2*d*x + 2*c + sin(2*
d*x + 2*c))*B*a*b^3 - 128*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a*b^3 + 24*(2
*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))
*B*b^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*b^4 +
```

```
48*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*A*a^3*b*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 384*C*a^3*b*sin(d*x + c) + 57
6*B*a^2*b^2*sin(d*x + c) + 384*A*a*b^3*sin(d*x + c) + 96*A*a^4*tan(d*x + c)
)/d
```

Fricas [A] time = 2.03138, size = 636, normalized size = 2.33

$$3\left(8\,Ca^4 + 32\,Ba^3b + 24\,(2\,A + C)a^2b^2 + 16\,Bab^3 + (4\,A + 3\,C)b^4\right)dx\cos\left(dx + c\right) + 12\left(Ba^4 + 4\,Aa^3b\right)\cos\left(dx + c\right)\log\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*C*a^4 + 32*B*a^3*b + 24*(2*A + C)*a^2*b^2 + 16*B*a*b^3 + (4*A +
3*C)*b^4)*d*x*cos(d*x + c) + 12*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)*log(sin(d*
x + c) + 1) - 12*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
(6*C*b^4*cos(d*x + c)^4 + 24*A*a^4 + 8*(4*C*a*b^3 + B*b^4)*cos(d*x + c)^3 +
 3*(24*C*a^2*b^2 + 16*B*a*b^3 + (4*A + 3*C)*b^4)*cos(d*x + c)^2 + 16*(6*C*a
^3*b + 9*B*a^2*b^2 + 2*(3*A + 2*C)*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31515, size = 1083, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x
, algorithm="giac")
```

```
[Out] -1/24*(48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(8*C*
a^4 + 32*B*a^3*b + 48*A*a^2*b^2 + 24*C*a^2*b^2 + 16*B*a*b^3 + 4*A*b^4 + 3*C
*b^4)*(d*x + c) - 24*(B*a^4 + 4*A*a^3*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 + 24*(B*a^4 + 4*A*a^3*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*C*a^3*
b*tan(1/2*d*x + 1/2*c)^7 + 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^2*
b^2*tan(1/2*d*x + 1/2*c)^7 + 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 48*B*a*b^3
*tan(1/2*d*x + 1/2*c)^7 + 96*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 12*A*b^4*tan(
1/2*d*x + 1/2*c)^7 + 24*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*C*b^4*tan(1/2*d*x
 + 1/2*c)^7 + 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 432*B*a^2*b^2*tan(1/2*d*
x + 1/2*c)^5 - 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 288*A*a*b^3*tan(1/2*d*
x + 1/2*c)^5 - 48*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*C*a*b^3*tan(1/2*d*x
+ 1/2*c)^5 - 12*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 40*B*b^4*tan(1/2*d*x + 1/2*c
)^5 + 9*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 +
 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 48*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 +
 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 40*
B*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 96*C*a^3*b*
tan(1/2*d*x + 1/2*c) + 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 72*C*a^2*b^2*ta
n(1/2*d*x + 1/2*c) + 96*A*a*b^3*tan(1/2*d*x + 1/2*c) + 48*B*a*b^3*tan(1/2*d
*x + 1/2*c) + 96*C*a*b^3*tan(1/2*d*x + 1/2*c) + 12*A*b^4*tan(1/2*d*x + 1/2*
c) + 24*B*b^4*tan(1/2*d*x + 1/2*c) + 15*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.968 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ s $dx)\,dx$

Optimal. Leaf size=274

$$-\frac{b\sin(c+dx)\left(a^2b(39A-34C)+12a^3B-24ab^2B-2b^3(3A+2C)\right)}{6d}+\frac{a^2\left(a^2(A+2C)+8abB+12Ab^2\right)\tanh^{-1}(\sin(c}{2d}$$

```
[Out] (b*(12*a^2*b*B + b^3*B + 8*a^3*C + 4*a*b^2*(2*A + C))*x)/2 + (a^2*(12*A*b^2
 + 8*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b*(12*a^3*B - 2
4*a*b^2*B + a^2*b*(39*A - 34*C) - 2*b^3*(3*A + 2*C))*Sin[c + d*x])/(6*d) -
(b^2*(6*a^2*B - 3*b^2*B + 2*a*b*(9*A - 4*C))*Cos[c + d*x]*Sin[c + d*x])/(6*
d) - (b*(15*A*b + 6*a*B - 2*b*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(6*d)
 + ((2*A*b + a*B)*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d + (A*(a + b*Cos[c
+ d*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.973756, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\sin(c+dx)\left(a^2b(39A-34C)+12a^3B-24ab^2B-2b^3(3A+2C)\right)}{6d}+\frac{a^2\left(a^2(A+2C)+8abB+12Ab^2\right)\tanh^{-1}(\sin(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```


```
[Out] (b*(12*a^2*b*B + b^3*B + 8*a^3*C + 4*a*b^2*(2*A + C))*x)/2 + (a^2*(12*A*b^2
 + 8*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b*(12*a^3*B - 2
4*a*b^2*B + a^2*b*(39*A - 34*C) - 2*b^3*(3*A + 2*C))*Sin[c + d*x])/(6*d) -
(b^2*(6*a^2*B - 3*b^2*B + 2*a*b*(9*A - 4*C))*Cos[c + d*x]*Sin[c + d*x])/(6*
d) - (b*(15*A*b + 6*a*B - 2*b*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(6*d)
 + ((2*A*b + a*B)*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d + (A*(a + b*Cos[c
+ d*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
```

```
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec(c+dx)\tan(c+}{2d}$$

$$= \frac{(2Ab+aB)(a+b\cos(c+dx))^3\tan(c+}{d}$$

$$= -\frac{b(15Ab+6aB-2bC)(a+b\cos(c+dx}{6d}$$

$$= -\frac{b^2\left(6a^2B-3b^2B+2ab(9A-4C)\right)\cos(}{6d}$$

$$= -\frac{b\left(12a^3B-24ab^2B+a^2b(39A-34C)-\right.}{6d}$$

$$= \frac{1}{2}b\left(12a^2bB+b^3B+8a^3C+4ab^2(2A+\right.$$

$$= \frac{1}{2}b\left(12a^2bB+b^3B+8a^3C+4ab^2(2A+\right.$$

Mathematica [A] time = 4.73348, size = 367, normalized size = 1.34

$$\frac{6b(c+dx)\left(12a^2bB+8a^3C+4ab^2(2A+C)+b^3B\right)+3b^2\sin(c+dx)\left(24a^2C+16abB+4Ab^2+3b^2C\right)-6a^2\left(a^2(A+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (6*b*(12*a^2*b*B + b^3*B + 8*a^3*C + 4*a*b^2*(2*A + C))*(c + d*x) - 6*a^2*(
12*A*b^2 + 8*a*b*B + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + 6*a^2*(12*A*b^2 + 8*a*b*B + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(
```

```
c + d*x)/2]] + (3*a^4*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*a^3*
(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (3*
a^4*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (12*a^3*(4*A*b + a*B)*Sin[
(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 3*b^2*(4*A*b^2 + 16*a
*b*B + 24*a^2*C + 3*b^2*C)*Sin[c + d*x] + 3*b^3*(b*B + 4*a*C)*Sin[2*(c + d*
x)] + b^4*C*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.079, size = 374, normalized size = 1.4

$$\frac{Ab^4\sin(dx+c)}{d}+\frac{b^4B\cos(dx+c)\sin(dx+c)}{2d}+\frac{b^4Bx}{2}+\frac{b^4Bc}{2d}+\frac{Cb^4\sin(dx+c)(\cos(dx+c))^2}{3d}+\frac{2Cb^4\sin(dx+c)}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*b^4*sin(d*x+c)+1/2/d*b^4*B*cos(d*x+c)*sin(d*x+c)+1/2*b^4*B*x+1/2/d*b^
4*B*c+1/3/d*C*b^4*sin(d*x+c)*cos(d*x+c)^2+2/3/d*C*b^4*sin(d*x+c)+4*a*A*b^3*
x+4/d*a*A*b^3*c+4/d*a*b^3*B*sin(d*x+c)+2/d*C*a*b^3*cos(d*x+c)*sin(d*x+c)+2*
a*b^3*C*x+2/d*C*a*b^3*c+6/d*a^2*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*B
*x+6/d*a^2*b^2*B*c+6/d*a^2*b^2*C*sin(d*x+c)+4/d*A*a^3*b*tan(d*x+c)+4/d*a^3*
b*B*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b*C*x+4/d*a^3*b*C*c+1/2/d*A*a^4*sec(d*x
+c)*tan(d*x+c)+1/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*B*tan(d*x+c)+1
/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01502, size = 420, normalized size = 1.53

$$\frac{48(dx+c)Ca^3b+72(dx+c)Ba^2b^2+48(dx+c)Aab^3+12(2dx+2c+\sin(2dx+2c))Cab^3+3(2dx+2c+\sin(2dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="maxima")
```

```
[Out] 1/12*(48*(d*x + c)*C*a^3*b + 72*(d*x + c)*B*a^2*b^2 + 48*(d*x + c)*A*a*b^3
+ 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b^3 + 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*b^4 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*b^4 - 3*A*a^4*(2*sin(
d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
```

```
1)) + 6*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*B*a^3*b*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*A*a^2*b^2*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 72*C*a^2*b^2*sin(d*x + c) + 48*B*a*b^
3*sin(d*x + c) + 12*A*b^4*sin(d*x + c) + 12*B*a^4*tan(d*x + c) + 48*A*a^3*b
*tan(d*x + c))/d
```

Fricas [A] time = 2.07179, size = 628, normalized size = 2.29

$$\frac{6\left(8\,Ca^3b + 12\,Ba^2b^2 + 4\,(2\,A + C)ab^3 + Bb^4\right)dx\cos\left(dx + c\right)^2 + 3\left((A + 2\,C)a^4 + 8\,Ba^3b + 12\,Aa^2b^2\right)\cos\left(dx + c\right)^2\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(8*C*a^3*b + 12*B*a^2*b^2 + 4*(2*A + C)*a*b^3 + B*b^4)*d*x*cos(d*x
+ c)^2 + 3*((A + 2*C)*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - 3*((A + 2*C)*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)
^2*log(-sin(d*x + c) + 1) + 2*(2*C*b^4*cos(d*x + c)^4 + 3*A*a^4 + 3*(4*C*a*
b^3 + B*b^4)*cos(d*x + c)^3 + 2*(18*C*a^2*b^2 + 12*B*a*b^3 + (3*A + 2*C)*b^
4)*cos(d*x + c)^2 + 6*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29781, size = 730, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(8*C*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3 + 4*C*a*b^3 + B*b^4)*(d*x + c)
 + 3*(A*a^4 + 2*C*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 3*(A*a^4 + 2*C*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) + 6*(A*a^4*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + A*a^4*tan(1/2*d*x + 1/2*c)
+ 2*B*a^4*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 - 1)^2 + 2*(36*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b^3*
tan(1/2*d*x + 1/2*c)^5 - 12*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^4*tan(1/
2*d*x + 1/2*c)^5 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^4*tan(1/2*d*x + 1
/2*c)^5 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 48*B*a*b^3*tan(1/2*d*x + 1/
2*c)^3 + 12*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^4*tan(1/2*d*x + 1/2*c)^3 +
 36*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*B*a*b^3*tan(1/2*d*x + 1/2*c) + 12*C
*a*b^3*tan(1/2*d*x + 1/2*c) + 6*A*b^4*tan(1/2*d*x + 1/2*c) + 3*B*b^4*tan(1/
2*d*x + 1/2*c) + 6*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^3)/d
```

3.969 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^4(c+dx)\,dx$

Optimal. Leaf size=303

$$-\frac{b\sin(c+dx)\left(4a^3(2A+3C)+39a^2bB+4ab^2(11A-6C)-6b^3B\right)}{6d}+\frac{a\left(4a^2b(A+2C)+a^3B+12ab^2B+8Ab^3\right)\tanh^{-1}}{2d}$$

```
[Out] (b^2*(2*A*b^2 + 8*a*b*B + 12*a^2*C + b^2*C)*x)/2 + (a*(8*A*b^3 + a^3*B + 12
*a*b^2*B + 4*a^2*b*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b*(39*a^2*b*B
 - 6*b^3*B + 4*a*b^2*(11*A - 6*C) + 4*a^3*(2*A + 3*C))*Sin[c + d*x])/(6*d)
- (b^2*(18*a*b*B + 3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Cos[c + d*x]*Sin[c +
d*x])/(6*d) + ((12*A*b^2 + 15*a*b*B + a^2*(4*A + 6*C))*(a + b*Cos[c + d*x])
^2*Tan[c + d*x])/(6*d) + ((4*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*
x]*Tan[c + d*x])/(6*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 1.07708, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3033, 3023, 2735, 3770}

$$-\frac{b\sin(c+dx)\left(4a^3(2A+3C)+39a^2bB+4ab^2(11A-6C)-6b^3B\right)}{6d}+\frac{a\left(4a^2b(A+2C)+a^3B+12ab^2B+8Ab^3\right)\tanh^{-1}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```


```
[Out] (b^2*(2*A*b^2 + 8*a*b*B + 12*a^2*C + b^2*C)*x)/2 + (a*(8*A*b^3 + a^3*B + 12
*a*b^2*B + 4*a^2*b*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*d) - (b*(39*a^2*b*B
 - 6*b^3*B + 4*a*b^2*(11*A - 6*C) + 4*a^3*(2*A + 3*C))*Sin[c + d*x])/(6*d)
- (b^2*(18*a*b*B + 3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Cos[c + d*x]*Sin[c +
d*x])/(6*d) + ((12*A*b^2 + 15*a*b*B + a^2*(4*A + 6*C))*(a + b*Cos[c + d*x])
^2*Tan[c + d*x])/(6*d) + ((4*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*
x]*Tan[c + d*x])/(6*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + b\cos(c + dx))^4 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)\sec^4(c + dx)\,dx = \frac{A(a + b\cos(c + dx))^4 \sec^2(c + dx)\tan(c}{3d}$$
$$= \frac{(4Ab + 3aB)(a + b\cos(c + dx))^3 \sec(c +}{6d}$$
$$= \frac{\left(12Ab^2 + 15abB + a^2(4A + 6C)\right)(a + b\,c}{6d}$$
$$= -\frac{b^2\left(18abB + 3b^2(6A - C) + a^2(4A + 6C\right.}{6d}$$
$$= -\frac{b\left(39a^2bB - 6b^3B + 4ab^2(11A - 6C) +\right.}{6d}$$
$$= \frac{1}{2}b^2\left(2Ab^2 + 8abB + 12a^2C + b^2C\right)x - \frac{b}{}$$
$$= \frac{1}{2}b^2\left(2Ab^2 + 8abB + 12a^2C + b^2C\right)x + \frac{a}{}$$

Mathematica [A] time = 2.25235, size = 351, normalized size = 1.16

$$\frac{\sec^3(c + dx)\left(36b^2(c + dx)\cos(c + dx)\left(12a^2C + 8abB + 2Ab^2 + b^2C\right) + 12b^2(c + dx)\cos(3(c + dx))\left(12a^2C + 8abB + 2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^3*(36*b^2*(2*A*b^2 + 8*a*b*B + 12*a^2*C + b^2*C)*(c + d*x)*Co
s[c + d*x] + 12*b^2*(2*A*b^2 + 8*a*b*B + 12*a^2*C + b^2*C)*(c + d*x)*Cos[3*
(c + d*x)] - 48*a*(8*A*b^3 + a^3*B + 12*a*b^2*B + 4*a^2*b*(A + 2*C))*Cos[c
+ d*x]^3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) + 2*(32*a^4*A + 144*a^2*A*b^2 + 96*a^3*b*B + 24*a^4*C +
 9*b^4*C + 12*(8*a^3*A*b + 2*a^4*B + 3*b^4*B + 12*a*b^3*C)*Cos[c + d*x] + 4
*(36*a^2*A*b^2 + 24*a^3*b*B + 3*b^4*C + a^4*(4*A + 6*C))*Cos[2*(c + d*x)] +
 12*b^4*B*Cos[3*(c + d*x)] + 48*a*b^3*C*Cos[3*(c + d*x)] + 3*b^4*C*Cos[4*(c
 + d*x)])*Sin[c + d*x]))/(96*d)
```

Maple [A] time = 0.081, size = 377, normalized size = 1.2

$$Ab^4x + \frac{Ab^4c}{d} + \frac{b^4B\sin(dx + c)}{d} + \frac{Cb^4\cos(dx + c)\sin(dx + c)}{2d} + \frac{b^4Cx}{2} + \frac{Cb^4c}{2d} + 4\frac{aAb^3\ln(\sec(dx + c) + \tan(dx +}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] A*b^4*x+1/d*A*b^4*c+1/d*b^4*B*sin(d*x+c)+1/2/d*C*b^4*cos(d*x+c)*sin(d*x+c)+
1/2*b^4*C*x+1/2/d*C*b^4*c+4/d*a*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+4*a*b^3*B*x
+4/d*a*b^3*B*c+4/d*C*a*b^3*sin(d*x+c)+6/d*a^2*A*b^2*tan(d*x+c)+6/d*a^2*b^2*
B*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*C*x+6/d*a^2*b^2*C*c+2/d*A*a^3*b*sec(d
*x+c)*tan(d*x+c)+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b*B*tan(d*x+
c)+4/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*A*a^4*tan(d*x+c)+1/3/d*A*a^4
*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)+1/2/d*a^4*B*ln(s
ec(d*x+c)+tan(d*x+c))+1/d*a^4*C*tan(d*x+c)
```

Maxima [A] time = 1.04641, size = 452, normalized size = 1.49

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{4}+72\left(dx+c\right)Ca^{2}b^{2}+48\left(dx+c\right)Bab^{3}+12\left(dx+c\right)Ab^{4}+3\left(2\,dx+2\,c+\sin\left(2\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 72*(d*x + c)*C*a^2*b^2 +
48*(d*x + c)*B*a*b^3 + 12*(d*x + c)*A*b^4 + 3*(2*d*x + 2*c + sin(2*d*x + 2*
c))*C*b^4 - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
 + 1) + log(sin(d*x + c) - 1)) - 12*A*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*a^3*b*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*B*a^2*b^2*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 24*A*a*b^3*(log(sin(d*x + c) + 1) - log(sin(
d*x + c) - 1)) + 48*C*a*b^3*sin(d*x + c) + 12*B*b^4*sin(d*x + c) + 12*C*a^4
*tan(d*x + c) + 48*B*a^3*b*tan(d*x + c) + 72*A*a^2*b^2*tan(d*x + c))/d
```

Fricas [A] time = 2.07981, size = 644, normalized size = 2.13

$$\frac{6\left(12\,Ca^{2}b^{2}+8\,Bab^{3}+\left(2\,A+C\right)b^{4}\right)dx\cos\left(dx+c\right)^{3}+3\left(Ba^{4}+4\left(A+2\,C\right)a^{3}b+12\,Ba^{2}b^{2}+8\,Aab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(12*C*a^2*b^2 + 8*B*a*b^3 + (2*A + C)*b^4)*d*x*cos(d*x + c)^3 + 3*(
B*a^4 + 4*(A + 2*C)*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(B*a^4 + 4*(A + 2*C)*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*
cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(3*C*b^4*cos(d*x + c)^4 + 2*A*a^4
 + 6*(4*C*a*b^3 + B*b^4)*cos(d*x + c)^3 + 2*((2*A + 3*C)*a^4 + 12*B*a^3*b +
 18*A*a^2*b^2)*cos(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38283, size = 743, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(12*C*a^2*b^2 + 8*B*a*b^3 + 2*A*b^4 + C*b^4)*(d*x + c) + 3*(B*a^4 +
4*A*a^3*b + 8*C*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 3*(B*a^4 + 4*A*a^3*b + 8*C*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 6*(8*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*
B*b^4*tan(1/2*d*x + 1/2*c)^3 - C*b^4*tan(1/2*d*x + 1/2*c)^3 + 8*C*a*b^3*tan
(1/2*d*x + 1/2*c) + 2*B*b^4*tan(1/2*d*x + 1/2*c) + C*b^4*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 2*(6*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*
B*a^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*b*
```

```
tan(1/2*d*x + 1/2*c)^5 + 24*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^5 - 4*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^4*tan(1/2*d
*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*
x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan(1/2*d*x + 1/2*c)
+ 6*C*a^4*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3
*b*tan(1/2*d*x + 1/2*c) + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^3)/d
```

3.970 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^5(c+dx)\,dx$

Optimal. Leaf size=293

$$-\frac{b^2\sin(c+dx)\left(3a^2(3A+4C)+32abB+2b^2(13A-12C)\right)}{24d}+\frac{a\tan(c+dx)\left(a^2b(23A+36C)+8a^3B+36ab^2B+12A\right.}{12d}$$

```
[Out] b^3*(b*B + 4*a*C)*x + ((8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B + 24*a^2*b^2*(A +
 2*C) + a^4*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*(32*a*b*B + 2*
b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Sin[c + d*x])/(24*d) + (a*(12*A*b^3
+ 8*a^3*B + 36*a*b^2*B + a^2*b*(23*A + 36*C))*Tan[c + d*x])/(12*d) + ((4*A*
b^2 + 8*a*b*B + a^2*(3*A + 4*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(8*d) + ((A*b + a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c +
d*x])/(3*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 1.06484, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3031, 3023, 2735, 3770}

$$-\frac{b^2\sin(c+dx)\left(3a^2(3A+4C)+32abB+2b^2(13A-12C)\right)}{24d}+\frac{a\tan(c+dx)\left(a^2b(23A+36C)+8a^3B+36ab^2B+12A\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```


```
[Out] b^3*(b*B + 4*a*C)*x + ((8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B + 24*a^2*b^2*(A +
 2*C) + a^4*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*(32*a*b*B + 2*
b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Sin[c + d*x])/(24*d) + (a*(12*A*b^3
+ 8*a^3*B + 36*a*b^2*B + a^2*b*(23*A + 36*C))*Tan[c + d*x])/(12*d) + ((4*A*
b^2 + 8*a*b*B + a^2*(3*A + 4*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(8*d) + ((A*b + a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c +
d*x])/(3*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{(Ab+aB)(a+b\cos(c+dx))^3\sec^2(c+d}{3d}$$
$$= \frac{\left(4Ab^2+8abB+a^2(3A+4C)\right)(a+b\cos}{8d}$$
$$= \frac{a\left(12Ab^3+8a^3B+36ab^2B+a^2b(23A+\right.}{12d}$$
$$= -\frac{b^2\left(32abB+2b^2(13A-12C)+3a^2(3A\right.}{24d}$$
$$= b^3(bB+4aC)x-\frac{b^2\left(32abB+2b^2(13A-\right.}{}$$
$$= b^3(bB+4aC)x+\frac{\left(8Ab^4+16a^3bB+32a\right.}{}$$

Mathematica [A] time = 2.45643, size = 462, normalized size = 1.58

$$\frac{3\tan(c+dx)\sec^3(c+dx)\left(24a^2Ab^2+a^4(11A+4C)+16a^3bB+4b^4C\right)+32a\tan(c+dx)\sec^2(c+dx)\left(a^2(8Ab+6bC)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (-12*(8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B + 24*a^2*b^2*(A + 2*C) + a^4*(3*A +
 4*C))*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c + d*x]^4*(36*b^4*B*c + 144*a*b^3*c*C + 36*b^4*B*d
*x + 144*a*b^3*C*d*x + 48*b^3*(b*B + 4*a*C)*(c + d*x)*Cos[2*(c + d*x)] + 12
*b^3*(b*B + 4*a*C)*(c + d*x)*Cos[4*(c + d*x)] + 9*a^4*A*Sin[3*(c + d*x)] +
72*a^2*A*b^2*Sin[3*(c + d*x)] + 48*a^3*b*B*Sin[3*(c + d*x)] + 12*a^4*C*Sin[
3*(c + d*x)] + 18*b^4*C*Sin[3*(c + d*x)] + 32*a^3*A*b*Sin[4*(c + d*x)] + 48
*a*A*b^3*Sin[4*(c + d*x)] + 8*a^4*B*Sin[4*(c + d*x)] + 72*a^2*b^2*B*Sin[4*(
c + d*x)] + 48*a^3*b*C*Sin[4*(c + d*x)] + 6*b^4*C*Sin[5*(c + d*x)]) + 32*a*
(6*A*b^3 + 2*a^3*B + 9*a*b^2*B + a^2*(8*A*b + 6*b*C))*Sec[c + d*x]^2*Tan[c
+ d*x] + 3*(24*a^2*A*b^2 + 16*a^3*b*B + 4*b^4*C + a^4*(11*A + 4*C))*Sec[c +
 d*x]^3*Tan[c + d*x])/(96*d)
```

Maple [A] time = 0.082, size = 457, normalized size = 1.6

$$\frac{Ab^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+b^4Bx+\frac{Bb^4c}{d}+\frac{Cb^4\sin(dx+c)}{d}+4\,\frac{aAb^3\tan(dx+c)}{d}+4\,\frac{ab^3B\ln(\sec(dx+c)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 1/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+b^4*B*x+1/d*b^4*B*c+1/d*C*b^4*sin(d*x+c
)+4/d*a*A*b^3*tan(d*x+c)+4/d*a*b^3*B*ln(sec(d*x+c)+tan(d*x+c))+4*a*b^3*C*x+
4/d*C*a*b^3*c+3/d*a^2*A*b^2*sec(d*x+c)*tan(d*x+c)+3/d*a^2*A*b^2*ln(sec(d*x+
c)+tan(d*x+c))+6/d*a^2*b^2*B*tan(d*x+c)+6/d*a^2*b^2*C*ln(sec(d*x+c)+tan(d*x
+c))+8/3/d*A*a^3*b*tan(d*x+c)+4/3/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^2+2/d*a^3
*b*B*sec(d*x+c)*tan(d*x+c)+2/d*a^3*b*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b*
C*tan(d*x+c)+1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^4*sec(d*x+c)*tan
(d*x+c)+3/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^4*B*tan(d*x+c)+1/3/d*
a^4*B*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^4*C
*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00991, size = 582, normalized size = 1.99

$$\frac{16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^4+64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3b+192\,(dx+c)Cab^3+48\,(dx+c)Bb^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 64*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^3*b + 192*(d*x + c)*C*a*b^3 + 48*(d*x + c)*B*b^4 - 3*A*a^4
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) - 48*B*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) - 72*A*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 144*C*a^2*b^2*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*B*a*b^3*(log(sin(d*x + c)
+ 1) - log(sin(d*x + c) - 1)) + 24*A*b^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 48*C*b^4*sin(d*x + c) + 192*C*a^3*b*tan(d*x + c) + 288*B*a^
```

```
2*b^2*tan(d*x + c) + 192*A*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 2.0737, size = 725, normalized size = 2.47

$$\frac{48\left(4\,Cab^{3}+Bb^{4}\right)dx\cos\left(dx+c\right)^{4}+3\left((3\,A+4\,C)a^{4}+16\,Ba^{3}b+24\,(A+2\,C)a^{2}b^{2}+32\,Bab^{3}+8\,Ab^{4}\right)\cos\left(dx+c\right)^{4}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="fricas")
```

```
[Out] 1/48*(48*(4*C*a*b^3 + B*b^4)*d*x*cos(d*x + c)^4 + 3*((3*A + 4*C)*a^4 + 16*B
*a^3*b + 24*(A + 2*C)*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*((3*A + 4*C)*a^4 + 16*B*a^3*b + 24*(A + 2*C)*a^2*b^2 +
32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(24*C*b^4*c
os(d*x + c)^4 + 6*A*a^4 + 16*(B*a^4 + 2*(2*A + 3*C)*a^3*b + 9*B*a^2*b^2 + 6
*A*a*b^3)*cos(d*x + c)^3 + 3*((3*A + 4*C)*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*
cos(d*x + c)^2 + 8*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43607, size = 1134, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x
, algorithm="giac")
```

```
[Out] 1/24*(48*C*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 24*(4*C*
a*b^3 + B*b^4)*(d*x + c) + 3*(3*A*a^4 + 4*C*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
 + 48*C*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 3*(3*A*a^4 + 4*C*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 48*C*a^2*b^2 + 32*B*a*
b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^4*tan(1/2*d*x
 + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^4*tan(1/2*d*x + 1/2*
c)^7 - 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^
7 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7
 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7
 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*C*
a^4*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 48*B*a^3*
b*tan(1/2*d*x + 1/2*c)^5 + 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*A*a^2*b^
2*tan(1/2*d*x + 1/2*c)^5 + 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 288*A*a*b
^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1
/2*d*x + 1/2*c)^3 - 12*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 288*C*a^3*b*tan(1/2*d*x
 + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d
*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x +
 1/2*c) + 24*B*a^4*tan(1/2*d*x + 1/2*c) + 12*C*a^4*tan(1/2*d*x + 1/2*c) + 9
6*A*a^3*b*tan(1/2*d*x + 1/2*c) + 48*B*a^3*b*tan(1/2*d*x + 1/2*c) + 96*C*a^3
*b*tan(1/2*d*x + 1/2*c) + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 144*B*a^2*b^2
*tan(1/2*d*x + 1/2*c) + 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^4)/d
```

3.971 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{s}$ $dx)\,dx$

Optimal. Leaf size=314

$$\frac{\tan(c+dx)\left(2a^2b^2(56A+85C)+4a^4(4A+5C)+80a^3bB+95ab^3B+12Ab^4\right)}{30d}+\frac{\left(4a^3b(3A+4C)+24a^2b^2B+3a^4B+\right.}{}$$

```
[Out] b^4*C*x + ((3*a^4*B + 24*a^2*b^2*B + 8*b^4*B + 16*a*b^3*(A + 2*C) + 4*a^3*b
*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) + ((12*A*b^4 + 80*a^3*b*B + 95*a
*b^3*B + 4*a^4*(4*A + 5*C) + 2*a^2*b^2*(56*A + 85*C))*Tan[c + d*x])/(30*d)
+ (a*(24*A*b^3 + 45*a^3*B + 130*a*b^2*B + 4*a^2*b*(29*A + 40*C))*Sec[c + d*
x]*Tan[c + d*x])/(120*d) + ((12*A*b^2 + 35*a*b*B + 4*a^2*(4*A + 5*C))*(a +
b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((4*A*b + 5*a*B)*(a
 + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*Cos[c
+ d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 1.04766, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3047, 3031, 3021, 2735, 3770}

$$\frac{\tan(c+dx)\left(2a^2b^2(56A+85C)+4a^4(4A+5C)+80a^3bB+95ab^3B+12Ab^4\right)}{30d}+\frac{\left(4a^3b(3A+4C)+24a^2b^2B+3a^4B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```


```
[Out] b^4*C*x + ((3*a^4*B + 24*a^2*b^2*B + 8*b^4*B + 16*a*b^3*(A + 2*C) + 4*a^3*b
*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*d) + ((12*A*b^4 + 80*a^3*b*B + 95*a
*b^3*B + 4*a^4*(4*A + 5*C) + 2*a^2*b^2*(56*A + 85*C))*Tan[c + d*x])/(30*d)
+ (a*(24*A*b^3 + 45*a^3*B + 130*a*b^2*B + 4*a^2*b*(29*A + 40*C))*Sec[c + d*
x]*Tan[c + d*x])/(120*d) + ((12*A*b^2 + 35*a*b*B + 4*a^2*(4*A + 5*C))*(a +
b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((4*A*b + 5*a*B)*(a
 + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (A*(a + b*Cos[c
+ d*x])^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^4(c+dx)\tan(c}{5d}$$

$$= \frac{(4Ab+5aB)(a+b\cos(c+dx))^3\sec^3(c+}{20d}$$

$$= \frac{\left(12Ab^2+35abB+4a^2(4A+5C)\right)(a+b}{60}$$

$$= \frac{a\left(24Ab^3+45a^3B+130ab^2B+4a^2b(29\right.}{120d}$$

$$= \frac{\left(12Ab^4+80a^3bB+95ab^3B+4a^4(4A+\right.}{30}$$

$$= b^4Cx + \frac{\left(12Ab^4+80a^3bB+95ab^3B+4a\right.}{}$$

$$= b^4Cx + \frac{\left(3a^4B+24a^2b^2B+8b^4B+16ab\right.}{}$$

Mathematica [A] time = 1.80237, size = 230, normalized size = 0.73

$$\frac{40a^2\tan^3(c+dx)\left(a^2(2A+C)+4abB+6Ab^2\right)+15\left(4a^3b(3A+4C)+24a^2b^2B+3a^4B+16ab^3(A+2C)+8b^4B\right)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (120*b^4*C*d*x + 15*(3*a^4*B + 24*a^2*b^2*B + 8*b^4*B + 16*a*b^3*(A + 2*C)
+ 4*a^3*b*(3*A + 4*C))*ArcTanh[Sin[c + d*x]] + 15*(8*(A*b^4 + 4*a^3*b*B + 4
*a*b^3*B + a^4*(A + C) + 6*a^2*b^2*(A + C)) + a*(16*A*b^3 + 3*a^3*B + 24*a*
b^2*B + 4*a^2*b*(3*A + 4*C))*Sec[c + d*x] + 2*a^3*(4*A*b + a*B)*Sec[c + d*x
]^3)*Tan[c + d*x] + 40*a^2*(6*A*b^2 + 4*a*b*B + a^2*(2*A + C))*Tan[c + d*x]
^3 + 24*a^4*A*Tan[c + d*x]^5)/(120*d)
```

Maple [A] time = 0.086, size = 572, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 1/d*b^4*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*A*b^3*sec(d*x+c)*tan(d*x+c)+2/d*a
^2*A*b^2*tan(d*x+c)*sec(d*x+c)^2+1/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^3+2/d*a^
3*b*C*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+1/d*A*b^4*t
an(d*x+c)+1/d*C*b^4*c+2/3/d*a^4*C*tan(d*x+c)+3/8/d*a^4*B*ln(sec(d*x+c)+tan(
d*x+c))+8/15/d*A*a^4*tan(d*x+c)+b^4*C*x+3/d*a^2*b^2*B*ln(sec(d*x+c)+tan(d*x
+c))+8/3/d*a^3*b*B*tan(d*x+c)+4/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2
*b^2*C*tan(d*x+c)+2/d*a*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*A*b^2*tan(d
*x+c)+2/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^3*b*ln(sec(d*x+c)+tan
(d*x+c))+1/4/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^4*B*sec(d*x+c)*tan(d*x
+c)+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2
+4/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+3/d*a^2*b^2*B*sec(d*x+c)*tan(d*x+c)+4
/3/d*a^3*b*B*tan(d*x+c)*sec(d*x+c)^2+4/d*a*b^3*B*tan(d*x+c)
```

Maxima [A] time = 1.03842, size = 690, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 320*(tan(d*x + c)^3 + 3*tan(d*
x + c))*B*a^3*b + 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2*b^2 + 240*(d*
x + c)*C*b^4 - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1)) - 60*A*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 240*C*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
 + log(sin(d*x + c) - 1)) - 360*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 -
 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 240*A*a*b^3*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 480*C*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 120*B*b^
4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 1440*C*a^2*b^2*tan(d*x
+ c) + 960*B*a*b^3*tan(d*x + c) + 240*A*b^4*tan(d*x + c))/d
```

Fricas [A] time = 2.01, size = 824, normalized size = 2.62

$$\frac{240\,Cb^4dx\cos\left(dx+c\right)^5+15\left(3\,Ba^4+4\,(3\,A+4\,C)a^3b+24\,Ba^2b^2+16\,(A+2\,C)ab^3+8\,Bb^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="fricas")
```

```
[Out] 1/240*(240*C*b^4*d*x*cos(d*x + c)^5 + 15*(3*B*a^4 + 4*(3*A + 4*C)*a^3*b + 2
4*B*a^2*b^2 + 16*(A + 2*C)*a*b^3 + 8*B*b^4)*cos(d*x + c)^5*log(sin(d*x + c)
 + 1) - 15*(3*B*a^4 + 4*(3*A + 4*C)*a^3*b + 24*B*a^2*b^2 + 16*(A + 2*C)*a*b
^3 + 8*B*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*A*a^4 + 8*(2*(4
*A + 5*C)*a^4 + 40*B*a^3*b + 30*(2*A + 3*C)*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4
)*cos(d*x + c)^4 + 15*(3*B*a^4 + 4*(3*A + 4*C)*a^3*b + 24*B*a^2*b^2 + 16*A*
a*b^3)*cos(d*x + c)^3 + 8*((4*A + 5*C)*a^4 + 20*B*a^3*b + 30*A*a^2*b^2)*cos
(d*x + c)^2 + 30*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4293, size = 1539, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x
, algorithm="giac")
```

```
[Out] 1/120*(120*(d*x + c)*C*b^4 + 15*(3*B*a^4 + 12*A*a^3*b + 16*C*a^3*b + 24*B*a
^2*b^2 + 16*A*a*b^3 + 32*C*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 15*(3*B*a^4 + 12*A*a^3*b + 16*C*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3
2*C*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^4*tan(
1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 120*C*a^4*tan(1/2*d*
x + 1/2*c)^9 - 300*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x
 + 1/2*c)^9 - 240*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/2*d*
x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 720*C*a^2*b^2*tan(1/2
*d*x + 1/2*c)^9 - 240*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*
d*x + 1/2*c)^9 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 160*A*a^4*tan(1/2*d*x +
 1/2*c)^7 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 320*C*a^4*tan(1/2*d*x + 1/2*c
)^7 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 1280*B*a^3*b*tan(1/2*d*x + 1/2*c
)^7 + 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^7 + 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 2880*C*a^2*b^2*tan(1/2*d*x +
 1/2*c)^7 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 1920*B*a*b^3*tan(1/2*d*x +
 1/2*c)^7 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*tan(1/2*d*x + 1/2*
c)^5 + 400*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)
^5 + 2400*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4320*C*a^2*b^2*tan(1/2*d*x + 1
/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*tan(1/2*d*x + 1/2
*c)^5 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^4*tan(1/2*d*x + 1/2*c)^3
- 320*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1
280*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1
920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
 - 2880*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)
^3 - 1920*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^3
 + 120*A*a^4*tan(1/2*d*x + 1/2*c) + 75*B*a^4*tan(1/2*d*x + 1/2*c) + 120*C*a
^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1/2*d*x + 1/2*c) + 480*B*a^3*b*ta
n(1/2*d*x + 1/2*c) + 240*C*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1
/2*d*x + 1/2*c) + 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b^2*tan(1/
2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x
 + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)
/d
```

3.972 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{s}$ $dx)\,dx$

Optimal. Leaf size=381

$$\frac{\tan(c+dx)\left(8a^3b(4A+5C)+60a^2b^2B+8a^4B+20ab^3(2A+3C)+15b^4B\right)}{15d}+\frac{\left(12a^2b^2(3A+4C)+a^4(5A+6C)+24a\right.}{}$$

```
[Out] ((24*a^3*b*B + 32*a*b^3*B + 8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*
(5*A + 6*C))*ArcTanh[Sin[c + d*x]])/(16*d) + ((8*a^4*B + 60*a^2*b^2*B + 15*
b^4*B + 20*a*b^3*(2*A + 3*C) + 8*a^3*b*(4*A + 5*C))*Tan[c + d*x])/(15*d) +
((24*A*b^4 + 360*a^3*b*B + 336*a*b^3*B + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(4
9*A + 66*C))*Sec[c + d*x]*Tan[c + d*x])/(240*d) + (a*(4*A*b^3 + 16*a^3*B +
36*a*b^2*B + a^2*b*(39*A + 50*C))*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((1
2*A*b^2 + 48*a*b*B + 5*a^2*(5*A + 6*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]
^3*Tan[c + d*x])/(120*d) + ((2*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sec[c +
d*x]^4*Tan[c + d*x])/(15*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5*Tan[
c + d*x])/(6*d)
```

Rubi [A] time = 1.32289, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\tan(c+dx)\left(8a^3b(4A+5C)+60a^2b^2B+8a^4B+20ab^3(2A+3C)+15b^4B\right)}{15d}+\frac{\left(12a^2b^2(3A+4C)+a^4(5A+6C)+24a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^7,x]
```


```
[Out] ((24*a^3*b*B + 32*a*b^3*B + 8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*
(5*A + 6*C))*ArcTanh[Sin[c + d*x]])/(16*d) + ((8*a^4*B + 60*a^2*b^2*B + 15*
b^4*B + 20*a*b^3*(2*A + 3*C) + 8*a^3*b*(4*A + 5*C))*Tan[c + d*x])/(15*d) +
((24*A*b^4 + 360*a^3*b*B + 336*a*b^3*B + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(4
9*A + 66*C))*Sec[c + d*x]*Tan[c + d*x])/(240*d) + (a*(4*A*b^3 + 16*a^3*B +
36*a*b^2*B + a^2*b*(39*A + 50*C))*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((1
2*A*b^2 + 48*a*b*B + 5*a^2*(5*A + 6*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]
^3*Tan[c + d*x])/(120*d) + ((2*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sec[c +
d*x]^4*Tan[c + d*x])/(15*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5*Tan[
```

c + d*x])/(6*d)

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^5(c+dx)\tan(c}{6d}$$

$$= \frac{(2Ab+3aB)(a+b\cos(c+dx))^3\sec^4(c+}{15d}$$

$$= \frac{\left(12Ab^2+48abB+5a^2(5A+6C)\right)(a+b}{120}$$

$$= \frac{a\left(4Ab^3+16a^3B+36ab^2B+a^2b(39A+\right.}{60d}$$

$$= \frac{\left(24Ab^4+360a^3bB+336ab^3B+15a^4(5\right.}{}$$

$$= \frac{\left(24Ab^4+360a^3bB+336ab^3B+15a^4(5\right.}{}$$

$$= \frac{\left(24a^3bB+32ab^3B+8b^4(A+2C)+12a\right.}{}$$

$$= \frac{\left(24a^3bB+32ab^3B+8b^4(A+2C)+12a\right.}{}$$

Mathematica [A] time = 6.30681, size = 366, normalized size = 0.96

$$\frac{b^2\left(6a^2C+4abB+Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)\left(a(aC+4bB)+6Ab^2\right)}{4d}+\frac{b^2\tan(c+dx)\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^7,x]
```

```
[Out] (b^4*C*ArcTanh[Sin[c + d*x]])/d + (b^2*(A*b^2 + 4*a*b*B + 6*a^2*C)*ArcTanh[
Sin[c + d*x]])/(2*d) + (b^3*(b*B + 4*a*C)*Tan[c + d*x])/d + (b^2*(A*b^2 + 4
*a*b*B + 6*a^2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^2*(6*A*b^2 + a*(4*b
*B + a*C))*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a^4*A*Sec[c + d*x]^5*Tan[c
 + d*x])/(6*d) + (3*a^2*(6*A*b^2 + a*(4*b*B + a*C))*(ArcTanh[Sin[c + d*x]]
+ Sec[c + d*x]*Tan[c + d*x]))/(8*d) + (2*a*b*(2*A*b^2 + a*(3*b*B + 2*a*C))*
(3*Tan[c + d*x] + Tan[c + d*x]^3))/(3*d) + (a^3*(4*A*b + a*B)*(15*Tan[c + d
*x] + 10*Tan[c + d*x]^3 + 3*Tan[c + d*x]^5))/(15*d) + (5*a^4*A*(2*Sec[c + d
*x]^3*Tan[c + d*x] + 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))
)/(48*d)
```

Maple [B] time = 0.087, size = 745, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] 1/d*b^4*B*tan(d*x+c)+1/d*C*b^4*ln(sec(d*x+c)+tan(d*x+c))+9/4/d*a^2*A*b^2*se
c(d*x+c)*tan(d*x+c)+16/15/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^2+1/2/d*A*b^4*ln(
sec(d*x+c)+tan(d*x+c))+3/8/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*a^4*B*t
an(d*x+c)+5/16/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3*b*B*ln(sec(d*x+c
)+tan(d*x+c))+2/d*a*b^3*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*b^2*B*tan(d*x+c
)+3/d*a^2*b^2*C*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*a^3*b*C*tan(d*x+c)+8/3/d*a*
A*b^3*tan(d*x+c)+9/4/d*a^2*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+32/15/d*A*a^3*b*
tan(d*x+c)+4/15/d*a^4*B*tan(d*x+c)*sec(d*x+c)^2+1/5/d*a^4*B*tan(d*x+c)*sec(
d*x+c)^4+1/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^5+1/4/d*a^4*C*tan(d*x+c)*sec(d*x
+c)^3+5/24/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^4*C*sec(d*x+c)*tan(d*x+c
)+5/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^4*sec(d*x+c)*tan(d*x+c)+4/d*
C*a*b^3*tan(d*x+c)+4/5/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^4+4/3/d*a^3*b*C*tan(
d*x+c)*sec(d*x+c)^2+2/d*a*b^3*B*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*A*b^2*tan(d
*x+c)*sec(d*x+c)^3+3/d*a^2*b^2*C*sec(d*x+c)*tan(d*x+c)+1/d*a^3*b*B*tan(d*x+
c)*sec(d*x+c)^3+4/3/d*a*A*b^3*tan(d*x+c)*sec(d*x+c)^2+2/d*a^2*b^2*B*tan(d*x
+c)*sec(d*x+c)^2+3/2/d*a^3*b*B*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.05716, size = 891, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 +
128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3*b + 640*
(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3*b + 960*(tan(d*x + c)^3 + 3*tan(d*x
 + c))*B*a^2*b^2 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a*b^3 - 5*A*a^4*
(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^
6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 1
5*log(sin(d*x + c) - 1)) - 30*C*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(s
in(d*x + c) - 1)) - 120*B*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 180*A*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 720*C*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*B*a*b^3*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*A*b^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) + 240*C*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 1
920*C*a*b^3*tan(d*x + c) + 480*B*b^4*tan(d*x + c))/d
```

Fricas [A] time = 2.14928, size = 941, normalized size = 2.47

$$\frac{15\left((5A+6C)a^4+24Ba^3b+12(3A+4C)a^2b^2+32Bab^3+8(A+2C)b^4\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left((\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="fricas")
```

```
[Out] 1/480*(15*((5*A + 6*C)*a^4 + 24*B*a^3*b + 12*(3*A + 4*C)*a^2*b^2 + 32*B*a*b
^3 + 8*(A + 2*C)*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*((5*A + 6*C
)*a^4 + 24*B*a^3*b + 12*(3*A + 4*C)*a^2*b^2 + 32*B*a*b^3 + 8*(A + 2*C)*b^4)
```

```
*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(8*B*a^4 + 8*(4*A + 5*C)*a^3
*b + 60*B*a^2*b^2 + 20*(2*A + 3*C)*a*b^3 + 15*B*b^4)*cos(d*x + c)^5 + 40*A*
a^4 + 15*((5*A + 6*C)*a^4 + 24*B*a^3*b + 12*(3*A + 4*C)*a^2*b^2 + 32*B*a*b^
3 + 8*A*b^4)*cos(d*x + c)^4 + 32*(2*B*a^4 + 2*(4*A + 5*C)*a^3*b + 15*B*a^2*
b^2 + 10*A*a*b^3)*cos(d*x + c)^3 + 10*((5*A + 6*C)*a^4 + 24*B*a^3*b + 36*A*
a^2*b^2)*cos(d*x + c)^2 + 48*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36718, size = 2238, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x
, algorithm="giac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 6*C*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 48*C*a^2*b^2 + 3
2*B*a*b^3 + 8*A*b^4 + 16*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*
A*a^4 + 6*C*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 48*C*a^2*b^2 + 32*B*a*b^3 + 8
*A*b^4 + 16*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*A*a^4*tan(1/
2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 150*C*a^4*tan(1/2*d
*x + 1/2*c)^11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*
d*x + 1/2*c)^11 - 960*C*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*b^2*tan(1
/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 720*C*a^2*b^2
*tan(1/2*d*x + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^
3*tan(1/2*d*x + 1/2*c)^11 - 960*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*A*b^4
*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*A*a^4*tan
```

(1/2*d*x + 1/2*c)^9 + 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 210*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 2160*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9600*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9600*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2160*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*A*a^4*tan(1/2*d*x + 1/2*c) + 240*B*a^4*tan(1/2*d*x + 1/2*c) + 150*C*a^4*tan(1/2*d*x + 1/2*c) + 960*A*a^3*b*tan(1/2*d*x + 1/2*c) + 600*B*a^3*b*tan(1/2*d*x + 1/2*c) + 960*C*a^3*b*tan(1/2*d*x + 1/2*c) + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 960*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x + 1/2*c) + 960*C*a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c) + 240*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

3.973 $\int (a+b\cos(c+dx))^4 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ se dx) dx

Optimal. Leaf size=454

$$\frac{\tan(c+dx)\left(84a^2b^2(4A+5C)+8a^4(6A+7C)+224a^3bB+280ab^3B+35b^4(2A+3C)\right)}{105d}+\frac{\left(4a^3b(5A+6C)+36a^2b^2B\right.}{}$$

```
[Out] ((5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6
*C))*ArcTanh[Sin[c + d*x]])/(16*d) + ((224*a^3*b*B + 280*a*b^3*B + 35*b^4*(
2*A + 3*C) + 84*a^2*b^2*(4*A + 5*C) + 8*a^4*(6*A + 7*C))*Tan[c + d*x])/(105
*d) + ((5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5
*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((4*A*b^4 + 112*a^3*b*B + 91
*a*b^3*B + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A + 63*C))*Sec[c + d*x]^2*Tan[
c + d*x])/(105*d) + (a*(24*A*b^3 + 175*a^3*B + 336*a*b^2*B + a^2*(412*A*b +
 504*b*C))*Sec[c + d*x]^3*Tan[c + d*x])/(840*d) + ((4*A*b^2 + 21*a*b*B + 2*
a^2*(6*A + 7*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(70*d)
 + ((4*A*b + 7*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(42
*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 1.39733, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(84a^2b^2(4A+5C)+8a^4(6A+7C)+224a^3bB+280ab^3B+35b^4(2A+3C)\right)}{105d}+\frac{\left(4a^3b(5A+6C)+36a^2b^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^8,x]
```


```
[Out] ((5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6
*C))*ArcTanh[Sin[c + d*x]])/(16*d) + ((224*a^3*b*B + 280*a*b^3*B + 35*b^4*(
2*A + 3*C) + 84*a^2*b^2*(4*A + 5*C) + 8*a^4*(6*A + 7*C))*Tan[c + d*x])/(105
*d) + ((5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5
*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((4*A*b^4 + 112*a^3*b*B + 91
*a*b^3*B + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A + 63*C))*Sec[c + d*x]^2*Tan[
c + d*x])/(105*d) + (a*(24*A*b^3 + 175*a^3*B + 336*a*b^2*B + a^2*(412*A*b +
 504*b*C))*Sec[c + d*x]^3*Tan[c + d*x])/(840*d) + ((4*A*b^2 + 21*a*b*B + 2*
a^2*(6*A + 7*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(70*d)
```

```
 + ((4*A*b + 7*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(42
*d) + (A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6*Tan[c + d*x])/(7*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^4\sec^6(c+dx)\tan(c}{7d}$$

$$= \frac{(4Ab+7aB)(a+b\cos(c+dx))^3\sec^5(c+}{42d}$$

$$= \frac{\left(4Ab^2+21abB+2a^2(6A+7C)\right)(a+b\,c}{70a}$$

$$= \frac{a\left(24Ab^3+175a^3B+336ab^2B+a^2(412}{840}$$

$$= \frac{\left(4Ab^4+112a^3bB+91ab^3B+4a^4(6A+\right.}{}$$

$$= \frac{\left(4Ab^4+112a^3bB+91ab^3B+4a^4(6A+\right.}{}$$

$$= \frac{\left(5a^4B+36a^2b^2B+8b^4B+8ab^3(3A+4\right.}{}$$

$$= \frac{\left(5a^4B+36a^2b^2B+8b^4B+8ab^3(3A+4\right.}{1}$$

Mathematica [A] time = 3.50877, size = 341, normalized size = 0.75

$$\frac{105\left(4a^3b(5A+6C)+36a^2b^2B+5a^4B+8ab^3(3A+4C)+8b^4B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(16\left(21a^2\tan^4(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^8,x]
```

```
[Out] (105*(5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A
 + 6*C))*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(105*(5*a^4*B + 36*a^2*b^2*B
+ 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6*C))*Sec[c + d*x] + 70*a*
(24*A*b^3 + 5*a^3*B + 36*a*b^2*B + 4*a^2*b*(5*A + 6*C))*Sec[c + d*x]^3 + 28
0*a^3*(4*A*b + a*B)*Sec[c + d*x]^5 + 16*(105*(4*a^3*b*B + 4*a*b^3*B + a^4*(
A + C) + 6*a^2*b^2*(A + C) + b^4*(A + C)) + 35*(A*b^4 + 8*a^3*b*B + 4*a*b^3
*B + 6*a^2*b^2*(2*A + C) + a^4*(3*A + 2*C))*Tan[c + d*x]^2 + 21*a^2*(6*A*b^
2 + 4*a*b*B + a^2*(3*A + C))*Tan[c + d*x]^4 + 15*a^4*A*Tan[c + d*x]^6)))/(1
680*d)
```

Maple [B] time = 0.1, size = 905, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x)
```

```
[Out] 1/2/d*b^4*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*b^2*C*tan(d*x+c)*sec(d*x+c)^2
+2/3/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^5+1/d*a^3*b*C*tan(d*x+c)*sec(d*x+c)^3+
1/d*a*A*b^3*tan(d*x+c)*sec(d*x+c)^3+2/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)+6/5/d
*a^2*A*b^2*tan(d*x+c)*sec(d*x+c)^4+3/2/d*a*A*b^3*sec(d*x+c)*tan(d*x+c)+8/5/
d*a^2*A*b^2*tan(d*x+c)*sec(d*x+c)^2+5/6/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^3+3
/2/d*a^3*b*C*sec(d*x+c)*tan(d*x+c)+5/4/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+1/d*
C*b^4*tan(d*x+c)+2/3/d*A*b^4*tan(d*x+c)+8/15/d*a^4*C*tan(d*x+c)+5/16/d*a^4*
B*ln(sec(d*x+c)+tan(d*x+c))+16/35/d*A*a^4*tan(d*x+c)+9/4/d*a^2*b^2*B*ln(sec
(d*x+c)+tan(d*x+c))+32/15/d*a^3*b*B*tan(d*x+c)+1/3/d*A*b^4*tan(d*x+c)*sec(d
*x+c)^2+1/7/d*A*a^4*tan(d*x+c)*sec(d*x+c)^6+1/5/d*a^4*C*tan(d*x+c)*sec(d*x+
c)^4+2/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*b^2*C*tan(d*x+c)+3/2/d*a
*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+16/5/d*a^2*A*b^2*tan(d*x+c)+3/2/d*a^3*b*C*
ln(sec(d*x+c)+tan(d*x+c))+5/4/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+5/24/d*a^
4*B*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a^4*B*sec(d*x+c)*tan(d*x+c)+6/35/d*A*a^4
*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+8/35/d*A*a^4*
tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^2*b^2*B*tan(d*x+c)*sec(d*x+c)^3+4/3/d*a*b^3
*B*tan(d*x+c)*sec(d*x+c)^2+4/5/d*a^3*b*B*tan(d*x+c)*sec(d*x+c)^4+9/4/d*a^2*
b^2*B*sec(d*x+c)*tan(d*x+c)+16/15/d*a^3*b*B*tan(d*x+c)*sec(d*x+c)^2+8/3/d*a
*b^3*B*tan(d*x+c)+1/6/d*a^4*B*tan(d*x+c)*sec(d*x+c)^5+1/2/d*b^4*B*sec(d*x+c
)*tan(d*x+c)
```

Maxima [A] time = 1.04805, size = 1007, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x
, algorithm="maxima")
```

```
[Out] 1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*A*a^4 + 224*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
 + c))*C*a^4 + 896*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))
*B*a^3*b + 1344*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*
```

```
a^2*b^2 + 6720*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b^2 + 4480*(tan(d*x
+ c)^3 + 3*tan(d*x + c))*B*a*b^3 + 1120*(tan(d*x + c)^3 + 3*tan(d*x + c))*A
*b^4 - 35*B*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d
*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 140*A*a^3*b*(2*(15*sin(d*x + c)^
5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4
 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) -
 1)) - 840*C*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 1260*B*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 840*A*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 33
60*C*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 840*B*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 3360*C*b^4*tan(d*x + c))/d
```

Fricas [A] time = 2.23407, size = 1085, normalized size = 2.39

$$\frac{105\left(5\,Ba^4+4\,(5\,A+6\,C)a^3b+36\,Ba^2b^2+8\,(3\,A+4\,C)ab^3+8\,Bb^4\right)\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\left(5\,Ba^4+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x
, algorithm="fricas")
```

```
[Out] 1/3360*(105*(5*B*a^4 + 4*(5*A + 6*C)*a^3*b + 36*B*a^2*b^2 + 8*(3*A + 4*C)*a
*b^3 + 8*B*b^4)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(5*B*a^4 + 4*(5*
A + 6*C)*a^3*b + 36*B*a^2*b^2 + 8*(3*A + 4*C)*a*b^3 + 8*B*b^4)*cos(d*x + c)
^7*log(-sin(d*x + c) + 1) + 2*(16*(8*(6*A + 7*C)*a^4 + 224*B*a^3*b + 84*(4*
A + 5*C)*a^2*b^2 + 280*B*a*b^3 + 35*(2*A + 3*C)*b^4)*cos(d*x + c)^6 + 105*(
5*B*a^4 + 4*(5*A + 6*C)*a^3*b + 36*B*a^2*b^2 + 8*(3*A + 4*C)*a*b^3 + 8*B*b^
4)*cos(d*x + c)^5 + 240*A*a^4 + 16*(4*(6*A + 7*C)*a^4 + 112*B*a^3*b + 42*(4
*A + 5*C)*a^2*b^2 + 140*B*a*b^3 + 35*A*b^4)*cos(d*x + c)^4 + 70*(5*B*a^4 +
4*(5*A + 6*C)*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3)*cos(d*x + c)^3 + 48*((6*A
+ 7*C)*a^4 + 28*B*a^3*b + 42*A*a^2*b^2)*cos(d*x + c)^2 + 280*(B*a^4 + 4*A*a
^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33971, size = 2549, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^8,x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(5*B*a^4 + 20*A*a^3*b + 24*C*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3
+ 32*C*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(5*B*a^4 +
 20*A*a^3*b + 24*C*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 32*C*a*b^3 + 8*B*b^4
)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(1680*A*a^4*tan(1/2*d*x + 1/2*c)^1
3 - 1155*B*a^4*tan(1/2*d*x + 1/2*c)^13 + 1680*C*a^4*tan(1/2*d*x + 1/2*c)^13
 - 4620*A*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 6720*B*a^3*b*tan(1/2*d*x + 1/2*c)
^13 - 4200*C*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 10080*A*a^2*b^2*tan(1/2*d*x +
1/2*c)^13 - 6300*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 10080*C*a^2*b^2*tan(1/
2*d*x + 1/2*c)^13 - 4200*A*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 6720*B*a*b^3*tan
(1/2*d*x + 1/2*c)^13 - 3360*C*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 1680*A*b^4*ta
n(1/2*d*x + 1/2*c)^13 - 840*B*b^4*tan(1/2*d*x + 1/2*c)^13 + 1680*C*b^4*tan(
1/2*d*x + 1/2*c)^13 - 3360*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 980*B*a^4*tan(1/
2*d*x + 1/2*c)^11 - 5600*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 3920*A*a^3*b*tan(1
/2*d*x + 1/2*c)^11 - 22400*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 10080*C*a^3*b*
tan(1/2*d*x + 1/2*c)^11 - 33600*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 15120*B
*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 47040*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^11
+ 10080*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 31360*B*a*b^3*tan(1/2*d*x + 1/2*c
)^11 + 13440*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 7840*A*b^4*tan(1/2*d*x + 1/2
*c)^11 + 3360*B*b^4*tan(1/2*d*x + 1/2*c)^11 - 10080*C*b^4*tan(1/2*d*x + 1/2
*c)^11 + 14448*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 2975*B*a^4*tan(1/2*d*x + 1/2*
c)^9 + 12656*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 11900*A*a^3*b*tan(1/2*d*x + 1/2
*c)^9 + 50624*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 7560*C*a^3*b*tan(1/2*d*x + 1
```

/2*c)^9 + 75936*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 11340*B*a^2*b^2*tan(1/2*
d*x + 1/2*c)^9 + 97440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 7560*A*a*b^3*tan(
1/2*d*x + 1/2*c)^9 + 64960*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 16800*C*a*b^3*t
an(1/2*d*x + 1/2*c)^9 + 16240*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 4200*B*b^4*tan
(1/2*d*x + 1/2*c)^9 + 25200*C*b^4*tan(1/2*d*x + 1/2*c)^9 - 10176*A*a^4*tan(
1/2*d*x + 1/2*c)^7 - 17472*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 69888*B*a^3*b*tan
(1/2*d*x + 1/2*c)^7 - 104832*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 120960*C*a^
2*b^2*tan(1/2*d*x + 1/2*c)^7 - 80640*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 20160
*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 33600*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 14448*
A*a^4*tan(1/2*d*x + 1/2*c)^5 + 2975*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 12656*C*
a^4*tan(1/2*d*x + 1/2*c)^5 + 11900*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 50624*B
*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 7560*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 75936
*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 11340*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5
+ 97440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 7560*A*a*b^3*tan(1/2*d*x + 1/2*c
)^5 + 64960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 16800*C*a*b^3*tan(1/2*d*x + 1/
2*c)^5 + 16240*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 4200*B*b^4*tan(1/2*d*x + 1/2*
c)^5 + 25200*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 3360*A*a^4*tan(1/2*d*x + 1/2*c)
^3 - 980*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 5600*C*a^4*tan(1/2*d*x + 1/2*c)^3 -
3920*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 22400*B*a^3*b*tan(1/2*d*x + 1/2*c)^3
- 10080*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 33600*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^3 - 15120*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 47040*C*a^2*b^2*tan(1/2*d*
x + 1/2*c)^3 - 10080*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 31360*B*a*b^3*tan(1/2
*d*x + 1/2*c)^3 - 13440*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 7840*A*b^4*tan(1/2
*d*x + 1/2*c)^3 - 3360*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 10080*C*b^4*tan(1/2*d
*x + 1/2*c)^3 + 1680*A*a^4*tan(1/2*d*x + 1/2*c) + 1155*B*a^4*tan(1/2*d*x +
1/2*c) + 1680*C*a^4*tan(1/2*d*x + 1/2*c) + 4620*A*a^3*b*tan(1/2*d*x + 1/2*c
) + 6720*B*a^3*b*tan(1/2*d*x + 1/2*c) + 4200*C*a^3*b*tan(1/2*d*x + 1/2*c) +
10080*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6300*B*a^2*b^2*tan(1/2*d*x + 1/2*c)
+ 10080*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 4200*A*a*b^3*tan(1/2*d*x + 1/2*c)
+ 6720*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3360*C*a*b^3*tan(1/2*d*x + 1/2*c) +
1680*A*b^4*tan(1/2*d*x + 1/2*c) + 840*B*b^4*tan(1/2*d*x + 1/2*c) + 1680*C*b
^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d

3.974 $\int (a+b\cos(c+dx))^3 \left(abB - a^2C + b^2B\cos(c+dx) + b^2C\cos\right.$

Optimal. Leaf size=256

$$\frac{b\left(32a^2b^2C + 95a^3bB - 83a^4C + 80ab^3B + 16b^4C\right)\sin(c+dx)}{30d} + \frac{b\left(-23a^2C + 35abB + 16b^2C\right)\sin(c+dx)(a+b\cos(c+}{60d}$$

```
[Out] ((8*a^4*b*B + 24*a^2*b^3*B + 3*b^5*B - 8*a^5*C - 8*a^3*b^2*C + 9*a*b^4*C)*x
)/8 + (b*(95*a^3*b*B + 80*a*b^3*B - 83*a^4*C + 32*a^2*b^2*C + 16*b^4*C)*Sin
[c + d*x])/(30*d) + (b^2*(130*a^2*b*B + 45*b^3*B - 106*a^3*C + 71*a*b^2*C)*
Cos[c + d*x]*Sin[c + d*x])/(120*d) + (b*(35*a*b*B - 23*a^2*C + 16*b^2*C)*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + (b*(5*b*B - a*C)*(a + b*Cos[c +
 d*x])^3*Sin[c + d*x])/(20*d) + (b*C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(
5*d)
```

Rubi [A] time = 0.551879, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3015, 2753, 2734}

$$\frac{b\left(32a^2b^2C + 95a^3bB - 83a^4C + 80ab^3B + 16b^4C\right)\sin(c+dx)}{30d} + \frac{b\left(-23a^2C + 35abB + 16b^2C\right)\sin(c+dx)(a+b\cos(c+}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[
c + d*x]^2),x]
```


```
[Out] ((8*a^4*b*B + 24*a^2*b^3*B + 3*b^5*B - 8*a^5*C - 8*a^3*b^2*C + 9*a*b^4*C)*x
)/8 + (b*(95*a^3*b*B + 80*a*b^3*B - 83*a^4*C + 32*a^2*b^2*C + 16*b^4*C)*Sin
[c + d*x])/(30*d) + (b^2*(130*a^2*b*B + 45*b^3*B - 106*a^3*C + 71*a*b^2*C)*
Cos[c + d*x]*Sin[c + d*x])/(120*d) + (b*(35*a*b*B - 23*a^2*C + 16*b^2*C)*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*d) + (b*(5*b*B - a*C)*(a + b*Cos[c +
 d*x])^3*Sin[c + d*x])/(20*d) + (b*C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(
5*d)
```


Rule 3015

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)\right)dx = \frac{\int (a+b\cos(c+dx))^4\left(b^2(bB-aC)+b^3\right.}{b^2}$$

$$= \frac{bC(a+b\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{\int(}{}$$

$$= \frac{b(5bB-aC)(a+b\cos(c+dx))^3\sin(c+}{20d}$$

$$= \frac{b\left(35abB-23a^2C+16b^2C\right)(a+b\cos(c}{60d}$$

$$= \frac{1}{8}\left(8a^4bB+24a^2b^3B+3b^5B-8a^5C-8\right.$$

Mathematica [A] time = 1.10825, size = 287, normalized size = 1.12

$$\frac{60b\left(12a^2b^2C+32a^3bB-24a^4C+24ab^3B+5b^4C\right)\sin(c+dx)+120b^2\left(6a^2bB-2a^3C+3ab^2C+b^3B\right)\sin(2(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*
C*Cos[c + d*x]^2),x]
```

```
[Out] (480*a^4*b*B*c + 1440*a^2*b^3*B*c + 180*b^5*B*c - 480*a^5*c*C - 480*a^3*b^2
*c*C + 540*a*b^4*c*C + 480*a^4*b*B*d*x + 1440*a^2*b^3*B*d*x + 180*b^5*B*d*x
```

```
 - 480*a^5*C*d*x - 480*a^3*b^2*C*d*x + 540*a*b^4*C*d*x + 60*b*(32*a^3*b*B +
 24*a*b^3*B - 24*a^4*C + 12*a^2*b^2*C + 5*b^4*C)*Sin[c + d*x] + 120*b^2*(6*
a^2*b*B + b^3*B - 2*a^3*C + 3*a*b^2*C)*Sin[2*(c + d*x)] + 160*a*b^4*B*Sin[3
*(c + d*x)] + 80*a^2*b^3*C*Sin[3*(c + d*x)] + 50*b^5*C*Sin[3*(c + d*x)] + 1
5*b^5*B*Sin[4*(c + d*x)] + 45*a*b^4*C*Sin[4*(c + d*x)] + 6*b^5*C*Sin[5*(c +
 d*x)])/(480*d)
```

Maple [A] time = 0.021, size = 276, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Cb^5\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+b^5B\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*C*b^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^5*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*C*a*b^4*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a*b^4*B*(2+cos(d*x+c)
^2)*sin(d*x+c)+2/3*C*a^2*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^2*b^3*B*(1/2*c
os(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-2*a^3*b^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+
1/2*d*x+1/2*c)+4*a^3*b^2*B*sin(d*x+c)-3*C*a^4*b*sin(d*x+c)+a^4*b*B*(d*x+c)-
a^5*C*(d*x+c))
```

Maxima [A] time = 1.01086, size = 355, normalized size = 1.39

$$-\frac{480(dx+c)Ca^5-480(dx+c)Ba^4b+240(2dx+2c+\sin(2dx+2c))Ca^3b^2-720(2dx+2c+\sin(2dx+2c))Ba^2b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] -1/480*(480*(d*x + c)*C*a^5 - 480*(d*x + c)*B*a^4*b + 240*(2*d*x + 2*c + si
n(2*d*x + 2*c))*C*a^3*b^2 - 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b^3
+ 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b^3 + 640*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a*b^4 - 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*C*a*b^4 - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c
```

```
))*B*b^5 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*b^
5 + 1440*C*a^4*b*sin(d*x + c) - 1920*B*a^3*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.87124, size = 500, normalized size = 1.95

$$-\frac{15\left(8\,Ca^5-8\,Ba^4b+8\,Ca^3b^2-24\,Ba^2b^3-9\,Cab^4-3\,Bb^5\right)dx-\left(24\,Cb^5\cos\left(dx+c\right)^4-360\,Ca^4b+480\,Ba^3b^2+160\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] -1/120*(15*(8*C*a^5 - 8*B*a^4*b + 8*C*a^3*b^2 - 24*B*a^2*b^3 - 9*C*a*b^4 -
3*B*b^5)*d*x - (24*C*b^5*cos(d*x + c)^4 - 360*C*a^4*b + 480*B*a^3*b^2 + 160
*C*a^2*b^3 + 320*B*a*b^4 + 64*C*b^5 + 30*(3*C*a*b^4 + B*b^5)*cos(d*x + c)^3
 + 16*(5*C*a^2*b^3 + 10*B*a*b^4 + 2*C*b^5)*cos(d*x + c)^2 - 15*(8*C*a^3*b^2
 - 24*B*a^2*b^3 - 9*C*a*b^4 - 3*B*b^5)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.848, size = 619, normalized size = 2.42

$$\begin{cases} Ba^4bx+\frac{4Ba^3b^2\sin\left(c+dx\right)}{d}+3Ba^2b^3x\sin^2\left(c+dx\right)+3Ba^2b^3x\cos^2\left(c+dx\right)+\frac{3Ba^2b^3\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{8Bab^4\sin^3\left(c+dx\right)}{3d}+\frac{4B}{} \\ x\left(a+b\cos\left(c\right)\right)^3\left(Bab+Bb^2\cos\left(c\right)-Ca^2+Cb^2\cos^2\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*
x+c)**2),x)
```

```
[Out] Piecewise((B*a**4*b*x + 4*B*a**3*b**2*sin(c + d*x)/d + 3*B*a**2*b**3*x*sin(
c + d*x)**2 + 3*B*a**2*b**3*x*cos(c + d*x)**2 + 3*B*a**2*b**3*sin(c + d*x)*
cos(c + d*x)/d + 8*B*a*b**4*sin(c + d*x)**3/(3*d) + 4*B*a*b**4*sin(c + d*x)
*cos(c + d*x)**2/d + 3*B*b**5*x*sin(c + d*x)**4/8 + 3*B*b**5*x*sin(c + d*x)
**2*cos(c + d*x)**2/4 + 3*B*b**5*x*cos(c + d*x)**4/8 + 3*B*b**5*sin(c + d*x
)**3*cos(c + d*x)/(8*d) + 5*B*b**5*sin(c + d*x)*cos(c + d*x)**3/(8*d) - C*a
**5*x - 3*C*a**4*b*sin(c + d*x)/d - C*a**3*b**2*x*sin(c + d*x)**2 - C*a**3*
b**2*x*cos(c + d*x)**2 - C*a**3*b**2*sin(c + d*x)*cos(c + d*x)/d + 4*C*a**2
*b**3*sin(c + d*x)**3/(3*d) + 2*C*a**2*b**3*sin(c + d*x)*cos(c + d*x)**2/d
+ 9*C*a*b**4*x*sin(c + d*x)**4/8 + 9*C*a*b**4*x*sin(c + d*x)**2*cos(c + d*x
```

```
)**2/4 + 9*C*a*b**4*x*cos(c + d*x)**4/8 + 9*C*a*b**4*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) + 15*C*a*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*C*b**5*si
n(c + d*x)**5/(15*d) + 4*C*b**5*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*b
**5*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**3*(B*a*b
+ B*b**2*cos(c) - C*a**2 + C*b**2*cos(c)**2), True))
```

Giac [A] time = 1.18481, size = 306, normalized size = 1.2

$$\frac{Cb^5\sin(5\,dx+5\,c)}{80\,d} - \frac{1}{8}\left(8\,Ca^5 - 8\,Ba^4b + 8\,Ca^3b^2 - 24\,Ba^2b^3 - 9\,Cab^4 - 3\,Bb^5\right)x + \frac{\left(3\,Cab^4 + Bb^5\right)\sin(4\,dx+4\,c)}{32\,d} + \frac{(8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] 1/80*C*b^5*sin(5*d*x + 5*c)/d - 1/8*(8*C*a^5 - 8*B*a^4*b + 8*C*a^3*b^2 - 24
*B*a^2*b^3 - 9*C*a*b^4 - 3*B*b^5)*x + 1/32*(3*C*a*b^4 + B*b^5)*sin(4*d*x +
4*c)/d + 1/48*(8*C*a^2*b^3 + 16*B*a*b^4 + 5*C*b^5)*sin(3*d*x + 3*c)/d - 1/4
*(2*C*a^3*b^2 - 6*B*a^2*b^3 - 3*C*a*b^4 - B*b^5)*sin(2*d*x + 2*c)/d - 1/8*(
24*C*a^4*b - 32*B*a^3*b^2 - 12*C*a^2*b^3 - 24*B*a*b^4 - 5*C*b^5)*sin(d*x +
c)/d
```

3.975 $\int (a+b\cos(c+dx))^2 \left(abB-a^2C+b^2B\cos(c+dx)+b^2C\cos\right.$

Optimal. Leaf size=176

$$\frac{b\left(16a^2bB-13a^3C+8ab^2C+4b^3B\right)\sin(c+dx)}{6d}+\frac{b^2\left(-14a^2C+20abB+9b^2C\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^3bB-\right.$$

```
[Out] ((8*a^3*b*B + 12*a*b^3*B - 8*a^4*C + 3*b^4*C)*x)/8 + (b*(16*a^2*b*B + 4*b^3
*B - 13*a^3*C + 8*a*b^2*C)*Sin[c + d*x])/(6*d) + (b^2*(20*a*b*B - 14*a^2*C
+ 9*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (b*(4*b*B - a*C)*(a + b*Cos[
c + d*x])^2*Sin[c + d*x])/(12*d) + (b*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x]
)/(4*d)
```

Rubi [A] time = 0.350412, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3015, 2753, 2734}

$$\frac{b\left(16a^2bB-13a^3C+8ab^2C+4b^3B\right)\sin(c+dx)}{6d}+\frac{b^2\left(-14a^2C+20abB+9b^2C\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^3bB-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[
c + d*x]^2),x]
```

```
[Out] ((8*a^3*b*B + 12*a*b^3*B - 8*a^4*C + 3*b^4*C)*x)/8 + (b*(16*a^2*b*B + 4*b^3
*B - 13*a^3*C + 8*a*b^2*C)*Sin[c + d*x])/(6*d) + (b^2*(20*a*b*B - 14*a^2*C
+ 9*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (b*(4*b*B - a*C)*(a + b*Cos[
c + d*x])^2*Sin[c + d*x])/(12*d) + (b*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x]
)/(4*d)
```

Rule 3015

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)\right)dx = \frac{\int (a+b\cos(c+dx))^3\left(b^2(bB-aC)+b^3C\right.}{b^2}$$

$$= \frac{bC(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{\int (a}{}$$

$$= \frac{b(4bB-aC)(a+b\cos(c+dx))^2\sin(c+d}{12d}$$

$$= \frac{1}{8}\left(8a^3bB+12ab^3B-8a^4C+3b^4C\right)x + \frac{b}{}$$

Mathematica [A] time = 0.578179, size = 134, normalized size = 0.76

$$\frac{-12(c+dx)\left(-8a^3bB+8a^4C-12ab^3B-3b^4C\right)+24b\left(12a^2bB-8a^3C+6ab^2C+3b^3B\right)\sin(c+dx)+24b^3(3aB+bC)\sin}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*
C*Cos[c + d*x]^2),x]
```

```
[Out] (-12*(-8*a^3*b*B - 12*a*b^3*B + 8*a^4*C - 3*b^4*C)*(c + d*x) + 24*b*(12*a^2
*b*B + 3*b^3*B - 8*a^3*C + 6*a*b^2*C)*Sin[c + d*x] + 24*b^3*(3*a*B + b*C)*S
in[2*(c + d*x)] + 8*b^3*(b*B + 2*a*C)*Sin[3*(c + d*x)] + 3*b^4*C*Sin[4*(c +
 d*x)])/(96*d)
```

Maple [A] time = 0.022, size = 168, normalized size = 1.

$$\frac{1}{d}\left(Cb^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^4B\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2\,Cab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(C*b^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*b^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*C*a*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+
3*a*b^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b^2*B*sin(d*x+c)-
2*a^3*b*C*sin(d*x+c)+a^3*b*B*(d*x+c)-a^4*C*(d*x+c))
```

Maxima [A] time = 0.994064, size = 219, normalized size = 1.24

$$-\frac{96\,(dx+c)Ca^4-96\,(dx+c)Ba^3b-72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^3+64\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Cab^3+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] -1/96*(96*(d*x + c)*C*a^4 - 96*(d*x + c)*B*a^3*b - 72*(2*d*x + 2*c + sin(2*
d*x + 2*c))*B*a*b^3 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a*b^3 + 32*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*b^4 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c)
+ 8*sin(2*d*x + 2*c))*C*b^4 + 192*C*a^3*b*sin(d*x + c) - 288*B*a^2*b^2*sin(
d*x + c))/d
```

Fricas [A] time = 1.66888, size = 311, normalized size = 1.77

$$-\frac{3\left(8\,Ca^4-8\,Ba^3b-12\,Bab^3-3\,Cb^4\right)dx-\left(6\,Cb^4\cos(dx+c)^3-48\,Ca^3b+72\,Ba^2b^2+32\,Cab^3+16\,Bb^4+8\left(2\,Cab^3+\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(8*C*a^4 - 8*B*a^3*b - 12*B*a*b^3 - 3*C*b^4)*d*x - (6*C*b^4*cos(d*
x + c)^3 - 48*C*a^3*b + 72*B*a^2*b^2 + 32*C*a*b^3 + 16*B*b^4 + 8*(2*C*a*b^3
 + B*b^4)*cos(d*x + c)^2 + 9*(4*B*a*b^3 + C*b^4)*cos(d*x + c))*sin(d*x + c)
)/d
```

Sympy [A] time = 1.71629, size = 357, normalized size = 2.03

$$\begin{cases} Ba^3bx + \frac{3Ba^2b^2\sin(c+dx)}{d} + \frac{3Bab^3x\sin^2(c+dx)}{2} + \frac{3Bab^3x\cos^2(c+dx)}{2} + \frac{3Bab^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Bb^4\sin^3(c+dx)}{3d} + \frac{Bb^4\sin(c+dx)\cos^2(c+ \\ x(a+b\cos(c))^2\left(Bab + Bb^2\cos(c) - Ca^2 + Cb^2\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*
x+c)**2),x)
```

```
[Out] Piecewise((B*a**3*b*x + 3*B*a**2*b**2*sin(c + d*x)/d + 3*B*a*b**3*x*sin(c +
 d*x)**2/2 + 3*B*a*b**3*x*cos(c + d*x)**2/2 + 3*B*a*b**3*sin(c + d*x)*cos(c
 + d*x)/(2*d) + 2*B*b**4*sin(c + d*x)**3/(3*d) + B*b**4*sin(c + d*x)*cos(c
+ d*x)**2/d - C*a**4*x - 2*C*a**3*b*sin(c + d*x)/d + 4*C*a*b**3*sin(c + d*x
)**3/(3*d) + 2*C*a*b**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*C*b**4*x*sin(c +
 d*x)**4/8 + 3*C*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*b**4*x*cos(
c + d*x)**4/8 + 3*C*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*b**4*sin(
c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**2*(B*a*b + B*
b**2*cos(c) - C*a**2 + C*b**2*cos(c)**2), True))
```

Giac [A] time = 1.14865, size = 194, normalized size = 1.1

$$\frac{Cb^4\sin(4\,dx+4\,c)}{32\,d} - \frac{1}{8}\left(8\,Ca^4 - 8\,Ba^3b - 12\,Bab^3 - 3\,Cb^4\right)x + \frac{\left(2\,Cab^3 + Bb^4\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(3\,Bab^3 + Cb^4\right)\sin(2\,d}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] 1/32*C*b^4*sin(4*d*x + 4*c)/d - 1/8*(8*C*a^4 - 8*B*a^3*b - 12*B*a*b^3 - 3*C
*b^4)*x + 1/12*(2*C*a*b^3 + B*b^4)*sin(3*d*x + 3*c)/d + 1/4*(3*B*a*b^3 + C*
b^4)*sin(2*d*x + 2*c)/d - 1/4*(8*C*a^3*b - 12*B*a^2*b^2 - 6*C*a*b^3 - 3*B*b
^4)*sin(d*x + c)/d
```

3.976 $\int (a+b\cos(c+dx)) \left(abB - a^2C + b^2B\cos(c+dx) + b^2C\cos\right.$

Optimal. Leaf size=120

$$\frac{2b\left(-2a^2C+3abB+b^2C\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2bB-2a^3C+ab^2C+b^3B\right)+\frac{b^2(3bB-aC)\sin(c+dx)\cos(c+dx)}{6d}+\frac{bC}{}$$

```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C + a*b^2*C)*x)/2 + (2*b*(3*a*b*B - 2*a^2*C + b
^2*C)*Sin[c + d*x])/(3*d) + (b^2*(3*b*B - a*C)*Cos[c + d*x]*Sin[c + d*x])/(
6*d) + (b*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.214151, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3015, 2753, 2734}

$$\frac{2b\left(-2a^2C+3abB+b^2C\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^2bB-2a^3C+ab^2C+b^3B\right)+\frac{b^2(3bB-aC)\sin(c+dx)\cos(c+dx)}{6d}+\frac{bC}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c
+ d*x]^2),x]
```


```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C + a*b^2*C)*x)/2 + (2*b*(3*a*b*B - 2*a^2*C + b
^2*C)*Sin[c + d*x])/(3*d) + (b^2*(3*b*B - a*C)*Cos[c + d*x]*Sin[c + d*x])/(
6*d) + (b*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```


Rule 3015

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)\right)\,dx=\frac{\int (a+b\cos(c+dx))^2\left(b^2(bB-aC)+b^3C\right.}{b^2}$$

$$=\frac{bC(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{\int (a}{}$$

$$=\frac{1}{2}\left(2a^2bB+b^3B-2a^3C+ab^2C\right)x+\frac{2b\left(3a\right.}{}$$

Mathematica [A] time = 0.387372, size = 102, normalized size = 0.85

$$\frac{-6(c+dx)\left(-2a^2bB+2a^3C-ab^2C-b^3B\right)+3b\left(-4a^2C+8abB+3b^2C\right)\sin(c+dx)+3b^2(aC+bB)\sin(2(c+dx))+b^3C}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*
Cos[c + d*x]^2),x]
```

```
[Out] (-6*(-2*a^2*b*B - b^3*B + 2*a^3*C - a*b^2*C)*(c + d*x) + 3*b*(8*a*b*B - 4*a
^2*C + 3*b^2*C)*Sin[c + d*x] + 3*b^2*(b*B + a*C)*Sin[2*(c + d*x)] + b^3*C*S
in[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.018, size = 131, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Cb^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+b^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Cab^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*C*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+b^3*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+C*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b^2*B
*sin(d*x+c)-a^2*b*C*sin(d*x+c)+a^2*b*B*(d*x+c)-a^3*C*(d*x+c))
```

Maxima [A] time = 1.03424, size = 169, normalized size = 1.41

$$-\frac{12\,(dx+c)Ca^3-12\,(dx+c)Ba^2b-3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Cab^2-3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Bb^3+4\left(\sin\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2
),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(d*x + c)*C*a^3 - 12*(d*x + c)*B*a^2*b - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*C*a*b^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^3 + 4*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*C*b^3 + 12*C*a^2*b*sin(d*x + c) - 24*B*a*b^2*sin(d
*x + c))/d
```

Fricas [A] time = 1.64934, size = 224, normalized size = 1.87

$$-\frac{3\left(2\,Ca^3-2\,Ba^2b-Cab^2-Bb^3\right)dx-\left(2\,Cb^3\cos\left(dx+c\right)^2-6\,Ca^2b+12\,Bab^2+4\,Cb^3+3\left(Cab^2+Bb^3\right)\cos\left(dx+c\right)\right)\mathrm{s}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2
),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(2*C*a^3 - 2*B*a^2*b - C*a*b^2 - B*b^3)*d*x - (2*C*b^3*cos(d*x + c)
^2 - 6*C*a^2*b + 12*B*a*b^2 + 4*C*b^3 + 3*(C*a*b^2 + B*b^3)*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [A] time = 0.86374, size = 241, normalized size = 2.01

$$\begin{cases} Ba^2bx + \frac{2Bab^2\sin(c+dx)}{d} + \frac{Bb^3x\sin^2(c+dx)}{2} + \frac{Bb^3x\cos^2(c+dx)}{2} + \frac{Bb^3\sin(c+dx)\cos(c+dx)}{2d} - Ca^3x - \frac{Ca^2b\sin(c+dx)}{d} + \frac{Cab^2x\sin^2(c+dx)}{2} + \\ x(a+b\cos(c))\left(Bab+Bb^2\cos(c)-Ca^2+Cb^2\cos^2(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c
)**2),x)
```

```
[Out] Piecewise((B*a**2*b*x + 2*B*a*b**2*sin(c + d*x)/d + B*b**3*x*sin(c + d*x)**
2/2 + B*b**3*x*cos(c + d*x)**2/2 + B*b**3*sin(c + d*x)*cos(c + d*x)/(2*d) -
 C*a**3*x - C*a**2*b*sin(c + d*x)/d + C*a*b**2*x*sin(c + d*x)**2/2 + C*a*b*
*2*x*cos(c + d*x)**2/2 + C*a*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*C*b**
3*sin(c + d*x)**3/(3*d) + C*b**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)),
 (x*(a + b*cos(c))*(B*a*b + B*b**2*cos(c) - C*a**2 + C*b**2*cos(c)**2), Tru
e))
```

Giac [A] time = 1.13797, size = 144, normalized size = 1.2

$$\frac{Cb^3\sin(3\,dx+3\,c)}{12\,d} - \frac{1}{2}\left(2\,Ca^3 - 2\,Ba^2b - Cab^2 - Bb^3\right)x + \frac{\left(Cab^2+Bb^3\right)\sin(2\,dx+2\,c)}{4\,d} - \frac{\left(4\,Ca^2b - 8\,Bab^2 - 3\,Cb^3\right)\sin}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2
),x, algorithm="giac")
```

```
[Out] 1/12*C*b^3*sin(3*d*x + 3*c)/d - 1/2*(2*C*a^3 - 2*B*a^2*b - C*a*b^2 - B*b^3)
*x + 1/4*(C*a*b^2 + B*b^3)*sin(2*d*x + 2*c)/d - 1/4*(4*C*a^2*b - 8*B*a*b^2
- 3*C*b^3)*sin(d*x + c)/d
```

3.977 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{\sin(c+dx)\left(3a^2bB-3a^3C-ab^2(3A+2C)+2b^3B\right)}{3b^4d}-\frac{2a^3\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{\sin(c+dx)\,\mathrm{c}}{}$$

```
[Out] -((8*a^3*b*B + 4*a*b^3*B - 8*a^4*C - 4*a^2*b^2*(2*A + C) - b^4*(4*A + 3*C))
*x)/(8*b^5) - (2*a^3*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) + ((3*a^2*b*B + 2*b^3
*B - 3*a^3*C - a*b^2*(3*A + 2*C))*Sin[c + d*x])/(3*b^4*d) + ((4*A*b^2 - 4*a
*b*B + 4*a^2*C + 3*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) + ((b*B - a*
C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (C*Cos[c + d*x]^3*Sin[c + d*x])
/(4*b*d)
```

Rubi [A] time = 0.935385, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3049, 3023, 2735, 2659, 205}

$$\frac{\sin(c+dx)\left(3a^2bB-3a^3C-ab^2(3A+2C)+2b^3B\right)}{3b^4d}-\frac{2a^3\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{\sin(c+dx)\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] -((8*a^3*b*B + 4*a*b^3*B - 8*a^4*C - 4*a^2*b^2*(2*A + C) - b^4*(4*A + 3*C))
*x)/(8*b^5) - (2*a^3*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) + ((3*a^2*b*B + 2*b^3
*B - 3*a^3*C - a*b^2*(3*A + 2*C))*Sin[c + d*x])/(3*b^4*d) + ((4*A*b^2 - 4*a
*b*B + 4*a^2*C + 3*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) + ((b*B - a*
C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (C*Cos[c + d*x]^3*Sin[c + d*x])
/(4*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx = \frac{C\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos^2(c+dx)\left(3aC+b(4A+3C)\cos(c+d\right.}{a+b\cos(c+d}}{4b}$$

$$= \frac{(bB-aC)\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{C\cos^3(c+dx)\sin(c+}{4bd}$$

$$= \frac{\left(4Ab^2-4abB+4a^2C+3b^2C\right)\cos(c+dx)\sin(c+dx)}{8b^3d} + \frac{(b}{}$$

$$= \frac{\left(3a^2bB+2b^3B-3a^3C-ab^2(3A+2C)\right)\sin(c+dx)}{3b^4d} + \frac{(4A}{}$$

$$= -\frac{\left(8a^3bB+4ab^3B-8a^4C-4a^2b^2(2A+C)-b^4(4A+3C)\right)}{8b^5}$$

$$= -\frac{\left(8a^3bB+4ab^3B-8a^4C-4a^2b^2(2A+C)-b^4(4A+3C)\right)}{8b^5}$$

$$= -\frac{\left(8a^3bB+4ab^3B-8a^4C-4a^2b^2(2A+C)-b^4(4A+3C)\right)}{8b^5}$$

<u>Mathematica</u> [A] time = 0.87444, size = 238, normalized size = 0.85

$$\frac{12(c+dx)\left(4a^2b^2(2A+C)-8a^3bB+8a^4C-4ab^3B+b^4(4A+3C)\right)+24b^2\sin(2(c+dx))\left(a^2C-abB+Ab^2+b^2C\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x]),x]
```

```
[Out] (12*(-8*a^3*b*B - 4*a*b^3*B + 8*a^4*C + 4*a^2*b^2*(2*A + C) + b^4*(4*A + 3*
C))*(c + d*x) + (192*a^3*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((a - b)*Tan[(c
 + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 24*b*(4*a^2*b*B + 3*b^3*B
 - 4*a^3*C - a*b^2*(4*A + 3*C))*Sin[c + d*x] + 24*b^2*(A*b^2 - a*b*B + a^2*
C + b^2*C)*Sin[2*(c + d*x)] + 8*b^3*(b*B - a*C)*Sin[3*(c + d*x)] + 3*b^4*C*
Sin[4*(c + d*x)])/(96*b^5*d)
```

Maple [B] time = 0.042, size = 1580, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^2*B+1/d/b^2/(tan(
1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a*B+1/d/b^2/(tan(1/2*d*x+1/2*c)^
2+1)^4*tan(1/2*d*x+1/2*c)^5*a*B+6/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*
d*x+1/2*c)^3*a^2*B-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^3*
a*B+6/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a^2*B-1/d/b^2/(
tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*a*B+2/d/b^3/(tan(1/2*d*x+1/2*c
)^2+1)^4*tan(1/2*d*x+1/2*c)*a^2*B+2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-10/3/d/b^2/(tan(1/2*d*x+1/2*c
)^2+1)^4*tan(1/2*d*x+1/2*c)^5*C*a-10/3/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan
(1/2*d*x+1/2*c)^3*C*a-6/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)
^5*a*A-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*a^3*C-1/d/b^
3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^2*C-2/d/b^2/(tan(1/2*d*
x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*C*a-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4
*tan(1/2*d*x+1/2*c)^3*a^3*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+
1/2*c)*C*a-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*A-6/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c
)^3*a*A-2/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a^3*C+1/d/b
^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*a^2*C-2/d/b^2/(tan(1/2*d*x
+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*a*A-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*ta
n(1/2*d*x+1/2*c)^5*a^2*C+1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2
*c)^3*a^2*C-2/d*a^5/b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*C-2/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*
c)*a^3*C-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*a*A+1/d/b*
arctan(tan(1/2*d*x+1/2*c))*A+3/4/d/b*arctan(tan(1/2*d*x+1/2*c))*C-1/d/b/(ta
n(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*A+1/d/b/(tan(1/2*d*x+1/2*c)^2+
1)^4*tan(1/2*d*x+1/2*c)^3*A-3/4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+
1/2*c)^3*C+1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*A+5/4/d/b/(t
an(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)*C-1/d/b/(tan(1/2*d*x+1/2*c)^2+1
)^4*tan(1/2*d*x+1/2*c)^7*A-5/4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1
/2*c)^7*C+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*C+2/d/b^5*arctan(tan(1/2*d
*x+1/2*c))*a^4*C+3/4/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*C+
2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*A-1/d/b^2*arctan(tan(1/2*d*x+1/2*c))
*a*B+10/3/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^5*B+2/d/b/(tan(
1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1/2*c)^7*B+10/3/d/b/(tan(1/2*d*x+1/2*c)^2
+1)^4*tan(1/2*d*x+1/2*c)^3*B+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^4*tan(1/2*d*x+1
/2*c)*B-2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23369, size = 1674, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [1/24*(3*(8*C*a^6 - 8*B*a^5*b + 4*(2*A - C)*a^4*b^2 + 4*B*a^3*b^3 - (4*A +
C)*a^2*b^4 + 4*B*a*b^5 - (4*A + 3*C)*b^6)*d*x - 12*(C*a^5 - B*a^4*b + A*a^3
*b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)
^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (24*C*a^5*b - 24*B*a^4*b^2
 + 8*(3*A - C)*a^3*b^3 + 8*B*a^2*b^4 - 8*(3*A + 2*C)*a*b^5 + 16*B*b^6 - 6*(
C*a^2*b^4 - C*b^6)*cos(d*x + c)^3 + 8*(C*a^3*b^3 - B*a^2*b^4 - C*a*b^5 + B*
b^6)*cos(d*x + c)^2 - 3*(4*C*a^4*b^2 - 4*B*a^3*b^3 + (4*A - C)*a^2*b^4 + 4*
B*a*b^5 - (4*A + 3*C)*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d),
 1/24*(3*(8*C*a^6 - 8*B*a^5*b + 4*(2*A - C)*a^4*b^2 + 4*B*a^3*b^3 - (4*A +
C)*a^2*b^4 + 4*B*a*b^5 - (4*A + 3*C)*b^6)*d*x - 24*(C*a^5 - B*a^4*b + A*a^3
*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
 + c))) - (24*C*a^5*b - 24*B*a^4*b^2 + 8*(3*A - C)*a^3*b^3 + 8*B*a^2*b^4 -
8*(3*A + 2*C)*a*b^5 + 16*B*b^6 - 6*(C*a^2*b^4 - C*b^6)*cos(d*x + c)^3 + 8*(
C*a^3*b^3 - B*a^2*b^4 - C*a*b^5 + B*b^6)*cos(d*x + c)^2 - 3*(4*C*a^4*b^2 -
4*B*a^3*b^3 + (4*A - C)*a^2*b^4 + 4*B*a*b^5 - (4*A + 3*C)*b^6)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^5 - b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.22737, size = 1081, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(8*C*a^4 - 8*B*a^3*b + 8*A*a^2*b^2 + 4*C*a^2*b^2 - 4*B*a*b^3 + 4*A*
b^4 + 3*C*b^4)*(d*x + c)/b^5 + 48*(C*a^5 - B*a^4*b + A*a^3*b^2)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) - 2*(24*C*a
^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^2*b*
tan(1/2*d*x + 1/2*c)^7 + 24*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*B*a*b^2*tan
(1/2*d*x + 1/2*c)^7 + 24*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^3*tan(1/2*
d*x + 1/2*c)^7 - 24*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 15*C*b^3*tan(1/2*d*x + 1
/2*c)^7 + 72*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 72*B*a^2*b*tan(1/2*d*x + 1/2*c)
^5 + 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*a*b^2*tan(1/2*d*x + 1/2*c)^5
- 12*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 40*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 1
2*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*C*b^3*
tan(1/2*d*x + 1/2*c)^5 + 72*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 72*B*a^2*b*tan(1
/2*d*x + 1/2*c)^3 - 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*A*a*b^2*tan(1/2*
d*x + 1/2*c)^3 + 12*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 40*C*a*b^2*tan(1/2*d*x
 + 1/2*c)^3 - 12*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*b^3*tan(1/2*d*x + 1/2*
c)^3 + 9*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 24*C*a^3*tan(1/2*d*x + 1/2*c) - 24*
B*a^2*b*tan(1/2*d*x + 1/2*c) - 12*C*a^2*b*tan(1/2*d*x + 1/2*c) + 24*A*a*b^2
*tan(1/2*d*x + 1/2*c) + 12*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*C*a*b^2*tan(1/
2*d*x + 1/2*c) - 12*A*b^3*tan(1/2*d*x + 1/2*c) - 24*B*b^3*tan(1/2*d*x + 1/2
*c) - 15*C*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^4))/
d
```

3.978 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=206

$$\frac{\sin(c+dx)\left(3a^2C-3abB+3Ab^2+2b^2C\right)}{3b^3d}+\frac{2a^2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2bB-2a^3C-ab\right.}{2b^4}$$

```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C - a*b^2*(2*A + C))*x)/(2*b^4) + (2*a^2*(A*b^2
 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b^4*Sqrt[a + b]*d) + ((3*A*b^2 - 3*a*b*B + 3*a^2*C + 2*b^2*C)*Sin[c
 + d*x])/(3*b^3*d) + ((b*B - a*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (C
*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.567419, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3049, 3023, 2735, 2659, 205}

$$\frac{\sin(c+dx)\left(3a^2C-3abB+3Ab^2+2b^2C\right)}{3b^3d}+\frac{2a^2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2bB-2a^3C-ab\right.}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C - a*b^2*(2*A + C))*x)/(2*b^4) + (2*a^2*(A*b^2
 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b^4*Sqrt[a + b]*d) + ((3*A*b^2 - 3*a*b*B + 3*a^2*C + 2*b^2*C)*Sin[c
 + d*x])/(3*b^3*d) + ((b*B - a*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (C
*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{C\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2aC+b(3A+2C)\cos(c+dx\right.}{a+b\cos(c+d}}{3b}$$

$$= \frac{(bB-aC)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{C\cos^2(c+dx)\sin(c+}{3bd}$$

$$= \frac{\left(3Ab^2-3abB+3a^2C+2b^2C\right)\sin(c+dx)}{3b^3d} + \frac{(bB-aC)\cos}{}$$

$$= \frac{\left(2a^2bB+b^3B-2a^3C-ab^2(2A+C)\right)x}{2b^4} + \frac{\left(3Ab^2-3abB+\right.}{}$$

$$= \frac{\left(2a^2bB+b^3B-2a^3C-ab^2(2A+C)\right)x}{2b^4} + \frac{\left(3Ab^2-3abB+\right.}{}$$

$$= \frac{\left(2a^2bB+b^3B-2a^3C-ab^2(2A+C)\right)x}{2b^4} + \frac{2a^2\left(Ab^2-a(bB\right.}{\sqrt{}}$$

Mathematica [A] time = 0.650606, size = 179, normalized size = 0.87

$$\frac{-6(c+dx)\left(-2a^2bB+2a^3C+ab^2(2A+C)-b^3B\right)+3b\sin(c+dx)\left(4a^2C-4abB+4Ab^2+3b^2C\right)-\frac{24a^2\left(a(aC-bB)+Ab^2\right)\tan}{\sqrt{b}}}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x]),x]
```

```
[Out] (-6*(-2*a^2*b*B - b^3*B + 2*a^3*C + a*b^2*(2*A + C))*(c + d*x) - (24*a^2*(A
*b^2 + a*(-(b*B) + a*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + 3*b*(4*A*b^2 - 4*a*b*B + 4*a^2*C + 3*b^2*C)*Sin[c +
d*x] + 3*b^2*(b*B - a*C)*Sin[2*(c + d*x)] + b^3*C*Sin[3*(c + d*x)])/(12*b^4
*d)
```

Maple [B] time = 0.036, size = 814, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*A-2/d/b^2/(tan(1/2*d*
x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a*B-1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*t
an(1/2*d*x+1/2*c)^5*B+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)
^5*a^2*C+1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*C*a+2/d/b/
(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*C+4/d/b/(tan(1/2*d*x+1/2*c)
^2+1)^3*tan(1/2*d*x+1/2*c)^3*A-4/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d
*x+1/2*c)^3*a*B+4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a^2
*C+4/3/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/b/(tan(1/2
*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*A-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*
tan(1/2*d*x+1/2*c)*a*B+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c
)*a^2*C+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*C+1/d/b/(tan(1/
2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*B-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3
*tan(1/2*d*x+1/2*c)*C*a-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*a*A+2/d/b^3*arct
an(tan(1/2*d*x+1/2*c))*a^2*B+1/d/b*arctan(tan(1/2*d*x+1/2*c))*B-2/d/b^4*arc
tan(tan(1/2*d*x+1/2*c))*a^3*C-1/d/b^2*C*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a^
2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))*A-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*B+2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00078, size = 1288, normalized size = 6.25

$$\left[-\frac{3\left(2\,Ca^5-2\,Ba^4b+(2\,A-C)a^3b^2+Ba^2b^3-(2\,A+C)ab^4+Bb^5\right)dx+3\left(Ca^4-Ba^3b+Aa^2b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(\ldots}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [-1/6*(3*(2*C*a^5 - 2*B*a^4*b + (2*A - C)*a^3*b^2 + B*a^2*b^3 - (2*A + C)*a
*b^4 + B*b^5)*d*x + 3*(C*a^4 - B*a^3*b + A*a^2*b^2)*sqrt(-a^2 + b^2)*log((2
*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*co
s(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + a^2)) - (6*C*a^4*b - 6*B*a^3*b^2 + 2*(3*A - C)*a^2*b^3 + 6*B*a*
b^4 - 2*(3*A + 2*C)*b^5 + 2*(C*a^2*b^3 - C*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b
^2 - B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b
^6)*d), -1/6*(3*(2*C*a^5 - 2*B*a^4*b + (2*A - C)*a^3*b^2 + B*a^2*b^3 - (2*A
 + C)*a*b^4 + B*b^5)*d*x - 6*(C*a^4 - B*a^3*b + A*a^2*b^2)*sqrt(a^2 - b^2)*
arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^4*b -
 6*B*a^3*b^2 + 2*(3*A - C)*a^2*b^3 + 6*B*a*b^4 - 2*(3*A + 2*C)*b^5 + 2*(C*a
^2*b^3 - C*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 - B*a^2*b^3 - C*a*b^4 + B*b^5
)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.30182, size = 572, normalized size = 2.78

$$-\frac{\frac{3\left(2\,Ca^3-2\,Ba^2b+2\,Aab^2+Cab^2-Bb^3\right)(dx+c)}{b^4}+\frac{12\left(Ca^4-Ba^3b+Aa^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}-\frac{2\left(6\,Ca^2\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] -1/6*(3*(2*C*a^3 - 2*B*a^2*b + 2*A*a*b^2 + C*a*b^2 - B*b^3)*(d*x + c)/b^4 +
 12*(C*a^4 - B*a^3*b + A*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*
a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a
^2 - b^2)))/(sqrt(a^2 - b^2)*b^4) - 2*(6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*B
*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(
1/2*d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x +
 1/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*b*tan(1/2*d*x + 1/2*c)
^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C
*a^2*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) - 3*C*a*b*tan(1/2*
d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c)
+ 6*C*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.979 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{2a\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(b^2(2A+C)-2a(bB-aC)\right)}{2b^3}+\frac{(bB-aC)\sin(c+dx)}{b^2d}+\frac{C\sin(c+}{}$$

```
[Out] ((b^2*(2*A + C) - 2*a*(b*B - a*C))*x)/(2*b^3) - (2*a*(A*b^2 - a*(b*B - a*C)
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt
[a + b]*d) + ((b*B - a*C)*Sin[c + d*x])/(b^2*d) + (C*Cos[c + d*x]*Sin[c + d
*x])/(2*b*d)
```

Rubi [A] time = 0.334223, antiderivative size = 142, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3049, 3023, 2735, 2659, 205}

$$-\frac{2a\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(-\frac{2a(bB-aC)}{b^2}+2A+C\right)}{2b}+\frac{(bB-aC)\sin(c+dx)}{b^2d}+\frac{C\sin(c+dx)c}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d
*x]),x]
```

```
[Out] ((2*A + C - (2*a*(b*B - a*C))/b^2)*x)/(2*b) - (2*a*(A*b^2 - a*(b*B - a*C))*
ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a
 + b]*d) + ((b*B - a*C)*Sin[c + d*x])/(b^2*d) + (C*Cos[c + d*x]*Sin[c + d*x
])/(2*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{C\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{aC+b(2A+C)\cos(c+dx)+2(bB-aC)\cos}{a+b\cos(c+dx)}}{2b}$$

$$= \frac{(bB-aC)\sin(c+dx)}{b^2d} + \frac{C\cos(c+dx)\sin(c+dx)}{2bd} + \int \frac{abC+(}{}$$

$$= \frac{\left(b^2(2A+C)-2a(bB-aC)\right)x}{2b^3} + \frac{(bB-aC)\sin(c+dx)}{b^2d} + C$$

$$= \frac{\left(b^2(2A+C)-2a(bB-aC)\right)x}{2b^3} + \frac{(bB-aC)\sin(c+dx)}{b^2d} + C$$

$$= \frac{\left(b^2(2A+C)-2a(bB-aC)\right)x}{2b^3} - \frac{2a\left(Ab^2-a(bB-aC)\right)\tan}{\sqrt{a-b}\,b^3\sqrt{a}}$$

Mathematica [A] time = 0.404276, size = 133, normalized size = 0.92

$$\frac{2(c+dx)\left(2a^2C-2abB+2Ab^2+b^2C\right)+\frac{8a\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+4b(bB-aC)\sin(c+dx)+b^2C\sin(2(c}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x]),x]
```

```
[Out] (2*(2*A*b^2 - 2*a*b*B + 2*a^2*C + b^2*C)*(c + d*x) + (8*a*(A*b^2 + a*(-(b*B
) + a*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
 b^2] + 4*b*(b*B - a*C)*Sin[c + d*x] + b^2*C*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [B] time = 0.034, size = 434, normalized size = 3.

$$2\,\frac{(\tan(1/2\,dx+c/2))^3 B}{db\left((\tan(1/2\,dx+c/2))^2+1\right)^2} - 2\,\frac{(\tan(1/2\,dx+c/2))^3 aC}{db^2\left((\tan(1/2\,dx+c/2))^2+1\right)^2} - \frac{C}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)^{-2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-2/d/b^2/(tan(1/2*d*
x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-1/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*t
an(1/2*d*x+1/2*c)^3*C+2/d/b/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*B
-2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C+1/d/b/(tan(1/2*d
*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C+2/d/b*arctan(tan(1/2*d*x+1/2*c))*A-2/
d/b^2*arctan(tan(1/2*d*x+1/2*c))*a*B+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2
*C+1/d/b*arctan(tan(1/2*d*x+1/2*c))*C-2/d*a/b/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*a^2/b^2/((a+b)*(a-b))^(1/
2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*a^3/b^3/((a+b
)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94794, size = 988, normalized size = 6.86

$$\left[\frac{\left(2\,Ca^4 - 2\,Ba^3b + (2\,A - C)a^2b^2 + 2\,Bab^3 - (2\,A + C)b^4\right)dx - \left(Ca^3 - Ba^2b + Aab^2\right)\sqrt{-a^2 + b^2}\log\left(\frac{2\,ab\cos(dx+c) + \left(2\,a^2 - b^2\right)\cdots}{\cdots}\right.}{2\left(a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*((2*C*a^4 - 2*B*a^3*b + (2*A - C)*a^2*b^2 + 2*B*a*b^3 - (2*A + C)*b^4)
*d*x - (C*a^3 - B*a^2*b + A*a*b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c)
 + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
```

```
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
 - (2*C*a^3*b - 2*B*a^2*b^2 - 2*C*a*b^3 + 2*B*b^4 - (C*a^2*b^2 - C*b^4)*cos
(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d), 1/2*((2*C*a^4 - 2*B*a^3*b + (
2*A - C)*a^2*b^2 + 2*B*a*b^3 - (2*A + C)*b^4)*d*x - 2*(C*a^3 - B*a^2*b + A*
a*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (2*C*a^3*b - 2*B*a^2*b^2 - 2*C*a*b^3 + 2*B*b^4 - (C*a^2*b^2 - C*
b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24016, size = 323, normalized size = 2.24

$$\frac{\frac{(2\,Ca^2-2\,Bab+2\,Ab^2+Cb^2)(dx+c)}{b^3}+\frac{4\,(Ca^3-Ba^2b+Aab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{2\left(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((2*C*a^2 - 2*B*a*b + 2*A*b^2 + C*b^2)*(d*x + c)/b^3 + 4*(C*a^3 - B*a^2
*b + A*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^
2 - b^2)*b^3) - 2*(2*C*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c
)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*tan(1/2*d*x + 1/2*c) - 2*B*b*tan(1
/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2
*b^2))/d
```

3.980 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{x(bB-aC)}{b^2}+\frac{C\sin(c+dx)}{bd}$$

```
[Out] ((b*B - a*C)*x)/b^2 + (2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*Sin[c + d*x]
)/(b*d)
```

Rubi [A] time = 0.147348, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3023, 2735, 2659, 205}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{x(bB-aC)}{b^2}+\frac{C\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```


```
[Out] ((b*B - a*C)*x)/b^2 + (2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*Sin[c + d*x]
)/(b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{C\sin(c+dx)}{bd}+\frac{\int \frac{Ab+(bB-aC)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{(bB-aC)x}{b^2}+\frac{C\sin(c+dx)}{bd}-\left(-A+\frac{a(bB-aC)}{b^2}\right)\int \frac{1}{a+b\cos(c+dx)}\,dx\\ &= \frac{(bB-aC)x}{b^2}+\frac{C\sin(c+dx)}{bd}+\frac{\left(2\left(A-\frac{a(bB-aC)}{b^2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,\right.}{d}\\ &= \frac{(bB-aC)x}{b^2}+\frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}+\frac{C\sin(c+dx)}{bd}\end{aligned}$$

Mathematica [A] time = 0.224974, size = 92, normalized size = 0.95

$$\frac{-\frac{2\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+(c+dx)(bB-aC)+bC\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((b*B - a*C)*(c + d*x) - (2*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((a - b)*Tan
[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*C*Sin[c + d*x])/(b^2
```

*d)

Maple [B] time = 0.03, size = 216, normalized size = 2.2

$$2\frac{C\tan\left(1/2\,dx+c/2\right)}{db\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}+1\right)}+2\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)B}{db}-2\frac{C\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)a}{db^{2}}+2\frac{A}{d\sqrt{(a+b)\,(a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*C/b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/b*arctan(tan(1/2*d*
x+1/2*c))*B-2/d/b^2*C*arctan(tan(1/2*d*x+1/2*c))*a+2/d/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a/b/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d/b^2/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86926, size = 710, normalized size = 7.32

$$\left[-\frac{2\left(Ca^{3}-Ba^{2}b-Cab^{2}+Bb^{3}\right)dx+\left(Ca^{2}-Bab+Ab^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^{2}-b^{2}\right)\cos(dx+c)^{2}+2\,\sqrt{-a^{2}+b^{2}}(a\cos(dx+c)+}{b^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+a^{2}}\right.}{2\left(a^{2}b^{2}-b^{4}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x + (C*a^2 - B*a*b + A*b^2)*
sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2
*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(C*a^2*b - C*b^3)*sin(d*x + c)
)/((a^2*b^2 - b^4)*d), -((C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x - (C*a^2 -
 B*a*b + A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^
2)*sin(d*x + c))) - (C*a^2*b - C*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18304, size = 198, normalized size = 2.04

$$-\frac{\frac{(Ca-Bb)(dx+c)}{b^2}-\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}+\frac{2\left(Ca^2-Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((C*a - B*b)*(d*x + c)/b^2 - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*b) + 2*(C*a^2 - B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))
/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^2))/d
```

3.981 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{b}$$

```
[Out] (C*x)/b - (2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(
a*d)
```

Rubi [A] time = 0.137051, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3057, 2659, 205, 3770}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}+\frac{Cx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d
*x]),x]
```


```
[Out] (C*x)/b - (2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(
a*d)
```


Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx = \frac{Cx}{b}+\frac{A\int\sec(c+dx)\,dx}{a}-\left(\frac{Ab}{a}-B+\frac{aC}{b}\right)\int\frac{1}{a+b\cos(c+$$

$$= \frac{Cx}{b}+\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\left(2\left(\frac{Ab}{a}-B+\frac{aC}{b}\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{Cx}{b}-\frac{2\left(\frac{Ab}{a}-B+\frac{aC}{b}\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}+\frac{A\tanh^{-1}(}{}$$

Mathematica [C] time = 0.608345, size = 256, normalized size = 2.72

$$\frac{2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\left(2(\sin(c)+i\cos(c))\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(b\cos(c)-a)+b\,s\right.}{\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\right.\right.}{abd\sqrt{\left(b^2-a^2\right)(\cos(2c)-i\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x]),x]
```

```
[Out] (2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*((a*C*d*x - A*b*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] + A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sq
rt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)] + 2*(A*b^2 + a*(-(b*B) + a*C))*Arc
Tan[((I*Cos[c] + Sin[c])*(b*Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-(
(a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]]*(I*Cos[c] + Sin[c])))/(a*b*d*(2*A + C
+ 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sqrt[(-a^2 + b^2)*(Cos[2*c] - I*Si
n[2*c])])
```

Maple [B] time = 0.067, size = 202, normalized size = 2.2

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{db}-\frac{A}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\,\frac{Ab}{ad\sqrt{(a+b)(a-b)}}\arctan\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))*C-1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-2/d/a*b
/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A
+2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*B-2/d*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*C+1/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 19.9063, size = 807, normalized size = 8.59

$$\left[\frac{2\left(Ca^3-Cab^2\right)dx-\left(Ca^2-Bab+Ab^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\left(a^3b-ab^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*(2*(C*a^3 - C*a*b^2)*d*x - (C*a^2 - B*a*b + A*b^2)*sqrt(-a^2 + b^2)*lo
g((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(
a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b
*cos(d*x + c) + a^2)) + (A*a^2*b - A*b^3)*log(sin(d*x + c) + 1) - (A*a^2*b
- A*b^3)*log(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d), 1/2*(2*(C*a^3 - C*a*b
^2)*d*x - 2*(C*a^2 - B*a*b + A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^2*b - A*b^3)*log(sin(d*x + c)
+ 1) - (A*a^2*b - A*b^3)*log(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c
 + d*x)), x)
```

Giac [A] time = 1.23375, size = 200, normalized size = 2.13

$$\frac{\frac{(dx+c)C}{b}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{2\left(Ca^2-Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\sqrt{a^2-b^2}}\right.\right.}{\sqrt{a^2-b^2}ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c)),x, al
gorithm="giac")
```

```
[Out] ((d*x + c)*C/b + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a - 2*(C*a^2 - B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b))/d
```

3.982 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

```
[Out] (2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(
a^2*d) + (A*Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.262122, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c +
 d*x]),x]
```

```
[Out] (2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(
a^2*d) + (A*Tan[c + d*x])/(a*d)
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\tan(c+dx)}{ad} + \frac{\int \frac{(-Ab+aB+aC\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{ad} - \frac{(Ab-aB)\int \sec(c+dx)\,dx}{a^2} + \frac{(-b(-Ab+a \ldots}{\ldots}$$

$$= -\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2 d} + \frac{A\tan(c+dx)}{ad} + \frac{(2(Ab \ldots}{\ldots}$$

$$= \frac{2(Ab^2-a(bB-aC))\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} - \frac{(Ab-aB) \ldots}{\ldots}$$

Mathematica [C] time = 2.62179, size = 339, normalized size = 3.17

$$\frac{2\cos^2(c+dx)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(-\frac{2i(\cos(c)-i\sin(c))\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(b\cos(c)-a)+b\sin(c)\right)}{\sqrt{-(a^2-b^2)(\cos(c)-i\sin(c))^2}}\right)}{\sqrt{(b^2-a^2)(\cos(c)-i\sin(c))^2}} \ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*C
os[c + d*x]),x]
```

```
[Out] (2*Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((A*b - a*B)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + (-(A*b) + a*B)*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] - ((2*I)*(A*b^2 + a*(-(b*B) + a*C))*ArcTan[((I*Cos[c] +
 Sin[c])*(b*Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos
[c] - I*Sin[c])^2)]]*(Cos[c] - I*Sin[c]))/Sqrt[(-a^2 + b^2)*(Cos[c] - I*Sin
[c])^2] + (a*A*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])) + (a*A*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))))/(a^2*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c +
 d*x)]))
```

Maple [B] time = 0.073, size = 272, normalized size = 2.5

$$-\frac{A}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{Ab}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - \frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + 2\frac{Ab^2}{da^2\sqrt{(a+b)(a-b)}}\arctan\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/d*A*b/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d*
B*ln(tan(1/2*d*x+1/2*c)-1)+2/d/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^2-2/d/a/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b*B+2/d/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/a/d*A/(tan(1/2*d*x+1/2
*c)+1)-1/d*A*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)+1/a/d*B*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 34.678, size = 1071, normalized size = 10.01

$$\left[-\frac{\left(Ca^2 - Bab + Ab^2\right)\sqrt{-a^2 + b^2}\cos(dx + c)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right) - (B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [-1/2*((C*a^2 - B*a*b + A*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos
(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x +
c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x +
```

```
c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c
) + 1) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c))
, 1/2*(2*(C*a^2 - B*a*b + A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) + (B*a^3 - A*a^2*b - B*a*b^
2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^3 - A*a^2*b - B*a*b^2
+ A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a^3 - A*a*b^2)*sin(d*x
+ c))/((a^4 - a^2*b^2)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)}{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c)),x
)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(a + b*co
s(c + d*x)), x)
```

Giac [A] time = 1.23587, size = 243, normalized size = 2.27

$$\frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}-\frac{2\left(Ca^2-Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(\cdots\right)\right)}{\sqrt{a^2-b^2}a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] ((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - (B*a - A*b)*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/a^2 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a) - 2*(C*a^2 - B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2))/d
```

3.983 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=154

$$-\frac{2b\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2(A+2C)-2abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(Ab-aB)\tan(c}{a^2d}$$

```
[Out] (-2*b*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*A*b^2 - 2*a*b*B + a^2*(A + 2*C
))*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - ((A*b - a*B)*Tan[c + d*x])/(a^2*d) +
(A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.54995, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{2b\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2(A+2C)-2abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(Ab-aB)\tan(c}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] (-2*b*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*A*b^2 - 2*a*b*B + a^2*(A + 2*C
))*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - ((A*b - a*B)*Tan[c + d*x])/(a^2*d) +
(A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{(-2(Ab-aB)+a(A+2C)\cos(c+dx)+Ab\,c}{a+b\cos(c+dx)}}{2a}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \int \frac{(2A}{}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{(2Ab}{}$$

$$= \frac{(2Ab^2 - 2abB + a^2(A+2C))\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(Ab-a}{}$$

$$= -\frac{2b(Ab^2 - a(bB-aC))\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{(2Ab^2-}{}$$

Mathematica [B] time = 2.06882, size = 314, normalized size = 2.04

$$\frac{\frac{8b(a(aC-bB)+Ab^2)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 2(a^2(A+2C) - 2abB + 2Ab^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2(a^2(A+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*C
os[c + d*x]),x]
```

```
[Out] ((8*b*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2] - 2*(2*A*b^2 - 2*a*b*B + a^2*(A + 2*C))*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 - 2*a*b*B + a^2*(A + 2*C))*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2 + (4*a*(-(A*b) + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]) - (a^2*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a
*(-(A*b) + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4
*a^3*d)
```

Maple [B] time = 0.079, size = 499, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/d*A/a
^2/(tan(1/2*d*x+1/2*c)-1)*b-1/a/d*B/(tan(1/2*d*x+1/2*c)-1)-1/2/a/d*A*ln(tan
(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+1/d/a^2*ln(tan(1/
2*d*x+1/2*c)-1)*b*B-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-2/d*b^3/a^3/((a+b)*(a-
b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*b^2/a^
2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
B-2/d*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*C-1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a/d*A/(tan(1/2*d*x+1/2*c)+
1)+1/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)*b-1/a/d*B/(tan(1/2*d*x+1/2*c)+1)+1/2/a/
d*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2-1/d/a^2
*ln(tan(1/2*d*x+1/2*c)+1)*b*B+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 76.9684, size = 1450, normalized size = 9.42

$$\left[-\frac{2\left(Ca^2b - Bab^2 + Ab^3\right)\sqrt{-a^2+b^2}\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [-1/4*(2*(C*a^2*b - B*a*b^2 + A*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2
*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*co
s(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + a^2)) - ((A + 2*C)*a^4 - 2*B*a^3*b + (A - 2*C)*a^2*b^2 + 2*B*a*
b^3 - 2*A*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + ((A + 2*C)*a^4 - 2*B*
a^3*b + (A - 2*C)*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) - 2*(A*a^4 - A*a^2*b^2 + 2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b
^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2), -1/4*(4
*(C*a^2*b - B*a*b^2 + A*b^3)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - ((A + 2*C)*a^4 - 2*B*a^3*b
+ (A - 2*C)*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)^2*log(sin(d*x + c)
+ 1) + ((A + 2*C)*a^4 - 2*B*a^3*b + (A - 2*C)*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4
)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(A*a^4 - A*a^2*b^2 + 2*(B*a^4 -
 A*a^3*b - B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2
)*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.27442, size = 387, normalized size = 2.51

$$\frac{\frac{(Aa^2+2\,Ca^2-2\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{(Aa^2+2\,Ca^2-2\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{4\,(Ca^2b-Bab^2+Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b\cdots\right.}{\sqrt{a^2-\cdots}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/2*((A*a^2 + 2*C*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/a^3 - (A*a^2 + 2*C*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/a^3 + 4*(C*a^2*b - B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3
- 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2
*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/((
tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.984 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{2b^2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\tan(c+dx)\left(a^2(2A+3C)-3abB+3Ab^2\right)}{3a^3d}-\frac{\left(a^2b(A+2C)+a^3(-B)\right.}{}$$

```
[Out] (2*b^2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) - ((2*A*b^3 - a^3*B - 2*a*b^2*B + a
^2*b*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + ((3*A*b^2 - 3*a*b*B + a^
2*(2*A + 3*C))*Tan[c + d*x])/(3*a^3*d) - ((A*b - a*B)*Sec[c + d*x]*Tan[c +
d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.882409, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$\frac{2b^2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\tan(c+dx)\left(a^2(2A+3C)-3abB+3Ab^2\right)}{3a^3d}-\frac{\left(a^2b(A+2C)+a^3(-B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] (2*b^2*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) - ((2*A*b^3 - a^3*B - 2*a*b^2*B + a
^2*b*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + ((3*A*b^2 - 3*a*b*B + a^
2*(2*A + 3*C))*Tan[c + d*x])/(3*a^3*d) - ((A*b - a*B)*Sec[c + d*x]*Tan[c +
d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3(Ab-aB)+a(2A+3C)\cos(c+dx)+2\ldots\right.}{a+b\cos(c+d\ldots}}{3a}$$

$$= -\frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(c+dx)\tan(c+\ldots}{3ad}$$

$$= \frac{\left(3Ab^2-3abB+a^2(2A+3C)\right)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c\ldots}{2\ldots}$$

$$= \frac{\left(3Ab^2-3abB+a^2(2A+3C)\right)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c\ldots}{2\ldots}$$

$$= -\frac{\left(2Ab^3-a^3B-2ab^2B+a^2b(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} \ldots$$

$$= \frac{2b^2\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{\left(2Ab^3-a\ldots\right.}{\ldots}$$

Mathematica [B] time = 2.73612, size = 466, normalized size = 2.18

$$\frac{\frac{4a\sin\left(\frac{1}{2}(c+dx)\right)\left(a^2(2A+3C)-3abB+3Ab^2\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a\sin\left(\frac{1}{2}(c+dx)\right)\left(a^2(2A+3C)-3abB+3Ab^2\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} - \frac{24b^2\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + 6\left(a^2b(A\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*C
os[c + d*x]),x]
```

```
[Out] ((-24*b^2*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 6*(2*A*b^3 - a^3*B - 2*a*b^2*B + a^2*b*(
A + 2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(-2*A*b^3 + a^3*B +
2*a*b^2*B - a^2*b*(A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^
2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*a^3*
A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (4*a*(3*A*b^2
 - 3*a*b*B + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3 - (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
 + (4*a*(3*A*b^2 - 3*a*b*B + a^2*(2*A + 3*C))*Sin[(c + d*x)/2])/(Cos[(c + d
```

```
*x)/2] + Sin[(c + d*x)/2]))/(12*a^4*d)
```

Maple [B] time = 0.091, size = 825, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/3/a/d*A/(tan(1/2*d*x+1/2*c)-1)^3-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-1/2/a/d*A/(tan(1/2*d*x+1/2*
c)-1)^2+1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*b^2/a^2/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/2/a/d*
B/(tan(1/2*d*x+1/2*c)-1)^2-1/2/a/d*B/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a/d*B/(ta
n(1/2*d*x+1/2*c)+1)+1/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)-1/2/a/d*B*ln(tan(1/2*d
*x+1/2*c)-1)+1/2/a/d*B*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d/(tan(1/2*d*x+1/2*c)-1
)*C-1/3/a/d*A/(tan(1/2*d*x+1/2*c)+1)^3-1/a/d/(tan(1/2*d*x+1/2*c)+1)*C-1/a/d
*A/(tan(1/2*d*x+1/2*c)+1)-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2/(tan(1/2*d
*x+1/2*c)+1)*b*B+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b^2*B-1/d/a^3*ln(tan(1/2*
d*x+1/2*c)-1)*b^2*B+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*b*B-1/d/a^3/(tan(1/2*d*x
+1/2*c)-1)*A*b^2-1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)^2*b+1/d*b^3/a^4*ln(tan(
1/2*d*x+1/2*c)-1)*A+1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-1/d/a^3/(tan(1/2*d
*x+1/2*c)+1)*A*b^2+1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)^2*b-1/d*b^3/a^4*ln(ta
n(1/2*d*x+1/2*c)+1)*A-1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d*A/a^2/(tan
(1/2*d*x+1/2*c)-1)*b-1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)*b-1/2/d*A*b/a^2*ln(
tan(1/2*d*x+1/2*c)+1)+1/2/d*A*b/a^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 160.163, size = 1796, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [-1/12*(6*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(-a^2 + b^2)*cos(d*x + c)^3*log
((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a
*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + a^2)) - 3*(B*a^5 - (A + 2*C)*a^4*b + B*a^3*b^2 - (A - 2*C)*a
^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(B*a
^5 - (A + 2*C)*a^4*b + B*a^3*b^2 - (A - 2*C)*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)
*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*((2*A
 + 3*C)*a^5 - 3*B*a^4*b + (A - 3*C)*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4)*cos(
d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(
d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), 1/12*(12*(C*a^2*b^2 - B*a*b^3
 + A*b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c)))*cos(d*x + c)^3 + 3*(B*a^5 - (A + 2*C)*a^4*b + B*a^3*b^2 - (A -
2*C)*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(B*a^5 - (A + 2*C)*a^4*b + B*a^3*b^2 - (A - 2*C)*a^2*b^3 - 2*B*a*b^4 + 2*
A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*A*a^5 - 2*A*a^3*b^2 + 2
*((2*A + 3*C)*a^5 - 3*B*a^4*b + (A - 3*C)*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4
)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c)
)*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.22968, size = 652, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b - 2*C*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*
d*x + 1/2*c) + 1))/a^4 - 3*(B*a^3 - A*a^2*b - 2*C*a^2*b + 2*B*a*b^2 - 2*A*b
^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 12*(C*a^2*b^2 - B*a*b^3 + A*b^
4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^
4) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6
*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*
c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B
*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*tan(1/2*
d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.985 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=285

$$-\frac{2b^3\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}-\frac{\tan(c+dx)\left(a^2b(2A+3C)-2a^3B-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{\left(4a^2b^2(A+\right.}{}$$

```
[Out] (-2*b^3*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) + ((8*A*b^4 - 4*a^3*b*B - 8*a*b^3*
B + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*a^5*d)
 - ((3*A*b^3 - 2*a^3*B - 3*a*b^2*B + a^2*b*(2*A + 3*C))*Tan[c + d*x])/(3*a^
4*d) + ((4*A*b^2 - 4*a*b*B + a^2*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(8
*a^3*d) - ((A*b - a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d) + (A*Sec[c +
d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rubi [A] time = 1.28194, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}-\frac{\tan(c+dx)\left(a^2b(2A+3C)-2a^3B-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{\left(4a^2b^2(A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] (-2*b^3*(A*b^2 - a*(b*B - a*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) + ((8*A*b^4 - 4*a^3*b*B - 8*a*b^3*
B + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*ArcTanh[Sin[c + d*x]])/(8*a^5*d)
 - ((3*A*b^3 - 2*a^3*B - 3*a*b^2*B + a^2*b*(2*A + 3*C))*Tan[c + d*x])/(3*a^
4*d) + ((4*A*b^2 - 4*a*b*B + a^2*(3*A + 4*C))*Sec[c + d*x]*Tan[c + d*x])/(8
*a^3*d) - ((A*b - a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d) + (A*Sec[c +
d*x]^3*Tan[c + d*x])/(4*a*d)
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{\int \frac{\left(-4(Ab-aB)+a(3A+4C)\cos(c+dx)+3.\right.}{a+b\cos(c+d}}{4a}$$

$$= -\frac{(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3a^2d} + \frac{A\sec^3(c+dx)\tan(c}{4ad}$$

$$= \frac{\left(4Ab^2-4abB+a^2(3A+4C)\right)\sec(c+dx)\tan(c+dx)}{8a^3d} - \frac{(Ab}{}$$

$$= -\frac{\left(3Ab^3-2a^3B-3ab^2B+a^2b(2A+3C)\right)\tan(c+dx)}{3a^4d} + \frac{(4A}{}$$

$$= -\frac{\left(3Ab^3-2a^3B-3ab^2B+a^2b(2A+3C)\right)\tan(c+dx)}{3a^4d} + \frac{(4A}{}$$

$$= \frac{\left(8Ab^4-4a^3bB-8ab^3B+4a^2b^2(A+2C)+a^4(3A+4C)\right)\tan}{8a^5d}$$

$$= -\frac{2b^3\left(Ab^2-a(bB-aC)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}d} + \frac{(8Ab^4-}{}$$

Mathematica [A] time = 1.42551, size = 406, normalized size = 1.42

$$\frac{\frac{96b^3\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 6\left(4a^2b^2(A+2C)+a^4(3A+4C)-4a^3bB-8ab^3B+8Ab^4\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/(a + b*C
os[c + d*x]),x]
```

```
[Out] ((96*b^3*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 6*(8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B + 4*a^
2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 6*(8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*
C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a*(21*a^3*A + 12*a*A*b^2 - 1
2*a^2*b*B + 12*a^3*C + 4*(-9*A*b^3 + 10*a^3*B + 9*a*b^2*B - a^2*b*(10*A + 9
*C))*Cos[c + d*x] + 3*a*(4*A*b^2 - 4*a*b*B + a^2*(3*A + 4*C))*Cos[2*(c + d*
x)] - 8*a^2*A*b*Cos[3*(c + d*x)] - 12*A*b^3*Cos[3*(c + d*x)] + 8*a^3*B*Cos[
```

```
3*(c + d*x)] + 12*a*b^2*B*Cos[3*(c + d*x)] - 12*a^2*b*C*Cos[3*(c + d*x)])*S
ec[c + d*x]^3*Tan[c + d*x])/(48*a^5*d)
```

Maple [B] time = 0.098, size = 1335, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)^3+1/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C+7/8/a
/d*A/(tan(1/2*d*x+1/2*c)-1)^2-1/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-7/8/a/d*A/
(tan(1/2*d*x+1/2*c)+1)^2+1/4/d*A/a/(tan(1/2*d*x+1/2*c)-1)^4-1/4/d*A/a/(tan(
1/2*d*x+1/2*c)+1)^4-1/2/a/d*B/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d*B/(tan(1/2*d
*x+1/2*c)+1)^2-1/3/a/d*B/(tan(1/2*d*x+1/2*c)-1)^3+1/2/a/d/(tan(1/2*d*x+1/2*
c)-1)^2*C-1/3/a/d*B/(tan(1/2*d*x+1/2*c)+1)^3-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)
^2*C-1/a/d*B/(tan(1/2*d*x+1/2*c)+1)-1/a/d*B/(tan(1/2*d*x+1/2*c)-1)+1/2/a/d/
(tan(1/2*d*x+1/2*c)-1)*C+1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)^3+1/2/a/d/(tan(1/
2*d*x+1/2*c)+1)*C+5/8/a/d*A/(tan(1/2*d*x+1/2*c)+1)+5/8/a/d*A/(tan(1/2*d*x+1
/2*c)-1)-3/8/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)+3/8/a/d*A*ln(tan(1/2*d*x+1/2*c)
+1)+1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*b*B-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)
*b*B-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*b*B+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)*
A*b^2+1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)-1)^2*b+1/2/d/a^3/(tan(1/2*d*x+1/2*c)+
1)*A*b^2-1/2/d*A/a^2/(tan(1/2*d*x+1/2*c)+1)^2*b+1/d*A/a^2/(tan(1/2*d*x+1/2*
c)-1)*b+1/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2+1/d*A/a^2/(tan(1/2*d*x+1/2
*c)+1)*b-1/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+1/d/a^4/(tan(1/2*d*x+1/2*
c)+1)*A*b^3-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b^2*B+1/d/a^2/(tan(1/2*d*x+1/2*c
)+1)*b*C+1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*A*b^3-1/d/a^3/(tan(1/2*d*x+1/2*c)-1
)*b^2*B+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*b*C+1/3/d/a^2/(tan(1/2*d*x+1/2*c)+1)
^3*A*b-1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2*A*b^2+1/2/d/a^2/(tan(1/2*d*x+1/2*
c)+1)^2*b*B+1/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*A*b^4-1/d/a^4*ln(tan(1/2*d*x+1
/2*c)+1)*b^3*B+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b^2*C+1/3/d/a^2/(tan(1/2*d*
x+1/2*c)-1)^3*A*b+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2*A*b^2-1/2/d/a^2/(tan(1
/2*d*x+1/2*c)-1)^2*b*B-1/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)*A*b^4+1/d/a^4*ln(ta
n(1/2*d*x+1/2*c)-1)*b^3*B-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*b^2*C-2/d*b^5/a^
5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
A+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*B-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5/(a+b*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.30812, size = 1185, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a^4 + 4*C*a^4 - 4*B*a^3*b + 4*A*a^2*b^2 + 8*C*a^2*b^2 - 8*B*a*
b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(3*A*a^4 + 4*C*a^
4 - 4*B*a^3*b + 4*A*a^2*b^2 + 8*C*a^2*b^2 - 8*B*a*b^3 + 8*A*b^4)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/a^5 + 48*(C*a^2*b^3 - B*a*b^4 + A*b^5)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + 2*(15*A*a
^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^3*tan(
1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*b*tan(1/2
*d*x + 1/2*c)^7 + 24*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*tan(1/2*d*
x + 1/2*c)^7 - 24*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan(1/2*d*x + 1
/2*c)^7 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^5
- 12*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*
B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 72*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*a
*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*A*b^3*
tan(1/2*d*x + 1/2*c)^5 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*
d*x + 1/2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^2*b*tan(1/2*d*x +
 1/2*c)^3 + 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b*tan(1/2*d*x + 1/
2*c)^3 - 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c
)^3 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^3*tan(1/2*d*x + 1/2*c) + 24*
B*a^3*tan(1/2*d*x + 1/2*c) + 12*C*a^3*tan(1/2*d*x + 1/2*c) - 24*A*a^2*b*tan
(1/2*d*x + 1/2*c) - 12*B*a^2*b*tan(1/2*d*x + 1/2*c) - 24*C*a^2*b*tan(1/2*d*
x + 1/2*c) + 12*A*a*b^2*tan(1/2*d*x + 1/2*c) + 24*B*a*b^2*tan(1/2*d*x + 1/2
*c) - 24*A*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^4))/
d
```

3.986 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=398

$$-\frac{\sin(c+dx)\left(-a^2b^2(6A-7C)+9a^3bB-12a^4C-6ab^3B+b^4(3A+2C)\right)}{3b^4d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab^2-5a^2b^2C-3a^3bB+4a^4C+4a\right.}{b^5d(a-b)^{3/2}(a}$$

```
[Out] ((6*a^2*b*B + b^3*B - 8*a^3*C - 2*a*b^2*(2*A + C))*x)/(2*b^5) + (2*a^2*(2*a
^2*A*b^2 - 3*A*b^4 - 3*a^3*b*B + 4*a*b^3*B + 4*a^4*C - 5*a^2*b^2*C)*ArcTan[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/
2)*d) - ((9*a^3*b*B - 6*a*b^3*B - a^2*b^2*(6*A - 7*C) - 12*a^4*C + b^4*(3*A
 + 2*C))*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) + ((3*a^2*b*B - b^3*B - 2*a*b^
2*(A - C) - 4*a^3*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) + ((3
*A*b^2 - 3*a*b*B + 4*a^2*C - b^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^
2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2
 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.59703, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3023, 2735, 2659, 205}

$$-\frac{\sin(c+dx)\left(-a^2b^2(6A-7C)+9a^3bB-12a^4C-6ab^3B+b^4(3A+2C)\right)}{3b^4d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab^2-5a^2b^2C-3a^3bB+4a^4C+4a\right.}{b^5d(a-b)^{3/2}(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] ((6*a^2*b*B + b^3*B - 8*a^3*C - 2*a*b^2*(2*A + C))*x)/(2*b^5) + (2*a^2*(2*a
^2*A*b^2 - 3*A*b^4 - 3*a^3*b*B + 4*a*b^3*B + 4*a^4*C - 5*a^2*b^2*C)*ArcTan[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/
2)*d) - ((9*a^3*b*B - 6*a*b^3*B - a^2*b^2*(6*A - 7*C) - 12*a^4*C + b^4*(3*A
 + 2*C))*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) + ((3*a^2*b*B - b^3*B - 2*a*b^
2*(A - C) - 4*a^3*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) + ((3
*A*b^2 - 3*a*b*B + 4*a^2*C - b^2*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^
2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2
```

```
- b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)(3}{}}{}$$

$$= \frac{\left(3Ab^2-3abB+4a^2C-b^2C\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{\left(Ab\right.}{}$$

$$= \frac{\left(3a^2bB-b^3B-2ab^2(A-C)-4a^3C\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(9a^3bB-6ab^3B-a^2b^2(6A-7C)-12a^4C+b^4(3A+2C)\right)s}{3b^4\left(a^2-b^2\right)d}$$

$$= \frac{\left(6a^2bB+b^3B-8a^3C-2ab^2(2A+C)\right)x}{2b^5} - \frac{\left(9a^3bB-6ab^3B-\right.}{}$$

$$= \frac{\left(6a^2bB+b^3B-8a^3C-2ab^2(2A+C)\right)x}{2b^5} - \frac{\left(9a^3bB-6ab^3B-\right.}{}$$

$$= \frac{\left(6a^2bB+b^3B-8a^3C-2ab^2(2A+C)\right)x}{2b^5} + \frac{2a^2\left(2a^2Ab^2-3A\right.}{}$$

Mathematica [A] time = 1.74533, size = 256, normalized size = 0.64

$$\frac{6(c+dx)\left(6a^2bB-8a^3C-2ab^2(2A+C)+b^3B\right)+3b\sin(c+dx)\left(12a^2C-8abB+4Ab^2+3b^2C\right)+\frac{12a^3b\sin(c+dx)\left(a(aC-b\right.}{(a-b)(a+b)(a+b\cos(c}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] (6*(6*a^2*b*B + b^3*B - 8*a^3*C - 2*a*b^2*(2*A + C))*(c + d*x) + (24*a^2*(-
3*A*b^4 - 3*a^3*b*B + 4*a*b^3*B + a^2*b^2*(2*A - 5*C) + 4*a^4*C)*ArcTanh[((
a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 3*b*(4*A*b
^2 - 8*a*b*B + 12*a^2*C + 3*b^2*C)*Sin[c + d*x] + (12*a^3*b*(A*b^2 + a*(-(b
*B) + a*C))*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + 3*b^2*(b
*B - 2*a*C)*Sin[2*(c + d*x)] + b^3*C*Sin[3*(c + d*x)])/(12*b^5*d)
```

Maple [B] time = 0.047, size = 1229, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 8/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B-10/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d/b^3/(tan(1/2*d*x+1/2*c
)^2+1)^3*tan(1/2*d*x+1/2*c)*C*a+6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*
d*x+1/2*c)^5*a^2*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*
C*a+12/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a^2*C-8/d/b^3/
(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^3*a*B-4/d/b^3/(tan(1/2*d*x+1/
2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*a*B+6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^3*tan
(1/2*d*x+1/2*c)*a^2*C-4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)
*a*B+1/d/b^2*B*arctan(tan(1/2*d*x+1/2*c))+4/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^
3*tan(1/2*d*x+1/2*c)^3*A+1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2
*c)*B-4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a*A+4/3/d/b^2/(tan(1/2*d*x+1/2*c)^
2+1)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*
x+1/2*c)*A+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)*C+2/d/b^2/
```

```
(tan(1/2*d*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*A-8/d/b^5*arctan(tan(1/2*d*
x+1/2*c))*a^3*C+6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2*B+2/d/b^2/(tan(1/2*d
*x+1/2*c)^2+1)^3*tan(1/2*d*x+1/2*c)^5*C-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^3*
tan(1/2*d*x+1/2*c)^5*B-2/d/b^3*C*arctan(tan(1/2*d*x+1/2*c))*a+8/d*a^6/b^5/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*C-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4/d*a^4/b^3/(a+b)/
(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))*A+2/d*a^5/b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)*C+2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A-2/d*a^4/b^3/(a^2-b^2)*tan(1/2
*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.88757, size = 2986, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [-1/6*(3*(8*C*a^7*b - 6*B*a^6*b^2 + 2*(2*A - 7*C)*a^5*b^3 + 11*B*a^4*b^4 -
4*(2*A - C)*a^3*b^5 - 4*B*a^2*b^6 + 2*(2*A + C)*a*b^7 - B*b^8)*d*x*cos(d*x
+ c) + 3*(8*C*a^8 - 6*B*a^7*b + 2*(2*A - 7*C)*a^6*b^2 + 11*B*a^5*b^3 - 4*(2
*A - C)*a^4*b^4 - 4*B*a^3*b^5 + 2*(2*A + C)*a^2*b^6 - B*a*b^7)*d*x + 3*(4*C
*a^7 - 3*B*a^6*b + (2*A - 5*C)*a^5*b^2 + 4*B*a^4*b^3 - 3*A*a^3*b^4 + (4*C*a
^6*b - 3*B*a^5*b^2 + (2*A - 5*C)*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d
```

```
*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (24*C*a^7*b - 18*B*a^6*
b^2 + 2*(6*A - 19*C)*a^5*b^3 + 30*B*a^4*b^4 - 2*(9*A - 5*C)*a^3*b^5 - 12*B*
a^2*b^6 + 2*(3*A + 2*C)*a*b^7 + 2*(C*a^4*b^4 - 2*C*a^2*b^6 + C*b^8)*cos(d*x
 + c)^3 - (4*C*a^5*b^3 - 3*B*a^4*b^4 - 8*C*a^3*b^5 + 6*B*a^2*b^6 + 4*C*a*b^
7 - 3*B*b^8)*cos(d*x + c)^2 + (12*C*a^6*b^2 - 9*B*a^5*b^3 + 2*(3*A - 10*C)*
a^4*b^4 + 18*B*a^3*b^5 - 4*(3*A - C)*a^2*b^6 - 9*B*a*b^7 + 2*(3*A + 2*C)*b^
8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c)
 + (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d), -1/6*(3*(8*C*a^7*b - 6*B*a^6*b^2 + 2*(
2*A - 7*C)*a^5*b^3 + 11*B*a^4*b^4 - 4*(2*A - C)*a^3*b^5 - 4*B*a^2*b^6 + 2*(
2*A + C)*a*b^7 - B*b^8)*d*x*cos(d*x + c) + 3*(8*C*a^8 - 6*B*a^7*b + 2*(2*A
- 7*C)*a^6*b^2 + 11*B*a^5*b^3 - 4*(2*A - C)*a^4*b^4 - 4*B*a^3*b^5 + 2*(2*A
+ C)*a^2*b^6 - B*a*b^7)*d*x - 6*(4*C*a^7 - 3*B*a^6*b + (2*A - 5*C)*a^5*b^2
+ 4*B*a^4*b^3 - 3*A*a^3*b^4 + (4*C*a^6*b - 3*B*a^5*b^2 + (2*A - 5*C)*a^4*b^
3 + 4*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos
(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (24*C*a^7*b - 18*B*a^6*b^2
 + 2*(6*A - 19*C)*a^5*b^3 + 30*B*a^4*b^4 - 2*(9*A - 5*C)*a^3*b^5 - 12*B*a^2
*b^6 + 2*(3*A + 2*C)*a*b^7 + 2*(C*a^4*b^4 - 2*C*a^2*b^6 + C*b^8)*cos(d*x +
c)^3 - (4*C*a^5*b^3 - 3*B*a^4*b^4 - 8*C*a^3*b^5 + 6*B*a^2*b^6 + 4*C*a*b^7 -
 3*B*b^8)*cos(d*x + c)^2 + (12*C*a^6*b^2 - 9*B*a^5*b^3 + 2*(3*A - 10*C)*a^4
*b^4 + 18*B*a^3*b^5 - 4*(3*A - C)*a^2*b^6 - 9*B*a*b^7 + 2*(3*A + 2*C)*b^8)*
cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) +
(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24514, size = 760, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(12*(4*C*a^6 - 3*B*a^5*b + 2*A*a^4*b^2 - 5*C*a^4*b^2 + 4*B*a^3*b^3 - 3
*A*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^5
- b^7)*sqrt(a^2 - b^2)) - 12*(C*a^5*tan(1/2*d*x + 1/2*c) - B*a^4*b*tan(1/2*
d*x + 1/2*c) + A*a^3*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^4 - b^6)*(a*tan(1/2*
d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 3*(8*C*a^3 - 6*B*a^2*
b + 4*A*a*b^2 + 2*C*a*b^2 - B*b^3)*(d*x + c)/b^5 - 2*(18*C*a^2*tan(1/2*d*x
+ 1/2*c)^5 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*b*tan(1/2*d*x + 1/2*c)
^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*
b^2*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan
(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 18*C*a^2*tan(1/2*d*x + 1/2*c) - 12*B*a*b*tan(1/2*d*x + 1/2*c)
 - 6*C*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 3*B*b^2*ta
n(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 + 1)^3*b^4))/d
```

3.987 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$\frac{\sin(c+dx)\left(2a^2bB-3a^3C-ab^2(A-2C)-b^3B\right)}{b^3d\left(a^2-b^2\right)}-\frac{2a\left(a^2Ab^2-4a^2b^2C-2a^3bB+3a^4C+3ab^3B-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\ldots}{\ldots}\right.}{b^4d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] ((2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*x)/(2*b^4) - (2*a*(a^2*A*b^2 - 2*A*b
^4 - 2*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 4*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*
b*B - b^3*B - a*b^2*(A - 2*C) - 3*a^3*C)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d)
+ ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*
(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.11738, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3023, 2735, 2659, 205}

$$\frac{\sin(c+dx)\left(2a^2bB-3a^3C-ab^2(A-2C)-b^3B\right)}{b^3d\left(a^2-b^2\right)}-\frac{2a\left(a^2Ab^2-4a^2b^2C-2a^3bB+3a^4C+3ab^3B-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\ldots}{\ldots}\right.}{b^4d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```

```
[Out] ((2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*x)/(2*b^4) - (2*a*(a^2*A*b^2 - 2*A*b
^4 - 2*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 4*a^2*b^2*C)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*
b*B - b^3*B - a*b^2*(A - 2*C) - 3*a^3*C)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d)
+ ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*
(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \int \frac{\cos(c+dx)(\ldots}{\ldots}$$

$$= \frac{\left(2Ab^2-2abB+3a^2C-b^2C\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{\left(Ab\ldots\right.}{\ldots}$$

$$= \frac{\left(2a^2bB-b^3B-ab^2(A-2C)-3a^3C\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2\ldots\right.}{\ldots}$$

$$= \frac{\left(2Ab^2-4abB+6a^2C+b^2C\right)x}{2b^4} + \frac{\left(2a^2bB-b^3B-ab^2(A-\ldots\right.}{b^3\left(a^2-\ldots\right.}$$

$$= \frac{\left(2Ab^2-4abB+6a^2C+b^2C\right)x}{2b^4} + \frac{\left(2a^2bB-b^3B-ab^2(A-\ldots\right.}{b^3\left(a^2-\ldots\right.}$$

$$= \frac{\left(2Ab^2-4abB+6a^2C+b^2C\right)x}{2b^4} - \frac{2a\left(a^2Ab^2-2Ab^4-2a^3b\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.3755, size = 208, normalized size = 0.69

$$\frac{2(c+dx)\left(6a^2C-4abB+2Ab^2+b^2C\right)-\frac{4a^2b\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{(a-b)(a+b)(a+b\cos(c+dx))}-\frac{8a\left(a^2b^2(A-4C)-2a^3bB+3a^4C+3ab^3B-2Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(\ldots\right.}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}\ldots}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] (2*(2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*(c + d*x) - (8*a*(-2*A*b^4 - 2*a^3
*b*B + 3*a*b^3*B + a^2*b^2*(A - 4*C) + 3*a^4*C)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 4*b*(b*B - 2*a*C)*Sin[c + d
*x] - (4*a^2*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a - b)*(a + b)*(a
 + b*Cos[c + d*x])) + b^2*C*Sin[2*(c + d*x)])/(4*b^4*d)
```

Maple [B] time = 0.043, size = 845, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-4/d/b^3/(tan(1/2*
d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C-1/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)
^2*tan(1/2*d*x+1/2*c)^3*C+2/d/b^2/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/
2*c)*B-4/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C+1/d/b^2/(t
an(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C+2/d/b^2*arctan(tan(1/2*d*x+1/
2*c))*A-4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a*B+6/d/b^4*arctan(tan(1/2*d*x+1
/2*c))*a^2*C+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))*C-2/d*a^2/b/(a^2-b^2)*tan(1
/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A+2/d*a^3
/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)*B-2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d*a/(a+b)/(a-b)/
((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+
4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*a^5/b^4/(a+b)/(a-b)/((a+b
)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+8/d*a
^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54595, size = 2381, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [1/2*((6*C*a^6*b - 4*B*a^5*b^2 + (2*A - 11*C)*a^4*b^3 + 8*B*a^3*b^4 - 4*(A
- C)*a^2*b^5 - 4*B*a*b^6 + (2*A + C)*b^7)*d*x*cos(d*x + c) + (6*C*a^7 - 4*B
*a^6*b + (2*A - 11*C)*a^5*b^2 + 8*B*a^4*b^3 - 4*(A - C)*a^3*b^4 - 4*B*a^2*b
^5 + (2*A + C)*a*b^6)*d*x - (3*C*a^6 - 2*B*a^5*b + (A - 4*C)*a^4*b^2 + 3*B*
a^3*b^3 - 2*A*a^2*b^4 + (3*C*a^5*b - 2*B*a^4*b^2 + (A - 4*C)*a^3*b^3 + 3*B*
a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c)
 + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
 - (6*C*a^6*b - 4*B*a^5*b^2 + 2*(A - 5*C)*a^4*b^3 + 6*B*a^3*b^4 - 2*(A - 2*
C)*a^2*b^5 - 2*B*a*b^6 - (C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7)*cos(d*x + c)^2 +
 (3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*C*a^3*b^4 + 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b
^7)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)
 + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*C*a^6*b - 4*B*a^5*b^2 + (2*A -
 11*C)*a^4*b^3 + 8*B*a^3*b^4 - 4*(A - C)*a^2*b^5 - 4*B*a*b^6 + (2*A + C)*b^
7)*d*x*cos(d*x + c) + (6*C*a^7 - 4*B*a^6*b + (2*A - 11*C)*a^5*b^2 + 8*B*a^4
*b^3 - 4*(A - C)*a^3*b^4 - 4*B*a^2*b^5 + (2*A + C)*a*b^6)*d*x - 2*(3*C*a^6
- 2*B*a^5*b + (A - 4*C)*a^4*b^2 + 3*B*a^3*b^3 - 2*A*a^2*b^4 + (3*C*a^5*b -
2*B*a^4*b^2 + (A - 4*C)*a^3*b^3 + 3*B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*sq
rt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))
- (6*C*a^6*b - 4*B*a^5*b^2 + 2*(A - 5*C)*a^4*b^3 + 6*B*a^3*b^4 - 2*(A - 2*C
)*a^2*b^5 - 2*B*a*b^6 - (C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7)*cos(d*x + c)^2 +
(3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*C*a^3*b^4 + 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^
7)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)
+ (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24799, size = 508, normalized size = 1.68

$$\frac{\frac{4\left(3\,Ca^{5}-2\,Ba^{4}b+Aa^{3}b^{2}-4\,Ca^{3}b^{2}+3\,Ba^{2}b^{3}-2\,Aab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}-\frac{4\left(Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ba^{3}b\tan\right.}{\left(a^{2}b^{3}-b^{5}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/2*(4*(3*C*a^5 - 2*B*a^4*b + A*a^3*b^2 - 4*C*a^3*b^2 + 3*B*a^2*b^3 - 2*A*a
*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)
*sqrt(a^2 - b^2)) - 4*(C*a^4*tan(1/2*d*x + 1/2*c) - B*a^3*b*tan(1/2*d*x + 1
/2*c) + A*a^2*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*C*a^2 - 4*B*a*b + 2*A*b^2
 + C*b^2)*(d*x + c)/b^4 - 2*(4*C*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d
*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 + 4*C*a*tan(1/2*d*x + 1/2*c) - 2
*B*b*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 + 1)^2*b^3))/d
```

3.988 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{2\left(3a^2b^2C+a^3bB-2a^4C-2ab^3B+Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(bB-2aC}{b^3}$$

```
[Out] ((b*B - 2*a*C)*x)/b^3 - (2*(A*b^4 + a^3*b*B - 2*a*b^3*B - 2*a^4*C + 3*a^2*b
^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^
3*(a + b)^(3/2)*d) + (C*Sin[c + d*x])/(b^2*d) + (a*(A*b^2 - a*(b*B - a*C))*
Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.461326, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3031, 3023, 2735, 2659, 205}

$$-\frac{2\left(3a^2b^2C+a^3bB-2a^4C-2ab^3B+Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(bB-2aC}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d
*x])^2,x]
```


```
[Out] ((b*B - 2*a*C)*x)/b^3 - (2*(A*b^4 + a^3*b*B - 2*a*b^3*B - 2*a^4*C + 3*a^2*b
^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^
3*(a + b)^(3/2)*d) + (C*Sin[c + d*x])/(b^2*d) + (a*(A*b^2 - a*(b*B - a*C))*
Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
```

```
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx = \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \int \frac{-b\left(Ab^2-a(bB-aC)\right)+\left(a^2-b^2\right)}{}$$

$$= \frac{C\sin(c+dx)}{b^2d} + \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \int \frac{-b^2(A}{}$$

$$= \frac{(bB-2aC)x}{b^3} + \frac{C\sin(c+dx)}{b^2d} + \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c}{b^2\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{(bB-2aC)x}{b^3} + \frac{C\sin(c+dx)}{b^2d} + \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c}{b^2\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{(bB-2aC)x}{b^3} - \frac{2\left(Ab^4+a^3bB-2ab^3B-2a^4C+3a^2b^2C\right)\text{ta}}{(a-b)^{3/2}b^3(a+b)^{3/2}d}$$

Mathematica [A] time = 0.959694, size = 159, normalized size = 0.95

$$\frac{-\frac{2\left(a\left(a^2bB-2a^3C+3ab^2C-2b^3B\right)+Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{ab\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{(a-b)(a+b)(a+b\cos(c+dx))}+(c+dx)(bB-2aC)+bC\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^2,x]
```

```
[Out] ((b*B - 2*a*C)*(c + d*x) - (2*(A*b^4 + a*(a^2*b*B - 2*b^3*B - 2*a^3*C + 3*a
*b^2*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)
^(3/2) + b*C*Sin[c + d*x] + (a*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/(
(a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^3*d)
```

Maple [B] time = 0.042, size = 561, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*C/b^2*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1)+2/d/b^2*B*arctan(tan(
1/2*d*x+1/2*c))-4/d/b^3*C*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A-2/d*a^
2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)*B+2/d/b^2*a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^3/b^2/(a+b)/(a-b)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4
/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*B+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-6/d/b/(a+b)/(a-b)/((a+b)*(a-b))^(1
/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31104, size = 1782, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B
*b^6)*d*x*cos(d*x + c) + 2*(2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 +
 2*C*a^2*b^4 - B*a*b^5)*d*x + (2*C*a^5 - B*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^
3 - A*a*b^4 + (2*C*a^4*b - B*a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4 - A*b^5)*cos
```

```
(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*C*a^5*b - B*a^4*
b^2 + (A - 3*C)*a^3*b^3 + B*a^2*b^4 - (A - C)*a*b^5 + (C*a^4*b^2 - 2*C*a^2*
b^4 + C*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos
(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d), -((2*C*a^5*b - B*a^4*b^2 - 4*
C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*d*x*cos(d*x + c) + (2*C*a^6 -
B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 + 2*C*a^2*b^4 - B*a*b^5)*d*x - (2*C*a^5
 - B*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^3 - A*a*b^4 + (2*C*a^4*b - B*a^3*b^2 -
 3*C*a^2*b^3 + 2*B*a*b^4 - A*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*C*a^5*b - B*a^4*b^2
+ (A - 3*C)*a^3*b^3 + B*a^2*b^4 - (A - C)*a*b^5 + (C*a^4*b^2 - 2*C*a^2*b^4
+ C*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x
 + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.21048, size = 554, normalized size = 3.3

$$-\frac{\frac{2\left(2\,Ca^4-Ba^3b-3\,Ca^2b^2+2\,Bab^3-Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\left(2\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -(2*(2*C*a^4 - B*a^3*b - 3*C*a^2*b^2 + 2*B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
```

```
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(
2*C*a^3*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*t
an(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d*
x + 1/2*c)^3 + C*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*C*a^3*tan(1/2*d*x + 1/2*c)
- B*a^2*b*tan(1/2*d*x + 1/2*c) + C*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan
(1/2*d*x + 1/2*c) - C*a*b^2*tan(1/2*d*x + 1/2*c) - C*b^3*tan(1/2*d*x + 1/2*
c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x
 + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) + (2*C*a - B*b)*(d*x + c)/b^3)/d
```

3.989 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=139

$$\frac{2\left(a^3(-C)+aAb^2+2ab^2C-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Cx}{b^2}$$

```
[Out] (C*x)/b^2 + (2*(a*A*b^2 - b^3*B - a^3*C + 2*a*b^2*C)*ArcTan[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - ((A*b^2
 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.226295, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3021, 2735, 2659, 205}

$$\frac{2\left(a^3(-C)+aAb^2+2ab^2C-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Cx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (C*x)/b^2 + (2*(a*A*b^2 - b^3*B - a^3*C + 2*a*b^2*C)*ArcTan[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - ((A*b^2
 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b(bB-a(A+C))-\left(a^2-b^2\right)C\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{Cx}{b^2} - \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(b^3B+a^3C-ab^2(A+2C)\right)\int \frac{\cdots}{a+b\,\mathrm{co}\cdots}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{Cx}{b^2} - \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(b^3B+a^3C-ab^2(A+2C)\right)\right)\mathrm{Sub}\cdots}{b^2\left(a\cdots\right.}$$

$$= \frac{Cx}{b^2} + \frac{2\left(aAb^2-b^3B-a^3C+2ab^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \frac{\left(Ab^2-a(b\cdots\right.}{b\left(a^2-b^2\right)\cdots}$$

Mathematica [A] time = 0.658052, size = 131, normalized size = 0.94

$$\frac{-\frac{2\left(a^3C-ab^2(A+2C)+b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - \frac{b\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{(a-b)(a+b)(a+b\cos(c+dx))} + C(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (C*(c + d*x) - (2*(b^3*B + a^3*C - a*b^2*(A + 2*C))*ArcTanh[((a - b)*Tan[(c
 + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - (b*(A*b^2 + a*(-(b*B) +
 a*C))*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^2*d)
```

Maple [B] time = 0.034, size = 436, normalized size = 3.1

$$2\,{\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{d{b}^{2}}}-2\,{\frac{b\tan\left(1/2\,dx+c/2\right)A}{d\left({a}^{2}-{b}^{2}\right)\left(a\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b+a+b\right)}}+2\,{\frac{}{d\left({a}^{2}-{b}^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A+2/d*a/(a^2-b^2)*tan(1/2*
d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d/b*a^2/
(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)*C+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*A-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*
(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+4/d*a/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72109, size = 1280, normalized size = 9.21

$$\left[\frac{2\left(Ca^4b-2\,Ca^2b^3+Cb^5\right)dx\cos\left(dx+c\right)+2\left(Ca^5-2\,Ca^3b^2+Cab^4\right)dx-\left(Ca^4-(A+2\,C)a^2b^2+Bab^3+\left(Ca^3b-(A+\right.\right.}{2\left(\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/2*(2*(C*a^4*b - 2*C*a^2*b^3 + C*b^5)*d*x*cos(d*x + c) + 2*(C*a^5 - 2*C*a
^3*b^2 + C*a*b^4)*d*x - (C*a^4 - (A + 2*C)*a^2*b^2 + B*a*b^3 + (C*a^3*b - (
A + 2*C)*a*b^3 + B*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
 c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(C*a^4*b - B*a^3*b^2 + (A - C)*a^2*b^3 + B*a*b^4 - A*b^5)*sin(d*x +
 c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a
*b^6)*d), ((C*a^4*b - 2*C*a^2*b^3 + C*b^5)*d*x*cos(d*x + c) + (C*a^5 - 2*C*
a^3*b^2 + C*a*b^4)*d*x - (C*a^4 - (A + 2*C)*a^2*b^2 + B*a*b^3 + (C*a^3*b -
(A + 2*C)*a*b^3 + B*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
 c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (C*a^4*b - B*a^3*b^2 + (A - C)*a
^2*b^3 + B*a*b^4 - A*b^5)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(
d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20485, size = 297, normalized size = 2.14

$$\frac{\frac{2\left(Ca^3-Aab^2-2\,Cab^2+Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{(dx+c)C}{b^2}-\frac{2\left(Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] (2*(C*a^3 - A*a*b^2 - 2*C*a*b^2 + B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))
/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*C/b^2 - 2*
(C*a^2*tan(1/2*d*x + 1/2*c) - B*a*b*tan(1/2*d*x + 1/2*c) + A*b^2*tan(1/2*d*
x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*
c)^2 + a + b)))/d
```

3.990 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$-\frac{2(2a^2Ab+a^2bC+a^3(-B)-Ab^3)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\sin(c+dx)(Ab^2-a(bB-aC))}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] (-2*(2*a^2*A*b - A*b^3 - a^3*B + a^2*b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c +
 d*x]])/(a^2*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.351364, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{2(2a^2Ab+a^2bC+a^3(-B)-Ab^3)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\sin(c+dx)(Ab^2-a(bB-aC))}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d
*x])^2,x]
```

```
[Out] (-2*(2*a^2*A*b - A*b^3 - a^3*B + a^2*b*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c +
 d*x]])/(a^2*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(A\left(a^2-b^2\right)-a(Ab-aB+bC)\cos(c+\cdots}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{A\int\sec(c+dx)\,dx}{a^2} + \frac{\left(Ab^3\cdots\right.}{\cdots}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(2\cdots\right.}{\cdots}$$

$$= -\frac{2\left(2a^2Ab-Ab^3-a^3B+a^2bC\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{A\cdots}{\cdots}$$

Mathematica [C] time = 2.82591, size = 319, normalized size = 2.17

$$\frac{2\cos(c+dx)(A\sec(c+dx)+B+C\cos(c+dx))\left(\frac{2i(\cos(c)-i\sin(c))^3\left(-a^2b(2A+C)+a^3B+Ab^3\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(b\cos(c)-a)+b\sin(c\cdots\right)}{\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\right)}{\left(\left(b^2-a^2\right)(\cos(c)-i\sin(c))^2\right)^{3/2}}\cdots\right)}{a^2d(2A+2B\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*(-(A*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]]) + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + ((
2*I)*(A*b^3 + a^3*B - a^2*b*(2*A + C))*ArcTan[((I*Cos[c] + Sin[c])*(b*Sin[c
] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2
)]]*(Cos[c] - I*Sin[c])^3)/((-a^2 + b^2)*(Cos[c] - I*Sin[c])^2)^(3/2) + (a*
(A*b^2 + a*(-(b*B) + a*C))*(-(a*Sin[c]) + b*Sin[d*x]))/((a - b)*b*(a + b)*(
a + b*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))))/(a^2*d*(
2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.077, size = 458, normalized size = 3.1

$$-\frac{A}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\,\frac{A\tan(1/2\,dx+c/2)\,b^2}{da\left(a^2-b^2\right)\left(a(\tan(1/2\,dx+c/2))^2-(\tan(1/2\,dx+c/2))^2b+a+b\right)}-2\,\frac{\cdots}{d\left(a^2-b^2\right)\left(a\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)+2/d/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A*b^2-2/d/(a^2-b^2)*tan(1/2
*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*b*B+2/d/(a^
2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)*a*C-4/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*A+2/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^3+2/d*a/(a+b)/(a-b)/((a+b)*(
a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d/(a+b
)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*C*b+1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 61.1697, size = 1605, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [-1/2*((B*a^4 - (2*A + C)*a^3*b + A*a*b^3 + (B*a^3*b - (2*A + C)*a^2*b^2 +
A*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^
2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) -
a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (A*a^5 - 2*
```

```
A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin
(d*x + c) + 1) + (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 +
A*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(C*a^5 - B*a^4*b + (A - C)*
a^3*b^2 + B*a^2*b^3 - A*a*b^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)
*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d), 1/2*(2*(B*a^4 - (2*A + C)
*a^3*b + A*a*b^3 + (B*a^3*b - (2*A + C)*a^2*b^2 + A*b^4)*cos(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) +
(A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x +
c))*log(sin(d*x + c) + 1) - (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A
*a^2*b^3 + A*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(C*a^5 - B*a^4*b
 + (A - C)*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3
 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\cos\left(c + dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**2,x
)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c
 + d*x))**2, x)
```

Giac [A] time = 1.26244, size = 329, normalized size = 2.24

$$\frac{\frac{2\left(Ba^{3}-2\,Aa^{2}b-Ca^{2}b+Ab^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b - C*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + A*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/a^2 - A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(C*a^2*ta
n(1/2*d*x + 1/2*c) - B*a*b*tan(1/2*d*x + 1/2*c) + A*b^2*tan(1/2*d*x + 1/2*c
))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
+ b)))/d
```

3.991 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{2\left(3a^2Ab^2-2a^3bB+a^4C+ab^3B-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\tan(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\tan(c}{ad\,(a}$$

```
[Out] (2*(3*a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + a*b^3*B + a^4*C)*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2
*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((2*A*b^2 - a*b*B - a^2*(A - C
))*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c + d*x
])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.767825, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$\frac{2\left(3a^2Ab^2-2a^3bB+a^4C+ab^3B-2Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\tan(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\tan(c}{ad\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] (2*(3*a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + a*b^3*B + a^4*C)*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2
*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((2*A*b^2 - a*b*B - a^2*(A - C
))*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c + d*x
])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-2Ab^2+abB+a^2(A-C)-a(Ab-\right.}{}}{}$$

$$= -\frac{\left(2Ab^2-abB-a^2(A-C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2-a(bB-aC\right.}{a\left(a^2-b^2\right)d(a+}$$

$$= -\frac{\left(2Ab^2-abB-a^2(A-C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2-a(bB-aC\right.}{a\left(a^2-b^2\right)d(a+}$$

$$= -\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\left(2Ab^2-abB-a^2(A-C\right.}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\left(3a^2Ab^2-2Ab^4-2a^3bB+ab^3B+a^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c\right.}{\sqrt{a+b}}\right.}{a^3(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 1.68695, size = 331, normalized size = 1.57

$$\frac{2\cos^2(c+dx)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(\frac{2\left(3a^2Ab^2-2a^3bB+a^4C+ab^3B-2Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - \frac{ab\sin(c+dx)\left(a(aC-b\right.}{(a-b)(a+b)(a+b\cos}\right.}{a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((2*(3*a^2*A*b^2
- 2*A*b^4 - 2*a^3*b*B + a*b^3*B + a^4*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (2*A*b - a*B)*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + (-2*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + (a*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (a*A*S
in[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - (a*b*(A*b^2 + a*(-
(b*B) + a*C))*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]))))/(a^3*d
*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.088, size = 618, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)+2/d*A*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/d/
a^2*B*ln(tan(1/2*d*x+1/2*c)-1)-2/d/a^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*A+2/d/a*b^2/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d*b/
(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)*C+6/d/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*A*b^2-4/d/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^4-4/d*b/(a+b)/(a-b)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d
/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*b^3*B+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-2/d
*A*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 175.453, size = 2498, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(((C*a^4*b - 2*B*a^3*b^2 + 3*A*a^2*b^3 + B*a*b^4 - 2*A*b^5)*cos(d*x +
c)^2 + (C*a^5 - 2*B*a^4*b + 3*A*a^3*b^2 + B*a^2*b^3 - 2*A*a*b^4)*cos(d*x +
c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + ((B*a^5*b - 2*A*a^4*b^2 - 2*
B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*
a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*lo
g(sin(d*x + c) + 1) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 +
 B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A
*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*
(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4 + ((A - C)*a^5*b + B*a^4*b^2 - (3*A - C)*a
^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5
*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x +
c)), 1/2*(2*((C*a^4*b - 2*B*a^3*b^2 + 3*A*a^2*b^3 + B*a*b^4 - 2*A*b^5)*cos(
d*x + c)^2 + (C*a^5 - 2*B*a^4*b + 3*A*a^3*b^2 + B*a^2*b^3 - 2*A*a*b^4)*cos(
d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c))) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5
 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3
 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) - ((B*a^5*b -
 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^
2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)
*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4
+ ((A - C)*a^5*b + B*a^4*b^2 - (3*A - C)*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*c
os(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2
+ (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27633, size = 597, normalized size = 2.83

$$-\frac{\frac{2\left(Ca^4-2\,Ba^3b+3\,Aa^2b^2+Bab^3-2\,Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -(2*(C*a^4 - 2*B*a^3*b + 3*A*a^2*b^2 + B*a*b^3 - 2*A*b^4)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 2*(
A*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + C*a^2*b*tan
(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 + A*a^3*tan(1/2*d*x + 1/2*c) +
A*a^2*b*tan(1/2*d*x + 1/2*c) - C*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*tan(1
/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/2*
c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x
 + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) - (B*a - 2*A*b)*log(abs(tan(1/2*d*x +
 1/2*c) + 1))/a^3 + (B*a - 2*A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d
```

3.992 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=307

$$-\frac{2b\left(4a^2Ab^2-a^2b^2C-3a^3bB+2a^4C+2ab^3B-3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\tan(c+dx)\left(-a^2b(2A-C)+a^3B-2\right.}{a^3d\left(a^2-b^2\right)}$$

```
[Out] (-2*b*(4*a^2*A*b^2 - 3*A*b^4 - 3*a^3*b*B + 2*a*b^3*B + 2*a^4*C - a^2*b^2*C)
*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a
+ b)^(3/2)*d) + ((6*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])
/(2*a^4*d) + ((3*A*b^3 + a^3*B - 2*a*b^2*B - a^2*b*(2*A - C))*Tan[c + d*x])
/(a^3*(a^2 - b^2)*d) - ((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Sec[c + d*x]*Ta
n[c + d*x])/(2*a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.40245, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{2b\left(4a^2Ab^2-a^2b^2C-3a^3bB+2a^4C+2ab^3B-3Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\tan(c+dx)\left(-a^2b(2A-C)+a^3B-2\right.}{a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] (-2*b*(4*a^2*A*b^2 - 3*A*b^4 - 3*a^3*b*B + 2*a*b^3*B + 2*a^4*C - a^2*b^2*C)
*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a
+ b)^(3/2)*d) + ((6*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])
/(2*a^4*d) + ((3*A*b^3 + a^3*B - 2*a*b^2*B - a^2*b*(2*A - C))*Tan[c + d*x])
/(a^3*(a^2 - b^2)*d) - ((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Sec[c + d*x]*Ta
n[c + d*x])/(2*a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-3Ab^2+2abB+\right.}{}}{}$$

$$= -\frac{\left(3Ab^2-2abB-a^2(A-2C)\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{(Ab}{}$$

$$= \frac{\left(3Ab^3+a^3B-2ab^2B-a^2b(2A-C)\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{(3Ab^2-}{}$$

$$= \frac{\left(3Ab^3+a^3B-2ab^2B-a^2b(2A-C)\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{(3Ab^2-}{}$$

$$= \frac{\left(6Ab^2-4abB+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{(3Ab^3+}{}$$

$$= -\frac{2b\left(4a^2Ab^2-3Ab^4-3a^3bB+2ab^3B+2a^4C-a^2b^2C\right)\tan^{-}}{a^4(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 5.23801, size = 389, normalized size = 1.27

$$\frac{\frac{8b\left(a^2b^2(C-4A)+3a^3bB-2a^4C-2ab^3B+3Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - 2\left(a^2(A+2C)-4abB+6Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] ((8*b*(3*A*b^4 + 3*a^3*b*B - 2*a*b^3*B - 2*a^4*C + a^2*b^2*(-4*A + C))*ArcT
anh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 2*(6
*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 2*(6*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a*(-2*A*b + a*
B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^2*A)/(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(-2*A*b + a*B)*Sin[(c + d*x)/2])/(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (4*a*b^2*(A*b^2 + a*(-(b*B) + a*C))*
```

```
Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(4*a^4*d)
```

Maple [B] time = 0.101, size = 914, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2*B/(tan(1/2*d*x+1/2*c)-1)-1/d/a^2*
ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)^2+1/d/a^2*ln(
tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2*A/(t
an(1/2*d*x+1/2*c)-1)+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^2*A*ln(tan(
1/2*d*x+1/2*c)+1)-1/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)-2/d*b^3/a^2/(a^2-b^2
)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-
8/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*A*b^3+2/d*b^3/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/d/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*b+6/d*b^2/a
/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*B-4/d*b^4/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d/a^4*b^5/(a+b)/(a-b)/((a+b)*(a-b))^
(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+3/d/a^4*ln(tan
(1/2*d*x+1/2*c)+1)*A*b^2+2/d/a^3/(tan(1/2*d*x+1/2*c)-1)*A*b-3/d/a^4*ln(tan(
1/2*d*x+1/2*c)-1)*A*b^2+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b+2/d/a^3*ln(tan(1
/2*d*x+1/2*c)-1)*b*B-2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b*B+2/d*b^2/a/(a^2-b^
2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C
+2/d/a^3*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27294, size = 571, normalized size = 1.86

$$\frac{\frac{4\left(2\,Ca^4b-3\,Ba^3b^2+4\,Aa^2b^3-Ca^2b^3+2\,Bab^4-3\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{4\left(Ca^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bab^3\,\mathrm{ta}}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/2*(4*(2*C*a^4*b - 3*B*a^3*b^2 + 4*A*a^2*b^3 - C*a^2*b^3 + 2*B*a*b^4 - 3*A
*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)
*sqrt(a^2 - b^2)) + 4*(C*a^2*b^2*tan(1/2*d*x + 1/2*c) - B*a*b^3*tan(1/2*d*x
 + 1/2*c) + A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (A*a^2 + 2*C*a^2 - 4*B*a*b +
 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (A*a^2 + 2*C*a^2 - 4*B*a
*b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(A*a*tan(1/2*d*x +
 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 + A
*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 4*A*b*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.993 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$\frac{2b^2\left(5a^2Ab^2-2a^2b^2C-4a^3bB+3a^4C+3ab^3B-4Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\tan(c+dx)\left(-a^2b^2(7A-6C)+a^4(-\right.}{3a^4d}$$

```
[Out] (2*b^2*(5*a^2*A*b^2 - 4*A*b^4 - 4*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 2*a^2*b^2
*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*
(a + b)^(3/2)*d) - ((8*A*b^3 - a^3*B - 6*a*b^2*B + 2*a^2*b*(A + 2*C))*ArcTa
nh[Sin[c + d*x]])/(2*a^5*d) - ((12*A*b^4 + 6*a^3*b*B - 9*a*b^3*B - a^2*b^2*
(7*A - 6*C) - a^4*(2*A + 3*C))*Tan[c + d*x])/(3*a^4*(a^2 - b^2)*d) + ((4*A*
b^3 + a^3*B - 3*a*b^2*B - 2*a^2*b*(A - C))*Sec[c + d*x]*Tan[c + d*x])/(2*a^
3*(a^2 - b^2)*d) - ((4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sec[c + d*x]^2*Tan[
c + d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.00368, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$\frac{2b^2\left(5a^2Ab^2-2a^2b^2C-4a^3bB+3a^4C+3ab^3B-4Ab^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\tan(c+dx)\left(-a^2b^2(7A-6C)+a^4(-\right.}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] (2*b^2*(5*a^2*A*b^2 - 4*A*b^4 - 4*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 2*a^2*b^2
*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*
(a + b)^(3/2)*d) - ((8*A*b^3 - a^3*B - 6*a*b^2*B + 2*a^2*b*(A + 2*C))*ArcTa
nh[Sin[c + d*x]])/(2*a^5*d) - ((12*A*b^4 + 6*a^3*b*B - 9*a*b^3*B - a^2*b^2*
(7*A - 6*C) - a^4*(2*A + 3*C))*Tan[c + d*x])/(3*a^4*(a^2 - b^2)*d) + ((4*A*
b^3 + a^3*B - 3*a*b^2*B - 2*a^2*b*(A - C))*Sec[c + d*x]*Tan[c + d*x])/(2*a^
3*(a^2 - b^2)*d) - ((4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sec[c + d*x]^2*Tan[
c + d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*T
```

```
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{(a+b\cos(c+dx))^2}dx = \frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-4Ab^2+3abB\right.}{}}{}$$

$$= -\frac{\left(4Ab^2-3abB-a^2(A-3C)\right)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{(A}{}$$

$$= \frac{\left(4Ab^3+a^3B-3ab^2B-2a^2b(A-C)\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12Ab^4+6a^3bB-9ab^3B-a^2b^2(7A-6C)-a^4(2A+3C)\right)}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12Ab^4+6a^3bB-9ab^3B-a^2b^2(7A-6C)-a^4(2A+3C)\right)}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(8Ab^3-a^3B-6ab^2B+2a^2b(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^5d}$$

$$= \frac{2b^2\left(5a^2Ab^2-4Ab^4-4a^3bB+3ab^3B+3a^4C-2a^2b^2C\right)\tan}{a^5(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 3.0485, size = 519, normalized size = 1.28

$$\frac{-\frac{24b^2\left(a^2b^2(2C-5A)+4a^3bB-3a^4C-3ab^3B+4Ab^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+6\left(2a^2b(A+2C)+a^3(-B)-6ab^2B+8Ab^3\right)\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] ((-24*b^2*(4*A*b^4 + 4*a^3*b*B - 3*a*b^3*B - 3*a^4*C + a^2*b^2*(-5*A + 2*C)
)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2)
+ 6*(8*A*b^3 - a^3*B - 6*a*b^2*B + 2*a^2*b*(A + 2*C))*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 6*(-8*A*b^3 + a^3*B + 6*a*b^2*B - 2*a^2*b*(A + 2*C))*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*(8*a^5*A + 4*a^3*A*b^2 - 12*a
```

```
*A*b^4 - 9*a^4*b*B + 9*a^2*b^3*B + 6*a^5*C - 6*a^3*b^2*C + (-36*A*b^5 + 6*a
^5*B - 24*a^3*b^2*B + 27*a*b^4*B + a^2*b^3*(29*A - 18*C) + a^4*(-2*A*b + 9*
b*C))*Cos[c + d*x] + a*(a^2 - b^2)*(12*A*b^2 - 9*a*b*B + a^2*(4*A + 6*C))*C
os[2*(c + d*x)] + 2*a^4*A*b*Cos[3*(c + d*x)] + 7*a^2*A*b^3*Cos[3*(c + d*x)]
 - 12*A*b^5*Cos[3*(c + d*x)] - 6*a^3*b^2*B*Cos[3*(c + d*x)] + 9*a*b^4*B*Cos
[3*(c + d*x)] + 3*a^4*b*C*Cos[3*(c + d*x)] - 6*a^2*b^3*C*Cos[3*(c + d*x)])*
Sec[c + d*x]^2*Tan[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])))/(12*a^5*d)
```

Maple [B] time = 0.109, size = 1242, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^2*B/(tan(1/2*d*x+1/2*c)-1)+1/2/d
/a^2*B/(tan(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2*B/(tan(1/2*d*x+1/2*c)+1)^2-1/2/d/
a^2*B*ln(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^2*B*ln(tan(1/2*d*x+1/2*c)+1)-1/3/d/a
^2*A/(tan(1/2*d*x+1/2*c)+1)^3-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-1/3/d/a^2*A/
(tan(1/2*d*x+1/2*c)-1)^3-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C-1/2/d/a^2*A/(tan(
1/2*d*x+1/2*c)-1)^2+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2-1/d/a^2*A/(tan(1/2
*d*x+1/2*c)-1)-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)+2/d/a^3*b^4/(a^2-b^2)*tan(1
/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-1/d*A*b
/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/d*A*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)-8/d/a^5*
b^6/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*A+6/d/a^4*b^5/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d/a*b^2/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/d/a^3*b^4/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*C+10/d/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^4-8/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^3*B-1/d/a^3/(tan(1/
2*d*x+1/2*c)-1)*A*b-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b+1/d/a^3/(tan(1/2*d*x
+1/2*c)+1)^2*A*b-4/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*A*b^3+3/d/a^4*ln(tan(1/2*
d*x+1/2*c)+1)*b^2*B-2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b*C-3/d/a^4/(tan(1/2*d
*x+1/2*c)+1)*A*b^2+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b*B-1/d/a^3/(tan(1/2*d*x+
1/2*c)-1)^2*A*b+4/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)*A*b^3-3/d/a^4*ln(tan(1/2*d
*x+1/2*c)-1)*b^2*B+2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*b*C-3/d/a^4/(tan(1/2*d*
x+1/2*c)-1)*A*b^2+2/d/a^3/(tan(1/2*d*x+1/2*c)-1)*b*B-2/d/a^2*b^3/(a^2-b^2)*
tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*C-2/
d/a^4*b^5/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
```

```
1/2*c)^2*b+a+b)*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27219, size = 836, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(12*(3*C*a^4*b^2 - 4*B*a^3*b^3 + 5*A*a^2*b^4 - 2*C*a^2*b^4 + 3*B*a*b^5
 - 4*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - a^
5*b^2)*sqrt(a^2 - b^2)) + 12*(C*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a*b^4*tan(
1/2*d*x + 1/2*c) + A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*
d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 3*(B*a^3 - 2*A*a^2*b
- 4*C*a^2*b + 6*B*a*b^2 - 8*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 +
 3*(B*a^3 - 2*A*a^2*b - 4*C*a^2*b + 6*B*a*b^2 - 8*A*b^3)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^5 + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*
d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a*b*tan(1/2*d*x + 1/2
*c)^5 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^2*tan(1/2*d*x + 1/2*c)^5 -
 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a*
b*tan(1/2*d*x + 1/2*c)^3 - 36*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/
2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x + 1/2*c
) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 12*B*a*b*tan(1/2*d*x + 1/2*c) + 18*A*b^2
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4))/d
```

3.994 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=456

$$\frac{\sin(c+dx)\left(-a^3b^2(2A-21C)-11a^2b^3B+6a^4bB-12a^5C+ab^4(5A-6C)+2b^5B\right)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a\left(a^4b^2(2A-29C)-5a^2b^4(A-4C\right.}{}$$

```
[Out] ((2*A*b^2 - 6*a*b*B + 12*a^2*C + b^2*C)*x)/(2*b^5) - (a*(6*A*b^6 - 6*a^5*b*
B + 15*a^3*b^3*B - 12*a*b^5*B + a^4*b^2*(2*A - 29*C) - 5*a^2*b^4*(A - 4*C)
+ 12*a^6*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5
/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*b*B - 11*a^2*b^3*B + 2*b^5*B - a^3*b^2*(
2*A - 21*C) + a*b^4*(5*A - 6*C) - 12*a^5*C)*Sin[c + d*x])/(2*b^4*(a^2 - b^2
)^2*d) - ((3*a^3*b*B - 6*a*b^3*B - a^2*b^2*(A - 10*C) + b^4*(4*A - C) - 6*a
^4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B
 - a*C))*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) + ((3*A*b^4 + a*(2*a^2*b*B - 5*b^3*B - 4*a^3*C + 7*a*b^2*C))*Cos[c +
d*x]^2*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 4.63685, antiderivative size = 456, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3023, 2735, 2659, 205}

$$\frac{\sin(c+dx)\left(-a^3b^2(2A-21C)-11a^2b^3B+6a^4bB-12a^5C+ab^4(5A-6C)+2b^5B\right)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a\left(a^4b^2(2A-29C)-5a^2b^4(A-4C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^3,x]
```

```
[Out] ((2*A*b^2 - 6*a*b*B + 12*a^2*C + b^2*C)*x)/(2*b^5) - (a*(6*A*b^6 - 6*a^5*b*
B + 15*a^3*b^3*B - 12*a*b^5*B + a^4*b^2*(2*A - 29*C) - 5*a^2*b^4*(A - 4*C)
+ 12*a^6*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5
/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*b*B - 11*a^2*b^3*B + 2*b^5*B - a^3*b^2*(
2*A - 21*C) + a*b^4*(5*A - 6*C) - 12*a^5*C)*Sin[c + d*x])/(2*b^4*(a^2 - b^2
)^2*d) - ((3*a^3*b*B - 6*a*b^3*B - a^2*b^2*(A - 10*C) + b^4*(4*A - C) - 6*a
^4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B
```

```
 - a*C))*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) + ((3*A*b^4 + a*(2*a^2*b*B - 5*b^3*B - 4*a^3*C + 7*a*b^2*C))*Cos[c +
d*x]^2*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(3}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(3Ab^4+a\left(\right.\right.}{}$$

$$= -\frac{\left(3a^3bB-6ab^3B-a^2b^2(A-10C)+b^4(4A-C)-6a^4C\right)\cos}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(6a^4bB-11a^2b^3B+2b^5B-a^3b^2(2A-21C)+ab^4(5A-6C)\right.}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(2Ab^2-6abB+12a^2C+b^2C\right)x}{2b^5} + \frac{\left(6a^4bB-11a^2b^3B+2b^5B\right.}{}$$

$$= \frac{\left(2Ab^2-6abB+12a^2C+b^2C\right)x}{2b^5} + \frac{\left(6a^4bB-11a^2b^3B+2b^5B\right.}{}$$

$$= \frac{\left(2Ab^2-6abB+12a^2C+b^2C\right)x}{2b^5} - \frac{a\left(2a^4Ab^2-5a^2Ab^4+6A\right.}{}$$

Mathematica [A] time = 4.41081, size = 883, normalized size = 1.94

$$\frac{\frac{16a\left(12Ca^6-6bBa^5+b^2(2A-29C)a^4+15b^3Ba^3-5b^4(A-4C)a^2-12b^5Ba+6Ab^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{96cCa^8+96Cdxa^8-48bBca^7-48bBdxa^7-96bC\sin(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] ((16*a*(6*A*b^6 - 6*a^5*b*B + 15*a^3*b^3*B - 12*a*b^5*B + a^4*b^2*(2*A - 29
*C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sq
rt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (16*a^6*A*b^2*c - 24*a^4*A*b^4*c + 8*
A*b^8*c - 48*a^7*b*B*c + 72*a^5*b^3*B*c - 24*a*b^7*B*c + 96*a^8*c*C - 136*a
^6*b^2*c*C - 12*a^4*b^4*c*C + 48*a^2*b^6*c*C + 4*b^8*c*C + 16*a^6*A*b^2*d*x
 - 24*a^4*A*b^4*d*x + 8*A*b^8*d*x - 48*a^7*b*B*d*x + 72*a^5*b^3*B*d*x - 24*
a*b^7*B*d*x + 96*a^8*C*d*x - 136*a^6*b^2*C*d*x - 12*a^4*b^4*C*d*x + 48*a^2*
b^6*C*d*x + 4*b^8*C*d*x + 16*a*b*(a^2 - b^2)^2*(2*A*b^2 - 6*a*b*B + 12*a^2*
C + b^2*C)*(c + d*x)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*(2*A*b^2 - 6*a*b*B
 + 12*a^2*C + b^2*C)*(c + d*x)*Cos[2*(c + d*x)] - 16*a^5*A*b^3*Sin[c + d*x]
 + 40*a^3*A*b^5*Sin[c + d*x] + 48*a^6*b^2*B*Sin[c + d*x] - 84*a^4*b^4*B*Sin
[c + d*x] + 8*a^2*b^6*B*Sin[c + d*x] + 4*b^8*B*Sin[c + d*x] - 96*a^7*b*C*Si
n[c + d*x] + 160*a^5*b^3*C*Sin[c + d*x] - 32*a^3*b^5*C*Sin[c + d*x] - 8*a*b
^7*C*Sin[c + d*x] - 12*a^4*A*b^4*Sin[2*(c + d*x)] + 24*a^2*A*b^6*Sin[2*(c +
 d*x)] + 36*a^5*b^3*B*Sin[2*(c + d*x)] - 64*a^3*b^5*B*Sin[2*(c + d*x)] + 16
*a*b^7*B*Sin[2*(c + d*x)] - 72*a^6*b^2*C*Sin[2*(c + d*x)] + 130*a^4*b^4*C*S
in[2*(c + d*x)] - 48*a^2*b^6*C*Sin[2*(c + d*x)] + 2*b^8*C*Sin[2*(c + d*x)]
+ 4*a^4*b^4*B*Sin[3*(c + d*x)] - 8*a^2*b^6*B*Sin[3*(c + d*x)] + 4*b^8*B*Sin
[3*(c + d*x)] - 8*a^5*b^3*C*Sin[3*(c + d*x)] + 16*a^3*b^5*C*Sin[3*(c + d*x)
] - 8*a*b^7*C*Sin[3*(c + d*x)] + a^4*b^4*C*Sin[4*(c + d*x)] - 2*a^2*b^6*C*S
in[4*(c + d*x)] + b^8*C*Sin[4*(c + d*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x
])^2))/(16*b^5*d)
```

Maple [B] time = 0.053, size = 2133, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

[Out] 4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a*C-6/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*a*C+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*A+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*C+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*C-6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a*B+12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*B-1/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*B+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-12/d*a^7/b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.99492, size = 4652, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] [1/4*(2*(12*C*a^8*b^2 - 6*B*a^7*b^3 + (2*A - 35*C)*a^6*b^4 + 18*B*a^5*b^5 -
 3*(2*A - 11*C)*a^4*b^6 - 18*B*a^3*b^7 + 3*(2*A - 3*C)*a^2*b^8 + 6*B*a*b^9
- (2*A + C)*b^10)*d*x*cos(d*x + c)^2 + 4*(12*C*a^9*b - 6*B*a^8*b^2 + (2*A -
 35*C)*a^7*b^3 + 18*B*a^6*b^4 - 3*(2*A - 11*C)*a^5*b^5 - 18*B*a^4*b^6 + 3*(
2*A - 3*C)*a^3*b^7 + 6*B*a^2*b^8 - (2*A + C)*a*b^9)*d*x*cos(d*x + c) + 2*(1
2*C*a^10 - 6*B*a^9*b + (2*A - 35*C)*a^8*b^2 + 18*B*a^7*b^3 - 3*(2*A - 11*C)
*a^6*b^4 - 18*B*a^5*b^5 + 3*(2*A - 3*C)*a^4*b^6 + 6*B*a^3*b^7 - (2*A + C)*a
^2*b^8)*d*x - (12*C*a^9 - 6*B*a^8*b + (2*A - 29*C)*a^7*b^2 + 15*B*a^6*b^3 -
 5*(A - 4*C)*a^5*b^4 - 12*B*a^4*b^5 + 6*A*a^3*b^6 + (12*C*a^7*b^2 - 6*B*a^6
*b^3 + (2*A - 29*C)*a^5*b^4 + 15*B*a^4*b^5 - 5*(A - 4*C)*a^3*b^6 - 12*B*a^2
*b^7 + 6*A*a*b^8)*cos(d*x + c)^2 + 2*(12*C*a^8*b - 6*B*a^7*b^2 + (2*A - 29*
C)*a^6*b^3 + 15*B*a^5*b^4 - 5*(A - 4*C)*a^4*b^5 - 12*B*a^3*b^6 + 6*A*a^2*b^
7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
+ 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*C*a^9*b -
 6*B*a^8*b^2 + (2*A - 33*C)*a^7*b^3 + 17*B*a^6*b^4 - (7*A - 27*C)*a^5*b^5 -
 13*B*a^4*b^6 + (5*A - 6*C)*a^3*b^7 + 2*B*a^2*b^8 - (C*a^6*b^4 - 3*C*a^4*b^
6 + 3*C*a^2*b^8 - C*b^10)*cos(d*x + c)^3 + 2*(2*C*a^7*b^3 - B*a^6*b^4 - 6*C
*a^5*b^5 + 3*B*a^4*b^6 + 6*C*a^3*b^7 - 3*B*a^2*b^8 - 2*C*a*b^9 + B*b^10)*co
```

```
s(d*x + c)^2 + (18*C*a^8*b^2 - 9*B*a^7*b^3 + (3*A - 50*C)*a^6*b^4 + 25*B*a^
5*b^5 - (9*A - 43*C)*a^4*b^6 - 20*B*a^3*b^7 + (6*A - 11*C)*a^2*b^8 + 4*B*a*
b^9)*cos(d*x + c))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)
*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x
 + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*C*a^8*b^2
 - 6*B*a^7*b^3 + (2*A - 35*C)*a^6*b^4 + 18*B*a^5*b^5 - 3*(2*A - 11*C)*a^4*b
^6 - 18*B*a^3*b^7 + 3*(2*A - 3*C)*a^2*b^8 + 6*B*a*b^9 - (2*A + C)*b^10)*d*x
*cos(d*x + c)^2 + 2*(12*C*a^9*b - 6*B*a^8*b^2 + (2*A - 35*C)*a^7*b^3 + 18*B
*a^6*b^4 - 3*(2*A - 11*C)*a^5*b^5 - 18*B*a^4*b^6 + 3*(2*A - 3*C)*a^3*b^7 +
6*B*a^2*b^8 - (2*A + C)*a*b^9)*d*x*cos(d*x + c) + (12*C*a^10 - 6*B*a^9*b +
(2*A - 35*C)*a^8*b^2 + 18*B*a^7*b^3 - 3*(2*A - 11*C)*a^6*b^4 - 18*B*a^5*b^5
 + 3*(2*A - 3*C)*a^4*b^6 + 6*B*a^3*b^7 - (2*A + C)*a^2*b^8)*d*x - (12*C*a^9
 - 6*B*a^8*b + (2*A - 29*C)*a^7*b^2 + 15*B*a^6*b^3 - 5*(A - 4*C)*a^5*b^4 -
12*B*a^4*b^5 + 6*A*a^3*b^6 + (12*C*a^7*b^2 - 6*B*a^6*b^3 + (2*A - 29*C)*a^5
*b^4 + 15*B*a^4*b^5 - 5*(A - 4*C)*a^3*b^6 - 12*B*a^2*b^7 + 6*A*a*b^8)*cos(d
*x + c)^2 + 2*(12*C*a^8*b - 6*B*a^7*b^2 + (2*A - 29*C)*a^6*b^3 + 15*B*a^5*b
^4 - 5*(A - 4*C)*a^4*b^5 - 12*B*a^3*b^6 + 6*A*a^2*b^7)*cos(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (1
2*C*a^9*b - 6*B*a^8*b^2 + (2*A - 33*C)*a^7*b^3 + 17*B*a^6*b^4 - (7*A - 27*C
)*a^5*b^5 - 13*B*a^4*b^6 + (5*A - 6*C)*a^3*b^7 + 2*B*a^2*b^8 - (C*a^6*b^4 -
 3*C*a^4*b^6 + 3*C*a^2*b^8 - C*b^10)*cos(d*x + c)^3 + 2*(2*C*a^7*b^3 - B*a^
6*b^4 - 6*C*a^5*b^5 + 3*B*a^4*b^6 + 6*C*a^3*b^7 - 3*B*a^2*b^8 - 2*C*a*b^9 +
 B*b^10)*cos(d*x + c)^2 + (18*C*a^8*b^2 - 9*B*a^7*b^3 + (3*A - 50*C)*a^6*b^
4 + 25*B*a^5*b^5 - (9*A - 43*C)*a^4*b^6 - 20*B*a^3*b^7 + (6*A - 11*C)*a^2*b
^8 + 4*B*a*b^9)*cos(d*x + c))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b
^11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12
)*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35695, size = 2284, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/2*(2*(12*C*a^7 - 6*B*a^6*b + 2*A*a^5*b^2 - 29*C*a^5*b^2 + 15*B*a^4*b^3 -
5*A*a^3*b^4 + 20*C*a^3*b^4 - 12*B*a^2*b^5 + 6*A*a*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(a^2 - b^
2)) - 2*(12*C*a^7*tan(1/2*d*x + 1/2*c)^7 - 6*B*a^6*b*tan(1/2*d*x + 1/2*c)^7
 - 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 2*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 +
 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 -
 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 +
33*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 -
16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 +
6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 1
3*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*C
*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*B*b^7*tan(1/2*d*x + 1/2*c)^7 + C*b^7*tan(
1/2*d*x + 1/2*c)^7 + 36*C*a^7*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^6*b*tan(1/2*d
*x + 1/2*c)^5 - 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 67*C*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^5 - 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 35*B*a^4*b^3*tan(1/2*d*x
 + 1/2*c)^5 + 29*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 15*A*a^3*b^4*tan(1/2*d*
x + 1/2*c)^5 - 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 26*C*a^3*b^4*tan(1/2*d
*x + 1/2*c)^5 + 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 10*B*a^2*b^5*tan(1/2*d
*x + 1/2*c)^5 - 5*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 4*B*a*b^6*tan(1/2*d*x
+ 1/2*c)^5 - 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 2*B*b^7*tan(1/2*d*x + 1/2*c
)^5 - 3*C*b^7*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^7*tan(1/2*d*x + 1/2*c)^3 - 18
*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*a
^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*C*a
^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*B*a
^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*
a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 26*C
*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 10*B
*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*
a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^7*t
an(1/2*d*x + 1/2*c)^3 + 3*C*b^7*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^7*tan(1/2*d
*x + 1/2*c) - 6*B*a^6*b*tan(1/2*d*x + 1/2*c) + 18*C*a^6*b*tan(1/2*d*x + 1/2
*c) + 2*A*a^5*b^2*tan(1/2*d*x + 1/2*c) - 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c) -
 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c) + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c) + 9*B
```

```
*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c) - 5*A*a^3
*b^4*tan(1/2*d*x + 1/2*c) + 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*C*a^3*b^4
*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c) + 2*B*a^2*b^5*tan(
1/2*d*x + 1/2*c) + 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c) - 4*B*a*b^6*tan(1/2*d*
x + 1/2*c) + 4*C*a*b^6*tan(1/2*d*x + 1/2*c) - 2*B*b^7*tan(1/2*d*x + 1/2*c)
- C*b^7*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^
2) + (12*C*a^2 - 6*B*a*b + 2*A*b^2 + C*b^2)*(d*x + c)/b^5)/d
```

3.995 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=314

$$\frac{\sin(c+dx)\left(3a^2C-abB+Ab^2-2b^2C\right)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)+5a^3b^3B-15a^4b^2C-2a^5bB+6a^6C-6ab^5B+2Ab^6\right)\tan}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((b*B - 3*a*C)*x)/b^4 + ((2*A*b^6 - 2*a^5*b*B + 5*a^3*b^3*B - 6*a*b^5*B + 6
*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((A*b^2 - a*b*B
+ 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B
 - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) - (a*(2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 6*C))*Si
n[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.82227, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3031, 3023, 2735, 2659, 205}

$$\frac{\sin(c+dx)\left(3a^2C-abB+Ab^2-2b^2C\right)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)+5a^3b^3B-15a^4b^2C-2a^5bB+6a^6C-6ab^5B+2Ab^6\right)\tan}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^3,x]
```

```
[Out] ((b*B - 3*a*C)*x)/b^4 + ((2*A*b^6 - 2*a^5*b*B + 5*a^3*b^3*B - 6*a*b^5*B + 6
*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((A*b^2 - a*b*B
+ 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B
 - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) - (a*(2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 6*C))*Si
n[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \int \frac{\cos(c+dx)(\ldots}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a\left(2Ab^4+\ldots\right.}{\ldots}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a(bB-a\ldots\right.}{2b\left(a^2-b^2\ldots\right.}$$

$$= \frac{(bB-3aC)x}{b^4} + \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \ldots$$

$$= \frac{(bB-3aC)x}{b^4} + \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \ldots$$

$$= \frac{(bB-3aC)x}{b^4} + \frac{\left(a^2Ab^4+2Ab^6-2a^5bB+5a^3b^3B-6ab^5B\ldots\right.}{(a-b\ldots}$$

Mathematica [A] time = 2.79907, size = 573, normalized size = 1.82

$$\frac{a^3Ab^4\sin(2(c+dx))-6a^2Ab^5\sin(c+dx)-8ab\left(a^2-b^2\right)^2(c+dx)(3aC-bB)\cos(c+dx)+2\left(b^3-a^2b\right)^2(c+dx)(bB-3aC)\cos(2(c+dx))-4a^5b^2B\sin(c+dx)-3a^4b^3B\sin(2(c\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] ((-4*(2*A*b^6 - 2*a^5*b*B + 5*a^3*b^3*B - 6*a*b^5*B + 6*a^6*C - 15*a^4*b^2*
C + a^2*b^4*(A + 12*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(-a^2 + b^2)^(5/2) + (4*a^6*b*B*c - 6*a^4*b^3*B*c + 2*b^7*B*c - 12*a^7*c
*C + 18*a^5*b^2*c*C - 6*a*b^6*c*C + 4*a^6*b*B*d*x - 6*a^4*b^3*B*d*x + 2*b^7
*B*d*x - 12*a^7*C*d*x + 18*a^5*b^2*C*d*x - 6*a*b^6*C*d*x - 8*a*b*(a^2 - b^2
)^2*(-(b*B) + 3*a*C)*(c + d*x)*Cos[c + d*x] + 2*(-(a^2*b) + b^3)^2*(b*B - 3
*a*C)*(c + d*x)*Cos[2*(c + d*x)] - 6*a^2*A*b^5*Sin[c + d*x] - 4*a^5*b^2*B*S
in[c + d*x] + 10*a^3*b^4*B*Sin[c + d*x] + 12*a^6*b*C*Sin[c + d*x] - 21*a^4*
b^3*C*Sin[c + d*x] + 2*a^2*b^5*C*Sin[c + d*x] + b^7*C*Sin[c + d*x] + a^3*A*
b^4*Sin[2*(c + d*x)] - 4*a*A*b^6*Sin[2*(c + d*x)] - 3*a^4*b^3*B*Sin[2*(c +
d*x)] + 6*a^2*b^5*B*Sin[2*(c + d*x)] + 9*a^5*b^2*C*Sin[2*(c + d*x)] - 16*a^
3*b^4*C*Sin[2*(c + d*x)] + 4*a*b^6*C*Sin[2*(c + d*x)] + a^4*b^3*C*Sin[3*(c
+ d*x)] - 2*a^2*b^5*C*Sin[3*(c + d*x)] + b^7*C*Sin[3*(c + d*x)])/((a^2 - b^
2)^2*(a + b*Cos[c + d*x])^2))/(4*b^4*d)
```

Maple [B] time = 0.049, size = 1693, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a
-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+4/d*a^5/b^3/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*
C-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d
/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*a^2*A+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+12/d/(a^4-2*a^2*b^2+b^
4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*C*a^2+2/d/b^3*B*arctan(tan(1/2*d*x+1/2*c))+1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d*a
^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+2/d/b^3*C*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/
2*c)^2+1)-6/d/b^4*C*arctan(tan(1/2*d*x+1/2*c))*a-8/d/b/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*C+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan
```

```
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*a*b/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*B-4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*
a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-4/
d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3*A-8/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^3*C-1/d*a^2/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*A-15/d/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^4*C+6/d/b^4/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C
+5/d/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*a^3*B-2/d/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^5*B+1/d*a^2/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x
+1/2*c)*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.1971, size = 3621, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*C*a^7*b^2 - B*a^6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6
 - 3*B*a^2*b^7 - 3*C*a*b^8 + B*b^9)*d*x*cos(d*x + c)^2 + 8*(3*C*a^8*b - B*a
```

```
^7*b^2 - 9*C*a^6*b^3 + 3*B*a^5*b^4 + 9*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^
7 + B*a*b^8)*d*x*cos(d*x + c) + 4*(3*C*a^9 - B*a^8*b - 9*C*a^7*b^2 + 3*B*a^
6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^3*b^6 + B*a^2*b^7)*d*x + (6*C*a^8
 - 2*B*a^7*b - 15*C*a^6*b^2 + 5*B*a^5*b^3 + (A + 12*C)*a^4*b^4 - 6*B*a^3*b^
5 + 2*A*a^2*b^6 + (6*C*a^6*b^2 - 2*B*a^5*b^3 - 15*C*a^4*b^4 + 5*B*a^3*b^5 +
 (A + 12*C)*a^2*b^6 - 6*B*a*b^7 + 2*A*b^8)*cos(d*x + c)^2 + 2*(6*C*a^7*b -
2*B*a^6*b^2 - 15*C*a^5*b^3 + 5*B*a^4*b^4 + (A + 12*C)*a^3*b^5 - 6*B*a^2*b^6
 + 2*A*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6
*C*a^8*b - 2*B*a^7*b^2 - 17*C*a^6*b^3 + 7*B*a^5*b^4 - (3*A - 13*C)*a^4*b^5
- 5*B*a^3*b^6 + (3*A - 2*C)*a^2*b^7 + 2*(C*a^6*b^3 - 3*C*a^4*b^5 + 3*C*a^2*
b^7 - C*b^9)*cos(d*x + c)^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 + (A - 25*C)*a^5*b
^4 + 9*B*a^4*b^5 - 5*(A - 4*C)*a^3*b^6 - 6*B*a^2*b^7 + 4*(A - C)*a*b^8)*cos
(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d
*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (
a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(2*(3*C*a^7*b^2 - B*a^
6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 - 3*C*a*b^8 +
 B*b^9)*d*x*cos(d*x + c)^2 + 4*(3*C*a^8*b - B*a^7*b^2 - 9*C*a^6*b^3 + 3*B*a
^5*b^4 + 9*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^7 + B*a*b^8)*d*x*cos(d*x + c
) + 2*(3*C*a^9 - B*a^8*b - 9*C*a^7*b^2 + 3*B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^
4*b^5 - 3*C*a^3*b^6 + B*a^2*b^7)*d*x - (6*C*a^8 - 2*B*a^7*b - 15*C*a^6*b^2
+ 5*B*a^5*b^3 + (A + 12*C)*a^4*b^4 - 6*B*a^3*b^5 + 2*A*a^2*b^6 + (6*C*a^6*b
^2 - 2*B*a^5*b^3 - 15*C*a^4*b^4 + 5*B*a^3*b^5 + (A + 12*C)*a^2*b^6 - 6*B*a*
b^7 + 2*A*b^8)*cos(d*x + c)^2 + 2*(6*C*a^7*b - 2*B*a^6*b^2 - 15*C*a^5*b^3 +
 5*B*a^4*b^4 + (A + 12*C)*a^3*b^5 - 6*B*a^2*b^6 + 2*A*a*b^7)*cos(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (6*C*a^8*b - 2*B*a^7*b^2 - 17*C*a^6*b^3 + 7*B*a^5*b^4 - (3*A - 13*C)*a^
4*b^5 - 5*B*a^3*b^6 + (3*A - 2*C)*a^2*b^7 + 2*(C*a^6*b^3 - 3*C*a^4*b^5 + 3*
C*a^2*b^7 - C*b^9)*cos(d*x + c)^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 + (A - 25*C)
*a^5*b^4 + 9*B*a^4*b^5 - 5*(A - 4*C)*a^3*b^6 - 6*B*a^2*b^7 + 4*(A - C)*a*b^
8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d
*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x +
c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37204, size = 899, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -((6*C*a^6 - 2*B*a^5*b - 15*C*a^4*b^2 + 5*B*a^3*b^3 + A*a^2*b^4 + 12*C*a^2*
b^4 - 6*B*a*b^5 + 2*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b
) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*C*a^6*tan(1/2*d*x +
 1/2*c)^3 - 2*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^5*b*tan(1/2*d*x + 1/2*
c)^3 + 3*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c
)^3 - A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3
 + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^2*b^4*tan(1/2*d*x + 1/2*c)^3
- 6*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 4
*C*a^6*tan(1/2*d*x + 1/2*c) - 2*B*a^5*b*tan(1/2*d*x + 1/2*c) + 5*C*a^5*b*ta
n(1/2*d*x + 1/2*c) - 3*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 7*C*a^4*b^2*tan(1/2
*d*x + 1/2*c) + A*a^3*b^3*tan(1/2*d*x + 1/2*c) + 5*B*a^3*b^3*tan(1/2*d*x +
1/2*c) - 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^2*b^4*tan(1/2*d*x + 1/2*c
) + 6*B*a^2*b^4*tan(1/2*d*x + 1/2*c) - 4*A*a*b^5*tan(1/2*d*x + 1/2*c))/((a^
4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)
^2 + a + b)^2) + (3*C*a - B*b)*(d*x + c)/b^4 - 2*C*tan(1/2*d*x + 1/2*c)/((t
an(1/2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

3.996 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=233

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-3ab^4(A+2C)+2b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\sin(c+dx)\left(a^2b^2(A+6C)+a^3bB-3a^4C\right.}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+}$$

```
[Out] (C*x)/b^3 + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3*b^2*C - 3*a*b^4*(A + 2*
C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*
(a + b)^(5/2)*d) + (a*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b^2*(a^2 - b
^2)*d*(a + b*Cos[c + d*x])^2) + ((2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*C +
 a^2*b^2*(A + 6*C))*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
]))
```

Rubi [A] time = 0.770619, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3031, 3021, 2735, 2659, 205}

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-3ab^4(A+2C)+2b^5B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\sin(c+dx)\left(a^2b^2(A+6C)+a^3bB-3a^4C\right.}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d
*x])^3,x]
```


```
[Out] (C*x)/b^3 + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3*b^2*C - 3*a*b^4*(A + 2*
C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*
(a + b)^(5/2)*d) + (a*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b^2*(a^2 - b
^2)*d*(a + b*Cos[c + d*x])^2) + ((2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*C +
 a^2*b^2*(A + 6*C))*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
]))
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}dx = \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \int \frac{-2b\left(Ab^2-a(bB-aC)\right)+\left(a^2bB-\cdots\right.}{\cdots}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(2Ab^4+a^3bB-4ab^3B-\cdots\right.}{2b^2\left(a^2-b^2\cdots\right.}$$

$$= \frac{Cx}{b^3} + \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(2Ab^4+a^3bB-4\cdots\right.}{2b^2\left(\cdots\right.}$$

$$= \frac{Cx}{b^3} + \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(2Ab^4+a^3bB-4\cdots\right.}{2b^2\left(\cdots\right.}$$

$$= \frac{Cx}{b^3} - \frac{\left(3aAb^4-a^2b^3B-2b^5B+2a^5C-5a^3b^2C+6ab^4C\right)\tan\cdots}{(a-b)^{5/2}b^3(a+b)^{5/2}d}$$

Mathematica [A] time = 1.6684, size = 225, normalized size = 0.97

$$\frac{\frac{b\sin(c+dx)\left(a^2b^2(A+6C)+a^3bB-3a^4C-4ab^3B+2Ab^4\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{2\left(-a^2b^3B-5a^3b^2C+2a^5C+3ab^4(A+2C)-2b^5B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{ab\sin(c+dx)\left(a(aC-bB)\cdots\right.}{(a-b)(a+b)(a+b\cos(c+\cdots}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^3,x]
```

```
[Out] (2*C*(c + d*x) + (2*(-(a^2*b^3*B) - 2*b^5*B + 2*a^5*C - 5*a^3*b^2*C + 3*a*b
^4*(A + 2*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 +
 b^2)^(5/2) + (a*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a - b)*(a + b
)*(a + b*Cos[c + d*x])^2) + (b*(2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*C + a
^2*b^2*(A + 6*C))*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))
/(2*b^3*d)
```

Maple [B] time = 0.044, size = 1485, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*C+2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*A+2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*a^2/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*
d*x+1/2*c)^3*B-4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*
a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^4/b^2/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*C+1/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a-
b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2*C+2
/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*
tan(1/2*d*x+1/2*c)*A-1/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*a^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*B-4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(
a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d/b/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*
c)*a^3*C+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a
-b)^2*tan(1/2*d*x+1/2*c)*a^2*C-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+1/d*a^2/(a^4-2*a
^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*B+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/d
*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*C-6/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
```

```
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.5683, size = 2651, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [1/4*(4*(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8)*d*x*cos(d*x + c)^2
+ 8*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*d*x*cos(d*x + c) + 4*(C
*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*d*x + (2*C*a^7 - 5*C*a^5*b^2
- B*a^4*b^3 + 3*(A + 2*C)*a^3*b^4 - 2*B*a^2*b^5 + (2*C*a^5*b^2 - 5*C*a^3*b^
4 - B*a^2*b^5 + 3*(A + 2*C)*a*b^6 - 2*B*b^7)*cos(d*x + c)^2 + 2*(2*C*a^6*b
- 5*C*a^4*b^3 - B*a^3*b^4 + 3*(A + 2*C)*a^2*b^5 - 2*B*a*b^6)*cos(d*x + c))*
sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2
*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*C*a^7*b - (2*A + 7*C)*a^5*b
^3 + 3*B*a^4*b^4 + (A + 5*C)*a^3*b^5 - 3*B*a^2*b^6 + A*a*b^7 + (3*C*a^6*b^2
 - B*a^5*b^3 - (A + 9*C)*a^4*b^4 + 5*B*a^3*b^5 - (A - 6*C)*a^2*b^6 - 4*B*a*
b^7 + 2*A*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^
9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d
*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2*(C*a
^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8)*d*x*cos(d*x + c)^2 + 4*(C*a^7*b
 - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*d*x*cos(d*x + c) + 2*(C*a^8 - 3*C*a
^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*d*x - (2*C*a^7 - 5*C*a^5*b^2 - B*a^4*b^3
+ 3*(A + 2*C)*a^3*b^4 - 2*B*a^2*b^5 + (2*C*a^5*b^2 - 5*C*a^3*b^4 - B*a^2*b^
5 + 3*(A + 2*C)*a*b^6 - 2*B*b^7)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^
3 - B*a^3*b^4 + 3*(A + 2*C)*a^2*b^5 - 2*B*a*b^6)*cos(d*x + c))*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*C*a^7
*b - (2*A + 7*C)*a^5*b^3 + 3*B*a^4*b^4 + (A + 5*C)*a^3*b^5 - 3*B*a^2*b^6 +
A*a*b^7 + (3*C*a^6*b^2 - B*a^5*b^3 - (A + 9*C)*a^4*b^4 + 5*B*a^3*b^5 - (A -
 6*C)*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5
- 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 +
 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^
2*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.26869, size = 814, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -((2*C*a^5 - 5*C*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4 + 6*C*a*b^4 - 2*B*b^5)*(pi
*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2
*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7
)*sqrt(a^2 - b^2)) - (d*x + c)*C/b^3 + (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*
C*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a^3
*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*b^
3*tan(1/2*d*x + 1/2*c)^3 + 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^3
*tan(1/2*d*x + 1/2*c)^3 - A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b^4*tan(1/
2*d*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*C*a^5*tan(1/2*d*x + 1
/2*c) + 3*C*a^4*b*tan(1/2*d*x + 1/2*c) - 2*A*a^3*b^2*tan(1/2*d*x + 1/2*c) -
 B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*C*a^3*b^2*tan(1/2*d*x + 1/2*c) - A*a^2*
b^3*tan(1/2*d*x + 1/2*c) + 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 6*C*a^2*b^3*t
an(1/2*d*x + 1/2*c) - A*a*b^4*tan(1/2*d*x + 1/2*c) + 4*B*a*b^4*tan(1/2*d*x
+ 1/2*c) - 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*ta
n(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.997 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(a^2(-(2A+C))+3abB-b^2(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\sin(c+dx)\left(a^2bB+a^3C-ab^2(3A+4C)+2b^3B\right)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-$$

```
[Out] -(((3*a*b*B - a^2*(2*A + C) - b^2*(A + 2*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) - ((A*b^2 - a*(b*B -
 a*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*b*B
 + 2*b^3*B + a^3*C - a*b^2*(3*A + 4*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*
(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.359285, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3021, 2754, 12, 2659, 205}

$$-\frac{\left(a^2(-(2A+C))+3abB-b^2(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\sin(c+dx)\left(a^2bB+a^3C-ab^2(3A+4C)+2b^3B\right)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -(((3*a*b*B - a^2*(2*A + C) - b^2*(A + 2*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) - ((A*b^2 - a*(b*B -
 a*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*b*B
 + 2*b^3*B + a^3*C - a*b^2*(3*A + 4*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*
(a + b*Cos[c + d*x]))
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
```

```
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b(bB-a(A+C))+\left(Ab^2-abB-a^2C+2b^2C\right)\cos(c+}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-ab^2(3A+4C)\right)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-ab^2(3A+4C)\right)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-ab^2(3A+4C)\right)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+a}$$

$$= \frac{\left(2a^2A+Ab^2-3abB+a^2C+2b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\left(Ab^2-a(b\right.}{2b\left(a^2-b^2\right)}$$

Mathematica [A] time = 1.04403, size = 192, normalized size = 0.95

$$\frac{\frac{\sin(c+dx)\left(a^2bB+a^3C-ab^2(3A+4C)+2b^3B\right)}{b(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(a^2(2A+C)-3abB+b^2(A+2C)\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{b(b-a)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*b*B + a^2*(2*A + C) + b^2*(A + 2*C))*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((A*b^2 + a*(-(b*B) + a*C))
*Sin[c + d*x])/(b*(-a + b)*(a + b)*(a + b*Cos[c + d*x])^2) + ((a^2*b*B + 2*
b^3*B + a^3*C - a*b^2*(3*A + 4*C))*Sin[c + d*x])/((a - b)^2*b*(a + b)^2*(a
+ b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.035, size = 1290, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d*a^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*B+1/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*b^2*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/
(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*b*C-
4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+
b^2)*tan(1/2*d*x+1/2*c)*a*A*b+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*b^2+2/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)*a^2*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*b*B+2/d/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b^2*B+
1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+
b^2)*tan(1/2*d*x+1/2*c)*a^2*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*b*C+2/d/(a^4-2*a^2*
b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*a^2*A+1/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)
*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d/(a
^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*C*a^2+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17451, size = 1837, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [-1/4*((((2*A + C)*a^4 - 3*B*a^3*b + (A + 2*C)*a^2*b^2 + ((2*A + C)*a^2*b^2
- 3*B*a*b^3 + (A + 2*C)*b^4)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b - 3*B*a^2*
b^2 + (A + 2*C)*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)
*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2
)) - 2*(2*B*a^5 - (4*A + 3*C)*a^4*b - B*a^3*b^2 + (5*A + 3*C)*a^2*b^3 - B*a
*b^4 - A*b^5 + (C*a^5 + B*a^4*b - (3*A + 5*C)*a^3*b^2 + B*a^2*b^3 + (3*A +
4*C)*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3
*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*(((2*A +
 C)*a^4 - 3*B*a^3*b + (A + 2*C)*a^2*b^2 + ((2*A + C)*a^2*b^2 - 3*B*a*b^3 +
(A + 2*C)*b^4)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b - 3*B*a^2*b^2 + (A + 2*C
)*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c))) + (2*B*a^5 - (4*A + 3*C)*a^4*b - B*a^3*b^2 + (5*A
+ 3*C)*a^2*b^3 - B*a*b^4 - A*b^5 + (C*a^5 + B*a^4*b - (3*A + 5*C)*a^3*b^2 +
 B*a^2*b^3 + (3*A + 4*C)*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3
 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b
^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3,x)
```

[Out] Timed out

Giac [B] time = 1.26574, size = 675, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] ((2*A*a^2 + C*a^2 - 3*B*a*b + A*b^2 + 2*C*b^2)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2
*c))/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*B*a^3
*tan(1/2*d*x + 1/2*c)^3 - C*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^2*b*tan(1/2*
d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*b*tan(1/2*d*x + 1
/2*c)^3 + 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3
 + 4*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*b^
3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c) + C*a^3*tan(1/2*d*x
 + 1/2*c) - 4*A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c) -
 3*C*a^2*b*tan(1/2*d*x + 1/2*c) - 3*A*a*b^2*tan(1/2*d*x + 1/2*c) + B*a*b^2*
tan(1/2*d*x + 1/2*c) - 4*C*a*b^2*tan(1/2*d*x + 1/2*c) + A*b^3*tan(1/2*d*x +
 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.998 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=238

$$\frac{\left(5a^2Ab^3-3a^4b(2A+C)+a^3b^2B+2a^5B-2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)\left(-a^2b^2(5A+2C)+3a^3bB+a^4\right.}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+d\right.}$$

```
[Out] ((5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B - 3*a^4*b*(2*A + C))*ArcTan[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2
)*d) + (A*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c +
 d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((2*A*b^4 + 3*a^3*b*B -
 a^4*C - a^2*b^2*(5*A + 2*C))*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x]))
```

Rubi [A] time = 0.978207, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3055, 3001, 3770, 2659, 205}

$$\frac{\left(5a^2Ab^3-3a^4b(2A+C)+a^3b^2B+2a^5B-2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)\left(-a^2b^2(5A+2C)+3a^3bB+a^4\right.}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d
*x])^3,x]
```


```
[Out] ((5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B - 3*a^4*b*(2*A + C))*ArcTan[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2
)*d) + (A*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c +
 d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((2*A*b^4 + 3*a^3*b*B -
 a^4*C - a^2*b^2*(5*A + 2*C))*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x]))
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2A\left(a^2-b^2\right)-2a(Ab-aB+bC)\,\mathrm{co}\ldots}{\ldots}}{\ldots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(2Ab^4+3a^3bB-a^4C-a\ldots\right.}{2a^2\left(a^2-b^2\right)^2 d(\ldots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(2Ab^4+3a^3bB-a^4C-a\ldots\right.}{2a^2\left(a^2-b^2\right)^2 d(\ldots}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \ldots$$

$$= -\frac{\left(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B+3a^4bC\right)\tan^{-1}\left(\ldots\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [C] time = 4.31554, size = 473, normalized size = 1.99

$$\frac{\cos(c+dx)(A\sec(c+dx)+B+C\cos(c+dx))\left(-\frac{4i(\cos(c)-i\sin(c))\left(5a^2Ab^3-3a^4b(2A+C)+a^3b^2B+2a^5B-2Ab^5\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)\ldots\right.}{\sqrt{-\left(a^2-b^2\right)(\cos(\ldots}}\right)}{\left(a^2-b^2\right)^2\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*(-4*A*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + 4*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((
4*I)*(5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B - 3*a^4*b*(2*A + C))*ArcT
an[((I*Cos[c] + Sin[c])*(b*Sin[c] + (-a + b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((
a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]]*(Cos[c] - I*Sin[c]))/((a^2 - b^2)^2*Sqr
t[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]) + (a*(b*Sec[c]*(a*(-7*A*b^4 - 10*a
^3*b*B + a*b^3*B + 4*a^4*C + a^2*b^2*(16*A + 5*C))*Sin[d*x] + b*(a*(A*b^3 +
 2*a^3*B + a*b^2*B - a^2*b*(4*A + 3*C))*Sin[2*c + d*x] + (-2*A*b^4 - 3*a^3*
b*B + a^4*C + a^2*b^2*(5*A + 2*C))*Sin[c + 2*d*x])) - (2*a^2 + b^2)*(-2*A*b
```

```
^4 - 3*a^3*b*B + a^4*C + a^2*b^2*(5*A + 2*C))*Tan[c]))/(b*(a^2 - b^2)^2*(a
+ b*Cos[c + d*x])^2)))/(2*a^3*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c +
d*x)]))
```

Maple [B] time = 0.087, size = 1507, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d/a/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*A*b^3-2/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-4/d*b/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B+2/d/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)
^3*a^2*C+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*b*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C+6/
d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*t
an(1/2*d*x+1/2*c)*A-1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^3-2/d/a^2/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4-4/d*b
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a-b)^2*tan(
1/2*d*x+1/2*c)*B+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*
b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*a^2*C-1/d*a/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*b*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-
b)^2*tan(1/2*d*x+1/2*c)*b^2*C-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/
2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+5/d/a*b^3/(a^4-2*
a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A-2/d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/
d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*B-3/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*a
```

```
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/a^3*A*ln(tan(1/2*
d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**3,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.30691, size = 842, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b - 3*C*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*
floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)
*sqrt(a^2 - b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - A*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/a^3 + (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^4*b*
tan(1/2*d*x + 1/2*c)^3 - C*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*b^2*tan(1
/2*d*x + 1/2*c)^3 + 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + C*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 - 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + B*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^3 - 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*A*a*b^4*tan(1/2*d*x +
1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*C*a^5*tan(1/2*d*x + 1/2*c) -
4*B*a^4*b*tan(1/2*d*x + 1/2*c) + C*a^4*b*tan(1/2*d*x + 1/2*c) + 6*A*a^3*b^2
*tan(1/2*d*x + 1/2*c) - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + C*a^3*b^2*tan(1/
2*d*x + 1/2*c) + 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + B*a^2*b^3*tan(1/2*d*x +
 1/2*c) + 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c)
 - 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.999 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-5a^3b^3B+6a^5bB-2a^6C+2ab^5B-6Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\tan(c+dx)\left(11a^2A\right.}{}$$

```
[Out] -(((15*a^2*A*b^4 - 6*A*b^6 + 6*a^5*b*B - 5*a^3*b^3*B + 2*a*b^5*B - 2*a^6*C
- a^4*b^2*(12*A + C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^4*(a - b)^(5/2)*(a + b)^(5/2)*d)) - ((3*A*b - a*B)*ArcTanh[Sin[c + d*x]])
/(a^4*d) - ((11*a^2*A*b^2 - 6*A*b^4 - 5*a^3*b*B + 2*a*b^3*B - a^4*(2*A - 3*
C))*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c
+ d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b^4 + 4*a^3*b*B
- a*b^3*B - 2*a^4*C - a^2*b^2*(6*A + C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 3.41858, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-5a^3b^3B+6a^5bB-2a^6C+2ab^5B-6Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\tan(c+dx)\left(11a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c +
 d*x])^3,x]
```


```
[Out] -(((15*a^2*A*b^4 - 6*A*b^6 + 6*a^5*b*B - 5*a^3*b^3*B + 2*a*b^5*B - 2*a^6*C
- a^4*b^2*(12*A + C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^4*(a - b)^(5/2)*(a + b)^(5/2)*d)) - ((3*A*b - a*B)*ArcTanh[Sin[c + d*x]])
/(a^4*d) - ((11*a^2*A*b^2 - 6*A*b^4 - 5*a^3*b*B + 2*a*b^3*B - a^4*(2*A - 3*
C))*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c
+ d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b^4 + 4*a^3*b*B
- a*b^3*B - 2*a^4*C - a^2*b^2*(6*A + C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2 - a(bB - aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \int \frac{\left(-3Ab^2+abB+a^2(2A-C)-2a(\ldots\right.}{\ldots}$$

$$= \frac{\left(Ab^2 - a(bB - aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3Ab^4 + 4a^3bB - ab^3B - \ldots\right.}{2a^2\left(a^2-b^2 \ldots\right.}$$

$$= -\frac{\left(11a^2Ab^2 - 6Ab^4 - 5a^3bB + 2ab^3B - a^4(2A-3C)\right)\tan(c+\ldots}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(11a^2Ab^2 - 6Ab^4 - 5a^3bB + 2ab^3B - a^4(2A-3C)\right)\tan(c+\ldots}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(3Ab - aB)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{\left(11a^2Ab^2 - 6Ab^4 - 5a^3b\ldots\right.}{2\ldots}$$

$$= \frac{\left(12a^4Ab^2 - 15a^2Ab^4 + 6Ab^6 - 6a^5bB + 5a^3b^3B - 2ab^5B + 2a\ldots\right.}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

<u>Mathematica</u> [A] time = 2.40768, size = 444, normalized size = 1.31

$$\frac{\cos(c+dx)\left(A\sec^2(c+dx) + B\sec(c+dx) + C\right)\left(\frac{2a\sin(c+dx)\left(2ab\cos(c+dx)\left(a^2b^2(C-16A)+4a^4(A-C)+6a^3bB-3ab^3B+9Ab^4\right)+b^2\cos(2(c+dx)\ldots\right.}{\left(a^2-b^2\ldots\right.}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((-8*(-15*a^2*A*b^4 +
 6*A*b^6 - 6*a^5*b*B + 5*a^3*b^3*B - 2*a*b^5*B + 2*a^6*C + a^4*b^2*(12*A +
C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]*Cos[c + d*x])/(-a^
2 + b^2)^(5/2) + 8*(3*A*b - a*B)*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 8*(-3*A*b + a*B)*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + (2*a*(4*a^6*A - 6*a^4*A*b^2 - 7*a^2*A*b^4 + 6*A*b^6 + 5*a^3*b^3
```

```
*B - 2*a*b^5*B - 3*a^4*b^2*C + 2*a*b*(9*A*b^4 + 6*a^3*b*B - 3*a*b^3*B + 4*a
^4*(A - C) + a^2*b^2*(-16*A + C))*Cos[c + d*x] + b^2*(-11*a^2*A*b^2 + 6*A*b
^4 + 5*a^3*b*B - 2*a*b^3*B + a^4*(2*A - 3*C))*Cos[2*(c + d*x)])*Sin[c + d*x
])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2)))/(4*a^4*d*(2*A + C + 2*B*Cos[c +
 d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.104, size = 1750, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b
+b^2)*tan(1/2*d*x+1/2*c)^3*a*b*C+1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d/a^2/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d/a^2/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)
*B+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2*b^5/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*
x+1/2*c)*b^2*C+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+1/d/a^3*B*ln(tan(1/2*d*x+1/2*c)+
1)-1/d/a^3*B*ln(tan(1/2*d*x+1/2*c)-1)+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2*C-1/d/a^3*A
/(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-6/d*a*b/(a^4-2*a^2
*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B-8
/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*ta
n(1/2*d*x+1/2*c)*A*b^3+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-8/d/a/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*A*b^3-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4-4/d*a
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/
```

```
2*d*x+1/2*c)*b*C-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^5*B+3/d/a^4*ln(tan(1/2*d*x+1/2
*c)-1)*A*b+5/d/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^3*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-15/d/a^2/(a^
4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*A*b^4+6/d/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6-3/d/a^4*ln(tan(1/2*d*x
+1/2*c)+1)*A*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34014, size = 944, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -((2*C*a^6 - 6*B*a^5*b + 12*A*a^4*b^2 + C*a^4*b^2 + 5*B*a^3*b^3 - 15*A*a^2*
b^4 - 2*B*a*b^5 + 6*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b
) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2 - b^2)) + (4*C*a^5*b*tan(1/2*d*x
 + 1/2*c)^3 - 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^4*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*b^3*tan(1/2*d*x +
 1/2*c)^3 - C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^2*b^4*tan(1/2*d*x + 1/
2*c)^3 + 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*A*a*b^5*tan(1/2*d*x + 1/2*c
)^3 - 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^6*tan(1/2*d*x + 1/2*c)^3 + 4
*C*a^5*b*tan(1/2*d*x + 1/2*c) - 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 3*C*a^4*
b^2*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 5*B*a^3*b^3*t
an(1/2*d*x + 1/2*c) - C*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*A*a^2*b^4*tan(1/2*
d*x + 1/2*c) + 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*A*a*b^5*tan(1/2*d*x + 1
/2*c) + 2*B*a*b^5*tan(1/2*d*x + 1/2*c) - 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^
7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)
^2 + a + b)^2) - (B*a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 + (B*
a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*A*tan(1/2*d*x + 1/2*c
)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

3.1000 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=462

$$-\frac{b\left(5a^4b^2(4A-C)-a^2b^4(29A-2C)+15a^3b^3B-12a^5bB+6a^6C-6ab^5B+12Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\tan(c+}{}$$

```
[Out] -((b*(12*A*b^6 - 12*a^5*b*B + 15*a^3*b^3*B - 6*a*b^5*B - a^2*b^4*(29*A - 2*
C) + 5*a^4*b^2*(4*A - C) + 6*a^6*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ((12*A*b^2 - 6*a*b*B +
a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^5*d) - ((12*A*b^5 - 2*a^5*B + 11
*a^3*b^2*B - 6*a*b^4*B + a^4*b*(6*A - 5*C) - a^2*b^3*(21*A - 2*C))*Tan[c +
d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B + a^4*(A
- 4*C) - a^2*b^2*(10*A - C))*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^
2*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^2) + ((7*a^2*A*b^2 - 4*A*b^4 - 5*a^3*b*B + 2*a*b^3*
B + 3*a^4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c
 + d*x]))
```

Rubi [A] time = 5.1589, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(5a^4b^2(4A-C)-a^2b^4(29A-2C)+15a^3b^3B-12a^5bB+6a^6C-6ab^5B+12Ab^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\tan(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c +
 d*x])^3,x]
```



```
[Out] -((b*(12*A*b^6 - 12*a^5*b*B + 15*a^3*b^3*B - 6*a*b^5*B - a^2*b^4*(29*A - 2*
C) + 5*a^4*b^2*(4*A - C) + 6*a^6*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ((12*A*b^2 - 6*a*b*B +
a^2*(A + 2*C))*ArcTanh[Sin[c + d*x]])/(2*a^5*d) - ((12*A*b^5 - 2*a^5*B + 11
*a^3*b^2*B - 6*a*b^4*B + a^4*b*(6*A - 5*C) - a^2*b^3*(21*A - 2*C))*Tan[c +
d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B + a^4*(A
```

```
- 4*C) - a^2*b^2*(10*A - C))*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^
2*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^2) + ((7*a^2*A*b^2 - 4*A*b^4 - 5*a^3*b*B + 2*a*b^3*
B + 3*a^4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c
 + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \int \frac{\left(-2\left(2Ab^2-abB\right.\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(7a^2Ab^2-4A\right.}{}$$

$$= \frac{\left(6Ab^4+6a^3bB-3ab^3B+a^4(A-4C)-a^2b^2(10A-C)\right)\sec(}{2a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(12Ab^5-2a^5B+11a^3b^2B-6ab^4B+a^4b(6A-5C)-a^2b^3(2\right.}{2a^4\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(12Ab^5-2a^5B+11a^3b^2B-6ab^4B+a^4b(6A-5C)-a^2b^3(2\right.}{2a^4\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(12Ab^2-6abB+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{\left(12Ab^5\right.}{}$$

$$= -\frac{b\left(20a^4Ab^2-29a^2Ab^4+12Ab^6-12a^5bB+15a^3b^3B-6ab^5\right.}{a^5(a-b)^{5/2}(a}$$

Mathematica [A] time = 3.91707, size = 606, normalized size = 1.31

$$\frac{\frac{16b\left(5a^4b^2(4A-C)+a^2b^4(2C-29A)+15a^3b^3B-12a^5bB+6a^6C-6ab^5B+12Ab^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} - 8\left(a^2(A+2C)-6abB+12Ab^2\right)\log\left(\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*C
os[c + d*x])^3,x]
```

```
[Out] ((16*b*(12*A*b^6 - 12*a^5*b*B + 15*a^3*b^3*B - 6*a*b^5*B + 5*a^4*b^2*(4*A -
 C) + 6*a^6*C + a^2*b^4*(-29*A + 2*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 8*(12*A*b^2 - 6*a*b*B + a^2*(A + 2*C
))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 8*(12*A*b^2 - 6*a*b*B + a^2*(
A + 2*C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(4*a^7*A - 30*a^5
*A*b^2 + 68*a^3*A*b^4 - 36*a*A*b^6 + 8*a^6*b*B - 32*a^4*b^3*B + 18*a^2*b^5*
B + 12*a^5*b^2*C - 6*a^3*b^4*C + (-16*a^6*A*b - 36*A*b^7 + 8*a^7*B - 10*a^5
*b^2*B - 25*a^3*b^4*B + 18*a*b^6*B + a^2*b^5*(47*A - 6*C) + a^4*b^3*(14*A +
 15*C))*Cos[c + d*x] + 2*a*b*(-18*A*b^5 + 4*a^5*B - 16*a^3*b^2*B + 9*a*b^4*
B + a^2*b^3*(32*A - 3*C) + a^4*(-11*A*b + 6*b*C))*Cos[2*(c + d*x)] - 6*a^4*
A*b^3*Cos[3*(c + d*x)] + 21*a^2*A*b^5*Cos[3*(c + d*x)] - 12*A*b^7*Cos[3*(c
+ d*x)] + 2*a^5*b^2*B*Cos[3*(c + d*x)] - 11*a^3*b^4*B*Cos[3*(c + d*x)] + 6*
a*b^6*B*Cos[3*(c + d*x)] + 5*a^4*b^3*C*Cos[3*(c + d*x)] - 2*a^2*b^5*C*Cos[3
*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])
^2))/(16*a^5*d)
```

Maple [B] time = 0.121, size = 2202, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -8/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d/
a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a-b)^2*t
an(1/2*d*x+1/2*c)*B+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d/a^3/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^5/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*A+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b
)^2*tan(1/2*d*x+1/2*c)*b^2*C-20/d/a*b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+29/d/a^3*b^5/(a
^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*A+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-1/d/a^3/(tan(1/2*d*x+1/2*c)-
1)*B-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C+1/
2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^2+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d
/a^3*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^
3*A/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^3*A
*ln(tan(1/2*d*x+1/2*c)+1)+3/d*A/a^4/(tan(1/2*d*x+1/2*c)+1)*b-6/d/a^5*ln(tan
```

```
(1/2*d*x+1/2*c)-1)*A*b^2+3/d*A/a^4/(tan(1/2*d*x+1/2*c)-1)*b+6/d/a^5*ln(tan(
1/2*d*x+1/2*c)+1)*A*b^2+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-12/d*b^7/a^5/(a^4-2*a
^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C
-6/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*C+3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*b*B-3/d/a^4*l
n(tan(1/2*d*x+1/2*c)+1)*b*B-2/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d*b^5/a^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/
2*d*x+1/2*c)*A-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-2/d*b^4/a^
2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1
/2*d*x+1/2*c)*C-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b^3/a/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*C+10/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+10/d/a^2/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4+
6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4/d*b^5/a^
3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1
/2*d*x+1/2*c)*B+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3886, size = 2354, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/2*(2*(6*C*a^6*b - 12*B*a^5*b^2 + 20*A*a^4*b^3 - 5*C*a^4*b^3 + 15*B*a^3*b^
4 - 29*A*a^2*b^5 + 2*C*a^2*b^5 - 6*B*a*b^6 + 12*A*b^7)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^
2)) + 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*
A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^
5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 6*C*a^5
*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4
```

```
*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3
*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*C*a^3*
b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*
b^5*tan(1/2*d*x + 1/2*c)^7 + 2*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^
6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(
1/2*d*x + 1/2*c)^7 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^5 - 2*B*a^7*tan(1/2*d*x +
 1/2*c)^5 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^6*b*tan(1/2*d*x + 1/2*
c)^5 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 10*B*a^5*b^2*tan(1/2*d*x + 1/2*
c)^5 - 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*A*a^4*b^3*tan(1/2*d*x + 1/2*
c)^5 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 15*C*a^4*b^3*tan(1/2*d*x + 1/2
*c)^5 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 35*B*a^3*b^4*tan(1/2*d*x + 1/
2*c)^5 + 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*A*a^2*b^5*tan(1/2*d*x + 1/
2*c)^5 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^2*b^5*tan(1/2*d*x + 1/2
*c)^5 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^6*tan(1/2*d*x + 1/2*c)
^5 - 36*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B
*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*
tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*
tan(1/2*d*x + 1/2*c)^3 - 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b^3*
tan(1/2*d*x + 1/2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*C*a^4*b^3
*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^3*b^
4*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2*b^
5*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^5
*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 18*B*a*b^6*ta
n(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^3 + A*a^7*tan(1/2*d*x
+ 1/2*c) + 2*B*a^7*tan(1/2*d*x + 1/2*c) - 4*A*a^6*b*tan(1/2*d*x + 1/2*c) +
4*B*a^6*b*tan(1/2*d*x + 1/2*c) - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*B*a^
5*b^2*tan(1/2*d*x + 1/2*c) + 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*A*a^4*b^3
*tan(1/2*d*x + 1/2*c) - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 5*C*a^4*b^3*tan
(1/2*d*x + 1/2*c) + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c) - 9*B*a^3*b^4*tan(1/2
*d*x + 1/2*c) - 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*A*a^2*b^5*tan(1/2*d*x
 + 1/2*c) + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) - 2*C*a^2*b^5*tan(1/2*d*x + 1/
2*c) - 18*A*a*b^6*tan(1/2*d*x + 1/2*c) + 6*B*a*b^6*tan(1/2*d*x + 1/2*c) - 1
2*A*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^
2) + (A*a^2 + 2*C*a^2 - 6*B*a*b + 12*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/a^5 - (A*a^2 + 2*C*a^2 - 6*B*a*b + 12*A*b^2)*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a^5)/d
```

3.1001 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=649

$$\frac{\sin(c+dx)\left(-a^5b^2(6A-167C)+a^3b^4(17A-146C)-68a^4b^3B+65a^2b^5B+24a^6bB-60a^7C-2ab^6(13A-12C)-6b^7\right.}{6b^5d\left(a^2-b^2\right)^3}$$

```
[Out] ((2*A*b^2 - 8*a*b*B + 20*a^2*C + b^2*C)*x)/(2*b^6) + (a*(8*A*b^8 + 8*a^7*b*
B - 28*a^5*b^3*B + 35*a^3*b^5*B - 20*a*b^7*B - a^6*b^2*(2*A - 69*C) + 7*a^4
*b^4*(A - 12*C) - 8*a^2*b^6*(A - 5*C) - 20*a^8*C)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^6*Sqrt[a + b]*(a^2 - b^2)^3*d) +
((24*a^6*b*B - 68*a^4*b^3*B + 65*a^2*b^5*B - 6*b^7*B - a^5*b^2*(6*A - 167*C
) + a^3*b^4*(17*A - 146*C) - 2*a*b^6*(13*A - 12*C) - 60*a^7*C)*Sin[c + d*x]
)/(6*b^5*(a^2 - b^2)^3*d) - ((4*a^5*b*B - 11*a^3*b^3*B + 12*a*b^5*B - a^4*b
^2*(A - 27*C) + a^2*b^4*(2*A - 23*C) - b^6*(6*A - C) - 10*a^6*C)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c +
 d*x]^4*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((4*A*b^
4 + 2*a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 10*C))*Cos[c + d*x]^3*Si
n[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((12*A*b^6 - 8
*a^5*b*B + 20*a^3*b^3*B - 27*a*b^5*B + a^4*b^2*(2*A - 53*C) + 20*a^6*C + a^
2*b^4*(A + 48*C))*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rubi [A] time = 12.1992, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3049, 3023, 2735, 2659, 205}

$$\frac{\sin(c+dx)\left(-a^5b^2(6A-167C)+a^3b^4(17A-146C)-68a^4b^3B+65a^2b^5B+24a^6bB-60a^7C-2ab^6(13A-12C)-6b^7\right.}{6b^5d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^4,x]
```

```
[Out] ((2*A*b^2 - 8*a*b*B + 20*a^2*C + b^2*C)*x)/(2*b^6) + (a*(8*A*b^8 + 8*a^7*b*
B - 28*a^5*b^3*B + 35*a^3*b^5*B - 20*a*b^7*B - a^6*b^2*(2*A - 69*C) + 7*a^4
```

```
*b^4*(A - 12*C) - 8*a^2*b^6*(A - 5*C) - 20*a^8*C)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^6*Sqrt[a + b]*(a^2 - b^2)^3*d) +
((24*a^6*b*B - 68*a^4*b^3*B + 65*a^2*b^5*B - 6*b^7*B - a^5*b^2*(6*A - 167*C
) + a^3*b^4*(17*A - 146*C) - 2*a*b^6*(13*A - 12*C) - 60*a^7*C)*Sin[c + d*x]
)/(6*b^5*(a^2 - b^2)^3*d) - ((4*a^5*b*B - 11*a^3*b^3*B + 12*a*b^5*B - a^4*b
^2*(A - 27*C) + a^2*b^4*(2*A - 23*C) - b^6*(6*A - C) - 10*a^6*C)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c +
 d*x]^4*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((4*A*b^
4 + 2*a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 10*C))*Cos[c + d*x]^3*Si
n[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((12*A*b^6 - 8
*a^5*b*B + 20*a^3*b^3*B - 27*a*b^5*B + a^4*b^2*(2*A - 53*C) + 20*a^6*C + a^
2*b^4*(A + 48*C))*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^4(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)(4}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^4(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(4Ab^4+2a^3\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^4(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(4Ab^4+2a^3\right.}{}$$

$$= -\frac{\left(4a^5bB-11a^3b^3B+12ab^5B-a^4b^2(A-27C)+a^2b^4(2A-2\right.}{2b^4\left(a^2-b^2\right.}$$

$$= \frac{\left(24a^6bB-68a^4b^3B+65a^2b^5B-6b^7B-a^5b^2(6A-167C)+a\right.}{6b^5\left(a^2-\right.}$$

$$= \frac{\left(2Ab^2-8abB+20a^2C+b^2C\right)x}{2b^6} + \frac{\left(24a^6bB-68a^4b^3B+65a\right.}{}$$

$$= \frac{\left(2Ab^2-8abB+20a^2C+b^2C\right)x}{2b^6} + \frac{\left(24a^6bB-68a^4b^3B+65a\right.}{}$$

$$= \frac{\left(2Ab^2-8abB+20a^2C+b^2C\right)x}{2b^6} + \frac{a\left(8Ab^8+8a^7bB-28a^5b\right.}{}$$

Mathematica [C] time = 6.90157, size = 658, normalized size = 1.01

$$\frac{(c+dx)\left(20a^2C-8abB+2Ab^2+b^2C\right)}{2b^6d} + \frac{a^4Ab^2\sin(c+dx)-a^5bB\sin(c+dx)+a^6C\sin(c+dx)}{3b^5d\left(b^2-a^2\right)(a+b\cos(c+dx))^3} + \frac{7a^5Ab^2\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^4,x]
```

```
[Out] ((2*A*b^2 - 8*a*b*B + 20*a^2*C + b^2*C)*(c + d*x))/(2*b^6*d) + (a*(2*a^6*A*
b^2 - 7*a^4*A*b^4 + 8*a^2*A*b^6 - 8*A*b^8 - 8*a^7*b*B + 28*a^5*b^3*B - 35*a
^3*b^5*B + 20*a*b^7*B + 20*a^8*C - 69*a^6*b^2*C + 84*a^4*b^4*C - 40*a^2*b^6
*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(b^6*(a^2 - b^2)^
3*Sqrt[-a^2 + b^2]*d) + ((-(b*B) + 4*a*C)*(((-I/2)*Cos[c + d*x])/b^5 - Sin[
c + d*x]/(2*b^5)))/d + ((-(b*B) + 4*a*C)*(((I/2)*Cos[c + d*x])/b^5 - Sin[c
+ d*x]/(2*b^5)))/d + (a^4*A*b^2*Sin[c + d*x] - a^5*b*B*Sin[c + d*x] + a^6*C
*Sin[c + d*x])/(3*b^5*(-a^2 + b^2)*d*(a + b*Cos[c + d*x])^3) + (7*a^5*A*b^2
*Sin[c + d*x] - 12*a^3*A*b^4*Sin[c + d*x] - 10*a^6*b*B*Sin[c + d*x] + 15*a^
4*b^3*B*Sin[c + d*x] + 13*a^7*C*Sin[c + d*x] - 18*a^5*b^2*C*Sin[c + d*x])/(
6*b^5*(-a^2 + b^2)^2*d*(a + b*Cos[c + d*x])^2) + (11*a^6*A*b^2*Sin[c + d*x]
 - 32*a^4*A*b^4*Sin[c + d*x] + 36*a^2*A*b^6*Sin[c + d*x] - 26*a^7*b*B*Sin[c
 + d*x] + 71*a^5*b^3*B*Sin[c + d*x] - 60*a^3*b^5*B*Sin[c + d*x] + 47*a^8*C*
Sin[c + d*x] - 122*a^6*b^2*C*Sin[c + d*x] + 90*a^4*b^4*C*Sin[c + d*x])/(6*b
^5*(-a^2 + b^2)^3*d*(a + b*Cos[c + d*x])) + (C*Sin[2*(c + d*x)])/(4*b^4*d)
```

Maple [B] time = 0.059, size = 4367, normalized size = 6.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+40/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*C+2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*A+1/d/b^4*arctan(tan(1/2*d*x+1/2*c))*
C-1/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)^3*C+2/d/b^4/(tan(1/
2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*B+1/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2
*tan(1/2*d*x+1/2*c)*C+2/d/b^4/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)
^3*B-8/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a*B+20/d/b^6*arctan(tan(1/2*d*x+1/2
*c))*a^2*C-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d*a^5/b^2/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+
b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-24/
d*a^8/b^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+
b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-12/d*a^8/b^5/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*C+12/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d*a^6/b^3/(a
```

*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5
*B-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/
(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-116/3/d*a^5/b^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*B+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-1/d*a^5
/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*A-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+44/3/d*a^4/b/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*A-5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d*a^3/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5*A+8/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)
*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+69/d*a
^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-84/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-
b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*C+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-30/d*a^4/b/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5*C-24/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a^5/b^2/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b
^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A
-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+34/d*a^6/b^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*C+6/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+212/3/d*a^6/b^3/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+34/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5*C-30/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-60/d*a^4/b/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*C-3/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-18/d*a^5/b^2/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2

```
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B
+3/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-12/d*a^8/b^5/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(
1/2*d*x+1/2*c)^5*C+5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-18/d*a^5/b^2
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-8/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1
/2*d*x+1/2*c)^3*a*C-8/d/b^5/(tan(1/2*d*x+1/2*c)^2+1)^2*tan(1/2*d*x+1/2*c)*a
*C-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-28/d*a^6/b^3/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*B+35/d*a^4/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+8/d*a^8/b^5/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*B-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+7/d*a^5/b^2/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*A+20/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+40/
d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+20/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*B-20/d*a^9/b^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 5.50197, size = 7788, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] [1/12*(6*(20*C*a^10*b^3 - 8*B*a^9*b^4 + (2*A - 79*C)*a^8*b^5 + 32*B*a^7*b^6
 - 4*(2*A - 29*C)*a^6*b^7 - 48*B*a^5*b^8 + 2*(6*A - 37*C)*a^4*b^9 + 32*B*a^
3*b^10 - 8*(A - 2*C)*a^2*b^11 - 8*B*a*b^12 + (2*A + C)*b^13)*d*x*cos(d*x +
c)^3 + 18*(20*C*a^11*b^2 - 8*B*a^10*b^3 + (2*A - 79*C)*a^9*b^4 + 32*B*a^8*b
^5 - 4*(2*A - 29*C)*a^7*b^6 - 48*B*a^6*b^7 + 2*(6*A - 37*C)*a^5*b^8 + 32*B*
a^4*b^9 - 8*(A - 2*C)*a^3*b^10 - 8*B*a^2*b^11 + (2*A + C)*a*b^12)*d*x*cos(d
*x + c)^2 + 18*(20*C*a^12*b - 8*B*a^11*b^2 + (2*A - 79*C)*a^10*b^3 + 32*B*a
^9*b^4 - 4*(2*A - 29*C)*a^8*b^5 - 48*B*a^7*b^6 + 2*(6*A - 37*C)*a^6*b^7 + 3
2*B*a^5*b^8 - 8*(A - 2*C)*a^4*b^9 - 8*B*a^3*b^10 + (2*A + C)*a^2*b^11)*d*x*
cos(d*x + c) + 6*(20*C*a^13 - 8*B*a^12*b + (2*A - 79*C)*a^11*b^2 + 32*B*a^1
0*b^3 - 4*(2*A - 29*C)*a^9*b^4 - 48*B*a^8*b^5 + 2*(6*A - 37*C)*a^7*b^6 + 32
*B*a^6*b^7 - 8*(A - 2*C)*a^5*b^8 - 8*B*a^4*b^9 + (2*A + C)*a^3*b^10)*d*x -
3*(20*C*a^12 - 8*B*a^11*b + (2*A - 69*C)*a^10*b^2 + 28*B*a^9*b^3 - 7*(A - 1
2*C)*a^8*b^4 - 35*B*a^7*b^5 + 8*(A - 5*C)*a^6*b^6 + 20*B*a^5*b^7 - 8*A*a^4*
b^8 + (20*C*a^9*b^3 - 8*B*a^8*b^4 + (2*A - 69*C)*a^7*b^5 + 28*B*a^6*b^6 - 7
*(A - 12*C)*a^5*b^7 - 35*B*a^4*b^8 + 8*(A - 5*C)*a^3*b^9 + 20*B*a^2*b^10 -
8*A*a*b^11)*cos(d*x + c)^3 + 3*(20*C*a^10*b^2 - 8*B*a^9*b^3 + (2*A - 69*C)*
a^8*b^4 + 28*B*a^7*b^5 - 7*(A - 12*C)*a^6*b^6 - 35*B*a^5*b^7 + 8*(A - 5*C)*
a^4*b^8 + 20*B*a^3*b^9 - 8*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20*C*a^11*b - 8*
B*a^10*b^2 + (2*A - 69*C)*a^9*b^3 + 28*B*a^8*b^4 - 7*(A - 12*C)*a^7*b^5 - 3
5*B*a^6*b^6 + 8*(A - 5*C)*a^5*b^7 + 20*B*a^4*b^8 - 8*A*a^3*b^9)*cos(d*x + c
))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(60*C*a^12*b - 24*B*a^11*b^
2 + (6*A - 227*C)*a^10*b^3 + 92*B*a^9*b^4 - (23*A - 313*C)*a^8*b^5 - 133*B*
a^7*b^6 + (43*A - 170*C)*a^6*b^7 + 71*B*a^5*b^8 - 2*(13*A - 12*C)*a^4*b^9 -
 6*B*a^3*b^10 - 3*(C*a^8*b^5 - 4*C*a^6*b^7 + 6*C*a^4*b^9 - 4*C*a^2*b^11 + C
*b^13)*cos(d*x + c)^4 + 3*(5*C*a^9*b^4 - 2*B*a^8*b^5 - 20*C*a^7*b^6 + 8*B*a
^6*b^7 + 30*C*a^5*b^8 - 12*B*a^4*b^9 - 20*C*a^3*b^10 + 8*B*a^2*b^11 + 5*C*a
*b^12 - 2*B*b^13)*cos(d*x + c)^3 + (110*C*a^10*b^3 - 44*B*a^9*b^4 + (11*A -
 421*C)*a^8*b^5 + 169*B*a^7*b^6 - (43*A - 590*C)*a^6*b^7 - 239*B*a^5*b^8 +
2*(34*A - 171*C)*a^4*b^9 + 132*B*a^3*b^10 - 9*(4*A - 7*C)*a^2*b^11 - 18*B*a
*b^12)*cos(d*x + c)^2 + 3*(50*C*a^11*b^2 - 20*B*a^10*b^3 + 5*(A - 38*C)*a^9
*b^4 + 77*B*a^8*b^5 - (20*A - 263*C)*a^7*b^6 - 110*B*a^6*b^7 + (35*A - 146*
C)*a^5*b^8 + 59*B*a^4*b^9 - (20*A - 23*C)*a^3*b^10 - 6*B*a^2*b^11)*cos(d*x
+ c))*sin(d*x + c))/((a^8*b^9 - 4*a^6*b^11 + 6*a^4*b^13 - 4*a^2*b^15 + b^17
)*d*cos(d*x + c)^3 + 3*(a^9*b^8 - 4*a^7*b^10 + 6*a^5*b^12 - 4*a^3*b^14 + a*
b^16)*d*cos(d*x + c)^2 + 3*(a^10*b^7 - 4*a^8*b^9 + 6*a^6*b^11 - 4*a^4*b^13
+ a^2*b^15)*d*cos(d*x + c) + (a^11*b^6 - 4*a^9*b^8 + 6*a^7*b^10 - 4*a^5*b^1
```

```
2 + a^3*b^14)*d), 1/6*(3*(20*C*a^10*b^3 - 8*B*a^9*b^4 + (2*A - 79*C)*a^8*b^
5 + 32*B*a^7*b^6 - 4*(2*A - 29*C)*a^6*b^7 - 48*B*a^5*b^8 + 2*(6*A - 37*C)*a
^4*b^9 + 32*B*a^3*b^10 - 8*(A - 2*C)*a^2*b^11 - 8*B*a*b^12 + (2*A + C)*b^13
)*d*x*cos(d*x + c)^3 + 9*(20*C*a^11*b^2 - 8*B*a^10*b^3 + (2*A - 79*C)*a^9*b
^4 + 32*B*a^8*b^5 - 4*(2*A - 29*C)*a^7*b^6 - 48*B*a^6*b^7 + 2*(6*A - 37*C)*
a^5*b^8 + 32*B*a^4*b^9 - 8*(A - 2*C)*a^3*b^10 - 8*B*a^2*b^11 + (2*A + C)*a*
b^12)*d*x*cos(d*x + c)^2 + 9*(20*C*a^12*b - 8*B*a^11*b^2 + (2*A - 79*C)*a^1
0*b^3 + 32*B*a^9*b^4 - 4*(2*A - 29*C)*a^8*b^5 - 48*B*a^7*b^6 + 2*(6*A - 37*
C)*a^6*b^7 + 32*B*a^5*b^8 - 8*(A - 2*C)*a^4*b^9 - 8*B*a^3*b^10 + (2*A + C)*
a^2*b^11)*d*x*cos(d*x + c) + 3*(20*C*a^13 - 8*B*a^12*b + (2*A - 79*C)*a^11*
b^2 + 32*B*a^10*b^3 - 4*(2*A - 29*C)*a^9*b^4 - 48*B*a^8*b^5 + 2*(6*A - 37*C
)*a^7*b^6 + 32*B*a^6*b^7 - 8*(A - 2*C)*a^5*b^8 - 8*B*a^4*b^9 + (2*A + C)*a^
3*b^10)*d*x - 3*(20*C*a^12 - 8*B*a^11*b + (2*A - 69*C)*a^10*b^2 + 28*B*a^9*
b^3 - 7*(A - 12*C)*a^8*b^4 - 35*B*a^7*b^5 + 8*(A - 5*C)*a^6*b^6 + 20*B*a^5*
b^7 - 8*A*a^4*b^8 + (20*C*a^9*b^3 - 8*B*a^8*b^4 + (2*A - 69*C)*a^7*b^5 + 28
*B*a^6*b^6 - 7*(A - 12*C)*a^5*b^7 - 35*B*a^4*b^8 + 8*(A - 5*C)*a^3*b^9 + 20
*B*a^2*b^10 - 8*A*a*b^11)*cos(d*x + c)^3 + 3*(20*C*a^10*b^2 - 8*B*a^9*b^3 +
 (2*A - 69*C)*a^8*b^4 + 28*B*a^7*b^5 - 7*(A - 12*C)*a^6*b^6 - 35*B*a^5*b^7
+ 8*(A - 5*C)*a^4*b^8 + 20*B*a^3*b^9 - 8*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20
*C*a^11*b - 8*B*a^10*b^2 + (2*A - 69*C)*a^9*b^3 + 28*B*a^8*b^4 - 7*(A - 12*
C)*a^7*b^5 - 35*B*a^6*b^6 + 8*(A - 5*C)*a^5*b^7 + 20*B*a^4*b^8 - 8*A*a^3*b^
9)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c))) - (60*C*a^12*b - 24*B*a^11*b^2 + (6*A - 227*C)*a^10*b^3
+ 92*B*a^9*b^4 - (23*A - 313*C)*a^8*b^5 - 133*B*a^7*b^6 + (43*A - 170*C)*a^
6*b^7 + 71*B*a^5*b^8 - 2*(13*A - 12*C)*a^4*b^9 - 6*B*a^3*b^10 - 3*(C*a^8*b^
5 - 4*C*a^6*b^7 + 6*C*a^4*b^9 - 4*C*a^2*b^11 + C*b^13)*cos(d*x + c)^4 + 3*(
5*C*a^9*b^4 - 2*B*a^8*b^5 - 20*C*a^7*b^6 + 8*B*a^6*b^7 + 30*C*a^5*b^8 - 12*
B*a^4*b^9 - 20*C*a^3*b^10 + 8*B*a^2*b^11 + 5*C*a*b^12 - 2*B*b^13)*cos(d*x +
 c)^3 + (110*C*a^10*b^3 - 44*B*a^9*b^4 + (11*A - 421*C)*a^8*b^5 + 169*B*a^7
*b^6 - (43*A - 590*C)*a^6*b^7 - 239*B*a^5*b^8 + 2*(34*A - 171*C)*a^4*b^9 +
132*B*a^3*b^10 - 9*(4*A - 7*C)*a^2*b^11 - 18*B*a*b^12)*cos(d*x + c)^2 + 3*(
50*C*a^11*b^2 - 20*B*a^10*b^3 + 5*(A - 38*C)*a^9*b^4 + 77*B*a^8*b^5 - (20*A
 - 263*C)*a^7*b^6 - 110*B*a^6*b^7 + (35*A - 146*C)*a^5*b^8 + 59*B*a^4*b^9 -
 (20*A - 23*C)*a^3*b^10 - 6*B*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^8*b
^9 - 4*a^6*b^11 + 6*a^4*b^13 - 4*a^2*b^15 + b^17)*d*cos(d*x + c)^3 + 3*(a^9
*b^8 - 4*a^7*b^10 + 6*a^5*b^12 - 4*a^3*b^14 + a*b^16)*d*cos(d*x + c)^2 + 3*
(a^10*b^7 - 4*a^8*b^9 + 6*a^6*b^11 - 4*a^4*b^13 + a^2*b^15)*d*cos(d*x + c)
+ (a^11*b^6 - 4*a^9*b^8 + 6*a^7*b^10 - 4*a^5*b^12 + a^3*b^14)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35508, size = 1939, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/6*(6*(20*C*a^9 - 8*B*a^8*b + 2*A*a^7*b^2 - 69*C*a^7*b^2 + 28*B*a^6*b^3 -
7*A*a^5*b^4 + 84*C*a^5*b^4 - 35*B*a^4*b^5 + 8*A*a^3*b^6 - 40*C*a^3*b^6 + 20
*B*a^2*b^7 - 8*A*a*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
)/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*sqrt(a^2 - b^2)) - 2*(36*C*a^1
0*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^9*b*tan(1/2*d*x + 1/2*c)^5 - 81*C*a^9*b*t
an(1/2*d*x + 1/2*c)^5 + 6*A*a^8*b^2*tan(1/2*d*x + 1/2*c)^5 + 42*B*a^8*b^2*t
an(1/2*d*x + 1/2*c)^5 - 48*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^5 - 15*A*a^7*b^3*
tan(1/2*d*x + 1/2*c)^5 + 24*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 + 213*C*a^7*b^
3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 117*B*a^6*b
^4*tan(1/2*d*x + 1/2*c)^5 - 48*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*A*a^5*
b^5*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 - 162*C*a^
5*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 105*B*a
^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 90*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 60*A*
a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 36*A
*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^10*tan(1/2*d*x + 1/2*c)^3 - 36*B*a
^9*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 - 284*C*a
^8*b^2*tan(1/2*d*x + 1/2*c)^3 + 152*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 - 56*A
*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 392*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 - 23
6*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 -
 180*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^
3 - 72*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^10*tan(1/2*d*x + 1/2*c) -
18*B*a^9*b*tan(1/2*d*x + 1/2*c) + 81*C*a^9*b*tan(1/2*d*x + 1/2*c) + 6*A*a^8
*b^2*tan(1/2*d*x + 1/2*c) - 42*B*a^8*b^2*tan(1/2*d*x + 1/2*c) - 48*C*a^8*b^
2*tan(1/2*d*x + 1/2*c) + 15*A*a^7*b^3*tan(1/2*d*x + 1/2*c) + 24*B*a^7*b^3*t
an(1/2*d*x + 1/2*c) - 213*C*a^7*b^3*tan(1/2*d*x + 1/2*c) - 6*A*a^6*b^4*tan(
```

```
1/2*d*x + 1/2*c) + 117*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 48*C*a^6*b^4*tan(1/
2*d*x + 1/2*c) - 45*A*a^5*b^5*tan(1/2*d*x + 1/2*c) + 24*B*a^5*b^5*tan(1/2*d
*x + 1/2*c) + 162*C*a^5*b^5*tan(1/2*d*x + 1/2*c) - 6*A*a^4*b^6*tan(1/2*d*x
+ 1/2*c) - 105*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 90*C*a^4*b^6*tan(1/2*d*x +
1/2*c) + 60*A*a^3*b^7*tan(1/2*d*x + 1/2*c) - 60*B*a^3*b^7*tan(1/2*d*x + 1/2
*c) + 36*A*a^2*b^8*tan(1/2*d*x + 1/2*c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9
- b^11)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) +
3*(20*C*a^2 - 8*B*a*b + 2*A*b^2 + C*b^2)*(d*x + c)/b^6 - 6*(8*C*a*tan(1/2*d
*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 +
 8*C*a*tan(1/2*d*x + 1/2*c) - 2*B*b*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^5))/d
```

3.1002 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=461

$$-\frac{\sin(c+dx)\left(23a^2b^2C+3a^3bB-12a^4C-8ab^3B+5Ab^4-6b^4C\right)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)-7a^5b^3B+8a^3b^5B+28a^6b^2C-3\right.}{b^5d}$$

```
[Out] ((b*B - 4*a*C)*x)/b^5 - ((2*A*b^8 + 2*a^7*b*B - 7*a^5*b^3*B + 8*a^3*b^5*B -
 8*a*b^7*B - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C + a^2*b^6*(3*A + 20*C))*
ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a +
 b)^(7/2)*d) - ((5*A*b^4 + 3*a^3*b*B - 8*a*b^3*B - 12*a^4*C + 23*a^2*b^2*C
- 6*b^4*C)*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C))
*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Cos[c + d*
x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a*(2*A
*b^6 - a^5*b*B + 2*a^3*b^3*B - 6*a*b^5*B + 4*a^6*C - 11*a^4*b^2*C + 3*a^2*b
^4*(A + 4*C))*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 9.76056, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3031, 3023, 2735, 2659, 205}

$$-\frac{\sin(c+dx)\left(23a^2b^2C+3a^3bB-12a^4C-8ab^3B+5Ab^4-6b^4C\right)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)-7a^5b^3B+8a^3b^5B+28a^6b^2C-3\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^4,x]
```


```
[Out] ((b*B - 4*a*C)*x)/b^5 - ((2*A*b^8 + 2*a^7*b*B - 7*a^5*b^3*B + 8*a^3*b^5*B -
 8*a*b^7*B - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C + a^2*b^6*(3*A + 20*C))*
ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a +
 b)^(7/2)*d) - ((5*A*b^4 + 3*a^3*b*B - 8*a*b^3*B - 12*a^4*C + 23*a^2*b^2*C
- 6*b^4*C)*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C))
*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Cos[c + d*
```

```
x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a*(2*A
*b^6 - a^5*b*B + 2*a^3*b^3*B - 6*a*b^5*B + 4*a^6*C - 11*a^4*b^2*C + 3*a^2*b
^4*(A + 4*C))*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(3}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4+a^3b\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4+a^3b\right.}{}$$

$$= -\frac{\left(5Ab^4+3a^3bB-8ab^3B-12a^4C+23a^2b^2C-6b^4C\right)\sin(c}{6b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{(bB-4aC)x}{b^5} - \frac{\left(5Ab^4+3a^3bB-8ab^3B-12a^4C+23a^2b^2C-\right.}{6b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{(bB-4aC)x}{b^5} - \frac{\left(5Ab^4+3a^3bB-8ab^3B-12a^4C+23a^2b^2C-\right.}{6b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{(bB-4aC)x}{b^5} - \frac{\left(3a^2Ab^6+2Ab^8+2a^7bB-7a^5b^3B+8a^3b^5B\right.}{}$$

Mathematica [A] time = 6.83562, size = 532, normalized size = 1.15

$$\frac{-a^3Ab^2\sin(c+dx)+a^4bB\sin(c+dx)+a^5(-C)\sin(c+dx)}{3b^4d\left(b^2-a^2\right)(a+b\cos(c+dx))^3}+\frac{-4a^4Ab^2\sin(c+dx)+9a^2Ab^4\sin(c+dx)-12a^3b^3B\sin}{6b^4d\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^4,x]
```

```
[Out] ((b*B - 4*a*C)*(c + d*x))/(b^5*d) - ((-3*a^2*A*b^6 - 2*A*b^8 - 2*a^7*b*B +
7*a^5*b^3*B - 8*a^3*b^5*B + 8*a*b^7*B + 8*a^8*C - 28*a^6*b^2*C + 35*a^4*b^4
*C - 20*a^2*b^6*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(b
^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d) + (C*Sin[c + d*x])/(b^4*d) + (-(a^3*A*
b^2*Sin[c + d*x]) + a^4*b*B*Sin[c + d*x] - a^5*C*Sin[c + d*x])/(3*b^4*(-a^2
 + b^2)*d*(a + b*Cos[c + d*x])^3) + (-4*a^4*A*b^2*Sin[c + d*x] + 9*a^2*A*b^
4*Sin[c + d*x] + 7*a^5*b*B*Sin[c + d*x] - 12*a^3*b^3*B*Sin[c + d*x] - 10*a^
6*C*Sin[c + d*x] + 15*a^4*b^2*C*Sin[c + d*x])/(6*b^4*(-a^2 + b^2)^2*d*(a +
b*Cos[c + d*x])^2) + (-2*a^5*A*b^2*Sin[c + d*x] + 5*a^3*A*b^4*Sin[c + d*x]
- 18*a*A*b^6*Sin[c + d*x] + 11*a^6*b*B*Sin[c + d*x] - 32*a^4*b^3*B*Sin[c +
d*x] + 36*a^2*b^5*B*Sin[c + d*x] - 26*a^7*C*Sin[c + d*x] + 71*a^5*b^2*C*Sin
[c + d*x] - 60*a^3*b^4*C*Sin[c + d*x])/(6*b^4*(-a^2 + b^2)^3*d*(a + b*Cos[c
 + d*x]))
```

Maple [B] time = 0.054, size = 3571, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A+2/d/b^4*B*arctan(tan(1/2*d*x+1/2*c))+2/d/b^4*C*tan(1/2*d*x+1/2*c
)/(tan(1/2*d*x+1/2*c)^2+1)-8/d/b^5*C*arctan(tan(1/2*d*x+1/2*c))*a-12/d*b/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
```

C+20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5*B+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+2/d*a^3/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+5/d*a^4/b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-18/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c
)^5*C+3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-3/d*a^2*b/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*A+2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-18/d*a^5/b^2/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*
C-5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*C-1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d*a^6/b^3/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)*B+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+6/d
*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a
^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*B-8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^3*B-2/d*b^3/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A+12/d/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
*a^7/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-24/d*b/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3*B+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-
4/d/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^6/(a^2+2*a*
b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+44/3/d/b/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^4/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*B-116/3/d/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3*a^5/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-12/d*b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2

```
*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5
*B+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+40/d/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*C+4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/
(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d/b^4/(a^6-3*a^4*b
^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*a^7*B-3/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/
2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*A+8/d/b^5/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*a^8*C-20/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a
-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2-28/d/
b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C+8/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)
/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a
*B+35/d/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^4*C+7/d/b^2/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*a^5*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.346, size = 6165, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="fricas")
```

[Out] [-1/12*(12*(4*C*a^9*b^3 - B*a^8*b^4 - 16*C*a^7*b^5 + 4*B*a^6*b^6 + 24*C*a^5
*b^7 - 6*B*a^4*b^8 - 16*C*a^3*b^9 + 4*B*a^2*b^10 + 4*C*a*b^11 - B*b^12)*d*x
*cos(d*x + c)^3 + 36*(4*C*a^10*b^2 - B*a^9*b^3 - 16*C*a^8*b^4 + 4*B*a^7*b^5
 + 24*C*a^6*b^6 - 6*B*a^5*b^7 - 16*C*a^4*b^8 + 4*B*a^3*b^9 + 4*C*a^2*b^10 -
 B*a*b^11)*d*x*cos(d*x + c)^2 + 36*(4*C*a^11*b - B*a^10*b^2 - 16*C*a^9*b^3
+ 4*B*a^8*b^4 + 24*C*a^7*b^5 - 6*B*a^6*b^6 - 16*C*a^5*b^7 + 4*B*a^4*b^8 + 4
*C*a^3*b^9 - B*a^2*b^10)*d*x*cos(d*x + c) + 12*(4*C*a^12 - B*a^11*b - 16*C*
a^10*b^2 + 4*B*a^9*b^3 + 24*C*a^8*b^4 - 6*B*a^7*b^5 - 16*C*a^6*b^6 + 4*B*a^
5*b^7 + 4*C*a^4*b^8 - B*a^3*b^9)*d*x + 3*(8*C*a^11 - 2*B*a^10*b - 28*C*a^9*
b^2 + 7*B*a^8*b^3 + 35*C*a^7*b^4 - 8*B*a^6*b^5 - (3*A + 20*C)*a^5*b^6 + 8*B
*a^4*b^7 - 2*A*a^3*b^8 + (8*C*a^8*b^3 - 2*B*a^7*b^4 - 28*C*a^6*b^5 + 7*B*a^
5*b^6 + 35*C*a^4*b^7 - 8*B*a^3*b^8 - (3*A + 20*C)*a^2*b^9 + 8*B*a*b^10 - 2*
A*b^11)*cos(d*x + c)^3 + 3*(8*C*a^9*b^2 - 2*B*a^8*b^3 - 28*C*a^7*b^4 + 7*B*
a^6*b^5 + 35*C*a^5*b^6 - 8*B*a^4*b^7 - (3*A + 20*C)*a^3*b^8 + 8*B*a^2*b^9 -
 2*A*a*b^10)*cos(d*x + c)^2 + 3*(8*C*a^10*b - 2*B*a^9*b^2 - 28*C*a^8*b^3 +
7*B*a^7*b^4 + 35*C*a^6*b^5 - 8*B*a^5*b^6 - (3*A + 20*C)*a^4*b^7 + 8*B*a^3*b
^8 - 2*A*a^2*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
(2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(
d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) -
2*(24*C*a^11*b - 6*B*a^10*b^2 - 92*C*a^9*b^3 + 23*B*a^8*b^4 + (4*A + 133*C)
*a^7*b^5 - 43*B*a^6*b^6 + (7*A - 71*C)*a^5*b^7 + 26*B*a^4*b^8 - (11*A - 6*C
)*a^3*b^9 + 6*(C*a^8*b^4 - 4*C*a^6*b^6 + 6*C*a^4*b^8 - 4*C*a^2*b^10 + C*b^1
2)*cos(d*x + c)^3 + (44*C*a^9*b^3 - 11*B*a^8*b^4 + (2*A - 169*C)*a^7*b^5 +
43*B*a^6*b^6 - (7*A - 239*C)*a^5*b^7 - 68*B*a^4*b^8 + (23*A - 132*C)*a^3*b^
9 + 36*B*a^2*b^10 - 18*(A - C)*a*b^11)*cos(d*x + c)^2 + 3*(20*C*a^10*b^2 -
5*B*a^9*b^3 - 77*C*a^8*b^4 + 20*B*a^7*b^5 + (A + 110*C)*a^6*b^6 - 35*B*a^5*
b^7 + (8*A - 59*C)*a^4*b^8 + 20*B*a^3*b^9 - 3*(3*A - 2*C)*a^2*b^10)*cos(d*x
 + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^1
6)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*
b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12
+ a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11
 + a^3*b^13)*d), -1/6*(6*(4*C*a^9*b^3 - B*a^8*b^4 - 16*C*a^7*b^5 + 4*B*a^6*
b^6 + 24*C*a^5*b^7 - 6*B*a^4*b^8 - 16*C*a^3*b^9 + 4*B*a^2*b^10 + 4*C*a*b^11
 - B*b^12)*d*x*cos(d*x + c)^3 + 18*(4*C*a^10*b^2 - B*a^9*b^3 - 16*C*a^8*b^4
 + 4*B*a^7*b^5 + 24*C*a^6*b^6 - 6*B*a^5*b^7 - 16*C*a^4*b^8 + 4*B*a^3*b^9 +
4*C*a^2*b^10 - B*a*b^11)*d*x*cos(d*x + c)^2 + 18*(4*C*a^11*b - B*a^10*b^2 -
 16*C*a^9*b^3 + 4*B*a^8*b^4 + 24*C*a^7*b^5 - 6*B*a^6*b^6 - 16*C*a^5*b^7 + 4
*B*a^4*b^8 + 4*C*a^3*b^9 - B*a^2*b^10)*d*x*cos(d*x + c) + 6*(4*C*a^12 - B*a
^11*b - 16*C*a^10*b^2 + 4*B*a^9*b^3 + 24*C*a^8*b^4 - 6*B*a^7*b^5 - 16*C*a^6
*b^6 + 4*B*a^5*b^7 + 4*C*a^4*b^8 - B*a^3*b^9)*d*x - 3*(8*C*a^11 - 2*B*a^10*
b - 28*C*a^9*b^2 + 7*B*a^8*b^3 + 35*C*a^7*b^4 - 8*B*a^6*b^5 - (3*A + 20*C)*
a^5*b^6 + 8*B*a^4*b^7 - 2*A*a^3*b^8 + (8*C*a^8*b^3 - 2*B*a^7*b^4 - 28*C*a^6
*b^5 + 7*B*a^5*b^6 + 35*C*a^4*b^7 - 8*B*a^3*b^8 - (3*A + 20*C)*a^2*b^9 + 8*
B*a*b^10 - 2*A*b^11)*cos(d*x + c)^3 + 3*(8*C*a^9*b^2 - 2*B*a^8*b^3 - 28*C*a
^7*b^4 + 7*B*a^6*b^5 + 35*C*a^5*b^6 - 8*B*a^4*b^7 - (3*A + 20*C)*a^3*b^8 +

```
8*B*a^2*b^9 - 2*A*a*b^10)*cos(d*x + c)^2 + 3*(8*C*a^10*b - 2*B*a^9*b^2 - 28
*C*a^8*b^3 + 7*B*a^7*b^4 + 35*C*a^6*b^5 - 8*B*a^5*b^6 - (3*A + 20*C)*a^4*b^
7 + 8*B*a^3*b^8 - 2*A*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos
(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (24*C*a^11*b - 6*B*a^10*b^
2 - 92*C*a^9*b^3 + 23*B*a^8*b^4 + (4*A + 133*C)*a^7*b^5 - 43*B*a^6*b^6 + (7
*A - 71*C)*a^5*b^7 + 26*B*a^4*b^8 - (11*A - 6*C)*a^3*b^9 + 6*(C*a^8*b^4 - 4
*C*a^6*b^6 + 6*C*a^4*b^8 - 4*C*a^2*b^10 + C*b^12)*cos(d*x + c)^3 + (44*C*a^
9*b^3 - 11*B*a^8*b^4 + (2*A - 169*C)*a^7*b^5 + 43*B*a^6*b^6 - (7*A - 239*C)
*a^5*b^7 - 68*B*a^4*b^8 + (23*A - 132*C)*a^3*b^9 + 36*B*a^2*b^10 - 18*(A -
C)*a*b^11)*cos(d*x + c)^2 + 3*(20*C*a^10*b^2 - 5*B*a^9*b^3 - 77*C*a^8*b^4 +
 20*B*a^7*b^5 + (A + 110*C)*a^6*b^6 - 35*B*a^5*b^7 + (8*A - 59*C)*a^4*b^8 +
 20*B*a^3*b^9 - 3*(3*A - 2*C)*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b
^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9
*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(
a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) +
 (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34957, size = 1654, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] -1/3*(3*(8*C*a^8 - 2*B*a^7*b - 28*C*a^6*b^2 + 7*B*a^5*b^3 + 35*C*a^4*b^4 -
8*B*a^3*b^5 - 3*A*a^2*b^6 - 20*C*a^2*b^6 + 8*B*a*b^7 - 2*A*b^8)*(pi*floor(1
```

```
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9
 - b^11)*sqrt(a^2 - b^2)) - (18*C*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^8*b*ta
n(1/2*d*x + 1/2*c)^5 - 42*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^7*b^2*tan
(1/2*d*x + 1/2*c)^5 - 24*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^6*b^3*tan
(1/2*d*x + 1/2*c)^5 + 117*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^5*b^4*ta
n(1/2*d*x + 1/2*c)^5 - 45*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*C*a^5*b^4*t
an(1/2*d*x + 1/2*c)^5 - 3*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^4*b^5*ta
n(1/2*d*x + 1/2*c)^5 - 105*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^3*b^6*t
an(1/2*d*x + 1/2*c)^5 + 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^3*b^6*
tan(1/2*d*x + 1/2*c)^5 - 27*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^2*b^7
*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^9*tan(
1/2*d*x + 1/2*c)^3 - 12*B*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 152*C*a^7*b^2*tan(
1/2*d*x + 1/2*c)^3 + 56*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*A*a^5*b^4*tan(
1/2*d*x + 1/2*c)^3 + 236*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*B*a^4*b^5*t
an(1/2*d*x + 1/2*c)^3 + 32*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 120*C*a^3*b^6
*tan(1/2*d*x + 1/2*c)^3 + 72*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^8*
tan(1/2*d*x + 1/2*c)^3 + 18*C*a^9*tan(1/2*d*x + 1/2*c) - 6*B*a^8*b*tan(1/2*
d*x + 1/2*c) + 42*C*a^8*b*tan(1/2*d*x + 1/2*c) - 15*B*a^7*b^2*tan(1/2*d*x +
 1/2*c) - 24*C*a^7*b^2*tan(1/2*d*x + 1/2*c) + 6*B*a^6*b^3*tan(1/2*d*x + 1/2
*c) - 117*C*a^6*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^5*b^4*tan(1/2*d*x + 1/2*c)
 + 45*B*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*C*a^5*b^4*tan(1/2*d*x + 1/2*c) +
3*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*C
*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 60*B*a^3
*b^6*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b^6*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^
7*tan(1/2*d*x + 1/2*c) - 36*B*a^2*b^7*tan(1/2*d*x + 1/2*c) + 18*A*a*b^8*tan
(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x
 + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(4*C*a - B*b)*(d*x +
 c)/b^5 - 6*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^4))/d
```

3.1003 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=349

$$-\frac{\left(-a^3b^4(A-8C)+3a^2b^5B-7a^5b^2C+2a^7C-4ab^6(A+2C)+2b^7B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sin(c+dx)\cos^2(c+}{3bd\left(a^2-b^2\right)(a}$$

```
[Out] (C*x)/b^4 - ((3*a^2*b^5*B + 2*b^7*B - a^3*b^4*(A - 8*C) + 2*a^7*C - 7*a^5*b
^2*C - 4*a*b^6*(A + 2*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d
*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (a*(2*A*b^
4 - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 8*C))*Sin[c + d*x])/(6*b^3*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((4*A*b^6 + a^3*b^3*B - 16*a*b^5*B + 9*a
^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A + 28*C))*Sin[c + d*x])/(6*b^3*
(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.39355, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3047, 3031, 3021, 2735, 2659, 205}

$$-\frac{\left(-a^3b^4(A-8C)+3a^2b^5B-7a^5b^2C+2a^7C-4ab^6(A+2C)+2b^7B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sin(c+dx)\cos^2(c+}{3bd\left(a^2-b^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^4,x]
```

```
[Out] (C*x)/b^4 - ((3*a^2*b^5*B + 2*b^7*B - a^3*b^4*(A - 8*C) + 2*a^7*C - 7*a^5*b
^2*C - 4*a*b^6*(A + 2*C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d
*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (a*(2*A*b^
4 - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 8*C))*Sin[c + d*x])/(6*b^3*(a^2 -
b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((4*A*b^6 + a^3*b^3*B - 16*a*b^5*B + 9*a
^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A + 28*C))*Sin[c + d*x])/(6*b^3*
(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2Ab^4-\right.}{6}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2Ab^4-\right.}{6}$$

$$= \frac{Cx}{b^4} - \frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2A\right.}{}$$

$$= \frac{Cx}{b^4} - \frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a\left(2A\right.}{}$$

$$= \frac{Cx}{b^4} + \frac{\left(a^3Ab^4+4aAb^6-3a^2b^5B-2b^7B-2a^7C+7a^5b^2C-\right.}{(a-b)^{7/2}b^4(a+}$$

Mathematica [B] time = 4.5894, size = 863, normalized size = 2.47

$$\frac{24cCa^9+24Cdxa^9-24bC\sin(c+dx)a^8-36b^2cCa^7-36b^2Cdxa^7-30b^2C\sin(2(c+dx))a^7+6b^3cC\cos(3(c+dx))a^6+6b^3Cdx\cos(3(c+dx))a^6+57b^3C\sin(c+dx)a^6-11b^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^4,x]
```

```
[Out] ((-24*(3*a^2*b^5*B + 2*b^7*B - a^3*b^4*(A - 8*C) + 2*a^7*C - 7*a^5*b^2*C -
4*a*b^6*(A + 2*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(7/2) + (24*a^9*c*C - 36*a^7*b^2*c*C - 36*a^5*b^4*c*C + 84*a^3*b
^6*c*C - 36*a*b^8*c*C + 24*a^9*C*d*x - 36*a^7*b^2*C*d*x - 36*a^5*b^4*C*d*x
+ 84*a^3*b^6*C*d*x - 36*a*b^8*C*d*x + 18*b*(a^2 - b^2)^3*(4*a^2 + b^2)*C*(c
 + d*x)*Cos[c + d*x] + 36*a*b^2*(a^2 - b^2)^3*C*(c + d*x)*Cos[2*(c + d*x)]
+ 6*a^6*b^3*c*C*Cos[3*(c + d*x)] - 18*a^4*b^5*c*C*Cos[3*(c + d*x)] + 18*a^2
*b^7*c*C*Cos[3*(c + d*x)] - 6*b^9*c*C*Cos[3*(c + d*x)] + 6*a^6*b^3*C*d*x*Co
s[3*(c + d*x)] - 18*a^4*b^5*C*d*x*Cos[3*(c + d*x)] + 18*a^2*b^7*C*d*x*Cos[3
*(c + d*x)] - 6*b^9*C*d*x*Cos[3*(c + d*x)] - 51*a^4*A*b^5*Sin[c + d*x] - 18
*a^2*A*b^7*Sin[c + d*x] - 6*A*b^9*Sin[c + d*x] + 18*a^5*b^4*B*Sin[c + d*x]
+ 39*a^3*b^6*B*Sin[c + d*x] + 18*a*b^8*B*Sin[c + d*x] - 24*a^8*b*C*Sin[c +
d*x] + 57*a^6*b^3*C*Sin[c + d*x] - 72*a^4*b^5*C*Sin[c + d*x] - 36*a^2*b^7*C
*Sin[c + d*x] + 6*a^5*A*b^4*Sin[2*(c + d*x)] - 54*a^3*A*b^6*Sin[2*(c + d*x)
] - 12*a*A*b^8*Sin[2*(c + d*x)] + 6*a^4*b^5*B*Sin[2*(c + d*x)] + 54*a^2*b^7
*B*Sin[2*(c + d*x)] - 30*a^7*b^2*C*Sin[2*(c + d*x)] + 90*a^5*b^4*C*Sin[2*(c
 + d*x)] - 120*a^3*b^6*C*Sin[2*(c + d*x)] + a^4*A*b^5*Sin[3*(c + d*x)] - 10
*a^2*A*b^7*Sin[3*(c + d*x)] - 6*A*b^9*Sin[3*(c + d*x)] + 2*a^5*b^4*B*Sin[3*
(c + d*x)] - 5*a^3*b^6*B*Sin[3*(c + d*x)] + 18*a*b^8*B*Sin[3*(c + d*x)] - 1
1*a^6*b^3*C*Sin[3*(c + d*x)] + 32*a^4*b^5*C*Sin[3*(c + d*x)] - 36*a^2*b^7*C
*Sin[3*(c + d*x)])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(24*b^4*d)
```

Maple [B] time = 0.048, size = 3098, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-8/d*a^3/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*C+2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b^3/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*B+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/
```

(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*B+6/d*b^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*a*B+3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+4/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-
2/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5*A+8/d*b^2*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/d*b^3/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a*B-1/d*a^3/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5*A+4/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-
b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^7/b^
4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*C+7/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1
/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*C-28/3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/
(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-24/d*b/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*C*a^2-12/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2-12/d*b/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*C-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*a^2*b/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)*A-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-1/d*a^5/b^2
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-4/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*C-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+6/d*a^4/b/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*C+44/3/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-3/d*b/(a*ta

```
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-3/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*
a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+4/3/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*B+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.8336, size = 4518, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] [1/12*(12*(C*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4*C*a^2*b^9 + C*b^11)*d*
x*cos(d*x + c)^3 + 36*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^6 - 4*C*a^3*b^8
+ C*a*b^10)*d*x*cos(d*x + c)^2 + 36*(C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 -
 4*C*a^4*b^7 + C*a^2*b^9)*d*x*cos(d*x + c) + 12*(C*a^11 - 4*C*a^9*b^2 + 6*C
*a^7*b^4 - 4*C*a^5*b^6 + C*a^3*b^8)*d*x + 3*(2*C*a^10 - 7*C*a^8*b^2 - (A -
8*C)*a^6*b^4 + 3*B*a^5*b^5 - 4*(A + 2*C)*a^4*b^6 + 2*B*a^3*b^7 + (2*C*a^7*b
^3 - 7*C*a^5*b^5 - (A - 8*C)*a^3*b^7 + 3*B*a^2*b^8 - 4*(A + 2*C)*a*b^9 + 2*
B*b^10)*cos(d*x + c)^3 + 3*(2*C*a^8*b^2 - 7*C*a^6*b^4 - (A - 8*C)*a^4*b^6 +
 3*B*a^3*b^7 - 4*(A + 2*C)*a^2*b^8 + 2*B*a*b^9)*cos(d*x + c)^2 + 3*(2*C*a^9
*b - 7*C*a^7*b^3 - (A - 8*C)*a^5*b^5 + 3*B*a^4*b^6 - 4*(A + 2*C)*a^3*b^7 +
2*B*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^
```

```
2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
 c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*
C*a^10*b - 23*C*a^8*b^3 - 4*B*a^7*b^4 + (13*A + 43*C)*a^6*b^5 - 7*B*a^5*b^6
 - (11*A + 26*C)*a^4*b^7 + 11*B*a^3*b^8 - 2*A*a^2*b^9 + (11*C*a^8*b^3 - 2*B
*a^7*b^4 - (A + 43*C)*a^6*b^5 + 7*B*a^5*b^6 + (11*A + 68*C)*a^4*b^7 - 23*B*
a^3*b^8 - 4*(A + 9*C)*a^2*b^9 + 18*B*a*b^10 - 6*A*b^11)*cos(d*x + c)^2 + 3*
(5*C*a^9*b^2 - (A + 20*C)*a^7*b^4 - B*a^6*b^5 + 5*(2*A + 7*C)*a^5*b^6 - 8*B
*a^4*b^7 - (7*A + 20*C)*a^3*b^8 + 9*B*a^2*b^9 - 2*A*a*b^10)*cos(d*x + c))*s
in(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(
d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*c
os(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13
)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^1
2)*d), 1/6*(6*(C*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4*C*a^2*b^9 + C*b^11
)*d*x*cos(d*x + c)^3 + 18*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^6 - 4*C*a^3*
b^8 + C*a*b^10)*d*x*cos(d*x + c)^2 + 18*(C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b
^5 - 4*C*a^4*b^7 + C*a^2*b^9)*d*x*cos(d*x + c) + 6*(C*a^11 - 4*C*a^9*b^2 +
6*C*a^7*b^4 - 4*C*a^5*b^6 + C*a^3*b^8)*d*x - 3*(2*C*a^10 - 7*C*a^8*b^2 - (A
 - 8*C)*a^6*b^4 + 3*B*a^5*b^5 - 4*(A + 2*C)*a^4*b^6 + 2*B*a^3*b^7 + (2*C*a^
7*b^3 - 7*C*a^5*b^5 - (A - 8*C)*a^3*b^7 + 3*B*a^2*b^8 - 4*(A + 2*C)*a*b^9 +
 2*B*b^10)*cos(d*x + c)^3 + 3*(2*C*a^8*b^2 - 7*C*a^6*b^4 - (A - 8*C)*a^4*b^
6 + 3*B*a^3*b^7 - 4*(A + 2*C)*a^2*b^8 + 2*B*a*b^9)*cos(d*x + c)^2 + 3*(2*C*
a^9*b - 7*C*a^7*b^3 - (A - 8*C)*a^5*b^5 + 3*B*a^4*b^6 - 4*(A + 2*C)*a^3*b^7
 + 2*B*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^10*b - 23*C*a^8*b^3 - 4*B*a^7*b^4
+ (13*A + 43*C)*a^6*b^5 - 7*B*a^5*b^6 - (11*A + 26*C)*a^4*b^7 + 11*B*a^3*b^
8 - 2*A*a^2*b^9 + (11*C*a^8*b^3 - 2*B*a^7*b^4 - (A + 43*C)*a^6*b^5 + 7*B*a^
5*b^6 + (11*A + 68*C)*a^4*b^7 - 23*B*a^3*b^8 - 4*(A + 9*C)*a^2*b^9 + 18*B*a
*b^10 - 6*A*b^11)*cos(d*x + c)^2 + 3*(5*C*a^9*b^2 - (A + 20*C)*a^7*b^4 - B*
a^6*b^5 + 5*(2*A + 7*C)*a^5*b^6 - 8*B*a^4*b^7 - (7*A + 20*C)*a^3*b^8 + 9*B*
a^2*b^9 - 2*A*a*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6
*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 +
6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^
7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b
^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33147, size = 1490, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/3*(3*(2*C*a^7 - 7*C*a^5*b^2 - A*a^3*b^4 + 8*C*a^3*b^4 + 3*B*a^2*b^5 - 4*A
*a*b^6 - 8*C*a*b^6 + 2*B*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
 b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) + 3*(d*x
 + c)*C/b^4 - (6*C*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^7*b*tan(1/2*d*x + 1/
2*c)^5 - 6*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^5*b^3*tan(1/2*d*x + 1/2
*c)^5 - 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^3*tan(1/2*d*x + 1/2
*c)^5 + 12*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b^4*tan(1/2*d*x + 1/2
*c)^5 - 6*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^3*b^5*tan(1/2*d*x + 1/2
*c)^5 - 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^3*b^5*tan(1/2*d*x + 1/2
*c)^5 + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^6*tan(1/2*d*x + 1/
2*c)^5 + 36*C*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b^7*tan(1/2*d*x + 1/2*
c)^5 - 18*B*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 +
 12*C*a^8*tan(1/2*d*x + 1/2*c)^3 - 56*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*
B*a^5*b^3*tan(1/2*d*x + 1/2*c)^3 + 28*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 11
6*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 -
16*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 72*C*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 +
 36*B*a*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*
a^8*tan(1/2*d*x + 1/2*c) + 15*C*a^7*b*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^2*ta
n(1/2*d*x + 1/2*c) - 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^5*b^3*tan(1/2
*d*x + 1/2*c) - 45*C*a^5*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a^4*b^4*tan(1/2*d*
x + 1/2*c) - 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^4*tan(1/2*d*x + 1
/2*c) + 27*A*a^3*b^5*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c
) + 60*C*a^3*b^5*tan(1/2*d*x + 1/2*c) + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c) -
 27*B*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b^6*tan(1/2*d*x + 1/2*c) + 6*
A*a*b^7*tan(1/2*d*x + 1/2*c) - 18*B*a*b^7*tan(1/2*d*x + 1/2*c) + 6*A*b^8*ta
n(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x
 + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.1004 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=314

$$\frac{\left(-a^2b(4A+3C)+a^3B+4ab^2B-b^3(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\sin(c+dx)\left(a^3b^2(2A-5C)-10a^2b^3B+a^4b\right.}{6b^2d\left(a^2-b^2\right)^3(a+b}$$

```
[Out] ((a^3*B + 4*a*b^2*B - b^3*(A + 2*C) - a^2*b*(4*A + 3*C))*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) + (a*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^3) + ((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Sin[
c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((a^4*b*B - 10*a
^2*b^3*B - 6*b^5*B + a^3*b^2*(2*A - 5*C) + 2*a^5*C + a*b^4*(13*A + 18*C))*S
in[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.918785, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3031, 3021, 2754, 12, 2659, 205}

$$\frac{\left(-a^2b(4A+3C)+a^3B+4ab^2B-b^3(A+2C)\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\sin(c+dx)\left(a^3b^2(2A-5C)-10a^2b^3B+a^4b\right.}{6b^2d\left(a^2-b^2\right)^3(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d
*x])^4,x]
```


```
[Out] ((a^3*B + 4*a*b^2*B - b^3*(A + 2*C) - a^2*b*(4*A + 3*C))*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) + (a*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^3) + ((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Sin[
c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((a^4*b*B - 10*a
^2*b^3*B - 6*b^5*B + a^3*b^2*(2*A - 5*C) + 2*a^5*C + a*b^4*(13*A + 18*C))*S
in[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^4}\,dx = \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{-3b\left(Ab^2-a(bB-aC)\right)+\left(a^2b\right.}{}}{}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3B\right.}{6b^2\left(a^2-\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3B\right.}{6b^2\left(a^2-\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3B\right.}{6b^2\left(a^2-\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3B\right.}{6b^2\left(a^2-\right.}$$

$$= -\frac{\left(4a^2Ab+Ab^3-a^3B-4ab^2B+3a^2bC+2b^3C\right)\tan^{-1}\left(\frac{\sqrt{a-}}{}\right.}{(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 1.70488, size = 307, normalized size = 0.98

$$\frac{\frac{2\sin(c+dx)\left(6\cos(c+dx)\left(9a^2b^3(A+C)+a^4b(2A+C)-9a^3b^2B+a^5B-2ab^4B-Ab^5\right)+\cos(2(c+dx))\left(a^3b^2(2A-5C)-10a^2b^3B+a^4bB+2a^5C+ab^4(13A+18C)-6b^5B\right)+22\right.}{(a+b\cos(c+dx))^3}}{24d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^4,x]
```

```
[Out] ((-24*(a^3*B + 4*a*b^2*B - b^3*(A + 2*C) - a^2*b*(4*A + 3*C))*ArcTanh[((a -
 b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(12*a^5*A +
22*a^3*A*b^2 + 11*a*A*b^4 - 25*a^4*b*B - 14*a^2*b^3*B - 6*b^5*B + 10*a^5*C
```

```
+ 17*a^3*b^2*C + 18*a*b^4*C + 6*(-(A*b^5) + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B
 + 9*a^2*b^3*(A + C) + a^4*b*(2*A + C))*Cos[c + d*x] + (a^4*b*B - 10*a^2*b^
3*B - 6*b^5*B + a^3*b^2*(2*A - 5*C) + 2*a^5*C + a*b^4*(13*A + 18*C))*Cos[2*
(c + d*x)])*Sin[c + d*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [B] time = 0.042, size = 2667, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -2/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*b^3+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5*C*a*b^2+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a*b^2+6/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*C*a*b^2+2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a*B-6/
d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a-b)/
(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-1/d*b^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*A+1/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*b^2/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*a*B+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^3/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2+3
/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+2/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*A-2/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
```

```
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+1/d/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*a^3*B-1/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-4/d/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*b^3*B-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B-2/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*b^3*B-28/3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B
+28/3/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*b/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)*B-1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+1/d*a^3/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*B+4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+4/d/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)
*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*A-3/
d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+4/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)
/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a
*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38875, size = 3094, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] [-1/12*(3*(B*a^6 - (4*A + 3*C)*a^5*b + 4*B*a^4*b^2 - (A + 2*C)*a^3*b^3 + (B
*a^3*b^3 - (4*A + 3*C)*a^2*b^4 + 4*B*a*b^5 - (A + 2*C)*b^6)*cos(d*x + c)^3
+ 3*(B*a^4*b^2 - (4*A + 3*C)*a^3*b^3 + 4*B*a^2*b^4 - (A + 2*C)*a*b^5)*cos(d
*x + c)^2 + 3*(B*a^5*b - (4*A + 3*C)*a^4*b^2 + 4*B*a^3*b^3 - (A + 2*C)*a^2*
b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)
*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^
2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*(3*A + 2
*C)*a^7 - 13*B*a^6*b + (4*A + 7*C)*a^5*b^2 + 11*B*a^4*b^3 - 11*(A + C)*a^3*
b^4 + 2*B*a^2*b^5 + A*a*b^6 + (2*C*a^7 + B*a^6*b + (2*A - 7*C)*a^5*b^2 - 11
*B*a^4*b^3 + (11*A + 23*C)*a^3*b^4 + 4*B*a^2*b^5 - (13*A + 18*C)*a*b^6 + 6*
B*b^7)*cos(d*x + c)^2 + 3*(B*a^7 + (2*A + C)*a^6*b - 10*B*a^5*b^2 + (7*A +
8*C)*a^4*b^3 + 7*B*a^3*b^4 - (10*A + 9*C)*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(
d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^1
1)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^
10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*
b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*
d), 1/6*(3*(B*a^6 - (4*A + 3*C)*a^5*b + 4*B*a^4*b^2 - (A + 2*C)*a^3*b^3 + (
B*a^3*b^3 - (4*A + 3*C)*a^2*b^4 + 4*B*a*b^5 - (A + 2*C)*b^6)*cos(d*x + c)^3
 + 3*(B*a^4*b^2 - (4*A + 3*C)*a^3*b^3 + 4*B*a^2*b^4 - (A + 2*C)*a*b^5)*cos(
d*x + c)^2 + 3*(B*a^5*b - (4*A + 3*C)*a^4*b^2 + 4*B*a^3*b^3 - (A + 2*C)*a^2
*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) + (2*(3*A + 2*C)*a^7 - 13*B*a^6*b + (4*A + 7*C)*a^5*b
^2 + 11*B*a^4*b^3 - 11*(A + C)*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6 + (2*C*a^7 +
 B*a^6*b + (2*A - 7*C)*a^5*b^2 - 11*B*a^4*b^3 + (11*A + 23*C)*a^3*b^4 + 4*B
*a^2*b^5 - (13*A + 18*C)*a*b^6 + 6*B*b^7)*cos(d*x + c)^2 + 3*(B*a^7 + (2*A
+ C)*a^6*b - 10*B*a^5*b^2 + (7*A + 8*C)*a^4*b^3 + 7*B*a^3*b^4 - (10*A + 9*C
)*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7
*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2
+ 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.27686, size = 1304, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] -1/3*(3*(B*a^3 - 4*A*a^2*b - 3*C*a^2*b + 4*B*a*b^2 - A*b^3 - 2*C*b^3)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2
*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*sqrt(a^2 - b^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(
1/2*d*x + 1/2*c)^5 + 6*C*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b*tan(1/2*d*x
 + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^4*b*tan(1/2*d*x + 1
/2*c)^5 + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x +
1/2*c)^5 + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^2*b^3*tan(1/2*d*x +
1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^2*b^3*tan(1/2*d*x +
 1/2*c)^5 + 12*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2
*c)^5 + 18*C*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5*tan(1/2*d*x + 1/2*c)^5
- 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^5*tan(1/2*d*x + 1/2*c)^3 + 4*C*a^
5*tan(1/2*d*x + 1/2*c)^3 - 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^2
*tan(1/2*d*x + 1/2*c)^3 + 32*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^2*b^
3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 36*C*a*b^4*t
an(1/2*d*x + 1/2*c)^3 + 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d
*x + 1/2*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*C*a^5*tan(1/2*d*x + 1/2*c) +
 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B*a^4*b*tan(1/2*d*x + 1/2*c) + 3*C*a^4
*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*b^2*
tan(1/2*d*x + 1/2*c) + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(
1/2*d*x + 1/2*c) - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*C*a^2*b^3*tan(1/2
*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2*d*x + 1
/2*c) + 18*C*a*b^4*tan(1/2*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*
B*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.1005 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\left(a^3(-(2A+C))+4a^2bB-ab^2(3A+4C)+b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\sin(c+dx)\left(-a^2b^2(11A+10C)+2a^3bB+\right.}{6bd\left(a^2-b^2\right)^3(a+b\,\mathrm{c}}$$

```
[Out] -(((4*a^2*b*B + b^3*B - a^3*(2*A + C) - a*b^2*(3*A + 4*C))*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) - ((A*
b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
3) + ((2*a^2*b*B + 3*b^3*B + a^3*C - a*b^2*(5*A + 6*C))*Sin[c + d*x])/(6*b*
(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((2*a^3*b*B + 13*a*b^3*B + a^4*C
- 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Sin[c + d*x])/(6*b*(a^2 - b^2)
^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.775274, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3021, 2754, 12, 2659, 205}

$$-\frac{\left(a^3(-(2A+C))+4a^2bB-ab^2(3A+4C)+b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\sin(c+dx)\left(-a^2b^2(11A+10C)+2a^3bB+\right.}{6bd\left(a^2-b^2\right)^3(a+b\,\mathrm{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```



```
[Out] -(((4*a^2*b*B + b^3*B - a^3*(2*A + C) - a*b^2*(3*A + 4*C))*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) - ((A*
b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
3) + ((2*a^2*b*B + 3*b^3*B + a^3*C - a*b^2*(5*A + 6*C))*Sin[c + d*x])/(6*b*
(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((2*a^3*b*B + 13*a*b^3*B + a^4*C
- 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Sin[c + d*x])/(6*b*(a^2 - b^2)
^3*d*(a + b*Cos[c + d*x]))
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b(bB-a(A+C))+\left(2Ab^2-2abB-a^2C+3b^2C\right)\cos}{(a+b\cos(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-ab^2(5A+6C)\right.}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-ab^2(5A+6C)\right.}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-ab^2(5A+6C)\right.}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-ab^2(5A+6C)\right.}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B+a^3C+4ab^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \cdots$$

Mathematica [A] time = 1.54055, size = 301, normalized size = 1.01

$$\frac{\frac{2\sin(c+dx)\left(6\cos(c+dx)\left(-9a^3b^2(A+C)+9a^2b^3B+2a^4bB+a^5C-ab^4(A+2C)-b^5B\right)+b\cos(2(c+dx))\left(-a^2b^2(11A+10C)+2a^3bB+a^4C+13ab^3B-2b^4(2A+3C)\right)-a^2Ab^3\cdots}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(-4*a^2*b*B - b^3*B + a^3*(2*A + C) + a*b^2*(3*A + 4*C))*ArcTanh[((a
- b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-36*a^4*A*
b - a^2*A*b^3 - 8*A*b^5 + 12*a^5*B + 22*a^3*b^2*B + 11*a*b^4*B - 25*a^4*b*C
 - 14*a^2*b^3*C - 6*b^5*C + 6*(2*a^4*b*B + 9*a^2*b^3*B - b^5*B + a^5*C - 9*
a^3*b^2*(A + C) - a*b^4*(A + 2*C))*Cos[c + d*x] + b*(2*a^3*b*B + 13*a*b^3*B
 + a^4*C - 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Cos[2*(c + d*x)])*Sin
[c + d*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [B] time = 0.039, size = 2667, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1
/2*c)^3*C-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3+3/d*b^2/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*A-3/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^3/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*A+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5*C*a*b^2-1/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)
*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a*b^2+28/3/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*a*B+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a*B+2/d*b/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(
1/2*d*x+1/2*c)^5*B+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-1/d/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^
3)*tan(1/2*d*x+1/2*c)^5*C-2/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*b^3/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a
*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+4/d*b^2*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/
3/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^
2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*a*B+3/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-12/d*a^2*b/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*A-28/3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
```

```
b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2-6/d*b/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*
a^2-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*a^2*b/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)*A+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B-1/d/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)*b^3*B-2/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d*a^2*b/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+4/d*
a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(
a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15511, size = 3094, normalized size = 10.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [-1/12*(3*((2*A + C)*a^6 - 4*B*a^5*b + (3*A + 4*C)*a^4*b^2 - B*a^3*b^3 + ((
2*A + C)*a^3*b^3 - 4*B*a^2*b^4 + (3*A + 4*C)*a*b^5 - B*b^6)*cos(d*x + c)^3
+ 3*((2*A + C)*a^4*b^2 - 4*B*a^3*b^3 + (3*A + 4*C)*a^2*b^4 - B*a*b^5)*cos(d
*x + c)^2 + 3*((2*A + C)*a^5*b - 4*B*a^4*b^2 + (3*A + 4*C)*a^3*b^3 - B*a^2*
b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)
*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^
2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*B*a^7 -
(18*A + 13*C)*a^6*b + 4*B*a^5*b^2 + (23*A + 11*C)*a^4*b^3 - 11*B*a^3*b^4 -
(7*A - 2*C)*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (C*a^6*b + 2*B*a^5*b^2 - 11*(A +
C)*a^4*b^3 + 11*B*a^3*b^4 + (7*A + 4*C)*a^2*b^5 - 13*B*a*b^6 + 2*(2*A + 3*C
)*b^7)*cos(d*x + c)^2 + 3*(C*a^7 + 2*B*a^6*b - (9*A + 10*C)*a^5*b^2 + 7*B*a
^4*b^3 + (8*A + 7*C)*a^3*b^4 - 10*B*a^2*b^5 + (A + 2*C)*a*b^6 + B*b^7)*cos(
d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^1
1)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^
10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*
b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*
d), 1/6*(3*((2*A + C)*a^6 - 4*B*a^5*b + (3*A + 4*C)*a^4*b^2 - B*a^3*b^3 + (
(2*A + C)*a^3*b^3 - 4*B*a^2*b^4 + (3*A + 4*C)*a*b^5 - B*b^6)*cos(d*x + c)^3
 + 3*((2*A + C)*a^4*b^2 - 4*B*a^3*b^3 + (3*A + 4*C)*a^2*b^4 - B*a*b^5)*cos(
d*x + c)^2 + 3*((2*A + C)*a^5*b - 4*B*a^4*b^2 + (3*A + 4*C)*a^3*b^3 - B*a^2
*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) + (6*B*a^7 - (18*A + 13*C)*a^6*b + 4*B*a^5*b^2 + (23*
A + 11*C)*a^4*b^3 - 11*B*a^3*b^4 - (7*A - 2*C)*a^2*b^5 + B*a*b^6 + 2*A*b^7
+ (C*a^6*b + 2*B*a^5*b^2 - 11*(A + C)*a^4*b^3 + 11*B*a^3*b^4 + (7*A + 4*C)*
a^2*b^5 - 13*B*a*b^6 + 2*(2*A + 3*C)*b^7)*cos(d*x + c)^2 + 3*(C*a^7 + 2*B*a
^6*b - (9*A + 10*C)*a^5*b^2 + 7*B*a^4*b^3 + (8*A + 7*C)*a^3*b^4 - 10*B*a^2*
b^5 + (A + 2*C)*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7
*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2
+ 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34372, size = 1304, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/3*(3*(2*A*a^3 + C*a^3 - 4*B*a^2*b + 3*A*a*b^2 + 4*C*a*b^2 - B*b^3)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2
*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*sqrt(a^2 - b^2)) - (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^5*tan(
1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b*tan(1/2*
d*x + 1/2*c)^5 - 12*C*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d
*x + 1/2*c)^5 + 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*C*a^3*b^2*tan(1/2*
d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^2*b^3*tan(1/2*
d*x + 1/2*c)^5 - 12*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*
x + 1/2*c)^5 + 12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*b^4*tan(1/2*d*x +
1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^5*tan(1/2*d*x + 1/2*c)^5
- 6*C*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 36*A*a
^4*b*tan(1/2*d*x + 1/2*c)^3 - 28*C*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^3*
b^2*tan(1/2*d*x + 1/2*c)^3 + 32*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*C*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^5*t
an(1/2*d*x + 1/2*c)^3 + 12*C*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d
*x + 1/2*c) + 3*C*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/2*c
) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 12*C*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A
*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*C*a^
3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*B*a^2*b^
3*tan(1/2*d*x + 1/2*c) - 12*C*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(
1/2*d*x + 1/2*c) + 12*B*a*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a*b^4*tan(1/2*d*x
+ 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x + 1/2*c) - 6*
C*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.1006 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(-a^4b^3(8A-C)+7a^2Ab^5+4a^6b(2A+C)-3a^5b^2B-2a^7B-2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sin(c+dx)\left(-13a^4b^2\right.}{}$$

```
[Out] -(((7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B - a^4*b^3*(8*A - C) + 4*a
^6*b*(2*A + C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a
 - b)^(7/2)*(a + b)^(7/2)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((A*b^2
 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3)
- ((3*A*b^4 + 5*a^3*b*B - 2*a^4*C - a^2*b^2*(8*A + 3*C))*Sin[c + d*x])/(6*a
^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((17*a^2*A*b^4 - 6*A*b^6 + 11*
a^5*b*B + 4*a^3*b^3*B - 2*a^6*C - 13*a^4*b^2*(2*A + C))*Sin[c + d*x])/(6*a^
3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.55045, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-a^4b^3(8A-C)+7a^2Ab^5+4a^6b(2A+C)-3a^5b^2B-2a^7B-2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sin(c+dx)\left(-13a^4b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d
*x])^4,x]
```


```
[Out] -(((7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B - a^4*b^3*(8*A - C) + 4*a
^6*b*(2*A + C))*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a
 - b)^(7/2)*(a + b)^(7/2)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((A*b^2
 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3)
- ((3*A*b^4 + 5*a^3*b*B - 2*a^4*C - a^2*b^2*(8*A + 3*C))*Sin[c + d*x])/(6*a
^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((17*a^2*A*b^4 - 6*A*b^6 + 11*
a^5*b*B + 4*a^3*b^3*B - 2*a^6*C - 13*a^4*b^2*(2*A + C))*Sin[c + d*x])/(6*a^
3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3A\left(a^2-b^2\right)-3a(Ab-aB+bC)}{}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C\right.}{6a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C\right.}{6a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C\right.}{6a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}$$

$$= -\frac{\left(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B+4a^6\right.}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [C] time = 6.7203, size = 587, normalized size = 1.7

$$\frac{\cos(c+dx)(A\sec(c+dx)+B+C\cos(c+dx))\left(-\frac{6i(\cos(c)-i\sin(c))\left(a^4b^3(8A-C)-7a^2Ab^5-4a^6b(2A+C)+3a^5b^2B+2a^7B+2Ab^7\right)\tan^{-1}\left(\frac{(\sin(c)}{}\right.}{a^4\left(a^2-b^2\right)^3\sqrt{-\left(a^2-b^2\right)(\cos(c)-i\sin(c))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-6*A*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/a^4 + (6*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]])/a^4 - ((6*I)*(-7*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B + a^4*b^3*
(8*A - C) - 4*a^6*b*(2*A + C))*ArcTan[((I*Cos[c] + Sin[c])*(b*Sin[c] + (-a
+ b*Cos[c])*Tan[(d*x)/2]))/Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^2)]]*(Cos
```

```
[c] - I*Sin[c]))/(a^4*(a^2 - b^2)^3*Sqrt[-((a^2 - b^2)*(Cos[c] - I*Sin[c])^
2)]) + (2*(A*b^2 + a*(-(b*B) + a*C))*Sec[c]*(-(a*Sin[c]) + b*Sin[d*x]))/(a*
b*(a^2 - b^2)*(a + b*Cos[c + d*x])^3) + ((-17*a^2*A*b^4 + 6*A*b^6 - 11*a^5*
b*B - 4*a^3*b^3*B + 2*a^6*C + 13*a^4*b^2*(2*A + C))*Sec[c]*Sin[d*x] + 3*a*(
-(A*b^5) + 2*a^5*B + 3*a^3*b^2*B + a^2*b^3*(2*A - C) - 2*a^4*b*(3*A + 2*C))
*Tan[c])/((a^3 - a*b^2)^3*(a + b*Cos[c + d*x])) + ((-3*A*b^4 - 5*a^3*b*B +
2*a^4*C + a^2*b^2*(8*A + 3*C))*Sec[c]*Sin[d*x] + a*(A*b^3 + 3*a^3*B + 2*a*b
^2*B - a^2*b*(6*A + 5*C))*Tan[c])/(a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2
)))/(3*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.098, size = 3121, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*C*b^3-1/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*b^3+12/d*b^2/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*A+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d/a^4*A*ln(tan(
1/2*d*x+1/2*c)+1)-1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)+6/d/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5*C*a*b^2+28/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a*b^2+6/d/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a*b^2-7/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
(a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b
^5+3/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a*B-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*B+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+2/d/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5*C-4/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+4/d*b^3/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+
```

```
b^3)*tan(1/2*d*x+1/2*c)^5*A-3/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a*B-2/d
*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2+2/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^
5*C*a^2+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^6-6/d/a/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*A*b^4+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5+2/d/a^3/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6-6/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b
^4-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5+2/d/(a^6-3*a^4*b^2+3*a^2*b
^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*a^3*B+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-44/3/d/a/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*A*b^4+4/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^6+2/d/a^4
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^7-4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^
3*B-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B-2/d/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*b^3*B-12/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2
/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+24/d*b^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*A-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+4/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2
-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*A-
4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+3/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*a*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**4,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.3989, size = 1521, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b - 4*C*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - C*a^4
*b^3 - 7*A*a^2*b^5 + 2*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2
*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b
^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) + 3*A*log(
abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3*A*log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a^4 + (6*C*a^8*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^7*b*tan(1/2*d*x + 1/2*c)^5
- 6*C*a^7*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 +
27*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 -
 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 -
 27*C*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 +
 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 +
 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 +
 3*C*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 -
15*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 + 12*C*a
^8*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^7*b*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^6*b^
2*tan(1/2*d*x + 1/2*c)^3 + 16*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*B*a^5*b
^3*tan(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 28*C*a^4
*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 + 56*A*a^2
*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^8*tan
(1/2*d*x + 1/2*c) - 18*B*a^7*b*tan(1/2*d*x + 1/2*c) + 6*C*a^7*b*tan(1/2*d*x
 + 1/2*c) + 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^6*b^2*tan(1/2*d*x +
1/2*c) + 12*C*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^5*b^3*tan(1/2*d*x + 1/2
*c) - 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 27*C*a^5*b^3*tan(1/2*d*x + 1/2*c)
- 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 12*
C*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c) - 6*B*a^
3*b^5*tan(1/2*d*x + 1/2*c) - 3*C*a^3*b^5*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^6
*tan(1/2*d*x + 1/2*c) + 15*A*a*b^7*tan(1/2*d*x + 1/2*c) + 6*A*b^8*tan(1/2*d
*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.1007 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=480

$$-\frac{\left(-a^6b^2(20A+3C)+35a^4Ab^4-28a^2Ab^6-8a^5b^3B+7a^3b^5B+8a^7bB-2a^8C-2ab^7B+8Ab^8\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}$$

```
[Out] -(((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 + 8*a^7*b*B - 8*a^5*b^3*B + 7*a^3
*b^5*B - 2*a*b^7*B - 2*a^8*C - a^6*b^2*(20*A + 3*C))*ArcTan[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d)) - ((4*A*
b - a*B)*ArcTanh[Sin[c + d*x]])/(a^5*d) + ((68*a^2*A*b^4 - 24*A*b^6 + 26*a^
5*b*B - 17*a^3*b^3*B + 6*a*b^5*B + a^6*(6*A - 11*C) - a^4*b^2*(65*A + 4*C))
*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((4*A*b^4 + 6*a^3*b*B - a
*b^3*B - 3*a^4*C - a^2*b^2*(9*A + 2*C))*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*
d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*b^4 - 4*A*b^6 + 6*a^5*b*B - 2*a^3*b^
3*B + a*b^5*B - 2*a^6*C - 3*a^4*b^2*(4*A + C))*Tan[c + d*x])/(2*a^3*(a^2 -
b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 10.5378, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-a^6b^2(20A+3C)+35a^4Ab^4-28a^2Ab^6-8a^5b^3B+7a^3b^5B+8a^7bB-2a^8C-2ab^7B+8Ab^8\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c +
 d*x])^4,x]
```


```
[Out] -(((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 + 8*a^7*b*B - 8*a^5*b^3*B + 7*a^3
*b^5*B - 2*a*b^7*B - 2*a^8*C - a^6*b^2*(20*A + 3*C))*ArcTan[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d)) - ((4*A*
b - a*B)*ArcTanh[Sin[c + d*x]])/(a^5*d) + ((68*a^2*A*b^4 - 24*A*b^6 + 26*a^
5*b*B - 17*a^3*b^3*B + 6*a*b^5*B + a^6*(6*A - 11*C) - a^4*b^2*(65*A + 4*C))
*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c + d
```

```
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((4*A*b^4 + 6*a^3*b*B - a
*b^3*B - 3*a^4*C - a^2*b^2*(9*A + 2*C))*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*
d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*b^4 - 4*A*b^6 + 6*a^5*b*B - 2*a^3*b^
3*B + a*b^5*B - 2*a^6*C - 3*a^4*b^2*(4*A + C))*Tan[c + d*x])/(2*a^3*(a^2 -
b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \left(-4Ab^2+abB+a^2(3A-C)-3a(\ldots\right.}{\ldots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(4Ab^4+6a^3bB-ab^3B-\ldots\right.}{6a^2\left(a^2-b^2\ldots\right.}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(4Ab^4+6a^3bB-ab^3B-\ldots\right.}{6a^2\left(a^2-b^2\ldots\right.}$$

$$= \frac{\left(68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B+a^6(6A-1\ldots\right.}{6a^4\left(a^2-b^2\right)^3d}$$

$$= \frac{\left(68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B+a^6(6A-1\ldots\right.}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(68a^2Ab^4-24Ab^6+26\ldots\right.}{\ldots}$$

$$= \frac{\left(20a^6Ab^2-35a^4Ab^4+28a^2Ab^6-8Ab^8-8a^7bB+8a^5b^3B-\ldots\right.}{a^5(a-b)^{7/2}\ldots}$$

Mathematica [A] time = 4.45901, size = 709, normalized size = 1.48

$$\frac{\cos(c+dx)\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(\frac{2a\sin(c+dx)\left(6ab^2\cos(2(c+dx))\left(-a^4b^2(53A+C)+57a^2Ab^4+a^6(6A-9C)-15a^3b^3B+20a^5bB+5a\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*C
os[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((48*(-35*a^4*A*b^4 +
 28*a^2*A*b^6 - 8*A*b^8 - 8*a^7*b*B + 8*a^5*b^3*B - 7*a^3*b^5*B + 2*a*b^7*B
 + 2*a^8*C + a^6*b^2*(20*A + 3*C))*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[
-a^2 + b^2]]*Cos[c + d*x])/(-a^2 + b^2)^(7/2) + 48*(4*A*b - a*B)*Cos[c + d*
x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*(-4*A*b + a*B)*Cos[c + d*x
]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(24*a^9*A - 36*a^7*A*b^2
- 246*a^5*A*b^4 + 318*a^3*A*b^6 - 120*a*A*b^8 + 120*a^6*b^3*B - 90*a^4*b^5*
B + 30*a^2*b^7*B - 54*a^7*b^2*C - 6*a^5*b^4*C - b*(-28*a^2*A*b^6 + 72*A*b^8
 - 144*a^7*b*B + 50*a^5*b^3*B + 7*a^3*b^5*B - 18*a*b^7*B - 5*a^4*b^4*(61*A
- 4*C) - 72*a^8*(A - C) + a^6*b^2*(438*A + 13*C))*Cos[c + d*x] + 6*a*b^2*(5
7*a^2*A*b^4 - 20*A*b^6 + 20*a^5*b*B - 15*a^3*b^3*B + 5*a*b^5*B + a^6*(6*A -
 9*C) - a^4*b^2*(53*A + C))*Cos[2*(c + d*x)] + 6*a^6*A*b^3*Cos[3*(c + d*x)]
 - 65*a^4*A*b^5*Cos[3*(c + d*x)] + 68*a^2*A*b^7*Cos[3*(c + d*x)] - 24*A*b^9
*Cos[3*(c + d*x)] + 26*a^5*b^4*B*Cos[3*(c + d*x)] - 17*a^3*b^6*B*Cos[3*(c +
 d*x)] + 6*a*b^8*B*Cos[3*(c + d*x)] - 11*a^6*b^3*C*Cos[3*(c + d*x)] - 4*a^4
*b^5*C*Cos[3*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^
3)))/(24*a^5*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.115, size = 3628, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*C*b^3-4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3
/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*C*b^3+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1
/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/d/a^4*ln(tan(1
/2*d*x+1/2*c)-1)*B+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B+4/d/a^3/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-1/d/a^4*A/(tan(1/
2*d*x+1/2*c)+1)-4/d*A*b/a^5*ln(tan(1/2*d*x+1/2*c)+1)+4/d*A*b/a^5*ln(tan(1/2
*d*x+1/2*c)-1)-3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a*b^2+8/d*b^3/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))*B+3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
```

b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a*b^2-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-44/3/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+24/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*B+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a*B-20/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-20/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+3/d*b^2*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-40/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a*B+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-12/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2-35/d/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^4+28/d/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6-8/d/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^8+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^6-6/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+116/3/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-12/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^4+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5-2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6-5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^4+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5+4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(

```
1/2*d*x+1/2*c)^5*b^3*B-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^3*B+2/d/a^4/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*b^7*B-7/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*
(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^5*B-8/d
*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*B+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a+
b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d/a/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36395, size = 1694, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] -1/3*(3*(2*C*a^8 - 8*B*a^7*b + 20*A*a^6*b^2 + 3*C*a^6*b^2 + 8*B*a^5*b^3 - 3
5*A*a^4*b^4 - 7*B*a^3*b^5 + 28*A*a^2*b^6 + 2*B*a*b^7 - 8*A*b^8)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 -
a^5*b^6)*sqrt(a^2 - b^2)) + (18*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^7*b
^2*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^6*
b^3*tan(1/2*d*x + 1/2*c)^5 + 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^6*
b^3*tan(1/2*d*x + 1/2*c)^5 - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5
*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^4
*b^5*tan(1/2*d*x + 1/2*c)^5 - 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^4
*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^
3*b^6*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 15*B*a
^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b
^8*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^9*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^8*b*ta
n(1/2*d*x + 1/2*c)^3 - 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^6*b^3*
tan(1/2*d*x + 1/2*c)^3 - 32*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 + 116*B*a^5*b^
4*tan(1/2*d*x + 1/2*c)^3 - 236*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^4*b
^5*tan(1/2*d*x + 1/2*c)^3 - 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 152*A*a^2
*b^7*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*A*b^9*
tan(1/2*d*x + 1/2*c)^3 + 18*C*a^8*b*tan(1/2*d*x + 1/2*c) - 36*B*a^7*b^2*tan
(1/2*d*x + 1/2*c) + 27*C*a^7*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^6*b^3*tan(1/
2*d*x + 1/2*c) - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) + 6*C*a^6*b^3*tan(1/2*d*
x + 1/2*c) + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) + 6*B*a^5*b^4*tan(1/2*d*x +
 1/2*c) + 3*C*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*A*a^4*b^5*tan(1/2*d*x + 1/2
*c) + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*C*a^4*b^5*tan(1/2*d*x + 1/2*c)
- 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c) - 2
```

```
4*A*a^2*b^7*tan(1/2*d*x + 1/2*c) - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*A
*a*b^8*tan(1/2*d*x + 1/2*c) - 6*B*a*b^8*tan(1/2*d*x + 1/2*c) + 18*A*b^9*tan
(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x
 + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) - 3*(B*a - 4*A*b)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/a^5 + 3*(B*a - 4*A*b)*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a^5 + 6*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4
))/d
```

3.1008 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=657

$$\frac{b\left(-8a^6b^2(5A-C)+7a^4b^4(12A-C)-a^2b^6(69A-2C)-35a^5b^3B+28a^3b^5B+20a^7bB-8a^8C-8ab^7B+20Ab^8\right)\tan^{-1}}{a^6d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)^3}$$

```
[Out] (b*(20*A*b^8 + 20*a^7*b*B - 35*a^5*b^3*B + 28*a^3*b^5*B - 8*a*b^7*B - a^2*b
^6*(69*A - 2*C) - 8*a^6*b^2*(5*A - C) + 7*a^4*b^4*(12*A - C) - 8*a^8*C)*Arc
Tan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*Sqrt[a - b]*Sqrt[a +
b]*(a^2 - b^2)^3*d) + ((20*A*b^2 - 8*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c +
 d*x]])/(2*a^6*d) + ((60*A*b^7 + 6*a^7*B - 65*a^5*b^2*B + 68*a^3*b^4*B - 24
*a*b^6*B + a^4*b^3*(146*A - 17*C) - a^2*b^5*(167*A - 6*C) - a^6*(24*A*b - 2
6*b*C))*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) - ((10*A*b^6 - 12*a^5*b*B + 1
1*a^3*b^3*B - 4*a*b^5*B - a^6*(A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(2
7*A - C))*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 - a*
(b*B - a*C))*Sec[c + d*x]*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^3) - ((5*A*b^4 + 7*a^3*b*B - 2*a*b^3*B - 4*a^4*C - a^2*b^2*(10*A + C))
*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2)
+ ((20*A*b^6 - 27*a^5*b*B + 20*a^3*b^3*B - 8*a*b^5*B - a^2*b^4*(53*A - 2*C)
 + 12*a^6*C + a^4*b^2*(48*A + C))*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 -
b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 12.8586, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-8a^6b^2(5A-C)+7a^4b^4(12A-C)-a^2b^6(69A-2C)-35a^5b^3B+28a^3b^5B+20a^7bB-8a^8C-8ab^7B+20Ab^8\right)\tan^{-1}}{a^6d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c +
 d*x])^4,x]
```

```
[Out] (b*(20*A*b^8 + 20*a^7*b*B - 35*a^5*b^3*B + 28*a^3*b^5*B - 8*a*b^7*B - a^2*b
^6*(69*A - 2*C) - 8*a^6*b^2*(5*A - C) + 7*a^4*b^4*(12*A - C) - 8*a^8*C)*Arc
```

```
Tan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*Sqrt[a - b]*Sqrt[a +
b]*(a^2 - b^2)^3*d) + ((20*A*b^2 - 8*a*b*B + a^2*(A + 2*C))*ArcTanh[Sin[c +
 d*x]])/(2*a^6*d) + ((60*A*b^7 + 6*a^7*B - 65*a^5*b^2*B + 68*a^3*b^4*B - 24
*a*b^6*B + a^4*b^3*(146*A - 17*C) - a^2*b^5*(167*A - 6*C) - a^6*(24*A*b - 2
6*b*C))*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) - ((10*A*b^6 - 12*a^5*b*B + 1
1*a^3*b^3*B - 4*a*b^5*B - a^6*(A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(2
7*A - C))*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 - a*
(b*B - a*C))*Sec[c + d*x]*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^3) - ((5*A*b^4 + 7*a^3*b*B - 2*a*b^3*B - 4*a^4*C - a^2*b^2*(10*A + C))
*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2)
+ ((20*A*b^6 - 27*a^5*b*B + 20*a^3*b^3*B - 8*a*b^5*B - a^2*b^4*(53*A - 2*C)
 + 12*a^6*C + a^4*b^2*(48*A + C))*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 -
b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(-5Ab^2+2abB+\right.}{}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5Ab^4+7a^3bB\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5Ab^4+7a^3bB\right.}{}$$

$$= -\frac{\left(10Ab^6-12a^5bB+11a^3b^3B-4ab^5B-a^6(A-6C)+a^4b^2(2\right.}{2a^4\left(a^2-b^2\right.}$$

$$= \frac{\left(60Ab^7+6a^7B-65a^5b^2B+68a^3b^4B-24ab^6B+a^4b^3(146A\right.}{6a^5\left(a^2-\right.}$$

$$= \frac{\left(60Ab^7+6a^7B-65a^5b^2B+68a^3b^4B-24ab^6B+a^4b^3(146A\right.}{6a^5\left(a^2-\right.}$$

$$= \frac{\left(20Ab^2-8abB+a^2(A+2C)\right)\tanh^{-1}(\sin(c+dx))}{2a^6d} + \frac{\left(60Ab\right.}{}$$

$$= \frac{b\left(20Ab^8+20a^7bB-35a^5b^3B+28a^3b^5B-8ab^7B-a^2b^6(69A\right.}{a^6\sqrt{a-}}$$

Mathematica [A] time = 6.51213, size = 686, normalized size = 1.04

$$\frac{a^2b^2C\sin(c+dx)-ab^3B\sin(c+dx)+Ab^4\sin(c+dx)}{3a^3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{14a^2Ab^4\sin(c+dx)-11a^3b^3B\sin(c+dx)-3a^2b^4C\sin(c+}{6a^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*C
os[c + d*x])^4,x]
```

```
[Out] (b*(40*a^6*A*b^2 - 84*a^4*A*b^4 + 69*a^2*A*b^6 - 20*A*b^8 - 20*a^7*b*B + 35
*a^5*b^3*B - 28*a^3*b^5*B + 8*a*b^7*B + 8*a^8*C - 8*a^6*b^2*C + 7*a^4*b^4*C
 - 2*a^2*b^6*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^6*
(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d) + ((-(a^2*A) - 20*A*b^2 + 8*a*b*B - 2*a^2
*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^6*d) + ((a^2*A + 20*A*b^
2 - 8*a*b*B + 2*a^2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^6*d)
+ (Sec[c + d*x]*(-4*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/(a^5*d) + (A*b^4*
Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x])/(3*a^3*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^3) + (14*a^2*A*b^4*Sin[c + d*x] - 9*A*b^6*Sin[
c + d*x] - 11*a^3*b^3*B*Sin[c + d*x] + 6*a*b^5*B*Sin[c + d*x] + 8*a^4*b^2*C
*Sin[c + d*x] - 3*a^2*b^4*C*Sin[c + d*x])/(6*a^4*(a^2 - b^2)^2*d*(a + b*Cos
[c + d*x])^2) + (74*a^4*A*b^4*Sin[c + d*x] - 95*a^2*A*b^6*Sin[c + d*x] + 36
*A*b^8*Sin[c + d*x] - 47*a^5*b^3*B*Sin[c + d*x] + 50*a^3*b^5*B*Sin[c + d*x]
 - 18*a*b^7*B*Sin[c + d*x] + 26*a^6*b^2*C*Sin[c + d*x] - 17*a^4*b^4*C*Sin[c
 + d*x] + 6*a^2*b^6*C*Sin[c + d*x]))/(6*a^5*(a^2 - b^2)^3*d*(a + b*Cos[c + d
*x])) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d)
```

Maple [B] time = 0.135, size = 4436, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -12/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+12/d*b^8/a^5/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan
(1/2*d*x+1/2*c)^5*A+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-44/3/d*b^4/
```

a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+4/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+116/3/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+12/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3+8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*b^3-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^2+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a*b^2+24/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a*b^2+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a*b^2+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+84/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^5+1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+24/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-34/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^6+3/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-3/d/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+30/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^4-6/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5-34/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*

```
x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^
6+30/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^4+6/d/a^2/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*A*b^5-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d/a^5/(tan(1/2*d
*x+1/2*c)-1)*A*b-10/d/a^6*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+4/d/a^5*ln(tan(1/2
*d*x+1/2*c)-1)*b*B+4/d/a^5/(tan(1/2*d*x+1/2*c)+1)*A*b+10/d/a^6*ln(tan(1/2*d
*x+1/2*c)+1)*A*b^2-4/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*b*B+60/d/a/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*A*b^4-212/3/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^6
-69/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^7-40/d/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*b^3*B-20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B-20/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*b^3*B-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+20/d*b^2/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*a*B-6/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/
d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*C+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d/a^3/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3
)*tan(1/2*d*x+1/2*c)*B-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b
))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+5/d/a/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+20
/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b
^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*B+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47239, size = 2001, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x
, algorithm="giac")
```

[Out] 1/6*(6*(8*C*a^8*b - 20*B*a^7*b^2 + 40*A*a^6*b^3 - 8*C*a^6*b^3 + 35*B*a^5*b^4 - 84*A*a^4*b^5 + 7*C*a^4*b^5 - 28*B*a^3*b^6 + 69*A*a^2*b^7 - 2*C*a^2*b^7 + 8*B*a*b^8 - 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(a^2 - b^2)) + 2*(36*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 81*A*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^10*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 + 180*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 - 392*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 + 56*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 + 284*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^10*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^8*b^2*tan(1/2*d*x + 1/2*c) - 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) + 60*C*a^7*b^3*tan(1/2*d*x + 1/2*c) + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c) - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^4*tan(1/2*d*x + 1/2*c) + 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c) + 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) - 45*C*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c) + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^6*tan(1/2*d*x + 1/2*c) - 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 15*C*a^3*b^7*tan(1/2*d*x + 1/2*c) - 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c) - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) + 6*C*a^2*b^8*tan(1/2*d*x + 1/2*c) + 81*A*a*b^9*tan(1/2*d*x + 1/2*c) - 18*B*a*b^9*tan(1/2*d*x + 1/2*c) + 36*A*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(A*a^2 + 2*C*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^6 - 3*(A*a^2 + 2*C*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^6 + 6*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^5))/d

3.1009 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=23

$$x(bB-aC)+\frac{bC\sin(c+dx)}{d}$$

```
[Out] (b*B - a*C)*x + (b*C*Sin[c + d*x])/d
```

Rubi [A] time = 0.0231874, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {24, 2637}

$$x(bB-aC)+\frac{bC\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x]),x]
```

```
[Out] (b*B - a*C)*x + (b*C*Sin[c + d*x])/d
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\int \left(b^2(bB-aC) + b^3C\cos(c+dx)\right)dx}{b^2}$$
$$= (bB-aC)x + (bC)\int \cos(c+dx)\,dx$$
$$= (bB-aC)x + \frac{bC\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0107633, size = 34, normalized size = 1.48

$$-aCx + bBx + \frac{bC\sin(c)\cos(dx)}{d} + \frac{bC\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x]),x]
```

```
[Out] b*B*x - a*C*x + (b*C*Cos[d*x]*Sin[c])/d + (b*C*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.024, size = 32, normalized size = 1.4

$$\frac{Cb\sin(dx+c) + bB(dx+c) - aC(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(C*b*sin(d*x+c)+b*B*(d*x+c)-a*C*(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56482, size = 55, normalized size = 2.39

$$-\frac{(Ca-Bb)dx-Cb\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
),x, algorithm="fricas")
```

```
[Out] -((C*a - B*b)*d*x - C*b*sin(d*x + c))/d
```

Sympy [A] time = 0.90777, size = 58, normalized size = 2.52

$$\begin{cases} Bbx-Cax+\frac{Cb\sin(c+dx)}{d} & \text{for } d\neq 0 \\ \frac{x\left(Bab+Bb^2\cos(c)-Ca^2+Cb^2\cos^2(c)\right)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c)),x)
```

```
[Out] Piecewise(((B*b*x - C*a*x + C*b*sin(c + d*x)/d, Ne(d, 0)), (x*(B*a*b + B*b**
2*cos(c) - C*a**2 + C*b**2*cos(c)**2)/(a + b*cos(c)), True))
```

Giac [B] time = 1.24834, size = 65, normalized size = 2.83

$$-\frac{(Ca-Bb)(dx+c)-\frac{2\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
),x, algorithm="giac")
```

```
[Out] -((C*a - B*b)*(d*x + c) - 2*C*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

3.1010 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{2(bB-2aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}+Cx$$

```
[Out] C*x + (2*(b*B - 2*a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(Sqrt[a - b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.115718, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {24, 2735, 2659, 205}

$$\frac{2(bB-2aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}+Cx$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^2,x]
```

```
[Out] C*x + (2*(b*B - 2*a*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b^2}$$
$$=Cx+(bB-2aC)\int \frac{1}{a+b\cos(c+dx)}\,dx$$
$$=Cx+\frac{(2(bB-2aC))\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$=Cx+\frac{2(bB-2aC)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.11916, size = 68, normalized size = 1.11

$$\frac{2(2aC-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}+\frac{C(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^2,x]
```

```
[Out] (C*(c + d*x))/d + (2*(-(b*B) + 2*a*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sq
rt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]*d)
```

Maple [B] time = 0.036, size = 108, normalized size = 1.8

$$2\,\frac{C\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{d}+2\,\frac{bB}{d\sqrt{\left(a+b\right)\left(a-b\right)}}\arctan\left(\frac{\left(a-b\right)\tan\left(1/2\,dx+c/2\right)}{\sqrt{\left(a+b\right)\left(a-b\right)}}\right)-4\,\frac{aC}{d\sqrt{\left(a+b\right)\left(a-b\right)}}\arctan\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*C*arctan(tan(1/2*d*x+1/2*c))+2/d/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b*B-4/d/((a+b)*(a-b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57174, size = 524, normalized size = 8.59

$$\left[\frac{2\left(Ca^2-Cb^2\right)dx+\left(2\,Ca-Bb\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2-b^2\right)d},\frac{\left(Ca^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(C*a^2 - C*b^2)*d*x + (2*C*a - B*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos
(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x +
c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
```

```
) + a^2)))/((a^2 - b^2)*d), ((C*a^2 - C*b^2)*d*x - (2*C*a - B*b)*sqrt(a^2 -
 b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2 -
 b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19159, size = 128, normalized size = 2.1

$$\frac{(dx+c)C-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(2Ca-Bb)}{\sqrt{a^2-b^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*C - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan(
(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(2*C*a
- B*b)/sqrt(a^2 - b^2))/d
```

3.1011 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=110

$$\frac{2\left(a^2(-C)+abB-b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b(bB-2aC)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (2*(a*b*B - a^2*C - b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(b*B - 2*a*C)*Sin[c + d*x])/((a^2
- b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.181053, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {24, 2754, 12, 2659, 205}

$$\frac{2\left(a^2(-C)+abB-b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b(bB-2aC)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^3,x]
```

```
[Out] (2*(a*b*B - a^2*C - b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(b*B - 2*a*C)*Sin[c + d*x])/((a^2
- b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{b^2}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{b^2\left(abB-a^2C-b^2C\right)}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(abB-a^2C-b^2C\right)\int \frac{1}{a+b\cos}}{a^2-b^2}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(2\left(abB-a^2C-b^2C\right)\right)\text{Sub}}{}$$

$$= \frac{2\left(abB-a^2C-b^2C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b(bB-2a}{\left(a^2-b^2\right)d(a}$$

Mathematica [A] time = 0.325613, size = 107, normalized size = 0.97

$$\frac{\frac{b(2aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-\frac{2\left(a^2C-abB+b^2C\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-(a*b*B) + a^2*C + b^2*C)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^
2 + b^2]])/(-a^2 + b^2)^(3/2) + (b*(-(b*B) + 2*a*C)*Sin[c + d*x])/((a - b)*
(a + b)*(a + b*Cos[c + d*x])))/d
```

Maple [B] time = 0.038, size = 299, normalized size = 2.7

$$-2\,\frac{b^2\tan(1/2\,dx+c/2)\,B}{d\left(a^2-b^2\right)\left(a\left(\tan(1/2\,dx+c/2)\right)^2-\left(\tan(1/2\,dx+c/2)\right)^2b+a+b\right)}+4\,\frac{b\tan(1/2\,dx+c/2)\,aC}{d\left(a^2-b^2\right)\left(a\left(\tan(1/2\,dx+c/2)\right)^2-(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)*B+4/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)*a*C+2/d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a*b*B-2/d/(a+b)/(a-b)/((a+b)
*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*C-2/
d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67824, size = 933, normalized size = 8.48

$$\left[\frac{\left(Ca^3 - Ba^2b + Cab^2 + \left(Ca^2b - Bab^2 + Cb^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\left(\left(a^4b - 2\,a^2b^3 + b^5\right)d\cos\left(dx+c\right) + \left(a^5 - 2\,a^3b^2 + a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^3,x, algorithm="fricas")
```

```
[Out] [1/2*((C*a^3 - B*a^2*b + C*a*b^2 + (C*a^2*b - B*a*b^2 + C*b^3)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 +
 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(2*C*a^3*b - B*a^2*b^2 - 2*C
*a*b^3 + B*b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (
a^5 - 2*a^3*b^2 + a*b^4)*d), -((C*a^3 - B*a^2*b + C*a*b^2 + (C*a^2*b - B*a*
b^2 + C*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sq
rt(a^2 - b^2)*sin(d*x + c))) - (2*C*a^3*b - B*a^2*b^2 - 2*C*a*b^3 + B*b^4)*
sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2
+ a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24567, size = 230, normalized size = 2.09

$$-\frac{2\left(\frac{\left(Ca^2-Bab+Cb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{2Cab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^3,x, algorithm="giac")
```

```
[Out] -2*((C*a^2 - B*a*b + C*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b
) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2
)))/(a^2 - b^2)^(3/2) - (2*C*a*b*tan(1/2*d*x + 1/2*c) - B*b^2*tan(1/2*d*x +
 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^
2 - b^2)))/d
```

3.1012 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=175

$$\frac{\left(2a^2bB-2a^3C-4ab^2C+b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{b\left(-4a^2C+3abB-2b^2C\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b(bB-2aC)\sin}{2d\left(a^2-b^2\right)(a+b\,c}$$

```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C - 4*a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (b*(b*B - 2*a*C)*Sin[
c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (b*(3*a*b*B - 4*a^2*C
- 2*b^2*C)*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.405926, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {24, 2754, 12, 2659, 205}

$$\frac{\left(2a^2bB-2a^3C-4ab^2C+b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{b\left(-4a^2C+3abB-2b^2C\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b(bB-2aC)\sin}{2d\left(a^2-b^2\right)(a+b\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^4,x]
```


```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C - 4*a*b^2*C)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (b*(b*B - 2*a*C)*Sin[
c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (b*(3*a*b*B - 4*a^2*C
- 2*b^2*C)*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{b^2}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2b^2(abB-a^2C-b^2C)+b^3(bB-}{(a+b\cos(c+dx)}}{2b^2(a^2-b^}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{b(3abB-4a^2C-2b^2C)}{2(a^2-b^2)^2\,d(a+b\cos}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{b(3abB-4a^2C-2b^2C)}{2(a^2-b^2)^2\,d(a+b\cos}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{b(3abB-4a^2C-2b^2C)}{2(a^2-b^2)^2\,d(a+b\cos}$$

$$= \frac{(2a^2bB + b^3B - 2a^3C - 4ab^2C)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{}{2(}$$

Mathematica [A] time = 0.71633, size = 171, normalized size = 0.98

$$\frac{\frac{b(4a^2C-3abB+2b^2C)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{2(-2a^2bB+2a^3C+4ab^2C-b^3B)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{5/2}} + \frac{b(2aC-bB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^4,x]
```

```
[Out] ((2*(-2*a^2*b*B - b^3*B + 2*a^3*C + 4*a*b^2*C)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (b*(-(b*B) + 2*a*C)*Sin[c +
d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (b*(-3*a*b*B + 4*a^2*C + 2
*b^2*C)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Maple [B] time = 0.041, size = 964, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*a^2*C+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*C+2/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*C-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*
b^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*B+1/d/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c
)*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/(a^2-
2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*C-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a*C+2/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a^2-2*a*b+b^2
)*tan(1/2*d*x+1/2*c)*C+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*b*B+1/d/(a^4-2*a^2*b^2+b^4
)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
b^3*B-2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*a^3*C-4/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/
2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94576, size = 1735, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^4,x, algorithm="fricas")
```

```
[Out] [1/4*((2*C*a^5 - 2*B*a^4*b + 4*C*a^3*b^2 - B*a^2*b^3 + (2*C*a^3*b^2 - 2*B*a
^2*b^3 + 4*C*a*b^4 - B*b^5)*cos(d*x + c)^2 + 2*(2*C*a^4*b - 2*B*a^3*b^2 + 4
*C*a^2*b^3 - B*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c
) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*
sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)
) + 2*(6*C*a^5*b - 4*B*a^4*b^2 - 6*C*a^3*b^3 + 5*B*a^2*b^4 - B*b^6 + (4*C*a
^4*b^2 - 3*B*a^3*b^3 - 2*C*a^2*b^4 + 3*B*a*b^5 - 2*C*b^6)*cos(d*x + c))*sin
(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a
^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*d), -1/2*((2*C*a^5 - 2*B*a^4*b + 4*C*a^3*b^2 - B*a^2*b^
3 + (2*C*a^3*b^2 - 2*B*a^2*b^3 + 4*C*a*b^4 - B*b^5)*cos(d*x + c)^2 + 2*(2*C
*a^4*b - 2*B*a^3*b^2 + 4*C*a^2*b^3 - B*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*C*a^5*b
- 4*B*a^4*b^2 - 6*C*a^3*b^3 + 5*B*a^2*b^4 - B*b^6 + (4*C*a^4*b^2 - 3*B*a^3*
b^3 - 2*C*a^2*b^4 + 3*B*a*b^5 - 2*C*b^6)*cos(d*x + c))*sin(d*x + c))/((a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3
+ 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27123, size = 514, normalized size = 2.94

$$-\frac{\frac{\left(2\,Ca^3-2\,Ba^2b+4\,Cab^2-Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{6\,Ca^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ca^2b}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^4,x, algorithm="giac")
```

```
[Out] -((2*C*a^3 - 2*B*a^2*b + 4*C*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) - (6*C*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 4*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 + 3*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + B*b^4*tan(1/2*d*
x + 1/2*c)^3 - 2*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^3*b*tan(1/2*d*x + 1/2
*c) - 4*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 4*C*a^2*b^2*tan(1/2*d*x + 1/2*c) -
 3*B*a*b^3*tan(1/2*d*x + 1/2*c) + B*b^4*tan(1/2*d*x + 1/2*c) + 2*C*b^4*tan(
1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.1013 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^5}\,dx$

Optimal. Leaf size=249

$$\frac{\left(-7a^2b^2C+2a^3bB-2a^4C+3ab^3B-b^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2bB-13a^3C-17ab^2C+4b^3B\right)\sin(c+dx}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}$$

```
[Out] ((2*a^3*b*B + 3*a*b^3*B - 2*a^4*C - 7*a^2*b^2*C - b^4*C)*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b*(b*B
 - 2*a*C)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (b*(5*a*
b*B - 7*a^2*C - 3*b^2*C)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*
x])^2) - (b*(11*a^2*b*B + 4*b^3*B - 13*a^3*C - 17*a*b^2*C)*Sin[c + d*x])/(6
*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.746219, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {24, 2754, 12, 2659, 205}

$$\frac{\left(-7a^2b^2C+2a^3bB-2a^4C+3ab^3B-b^4C\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2bB-13a^3C-17ab^2C+4b^3B\right)\sin(c+dx}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^5,x]
```


```
[Out] ((2*a^3*b*B + 3*a*b^3*B - 2*a^4*C - 7*a^2*b^2*C - b^4*C)*ArcTan[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b*(b*B
 - 2*a*C)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (b*(5*a*
b*B - 7*a^2*C - 3*b^2*C)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*
x])^2) - (b*(11*a^2*b*B + 4*b^3*B - 13*a^3*C - 17*a*b^2*C)*Sin[c + d*x])/(6
*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
```

```
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^5}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx}{b^2}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{\int \frac{-3b^2(abB-a^2C-b^2C)+2b^3(bB}{(a+b\cos(c+dx)}}{3b^2(a^2-b}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{b(5abB-7a^2C-3b^2C)}{6(a^2-b^2)^2\,d(a+b\cos}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{b(5abB-7a^2C-3b^2C)}{6(a^2-b^2)^2\,d(a+b\cos}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{b(5abB-7a^2C-3b^2C)}{6(a^2-b^2)^2\,d(a+b\cos}$$

$$= -\frac{b(bB-2aC)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{b(5abB-7a^2C-3b^2C)}{6(a^2-b^2)^2\,d(a+b\cos}$$

$$= \frac{(2a^3bB + 3ab^3B - 2a^4C - 7a^2b^2C - b^4C)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 1.00508, size = 246, normalized size = 0.99

$$\frac{\frac{24(7a^2b^2C-2a^3bB+2a^4C-3ab^3B+b^4C)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - \frac{2b\sin(c+dx)\left(6b\left(-10a^2b^2C+9a^3bB-11a^4C+ab^3B+b^4C\right)\cos(c+dx)+b^2\left(11a^2bB-13a^3C-\right.\right.}{(a+b\cos(c+}}{24d(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^5,x]
```

```
[Out] ((24*(-2*a^3*b*B - 3*a*b^3*B + 2*a^4*C + 7*a^2*b^2*C + b^4*C)*ArcTanh[((a -
 b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - (2*b*(36*a^4*b*
B + a^2*b^3*B + 8*b^5*B - 48*a^5*C - 23*a^3*b^2*C - 19*a*b^4*C + 6*b*(9*a^3
*b*B + a*b^3*B - 11*a^4*C - 10*a^2*b^2*C + b^4*C)*Cos[c + d*x] + b^2*(11*a^
```

```
2*b*B + 4*b^3*B - 13*a^3*C - 17*a*b^2*C)*Cos[2*(c + d*x)])*Sin[c + d*x])/(a
 + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Maple [B] time = 0.049, size = 1817, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^5,x)
```

```
[Out] -6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^2/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a^2*B-3/d/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*a*B-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b
^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+8/d/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*a^3*C+5/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3*b^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a^2*C+8/
d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*C-12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^2/(a^
2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^2*B-4/3/d/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B+16/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^3*C+32/3/
d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a-6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3*b^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*a^2*B+3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a*B-2/d/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)*B+8/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a^3*C-5/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*a^2*C+8/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3*b^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a-1/d/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+2/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^3*b*B
+3/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2
```

```
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a*b^3*B-2/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^
4*C-7/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2*b^2*C-1/d/(a^6-3*a^4*b^2+3*a^2*b^4-
b^6)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*C*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19836, size = 2859, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^5,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*C*a^7 - 2*B*a^6*b + 7*C*a^5*b^2 - 3*B*a^4*b^3 + C*a^3*b^4 + (2*
C*a^4*b^3 - 2*B*a^3*b^4 + 7*C*a^2*b^5 - 3*B*a*b^6 + C*b^7)*cos(d*x + c)^3 +
 3*(2*C*a^5*b^2 - 2*B*a^4*b^3 + 7*C*a^3*b^4 - 3*B*a^2*b^5 + C*a*b^6)*cos(d*
x + c)^2 + 3*(2*C*a^6*b - 2*B*a^5*b^2 + 7*C*a^4*b^3 - 3*B*a^3*b^4 + C*a^2*b
^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(24*C*a^7*b
- 18*B*a^6*b^2 - 19*C*a^5*b^3 + 23*B*a^4*b^4 - 4*C*a^3*b^5 - 7*B*a^2*b^6 -
C*a*b^7 + 2*B*b^8 + (13*C*a^5*b^3 - 11*B*a^4*b^4 + 4*C*a^3*b^5 + 7*B*a^2*b^
6 - 17*C*a*b^7 + 4*B*b^8)*cos(d*x + c)^2 + 3*(11*C*a^6*b^2 - 9*B*a^5*b^3 -
C*a^4*b^4 + 8*B*a^3*b^5 - 11*C*a^2*b^6 + B*a*b^7 + C*b^8)*cos(d*x + c))*sin
(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x
+ c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x
```

```
 + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*
x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(
2*C*a^7 - 2*B*a^6*b + 7*C*a^5*b^2 - 3*B*a^4*b^3 + C*a^3*b^4 + (2*C*a^4*b^3
- 2*B*a^3*b^4 + 7*C*a^2*b^5 - 3*B*a*b^6 + C*b^7)*cos(d*x + c)^3 + 3*(2*C*a^
5*b^2 - 2*B*a^4*b^3 + 7*C*a^3*b^4 - 3*B*a^2*b^5 + C*a*b^6)*cos(d*x + c)^2 +
 3*(2*C*a^6*b - 2*B*a^5*b^2 + 7*C*a^4*b^3 - 3*B*a^3*b^4 + C*a^2*b^5)*cos(d*
x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d
*x + c))) - (24*C*a^7*b - 18*B*a^6*b^2 - 19*C*a^5*b^3 + 23*B*a^4*b^4 - 4*C*
a^3*b^5 - 7*B*a^2*b^6 - C*a*b^7 + 2*B*b^8 + (13*C*a^5*b^3 - 11*B*a^4*b^4 +
4*C*a^3*b^5 + 7*B*a^2*b^6 - 17*C*a*b^7 + 4*B*b^8)*cos(d*x + c)^2 + 3*(11*C*
a^6*b^2 - 9*B*a^5*b^3 - C*a^4*b^4 + 8*B*a^3*b^5 - 11*C*a^2*b^6 + B*a*b^7 +
C*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^
2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3
*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4
*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6
+ a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44506, size = 960, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^5,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*C*a^4 - 2*B*a^3*b + 7*C*a^2*b^2 - 3*B*a*b^3 + C*b^4)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*
```

tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - (24*C*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^5 - 33*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^3*tan (1/2*d*x + 1/2*c)^5 + 18*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^2*b^4*tan (1/2*d*x + 1/2*c)^5 - 30*C*a^2*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*b^5*tan(1 /2*d*x + 1/2*c)^5 + 18*C*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^6*tan(1/2*d*x + 1/2*c)^5 + 3*C*b^6*tan(1/2*d*x + 1/2*c)^5 + 48*C*a^5*b*tan(1/2*d*x + 1/2 *c)^3 - 36*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*C*a^3*b^3*tan(1/2*d*x + 1/ 2*c)^3 + 32*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*C*a*b^5*tan(1/2*d*x + 1/2 *c)^3 + 4*B*b^6*tan(1/2*d*x + 1/2*c)^3 + 24*C*a^5*b*tan(1/2*d*x + 1/2*c) - 18*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 33*C*a^4*b^2*tan(1/2*d*x + 1/2*c) - 27* B*a^3*b^3*tan(1/2*d*x + 1/2*c) + 18*C*a^3*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^ 2*b^4*tan(1/2*d*x + 1/2*c) + 30*C*a^2*b^4*tan(1/2*d*x + 1/2*c) - 3*B*a*b^5* tan(1/2*d*x + 1/2*c) + 18*C*a*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^6*tan(1/2*d* x + 1/2*c) - 3*C*b^6*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d

3.1014 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=416

$$\frac{2\sin(c+dx)\left(24a^2C-36abB+63Ab^2+49b^2C\right)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2\sin(c+dx)\left(24a^2bB-16a^3C-6ab^2(7A+6C\right.}{315b^3d}$$

```
[Out] (2*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 6*a^2*b^2*(7*A + 4*C) + 21*b^4*(9*
A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
315*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(24*a^2*b*B
+ 75*b^3*B - 16*a^3*C - 6*a*b^2*(7*A + 6*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[a + b*Cos[c + d
*x]]) + (2*(24*a^2*b*B + 75*b^3*B - 16*a^3*C - 6*a*b^2*(7*A + 6*C))*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^3*d) + (2*(63*A*b^2 - 36*a*b*B + 24*
a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^3*d) + (2
*(3*b*B - 2*a*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(21*
b^2*d) + (2*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*b*
d)
```

Rubi [A] time = 0.930638, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(24a^2C-36abB+63Ab^2+49b^2C\right)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2\sin(c+dx)\left(24a^2bB-16a^3C-6ab^2(7A+6C\right.}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 6*a^2*b^2*(7*A + 4*C) + 21*b^4*(9*
A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
315*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(24*a^2*b*B
+ 75*b^3*B - 16*a^3*C - 6*a*b^2*(7*A + 6*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[a + b*Cos[c + d
*x]]) + (2*(24*a^2*b*B + 75*b^3*B - 16*a^3*C - 6*a*b^2*(7*A + 6*C))*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^3*d) + (2*(63*A*b^2 - 36*a*b*B + 24*
a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^3*d) + (2
```

```
*(3*b*B - 2*a*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(21*
b^2*d) + (2*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*b*
d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
```

```
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\sin}{9bd}$$

$$= \frac{2(3bB-2aC)\cos(c+dx)(a+b\cos(c+}{21b^2d}$$

$$= \frac{2\left(63Ab^2-36abB+24a^2C+49b^2C\right)(a}{315b^3d}$$

$$= \frac{2\left(24a^2bB+75b^3B-16a^3C-6ab^2(7A+\right.}{315b^3}$$

$$= \frac{2\left(24a^2bB+75b^3B-16a^3C-6ab^2(7A+\right.}{315b^3}$$

$$= \frac{2\left(24a^2bB+75b^3B-16a^3C-6ab^2(7A+\right.}{315b^3}$$

$$= \frac{2\left(24a^3bB+57ab^3B-16a^4C-6a^2b^2(7A\right.}{}$$

Mathematica [A] time = 1.7367, size = 321, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(2\sin(c+dx)\left(-48a^2bB+32a^3C+3ab^2(28A+19C)+345b^3B\right)+b\left(\sin(2(c+dx))\left(-24a^2C+36\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(6*a^2*b*B + 75*b^3*B - 4*a^3*C
+ 3*a*b^2*(49*A + 37*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-24*a^3*b
*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*((
a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])) + b*(a + b*Cos[c + d*x])*(2*(-48*a^2*b*B + 345*b^3*B + 32*a^3
*C + 3*a*b^2*(28*A + 19*C))*Sin[c + d*x] + b*((252*A*b^2 + 36*a*b*B - 24*a^
2*C + 266*b^2*C)*Sin[2*(c + d*x)] + 5*b*(2*(9*b*B + a*C)*Sin[3*(c + d*x)] +
 7*b*C*Sin[4*(c + d*x)]))))/(1260*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.063, size = 2143, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-42*a*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-42*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+42*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3*b^2-36*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*b^4+16*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^4*b-24*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
3*b^2+24*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+1
47*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-1120*C*b^
5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+42*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b^2+75*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*b^5+16*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-16*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-147*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5+(720*B*b^5+640*C*a*b^4+224
0*C*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^5-432*B*a*b^4-10
```

```
80*B*b^5+8*C*a^2*b^3-960*C*a*b^4-2072*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(336*A*a*b^4+504*A*b^5-12*B*a^2*b^3+432*B*a*b^4+840*B*b^5+8*C*a^3
*b^2-8*C*a^2*b^3+728*C*a*b^4+952*C*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+(-42*A*a^2*b^3-168*A*a*b^4-126*A*b^5+24*B*a^3*b^2+6*B*a^2*b^3-258*B*a*
b^4-240*B*b^5-16*C*a^4*b-4*C*a^3*b^2-24*C*a^2*b^3-204*C*a*b^4-168*C*b^5)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3*b^2+57*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2*b^3-57*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a*b^4-24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^4*b-51*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)/
b^4/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{4}+B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*co
s(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
cos(d*x + c)^2, x)
```

3.1015 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=321

$$\frac{2\sin(c+dx)\left(8a^2C-14abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(a^2-b^2\right)\left(8a^2C-14abB+35Ab^2+25b^2C\right)\sqrt{}}{105b^3d\sqrt{a+b\cos(c+}}$$

```
[Out] (-2*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - a*b^2*(35*A + 19*C))*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(35*A*b^2 - 14*a*b*B + 8*a^2*C + 25*b
^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(105*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(35*A*b^2 - 14*a*b*B + 8*a^2
*C + 25*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*b
*B - 4*a*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[
c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.574211, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(8a^2C-14abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(a^2-b^2\right)\left(8a^2C-14abB+35Ab^2+25b^2C\right)\sqrt{}}{105b^3d\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```


```
[Out] (-2*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - a*b^2*(35*A + 19*C))*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(35*A*b^2 - 14*a*b*B + 8*a^2*C + 25*b
^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(105*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(35*A*b^2 - 14*a*b*B + 8*a^2
*C + 25*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*b
*B - 4*a*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[
c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c}{7bd}$$

$$= \frac{2(7bB-4aC)(a+b\cos(c+dx))^{3/2}\sin(c+}{35b^2d}$$

$$= \frac{2\left(35Ab^2-14abB+8a^2C+25b^2C\right)\sqrt{a+}}{105b^2d}$$

$$= \frac{2\left(35Ab^2-14abB+8a^2C+25b^2C\right)\sqrt{a+}}{105b^2d}$$

$$= \frac{2\left(35Ab^2-14abB+8a^2C+25b^2C\right)\sqrt{a+}}{105b^2d}$$

$$= -\frac{2\left(14a^2bB-63b^3B-8a^3C-ab^2(35A+\right.}{105b^3d\sqrt{}}$$

Mathematica [A] time = 1.26433, size = 249, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(\sin(c+dx)\left(-16a^2C+28abB+140Ab^2+115b^2C\right)+3b(2(aC+7bB)\sin(2(c+dx))+5bC\sin(3(c\cdots\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(35*A*b^2 + 49*a*b*B + 2*a^2*C +
 25*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B
+ 8*a^3*C + a*b^2*(35*A + 19*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*(
(140*A*b^2 + 28*a*b*B - 16*a^2*C + 115*b^2*C)*Sin[c + d*x] + 3*b*(2*(7*b*B
+ a*C)*Sin[2*(c + d*x)] + 5*b*C*Sin[3*(c + d*x)])))/(210*b^3*d*Sqrt[a + b*C
os[c + d*x]])
```

Maple [B] time = 0.961, size = 1635, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-144*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4+112*B*a*b^3+168*B*b^4
-4*C*a^2*b^2+144*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+(-70*A*a*b^3-70*A*b^4-14*B*a^2*b^2-56*B*a*b^3-42*B*b^4+8*C*a^3*b+2*C*a^2*b
^2-86*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-35*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a*b^3+14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
```

```
/(a-b))^(1/2))*a^3*b-14*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*b^3-14*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a^3
*b+14*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a^2*b^2+63*B
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a*b^3-63*B*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^4-8*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^4-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+1
9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-19*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin
(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1016 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$

Optimal. Leaf size=237

$$-\frac{2\left(a^2-b^2\right)(5bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a(5bB-2aC)+3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(3*b^2*(5*A + 3*C) + a*(5*b*B - 2*a*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (2*(a^2 - b^2)*(5*b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2
*(5*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*C*(a
+ b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.369654, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(5bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a(5bB-2aC)+3b^2(5A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*(3*b^2*(5*A + 3*C) + a*(5*b*B - 2*a*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (2*(a^2 - b^2)*(5*b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2
*(5*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*C*(a
+ b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\sqrt{a+b\cos}}{}$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2C(a}{}$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2C(a}{}$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2C(a}{}$$

$$= \frac{2\left(15A+9C+\frac{a(5bB-2aC)}{b^2}\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c-\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.851911, size = 189, normalized size = 0.8

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(-2a^2C+5abB+15Ab^2+9b^2C\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(15aA+7aC+5\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),
x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(15*a*A + 5*b*B + 7*a*C)*Ellipti
cF[(c + d*x)/2, (2*b)/(a + b)] + (15*A*b^2 + 5*a*b*B - 2*a^2*C + 9*b^2*C)*(
(a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(5*b*B + a*C + 3*b*C*Cos[c + d*x]
)*Sin[c + d*x])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.908, size = 1187, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-2*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^2-10*B*cos(1/2*d*x+1/2*c)*a*b^2+10*B*cos(1/2*d*x+1/2*c)^3*a*b
^2-24*C*cos(1/2*d*x+1/2*c)^3*a*b^2-2*C*cos(1/2*d*x+1/2*c)*a^2*b+8*C*cos(1/2
*d*x+1/2*c)*a*b^2+16*C*cos(1/2*d*x+1/2*c)^5*a*b^2+2*C*cos(1/2*d*x+1/2*c)^3*
a^2*b+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+5*b^3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b^3+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^3-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-9*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-5*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a*b^2+24*C*cos(1/2*d*x+1/2*c)^7*b^3-48*C*cos(1/2*d*x
+1/2*c)^5*b^3+30*C*cos(1/2*d*x+1/2*c)^3*b^3-6*C*cos(1/2*d*x+1/2*c)*b^3+10*B
*cos(1/2*d*x+1/2*c)*b^3+20*B*cos(1/2*d*x+1/2*c)^5*b^3-30*B*cos(1/2*d*x+1/2*
c)^3*b^3-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b)/b^2/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
```

```
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.1017 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=240

$$\frac{2\left(3Ab^2-C\left(a^2-b^2\right)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(aC+3bB)\sqrt{a+b\cos(c+dx)}\ldots}{3bd\sqrt{\ldots}}$$

```
[Out] (2*(3*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*A*b^2 - (a^2 - b^2
)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*A*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.692933, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(3Ab^2-C\left(a^2-b^2\right)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(aC+3bB)\sqrt{a+b\cos(c+dx)}\ldots}{3bd\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (2*(3*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*A*b^2 - (a^2 - b^2
)*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*A*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int$$

$$=\frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}-\frac{2\int}{}$$

$$=\frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+(aA)$$

$$=\frac{2(3bB+aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=\frac{2(3bB+aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.5222, size = 393, normalized size = 1.64

$$\frac{\frac{4(3aB+3Ab+bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(a(6A+C)+3bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(aC+3bB)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+}{b-a}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x],x]
```

```
[Out] ((4*(3*A*b + 3*a*B + b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(3*b*B + a*(6*A + C)
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*b*B + a*C)*Sqrt[-((b*(-1 + Cos[c
+ d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a
*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqr
t[-(a + b)^(-1)]) + 4*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Maple [B] time = 0.953, size = 740, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b+3*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*b^2+2*C*cos(1/2*d*x+1/2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3
*b^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*C*
cos(1/2*d*x+1/2*c)*a*b+2*C*cos(1/2*d*x+1/2*c)*b^2)/b/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sqrt{b\cos\left(dx+c\right) + a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c), x)
```

3.1018 $\int \sqrt{a + b\cos(c + dx)} \left(A + B\cos(c + dx) + C\cos^2(c + dx\right.$ $\left.dx\right) dx$

Optimal. Leaf size=217

$$\frac{(aA + 2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c + dx)|\frac{2b}{a+b}\right)}{d\sqrt{a + b\cos(c + dx)}} + \frac{(2aB + Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2; \frac{1}{2}(c + dx)|\frac{2b}{a+b}\right)}{d\sqrt{a + b\cos(c + dx)}} - \frac{(A - 2C)\sqrt{a + b\cos(c + dx}}{d\sqrt{\frac{a+b\cos(c}{a+b}}}$$

```
[Out] -(((A - 2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a*A + 2*b*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos
[c + d*x]]) + ((A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a +
b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.666005, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(aA + 2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c + dx)|\frac{2b}{a+b}\right)}{d\sqrt{a + b\cos(c + dx)}} + \frac{(2aB + Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2; \frac{1}{2}(c + dx)|\frac{2b}{a+b}\right)}{d\sqrt{a + b\cos(c + dx)}} - \frac{(A - 2C)\sqrt{a + b\cos(c + dx}}{d\sqrt{\frac{a+b\cos(c}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] -(((A - 2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a*A + 2*b*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos
[c + d*x]]) + ((A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a +
b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} -$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}(-$$

$$= -\frac{(A-2C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(A-2C)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.58383, size = 385, normalized size = 1.77

$$\frac{\frac{2(4aB+Ab+2bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i(A-2C)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\right.\right.\right.}{ab\sqrt{-}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^2,x]
```

```
[Out] ((8*(b*B + a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(A*b + 4*a*B + 2*b*C)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[
a + b*Cos[c + d*x]] - ((2*I)*(A - 2*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a +
b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*Ellip
ticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a -
b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]
) + 4*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 0.993, size = 1035, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a-2*A*b)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+A*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*b-A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*b+2*b*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*B*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a+2*C*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a-2*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*b)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b-A*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*b*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(
-2*b/(a-b))^(1/2))*a+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(2*cos
(1/2*d*x+1/2*c)^2-1)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^2, x)
```

3.1019 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\ldots dx$

Optimal. Leaf size=299

$$-\frac{\left(-4a^2(A+2C)-4abB+Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+3Ab+8bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*A*b + 4*a*B + 8*b*C)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(
4*d*Sqrt[a + b*Cos[c + d*x]]) - ((A*b^2 - 4*a*b*B - 4*a^2*(A + 2*C))*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*
a*d*Sqrt[a + b*Cos[c + d*x]]) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan
[c + d*x])/(4*a*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])
/(2*d)
```

Rubi [A] time = 1.0572, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-4a^2(A+2C)-4abB+Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+3Ab+8bC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] -((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*A*b + 4*a*B + 8*b*C)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(
4*d*Sqrt[a + b*Cos[c + d*x]]) - ((A*b^2 - 4*a*b*B - 4*a^2*(A + 2*C))*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*
a*d*Sqrt[a + b*Cos[c + d*x]]) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan
[c + d*x])/(4*a*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])
/(2*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.41857, size = 428, normalized size = 1.43

$$\frac{\frac{2\left(8a^2(A+2C)+4abB-3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}} - \frac{2i(4aB+Ab)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos\right.\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(A + 4*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-3*A*b^2 + 4*a*b*B + 8*a^2*(A
 + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(a*Sqrt[a + b*Cos[c + d*x]]) - ((2*I)*(A*b + 4*a*B)*Sqrt[-((b*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c +
 d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[
c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-
```

```
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))
/(a^2*b*Sqrt[-(a + b)^(-1)]) + (4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (A*b +
4*a*B)*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/a)/(16*d)
```

Maple [B] time = 1.908, size = 1395, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*(A*b+B*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2)))-2*(B*b+C*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a*A*(-
1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^
2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^3, x)
```

3.1020 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=399

$$-\frac{\tan(c+dx)\left(-8a^2(2A+3C)-6abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24a^2d}-\frac{\left(-8a^2(2A+3C)-18abB+Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{24ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] ((3*A*b^2 - 6*a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
 - ((A*b^2 - 18*a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]])
 + ((A*b^3 + 8*a^3*B - 2*a*b^2*B + 4*a^2*b*(A + 2*C))*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^2*d*Sqrt[a +
b*Cos[c + d*x]]) - ((3*A*b^2 - 6*a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[
c + d*x]]*Tan[c + d*x])/(24*a^2*d) + ((A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]
]*Sec[c + d*x]*Tan[c + d*x])/(12*a*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.52968, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\tan(c+dx)\left(-8a^2(2A+3C)-6abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24a^2d}-\frac{\left(-8a^2(2A+3C)-18abB+Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{24ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```


```
[Out] ((3*A*b^2 - 6*a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
 - ((A*b^2 - 18*a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]])
 + ((A*b^3 + 8*a^3*B - 2*a*b^2*B + 4*a^2*b*(A + 2*C))*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^2*d*Sqrt[a +
b*Cos[c + d*x]]) - ((3*A*b^2 - 6*a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[
```

```
c + d*x]]*Tan[c + d*x])/(24*a^2*d) + ((A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]
]*Sec[c + d*x]*Tan[c + d*x])/(12*a*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c}{3d}$$

$$= \frac{(Ab+6aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx}{12ad}$$

$$= -\frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+b}}{24a^2d}$$

$$= -\frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+b}}{24a^2d}$$

$$= -\frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+b}}{24a^2d}$$

$$= \frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+b\,c}}{24a^2d\sqrt{\frac{a+b\cos(c+}{a+b}}}$$

$$= \frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+b\,c}}{24a^2d\sqrt{\frac{a+b\cos(c+}{a+b}}}$$

Mathematica [C] time = 6.64262, size = 661, normalized size = 1.66

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+24a^2C\sin(c+dx)+6abB\sin(c+dx)-3Ab^2\sin(c+dx)\right)}{24a^2}+\frac{\sec^2(c+dx)(6aB\sin(c+dx)+Ab\sin(c+dx))}{12a}+\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
```

```
Sec[c + d*x]^4,x]
```

```
[Out] ((2*(4*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A*b + 9*A*
b^3 + 48*a^3*B - 18*a*b^2*B + 24*a^2*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2
*I)*(-16*a^2*A*b + 3*A*b^3 - 6*a*b^2*B - 24*a^2*b*C)*Sqrt[(b - b*Cos[c + d*
x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a
 - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
 + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(
a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a +
 b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*
Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(96*a^2*d) + (Sqrt[a + b*Cos[c
+ d*x]]*((Sec[c + d*x]^2*(A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/(12*a) +
(Sec[c + d*x]*(16*a^2*A*Sin[c + d*x] - 3*A*b^2*Sin[c + d*x] + 6*a*b*B*Sin[c
 + d*x] + 24*a^2*C*Sin[c + d*x]))/(24*a^2) + (A*Sec[c + d*x]^2*Tan[c + d*x]
)/3))/d
```

Maple [B] time = 2.849, size = 2319, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(B*b+C*a)
*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*C*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
```

+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2))+2*a*A*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b
/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x
+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))
+2*(A*b+B*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*
x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(
a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos \left(dx+c\right)^{2}+B\cos \left(dx+c\right)+A\right)\sqrt{b\cos \left(dx+c\right)+a}\sec \left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^4, x)
```

3.1021 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\,\mathrm{c}\right.$

Optimal. Leaf size=518

$$\frac{2\sin(c+dx)\left(24a^2C-44abB+99Ab^2+81b^2C\right)(a+b\cos(c+dx))^{5/2}}{693b^3d}+\frac{2\sin(c+dx)\left(88a^2bB-48a^3C-6ab^2(33A+3\right.}{3465b^3d}$$

```
[Out] (2*(88*a^4*b*B + 363*a^2*b^3*B + 1617*b^5*B - 48*a^5*C - 18*a^3*b^2*(11*A +
 6*C) + 6*a*b^4*(451*A + 348*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3465*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*
(a^2 - b^2)*(88*a^3*b*B + 429*a*b^3*B - 48*a^4*C - 18*a^2*b^2*(11*A + 8*C)
+ 75*b^4*(11*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*b)/(a + b)])/(3465*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(88*a^3*b*
B + 429*a*b^3*B - 48*a^4*C - 18*a^2*b^2*(11*A + 8*C) + 75*b^4*(11*A + 9*C))
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^3*d) + (2*(88*a^2*b*B + 539
*b^3*B - 48*a^3*C - 6*a*b^2*(33*A + 34*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(3465*b^3*d) + (2*(99*A*b^2 - 44*a*b*B + 24*a^2*C + 81*b^2*C)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^3*d) + (2*(11*b*B - 6*a*C)*Cos[
c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c +
 d*x]^2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 1.28733, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(24a^2C-44abB+99Ab^2+81b^2C\right)(a+b\cos(c+dx))^{5/2}}{693b^3d}+\frac{2\sin(c+dx)\left(88a^2bB-48a^3C-6ab^2(33A+3\right.}{3465b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*(88*a^4*b*B + 363*a^2*b^3*B + 1617*b^5*B - 48*a^5*C - 18*a^3*b^2*(11*A +
 6*C) + 6*a*b^4*(451*A + 348*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3465*b^4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*
(a^2 - b^2)*(88*a^3*b*B + 429*a*b^3*B - 48*a^4*C - 18*a^2*b^2*(11*A + 8*C)
+ 75*b^4*(11*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*b)/(a + b)])/(3465*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(88*a^3*b*
```

```
B + 429*a*b^3*B - 48*a^4*C - 18*a^2*b^2*(11*A + 8*C) + 75*b^4*(11*A + 9*C))
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^3*d) + (2*(88*a^2*b*B + 539
*b^3*B - 48*a^3*C - 6*a*b^2*(33*A + 34*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(3465*b^3*d) + (2*(99*A*b^2 - 44*a*b*B + 24*a^2*C + 81*b^2*C)*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^3*d) + (2*(11*b*B - 6*a*C)*Cos[
c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c +
 d*x]^2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(11*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\mathrm{s}}{11bd}$$

$$= \frac{2(11bB-6aC)\cos(c+dx)(a+b\cos(c}{99b^2d}$$

$$= \frac{2\left(99Ab^2-44abB+24a^2C+81b^2C\right)(}{693b^3d}$$

$$= \frac{2\left(88a^2bB+539b^3B-48a^3C-6ab^2(3\right.}{3}$$

$$= \frac{2\left(88a^3bB+429ab^3B-48a^4C-18a^2b\right.}{}$$

$$= \frac{2\left(88a^3bB+429ab^3B-48a^4C-18a^2b\right.}{}$$

$$= \frac{2\left(88a^3bB+429ab^3B-48a^4C-18a^2b\right.}{}$$

$$= \frac{2\left(88a^4bB+363a^2b^3B+1617b^5B-48\right.}{}$$

Mathematica [A] time = 2.76781, size = 407, normalized size = 0.79

$$\frac{b(a+b\cos(c+dx))\left(2\sin(c+dx)\left(18a^2b^2(44A+27C)-352a^3bB+192a^4C+8844ab^3B+15b^4(506A+435C)\right)+b\left(4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(22*a^3*b*B + 2046*a*b^3*B - 12
*a^4*C + 75*b^4*(11*A + 9*C) + 9*a^2*b^2*(187*A + 141*C))*EllipticF[(c + d*
x)/2, (2*b)/(a + b)] - (-88*a^4*b*B - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C
 + 18*a^3*b^2*(11*A + 6*C) - 6*a*b^4*(451*A + 348*C))*((a + b)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(
```

```
a + b*Cos[c + d*x])*(2*(-352*a^3*b*B + 8844*a*b^3*B + 192*a^4*C + 18*a^2*b^
2*(44*A + 27*C) + 15*b^4*(506*A + 435*C))*Sin[c + d*x] + b*(4*(66*a^2*b*B +
 1463*b^3*B - 36*a^3*C + 48*a*b^2*(33*A + 34*C))*Sin[2*(c + d*x)] + 5*b*((3
96*A*b^2 + 440*a*b*B + 12*a^2*C + 513*b^2*C)*Sin[3*(c + d*x)] + 7*b*((22*b*
B + 24*a*C)*Sin[4*(c + d*x)] + 9*b*C*Sin[5*(c + d*x)])))))/(27720*b^4*d*Sqr
t[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.13, size = 2603, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+2088*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-2088*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5+96*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-819*a^2*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+198*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b^3+2706*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b^4-2706*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b^5+198*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4*b^2-1023*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*b^4+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b-
108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2+20160*
C*b^6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-198*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^6+675*b^6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))+825*A*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6
+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-363*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+(7920*A*b^6+1496
0*B*a*b^5+24640*B*b^6+6960*C*a^2*b^4+47040*C*a*b^5+56880*C*b^6)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-10296*A*a*b^5-11880*A*b^6-4664*B*a^2*b^4-224
40*B*a*b^5-22792*B*b^6+24*C*a^3*b^3-10440*C*a^2*b^4-43368*C*a*b^5-34920*C*b
^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(3564*A*a^2*b^4+10296*A*a*b^5+9
240*A*b^6-44*B*a^3*b^3+4664*B*a^2*b^4+17248*B*a*b^5+10472*B*b^6+24*C*a^4*b^
2-24*C*a^3*b^3+7872*C*a^2*b^4+19848*C*a*b^5+13860*C*b^6)*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+(-198*A*a^3*b^3-1782*A*a^2*b^4-4224*A*a*b^5-2640*A*b^
6+88*B*a^4*b^2+22*B*a^3*b^3-3102*B*a^2*b^4-4884*B*a*b^5-1848*B*b^6-48*C*a^5
*b-12*C*a^4*b^2-108*C*a^3*b^3-2196*C*a^2*b^4-4842*C*a*b^5-2790*C*b^6)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(-12320*B*b^6-23520*C*a*b^5-50400*C*b^6)
*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-88*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2+88*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^5*b+1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a*b^5-88*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^5*b-341*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
b^3+429*a*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+363*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3)/b^4/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{5}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{4}+Aa\cos\left(dx+c\right)^{2}+\left(Ba+Ab\right)\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^5 + (C*a + B*b)*cos(d*x + c)^4 + A*a*cos(d*x + c
)^2 + (B*a + A*b)*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*cos(d*x + c)^2, x)
```

3.1022 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=408

$$\frac{2\sin(c+dx)\left(8a^2C-18abB+63Ab^2+49b^2C\right)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\sin(c+dx)\left(18a^2bB-8a^3C-3ab^2(21A+13C\right.}{315b^2d}$$

```
[Out] (-2*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(2
1*A + 11*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(315*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*b
*B - 75*b^3*B - 8*a^3*C - 3*a*b^2*(21*A + 13*C))*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*(18*a^2*b*B - 75*b^3*B - 8*a^3*C - 3*a*b^2*(21*A + 13*C))*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(63*A*b^2 - 18*a*b*B
+ 8*a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d)
+ (2*(9*b*B - 4*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) +
(2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.820251, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(8a^2C-18abB+63Ab^2+49b^2C\right)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\sin(c+dx)\left(18a^2bB-8a^3C-3ab^2(21A+13C\right.}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (-2*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(2
1*A + 11*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(315*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*b
*B - 75*b^3*B - 8*a^3*C - 3*a*b^2*(21*A + 13*C))*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*(18*a^2*b*B - 75*b^3*B - 8*a^3*C - 3*a*b^2*(21*A + 13*C))*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(63*A*b^2 - 18*a*b*B
+ 8*a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d)
+ (2*(9*b*B - 4*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) +
```

```
(2*C*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin}{9bd}$$

$$= \frac{2(9bB-4aC)(a+b\cos(c+dx))^{5/2}\sin(}{63b^2d}$$

$$= \frac{2\left(63Ab^2-18abB+8a^2C+49b^2C\right)(a}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-3ab^2(21A\right.}{31}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-3ab^2(21A\right.}{31}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-3ab^2(21A\right.}{31}$$

$$= -\frac{2\left(18a^3bB-246ab^3B-8a^4C-21b^4(9\right.}{}$$

Mathematica [A] time = 1.71127, size = 321, normalized size = 0.79

$$\frac{b(a+b\cos(c+dx))\left(\sin(c+dx)\left(72a^2bB-32a^3C+12ab^2(84A+67C)+690b^3B\right)+b\left(2\sin(2(c+dx))\left(6a^2C+144ab\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(153*a^2*b*B + 75*b^3*B + 2*a^3*
C + 6*a*b^2*(42*A + 31*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-18*a^3
*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C)
)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((72*a^2*b*B + 690*b^3*B - 32*a^
3*C + 12*a*b^2*(84*A + 67*C))*Sin[c + d*x] + b*(2*(126*A*b^2 + 144*a*b*B +
6*a^2*C + 133*b^2*C)*Sin[2*(c + d*x)] + 5*b*(2*(9*b*B + 10*a*C)*Sin[3*(c +
d*x)] + 7*b*C*Sin[4*(c + d*x)]))))/(1260*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.161, size = 2143, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(63*a*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+63*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-63*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-31*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^3*b^2+39*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^4-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^4*b+33*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*
b^2-33*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-1120*C*b^5*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^4*b-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3*b^2+75*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
b^5-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+8*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-147*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5+(756*A*a*b^4+504*A*b^5+324*B*a
^2*b^3+936*B*a*b^4+840*B*b^5-4*C*a^3*b^2+424*C*a^2*b^3+1568*C*a*b^4+952*C*b
```

```
^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-252*A*a^2*b^3-378*A*a*b^4-126
*A*b^5-18*B*a^3*b^2-162*B*a^2*b^3-384*B*a*b^4-240*B*b^5+8*C*a^4*b+2*C*a^3*b
^2-282*C*a^2*b^3-444*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+(-504*A*b^5-936*B*a*b^4-1080*B*b^5-424*C*a^2*b^3-2040*C*a*b^4-2072*C*b
^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(720*B*b^5+1360*C*a*b^4+2240*C*
b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+18*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+246*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^2*b^3-246*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a*b^4+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^4*b-93*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*b^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^4 + (Ca+Bb)\cos(dx+c)^3 + Aa\cos(dx+c) + (Ba+Ab)\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + (C*a + B*b)*cos(d*x + c)^3 + A*a*cos(d*x + c
) + (B*a + A*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*cos(d*x + c), x)
```

3.1023 $\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$

Optimal. Leaf size=315

$$\frac{2\sin(c+dx)\left(-6a^2C+21abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2C+21abB+35Ab^2+25b^2C\right)}{105b^2d\sqrt{a+b\cos(c\cdots}}$$

```
[Out] (2*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 2*a*b^2*(70*A + 41*C))*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(35*A*b^2 + 21*a*b*B - 6*a^2*C + 25*
b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(105*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(35*A*b^2 + 21*a*b*B - 6*a^
2*C + 25*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*b*
B - 2*a*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b*d) + (2*C*(a + b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.518754, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(-6a^2C+21abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2C+21abB+35Ab^2+25b^2C\right)}{105b^2d\sqrt{a+b\cos(c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```


```
[Out] (2*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 2*a*b^2*(70*A + 41*C))*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(35*A*b^2 + 21*a*b*B - 6*a^2*C + 25*
b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(105*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(35*A*b^2 + 21*a*b*B - 6*a^
2*C + 25*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*b*
B - 2*a*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b*d) + (2*C*(a + b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int(a+b\cos$$

$$= \frac{2(7bB-2aC)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd} + \frac{2C$$

$$= \frac{2\left(35Ab^2+21abB-6a^2C+25b^2C\right)\sqrt{a+b\cos(c+}}{105bd}$$

$$= \frac{2\left(35Ab^2+21abB-6a^2C+25b^2C\right)\sqrt{a+b\cos(c+}}{105bd}$$

$$= \frac{2\left(35Ab^2+21abB-6a^2C+25b^2C\right)\sqrt{a+b\cos(c+}}{105bd}$$

$$= \frac{2\left(21a^2bB+63b^3B-6a^3C+2ab^2(70A+41C)\right)\sqrt{a}}{105b^2d\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

Mathematica [A] time = 1.26585, size = 257, normalized size = 0.82

$$\frac{b(a+b\cos(c+dx))\left(\sin(c+dx)\left(12a^2C+168abB+140Ab^2+115b^2C\right)+3b(2(8aC+7bB)\sin(2(c+dx))+5bC\sin(3($$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(84*a*b*B + 5*b^2*(7*A + 5*C) +
3*a^2*(35*A + 17*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (21*a^2*b*B +
63*b^3*B - 6*a^3*C + 2*a*b^2*(70*A + 41*C))*((a + b)*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[
c + d*x])*((140*A*b^2 + 168*a*b*B + 12*a^2*C + 115*b^2*C)*Sin[c + d*x] + 3*
b*(2*(7*b*B + 8*a*C)*Sin[2*(c + d*x)] + 5*b*C*Sin[3*(c + d*x)])))/(210*b^2*
```

```
d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.983, size = 1635, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-312*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4+252*B*a*b^3+168*B*b^4
+108*C*a^2*b^2+312*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+(-70*A*a*b^3-70*A*b^4-84*B*a^2*b^2-126*B*a*b^3-42*B*b^4-6*C*a^3*b-54*C*a
^2*b^2-128*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+140*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-140*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-35*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+21*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*a^3*b-21*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*a^2*b^2+63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*a*b^3-63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^4
-21*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+21*B*a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-6*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+6*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a^2*b^2-82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
```

```
(a-b))^(1/2))*a*b^3+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^4-31*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2
+25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^2/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.1024 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=306

$$-\frac{2\left(5a^2bB + 3a^3C - 3ab^2(5A+C) - 5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}} + \frac{2\left(3a^2C + 20abB + 15Ab^2 + 9b^2C\right)\sqrt{a+}}{15bd\sqrt{\frac{a+b\cos(c}{a+b}}}$$

```
[Out] (2*(15*A*b^2 + 20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (2*(5*a^2*b*B - 5*b^3*B + 3*a^3*C - 3*a*b^2*(5*A + C))*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a +
 b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*b*B +
3*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.01411, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(5a^2bB + 3a^3C - 3ab^2(5A+C) - 5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}} + \frac{2\left(3a^2C + 20abB + 15Ab^2 + 9b^2C\right)\sqrt{a+}}{15bd\sqrt{\frac{a+b\cos(c}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```


```
[Out] (2*(15*A*b^2 + 20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (2*(5*a^2*b*B - 5*b^3*B + 3*a^3*C - 3*a*b^2*(5*A + C))*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a +
 b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*b*B +
3*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{15d}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{15d}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{15d}$$

$$= \frac{2\left(15Ab^2+20abB+3a^2C+9b^2C\right)\sqrt{a+}}{15bd\sqrt{\frac{a+b\cos(c}{a+b}}}$$

$$= \frac{2\left(15Ab^2+20abB+3a^2C+9b^2C\right)\sqrt{a+}}{15bd\sqrt{\frac{a+b\cos(c}{a+b}}}$$

Mathematica [C] time = 3.31999, size = 455, normalized size = 1.49

$$\frac{\frac{4\left(15a^2B+6ab(5A+2C)+5b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2(10A+C)+20abB+3b^2(5A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i\csc(c+dx)\left(3a^2C\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x],x]
```

```
[Out] ((4*(15*a^2*B + 5*b^2*B + 6*a*b*(5*A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(
20*a*b*B + 3*a^2*(10*A + C) + 3*b^2*(5*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]
] + ((2*I)*(15*A*b^2 + 20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[-((b*(-1 + Cos[c
+ d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a
*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sq
```

```
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqr
t[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(5*b*B + 6*a*C + 3*b*C*Cos[c
 + d*x])*Sin[c + d*x])/(30*d)
```

Maple [B] time = 0.961, size = 1330, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-3*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+3*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*b^2-10*B*cos(1/2*d*x+1/2*c)*a*b^2+10*B*cos(1/2*d*x+1/2*c)^3*a*
b^2-54*C*cos(1/2*d*x+1/2*c)^3*a*b^2-12*C*cos(1/2*d*x+1/2*c)*a^2*b+18*C*cos(
1/2*d*x+1/2*c)*a*b^2+36*C*cos(1/2*d*x+1/2*c)^5*a*b^2+12*C*cos(1/2*d*x+1/2*c
)^3*a^2*b+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+5*b^3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^3-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^3+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-9*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-20*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+24*C*cos(1/2*d*x+1/2*c)^7*b^3-48*C*cos(1/
2*d*x+1/2*c)^5*b^3+30*C*cos(1/2*d*x+1/2*c)^3*b^3-6*C*cos(1/2*d*x+1/2*c)*b^3
+10*B*cos(1/2*d*x+1/2*c)*b^3+20*B*cos(1/2*d*x+1/2*c)^5*b^3-30*B*cos(1/2*d*x
+1/2*c)^3*b^3-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+20*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-15*A*a^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b+15*a*A*b^2*(sin(1/2*d*x+1/2*c)^2
```

```
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2)))/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)(b\cos{(dx+c)} + a)^{\frac{3}{2}}\sec{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c), x)
```

3.1025 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^2(c + dx)\, dx$

Optimal. Leaf size=286

$$\frac{\left(a^2(3A-2C)+6abB+2b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{(3aA-8aC-6bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((3*a*A - 6*b*B - 8*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((6*a*b*B + a^2*(
3*A - 2*C) + 2*b^2*(3*A + C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*A*b + 2
*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*A - 2*C)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*d) + (A*(a + b*Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rubi [A] time = 1.03435, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2(3A-2C)+6abB+2b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{(3aA-8aC-6bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```


```
[Out] -((3*a*A - 6*b*B - 8*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((6*a*b*B + a^2*(
3*A - 2*C) + 2*b^2*(3*A + C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*A*b + 2
*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*A - 2*C)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*d) + (A*(a + b*Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{A(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d}+$$

$$=-\frac{b(3A-2C)\sqrt{a+b\cos(c+dx)}\sin(c+}{3d}$$

$$=-\frac{b(3A-2C)\sqrt{a+b\cos(c+dx)}\sin(c+}{3d}$$

$$=-\frac{b(3A-2C)\sqrt{a+b\cos(c+dx)}\sin(c+}{3d}$$

$$=-\frac{(3aA-6bB-8aC)\sqrt{a+b\cos(c+dx}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=-\frac{(3aA-6bB-8aC)\sqrt{a+b\cos(c+dx}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 4.03069, size = 434, normalized size = 1.52

$$\frac{\frac{8\left(3a^2C+6abB+3Ab^2+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(12a^2B+ab(15A+8C)+6b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i\csc(c+dx)(-3aA+8aC+6}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^2,x]
```

```
[Out] ((8*(3*A*b^2 + 6*a*b*B + 3*a^2*C + b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(12*
a^2*B + 6*b^2*B + a*b*(15*A + 8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-
3*a*A + 6*b*B + 8*a*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1
+ Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[S
qrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*El
lipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
```

```
os[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b
*Cos[c + d*x]]*(3*a*A + 2*b*C*Cos[c + d*x])*Tan[c + d*x])/(12*d)
```

Maple [B] time = 1.118, size = 1635, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*A*a*b+8*C*a*b+16*C*b^2)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^2-6*A*a*b-4*C*a*b-4*C*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))*a*b-3*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^2-6*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+3*A*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-3*A*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b+6*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))*a^2-6*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-6*
B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+6*B*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2-2*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2
-8*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+8*C*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b+3*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+6*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a*b-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2+6*a*
b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-6*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
```

```
d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))+2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-8
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^2, x)
```

3.1026 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=307

$$\frac{\left(4a^2B + ab(7A+8C) + 8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4a^2(A+2C) + 12abB + 3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\ldots\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((5*A*b + 4*a*B - 8*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((4*a^2*B + 8*b^2
*B + a*b*(7*A + 8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 + 12*a*b*B
+ 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b + 4*a*B)*Sqr
t[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(3/2)*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 1.07579, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2B + ab(7A+8C) + 8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4a^2(A+2C) + 12abB + 3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\ldots\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```


```
[Out] -((5*A*b + 4*a*B - 8*b*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((4*a^2*B + 8*b^2
*B + a*b*(7*A + 8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 + 12*a*b*B
+ 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b + 4*a*B)*Sqr
t[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(3/2)*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c}{2d}$$

$$= \frac{(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+}{4d}$$

$$= \frac{(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+}{4d}$$

$$= \frac{(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+}{4d}$$

$$= -\frac{(5Ab+4aB-8bC)\sqrt{a+b\cos(c+dx)}E}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(5Ab+4aB-8bC)\sqrt{a+b\cos(c+dx)}E}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.2502, size = 438, normalized size = 1.43

$$\frac{\frac{2\left(8a^2(A+2C)+20abB+b^2(A+8C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{8b(a(A+8C)+4bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i\csc(c+dx)(4aB+5Ab-8bC)\sqrt{-\frac{b}{}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(4*b*B + a*(A + 8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(20*a*b*B + 8*a^2*
(A + 2*C) + b^2*(A + 8*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*A*b + 4*
a*B - 8*b*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c +
d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*
ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b
*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x
```

```
]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d
*x]]*(2*a*A + (5*A*b + 4*a*B)*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*
d)
```

Maple [B] time = 2.104, size = 1743, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*C*(a-b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2)))+2*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*a*b*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a*(2*A*b+B
*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*(A*b^2+2*B*a*b+C*
a^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*a^2*(-1/2/a*cos(1/2*d*x+1/2*c
)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2
/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)(b\cos{(dx+c)} + a)^{\frac{3}{2}} \sec{(dx+c)}^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^3, x)
```

3.1027 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \ldots dx$

Optimal. Leaf size=399

$$\frac{\tan(c+dx)\left(8a^2(2A+3C)+30abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(8a^2(2A+3C)+42abB+b^2(17A+48C)\right)\sqrt{\frac{a+b\cos(c\ldots}{a+b}}}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((3*A*b^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
 + ((42*a*b*B + 8*a^2*(2*A + 3*C) + b^2*(17*A + 48*C))*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[
c + d*x]]) - ((A*b^3 - 8*a^3*B - 6*a*b^2*B - 12*a^2*b*(A + 2*C))*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*
Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[
a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) + ((A*b + 2*a*B)*Sqrt[a + b*Cos[
c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.54994, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\left(8a^2(2A+3C)+30abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{\left(8a^2(2A+3C)+42abB+b^2(17A+48C)\right)\sqrt{\frac{a+b\cos(c\ldots}{a+b}}}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```


```
[Out] -((3*A*b^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
 + ((42*a*b*B + 8*a^2*(2*A + 3*C) + b^2*(17*A + 48*C))*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[
c + d*x]]) - ((A*b^3 - 8*a^3*B - 6*a*b^2*B - 12*a^2*b*(A + 2*C))*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*
Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[
```

```
a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) + ((A*b + 2*a*B)*Sqrt[a + b*Cos[
c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan}{3d}$$

$$=\frac{(Ab+2aB)\sqrt{a+b\cos(c+dx)}\sec(c+}{4d}$$

$$=\frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a+}}{24ad}$$

$$=\frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a+}}{24ad}$$

$$=\frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a+}}{24ad}$$

$$=-\frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a}}{24ad\sqrt{\frac{a+b\,c}{}}}$$

$$=-\frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a}}{24ad\sqrt{\frac{a+b\,c}{}}}$$

Mathematica [C] time = 6.85806, size = 667, normalized size = 1.67

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+24a^2C\sin(c+dx)+30abB\sin(c+dx)+3Ab^2\sin(c+dx)\right)}{24a}+\frac{1}{12}\sec^2(c+dx)(6aB\sin(c+dx)+\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(28*a*A*b^2 + 24*a^2*b*B + 96*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(56
*a^2*A*b - 9*A*b^3 + 48*a^3*B + 6*a*b^2*B + 120*a^2*b*C)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] - ((2*I)*(-16*a^2*A*b - 3*A*b^3 - 30*a*b^2*B - 24*a^2*b*C)*Sqrt[(
b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c
 + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a,
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))
*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2
- b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b
^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(96*a*d) + (Sqr
t[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(7*A*b*Sin[c + d*x] + 6*a*B*Sin[c +
d*x]))/12 + (Sec[c + d*x]*(16*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 3
0*a*b*B*Sin[c + d*x] + 24*a^2*C*Sin[c + d*x]))/(24*a) + (a*A*Sec[c + d*x]^2
*Tan[c + d*x])/3))/d
```

Maple [B] time = 3.121, size = 2441, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*(A*b^2+2*B*a*b+C*a^2)*(-1/a*cos(1/2*d*x+1/
2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*b*(B*b+2*C*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2
```

)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*a^2*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a*(2*A*b+B*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^4, x)
```

3.1028 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \ldots dx) \, dx$

Optimal. Leaf size=503

$$-\frac{\tan(c+dx)\left(-12a^2b(13A+20C)-128a^3B-24ab^2B+9Ab^3\right)\sqrt{a+b\cos(c+dx)}}{192a^2d}-\frac{\left(-12a^2b(19A+28C)-128a^3B-1\ldots\right.}{192a\ldots}$$

```
[Out] ((9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A + 20*C))*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - ((3*A*b^3 - 128*a^3*B - 136*a*b^2*B - 12*a^2*b*(19*
A + 28*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(192*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^4 + 96*a^3*b*B - 8*a
*b^3*B + 24*a^2*b^2*(A + 2*C) + 16*a^4*(3*A + 4*C))*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a^2*d*Sqrt[a + b
*Cos[c + d*x]]) - ((9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A + 20*
C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192*a^2*d) + ((3*A*b^2 + 56*a*b
*B + 12*a^2*(3*A + 4*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(96*a*d) + ((3*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c
 + d*x])/(24*d) + (A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x]
)/(4*d)
```

Rubi [A] time = 2.04257, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\tan(c+dx)\left(-12a^2b(13A+20C)-128a^3B-24ab^2B+9Ab^3\right)\sqrt{a+b\cos(c+dx)}}{192a^2d}-\frac{\left(-12a^2b(19A+28C)-128a^3B-1\ldots\right.}{192a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```


```
[Out] ((9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A + 20*C))*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - ((3*A*b^3 - 128*a^3*B - 136*a*b^2*B - 12*a^2*b*(19*
A + 28*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
```

```
(a + b)])/(192*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^4 + 96*a^3*b*B - 8*a
*b^3*B + 24*a^2*b^2*(A + 2*C) + 16*a^4*(3*A + 4*C))*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a^2*d*Sqrt[a + b
*Cos[c + d*x]]) - ((9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A + 20*
C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192*a^2*d) + ((3*A*b^2 + 56*a*b
*B + 12*a^2*(3*A + 4*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(96*a*d) + ((3*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c
 + d*x])/(24*d) + (A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x]
)/(4*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\tan}{4d}$$

$$= \frac{(3Ab+8aB)\sqrt{a+b\cos(c+dx)}\sec^2(c}{24d}$$

$$= \frac{\left(3Ab^2+56abB+12a^2(3A+4C)\right)\sqrt{a}}{96}$$

$$= -\frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13\right.}{1}$$

$$= -\frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13\right.}{1}$$

$$= -\frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13\right.}{1}$$

$$= \frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13A\right.}{192a^2}$$

$$= \frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13A\right.}{192a^2}$$

Mathematica [C] time = 6.96388, size = 783, normalized size = 1.56

$$\sqrt{a+b\cos(c+dx)}\left(\frac{\sec^2(c+dx)\left(36a^2A\sin(c+dx)+48a^2C\sin(c+dx)+56abB\sin(c+dx)+3Ab^2\sin(c+dx)\right)}{96a}+\frac{\sec(c+dx)\left(156a^2Ab\sin(c+dx)+240a^2bC\sin(\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^5,x]
```

```
[Out] ((2*(144*a^3*A*b + 12*a*A*b^3 + 224*a^2*b^2*B + 192*a^3*b*C)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] + (2*(288*a^4*A - 12*a^2*A*b^2 + 27*A*b^4 + 448*a^3*b*B - 72*a*b^
3*B + 384*a^4*C + 48*a^2*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
Pi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-156*
a^2*A*b^2 + 9*A*b^4 - 128*a^3*b*B - 24*a*b^3*B - 240*a^2*b^2*C)*Sqrt[(b - b
*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*
x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
 d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[
c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2
 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 -
4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(768*a^2*d) + (Sqrt[
a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(9*A*b*Sin[c + d*x] + 8*a*B*Sin[c + d*
x]))/24 + (Sec[c + d*x]^2*(36*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 5
6*a*b*B*Sin[c + d*x] + 48*a^2*C*Sin[c + d*x]))/(96*a) + (Sec[c + d*x]*(156*
a^2*A*b*Sin[c + d*x] - 9*A*b^3*Sin[c + d*x] + 128*a^3*B*Sin[c + d*x] + 24*a
*b^2*B*Sin[c + d*x] + 240*a^2*b*C*Sin[c + d*x]))/(192*a^2) + (a*A*Sec[c + d
*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] time = 4.403, size = 3551, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^2*(-1
/4/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^4+7/24*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^3-1/96*(36*a^2+35*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b
*(20*a^2+21*b^2)/a^4*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-35/384*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-25/96*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))+35/128/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
-3/16/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))*b^4)+2*b*(B*b+2*C*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2
c)^2)^(1/2)((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2)))-2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1

/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a*(2*A
*b+B*a)*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/
2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*(A*b^2+2*B*a*b+C
a^2)(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c
)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^5, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^5, x)
```

3.1029 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=629

$$\frac{2\sin(c+dx)\left(24a^2C-52abB+143Ab^2+121b^2C\right)(a+b\cos(c+dx))^{7/2}}{1287b^3d}+\frac{2\sin(c+dx)\left(104a^2bB-48a^3C-2ab^2(143\right.}{900}$$

```
[Out] (2*(520*a^5*b*B + 3315*a^3*b^3*B + 48165*a*b^5*B - 240*a^6*C + 1617*b^6*(13
*A + 11*C) - 10*a^4*b^2*(143*A + 76*C) + 3*a^2*b^4*(13299*A + 10223*C))*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(45045*b^4*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(520*a^4*b*B + 3705*a^2
*b^3*B + 8775*b^5*B - 240*a^5*C - 10*a^3*b^2*(143*A + 94*C) + 6*a*b^4*(2717
*A + 2174*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/(45045*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(520*a^4*b*B + 370
5*a^2*b^3*B + 8775*b^5*B - 240*a^5*C - 10*a^3*b^2*(143*A + 94*C) + 6*a*b^4*
(2717*A + 2174*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(45045*b^3*d) + (
2*(520*a^3*b*B + 4355*a*b^3*B - 240*a^4*C + 539*b^4*(13*A + 11*C) - 10*a^2*
b^2*(143*A + 124*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45045*b^3*d)
 + (2*(104*a^2*b*B + 1053*b^3*B - 48*a^3*C - 2*a*b^2*(143*A + 166*C))*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9009*b^3*d) + (2*(143*A*b^2 - 52*a*b*B
 + 24*a^2*C + 121*b^2*C)*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(1287*b^3
*d) + (2*(13*b*B - 6*a*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d
*x])/(143*b^2*d) + (2*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d
*x])/(13*b*d)
```

Rubi [A] time = 1.51451, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(24a^2C-52abB+143Ab^2+121b^2C\right)(a+b\cos(c+dx))^{7/2}}{1287b^3d}+\frac{2\sin(c+dx)\left(104a^2bB-48a^3C-2ab^2(143\right.}{900}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*(520*a^5*b*B + 3315*a^3*b^3*B + 48165*a*b^5*B - 240*a^6*C + 1617*b^6*(13
*A + 11*C) - 10*a^4*b^2*(143*A + 76*C) + 3*a^2*b^4*(13299*A + 10223*C))*Sqr
```

```
t[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(45045*b^4*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(520*a^4*b*B + 3705*a^2
*b^3*B + 8775*b^5*B - 240*a^5*C - 10*a^3*b^2*(143*A + 94*C) + 6*a*b^4*(2717
*A + 2174*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/(45045*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(520*a^4*b*B + 370
5*a^2*b^3*B + 8775*b^5*B - 240*a^5*C - 10*a^3*b^2*(143*A + 94*C) + 6*a*b^4*
(2717*A + 2174*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(45045*b^3*d) + (
2*(520*a^3*b*B + 4355*a*b^3*B - 240*a^4*C + 539*b^4*(13*A + 11*C) - 10*a^2*
b^2*(143*A + 124*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45045*b^3*d)
 + (2*(104*a^2*b*B + 1053*b^3*B - 48*a^3*C - 2*a*b^2*(143*A + 166*C))*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9009*b^3*d) + (2*(143*A*b^2 - 52*a*b*B
 + 24*a^2*C + 121*b^2*C)*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(1287*b^3
*d) + (2*(13*b*B - 6*a*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d
*x])/(143*b^2*d) + (2*C*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d
*x])/(13*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^2(c+dx)(a+b\cos(c+dx))^{7/2}\sin}{13bd}$$

$$= \frac{2(13bB-6aC)\cos(c+dx)(a+b\cos(c+}{143b^2d}$$

$$= \frac{2\left(143Ab^2-52abB+24a^2C+121b^2C\right)}{1287b^3d}$$

$$= \frac{2\left(104a^2bB+1053b^3B-48a^3C-2ab^2(1\right.}{}$$

$$= \frac{2\left(520a^3bB+4355ab^3B-240a^4C+539\right.}{}$$

$$= \frac{2\left(520a^4bB+3705a^2b^3B+8775b^5B-2\right.}{}$$

$$= \frac{2\left(520a^4bB+3705a^2b^3B+8775b^5B-2\right.}{}$$

$$= \frac{2\left(520a^4bB+3705a^2b^3B+8775b^5B-2\right.}{}$$

$$= \frac{2\left(520a^5bB+3315a^3b^3B+48165ab^5B-\right.}{}$$

Mathematica [A] time = 3.85216, size = 501, normalized size = 0.8

$$\frac{b(a+b\cos(c+dx))\left(4\sin(c+dx)\left(10a^3b^2(572A+331C)+121290a^2b^3B-2080a^4bB+960a^5C+3ab^4(71214A+60793\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (32*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(130*a^4*b*B + 43095*a^2*b^3*B
+ 8775*b^5*B - 60*a^5*C + 5*a^3*b^2*(4433*A + 3337*C) + 3*a*b^4*(12441*A +
10277*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-520*a^5*b*B - 3315*a^3*
```

```
b^3*B - 48165*a*b^5*B + 240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(14
3*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*((a + b)*EllipticE[(c + d*x)/2
, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos
[c + d*x])*(4*(-2080*a^4*b*B + 121290*a^2*b^3*B + 84825*b^5*B + 960*a^5*C +
 10*a^3*b^2*(572*A + 331*C) + 3*a*b^4*(71214*A + 60793*C))*Sin[c + d*x] + b
*((3120*a^3*b*B + 321880*a*b^3*B - 1440*a^4*C + 120*a^2*b^2*(1430*A + 1457*
C) + 77*b^4*(1976*A + 1897*C))*Sin[2*(c + d*x)] + 5*b*(2*(5876*a^2*b*B + 66
69*b^3*B + 60*a^3*C + a*b^2*(10868*A + 13939*C))*Sin[3*(c + d*x)] + 7*b*(4*
(143*A*b^2 + 299*a*b*B + 159*a^2*C + 220*b^2*C)*Sin[4*(c + d*x)] + 9*b*((26
*b*B + 54*a*C)*Sin[5*(c + d*x)] + 11*b*C*Sin[6*(c + d*x)]))))))/(720720*b^4
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.168, size = 3165, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4435
20*C*b^7*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14-520*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b^2+(262080*B*b^7+766080*C*a*b^6+13
30560*C*b^7)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^7-465920
*B*a*b^6-655200*B*b^7-450240*C*a^2*b^5-1915200*C*a*b^6-1798720*C*b^7)*sin(1
/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(297440*A*a*b^6+320320*A*b^7+284960*B*a
^2*b^5+931840*B*a*b^6+739440*B*b^7+90240*C*a^3*b^4+900480*C*a^2*b^5+2159680
*C*a*b^6+1379840*C*b^7)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-194480*A*
a^2*b^5-446160*A*a*b^6-296296*A*b^7-60320*B*a^3*b^4-427440*B*a^2*b^5-860080
*B*a*b^6-453960*B*b^7+120*C*a^4*b^3-135360*C*a^3*b^4-828880*C*a^2*b^5-13243
20*C*a*b^6-666512*C*b^7)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(45760*A*a
^3*b^4+194480*A*a^2*b^5+344344*A*a*b^6+136136*A*b^7-260*B*a^4*b^3+60320*B*a
^3*b^4+326560*B*a^2*b^5+394160*B*a*b^6+180180*B*b^7+120*C*a^5*b^2-120*C*a^4
*b^3+101840*C*a^3*b^4+378640*C*a^2*b^5+522368*C*a*b^6+198352*C*b^7)*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1430*A*a^4*b^3-22880*A*a^3*b^4-95238*A*a
^2*b^5-97812*A*a*b^6-24024*A*b^7+520*B*a^5*b^2+130*B*a^4*b^3-41730*B*a^3*b^
4-92040*B*a^2*b^5-86970*B*a*b^6-36270*B*b^7-240*C*a^6*b-60*C*a^5*b^2-760*C*
a^4*b^3-28360*C*a^3*b^4-104466*C*a^2*b^5-104304*C*a*b^6-27258*C*b^7)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-240*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^7+8775*b^7*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
```

*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))-17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^7-21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^7+2
40*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^7-17732*A*a^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+240*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6*b-520*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6*b+21021*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^6-39897*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^2*b^5+48165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2*b^5+13044*C*a*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-5070*a^2*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-3185*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4*b^3+39897*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3*b^4+3315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4
*b^3+17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^6-4
8165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^6+700*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b^2-3315*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^4-760*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b^2+520*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6*b+30669*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^4+1430*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^4*b^3+760*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),

```
(-2*b/(a-b))^(1/2))*a^4*b^3+1430*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^5*b^2-30669*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^2*b^5-13984*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^4-1430*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^5*b^2+16302*a*A*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2)))/b^4/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)(b\cos{(dx+c)} + a)^{\frac{5}{2}}\cos{(dx+c)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos{(dx+c)}^6 + \left(2\,Cab + Bb^2\right)\cos{(dx+c)}^5 + Aa^2\cos{(dx+c)}^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos{(dx+c)}^4 + \left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^6 + (2*C*a*b + B*b^2)*cos(d*x + c)^5 + A*a^2*c
os(d*x + c)^2 + (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^4 + (B*a^2 + 2*A*a*b
)*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1030 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=510

$$\frac{2\sin(c+dx)\left(8a^2C-22abB+99Ab^2+81b^2C\right)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\sin(c+dx)\left(110a^2bB-40a^3C-5ab^2(99A+\right.}{3465b^2d}$$

```
[Out] (-2*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*C - 15*a^3*b^2*(33*
A + 17*C) - 15*a*b^4*(319*A + 247*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ (2*(a^2 - b^2)*(110*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 75*b^4*(11*A + 9*
C) - 15*a^2*b^2*(33*A + 19*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(1
10*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 75*b^4*(11*A + 9*C) - 15*a^2*b^2*(33
*A + 19*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) - (2*(110*a
^2*b*B - 539*b^3*B - 40*a^3*C - 5*a*b^2*(99*A + 67*C))*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) + (2*(99*A*b^2 - 22*a*b*B + 8*a^2*C + 81*
b^2*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*b*B -
4*a*C)*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c + d
*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 1.09643, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(8a^2C-22abB+99Ab^2+81b^2C\right)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\sin(c+dx)\left(110a^2bB-40a^3C-5ab^2(99A+\right.}{3465b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```



```
[Out] (-2*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*C - 15*a^3*b^2*(33*
A + 17*C) - 15*a*b^4*(319*A + 247*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ (2*(a^2 - b^2)*(110*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 75*b^4*(11*A + 9*
C) - 15*a^2*b^2*(33*A + 19*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(1
```

```
10*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 75*b^4*(11*A + 9*C) - 15*a^2*b^2*(33
*A + 19*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) - (2*(110*a
^2*b*B - 539*b^3*B - 40*a^3*C - 5*a*b^2*(99*A + 67*C))*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) + (2*(99*A*b^2 - 22*a*b*B + 8*a^2*C + 81*
b^2*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*b*B -
4*a*C)*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*C*Cos[c + d
*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c}{11bd}$$

$$= \frac{2(11bB-4aC)(a+b\cos(c+dx))^{7/2}\sin(c}{99b^2d}$$

$$= \frac{2\left(99Ab^2-22abB+8a^2C+81b^2C\right)(a+}{693b^2d}$$

$$= -\frac{2\left(110a^2bB-539b^3B-40a^3C-5ab^2(9}{3}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-75b^4\right.}{}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-75b^4\right.}{}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-75b^4\right.}{}$$

$$= -\frac{2\left(110a^4bB-3069a^2b^3B-1617b^5B-4\right.}{}$$

Mathematica [A] time = 2.59852, size = 405, normalized size = 0.79

$$\frac{b(a+b\cos(c+dx))\left(\sin(c+dx)\left(60a^2b^2(396A+311C)+880a^3bB-320a^4C+32868ab^3B+30b^4(506A+435C)\right)+b\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(1705*a^3*b*B + 2871*a*b^3*B +
10*a^4*C + 75*b^4*(11*A + 9*C) + 15*a^2*b^2*(297*A + 221*C))*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)] + (-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*
a^5*C + 15*a^3*b^2*(33*A + 17*C) + 15*a*b^4*(319*A + 247*C))*((a + b)*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
```

```
) + b*(a + b*Cos[c + d*x])*((880*a^3*b*B + 32868*a*b^3*B - 320*a^4*C + 60*a
^2*b^2*(396*A + 311*C) + 30*b^4*(506*A + 435*C))*Sin[c + d*x] + b*(4*(1650*
a^2*b*B + 1463*b^3*B + 30*a^3*C + 5*a*b^2*(594*A + 619*C))*Sin[2*(c + d*x)]
 + 5*b*((396*A*b^2 + 836*a*b*B + 452*a^2*C + 513*b^2*C)*Sin[3*(c + d*x)] +
7*b*((22*b*B + 46*a*C)*Sin[4*(c + d*x)] + 9*b*C*Sin[5*(c + d*x)])))))/(2772
0*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.078, size = 2603, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((7920*
A*b^6+22880*B*a*b^5+24640*B*b^6+21920*C*a^2*b^4+71680*C*a*b^5+56880*C*b^6)*
sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-15840*A*a*b^5-11880*A*b^6-14960*B
*a^2*b^4-34320*B*a*b^5-22792*B*b^6-4640*C*a^3*b^3-32880*C*a^2*b^4-66160*C*a
*b^5-34920*C*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(11880*A*a^2*b^4+
15840*A*a*b^5+9240*A*b^6+3520*B*a^3*b^3+14960*B*a^2*b^4+26488*B*a*b^5+10472
*B*b^6-20*C*a^4*b^2+4640*C*a^3*b^3+25120*C*a^2*b^4+30320*C*a*b^5+13860*C*b^
6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2970*A*a^3*b^3-5940*A*a^2*b^4-
5610*A*a*b^5-2640*A*b^6-110*B*a^4*b^2-1760*B*a^3*b^3-7326*B*a^2*b^4-7524*B*
a*b^5-1848*B*b^6+40*C*a^5*b+10*C*a^4*b^2-3210*C*a^3*b^3-7080*C*a^2*b^4-6690
*C*a*b^5-2790*C*b^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-255*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+3705*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-3705*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-245*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-390*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b^4-495*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3*b^3+4785*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^2*b^4-4785*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a*b^5-495*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
```

```
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^4*b^2-330*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4
-40*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+255*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2+20160*C*b^6*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+495*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^6+675*b^6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))+825*A*b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))+40*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-40*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-3069*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+(-12320*B*b^6-35840*C*a
*b^5-50400*C*b^6)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+110*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-110*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+1617*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5+110*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^5*b-1364*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3+1254*a*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3069*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^3*b^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^5+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^4+Aa^2\cos\left(dx+c\right)+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^3+\left(B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5 + (2*C*a*b + B*b^2)*cos(d*x + c)^4 + A*a^2*c
os(d*x + c) + (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3 + (B*a^2 + 2*A*a*b)*
cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1031 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx$

Optimal. Leaf size=402

$$\frac{2\sin(c+dx)\left(-10a^2C+45abB+63Ab^2+49b^2C\right)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\sin(c+dx)\left(45a^2bB-10a^3C+6ab^2(28A\right.}{315bd}$$

```
[Out] (2*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(1
61*A + 93*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)
])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*
b*B + 75*b^3*B - 10*a^3*C + 6*a*b^2*(28*A + 19*C))*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 6*a*b^2*(28*A + 19*C))
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) + (2*(63*A*b^2 + 45*a*b*B
 - 10*a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d)
+ (2*(9*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2
*C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.755797, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(-10a^2C+45abB+63Ab^2+49b^2C\right)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\sin(c+dx)\left(45a^2bB-10a^3C+6ab^2(28A\right.}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(1
61*A + 93*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)
])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*
b*B + 75*b^3*B - 10*a^3*C + 6*a*b^2*(28*A + 19*C))*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 6*a*b^2*(28*A + 19*C))
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) + (2*(63*A*b^2 + 45*a*b*B
 - 10*a^2*C + 49*b^2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d)
+ (2*(9*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2
```

```
*C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx=\frac{2C(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}+\frac{2\int(a+b\,\mathrm{co}}{}$$

$$=\frac{2(9bB-2aC)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd}+\frac{2\mathrm{C}}{}$$

$$=\frac{2\left(63Ab^2+45abB-10a^2C+49b^2C\right)(a+b\cos(c+}{315bd}$$

$$=\frac{2\left(45a^2bB+75b^3B-10a^3C+6ab^2(28A+19C)\right)\sqrt{}}{315bd}$$

$$=\frac{2\left(45a^2bB+75b^3B-10a^3C+6ab^2(28A+19C)\right)\sqrt{}}{315bd}$$

$$=\frac{2\left(45a^2bB+75b^3B-10a^3C+6ab^2(28A+19C)\right)\sqrt{}}{315bd}$$

$$=\frac{2\left(45a^3bB+435ab^3B-10a^4C+21b^4(9A+7C)+\right.}{315b^2d}$$

Mathematica [A] time = 1.83605, size = 327, normalized size = 0.81

$$\frac{b(a+b\cos(c+dx))\left(2\sin(c+dx)\left(540a^2bB+20a^3C+3ab^2(308A+249C)+345b^3B\right)+b\left(\sin(2(c+dx))\left(300a^2C+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(405*a^2*b*B + 75*b^3*B + 5*a^3*
(63*A + 31*C) + 3*a*b^2*(119*A + 87*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b
)] + (45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*
(161*A + 93*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*(2*(540*a^2*b*B +
345*b^3*B + 20*a^3*C + 3*a*b^2*(308*A + 249*C))*Sin[c + d*x] + b*((252*A*b^
2 + 540*a*b*B + 300*a^2*C + 266*b^2*C)*Sin[2*(c + d*x)] + 5*b*(2*(9*b*B + 1
9*a*C)*Sin[3*(c + d*x)] + 7*b*C*Sin[4*(c + d*x)]))))/(1260*b^2*d*Sqrt[a + b
*Cos[c + d*x]])
```

Maple [B] time = 1.022, size = 2143, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(168*a*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+483*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-483*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+189*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-124*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^4+10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4*b+279*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^3*b^2-279*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2
*b^3+147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-112
0*C*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+45*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-168*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
```

```
/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+75*b^5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*b^5+10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
5-10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-147*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5+(720*B*b^5+2080*C*a
*b^4+2240*C*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^5-1440*B
*a*b^4-1080*B*b^5-1360*C*a^2*b^3-3120*C*a*b^4-2072*C*b^5)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(1176*A*a*b^4+504*A*b^5+1080*B*a^2*b^3+1440*B*a*b^4+
840*B*b^5+320*C*a^3*b^2+1360*C*a^2*b^3+2408*C*a*b^4+952*C*b^5)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-462*A*a^2*b^3-588*A*a*b^4-126*A*b^5-270*B*a^3
*b^2-540*B*a^2*b^3-510*B*a*b^4-240*B*b^5-10*C*a^4*b-160*C*a^3*b^2-666*C*a^2
*b^3-684*C*a*b^4-168*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-45*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+435*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-435*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-45*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-30*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b^3)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
), x)
```

3.1032 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=383

$$\frac{2\sin(c+dx)\left(15a^2C+56abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-10a^2b^2(7A-C)+56a^3bB+15a^4C-56ab^3\right.}{105bd\sqrt{a+}}$$

```
[Out] (2*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 5*a*b^2*(49*A + 29*C))*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*(56*a^3*b*B - 56*a*b^3*B - 10*a^2*b^2*(7*A - C)
+ 15*a^4*C - 5*b^4*(7*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^3*
A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*(35*A*b^2 + 56*a*b*B + 15*a^2*C + 25
*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*b*B + 5*a*C)
*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a + b*Cos[c + d*x]
)^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.3781, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\sin(c+dx)\left(15a^2C+56abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-10a^2b^2(7A-C)+56a^3bB+15a^4C-56ab^3\right.}{105bd\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```


```
[Out] (2*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 5*a*b^2*(49*A + 29*C))*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*(56*a^3*b*B - 56*a*b^3*B - 10*a^2*b^2*(7*A - C)
+ 15*a^4*C - 5*b^4*(7*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^3*
A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*(35*A*b^2 + 56*a*b*B + 15*a^2*C + 25
```

```
*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*b*B + 5*a*C)
*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*C*(a + b*Cos[c + d*x]
)^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{2}{7}$$

$$= \frac{2(7bB+5aC)(a+b\cos(c+dx))^{3/2}\sin(c}{35d}$$

$$= \frac{2\left(35Ab^2+56abB+15a^2C+25b^2C\right)\sqrt{a}}{105d}$$

$$= \frac{2\left(35Ab^2+56abB+15a^2C+25b^2C\right)\sqrt{a}}{105d}$$

$$= \frac{2\left(35Ab^2+56abB+15a^2C+25b^2C\right)\sqrt{a}}{105d}$$

$$= \frac{2\left(161a^2bB+63b^3B+15a^3C+5ab^2(49A\right.}{105bd\sqrt{}}$$

$$= \frac{2\left(161a^2bB+63b^3B+15a^3C+5ab^2(49A\right.}{105bd\sqrt{}}$$

Mathematica [C] time = 4.03676, size = 526, normalized size = 1.37

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}\left(90a^2C+6b(15aC+7bB)\cos(c+dx)+154abB+70Ab^2+15b^2C\cos(2(c+dx))+65b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x],x]
```

```
[Out] ((4*(105*a^3*B + 119*a*b^2*B + 45*a^2*b*(7*A + 3*C) + 5*b^3*(7*A + 5*C))*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqr
t[a + b*Cos[c + d*x]] + (2*(161*a^2*b*B + 63*b^3*B + 15*a^3*(14*A + C) + 5*
a*b^2*(49*A + 29*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(161*a^2*b*B + 63
*b^3*B + 15*a^3*C + 5*a*b^2*(49*A + 29*C))*Sqrt[-((b*(-1 + Cos[c + d*x]))/(
a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)
```

```
*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)
/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[
c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a +
 b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqrt[-(a +
b)^(-1)]) + 2*Sqrt[a + b*Cos[c + d*x]]*(70*A*b^2 + 154*a*b*B + 90*a^2*C + 6
5*b^2*C + 6*b*(7*b*B + 15*a*C)*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Si
n[c + d*x])/(210*d)
```

Maple [B] time = 1.082, size = 1713, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-480*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4+392*B*a*b^3+168*B*b^4
+360*C*a^2*b^2+480*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+(-70*A*a*b^3-70*A*b^4-154*B*a^2*b^2-196*B*a*b^3-42*B*b^4-90*C*a^3*b-180*
C*a^2*b^2-170*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+245*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-245*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-105*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b-56*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^3*b+56*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*b^3+161*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*a^3*b-161*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a^2*b
^2+63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a*b^3-63*B*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^4-15*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-10*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^4-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^3*b+145*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b
^2-145*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c), x)
```

3.1033 $\int (a+b\cos(c+dx))^{5/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^2(c + dx)\, dx$

Optimal. Leaf size=357

$$\frac{\left(a^3(15A-16C)+20a^2bB+4ab^2(15A+4C)+10b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{\left(a^2(-(15A-46C))+70abB+\cdots\right.}{}$$

```
[Out] ((70*a*b*B - a^2*(15*A - 46*C) + 6*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]) + ((20*a^2*b*B + 10*b^3*B + a^3*(15*A - 16*C) + 4*a*b^2*(15*A + 4*C
))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5*A*b + 2*a*B)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*C
os[c + d*x]]) - (b*(15*a*A - 10*b*B - 16*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(15*d) - (b*(5*A - 2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(5*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Tan[c + d*x])/d
```

Rubi [A] time = 1.40069, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^3(15A-16C)+20a^2bB+4ab^2(15A+4C)+10b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{\left(a^2(-(15A-46C))+70abB+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```


```
[Out] ((70*a*b*B - a^2*(15*A - 46*C) + 6*b^2*(5*A + 3*C))*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]) + ((20*a^2*b*B + 10*b^3*B + a^3*(15*A - 16*C) + 4*a*b^2*(15*A + 4*C
))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5*A*b + 2*a*B)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*C
os[c + d*x]]) - (b*(15*a*A - 10*b*B - 16*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(15*d) - (b*(5*A - 2*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
```

```
(5*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Tan[c + d*x])/d
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\tan(c+dx)}{d} + \int$$

$$= -\frac{b(5A-2C)(a+b\cos(c+dx))^{3/2}\sin(c}{5d}$$

$$= -\frac{b(15aA-10bB-16aC)\sqrt{a+b\cos(c}}{15d}$$

$$= -\frac{b(15aA-10bB-16aC)\sqrt{a+b\cos(c}}{15d}$$

$$= -\frac{b(15aA-10bB-16aC)\sqrt{a+b\cos(c}}{15d}$$

$$= \frac{\left(70abB-a^2(15A-46C)+6b^2(5A+3\right.}{15d\sqrt{\frac{a+}{}}}$$

$$= \frac{\left(70abB-a^2(15A-46C)+6b^2(5A+3\right.}{15d\sqrt{\frac{a+}{}}}$$

Mathematica [C] time = 4.15924, size = 502, normalized size = 1.41

$$\frac{\frac{8\left(45a^2bB+15a^3C+ab^2(45A+17C)+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2b(135A+46C)+60a^3B+70ab^2B+6b^3(5A+3C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{\sqrt{a+b\cos(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^2,x]
```

```
[Out] ((8*(45*a^2*b*B + 5*b^3*B + 15*a^3*C + a*b^2*(45*A + 17*C))*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] + (2*(60*a^3*B + 70*a*b^2*B + 6*b^3*(5*A + 3*C) + a^2*b*(135*A + 4
6*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(70*a*b*B + 6*b^2*(5*A + 3*C) +
```

```
a^2*(-15*A + 46*C))*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 +
 Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 60*Sqrt[a + b
*Cos[c + d*x]]*((2*b*(5*b*B + 11*a*C)*Sin[c + d*x])/15 + (b^2*C*Sin[2*(c +
d*x)])/5 + a^2*A*Tan[c + d*x]))/(60*d)
```

Maple [B] time = 1.233, size = 2274, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-80*B*b^3-224*C*a*b^2-144*C*b^3)*
sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(60*A*a^2*b+40*B*a*b^2+80*B*b^3+88*
C*a^2*b+224*C*a*b^2+72*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-30*
A*a^3-30*A*a^2*b-20*B*a*b^2-20*B*b^3-44*C*a^2*b-56*C*a*b^2-12*C*b^3)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^3+60*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b^2-15*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+15*A*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+30*A*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-30*A*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*b^3-75*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/
2))*a^2*b+20*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+10*B*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+70*B*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-70*B*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a*b^2-30*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*a^3-16*C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+16*C*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+46*C*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-46*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^2*b+18*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
*b^2-18*C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)*sin(1/2*d*x
+1/2*c)^2+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+6
0*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-15*A*(sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+15*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+30*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(
a-b))^(1/2))*a^2*b+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b+10*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+7
0*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-70*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-30*a^3*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-16*a^3*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+16*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))+46*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^3-46*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b+18*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-1
8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^2, x)
```

3.1034 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=372

$$\frac{\left(a^2b(33A+16C)+12a^3B+48ab^2B+8b^3(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12d\sqrt{a+b\cos(c+dx)}}-\frac{\left(12a^2B+ab(27A-56C)-24b^2\right.}{12d\sqrt{}}$$

```
[Out] -((12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*b)/(a + b)])/(12*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ ((12*a^3*B + 48*a*b^2*B + 8*b^3*(3*A + C) + a^2*b*(33*A + 16*C))*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(12*d*Sqr
t[a + b*Cos[c + d*x]]) + (a*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d
*Sqrt[a + b*Cos[c + d*x]]) - (b*(21*A*b + 12*a*B - 8*b*C)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(12*d) + ((5*A*b + 4*a*B)*(a + b*Cos[c + d*x])^(3/2)*
Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]*Tan[c + d*
x])/(2*d)
```

Rubi [A] time = 1.44759, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2b(33A+16C)+12a^3B+48ab^2B+8b^3(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{12d\sqrt{a+b\cos(c+dx)}}-\frac{\left(12a^2B+ab(27A-56C)-24b^2\right.}{12d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```


```
[Out] -((12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*b)/(a + b)])/(12*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ ((12*a^3*B + 48*a*b^2*B + 8*b^3*(3*A + C) + a^2*b*(33*A + 16*C))*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(12*d*Sqr
t[a + b*Cos[c + d*x]]) + (a*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d
*Sqrt[a + b*Cos[c + d*x]]) - (b*(21*A*b + 12*a*B - 8*b*C)*Sqrt[a + b*Cos[c
```

```
+ d*x]]*Sin[c + d*x])/(12*d) + ((5*A*b + 4*a*B)*(a + b*Cos[c + d*x])^(3/2)*
Tan[c + d*x])/(4*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]*Tan[c + d*
x])/(2*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec(c+dx)\tan(c}{2d}$$

$$= \frac{(5Ab+4aB)(a+b\cos(c+dx))^{3/2}\tan(c}{4d}$$

$$= -\frac{b(21Ab+12aB-8bC)\sqrt{a+b\cos(c+d}}{12d}$$

$$= -\frac{b(21Ab+12aB-8bC)\sqrt{a+b\cos(c+d}}{12d}$$

$$= -\frac{b(21Ab+12aB-8bC)\sqrt{a+b\cos(c+d}}{12d}$$

$$= -\frac{\left(27aAb+12a^2B-24b^2B-56abC\right)\sqrt{a}}{12d\sqrt{\frac{a+b\cos}{a+}}}$$

$$= -\frac{\left(27aAb+12a^2B-24b^2B-56abC\right)\sqrt{a}}{12d\sqrt{\frac{a+b\cos}{a+}}}$$

Mathematica [C] time = 5.0116, size = 492, normalized size = 1.32

$$\frac{\frac{8b\left(3a^2(A+12C)+36abB+4b^2(3A+C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(24a^3(A+2C)+108a^2bB+7ab^2(9A+8C)+24b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(36*a*b*B + 4*b^2*(3*A + C) + 3*a^2*(A + 12*C))*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x
]] + (2*(108*a^2*b*B + 24*b^3*B + 24*a^3*(A + 2*C) + 7*a*b^2*(9*A + 8*C))*S
```

```
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]
)/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-27*a*A*b - 12*a^2*B + 24*b^2*B + 56*a
*b*C)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/
(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*Ellipt
icPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
 + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Se
c[c + d*x]*(3*a*(9*A*b + 4*a*B)*Sin[c + d*x] + 4*b^2*C*Sin[2*(c + d*x)] + 6
*a^2*A*Tan[c + d*x]))/(48*d)
```

Maple [B] time = 2.752, size = 2375, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*b^3*(-1
/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6*b)
*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))))-(2*B*b^3+6*C*a*b^2-4*C*b^3)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*A*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
```

```
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*C*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))+2*a^2*(3*A*b+B*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2)))-2*a*(3*A*b^2+3*B*a*b+C*a^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2))+2*A*a^3*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*
d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b
/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^3, x)
```

3.1035 $\int (a+b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx)\, dx$

Optimal. Leaf size=407

$$\frac{\tan(c+dx)\left(8a^2(2A+3C)+42abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(8a^3(2A+3C)+66a^2bB+ab^2(59A+96C)+48b\right.}{24d\sqrt{a+b\cos(c+}}$$

```
[Out] -((54*a*b*B + 3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) + ((66*a^2*b*B + 48*b^3*B + 8*a^3*(2*A + 3*C) + a*b^2*(59*A + 96
*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*A*b^3 + 8*a^3*B + 30*a*b^2*B + 20
*a^2*b*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x
)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 + 42*a*b*B
 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d) + ((5*A
*b + 6*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(12*d) +
(A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.57094, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\left(8a^2(2A+3C)+42abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(8a^3(2A+3C)+66a^2bB+ab^2(59A+96C)+48b\right.}{24d\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```


```
[Out] -((54*a*b*B + 3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) + ((66*a^2*b*B + 48*b^3*B + 8*a^3*(2*A + 3*C) + a*b^2*(59*A + 96
*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*A*b^3 + 8*a^3*B + 30*a*b^2*B + 20
*a^2*b*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x
)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 + 42*a*b*B
```

```
 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d) + ((5*A
*b + 6*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(12*d) +
(A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\tan(}{3d}$$

$$= \frac{(5Ab+6aB)(a+b\cos(c+dx))^{3/2}\sec(c}{12d}$$

$$= \frac{\left(15Ab^2+42abB+8a^2(2A+3C)\right)\sqrt{a+}}{24d}$$

$$= \frac{\left(15Ab^2+42abB+8a^2(2A+3C)\right)\sqrt{a+}}{24d}$$

$$= \frac{\left(15Ab^2+42abB+8a^2(2A+3C)\right)\sqrt{a+}}{24d}$$

$$= -\frac{\left(54abB+3b^2(11A-16C)+8a^2(2A+\right.}{24d\sqrt{\frac{a-}{}}}$$

$$= -\frac{\left(54abB+3b^2(11A-16C)+8a^2(2A+\right.}{24d\sqrt{\frac{a-}{}}}$$

Mathematica [C] time = 6.1693, size = 519, normalized size = 1.28

$$\frac{\frac{8b\left(6a^2B+ab(13A+72C)+24b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2b(13A+27C)+48a^3B+126ab^2B-3b^3(A-16C)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^4,x]
```

```
[Out] ((8*b*(6*a^2*B + 24*b^2*B + a*b*(13*A + 72*C))*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2
*(48*a^3*B + 126*a*b^2*B - 3*b^3*(A - 16*C) + 8*a^2*b*(13*A + 27*C))*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqr
t[a + b*Cos[c + d*x]] - ((2*I)*(54*a*b*B + 3*b^2*(11*A - 16*C) + 8*a^2*(2*A
 + 3*C))*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]
))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
```

```
-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^2*(2*a*(13*A*b + 6*a*B)*Sin[c + d*x] + ((33*A*b^2)/2 + 27*a*b
*B + 4*a^2*(2*A + 3*C))*Sin[2*(c + d*x)] + 8*a^2*A*Tan[c + d*x]))/(96*d)
```

Maple [B] time = 3.318, size = 2791, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C*b^2*(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*C*a*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a*(3*A
*b^2+3*B*a*b+C*a^2)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*b
*(A*b^2+3*B*a*b+3*C*a^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*a^3*(-1/3
```

```
/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^
3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a^2*(3*A*b+B*a)*(-1/2/a*cos(1
/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*
b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2
*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^4,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^4, x)
```

3.1036 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=502

$$\frac{\tan(c+dx)\left(4a^2b(71A+108C)+128a^3B+264ab^2B+15Ab^3\right)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{\left(4a^2b(89A+132C)+128a^3B+\cdots\right.}{\cdots}$$

```
[Out] -((15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*b*(71*A + 108*C))*Sqrt[a + b*
Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) + ((128*a^3*B + 472*a*b^2*B + 4*a^2*b*(89*A + 132*C)
 + b^3*(133*A + 384*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(192*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b^4 - 160*a
^3*b*B - 40*a*b^3*B - 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a*
d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*
b*(71*A + 108*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192*a*d) + ((5*A*
b^2 + 24*a*b*B + 4*a^2*(3*A + 4*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*T
an[c + d*x])/(32*d) + ((5*A*b + 8*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d
*x]^2*Tan[c + d*x])/(24*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3*T
an[c + d*x])/(4*d)
```

Rubi [A] time = 2.10998, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\left(4a^2b(71A+108C)+128a^3B+264ab^2B+15Ab^3\right)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{\left(4a^2b(89A+132C)+128a^3B+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```


```
[Out] -((15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*b*(71*A + 108*C))*Sqrt[a + b*
Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) + ((128*a^3*B + 472*a*b^2*B + 4*a^2*b*(89*A + 132*C)
 + b^3*(133*A + 384*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
```

```
*x)/2, (2*b)/(a + b)])/(192*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b^4 - 160*a
^3*b*B - 40*a*b^3*B - 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(64*a*
d*Sqrt[a + b*Cos[c + d*x]]) + ((15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*
b*(71*A + 108*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192*a*d) + ((5*A*
b^2 + 24*a*b*B + 4*a^2*(3*A + 4*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*T
an[c + d*x])/(32*d) + ((5*A*b + 8*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d
*x]^2*Tan[c + d*x])/(24*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3*T
an[c + d*x])/(4*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\tan(}{4d}$$

$$= \frac{(5Ab+8aB)(a+b\cos(c+dx))^{3/2}\sec^2(c}{24d}$$

$$= \frac{\left(5Ab^2+24abB+4a^2(3A+4C)\right)\sqrt{a+b}}{32d}$$

$$= \frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(71A\right.}{19}$$

$$= \frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(71A\right.}{19}$$

$$= \frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(71A\right.}{19}$$

$$= -\frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(71\right.}{192a}$$

$$= -\frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(71\right.}{192a}$$

Mathematica [C] time = 7.0633, size = 792, normalized size = 1.58

$$\sqrt{a+b\cos(c+dx)}\left(\frac{1}{96}\sec^2(c+dx)\left(36a^2A\sin(c+dx)+48a^2C\sin(c+dx)+104abB\sin(c+dx)+59Ab^2\sin(c+dx)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^5,x]
```

```
[Out] ((2*(144*a^3*A*b + 236*a*A*b^3 + 416*a^2*b^2*B + 192*a^3*b*C + 768*a*b^3*C)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/
Sqrt[a + b*Cos[c + d*x]] + (2*(288*a^4*A + 436*a^2*A*b^2 - 45*A*b^4 + 832*a
^3*b*B - 24*a*b^3*B + 384*a^4*C + 1008*a^2*b^2*C)*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x
]] - ((2*I)*(-284*a^2*A*b^2 - 15*A*b^4 - 128*a^3*b*B - 264*a*b^3*B - 432*a^
2*b^2*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a
- b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ellipti
cPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
+ b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^
2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b
^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/
(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(17*a*A*b*Sin[c + d*
x] + 8*a^2*B*Sin[c + d*x]))/24 + (Sec[c + d*x]^2*(36*a^2*A*Sin[c + d*x] + 5
9*A*b^2*Sin[c + d*x] + 104*a*b*B*Sin[c + d*x] + 48*a^2*C*Sin[c + d*x]))/96
+ (Sec[c + d*x]*(284*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 128*a^3
*B*Sin[c + d*x] + 264*a*b^2*B*Sin[c + d*x] + 432*a^2*b*C*Sin[c + d*x]))/(19
2*a) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/4))/d
```

Maple [B] time = 4.656, size = 3673, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*A*a^3*(-1/4/a*cos(1/2*d*x+1/2*c)*(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^4+7/24*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3-1/96*(36*a^2+35*b^2)/a^3
*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b*(20*a^2+21*b^2)/a^4*cos(1/2*d*x+
1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-35/384*b^3/a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-25/96*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35/128/a^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/16/a^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/
(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^4)+2*b*
(A*b^2+3*B*a*b+3*C*a^2)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1

/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))
-2*b^2*(B*b+3*C*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^2*(3*A*b+B*a)*(
-1/3/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
c)^2-1)^2-1/24(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^
2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/1
6*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a*(3*A*b^2+3*B*a*b+C*a^2)
*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(

```
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**5,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^5,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^5, x)
```

3.1037 $\int (a+b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=624

$$-\frac{\tan(c+dx)\left(-12a^2b^2(141A+220C)-256a^4(4A+5C)-2840a^3bB-150ab^3B+45Ab^4\right)\sqrt{a+b\cos(c+dx)}}{1920a^2d} - \frac{\left(-4a^2b\right.}{}$$

```
[Out] ((45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*(4*A + 5*C) - 12*a^2*b^2*
(141*A + 220*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(1920*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((15*A*b^4 - 3560*a
^3*b*B - 1330*a*b^3*B - 256*a^4*(4*A + 5*C) - 4*a^2*b^2*(809*A + 1180*C))*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(1
920*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^5 + 96*a^5*B + 240*a^3*b^2*B -
10*a*b^4*B + 40*a^2*b^3*(A + 2*C) + 80*a^4*b*(3*A + 4*C))*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(128*a^2*d*Sqr
t[a + b*Cos[c + d*x]]) - ((45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*
(4*A + 5*C) - 12*a^2*b^2*(141*A + 220*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
d*x])/(1920*a^2*d) + ((15*A*b^3 + 360*a^3*B + 590*a*b^2*B + 4*a^2*b*(193*A
+ 260*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(960*a*d) + (
(15*A*b^2 + 110*a*b*B + 16*a^2*(4*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c
+ d*x]^2*Tan[c + d*x])/(240*d) + ((A*b + 2*a*B)*(a + b*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c +
d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 2.71027, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\tan(c+dx)\left(-12a^2b^2(141A+220C)-256a^4(4A+5C)-2840a^3bB-150ab^3B+45Ab^4\right)\sqrt{a+b\cos(c+dx)}}{1920a^2d} - \frac{\left(-4a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^6,x]
```

```
[Out] ((45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*(4*A + 5*C) - 12*a^2*b^2*
(141*A + 220*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
```

```
b)])/(1920*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((15*A*b^4 - 3560*a
^3*b*B - 1330*a*b^3*B - 256*a^4*(4*A + 5*C) - 4*a^2*b^2*(809*A + 1180*C))*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(1
920*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*A*b^5 + 96*a^5*B + 240*a^3*b^2*B -
10*a*b^4*B + 40*a^2*b^3*(A + 2*C) + 80*a^4*b*(3*A + 4*C))*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(128*a^2*d*Sqr
t[a + b*Cos[c + d*x]]) - ((45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*
(4*A + 5*C) - 12*a^2*b^2*(141*A + 220*C))*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
d*x])/(1920*a^2*d) + ((15*A*b^3 + 360*a^3*B + 590*a*b^2*B + 4*a^2*b*(193*A
+ 260*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(960*a*d) + (
(15*A*b^2 + 110*a*b*B + 16*a^2*(4*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c
+ d*x]^2*Tan[c + d*x])/(240*d) + ((A*b + 2*a*B)*(a + b*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (A*(a + b*Cos[c + d*x])^(5/2)*Sec[c +
d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{A(a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\tan(}{5d}$$

$$= \frac{(Ab+2aB)(a+b\cos(c+dx))^{3/2}\sec^3(c+}{8d}$$

$$= \frac{\left(15Ab^2+110abB+16a^2(4A+5C)\right)\sqrt{a}}{24}$$

$$= \frac{\left(15Ab^3+360a^3B+590ab^2B+4a^2b(193\right.}{}$$

$$= -\frac{\left(45Ab^4-2840a^3bB-150ab^3B-256a^4\right.}{}$$

$$= -\frac{\left(45Ab^4-2840a^3bB-150ab^3B-256a^4\right.}{}$$

$$= -\frac{\left(45Ab^4-2840a^3bB-150ab^3B-256a^4\right.}{}$$

$$= \frac{\left(45Ab^4-2840a^3bB-150ab^3B-256a^4(\right.}{}$$

$$= \frac{\left(45Ab^4-2840a^3bB-150ab^3B-256a^4(\right.}{}$$

Mathematica [C] time = 7.23387, size = 930, normalized size = 1.49

$$\frac{\frac{2\left(1440bBa^4+3088Ab^2a^3+4160b^2Ca^3+2360b^3Ba^2+60Ab^4a\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2880Ba^5+6176Aba^4+8320bCa^4+4360b^2Ba^3-492Ab^3a^2-240b^3\right.}{\sqrt{a+b\cos(c}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^6,x]
```

```
[Out] ((2*(3088*a^3*A*b^2 + 60*a*A*b^4 + 1440*a^4*b*B + 2360*a^2*b^3*B + 4160*a^3
*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6176*a^4*A*b - 492*a^2*A*b^3 + 135*A*
b^5 + 2880*a^5*B + 4360*a^3*b^2*B - 450*a*b^4*B + 8320*a^4*b*C - 240*a^2*b^
3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-1024*a^4*A*b - 1692*a^2*A*b^3 +
 45*A*b^5 - 2840*a^3*b^2*B - 150*a*b^4*B - 1280*a^4*b*C - 2640*a^2*b^3*C)*S
qrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos
[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
 b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b
)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a -
b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-(
(a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^
2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(7680*a^2*
d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^4*(21*a*A*b*Sin[c + d*x] + 10
*a^2*B*Sin[c + d*x]))/40 + (Sec[c + d*x]^3*(64*a^2*A*Sin[c + d*x] + 93*A*b^
2*Sin[c + d*x] + 170*a*b*B*Sin[c + d*x] + 80*a^2*C*Sin[c + d*x]))/240 + (Se
c[c + d*x]^2*(772*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 360*a^3*B*
Sin[c + d*x] + 590*a*b^2*B*Sin[c + d*x] + 1040*a^2*b*C*Sin[c + d*x]))/(960*
a) + (Sec[c + d*x]*(1024*a^4*A*Sin[c + d*x] + 1692*a^2*A*b^2*Sin[c + d*x] -
 45*A*b^4*Sin[c + d*x] + 2840*a^3*b*B*Sin[c + d*x] + 150*a*b^3*B*Sin[c + d*
x] + 1280*a^4*C*Sin[c + d*x] + 2640*a^2*b^2*C*Sin[c + d*x]))/(1920*a^2) + (
a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/5))/d
```

Maple [B] time = 6.298, size = 5171, normalized size = 8.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^6,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^6, x)
```

3.1038 $\int (a+b\cos(c+dx))^{3/2}\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\right.$

Optimal. Leaf size=285

$$\frac{2b\left(-41a^2C+56abB+25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(-41a^2C+56abB+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\Big(}{105d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(161*a^2*b*B + 63*b^3*B - 146*a^3*C + 82*a*b^2*C)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]) - (2*(a^2 - b^2)*(56*a*b*B - 41*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*d*Sqrt[a +
b*Cos[c + d*x]]) + (2*b*(56*a*b*B - 41*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*d) + (2*b*(7*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*b*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/
(7*d)
```

Rubi [A] time = 0.6555, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3015, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-41a^2C+56abB+25b^2C\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(-41a^2C+56abB+25b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\Big(}{105d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*
Cos[c + d*x]^2),x]
```


```
[Out] (2*(161*a^2*b*B + 63*b^3*B - 146*a^3*C + 82*a*b^2*C)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]) - (2*(a^2 - b^2)*(56*a*b*B - 41*a^2*C + 25*b^2*C)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*d*Sqrt[a +
b*Cos[c + d*x]]) + (2*b*(56*a*b*B - 41*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(105*d) + (2*b*(7*b*B - 2*a*C)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*b*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/
(7*d)
```


Rule 3015

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)\right)dx=\frac{\int (a+b\cos(c+dx))^{5/2}\left(b^2(bB-aC)+b\right.}{b^2}$$

$$=\frac{2bC(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+$$

$$=\frac{2b(7bB-2aC)(a+b\cos(c+dx))^{3/2}\sin(}{35d}$$

$$=\frac{2b\left(56abB-41a^2C+25b^2C\right)\sqrt{a+b\cos}}{105d}$$

$$=\frac{2b\left(56abB-41a^2C+25b^2C\right)\sqrt{a+b\cos}}{105d}$$

$$=\frac{2b\left(56abB-41a^2C+25b^2C\right)\sqrt{a+b\cos}}{105d}$$

$$=\frac{2\left(161a^2bB+63b^3B-146a^3C+82ab^2C\right.}{105d\sqrt{\frac{a+b}{}}}$$

Mathematica [A] time = 1.22072, size = 259, normalized size = 0.91

$$\frac{b\sin(c+dx)(a+b\cos(c+dx))\left(-64a^2C+6b(8aC+7bB)\cos(c+dx)+154abB+15b^2C\cos(2(c+dx))+65b^2C\right)+2\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] +
b^2*C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(105*a^3*b*B + 119*a*b^3*B - 105*a^4*C + 16*a^2*b^2*C + 25*b^4*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*(161
*a^2*b*B + 63*b^3*B - 146*a^3*C + 82*a*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x
)/2, (2*b)/(a + b)]) + b*(a + b*Cos[c + d*x])*(154*a*b*B - 64*a^2*C + 65*b^
```

```
2*C + 6*b*(7*b*B + 8*a*C)*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c +
 d*x])/(105*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.026, size = 1302, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2
),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4-312*C*a*b^3-360*C*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(392*B*a*b^3+168*B*b^4-32*C*a^2*
b^2+312*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-154*B*
a^2*b^2-196*B*a*b^3-42*B*b^4+64*C*a^3*b+16*C*a^2*b^2-128*C*a*b^3-80*C*b^4)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3*b+56*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*b^3+161*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*a^3*b-161*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
a^2*b^2+63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a*b^3-6
3*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^4+41*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-66*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*C*b^4*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-146*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^4+146*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^3*b+82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^2*b^2-82*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)
```

```
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^2 \cos(dx+c)^2 + Bb^2 \cos(dx+c) - Ca^2 + Bab\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)*(b*co
s(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3 \cos(dx+c)^3 - Ca^3 + Ba^2b + \left(Cab^2 + Bb^3\right)\cos(dx+c)^2 - \left(Ca^2b - 2\,Bab^2\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c) -}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^3 - C*a^3 + B*a^2*b + (C*a*b^2 + B*b^3)*cos(d*
x + c)^2 - (C*a^2*b - 2*B*a*b^2)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*co
s(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1039 $\int \sqrt{a+b\cos(c+dx)}\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\right.$

Optimal. Leaf size=221

$$-\frac{2\left(a^2-b^2\right)(5bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-17a^2C+20abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*(20*a*b*B - 17*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2
 - b^2)*(5*b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*b*B - 2*a*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*C*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.493857, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3015, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(5bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-17a^2C+20abB+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Co
s[c + d*x]^2),x]
```


```
[Out] (2*(20*a*b*B - 17*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2
 - b^2)*(5*b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*b*B - 2*a*
C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*C*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3015

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Sin[e + f*x], x], x],
```

```
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)\right)dx = \frac{\int (a+b\cos(c+dx))^{3/2}\left(b^2(bB-aC)+b^3C\right.}{b^2}$$

$$= \frac{2bC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{}$$

$$= \frac{2b(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+}{15d}$$

$$= \frac{2b(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+}{15d}$$

$$= \frac{2b(5bB-2aC)\sqrt{a+b\cos(c+dx)}\sin(c+}{15d}$$

$$= \frac{2\left(20abB-17a^2C+9b^2C\right)\sqrt{a+b\cos(c+}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.00803, size = 178, normalized size = 0.81

$$\frac{2\left(a^2-b^2\right)(2aC-5bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a+b)\left(17a^2C-20abB-9b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^
2*C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*(a + b)*(-20*a*b*B + 17*a^2*C - 9*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*(a^2 - b^2)*(-5*b*B + 2*a*C)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] +
 2*b*(a + b*Cos[c + d*x])*(5*b*B + a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x])/
(15*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.032, size = 990, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2
),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b^3+16*C*a*b^2+24*C*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*B*a*b^2-10*B*b^3-2*C*a^2*b-8*C*a
*b^2-6*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a*b^2+2*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-2*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-17*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+17*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^2\cos\left(dx+c\right)^2+Bb^2\cos\left(dx+c\right)-Ca^2+Bab\right)\sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)*sqrt(
b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^2+Bb^2\cos\left(dx+c\right)-Ca^2+Bab\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)*sqrt(b
*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*co
s(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1040 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=344

$$\frac{2\sin(c+dx)\left(24a^2C-28abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^3d}-\frac{2\left(-2a^2b^2(35A+16C)+56a^3bB-48a^4C+49a\right.}{105b^4d\sqrt{a}}$$

```
[Out] (2*(56*a^2*b*B + 63*b^3*B - 48*a^3*C - 2*a*b^2*(35*A + 22*C))*Sqrt[a + b*Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) - (2*(56*a^3*b*B + 49*a*b^3*B - 48*a^4*C - 5*b^4*(7*
A + 5*C) - 2*a^2*b^2*(35*A + 16*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*(35*A*b^2 - 28*a*b*B + 24*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(105*b^3*d) + (2*(7*b*B - 6*a*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.714605, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(24a^2C-28abB+35Ab^2+25b^2C\right)\sqrt{a+b\cos(c+dx)}}{105b^3d}-\frac{2\left(-2a^2b^2(35A+16C)+56a^3bB-48a^4C+49a\right.}{105b^4d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos
[c + d*x]],x]
```

```
[Out] (2*(56*a^2*b*B + 63*b^3*B - 48*a^3*C - 2*a*b^2*(35*A + 22*C))*Sqrt[a + b*Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) - (2*(56*a^3*b*B + 49*a*b^3*B - 48*a^4*C - 5*b^4*(7*
A + 5*C) - 2*a^2*b^2*(35*A + 16*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*(35*A*b^2 - 28*a*b*B + 24*a^2*C + 25*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(105*b^3*d) + (2*(7*b*B - 6*a*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*C*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x
```

```
]]*Sin[c + d*x])/(7*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos(c+}{}}{}$$

$$= \frac{2(7bB-6aC)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35b^2d} +$$

$$= \frac{2\left(35Ab^2-28abB+24a^2C+25b^2C\right)\sqrt{a+b\cos(c+dx)}\sin}{105b^3d}$$

$$= \frac{2\left(35Ab^2-28abB+24a^2C+25b^2C\right)\sqrt{a+b\cos(c+dx)}\sin}{105b^3d}$$

$$= \frac{2\left(35Ab^2-28abB+24a^2C+25b^2C\right)\sqrt{a+b\cos(c+dx)}\sin}{105b^3d}$$

$$= \frac{2\left(56a^2bB+63b^3B-48a^3C-2ab^2(35A+22C)\right)\sqrt{a+b\cos}}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.2991, size = 252, normalized size = 0.73

$$\frac{b(a+b\cos(c+dx))\left(\sin(c+dx)\left(96a^2C-112abB+140Ab^2+115b^2C\right)+3b(2(7bB-6aC)\sin(2(c+dx))+5bC\sin(3(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a +
 b*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(35*A*b^2 + 14*a*b*B - 12*a^2*C
+ 25*b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-56*a^2*b*B - 63*b^3*B
 + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/
(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x
])*((140*A*b^2 - 112*a*b*B + 96*a^2*C + 115*b^2*C)*Sin[c + d*x] + 3*b*(2*(7
*b*B - 6*a*C)*Sin[2*(c + d*x)] + 5*b*C*Sin[3*(c + d*x)])))/(210*b^4*d*Sqrt[
a + b*Cos[c + d*x]])
```

Maple [B] time = 1.027, size = 1635, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^4+24*C*a*b^3-360*C*b^4
)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^4-28*B*a*b^3+168*B*b^4+2
4*C*a^2*b^2-24*C*a*b^3+280*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(
-70*A*a*b^3-70*A*b^4+56*B*a^2*b^2+14*B*a*b^3-42*B*b^4-48*C*a^3*b-12*C*a^2*b
^2-44*C*a*b^3-80*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-70*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+70*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3+70*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+35*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+56*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*a^3*b-56*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*a^2*b^2+63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*a
*b^3-63*B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b^4-56*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b-49*B*a*(sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-48*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3*b-44*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2*b^2+44*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a*b^3+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^4+32*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*
C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^4/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*
sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^4+B\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)/sqrt(b*co
s(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos
(d*x + c) + a), x)
```

3.1041 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=258

$$\frac{2\left(10a^2bB-8a^3C-ab^2(15A+7C)+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C-10abB+15Ab^2+9b^2C\right)\sqrt{a+}}{15b^3d\sqrt{\frac{a+b\cos}{a+}}}$$

```
[Out] (2*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) + (2*(10*a^2*b*B + 5*b^3*B - 8*a^3*C - a*b^2*(15*A + 7*C))*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(15*b^2*d) + (2*C*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(5*b*d)
```

Rubi [A] time = 0.423549, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(10a^2bB-8a^3C-ab^2(15A+7C)+5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C-10abB+15Ab^2+9b^2C\right)\sqrt{a+}}{15b^3d\sqrt{\frac{a+b\cos}{a+}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b*Cos[c
 + d*x]],x]
```


```
[Out] (2*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) + (2*(10*a^2*b*B + 5*b^3*B - 8*a^3*C - a*b^2*(15*A + 7*C))*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(15*b^2*d) + (2*C*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(5*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{2\int \frac{aC+\frac{1}{2}b(5A}{}}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+d}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+d}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2C\cos(c+d}{}$$

$$= \frac{2\left(15Ab^2-10abB+8a^2C+9b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.06969, size = 186, normalized size = 0.72

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(8a^2C-10abB+15Ab^2+9b^2C\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(2aC+5bB)F\left(\frac{1}{2}\right.\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b
*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*b*B + 2*a*C)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*((a + b)*
EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])) + 2*b*(a + b*Cos[c + d*x])*(5*b*B - 4*a*C + 3*b*C*Cos[c + d*x])*Sin[
c + d*x])/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.251, size = 1258, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-9*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^2+7*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^2+10*B*cos(1/2*d*x+1/2*c)*a*b^2-10*B*cos(1/2*d*x+1/2*c)^3*a*b^
2-6*C*cos(1/2*d*x+1/2*c)^3*a*b^2-8*C*cos(1/2*d*x+1/2*c)*a^2*b+2*C*cos(1/2*d
*x+1/2*c)*a*b^2+4*C*cos(1/2*d*x+1/2*c)^5*a*b^2+8*C*cos(1/2*d*x+1/2*c)^3*a^2
*b-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-5*b^3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+9*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b^2-24*C*cos(1/2*d*x+1/2*c)^7*b^3+48*C*cos(1/2*d*x+1
/2*c)^5*b^3-30*C*cos(1/2*d*x+1/2*c)^3*b^3+6*C*cos(1/2*d*x+1/2*c)*b^3-10*B*c
os(1/2*d*x+1/2*c)*b^3-20*B*cos(1/2*d*x+1/2*c)^5*b^3+30*B*cos(1/2*d*x+1/2*c)
^3*b^3-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+10*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+15*a*A*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d
*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^3+B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))/sqrt(b*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d
*x + c) + a), x)
```

3.1042 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=188

$$\frac{2\left(2a^2C-3abB+3Ab^2+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+$$

```
[Out] (2*(3*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*A*b^2 - 3*a*b*
B + 2*a^2*C + b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.234596, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(2a^2C-3abB+3Ab^2+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*(3*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*A*b^2 - 3*a*b*
B + 2*a^2*C + b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(3*b*d)
```


Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\frac{1}{2}(3bB-2aC)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{(3bB-2aC)\int \sqrt{a+b\cos(c+dx)}\,dx}{3b^2}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left((3bB-2aC)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{}}{3b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(3A+C-\frac{a(3bB-2aC}{b^2}\right.}{3d\sqrt{}}$$

Mathematica [A] time = 0.754281, size = 160, normalized size = 0.85

$$\frac{2\left(2a^2C-3abB+3Ab^2+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a+b)(2aC-3bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],
x]
```

```
[Out] (-2*(a + b)*(-3*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] + 2*(3*A*b^2 - 3*a*b*B + 2*a^2*C + b^2*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*C*
(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.894, size = 740, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*b^2+2*C*cos(1/2*d*x+1/2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3*b^
2+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^2+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2
*C*cos(1/2*d*x+1/2*c)*a*b+2*C*cos(1/2*d*x+1/2*c)*b^2)/b^2/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a),
 x)
```

3.1043 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.449183, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b*Cos[c
 + d*x]],x]
```


```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(b*B - a*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{\int \frac{(-Ab-(bB-aC)\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}+\frac{C\int\sqrt{a+b\cos(c+dx)}$$

$$= A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx-\frac{(-bB+aC)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}+$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int}{\sqrt{a+b\cos}}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(bB-aC)\sqrt{\frac{a+b\cos}{a+}}}{bd\sqrt{a+b}}$$

Mathematica [F] time = 16.8939, size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b
*Cos[c + d*x]],x]
```

```
[Out] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[a + b
*Cos[c + d*x]], x]
```

Maple [A] time = 0.918, size = 275, normalized size = 1.5

$$2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b\sin\left(1/2\,dx+c/2\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}\sqrt{\frac{2\,(\cdots}{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b-b*B*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))+C*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{a + b\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(1/
2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(a + b*c
os(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)}{\sqrt{b\cos(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c) + a), x)
```

3.1044 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=220

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{(A+2C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}$$

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + ((A + 2*C)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]
) - ((A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.674512, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{(A+2C)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a + b*Cos
[c + d*x]],x]
```

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + ((A + 2*C)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]
) - ((A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/(a*d)
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
```

```
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{\int \frac{\left(\frac{1}{2}(-Ab+2aB)+aC\cos(c+dx)\cdots\right.}{\sqrt{a+b\cos\cdots}}}{a\cdots}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} - \frac{A\int\sqrt{a+b\cos(c+dx)}\,d\cdots}{2a}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} - \frac{(Ab-2aB)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+d\cdots}}}{2a}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b\cos(c+\cdots}}{ad\cdots}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(A+2C)\sqrt{\frac{a+b\cos(\cdots}{a+\cdots}}}{d\sqrt{a+b\cdots}}$$

Mathematica [C] time = 13.7629, size = 600, normalized size = 2.73

$$\frac{2A\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)}{ad(2A+2B\cos(c+dx)+C\cos(2c+2dx)+C)} + \frac{\cos^2(c+dx)\left(A\sec^2(c+dx)+B\,\mathrm{s}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[a +
 b*Cos[c + d*x]],x]
```

```
[Out] (2*A*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(C + B*Sec[c + d*x] + A*Sec[c +
d*x]^2)*Sin[c + d*x])/(a*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]
)) + (Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((8*a*C*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a
+ b*Cos[c + d*x]] + (2*(-3*A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I
)*A*b*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b
))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*S
qrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi
```

```
[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b
)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*
Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)
]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))))/(2
*a*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))
```

Maple [B] time = 1.425, size = 738, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))+2*A*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
(1/2)))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*si
n(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c) + a), x)
```

3.1045 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=303

$$\frac{\left(4a^2(A+2C)-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab}{}$$

```
[Out] ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b - 4*a*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 - 4*a*b*B + 4*a^2*(A + 2*C))*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2
*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*a^2*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*
x])/(2*a*d)
```

Rubi [A] time = 1.01973, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2(A+2C)-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a + b*Cos
[c + d*x]],x]
```


```
[Out] ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b - 4*a*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((3*A*b^2 - 4*a*b*B + 4*a^2*(A + 2*C))*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2
*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*a^2*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*
x])/(2*a*d)
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} + \int \frac{\left(\frac{1}{2}(-3Ab+4\right.}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos}}{}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(3Ab-}{}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(Ab-4}{}$$

Mathematica [C] time = 6.50177, size = 424, normalized size = 1.4

$$\frac{\frac{2\left(8a^2(A+2C)-12abB+9Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}((4aB-3Ab)\cos(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[a +
 b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(9*A*b^2 - 12*a*b*B + 8*a^2*(A + 2*C
))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b - 4*a*B)*Sqrt[-((b*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-
2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh
```

```
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sq
rt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (-3*A*b + 4*a*B)*C
os[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d)
```

Maple [B] time = 2.076, size = 1282, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/a*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*(-1/2/a*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/
2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
```

```
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,
(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos
(d*x + c) + a), x)
```

3.1046 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=405

$$\frac{\tan(c+dx)\left(8a^2(2A+3C)-18abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24a^3d}+\frac{\left(8a^2(2A+3C)-6abB+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{24a^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((15*A*b^2 - 18*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*b)/(a + b)])/(24*a^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]) + ((5*A*b^2 - 6*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[a + b*Cos[c +
d*x]]) - ((5*A*b^3 - 8*a^3*B - 6*a*b^2*B + 4*a^2*b*(A + 2*C))*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^3*d*S
qrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 - 18*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[
a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^3*d) - ((5*A*b - 6*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a^2*d) + (A*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 1.48571, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\left(8a^2(2A+3C)-18abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{24a^3d}+\frac{\left(8a^2(2A+3C)-6abB+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{24a^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a + b*Cos
[c + d*x]],x]
```



```
[Out] -((15*A*b^2 - 18*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*b)/(a + b)])/(24*a^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]) + ((5*A*b^2 - 6*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[a + b*Cos[c +
d*x]]) - ((5*A*b^3 - 8*a^3*B - 6*a*b^2*B + 4*a^2*b*(A + 2*C))*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^3*d*S
qrt[a + b*Cos[c + d*x]]) + ((15*A*b^2 - 18*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[
a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^3*d) - ((5*A*b - 6*a*B)*Sqrt[a + b*
```

```
Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a^2*d) + (A*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+6a\right.}{}}{}$$

$$= -\frac{(5Ab-6aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12a^2d} + \frac{A}{}$$

$$= \frac{\left(15Ab^2-18abB+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+}{24a^3d}$$

$$= \frac{\left(15Ab^2-18abB+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+}{24a^3d}$$

$$= \frac{\left(15Ab^2-18abB+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}\tan(c+}{24a^3d}$$

$$= -\frac{\left(15Ab^2-18abB+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{24a^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(15Ab^2-18abB+8a^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{24a^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.77977, size = 665, normalized size = 1.64

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+24a^2C\sin(c+dx)-18abB\sin(c+dx)+15Ab^2\sin(c+dx)\right)}{24a^3} + \frac{\sec^2(c+dx)(6aB\sin(c+dx)-5Ab\sin(c+dx))}{12a^2} + \right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[a +
 b*Cos[c + d*x]],x]
```

```
[Out] ((2*(-20*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-40*a^2*A*b -
45*A*b^3 + 48*a^3*B + 54*a*b^2*B - 72*a^2*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
```

```
- ((2*I)*(-16*a^2*A*b - 15*A*b^3 + 18*a*b^2*B - 24*a^2*b*C)*Sqrt[(b - b*Cos
[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*
(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x
]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c +
d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2
*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*
(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(96*a^3*d) + (Sqrt[a + b
*Cos[c + d*x]]*((Sec[c + d*x]^2*(-5*A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))
/(12*a^2) + (Sec[c + d*x]*(16*a^2*A*Sin[c + d*x] + 15*A*b^2*Sin[c + d*x] -
18*a*b*B*Sin[c + d*x] + 24*a^2*C*Sin[c + d*x]))/(24*a^3) + (A*Sec[c + d*x]^
2*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 2.885, size = 2205, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/a*c
os(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*B*(-1/2/a*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/
a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/
2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*A*(-1/3/a*cos(1/2*d
*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*
(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2
/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(a+b*cos(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{4}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+b*cos(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*cos
(d*x + c) + a), x)
```

3.1047 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=426

$$-\frac{2\sin(c+dx)\cos^2(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\cos(c+dx)\left(6a^2C-5abB+5Ab^2-b^2C\right)\sqrt{a+b\cos(c}}{5b^2d\left(a^2-b^2\right)}$$

```
[Out] (-2*(40*a^3*b*B - 25*a*b^3*B - 6*a^2*b^2*(5*A - 4*C) - 48*a^4*C + 3*b^4*(5*
A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
15*b^4*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*b*B +
 5*b^3*B - 48*a^3*C - 6*a*b^2*(5*A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]
]) - (2*(A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)
*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*a^2*b*B - 5*b^3*B - 3*a*b^2*(5*A - 3*
C) - 24*a^3*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)*d
) + (2*(5*A*b^2 - 5*a*b*B + 6*a^2*C - b^2*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.922358, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sin(c+dx)\cos^2(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\cos(c+dx)\left(6a^2C-5abB+5Ab^2-b^2C\right)\sqrt{a+b\cos(c}}{5b^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (-2*(40*a^3*b*B - 25*a*b^3*B - 6*a^2*b^2*(5*A - 4*C) - 48*a^4*C + 3*b^4*(5*
A + 3*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
15*b^4*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*b*B +
 5*b^3*B - 48*a^3*C - 6*a*b^2*(5*A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]
]) - (2*(A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)
*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*a^2*b*B - 5*b^3*B - 3*a*b^2*(5*A - 3*
C) - 24*a^3*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)*d
```

```
) + (2*(5*A*b^2 - 5*a*b*B + 6*a^2*C - b^2*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+d}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(5Ab^2-\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2bB\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2bB\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2bB\right.}{}$$

$$= -\frac{2\left(40a^3bB-25ab^3B-6a^2b^2(5A-4C)-48a^4C+3b^4(5A+\right.}{15b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos}{a}}}$$

Mathematica [A] time = 2.42844, size = 328, normalized size = 0.77

$$\frac{\frac{30a^2b\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{b^2-a^2}+\frac{2b^2\left(-10a^2bB+12a^3C+3ab^2(5A+C)-5b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a-b)(a+b)}+\frac{2\left(6a^2b^2(5A-4C)-40a^3bB+48a^4C+25ab^3B-\right.}{}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(3/2),x]
```

```
[Out] ((2*b^2*(-10*a^2*b*B - 5*b^3*B + 12*a^3*C + 3*a*b^2*(5*A + C))*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a +
 b)) + (2*(-40*a^3*b*B + 25*a*b^3*B + 6*a^2*b^2*(5*A - 4*C) + 48*a^4*C - 3*
b^4*(5*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/((a - b
)*(a + b)) + (30*a^2*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/(-a^2 + b^2
) + 2*b*(5*b*B - 9*a*C)*(a + b*Cos[c + d*x])*Sin[c + d*x] + 3*b^2*C*(a + b*
```

```
Cos[c + d*x])*Sin[2*(c + d*x)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.402, size = 1331, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C/b*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8/b
^2*(B*b-C*a-3*C*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^4*(A*b^2-B*a*b-2*B*b^2+C*a^2+2*
C*a*b+3*C*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2)))-2*(A*a*b^2+A*b^3-B*a^2*b-B*a*b^2-B*b^3+C*a^3
+C*a^2*b+C*a*b^2+C*b^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^2*(A*b^2-
B*a*b+C*a^2)/b^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-
b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^4 + B\cos\left(dx+c\right)^3 + A\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^2 + 2\,ab\cos\left(dx+c\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*co
s(d*x + c) + a)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c) + a)^(3/2), x)
```

3.1048 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{2a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C-6abB+3Ab^2+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2bB-8a^3C\right.}{}$$

```
[Out] (2*(6*a^2*b*B - 3*b^3*B - a*b^2*(3*A - 5*C) - 8*a^3*C)*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]) + (2*(3*A*b^2 - 6*a*b*B + 8*a^2*C + b^2*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*
d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b
^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.516079, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2C-6abB+3Ab^2+b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2bB-8a^3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d
*x])^(3/2),x]
```


```
[Out] (2*(6*a^2*b*B - 3*b^3*B - a*b^2*(3*A - 5*C) - 8*a^3*C)*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]) + (2*(3*A*b^2 - 6*a*b*B + 8*a^2*C + b^2*C)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*
d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b
^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*C*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(3*b^2*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}b\left(Ab^2-a(bB-aC)\right)+\frac{1}{2}(2a}{}}{}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin}{3b^2d}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin}{3b^2d}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2C\sqrt{a+b\cos(c+dx)}\sin}{3b^2d}$$

$$= \frac{2\left(6a^2bB-3b^3B-ab^2(3A-5C)-8a^3C\right)\sqrt{a+b\cos(c+dx)}E}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.72091, size = 236, normalized size = 0.84

$$\frac{2\left(b\sin(c+dx)\left(\frac{a\left(-4a^2C+3abB-3Ab^2+b^2C\right)}{b^2-a^2}+bC\cos(c+dx)\right)-\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2\left(2a^2C-3abB+3Ab^2+b^2C\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(-6a^2bB+8a^3C+a\right.\right.}{(a-b)(a+b)}}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(3*A*b^2 - 3*a*b*B + 2*a^2*C
 + b^2*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-6*a^2*b*B + 3*b^3*B + a
*b^2*(3*A - 5*C) + 8*a^3*C)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
```

```
- a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)*(a + b))) + b*((a*(-3
*A*b^2 + 3*a*b*B - 4*a^2*C + b^2*C))/(-a^2 + b^2) + b*C*Cos[c + d*x])*Sin[c
 + d*x]))/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 2.792, size = 1036, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^3*(4*
b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b-2*C*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-6*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*a^2*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a*(A
*b^2-B*a*b+C*a^2)/b^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/
(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/
2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c) + a)^(3/2), x)
```

3.1049 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2C-abB+Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(bB-2aC)\sqrt{}}{b^2d}$$

```
[Out] (2*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) + (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b
*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.289416, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2C-abB+Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(bB-2aC)\sqrt{}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```



```
[Out] (2*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) + (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b
*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}b(bB-a(A+C)) - \frac{1}{2}\left(Ab^2 - abB + 2a^2C - b^2C\right)\text{co}}{\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(bB-2aC)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b^2} + \frac{\left(Ab^2\right.}{}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\left(Ab^2 - abB + 2a^2C - b^2C\right)\sqrt{a+b\,\text{co}}\right.}{b^2\left(a^2-b^2\right)\sqrt{}}$$

$$= \frac{2\left(Ab^2 - abB + 2a^2C - b^2C\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(bB -}{}$$

Mathematica [A] time = 0.947786, size = 182, normalized size = 0.83

$$-\frac{2\left(-(a+b)\left(2a^2C - abB + Ab^2 - b^2C\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right) + \left(a^2-b^2\right)(2aC - bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right.\right.}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (-2*(-((a + b)*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*(-(b*B) + 2*a
*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x]))/((a - b)*b^2*(a + b)*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [A] time = 2.236, size = 522, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*C*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2*(A*b^2-B*a*b+C*a^2)/
b^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2
), x)
```

3.1050 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{abd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2\right.}{ad\sqrt{a+b\cos\left(\right.}}$$

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)])/(a*b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (
2*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
])/(b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]])
+ (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]])
```

Rubi [A] time = 0.787213, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{abd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2\right.}{ad\sqrt{a+b\cos\left(\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d
*x])^(3/2),x]
```


```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)])/(a*b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (
2*C*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
])/(b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]])
+ (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]])
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}A\left(a^2-b^2\right)-\frac{1}{2}a(Ab-aB+bC\right.}{}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\left(-\frac{1}{2}Ab\left(a^2-b^2\right)-\frac{1}{2}a\left(a^2-b^2\right)C\right.}{\sqrt{a+b\cos(c}}}{ab\left(a^2-b\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a} + \frac{C}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ab\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ab\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

Mathematica [F] time = 31.8937, size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^(3/2),x]
```

```
[Out] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^(3/2), x]
```

Maple [A] time = 1.847, size = 543, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*(-A*b^2+B*a*b-C*a^2)/a/b/sin(1/2*d*x+1/2*c)^
2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*A/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(3/
2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(a + b*cos(c
 + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c) + a)^(3/2), x)
```

3.1051 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{b\sin(c+dx)\left(a^2(-(A-2C))-2abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(a^2(-(A-2C))-2abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] ((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 2*a*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - (b*(3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Sin[c + d*x])
/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[
a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.07437, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\sin(c+dx)\left(a^2(-(A-2C))-2abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(a^2(-(A-2C))-2abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c +
 d*x])^(3/2),x]
```


```
[Out] ((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 2*a*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - (b*(3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Sin[c + d*x])
/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[
a + b*Cos[c + d*x]])
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+2aB)+aC\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+\cdots\right.}{(a+b\cos(c+dx))^{3/2}}}{a}$$

$$= -\frac{b\left(3Ab^2-2abB-a^2(A-2C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+\cdots}{ad\sqrt{a+b\cos(\cdots}}$$

$$= -\frac{b\left(3Ab^2-2abB-a^2(A-2C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+\cdots}{ad\sqrt{a+b\cos(\cdots}}$$

$$= -\frac{b\left(3Ab^2-2abB-a^2(A-2C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+\cdots}{ad\sqrt{a+b\cos(\cdots}}$$

$$= \frac{\left(3Ab^2-2abB-a^2(A-2C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|\cdots}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{\left(3Ab^2-2abB-a^2(A-2C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|\cdots}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.94605, size = 751, normalized size = 2.4

$$\frac{\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(\frac{2A\tan(c+dx)}{a^2} - \frac{4\left(a^2bC\sin(c+dx)-ab^2B\sin(c+dx)+Ab^3\sin(c+d\cdots\right.}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))}\cdots\right.}{d(2A+2B\cos(c+dx)+C\cos(2c+2dx)+C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((2*(4*a*A*b^2 - 4*
a^2*b*B + 4*a^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-7*a^2*A*b + 9*A*b^3 + 4*a
^3*B - 6*a*b^2*B + 2*a^2*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-(a^2*A
*b) + 3*A*b^3 - 2*a*b^2*B + 2*a^2*b*C)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*S
```

```
qrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*Elliptic
E[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
 + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]]
, (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b
)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x
]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x])
+ 2*(a + b*Cos[c + d*x])^2))))/(2*a^2*(a - b)*(a + b)*d*(2*A + C + 2*B*Cos[
c + d*x] + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*
(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((-4*(A*b^3*Sin[c + d*x] - a*b^2*B*
Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])
) + (2*A*Tan[c + d*x])/a^2))/(d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2
*d*x]))
```

Maple [B] time = 2.73, size = 915, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b^2-B*
a*b+C*a^2)/a^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^
2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2
*A/a*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*(-A*b+B*a)/a^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
```

```
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2
*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c) + a)^(3/2), x)
```

3.1052 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{b\sin(c+dx)\left(-a^2(7Ab-8bC)+4a^3B-12ab^2B+15Ab^3\right)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(-a^2(7Ab-8bC)+4a^3B-12ab^2B+15Ab^3\right)\sqrt{a+b\cos(c}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((15*A*b^3 + 4*a^3*B - 12*a*b^2*B - a^2*(7*A*b - 8*b*C))*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) - ((5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c +
d*x]]) + ((15*A*b^2 - 12*a*b*B + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*Co
s[c + d*x]]) + (b*(15*A*b^3 + 4*a^3*B - 12*a*b^2*B - a^2*(7*A*b - 8*b*C))*S
in[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 4*a
*B)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[
c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.586, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\sin(c+dx)\left(-a^2(7Ab-8bC)+4a^3B-12ab^2B+15Ab^3\right)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(-a^2(7Ab-8bC)+4a^3B-12ab^2B+15Ab^3\right)\sqrt{a+b\cos(c}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c +
 d*x])^(3/2),x]
```


```
[Out] -((15*A*b^3 + 4*a^3*B - 12*a*b^2*B - a^2*(7*A*b - 8*b*C))*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) - ((5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c +
d*x]]) + ((15*A*b^2 - 12*a*b*B + 4*a^2*(A + 2*C))*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*Co
s[c + d*x]]) + (b*(15*A*b^3 + 4*a^3*B - 12*a*b^2*B - a^2*(7*A*b - 8*b*C))*S
in[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 4*a
```

```
*B)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[
c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+4aB)+a(A+2C)\cos(c+dx)\right.}{(a+b\cos(c+d}}{2a}$$

$$= -\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} +$$

$$= \frac{b\left(15Ab^3+4a^3B-12ab^2B-a^2(7Ab-8bC)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} -$$

$$= \frac{b\left(15Ab^3+4a^3B-12ab^2B-a^2(7Ab-8bC)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} -$$

$$= \frac{b\left(15Ab^3+4a^3B-12ab^2B-a^2(7Ab-8bC)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} -$$

$$= -\frac{\left(15Ab^3+4a^3B-12ab^2B-a^2b(7A-8C)\right)\sqrt{a+b\cos(c+}}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(15Ab^3+4a^3B-12ab^2B-a^2b(7A-8C)\right)\sqrt{a+b\cos(c+}}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.10543, size = 723, normalized size = 1.74

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(a^2b^2C\sin(c+dx)-ab^3B\sin(c+dx)+Ab^4\sin(c+dx)\right)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\sec(c+dx)(4aB\sin(c+dx)-7Ab\sin(c+dx))}{4a^3}+\frac{A\tan(c+dx)\sec(c+dx)}{2a^2}\right)}{d} -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*C
os[c + d*x])^(3/2),x]
```

```
[Out] -((2*(4*a^3*A*b - 20*a*A*b^3 + 16*a^2*b^2*B - 16*a^3*b*C)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c +
 d*x]] + (2*(8*a^4*A + 29*a^2*A*b^2 - 45*A*b^4 - 28*a^3*b*B + 36*a*b^3*B +
```

```
16*a^4*C - 24*a^2*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(7*a^2*A*b^2
- 15*A*b^4 - 4*a^3*b*B + 12*a*b^3*B - 8*a^2*b^2*C)*Sqrt[(b - b*Cos[c + d*x]
)/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*
Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b
*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Co
s[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(16*a^3*(-a + b)*(a + b)*d) + (Sq
rt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(-7*A*b*Sin[c + d*x] + 4*a*B*Sin[c +
d*x]))/(4*a^3) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2*C*
Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan
[c + d*x])/(2*a^2)))/d
```

Maple [B] time = 3.339, size = 1577, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b^2-B
*a*b+C*a^2)*b/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2
-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)+2*(-A*b+B*a)/a^2*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2/a*
A*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
)*b^2)-2*(A*b^2-B*a*b+C*a^2)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(
1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c) + a)^(3/2), x)
```

3.1053 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=622

$$-\frac{2\sin(c+dx)\cos^3(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\sin(c+dx)\cos^2(c+dx)\left(-2a^2b^2(A-6C)+5a^3bB-8a^4C-9ab^3B\right.}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*(80*a^5*b*B - 140*a^3*b^3*B + 40*a*b^5*B - 4*a^4*b^2*(10*A - 53*C) + 5*
a^2*b^4*(15*A - 11*C) - 128*a^6*C - 3*b^6*(5*A + 3*C))*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(80*a^4*b*B - 80*a^2*b^3*B - 5*b^5*B - 4*
a^3*b^2*(10*A - 29*C) - 128*a^5*C + a*b^4*(45*A + 17*C))*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)
*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^3*Si
n[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(6*A*b^4 +
5*a^3*b*B - 9*a*b^3*B - 2*a^2*b^2*(A - 6*C) - 8*a^4*C)*Cos[c + d*x]^2*Sin[c
 + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(40*a^4*b*B
- 65*a^2*b^3*B + 5*b^5*B - 2*a^3*b^2*(10*A - 49*C) + 2*a*b^4*(20*A - 7*C) -
 64*a^5*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d)
- (2*(30*a^3*b*B - 50*a*b^3*B - a^2*b^2*(15*A - 71*C) + b^4*(35*A - 3*C) -
48*a^4*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2
- b^2)^2*d)
```

Rubi [A] time = 1.67499, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sin(c+dx)\cos^3(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\sin(c+dx)\cos^2(c+dx)\left(-2a^2b^2(A-6C)+5a^3bB-8a^4C-9ab^3B\right.}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (-2*(80*a^5*b*B - 140*a^3*b^3*B + 40*a*b^5*B - 4*a^4*b^2*(10*A - 53*C) + 5*
a^2*b^4*(15*A - 11*C) - 128*a^6*C - 3*b^6*(5*A + 3*C))*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a
```

```
 + b*Cos[c + d*x])/(a + b)]) + (2*(80*a^4*b*B - 80*a^2*b^3*B - 5*b^5*B - 4*
a^3*b^2*(10*A - 29*C) - 128*a^5*C + a*b^4*(45*A + 17*C))*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)
*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^3*Si
n[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(6*A*b^4 +
5*a^3*b*B - 9*a*b^3*B - 2*a^2*b^2*(A - 6*C) - 8*a^4*C)*Cos[c + d*x]^2*Sin[c
 + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(40*a^4*b*B
- 65*a^2*b^3*B + 5*b^5*B - 2*a^3*b^2*(10*A - 49*C) + 2*a*b^4*(20*A - 7*C) -
 64*a^5*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d)
- (2*(30*a^3*b*B - 50*a*b^3*B - a^2*b^2*(15*A - 71*C) + b^4*(35*A - 3*C) -
48*a^4*C)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2
- b^2)^2*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(6Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(6Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(6Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(6Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(6Ab^4+\right.}{}$$

$$= -\frac{2\left(80a^5bB-140a^3b^3B+40ab^5B-4a^4b^2(10A-53C)+5a^2\right.}{15b^5\,(}$$

Mathematica [A] time = 5.3527, size = 422, normalized size = 0.68

$$\frac{b\left(\frac{10a^3\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{a^2-b^2}-\frac{10a^2\sin(c+dx)\left(5a^2b^2(A-3C)-8a^3bB+11a^4C+12ab^3B-9Ab^4\right)(a+b\cos(c+dx))}{\left(a^2-b^2\right)^2}+2(5bB-14aC)\sin(c+dx)(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(5/2),x]
```

```
[Out] ((2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(-20*a^4*b*B + 35*a^2*b^3*B +
 5*b^5*B + 2*a^3*b^2*(5*A - 22*C) + 32*a^5*C - 2*a*b^4*(15*A + 4*C))*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)] + (-80*a^5*b*B + 140*a^3*b^3*B - 40*a*b^5*B
```

```
+ 4*a^4*b^2*(10*A - 53*C) + 128*a^6*C + 3*b^6*(5*A + 3*C) + 5*a^2*b^4*(-15
*A + 11*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + b*((10*a^3*(A*b^2 + a*(-
(b*B) + a*C))*Sin[c + d*x])/(a^2 - b^2) - (10*a^2*(-9*A*b^4 - 8*a^3*b*B + 1
2*a*b^3*B + 5*a^2*b^2*(A - 3*C) + 11*a^4*C)*(a + b*Cos[c + d*x])*Sin[c + d*
x])/(a^2 - b^2)^2 + 2*(5*b*B - 14*a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
+ 3*b*C*(a + b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*b^5*d*(a + b*Cos[c +
d*x])^(3/2))
```

Maple [B] time = 5.57, size = 1780, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C/b^2*(-
1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8
/b^3*(B*b-2*C*a-3*C*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^5*(A*b^2-2*B*a*b-2*B*b^2+3*
C*a^2+4*C*a*b+3*C*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))-2*(2*A*a*b^2+A*b^3-3*B*a^2*b-2*B*a*b
^2-B*b^3+4*C*a^3+3*C*a^2*b+2*C*a*b^2+C*b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
```

```
(1/2))+2*a^2/b^5*(3*A*b^2-4*B*a*b+5*C*a^2)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2)-2*a^3*(A*b^2-B*a*b+C*a^2)/b^5*(1/6/b/(a-b)/(a+b)
*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)
^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(
1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{5}+B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^5 + B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*co
s(d*x + c) + a)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(
d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c) + a)^(5/2), x)
```

3.1054 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=453

$$-\frac{2\sin(c+dx)\cos^2(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\sin(c+dx)\left(2a^2C-abB+Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}-2a\sin($$

```
[Out] (2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 2*a^3*b^2*(A - 14*C) + 2*a*b^4*(3*
A - 4*C) - 16*a^5*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(
8*a^3*b*B - 9*a*b^3*B - 2*a^2*b^2*(A - 8*C) - 16*a^4*C + b^4*(3*A + C))*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b
^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Cos
[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) -
(2*a*(4*A*b^4 + a*(3*a^2*b*B - 7*b^3*B - 6*a^3*C + 10*a*b^2*C))*Sin[c + d*x
])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*b^2 - a*b*B + 2
*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d
)
```

Rubi [A] time = 1.02471, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3047, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sin(c+dx)\cos^2(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\sin(c+dx)\left(2a^2C-abB+Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}-2a\sin($$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 2*a^3*b^2*(A - 14*C) + 2*a*b^4*(3*
A - 4*C) - 16*a^5*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(
8*a^3*b*B - 9*a*b^3*B - 2*a^2*b^2*(A - 8*C) - 16*a^4*C + b^4*(3*A + C))*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b
^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Cos
[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) -
```

```
(2*a*(4*A*b^4 + a*(3*a^2*b*B - 7*b^3*B - 6*a^3*C + 10*a*b^2*C))*Sin[c + d*x
])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(A*b^2 - a*b*B + 2
*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d
)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a\left(4Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a\left(4Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a\left(4Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a\left(4Ab^4\right.}{}$$

$$= \frac{2\left(8a^4bB-15a^2b^3B+3b^5B-2a^3b^2(A-14C)+2ab^4(3A-\right.}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a}{}}}$$

Mathematica [A] time = 3.72487, size = 377, normalized size = 0.83

$$2\left(\frac{b\sin(c+dx)\left(2ab\cos(c+dx)\left(2a^2b^2(A-8C)-5a^3bB+10a^4C+9ab^3B+2b^4(C-3A)\right)+2a^4Ab^2-10a^2Ab^4+16a^3b^3B+C\left(b^3-a^2b\right)^2\cos(2(c+dx))-25a^4b^2C-8a^5bB+1\right.}{2\left(a^2-b^2\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(2*a^3*b*B - 6*a*b^3*B - 4*a
^4*C + b^4*(3*A + C) + a^2*b^2*(A + 7*C))*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)] - (-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 2*a^3*b^2*(A - 14*C) + 16*a^5
*C + 2*a*b^4*(-3*A + 4*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] -
 a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + (b*(2*a^4
*A*b^2 - 10*a^2*A*b^4 - 8*a^5*b*B + 16*a^3*b^3*B + 16*a^6*C - 25*a^4*b^2*C
+ b^6*C + 2*a*b*(-5*a^3*b*B + 9*a*b^3*B + 2*a^2*b^2*(A - 8*C) + 10*a^4*C +
```

```
2*b^4*(-3*A + C))*Cos[c + d*x] + (-(a^2*b) + b^3)^2*C*Cos[2*(c + d*x)])*Sin
[c + d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 5.096, size = 1480, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^4*(4*
b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b-2*C*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-9*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+17*a^2*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2+8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a*b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a/b
^4*(2*A*b^2-3*B*a*b+4*C*a^2)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*
b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)+2*a^2*(A*b^2-B*a*b+C*a^2)/b^4*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1
/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos
(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
```

```
)-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{4}+B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*co
s(d*x + c) + a)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(
d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c) + a)^(5/2), x)
```

3.1055 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{2\sin(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+3Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2bB-8a^3\cdots\right.}{\cdots}$$

```
[Out] (-2*(2*a^3*b*B - 6*a*b^3*B + 3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*
Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2
 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*b*B - 3*b^3*B -
 8*a^3*C + a*b^2*(A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^3*b*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2
*(A + 9*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.629763, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3031, 3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+3Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2bB-8a^3\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c + d
*x])^(5/2),x]
```

```
[Out] (-2*(2*a^3*b*B - 6*a*b^3*B + 3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*
Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2
 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*b*B - 3*b^3*B -
 8*a^3*C + a*b^2*(A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^3*b*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2
*(A + 9*C))*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}b\left(Ab^2-a(bB-aC)\right)+\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4+2a^3bB-6a\cdots\right.}{3b^2\left(a^2\cdots\right.}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4+2a^3bB-6a\cdots\right.}{3b^2\left(a^2\cdots\right.}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4+2a^3bB-6a\cdots\right.}{3b^2\left(a^2\cdots\right.}$$

$$= -\frac{2\left(2a^3bB-6ab^3B+3b^4(A-C)-8a^4C+a^2b^2(A+15C)\right)\cdots}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+\cdots}{a+b}}}$$

Mathematica [A] time = 3.04326, size = 323, normalized size = 0.9

$$\frac{2\left(\frac{b\sin(c+dx)\left(b\cos(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+3Ab^4\right)+a\left(2a^2b^2(A+4C)+a^3bB-4a^4C-5ab^3B+2Ab^4\right)\right)}{\left(a^2-b^2\right)^2}+\frac{(-a-b\cos(c+dx))\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(a^2\cdots\right.\right.}{\cdots}\right.}{3b^3d(a+b\cos(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^(5/2),x]
```

```
[Out] (2*(((-a - b*Cos[c + d*x])*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(-(b^2*(a^2*b
*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*EllipticF[(c + d*x)/2, (2*b)/
(a + b)]) + (2*a^3*b*B - 6*a*b^3*B + 3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A +
 15*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)^2) + (b*(a*(2*A*b^4 + a^3*b*B -
 5*a*b^3*B - 4*a^4*C + 2*a^2*b^2*(A + 4*C)) + b*(3*A*b^4 + 2*a^3*b*B - 6*a*
b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2
)^2))/(3*b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 4.58, size = 963, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*b
*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*C*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2/b^3*(A*b^2-2*B*a*b+3
*C*a^2)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a*(A*b
^2-B*a*b+C*a^2)/b^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)
/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2
*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a
^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^3+B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*
x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c) + a)^(5/2), x)
```

3.1056 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=333

$$\frac{2\sin(c+dx)\left(a^2bB+2a^3C-2ab^2(2A+3C)+3b^3B\right)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-2a^2C-abB+Ab^2+\right.}{3b^2d\left(a^2-\right.}$$

```
[Out] (-2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) - (2*(A*b^2 - a*b*B - 2*a^2*C + 3*b^2*C)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2
*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c
 + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*b*B + 3*b
^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sq
rt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.485938, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(a^2bB+2a^3C-2ab^2(2A+3C)+3b^3B\right)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-2a^2C-abB+Ab^2+\right.}{3b^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) - (2*(A*b^2 - a*b*B - 2*a^2*C + 3*b^2*C)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2
*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c
 + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*b*B + 3*b
^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sq
rt[a + b*Cos[c + d*x]])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}b(bB-a(A+C))+\frac{1}{2}\left(Ab^2-abB-2a^2C+3b^2\right.}{(a+b\cos(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-2ab^2(2A\right.}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-2ab^2(2A\right.}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-2ab^2(2A\right.}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= -\frac{2\left(a^2bB+3b^3B+2a^3C-2ab^2(2A+3C)\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.39918, size = 278, normalized size = 0.83

$$\frac{2\left(\frac{b\sin(c+dx)\left(b\cos(c+dx)\left(a^2bB+2a^3C-2ab^2(2A+3C)+3b^3B\right)-5a^2b^2(A+C)+2a^3bB+a^4C+2ab^3B+Ab^4\right)}{\left(a^2-b^2\right)^2}+\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(a^2(3A+C)-4abB+b^2(A+3C\right.\right.}{}\right.}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(-4*a*b*B + a^2*(3*A + C) +
b^2*(A + 3*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (a^2*b*B + 3*b^3*B +
 2*a^3*C - 2*a*b^2*(2*A + 3*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a +
```

```
b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + (b*(
A*b^4 + 2*a^3*b*B + 2*a*b^3*B + a^4*C - 5*a^2*b^2*(A + C) + b*(a^2*b*B + 3*
b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b
^2)^2))/(3*b^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 3.935, size = 867, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2/b^2*(B*b-2*C*a)/sin(1/2*d*x+1/2*c)^2/(-2*s
in(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2/b^2*(A*b^2-B*a*b+C*a^2)*(1/6/b/(a-b)/(a+
b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-
b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*si
n(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2
), x)
```

3.1057 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=401

$$-\frac{2\sin(c+dx)\left(-a^2b^2(7A+3C)+4a^3bB+a^4(-C)+3Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(Ab^2-a(bB-a\right.}{3abd\left(a^2\right.}$$

```
[Out] (2*(3*A*b^4 + 4*a^3*b*B - a^4*C - a^2*b^2*(7*A + 3*C))*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*b*(a^2 - b^2)^2*d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a*b*(a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) +
(2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])^(3/2)) - (2*(3*A*b^4 + 4*a^3*b*B - a^4*C - a^2*b^2*(7*A + 3*C))*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.22946, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\sin(c+dx)\left(-a^2b^2(7A+3C)+4a^3bB+a^4(-C)+3Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(Ab^2-a(bB-a\right.}{3abd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos[c + d
*x])^(5/2),x]
```


```
[Out] (2*(3*A*b^4 + 4*a^3*b*B - a^4*C - a^2*b^2*(7*A + 3*C))*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*b*(a^2 - b^2)^2*d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a*b*(a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) +
(2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])^(3/2)) - (2*(3*A*b^4 + 4*a^3*b*B - a^4*C - a^2*b^2*(7*A + 3*C))*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}A\left(a^2-b^2\right)-\frac{3}{2}a(Ab-aB+b\right.}{}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(3Ab^4+4a^3bB-a^4C\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(3Ab^4+4a^3bB-a^4C\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(3Ab^4+4a^3bB-a^4C\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2\left(3Ab^4+4a^3bB-a^4C-a^2b^2(7A+3C)\right)\sqrt{a+b\cos(c+dx)}E}{3a^2b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(3Ab^4+4a^3bB-a^4C-a^2b^2(7A+3C)\right)\sqrt{a+b\cos(c+dx)}E}{3a^2b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [F] time = 48.0328, size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^(5/2),x]
```

```
[Out] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(a + b*Cos
[c + d*x])^(5/2), x]
```

Maple [A] time = 3.95, size = 879, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b^2+C
*a^2)/a^2/b/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(-
A*b^2+B*a*b-C*a^2)/a/b*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a
-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-
(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+
3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))))-2*A/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(a+b*cos(d*x+c))**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c) + a)^(5/2), x)
```

3.1058 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=461

$$-\frac{b\sin(c+dx)\left(26a^2Ab^2+a^4(-(3A-8C))-14a^3bB+6ab^3B-15Ab^4\right)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\sin(c+dx)\left(a^2(-(3A-2C))-2abB+5Ab\right.}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

```
[Out] ((26*a^2*A*b^2 - 15*A*b^4 - 14*a^3*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^
2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((5*A*b^2 - 2*a*b*B - a^2*(3*A
 - 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 2*a*B)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]
)/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(5*A*b^2 - 2*a*b*B - a^2*(3*A - 2*C
))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (b*(26*
a^2*A*b^2 - 15*A*b^4 - 14*a^3*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Sin[c + d*
x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*
d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 1.61309, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\sin(c+dx)\left(26a^2Ab^2+a^4(-(3A-8C))-14a^3bB+6ab^3B-15Ab^4\right)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\sin(c+dx)\left(a^2(-(3A-2C))-2abB+5Ab\right.}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*Cos[c +
 d*x])^(5/2),x]
```


```
[Out] ((26*a^2*A*b^2 - 15*A*b^4 - 14*a^3*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^
2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((5*A*b^2 - 2*a*b*B - a^2*(3*A
 - 2*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 2*a*B)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]
)/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(5*A*b^2 - 2*a*b*B - a^2*(3*A - 2*C
```

```
))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (b*(26*
a^2*A*b^2 - 15*A*b^4 - 14*a^3*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Sin[c + d*
x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*
d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+2aB)+aC\cos(c+dx)+\frac{3}{2}Ab\cos^2(\right.}{(a+b\cos(c+dx))^{5/2}}}{a}$$

$$= -\frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{A\tan(c}{ad(a+b\cos(}$$

$$= -\frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(26a^2Ab^2-\right.}{}$$

$$= -\frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(26a^2Ab^2-\right.}{}$$

$$= -\frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(26a^2Ab^2-\right.}{}$$

$$= \frac{\left(26a^2Ab^2-15Ab^4-14a^3bB+6ab^3B-a^4(3A-8C)\right)\sqrt{a+b}}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{\left(26a^2Ab^2-15Ab^4-14a^3bB+6ab^3B-a^4(3A-8C)\right)\sqrt{a+b}}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.2864, size = 915, normalized size = 1.98

$$\frac{\left(A\sec^2(c+dx)+B\sec(c+dx)+C\right)\left(\frac{2\left(12Ca^5-24bBa^4+36Ab^2a^3+4b^2Ca^3+8b^3Ba^2-20Ab^4a\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(12Ba^5-33Aba^4\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(a + b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((2*(36*a^3*A*b^2 -
 20*a*A*b^4 - 24*a^4*b*B + 8*a^2*b^3*B + 12*a^5*C + 4*a^3*b^2*C)*Sqrt[(a +
```

```
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*
Cos[c + d*x]] + (2*(-33*a^4*A*b + 86*a^2*A*b^3 - 45*A*b^5 + 12*a^5*B - 38*a
^3*b^2*B + 18*a*b^4*B + 8*a^4*b*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3
*a^4*A*b + 26*a^2*A*b^3 - 15*A*b^5 - 14*a^3*b^2*B + 6*a*b^4*B + 8*a^4*b*C)*
Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Co
s[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a +
b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a -
 b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-
((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a
^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))))/(6*a^3*(
-a + b)^2*(a + b)^2*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) +
(Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(C + B*Sec[c + d*x] + A*Sec[c + d*
x]^2)*((-4*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x
]))/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (4*(10*a^2*A*b^3*Sin[c + d
*x] - 6*A*b^5*Sin[c + d*x] - 7*a^3*b^2*B*Sin[c + d*x] + 3*a*b^4*B*Sin[c + d
*x] + 4*a^4*b*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) +
 (2*A*Tan[c + d*x])/a^3))/(d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*
x]))
```

Maple [B] time = 5.882, size = 1348, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*(2*A*b-
B*a)/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(A*b^
2-B*a*b+C*a^2)/a^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/
b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*
cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^
2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
```

```
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))))+2*A/a^2*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*(-2*
A*b+B*a)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*s
in(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(a+b*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c) + a)^(5/2), x)
```

3.1059 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=572

$$-\frac{b\sin(c+dx)\left(-2a^2b^3(85A-12C)+a^4b(33A-56C)+104a^3b^2B-12a^5B-60ab^4B+105Ab^5\right)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\sin(c+dx)\left(-a^2(27\right.}{12a^3d\left(a\right.}$$

```
[Out] ((105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B + a^4*b*(33*A - 56*C) -
 2*a^2*b^3*(85*A - 12*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(12*a^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ ((35*A*b^3 + 12*a^3*B - 20*a*b^2*B - a^2*(27*A*b - 8*b*C))*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(12*a^3*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((35*A*b^2 - 20*a*b*B + 4*a^2*(A + 2*C))
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b
)])/(4*a^4*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(35*A*b^3 + 12*a^3*B - 20*a*b^2
*B - a^2*(27*A*b - 8*b*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^(3/2)) - (b*(105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B + a
^4*b*(33*A - 56*C) - 2*a^2*b^3*(85*A - 12*C))*Sin[c + d*x])/(12*a^4*(a^2 -
b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((7*A*b - 4*a*B)*Tan[c + d*x])/(4*a^2*
d*(a + b*Cos[c + d*x])^(3/2)) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*(a + b
*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 2.31224, antiderivative size = 572, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\sin(c+dx)\left(-2a^2b^3(85A-12C)+a^4b(33A-56C)+104a^3b^2B-12a^5B-60ab^4B+105Ab^5\right)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\sin(c+dx)\left(-a^2(27\right.}{12a^3d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*Cos[c +
 d*x])^(5/2),x]
```


```
[Out] ((105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B + a^4*b*(33*A - 56*C) -
 2*a^2*b^3*(85*A - 12*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*b)/(a + b)])/(12*a^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])
+ ((35*A*b^3 + 12*a^3*B - 20*a*b^2*B - a^2*(27*A*b - 8*b*C))*Sqrt[(a + b*Co
```

```
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(12*a^3*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((35*A*b^2 - 20*a*b*B + 4*a^2*(A + 2*C))
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b
)])/(4*a^4*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(35*A*b^3 + 12*a^3*B - 20*a*b^2
*B - a^2*(27*A*b - 8*b*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^(3/2)) - (b*(105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B + a
^4*b*(33*A - 56*C) - 2*a^2*b^3*(85*A - 12*C))*Sin[c + d*x])/(12*a^4*(a^2 -
b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((7*A*b - 4*a*B)*Tan[c + d*x])/(4*a^2*
d*(a + b*Cos[c + d*x])^(3/2)) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*(a + b
*Cos[c + d*x])^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-7Ab+4aB)+a(A+2C)\cos(c+dx)\right.}{(a+b\cos(c+}}{2a}$$

$$= -\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} +$$

$$= \frac{b\left(35Ab^3+12a^3B-20ab^2B-a^2(27Ab-8bC)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{b\left(35Ab^3+12a^3B-20ab^2B-a^2(27Ab-8bC)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{b\left(35Ab^3+12a^3B-20ab^2B-a^2(27Ab-8bC)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{b\left(35Ab^3+12a^3B-20ab^2B-a^2(27Ab-8bC)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{\left(105Ab^5-12a^5B+104a^3b^2B-60ab^4B+a^4b(33A-56C)\right.}{12a^4\left(a^2-b^2\right)^2}$$

$$= \frac{\left(105Ab^5-12a^5B+104a^3b^2B-60ab^4B+a^4b(33A-56C)\right.}{12a^4\left(a^2-b^2\right)^2}$$

Mathematica [C] time = 7.76397, size = 922, normalized size = 1.61

$$\frac{\frac{2\left(12Aba^5-96bCa^5+144b^2Ba^4-216Ab^3a^3+32b^3Ca^3-80b^4Ba^2+140Ab^5a\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(24Aa^6+48Ca^6-132bBa^5+195Ab^2a^4-152b^2Ca^4\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(a + b*C
os[c + d*x])^(5/2),x]
```

```
[Out] ((2*(12*a^5*A*b - 216*a^3*A*b^3 + 140*a*A*b^5 + 144*a^4*b^2*B - 80*a^2*b^4*
B - 96*a^5*b*C + 32*a^3*b^3*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(24*a^6*A + 195
*a^4*A*b^2 - 566*a^2*A*b^4 + 315*A*b^6 - 132*a^5*b*B + 344*a^3*b^3*B - 180*
a*b^5*B + 48*a^6*C - 152*a^4*b^2*C + 72*a^2*b^4*C)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] - ((2*I)*(33*a^4*A*b^2 - 170*a^2*A*b^4 + 105*A*b^6 - 12*a^5*b*B + 104*a
^3*b^3*B - 60*a*b^5*B - 56*a^4*b^2*C + 24*a^2*b^4*C)*Sqrt[(b - b*Cos[c + d*
x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a
 - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
 + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(
a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a +
 b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*
Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(48*a^4*(a - b)^2*(a + b)^2*d)
+ (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(-11*A*b*Sin[c + d*x] + 4*a*B*Si
n[c + d*x]))/(4*a^4) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*
b^2*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(13*a^
2*A*b^4*Sin[c + d*x] - 9*A*b^6*Sin[c + d*x] - 10*a^3*b^3*B*Sin[c + d*x] + 6
*a*b^5*B*Sin[c + d*x] + 7*a^4*b^2*C*Sin[c + d*x] - 3*a^2*b^4*C*Sin[c + d*x]
))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan[c + d*x
])/(2*a^3)))/d
```

Maple [B] time = 7.628, size = 2019, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*(3*A*b
^2-2*B*a*b+C*a^2)/a^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/
(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2)-2*(A*b^2-B*a*b+C*a^2)*b/a^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+
```

1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a
-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(
a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))))+2*(-2*A*b+B*a)/a^3*(-1/a*cos(1/2*d*x+1/2*c)*(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(
-2*b/(a-b))^(1/2)))+2*A/a^2*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b
/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
b/(a-b))^(1/2))-1/2(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2*(3*A*b^2-2*B*a*b+C*a^2)/a^4*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(a+b*cos(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c) + a)^(5/2), x)
```

3.1060 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=449

$$\frac{2\sin(c+dx)\left(-a^2b^2(23A+19C)+3a^3bB+2a^4C+29ab^3B-3b^4(3A+5C)\right)}{15bd\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(3a^2bB+2a^3C-2ab^2(4A\right.}{15bd\left(a^2-b^2\right)^2(a+b\cos(c+d\right.}$$

```
[Out] (-2*(3*a^3*b*B + 29*a*b^3*B + 2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23*A +
 19*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15
*b^2*(a^2 - b^2)^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a^2*b*B +
5*b^3*B + 2*a^3*C - 2*a*b^2*(4*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*
Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(5*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(5/2)) + (2*(3*a^2*b*B + 5*b^3*B + 2*a^3*C - 2*a*b^2
*(4*A + 5*C))*Sin[c + d*x])/(15*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2
)) + (2*(3*a^3*b*B + 29*a*b^3*B + 2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23
*A + 19*C))*Sin[c + d*x])/(15*b*(a^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.73646, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sin(c+dx)\left(-a^2b^2(23A+19C)+3a^3bB+2a^4C+29ab^3B-3b^4(3A+5C)\right)}{15bd\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(3a^2bB+2a^3C-2ab^2(4A\right.}{15bd\left(a^2-b^2\right)^2(a+b\cos(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*a^3*b*B + 29*a*b^3*B + 2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23*A +
 19*C))*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15
*b^2*(a^2 - b^2)^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a^2*b*B +
5*b^3*B + 2*a^3*C - 2*a*b^2*(4*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*
Cos[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(5*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(5/2)) + (2*(3*a^2*b*B + 5*b^3*B + 2*a^3*C - 2*a*b^2
*(4*A + 5*C))*Sin[c + d*x])/(15*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2
)) + (2*(3*a^3*b*B + 29*a*b^3*B + 2*a^4*C - 3*b^4*(3*A + 5*C) - a^2*b^2*(23
```

```
*A + 19*C))*Sin[c + d*x])/(15*b*(a^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx = -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{2\int \frac{\frac{5}{2}b(bB-a(A+C))+\frac{1}{2}\left(3Ab^2-3abB-2a^2C+5b^2\right.}{(a+b\cos(c+dx))^{5/2}}}{5b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} + \frac{2\left(3a^2bB + 5b^3B + 2a^3C - 2ab^2(4A\right.}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} + \frac{2\left(3a^2bB + 5b^3B + 2a^3C - 2ab^2(4A\right.}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} + \frac{2\left(3a^2bB + 5b^3B + 2a^3C - 2ab^2(4A\right.}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{5b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} + \frac{2\left(3a^2bB + 5b^3B + 2a^3C - 2ab^2(4A\right.}{15b\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{2\left(3a^3bB + 29ab^3B + 2a^4C - 3b^4(3A+5C) - a^2b^2(23A+19C)\right)\sqrt{a+b\cos}}{15b^2\left(a^2-b^2\right)^3 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 3.62935, size = 433, normalized size = 0.96

$$2\left(\frac{b\sin(c+dx)\left(2b\cos(c+dx)\left(2a^3b^2(27A+25C)-60a^2b^3B-9a^4bB-6a^5C+10ab^4(A+2C)+5b^5B\right)+b^2\cos(2(c+dx))\left(a^2b^2(23A+19C)-3a^3bB-2a^4C-29ab^3B+3b^4(3A\right.}{2\left(b^2-a^2\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(7/2
),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(5/2)*(b^2*(-27*a^2*b*B - 5*b^3*B + a^3
*(15*A + 7*C) + a*b^2*(17*A + 25*C))*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
+ (-3*a^3*b*B - 29*a*b^3*B - 2*a^4*C + 3*b^4*(3*A + 5*C) + a^2*b^2*(23*A +
19*C))*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*
x)/2, (2*b)/(a + b)])))/((a - b)^3*(a + b)) + (b*(68*a^4*A*b^2 + 13*a^2*A*b
^4 + 15*A*b^6 - 18*a^5*b*B - 53*a^3*b^3*B - 25*a*b^5*B - 2*a^6*C + 48*a^4*b
^2*C + 35*a^2*b^4*C + 15*b^6*C + 2*b*(-9*a^4*b*B - 60*a^2*b^3*B + 5*b^5*B -
 6*a^5*C + 10*a*b^4*(A + 2*C) + 2*a^3*b^2*(27*A + 25*C))*Cos[c + d*x] + b^2
*(-3*a^3*b*B - 29*a*b^3*B - 2*a^4*C + 3*b^4*(3*A + 5*C) + a^2*b^2*(23*A + 1
9*C))*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(-a^2 + b^2)^3)))/(15*b^2*d*(a + b
*Cos[c + d*x])^(5/2))
```

Maple [B] time = 6.512, size = 1316, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2/sin
(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(B*b-2*C*a)/b^2*(
1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d
*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2
*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*(A*b^2-B*
a*b+C*a^2)/b^2*(1/20/b^2/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b
```

```
)^3+4/15*a/b/(a+b)^2/(a-b)^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+2/15
*b*sin(1/2*d*x+1/2*c)^2/(a-b)^3/(a+b)^3*cos(1/2*d*x+1/2*c)*(23*a^2+9*b^2)/(
-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(15*a^2-8*a*b+
9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4+15*b^5)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-1/15*(23*a^2+9*b^2)/(a+b)^3/(a-b)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}}{b^4\cos(dx+c)^4 + 4\,ab^3\cos(dx+c)^3 + 6\,a^2b^2\cos(dx+c)^2 + 4\,a^3b\cos(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(
b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c)^3 + 6*a^2*b^2*cos(d*x + c)^2 + 4*
```

```
a^3*b*cos(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(7/2
), x)
```

3.1061 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=167

$$-\frac{2C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2bC\sin(c+dx)\sqrt{a}}{3d}$$

```
[Out] (2*(3*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*C*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*b*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```

Rubi [A] time = 0.342166, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3015, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2bC\sin(c+dx)\sqrt{a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/Sqrt[a + b*
Cos[c + d*x]],x]
```


```
[Out] (2*(3*b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*C*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*b*C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```


Rule 3015

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\int \sqrt{a+b\cos(c+dx)}\left(b^2(bB-aC)+b^3C\cos(c+dx)\right)dx}{b^2}$$

$$= \frac{2bC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2\int \frac{\frac{1}{2}b^2\left(3abB-3a^2C+b^2C\right)+\frac{1}{2}b^3 \cdots}{\sqrt{a+b\cos(\cdots}}}{3b^2}$$

$$= \frac{2bC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\left(\left(a^2-b^2\right)C\right)\int \frac{\cdots}{\sqrt{a+b \cdots}}$$

$$= \frac{2bC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left((3bB-2aC)\sqrt{a+b\cos(\cdots}\right.}{3\sqrt{\cdots}}$$

$$= \frac{2(3bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(a^2-b^2\right.\cdots}{\cdots}$$

Mathematica [A] time = 0.633121, size = 144, normalized size = 0.86

$$\frac{-2C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right) - 2(a+b)(2aC-3bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right) + 2bC\sin(c+dx)(a \cdots}{3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/Sqrt[
a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a + b)*(-3*b*B + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*C*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*C*(a + b*Cos[c + d*x])*Sin
[c + d*x])/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 1.104, size = 598, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2
),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1
/2*d*x+1/2*c)^5*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*C*cos(1/2*d*x+1/
2*c)^3*a*b-6*C*cos(1/2*d*x+1/2*c)^3*b^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2+C*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+2*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*C*cos(1/2*d*x+1/2*c)*a*b+2*C*cos(1/2*d
*x+1/2*c)*b^2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \cos{(dx+c)}^2 + Bb^2 \cos{(dx+c)} - Ca^2 + Bab}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/sqrt(
b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((Cb\cos{(dx+c)} - Ca + Bb)\sqrt{b\cos{(dx+c)}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c) - C*a + B*b)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \cos{(dx+c)}^2 + Bb^2 \cos{(dx+c)} - Ca^2 + Bab}{\sqrt{b\cos{(dx+c)} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/sqrt(
b*cos(d*x + c) + a), x)
```

3.1062 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{2(bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*
x]])
```

Rubi [A] time = 0.164509, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {24, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^(3/2),x]
```

```
[Out] (2*C*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2}$$

$$= C\int \sqrt{a+b\cos(c+dx)}\,dx + (bB-2aC)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{\left(C\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left((bB-2aC)\right.}{}$$

$$= \frac{2C\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(bB-2aC)\sqrt{\frac{a+b\,c}{}}}{d\sqrt{a+}}$$

Mathematica [A] time = 0.233296, size = 90, normalized size = 0.73

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((bB-2aC)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+C(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*C*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + (b*B - 2*a*C)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.871, size = 246, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}b+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}\sqrt{\frac{2\,(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2
),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(b*B*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*C*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a+C*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-C
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \cos{(dx+c)}^2 + Bb^2 \cos{(dx+c)} - Ca^2 + Bab}{(b\cos{(dx+c)} + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/(b*co
s(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos{(dx+c)} - Ca + Bb}{\sqrt{b\cos{(dx+c)} + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c) - C*a + B*b)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \cos (dx+c)^2 + Bb^2 \cos (dx+c) - Ca^2 + Bab}{(b \cos (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/(b*co
s(d*x + c) + a)^(3/2), x)
```

3.1063 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{2b(bB-2aC)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/((a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*C*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
b*Cos[c + d*x]]) - (2*b*(b*B - 2*a*C)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Rubi [A] time = 0.289058, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {24, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b(bB-2aC)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2C\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^(5/2),x]
```


```
[Out] (2*(b*B - 2*a*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/((a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*C*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
b*Cos[c + d*x]]) - (2*b*(b*B - 2*a*C)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
 b*Cos[c + d*x]])
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{b^2}$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}b^2\left(abB-a^2C-b^2C\right)-\frac{1}{2}b^3(bB-2}{\sqrt{a+b\cos(c+dx)}}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + C\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx +$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((bB-2aC)\sqrt{a+b\cos(c+d}\right.}{\left(a^2-b^2\right)\sqrt{\frac{a}{}}}$$

$$= \frac{2(bB-2aC)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2C\sqrt{\frac{a+b\cos}{a+}}}{d\sqrt{a}}$$

Mathematica [A] time = 0.593566, size = 150, normalized size = 0.83

$$\frac{2\left(C\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right) + b(2aC-bB)\sin(c+dx) - (a+b)(2aC-bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.\right.}{d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((a + b)*(-(b*B) + 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE
[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*C*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-(b*B) + 2*a*C)*Sin[c + d*
x]))/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 2.326, size = 422, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2b-a+b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\frac{C\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}}{\sqrt{-2\,b\left(\sin(1/2\,dx+c/2)\right)^4+(a+b)\left(\sin(1/2\,dx+c/2)\right)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2
),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))+2*(B*b-2*C*a)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d
*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \cos{(dx+c)}^2 + Bb^2 \cos{(dx+c)} - Ca^2 + Bab}{(b\cos{(dx+c)} + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/(b*co
s(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cb\cos{(dx+c)} - Ca + Bb)\sqrt{b\cos{(dx+c)} + a}}{b^2\cos{(dx+c)}^2 + 2\,ab\cos{(dx+c)} + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c) - C*a + B*b)*sqrt(b*cos(d*x + c) + a)/(b^2*cos(d
*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2\cos\left(dx+c\right)^2+Bb^2\cos\left(dx+c\right)-Ca^2+Bab}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/(b*co
s(d*x + c) + a)^(5/2), x)
```

3.1064 $\int \frac{abB-a^2C+b^2B\cos(c+dx)+b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{2b\left(-5a^2C+4abB-3b^2C\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b(bB-2aC)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\right.}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(4*a*b*B - 5*a^2*C - 3*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*(b*B
- 2*a*C)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*
(4*a*b*B - 5*a^2*C - 3*b^2*C)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*C
os[c + d*x]])
```

Rubi [A] time = 0.451462, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {24, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b\left(-5a^2C+4abB-3b^2C\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b(bB-2aC)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(bB-2aC)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\right.}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a + b*Cos[
c + d*x])^(7/2),x]
```


```
[Out] (2*(4*a*b*B - 5*a^2*C - 3*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(b*B - 2*a*C)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*(b*B
- 2*a*C)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*
(4*a*b*B - 5*a^2*C - 3*b^2*C)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*C
os[c + d*x]])
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
```

```
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{7/2}}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{b^2}$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}b^2\left(abB-a^2C-b^2C\right)+\frac{1}{2}\cdots}{(a+b\cos(c\cdots}}{3b^2\left(a^2\cdots\right.}$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(4abB-5a^2C-3b^2\cdots\right.}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cdots}}$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(4abB-5a^2C-3b^2\cdots\right.}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cdots}}$$

$$= -\frac{2b(bB-2aC)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(4abB-5a^2C-3b^2\cdots\right.}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cdots}}$$

$$= \frac{2\left(4abB-5a^2C-3b^2C\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\cdots}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.54474, size = 193, normalized size = 0.71

$$\frac{2\left(\frac{b\sin(c+dx)\left(b\left(5a^2C-4abB+3b^2C\right)\cos(c+dx)-5a^2bB+7a^3C+ab^2C+b^3B\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(5a^2C-4abB+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-(a-b)(2aC-bB)F\left(\frac{1}{2}(c+\cdots\right.\right.}{(a-b)^2}\cdots}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Cos[c + d*x] + b^2*C*Cos[c + d*x]^2)/(a +
b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-4*a*b*B + 5*a^2*C + 3*b^2*C)
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(-(b*B) + 2*a*C)*EllipticF
[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (b*(-5*a^2*b*B + b^3*B + 7*a^3*
C + a*b^2*C + b*(-4*a*b*B + 5*a^2*C + 3*b^2*C)*Cos[c + d*x])*Sin[c + d*x])/
(a^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 3.672, size = 744, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2
),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/sin(1/2
*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(B*b-2*C*a)*(1/6/b/(a
-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c
)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a+b)^2/(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)
/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \cos(dx+c)^2 + Bb^2 \cos(dx+c) - Ca^2 + Bab}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/(b*co
s(d*x + c) + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cb\cos(dx+c)-Ca+Bb)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c) - C*a + B*b)*sqrt(b*cos(d*x + c) + a)/(b^3*cos(d
*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*cos(d*x+c)+b**2*C*cos(d*x+c)**2)/(a+b*cos(d*
x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2\cos(dx+c)^2+Bb^2\cos(dx+c)-Ca^2+Bab}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c)
)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*cos(d*x + c)^2 + B*b^2*cos(d*x + c) - C*a^2 + B*a*b)/(b*co
s(d*x + c) + a)^(7/2), x)
```

3.1065 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=190

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA+5aC+5bB)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(9aB+9Ab+7bC)}{15d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(9aB+9Ab+}{45d}$$

```
[Out] (2*(9*A*b + 9*a*B + 7*b*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*a*A +
5*b*B + 5*a*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*A + 5*b*B + 5*a*
C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A*b + 9*a*B + 7*b*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*B + a*C)*Cos[c + d*x]^(5/2)*Si
n[c + d*x])/(7*d) + (2*b*C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.25543, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA+5aC+5bB)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(9aB+9Ab+7bC)}{15d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(9aB+9Ab+}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*(9*A*b + 9*a*B + 7*b*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*a*A +
5*b*B + 5*a*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*A + 5*b*B + 5*a*
C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A*b + 9*a*B + 7*b*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*B + a*C)*Cos[c + d*x]^(5/2)*Si
n[c + d*x])/(7*d) + (2*b*C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2bC\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{3}{2}}$$
$$= \frac{2(bB+aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2b}{}$$
$$= \frac{2(bB+aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2b}{}$$
$$= \frac{2(7aA+5bB+5aC)\sqrt{\cos(c+dx)}\sin(c+}{21d}$$
$$= \frac{2(9Ab+9aB+7bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2(}{}$$

Mathematica [A] time = 0.994038, size = 143, normalized size = 0.75

$$\frac{60F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA+5aC+5bB)+84E\left(\frac{1}{2}(c+dx)\middle|2\right)(9aB+9Ab+7bC)+\sin(c+dx)\sqrt{\cos(c+dx)}(7\cos(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (84*(9*A*b + 9*a*B + 7*b*C)*EllipticE[(c + d*x)/2, 2] + 60*(7*a*A + 5*b*B +
 5*a*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A*b + 36*a*B
+ 43*b*C)*Cos[c + d*x] + 5*(84*a*A + 78*b*B + 78*a*C + 18*(b*B + a*C)*Cos[2
*(c + d*x)] + 7*b*C*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 0.929, size = 565, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b+720*C*a+2240*C*b)*sin(1/2*
```

```
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b-504*B*a-1080*B*b-1080*C*a-2072*C*
b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a+504*A*b+504*B*a+840*B*b
+840*C*a+952*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a-126*A*b
-126*B*a-240*B*b-240*C*a-168*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1
05*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+75*b*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+75*a*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^4+\left(Ca+Bb\right)\cos\left(dx+c\right)^3+Aa\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + (C*a + B*b)*cos(d*x + c)^3 + A*a*cos(d*x + c
) + (B*a + A*b)*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

3.1066 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=154

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aB+7Ab+5bC)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+3aC+3bB)}{5d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(7aB+7Ab}{21d}$$

```
[Out] (2*(5*a*A + 3*b*B + 3*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A*b + 7
*a*B + 5*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*A*b + 7*a*B + 5*b*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*B + a*C)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(5*d) + (2*b*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.238454, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aB+7Ab+5bC)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+3aC+3bB)}{5d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(7aB+7Ab}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```


```
[Out] (2*(5*a*A + 3*b*B + 3*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A*b + 7
*a*B + 5*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*A*b + 7*a*B + 5*b*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*B + a*C)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(5*d) + (2*b*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2bC\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int\sqrt{c}$$

$$= \frac{2(bB+aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+$$

$$= \frac{2(bB+aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+$$

$$= \frac{2(5aA+3bB+3aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

$$= \frac{2(5aA+3bB+3aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

Mathematica [A] time = 0.858339, size = 117, normalized size = 0.76

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aB+7Ab+5bC)+42E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+3aC+3bB)+\sin(c+dx)\sqrt{\cos(c+dx)}(42(aC+bB}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (42*(5*a*A + 3*b*B + 3*a*C)*EllipticE[(c + d*x)/2, 2] + 10*(7*A*b + 7*a*B +
 5*b*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A*b + 70*a*B + 6
5*b*C + 42*(b*B + a*C)*Cos[c + d*x] + 15*b*C*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

Maple [B] time = 1.096, size = 515, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b-168*C*a-360*C*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b+140*B*a+168*B*b+168*C*a+280*C*b)*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*b-70*B*a-42*B*b-42*C*a-80*C*b)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a+35*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b+25*C*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1067 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=116

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(3A+C)+bB)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+3bC)}{5d}+\frac{2(aC+bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b}{}$$

```
[Out] (2*(5*A*b + 5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*(
3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*(b*B + a*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(3*d) + (2*b*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.217469, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(3A+C)+bB)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+3bC)}{5d}+\frac{2(aC+bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```


```
[Out] (2*(5*A*b + 5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*(
3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*(b*B + a*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(3*d) + (2*b*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bC\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \frac{\frac{5aA}{2}+\frac{1}{2}(5Ab+$$

$$= \frac{2(bB+aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2bC\cos^{\frac{3}{2}}(c+$$

$$= \frac{2(bB+aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2bC\cos^{\frac{3}{2}}(c+$$

$$= \frac{2(5Ab+5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(bB+a(3A$$

Mathematica [A] time = 0.593086, size = 94, normalized size = 0.81

$$\frac{2\left(5F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+aC+bB)+3E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+3bC)+\sin(c+dx)\sqrt{\cos(c+dx)}(5aC+5bB+}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*(5*A*b + 5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2] + 5*(3*a*A + b*B +
a*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*b*B + 5*a*C + 3*b*C*
Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 0.844, size = 465, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b+20*C*a+24*C*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*B*b-10*C*a-6*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+15*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
+5*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5*a*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

3.1068 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a(A-C))}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sin(c+dx)\sqrt{\cos(c}}{3d}$$

```
[Out] (2*(b*B - a*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A*b + 3*a*B + b*C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]]) + (2*b*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.224238, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a(A-C))}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sin(c+dx)\sqrt{\cos(c}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```


```
[Out] (2*(b*B - a*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A*b + 3*a*B + b*C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]]) + (2*b*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-2\int\frac{\frac{1}{2}(-Ab-aB)-\frac{1}{2}(bB-a(A-C}{\sqrt{\cos}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}-\frac{4}{3}\int$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}-(-b$$

$$= \frac{2(bB-a(A-C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3Ab+3aB+bC}{3a}$$

Mathematica [A] time = 0.485783, size = 90, normalized size = 0.84

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)+E\left(\frac{1}{2}(c+dx)\middle|2\right)(-6aA+6aC+6bB)+\frac{2\sin(c+dx)(3aA+bC\cos(c+dx))}{\sqrt{\cos(c+dx)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] ((-6*a*A + 6*b*B + 6*a*C)*EllipticE[(c + d*x)/2, 2] + 2*(3*A*b + 3*a*B + b*
C)*EllipticF[(c + d*x)/2, 2] + (2*(3*a*A + b*C*Cos[c + d*x])*Sin[c + d*x])/
Sqrt[Cos[c + d*x]])/(3*d)
```

Maple [B] time = 1.392, size = 388, normalized size = 3.6

$$-\frac{2}{3\,d}\left(4\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+3\,Ab\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*C*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-6*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+3*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+C*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*C*b*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Cb\cos\left(dx+c\right)^3+\left(Ca+Bb\right)\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

3.1069 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab-bC)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(A*b + a*B - b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*b*B + a*(A + 3*C
))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.237622, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab-bC)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B - b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*b*B + a*(A + 3*C
))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{1}{2}(3bB+a(A$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{4}{3}\int \frac{\frac{1}{4}(}{}$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (Ab+a$$

$$= -\frac{2(Ab+aB-bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3bB+a(A+}{}$$

Mathematica [A] time = 0.600087, size = 115, normalized size = 1.04

$$\frac{2\left(\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(aA+3aC+3bB)-3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab-bC)+aA\tan(c+dx)+\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] (2*(-3*(A*b + a*B - b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (a*
A + 3*b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin
[c + d*x] + 3*a*B*Sin[c + d*x] + a*A*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]]
)
```

Maple [B] time = 1.873, size = 666, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*A*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
```

```
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^3 + \left(Ca+Bb\right)\cos\left(dx+c\right)^2 + Aa + \left(Ba+Ab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/
2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(5/2), x)
```

3.1070 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+5aC+5bB)}{5d}+\frac{2\sin(c+dx)(3aA+5aC+5bB)}{5d\sqrt{\cos(c+dx)}}+\frac{2(aB}{3}$$

```
[Out] (-2*(3*a*A + 5*b*B + 5*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*
B + 3*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos
[c + d*x]^(5/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) +
(2*(3*a*A + 5*b*B + 5*a*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.255427, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+5aC+5bB)}{5d}+\frac{2\sin(c+dx)(3aA+5aC+5bB)}{5d\sqrt{\cos(c+dx)}}+\frac{2(aB}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
```


```
[Out] (-2*(3*a*A + 5*b*B + 5*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*
B + 3*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos
[c + d*x]^(5/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) +
(2*(3*a*A + 5*b*B + 5*a*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
```

```
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{1}{2}(3aA+5bB}{\mathrm{c}}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int -$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}(-3aA$$

$$= \frac{2(Ab+aB+3bC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2(3aA+5bB+5aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB}{}$$

Mathematica [A] time = 1.38191, size = 136, normalized size = 0.89

$$\frac{3\sin(2(c+dx))(3aA+5aC+5bB)+10\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a*A + 5*b*B + 5*a*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] +
10*(A*b + a*B + 3*b*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*(A
*b + a*B)*Sin[c + d*x] + 3*(3*a*A + 5*b*B + 5*a*C)*Sin[2*(c + d*x)] + 6*a*A
*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 3.295, size = 742, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(A*
b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a*A/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^
2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*(B*b+C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin
(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^3+\left(Ca+Bb\right)\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(7/2), x)
```

3.1071 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=190

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+7aC+7bB)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC)}{5d}+\frac{2\sin(c+dx)(5aA+7aC+7bB)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin}{}$$

```
[Out] (-2*(3*A*b + 3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A +
7*b*B + 7*a*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (2*(5*a*A + 7*b*B + 7*a*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (
2*(3*A*b + 3*a*B + 5*b*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.281818, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+7aC+7bB)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC)}{5d}+\frac{2\sin(c+dx)(5aA+7aC+7bB)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```



```
[Out] (-2*(3*A*b + 3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A +
7*b*B + 7*a*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*A*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (2*(5*a*A + 7*b*B + 7*a*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (
2*(3*A*b + 3*a*B + 5*b*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
```

```
1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(5aA+7bB+}{\cos}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}}{}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{7}(-5aA$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(5aA+}{2}$$

$$= -\frac{2(3Ab+3aB+5bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5aA+7bB}{}$$

Mathematica [A] time = 4.18118, size = 173, normalized size = 0.91

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+7aC+7bB)-42E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC)+\frac{\sin(c+dx)(21\cos(c+dx)(13aB+13Ab+15bC)+10\cos}{}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```

```
[Out] (-42*(3*A*b + 3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2] + 10*(5*a*A + 7*b*B
+ 7*a*C)*EllipticF[(c + d*x)/2, 2] + ((110*a*A + 70*b*B + 70*a*C + 21*(13*A
*b + 13*a*B + 15*b*C)*Cos[c + d*x] + 10*(5*a*A + 7*b*B + 7*a*C)*Cos[2*(c +
d*x)] + 63*A*b*Cos[3*(c + d*x)] + 63*a*B*Cos[3*(c + d*x)] + 105*b*C*Cos[3*(
c + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(7/2)))/(105*d)
```

Maple [B] time = 3.511, size = 851, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*A*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b+C
*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*(A*b+B*a)/(8*sin(1/2*d*x+
1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*
c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*C*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos(dx+c)^3+(Ca+Bb)\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(
d*x + c)^(9/2), x)
```

3.1072 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos\right.$

Optimal. Leaf size=305

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(11a^2(7A+5C)+110abB+5b^2(11A+9C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2B+18aAb+14abC+7b^2B\right)}{15d}+$$

```
[Out] (2*(18*a*A*b + 9*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticE[(c + d*x)/2, 2])/(15
*d) + (2*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^2*(11*A +
 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(18*a*A*b + 9*a^2*B +
7*b^2*B + 14*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(11*A*b^2
+ 22*a*b*B + 4*a^2*C + 9*b^2*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (
2*b*(11*b*B + 4*a*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*C*Cos[c +
 d*x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.607657, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3049, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(11a^2(7A+5C)+110abB+5b^2(11A+9C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2B+18aAb+14abC+7b^2B\right)}{15d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*(18*a*A*b + 9*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticE[(c + d*x)/2, 2])/(15
*d) + (2*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^2*(11*A +
 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(18*a*A*b + 9*a^2*B +
7*b^2*B + 14*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(11*A*b^2
+ 22*a*b*B + 4*a^2*C + 9*b^2*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (
2*b*(11*b*B + 4*a*C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*C*Cos[c +
 d*x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin}{11d}$$

$$= \frac{2b(11bB+4aC)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{99d}$$

$$= \frac{2\left(11Ab^2+22abB+4a^2C+9b^2C\right)\cos}{77d}$$

$$= \frac{2\left(11Ab^2+22abB+4a^2C+9b^2C\right)\cos}{77d}$$

$$= \frac{2\left(110abB+11a^2(7A+5C)+5b^2(11A\right.}{231d}$$

$$= \frac{2\left(18aAb+9a^2B+7b^2B+14abC\right)E\left(\right.}{15d}$$

Mathematica [A] time = 1.60861, size = 239, normalized size = 0.78

$$10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(11a^2(7A+5C)+110abB+5b^2(11A+9C)\right)+154E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2B+2ab(9A+7C)+7b^2B\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (154*(9*a^2*B + 7*b^2*B + 2*a*b*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 10
*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*EllipticF[(c + d*x)/
2, 2] + (Sqrt[Cos[c + d*x]]*(154*(72*a*A*b + 36*a^2*B + 43*b^2*B + 86*a*b*C
)*Cos[c + d*x] + 5*(3432*a*b*B + 132*a^2*(14*A + 13*C) + 3*b^2*(572*A + 531
*C) + 36*(11*A*b^2 + 22*a*b*B + 11*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 154
```

```
*b*(b*B + 2*a*C)*Cos[3*(c + d*x)] + 63*b^2*C*Cos[4*(c + d*x)]))*Sin[c + d*x
])/12)/(1155*d)
```

Maple [B] time = 0.947, size = 863, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^2-24640*C*a*b-50400*
C*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^2+15840*B*a*b+246
40*B*b^2+7920*C*a^2+49280*C*a*b+56880*C*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-11088*A*a*b-11880*A*b^2-5544*B*a^2-23760*B*a*b-22792*B*b^2-1188
0*C*a^2-45584*C*a*b-34920*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4
620*A*a^2+11088*A*a*b+9240*A*b^2+5544*B*a^2+18480*B*a*b+10472*B*b^2+9240*C*
a^2+20944*C*a*b+13860*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310
*A*a^2-2772*A*a*b-2640*A*b^2-1386*B*a^2-5280*B*a*b-1848*B*b^2-2640*C*a^2-36
96*C*a*b-2790*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*A*a^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+825*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4158*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b+1650*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+825*a^2*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+675*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3234*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^5+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^4+Aa^2\cos\left(dx+c\right)+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^3+\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5 + (2*C*a*b + B*b^2)*cos(d*x + c)^4 + A*a^2*c
os(d*x + c) + (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3 + (B*a^2 + 2*A*a*b)*
cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1073 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=251

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2B+14aAb+10abC+5b^2B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2(5A+3C)+18abB+b^2(9A+7C)\right)}{15d}+\frac{2\,\mathrm{si}}{}$$

```
[Out] (2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2,
2])/(15*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A*b^2 + 18*a*b*B + 4*a^2*C + 7*b^2*C)*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*b*B + 4*a*C)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(63*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^
2*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.537068, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2B+14aAb+10abC+5b^2B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2(5A+3C)+18abB+b^2(9A+7C)\right)}{15d}+\frac{2\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2,
2])/(15*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A*b^2 + 18*a*b*B + 4*a^2*C + 7*b^2*C)*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*b*B + 4*a*C)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(63*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^
2*Sin[c + d*x])/(9*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

<u>Rule 3033</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

<u>Rule 3023</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

<u>Rule 2748</u>

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

<u>Rule 2639</u>

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

<u>Rule 2635</u>

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin}{9d}$$

$$= \frac{2b(9bB+4aC)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx}{63d}$$

$$= \frac{2\left(9Ab^2+18abB+4a^2C+7b^2C\right)\cos^{\frac{3}{2}}}{45d}$$

$$= \frac{2\left(9Ab^2+18abB+4a^2C+7b^2C\right)\cos^{\frac{3}{2}}}{45d}$$

$$= \frac{2\left(18abB+3a^2(5A+3C)+b^2(9A+7\right.}{15d}$$

$$= \frac{2\left(18abB+3a^2(5A+3C)+b^2(9A+7\right.}{15d}$$

Mathematica [A] time = 1.26707, size = 195, normalized size = 0.78

$$\frac{60F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2B+2ab(7A+5C)+5b^2B\right)+84E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2(5A+3C)+18abB+b^2(9A+7C)\right)+\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (84*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2,
 2] + 60*(7*a^2*B + 5*b^2*B + 2*a*b*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2]
+ Sqrt[Cos[c + d*x]]*(7*(36*A*b^2 + 72*a*b*B + 36*a^2*C + 43*b^2*C)*Cos[c +
 d*x] + 5*(168*a*A*b + 84*a^2*B + 78*b^2*B + 156*a*b*C + 18*b*(b*B + 2*a*C)
*Cos[2*(c + d*x)] + 7*b^2*C*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 1.02, size = 784, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^2+1440*C*a*b+2240*C*b^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^2-1008*B*a*b-1080*B*b^2-
504*C*a^2-2160*C*a*b-2072*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(8
40*A*a*b+504*A*b^2+420*B*a^2+1008*B*a*b+840*B*b^2+504*C*a^2+1680*C*a*b+952*
C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-420*A*a*b-126*A*b^2-210*B*
a^2-252*B*a*b-240*B*b^2-126*C*a^2-480*C*a*b-168*C*b^2)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2+210*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b+105*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*b^2*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+1
50*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1074 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=203

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+6abC+3b^2B\right)}{5d}+\frac{2\sin(c}{}$$

```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d
) + (2*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2
, 2])/(21*d) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(21*d) + (2*b*(7*b*B + 4*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/
(7*d)
```

Rubi [A] time = 0.505848, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+6abC+3b^2B\right)}{5d}+\frac{2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d
) + (2*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2
, 2])/(21*d) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(21*d) + (2*b*(7*b*B + 4*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/
(7*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d}+\frac{2}{7}$$

$$= \frac{2b(7bB+4aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2C\sqrt{\cos(c}{}$$

$$= \frac{2\left(7Ab^2+14abB+4a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c}{21d}$$

$$= \frac{2\left(7Ab^2+14abB+4a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c}{21d}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B+6abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

Mathematica [A] time = 1.30274, size = 160, normalized size = 0.79

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)+42E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+2ab(5A+3C)+3b^2B\right)+\sin(c+}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (42*(5*a^2*B + 3*b^2*B + 2*a*b*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 10*
(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2] +
Sqrt[Cos[c + d*x]]*(42*b*(b*B + 2*a*C)*Cos[c + d*x] + 5*(14*A*b^2 + 28*a*b*
B + 14*a^2*C + 13*b^2*C + 3*b^2*C*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] time = 0.999, size = 706, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^2-336*C*a*b-360*C*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^2+280*B*a*b+168*B*b^2+140*C*a
^2+336*C*a*b+280*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*b^2-
140*B*a*b-42*B*b^2-70*C*a^2-84*C*a*b-80*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-210*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+105*A*a^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+70*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b+35*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)}{\sqrt{\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sq
rt(cos(d*x + c)), x)
```

3.1075 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=189

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^2(A-C)+10abB+b^2(5A+3C)\right)}{5d}-\frac{2b\sin(c+\cdots}{}$$

```
[Out] (2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(
3*d) - (2*b*(6*a*A - b*B - 2*a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) -
(2*b^2*(5*A - C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*(a + b*Cos[c
 + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.519033, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^2(A-C)+10abB+b^2(5A+3C)\right)}{5d}-\frac{2b\sin(c+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```



```
[Out] (2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(
3*d) - (2*b*(6*a*A - b*B - 2*a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) -
(2*b^2*(5*A - C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*(a + b*Cos[c
 + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
```

```
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos}{}$$

$$= -\frac{2b^2(5A-C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2A(a+b}{}$$

$$= -\frac{2b(6aA-bB-2aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b}{}$$

$$= -\frac{2b(6aA-bB-2aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2b}{}$$

$$= \frac{2\left(10abB-5a^2(A-C)+b^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\right.}{5d}$$

Mathematica [A] time = 1.24664, size = 144, normalized size = 0.76

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)+6E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^2(A-C)+10abB+b^2(5A+3C)\right)+\frac{\sin(c+dx)(}{}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] (6*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] +
 10*(3*a^2*B + b^2*B + 2*a*b*(3*A + C))*EllipticF[(c + d*x)/2, 2] + ((10*b*
(b*B + 2*a*C)*Cos[c + d*x] + 3*(10*a^2*A + b^2*C + b^2*C*Cos[2*(c + d*x)]))
*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] time = 1.089, size = 932, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b*(5*B*b+10*C*a+6*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^2+5*B*b^2
+10*C*a*b+3*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*a*A*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*b^2+15*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+5*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-30*B*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*a*b*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^2\cos(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)}{\cos(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(3/2), x)
```

3.1076 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-C)-b^2B\right)}{d}+\frac{2a(3aB+4Ab)\,\mathrm{si}}{3d\sqrt{\cos(c+}}$$

```
[Out] (-2*(a^2*B - b^2*B + 2*a*b*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*
b*B + b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*
a*(4*A*b + 3*a*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(A - C)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.500862, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-C)-b^2B\right)}{d}+\frac{2a(3aB+4Ab)\,\mathrm{si}}{3d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```


```
[Out] (-2*(a^2*B - b^2*B + 2*a*b*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*
b*B + b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*
a*(4*A*b + 3*a*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(A - C)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{(a+b\cos(}{}$$

$$= \frac{2a(4Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2A(a+b\cos(c+dx))^2\,\mathrm{s}}{3d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a(4Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{2b^2(A-C)\sqrt{\cos(c+dx}}{3d}$$

$$= \frac{2a(4Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{2b^2(A-C)\sqrt{\cos(c+dx}}{3d}$$

$$= -\frac{2\left(a^2B-b^2B+2ab(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(6a\right.}{}$$

Mathematica [A] time = 1.23892, size = 157, normalized size = 0.87

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)-6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-C)-\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*(a^2*B - b^2*B + 2*a*b*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2] + 2*(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2] + 12*a*A*b*Sin[c + d*x] + 6*a^2*B*Sin[c + d*x] + b^2*
C*Sin[2*(c + d*x)] + 2*a^2*A*Tan[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 3.009, size = 1303, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-6*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-12*B*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+8*
b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*b^2*C*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^4+6*B*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*C*b^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+2*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2+2*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*
sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-
6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-24*A*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+12*A*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*a*b*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+12*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b*sin(1/2*d*x+1/2*c)^2+12*B*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b*sin(1/2*d*x+1/2*c)^2-12*C*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(5/2), x)
```

3.1077 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)+10abB+5b^2(A-C)\right)}{5d}+\frac{2\sin(c+dx}{}$$

```
[Out] (-2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])
/(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/
(3*d) + (2*a*(4*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*(
4*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]
) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.539373, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)+10abB+5b^2(A-C)\right)}{5d}+\frac{2\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```



```
[Out] (-2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])
/(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/
(3*d) + (2*a*(4*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*(
4*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]
) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c$$

$$= \frac{2a(4Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^2 \mathrm{s}}{5d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2a(4Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(4Ab^2+10abB+a^2(3\right.}{5d\sqrt{\cos(c}}$$

$$= \frac{2a(4Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(4Ab^2+10abB+a^2(3\right.}{5d\sqrt{\cos(c}}$$

$$= -\frac{2\left(10abB+5b^2(A-C)+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\right.}{5d}$$

Mathematica [A] time = 1.59025, size = 202, normalized size = 1.01

$$\frac{10\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)+10abB+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*x]^(3/2)*Ellipti
cE[(c + d*x)/2, 2] + 10*(a^2*B + 3*b^2*B + 2*a*b*(A + 3*C))*Cos[c + d*x]^(3
/2)*EllipticF[(c + d*x)/2, 2] + 20*a*A*b*Sin[c + d*x] + 10*a^2*B*Sin[c + d*
x] + 9*a^2*A*Sin[2*(c + d*x)] + 15*A*b^2*Sin[2*(c + d*x)] + 30*a*b*B*Sin[2*
(c + d*x)] + 15*a^2*C*Sin[2*(c + d*x)] + 6*a^2*A*Tan[c + d*x])/(15*d*Cos[c
+ d*x]^(3/2))
```

Maple [B] time = 2.845, size = 1000, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a*b*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*A*a^2/(8*sin(1/
2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*
x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*(A*b^2+2*B*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\mathrm{c}}{\cos(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\left(b\cos(dx+c) + a\right)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(7/2), x)
```

3.1078 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=248

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)+14abB+7b^2(A+3C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+6aAb+10abC+5b^2B\right)}{5d}+\frac{2\sin(c$$

```
[Out] (-2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*
d) + (2*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*a*(4*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)
) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*d*Cos[c + d
*x]^(3/2)) + (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x])/(5*d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c
 + d*x]^(7/2))
```

Rubi [A] time = 0.560113, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)+14abB+7b^2(A+3C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+6aAb+10abC+5b^2B\right)}{5d}+\frac{2\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```


```
[Out] (-2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*
d) + (2*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*a*(4*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)
) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*d*Cos[c + d
*x]^(3/2)) + (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x])/(5*d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c
 + d*x]^(7/2))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\cos(}{}$$

$$= \frac{2a(4Ab+7aB)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^2 \mathrm{s}}{7d\cos^{\frac{7}{2}}(c+dx}$$

$$= \frac{2a(4Ab+7aB)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(4Ab^2+14abB+a^2(5\right.}{21d\cos^{\frac{3}{2}}(}$$

$$= \frac{2a(4Ab+7aB)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(4Ab^2+14abB+a^2(5\right.}{21d\cos^{\frac{3}{2}}(}$$

$$= \frac{2\left(14abB+7b^2(A+3C)+a^2(5A+7C)\right)F\left(\frac{1}{2}(c+dx)\right.}{21d}$$

$$= -\frac{2\left(6aAb+3a^2B+5b^2B+10abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} +$$

Mathematica [A] time = 4.56026, size = 217, normalized size = 0.88

$$\frac{2\left(5F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)+14abB+7b^2(A+3C)\right)-21E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+5C)+5b^2B\right)+\frac{5\sin(c}{}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(-21*(3*a^2*B + 5*b^2*B + 2*a*b*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2] +
 5*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2]
 + (15*a^2*A*Sin[c + d*x])/Cos[c + d*x]^(7/2) + (21*a*(2*A*b + a*B)*Sin[c +
 d*x])/Cos[c + d*x]^(5/2) + (5*(7*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sin[c
 + d*x])/Cos[c + d*x]^(3/2) + (21*(3*a^2*B + 5*b^2*B + 2*a*b*(3*A + 5*C))*S
in[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)
```

Maple [B] time = 3.714, size = 947, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A
*a^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))+2*(A*b^2+2*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5
*a*(2*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x
+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(B*b+2*C*a)*(-(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
```

```
1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(9/2), x)
```

3.1079 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+14abC+7b^2B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(7A+9C)+18abB+3b^2(3A+5C)\right)}{15d}+\frac{2\sin(\ldots}{\ldots}$$

```
[Out] (-2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2,
 2])/(15*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d
*x)/2, 2])/(21*d) + (2*a*(4*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(
7/2)) + (2*(4*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Sin[c + d*x])/(45*d*Cos[c
 + d*x]^(5/2)) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])
/(21*d*Cos[c + d*x]^(3/2)) + (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A +
9*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2
*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.638018, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+14abC+7b^2B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(7A+9C)+18abB+3b^2(3A+5C)\right)}{15d}+\frac{2\sin(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```


```
[Out] (-2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2,
 2])/(15*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d
*x)/2, 2])/(21*d) + (2*a*(4*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(
7/2)) + (2*(4*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Sin[c + d*x])/(45*d*Cos[c
 + d*x]^(5/2)) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])
/(21*d*Cos[c + d*x]^(3/2)) + (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A +
9*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2
*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2}{9}\int \frac{(a+b\cos(}{}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{2A(a+b\cos(c+dx))^2\,\mathrm{s}}{9d\cos^{\frac{9}{2}}(c+dx}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\left(4Ab^2+18abB+a^2(7\right.}{45d\cos^{\frac{5}{2}}(}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\left(4Ab^2+18abB+a^2(7\right.}{45d\cos^{\frac{5}{2}}(}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\left(4Ab^2+18abB+a^2(7\right.}{45d\cos^{\frac{5}{2}}(}$$

$$= -\frac{2\left(18abB+3b^2(3A+5C)+a^2(7A+9C)\right)E\left(\frac{1}{2}(c+\right.}{15d}$$

Mathematica [A] time = 5.24569, size = 266, normalized size = 0.88

$$\frac{2\left(15F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+2ab(5A+7C)+7b^2B\right)-21E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(7A+9C)+18abB+3b^2(3A+5C)\right)+\frac{15\,\mathrm{s}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] (2*(-21*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x
)/2, 2] + 15*(5*a^2*B + 7*b^2*B + 2*a*b*(5*A + 7*C))*EllipticF[(c + d*x)/2,
 2] + (35*a^2*A*Sin[c + d*x])/Cos[c + d*x]^(9/2) + (45*a*(2*A*b + a*B)*Sin[
c + d*x])/Cos[c + d*x]^(7/2) + (7*(9*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Si
n[c + d*x])/Cos[c + d*x]^(5/2) + (15*(5*a^2*B + 7*b^2*B + 2*a*b*(5*A + 7*C)
)*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^
2*(7*A + 9*C))*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(315*d)
```

Maple [B] time = 4.865, size = 1196, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(2*A*b+B*a)
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+2*b*(B*b+2*C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*(A*b^2+2*B*
a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^2*(-1/144*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*b^2*C*(-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/co
s(d*x + c)^(11/2), x)
```

3.1080 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=361

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(33a^2b(7A+5C)+77a^3B+165ab^2B+5b^3(11A+9C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^3(5A+3C)+27a^2bB\right.}{15d}$$

```
[Out] (2*(27*a^2*b*B + 7*b^3*B + 3*a^3*(5*A + 3*C) + 3*a*b^2*(9*A + 7*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(77*a^3*B + 165*a*b^2*B + 33*a^2*b*(7*A + 5
*C) + 5*b^3*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (2*(77*a^3*B
 + 165*a*b^2*B + 33*a^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(231*d) + (2*(242*a^2*b*B + 77*b^3*B + 24*a^3*C + 33*a*b^
2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(495*d) + (2*b*(99*A*b^2 +
143*a*b*B + 24*a^2*C + 81*b^2*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d) +
 (2*(11*b*B + 6*a*C)*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(99*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(11
*d)
```

Rubi [A] time = 0.920162, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(33a^2b(7A+5C)+77a^3B+165ab^2B+5b^3(11A+9C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^3(5A+3C)+27a^2bB\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*(27*a^2*b*B + 7*b^3*B + 3*a^3*(5*A + 3*C) + 3*a*b^2*(9*A + 7*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(77*a^3*B + 165*a*b^2*B + 33*a^2*b*(7*A + 5
*C) + 5*b^3*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (2*(77*a^3*B
 + 165*a*b^2*B + 33*a^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(231*d) + (2*(242*a^2*b*B + 77*b^3*B + 24*a^3*C + 33*a*b^
2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(495*d) + (2*b*(99*A*b^2 +
143*a*b*B + 24*a^2*C + 81*b^2*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d) +
 (2*(11*b*B + 6*a*C)*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(99*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(11
*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\sin(}{11d}$$

$$= \frac{2(11bB+6aC)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c}{99d}$$

$$= \frac{2b\left(99Ab^2+143abB+24a^2C+81b^2C\right)}{693d}$$

$$= \frac{2\left(242a^2bB+77b^3B+24a^3C+33ab^2(9}{495d}$$

$$= \frac{2\left(242a^2bB+77b^3B+24a^3C+33ab^2(9}{495d}$$

$$= \frac{2\left(27a^2bB+7b^3B+3a^3(5A+3C)+3ab\right.}{15d}$$

$$= \frac{2\left(27a^2bB+7b^3B+3a^3(5A+3C)+3ab\right.}{15d}$$

Mathematica [A] time = 1.87997, size = 285, normalized size = 0.79

$$10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(33a^2b(7A+5C)+77a^3B+165ab^2B+5b^3(11A+9C)\right)+154E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^3(5A+3C)+27a^2b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (154*(27*a^2*b*B + 7*b^3*B + 3*a^3*(5*A + 3*C) + 3*a*b^2*(9*A + 7*C))*Ellip
ticE[(c + d*x)/2, 2] + 10*(77*a^3*B + 165*a*b^2*B + 33*a^2*b*(7*A + 5*C) +
5*b^3*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(154*(1
08*a^2*b*B + 43*b^3*B + 36*a^3*C + 3*a*b^2*(36*A + 43*C))*Cos[c + d*x] + 5*
(1848*a^3*B + 5148*a*b^2*B + 396*a^2*b*(14*A + 13*C) + 3*b^3*(572*A + 531*C
) + 36*b*(11*A*b^2 + 33*a*b*B + 33*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 154
*b^2*(b*B + 3*a*C)*Cos[3*(c + d*x)] + 63*b^3*C*Cos[4*(c + d*x)]))*Sin[c + d
*x])/12)/(1155*d)
```

Maple [B] time = 0.889, size = 1082, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^3-36960*C*a*b^2-5040
0*C*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^3+23760*B*a*b^2
+24640*B*b^3+23760*C*a^2*b+73920*C*a*b^2+56880*C*b^3)*sin(1/2*d*x+1/2*c)^8*
cos(1/2*d*x+1/2*c)+(-16632*A*a*b^2-11880*A*b^3-16632*B*a^2*b-35640*B*a*b^2-
22792*B*b^3-5544*C*a^3-35640*C*a^2*b-68376*C*a*b^2-34920*C*b^3)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A*a^2*b+16632*A*a*b^2+9240*A*b^3+4620*B
*a^3+16632*B*a^2*b+27720*B*a*b^2+10472*B*b^3+5544*C*a^3+27720*C*a^2*b+31416
*C*a*b^2+13860*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6930*A*a^2*
b-4158*A*a*b^2-2640*A*b^3-2310*B*a^3-4158*B*a^2*b-7920*B*a*b^2-1848*B*b^3-1
386*C*a^3-7920*C*a^2*b-5544*C*a*b^2-2790*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-3465*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6237*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b^2+3465*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-6237*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-1617*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*b^3+1155*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2475*a*b^2*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-2079*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-4851*C*(sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a*b^2+2475*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*C*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\cos(dx+c)^4 + Aa^3 + \left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + \left(Ca^3 + 3\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1081 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=296

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^3(3A+C)+21a^2bB+3ab^2(7A+5C)+5b^3B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2b(5A+3C)+15a^3B+27a\right.}{15d}$$

```
[Out] (2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Elliptic
E[(c + d*x)/2, 2])/(15*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*
a*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(54*a^2*b*B + 15*
b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63
*d) + (2*b*(63*A*b^2 + 99*a*b*B + 24*a^2*C + 49*b^2*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(315*d) + (2*(3*b*B + 2*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
 d*x])^2*Sin[c + d*x])/(21*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.838873, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^3(3A+C)+21a^2bB+3ab^2(7A+5C)+5b^3B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2b(5A+3C)+15a^3B+27a\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] (2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Elliptic
E[(c + d*x)/2, 2])/(15*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*
a*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(54*a^2*b*B + 15*
b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63
*d) + (2*b*(63*A*b^2 + 99*a*b*B + 24*a^2*C + 49*b^2*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(315*d) + (2*(3*b*B + 2*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
 d*x])^2*Sin[c + d*x])/(21*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(9*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\sin(c+dx)}{9d}+\frac{2}{9}\int$$

$$=\frac{2(3bB+2aC)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+}{21d}$$

$$=\frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{315d}$$

$$=\frac{2\left(54a^2bB+15b^3B+8a^3C+9ab^2(7A+5C)\right)\sqrt{\cos(c}}{63d}$$

$$=\frac{2\left(54a^2bB+15b^3B+8a^3C+9ab^2(7A+5C)\right)\sqrt{\cos(c}}{63d}$$

$$=\frac{2\left(15a^3B+27ab^2B+9a^2b(5A+3C)+b^3(9A+7C)\right)}{15d}$$

Mathematica [A] time = 2.01, size = 230, normalized size = 0.78

$$\frac{60F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^3(3A+C)+21a^2bB+3ab^2(7A+5C)+5b^3B\right)+84E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2b(5A+3C)+15a^3B+27\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (84*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Ellipti
cE[(c + d*x)/2, 2] + 60*(21*a^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*a*b^2*(
7*A + 5*C))*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(36*A*b^2 +
 108*a*b*B + 108*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*(252*a^2*b*B + 78*b^3*B
 + 84*a^3*C + 18*a*b^2*(14*A + 13*C) + 18*b^2*(b*B + 3*a*C)*Cos[2*(c + d*x)
] + 7*b^3*C*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 1.05, size = 975, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^3+2160*C*a*b^2+2240*C*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^3-1512*B*a*b^2-1080*B*
b^3-1512*C*a^2*b-3240*C*a*b^2-2072*C*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(1260*A*a*b^2+504*A*b^3+1260*B*a^2*b+1512*B*a*b^2+840*B*b^3+420*C*a^
3+1512*C*a^2*b+2520*C*a*b^2+952*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-630*A*a*b^2-126*A*b^3-630*B*a^2*b-378*B*a*b^2-240*B*b^3-210*C*a^3-378
*C*a^2*b-720*C*a*b^2-168*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+315
*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+315*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-945*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+315*a^2*b
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+75*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-315*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^3-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+105*a^3*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+225*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2*b-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\cos(dx+c)^4 + Aa^3 + \left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + \left(Ca^3 + 3\,Ba\right.}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sq
rt(cos(d*x + c)), x)
```

3.1082 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^3(A-C)+15a^2bB+3ab^2\right.}{5d}$$

```
[Out] (2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^
3*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(21*a*b*B - 6*a^2*(
7*A - 3*C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*
b^2*(35*a*A - 7*b*B - 11*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) - (2*
b*(7*A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) +
 (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.83112, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^3(A-C)+15a^2bB+3ab^2\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```


```
[Out] (2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^
3*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(21*a*b*B - 6*a^2*(
7*A - 3*C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*
b^2*(35*a*A - 7*b*B - 11*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) - (2*
b*(7*A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) +
 (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{(a+b\cos(c$$

$$= -\frac{2b(7A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+$$

$$= -\frac{2b^2(35aA-7bB-11aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} -$$

$$= \frac{2b\left(21abB-6a^2(7A-3C)+b^2(7A+5C)\right)\sqrt{\cos(c+}}{21d}$$

$$= \frac{2b\left(21abB-6a^2(7A-3C)+b^2(7A+5C)\right)\sqrt{\cos(c+}}{21d}$$

$$= \frac{2\left(15a^2bB+3b^3B-5a^3(A-C)+3ab^2(5A+3C)\right)E\Big(}{5d}$$

Mathematica [A] time = 1.85495, size = 212, normalized size = 0.76

$$\frac{20F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)-84E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^3(A-C)-15a^2bB-3ab^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] (-84*(-15*a^2*b*B - 3*b^3*B + 5*a^3*(A - C) - 3*a*b^2*(5*A + 3*C))*Elliptic
E[(c + d*x)/2, 2] + 20*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^3*(7
*A + 5*C))*EllipticF[(c + d*x)/2, 2] + ((420*a^3*A + 42*b^3*B + 126*a*b^2*C
 + 5*b*(28*A*b^2 + 84*a*b*B + 84*a^2*C + 29*b^2*C)*Cos[c + d*x] + 42*b^2*(b
*B + 3*a*C)*Cos[2*(c + d*x)] + 15*b^3*C*Cos[3*(c + d*x)])*Sin[c + d*x])/Sqr
t[Cos[c + d*x]])/(210*d)
```

Maple [B] time = 1.321, size = 1278, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/105*(240*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*b^2*(7*B*b+21*C*a+15*C*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+28*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(5*A*b^2+
15*B*a*b+6*B*b^2+15*C*a^2+18*C*a*b+10*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^3
+35*A*b^3+105*B*a*b^2+21*B*b^3+105*C*a^2*b+63*C*a*b^2+40*C*b^3)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+35*A*b^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+105*A*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-315*A*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+105*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-315*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*B*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
```

```
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+105*a^2*b*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)+25*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)-105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^3-189*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3+3\,Ba\right.}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/cos(d*x + c)^(3
```

```
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(3/2), x)
```

3.1083 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=271

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(15a^2b(A-C)+5a^3B-15ab^2B-b^3\right.}{5d}$$

```
[Out] (-2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(9*a^2*b*B + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A
+ 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) - (2*b*(6*a^2*B - b^2*B + 3*a*b*(5
*A - C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) - (2*b^2*(35*A*b + 15*a*B -
 3*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(2*A*b + a*B)*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d
*x])^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.858457, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(15a^2b(A-C)+5a^3B-15ab^2B-b^3\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```


```
[Out] (-2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(9*a^2*b*B + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A
+ 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) - (2*b*(6*a^2*B - b^2*B + 3*a*b*(5
*A - C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) - (2*b^2*(35*A*b + 15*a*B -
 3*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(2*A*b + a*B)*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d
*x])^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(}{}$$

$$= \frac{2(2Ab+aB)(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2A(a+}{}$$

$$= -\frac{2b^2(35Ab+15aB-3bC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} +$$

$$= -\frac{2b\left(6a^2B-b^2B+3ab(5A-C)\right)\sqrt{\cos(c+dx)}\sin(c+}{3d}$$

$$= -\frac{2b\left(6a^2B-b^2B+3ab(5A-C)\right)\sqrt{\cos(c+dx)}\sin(c+}{3d}$$

$$= -\frac{2\left(5a^3B-15ab^2B+15a^2b(A-C)-b^3(5A+3C)\right)E}{5d}$$

Mathematica [A] time = 2.32419, size = 186, normalized size = 0.69

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)+2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-45a^2b(A-C)-15a^3B+45ab^2B\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(-15*a^3*B + 45*a*b^2*B - 45*a^2*b*(A - C) + 3*b^3*(5*A + 3*C))*Elliptic
E[(c + d*x)/2, 2] + 10*(9*a^2*b*B + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A + 3*
C))*EllipticF[(c + d*x)/2, 2] + (6*(5*a^2*(3*A*b + a*B) + b^3*C*Cos[c + d*x
]^2)*Sin[c + d*x] + 5*(b^2*(b*B + 3*a*C)*Sin[2*(c + d*x)] + 2*a^3*A*Tan[c +
 d*x]))/Sqrt[Cos[c + d*x]])/(15*d)
```

Maple [B] time = 3.149, size = 1837, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(30*C*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-18*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*s
in(1/2*d*x+1/2*c)^2+10*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-
45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-45*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b-15*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+40*B*b
^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*B*a^3*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^4-40*B*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+10*B*b^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+10*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2+6*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+72*C*b^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+30*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48*C*b^3*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-30*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*
d*x+1/2*c)^2-5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^3-180*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-120*C*a*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+90*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+30*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+120*C*a*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-5*A
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-15*a^3*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-45*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))*a*b^2-90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2
+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b*sin(1/2*d*x+1/2*c)^2+30*C*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-90*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b
*sin(1/2*d*x+1/2*c)^2+90*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c
)^2+90*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+30*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3*
sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\,Ba\right.}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/cos(d*x + c)^(5
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(5/2), x)
```

3.1084 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=273

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(A+3C)+a^3B+9ab^2B+b^3(3A+C)\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(3A+5C)+15a^2bB+15ab^2(A-\right.}{5d}$$

```
[Out] (-2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(a^3*B + 9*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A
+ 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(24*A*b^2 + 35*a*b*B + 3*a^
2*(3*A + 5*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(9*A*b + 5*
a*B - 5*b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(6*A*b + 5*a*B)*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b
*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.823653, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(A+3C)+a^3B+9ab^2B+b^3(3A+C)\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(3A+5C)+15a^2bB+15ab^2(A-\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```


```
[Out] (-2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(a^3*B + 9*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A
+ 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(24*A*b^2 + 35*a*b*B + 3*a^
2*(3*A + 5*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(9*A*b + 5*
a*B - 5*b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(6*A*b + 5*a*B)*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b
*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(}{}$$

$$= \frac{2(6Ab+5aB)(a+b\cos(c+dx))^2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2a\left(24Ab^2+35abB+3a^2(3A+5C)\right)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2(}{}$$

$$= \frac{2a\left(24Ab^2+35abB+3a^2(3A+5C)\right)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} - \frac{2b}{}$$

$$= \frac{2a\left(24Ab^2+35abB+3a^2(3A+5C)\right)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} - \frac{2b}{}$$

$$= -\frac{2\left(15a^2bB-5b^3B+15ab^2(A-C)+a^3(3A+5C)\right)E}{5d}$$

Mathematica [A] time = 2.13405, size = 248, normalized size = 0.91

$$\frac{10\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(A+3C)+a^3B+9ab^2B+b^3(3A+C)\right)-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(3A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*Cos[c + d*x
]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(a^3*B + 9*a*b^2*B + b^3*(3*A + C) +
 3*a^2*b*(A + 3*C))*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 30*a^2*A
*b*Sin[c + d*x] + 10*a^3*B*Sin[c + d*x] + 10*b^3*C*Cos[c + d*x]^2*Sin[c + d
*x] + 9*a^3*A*Sin[2*(c + d*x)] + 45*a*A*b^2*Sin[2*(c + d*x)] + 45*a^2*b*B*S
in[2*(c + d*x)] + 15*a^3*C*Sin[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x])/(15*d*C
```

```
os[c + d*x]^(3/2))
```

Maple [B] time = 3.658, size = 1419, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^3*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*B*b^3+6*C*a*b^2-4*C*b^3)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a*b^2*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a^
2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(3*A*b^2+3*B
*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\,Ba\right.}{\cos\left(dx+c\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/cos(d*x + c)^(7
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(7/2), x)
```

3.1085 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(3A+5C)+3a^3B+15ab\right.}{5d}$$

```
[Out] (-2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*EllipticE[
(c + d*x)/2, 2])/(5*d) + (2*(21*a^2*b*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a
^3*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(24*A*b^2 + 63*a*b
*B + 5*a^2*(5*A + 7*C))*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(24*A
*b^3 + 21*a^3*B + 98*a*b^2*B + 21*a^2*b*(3*A + 5*C))*Sin[c + d*x])/(35*d*Sq
rt[Cos[c + d*x]]) + (2*(6*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d
*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.851041, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(3A+5C)+3a^3B+15ab\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (-2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*EllipticE[
(c + d*x)/2, 2])/(5*d) + (2*(21*a^2*b*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a
^3*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(24*A*b^2 + 63*a*b
*B + 5*a^2*(5*A + 7*C))*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(24*A
*b^3 + 21*a^3*B + 98*a*b^2*B + 21*a^2*b*(3*A + 5*C))*Sin[c + d*x])/(35*d*Sq
rt[Cos[c + d*x]]) + (2*(6*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d
*Cos[c + d*x]^(7/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int\frac{(a+b\cos(}{}$$

$$=\frac{2(6Ab+7aB)(a+b\cos(c+dx))^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A(a}{}$$

$$=\frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(}{}$$

$$=\frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{}$$

$$=\frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{}$$

$$=-\frac{2\left(3a^3B+15ab^2B+5b^3(A-C)+3a^2b(3A+5C)\right)E}{5d}$$

Mathematica [A] time = 4.86745, size = 251, normalized size = 0.85

$$\frac{2\left(5F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)-21E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(3A+5C)+3a^3B+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(-21*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*Ellipt
icE[(c + d*x)/2, 2] + 5*(21*a^2*b*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(
```

```
5*A + 7*C))*EllipticF[(c + d*x)/2, 2] + (15*a^3*A*Sin[c + d*x])/Cos[c + d*x
]^(7/2) + (21*a^2*(3*A*b + a*B)*Sin[c + d*x])/Cos[c + d*x]^(5/2) + (5*a*(21
*A*b^2 + 21*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21
*(5*A*b^3 + 3*a^3*B + 15*a*b^2*B + 3*a^2*b*(3*A + 5*C))*Sin[c + d*x])/Sqrt[
Cos[c + d*x]]))/(105*d)
```

Maple [B] time = 4.113, size = 1205, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*A*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2)))+2*a*(3*A*b^2+3*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))-2/5*a^2*(3*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*s
in(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b^2+3*B*a*b+3*C*
```

```
a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\,Ba\right.}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/cos(d*x + c)^(9
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(9/2), x)
```

3.1086 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=357

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(5A+7C)+5a^3B+21ab^2B+7b^3(A+3C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(7A+9C)+27a^2bB+9ab^2(\right.}{15d}$$

```
[Out] (-2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(5*a^3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3
*a^2*b*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(24*A*b^2 + 99
*a*b*B + 7*a^2*(7*A + 9*C))*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(
8*A*b^3 + 15*a^3*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*Sin[c + d*x])/(63*d*
Cos[c + d*x]^(3/2)) + (2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3
*(7*A + 9*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(2*A*b + 3*a*B)*
(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*A*(a +
b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.923861, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(5A+7C)+5a^3B+21ab^2B+7b^3(A+3C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(7A+9C)+27a^2bB+9ab^2(\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```


```
[Out] (-2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(5*a^3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3
*a^2*b*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(24*A*b^2 + 99
*a*b*B + 7*a^2*(7*A + 9*C))*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(
8*A*b^3 + 15*a^3*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*Sin[c + d*x])/(63*d*
Cos[c + d*x]^(3/2)) + (2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3
*(7*A + 9*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(2*A*b + 3*a*B)*
(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*A*(a +
b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\cos(}{}$$

$$= \frac{2(2Ab+3aB)(a+b\cos(c+dx))^2\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{}$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{}$$

$$= \frac{2\left(5a^3B+21ab^2B+7b^3(A+3C)+3a^2b(5A+7C)\right)F}{21d}$$

$$= -\frac{2\left(27a^2bB+15b^3B+9ab^2(3A+5C)+a^3(7A+9C)\right.}{15d}$$

Mathematica [A] time = 6.90144, size = 414, normalized size = 1.16

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(75a^2Ab+105a^2bC+25a^3B+105ab^2B+35Ab^3+105b^3C\right)+2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-49a^3A-189a^2bB\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] (2*(-49*a^3*A - 189*a*A*b^2 - 189*a^2*b*B - 105*b^3*B - 63*a^3*C - 315*a*b^
2*C)*EllipticE[(c + d*x)/2, 2] + 2*(75*a^2*A*b + 35*A*b^3 + 25*a^3*B + 105*
a*b^2*B + 105*a^2*b*C + 105*b^3*C)*EllipticF[(c + d*x)/2, 2])/(105*d) + (Sq
rt[Cos[c + d*x]]*((2*Sec[c + d*x]^4*(3*a^2*A*b*Sin[c + d*x] + a^3*B*Sin[c +
 d*x]))/7 + (2*Sec[c + d*x]^3*(7*a^3*A*Sin[c + d*x] + 27*a*A*b^2*Sin[c + d*
x] + 27*a^2*b*B*Sin[c + d*x] + 9*a^3*C*Sin[c + d*x]))/45 + (2*Sec[c + d*x]^
2*(15*a^2*A*b*Sin[c + d*x] + 7*A*b^3*Sin[c + d*x] + 5*a^3*B*Sin[c + d*x] +
21*a*b^2*B*Sin[c + d*x] + 21*a^2*b*C*Sin[c + d*x]))/21 + (2*Sec[c + d*x]*(7
*a^3*A*Sin[c + d*x] + 27*a*A*b^2*Sin[c + d*x] + 27*a^2*b*B*Sin[c + d*x] + 1
5*b^3*B*Sin[c + d*x] + 9*a^3*C*Sin[c + d*x] + 45*a*b^2*C*Sin[c + d*x]))/15
+ (2*a^3*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 4.806, size = 1292, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
^2*(3*A*b+B*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)
^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+2*b*(A*b^2+3*B*a*b+3*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
```

```
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))-2/5*a*(3*A*b^2+3*B*a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^
3*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(
1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))))+2*b^2*(B*b+3*C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos(dx+c)^5+\left(3\,Cab^2+Bb^3\right)\cos(dx+c)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\cos(dx+c)^3+\left(Ca^3+3\,I\right.}{\cos(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/cos(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/co
s(d*x + c)^(11/2), x)
```

3.1087 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4 \left(A + B\cos(c+dx) + C\right.$

Optimal. Leaf size=477

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(44a^3b(7A+5C)+330a^2b^2B+77a^4B+20ab^3(11A+9C)+45b^4B\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(78a^2b^2(9A+\right.}{}$$

```
[Out] (2*(468*a^3*b*B + 364*a*b^3*B + 39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C)
 + 7*b^4*(13*A + 11*C))*EllipticE[(c + d*x)/2, 2])/(195*d) + (2*(77*a^4*B +
 330*a^2*b^2*B + 45*b^4*B + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*E
llipticF[(c + d*x)/2, 2])/(231*d) + (2*(77*a^4*B + 330*a^2*b^2*B + 45*b^4*B
 + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(231*d) + (2*(3458*a^3*b*B + 4004*a*b^3*B + 192*a^4*C + 77*b^4*(13*A
 + 11*C) + 11*a^2*b^2*(637*A + 491*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(64
35*d) + (2*b*(2171*a^2*b*B + 1053*b^3*B + 192*a^3*C + 2*a*b^2*(1573*A + 125
9*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(9009*d) + (2*(143*A*b^2 + 221*a*b*B
 + 48*a^2*C + 121*b^2*C)*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c +
d*x])/(1287*d) + (2*(13*b*B + 8*a*C)*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(143*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^4*
Sin[c + d*x])/(13*d)
```

Rubi [A] time = 1.31604, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(44a^3b(7A+5C)+330a^2b^2B+77a^4B+20ab^3(11A+9C)+45b^4B\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(78a^2b^2(9A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```


```
[Out] (2*(468*a^3*b*B + 364*a*b^3*B + 39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C)
 + 7*b^4*(13*A + 11*C))*EllipticE[(c + d*x)/2, 2])/(195*d) + (2*(77*a^4*B +
 330*a^2*b^2*B + 45*b^4*B + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*E
llipticF[(c + d*x)/2, 2])/(231*d) + (2*(77*a^4*B + 330*a^2*b^2*B + 45*b^4*B
 + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(231*d) + (2*(3458*a^3*b*B + 4004*a*b^3*B + 192*a^4*C + 77*b^4*(13*A
 + 11*C) + 11*a^2*b^2*(637*A + 491*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(64
35*d) + (2*b*(2171*a^2*b*B + 1053*b^3*B + 192*a^3*C + 2*a*b^2*(1573*A + 125
```

```
9*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(9009*d) + (2*(143*A*b^2 + 221*a*b*B
 + 48*a^2*C + 121*b^2*C)*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c +
d*x])/(1287*d) + (2*(13*b*B + 8*a*C)*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(143*d) + (2*C*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^4*
Sin[c + d*x])/(13*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^4\sin(}{13d}$$

$$= \frac{2(13bB+8aC)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c}{143d}$$

$$= \frac{2\left(143Ab^2+221abB+48a^2C+121b^2C\right)}{1}$$

$$= \frac{2b\left(2171a^2bB+1053b^3B+192a^3C+2a\right.}{9}$$

$$= \frac{2\left(3458a^3bB+4004ab^3B+192a^4C+77\right.}{}$$

$$= \frac{2\left(3458a^3bB+4004ab^3B+192a^4C+77\right.}{}$$

$$= \frac{2\left(468a^3bB+364ab^3B+39a^4(5A+3C)\right.}{}$$

$$= \frac{2\left(468a^3bB+364ab^3B+39a^4(5A+3C)\right.}{}$$

Mathematica [A] time = 3.51933, size = 381, normalized size = 0.8

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(154\cos(c+dx)\left(156a^2b^2(36A+43C)+3744a^3bB+936a^4C+4472ab^3B+b^4(1118A+1171\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] (48*(77*(468*a^3*b*B + 364*a*b^3*B + 39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A +
 7*C) + 7*b^4*(13*A + 11*C))*EllipticE[(c + d*x)/2, 2] + 65*(77*a^4*B + 330
*a^2*b^2*B + 45*b^4*B + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Ellip
ticF[(c + d*x)/2, 2]) + Sqrt[Cos[c + d*x]]*(154*(3744*a^3*b*B + 4472*a*b^3*
B + 936*a^4*C + 156*a^2*b^2*(36*A + 43*C) + b^4*(1118*A + 1171*C))*Cos[c +
d*x] + 5*(78*(616*a^4*B + 3432*a^2*b^2*B + 531*b^4*B + 176*a^3*b*(14*A + 13
*C) + 4*a*b^3*(572*A + 531*C)) + 1872*b*(33*a^2*b*B + 8*b^3*B + 22*a^3*C +
2*a*b^2*(11*A + 16*C))*Cos[2*(c + d*x)] + 77*b^2*(52*A*b^2 + 208*a*b*B + 31
2*a^2*C + 89*b^2*C)*Cos[3*(c + d*x)] + 1638*b^3*(b*B + 4*a*C)*Cos[4*(c + d*
x)] + 693*b^4*C*Cos[5*(c + d*x)]))*Sin[c + d*x])/(360360*d)
```

Maple [B] time = 1.152, size = 1407, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*C
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(262080*B*b^4+1048320*C*a*b^3
+1330560*C*b^4)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^4-640
640*B*a*b^3-655200*B*b^4-960960*C*a^2*b^2-2620800*C*a*b^3-1798720*C*b^4)*si
n(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(411840*A*a*b^3+320320*A*b^4+617760*
B*a^2*b^2+1281280*B*a*b^3+739440*B*b^4+411840*C*a^3*b+1921920*C*a^2*b^2+295
7760*C*a*b^3+1379840*C*b^4)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-43243
2*A*a^2*b^2-617760*A*a*b^3-296296*A*b^4-288288*B*a^3*b-926640*B*a^2*b^2-118
5184*B*a*b^3-453960*B*b^4-72072*C*a^4-617760*C*a^3*b-1777776*C*a^2*b^2-1815
840*C*a*b^3-666512*C*b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(240240*A
*a^3*b+432432*A*a^2*b^2+480480*A*a*b^3+136136*A*b^4+60060*B*a^4+288288*B*a^
3*b+720720*B*a^2*b^2+544544*B*a*b^3+180180*B*b^4+72072*C*a^4+480480*C*a^3*b
```

```
+816816*C*a^2*b^2+720720*C*a*b^3+198352*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-120120*A*a^3*b-108108*A*a^2*b^2-137280*A*a*b^3-24024*A*b^4-30
030*B*a^4-72072*B*a^3*b-205920*B*a^2*b^2-96096*B*a*b^3-36270*B*b^4-18018*C*
a^4-137280*C*a^3*b-144144*C*a^2*b^2-145080*C*a*b^3-27258*C*b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-45045*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-162162*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+60060
*A*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+42900*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10
8108*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-84084*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^
3+15015*a^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+64350*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
*b^2+8775*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-27027*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4-126126*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-17787*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*b^4+42900*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35100*C*a*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos(dx+c)^4 + 2\left(2\,Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1088 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=404

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(7A+5C)+77a^4(3A+C)+308a^3bB+220ab^3B+5b^4(11A+9C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(12a^3\right.}{}$$

```
[Out] (2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*A
 + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(308*a^3*b*B + 220*a*b^3*B
+ 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*EllipticF
[(c + d*x)/2, 2])/(231*d) + (2*(682*a^3*b*B + 660*a*b^3*B + 64*a^4*C + 15*b
^4*(11*A + 9*C) + 9*a^2*b^2*(143*A + 101*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(693*d) + (2*b*(1353*a^2*b*B + 539*b^3*B + 192*a^3*C + 2*a*b^2*(891*A +
673*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d) + (2*(33*A*b^2 + 55*a*b*B
 + 16*a^2*C + 27*b^2*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(231*d) + (2*(11*b*B + 8*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
3*Sin[c + d*x])/(99*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*Sin
[c + d*x])/(11*d)
```

Rubi [A] time = 1.25902, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(7A+5C)+77a^4(3A+C)+308a^3bB+220ab^3B+5b^4(11A+9C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(12a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```

```
[Out] (2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*A
 + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(308*a^3*b*B + 220*a*b^3*B
+ 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*EllipticF
[(c + d*x)/2, 2])/(231*d) + (2*(682*a^3*b*B + 660*a*b^3*B + 64*a^4*C + 15*b
^4*(11*A + 9*C) + 9*a^2*b^2*(143*A + 101*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(693*d) + (2*b*(1353*a^2*b*B + 539*b^3*B + 192*a^3*C + 2*a*b^2*(891*A +
673*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d) + (2*(33*A*b^2 + 55*a*b*B
 + 16*a^2*C + 27*b^2*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(231*d) + (2*(11*b*B + 8*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
```

```
3*Sin[c + d*x])/(99*d) + (2*C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^4*Sin
[c + d*x])/(11*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4\sin(c+dx)}{11d} + \frac{2}{11}$$

$$= \frac{2(11bB+8aC)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\sin(c}{99d}$$

$$= \frac{2\left(33Ab^2+55abB+16a^2C+27b^2C\right)\sqrt{\cos(c+dx)}(a}{231d}$$

$$= \frac{2b\left(1353a^2bB+539b^3B+192a^3C+2ab^2(891A+673\right.}{3465d}$$

$$= \frac{2\left(682a^3bB+660ab^3B+64a^4C+15b^4(11A+9C)+9\right.}{693d}$$

$$= \frac{2\left(682a^3bB+660ab^3B+64a^4C+15b^4(11A+9C)+9\right.}{693d}$$

$$= \frac{2\left(15a^4B+54a^2b^2B+7b^4B+12a^3b(5A+3C)+4ab^3\right.}{15d}$$

Mathematica [A] time = 2.45618, size = 319, normalized size = 0.79

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(7A+5C)+77a^4(3A+C)+308a^3bB+220ab^3B+5b^4(11A+9C)\right)+154E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(12\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (154*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9
*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 10*(308*a^3*b*B + 220*a*b^3*B + 77*a
^4*(3*A + C) + 66*a^2*b^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*EllipticF[(c +
d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(154*b*(216*a^2*b*B + 43*b^3*B + 144*a^3*C
 + 4*a*b^2*(36*A + 43*C))*Cos[c + d*x] + 5*(7392*a^3*b*B + 6864*a*b^3*B + 1
848*a^4*C + 792*a^2*b^2*(14*A + 13*C) + 3*b^4*(572*A + 531*C) + 36*b^2*(11*
A*b^2 + 44*a*b*B + 66*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 154*b^3*(b*B + 4
*a*C)*Cos[3*(c + d*x)] + 63*b^4*C*Cos[4*(c + d*x)]))*Sin[c + d*x])/12)/(115
5*d)
```

Maple [B] time = 1.142, size = 1273, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^4-49280*C*a*b^3-5040
0*C*b^4)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^4+31680*B*a*b^3
+24640*B*b^4+47520*C*a^2*b^2+98560*C*a*b^3+56880*C*b^4)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-22176*A*a*b^3-11880*A*b^4-33264*B*a^2*b^2-47520*B*a*
b^3-22792*B*b^4-22176*C*a^3*b-71280*C*a^2*b^2-91168*C*a*b^3-34920*C*b^4)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(27720*A*a^2*b^2+22176*A*a*b^3+9240*A
*b^4+18480*B*a^3*b+33264*B*a^2*b^2+36960*B*a*b^3+10472*B*b^4+4620*C*a^4+221
76*C*a^3*b+55440*C*a^2*b^2+41888*C*a*b^3+13860*C*b^4)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-13860*A*a^2*b^2-5544*A*a*b^3-2640*A*b^4-9240*B*a^3*b-8
316*B*a^2*b^2-10560*B*a*b^3-1848*B*b^4-2310*C*a^4-5544*C*a^3*b-15840*C*a^2*
b^2-7392*C*a*b^3-2790*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3465*A
*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+6930*a^2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A
*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-13860*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b^3+4620*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+3300*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b^3-3465*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12474*
```

```
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1155
*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675
*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-8316*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-6468*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)(b\cos{(dx+c)} + a)^4}{\sqrt{\cos{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/sq
rt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^4\cos{(dx+c)}^6 + \left(4\,Cab^3 + Bb^4\right)\cos{(dx+c)}^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos{(dx+c)}^4 + 2\left(2\,Ca^3b\right.}{\sqrt{\cos{(dx}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
```

```
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/sq
rt(cos(d*x + c)), x)
```

3.1089 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=379

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(28a^3b(3A+C)+42a^2b^2B+21a^4B+4ab^3(7A+5C)+5b^4B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(5A+3C)\right.}{}$$

```
[Out] (2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^4
*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(21*a^4*B + 42*a^2*b^2
*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4*a*b^3*(7*A + 5*C))*EllipticF[(c + d*x
)/2, 2])/(21*d) + (2*b*(117*a^2*b*B + 15*b^3*B - a^3*(126*A - 62*C) + 12*a*
b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63*d) + (2*b^2*(162*a*b*
B - a^2*(315*A - 123*C) + 7*b^2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(315*d) - (2*b*(21*a*A - 3*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c
+ d*x])^2*Sin[c + d*x])/(21*d) - (2*b*(9*A - C)*Sqrt[Cos[c + d*x]]*(a + b*C
os[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*
x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.27097, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(28a^3b(3A+C)+42a^2b^2B+21a^4B+4ab^3(7A+5C)+5b^4B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(5A+3C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```


```
[Out] (2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^4
*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(21*a^4*B + 42*a^2*b^2
*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4*a*b^3*(7*A + 5*C))*EllipticF[(c + d*x
)/2, 2])/(21*d) + (2*b*(117*a^2*b*B + 15*b^3*B - a^3*(126*A - 62*C) + 12*a*
b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63*d) + (2*b^2*(162*a*b*
B - a^2*(315*A - 123*C) + 7*b^2*(9*A + 7*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(315*d) - (2*b*(21*a*A - 3*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c
+ d*x])^2*Sin[c + d*x])/(21*d) - (2*b*(9*A - C)*Sqrt[Cos[c + d*x]]*(a + b*C
os[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*
```

```
x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos}{}$$

$$= -\frac{2b(9A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\sin(c}{9d}$$

$$= -\frac{2b(21aA-3bB-5aC)\sqrt{\cos(c+dx)}(a+b\cos(c+}{21d}$$

$$= \frac{2b^2\left(162abB-a^2(315A-123C)+7b^2(9A+7C)\right)\mathrm{co}}{315d}$$

$$= \frac{2b\left(117a^2bB+15b^3B-a^3(126A-62C)+12ab^2(7A\right.}{63d}$$

$$= \frac{2b\left(117a^2bB+15b^3B-a^3(126A-62C)+12ab^2(7A\right.}{63d}$$

$$= \frac{2\left(60a^3bB+36ab^3B-15a^4(A-C)+18a^2b^2(5A+3\right.}{15d}$$

Mathematica [A] time = 4.31621, size = 275, normalized size = 0.73

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(28a^3b(3A+C)+42a^2b^2B+21a^4B+4ab^3(7A+5C)+5b^4B\right)-14E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-18a^2b^2(5A+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] (-14*(-60*a^3*b*B - 36*a*b^3*B + 15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) -
b^4*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 10*(21*a^4*B + 42*a^2*b^2*B +
5*b^4*B + 28*a^3*b*(3*A + C) + 4*a*b^3*(7*A + 5*C))*EllipticF[(c + d*x)/2,
2] + (Sqrt[Cos[c + d*x]]*(30*b*(168*a^2*b*B + 23*b^3*B + 112*a^3*C + 4*a*b^
2*(28*A + 23*C))*Sin[c + d*x] + 14*b^2*(18*A*b^2 + 72*a*b*B + 108*a^2*C + 1
9*b^2*C)*Sin[2*(c + d*x)] + 90*b^3*(b*B + 4*a*C)*Sin[3*(c + d*x)] + 35*(b^4
*C*Sin[4*(c + d*x)] + 72*a^4*A*Tan[c + d*x])))/12)/(105*d)
```

Maple [B] time = 1.459, size = 1652, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/315*(-1120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+80*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*b^3*(9*B*b+36*C*a+28*C*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*
d*x+1/2*c)-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(63*A
*b^2+252*B*a*b+135*B*b^2+378*C*a^2+540*C*a*b+259*C*b^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+56*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*b*(30*A*a*b^2+9*A*b^3+45*B*a^2*b+36*B*a*b^2+15*B*b^3+30*C*a^3+54*C*a^2*b+
60*C*a*b^2+17*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^4+140*A*a*b^3+21*A*b^4+21
0*B*a^2*b^2+84*B*a*b^3+40*B*b^4+140*C*a^3*b+126*C*a^2*b^2+160*C*a*b^3+28*C*
b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1260*A*a^3*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+420*a*A*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
+315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^4-1890*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-189*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+315*a^4*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+630*a^2*b^2*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+
75*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)-1260*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a^3*b-756*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+420*a^3*b*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+300*C*a*b^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
```

```
d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-31
5*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^4-1134*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^2-147*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos\left(dx+c\right)^6+\left(4\,Cab^3+Bb^4\right)\cos\left(dx+c\right)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos\left(dx+c\right)^4+2\left(2\,Ca^3b\right.}{\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(3/2), x)
```

3.1090 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=373

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(20a^3b(A-\right.}{}$$

```
[Out] (-2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3
*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^
2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2,
 2])/(21*d) - (2*b*(42*a^3*B - 28*a*b^2*B + 3*a^2*b*(49*A - 13*C) - b^3*(7*
A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*b^2*(350*a*A*b + 105
*a^2*B - 21*b^2*B - 54*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) - (2
*b*(21*A*b + 7*a*B - b*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(7*d) + (2*(8*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(3*d
*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2))
```

Rubi [A] time = 1.25774, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(20a^3b(A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```


```
[Out] (-2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3
*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^
2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2,
 2])/(21*d) - (2*b*(42*a^3*B - 28*a*b^2*B + 3*a^2*b*(49*A - 13*C) - b^3*(7*
A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*b^2*(350*a*A*b + 105
*a^2*B - 21*b^2*B - 54*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) - (2
*b*(21*A*b + 7*a*B - b*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(7*d) + (2*(8*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(3*d
```

```
*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(\ldots}{\ldots}$$

$$= \frac{2(8Ab+3aB)(a+b\cos(c+dx))^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A(a\ldots}{\ldots}$$

$$= -\frac{2b(21Ab+7aB-bC)\sqrt{\cos(c+dx)}(a+b\cos(c+dx\ldots}{7d}$$

$$= -\frac{2b^2\left(350aAb+105a^2B-21b^2B-54abC\right)\cos^{\frac{3}{2}}(c+\ldots}{105d}$$

$$= -\frac{2b\left(42a^3B-28ab^2B+3a^2b(49A-13C)-b^3(7A+5\ldots\right.}{21d}$$

$$= -\frac{2b\left(42a^3B-28ab^2B+3a^2b(49A-13C)-b^3(7A+5\ldots\right.}{21d}$$

$$= -\frac{2\left(5a^4B-30a^2b^2B-3b^4B+20a^3b(A-C)-4ab^3(5\ldots\right.}{5d}$$

Mathematica [A] time = 2.59591, size = 257, normalized size = 0.69

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5C)\right)-42E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(20a^3b(A-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] (-42*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A +
3*C))*EllipticE[(c + d*x)/2, 2] + 10*(84*a^3*b*B + 28*a*b^3*B + 42*a^2*b^2*
(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2] +
(168*(5*a^3*(4*A*b + a*B) + b^3*(b*B + 4*a*C)*Cos[c + d*x]^2)*Sin[c + d*x]
+ 5*(b^2*(28*A*b^2 + 112*a*b*B + 168*a^2*C + 23*b^2*C)*Sin[2*(c + d*x)] + 6
*b^4*C*Cos[c + d*x]*Sin[3*(c + d*x)] + 56*a^4*A*Tan[c + d*x]))/(4*Sqrt[Cos[
c + d*x]]))/(105*d)
```

Maple [B] time = 3.875, size = 2507, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(420*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b^3-630*a^2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+420*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^3*b+252*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-210*a^2*b^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+480*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10
-336*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+280*A*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6+504*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6
+920*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-280*A*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4-420*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
-252*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-440*C*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4+70*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
70*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+210*B*a^4*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+42*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+80
*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-960*C*b^4*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^8+630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-105*a^4*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-420*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-140*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^3+840*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)
^2+280*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2-840*C*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-504*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
a*b^3*sin(1/2*d*x+1/2*c)^2+420*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d
*x+1/2*c)^2+840*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
```

```
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-840*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2-35*A*a^4*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-35*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^4+63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
120*B*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+1680*C*a^2*b^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2016*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^6-1680*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1120*B*a*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1680*C*a^2*b^2*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^4-1008*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+8
40*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+280*B*a*b^3*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+420*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2+168*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1344*C*a*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-420*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+2
10*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+50*C*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/
2*d*x+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+210*B
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2-126*B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+1260*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin
(1/2*d*x+1/2*c)^2-1260*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c
)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos(dx+c)^4 + 2\left(2\,Ca^3b\right.}{\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)^4}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(5/2), x)
```

3.1091 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=386

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(A+3C)+18a^2b^2B+a^4B+4ab^3(3A+C)+b^4B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(30a^2b^2(A-C)+a^4(3A\right.}{5}$$

```
[Out] (-2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(
3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b
^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3
*d) - (2*b*(105*a^2*b*B - 5*b^3*B + 4*a*b^2*(33*A - 5*C) + 6*a^3*(3*A + 5*C
))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(50*a*b*B + b^2*(59*A -
 3*C) + 3*a^2*(3*A + 5*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(16
*A*b^2 + 15*a*b*B + a^2*(3*A + 5*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
5*d*Sqrt[Cos[c + d*x]]) + (2*(8*A*b + 5*a*B)*(a + b*Cos[c + d*x])^3*Sin[c +
 d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x]
)/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 1.295, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(A+3C)+18a^2b^2B+a^4B+4ab^3(3A+C)+b^4B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(30a^2b^2(A-C)+a^4(3A\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```


```
[Out] (-2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(
3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b
^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3
*d) - (2*b*(105*a^2*b*B - 5*b^3*B + 4*a*b^2*(33*A - 5*C) + 6*a^3*(3*A + 5*C
))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(50*a*b*B + b^2*(59*A -
 3*C) + 3*a^2*(3*A + 5*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(16
*A*b^2 + 15*a*b*B + a^2*(3*A + 5*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
5*d*Sqrt[Cos[c + d*x]]) + (2*(8*A*b + 5*a*B)*(a + b*Cos[c + d*x])^3*Sin[c +
```

```
d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x]
)/(5*d*Cos[c + d*x]^(5/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{(a+b\cos$$

$$= \frac{2(8Ab+5aB)(a+b\cos(c+dx))^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2A(}{}$$

$$= \frac{2\left(16Ab^2+15abB+a^2(3A+5C)\right)(a+b\cos(c+dx}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b^2\left(50abB+b^2(59A-3C)+3a^2(3A+5C)\right)\cos^{\frac{3}{2}}}{15d}$$

$$= -\frac{2b\left(105a^2bB-5b^3B+4ab^2(33A-5C)+6a^3(3A+\right.}{15d}$$

$$= -\frac{2b\left(105a^2bB-5b^3B+4ab^2(33A-5C)+6a^3(3A+\right.}{15d}$$

$$= -\frac{2\left(20a^3bB-20ab^3B+30a^2b^2(A-C)-b^4(5A+3\right.}{5d}$$

Mathematica [A] time = 2.36881, size = 316, normalized size = 0.82

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(20a^3Ab+90a^2b^2B+60a^3bC+5a^4B+60aAb^3+20ab^3C+5b^4B\right)+2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-90a^2Ab^2-\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(-9*a^4*A - 90*a^2*A*b^2 + 15*A*b^4 - 60*a^3*b*B + 60*a*b^3*B - 15*a^4*C
 + 90*a^2*b^2*C + 9*b^4*C)*EllipticE[(c + d*x)/2, 2] + 2*(20*a^3*A*b + 60*a
*A*b^3 + 5*a^4*B + 90*a^2*b^2*B + 5*b^4*B + 60*a^3*b*C + 20*a*b^3*C)*Ellipt
icF[(c + d*x)/2, 2])/(15*d) + (Sqrt[Cos[c + d*x]]*((2*b^3*(b*B + 4*a*C)*Sin
[c + d*x])/3 + (2*Sec[c + d*x]^2*(4*a^3*A*b*Sin[c + d*x] + a^4*B*Sin[c + d*
x]))/3 + (2*Sec[c + d*x]*(3*a^4*A*Sin[c + d*x] + 30*a^2*A*b^2*Sin[c + d*x]
+ 20*a^3*b*B*Sin[c + d*x] + 5*a^4*C*Sin[c + d*x]))/5 + (b^4*C*Sin[2*(c + d*
x)])/5 + (2*a^4*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 4.677, size = 1884, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C*b^4*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+1/3*(4*B*b^4+16*C*a*b^3-12*C*b^4)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+(2*A*b^4+8*B*a*b^3-4*B*b^4+12*C*a^2*b^2-16*C*a*b^3+6*C*b^4)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+8*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*b^4*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a^2*b^2*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-8*a*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
```

```
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+8*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*a^2*b^2*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*C*a*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-2/5*A*a^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin
(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^3*(4*A*b+B*a)*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
```

```
s(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos(dx+c)^4 + 2\left(2\,Ca^3b \right.}{\cos(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(7/2), x)
```

3.1092 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=383

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(A+3C)+a^4(5A+7C)+28a^3bB+84ab^3B+7b^4(3A+C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(3A+5\right.}{}$$

```
[Out] (-2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5
*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4
*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(c + d*x)/2,
 2])/(21*d) + (2*a*(192*A*b^3 + 63*a^3*B + 413*a*b^2*B + a^2*(202*A*b + 350
*b*C))*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(98*a*b*B + b^2*(8
7*A - 35*C) + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) +
 (2*(48*A*b^2 + 77*a*b*B + 5*a^2*(5*A + 7*C))*(a + b*Cos[c + d*x])^2*Sin[c
+ d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(8*A*b + 7*a*B)*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x])^4*S
in[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.27169, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(A+3C)+a^4(5A+7C)+28a^3bB+84ab^3B+7b^4(3A+C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(3A+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```


```
[Out] (-2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5
*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4
*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(c + d*x)/2,
 2])/(21*d) + (2*a*(192*A*b^3 + 63*a^3*B + 413*a*b^2*B + a^2*(202*A*b + 350
*b*C))*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(98*a*b*B + b^2*(8
7*A - 35*C) + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) +
 (2*(48*A*b^2 + 77*a*b*B + 5*a^2*(5*A + 7*C))*(a + b*Cos[c + d*x])^2*Sin[c
+ d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(8*A*b + 7*a*B)*(a + b*Cos[c + d*x]
```

```
)^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x])^4*S
in[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\cos(}{}$$

$$= \frac{2(8Ab+7aB)(a+b\cos(c+dx))^3\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2\left(48Ab^2+77abB+5a^2(5A+7C)\right)(a+b\cos(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+63a^3B+413ab^2B+a^2(202Ab+350bC\right.}{105d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a\left(192Ab^3+63a^3B+413ab^2B+a^2(202Ab+350bC\right.}{105d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a\left(192Ab^3+63a^3B+413ab^2B+a^2(202Ab+350bC\right.}{105d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\left(3a^4B+30a^2b^2B-5b^4B+20ab^3(A-C)+4a^3b(3\right.}{5d}$$

Mathematica [A] time = 5.13019, size = 271, normalized size = 0.71

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(A+3C)+a^4(5A+7C)+28a^3bB+84ab^3B+7b^4(3A+C)\right)-42E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(3A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] (-42*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A +
5*C))*EllipticE[(c + d*x)/2, 2] + 10*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*(3*A
+ C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2] +
(14*(3*a^3*(4*A*b + a*B) + 3*a*(20*A*b^3 + 3*a^3*B + 30*a*b^2*B + 4*a^2*b*(
3*A + 5*C))*Cos[c + d*x]^2 + 5*b^4*C*Cos[c + d*x]^3)*Sin[c + d*x] + 5*(a^2*
(42*A*b^2 + 28*a*b*B + a^2*(5*A + 7*C))*Sin[2*(c + d*x)] + 6*a^4*A*Tan[c +
d*x]))/Cos[c + d*x]^(5/2))/(105*d)
```

Maple [B] time = 4.952, size = 1624, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^4*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*B*b^4+8*C*a*b^3-4*C*b^4)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a*b^3*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^4*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a
^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*a^3*(4*A*b+B*a)/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
```

```
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*A*a^4*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1
/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2
*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos\left(dx+c\right)^6+\left(4\,Cab^3+Bb^4\right)\cos\left(dx+c\right)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos\left(dx+c\right)^4+2\left(2\,Ca^3b\right.}{\cos\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(9/2), x)
```

3.1093 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=401

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(5A+7C)+42a^2b^2B+5a^4B+28ab^3(A+3C)+21b^4B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(3A+5C)\right.}{}$$

```
[Out] (-2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^
4*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2
*B + 21*b^4*B + 28*a*b^3*(A + 3*C) + 4*a^3*b*(5*A + 7*C))*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*a*(64*A*b^3 + 75*a^3*B + 261*a*b^2*B + a^2*(202*A*b +
 294*b*C))*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)) + (2*(192*A*b^4 + 756*a
^3*b*B + 1098*a*b^3*B + 21*a^4*(7*A + 9*C) + 7*a^2*b^2*(155*A + 261*C))*Sin
[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]) + (2*(48*A*b^2 + 117*a*b*B + 7*a^2*(7
*A + 9*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2))
+ (2*(8*A*b + 9*a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c + d*x
]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2
))
```

Rubi [A] time = 1.30586, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(5A+7C)+42a^2b^2B+5a^4B+28ab^3(A+3C)+21b^4B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(3A+5C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```


```
[Out] (-2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^
4*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2
*B + 21*b^4*B + 28*a*b^3*(A + 3*C) + 4*a^3*b*(5*A + 7*C))*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*a*(64*A*b^3 + 75*a^3*B + 261*a*b^2*B + a^2*(202*A*b +
 294*b*C))*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)) + (2*(192*A*b^4 + 756*a
^3*b*B + 1098*a*b^3*B + 21*a^4*(7*A + 9*C) + 7*a^2*b^2*(155*A + 261*C))*Sin
[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]) + (2*(48*A*b^2 + 117*a*b*B + 7*a^2*(7
```

```
*A + 9*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2))
+ (2*(8*A*b + 9*a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c + d*x
]^(7/2)) + (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2
))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2}{9}\int \frac{(a+b\cos(}{}$$

$$= \frac{2(8Ab+9aB)(a+b\cos(c+dx))^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}+\frac{2A(a}{}$$

$$= \frac{2\left(48Ab^2+117abB+7a^2(7A+9C)\right)(a+b\cos(c+d}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(202Ab+294bC)\right.}{315d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(202Ab+294bC)\right.}{315d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(202Ab+294bC)\right.}{315d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\left(36a^3bB+60ab^3B+15b^4(A-C)+18a^2b^2(3A+5\right.}{15d}$$

Mathematica [A] time = 7.21694, size = 463, normalized size = 1.15

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(100a^3Ab+210a^2b^2B+140a^3bC+25a^4B+140aAb^3+420ab^3C+105b^4B\right)+2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-378\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] (2*(-49*a^4*A - 378*a^2*A*b^2 - 105*A*b^4 - 252*a^3*b*B - 420*a*b^3*B - 63*
a^4*C - 630*a^2*b^2*C + 105*b^4*C)*EllipticE[(c + d*x)/2, 2] + 2*(100*a^3*A
*b + 140*a*A*b^3 + 25*a^4*B + 210*a^2*b^2*B + 105*b^4*B + 140*a^3*b*C + 420
*a*b^3*C)*EllipticF[(c + d*x)/2, 2])/(105*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[
c + d*x]^4*(4*a^3*A*b*Sin[c + d*x] + a^4*B*Sin[c + d*x]))/7 + (2*Sec[c + d*
x]^3*(7*a^4*A*Sin[c + d*x] + 54*a^2*A*b^2*Sin[c + d*x] + 36*a^3*b*B*Sin[c +
 d*x] + 9*a^4*C*Sin[c + d*x]))/45 + (2*Sec[c + d*x]^2*(20*a^3*A*b*Sin[c + d
*x] + 28*a*A*b^3*Sin[c + d*x] + 5*a^4*B*Sin[c + d*x] + 42*a^2*b^2*B*Sin[c +
 d*x] + 28*a^3*b*C*Sin[c + d*x]))/21 + (2*Sec[c + d*x]*(7*a^4*A*Sin[c + d*x
] + 54*a^2*A*b^2*Sin[c + d*x] + 15*A*b^4*Sin[c + d*x] + 36*a^3*b*B*Sin[c +
d*x] + 60*a*b^3*B*Sin[c + d*x] + 9*a^4*C*Sin[c + d*x] + 90*a^2*b^2*C*Sin[c
+ d*x]))/15 + (2*a^4*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 5.803, size = 1550, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*C*a*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^4*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2/5*a^2*(6*A*b^2+4*B*a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*
c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2
```

```
*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a*b*(2*A*b^2+
3*B*a*b+2*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^4*(-1/144*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*
b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1)+2*a^3*(4*A*b+B*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2
)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos(dx+c)^6+\left(4\,Cab^3+Bb^4\right)\cos(dx+c)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos(dx+c)^4+2\left(2\,Ca^3b\right.}{\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(11/2), x)
```

3.1094 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=475

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A+3C)\right)}{231d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b\right.}{}$$

```
[Out] (-2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*
A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(220*a^3*b*B + 308*a*b^3*B
 + 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*Elliptic
F[(c + d*x)/2, 2])/(231*d) + (2*a*(192*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2
*a^2*b*(673*A + 891*C))*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(5/2)) + (2*(64*
A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a^4*(9*A + 11*C) + 9*a^2*b^2*(101*A
+ 143*C))*Sin[c + d*x])/(693*d*Cos[c + d*x]^(3/2)) + (2*(7*a^4*B + 54*a^2*b
^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*A + 9*C))*Sin[c + d*x])
/(15*d*Sqrt[Cos[c + d*x]]) + (2*(16*A*b^2 + 55*a*b*B + 3*a^2*(9*A + 11*C))*
(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) + (2*(8*A*b
 + 11*a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) +
 (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rubi [A] time = 1.3956, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A+3C)\right)}{231d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(13/2),x]
```


```
[Out] (-2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*
A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(220*a^3*b*B + 308*a*b^3*B
 + 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*Elliptic
F[(c + d*x)/2, 2])/(231*d) + (2*a*(192*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2
*a^2*b*(673*A + 891*C))*Sin[c + d*x])/(3465*d*Cos[c + d*x]^(5/2)) + (2*(64*
A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a^4*(9*A + 11*C) + 9*a^2*b^2*(101*A
```

```
+ 143*C))*Sin[c + d*x])/(693*d*Cos[c + d*x]^(3/2)) + (2*(7*a^4*B + 54*a^2*b
^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*A + 9*C))*Sin[c + d*x])
/(15*d*Sqrt[Cos[c + d*x]]) + (2*(16*A*b^2 + 55*a*b*B + 3*a^2*(9*A + 11*C))*
(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) + (2*(8*A*b
 + 11*a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) +
 (2*A*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^4\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2(8Ab+11aB)(a+b\cos(c+dx))^3\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2\left(16Ab^2+55abB+3a^2(9A+11C)\right)(a+b\cos(c+}{231d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b(673A+8\right.}{3465d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b(673A+8\right.}{3465d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b(673A+8\right.}{3465d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(220a^3bB+308ab^3B+77b^4(A+3C)+66a^2b^2(5A\right.}{231d}$$

$$= -\frac{2\left(7a^4B+54a^2b^2B+15b^4B+12ab^3(3A+5C)+4\right.}{15d}$$

Mathematica [A] time = 6.16263, size = 381, normalized size = 0.8

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A+3C)\right)-154E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] (-154*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(
7*A + 9*C))*EllipticE[(c + d*x)/2, 2] + 10*(220*a^3*b*B + 308*a*b^3*B + 77*
```

```
b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*EllipticF[(c +
 d*x)/2, 2] + (2*(385*a^3*(4*A*b + a*B) + 77*a*(36*A*b^3 + 7*a^3*B + 54*a*b
^2*B + 4*a^2*b*(7*A + 9*C))*Cos[c + d*x]^2 + 15*(77*A*b^4 + 220*a^3*b*B + 3
08*a*b^3*B + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*Cos[c + d*x]^3 +
231*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*
A + 9*C))*Cos[c + d*x]^4)*Sin[c + d*x] + 45*(a^2*(66*A*b^2 + 44*a*b*B + a^2
*(9*A + 11*C))*Sin[2*(c + d*x)] + 14*a^4*A*Tan[c + d*x]))/(3*Cos[c + d*x]^(
9/2)))/(1155*d)
```

Maple [B] time = 6.718, size = 1550, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5
*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^4*(-1/352*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/154*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-1/6*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/
2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))+2*a^3*(4*A*b+B*a)*(-1/144*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
```

```
)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*b^3*(B*b+4*C*a)*(-(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*a^2*(
6*A*b^2+4*B*a*b+C*a^2)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos\left(dx+c\right)^6+\left(4\,Cab^3+Bb^4\right)\cos\left(dx+c\right)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos\left(dx+c\right)^4+2\left(2\,Ca^3b\right.}{\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/cos(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**
(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/co
s(d*x + c)^(13/2), x)
```

3.1095 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=285

$$-\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(3A+C)+21a^3bB-21a^4C+7ab^3B-b^4(7A+5C)\right)}{21b^5d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2bB-5a^3C-\right.}{5b^4d}$$

```
[Out] (2*(5*a^2*b*B + 3*b^3*B - 5*a^3*C - a*b^2*(5*A + 3*C))*EllipticE[(c + d*x)/
2, 2])/(5*b^4*d) - (2*(21*a^3*b*B + 7*a*b^3*B - 21*a^4*C - 7*a^2*b^2*(3*A +
 C) - b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*b^5*d) - (2*a^3*(A*b^
2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^5*(a + b)*
d) + (2*(7*A*b^2 - 7*a*b*B + 7*a^2*C + 5*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*b^3*d) + (2*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b^2*d
) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 1.2858, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(3A+C)+21a^3bB-21a^4C+7ab^3B-b^4(7A+5C)\right)}{21b^5d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2bB-5a^3C-\right.}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x]),x]
```


```
[Out] (2*(5*a^2*b*B + 3*b^3*B - 5*a^3*C - a*b^2*(5*A + 3*C))*EllipticE[(c + d*x)/
2, 2])/(5*b^4*d) - (2*(21*a^3*b*B + 7*a*b^3*B - 21*a^4*C - 7*a^2*b^2*(3*A +
 C) - b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*b^5*d) - (2*a^3*(A*b^
2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^5*(a + b)*
d) + (2*(7*A*b^2 - 7*a*b*B + 7*a^2*C + 5*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*b^3*d) + (2*(b*B - a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b^2*d
) + (2*C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5aC}{2}+\frac{1}{2}b(7A+5C)\,c\right.}{a+b\,c}}{7}$$

$$= \frac{2(bB-aC)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5b^2d} + \frac{2C\cos^{\frac{5}{2}}(c+dx)\sin}{7bd}$$

$$= \frac{2\left(7Ab^2-7abB+7a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^3d}$$

$$= \frac{2\left(7Ab^2-7abB+7a^2C+5b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21b^3d}$$

$$= \frac{2\left(5a^2bB+3b^3B-5a^3C-ab^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d}$$

$$= \frac{2\left(5a^2bB+3b^3B-5a^3C-ab^2(5A+3C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d}$$

Mathematica [A] time = 2.60333, size = 339, normalized size = 1.19

$$-\frac{2\left(-35a^2bB+35a^3C+ab^2(35A+13C)-63b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 2\sin(c+dx)\sqrt{\cos(c+dx)}\left(70a^2C+42b(bB-aC)\cos(c+dx)-70\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] ((-2*(-35*a^2*b*B - 63*b^3*B + 35*a^3*C + a*b^2*(35*A + 13*C))*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*(35*A*b^2 + 28*a*b*B - 28*a^2*C
 + 25*b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2]))/(a + b) + 2*Sqrt[Cos[c + d*x]]*(70*A*b^2 - 70*a*b*B +
70*a^2*C + 65*b^2*C + 42*b*(b*B - a*C)*Cos[c + d*x] + 15*b^2*C*Cos[2*(c + d
*x)])*Sin[c + d*x] - (42*(-5*a^2*b*B - 3*b^3*B + 5*a^3*C + a*b^2*(5*A + 3*C
))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(21
```

0*b^3*d)

Maple [B] time = 2.984, size = 1097, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/105*C/b*(60*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-168*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4/5/b^2*(B*b-C*a-4
*C*b)*(-4*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+4/3/b^3*(A*b^2-B*a*b-3*B*b^2+C*a^2+3*C*a*b+6*C*b^2)*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/b^4*(A*a*b^2+2*A*b^3-B*a^2*b-2*B*a*b^
2-3*B*b^3+C*a^3+2*C*a^2*b+3*C*a*b^2+4*C*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*(A*a^2*b^2+A*a*b^3+A*b^4-B*a^3*b-B*a^2*b^2-B*a*b^3-B*b^4+C*
a^4+C*a^3*b+C*a^2*b^2+C*a*b^3+C*b^4)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a^3*(A*b^2-B*a*b+C*a^2)/b^4/
(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c) + a), x)
```

3.1096 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-3a^3C-ab^2(3A+C)+b^3B\right)}{3b^4d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d}+\frac{2a^2\left(Ab^2\right.}{}$$

```
[Out] (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b^
3*d) + (2*(3*a^2*b*B + b^3*B - 3*a^3*C - a*b^2*(3*A + C))*EllipticF[(c + d*
x)/2, 2])/(3*b^4*d) + (2*a^2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(3*b^2*d) + (2*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.878859, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-3a^3C-ab^2(3A+C)+b^3B\right)}{3b^4d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d}+\frac{2a^2\left(Ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x]),x]
```



```
[Out] (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b^
3*d) + (2*(3*a^2*b*B + b^3*B - 3*a^3*C - a*b^2*(3*A + C))*EllipticF[(c + d*
x)/2, 2])/(3*b^4*d) + (2*a^2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(3*b^2*d) + (2*C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
```

```
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx=\frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd}+\frac{2\int\frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+\frac{1}{2}b(5A+3C)\,c\ldots\right.}{a+b\,c\ldots}}{5\ldots}$$

$$=\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2C\cos^{\frac{3}{2}}(c+dx)\sin\ldots}{5bd}$$

$$=\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2C\cos^{\frac{3}{2}}(c+dx)\sin\ldots}{5bd}$$

$$=\frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2(bB-\ldots}{\ldots}$$

$$=\frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2\left(3a^2b\ldots\right.}{\ldots}$$

Mathematica [A] time = 2.3118, size = 276, normalized size = 1.31

$$\frac{2b^2\left(5a^2C-5abB+15Ab^2+9b^2C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{6\sin(c+dx)\left(5a^2C-5abB+5Ab^2+3b^2C\right)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{c\ldots}\right.\right.\right.}{a\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] ((2*b^2*(15*A*b^2 - 5*a*b*B + 5*a^2*C + 9*b^2*C)*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/(a + b) + 2*b^2*(5*b*B + 4*a*C)*(2*EllipticF[(c + d*x)/2,
2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*b^2*Sqrt[
Cos[c + d*x]]*(5*b*B - 5*a*C + 3*b*C*Cos[c + d*x])*Sin[c + d*x] + (6*(5*A*b
^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 -
 b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a
*Sqrt[Sin[c + d*x]^2]))/(30*b^4*d)
```

Maple [B] time = 2.404, size = 803, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b*(-4*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+4/3/b^2*(B*b-C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/b^3
*(A*b^2-B*a*b-2*B*b^2+C*a^2+2*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*(A*a*b^2+A*b^3-B*a^2*b-B*a*b^2-B*b^3+C*a^3+C*a^2*b+C*a*b^2+
C*b^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-4*a^2*(A*b^2-B*a*b+C*a^2)/b^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a), x)
```

3.1097 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(b^2(3A+C)-3a(bB-aC)\right)}{3b^3d}-\frac{2a\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|\right.}{b^2d}$$

```
[Out] (2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(b^2*(3*A + C) - 3*a
*(b*B - a*C))*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) - (2*a*(A*b^2 - a*(b*B -
 a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*C*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.610055, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(b^2(3A+C)-3a(bB-aC)\right)}{3b^3d}-\frac{2a\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x]),x]
```


```
[Out] (2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(b^2*(3*A + C) - 3*a
*(b*B - a*C))*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) - (2*a*(A*b^2 - a*(b*B -
 a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*C*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```


Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{aC}{2}+\frac{1}{2}b(3A+C)\cos(c+dx)+\frac{3}{2}(bB\cdots}{\sqrt{\cos(c+dx)}(a+b\cos(c\cdots}}{3b}$$

$$= \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{1}{2}abC-\frac{1}{2}\left(b^2(3A+C)-3a(bB-aC\cdots\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c+\cdots}}{3b^2}$$

$$= \frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \cdots$$

$$= \frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2\left(b^2(3A+C)-3a(bB-aC)\cdots\right.}{3b^3d}$$

Mathematica [A] time = 1.31793, size = 216, normalized size = 1.47

$$\frac{-\frac{6(bB-aC)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}+\frac{4(3A+C)\left((a+b)F\left(\frac{1}{2}(c\cdots\right.\right.}{\cdots}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] ((-2*(-3*b*B + a*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4
*(3*A + C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2]))/(a + b) + 4*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(b*B
- a*C)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -A
rcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))
/(6*b*d)
```

Maple [B] time = 1.157, size = 945, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*C*a*b^2-4*
C*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b^2+2*C*b^3)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b^2-3*
a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+3*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3+3*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2*b-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/b^3/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a), x)
```

3.1098 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(b*B - a*C)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(b^2*(a + b)*d)
```

Rubi [A] time = 0.313814, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[
c + d*x])),x]
```

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(b*B - a*C)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(b^2*(a + b)*d)
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = -\frac{\int \frac{-Ab-(bB-aC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b}+\frac{C\int\sqrt{\cos(c+dx)}\,dx}{b}$$
$$=\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{(bB-aC)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}+\left(A-\frac{a(bB-aC)}{b^2}\right)\int\frac{}{\sqrt{\mathrm{co}}}$$
$$=\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2(bB-aC)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\Pi\left(\frac{2b}{a+b}\right.}{(a+b)d}$$

Mathematica [A] time = 1.57332, size = 177, normalized size = 1.82

$$\frac{\frac{C\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}+\frac{(2A+C)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\cdots}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])),x]
```

```
[Out] (((2*A + C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (B*(2*Elli
pticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
+ b)))/b + (C*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b
/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*
x]^2]))/d
```

Maple [A] time = 1.196, size = 323, normalized size = 3.3

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{\left(a-b\right)b^{2}\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\Big(A\mathrm{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-B*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))*a*b-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a*b+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+C*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c) + a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

3.1099 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=118

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(b
*d) - (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a*b*(a + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.527507, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[
c + d*x])),x]
```


```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(b
*d) - (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a*b*(a + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{1}{2}a(A-C)\cos(c+dx)-\frac{1}{2}Ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}$$

$$= \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{A\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{2\int \frac{\frac{1}{2}b(Ab-aB)-\frac{1}{2}abC\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{ab}$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{C\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b} - \left(\frac{Ab}{a} - B + \frac{a}{b}\right.$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2\left(\frac{Ab}{a}-B+\frac{aC}{b}\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{(a+b)d}$$

Mathematica [A] time = 1.23216, size = 216, normalized size = 1.83

$$\frac{-\frac{2A\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}} + \frac{2(2aB-3Ab)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right.}{a+b}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Cos[c + d*x])),x]
```

```
[Out] ((2*(-3*A*b + 2*a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (
4*a*(A - C)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]))/(b*(a + b)) + (4*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (
2*A*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellipti
cF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcS
in[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a
*d)
```

Maple [B] time = 1.899, size = 411, normalized size = 3.5

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\frac{C\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\,\mathrm{EllipticF}}{b\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin(1/2\,dx+c\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*(-A
*b^2+B*a*b-C*a^2)/a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

3.1100 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=158

$$\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|\right.}{3ad}$$

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(a^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2
)) - (2*(A*b - a*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.837433, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[
c + d*x])),x]
```


```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(a^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2
)) - (2*(A*b - a*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3}{2}(Ab-aB)+\frac{1}{2}a(A+3C)\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(3Ab^2-3abB+a^2(A+3C)\right)+\frac{1}{4}a}{\sqrt{\cos(}}}{}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b\left(3Ab^2-3abB+a^2(A+3C)\right)-}{\sqrt{\cos(c+dx)}(a+b\cos(}}{3a^2b}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2\left(Ab^2-a(bB-aC)\right)}{a^2(a}$$

Mathematica [A] time = 2.46276, size = 266, normalized size = 1.68

$$\frac{\frac{2a\left(2a^2(A+3C)-9abB+9Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{6(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE}{b\sqrt{\sin^2(c+dx)}}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Cos[c + d*x])),x]
```

```
[Out] ((2*a*(9*A*b^2 - 9*a*b*B + 2*a^2*(A + 3*C))*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(a + b) + (a*(8*a*A*b - 6*a^2*B)*(2*EllipticF[(c + d*x)/2, 2] -
 (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (4*a^2*A*Sin
[c + d*x])/Cos[c + d*x]^(3/2) + (12*a*(-(A*b) + a*B)*Sin[c + d*x])/Sqrt[Cos
[c + d*x]] - (6*(A*b - a*B)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1
] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*
EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[
Sin[c + d*x]^2]))/(6*a^3*d)
```

Maple [B] time = 2.854, size = 474, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b^2-B*a*b
+C*a^2)/a^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

3.1101 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=234

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)-5abB+5Ab^2\right)}{5a^3d}-\frac{2b\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}+\frac{2\sin(c+dx)\left(a^2(3A\right.}{5a^3d\sqrt{c}}$$

```
[Out] (-2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^3
*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (2*b*(A*b^2 - a
*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) +
(2*A*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(A*b - a*B)*Sin[c + d*x]
)/(3*a^2*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*S
in[c + d*x])/(5*a^3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.23611, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)-5abB+5Ab^2\right)}{5a^3d}-\frac{2b\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}+\frac{2\sin(c+dx)\left(a^2(3A\right.}{5a^3d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[
c + d*x])),x]
```



```
[Out] (-2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^3
*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (2*b*(A*b^2 - a
*(b*B - a*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) +
(2*A*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(A*b - a*B)*Sin[c + d*x]
)/(3*a^2*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*S
in[c + d*x])/(5*a^3*d*Sqrt[Cos[c + d*x]])
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{-\frac{5}{2}(Ab-aB)+\frac{1}{2}a(3A+5C)\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{3}{4}\left(5Ab^2-5abB+a^2(3A+5C)\right)+\frac{1}{4}a(\cdots}{\cos^{\frac{3}{2}}(c+\cdots}}{\cdots}$$

$$= \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5Ab^2-5abB+a^2(3A+5C\cdots\right.}{5a^3d\sqrt{\cos(c+dx\cdots}}$$

$$= \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5Ab^2-5abB+a^2(3A+5C\cdots\right.}{5a^3d\sqrt{\cos(c+dx\cdots}}$$

$$= -\frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} + \frac{2A\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(\cdots}{\cdots}$$

$$= -\frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx\cdots\right.}{3a^2d}$$

Mathematica [A] time = 4.92415, size = 336, normalized size = 1.44

$$-\frac{\frac{2\left(a^2b(19A+45C)-10a^3B-45ab^2B+45Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{4a\left(3a^2(3A+5C)-20abB+20Ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)} - \frac{2\left(3\sin(2(c+\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
b*Cos[c + d*x])),x]
```

```
[Out] -((2*(45*A*b^3 - 10*a^3*B - 45*a*b^2*B + a^2*b*(19*A + 45*C))*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*a*(20*A*b^2 - 20*a*b*B + 3*a^2*(
3*A + 5*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b)
```

```
, (c + d*x)/2, 2]))/(b*(a + b)) + (6*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*
(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[S
qrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]) - (2*(10*
a*(-(A*b) + a*B)*Sin[c + d*x] + 3*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*Sin
[2*(c + d*x)] + 6*a^2*A*Tan[c + d*x]))/Cos[c + d*x]^(3/2))/(30*a^3*d)
```

Maple [B] time = 3.932, size = 802, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(A*b^2-B*a*b+
C*a^2)*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/5*A/a/(8*sin(1/2*d*x+1/2*c)
^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*(A*b^2-B*a*b+C*a^2)/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*
c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b+B*a)/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos
(d*x + c)^(7/2)), x)
```

3.1102 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=318

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)-7abB+7Ab^2\right)}{21a^3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b(3A+5C)-3a^3B-5ab^2B+5Ab^3\right)}{5a^4d}+\frac{2b^2\left(Ab\right.}{}$$

```
[Out] (2*(5*A*b^3 - 3*a^3*B - 5*a*b^2*B + a^2*b*(3*A + 5*C))*EllipticE[(c + d*x)/
2, 2])/(5*a^4*d) + (2*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*EllipticF[(c +
d*x)/2, 2])/(21*a^3*d) + (2*b^2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a^4*(a + b)*d) + (2*A*Sin[c + d*x])/(7*a*d*Cos[c +
 d*x]^(7/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(5*a^2*d*Cos[c + d*x]^(5/2)) +
(2*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*a^3*d*Cos[c + d*
x]^(3/2)) - (2*(5*A*b^3 - 3*a^3*B - 5*a*b^2*B + a^2*b*(3*A + 5*C))*Sin[c +
d*x])/(5*a^4*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.74405, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)-7abB+7Ab^2\right)}{21a^3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b(3A+5C)-3a^3B-5ab^2B+5Ab^3\right)}{5a^4d}+\frac{2b^2\left(Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*(a + b*Cos[
c + d*x])),x]
```


```
[Out] (2*(5*A*b^3 - 3*a^3*B - 5*a*b^2*B + a^2*b*(3*A + 5*C))*EllipticE[(c + d*x)/
2, 2])/(5*a^4*d) + (2*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*EllipticF[(c +
d*x)/2, 2])/(21*a^3*d) + (2*b^2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a^4*(a + b)*d) + (2*A*Sin[c + d*x])/(7*a*d*Cos[c +
 d*x]^(7/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(5*a^2*d*Cos[c + d*x]^(5/2)) +
(2*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*a^3*d*Cos[c + d*
x]^(3/2)) - (2*(5*A*b^3 - 3*a^3*B - 5*a*b^2*B + a^2*b*(3*A + 5*C))*Sin[c +
d*x])/(5*a^4*d*Sqrt[Cos[c + d*x]])
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{-\frac{7}{2}(Ab-aB)+\frac{1}{2}a(5A+7C)\cos(c+dx)+\frac{5}{2}Ab\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{5}{4}\left(7Ab^2-7abB+a^2(5A+7C)\right)+\frac{1}{4}a(\ldots}{\cos^{\frac{5}{2}}(c\ldots}}{\ldots}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+a^2(5A+7C\ldots\right.}{21a^3d\cos^{\frac{3}{2}}(c+d\ldots}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+a^2(5A+7C\ldots\right.}{21a^3d\cos^{\frac{3}{2}}(c+d\ldots}$$

$$= \frac{2A\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+a^2(5A+7C\ldots\right.}{21a^3d\cos^{\frac{3}{2}}(c+d\ldots}$$

$$= \frac{2\left(5Ab^3-3a^3B-5ab^2B+a^2b(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d} + \frac{2A\sin(c+d\ldots}{7ad\cos^{\frac{7}{2}}(c+\ldots}$$

$$= \frac{2\left(5Ab^3-3a^3B-5ab^2B+a^2b(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d} + \frac{2\left(7Ab^2-7ab\ldots\right.}{\ldots}$$

Mathematica [A] time = 4.79384, size = 420, normalized size = 1.32

$$\frac{\frac{2\left(7a^2b^2(19A+45C)+10a^4(5A+7C)-133a^3bB-315ab^3B+315Ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{4a\left(4a^2b(22A+35C)-63a^3B-140ab^2B+140Ab^3\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Gamma\ldots\right.}{b(a+b)}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*(a +
b*Cos[c + d*x])),x]
```

```
[Out] ((2*(315*A*b^4 - 133*a^3*b*B - 315*a*b^3*B + 10*a^4*(5*A + 7*C) + 7*a^2*b^2
*(19*A + 45*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*a*(
140*A*b^3 - 63*a^3*B - 140*a*b^2*B + 4*a^2*b*(22*A + 35*C))*((a + b)*Ellipt
icF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a +
 b)) - (42*(-5*A*b^3 + 3*a^3*B + 5*a*b^2*B - a^2*b*(3*A + 5*C))*(-2*a*b*Ell
ipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]) + (2*(42*(a^2*(-(A*b
) + a*B) + (-5*A*b^3 + 3*a^3*B + 5*a*b^2*B - a^2*b*(3*A + 5*C))*Cos[c + d*x
]^2)*Sin[c + d*x] + 5*(a*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*Sin[2*(c + d
*x)] + 6*a^3*A*Tan[c + d*x])))/Cos[c + d*x]^(5/2))/(210*a^4*d)
```

Maple [B] time = 5.277, size = 1003, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b^2-B*a*b
+C*a^2)*b^3/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/5*(-A*b+B*a)/a^2/(8*sin(1/
2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*
x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)-2*(A*b^2-B*a*b+C*a^2)/a^4*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(A*b^2-B*a*b+C*a^2)/a^3*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
```

```
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A/a*(-1/56*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2
)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos
(d*x + c)^(9/2)), x)
```

3.1103 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=445

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^3b^2(9A-20C)-16a^2b^3B+15a^4bB-21a^5C+4ab^4(3A+C)-2b^5B\right)}{3b^5d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-3a^2b^2(5A\right.}{}$$

```
[Out] -((25*a^3*b*B - 20*a*b^3*B - 3*a^2*b^2*(5*A - 8*C) - 35*a^4*C + 2*b^4*(5*A
+ 3*C))*EllipticE[(c + d*x)/2, 2])/(5*b^4*(a^2 - b^2)*d) + ((15*a^4*b*B - 1
6*a^2*b^3*B - 2*b^5*B - a^3*b^2*(9*A - 20*C) - 21*a^5*C + 4*a*b^4*(3*A + C)
)*EllipticF[(c + d*x)/2, 2])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b^4 + 5*a^3*
b*B - 7*a*b^3*B - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/((a - b)*b^5*(a + b)^2*d) + ((5*a^2*b*B - 2*b^3*B - a*b^2*
(3*A - 4*C) - 7*a^3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*
d) + ((5*A*b^2 - 5*a*b*B + 7*a^2*C - 2*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(5*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.5921, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^3b^2(9A-20C)-16a^2b^3B+15a^4bB-21a^5C+4ab^4(3A+C)-2b^5B\right)}{3b^5d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-3a^2b^2(5A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^2,x]
```


```
[Out] -((25*a^3*b*B - 20*a*b^3*B - 3*a^2*b^2*(5*A - 8*C) - 35*a^4*C + 2*b^4*(5*A
+ 3*C))*EllipticE[(c + d*x)/2, 2])/(5*b^4*(a^2 - b^2)*d) + ((15*a^4*b*B - 1
6*a^2*b^3*B - 2*b^5*B - a^3*b^2*(9*A - 20*C) - 21*a^5*C + 4*a*b^4*(3*A + C)
)*EllipticF[(c + d*x)/2, 2])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b^4 + 5*a^3*
b*B - 7*a*b^3*B - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/((a - b)*b^5*(a + b)^2*d) + ((5*a^2*b*B - 2*b^3*B - a*b^2*
(3*A - 4*C) - 7*a^3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*
d) + ((5*A*b^2 - 5*a*b*B + 7*a^2*C - 2*b^2*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
```

```
x])/(5*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \int \frac{\cos^{\frac{3}{2}}(c+d\ldots}{\ldots}$$

$$= \frac{\left(5Ab^2-5abB+7a^2C-2b^2C\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5b^2\left(a^2-b^2\right)d} - \ldots$$

$$= \frac{\left(5a^2bB-2b^3B-ab^2(3A-4C)-7a^3C\right)\sqrt{\cos(c+dx)}\sin(c\ldots}{3b^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(5a^2bB-2b^3B-ab^2(3A-4C)-7a^3C\right)\sqrt{\cos(c+dx)}\sin(c\ldots}{3b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(25a^3bB-20ab^3B-3a^2b^2(5A-8C)-35a^4C+2b^4(5A\ldots\right.}{5b^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(25a^3bB-20ab^3B-3a^2b^2(5A-8C)-35a^4C+2b^4(5A\ldots\right.}{5b^4\left(a^2-b^2\right)d}$$

Mathematica [A] time = 5.49652, size = 408, normalized size = 0.92

$$\frac{4\sqrt{\cos(c+dx)}\left(-\frac{15a^2\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+10(bB-2aC)\sin(c+dx)+3bC\sin(2(c+dx))\right)+\frac{2\left(a^2b^2(15A-32C)-25a^3bB+35a^4C+\ldots\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^2,x]
```

```
[Out] (((2*(-25*a^3*b*B + 40*a*b^3*B + a^2*b^2*(15*A - 32*C) + 35*a^4*C - 6*b^4*(
5*A + 3*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-10*a^
2*b*B - 5*b^3*B + 14*a^3*C + a*b^2*(15*A + C))*((a + b)*EllipticF[(c + d*x)
/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(-25*a^
3*b*B + 20*a*b^3*B + 3*a^2*b^2*(5*A - 8*C) + 35*a^4*C - 2*b^4*(5*A + 3*C))*
(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[S
qrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a -
```

```
b)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*(10*(b*B - 2*a*C)*Sin[c + d*x] - (15*a^2
*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])
) + 3*b*C*Sin[2*(c + d*x)]))/(60*b^3*d)
```

Maple [B] time = 4.115, size = 1382, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^2*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+4/3/b^3*(B*b-2*C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
/b^4*(A*b^2-2*B*a*b-2*B*b^2+3*C*a^2+4*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-2*(2*A*a*b^2+A*b^3-3*B*a^2*b-2*B*a*b^2-B*b^3+4*C*a^3+
3*C*a^2*b+2*C*a*b^2+C*b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*a^2/b^4*(3*A*b^2-4*B*a*b+5*C*a^2)/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2-B*a*b+C*a^2)/b^5*(-1/a*b^2/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
```

```
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)
/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^2, x)
```

3.1104 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=343

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(3A-16C)+9a^3bB-15a^4C-12ab^3B+2b^4(3A+C)\right)}{3b^4d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-5a^3C-a\right.}{b^3d\left(a^2-b^2\right)}$$

```
[Out] ((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*EllipticE[(c + d*x)/2, 2
])/(b^3*(a^2 - b^2)*d) - ((9*a^3*b*B - 12*a*b^3*B - a^2*b^2*(3*A - 16*C) -
15*a^4*C + 2*b^4*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d
) + (a*(3*A*b^4 + 3*a^3*b*B - 5*a*b^3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^4*(a + b)^2*d) + ((3*A*b^
2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a
^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.12625, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(3A-16C)+9a^3bB-15a^4C-12ab^3B+2b^4(3A+C)\right)}{3b^4d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-5a^3C-a\right.}{b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^2,x]
```



```
[Out] ((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*EllipticE[(c + d*x)/2, 2
])/(b^3*(a^2 - b^2)*d) - ((9*a^3*b*B - 12*a*b^3*B - a^2*b^2*(3*A - 16*C) -
15*a^4*C + 2*b^4*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d
) + (a*(3*A*b^4 + 3*a^3*b*B - 5*a*b^3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^4*(a + b)^2*d) + ((3*A*b^
2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a
^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \sqrt{\cos(c+d}}{}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C-2b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} -$$

$$= \frac{\left(3Ab^2-3abB+5a^2C-2b^2C\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} -$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{(}{}$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{(}{}$$

Mathematica [A] time = 3.62617, size = 343, normalized size = 1.

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a\left(a(aC-bB)+Ab^2\right)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2C\right)-\frac{\frac{2\left(-3a^2bB+5a^3C-ab^2(3A+8C)+6b^3B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(2a^2C-3abB+3Ab^2+b^2C\right)\left((a+b)F\left(\frac{1}{2}(\right.\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*C + (3*a*(A*b^2 + a*(-(b*B) + a*C)))/((a^2 - b^2)*
(a + b*Cos[c + d*x])))*Sin[c + d*x] - ((2*(-3*a^2*b*B + 6*b^3*B + 5*a^3*C -
 a*b^2*(3*A + 8*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8
*(3*A*b^2 - 3*a*b*B + 2*a^2*C + b^2*C)*((a + b)*EllipticF[(c + d*x)/2, 2] -
 a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(-3*a^2*b*B + 2
*b^3*B + a*b^2*(A - 4*C) + 5*a^3*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2
 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/
(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(12*b^2*d)
```

Maple [B] time = 3.989, size = 1129, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^4*(4*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
6*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+9*a^2*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b-2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a/b^3*(2*A*b^2-3*B*a*b+4*C*a^2)/(
-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b^2-B*a*b+C*a^2)/b^4*(-1/a*b^2/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/
a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^2, x)
```

3.1105 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=251

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2bB-3a^3C+ab^2(A+4C)-2b^3B\right)}{b^3d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2C-abB+Ab^2-2b^2C\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2b^2(A+5C\right.}{}$$

```
[Out] ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 -
 b^2)*d) + ((a^2*b*B - 2*b^3*B - 3*a^3*C + a*b^2*(A + 4*C))*EllipticF[(c +
d*x)/2, 2])/(b^3*(a^2 - b^2)*d) - ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C +
 a^2*b^2*(A + 5*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3
*(a + b)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.719886, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2bB-3a^3C+ab^2(A+4C)-2b^3B\right)}{b^3d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2C-abB+Ab^2-2b^2C\right)}{b^2d\left(a^2-b^2\right)}-\frac{\left(a^2b^2(A+5C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^2,x]
```



```
[Out] ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 -
 b^2)*d) + ((a^2*b*B - 2*b^3*B - 3*a^3*C + a*b^2*(A + 4*C))*EllipticF[(c +
d*x)/2, 2])/(b^3*(a^2 - b^2)*d) - ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C +
 a^2*b^2*(A + 5*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3
*(a + b)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(Ab^2-\right.}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}b\left(Ab\right.}{}}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a(b\right.}{b\left(a\right.}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{\left(a^2bB-2b\right.}{}$$

Mathematica [A] time = 3.8957, size = 304, normalized size = 1.21

$$-\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(a(aC-bB)+Ab^2\right)}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(a^2C+abB-Ab^2-2b^2C\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\sin(c+dx)\left(3a^2C-abB+Ab^2-2b^2C\right)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(b-a)(a}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^2,x]
```

```
[Out] -((4*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^
2)*(a + b*Cos[c + d*x])) + ((2*(-(A*b^2) + a*b*B + a^2*C - 2*b^2*C)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-(b*B) + a*(A + C))*((a +
 b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
))/(a + b) + (2*(A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/(4*b*d)
```

Maple [B] time = 3.278, size = 862, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)-4/b^2*(A*b^2-2*B*a*b+3*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a
*(A*b^2-B*a*b+C*a^2)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/
2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^2, x)
```

3.1106 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=243

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-C)-abB+Ab^2+2b^2C\right)}{b^2d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}-\frac{\left(-3a^2b^2(A+C)+a^3bB+a^4C\right.}{ab^2d(}$$

```
[Out] -(((A*b^2 - a*(b*B - a*C))*EllipticE[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d))
- ((A*b^2 - a*b*B - a^2*C + 2*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 -
 b^2)*d) - ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 -
 a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Co
s[c + d*x]))
```

Rubi [A] time = 0.708097, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-C)-abB+Ab^2+2b^2C\right)}{b^2d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}-\frac{\left(-3a^2b^2(A+C)+a^3bB+a^4C\right.}{ab^2d(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[
c + d*x])^2),x]
```

```
[Out] -(((A*b^2 - a*(b*B - a*C))*EllipticE[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d))
- ((A*b^2 - a*b*B - a^2*C + 2*b^2*C)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 -
 b^2)*d) - ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 -
 a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Co
s[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-Ab^2-abB+a^2(2A+C)\right)-a(Ab-\ldots}{\sqrt{\cos\ldots}}}{\ldots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}b\left(Ab^2+abB-a^2(2A+C)\right)+\frac{1}{2}a\left(A\ldots\right.}{\sqrt{\cos(c+dx)}(a+\ldots}}{ab\left(a^2\ldots\right.}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} + \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\,\mathrm{s}\ldots}{a\left(a^2-b^2\right)d(a+b\cos(c+d\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} - \frac{\left(Ab^2-abB-a^2C+2b^2C\right)F\left(\frac{1}{2}(c\ldots\right.}{b^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 3.35444, size = 299, normalized size = 1.23

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(a(aC-bB)+Ab^2\right)}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(a^2(4A+C)-abB-3Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\sin(c+dx)\left(a(aC-bB)+Ab^2\right)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin\ldots\right.\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2
)*(a + b*Cos[c + d*x])) + ((2*(-3*A*b^2 - a*b*B + a^2*(4*A + C))*EllipticPi
[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*a*(A*b - a*B + b*C)*((a + b)*
EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(
b*(a + b)) + (2*(A*b^2 + a*(-(b*B) + a*C))*(2*a*b*EllipticE[ArcSin[Sqrt[Cos
[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
(-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c +
 d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [B] time = 2.797, size = 815, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(B*b-2*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^2*(A*b^2-B*a*b+C*a^2)*(-1/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+
1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*s
qrt(cos(d*x + c))), x)
```

3.1107 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=306

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A-C))-abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\left(-a^2b^2(5A+C)+3a^3bB+a\right.}{a^2}$$

```
[Out] ((3*A*b^2 - a*b*B - a^2*(2*A - C))*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b
^2)*d) + ((A*b^2 - a*(b*B - a*C))*EllipticF[(c + d*x)/2, 2])/(a*b*(a^2 - b^
2)*d) + ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C - a^2*b^2*(5*A + C))*Ellipt
icPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b^
2 - a*b*B - a^2*(2*A - C))*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*
x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.08942, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A-C))-abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\left(-a^2b^2(5A+C)+3a^3bB+a\right.}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[
c + d*x])^2),x]
```



```
[Out] ((3*A*b^2 - a*b*B - a^2*(2*A - C))*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b
^2)*d) + ((A*b^2 - a*(b*B - a*C))*EllipticF[(c + d*x)/2, 2])/(a*b*(a^2 - b^
2)*d) + ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C - a^2*b^2*(5*A + C))*Ellipt
icPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b^
2 - a*b*B - a^2*(2*A - C))*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*
x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]*(a + b*Cos[c + d*x]))
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-3Ab^2+abB+a^2(2A-C)\right)-a(A}{\cos}}{}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2-a(bB-aC)\right)\sin}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2-a(bB-aC)\right)\sin}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+}$$

$$= \frac{\left(3Ab^2-abB-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-abB-a^2(2A-C)\right)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c}}$$

$$= \frac{\left(3Ab^2-abB-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2-a(bB-aC)\right)F\left(\frac{1}{2}(c\right.}{ab\left(a^2-b^2\right)d}$$

Mathematica [A] time = 4.33729, size = 355, normalized size = 1.16

$$\frac{\frac{4\sin(c+dx)\left(b\cos(c+dx)\left(a^2(2A-C)+abB-3Ab^2\right)+2aA\left(a^2-b^2\right)\right)}{\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} - \frac{\frac{2\left(-a^2b(10A+C)+4a^3B-3ab^2B+9Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{8a\left(a^2(A-C)+abB-2Ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\right.}{b(a+b)}}{}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*(2*a*A*(a^2 - b^2) + b*(-3*A*b^2 + a*b*B + a^2*(2*A - C))*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])) - ((2*
(9*A*b^3 + 4*a^3*B - 3*a*b^2*B - a^2*b*(10*A + C))*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2])/(a + b) - (8*a*(-2*A*b^2 + a*b*B + a^2*(A - C))*((a + b)
*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/
(b*(a + b)) - (2*(-3*A*b^2 + a*b*B + a^2*(2*A - C))*(-2*a*b*EllipticE[ArcSi
```

```
n[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/(4*a^2*d)
```

Maple [B] time = 3.482, size = 903, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b^2+C*a^
2)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin
(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b^2+B*a*b-C*a^2)/a/b*(-1
/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*c
os(d*x + c)^(3/2)), x)
```

3.1108 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=392

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A-3C))-3abB+5Ab^2\right)}{3a^2d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b(4A-C)+2a^3B-3ab^2B+5Ab^3\right)}{a^3d\left(a^2-b^2\right)}-\frac{\left(-a^2\right.}{}$$

```
[Out] -(((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*EllipticE[(c + d*x)/2,
 2])/(a^3*(a^2 - b^2)*d)) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Elliptic
F[(c + d*x)/2, 2])/(3*a^2*(a^2 - b^2)*d) - ((5*A*b^4 + 5*a^3*b*B - 3*a*b^3*
B - a^2*b^2*(7*A - C) - 3*a^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a^3*(a - b)*(a + b)^2*d) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Sin[c +
 d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((5*A*b^3 + 2*a^3*B - 3*a
*b^2*B - a^2*b*(4*A - C))*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x
]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^
(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.56235, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A-3C))-3abB+5Ab^2\right)}{3a^2d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b(4A-C)+2a^3B-3ab^2B+5Ab^3\right)}{a^3d\left(a^2-b^2\right)}-\frac{\left(-a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[
c + d*x])^2),x]
```


```
[Out] -(((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*EllipticE[(c + d*x)/2,
 2])/(a^3*(a^2 - b^2)*d)) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Elliptic
F[(c + d*x)/2, 2])/(3*a^2*(a^2 - b^2)*d) - ((5*A*b^4 + 5*a^3*b*B - 3*a*b^3*
B - a^2*b^2*(7*A - C) - 3*a^4*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a^3*(a - b)*(a + b)^2*d) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Sin[c +
 d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((5*A*b^3 + 2*a^3*B - 3*a
*b^2*B - a^2*b*(4*A - C))*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x
]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^
```

```
(3/2)*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-5Ab^2+3abB+a^2(2A-3C)\right)-a}{\cos}}{}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(Ab^2-a(bB-aC)\right)\mathrm{s}}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(5Ab^3+2a^3B-3ab^2B-a^2\right.}{a^3\left(a^2-b^2\right)d\sqrt{c}}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(5Ab^3+2a^3B-3ab^2B-a^2\right.}{a^3\left(a^2-b^2\right)d\sqrt{c}}$$

$$= -\frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} - \frac{\left(5Ab^2-3abB\right.}{3a^2\left(a^2\right.}$$

$$= -\frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} - \frac{\left(5Ab^2-3abB\right.}{}$$

Mathematica [A] time = 7.08061, size = 428, normalized size = 1.09

$$\frac{\frac{6b\sin(2(c+dx))\left(a^2b(C-4A)+2a^3B-3ab^2B+5Ab^3\right)+8a\left(a^2-b^2\right)(3aB-5Ab)\sin(c+dx)+8a^2A\left(a^2-b^2\right)\tan(c+dx)}{\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{\frac{2\left(a^2b^2(44A-9C)+4a^4(A+3C)-30a^3bB+27ab^3B-45A\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Cos[c + d*x])^2),x]
```

```
[Out] (((2*(-45*A*b^4 - 30*a^3*b*B + 27*a*b^3*B + a^2*b^2*(44*A - 9*C) + 4*a^4*(A
 + 3*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*a*(10*A*b^
3 + 3*a^3*B - 6*a*b^2*B + a^2*(-7*A*b + 3*b*C))*((a + b)*EllipticF[(c + d*x
)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) - (6*(5
*A*b^3 + 2*a^3*B - 3*a*b^2*B + a^2*b*(-4*A + C))*(-2*a*b*EllipticE[ArcSin[S
qrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*S
in[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)) + (8*a*(a^2 - b^
2)*(-5*A*b + 3*a*B)*Sin[c + d*x] + 6*b*(5*A*b^3 + 2*a^3*B - 3*a*b^2*B + a^2
*b*(-4*A + C))*Sin[2*(c + d*x)] + 8*a^2*A*(a^2 - b^2)*Tan[c + d*x])/((a^2 -
 b^2)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])))/(12*a^3*d)
```

Maple [B] time = 5.498, size = 1038, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*A*b-B
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-2*A*b+B*a)/a^3*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a^2*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+c
os(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2-B*a*b+C*a^2)/a^2*(-1/a*b^2/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
```

```
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2
)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*c
os(d*x + c)^(5/2)), x)
```

3.1109 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=654

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-9a^5b^2(5A-43C)+3a^3b^4(33A-64C)-223a^4b^3B+128a^2b^5B+105a^6bB-189a^7C-24ab^6(3A+C\right.}{12b^6d\left(a^2-b^2\right)^2}$$

```
[Out] -((175*a^5*b*B - 325*a^3*b^3*B + 120*a*b^5*B + a^2*b^4*(145*A - 192*C) - 3*
a^4*b^2*(25*A - 187*C) - 315*a^6*C - 8*b^6*(5*A + 3*C))*EllipticE[(c + d*x)
/2, 2])/(20*b^5*(a^2 - b^2)^2*d) + ((105*a^6*b*B - 223*a^4*b^3*B + 128*a^2*
b^5*B + 8*b^7*B + 3*a^3*b^4*(33*A - 64*C) - 9*a^5*b^2*(5*A - 43*C) - 189*a^
7*C - 24*a*b^6*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(12*b^6*(a^2 - b^2)^2*
d) + (a^2*(35*A*b^6 - 35*a^5*b*B + 86*a^3*b^3*B - 63*a*b^5*B - a^2*b^4*(38*
A - 99*C) + 15*a^4*b^2*(A - 10*C) + 63*a^6*C)*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(4*(a - b)^2*b^6*(a + b)^3*d) + ((35*a^4*b*B - 61*a^2*b^3*B +
 8*b^5*B + 3*a*b^4*(11*A - 8*C) - 15*a^3*b^2*(A - 7*C) - 63*a^5*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(12*b^4*(a^2 - b^2)^2*d) - ((35*a^3*b*B - 65*a*b^3
*B - a^2*b^2*(15*A - 101*C) + b^4*(45*A - 8*C) - 63*a^4*C)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(20*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c
+ d*x]^(7/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((7
*A*b^4 + 5*a^3*b*B - 11*a*b^3*B - a^2*b^2*(A - 15*C) - 9*a^4*C)*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.63888, antiderivative size = 654, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-9a^5b^2(5A-43C)+3a^3b^4(33A-64C)-223a^4b^3B+128a^2b^5B+105a^6bB-189a^7C-24ab^6(3A+C\right.}{12b^6d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^3,x]
```


```
[Out] -((175*a^5*b*B - 325*a^3*b^3*B + 120*a*b^5*B + a^2*b^4*(145*A - 192*C) - 3*
a^4*b^2*(25*A - 187*C) - 315*a^6*C - 8*b^6*(5*A + 3*C))*EllipticE[(c + d*x)
/2, 2])/(20*b^5*(a^2 - b^2)^2*d) + ((105*a^6*b*B - 223*a^4*b^3*B + 128*a^2*
```

```
b^5*B + 8*b^7*B + 3*a^3*b^4*(33*A - 64*C) - 9*a^5*b^2*(5*A - 43*C) - 189*a^
7*C - 24*a*b^6*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(12*b^6*(a^2 - b^2)^2*
d) + (a^2*(35*A*b^6 - 35*a^5*b*B + 86*a^3*b^3*B - 63*a*b^5*B - a^2*b^4*(38*
A - 99*C) + 15*a^4*b^2*(A - 10*C) + 63*a^6*C)*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(4*(a - b)^2*b^6*(a + b)^3*d) + ((35*a^4*b*B - 61*a^2*b^3*B +
 8*b^5*B + 3*a*b^4*(11*A - 8*C) - 15*a^3*b^2*(A - 7*C) - 63*a^5*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(12*b^4*(a^2 - b^2)^2*d) - ((35*a^3*b*B - 65*a*b^3
*B - a^2*b^2*(15*A - 101*C) + b^4*(45*A - 8*C) - 63*a^4*C)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(20*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C))*Cos[c
+ d*x]^(7/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((7
*A*b^4 + 5*a^3*b*B - 11*a*b^3*B - a^2*b^2*(A - 15*C) - 9*a^4*C)*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \int \frac{\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(7Ab^4+5a\right.}{}$$

$$= -\frac{\left(35a^3bB-65ab^3B-a^2b^2(15A-101C)+b^4(45A-8C)-6\right.}{20b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(35a^4bB-61a^2b^3B+8b^5B+3ab^4(11A-8C)-15a^3b^2(A-\right.}{12b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(35a^4bB-61a^2b^3B+8b^5B+3ab^4(11A-8C)-15a^3b^2(A-\right.}{12b^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(175a^5bB-325a^3b^3B+120ab^5B+a^2b^4(145A-192C)-3\right.}{20b^5\left(a\right.}$$

$$= -\frac{\left(175a^5bB-325a^3b^3B+120ab^5B+a^2b^4(145A-192C)-3\right.}{20b^5\left(a\right.}$$

Mathematica [A] time = 7.4406, size = 555, normalized size = 0.85

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{30a^3\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{\left(a^2-b^2\right)(a+b\cos(c+dx))^2} - \frac{15a^2\sin(c+dx)\left(7a^2b^2(A-3C)-11a^3bB+15a^4C+17ab^3B-13Ab^4\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))} + 40(bB-3aC)\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^3,x]
```

```
[Out] (((2*(-175*a^5*b*B + 365*a^3*b^3*B - 280*a*b^5*B + 3*a^4*b^2*(25*A - 211*C)
 - 21*a^2*b^4*(5*A - 16*C) + 315*a^6*C + 24*b^6*(5*A + 3*C))*EllipticPi[(2*
```

```
b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(-35*a^4*b*B + 70*a^2*b^3*B + 10
*b^5*B + 3*a^3*b^2*(5*A - 32*C) + 63*a^5*C - 12*a*b^4*(5*A + C))*((a + b)*E
llipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a
 + b) + (6*(-175*a^5*b*B + 325*a^3*b^3*B - 120*a*b^5*B + 3*a^4*b^2*(25*A -
187*C) + 315*a^6*C + 8*b^6*(5*A + 3*C) + a^2*b^4*(-145*A + 192*C))*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a
+ b)^2) + 4*Sqrt[Cos[c + d*x]]*(40*(b*B - 3*a*C)*Sin[c + d*x] + (30*a^3*(A*
b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])^2)
- (15*a^2*(-13*A*b^4 - 11*a^3*b*B + 17*a*b^3*B + 7*a^2*b^2*(A - 3*C) + 15*a
^4*C)*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])) + 12*b*C*Sin[2*(c
+ d*x)]))/(240*b^4*d)
```

Maple [B] time = 6.424, size = 2520, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^3*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+4/3/b^4*(B*b-3*C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
/b^5*(A*b^2-3*B*a*b-2*B*b^2+6*C*a^2+6*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-2*(3*A*a*b^2+A*b^3-6*B*a^2*b-3*B*a*b^2-B*b^3+10*C*a^3
+6*C*a^2*b+3*C*a*b^2+C*b^3)/b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b^5*a^2*(6*A*b^2-10*B*a*b+15*C*a^2)/(
-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
```

*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(A*b^2-B*a*b+C*a^2)/b^6*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2/b^6*a^3*(4*A*b^2-5*B*a*b+6*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

---

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos
(d*x + c) + a)^3, x)
```

3.1110 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=536

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^4b^2(9A-223C)+a^2b^4(15A-128C)-99a^3b^3B+45a^5bB-105a^6C+72ab^5B-8b^6(3A+C)\right)}{12b^5d\left(a^2-b^2\right)^2}+$$

```
[Out] ((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*A
 - 8*C) - 35*a^5*C)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) - ((
45*a^5*b*B - 99*a^3*b^3*B + 72*a*b^5*B - a^4*b^2*(9*A - 223*C) + a^2*b^4*(1
5*A - 128*C) - 105*a^6*C - 8*b^6*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(12*
b^5*(a^2 - b^2)^2*d) - (a*(15*A*b^6 - 15*a^5*b*B + 38*a^3*b^3*B - 35*a*b^5*
B + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A - 21*C) + 35*a^6*C)*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*b*B
- 33*a*b^3*B - a^2*b^2*(3*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C)
)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
2) + ((5*A*b^4 + 3*a^3*b*B - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A + 13*C))*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.90138, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^4b^2(9A-223C)+a^2b^4(15A-128C)-99a^3b^3B+45a^5bB-105a^6C+72ab^5B-8b^6(3A+C)\right)}{12b^5d\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^3,x]
```


```
[Out] ((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*A
 - 8*C) - 35*a^5*C)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) - ((
45*a^5*b*B - 99*a^3*b^3*B + 72*a*b^5*B - a^4*b^2*(9*A - 223*C) + a^2*b^4*(1
5*A - 128*C) - 105*a^6*C - 8*b^6*(3*A + C))*EllipticF[(c + d*x)/2, 2])/(12*
b^5*(a^2 - b^2)^2*d) - (a*(15*A*b^6 - 15*a^5*b*B + 38*a^3*b^3*B - 35*a*b^5*
B + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A - 21*C) + 35*a^6*C)*EllipticPi[(2
```

```
*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*b*B
- 33*a*b^3*B - a^2*b^2*(3*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C)
)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
2) + ((5*A*b^4 + 3*a^3*b*B - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A + 13*C))*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \int \frac{\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(5Ab^4+3a\right.}{}$$

$$= -\frac{\left(15a^3bB-33ab^3B-a^2b^2(3A-61C)+b^4(21A-8C)-35a\right.}{12b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(15a^3bB-33ab^3B-a^2b^2(3A-61C)+b^4(21A-8C)-35a\right.}{12b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A-8\right.}{4b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A-8\right.}{4b^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 5.85132, size = 524, normalized size = 0.98

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(ab\cos(c+dx)\left(a^2b^2(9A-83C)-21a^3bB+49a^4C+39ab^3B+b^4(16C-27A)\right)+3a^4Ab^2-21a^2Ab^4+33a^3b^3B+4C\left(b^3-a^2b\right)^2\cos(2(c+dx))-57a^4b^2\right.}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*a^4*A*b^2 - 21*a^2*A*b^4 - 15*a^5*b*B + 33*a^3*b^
3*B + 35*a^6*C - 57*a^4*b^2*C + 4*b^6*C + a*b*(-21*a^3*b*B + 39*a*b^3*B + a
^2*b^2*(9*A - 83*C) + 49*a^4*C + b^4*(-27*A + 16*C))*Cos[c + d*x] + 4*(-(a^
2*b) + b^3)^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c
 + d*x])^2) - ((2*(-15*a^4*b*B + 21*a^2*b^3*B - 24*b^5*B + a^3*b^2*(3*A - 7
3*C) + 35*a^5*C + a*b^4*(15*A + 56*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/
```

```
2, 2])/(a + b) - (16*(3*a^3*b*B - 12*a*b^3*B - 7*a^4*C + 2*b^4*(3*A + C) +
a^2*b^2*(3*A + 14*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(-15*a^4*b*B + 29*a^2*b^3*B - 8*
b^5*B + a^3*b^2*(3*A - 65*C) + 35*a^5*C + 3*a*b^4*(-3*A + 8*C))*(-2*a*b*Ell
ipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b
)^2))/(48*b^3*d)
```

Maple [B] time = 6.158, size = 2267, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^5*(4*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+18*a^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b-2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a/b^4*(3*A*b^2-6*B*a*b+10*C*a^2)
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2-B*a*b+C*a^2)/b^5*(-1/2/a*b^2/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
```

```
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2
-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*
a^2/b^5*(3*A*b^2-4*B*a*b+5*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*
b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2
-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^3, x)
```

3.1111 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=423

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3b^2(A+33C)-5a^2b^3B+3a^4bB-15a^5C-ab^4(7A+24C)+8b^5B\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(A+29C\right.}{4}$$

```
[Out] -((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))
*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*b*B - 5*a^2*b
^3*B + 8*b^5*B - 15*a^5*C - a*b^4*(7*A + 24*C) + a^3*b^2*(A + 33*C))*Ellipt
icF[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^5*b*B + 6*a^
3*b^3*B - 15*a*b^5*B + 15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C
))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d)
 - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Cos[c + d*x])^2) + ((3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C +
a^2*b^2*(3*A + 11*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.36311, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3b^2(A+33C)-5a^2b^3B+3a^4bB-15a^5C-ab^4(7A+24C)+8b^5B\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(A+29C\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^3,x]
```


```
[Out] -((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))
*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*b*B - 5*a^2*b
^3*B + 8*b^5*B - 15*a^5*C - a*b^4*(7*A + 24*C) + a^3*b^2*(A + 33*C))*Ellipt
icF[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^5*b*B + 6*a^
3*b^3*B - 15*a*b^5*B + 15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C
))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d)
 - ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Cos[c + d*x])^2) + ((3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C +
```

```
a^2*b^2*(3*A + 11*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \int \frac{\sqrt{\cos(c+dx)}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(3Ab^4+a^3\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(3Ab^4+a^3\right.}{}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)}{4b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)}{4b^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 6.30762, size = 441, normalized size = 1.04

$$\frac{\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\cos(c+dx)\left(a^2b^2(A+13C)+3a^3bB-7a^4C-9ab^3B+5Ab^4\right)+a\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+3Ab^4\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\left(a^2b^2(5A-7C)-a^3bB+5a^4C-5ab\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*(3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^
2*(3*A + 11*C)) + b*(5*A*b^4 + 3*a^3*b*B - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A
```

```
 + 13*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2
) + (((-(a^3*b*B) - 5*a*b^3*B + a^2*b^2*(5*A - 7*C) + 5*a^4*C + b^4*(A + 8*
C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^2*b*B + 2*b^
3*B + a^3*C - a*b^2*(3*A + 4*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*Ell
ipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((-3*a^3*b*B + 9*a*b^3*B
 + 15*a^4*C + b^4*(-5*A + 8*C) - a^2*b^2*(A + 29*C))*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1
])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*b^
2*d)
```

Maple [B] time = 5.379, size = 2000, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)-4/b^3*(A*b^2-3*B*a*b+6*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a
^2*(A*b^2-B*a*b+C*a^2)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b
)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+
b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^
2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^
2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a/b^4*(2*A*b^2-3*B*a*b+4*C*a^2)*(-1
/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^3, x)
```

3.1112 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=418

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(3A-5C)+a^3bB+3a^4C-7ab^3B+b^4(3A+8C)\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(5A+9C)-a^3bB-3a\right.}{4ab^2d\left(a^2-b^2\right)^2}$$

```
[Out] ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*EllipticE[(c
 + d*x)/2, 2])/(4*a*b^2*(a^2 - b^2)^2*d) + ((a^3*b*B - 7*a*b^3*B + a^2*b^2*
(3*A - 5*C) + 3*a^4*C + b^4*(3*A + 8*C))*EllipticF[(c + d*x)/2, 2])/(4*b^3*
(a^2 - b^2)^2*d) + ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^2
*(A - 2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(4*a*(a - b)^2*b^3*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) -
 ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.33308, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3047, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(3A-5C)+a^3bB+3a^4C-7ab^3B+b^4(3A+8C)\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(5A+9C)-a^3bB-3a\right.}{4ab^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^3,x]
```


```
[Out] ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*EllipticE[(c
 + d*x)/2, 2])/(4*a*b^2*(a^2 - b^2)^2*d) + ((a^3*b*B - 7*a*b^3*B + a^2*b^2*
(3*A - 5*C) + 3*a^4*C + b^4*(3*A + 8*C))*EllipticF[(c + d*x)/2, 2])/(4*b^3*
(a^2 - b^2)^2*d) + ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^2
*(A - 2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/(4*a*(a - b)^2*b^3*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) -
 ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{1}{2}\left(Ab^2-a(\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(Ab^4-a^3\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(Ab^4-a^3\ldots\right.}{\ldots}$$

$$= \frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)E\left(\frac{1}{2}(c+dx\ldots\right.}{4ab^2\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)E\left(\frac{1}{2}(c+dx\ldots\right.}{4ab^2\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 4.41299, size = 429, normalized size = 1.03

$$\frac{\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\cos(c+dx)\left(-a^2b^2(5A+9C)+a^3bB+3a^4C+5ab^3B-Ab^4\right)+a\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{\frac{\left(a^2b^2(9A+5C)-5a^3bB+a^4C-ab^3\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*(A*b^4 + 3*a^3*b*B + 3*a*b^3*B + a^4*C - 7*a^2*b^
2*(A + C)) + b*(-(A*b^4) + a^3*b*B + 5*a*b^3*B + 3*a^4*C - a^2*b^2*(5*A + 9
*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - (
((-3*A*b^4 - 5*a^3*b*B - a*b^3*B + a^4*C + a^2*b^2*(9*A + 5*C))*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*a*(-3*a*b*B + a^2*(2*A + C) +
b^2*(A + 2*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]))/(a + b) + ((-(A*b^4) + a^3*b*B + 5*a*b^3*B + 3*a^4*C
 - a^2*b^2*(5*A + 9*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[
Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a*b*d)
```

Maple [B] time = 4.97, size = 1950, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(B*b-3*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b^2-B*a*b+C*a^2)/b^3*(
-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
```

2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(
a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*
b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
5/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2)))+2/b^3*(A*b^2-2*B*a*b+3*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2
c)(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*
a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^3, x)
```

3.1113 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=413

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(9A+5C)+5a^3bB+a^4(-C\right.}{4a^2bd\left(a^2-b^2\right)^2}$$

```
[Out] ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*EllipticE[(c
 + d*x)/2, 2])/(4*a^2*b*(a^2 - b^2)^2*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B
+ a^4*C - 7*a^2*b^2*(A + C))*EllipticF[(c + d*x)/2, 2])/(4*a*b^2*(a^2 - b^2
)^2*d) + ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2*b^4*(2*A -
C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b
^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.2755, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(9A+5C)+5a^3bB+a^4(-C\right.}{4a^2bd\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[
c + d*x])^3),x]
```


```
[Out] ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*EllipticE[(c
 + d*x)/2, 2])/(4*a^2*b*(a^2 - b^2)^2*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B
+ a^4*C - 7*a^2*b^2*(A + C))*EllipticF[(c + d*x)/2, 2])/(4*a*b^2*(a^2 - b^2
)^2*d) + ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2*b^4*(2*A -
C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*A*b
^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/((((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-3Ab^2-abB+a^2(4A+C)\right)-2a}{\sqrt{c}}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3Ab^4+5a^3bB+ab^3B-\right.}{4a^2\left(a^2\right.}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3Ab^4+5a^3bB+ab^3B-\right.}{4a^2\left(a^2\right.}$$

$$= \frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2b\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2-\right.}{2}$$

$$= \frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2b\left(a^2-b^2\right)^2d} + \frac{\left(Ab^4+\right.}{}$$

Mathematica [A] time = 6.12814, size = 443, normalized size = 1.07

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\cos(c+dx)\left(a^2b^2(9A+5C)-5a^3bB+a^4C-ab^3B-3Ab^4\right)+a\left(a^2b^2(11A+3C)-7a^3bB+3a^4C+ab^3B-5Ab^4\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{2\left(a^2b^2(C-19A)+a^4(16A+5C)-9\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^3),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(a*(-5*A*b^4 - 7*a^3*b*B + a*b^3*B + 3*a^4*C + a^2*b
^2*(11*A + 3*C)) + b*(-3*A*b^4 - 5*a^3*b*B - a*b^3*B + a^4*C + a^2*b^2*(9*A
 + 5*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2)
```

```
 + ((2*(9*A*b^4 - 9*a^3*b*B + 3*a*b^3*B + a^2*b^2*(-19*A + C) + a^4*(16*A +
 5*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*a*(A*b^3 +
2*a^3*B + a*b^2*B - a^2*b*(4*A + 3*C))*((a + b)*EllipticF[(c + d*x)/2, 2] -
 a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) - (2*(-3*A*b^4 -
 5*a^3*b*B - a*b^3*B + a^4*C + a^2*b^2*(9*A + 5*C))*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a^
2*d)
```

Maple [B] time = 4.942, size = 1857, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b^2-B*a*b+C*a^2)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b
)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9
/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a
```

```
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(B*b-2*C*a)/b
^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*s
qrt(cos(d*x + c))), x)
```

3.1114 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=502

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(11A+3C)+7a^3bB-3a^4C-ab^3B+5Ab^4\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(29A+C)+a^4(8A-5\right.}{4a^3d\left(a^2-b^2\right)^2}$$

```
[Out] -((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))
*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((5*A*b^4 + 7*a^3*b*B
 - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A + 3*C))*EllipticF[(c + d*x)/2, 2])/(4*
a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 - 15*a^5*b*B + 6*a^3*b^3*B - 3*a*b^5*B
+ 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*b*(a + b)^3*d) + ((15*A*b^4 + 9*a^
3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*Sin[c + d*x])/(4*
a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c +
d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) - ((5*A
*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sin[c + d*x])/
(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.85821, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(11A+3C)+7a^3bB-3a^4C-ab^3B+5Ab^4\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(29A+C)+a^4(8A-5\right.}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[
c + d*x])^3),x]
```


```
[Out] -((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))
*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((5*A*b^4 + 7*a^3*b*B
 - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A + 3*C))*EllipticF[(c + d*x)/2, 2])/(4*
a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 - 15*a^5*b*B + 6*a^3*b^3*B - 3*a*b^5*B
+ 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*b*(a + b)^3*d) + ((15*A*b^4 + 9*a^
3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*Sin[c + d*x])/(4*
```

```
a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c +
d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) - ((5*A
*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sin[c + d*x])/
(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}+\frac{\int\frac{\frac{1}{2}\left(-5Ab^2+abB+a^2(4A-C)\right)-2}{}}{}$$

$$=\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}-\frac{\left(5Ab^4+7a^3bB-ab^3B-\right.}{4a^2\left(a^2-b^2\right)^2d\sqrt{}}$$

$$=\frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}}+$$

$$=\frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}}+$$

$$=-\frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)E\left(\frac{1}{2}(c+dx)\right|}{4a^3\left(a^2-b^2\right)^2d}$$

$$=-\frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)E\left(\frac{1}{2}(c+dx)\right|}{4a^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.03742, size = 510, normalized size = 1.02

$$\frac{2\sqrt{\cos(c+dx)}\left(b^2\sin(2(c+dx))\left(-a^2b^2(29A+C)+a^4(8A-5C)+9a^3bB-3ab^3B+15Ab^4\right)+2ab\sin(c+dx)\left(a^2b^2(C-47A)+a^4(16A-7C)+11a^3bB-5ab^3B+25Ab^4\right)+16A\left(a\right.\right.}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Cos[c + d*x])^3),x]
```

```
[Out] (-(((-2*(-45*A*b^5 + 16*a^5*B - 19*a^3*b^2*B + 9*a*b^4*B + a^2*b^3*(95*A +
3*C) - a^4*b*(56*A + 9*C))*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
b) + (16*a*(5*A*b^4 + 4*a^3*b*B - a*b^3*B + 2*a^4*(A - C) - a^2*b^2*(10*A +
 C))*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2]))/(b*(a + b)) + (2*(15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A
- 5*C) - a^2*b^2*(29*A + C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]],
-1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)
*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sq
rt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + (2*Sqrt[Cos[c + d*x]]*(2*a*b*
(25*A*b^4 + 11*a^3*b*B - 5*a*b^3*B + a^4*(16*A - 7*C) + a^2*b^2*(-47*A + C)
)*Sin[c + d*x] + b^2*(15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) -
a^2*b^2*(29*A + C))*Sin[2*(c + d*x)] + 16*A*(a^3 - a*b^2)^2*Tan[c + d*x]))/
((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*a^3*d)
```

Maple [B] time = 5.839, size = 2027, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
```

/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b^2+B*a*b-C*a^2)/a/b*(-1/2/a*b^2/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2
-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*
(-A*b^2+C*a^2)/a^2/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a
+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b
+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2
c)^2)^(1/2)(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*c
os(d*x + c)^(3/2)), x)
```

3.1115 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=609

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(61A-3C)+a^4(8A-21C)+33a^3bB-15ab^3B+35Ab^4\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^3(65A-3C)\right.}{}$$

```
[Out] ((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a^
2*b^3*(65*A - 3*C))*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((
35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C
))*EllipticF[(c + d*x)/2, 2])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A*b^6 - 35*a^
5*b*B + 38*a^3*b^3*B - 15*a*b^5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A
- 2*C) + 15*a^6*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b
)^2*(a + b)^3*d) + ((35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C)
- a^2*b^2*(61*A - 3*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^
(3/2)) - ((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A -
3*C) - a^2*b^3*(65*A - 3*C))*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[
c + d*x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[
c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^3*B
- 5*a^4*C - a^2*b^2*(13*A + C))*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Cos[c
+ d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.49354, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(61A-3C)+a^4(8A-21C)+33a^3bB-15ab^3B+35Ab^4\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^3(65A-3C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[
c + d*x])^3),x]
```


```
[Out] ((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a^
2*b^3*(65*A - 3*C))*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((
35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C
))*EllipticF[(c + d*x)/2, 2])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A*b^6 - 35*a^
```

```
5*b*B + 38*a^3*b^3*B - 15*a*b^5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A
- 2*C) + 15*a^6*C)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b
)^2*(a + b)^3*d) + ((35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C)
- a^2*b^2*(61*A - 3*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^
(3/2)) - ((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A -
3*C) - a^2*b^3*(65*A - 3*C))*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[
c + d*x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[
c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^3*B
- 5*a^4*C - a^2*b^2*(13*A + C))*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Cos[c
+ d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-7Ab^2+3abB+a^2(4A-3C)\right)}{\cdots}}{\cdots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} - \frac{\left(7Ab^4+9a^3bB-3ab^3B \cdots\right.}{4a^2\left(a^2-b^2\right)^2 d\,\mathrm{co}\cdots}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx\cdots}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx\cdots}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx\cdots}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(35Ab^5-8a^5B+29a^3b^2B-15ab^4B+3a^4b(8A-3C)-a^2b^3(65A-3C)\right)\cdots}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(35Ab^5-8a^5B+29a^3b^2B-15ab^4B+3a^4b(8A-3C)-a^2b^3(65A-3C)\right)\cdots}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.70384, size = 676, normalized size = 1.11

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{a^2b^2C\sin(c+dx)-ab^3B\sin(c+dx)+Ab^4\sin(c+dx)}{2a^3\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{17a^2Ab^4\sin(c+dx)-13a^3b^3B\sin(c+dx)-3a^2b^4C\sin(c+dx)+9a^4b^2C\sin(c+dx)+7ab^5B\cdots}{4a^4\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\cdots\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Cos[c + d*x])^3),x]
```

```
[Out] ((2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 - 168*a^5*b*B + 2
85*a^3*b^3*B - 135*a*b^5*B + 48*a^6*C - 57*a^4*b^2*C + 27*a^2*b^4*C)*Ellipt
```

```
icPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + ((160*a^5*A*b - 512*a^3*A*b^
3 + 280*a*A*b^5 - 48*a^6*B + 240*a^4*b^2*B - 120*a^2*b^4*B - 96*a^5*b*C + 2
4*a^3*b^3*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/(a + b)))/b + (2*(72*a^4*A*b^2 - 195*a^2*A*b^4 + 105*A*b^6
 - 24*a^5*b*B + 87*a^3*b^3*B - 45*a*b^5*B - 27*a^4*b^2*C + 9*a^2*b^4*C)*Cos
[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b
/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[1 - Cos[c
+ d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[
Cos[c + d*x]]*((2*Sec[c + d*x]*(-3*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/a^
4 + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x])/(2
*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (17*a^2*A*b^4*Sin[c + d*x] - 11*
A*b^6*Sin[c + d*x] - 13*a^3*b^3*B*Sin[c + d*x] + 7*a*b^5*B*Sin[c + d*x] + 9
*a^4*b^2*C*Sin[c + d*x] - 3*a^2*b^4*C*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(a
 + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 10.188, size = 2165, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(3*A*b-B
*a)/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-3*A*b+B*a)/a^4*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(A*b^2-B*a*b+C*a
^2)/a^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-
b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)
^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2)))+2*A/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(2
*A*b-B*a)/a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)
*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*c
os(d*x + c)^(5/2)), x)
```

3.1116 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=586

$$\frac{\sqrt{a+b}\cot(c+dx)\left(2a^2bB+a^3(-C)-4ab^2(2A+C)-8b^3B\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{8b^3d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(8*b^2*(3*A + 2*C) + 3*a*(2*b*B - a*C))*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(24*A*b^2 + (a + 2*b)*(6*
b*B - 3*a*C + 8*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt
[a + b]*(2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(8*b^3*d) + ((8*b^2*(3*A + 2*C) + 3*a*(2*b*B
- a*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]
) + ((2*b*B - a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(3*b*d)
```

Rubi [A] time = 1.79468, antiderivative size = 586, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\cot(c+dx)\left(2a^2bB+a^3(-C)-4ab^2(2A+C)-8b^3B\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(8*b^2*(3*A + 2*C) + 3*a*(2*b*B - a*C))*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(24*A*b^2 + (a + 2*b)*(6*
b*B - 3*a*C + 8*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
```

```
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt
[a + b]*(2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(8*b^3*d) + ((8*b^2*(3*A + 2*C) + 3*a*(2*b*B
- a*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]
) + ((2*b*B - a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(3*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin}{3bd}$$

$$= \frac{(2bB-aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}{4bd}$$

$$= \frac{\left(8b^2(3A+2C)+3a(2bB-aC)\right)\sqrt{a+b}}{24b^2d\sqrt{\cos(c+dx}}$$

$$= \frac{\left(8b^2(3A+2C)+3a(2bB-aC)\right)\sqrt{a+b}}{24b^2d\sqrt{\cos(c+dx}}$$

$$= \frac{\sqrt{a+b}\left(2a^2bB-8b^3B-a^3C-4ab^2(2A\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(8b^2(3A+2C)+3a(2bB\right.}{}$$

Mathematica [C] time = 6.41478, size = 1242, normalized size = 2.12

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] +
 C*Cos[c + d*x]^2),x]
```

```
[Out] ((-4*a*(24*A*b^2 + 18*a*b*B - a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - 4*a*(48*a*A*b + 24*b^2*B + 28*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
```

```
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(24*A*b^2 + 6*a*b*B - 3*a^2*C + 16*b^2*C)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
48*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((6*b*B + a*C)*Sin[
c + d*x])/(12*b) + (C*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.286, size = 3765, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b
^2+10*C*cos(d*x+c)^4*a*b^2-C*cos(d*x+c)^3*a^2*b+3*C*cos(d*x+c)^2*a^2*b+6*C*
cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+6*B*cos(d*x+c
)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2-6*B*cos(d*x+c)*a^2*b-12*B*cos(d*x+c)*a*b^2
+18*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^3+8*C*cos(d*x+c)^5*b^3+6*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
```

d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*a^2*b-28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*a*b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*b^3+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b-28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+6*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*a^3-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-12*B*cos(d*x+c)^2*b^3+24*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*a^3-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c

```
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3-3*C*cos(d*x+c)^2*a^3-1
6*C*cos(d*x+c)^2*b^3+3*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+
c)^2*b^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2+48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2-12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b
^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
```

```
sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1117 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=483

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(8*A*b + a*C + 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b*d) - (Sqrt[a + b]*(8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Cot
[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*b*B + a*C)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.15338, antiderivative size = 483, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(8*A*b + a*C + 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b*d) - (Sqrt[a + b]*(8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Cot
```

```
[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*b*B + a*C)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
```

```
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int$$

$$= \frac{(4bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(}{}$$

$$= \frac{(4bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+4abB-a^2C+4b^2C\right)\cot(c+dx)\Pi}{}$$

$$= -\frac{(a-b)\sqrt{a+b}(4bB+aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{4aba}$$

Mathematica [C] time = 6.3451, size = 1183, normalized size = 2.45

$$-\frac{4a(8aA+4bB+3aC)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(8*a*A + 4*b*B + 3*a*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
```

```
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*A*b + 8*a*B +
 4*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
2*(4*b*B + a*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*A
rcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])
/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
+ d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos
[c + d*x]])))/(8*d)
```

Maple [B] time = 0.19, size = 2999, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-4*B*cos(d*x+c)^2*a*b-C*cos(d*x+c)^3*a*b-2*C
*cos(d*x+c)^2*a*b+3*C*cos(d*x+c)^4*a*b+4*B*cos(d*x+c)^3*a*b+C*cos(d*x+c)^3*
a^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
```

(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+4*B*cos(d*x+c)^4*b^2-16*A*c
os(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*b^2+32*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-4*C*cos(d*x+c)^2*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-
2*C*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+C*cos(d*x+c)^2*sin(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-
4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*b^2-2*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+C*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2-8*A*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+16*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+8*A*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a*b-4*B*cos(d*x+c)^3*b^2+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2-C*cos(d*x+c)^2*a^
2+2*C*cos(d*x+c)^5*b^2-2*C*cos(d*x+c)^3*b^2+16*A*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+2*C*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a*b+C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/

```
2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+
8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+16*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-8*B*
cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*a*b)/b/sin(d
*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/s
qrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
sqrt(cos(d*x + c)), x)
```

3.1118 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt{a+b}\cot(c+dx)(2Ab-a(2A-2B-C))\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{(2A-C)\sin}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (
Sqrt[a + b]*(2*A*b - a*(2*A - 2*B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(a*d) - (Sqrt[a + b]*(2*b*B + a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(b*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*A - C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rubi [A] time = 1.13531, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\cot(c+dx)(2Ab-a(2A-2B-C))\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{(2A-C)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (
Sqrt[a + b]*(2*A*b - a*(2*A - 2*B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
```

```
(a*d) - (Sqrt[a + b]*(2*b*B + a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(b*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*A - C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
```

```
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}(Ab+aB)}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2A-C)\sqrt{a+b}}{d\sqrt{c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2A-C)\sqrt{a+b}}{d\sqrt{c}}$$

$$= -\frac{\sqrt{a+b}(2bB+aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}(2A-C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{ad}$$

Mathematica [C] time = 20.8101, size = 1176, normalized size = 2.62

$$-\frac{4a(2aB+bC)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)F}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((-4*a
*(2*a*B + b*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
```

```
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-2*a*A + 2*b*B + 2*a*C)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*A*b
 + b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[S
in[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/
(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x
]])))/(2*d)
```

Maple [B] time = 0.337, size = 2507, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/d/(a+b*cos(d*x+c))^(1/2)*(-C*cos(d*x+c)^3*b+4*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-C*cos(d*x+c)^2*a
+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
```

(a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b+2*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-2*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+
C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)^2*b-2*A*cos(d*x+c)*a+C*cos(d*x+c)^4*b+2
*A*cos(d*x+c)^3*b+C*cos(d*x+c)^3*a+2*A*cos(d*x+c)^2*a+2*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*B*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*b+2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d

```
*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+8*B*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*b+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b)/cos(d*x+c)^(3/2)/sin(d*x+c
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/
cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(3/2), x)
```

3.1119 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=407

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2\sqrt{a+b}\cot(c+a}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) - (2*Sqrt[a + b]*(b*(A - 3*B) - a*(A - 3*B + 3*C))*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(3*a*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/d + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2))
```

Rubi [A] time = 0.842616, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2\sqrt{a+b}\cot(c+a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) - (2*Sqrt[a + b]*(b*(A - 3*B) - a*(A - 3*B + 3*C))*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(3*a*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b
```

```
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/d + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)}{\cdots}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)}{\cos^{\frac{3}{2}}\cdots}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\cdots\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}(Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos\cdots}}{\sqrt{a+b}\sqrt{\cdots}}\right.\right.}{3a^2d}$$

Mathematica [C] time = 6.38877, size = 1240, normalized size = 3.05

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((-4*a*(a^2*A - A*b^2 + 3*a^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-(a*A*b
) - 3*a^2*B + 3*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(-(A*b^2) - 3*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(
3*a) + (2*A*Sec[c + d*x]*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.385, size = 2585, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(1/2)*(-2*A*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b+A*cos
(d*x+c)^2*a*b-3*B*cos(d*x+c)^2*a*b+3*B*cos(d*x+c)^3*a*b-A*a^2+3*B*cos(d*x+c
)^2*a^2+A*cos(d*x+c)^2*a^2-A*cos(d*x+c)^2*b^2+A*cos(d*x+c)^3*b^2-3*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)*a^2-3*B*cos(d*x+c)*a^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*C*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*a*b-3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+12*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-6*C*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*a*b+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*C*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+6*C
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
```

```
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2-3*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*a^2+6*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*a*b-3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2-A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2)/a/si
n(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/c
os(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(5/2), x)
```

3.1120 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=360

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{15a^3d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(2*A*b + a*(9*A
 - 5*B + 15*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(A*b + 5*a*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.931893, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3047, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(7/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(2*A*b + a*(9*A
 - 5*B + 15*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(A*b + 5*a*B
```

```
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5a\ldots}{\ldots}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a\ldots}}{15a\ldots}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a\ldots}}{15a\ldots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-5abB-3a^2(3A+5C)\right)\cot(\ldots}{\ldots}$$

Mathematica [C] time = 6.50084, size = 1340, normalized size = 3.72

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(7/2),x]
```

```
[Out] -((-4*a*(2*a^2*A*b - 2*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(9*a^3*A - 2*a*A*b^2 + 5*a^2*b*B + 15*a^3*C)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b - 2*A*b^3 + 5*
a*b^2*B + 15*a^2*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c +
 d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*
Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sq
rt[Cos[c + d*x]])))/(15*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]*((2*Sec[c + d*x]^2*(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a) + (2*
Sec[c + d*x]*(9*a^2*A*Sin[c + d*x] - 2*A*b^2*Sin[c + d*x] + 5*a*b*B*Sin[c +
 d*x] + 15*a^2*C*Sin[c + d*x]))/(15*a^2) + (2*A*Sec[c + d*x]^2*Tan[c + d*x]
)/5))/d
```

Maple [B] time = 0.2, size = 3333, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)
,x)
```

[Out] -2/15/d*(-3*A*a^3+A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos(d*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b+A*cos(d*x+c)^4*a*b^2-2*A*cos(d*x+c)^3*a*b^2-4*A*cos(d*x+c)*a^2*b-10*B*cos(d*x+c)^2*a^2*b-5*B*cos(d*x+c)^3*a*b^2-5*A*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b+5*B*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)*a^3+5*B*cos(d*x+c)^3*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-5*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-5*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-2*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15*C*cos(d*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3+2*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)

```
^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin
(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2+15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*A*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+5*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*a^3)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/c
os(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(7/2), x)
```

3.1121 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=447

$$-\frac{2\sin(c+dx)\left(-5a^2(5A+7C)-7abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(a^2(25A-63B+35C\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*
C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]
*(8*A*b^2 + 2*a*b*(3*A - 7*B) + a^2*(25*A - 63*B + 35*C))*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(105*a^3*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(7*d*Cos[c + d*x]^(7/2)) + (2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A +
 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2)
)
```

Rubi [A] time = 1.33156, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3047, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\left(-5a^2(5A+7C)-7abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(a^2(25A-63B+35C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(9/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*
C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]
*(8*A*b^2 + 2*a*b*(3*A - 7*B) + a^2*(25*A - 63*B + 35*C))*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
```

```
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(105*a^3*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(7*d*Cos[c + d*x]^(7/2)) + (2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A +
 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2)
)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7a$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35a}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^3+63a^3B-14ab^2B+a^2b(19A-\right.}{}$$

Mathematica [C] time = 6.65735, size = 1464, normalized size = 3.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 17*a^2*A*b^2 - 8*A*b^4 - 14*a^3*b*B + 14*a*b^3*B + 35*a^
4*C - 35*a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*A*b - 8*a*A*
b^3 - 63*a^4*B + 14*a^2*b^2*B - 35*a^3*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-19*a^2*A*b^2 - 8*A*b^4 - 63*a^3*b*B
 + 14*a*b^3*B - 35*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]
*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
+ d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a
*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*((2*Sec[c + d*x]^3*(A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/(35
*a) + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] - 4*A*b^2*Sin[c + d*x] + 7*a
*b*B*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a^2) + (2*Sec[c + d*x]*(19
*a^2*A*b*Sin[c + d*x] + 8*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] - 14*a
*b^2*B*Sin[c + d*x] + 35*a^2*b*C*Sin[c + d*x]))/(105*a^3) + (2*A*Sec[c + d*
```

```
x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.316, size = 4337, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
,x)
```

```
[Out] 2/105/d*(15*A*a^4+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-2*A*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+35*
C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b+19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b-2*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+42*B*cos(d*x+c)^3*a^4+21*B*cos(d*x+c)*a^4-
63*B*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
```

(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*cos(d*x+c)^5*a^3*b-7*B*cos(d*x+c)^5*a^2*b^2+14*B*cos(d*x+c)^5*a*b^3+35*B*cos(d*x+c)^4*a^3*b+14*B*cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+35*C*cos(d*x+c)^4*a^2*b^2+70*C*cos(d*x+c)^3*a^3*b-35*C*cos(d*x+c)^5*a^3*b-35*C*cos(d*x+c)^5*a^2*b^2-35*C*cos(d*x+c)^4*a^3*b-7*B*cos(d*x+c)^3*a^2*b^2+28*B*cos(d*x+c)^2*a^3*b+4*A*cos(d*x+c)^3*a*b^3-A*cos(d*x+c)^2*a^2*b^2+18*A*cos(d*x+c)*a^3*b-25*A*cos(d*x+c)^5*a^3*b-19*A*cos(d*x+c)^5*a^2*b^2+4*A*cos(d*x+c)^5*a*b^3-19*A*cos(d*x+c)^4*a^3*b+20*A*cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^4*a*b^3+26*A*cos(d*x+c)^3*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-25*A*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+35*C*cos(d*x+c)^2*a^4-8*A*cos(d*x+c)^5*b^4+8*A*cos(d*x+c)^4*b^4+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-14*B*cos(d*x+c)^4*a*b^3+63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d*x+c

```
)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*a^4-14*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-49*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+14*B*sin(d*x+
c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*a^2*b^2+63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^3*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-14*B*s
in(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a*b^3-49*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+14*B*sin(d*x+c)*cos(d*x+c)^3*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+6
3*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a^3*b)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^
(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/c
os(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/cos(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
cos(d*x + c)^(9/2), x)
```

3.1122 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+\right.$

Optimal. Leaf size=704

$$\frac{\sin(c+dx)\left(24a^2bB-9a^3C+12ab^2(20A+13C)+128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192b^2d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\cot(c+dx)\left(-6a^2b(4B+C)\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A +
13*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a
^3*C - 6*a^2*b*(4*B + C) - 8*b^3*(12*A + 16*B + 9*C) - 4*a*b^2*(60*A + 28*B
 + 39*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*
a^3*b*B - 96*a*b^3*B - 3*a^4*C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))
*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3*d) + ((24*a^2*b*B +
 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((4*b^2*(4*A + 3*C) + a*(8*b*B
- 3*a*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d
) + ((8*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c +
d*x])/(24*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d
*x])/(4*b*d)
```

Rubi [A] time = 2.48526, antiderivative size = 704, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\left(24a^2bB-9a^3C+12ab^2(20A+13C)+128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192b^2d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\cot(c+dx)\left(-6a^2b(4B+C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A +
13*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
```

Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a^3*C - 6*a^2*b*(4*B + C) - 8*b^3*(12*A + 16*B + 9*C) - 4*a*b^2*(60*A + 28*B + 39*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*b*B - 96*a*b^3*B - 3*a^4*C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3*d) + ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((4*b^2*(4*A + 3*C) + a*(8*b*B - 3*a*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d) + ((8*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d)

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin}{4bd}$$

$$= \frac{(8bB-3aC)\sqrt{\cos(c+dx)}(a+b\cos(c+}{24bd}$$

$$= \frac{\left(4b^2(4A+3C)+a(8bB-3aC)\right)\sqrt{\cos(}}{32b}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+12ab^2(20A\right.}{192b^2d\sqrt{c}}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+12ab^2(20A\right.}{192b^2d\sqrt{c}}$$

$$= \frac{\sqrt{a+b}\left(8a^3bB-96ab^3B-3a^4C-24a^2\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24a^2bB+128b^3B-9a\right.}{}$$

Mathematica [C] time = 6.55809, size = 1317, normalized size = 1.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
 + C*Cos[c + d*x]^2),x]
```

```
[Out] -((-4*a*(-336*a*A*b^2 - 136*a^2*b*B - 128*b^3*B + 3*a^3*C - 228*a*b^2*C)*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-384*a^2*A*b - 192*A*b^3 - 416*a*b^2*B - 2
28*a^2*b*C - 144*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
```

```
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])) + 2*(-240*a*A*b^2 - 24*a^2*b*B - 128*b^3*B + 9*a^3*C - 156*a*
b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Si
n[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(
a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]
])))/(384*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((48*A*b^2 +
 56*a*b*B + 3*a^2*C + 42*b^2*C)*Sin[c + d*x])/(96*b) + ((8*b*B + 9*a*C)*Sin
[2*(c + d*x)])/48 + (b*C*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.479, size = 5493, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1123 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=587

$$\frac{\sin(c+dx)\left(3a^2C+30abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\cot(c+dx)\left(3a^2C+2ab(24A+15B+7C)+\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(3*a^2*C + 4*b^2*(6*A +
3*B + 4*C) + 2*a*b*(24*A + 15*B + 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(24*b*d) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*
Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d) + ((24*A*b^2 + 30*
a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*b*d*
Sqrt[Cos[c + d*x]]) + ((2*b*B + a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.78689, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\left(3a^2C+30abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\cot(c+dx)\left(3a^2C+2ab(24A+15B+7C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(3*a^2*C + 4*b^2*(6*A +
```

```
3*B + 4*C) + 2*a*b*(24*A + 15*B + 7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(24*b*d) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*
Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d) + ((24*A*b^2 + 30*
a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*b*d*
Sqrt[Cos[c + d*x]]) + ((2*b*B + a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(3*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} +$$

$$= \frac{(2bB+aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+}{4d}$$

$$= \frac{\left(24Ab^2+30abB+3a^2C+16b^2C\right)\sqrt{a+b\cos(c+}}{24bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(24Ab^2+30abB+3a^2C+16b^2C\right)\sqrt{a+b\cos(c+}}{24bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(6a^2bB+8b^3B-a^3C+12ab^2(2A+C)\right)\mathrm{c}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24Ab^2+30abB+3a^2C+16b^2C\right)}{}$$

Mathematica [C] time = 6.57981, size = 1250, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((-4*a*(48*a^2*A + 24*A*b^2 + 42*a*b*B + 17*a^2*C + 16*b^2*C)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(96*a*A*b + 48*a^2*B + 24*b^2*B + 52*a*b*C)*((Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
```

```
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(24*A*b^2 + 30*a
*b*B + 3*a^2*C + 16*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*El
lipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Se
c[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d
*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(b*Sqrt[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]*(((6*b*B + 7*a*C)*Sin[c + d*x])/12 + (b*C*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.461, size = 4145, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b
^2+22*C*cos(d*x+c)^4*a*b^2+17*C*cos(d*x+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-6
*C*cos(d*x+c)^2*a*b^2-14*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+30*B*cos(
d*x+c)^2*a^2*b-30*B*cos(d*x+c)^2*a*b^2-30*B*cos(d*x+c)*a^2*b-12*B*cos(d*x+c
)*a*b^2+42*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^3+48*A*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*C*cos(
d*x+c)^5*b^3+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+72*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos
```

(d*x+c)*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-12*B*cos(d*x+c)^2*b^3-48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si

```
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+24*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3+3*C*cos(d*x+c)^2*a^3-16*
C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+c)
^2*b^3+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2-48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b+48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b+30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2+36*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3
)/b/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/sqrt(cos(d*x + c)), x)
```

3.1124 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=535

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(3a^2C+12abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{4bd}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*a*A - 4*b*B - 5*a*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(4*a*d) - (Sqrt[a + b]*(a*(8*A - 8*B - 5*C) - 2*b*(8*A + 2*B + C))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(8*A*b^2 + 12*a*b*B +
3*a^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) -
 ((8*a*A - 4*b*B - 5*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[
Cos[c + d*x]]) - (b*(4*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(2*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 1.79305, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(3a^2C+12abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a*A - 4*b*B - 5*a*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(4*a*d) - (Sqrt[a + b]*(a*(8*A - 8*B - 5*C) - 2*b*(8*A + 2*B + C))*Cot
```

```
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(8*A*b^2 + 12*a*b*B +
3*a^2*C + 4*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) -
 ((8*a*A - 4*b*B - 5*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[
Cos[c + d*x]]) - (b*(4*A - C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(2*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\sqrt{a+b\,\mathrm{c}}}{}$$

$$= -\frac{b(4A-C)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+}{2d}$$

$$= -\frac{(8aA-4bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$= -\frac{(8aA-4bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+12abB+3a^2C+4b^2C\right)\cot(c+dx}{}$$

$$= \frac{(a-b)\sqrt{a+b}(8aA-4bB-5aC)\cot(c+dx)E\left(\sin^{-}\right.}{4}$$

Mathematica [C] time = 6.55536, size = 1232, normalized size = 2.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((4*a*(-8*a*A*b - 8*a^2*B - 4*b^2*B - 7*a*b*C)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
```

```
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
+ 4*a*(8*a^2*A - 8*A*b^2 - 16*a*b*B - 8*a^2*C - 4*b^2*C)*((Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(8*a*A*b - 4*b^2*B - 5*a
*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(8*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b*C*Sin[c + d*x]
)/2 + 2*a*A*Tan[c + d*x]))/d
```

Maple [B] time = 0.328, size = 3598, normalized size = 6.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/4/d*(-8*A*cos(d*x+c)*a*b+8*A*cos(d*x+c)^2*a*b-2*C*cos(d*x+c)*a*b+4*B*cos
(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b+7*C*cos(d*x+c)^3*a*b-5*C*cos(d*x+c)^2*a*b+
4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2-8*A*a^2-2*b^2*C*cos(d*x+c)^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
```

)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+16*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-
8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*C*cos
(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+5*C*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-8*C*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^3*b^2-4*B*cos(d*x+c)^2*b^2+16*A*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co

```
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+5*C*cos(d*x+c)^2*a^2+2
*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a*b+5*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(
d*x+c)^4*b^2-5*C*cos(d*x+c)*a^2-16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*cos(d*x+c)*a^2+16
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+24*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*a*b-16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2+8*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)/
(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(3/2), x)
```

3.1125 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=528

$$\frac{\sqrt{a+b}\cot(c+dx)\left(2a^2(A-3B+3C)-ab(8A-3(4B+C))+6Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{3ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*A*b + 6*a*B - 3*b*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a*d) + (Sqrt[a + b]*(6*A*b^2 + 2*a^2*(A - 3*B + 3*C) - a*b*(8*A - 3
*(4*B + C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*b
*B + 3*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*(A*b + a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((8*A*b
+ 6*a*B - 3*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]]) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/
2))
```

Rubi [A] time = 1.66688, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\cot(c+dx)\left(2a^2(A-3B+3C)-ab(8A-3(4B+C))+6Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*A*b + 6*a*B - 3*b*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a*d) + (Sqrt[a + b]*(6*A*b^2 + 2*a^2*(A - 3*B + 3*C) - a*b*(8*A - 3
```

```
*(4*B + C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*b
*B + 3*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*(A*b + a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((8*A*b
+ 6*a*B - 3*b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]]) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/
2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+b}}{}$$

$$= \frac{2(Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2A(a}{}$$

$$= \frac{2(Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(8Ab}{}$$

$$= \frac{2(Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(8Ab}{}$$

$$= -\frac{\sqrt{a+b}(2bB+3aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a-}}\right.\right.}{a}$$

$$= \frac{(a-b)\sqrt{a+b}(8Ab+6aB-3bC)\cot(c+dx)E\left(\sin\right.}{}$$

Mathematica [C] time = 6.54768, size = 1260, normalized size = 2.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((-4*a*(2*a^2*A - 2*A*b^2 + 6*a*b*B + 6*a^2*C + 3*b^2*C)*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) - 4*a*(-8*a*A*b - 6*a^2*B + 6*b^2*B + 12*a*b*C)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
```

```
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-8*A*b^2 - 6*a*b*B + 3
*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[S
in[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/
(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x
]])))/(6*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]
*(4*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/3 + (2*a*A*Sec[c + d*x]*Tan[c +
 d*x])/3))/d
```

Maple [B] time = 0.2, size = 3352, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] 1/3/d/(a+b*cos(d*x+c))^(1/2)*(10*A*cos(d*x+c)*a*b-2*A*cos(d*x+c)^3*a*b-8*A*
cos(d*x+c)^2*a*b+6*B*cos(d*x+c)^2*a*b-3*C*cos(d*x+c)^3*a*b+3*C*cos(d*x+c)^2
*a*b-6*B*cos(d*x+c)^3*a*b+2*A*a^2-6*B*cos(d*x+c)^2*a^2-2*A*cos(d*x+c)^2*a^2
+8*A*cos(d*x+c)^2*b^2-8*A*cos(d*x+c)^3*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*a*b-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)^2*a^2-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
```

+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-6*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
2-6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2+6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-12*B*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+6*B*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^2-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+6*B*cos(d*x+c)*
a^2+3*C*cos(d*x+c)^3*b^2+12*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-3*C*cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+12*
C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a*b-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-3*C*cos(d
*x+c)^4*b^2-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-18*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-8*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+8*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-
12*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+

```
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-12*B*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*b^2-3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+8*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^2*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2+8*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*b^2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(5/2), x)
```

3.1126 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=490

$$-\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2(9A-5B+15C)-2ab(6A-10B+15C)+3b^2(A-5B)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{15ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(15*a^2*d) - (2*Sqrt[a + b]*(3*b^2*(A - 5*B) - 2*a*
b*(6*A - 10*B + 15*C) + a^2*(9*A - 5*B + 15*C))*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(15*a*d) - (2*b*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/d + (2*(3*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 1.29115, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3047, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2(9A-5B+15C)-2ab(6A-10B+15C)+3b^2(A-5B)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(15*a^2*d) - (2*Sqrt[a + b]*(3*b^2*(A - 5*B) - 2*a*
b*(6*A - 10*B + 15*C) + a^2*(9*A - 5*B + 15*C))*Cot[c + d*x]*EllipticF[ArcS
```

```
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(15*a*d) - (2*b*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/d + (2*(3*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
*Cos[c + d*x]^(5/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\sqrt{a+b}}{}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A}{}$$

$$= -\frac{2b\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3Ab^2+20abB+3a^2(3A+5C)\right)\text{co}}{}$$

Mathematica [C] time = 6.65322, size = 1353, normalized size = 2.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] -((-4*a*(-3*a^2*A*b + 3*A*b^3 - 5*a^3*B + 5*a*b^2*B - 15*a^2*b*C)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(9*a^3*A + 3*a*A*b^2 + 20*a^2*b*B + 15*a^3*C - 15*
a*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
```

```
 2*(9*a^2*A*b + 3*A*b^3 + 20*a*b^2*B + 15*a^2*b*C)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(15*a*d) + (Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(6*A*b*Sin[c + d*x] + 5*a*
B*Sin[c + d*x]))/15 + (2*Sec[c + d*x]*(9*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c
 + d*x] + 20*a*b*B*Sin[c + d*x] + 15*a^2*C*Sin[c + d*x]))/(15*a) + (2*a*A*S
ec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.215, size = 3922, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x)
```

```
[Out] -2/15/d*(-3*A*a^3-9*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos(d
*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+c)^4*a*b^2+3*A*cos(d*x+c)^
3*a*b^2-9*A*cos(d*x+c)*a^2*b-25*B*cos(d*x+c)^2*a^2*b-20*B*cos(d*x+c)^3*a*b^
2+20*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b+20*B*cos(d*x+c)^4*a*b^2-5*
B*cos(d*x+c)*a^3+5*B*cos(d*x+c)^3*a^3-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*sin(d*x
+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+30*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+30*C*
```

cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+15*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^
2-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^3*a^2*b-20*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-20*B*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*a^2*b+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b^2+3*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15*C
*cos(d*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3-3*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*co
s(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3
A(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*

```
x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2+30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+12*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*
A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+
12*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+5
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^2*a^3)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(7/2), x)
```

3.1127 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=450

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)+42abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(a^2(-(25A-63B+35\right.}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B - 2*a^2*b*(41*A +
70*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a +
 b]*(6*A*b^2 - a^2*(25*A - 63*B + 35*C) + 3*a*b*(19*A - 7*B + 35*C))*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(3*A*b + 7*a*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(3*A*b^2 + 42*a*b*
B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[
c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c
+ d*x]^(7/2))
```

Rubi [A] time = 1.43189, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3047, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)+42abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(a^2(-(25A-63B+35\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B - 2*a^2*b*(41*A +
70*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a +
 b]*(6*A*b^2 - a^2*(25*A - 63*B + 35*C) + 3*a*b*(19*A - 7*B + 35*C))*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
```

```
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(3*A*b + 7*a*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(3*A*b^2 + 42*a*b*
B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[
c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c
+ d*x]^(7/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int\frac{\sqrt{a+b}}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2A}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(3\right.}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(3\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(6Ab^3-63a^3B-21ab^2B-2a^2b(4\right.}{}$$

Mathematica [C] time = 6.73109, size = 1463, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 31*a^2*A*b^2 + 6*A*b^4 + 21*a^3*b*B - 21*a*b^3*B + 35*a^
4*C - 35*a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*A*b + 6*a*A*
b^3 - 63*a^4*B - 21*a^2*b^2*B - 140*a^3*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-82*a^2*A*b^2 + 6*A*b^4 - 63*a^3*b*
B - 21*a*b^3*B - 140*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]]))))/(105*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]^3*(8*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))
/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 42*
a*b*B*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a) + (2*Sec[c + d*x]*(82*
a^2*A*b*Sin[c + d*x] - 6*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 21*a*
b^2*B*Sin[c + d*x] + 140*a^2*b*C*Sin[c + d*x]))/(105*a^2) + (2*a*A*Sec[c +
```

```
d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.332, size = 4526, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x)
```

```
[Out] -2/105/d*(-15*A*a^4+6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*A
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-140*C*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^2+140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*cos(d*x+c)^3*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*b-82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3*b+51*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+
c)*a^4+63*B*cos(d*x+c)^4*a^4+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
```

(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x+c)^4*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4
+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^4+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x+c)^3*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*
A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^4+63*B*cos(d*x+c)^5*a^3*b+42*B*cos(d*x+c)^5*a^2*b^2+21*B
*cos(d*x+c)^5*a*b^3+21*B*cos(d*x+c)^4*a^2*b^2-6*A*cos(d*x+c)^3*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-140*
C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3*b-140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+140*C*cos(d*x+c)^3*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b-82*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b-140*C*cos(d*x+c)^4*a^2*b^2-175*C*cos(d*x+c)^3
*a^3*b+35*C*cos(d*x+c)^5*a^3*b+140*C*cos(d*x+c)^5*a^2*b^2+140*C*cos(d*x+c)^
4*a^3*b+105*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+105*C*sin(d*x+c)*cos(d*x+c)^4*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-63*B*cos(
d*x+c)^3*a^2*b^2-63*B*cos(d*x+c)^2*a^3*b+3*A*cos(d*x+c)^3*a*b^3-27*A*cos(d*
x+c)^2*a^2*b^2-39*A*cos(d*x+c)*a^3*b+25*A*cos(d*x+c)^5*a^3*b+82*A*cos(d*x+c
)^5*a^2*b^2+3*A*cos(d*x+c)^5*a*b^3+82*A*cos(d*x+c)^4*a^3*b-55*A*cos(d*x+c)^
4*a^2*b^2-6*A*cos(d*x+c)^4*a*b^3-68*A*cos(d*x+c)^3*a^3*b-21*B*sin(d*x+c)*co
s(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a^2*b^2+25*A*cos(d*x+c)^4*a^4+35*C*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4
-35*C*cos(d*x+c)^2*a^4-6*A*cos(d*x+c)^5*b^4+6*A*cos(d*x+c)^4*b^4-82*A*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b^2-21*B*cos(d*x+c)^4*a*b^3-63*B*sin(d*x+c)*cos(d*x+c)^4*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(

```
d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-21*B*sin(d*x
+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a*b^3+84*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^4*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-63*B*si
n(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^3*b-21*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-21*B*sin(d*x+c)*cos(d*x+c)^3
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+84
*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x
+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3
*b)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(9/2), x)
```

3.1128 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=550

$$-\frac{2\sin(c+dx)\left(-2a^2b(44A+63C)-75a^3B-9ab^2B+4Ab^3\right)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(7a^2(7A+9C)+72a\right.}{315ad\cos^{\frac{5}{2}}(}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A +
9*C) + 3*a^2*b^2*(11*A + 21*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4
*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 + 6*a*b^2*(A - 3*B) + 3*a^2*b*(13*A -
 57*B + 21*C) - 3*a^3*(49*A - 25*B + 63*C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(315*a^3*d) + (2*(A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
21*d*Cos[c + d*x]^(7/2)) + (2*(3*A*b^2 + 72*a*b*B + 7*a^2*(7*A + 9*C))*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b
^3 - 75*a^3*B - 9*a*b^2*B - 2*a^2*b*(44*A + 63*C))*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 2.01615, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3047, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\left(-2a^2b(44A+63C)-75a^3B-9ab^2B+4Ab^3\right)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(7a^2(7A+9C)+72a\right.}{315ad\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A +
9*C) + 3*a^2*b^2*(11*A + 21*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4
```

```
*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 + 6*a*b^2*(A - 3*B) + 3*a^2*b*(13*A -
 57*B + 21*C) - 3*a^3*(49*A - 25*B + 63*C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(315*a^3*d) + (2*(A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
21*d*Cos[c + d*x]^(7/2)) + (2*(3*A*b^2 + 72*a*b*B + 7*a^2*(7*A + 9*C))*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b
^3 - 75*a^3*B - 9*a*b^2*B - 2*a^2*b*(44*A + 63*C))*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2)) + (2*A*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\,c}}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(3A\right.}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(3A\right.}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(3A\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^4+246a^3bB-18ab^3B+21a^4(7\right.}{}$$

Mathematica [C] time = 6.95763, size = 1614, normalized size = 2.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] -((-4*a*(-39*a^4*A*b + 31*a^2*A*b^3 + 8*A*b^5 - 75*a^5*B + 93*a^3*b^2*B - 1
8*a*b^4*B - 63*a^4*b*C + 63*a^2*b^3*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(1
47*a^5*A + 33*a^3*A*b^2 + 8*a*A*b^4 + 246*a^4*b*B - 18*a^2*b^3*B + 189*a^5*
C + 63*a^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
```

```
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(147*a^4*A*b + 33*a^2*A*b^3 + 8*A*b^5 + 246*a^3*b^2*B - 18*a*b^
4*B + 189*a^4*b*C + 63*a^2*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(315*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^4*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x
]))/63 + (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] +
72*a*b*B*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]^2
*(88*a^2*A*b*Sin[c + d*x] - 4*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] +
9*a*b^2*B*Sin[c + d*x] + 126*a^2*b*C*Sin[c + d*x]))/(315*a^2) + (2*Sec[c +
d*x]*(147*a^4*A*Sin[c + d*x] + 33*a^2*A*b^2*Sin[c + d*x] + 8*A*b^4*Sin[c +
d*x] + 246*a^3*b*B*Sin[c + d*x] - 18*a*b^3*B*Sin[c + d*x] + 189*a^4*C*Sin[c
 + d*x] + 63*a^2*b^2*C*Sin[c + d*x]))/(315*a^3) + (2*a*A*Sec[c + d*x]^4*Tan
[c + d*x])/9))/d
```

Maple [B] time = 0.566, size = 5956, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/cos(d*x + c)^(11/2), x)
```

3.1129 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=834

$$\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd}+\frac{(10bB-3aC)\sqrt{\cos(c+dx)}\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd}+\frac{(-15C\ldots}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A
 + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1
920*a*b^2*d) - (Sqrt[a + b]*(45*a^4*C - 30*a^3*b*(5*B + C) - 16*b^4*(80*A +
 45*B + 64*C) - 8*a*b^3*(260*A + 355*B + 193*C) - 4*a^2*b^2*(660*A + 295*B
+ 423*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d) + (Sqrt[a + b]*(1
0*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5*C - 40*a^3*b^2*(2*A + C) - 80*
a*b^4*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(128*b^3*
d) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a^2
*b^2*(220*A + 141*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sq
rt[Cos[c + d*x]]) + ((50*a^2*b*B + 120*b^3*B - 15*a^3*C + 4*a*b^2*(60*A + 4
3*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) +
 ((80*A*b^2 + 50*a*b*B - 15*a^2*C + 64*b^2*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d) + ((10*b*B - 3*a*C)*Sqrt[Cos[c + d
*x]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d) + (C*Sqrt[Cos[c + d*
x]]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 3.79186, antiderivative size = 834, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd}+\frac{(10bB-3aC)\sqrt{\cos(c+dx)}\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd}+\frac{(-15C\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A
 + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1
920*a*b^2*d) - (Sqrt[a + b]*(45*a^4*C - 30*a^3*b*(5*B + C) - 16*b^4*(80*A +
 45*B + 64*C) - 8*a*b^3*(260*A + 355*B + 193*C) - 4*a^2*b^2*(660*A + 295*B
+ 423*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d) + (Sqrt[a + b]*(1
0*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5*C - 40*a^3*b^2*(2*A + C) - 80*
a*b^4*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(128*b^3*
d) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a^2
*b^2*(220*A + 141*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sq
rt[Cos[c + d*x]]) + ((50*a^2*b*B + 120*b^3*B - 15*a^3*C + 4*a*b^2*(60*A + 4
3*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) +
 ((80*A*b^2 + 50*a*b*B - 15*a^2*C + 64*b^2*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d) + ((10*b*B - 3*a*C)*Sqrt[Cos[c + d
*x]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d) + (C*Sqrt[Cos[c + d*
x]]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d)
```

<u>Rule 3049</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

<u>Rule 3061</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}\,\mathrm{s}}{5bd}$$

$$= \frac{(10bB-3aC)\sqrt{\cos(c+dx)}(a+b\cos(}{40bd}$$

$$= \frac{\left(80Ab^2+50abB-15a^2C+64b^2C\right)\sqrt{}}{}$$

$$= \frac{\left(50a^2bB+120b^3B-15a^3C+4ab^2(60\right.}{}$$

$$= \frac{\left(150a^3bB+2840ab^3B-45a^4C+256\right.}{}$$

$$= \frac{\left(150a^3bB+2840ab^3B-45a^4C+256\right.}{}$$

$$= \frac{\sqrt{a+b}\left(10a^4bB-240a^2b^3B-96b^5B\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(150a^3bB+2840ab^3B\right.}{}$$

Mathematica [C] time = 6.71755, size = 1410, normalized size = 1.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
 + C*Cos[c + d*x]^2),x]
```

```
[Out] -((-4*a*(-4720*a^2*A*b^2 - 1280*A*b^4 - 1330*a^3*b*B - 3560*a*b^3*B + 15*a^
4*C - 3236*a^2*b^2*C - 1024*b^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3840
*a^3*A*b - 6080*a*A*b^3 - 6440*a^2*b^2*B - 1440*b^4*B - 2292*a^3*b*C - 4624
*a*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
+ 2*(-2640*a^2*A*b^2 - 1280*A*b^4 - 150*a^3*b*B - 2840*a*b^3*B + 45*a^4*C -
 1692*a^2*b^2*C - 1024*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]
*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
+ d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a
*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(b*Sqrt[Cos[c + d*x]])))/(3840*b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]*(((1040*a*A*b^2 + 590*a^2*b*B + 420*b^3*B + 15*a^3*C + 898*a*b^2
*C)*Sin[c + d*x])/(960*b) + ((80*A*b^2 + 170*a*b*B + 93*a^2*C + 88*b^2*C)*S
in[2*(c + d*x)])/480 + (b*(10*b*B + 21*a*C)*Sin[3*(c + d*x)])/160 + (b^2*C*
Sin[4*(c + d*x)])/40))/d
```

Maple [B] time = 0.873, size = 7062, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
```

```
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1130 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=700

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2C+24abB+16Ab^2+12b^2C\right)\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\sin(c+dx)\left(264a^2bB+15a^3C+4ab^2\right.}{192ba}$$

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A
+ 71*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*
a^3*C + 8*b^3*(12*A + 16*B + 9*C) + 2*a^2*b*(192*A + 132*B + 59*C) + 4*a*b^
2*(108*A + 52*B + 71*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sq
rt[a + b]*(40*a^3*b*B + 160*a*b^3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16*
b^4*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d)
+ ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A + 71*C))*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((16*A*b^2 + 24
*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(32*d) + ((8*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 2.53379, antiderivative size = 700, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2C+24abB+16Ab^2+12b^2C\right)\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\sin(c+dx)\left(264a^2bB+15a^3C+4ab^2\right.}{192ba}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A
+ 71*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
```

```
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*
a^3*C + 8*b^3*(12*A + 16*B + 9*C) + 2*a^2*b*(192*A + 132*B + 59*C) + 4*a*b^
2*(108*A + 52*B + 71*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sq
rt[a + b]*(40*a^3*b*B + 160*a*b^3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16*
b^4*(4*A + 3*C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d)
+ ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A + 71*C))*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((16*A*b^2 + 24
*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(32*d) + ((8*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(4*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4d} + \frac{1}{4}$$

$$= \frac{(8bB+5aC)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c}{24d}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{\cos(c+dx)}\sqrt{a}}{32d}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+4ab^2(108A+71C)\right)}{192bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+4ab^2(108A+71C)\right)}{192bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(40a^3bB+160ab^3B-5a^4C+120a^2b^2(2A\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(264a^2bB+128b^3B+15a^3C+4ab^2(\right.}{}$$

Mathematica [C] time = 6.81495, size = 1326, normalized size = 1.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((-4*a*(384*a^3*A + 528*a*A*b^2 + 472*a^2*b*B + 128*b^3*B + 133*a^3*C + 356
*a*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(1152*a^2*A*b + 192*A*b^3 + 384
*a^3*B + 608*a*b^2*B + 644*a^2*b*C + 144*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*
```

```
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(432*a*A*b^2 + 264*a^2*b*B + 128*
b^3*B + 15*a^3*C + 284*a*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(384*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*(((48*A*b^2 + 104*a*b*B + 59*a^2*C + 42*b^2*C)*Sin[c + d*x])/96 +
 (b*(8*b*B + 17*a*C)*Sin[2*(c + d*x)])/48 + (b^2*C*Sin[3*(c + d*x)])/16))/d
```

Maple [B] time = 0.902, size = 5873, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/sqrt(cos(d*x + c)), x)
```

3.1131 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=647

$$\frac{\sin(c+dx)\left(a^2(-(48A-33C))+54abB+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\cot(c+dx)\left(a^2(48A-48B-33C\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d) - (Sqrt[a + b]*(a^2*(48*A - 48*B
- 33*C) - 4*b^2*(6*A + 3*B + 4*C) - 2*a*b*(72*A + 27*B + 13*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(24*d) - (Sqrt[a + b]*(30*a^2*b*B + 8*b^3*B + 5*a^3
*C + 20*a*b^2*(2*A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*
b*d) + ((54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) - (b*(8*a*A - 2*b*B - 3*a*
C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*(6*
A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) +
(2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 2.36875, antiderivative size = 647, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\left(a^2(-(48A-33C))+54abB+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\cot(c+dx)\left(a^2(48A-48B-33C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
```

```
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d) - (Sqrt[a + b]*(a^2*(48*A - 48*B
- 33*C) - 4*b^2*(6*A + 3*B + 4*C) - 2*a*b*(72*A + 27*B + 13*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(24*d) - (Sqrt[a + b]*(30*a^2*b*B + 8*b^3*B + 5*a^3
*C + 20*a*b^2*(2*A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*
b*d) + ((54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) - (b*(8*a*A - 2*b*B - 3*a*
C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*(6*
A - C)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) +
(2*A*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

<u>Rule 3047</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

<u>Rule 3049</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

<u>Rule 3061</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\,\mathrm{c}}{}$$

$$= -\frac{b(6A-C)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(}{3d}$$

$$= -\frac{b(8aA-2bB-3aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}{4d}$$

$$= \frac{\left(54abB-a^2(48A-33C)+8b^2(3A+2C)\right)\sqrt{a+b}}{24d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(54abB-a^2(48A-33C)+8b^2(3A+2C)\right)\sqrt{a+b}}{24d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(30a^2bB+8b^3B+5a^3C+20ab^2(2A+C)\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(54abB-a^2(48A-33C)+8b^2(3A\right.}{}$$

Mathematica [C] time = 6.81999, size = 1302, normalized size = 2.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((4*a*(-96*a^2*A*b - 24*A*b^3 - 48*a^3*B - 66*a*b^2*B - 59*a^2*b*C - 16*b^3
*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(48*a^3*A - 144*a*A*b^2 - 144*a^2*b*B
 - 24*b^3*B - 48*a^3*C - 76*a*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])) - 2*(48*a^2*A*b - 24*A*b^3 - 54*a*b^2*B - 33*a^2
*b*C - 16*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
 + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Co
s[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b*(6
*b*B + 13*a*C)*Sin[c + d*x])/12 + (b^2*C*Sin[2*(c + d*x)])/6 + 2*a^2*A*Tan[
c + d*x]))/d
```

Maple [B] time = 0.513, size = 5130, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right.}{\cos\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(3/2), x)
```

3.1132 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=622

$$-\frac{\sin(c+dx)\left(24a^2B+ab(56A-27C)-12b^2B\right)\sqrt{a+b\cos(c+dx)}}{12d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\cot(c+dx)\left(-8a^2(A-3B+3C)+ab(56\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(12*a*d) - (Sqrt[a + b]*(a*b*(56*A - 72*B - 27*C) -
6*b^2*(12*A + 2*B + C) - 8*a^2*(A - 3*B + 3*C))*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(12*d) - (Sqrt[a + b]*(8*A*b^2 + 20*a*b*B + 15*a^2*C + 4*b^2*C)*Cot
[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - ((24*a^2*B - 12*b^2*B +
 a*b*(56*A - 27*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Cos[c
 + d*x]]) - (b*(8*A*b + 4*a*B - b*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(2*d) + (2*(5*A*b + 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sin
[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 2.20662, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sin(c+dx)\left(24a^2B+ab(56A-27C)-12b^2B\right)\sqrt{a+b\cos(c+dx)}}{12d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\cot(c+dx)\left(-8a^2(A-3B+3C)+ab(56\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
```

```
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(12*a*d) - (Sqrt[a + b]*(a*b*(56*A - 72*B - 27*C) -
6*b^2*(12*A + 2*B + C) - 8*a^2*(A - 3*B + 3*C))*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(12*d) - (Sqrt[a + b]*(8*A*b^2 + 20*a*b*B + 15*a^2*C + 4*b^2*C)*Cot
[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - ((24*a^2*B - 12*b^2*B +
 a*b*(56*A - 27*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Cos[c
 + d*x]]) - (b*(8*A*b + 4*a*B - b*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(2*d) + (2*(5*A*b + 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sin
[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2(5Ab+3aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A}{}$$

$$= -\frac{b(8Ab+4aB-bC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}{2d}$$

$$= -\frac{\left(24a^2B-12b^2B+ab(56A-27C)\right)\sqrt{a+b\cos(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(24a^2B-12b^2B+ab(56A-27C)\right)\sqrt{a+b\cos(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+20abB+15a^2C+4b^2C\right)\cot(c+d}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(24a^2B-12b^2B+ab(56A-27C)\right)\mathrm{cot}}{}$$

Mathematica [C] time = 6.78893, size = 1316, normalized size = 2.12

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((-4*a*(8*a^3*A + 16*a*A*b^2 + 48*a^2*b*B + 12*b^3*B + 24*a^3*C + 33*a*b^2*
C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-56*a^2*A*b + 24*A*b^3 - 24*a^3*B + 7
2*a*b^2*B + 72*a^2*b*C + 12*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])) + 2*(-56*a*A*b^2 - 24*a^2*b*B + 12*b^3*B + 27*a*b^
2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[
(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Co
s[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
+ b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])
))/(24*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b^2*C*Sin[c + d*
x])/2 + (2*Sec[c + d*x]*(7*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x]))/3 +
(2*a^2*A*Sec[c + d*x]*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.247, size = 4889, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x)

[Out] 1/12/d*(8*A*a^3+56*A*cos(d*x+c)^2*a*b^2-33*C*cos(d*x+c)^4*a*b^2-27*C*cos(d*
x+c)^3*a^2*b+27*C*cos(d*x+c)^2*a^2*b+6*C*cos(d*x+c)^2*a*b^2-56*A*cos(d*x+c)
^3*a*b^2+64*A*cos(d*x+c)*a^2*b+24*B*cos(d*x+c)^2*a^2*b+12*B*cos(d*x+c)^2*a*
b^2-12*B*cos(d*x+c)^3*a*b^2-8*A*cos(d*x+c)^3*a^2*b-24*B*cos(d*x+c)^3*a^2*b+
27*C*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^3*b^3+24*B*cos(d*x+c)*a^3-12*B*cos(
d*x+c)^4*b^3-6*C*cos(d*x+c)^5*b^3-24*B*cos(d*x+c)^2*a^3+24*B*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b+56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-6*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2
-27*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*a^2*b-27*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-72*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-6*
C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*a*b^2-56*A*cos(d*x+c)^2*a^2*b+24*B*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-12
*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a*b^2-72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+72*B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2
-56*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b-8*A*cos(d*x+c)^2*a^3-72*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+12*C*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)

/(a+b))^(1/2))*b^3-24*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+24*A*sin(d*x+c)*cos(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3-48*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*b^3+24*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3-48*A*sin(d*x+c)*cos(d*x+c)^2*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+6*C
*cos(d*x+c)^3*b^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-24*C*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^
2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-56*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b-72*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+56*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+56*A*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2+72*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-27*C*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-12*B
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2+72*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*C*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3+24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3-12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^3-24*B*(cos(d*x+c

```
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*a^3+56*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-90*C*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*a^2*b-27*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+12*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-24*C*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*b^3+24*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^3-12*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-120*B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a
*b^2-120*B*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-90*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b)/(a+b*cos
(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right.\right.}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(5/2), x)
```

3.1133 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=643

$$\frac{2\sin(c+dx)\left(a^2(3A+5C)+10abB+5Ab^2\right)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)\left(6a^2(3A+5C)+70abB+b^2(46A-15C\right.}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(70*a*b*B + b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (Sqrt[a + b]*(30*A*b^3 - 2*a^3*(
9*A - 5*B + 15*C) + 2*a^2*b*(17*A - 35*B + 45*C) - a*b^2*(46*A - 15*(6*B +
C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) - (b*Sqrt[a + b]*(2*b*B +
5*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*(5*A*b^2 + 10*
a*b*B + a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[C
os[c + d*x]]) - ((70*a*b*B + b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(A*b + a*B)*
(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*(a
 + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 2.36182, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(a^2(3A+5C)+10abB+5Ab^2\right)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)\left(6a^2(3A+5C)+70abB+b^2(46A-15C\right.}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(70*a*b*B + b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
```

```
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (Sqrt[a + b]*(30*A*b^3 - 2*a^3*(
9*A - 5*B + 15*C) + 2*a^2*b*(17*A - 35*B + 45*C) - a*b^2*(46*A - 15*(6*B +
C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) - (b*Sqrt[a + b]*(2*b*B +
5*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*(5*A*b^2 + 10*
a*b*B + a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[C
os[c + d*x]]) - ((70*a*b*B + b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(A*b + a*B)*
(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*A*(a
 + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2(Ab+aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2\left(5Ab^2+10abB+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2\left(5Ab^2+10abB+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2\left(5Ab^2+10abB+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b\sqrt{a+b}(2bB+5aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(70abB+b^2(46A-15C)+6a^2(3A+\right.}{}$$

<u>Mathematica</u> [C] time = 6.85658, size = 1370, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] ((4*a*(-16*a^2*A*b + 16*A*b^3 - 10*a^3*B - 20*a*b^2*B - 60*a^2*b*C - 15*b^3
*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(18*a^3*A + 46*a*A*b^2 + 70*a^2*b*B -
 30*b^3*B + 30*a^3*C - 90*a*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
```

```
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])) - 2*(18*a^2*A*b + 46*A*b^3 + 70*a*b^2*B + 30*a^2*b
*C - 15*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Ar
cSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/
(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c +
 d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[
c + d*x]])))/(30*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[
c + d*x]^2*(11*a*A*b*Sin[c + d*x] + 5*a^2*B*Sin[c + d*x]))/15 + (2*Sec[c +
d*x]*(9*a^2*A*Sin[c + d*x] + 23*A*b^2*Sin[c + d*x] + 35*a*b*B*Sin[c + d*x]
+ 15*a^2*C*Sin[c + d*x]))/15 + (2*a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.268, size = 4986, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x)
```

```
[Out] 1/15/d*(6*A*a^3+68*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^4*a*b^2+30*C*cos(d*
x+c)^3*a^2*b-30*C*cos(d*x+c)^4*a^2*b-18*A*cos(d*x+c)^4*a^2*b-22*A*cos(d*x+c
)^4*a*b^2-46*A*cos(d*x+c)^3*a*b^2+28*A*cos(d*x+c)*a^2*b+80*B*cos(d*x+c)^2*a
^2*b+70*B*cos(d*x+c)^3*a*b^2-10*A*cos(d*x+c)^3*a^2*b-70*B*cos(d*x+c)^3*a^2*
b+15*C*cos(d*x+c)^3*a*b^2-10*B*cos(d*x+c)^4*a^2*b-70*B*cos(d*x+c)^4*a*b^2+1
0*B*cos(d*x+c)*a^3-10*B*cos(d*x+c)^3*a^3-60*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
```

1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-30*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-60*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-15*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-15*C*cos(d*x+c)^5*b^3+70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+18*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-150*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+90*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-150*C*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-90*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+70*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+70*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b-90*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+90*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*

```
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*C*cos(d*x+
c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-46*A*cos(d*x+c)^4*b^3-18*A*cos(d*x+c)^3*a^3+12*A*cos(d*x+c)^2*
a^3-30*C*cos(d*x+c)^3*a^3-30*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+15*C*cos(d*x+c)^4*b^3+30*
C*cos(d*x+c)^2*a^3+46*A*cos(d*x+c)^3*b^3-18*A*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3+46*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+30*C*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-18*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+46*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3-30*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3+30*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+46*A*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2-90*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-34*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b-46*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
```

```
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+46*A*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-90*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-34*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b-46*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-10*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*a^3-15*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(7/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(7/2), x)
```

3.1134 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=580

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)+56abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2b(145A-119B+245C)+\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(15*A*b^3 + 63*a^3*B + 161*a*b^2*B + 5*a^2*b*(29*A +
 49*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) - (2*Sqrt[a + b]*(15
*b^3*(A - 7*B) - a^3*(25*A - 63*B + 35*C) + a^2*b*(145*A - 119*B + 245*C) -
 a*b^2*(135*A - 161*B + 315*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a*d
) - (2*b^2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d +
 (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b + 7*a*B)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x
])^(5/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.79793, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)+56abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2b(145A-119B+245C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(15*A*b^3 + 63*a^3*B + 161*a*b^2*B + 5*a^2*b*(29*A +
 49*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
```

```
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) - (2*Sqrt[a + b]*(15
*b^3*(A - 7*B) - a^3*(25*A - 63*B + 35*C) + a^2*b*(145*A - 119*B + 245*C) -
 a*b^2*(135*A - 161*B + 315*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a*d
) - (2*b^2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d +
 (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b + 7*a*B)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*A*(a + b*Cos[c + d*x
])^(5/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b}{}$$

$$= \frac{2(5Ab+7aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{a+b\cos(c+}}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{a+b\cos(c+}}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(15Ab^3+63a^3B+161ab^2B+5a^2b(\right.}{}$$

Mathematica [C] time = 6.9613, size = 1472, normalized size = 2.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 10*a^2*A*b^2 - 15*A*b^4 + 56*a^3*b*B - 56*a*b^3*B + 35*a
^4*C + 70*a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-145*a^3*A*b - 15*a
*A*b^3 - 63*a^4*B - 161*a^2*b^2*B - 245*a^3*b*C + 105*a*b^3*C)*((Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
```

```
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-145*a^2*A*b^2 -
15*A*b^4 - 63*a^3*b*B - 161*a*b^3*B - 245*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d
*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(105*a*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(15*a*A*b*Sin[c + d*x]
+ 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 45
*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/105 +
 (2*Sec[c + d*x]*(145*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 63*a^3
*B*Sin[c + d*x] + 161*a*b^2*B*Sin[c + d*x] + 245*a^2*b*C*Sin[c + d*x]))/(10
5*a) + (2*a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.348, size = 5143, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right.\right.}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(9/2), x)
```

3.1135 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=552

$$\frac{2\sin(c+dx)\left(a^2b(163A+231C)+75a^3B+135ab^2B+5Ab^3\right)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(7a^2(7A+9C)+90a\right.}{315d\cos^{\frac{5}{2}}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 435*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A
+ 9*C) - 3*a^2*b^2*(93*A + 161*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*
a^3*d) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 + 15*a*b^2*(11*A - 3*B + 21*C) -
6*a^2*b*(19*A - 60*B + 28*C) + 3*a^3*(49*A - 25*B + 63*C))*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(315*a^2*d) + (2*(15*A*b^2 + 90*a*b*B + 7*a^2*(7*A + 9*C
))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(
5*A*b^3 + 75*a^3*B + 135*a*b^2*B + a^2*b*(163*A + 231*C))*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b + 9*a*B)*(a
+ b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*A*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 2.04865, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3047, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(a^2b(163A+231C)+75a^3B+135ab^2B+5Ab^3\right)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(7a^2(7A+9C)+90a\right.}{315d\cos^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(11/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 435*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A
+ 9*C) - 3*a^2*b^2*(93*A + 161*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*
```

```
a^3*d) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 + 15*a*b^2*(11*A - 3*B + 21*C) -
6*a^2*b*(19*A - 60*B + 28*C) + 3*a^3*(49*A - 25*B + 63*C))*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(315*a^2*d) + (2*(15*A*b^2 + 90*a*b*B + 7*a^2*(7*A + 9*C
))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(
5*A*b^3 + 75*a^3*B + 135*a*b^2*B + a^2*b*(163*A + 231*C))*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b + 9*a*B)*(a
+ b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*A*(a +
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2(5Ab+9aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+}}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(10Ab^4-435a^3bB-45ab^3B-21a^4\right.}{}$$

Mathematica [C] time = 7.101, size = 1616, normalized size = 2.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] -((-4*a*(-114*a^4*A*b + 124*a^2*A*b^3 - 10*A*b^5 - 75*a^5*B + 30*a^3*b^2*B
+ 45*a*b^4*B - 168*a^4*b*C + 168*a^2*b^3*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4
*a*(147*a^5*A + 279*a^3*A*b^2 - 10*a*A*b^4 + 435*a^4*b*B + 45*a^2*b^3*B + 1
89*a^5*C + 483*a^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
```

```
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])) + 2*(147*a^4*A*b + 279*a^2*A*b^3 - 10*A*b^5 + 435*a^3*b^2*B
 + 45*a*b^4*B + 189*a^4*b*C + 483*a^2*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (
-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt
[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*
x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(315*a^2*d) + (Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^4*(19*a*A*b*Sin[c + d*x] + 9*a^
2*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 75*A*b^2
*Sin[c + d*x] + 135*a*b*B*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x]))/315 + (2*S
ec[c + d*x]^2*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x] + 75*a^3*B*S
in[c + d*x] + 135*a*b^2*B*Sin[c + d*x] + 231*a^2*b*C*Sin[c + d*x]))/(315*a)
 + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 279*a^2*A*b^2*Sin[c + d*x] - 1
0*A*b^4*Sin[c + d*x] + 435*a^3*b*B*Sin[c + d*x] + 45*a*b^3*B*Sin[c + d*x] +
 189*a^4*C*Sin[c + d*x] + 483*a^2*b^2*C*Sin[c + d*x]))/(315*a^2) + (2*a^2*A
*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.509, size = 6176, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2
),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/cos(d*x + c)^(11/2), x)
```

3.1136 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=593

$$\frac{\sin(c+dx)\left(15a^2C-18abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24b^3d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\cot(c+dx)\left(15a^2C-18abB-10abC+24A\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(24*a*b^3*d) + (Sqrt[a + b]*(24*A*b^2 - 18*a*b*B
+ 12*b^2*B + 15*a^2*C - 10*a*b*C + 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(24*b^3*d) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - 5*a^3*C - 4*a*b^2*(2*
A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^4*d) + ((24*A*b
^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(24*b^3*d*Sqrt[Cos[c + d*x]]) + ((6*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(12*b^2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 1.86086, antiderivative size = 593, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\left(15a^2C-18abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24b^3d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\cot(c+dx)\left(15a^2C-18abB-10abC+24A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b
*Cos[c + d*x]],x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
```

```
 Sec[c + d*x]))/(a - b)])/(24*a*b^3*d) + (Sqrt[a + b]*(24*A*b^2 - 18*a*b*B
+ 12*b^2*B + 15*a^2*C - 10*a*b*C + 16*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(24*b^3*d) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - 5*a^3*C - 4*a*b^2*(2*
A + C))*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^4*d) + ((24*A*b
^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(24*b^3*d*Sqrt[Cos[c + d*x]]) + ((6*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(12*b^2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\int \frac{\sqrt{\cos(c+d}}{}}{}$$

$$= \frac{(6bB-5aC)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12b^2d} +$$

$$= \frac{\left(24Ab^2-18abB+15a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(}{24b^3d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(24Ab^2-18abB+15a^2C+16b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(}{24b^3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(6a^2bB+8b^3B-5a^3C-4ab^2(2A+C)\right)\cot(c+d}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24Ab^2-18abB+15a^2C+16b^2C\right)\cot(c+}{}$$

Mathematica [C] time = 6.52373, size = 1241, normalized size = 2.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[a + b*Cos[c + d*x]],x]
```

```
[Out] ((-4*a*(24*A*b^2 - 6*a*b*B + 5*a^2*C + 16*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
) - 4*a*(24*b^2*B + 4*a*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
```

```
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]])) + 2*(24*A*b^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*((I*Cos[
(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/
Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a
*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
(Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(48*b^2*d)
 + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((6*b*B - 5*a*C)*Sin[c + d
*x])/(12*b^2) + (C*Sin[2*(c + d*x)])/(6*b)))/d
```

Maple [B] time = 0.309, size = 3575, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b
^2-2*C*cos(d*x+c)^4*a*b^2+5*C*cos(d*x+c)^3*a^2*b-15*C*cos(d*x+c)^2*a^2*b+18
*C*cos(d*x+c)^2*a*b^2+10*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2-18*B*cos(
d*x+c)^2*a^2*b+18*B*cos(d*x+c)^2*a*b^2+18*B*cos(d*x+c)*a^2*b-12*B*cos(d*x+c
)*a*b^2-6*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^3+8*C*cos(d*x+c)^5*b^3-1
8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-24*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-
10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
```

/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*a^2*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-48*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*a*b^2-24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-10*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-4*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+15*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+15*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^3-12*B*cos(d*x+c)^2*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-30*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+15*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3+8*C*cos(d*x+c)^3*b^3+15*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-15

```
*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+c)^2*b^3-18*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2+36*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-18*B*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^
2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b^3/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}+B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*
x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b
*cos(d*x + c) + a), x)
```

3.1137 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=485

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B - 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) - (Sqrt[a + b]*(3*a*C - 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) - (Sqrt[a + b]*(8*A*b^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*C
ot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*b*B - 3*a*C)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (C*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 1.12251, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[a + b
*Cos[c + d*x]],x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B - 3*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) - (Sqrt[a + b]*(3*a*C - 2*b*(2*B + C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) - (Sqrt[a + b]*(8*A*b^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*C
```

```
ot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*b*B - 3*a*C)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (C*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
```

```
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \int \frac{\frac{aC}{2}+b(2A}{}$$

$$= \frac{(4bB-3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+}}{}$$

$$= \frac{(4bB-3aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+}}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2-4abB+3a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b}\right.}{4}$$

$$= -\frac{(a-b)\sqrt{a+b}(4bB-3aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{4ab^2d}$$

Mathematica [C] time = 12.7192, size = 1182, normalized size = 2.44

$$-\frac{4a(4bB-aC)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+\right.}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$\frac{C\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \frac{}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[a + b*Cos[c + d*x]],x]
```

```
[Out] (C*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + ((-4
*a*(4*b*B - a*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
```

```
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*A*b + 4*b*C)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*b*B - 3*a*C
)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
+ d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b
)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b)
, ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/
(8*b*d)
```

Maple [B] time = 0.173, size = 2248, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/4/d/(a+b*cos(d*x+c))^(1/2)*(2*C*cos(d*x+c)*a*b-4*B*cos(d*x+c)^2*a*b+4*B*c
os(d*x+c)*a*b+C*cos(d*x+c)^3*a*b-3*C*cos(d*x+c)^2*a*b-4*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*b^2*C*cos
(d*x+c)^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
```

+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b-16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+3*C*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-6*C*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^2-8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*b^2+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^2+4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-6*C*cos(d*x+c)*sin(d*x+c)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-
8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*b^2+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-16*A*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)^3*b^2+4*B*cos(d*x+c)^2
*b^2+3*C*cos(d*x+c)^2*a^2-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+3*C*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a*b-2*C*cos(d*x+c)^4*b^2-3*C*cos(d*x+c)*a^2-4*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b

```
))^(1/2))*a*b+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a*b)/sin(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b
*cos(d*x + c) + a), x)
```

3.1138 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=401

$$\frac{\sqrt{a+b}(aC+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd}-\frac{\sqrt{a+b}(2bB-aC)\cot(c+}{}$$

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*(2*A*b + a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d) - (Sqrt[a
 + b]*(2*b*B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d)
+ (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.792452, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(aC+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd}-\frac{\sqrt{a+b}(2bB-aC)\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*(2*A*b + a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d) - (Sqrt[a
 + b]*(2*b*B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d)
```

```
+ (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\int \frac{-aC+2Ab\cos(c+dx)+(2bB-aC)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\int \frac{-aC+2Ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{2b} + \frac{(2bB-aC}{}$$

$$= -\frac{\sqrt{a+b}(2bB-aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{abd}$$

Mathematica [C] time = 18.9797, size = 1117, normalized size = 2.79

$$-\frac{4a(2A+C)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}}{\sqrt{2}}\right)\middle|-\frac{2a}{b-a}\right)\sin^4(}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]),x]
```

```
[Out] ((-4*a*(2*A + C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 8*a*B*((Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*C*((I*Cos[(c + d*x)/2]*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]
], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
```

```
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*d)
```

Maple [B] time = 0.137, size = 1325, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d*(2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*b+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*b-2*B*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+
4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*b+C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*a-2*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*b+4*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*b+C*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
```

```
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)^4*b+C*cos(d*x+c)^3*a-C*co
s(d*x+c)^3*b-C*cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(1/2)/b/sin(d*x+c)/cos(d*x+
c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A}{\sqrt{b\cos{(dx+c)} + a}\sqrt{\cos{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)\sqrt{b\cos{(dx+c)} + a}\sqrt{\cos{(dx+c)}}}{b\cos{(dx+c)}^2 + a\cos{(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos{(c+dx)} + C\cos^2{(c+dx)}}{\sqrt{a + b\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x))
*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*sqrt(cos(d*x + c))), x)
```

3.1139 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=347

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(A-B)\cot(c+d}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]
*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rubi [A] time = 0.535276, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3053, 2809, 2998, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(A-B)\cot(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + b*
Cos[c + d*x]]),x]
```


```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]
*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = C\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{bd}$$

$$= \frac{2A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{a^2d}$$

Mathematica [C] time = 19.0126, size = 1169, normalized size = 3.37

$$-\frac{4a(Ab-aB)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)F\left(\mathrm{s}\right.}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[
a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((-4
*a*(A*b - a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
```

```
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(a*A - a*C)*((Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*A*b*((I*Cos[(c + d*x
)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos
[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a*d)
```

Maple [B] time = 0.192, size = 1321, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*B*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a-A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
```

```
*x+c),(-(a-b)/(a+b))^(1/2))*b+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-C*cos(d*x+c)^2*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2
*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-
C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+A*cos(d*x+c)^3*b+A*cos(d*x+c)^2*a-A
*cos(d*x+c)^2*b-A*cos(d*x+c)*a)/a/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}{b\cos{(dx+c)}^3 + a\cos{(dx+c)}^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos{(c + dx)} + C\cos^{2}{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}\cos^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x))
*cos(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx + c)^{2} + B\cos(dx + c) + A}{\sqrt{b\cos(dx + c) + a}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(3/2)), x)
```

3.1140 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt{a+b}\cot(c+dx)(a(A-3B+3C)+2Ab)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2(a-b)\sqrt{}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B + 3*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(3*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a
*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.570914, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\cot(c+dx)(a(A-3B+3C)+2Ab)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2(a-b)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + b*
Cos[c + d*x]]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B + 3*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(3*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a
*d*Cos[c + d*x]^(3/2))
```


Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{a + b\cos(c + dx)}}\,dx = \frac{2A\sqrt{a + b\cos(c + dx)}\sin(c + dx)}{3ad\cos^{\frac{3}{2}}(c + dx)} + \frac{2\int \frac{\frac{1}{2}(-2Ab+3aB)+\frac{1}{2}a(A+3C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sqrt{a + b\cos(c + dx)}\sin(c + dx)}{3ad\cos^{\frac{3}{2}}(c + dx)} - \frac{(2Ab - 3aB)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a}$$

$$= -\frac{2(a - b)\sqrt{a + b}(2Ab - 3aB)\cot(c + dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) | -\frac{a+b}{a-b}\right.}{3a^3d}$$

<u>Mathematica</u> [C] time = 6.47571, size = 1244, normalized size = 4.25

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[
a + b*Cos[c + d*x]]),x]
```

```
[Out] ((-4*a*(a^2*A + 2*A*b^2 - 3*a*b*B + 3*a^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
4*a*(2*a*A*b - 3*a^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])) + 2*(2*A*b^2 - 3*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
```

```
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x
]))/(3*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 0.24, size = 1823, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(1/2)*(A*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b-2*A*cos(
d*x+c)^2*a*b-3*B*cos(d*x+c)^2*a*b+3*B*cos(d*x+c)^3*a*b-A*a^2+3*B*cos(d*x+c)
^2*a^2+A*cos(d*x+c)^2*a^2+2*A*cos(d*x+c)^2*b^2-2*A*cos(d*x+c)^3*b^2-3*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^2-3*B*cos(d*x+c)*a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+2*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^2*a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

```
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+3*C*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*C*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^2*a^2+6*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+2*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^2*b^2-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^2*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2+2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2)/a^2/
sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^4 + a\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(5/2)), x)
```

3.1141 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2(9A-5B+15C)-2ab(A+5B)+8Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{15a^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*A*b^2 - 2*a*b*(A + 5
*B) + a^2*(9*A - 5*B + 15*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d)
 + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) -
 (2*(4*A*b - 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c
+ d*x]^(3/2))
```

Rubi [A] time = 0.933386, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2(9A-5B+15C)-2ab(A+5B)+8Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[a + b*
Cos[c + d*x]]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*A*b^2 - 2*a*b*(A + 5
*B) + a^2*(9*A - 5*B + 15*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d)
 + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) -
```

```
(2*(4*A*b - 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c
+ d*x]^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-4Ab+5aB)+\frac{1}{2}a(3A+5C)\cos(c+dx)+Ab\cos}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sin(}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sin(}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^2-10abB+3a^2(3A+5C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{15a^4d}$$

Mathematica [C] time = 6.5785, size = 1351, normalized size = 3.63

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[
a + b*Cos[c + d*x]]),x]
```

```
[Out] -((-4*a*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B + 15*a^2*b*C)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(9*a^3*A + 8*a*A*b^2 - 10*a^2*b*B + 15*a^3*C)*((Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
```

```
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b
 + 8*A*b^3 - 10*a*b^2*B + 15*a^2*b*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(15*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(-4*A*b*Sin[c + d*x] + 5*a*B*Sin[c +
d*x]))/(15*a^2) + (2*Sec[c + d*x]*(9*a^2*A*Sin[c + d*x] + 8*A*b^2*Sin[c + d
*x] - 10*a*b*B*Sin[c + d*x] + 15*a^2*C*Sin[c + d*x]))/(15*a^3) + (2*A*Sec[c
 + d*x]^2*Tan[c + d*x])/(5*a)))/d
```

Maple [B] time = 0.214, size = 3134, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/15/d*(-3*A*a^3-4*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos(d
*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2+8*A*cos(d*x+c)^
3*a*b^2+A*cos(d*x+c)*a^2*b+5*B*cos(d*x+c)^2*a^2*b+10*B*cos(d*x+c)^3*a*b^2-1
0*A*cos(d*x+c)^3*a^2*b-10*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b-10*B*
cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)*a^3+5*B*cos(d*x+c)^3*a^3+10*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^3*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
```

1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+10*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^3*a^2*b+10*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+10*B*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-10*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*a^2*b+8*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a
^3+15*C*cos(d*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3-8*A*cos(d*x+c)^3*b^3+9*A*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*
cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b+8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),

```
(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^2-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^3*a^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3)/(a+b*cos(d*x+c))^(1/2)/a
^3/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A\right)\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}{b\cos{(dx+c)}^5 + a\cos{(dx+c)}^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(7/2)), x)
```

3.1142 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=466

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)-28abB+24Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sqrt{a+b}\cot(c+dx)\left(2a^2b(22A+7(B+5C))+a^3(\right.}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 7
0*b*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^5*d) + (2*Sqrt[a + b]*(48
*A*b^3 - 4*a*b^2*(3*A + 14*B) + a^3*(25*A - 63*B + 35*C) + 2*a^2*b*(22*A +
7*(B + 5*C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*A*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(7*a*d*Cos[c + d*x]^(7/2)) - (2*(6*A*b - 7*
a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a^2*d*Cos[c + d*x]^(5/2)) +
 (2*(24*A*b^2 - 28*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(105*a^3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.41456, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)-28abB+24Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sqrt{a+b}\cot(c+dx)\left(2a^2b(22A+7(B+5C))+a^3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[a + b*
Cos[c + d*x]]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 7
0*b*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^5*d) + (2*Sqrt[a + b]*(48
*A*b^3 - 4*a*b^2*(3*A + 14*B) + a^3*(25*A - 63*B + 35*C) + 2*a^2*b*(22*A +
7*(B + 5*C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
```

```
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*A*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(7*a*d*Cos[c + d*x]^(7/2)) - (2*(6*A*b - 7*
a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a^2*d*Cos[c + d*x]^(5/2)) +
 (2*(24*A*b^2 - 28*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(105*a^3*d*Cos[c + d*x]^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-6Ab+7aB)+\frac{1}{2}a(5A+7C)\cos(c+dx)+2Ab\,\mathrm{co}}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{7a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\cos(c+dx)}\sin(c+}{35a^2d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\cos(c+dx)}\sin(c+}{35a^2d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7ad\cos^{\frac{7}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\cos(c+dx)}\sin(c+}{35a^2d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(48Ab^3-63a^3B-56ab^2B+a^2(44Ab+70bC)\right)\cot(c+dx)}{105a^5d}$$

Mathematica [C] time = 6.70815, size = 1468, normalized size = 3.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[
a + b*Cos[c + d*x]]),x]
```

```
[Out] ((-4*a*(25*a^4*A + 32*a^2*A*b^2 + 48*A*b^4 - 49*a^3*b*B - 56*a*b^3*B + 35*a
^4*C + 70*a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
```

```
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(44*a^3*A*b + 48*a*A
*b^3 - 63*a^4*B - 56*a^2*b^2*B + 70*a^3*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(44*a^2*A*b^2 + 48*A*b^4 - 63*a^3*b*
B - 56*a*b^3*B + 70*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)
]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^4*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]*((2*Sec[c + d*x]^3*(-6*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))
/(35*a^2) + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 24*A*b^2*Sin[c + d*x
] - 28*a*b*B*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a^3) + (2*Sec[c +
d*x]*(-44*a^2*A*b*Sin[c + d*x] - 48*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d
*x] + 56*a*b^2*B*Sin[c + d*x] - 70*a^2*b*C*Sin[c + d*x]))/(105*a^4) + (2*A*
Sec[c + d*x]^3*Tan[c + d*x])/(7*a)))/d
```

Maple [B] time = 0.315, size = 4337, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

[Out] -2/105/d*(-15*A*a^4+48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-12*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-12*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4+63*B*cos(d*x+c)^4*a^4+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*cos(d*x+c)^5*a^3*b-28*B*cos(d*x+c)^5*a^2*b^2+56*B*cos(d*x+c)^5*a*b^3-70*B*cos(d*x+c)^4*a^3*b+56*B*cos(d*x+c)^4*a^2*b^2-48*

A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^
2-70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^4*a^2
*b^2+35*C*cos(d*x+c)^3*a^3*b+35*C*cos(d*x+c)^5*a^3*b-70*C*cos(d*x+c)^5*a^2*
b^2-70*C*cos(d*x+c)^4*a^3*b-28*B*cos(d*x+c)^3*a^2*b^2+7*B*cos(d*x+c)^2*a^3*
b+24*A*cos(d*x+c)^3*a*b^3-6*A*cos(d*x+c)^2*a^2*b^2+3*A*cos(d*x+c)*a^3*b+25*
A*cos(d*x+c)^5*a^3*b-44*A*cos(d*x+c)^5*a^2*b^2+24*A*cos(d*x+c)^5*a*b^3-44*A
*cos(d*x+c)^4*a^3*b+50*A*cos(d*x+c)^4*a^2*b^2-48*A*cos(d*x+c)^4*a*b^3+16*A*
cos(d*x+c)^3*a^3*b-56*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+25*A*cos(d*x+c)^4*a^4+35*C*c
os(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4-35*C*cos(d*x+c)^2*a^4-48*A*cos(d*x+c)
^5*b^4+48*A*cos(d*x+c)^4*b^4+44*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-56*B*cos(d*x+c)^4*
a*b^3-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d*x+c)*cos(d
*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
^4+63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*a^4-56*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+14*B*sin(d*x+c)*cos(d*
x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^
3*b+56*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-56*B*sin(d*x+c)*c
os(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/

```
2)*a^2*b^2-56*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+14*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+56*B*sin(d*x
+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b)/(a+b*cos(d*x+c))^(1/2)/a^4/sin(d
*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)^{6}+a\cos\left(dx+c\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b*cos(d*x + c)^6 + a*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*cos(d*x + c)^(9/2)), x)
```

3.1143 $\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=473

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{(aB+$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.47574, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3029, 3003, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{(aB+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^
2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Elliptic
```

```
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}dx = \frac{\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-abB+b(A\right.}{b^2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} +$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{B}{}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{B}{}$$

$$= -\frac{\sqrt{a+b}\left(4aAb-a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a}{b}\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab+aB)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{}$$

Mathematica [C] time = 6.20388, size = 1175, normalized size = 2.48

$$-\frac{4a(4Ab+3aB)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c\right.}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c +
 d*x]^2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(4*A*b + 3*a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
```

```
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a*A + 4*b*B)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b + a*B)*
((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]
) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8
*d)
```

Maple [B] time = 0.158, size = 2052, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d
*x+c))^(1/2),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-4*A*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-B*c
os(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b+8*B*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+B*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x
```

```
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
^2+B*cos(d*x+c)^2*a^2-4*A*cos(d*x+c)^2*b^2+4*A*cos(d*x+c)^3*b^2+B*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2-8*A*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^2+8*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-4*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-B*cos(d*x+c)*a^
2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-4*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*A*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-2*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*a^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2)/sin(d*x+c)/b/cos(d*x+c
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(cos(d*
x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a
+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(cos(d*
x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.1144 $\int \frac{a+a\cos(c+dx)+2b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=256

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{4\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (4*Sqrt[
a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.495007, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3061, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{4\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x] + 2*b*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]),x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (4*Sqrt[
a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{a+a\cos(c+dx)+2b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\int \frac{-2ab+2ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx +$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad}$$

Mathematica [A] time = 4.75302, size = 160, normalized size = 0.62

$$\frac{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}\left(\frac{2E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}}+\frac{\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x] + 2*b*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]]*((2*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))] + (Sec[(c + d*x)/2]*(-Sin[(c + d*x)/2] + Sin[(
3*(c + d*x))/2]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]))/(d*Sqrt[Cos[c + d
*x]])
```

Maple [B] time = 0.153, size = 919, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c)+2*b*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/
2),x)
```

```
[Out] -2/d*(sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(
```

```
1+cos(d*x+c)))^(1/2)*a+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-s
in(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a+a*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a+cos(d*x+c)
^4*b+cos(d*x+c)^3*a-cos(d*x+c)^3*b-cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(1/2)/s
in(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+a}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c)+2*b*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c
))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*b*cos(d*x + c)^2 + a*cos(d*x + c) + a)/(sqrt(b*cos(d*x + c) +
a)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(2\,b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+a\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c)+2*b*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c
))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((2*b*cos(d*x + c)^2 + a*cos(d*x + c) + a)*sqrt(b*cos(d*x + c) + a)
*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos\left(c+dx\right)+a+2b\cos^{2}\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c)+2*b*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*cos(d*x
+c))**(1/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a + 2*b*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x
))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+a}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c)+2*b*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c
))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*b*cos(d*x + c)^2 + a*cos(d*x + c) + a)/(sqrt(b*cos(d*x + c) +
a)*sqrt(cos(d*x + c))), x)
```

3.1145 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=660

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2C-4abB+4Ab^2-b^2C\right)\sqrt{a+b\cos(c}{2b^2d\left(a^2-b^2\right)}$$

```
[Out] -((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(4*a*b^3*Sqrt[a + b]*d) - ((8*A*b^2 - a*b*(12*B - 5*C) + 15
*a^2*C - 2*b^2*(2*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*Sqrt[a
+ b]*d) - (Sqrt[a + b]*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^4*d) - (2*(A*b^2 - a*(b*B - a*C))
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
) + ((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((4*A*b^2
 - 4*a*b*B + 5*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(2*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 2.04702, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2C-4abB+4Ab^2-b^2C\right)\sqrt{a+b\cos(c}{2b^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^(3/2),x]
```

```
[Out] -((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
```

```
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(4*a*b^3*Sqrt[a + b]*d) - ((8*A*b^2 - a*b*(12*B - 5*C) + 15
*a^2*C - 2*b^2*(2*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*Sqrt[a
+ b]*d) - (Sqrt[a + b]*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^4*d) - (2*(A*b^2 - a*(b*B - a*C))
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
) + ((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + ((4*A*b^2
 - 4*a*b*B + 5*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(2*b^2*(a^2 - b^2)*d)
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\sqrt{\cos(c}}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4Ab^2-4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(12a^2bB-\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(12a^2bB-\right.}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2-12abB+15a^2C+4b^2C\right)\cot(c+dx)\Pi\left(\frac{a+b}{b}\right.}{4b}$$

$$= -\frac{\left(12a^2bB-4b^3B-ab^2(8A-7C)-15a^3C\right)\cot(c+dx)E\left(\sin\right.}{4ab^3\sqrt{a}}$$

Mathematica [C] time = 6.72388, size = 1322, normalized size = 2.

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((C*Sin[c + d*x])/(2*b^2) - (2
*(a*A*b^2*Sin[c + d*x] - a^2*b*B*Sin[c + d*x] + a^3*C*Sin[c + d*x]))/(b^2*(
-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - ((-4*a*(-4*a^2*b*B + 4*b^3*B + 5*a^
3*C - 5*a*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*A*b^3 - 8*a*b^2*B + 4
*a^2*b*C + 4*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) + 2*(8*a*A*b^2 - 12*a^2*b*B + 4*b^3*B + 15*a^3*C - 7*a*b^2*C)*((I
*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x
)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) +
 (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))
/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(8*(a
 - b)*b^2*(a + b)*d)
```

Maple [B] time = 0.259, size = 5209, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

3.1146 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=535

$$\frac{\sin(c+dx)\left(3a^2C-2abB+2Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\cot(c+}{}$$

```
[Out] -(((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(a*b^2*Sqrt[a + b]*d)) + ((2*A*b^2 - a*(b*(2*B - C) - 3*a*C))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*b*B - 3*a*C)
*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*(A*b^2 - a*(b*
B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.45611, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\left(3a^2C-2abB+2Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^(3/2),x]
```

```
[Out] -(((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(a*b^2*Sqrt[a + b]*d)) + ((2*A*b^2 - a*(b*(2*B - C) - 3*a*C))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
```

```
c[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*b*B - 3*a*C)
*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*(A*b^2 - a*(b*
B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{1}{2}(A \cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2Ab^2 \cdots\right.}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2Ab^2 \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a+b}(2bB-3aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+\cdots}}{\sqrt{a+b}\sqrt{\cos(c+\cdots}}\right.\right.}{b^3d}$$

$$= -\frac{\left(2Ab^2-2abB+3a^2C-b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cdots}}{\sqrt{a+b}\sqrt{\cdots}}\right.\right.}{ab^2\sqrt{a+b\cdots}}$$

Mathematica [C] time = 6.49252, size = 1256, normalized size = 2.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[
c + d*x]))/(b*(-a^2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*C - b^
2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(2*a*A*b - 2*b^2*B + 2*a*b*C)*((Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
```

```
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(2*A*b^2 - 2
*a*b*B + 3*a^2*C - b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec
[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*
x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
b*Sqrt[Cos[c + d*x]])))/(2*(a - b)*b*(a + b)*d)
```

Maple [B] time = 0.21, size = 3691, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 1/d*(-2*A*cos(d*x+c)^2*a*b^2+2*A*cos(d*x+c)*a*b^2-C*cos(d*x+c)^3*a^2*b+3*C*
cos(d*x+c)^2*a^2*b+C*cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)*a^2*b-C*cos(d*x+c)*a
*b^2+2*B*cos(d*x+c)^2*a^2*b-2*B*cos(d*x+c)^2*a*b^2-2*B*cos(d*x+c)*a^2*b+2*B
*cos(d*x+c)*a*b^2+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)*b^3-2*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a*b^2-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+2*C*sin
```

```
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-6*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+6*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*a^3-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*a^3+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3-2*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-
3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^3+C*cos(d*x+c)^3*b^3-3*C*cos(d*x+c)^2*a^3-C*cos(d*x+c)^2*b^
```

```
3+3*C*cos(d*x+c)*a^3+2*A*cos(d*x+c)^2*b^3+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*a^2*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-2*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^3+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-2*B*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^
3-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b^2/(a^2-b^2)/cos(d*x+c)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^(3/2), x)
```

3.1147 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=436

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\cot(c+dx)\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\right.}{a^2bd\sqrt{a+b}}$$

```
[Out] (2*(A*b^2 - a*(b*B - a*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*b*Sqrt[a
 + b]*d) + (2*(A*b + b*B - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqr
t[a + b]*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(b^2*d) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.899601, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\cot(c+dx)\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\right.}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[
c + d*x])^(3/2)),x]
```


```
[Out] (2*(A*b^2 - a*(b*B - a*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*b*Sqrt[a
 + b]*d) + (2*(A*b + b*B - a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqr
t[a + b]*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
```

```
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(b^2*d) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{Ab+(bB-aC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{b} + \frac{C\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+b}d}$$

Mathematica [C] time = 6.52313, size = 1245, normalized size = 2.86

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[
c + d*x]))/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*A - A*b
```

```
^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-(a*A*b) + a^2*B - a*b*C)*((Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-(A*b^2) + a*b*
B - a^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSin
h[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x
])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
d*x]])))/(a*(a - b)*(a + b)*d)
```

Maple [B] time = 0.276, size = 2856, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] -2/d*(-A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a*b^2+C*cos(d*x+c)^2*a^2*b-C*cos(d
*x+c)*a^2*b+B*cos(d*x+c)^2*a^2*b-B*cos(d*x+c)^2*a*b^2-B*cos(d*x+c)*a^2*b+B*
cos(d*x+c)*a*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
```

```
-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*cos(d*x+c)*b^3-A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a
*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a^2*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+2*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*a^3+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3
+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
```

```
b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3-C*cos(d*x+c)^2*a^3+C*cos(d*x+c)*a^3+A*cos(d*x+c)^2*b
^3+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/a/b/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^3 + 2\,ab\cos(dx+c)^2 + a^2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*sqrt(cos(d*x + c))), x)
```

3.1148 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=322

$$\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\cot(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{a^3d\sqrt{a+b}}$$

```
[Out] (-2*(2*A*b^2 - a*b*B - a^2*(A - C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3
*Sqrt[a + b]*d) - (2*(2*A*b + a*(A - B - C))*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(a^2*Sqrt[a + b]*d) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.683155, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\cot(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[
c + d*x])^(3/2)),x]
```

```
[Out] (-2*(2*A*b^2 - a*b*B - a^2*(A - C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3
*Sqrt[a + b]*d) - (2*(2*A*b + a*(A - B - C))*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(a^2*Sqrt[a + b]*d) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(-2Ab^2+abB+a^2(A-C)\right)-\frac{1}{2}}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}}{a\left(a^2-\right.}$$

$$= \frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{\left(2Ab^2 - abB - a^2(A-C)\right)}{a\left(a^2\right.}$$

$$= -\frac{2\left(2Ab^2 - abB - a^2(A-C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle| -\frac{a+b}{a-b}\right)}{a^3\sqrt{a+b}d}$$

<u>Mathematica</u> [C] time = 6.67253, size = 1306, normalized size = 4.06

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(2*a^2*A*b - 2*A*b^3 - a^3*B + a*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(a^3*A - 2*a*A*b^2 + a^2*b*B - a^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(a^2*A*b - 2*A*b^3 + a*b^2*B - a^2*b*
C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
 + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a +
b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
```

```
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))
/(a^2*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((
-2*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(a^2
*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/a^2))/d
```

Maple [B] time = 0.403, size = 3086, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(a*A*b^2-A*a^3+A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x
+c)*a*b^2-C*cos(d*x+c)^2*a^2*b+C*cos(d*x+c)*a^2*b-A*cos(d*x+c)*a^2*b-B*cos(
d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)*a^2*b-B*cos(d*x+c)*a*b^2-A
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*cos(d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+A*cos(d*x+c)*
a^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*a*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
```

```
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^3+A*cos(d*x+c)^2*a^2*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b
+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*b^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3+C*cos(d*x+c)^2*a^3-C*cos(d*x+c)*a^3-2*A*cos(d*x+c)^2*b^3
+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
```

```
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/a^2/(a^2-b^2)/sin(d*x+c)/cos(d*x+
c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^4+2\,ab\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x
+ c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(3/2)), x)
```

3.1149 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=424

$$-\frac{2\sin(c+dx)\left(a^2(-(A-3C))-3abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\,\mathrm{co}\ldots}{\ldots}$$

```
[Out] (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(8*A*b^2 + 6*a*b*(A - B) + a^2*
(A - 3*B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) +
 (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/
2)*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.11029, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\left(a^2(-(A-3C))-3abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\,\mathrm{co}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[
c + d*x])^(3/2)),x]
```

```
[Out] (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(8*A*b^2 + 6*a*b*(A - B) + a^2*
(A - 3*B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) +
 (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/
```

```
2)*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(-4Ab^2+3abB+a^2(A-3C)\right)-\frac{1}{2}\cdots}{\cos\cdots}}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(4Ab^2-3abB-a^2(A-3C\cdots\right.}{3a^2\left(a^2-b^2\cdots\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(4Ab^2-3abB-a^2(A-3C\cdots\right.}{3a^2\left(a^2-b^2\cdots\right.}$$

$$= \frac{2\left(8Ab^3+3a^3B-6ab^2B-a^2(5Ab-3bC)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+\cdots}}{\sqrt{a+b}\sqrt{\cos(c+\cdots}}\cdots\right.\right.}{3a^4\sqrt{a+b}d}$$

Mathematica [C] time = 6.86999, size = 1402, normalized size = 3.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(a^4*A + 7*a^2*A*b^2 - 8*A*b^4 - 6*a^3*b*B + 6*a*b^3*B + 3*a^4*C - 3
*a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(5*a^3*A*b - 8*a*A*b^3 - 3*a^
4*B + 6*a^2*b^2*B - 3*a^3*b*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
```

```
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])) + 2*(5*a^2*A*b^2 - 8*A*b^4 - 3*a^3*b*B + 6*a*b^3*B - 3
*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSi
nh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*
x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
 d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]*((2*Sec[c + d*x]*(-5*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^3
) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x])
)/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/
(3*a^2)))/d
```

Maple [B] time = 0.423, size = 4192, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(1/2)*(a^2*A*b^2-A*a^4+3*C*cos(d*x+c)^3*a^2*b^2-5*A
*cos(d*x+c)^3*a^2*b^2-5*A*cos(d*x+c)^2*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+8*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a*b^3+5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d
```

*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+3*C*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*B*cos(d*x+c
)*a^4-8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^4+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*
A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^4+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*C*cos(d*x+c)^3
*a^3*b+3*B*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^2*a^3*b+6*B*cos(d*x+c)^2*a*b
^3+3*B*cos(d*x+c)*a^2*b^2+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^4-5*A*cos
(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a^3*b-4*A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x
+c)*a^3*b+A*cos(d*x+c)^3*a^3*b+A*cos(d*x+c)^2*a^4+8*A*cos(d*x+c)^2*a*b^3-4*
A*cos(d*x+c)*a*b^3+3*B*cos(d*x+c)^2*a^4+3*B*cos(d*x+c)^3*a^3*b-6*B*cos(d*x+
c)^3*a*b^3-6*B*cos(d*x+c)^2*a^2*b^2+3*C*cos(d*x+c)^2*a^3*b-3*C*cos(d*x+c)^2
*a^2*b^2+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)
^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^3-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^

```
(1/2))*a*b^3-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*C*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b+5*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*B*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4-3*B*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)*a^4+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^4+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*B*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2+6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-6*B*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)/
a^3/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^5 + 2\,ab\cos(dx+c)^4 + a^2\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x
+ c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(5/2)), x)
```

3.1150 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=545

$$\frac{2\sin(c+dx)\left(-a^2(9Ab-15bC)+5a^3B-20ab^2B+24Ab^3\right)\sqrt{a+b\cos(c+dx)}}{15a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sin(c+dx)\left(a^2(-(A-5C))-5abB+\cdots\right.}{5a^2d\left(a^2-b^2\right)\cos^{\frac{5}{2}}(\cdots}$$

```
[Out] (-2*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*
A + 5*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^5*Sqrt[a + b]*d) - (2*(4
8*A*b^3 + 4*a*b^2*(9*A - 10*B) + 6*a^2*b*(2*A - 5*B + 5*C) + a^3*(9*A - 5*B
 + 15*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*Sqrt[a + b]*d) + (2*(A
*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqr
t[a + b*Cos[c + d*x]]) - (2*(6*A*b^2 - 5*a*b*B - a^2*(A - 5*C))*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) + (2*(
24*A*b^3 + 5*a^3*B - 20*a*b^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.73801, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(-a^2(9Ab-15bC)+5a^3B-20ab^2B+24Ab^3\right)\sqrt{a+b\cos(c+dx)}}{15a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2\sin(c+dx)\left(a^2(-(A-5C))-5abB+\cdots\right.}{5a^2d\left(a^2-b^2\right)\cos^{\frac{5}{2}}(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + b*Cos[
c + d*x])^(3/2)),x]
```

```
[Out] (-2*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*
A + 5*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^5*Sqrt[a + b]*d) - (2*(4
8*A*b^3 + 4*a*b^2*(9*A - 10*B) + 6*a^2*b*(2*A - 5*B + 5*C) + a^3*(9*A - 5*B
```

```
 + 15*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*Sqrt[a + b]*d) + (2*(A
*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqr
t[a + b*Cos[c + d*x]]) - (2*(6*A*b^2 - 5*a*b*B - a^2*(A - 5*C))*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) + (2*(
24*A*b^3 + 5*a^3*B - 20*a*b^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(-6Ab^2+5abB+a^2(A-5C)\right)-\frac{1}{2}}{\text{co}}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-5C\right.}{5a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-5C\right.}{5a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-5C\right.}{5a^2\left(a^2-b^2\right.}$$

$$= -\frac{2\left(48Ab^4+25a^3bB-40ab^3B-6a^2b^2(4A-5C)-3a^4(3A+5C)\right)\cot(c+}{15a^5\sqrt{a+b}}$$

Mathematica [C] time = 7.0898, size = 1511, normalized size = 2.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(12*a^4*A*b + 36*a^2*A*b^3 - 48*A*b^5 - 5*a^5*B - 35*a^3*b^2*B + 40*
a*b^4*B + 30*a^4*b*C - 30*a^2*b^3*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(9*a
^5*A + 24*a^3*A*b^2 - 48*a*A*b^4 - 25*a^4*b*B + 40*a^2*b^3*B + 15*a^5*C - 3
0*a^3*b^2*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)) + 2*(9*a^4*A*b + 24*a^2*A*b^3 - 48*A*b^5 - 25*a^3*b^2*B + 40*a*b^4*B + 1
5*a^4*b*C - 30*a^2*b^3*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec
[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*
x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
b*Sqrt[Cos[c + d*x]])))/(15*a^4*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(-9*A*b*Sin[c + d*x] + 5*a*B*Sin
[c + d*x]))/(15*a^3) + (2*Sec[c + d*x]*(9*a^2*A*Sin[c + d*x] + 33*A*b^2*Sin
[c + d*x] - 25*a*b*B*Sin[c + d*x] + 15*a^2*C*Sin[c + d*x]))/(15*a^4) - (2*(
A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x]))/(a^4*(
a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5*a^2
)))/d
```

Maple [B] time = 0.299, size = 5884, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^6 + 2\,ab\cos(dx+c)^5 + a^2\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^2*cos(d*x + c)^6 + 2*a*b*cos(d*x + c)^5 + a^2*cos(d*x
+ c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*cos(d*x + c)^(7/2)), x)
```

3.1151 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=723

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+\right.}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}$$

```
[Out] ((8*A*b^4 + 6*a^3*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((6*A*b
^4 - a*b^3*(2*A + 3*(4*B - C)) + a^3*b*(6*B - 5*C) - 15*a^4*C + a^2*b^2*(2*
B + 21*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*b^3*Sqrt[a + b]*(a^2 - b
^2)*d) - (Sqrt[a + b]*(2*b*B - 5*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(b^4*d) - (2*(A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^3*b
*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((8*A*b^4 + 6*a^3*b*
B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 2.63006, antiderivative size = 723, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+\right.}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^(5/2),x]
```

```
[Out] ((8*A*b^4 + 6*a^3*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((6*A*b
^4 - a*b^3*(2*A + 3*(4*B - C)) + a^3*b*(6*B - 5*C) - 15*a^4*C + a^2*b^2*(2*
B + 21*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*b^3*Sqrt[a + b]*(a^2 - b
^2)*d) - (Sqrt[a + b]*(2*b*B - 5*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(b^4*d) - (2*(A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^3*b
*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((8*A*b^4 + 6*a^3*b*
B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\cos}}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3Ab^4\right.}{}$$

$$= -\frac{\sqrt{a+b}(2bB-5aC)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+d}}\right.\right.}{b^4d}$$

$$= \frac{\left(8Ab^4+6a^3bB-14ab^3B-15a^4C+26a^2b^2C-3b^4C\right)\cot(}{3a(a-}$$

Mathematica [C] time = 6.81725, size = 1448, normalized size = 2.

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(a*A*b^2*Sin[c + d*x] - a
^2*b*B*Sin[c + d*x] + a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c
 + d*x])^2) + (2*(4*A*b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[c + d*x] - 7*a*b^3*B
*Sin[c + d*x] - 6*a^4*C*Sin[c + d*x] + 10*a^2*b^2*C*Sin[c + d*x]))/(3*b^2*(
-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(2*a^2*A*b^2 - 2*A*b^4 - 2
```

```
*a^3*b*B + 2*a*b^3*B + 5*a^4*C - 8*a^2*b^2*C + 3*b^4*C)*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]) - 4*a*(-8*a*A*b^3 + 2*a^2*b^2*B + 6*b^4*B + 4*a^3*b*C - 12*a*b^3*C
)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-8*
A*b^4 - 6*a^3*b*B + 14*a*b^3*B + 15*a^4*C - 26*a^2*b^2*C + 3*b^4*C)*((I*Cos
[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*
a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
 (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(6*(a - b
)^2*b^2*(a + b)^2*d)
```

Maple [B] time = 0.516, size = 10402, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(5/2), x)
```

3.1152 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=589

$$\frac{2\sin(c+dx)\left(a^2b^2(3A+7C)-3a^4C-4ab^3B+Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\cot(c+dx)}{}$$

```
[Out] (-2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*b^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*(b^3*(A + 3*B) +
 3*a^3*C + a^2*b*C - a*b^2*(3*A + B + 6*C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a*b^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) +
(2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sin[c + d*x])/(3*b^2
*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.61775, antiderivative size = 589, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3047, 3051, 2809, 2993, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(a^2b^2(3A+7C)-3a^4C-4ab^3B+Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos
[c + d*x])^(5/2),x]
```


```
[Out] (-2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*b^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*(b^3*(A + 3*B) +
```

```
 3*a^3*C + a^2*b*C - a*b^2*(3*A + B + 6*C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a*b^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*Sqrt[a + b]*C*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) +
(2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sin[c + d*x])/(3*b^2
*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(Ab^2\right.}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}b\left(Ab\right.}{}}{}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^3d}$$

$$= -\frac{2\sqrt{a+b}C\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^3d}$$

$$= -\frac{2\left(Ab^4-4ab^3B-3a^4C+a^2b^2(3A+7C)\right)\cot(c+dx)E\left(\mathrm{si}\right.}{3a^2(a-b)b^2}$$

<u>Mathematica</u> [C] time = 6.77726, size = 1441, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a +
 b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(A*b^2*Sin[c + d*x] - a*b*
B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]
)^2) + (2*(-3*a^2*A*b^2*Sin[c + d*x] - A*b^4*Sin[c + d*x] + 4*a*b^3*B*Sin[c
 + d*x] + 3*a^4*C*Sin[c + d*x] - 7*a^2*b^2*C*Sin[c + d*x]))/(3*a*b*(a^2 - b
^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(a^2*A*b^2 - A*b^4 - a^3*b*B + a*b
^3*B + a^4*C - a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3*a^3*A*b - a
*A*b^3 + 4*a^2*b^2*B - a^3*b*C - 3*a*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
```

```
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-3*a^2*A*b^2 - A*b^4 + 4*a*b^3*B + 3
*a^4*C - 7*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c +
 d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*
Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sq
rt[Cos[c + d*x]])))/(3*a*(a - b)^2*b*(a + b)^2*d)
```

Maple [B] time = 0.386, size = 8239, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*co
s(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c) + a)^(5/2), x)
```

3.1153 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=457

$$\frac{2\sin(c+dx)\left(-2a^2b(3A+2C)+3a^3B+ab^2B+2Ab^3\right)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\cot(c+dx}{}$$

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B + 4*a^2*b*C)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 - a^2*(3*A +
 3*B + C) + a*b*(3*A + B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*
Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(2*A*b^3
+ 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)^2
*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.14745, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3055, 2993, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(-2a^2b(3A+2C)+3a^3B+ab^2B+2Ab^3\right)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Cos[
c + d*x])^(5/2)),x]
```

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B + 4*a^2*b*C)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 - a^2*(3*A +
 3*B + C) + a*b*(3*A + B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*
```

```
Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(2*A*b^3
+ 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)^2
*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(-2Ab^2-abB+a^2(3A+C)\right)-}{\sqrt{\cos(c+dx)}(a+b}}{3a\left(a^2-\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(2Ab^3+3a^3B+ab^2B-\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(2Ab^3+3a^3B+ab^2B-\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c}}$$

$$= -\frac{2\left(2Ab^3+3a^3B+ab^2B-2a^2b(3A+2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.70008, size = 1440, normalized size = 3.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(A*b^2*Sin[c + d*x] - a*b*
B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])
^2) - (2*(-6*a^2*A*b^2*Sin[c + d*x] + 2*A*b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[
c + d*x] + a*b^3*B*Sin[c + d*x] - 4*a^2*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2 -
b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3*a^4*A - 5*a^2*A*b^2 + 2*A*b^4
- a^3*b*B + a*b^3*B + a^4*C - a^2*b^2*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*
(-6*a^3*A*b + 2*a*A*b^3 + 3*a^4*B + a^2*b^2*B - 4*a^3*b*C)*((Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*A*b^2 + 2*A*b^
4 + 3*a^3*b*B + a*b^3*B - 4*a^2*b^2*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/
(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
 b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 1.071, size = 7003, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/
2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^4 + 3\,ab^2\cos(dx+c)^3 + 3\,a^2b\cos(dx+c)^2 + a^3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*co
s(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/
2)*sqrt(cos(d*x + c))), x)
```

3.1154 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$-\frac{2\sin(c+dx)\left(-2a^2b^2(4A+C)+5a^3bB-2a^4C-ab^3B+4Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+$$

```
[Out] (2*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(8
*A*b^3 + 2*a*b^2*(3*A - B) - 3*a^3*(A - B - C) - a^2*b*(9*A + 3*B + C))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a
+ b*Cos[c + d*x])^(3/2)) - (2*(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2*
a^2*b^2*(4*A + C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.31959, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\left(-2a^2b^2(4A+C)+5a^3bB-2a^4C-ab^3B+4Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Cos[
c + d*x])^(5/2)),x]
```

```
[Out] (2*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(8
*A*b^3 + 2*a*b^2*(3*A - B) - 3*a^3*(A - B - C) - a^2*b*(9*A + 3*B + C))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
```

```
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a
+ b*Cos[c + d*x])^(3/2)) - (2*(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2*
a^2*b^2*(4*A + C))*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(-4Ab^2+abB+a^2(3A-C)\right)}{}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4Ab^4+5a^3bB-ab^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4Ab^4+5a^3bB-ab^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2\left(8Ab^4+6a^3bB-2ab^3B+3a^4(A-C)-a^2b^2(15A+C)\right)\cot(c+dx)E\left(\sin\right.}{3a^4(a-b)(a+b)^{3/2}a}$$

Mathematica [C] time = 6.92646, size = 1516, normalized size = 3.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -((-4*a*(9*a^4*A*b - 17*a^2*A*b^3 + 8*A*b^5 - 3*a^5*B + 5*a^3*b^2*B - 2*a*b
^4*B + a^4*b*C - a^2*b^3*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*A - 15
*a^3*A*b^2 + 8*a*A*b^4 + 6*a^4*b*B - 2*a^2*b^3*B - 3*a^5*C - a^3*b^2*C)*((S
```

```
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^4*A*
b - 15*a^2*A*b^3 + 8*A*b^5 + 6*a^3*b^2*B - 2*a*b^4*B - 3*a^4*b*C - a^2*b^3*
C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
 + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a +
b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))
/(3*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]*((-2*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))
/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(9*a^2*A*b^3*Sin[c + d*x]
- 5*A*b^5*Sin[c + d*x] - 6*a^3*b^2*B*Sin[c + d*x] + 2*a*b^4*B*Sin[c + d*x]
+ 3*a^4*b*C*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a
 + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/a^3))/d
```

Maple [B] time = 1.44, size = 8926, normalized size = 18.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/
2)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{5}+3\,ab^{2}\cos\left(dx+c\right)^{4}+3\,a^{2}b\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*co
s(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/
2)*cos(d*x + c)^(3/2)), x)
```

3.1155 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=620

$$\frac{2\sin(c+dx)\left(-a^2b^2(13A-C)+a^4(A-5C)+8a^3bB-4ab^3B+8Ab^4\right)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(10a^2Ab^2-7\right.}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c}$$

```
[Out] (-2*(16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) +
 a^4*(8*A*b - 6*b*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*Sqrt[a + b]
*(a^2 - b^2)*d) - (2*(16*A*b^4 + 4*a*b^3*(3*A - 2*B) - 3*a^3*b*(3*A - 3*B -
 C) - 2*a^2*b^2*(8*A + 3*B - C) - a^4*(A - 3*B + 3*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 - a*(b*B - a*C
))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a^3*b*B + 3*a*b^3*B + 4*a^4*C)*Sin
[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x
]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B + a^4*(A - 5*C) - a^2*b^2*(13*A -
 C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c +
d*x]^(3/2))
```

Rubi [A] time = 2.30145, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\left(-a^2b^2(13A-C)+a^4(A-5C)+8a^3bB-4ab^3B+8Ab^4\right)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(10a^2Ab^2-7\right.}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[
c + d*x])^(5/2)),x]
```

```
[Out] (-2*(16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) +
 a^4*(8*A*b - 6*b*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
```

```
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*Sqrt[a + b]
*(a^2 - b^2)*d) - (2*(16*A*b^4 + 4*a*b^3*(3*A - 2*B) - 3*a^3*b*(3*A - 3*B -
 C) - 2*a^2*b^2*(8*A + 3*B - C) - a^4*(A - 3*B + 3*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(A*b^2 - a*(b*B - a*C
))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a^3*b*B + 3*a*b^3*B + 4*a^4*C)*Sin
[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x
]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B + a^4*(A - 5*C) - a^2*b^2*(13*A -
 C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c +
d*x]^(3/2))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
```

```
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}\left(2Ab^2-abB-a^2(A-C)\right)-}{}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\right.}{3a^2\left(a^2-b^2\right)^2d\cos}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\right.}{3a^2\left(a^2-b^2\right)^2d\cos}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\right.}{3a^2\left(a^2-b^2\right)^2d\cos}$$

$$= -\frac{2\left(16Ab^5-3a^5B+15a^3b^2B-8ab^4B-2a^2b^3(14A-C)+a^4(8Ab-6bC)\right)}{3a^5(a-b)(}$$

Mathematica [C] time = 7.14604, size = 1601, normalized size = 2.58

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]

[Out] ((-4*a*(a^6*A + 15*a^4*A*b^2 - 32*a^2*A*b^4 + 16*A*b^6 - 9*a^5*b*B + 17*a^3*b^3*B - 8*a*b^5*B + 3*a^6*C - 5*a^4*b^2*C + 2*a^2*b^4*C)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*A*b - 28*a^3*A*b^3 + 16*a*A*b^5 - 3*a^6*B + 15*a^4*b^2*B - 8*a^2*b^4*B - 6*a^5*b*C + 2*a^3*b^3*C)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*A*b^2 - 28*a^2*A*b^4 + 16*A*b^6 - 3*a^5*b*B + 15*a^3*b^3*B - 8*a*b^5*B - 6*a^4*b^2*C + 2*a^2*b^4*C)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(3*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-8*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^4) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(12*a^2*A*b^4*Sin[c + d*x] - 8*A*b^6*Sin[c + d*x] - 9*a^3*b^3*B*Sin[c + d*x] + 5*a*b^5*B*Sin[c + d*x] + 6*a^4*b^2*C*Sin[c + d*x] - 2*a^2*b^4*C*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d

Maple [B] time = 0.454, size = 10927, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/
2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^6 + 3\,ab^2\cos(dx+c)^5 + 3\,a^2b\cos(dx+c)^4 + a^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*co
s(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/
2)*cos(d*x + c)^(5/2)), x)
```

3.1156 $\int \cos^m(c+dx)(a+b\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos\right.$

Optimal. Leaf size=367

$$-\frac{\sin(c+dx)\cos^{m+1}(c+dx)\left(a^2(m+4)(A(m+2)+C(m+1))+2abB\left(m^2+5m+4\right)+b^2(m+1)(A(m+4)+C(m+3)\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] ((2*a^2*C + b^2*C*(3 + m) + A*b^2*(4 + m) + 2*a*b*B*(4 + m))*Cos[c + d*x]^(
1 + m)*Sin[c + d*x])/(d*(2 + m)*(4 + m)) + (b*(2*a*C + b*B*(4 + m))*Cos[c +
 d*x]^(2 + m)*Sin[c + d*x])/(d*(3 + m)*(4 + m)) + (C*Cos[c + d*x]^(1 + m)*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*(4 + m)) - ((2*a*b*B*(4 + 5*m + m^2)
 + a^2*(4 + m)*(C*(1 + m) + A*(2 + m)) + b^2*(1 + m)*(C*(3 + m) + A*(4 + m)
))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2]) -
((b^2*B*(2 + m) + a^2*B*(3 + m) + 2*a*b*(C*(2 + m) + A*(3 + m)))*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.939783, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3049, 3033, 3023, 2748, 2643}

$$-\frac{\sin(c+dx)\cos^{m+1}(c+dx)\left(a^2(m+4)(A(m+2)+C(m+1))+2abB\left(m^2+5m+4\right)+b^2(m+1)(A(m+4)+C(m+3)\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```



```
[Out] ((2*a^2*C + b^2*C*(3 + m) + A*b^2*(4 + m) + 2*a*b*B*(4 + m))*Cos[c + d*x]^(
1 + m)*Sin[c + d*x])/(d*(2 + m)*(4 + m)) + (b*(2*a*C + b*B*(4 + m))*Cos[c +
 d*x]^(2 + m)*Sin[c + d*x])/(d*(3 + m)*(4 + m)) + (C*Cos[c + d*x]^(1 + m)*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*(4 + m)) - ((2*a*b*B*(4 + 5*m + m^2)
 + a^2*(4 + m)*(C*(1 + m) + A*(2 + m)) + b^2*(1 + m)*(C*(3 + m) + A*(4 + m)
))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2]) -
((b^2*B*(2 + m) + a^2*B*(3 + m) + 2*a*b*(C*(2 + m) + A*(3 + m)))*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
```

```
[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{C\cos^{1+m}(c+dx)(a+b\cos(c+dx))^2\sin(\ldots}{d(4+m)}$$

$$= \frac{b(2aC+bB(4+m))\cos^{2+m}(c+dx)\sin(\ldots}{d(3+m)(4+m)}$$

$$= \frac{\left(2a^2C+b^2C(3+m)+Ab^2(4+m)+2ab\ldots\right.}{d(2+m)(\ldots}$$

$$= \frac{\left(2a^2C+b^2C(3+m)+Ab^2(4+m)+2ab\ldots\right.}{d(2+m)(\ldots}$$

$$= \frac{\left(2a^2C+b^2C(3+m)+Ab^2(4+m)+2ab\ldots\right.}{d(2+m)(\ldots}$$

Mathematica [A] time = 3.26577, size = 268, normalized size = 0.73

$$\frac{\sin(c+dx)\cos^{m+1}(c+dx)\left(\cos(c+dx)\left(\cos(c+dx)\left(b\cos(c+dx)\left(-\frac{(2aC+bB)\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(c+dx)\right)}{m+4}-\frac{bC\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\ldots\right)}{m+\ldots}\right.\right.\right.\right.}{d\sqrt{\sin^{\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*(-((a^2*A*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2
, Cos[c + d*x]^2])/(1 + m)) + Cos[c + d*x]*(-((a*(2*A*b + a*B)*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m)) + Cos[c + d*x]*(
-(((A*b^2 + a*(2*b*B + a*C))*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, C
os[c + d*x]^2])/(3 + m)) + b*Cos[c + d*x]*(-(((b*B + 2*a*C)*Hypergeometric2
F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c + d*x]^2])/(4 + m)) - (b*C*Cos[c + d*x]
*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Cos[c + d*x]^2])/(5 + m)))))*
Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.783, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^m (a+b\cos (dx+c))^2 \left(A+B\cos (dx+c)+C(\cos (dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c)+a)^2\cos (dx+c)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*co
s(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Cb^2\cos (dx+c)^4+\left(2\,Cab+Bb^2\right)\cos (dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos (dx+c)^2+\left(Ba^2+2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*co
s(d*x + c)^m, x)
```

3.1157 $\int \cos^m(c+dx)(a+b\cos(c+dx)) \left(A + B\cos(c+dx) + C\cos\right.$

Optimal. Leaf size=235

$$-\frac{\sin(c+dx)\cos^{m+1}(c+dx)(aA(m+2)+(m+1)(aC+bB))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}} - \frac{\sin(c+dx)\cos^{m+2}(c+}{}$$

```
[Out] ((b*B + a*C)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (b*C*Cos[c +
d*x]^(2 + m)*Sin[c + d*x])/(d*(3 + m)) - (((b*B + a*C)*(1 + m) + a*A*(2 + m
))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - ((b*C*(2
 + m) + A*b*(3 + m) + a*B*(3 + m))*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1
/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*
Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.373575, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3033, 3023, 2748, 2643}

$$-\frac{\sin(c+dx)\cos^{m+1}(c+dx)(aA(m+2)+(m+1)(aC+bB))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}} - \frac{\sin(c+dx)\cos^{m+2}(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```


```
[Out] ((b*B + a*C)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (b*C*Cos[c +
d*x]^(2 + m)*Sin[c + d*x])/(d*(3 + m)) - (((b*B + a*C)*(1 + m) + a*A*(2 + m
))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - ((b*C*(2
 + m) + A*b*(3 + m) + a*B*(3 + m))*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1
/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*
Sqrt[Sin[c + d*x]^2])
```


Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{bC\cos^{2+m}(c+dx)\sin(c+dx)}{d(3+m)} + \frac{\int \cos^m(c}{}$$

$$= \frac{(bB+aC)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \frac{b}{}$$

$$= \frac{(bB+aC)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \frac{b}{}$$

$$= \frac{(bB+aC)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \frac{b}{}$$

Mathematica [A] time = 1.85481, size = 205, normalized size = 0.87

$$\frac{\sin(c+dx)\cos^{m+1}(c+dx)\left(\cos(c+dx)\left(\cos(c+dx)\left(-\frac{(aC+bB)\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(c+dx)\right)}{m+3}-\frac{bC\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(c+dx\right.}{m+4}\right.\right.\right.}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*(-((a*A*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Cos[c + d*x]^2])/(1 + m)) + Cos[c + d*x]*(-(((A*b + a*B)*Hypergeometric2F1[
1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m)) + Cos[c + d*x]*(-(((b*
B + a*C)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3 +
 m)) - (b*C*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c
 + d*x]^2])/(4 + m))))*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.175, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a+b\cos(dx+c)) \left(A+B\cos(dx+c)+C(\cos(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(
d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(
d*x + c)^m, x)
```

3.1158 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=372

$$\frac{a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{b^2d\left(a^2-b^2\right)}-\frac{\sin(c+d}{}$$

```
[Out] (a*(A*b^2 - a*(b*B - a*C))*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2,
 -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2
)^((1 - m)/2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*
AppellF1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b
^2))]*Cos[c + d*x]^m*Sin[c + d*x])/(b*(a^2 - b^2)*d*(Cos[c + d*x]^2)^(m/2))
 - ((b*B - a*C)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) -
 (C*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.420838, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3063, 2643, 2823, 3189, 429}

$$\frac{a\sin(c+dx)\left(Ab^2-a(bB-aC)\right)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{b^2d\left(a^2-b^2\right)}-\frac{\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x]),x]
```


```
[Out] (a*(A*b^2 - a*(b*B - a*C))*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2,
 -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2
)^((1 - m)/2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*
AppellF1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b
^2))]*Cos[c + d*x]^m*Sin[c + d*x])/(b*(a^2 - b^2)*d*(Cos[c + d*x]^2)^(m/2))
 - ((b*B - a*C)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) -
 (C*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3063

```
Int[(((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)
*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2))/((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Dist[(b*B - a*C)/b^2, Int[(d*Sin[e + f*x])^n, x], x] +
 (Dist[(A*b^2 - a*b*B + a^2*C)/b^2, Int[(d*Sin[e + f*x])^n/(a + b*Sin[e + f
*x]), x], x] + Dist[C/(b*d), Int[(d*Sin[e + f*x])^(n + 1), x], x]) /; FreeQ
[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx = \frac{C\int \cos^{1+m}(c+dx)\,dx}{b} + \frac{(bB-aC)\int \cos^m(c+dx)\,dx}{b^2} + \Big(A$$

$$= -\frac{(bB-aC)\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)}{b^2d(1+m)\sqrt{\sin^2(c+dx)}}$$

$$= -\frac{(bB-aC)\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)}{b^2d(1+m)\sqrt{\sin^2(c+dx)}}$$

$$= \frac{a\left(A-\frac{a(bB-aC)}{b^2}\right)F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 30.1837, size = 15557, normalized size = 41.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.754, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m\left(A+B\cos(dx+c)+C(\cos(dx+c))^2\right)}{a+b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{m}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{m}}{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{m}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c)),x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c) + a), x)
```

3.1159 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=564

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\left(a^2b^2(A(-m)+A+C(m+2))+a^3bBm+a^4(-C)(m+1)-ab^3B(m+1)+A\right.}{b^2d\left(a^2-b^2\right)^2}$$

```
[Out] ((A*b^4*m + a^3*b*B*m - a*b^3*B*(1 + m) - a^4*C*(1 + m) + a^2*b^2*(A - A*m
+ C*(2 + m)))*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c
 + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)
*Sin[c + d*x])/(b^2*(a^2 - b^2)^2*d) - ((A*b^4*m + a^3*b*B*m - a*b^3*B*(1 +
 m) - a^4*C*(1 + m) + a^2*b^2*(A - A*m + C*(2 + m)))*AppellF1[1/2, -m/2, 1,
 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*S
in[c + d*x])/(a*b*(a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2)) + ((A*b^2 - a*(b*
B - a*C))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])) + ((a*b*B*m - a^2*C*(1 + m) + b^2*(C - A*m))*Cos[c + d*x]^(1 + m)*
Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(b^2*(a^2 - b^2)*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) + ((A*b^2 - a*(b*B - a*C))
*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(a*b*(a^2 - b^2)*d*(2 + m)*Sqrt[Sin[c + d*x]^
2])
```

Rubi [A] time = 1.02937, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3055, 3063, 2643, 2823, 3189, 429}

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\left(a^2b^2(A(-m)+A+C(m+2))+a^3bBm+a^4(-C)(m+1)-ab^3B(m+1)+A\right.}{b^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*Cos[c +
 d*x])^2,x]
```


```
[Out] ((A*b^4*m + a^3*b*B*m - a*b^3*B*(1 + m) - a^4*C*(1 + m) + a^2*b^2*(A - A*m
+ C*(2 + m)))*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c
 + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)
*Sin[c + d*x])/(b^2*(a^2 - b^2)^2*d) - ((A*b^4*m + a^3*b*B*m - a*b^3*B*(1 +
```

```
 m) - a^4*C*(1 + m) + a^2*b^2*(A - A*m + C*(2 + m)))*AppellF1[1/2, -m/2, 1,
 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*S
in[c + d*x])/(a*b*(a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2)) + ((A*b^2 - a*(b*
B - a*C))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])) + ((a*b*B*m - a^2*C*(1 + m) + b^2*(C - A*m))*Cos[c + d*x]^(1 + m)*
Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(b^2*(a^2 - b^2)*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) + ((A*b^2 - a*(b*B - a*C))
*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(a*b*(a^2 - b^2)*d*(2 + m)*Sqrt[Sin[c + d*x]^
2])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3063

```
Int[(((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)
*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2))/((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Dist[(b*B - a*C)/b^2, Int[(d*Sin[e + f*x])^n, x], x] +
 (Dist[(A*b^2 - a*b*B + a^2*C)/b^2, Int[(d*Sin[e + f*x])^n/(a + b*Sin[e + f
*x]), x], x] + Dist[C/(b*d), Int[(d*Sin[e + f*x])^(n + 1), x], x]) /; FreeQ
[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\cos^m(c+dx)}{}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(\left(Ab^2-a(\right.\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(abBm-a\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(abBm-a\right.}{}$$

$$= \frac{\left(Ab^4m+a^3bBm-ab^3B(1+m)-a^4C(1+m)+a^2b^2(A-Am\right.}{}$$

Mathematica [B] time = 35.4635, size = 25789, normalized size = 45.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(a + b*C
os[c + d*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.608, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m \left(A + B\cos(dx+c) + C(\cos(dx+c))^2\right)}{(a+b\cos(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\cos(dx+c)^m}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{m}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{m}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c) + a)^2, x)
```

3.1160 $\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=205

$$\frac{2a(5A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{21d}$$

```
[Out] (-2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(5*d) + (2*a*(5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (
2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*A*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(7*d)
```

Rubi [A] time = 0.285331, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3032, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(5A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(5*d) + (2*a*(5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (
2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*A*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))\left(A+C\right.}{\cos^{\frac{9}{2}}(c+dx}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$= \frac{2a(3A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a(5A+7C)\sec}{}$$

$$= -\frac{2a(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 3.97304, size = 392, normalized size = 1.91

$$\frac{a\csc(c)e^{-idx}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)(3A+5C)e^{2idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Csc[c]*Sec[(c + d*x)/2]^2*(7*Sqrt[2]*(3*A + 5*C)*E^((
2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))] - ((-1 + E^((2*I)*c))*(35*C*(1 + E^((2*I)*(c + d*x)))^2*(-1 + 3
*E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))) + A*(-25 +
21*E^(I*(c + d*x)) - 85*E^((2*I)*(c + d*x)) + 189*E^((3*I)*(c + d*x)) + 85*
E^((4*I)*(c + d*x)) + 231*E^((5*I)*(c + d*x)) + 25*E^((6*I)*(c + d*x)) + 63
*E^((7*I)*(c + d*x))))*Sqrt[Sec[c + d*x]])/(E^(I*(c - d*x))*(1 + E^((2*I)*(
c + d*x)))^3) + 10*(5*A + 7*C)*E^(I*d*x)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]]*Sin[c]))/(210*d*E^(I*d*x))
```

Maple [B] time = 3.601, size = 838, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(-1/6
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-
1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2
*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)+1/2*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x
)
```

3.1161 $\int (a+a\cos(c+dx)) \left(A+C\cos^2(c+dx)\right) \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=172

$$\frac{2a(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5C)\sqrt{\cos(c+}{}$$

```
[Out] (-2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.263245, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3032, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5C)\sqrt{\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))\left(A+C\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{2a(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2}{}$$

$$= -\frac{2a(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 1.73715, size = 277, normalized size = 1.61

$$\frac{ae^{-ic}\left(-1+e^{2ic}\right)\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((3A+5C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])*Csc[c]*(5*A - 3*A*E^(I*(c + d*x))
- 15*C*E^(I*(c + d*x)) - 24*A*E^((3*I)*(c + d*x)) - 30*C*E^((3*I)*(c + d*x)
) - 5*A*E^((4*I)*(c + d*x)) - 9*A*E^((5*I)*(c + d*x)) - 15*C*E^((5*I)*(c +
d*x)) - (5*I)*(A + 3*C)*(1 + E^((2*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2] + (3*A + 5*C)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)
))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[(c + d
*x)/2]^2*Sqrt[Sec[c + d*x]])/(30*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] time = 3.153, size = 729, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/10*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4
-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+1/2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x
)
```

3.1162 $\int (a+a\cos(c+dx)) \left(A+C\cos^2(c+dx)\right) \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\,\mathrm{s}}{}$$

```
[Out] (-2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.235549, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4221, 3032, 3021, 2748, 2641, 2639}

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
```

```
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))\left(A+C\right.}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= -\frac{2a(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [C] time = 1.13962, size = 173, normalized size = 1.28

$$\frac{ae^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(i(A-C)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+2(A+3C)\cos^{\frac{3}{2}}(c+dx)F\left(\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-3*I)*A + (3*I)*C - (3*I)*A
*Cos[2*(c + d*x)] + (3*I)*C*Cos[2*(c + d*x)] + 2*(A + 3*C)*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + I*(A - C)*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2*A*Sin[c + d*x] + 3
*A*Sin[2*(c + d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 2.268, size = 437, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2
*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x
)
```

3.1163 $\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin}{}$$

```
[Out] (-2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*A
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.238345, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4221, 3032, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*A
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
```

```
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))\left(A + C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a + a\cos(c+dx))\left(A + C\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{3}\left(4\sqrt{c\right.$$

$$= \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(a(A-\right.$$

$$= -\frac{2a(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \cdots$$

Mathematica [C] time = 1.13892, size = 169, normalized size = 1.25

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(2i(A-C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+2(3A+C)\sqrt{\cos(c+dx)}F\cdots\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-6*I)*A*Cos[c + d*x] + (6*I
)*C*Cos[c + d*x] + 2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + (2*I)*(A - C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 6*A*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 1.201, size = 458, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*a*(4*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(3*A+C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x
)
```

3.1164 $\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a}{5d}$$

```
[Out] (2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.232775, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4221, 3034, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
```

```
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))\left(A+\right.}{\sqrt{\cos(c+d}}$$

$$= \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{5a}{2}$$

$$= \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{15}\left(4\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a(3A+C)\sqrt{\cos}\right.$$

$$= \frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 1.47329, size = 169, normalized size = 1.2

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2i(5A+3C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(6i(5A\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(10*(3*A + C)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] - (2*I)*(5*A + 3*C)*E^(I*(c + d*x))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
 + Cos[c + d*x]*((6*I)*(5*A + 3*C) + 10*C*Sin[c + d*x] + 3*C*Sin[2*(c + d*x
)])))/(15*d*E^(I*d*x))
```

Maple [A] time = 0.865, size = 345, normalized size = 2.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,C\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+44\,C\left(\sin(1/2\,dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+44*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*C*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)+Aa\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x
)
```

3.1165 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{2a(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5d}$$

```
[Out] (2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*C)*Sin[c
+ d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.25672, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3034, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*C)*Sin[c
+ d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(A+C\right.$$

$$= \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\left(\frac{7}{}\right.$$

$$= \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{35}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec}\right.$$

$$= \frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2aC\sin(c+}{7d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(7A+5C}{}$$

Mathematica [C] time = 1.98733, size = 188, normalized size = 1.08

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-28i(5A+3C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(5(28A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(7*A + 5*C)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] - (28*I)*(5*A + 3*C)*E^(I*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))] + Cos[c + d*x]*((84*I)*(5*A + 3*C) + 5*(28*A + 23*C)*Sin[c + d*x] + 42*
C*Sin[2*(c + d*x)] + 15*C*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Maple [A] time = 1.145, size = 378, normalized size = 2.2

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,C\left(\sin(1/2\,dx+c/2)\right)^8\cos(1/2\,dx+c/2)-528\,C\left(\sin(1/2\,dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*C*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-528*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(140*A+448*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A-122*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)+Aa}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{A\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{C\cos^{2}\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{C\cos^{3}\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)/sqrt(sec(c +
 d*x)), x) + Integral(C*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integral(C
*cos(c + d*x)**3/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x
)
```

3.1166 $\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{2a(9A+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9$$

```
[Out] (2*a*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(15*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
)) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(9*A + 7*C)*Sin[c
 + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*C)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.279674, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3034, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(9A+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*a*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(15*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
)) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(9*A + 7*C)*Sin[c
 + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*C)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(A+\right.$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\left(\right.$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{63}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{s\right.$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(9A+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(}{}$$

$$= \frac{2a(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2a(7A+5}{}$$

Mathematica [C] time = 2.59975, size = 204, normalized size = 1.

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(9A+7C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(30(28\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(120*(7*A + 5*C)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(9*A + 7*C)*E^(I*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))] + Cos[c + d*x]*((1512*I)*A + (1176*I)*C + 30*(28*A + 23*C)*Sin[c + d*x
] + 14*(18*A + 19*C)*Sin[2*(c + d*x)] + 90*C*Sin[3*(c + d*x)] + 35*C*Sin[4*
(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 0.949, size = 406, normalized size = 2.

$$-\frac{2a}{315d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-1120\,C\left(\sin(1/2\,dx+c/2)\right)^{10}\cos(1/2\,dx+c/2)+2960\,C\left(\sin(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*C*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2960*C*sin(1/2*d*x+1/2*c)^8*cos(1/2
*d*x+1/2*c)+(-504*A-3152*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(924*A+
1792*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-336*A-408*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*a*cos(d*x + c) + A*a)
/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x
)
```

3.1167 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=270

$$\frac{2a^2(19A+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(5A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{16a^2(2A+3C)\sin(c+dx)\sqrt{\sec(c}}{15d}$$

```
[Out] (-16*a^2*(2*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (16*a^2*(2*A + 3*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d) + (4*a^2*(5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(21*d) + (2*a^2*(19*A + 21*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d) +
(8*A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*
A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.602894, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3044, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a^2(19A+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(5A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{16a^2(2A+3C)\sin(c+dx)\sqrt{\sec(c}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (-16*a^2*(2*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (16*a^2*(2*A + 3*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d) + (4*a^2*(5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(21*d) + (2*a^2*(19*A + 21*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d) +
(8*A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*
A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^2 \left(A \right.}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A}{}$$

$$= \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A}{}$$

$$= \frac{2a^2(19A+21C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{8A\left(a^2+a^2\right.}{}$$

$$= \frac{2a^2(19A+21C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{8A\left(a^2+a^2\right.}{}$$

$$= \frac{16a^2(2A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{4a^2(5A+7C}{}$$

$$= -\frac{16a^2(2A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{15d}$$

Mathematica [C] time = 6.77802, size = 655, normalized size = 2.43

$$\sqrt{\sec(c+dx)}\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{4(2A+3C)\csc(c)\cos(dx)}{15d} + \frac{\sec(c)\sec^2(c+dx)(90A\sin(c)+112A\sin}{630d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(a + a*Cos[c + d*x])^2*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^4)/(45*d*E^(I*d*x)) + (2*Sqrt[2]*C*
```

```
Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x)
)]*(a + a*Cos[c + d*x])^2*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*
I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])*Sec[c/2 + (d*x)/2]^4)/(15*d*E^(I*d*x)) + (5*A*Sqrt[Cos[c + d*x]]*(a
 + a*Cos[c + d*x])^2*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*Sqrt[Se
c[c + d*x]])/(21*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Elliptic
F[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*Sqrt[Sec[c + d*x]])/(3*d) + (a + a*C
os[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sqrt[Sec[c + d*x]]*((4*(2*A + 3*C)*Cos[
d*x]*Csc[c])/(15*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(18*d) + (Sec[c]*S
ec[c + d*x]^3*(7*A*Sin[c] + 18*A*Sin[d*x]))/(126*d) + (Sec[c]*Sec[c + d*x]^
2*(90*A*Sin[c] + 112*A*Sin[d*x] + 63*C*Sin[d*x]))/(630*d) + (Sec[c]*Sec[c +
 d*x]*(112*A*Sin[c] + 63*C*Sin[c] + 150*A*Sin[d*x] + 210*C*Sin[d*x]))/(630*
d) + ((5*A + 7*C)*Tan[c])/(21*d))
```

Maple [B] time = 4.419, size = 1168, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/2*C*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/4*A+1/4*C)/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^
2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+1/4*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1)+1/2*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+
```

```
1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))+1/4*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*co
s(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3+(A+C)a^2\cos\left(dx+c\right)^2+2\,Aa^2\cos\left(dx+c\right)+Aa^2\right)\sec\left(dx+c\right)^{\frac{11}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(11/2)
, x)
```

3.1168 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=237

$$\frac{2a^2(33A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2(3A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{21d}$$

```
[Out] (-4*a^2*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (8*a^2*(3*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (2*a^2*(33*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(105*d) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3
5*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.562733, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3044, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a^2(33A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2(3A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-4*a^2*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (8*a^2*(3*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (2*a^2*(33*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(105*d) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3
5*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^2\left(A+\right.}{\cos^{\frac{9}{2}}(c+dx}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{\left(2\sqrt{\cos\right.}{}$$

$$= \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A(a}{}$$

$$= \frac{8A\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A(a}{}$$

$$= \frac{2a^2(33A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{8A\left(a^2+a^2\,\mathrm{c}\right.}{}$$

$$= \frac{2a^2(33A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{8A\left(a^2+a^2\,\mathrm{c}\right.}{}$$

$$= \frac{8a^2(3A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}$$

$$= -\frac{4a^2(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}$$

Mathematica [C] time = 4.20427, size = 399, normalized size = 1.68

$$\frac{a^2\csc(c)e^{-idx}(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)(3A+5C)e^{2idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Csc[c]*Sec[(c + d*x)/2]^4*(7*Sqrt[2]*(3*A + 5*C)*
E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
```

```
*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(35*C*(1 + E^((2*I)*(c + d*x)))^2*(-1
 + 6*E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + 6*E^((3*I)*(c + d*x))) + 6*A*(
-10 + 7*E^(I*(c + d*x)) - 20*E^((2*I)*(c + d*x)) + 63*E^((3*I)*(c + d*x)) +
 20*E^((4*I)*(c + d*x)) + 77*E^((5*I)*(c + d*x)) + 10*E^((6*I)*(c + d*x)) +
 21*E^((7*I)*(c + d*x))))*Sqrt[Sec[c + d*x]])/(2*E^(I*(c - d*x))*(1 + E^((2
*I)*(c + d*x)))^3) + 20*(3*A + 7*C)*E^(I*d*x)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*Sin[c]))/(210*d*E^(I*d*x))
```

Maple [B] time = 3.711, size = 918, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/10*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/4*C)*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*
c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+1/2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1)+1/4*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^4 + 2\,Ca^2\cos(dx+c)^3 + (A+C)a^2\cos(dx+c)^2 + 2\,Aa^2\cos(dx+c) + Aa^2\right)\sec(dx+c)^{\frac{9}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2),
 x)
```

3.1169 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=196

$$\frac{2a^2(17A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{8A\sin(c+dx)\sec$$

```
[Out] (-16*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (4*a^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a^2*(17*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*d) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.541903, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3044, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{2a^2(17A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{8A\sin(c+dx)\sec$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-16*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (4*a^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a^2*(17*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*d) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^2\left(A+\right.}{\cos^{\frac{7}{2}}(c+dx}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{\left(2\sqrt{\cos}\right.}{}$$

$$=\frac{8A\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2A(a}{}$$

$$=\frac{8A\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2A(a}{}$$

$$=\frac{2a^2(17A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{8A\left(a^2+a^2\,\mathrm{c}\right.}{}$$

$$=\frac{2a^2(17A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{8A\left(a^2+a^2\,\mathrm{c}\right.}{}$$

$$=-\frac{16a^2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2}{}$$

Mathematica [C] time = 2.65281, size = 301, normalized size = 1.54

$$\frac{1}{15}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan(c)\sqrt{\sec(c+dx)}\left(3\cot(c)\csc(c)\cos(dx)(16A-5C\cos(2c)+5C)+6A\csc(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-I)*Sqrt[2]*(12*A*Sqrt[1 +
E^((2*I)*(c + d*x))] + 12*A*(-1 + E^((2*I)*c))*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 5*(A + 3*C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c)
)*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*(-1 + E^((2*I
)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c
+ d*x))]) + (Sqrt[Sec[c + d*x]]*(20*A + 3*(16*A + 5*C - 5*C*Cos[2*c])*Cos[d
*x]*Cot[c]*Csc[c] + 30*C*Cos[c]*Cot[c]*Sin[d*x] + 6*A*Csc[c]*Sec[c + d*x]^2
*Sin[d*x] + 2*A*Sec[c + d*x]*(3 + 10*Csc[c]*Sin[d*x]))*Tan[c])/(4*d)))/15
```

Maple [B] time = 3.109, size = 756, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+48*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-96*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^6+60*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*C*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-20*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-48*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+116*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2+60*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-37*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
```

```
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3+\left(A+C\right)a^2\cos\left(dx+c\right)^2+2\,Aa^2\cos\left(dx+c\right)+Aa^2\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
 x)
```

3.1170 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=196

$$-\frac{2a^2(5A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+$$

```
[Out] (-4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (8*a^2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) - (2*a^2*(5*A - C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]]) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.543133, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3044, 2975, 2968, 3023, 2748, 2641, 2639}

$$-\frac{2a^2(5A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (8*a^2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) - (2*a^2*(5*A - C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]]) + (8*A*(a^2 + a^2*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^2\left(A+\right.}{\cos^{\frac{5}{2}}(c+dx}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{\left(2\sqrt{\cos\right.}{}$$

$$=\frac{8A\left(a^2+a^2\cos(c+dx)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A(a}{}$$

$$=\frac{8A\left(a^2+a^2\cos(c+dx)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A(a}{}$$

$$=-\frac{2a^2(5A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8A\left(a^2+a^2\cos(c+dx)\right)\sqrt{\mathrm{se}}}{3d}$$

$$=-\frac{2a^2(5A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8A\left(a^2+a^2\cos(c+dx)\right)\sqrt{\mathrm{se}}}{3d}$$

$$=-\frac{4a^2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+$$

Mathematica [C] time = 1.67731, size = 191, normalized size = 0.97

$$\frac{a^2e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(4i(A-C)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+16(A+C)\cos^{\frac{3}{2}}(c+dx)F\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a^2*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-12*I)*A + (12*I)*C - (12
*I)*A*Cos[2*(c + d*x)] + (12*I)*C*Cos[2*(c + d*x)] + 16*(A + C)*Cos[c + d*x
]^(3/2)*EllipticF[(c + d*x)/2, 2] + (4*I)*(A - C)*(1 + E^((2*I)*(c + d*x)))
^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 4*A*Sin[c +
 d*x] + C*Sin[c + d*x] + 12*A*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(6*d*
E^(I*d*x))
```

Maple [B] time = 2.576, size = 651, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+4*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+4*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-4*C*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+7*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3+\left(A+C\right)a^2\cos\left(dx+c\right)^2+2\,Aa^2\cos\left(dx+c\right)+Aa^2\right)\sec\left(dx+c\right)^{\frac{5}{2}},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
 x)
```

3.1171 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=200

$$-\frac{2a^2(15A-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(5A-C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\sqrt{\sec(c+dx)}}+\frac{4a^2(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\right.}{3d}$$

```
[Out] (16*a^2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) - (2*a^2*(15*A - 7*C)*Sin[c + d*x])/(15*d*Sqrt[Sec[c +
 d*x]]) - (2*(5*A - C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Sqrt[Sec
[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d
```

Rubi [A] time = 0.504828, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3044, 2976, 2968, 3023, 2748, 2641, 2639}

$$-\frac{2a^2(15A-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(5A-C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\sqrt{\sec(c+dx)}}+\frac{4a^2(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (16*a^2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) - (2*a^2*(15*A - 7*C)*Sin[c + d*x])/(15*d*Sqrt[Sec[c +
 d*x]]) - (2*(5*A - C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Sqrt[Sec
[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^2\left(A+\right.}{\cos^{\frac{3}{2}}(c+dx}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{\left(2\sqrt{\cos}\right.}{}$$

$$=-\frac{2(5A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2A(a+a}{}$$

$$=-\frac{2(5A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2A(a+a}{}$$

$$=-\frac{2a^2(15A-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(5A-C)\left(a^2+a^2\cos(c+\right.}{5d\sqrt{\sec(c+}}$$

$$=-\frac{2a^2(15A-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(5A-C)\left(a^2+a^2\cos(c+\right.}{5d\sqrt{\sec(c+}}$$

$$=\frac{16a^2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(3}{}$$

Mathematica [C] time = 2.30683, size = 281, normalized size = 1.4

$$\frac{1}{15}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{\sec(c+dx)}(3(20A-31C)\csc(c)\cos(dx)-3(20A+33C)\csc(c)\cos(2c+dx)}{16d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((I*Sqrt[2]*(12*C*Sqrt[1 + E^(
(2*I)*(c + d*x))] + 12*C*(-1 + E^((2*I)*c))*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] - 5*(3*A + C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*(-1 + E^((2*I)*c
))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d
*x))]) + (Sqrt[Sec[c + d*x]]*(3*(20*A - 31*C)*Cos[d*x]*Csc[c] - 3*(20*A + 3
3*C)*Cos[2*c + d*x]*Csc[c] + 40*C*Sin[2*(c + d*x)] + 6*C*Sin[3*(c + d*x)]))
/(16*d)))/15
```

Maple [A] time = 1.26, size = 440, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^2*(-12*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A+13*C)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-12*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^4 + 2\,Ca^2\cos(dx+c)^3 + (A+C)a^2\cos(dx+c)^2 + 2\,Aa^2\cos(dx+c) + Aa^2\right)\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c) + a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
 x)
```

3.1172 $\int (a+a\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{2a^2(35A+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{8a^2(7A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\text{s}}}{5d}$$

```
[Out] (4*a^2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (8*a^2*(7*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(35*A + 33*C)*Sin[c + d*x])/(105*
d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt
[Sec[c + d*x]]) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d*Sqrt[Se
c[c + d*x]])
```

Rubi [A] time = 0.509097, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3046, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{2a^2(35A+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{8a^2(7A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\text{s}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a^2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (8*a^2*(7*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(35*A + 33*C)*Sin[c + d*x])/(105*
d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt
[Sec[c + d*x]]) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d*Sqrt[Se
c[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^2\left(A+\right.}{\sqrt{\cos(c+d}}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8C\left(a^2+a^2\cos(c+\right.}{35d\sqrt{\sec(}}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8C\left(a^2+a^2\cos(c+\right.}{35d\sqrt{\sec(}}$$

$$=\frac{2a^2(35A+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2C(a+a\cos(c+dx))^2\sin}{7d\sqrt{\sec(c+dx)}}$$

$$=\frac{2a^2(35A+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2C(a+a\cos(c+dx))^2\sin}{7d\sqrt{\sec(c+dx)}}$$

$$=\frac{4a^2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 1.83422, size = 189, normalized size = 0.93

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(5A+3C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(5(28A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(80*(7*A + 3*C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(5*A + 3*C)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((840*I)*A + (504*I)*C + 5*(28*A + 51*C)*Sin[c + d*x]
+ 84*C*Sin[2*(c + d*x)] + 15*C*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Maple [A] time = 0.911, size = 380, normalized size = 1.9

$$-\frac{4a^2}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^8\cos\left(1/2\,dx+c/2\right)-348\,C\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-348*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(70*A+378*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-35*A-117*C
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+30*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^4 + 2\,Ca^2\cos(dx+c)^3 + (A+C)a^2\cos(dx+c)^2 + 2\,Aa^2\cos(dx+c) + Aa^2\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
 x)
```

3.1173 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=237

$$\frac{2a^2(21A+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+$$

```
[Out] (16*a^2*(3*A + 2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(15*d) + (4*a^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(21*A + 19*C)*Sin[c + d*x])/(10
5*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Se
c[c + d*x]^(3/2)) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d*Sec[c
 + d*x]^(3/2)) + (4*a^2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.530604, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3046, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a^2(21A+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (16*a^2*(3*A + 2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(15*d) + (4*a^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(21*A + 19*C)*Sin[c + d*x])/(10
5*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Se
c[c + d*x]^(3/2)) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d*Sec[c
 + d*x]^(3/2)) + (4*a^2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\left(A+\right.$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{63d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{63d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(21A+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(21A+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{16a^2(3A+2C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2a^2(21A}{105}$$

$$= \frac{16a^2(3A+2C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^2(7A}{}$$

<u>**Mathematica**</u> **[C]** time = 2.57938, size = 206, normalized size = 0.87

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-448i(3A+2C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(60(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(7*A + 5*C)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] - (448*I)*(3*A + 2*C)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((4032*I)*A + (2688*I)*C + 60*(28*A + 23*C)*Sin[c +
d*x] + 14*(18*A + 37*C)*Sin[2*(c + d*x)] + 180*C*Sin[3*(c + d*x)] + 35*C*Si
```

```
n[4*(c + d*x)]))))/(1260*d*E^(I*d*x))
```

Maple [A] time = 0.939, size = 408, normalized size = 1.7

$$-\frac{4a^2}{315d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-560\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}\cos\left(1/2\,dx+c/2\right)+1840\,C\left(\sin\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1840*C*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-252*A-2368*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(672*A
+1568*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-273*A-387*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos(dx+c)^4+2\,Ca^2\cos(dx+c)^3+(A+C)a^2\cos(dx+c)^2+2\,Aa^2\cos(dx+c)+Aa^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
 x)
```

3.1174 $\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=270

$$\frac{4a^2(9A+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(99A+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2(33A+25C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{8a^2(33A+25C)\sqrt{\cos(c+d}}{2}$$

```
[Out] (4*a^2*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (8*a^2*(33*A + 25*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(99*A + 89*C)*Sin[c + d*x])/(
693*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d
*Sec[c + d*x]^(5/2)) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(99*d*Se
c[c + d*x]^(5/2)) + (4*a^2*(9*A + 7*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/
2)) + (8*a^2*(33*A + 25*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.587857, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3046, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(9A+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(99A+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2(33A+25C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{8a^2(33A+25C)\sqrt{\cos(c+d}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```



```
[Out] (4*a^2*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (8*a^2*(33*A + 25*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(99*A + 89*C)*Sin[c + d*x])/(
693*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d
*Sec[c + d*x]^(5/2)) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(99*d*Se
c[c + d*x]^(5/2)) + (4*a^2*(9*A + 7*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/
2)) + (8*a^2*(33*A + 25*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(A+\right.$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{99d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{99d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(99A+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(99A+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(99A+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{4a^2(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{8a^2(33A}{}$$

Mathematica [C] time = 2.90494, size = 228, normalized size = 0.84

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2464i(9A+7C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(30(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(960*(33*A + 25*C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(9*A + 7*C)*E^(I*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))] + Cos[c + d*x]*((66528*I)*A + (51744*I)*C + 30*(1122*A + 941*C)*
Sin[c + d*x] + 616*(18*A + 19*C)*Sin[2*(c + d*x)] + 1980*A*Sin[3*(c + d*x)]
```

```
 + 4545*C*Sin[3*(c + d*x)] + 1540*C*Sin[4*(c + d*x)] + 315*C*Sin[5*(c + d*x
)])))/(27720*d*E^(I*d*x))
```

Maple [A] time = 0.916, size = 436, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-37520*C*sin(1/2*d*x+1/2*c)^10*co
s(1/2*d*x+1/2*c)+(3960*A+57040*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-11484*A-46192*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12474*A+22022*C)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-3861*A-4563*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+990*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+750*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos(dx+c)^4+2\,Ca^2\cos(dx+c)^3+(A+C)a^2\cos(dx+c)^2+2\,Aa^2\cos(dx+c)+Aa^2}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3 + (A + C)*a^2*cos(d
*x + c)^2 + 2*A*a^2*cos(d*x + c) + A*a^2)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2),
 x)
```

3.1175 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=319

$$\frac{8a^3(35A+44C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{385d}+\frac{4a^3(105A+143C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{231d}+\frac{4a^3(5A+7C)\sin(c+dx)\sqrt{s}}{5d}$$

```
[Out] (-4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(105*A + 143*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(5*A + 7*C)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(5*d) + (4*a^3*(105*A + 143*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(231*d) + (8*a^3*(35*A + 44*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(385
*d) + (2*(35*A + 33*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(7/2)*Sin[c +
d*x])/(231*d) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*Sin[c +
d*x])/(33*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(11/2)*Sin[c + d*
x])/(11*d)
```

Rubi [A] time = 0.768622, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3044, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{8a^3(35A+44C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{385d}+\frac{4a^3(105A+143C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{231d}+\frac{4a^3(5A+7C)\sin(c+dx)\sqrt{s}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2),x]
```


```
[Out] (-4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(105*A + 143*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(5*A + 7*C)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(5*d) + (4*a^3*(105*A + 143*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(231*d) + (8*a^3*(35*A + 44*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(385
*d) + (2*(35*A + 33*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(7/2)*Sin[c +
d*x])/(231*d) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*Sin[c +
d*x])/(33*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(11/2)*Sin[c + d*
x])/(11*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^3\left(A+\right.}{\cos^{\frac{13}{2}}(c+d}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{\left(2\sqrt{c\right.}}{}$$

$$= \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{33ad} + \frac{2A}{}$$

$$= \frac{2(35A+33C)\left(a^3+a^3\cos(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{231d}$$

$$= \frac{2(35A+33C)\left(a^3+a^3\cos(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{231d}$$

$$= \frac{8a^3(35A+44C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{385d} + \frac{2(35A+33C}{}$$

$$= \frac{8a^3(35A+44C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{385d} + \frac{2(35A+33C}{}$$

$$= \frac{4a^3(5A+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^3(105A+14}{}$$

$$= -\frac{4a^3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [C] time = 6.92472, size = 697, normalized size = 2.18

$$\sqrt{\sec(c+dx)}\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{(5A+7C)\csc(c)\cos(dx)}{10d}+\frac{\sec(c)\sec^3(c+dx)(77A\sin(c)+126A\sin}{924d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2)
,x]
```

```
[Out] (A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*(a + a*Cos[c + d*x])^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(
(2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])*Sec[c/2 + (d*x)/2]^6)/(6*Sqrt[2]*d*E^(I*d*x)) + (7*C*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(a + a*C
os[c + d*x])^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2 + (d*x)/2]^6)/(30*Sqrt[2]*d*E^(I*d*x)) + (5*A*Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^3*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*Sqrt[Sec[c
 + d*x]])/(22*d) + (13*C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Elliptic
F[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]])/(42*d) + (a + a*
Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]]*(((5*A + 7*C)*Cos[d
*x]*Csc[c])/(10*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(44*d) + (Sec[c]*Se
c[c + d*x]^4*(3*A*Sin[c] + 11*A*Sin[d*x]))/(132*d) + (Sec[c]*Sec[c + d*x]^3
*(77*A*Sin[c] + 126*A*Sin[d*x] + 33*C*Sin[d*x]))/(924*d) + (Sec[c]*Sec[c +
d*x]^2*(630*A*Sin[c] + 165*C*Sin[c] + 770*A*Sin[d*x] + 693*C*Sin[d*x]))/(46
20*d) + (Sec[c]*Sec[c + d*x]*(770*A*Sin[c] + 693*C*Sin[c] + 1050*A*Sin[d*x]
 + 1430*C*Sin[d*x]))/(4620*d) + ((105*A + 143*C)*Tan[c])/(462*d))
```

Maple [B] time = 5.112, size = 1408, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/5*(1/
8*A+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(
1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/8*C+3/8*A)*(-1/56*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
```

```
*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+3/8*A*(-1/144*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*
c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+3/8*
C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/352*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/154*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos{(dx+c)}^2+A\right)(a\cos{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(13/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\big(Ca^3\cos{(dx+c)}^5+3\,Ca^3\cos{(dx+c)}^4+(A+3\,C)a^3\cos{(dx+c)}^3+(3\,A+C)a^3\cos{(dx+c)}^2+3\,Aa^3\cos{(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(13/2)
, x)
```

3.1176 $\int (a+a\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right) \sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=286

$$\frac{8a^3(16A+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^3(17A+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(73A+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c}{315c}$$

```
[Out] (-4*a^3*(17*A + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(11*A + 21*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(17*A + 27*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (8*a^3*(16*A + 21*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d) + (2*(73*A + 63*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(5
/2)*Sin[c + d*x])/(315*d) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(7
/2)*Sin[c + d*x])/(21*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)
*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.727919, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3044, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{8a^3(16A+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^3(17A+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(73A+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c}{315c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```



```
[Out] (-4*a^3*(17*A + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(11*A + 21*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(17*A + 27*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (8*a^3*(16*A + 21*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d) + (2*(73*A + 63*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(5
/2)*Sin[c + d*x])/(315*d) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(7
/2)*Sin[c + d*x])/(21*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)
*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^3 \left(A\right.}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21ad} + \frac{2A}{}$$

$$= \frac{2(73A+63C)\left(a^3+a^3\cos(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{315d}$$

$$= \frac{2(73A+63C)\left(a^3+a^3\cos(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{315d}$$

$$= \frac{8a^3(16A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2(73A+63}{}$$

$$= \frac{8a^3(16A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2(73A+63}{}$$

$$= \frac{4a^3(11A+21C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{21d}$$

$$= -\frac{4a^3(17A+27C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{15d}$$

Mathematica [C] time = 6.8194, size = 655, normalized size = 2.29

$$\sqrt{\sec(c+dx)}\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{(17A+27C)\csc(c)\cos(dx)}{30d} + \frac{\sec(c)\sec^2(c+dx)(135A\sin(c)+238A}{1260d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2)
,x]
```

[Out] (17*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(a + a*Cos[c + d*x])^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6)/(90*Sqrt[2]*d*E^(I*d*x)) + (3*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(a + a*Cos[c + d*x])^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6)/(10*Sqrt[2]*d*E^(I*d*x)) + (11*A*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]])/(42*d) + (C*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]])/(2*d) + (a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]]*(((17*A + 27*C)*Cos[d*x]*Csc[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(36*d) + (Sec[c]*Sec[c + d*x]^3*(7*A*Sin[c] + 27*A*Sin[d*x]))/(252*d) + (Sec[c]*Sec[c + d*x]^2*(135*A*Sin[c] + 238*A*Sin[d*x] + 63*C*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]*(238*A*Sin[c] + 63*C*Sin[c] + 330*A*Sin[d*x] + 315*C*Sin[d*x]))/(1260*d) + ((22*A + 21*C)*Tan[c])/(84*d))

Maple [B] time = 4.206, size = 1246, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)

[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/5*(1/8*C+3/8*A)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3/8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+3/8*A*(-1/56*cos(1/2*d

```
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/
2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/144*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+(1/
8*A+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^4+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^3+\left(3\,A+C\right)a^3\cos\left(dx+c\right)^2+3\,Aa^3\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos (dx+c)^2 + A\right)(a\cos (dx+c)+a)^3 \sec (dx+c)^{\frac{11}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(11/2)
, x)
```

3.1177 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=253

$$\frac{8a^3(53A+70C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d}+\frac{4a^3(13A+35$$

[Out] (-4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*(13*A + 35*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(53*A + 70*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (12*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rubi [A] time = 0.698361, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3044, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{8a^3(53A+70C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d}+\frac{4a^3(13A+35$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]

[Out] (-4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*(13*A + 35*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(53*A + 70*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (12*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^3\left(A\right.}{\cos^{\frac{9}{2}}(c+}$$

$$=\frac{2A(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{\left(2\sqrt{c}\right.}{}$$

$$=\frac{12A\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35ad}+\frac{2}{}$$

$$=\frac{2(7A+5C)\left(a^3+a^3\cos(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{15d}$$

$$=\frac{2(7A+5C)\left(a^3+a^3\cos(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{15d}$$

$$=\frac{8a^3(53A+70C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d}+\frac{2(7A+5C)}{}$$

$$=\frac{8a^3(53A+70C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d}+\frac{2(7A+5C)}{}$$

$$=-\frac{4a^3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [C] time = 4.38499, size = 302, normalized size = 1.19

$$\frac{a^3\csc(c)\sec(c)e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(14\left(-1+e^{4ic}\right)(7A+5C)e^{-i(c-dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a^3*Csc[c]*Sec[c]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((14*(7*A + 5
*C)*(-1 + E^((4*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c - d*x)) + (Sec[c + d*x]^3*Sin[2*c
]*((-882*I)*A - (630*I)*C - (168*I)*(7*A + 5*C)*Cos[2*(c + d*x)] - (294*I)*
A*Cos[4*(c + d*x)] - (210*I)*C*Cos[4*(c + d*x)] + 80*(13*A + 35*C)*Cos[c +
d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 380*A*Sin[c + d*x] + 70*C*Sin[c + d*
x] + 840*A*Sin[2*(c + d*x)] + 630*C*Sin[2*(c + d*x)] + 260*A*Sin[3*(c + d*x
)] + 70*C*Sin[3*(c + d*x)] + 294*A*Sin[4*(c + d*x)] + 315*C*Sin[4*(c + d*x)
]))/4))/(210*d*E^(I*d*x))
```

Maple [B] time = 3.586, size = 1012, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/40*A/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2
```

```
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+(1/8*A+3/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1)+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*C+3/8*A)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\Big(Ca^{3}\cos\left(dx+c\right)^{5}+3\,Ca^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+C\right)a^{3}\cos\left(dx+c\right)^{2}+3\,Aa^{3}\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
```

```
sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
 x)
```

3.1178 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=253

$$-\frac{4a^3(21A+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(11A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{4a^3(3A+5C)\sqrt{\cos(c+d}}{}$$

```
[Out] (-4*a^3*(9*A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(21*A + 5*C)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]) + (2*(11*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(5*d) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(5*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.682042, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3044, 2975, 2968, 3023, 2748, 2641, 2639}

$$-\frac{4a^3(21A+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(11A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{4a^3(3A+5C)\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-4*a^3*(9*A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(21*A + 5*C)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]) + (2*(11*A + 5*C)*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(5*d) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(5*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^3\left(A\ \cdots\right.}{\cos^{\frac{7}{2}}(c+\cdots}$$

$$=\frac{2A(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{\left(2\sqrt{c\cdots}\right.}{\cdots}$$

$$=\frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}+\frac{2A\cdots}{\cdots}$$

$$=\frac{2(11A+5C)\left(a^3+a^3\cos(c+dx)\right)\sqrt{\sec(c+dx)}\sin(c+\cdots}{5d}$$

$$=\frac{2(11A+5C)\left(a^3+a^3\cos(c+dx)\right)\sqrt{\sec(c+dx)}\sin(c+\cdots}{5d}$$

$$=-\frac{4a^3(21A+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(11A+5C)\left(a^3+a^3\cos\cdots\right.}{\cdots}$$

$$=-\frac{4a^3(21A+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(11A+5C)\left(a^3+a^3\cos\cdots\right.}{\cdots}$$

$$=-\frac{4a^3(9A-5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d\cdots}}{5d}$$

Mathematica [C] time = 3.03102, size = 279, normalized size = 1.1

$$\frac{a^3\csc(c)\sec(c)e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(4\left(-1+e^{4ic}\right)(9A-5C)e^{-i(c-dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a^3*Csc[c]*Sec[c]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((4*(9*A - 5*
C)*(-1 + E^((4*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c - d*x)) + (Sec[c + d*x]^2*Sin[2*c]
*((-36*I)*(9*A - 5*C)*Cos[c + d*x] - (108*I)*A*Cos[3*(c + d*x)] + (60*I)*C*
Cos[3*(c + d*x)] + 80*(3*A + 5*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2,
 2] + 132*A*Sin[c + d*x] + 30*C*Sin[c + d*x] + 60*A*Sin[2*(c + d*x)] + 10*C
*Sin[2*(c + d*x)] + 108*A*Sin[3*(c + d*x)] + 30*C*Sin[3*(c + d*x)] + 5*C*Si
n[4*(c + d*x)]))/2))/(60*d*E^(I*d*x))
```

Maple [B] time = 3.352, size = 939, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(40*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+60*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+108*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^4-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+100*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*A*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
```

```
1/2*c)^2+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-100*C*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1
/2*c)^2+90*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*C*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{5}+3\,Ca^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+C\right)a^{3}\cos\left(dx+c\right)^{2}+3\,Aa^{3}\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
 x)
```

3.1179 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=251

$$-\frac{8a^3(10A-3C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(35A-3C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3d}$$

```
[Out] (-4*a^3*(5*A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (8*a^3*(10*A - 3*C)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]) - (2*(35*A - 3*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x]
)/(15*d*Sqrt[Sec[c + d*x]]) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.689332, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3044, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$-\frac{8a^3(10A-3C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(35A-3C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-4*a^3*(5*A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (8*a^3*(10*A - 3*C)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]) - (2*(35*A - 3*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x]
)/(15*d*Sqrt[Sec[c + d*x]]) + (4*A*(a^2 + a^2*Cos[c + d*x])^2*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^3 \left(A+\right.}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{\left(2\sqrt{\cos\right.}{}$$

$$= \frac{4A\left(a^2+a^2\cos(c+dx)\right)^2\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{2A(}{}$$

$$= -\frac{2(35A-3C)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{4A\left(a^2\right.}{}$$

$$= -\frac{2(35A-3C)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{4A\left(a^2\right.}{}$$

$$= -\frac{8a^3(10A-3C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2(35A-3C)\left(a^3+a^3\cos(\right.}{15d\sqrt{\sec(c-}}$$

$$= -\frac{8a^3(10A-3C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2(35A-3C)\left(a^3+a^3\cos(\right.}{15d\sqrt{\sec(c-}}$$

$$= -\frac{4a^3(5A-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}$$

Mathematica [C] time = 2.18582, size = 221, normalized size = 0.88

$$\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(8i(5A-9C)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+80(5A+3C)\cos^{\frac{3}{2}}(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-120*I)*A + (216*I)*C - (
120*I)*A*Cos[2*(c + d*x)] + (216*I)*C*Cos[2*(c + d*x)] + 80*(5*A + 3*C)*Cos
[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + (8*I)*(5*A - 9*C)*(1 + E^((2*I)
*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] +
```

```
 40*A*Sin[c + d*x] + 30*C*Sin[c + d*x] + 180*A*Sin[2*(c + d*x)] + 6*C*Sin[2
*(c + d*x)] + 30*C*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/(60*d*E^(I*d*x
))
```

Maple [B] time = 1.125, size = 704, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*(24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A+13*C)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A+9*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(25*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+15*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*C*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+25*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-27*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^5 + 3\,Ca^3\cos(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^3 + (3\,A+C)a^3\cos(dx+c)^2 + 3\,Aa^3\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
 x)
```

3.1180 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=257

$$-\frac{4a^3(35A-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2(35A-11C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{35d\sqrt{\sec(c+dx)}}-\frac{2(7A-C)\sin(c+dx)\left(a^2\cos(c+dx)+\right.}{7ad\sqrt{\sec(c+dx)}}$$

```
[Out] (4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(35*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(35*A - 41*C)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) - (2*(7*A - C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d
*x])/(7*a*d*Sqrt[Sec[c + d*x]]) - (2*(35*A - 11*C)*(a^3 + a^3*Cos[c + d*x])
*Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.679621, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3044, 2976, 2968, 3023, 2748, 2641, 2639}

$$-\frac{4a^3(35A-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2(35A-11C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{35d\sqrt{\sec(c+dx)}}-\frac{2(7A-C)\sin(c+dx)\left(a^2\cos(c+dx)+\right.}{7ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(35*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(35*A - 41*C)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) - (2*(7*A - C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d
*x])/(7*a*d*Sqrt[Sec[c + d*x]]) - (2*(35*A - 11*C)*(a^3 + a^3*Cos[c + d*x])
*Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^3*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^3\left(A+\right.}{\cos^{\frac{3}{2}}(c+dx}$$

$$=\frac{2A(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{\left(2\sqrt{\cos}\right.}{}$$

$$=-\frac{2(7A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{7ad\sqrt{\sec(c+dx)}}+\frac{2A(a+}{}$$

$$=-\frac{2(7A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{7ad\sqrt{\sec(c+dx)}}-\frac{2(35A-}{}$$

$$=-\frac{2(7A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{7ad\sqrt{\sec(c+dx)}}-\frac{2(35A-}{}$$

$$=-\frac{4a^3(35A-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2(7A-C)\left(a^2+a^2\cos(c\right.}{7ad\sqrt{\sec(c}}$$

$$=-\frac{4a^3(35A-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2(7A-C)\left(a^2+a^2\cos(c\right.}{7ad\sqrt{\sec(c}}$$

$$=\frac{4a^3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 1.77697, size = 218, normalized size = 0.85

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(5A+7C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+80(35A+13C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((1680*I)*A*Cos[c + d*x] +
(2352*I)*C*Cos[c + d*x] + 80*(35*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(5*A + 7*C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 840*A*Sin[c
+ d*x] + 126*C*Sin[c + d*x] + 140*A*Sin[2*(c + d*x)] + 550*C*Sin[2*(c + d*x
)] + 126*C*Sin[3*(c + d*x)] + 15*C*Sin[4*(c + d*x)]))/(420*d*E^(I*d*x))
```

Maple [B] time = 1.214, size = 569, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/105*a^3*(120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+43*C)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)-4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(35
*A+52*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+175*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-105*A*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^4+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^3+\left(3\,A+C\right)a^3\cos\left(dx+c\right)^2+3\,Aa^3\cos\left(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
 x)
```

3.1181 $\int (a+a\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=253

$$\frac{8a^3(21A+16C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2(63A+73C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{315d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{21d}$$

```
[Out] (4*a^3*(27*A + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(21*A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(21*A + 16*C)*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d
*Sqrt[Sec[c + d*x]]) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*
d*Sqrt[Sec[c + d*x]]) + (2*(63*A + 73*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d
*x])/(315*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.66961, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3046, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^3(21A+16C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2(63A+73C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{315d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a^3*(27*A + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(21*A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(21*A + 16*C)*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d
*Sqrt[Sec[c + d*x]]) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*
d*Sqrt[Sec[c + d*x]]) + (2*(63*A + 73*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d
*x])/(315*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^3\left(A+\right.}{\sqrt{\cos(c+d}}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{4C\left(a^2+a^2\cos(c+\right.}{21ad\sqrt{\sec}}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{4C\left(a^2+a^2\cos(c+\right.}{21ad\sqrt{\sec}}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{4C\left(a^2+a^2\cos(c+\right.}{21ad\sqrt{\sec}}$$

$$=\frac{8a^3(21A+16C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2C(a+a\cos(c+dx))^3\sin}{9d\sqrt{\sec(c+dx)}}$$

$$=\frac{8a^3(21A+16C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2C(a+a\cos(c+dx))^3\sin}{9d\sqrt{\sec(c+dx)}}$$

$$=\frac{4a^3(27A+17C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{15d}$$

Mathematica [C] time = 2.58173, size = 206, normalized size = 0.81

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(27A+17C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(30\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(21*A + 11*C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(27*A + 17*C)*E^(I*(c + d*x)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))] + Cos[c + d*x]*((9072*I)*A + (5712*I)*C + 30*(84*A + 97*C)*Sin[
c + d*x] + 14*(18*A + 73*C)*Sin[2*(c + d*x)] + 270*C*Sin[3*(c + d*x)] + 35*
C*Sin[4*(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 1.328, size = 408, normalized size = 1.6

$$-\frac{4a^3}{315d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-560\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}\cos\left(1/2\,dx+c/2\right)+2200\,C\left(\sin\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2200*C*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-252*A-3412*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(882*A
+2702*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-378*A-738*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+165*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^5 + 3\,Ca^3\cos(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^3 + (3\,A+C)a^3\cos(dx+c)^2 + 3\,Aa^3\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)*
sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
 x)
```

3.1182 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=286

$$\frac{8a^3(44A+35C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(143A+105C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(33A+35C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{231d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(1}{}$$

```
[Out] (4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(143*A + 105*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a^3*(44*A + 35*C)*Sin[c + d*x])/
(385*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*
d*Sec[c + d*x]^(3/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(33*a
*d*Sec[c + d*x]^(3/2)) + (2*(33*A + 35*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c +
d*x])/(231*d*Sec[c + d*x]^(3/2)) + (4*a^3*(143*A + 105*C)*Sin[c + d*x])/(23
1*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.707347, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3046, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{8a^3(44A+35C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(143A+105C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(33A+35C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{231d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(143*A + 105*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a^3*(44*A + 35*C)*Sin[c + d*x])/
(385*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*
d*Sec[c + d*x]^(3/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(33*a
*d*Sec[c + d*x]^(3/2)) + (2*(33*A + 35*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c +
d*x])/(231*d*Sec[c + d*x]^(3/2)) + (4*a^3*(143*A + 105*C)*Sin[c + d*x])/(23
1*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(A\right.$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{33ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{33ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{33ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8a^3(44A+35C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{8a^3(44A+35C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{4a^3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^3(44A}{385}$$

$$= \frac{4a^3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(143}{}$$

Mathematica [C] time = 2.7254, size = 228, normalized size = 0.8

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2464i(7A+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(160*(143*A + 105*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(7*A + 5*C)*E^(I*(c + d*x
```

```
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)
*(c + d*x))] + Cos[c + d*x]*((51744*I)*A + (36960*I)*C + 10*(2354*A + 1953*
C)*Sin[c + d*x] + 308*(18*A + 25*C)*Sin[2*(c + d*x)] + 660*A*Sin[3*(c + d*x
)] + 2835*C*Sin[3*(c + d*x)] + 770*C*Sin[4*(c + d*x)] + 105*C*Sin[5*(c + d*
x)])))/(9240*d*E^(I*d*x))
```

Maple [A] time = 1.101, size = 436, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/1155*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(3360*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-14560*C*sin(1/2*d*x+1/2*c)^10*cos
(1/2*d*x+1/2*c)+(1320*A+25760*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-
4752*A-24080*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(6622*A+13090*C)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2288*A-2940*C)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+715*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1155*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+525*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5 + 3\,Ca^3\cos(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^3 + (3\,A+C)a^3\cos(dx+c)^2 + 3\,Aa^3\cos(d$$

$$\sqrt{\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
 x)
```

3.1183 $\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{4a^3(221A+175C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{40a^3(143A+118C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(121A+95C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(143A+145C)\,\mathrm{si}}{128}$$

```
[Out] (4*a^3*(221*A + 175*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(195*d) + (4*a^3*(121*A + 95*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (40*a^3*(143*A + 118*C)*Sin[c
+ d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(13*d*Sec[c + d*x]^(5/2)) + (12*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d
*x])/(143*a*d*Sec[c + d*x]^(5/2)) + (2*(143*A + 145*C)*(a^3 + a^3*Cos[c + d
*x])*Sin[c + d*x])/(1287*d*Sec[c + d*x]^(5/2)) + (4*a^3*(221*A + 175*C)*Sin
[c + d*x])/(585*d*Sec[c + d*x]^(3/2)) + (4*a^3*(121*A + 95*C)*Sin[c + d*x])
/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.75881, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3046, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(221A+175C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{40a^3(143A+118C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(121A+95C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(143A+145C)\,\mathrm{si}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (4*a^3*(221*A + 175*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(195*d) + (4*a^3*(121*A + 95*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (40*a^3*(143*A + 118*C)*Sin[c
+ d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(13*d*Sec[c + d*x]^(5/2)) + (12*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d
*x])/(143*a*d*Sec[c + d*x]^(5/2)) + (2*(143*A + 145*C)*(a^3 + a^3*Cos[c + d
*x])*Sin[c + d*x])/(1287*d*Sec[c + d*x]^(5/2)) + (4*a^3*(221*A + 175*C)*Sin
[c + d*x])/(585*d*Sec[c + d*x]^(3/2)) + (4*a^3*(121*A + 95*C)*Sin[c + d*x])
/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\left(A+\right.$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}+\frac{12C\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{143ad\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}+\frac{12C\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{143ad\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}+\frac{12C\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{143ad\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{40a^3(143A+118C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{40a^3(143A+118C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{40a^3(143A+118C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{4a^3(221A+175C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{195d}+\frac{4a^3(1}{}$$

Mathematica [C] time = 3.15354, size = 250, normalized size = 0.78

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-4928i(221A+175C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(12480*(121*A + 95*C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (4928*I)*(221*A + 175*C)*E^(I*(c
```

```
+ d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))] + Cos[c + d*x]*((3267264*I)*A + (2587200*I)*C + 780*(2134
*A + 1811*C)*Sin[c + d*x] + 77*(7592*A + 7825*C)*Sin[2*(c + d*x)] + 154440*
A*Sin[3*(c + d*x)] + 251550*C*Sin[3*(c + d*x)] + 20020*A*Sin[4*(c + d*x)] +
 90860*C*Sin[4*(c + d*x)] + 24570*C*Sin[5*(c + d*x)] + 3465*C*Sin[6*(c + d*
x)])))/(720720*d*E^(I*d*x))
```

Maple [A] time = 1.006, size = 464, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+1058400*C*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^12+(-80080*A-2122400*C)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*
x+1/2*c)+(314600*A+2331040*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-487
916*A-1535860*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(386386*A+633710*C
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-105534*A-121230*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+23595*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-51051*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+18525*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-40425*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5 + 3\,Ca^3\cos(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^3 + (3\,A+C)a^3\cos(dx+c)^2 + 3\,Aa^3\cos(dx}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4 + (A + 3*C)*a^3*cos
(d*x + c)^3 + (3*A + C)*a^3*cos(d*x + c)^2 + 3*A*a^3*cos(d*x + c) + A*a^3)/
sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2),
 x)
```

3.1184 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{(7A+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}-\frac{(5A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(7A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5ad}-\frac{(A+$$

```
[Out] (-3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) - ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a*d) + (3*(7*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*a*d) - ((5*A + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((7*A +
 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.326356, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2748, 2636, 2639, 2641}

$$\frac{(7A+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}-\frac{(5A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(7A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5ad}-\frac{(A+$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x]),x]
```


```
[Out] (-3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) - ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a*d) + (3*(7*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*a*d) - ((5*A + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((7*A +
 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$=-\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(7A+5C}{}}{a^2}$$

$$=-\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{\left((5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{2a}$$

$$=-\frac{(5A+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{(7A+5C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad}-$$

$$=-\frac{(5A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}+\frac{3(7A+5C)\sqrt{\sec(c}}{5a}$$

$$=-\frac{3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}-\frac{(5A+3C)\sqrt{\cos(c}}{}$$

<u>Mathematica</u> [C] time = 7.32456, size = 685, normalized size = 2.95

$$\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+C\sin\left(\frac{dx}{2}\right)\right)}{d}+\frac{3(7A+5C)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos(dx)}{5d}-\frac{2\tan\left(\frac{c}{2}\right)\sec(c)(5A\cos(c)+2A+3}{3d}\right.}{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x]),
x]
```

```
[Out] (7*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2])/(5*Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) + (C*Sq
rt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]
*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)
*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) - (5*A*Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*
Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])) - (C*Cos[c/2 + (d*x)/
```

```
2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Se
c[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[S
ec[c + d*x]]*((3*(7*A + 5*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (2*Sec[c/2
]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec
[c + d*x]^2*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]*(3*A*Sin[c] - 5*A*Sin[
d*x]))/(15*d) - (2*(2*A + 5*A*Cos[c] + 3*C*Cos[c])*Sec[c]*Tan[c/2])/(3*d)))
/(a + a*Cos[c + d*x])
```

Maple [B] time = 3.63, size = 803, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-2/5*A/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+(2*A+2*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(-A-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)-2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x
)
```

3.1185 $\int \frac{(A+C\cos^2(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=190

$$\frac{(5A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{(3A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A+3C}{}$$

```
[Out] ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((3*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(5*A + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A + C)*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.294969, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2748, 2636, 2641, 2639}

$$\frac{(5A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{(3A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A+3C}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```


```
[Out] ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((3*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(5*A + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A + C)*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(5A+3C}{}}{a^2}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\left((3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= -\frac{(3A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(5A+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} -$$

$$= \frac{(3A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A+3C)\sqrt{\cos(c+a}}{}$$

Mathematica [C] time = 7.10924, size = 651, normalized size = 3.43

$$\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+C\sin\left(\frac{dx}{2}\right)\right)}{d} - \frac{(3A+C)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos(dx)}{d} + \frac{2\tan\left(\frac{c}{2}\right)\sec(c)(5A\cos(c)+2A+3C\cos}{3d}\right.}{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),
x]
```

```
[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x]))) - (C*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2])/(3*Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) + (5*A*Cos[c/2
+ (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]
*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])) + (C*Cos[c/2 + (d*x)
/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[S
ec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[
Sec[c + d*x]]*(-(((3*A + C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d) + (2*Sec[c/2]*Se
```

```
c[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c +
 d*x]*Sin[d*x])/(3*d) + (2*(2*A + 5*A*Cos[c] + 3*C*Cos[c])*Sec[c]*Tan[c/2])
/(3*d)))/(a + a*Cos[c + d*x])
```

Maple [B] time = 2.984, size = 486, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-2*A*(-(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(A+C)*(co
s(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1
/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x
)
```

3.1186 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{(3A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\cos(c+dx)+a)}-\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{ad}$$

```
[Out] -(((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) - ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((3*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
+ C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.278663, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2748, 2636, 2639, 2641}

$$\frac{(3A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\cos(c+dx)+a)}-\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```


```
[Out] -(((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) - ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((3*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
+ C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(3A+}{}}{a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\left((A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A+C)\sqrt{\sec(c+}}{ad}$$

$$= -\frac{(3A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 2.29514, size = 396, normalized size = 2.59

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(6\sqrt{\sec(c+dx)}\left(2(3A+C)\csc(c)\cos(dx)-2(A+C)\tan\left(\frac{1}{2}(c+dx)\right)\right)+6\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),
x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) + (2*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 12*C*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 6*Sqrt[Sec[c + d*x]]*(2*(3*A + C)*Cos[d*
x]*Csc[c] - 2*(A + C)*Tan[(c + d*x)/2])))/(6*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.277, size = 316, normalized size = 2.1

$$-\frac{1}{ad}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{\left(\mathrm{si}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A+C)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+C)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x
)
```

3.1187 $\int \frac{(A+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)}+\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{ad}$$

```
[Out] ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.24416, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4221, 3042, 2748, 2641, 2639}

$$-\frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)}+\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```


```
[Out] ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*Sqrt[Sec[c +
 d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(A-}{}}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left((A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{2a}$$

$$= \frac{(A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-C)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 3.34209, size = 421, normalized size = 3.42

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{6\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((A+2C)\cos\left(\frac{1}{2}(c-dx)\right)+C\cos\left(\frac{1}{2}(3c+dx)\right)\right)}{\sqrt{\sec(c+dx)}} + 2\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),
x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*((A + 2*C)*Cos[(c - d*x)/2]
+ C*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*
x]] - 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
+ 12*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))/(6
*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.91, size = 247, normalized size = 2.

$$\frac{1}{ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A+2*C)*sin(1/2*d*x+1/2*c)^4+(-A-C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx+\int \frac{C\cos^{2}\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(C*cos(c +
d*x)**2*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x
)
```

3.1188 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{(3A+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(A}{}$$

```
[Out] -(((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*Sec
[c + d*x]^(3/2)) + ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.270268, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2748, 2639, 2635, 2641}

$$\frac{(3A+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] -(((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*Sec
[c + d*x]^(3/2)) + ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+}}{}$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} - \frac{\left((A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= -\frac{(A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A+C)\sin(c}{d(a+a\cos(c+dx))\,\mathrm{s}}$$

$$= -\frac{(A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A+5C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 4.01069, size = 439, normalized size = 2.71

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((6A+13C)\cos\left(\frac{1}{2}(c-dx)\right)+C\left(2\sin(c)\sin\left(\frac{3}{2}(c+dx)\right)+5\cos\left(\frac{1}{2}(3c+dx)\right)\right)\right)}{\sqrt{\sec(c+dx)}} + 2\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e}{1+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 12*A*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 20*C*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + (Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*((6*
A + 13*C)*Cos[(c - d*x)/2] + C*(5*Cos[(3*c + d*x)/2] + 2*Sin[c]*Sin[(3*(c +
 d*x))/2])))/Sqrt[Sec[c + d*x]]))/(6*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 1.002, size = 262, normalized size = 1.6

$$-\frac{1}{3da}\sqrt{\left(2\ (\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\ (\sin(1/2\,dx+c/2))^2-1}\left(3\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*C*sin(1/2*d*x+1/2*c)^6+(6*A+18*C)*sin(1/2*d*x+1/2*c)^4+(-3*A
-7*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx + \int \frac{C\cos^2(c+dx)}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x) + In
tegral(C*cos(c + d*x)**2/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*
x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

3.1189 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{(5A+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{(3A+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{3ad}$$

```
[Out] (3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*
Sec[c + d*x]^(5/2)) + ((5*A + 7*C)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2))
 - ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.290666, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2748, 2635, 2641, 2639}

$$\frac{(5A+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{(3A+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```


```
[Out] (3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*
Sec[c + d*x]^(5/2)) + ((5*A + 7*C)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2))
 - ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+a}{}$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} - \frac{\left((3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(3A+5C)\sin(c}{3ad\sqrt{\sec(c+}}$$

$$= \frac{3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(3A+5C)\sqrt{\cos(c+d}}{}$$

<u>Mathematica</u> [C] time = 2.88615, size = 458, normalized size = 2.3

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((60A+83C)\cos\left(\frac{1}{2}(c-dx)\right)+(30A+43C)\cos\left(\frac{1}{2}(3c+dx)\right)+C\sin(c)\left(7\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)\right)\right)}{\sqrt{\sec(c+dx)}}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*((60*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))]))/E^(I*d*x) + (84*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((
2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 120*A*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 200*C*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + (2*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/
2]*((60*A + 83*C)*Cos[(c - d*x)/2] + (30*A + 43*C)*Cos[(3*c + d*x)/2] + C*S
in[c]*(7*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2])))/Sqrt[Sec[c + d*x]
]))/(60*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 1.223, size = 276, normalized size = 1.4

$$\frac{1}{15\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(15\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(15*A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*C*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))-48*C*sin(1/2*d*x+1/2*c)^8+56*C*sin(1/2*d*x+1/2*c)^6+(30*A+
30*C)*sin(1/2*d*x+1/2*c)^4+(-15*A-23*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
 x)
```

3.1190 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=232

$$-\frac{(5A+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(7A+9C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(7A+9C)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{5(7A}{}$$

```
[Out] (-3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) + (5*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(21*a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*
x])*Sec[c + d*x]^(7/2)) + ((7*A + 9*C)*Sin[c + d*x])/(7*a*d*Sec[c + d*x]^(5
/2)) - ((5*A + 7*C)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (5*(7*A + 9*
C)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.314185, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2748, 2635, 2639, 2641}

$$-\frac{(5A+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(7A+9C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(7A+9C)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{5(7A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) + (5*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(21*a*d) - ((A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*
x])*Sec[c + d*x]^(7/2)) + ((7*A + 9*C)*Sin[c + d*x])/(7*a*d*Sec[c + d*x]^(5
/2)) - ((5*A + 7*C)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (5*(7*A + 9*
C)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{5}{2}}(c+\cdots}{\cdots}$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)} - \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\cdots}{2a}$$

$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)} + \frac{(7A+9C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{(5A+7C)\sin(\cdots}{5ad\sec^{\frac{3}{2}}(c+\cdots}$$

$$= -\frac{3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(A+C)\sin\cdots}{d(a+a\cos(c+dx\cdots}$$

$$= -\frac{3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(7A+9C)\sqrt{\cos(\cdots}}{\cdots}$$

Mathematica [C] time = 3.8355, size = 542, normalized size = 2.34

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\left(20(14A+27C)\sin(2c)\cos(2dx)-84(20A+33C)\cos(c)\sin(dx)+20(14A+27C)\cos\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((420*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))]))/E^(I*d*x) + (588*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 1400*A*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 1800*C*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d*x]]*(21*(40*A + 5
1*C + (20*A + 33*C)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 20*(14*A + 27*C)
```

```
*Cos[2*d*x]*Sin[2*c] - 84*C*Cos[3*d*x]*Sin[3*c] + 30*C*Cos[4*d*x]*Sin[4*c]
- 840*(A + C)*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 84*(20*A + 33*C)*Cos
[c]*Sin[d*x] + 20*(14*A + 27*C)*Cos[2*c]*Sin[2*d*x] - 84*C*Cos[3*c]*Sin[3*d
*x] + 30*C*Cos[4*c]*Sin[4*d*x] - 840*(A + C)*Tan[c/2])))/(420*a*d*(1 + Cos[
c + d*x]))
```

Maple [A] time = 1.041, size = 295, normalized size = 1.3

$$-\frac{1}{105\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(17\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(175*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+225*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*C*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))-480*C*sin(1/2*d*x+1/2*c)^10+864*C*sin(1/2*d*x+1/2*c)
^8+(-280*A-888*C)*sin(1/2*d*x+1/2*c)^6+(630*A+930*C)*sin(1/2*d*x+1/2*c)^4+(
-245*A-321*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos{(dx+c)}^2+A}{(a\cos{(dx+c)}+a)\sec{(dx+c)}^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos{(dx+c)}^2+A}{(a\cos{(dx+c)}+a)\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
 x)
```

3.1191 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{2(5A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{(7A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(7A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{2(5A+$$

```
[Out] ((7*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + (2*(5*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((7*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + (2*(5*A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((7*A + C)*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.453344, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2978, 2748, 2636, 2641, 2639}

$$\frac{2(5A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{(7A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(7A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{2(5A+$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] ((7*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + (2*(5*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((7*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + (2*(5*A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((7*A + C)*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3}{2}a(3A+C)}{\cos^{\frac{5}{2}}(c+}}{3a^2}$$

$$=-\frac{(7A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+$$

$$=-\frac{(7A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+$$

$$=-\frac{(7A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2(5A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d}-$$

$$=\frac{(7A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(5A+C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 7.41319, size = 444, normalized size = 1.94

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-2\sqrt{\sec(c+dx)}\left(6(7A+C)\csc(c)\cos(dx)-\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)((13A+3C)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^
2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*((-14*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))]))/E^(I*d*x) - (2*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 40*A*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 8*C*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 2*Sqrt[Sec[c + d*x]]*(6*(7*A + C)*Cos[d
```

```
*x]*Csc[c] - (7*A + C + (13*A + 3*C)*Cos[c + d*x] + (5*A + C)*Cos[2*(c + d*
x)])*Sec[(c + d*x)/2]^2*Sec[c + d*x]*Tan[(c + d*x)/2])))/(3*a^2*d*(1 + Cos[
c + d*x])^2)
```

Maple [B] time = 3.254, size = 738, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A+
C)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c
)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+4
*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-8*A*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+4*A*(cos(1/2*d*x+1/2*c)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a^2*cos(d*x + c)^2 + 2*
a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2,
 x)
```

3.1192 $\int \frac{(A+C\cos^2(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=195

$$-\frac{(5A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(5A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4A\sin(c+dx)\sqrt{\sec(c+dx}}{a^2d}$$

```
[Out] (-4*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) - ((5*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + (4*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - ((5*A -
C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.41414, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2978, 2748, 2636, 2639, 2641}

$$-\frac{(5A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(5A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4A\sin(c+dx)\sqrt{\sec(c+dx}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-4*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) - ((5*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + (4*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - ((5*A -
C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A + C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(7A+C)}{\cos^{\frac{3}{2}}(c+}}{3a^2}$$

$$= -\frac{(5A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(\sqrt{}}{}$$

$$= -\frac{(5A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2}{}$$

$$= -\frac{(5A-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4A\sqrt{\sec(c+dx)}\sin}{a^2d}$$

$$= -\frac{4A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(5A-C)\sqrt{\cos(c+dx)}F\left(\right.}{3a}$$

Mathematica [C] time = 1.37652, size = 275, normalized size = 1.41

$$\frac{e^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(i\left(4Ae^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-19Ae^{i(c+dx)}-29Ae^{2}\right.\right.}{12a^{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^
2,x]
```

```
[Out] ((1 + E^(I*(c + d*x)))*(-((5*A - C)*(1 + E^(I*(c + d*x)))^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]) + I*(-5*A + C - 19*A*E^(I*(c + d*x)) - C*E^(
I*(c + d*x)) - 29*A*E^((2*I)*(c + d*x)) + C*E^((2*I)*(c + d*x)) - 31*A*E^((
3*I)*(c + d*x)) - C*E^((3*I)*(c + d*x)) - 12*A*E^((4*I)*(c + d*x)) + 4*A*E^
(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*Sqrt[Sec[c + d*x]])/(12*a
^2*d*E^((2*I)*(c + d*x))*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 1.206, size = 452, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)-48*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(43*A+C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(37*A+C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*
a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2,
 x)
```

3.1193 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$-\frac{(A-C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{a^2}$$

```
[Out] ((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a^2*d) - ((A - C)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])*Sqrt
[Sec[c + d*x]]) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.394108, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2978, 2748, 2641, 2639}

$$-\frac{(A-C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a^2*d) - ((A - C)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])*Sqrt
[Sec[c + d*x]]) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[S
ec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(5}{\sqrt{}}}{3a^2}$$

$$= -\frac{(A-C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

$$= \frac{(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(A+C)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 4.76956, size = 450, normalized size = 2.73

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left((7A-5C)\cos\left(\frac{1}{2}(c-dx)\right)+2(A-2C)\cos\left(\frac{1}{2}(3c+dx)\right)+3(A-C)\cos\left(\frac{1}{2}(c+3dx)\right)\right)}{2\sqrt{\sec(c+dx)}} - 2\sqrt{2}A\csc(c)e^{-id}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^
2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*((-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (2*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (((7*A - 5*C)*Cos[(c - d*x)/2]
+ 2*(A - 2*C)*Cos[(3*c + d*x)/2] + 3*(A - C)*Cos[(c + 3*d*x)/2])*Csc[c/2]*S
ec[c/2]*Sec[(c + d*x)/2]^3)/(2*Sqrt[Sec[c + d*x]]) + 8*A*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 8*C*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 1.344, size = 419, normalized size = 2.5

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6-4\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^6-4*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-16*A*cos(1/2*d*x+1/2*c)^4+20*C*cos(1/2*d*x+1/2*c)^4+3*A*co
s(1/2*d*x+1/2*c)^2-9*C*cos(1/2*d*x+1/2*c)^2+A+C)/a^2/cos(1/2*d*x+1/2*c)^3/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^2*cos(d*x + c)^2 + 2*
a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2,
 x)
```

3.1194 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=166

$$\frac{(A-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{(A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4C\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{a^2d}$$

```
[Out] (4*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) + ((A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*a^2*d) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2*Sec[c
+ d*x]^(3/2)) + ((A - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.388971, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2977, 2748, 2641, 2639}

$$\frac{(A-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{(A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4C\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (4*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) + ((A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*a^2*d) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2*Sec[c
+ d*x]^(3/2)) + ((A - 5*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])*Sqrt[S
ec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+}}{}}{3a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-5C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}} +$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-5C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}} +$$

$$= \frac{4C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A-5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{3a^2d}$$

Mathematica [C] time = 1.40048, size = 267, normalized size = 1.61

$$\frac{e^{-3i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(i\left(1+e^{2i(c+dx)}\right)\left(C\left(16e^{i(c+dx)}+20e^{2i(c+dx)}+9e^{3i(c+dx)}+3\right)-Ae^{i(c+dx)}\left(-1+e^{i(c+dx)}\right)\right)\right.}{12a^2d(\text{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] ((1 + E^(I*(c + d*x)))*(I*(1 + E^((2*I)*(c + d*x)))*(-(A*E^(I*(c + d*x))*(-
1 + E^(I*(c + d*x)))) + C*(3 + 16*E^(I*(c + d*x)) + 20*E^((2*I)*(c + d*x))
+ 9*E^((3*I)*(c + d*x)))) + (A - 5*C)*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))
^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (4*I)*C*E^((2*I)*(c + d*x
))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]])/(12*a^2*d*E^((3*I
)*(c + d*x))*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 1.096, size = 348, normalized size = 2.1

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*C*cos(1/2*d*x+1/2*c)^6-10*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+38*C*cos(1/2*d*x+1/2*c)^4-3*A*co
s(1/2*d*x+1/2*c)^2-15*C*cos(1/2*d*x+1/2*c)^2+A+C)/a^2/cos(1/2*d*x+1/2*c)^3/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2+A}{(a\cos{(dx+c)}+a)^2\sqrt{\sec{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos{(dx+c)}^2+A}{\left(a^2\cos{(dx+c)}^2+2\,a^2\cos{(dx+c)}+a^2\right)\sqrt{\sec{(dx+c)}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) +
 a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))
), x)
```

3.1195 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{2(A+5C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A+7C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sec^{\frac{3}{2}}(c+dx)}+\frac{2(A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-$$

```
[Out] -(((A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + (2*(A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x
])^2*Sec[c + d*x]^(5/2)) - ((A + 7*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d
*x])*Sec[c + d*x]^(3/2)) + (2*(A + 5*C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.432473, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2977, 2748, 2639, 2635, 2641}

$$\frac{2(A+5C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A+7C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sec^{\frac{3}{2}}(c+dx)}+\frac{2(A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -(((A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + (2*(A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x
])^2*Sec[c + d*x]^(5/2)) - ((A + 7*C)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d
*x])*Sec[c + d*x]^(3/2)) + (2*(A + 5*C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c +
 d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c}{}}{3a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} - \frac{(A+7C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} - \frac{(A+7C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{(A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(A+C)\sin(}{3d(a+a\cos(c+dx)}$$

$$= -\frac{(A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(A+5C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.76605, size = 762, normalized size = 3.79

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+C\sin\left(\frac{dx}{2}\right)\right)}{3d}+\frac{2(A+C)\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{8\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2A\sin\left(\frac{dx}{2}\right)+5C\right.}{3d}\right.}{(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) + (7*Sq
rt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
```

```
c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
+ E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) + (4*A*Cos
[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (20*C*Cos[c
/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c
/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (Cos[c/2 + (d
*x)/2]^4*Sqrt[Sec[c + d*x]]*((2*(A + 5*C + 2*C*Cos[2*c])*Cos[d*x]*Csc[c/2]*
Sec[c/2])/d + (4*C*Cos[2*d*x]*Sin[2*c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (8*Sec[c/2]*Sec[c/2 + (d*x
)/2]*(2*A*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2]))/(3*d) - (16*C*Cos[c]*Sin[d*x])/
d + (4*C*Cos[2*c]*Sin[2*d*x])/(3*d) - (8*(2*A + 5*C)*Tan[c/2])/(3*d) + (2*(
A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A] time = 1.247, size = 437, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*C*cos(1/2*
d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3+6*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*cos(
1/2*d*x+1/2*c)^6+20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+42*C*c
os(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4-48*
C*cos(1/2*d*x+1/2*c)^4+9*A*cos(1/2*d*x+1/2*c)^2+21*C*cos(1/2*d*x+1/2*c)^2-A
-C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c) + a)^2 \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+A}{\left(a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) +
 a^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)
), x)
```

3.1196 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{4(5A+14C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A+3C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A+3C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sec^{\frac{5}{2}}(c+dx)}-\frac{5(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{3a^2d}$$

```
[Out] (4*(5*A + 14*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a^2*d) - (5*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c +
 d*x])^2*Sec[c + d*x]^(7/2)) - ((A + 3*C)*Sin[c + d*x])/(a^2*d*(1 + Cos[c +
 d*x])*Sec[c + d*x]^(5/2)) + (4*(5*A + 14*C)*Sin[c + d*x])/(15*a^2*d*Sec[c
+ d*x]^(3/2)) - (5*(A + 3*C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.461364, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2977, 2748, 2635, 2641, 2639}

$$\frac{4(5A+14C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A+3C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A+3C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sec^{\frac{5}{2}}(c+dx)}-\frac{5(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (4*(5*A + 14*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a^2*d) - (5*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c +
 d*x])^2*Sec[c + d*x]^(7/2)) - ((A + 3*C)*Sin[c + d*x])/(a^2*d*(1 + Cos[c +
 d*x])*Sec[c + d*x]^(5/2)) + (4*(5*A + 14*C)*Sin[c + d*x])/(15*a^2*d*Sec[c
+ d*x]^(3/2)) - (5*(A + 3*C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c}{}}{3a}$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{(A+3C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} +$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{(A+3C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} -$$

$$= -\frac{(A+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{(A+3C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{4(5A+14C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5(A+3C)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 6.86583, size = 813, normalized size = 3.44

$$-\frac{4\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)\cos^4\left(\frac{c}{2}\right.}{3d(\cos(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (5
6*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
```

```
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (10
*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) - (10*C
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^2) + (Cos[c/2
+ (d*x)/2]^4*Sqrt[Sec[c + d*x]]*(-((60*A + 151*C + 20*A*Cos[2*c] + 73*C*Cos
[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(10*d) - (8*C*Cos[2*d*x]*Sin[2*c])/(3*d)
 + (2*C*Cos[3*d*x]*Sin[3*c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Si
n[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(7*A*S
in[(d*x)/2] + 13*C*Sin[(d*x)/2]))/(3*d) + (2*(20*A + 73*C)*Cos[c]*Sin[d*x])
/(5*d) - (8*C*Cos[2*c]*Sin[2*d*x])/(3*d) + (2*C*Cos[3*c]*Sin[3*d*x])/(5*d)
+ (4*(7*A + 13*C)*Tan[c/2])/(3*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2
])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Maple [A] time = 1.15, size = 451, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*C*cos(1/2
*d*x+1/2*c)^10+352*C*cos(1/2*d*x+1/2*c)^8+120*A*cos(1/2*d*x+1/2*c)^6+50*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+120*A*cos(1/2*d*x+1/2*c)^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-120*C*cos(1/2*d*x+1/2*c)^6+150*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*cos(1/2*d*x+1/2*c)^3+336*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-190*A*cos(1/2*d*x+1/2*c)^4-266*C*cos(1/2*d*x+1/2*c)^4+75*A*cos(1/2*d
*x+1/2*c)^2+135*C*cos(1/2*d*x+1/2*c)^2-5*A-5*C)/a^2/cos(1/2*d*x+1/2*c)^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+A}{\left(a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) +
 a^2)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)
), x)
```

3.1197 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$\frac{(11A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{(119A+9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(119A+9C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+$$

```
[Out] ((119*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((11*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(2*a^3*d) - ((119*A + 9*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((11*A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) -
((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2
*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((119*
A + 9*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.6188, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2978, 2748, 2636, 2641, 2639}

$$\frac{(11A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{(119A+9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(119A+9C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] ((119*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((11*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(2*a^3*d) - ((119*A + 9*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((11*A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) -
((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2
*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((119*
A + 9*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$=-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(13A\cdots}{\cos^{\frac{5}{2}}\cdots}}{5a^2}$$

$$=-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}+\frac{\left(\sqrt{\cos(\cdots}\right.}{\cdots}$$

$$=-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}-\frac{(119A\cdots}{3\cdots}$$

$$=-\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}-\frac{(119A\cdots}{3\cdots}$$

$$=-\frac{(119A+9C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}+\frac{(11A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+d\cdots}{2a^3d}$$

$$=\frac{(119A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(11A+C)\sqrt{\cos(\cdots}}{\cdots}$$

Mathematica [C] time = 7.81563, size = 822, normalized size = 2.91

$$-\frac{119\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\cdots\right.}{15d(\cos(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] (-119*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) -
 (3*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2
2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (2*C*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
(d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(119*A + 9*C)*Cos[d*x]*Csc[c/2]*Sec[c/2]
)/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]
))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(13*A*Sin[(d*x)/2] + 3*C*Sin[(d
*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] + 3*C*S
in[(d*x)/2]))/(3*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(4*A +
 33*A*Cos[c] + 3*C*Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (4*(13*A + 3*C)*Sec[c/2
 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/
(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [B] time = 1.612, size = 876, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(55*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-119*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-30*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(55*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+24*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(55*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-119*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-9*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*
```

```
c)^2*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(55*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(119*A+9*C)*sin(1/2*d*x+1/2*c)^10+24*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(389*A+29*C)*sin(1/2*d*x+1/2*c)^8-10*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1111*A+81*C)*sin(1/2*d*x+1/2*c
)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1414*A+99*C)*si
n(1/2*d*x+1/2*c)^4-3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
343*A+23*C)*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/a^3/sin(1
/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3,
 x)
```

3.1198 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=259

$$\frac{(49A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(\right.}{6a^3d}$$

```
[Out] -((49*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(10*a^3*d) - ((13*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(10*a^3*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c +
d*x])^3) - (2*(4*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos
[c + d*x])^2) - ((13*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^
3*Cos[c + d*x]))
```

Rubi [A] time = 0.616024, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2978, 2748, 2636, 2639, 2641}

$$\frac{(49A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(\right.}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -((49*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(10*a^3*d) - ((13*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(10*a^3*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c +
d*x])^3) - (2*(4*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos
[c + d*x])^2) - ((13*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^
3*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$=-\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(11A\cdots}{\cos^{\frac{3}{2}}(\cdots}}{5a^2}$$

$$=-\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\cdots$$

$$=-\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\cdots$$

$$=-\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\cdots$$

$$=-\frac{(13A-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}+\frac{(49A-C)\sqrt{\sec(c\cdots}}{10\cdots}$$

$$=-\frac{(49A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-C)\sqrt{\cos(c\cdots}}{\cdots}$$

Mathematica [C] time = 5.1512, size = 359, normalized size = 1.39

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-i(49A-C)e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(49*A - C)*(1 + E^(I*(c + d*x)
))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 160*(13*A - C)*Cos[(c + d*x)/2]^5*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*
x)/2]) + (2*I)*(642*A - 18*C + 2*(541*A - 4*C)*Cos[c + d*x] + 18*(29*A - C)
*Cos[2*(c + d*x)] + 106*A*Cos[3*(c + d*x)] - 4*C*Cos[3*(c + d*x)] + (161*I)
*A*Sin[c + d*x] + I*C*Sin[c + d*x] + (148*I)*A*Sin[2*(c + d*x)] + (8*I)*C*S
in[2*(c + d*x)] + (41*I)*A*Sin[3*(c + d*x)] + I*C*Sin[3*(c + d*x)]))*(Cos[(
c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(120*a^3*d*E^(I*d*x)*(1 + Cos[c + d*
x])^3)
```

Maple [B] time = 1.28, size = 685, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c
)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-C)*sin(1/2*
d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(817*A-
13*C)*sin(1/2*d*x+1/2*c)^6+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(124*A-C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(439*A-C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3,
 x)
```

3.1199 $\int \frac{(A+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=224

$$-\frac{(9A-C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-C)\sqrt{\cos(c+d}}{}$$

```
[Out] ((9*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3*Sqrt[Sec[c + d*x]]) - (2*(3*A - 2*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2*Sqrt[Sec[c + d*x]]) - ((9*A - C)*Sin[c + d*x])/(10*d*(a^3 + a^3*C
os[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.572126, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2978, 2748, 2641, 2639}

$$-\frac{(9A-C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-C)\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] ((9*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3*Sqrt[Sec[c + d*x]]) - (2*(3*A - 2*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2*Sqrt[Sec[c + d*x]]) - ((9*A - C)*Sin[c + d*x])/(10*d*(a^3 + a^3*C
os[c + d*x])*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9}{\sqrt{}}}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} - \frac{2(3A-2C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} - \frac{2(3A-2C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} - \frac{2(3A-2C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+}}$$

$$= \frac{(9A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.89026, size = 792, normalized size = 3.54

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+C\sin\left(\frac{dx}{2}\right)\right)}{5d} + \frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3A\sin\left(\frac{dx}{2}\right)-7C\sin\left(\frac{dx}{2}\right)\right)}{15d} + \frac{2(A+C)\tan\left(\frac{c}{2}\right)s}{5d}\right.}{(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^
3,x]
```

```
[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (S
qrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (2*C*Cos[c/
```

```
2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*
x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(9*A - C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d)
 + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(3*A*Sin[(d*x)/2] - 7*C*Sin[(d*x)/2]))/
(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2])
)/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]
))/(3*d) + (4*(3*A + C)*Tan[c/2])/(3*d) + (4*(3*A - 7*C)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

Maple [A] time = 1.224, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+1/2*c)^6+2
4*A*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
-17*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3,
 x)
```

3.1200 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=220

$$-\frac{(A-9C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-9C)\sqrt{\cos(c+dx)}}{}$$

```
[Out] ((A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3*Sec[c + d*x]^(3/2)) + (2*(2*A - 3*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2*Sqrt[Sec[c + d*x]]) - ((A - 9*C)*Sin[c + d*x])/(10*d*(a^3 + a^3*C
os[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.564981, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2977, 2978, 2748, 2641, 2639}

$$-\frac{(A-9C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-9C)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3*Sec[c + d*x]^(3/2)) + (2*(2*A - 3*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2*Sqrt[Sec[c + d*x]]) - ((A - 9*C)*Sin[c + d*x])/(10*d*(a^3 + a^3*C
os[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+}}{}}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{2(2A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{2(2A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{2(2A-3C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

$$= \frac{(A-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+3C)\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 6.97093, size = 787, normalized size = 3.58

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+C\sin\left(\frac{dx}{2}\right)\right)}{5d} + \frac{8\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+6C\sin\left(\frac{dx}{2}\right)\right)}{15d} - \frac{2(A+C)\tan\left(\frac{c}{2}\right)\text{se}}{5d}\right.}{(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] -(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
```

```
((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (3*
Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*C*Cos[
c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[
c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*
x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(A - 9*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d)
 + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 9*C*Sin[(d*x)/2]))/(3*d
) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*
d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + 6*C*Sin[(d*x)/2]))/
(15*d) + (4*(A - 9*C)*Tan[c/2])/(3*d) + (8*(A + 6*C)*Sec[c/2 + (d*x)/2]^2*T
an[c/2])/(15*d) - (2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*
Cos[c + d*x])^3
```

Maple [A] time = 1.468, size = 451, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8-30*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+198*C*cos(1/2*d*x+1/2*c)^6+
6*A*cos(1/2*d*x+1/2*c)^4-114*C*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^
2+27*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + A}{\left(a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3\right)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2
 + 3*a^3*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))
), x)
```

3.1201 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=218

$$\frac{(A-13C)\sin(c+dx)}{6d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-49C)\sqrt{\cos(c+dx)}}{1}$$

```
[Out] -((A - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(10*a^3*d) + ((A - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*
x])^3*Sec[c + d*x]^(5/2)) + (2*(A - 4*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(3/2)) + ((A - 13*C)*Sin[c + d*x])/(6*d*(a^3 + a^3*
Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.575059, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3042, 2977, 2748, 2641, 2639}

$$\frac{(A-13C)\sin(c+dx)}{6d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-49C)\sqrt{\cos(c+dx)}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((A - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(10*a^3*d) + ((A - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*
x])^3*Sec[c + d*x]^(5/2)) + (2*(A - 4*C)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(3/2)) + ((A - 13*C)*Sin[c + d*x])/(6*d*(a^3 + a^3*
Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+}{}}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{2(A-4C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{2(A-4C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{2(A-4C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A-49C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A-13C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.95723, size = 813, normalized size = 3.73

$$\frac{\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right.}{15d(\cos(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (49*
Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*
```

```
Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) - (26*C*Co
s[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
 (d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(-A + 39*C + 10*C*Cos[2*c])*Cos[d*x]*Cs
c[c/2]*Sec[c/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] +
 C*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2
] + 17*C*Sin[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x
)/2] + 23*C*Sin[(d*x)/2]))/(3*d) + (16*C*Cos[c]*Sin[d*x])/d + (4*(A + 23*C)
*Tan[c/2])/(3*d) - (4*(7*A + 17*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) +
(2*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 1.124, size = 451, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*C*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+578*C*cos(1/2*d*x+1/2*c)^
6-24*A*cos(1/2*d*x+1/2*c)^4-264*C*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2
*c)^2+37*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos \left(dx + c\right)^{2} + A}{\left(a \cos \left(dx + c\right) + a\right)^{3} \sec \left(dx + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2
 + 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)
), x)
```

3.1202 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=249

$$\frac{(A+11C)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{(9A+119C)\sin(c+dx)}{30d\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}$$

```
[Out] -((9*A + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(2*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c +
 d*x])^3*Sec[c + d*x]^(7/2)) - (2*C*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x
])^2*Sec[c + d*x]^(5/2)) - ((9*A + 119*C)*Sin[c + d*x])/(30*d*(a^3 + a^3*Co
s[c + d*x])*Sec[c + d*x]^(3/2)) + ((A + 11*C)*Sin[c + d*x])/(2*a^3*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.597213, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2977, 2748, 2639, 2635, 2641}

$$\frac{(A+11C)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{(9A+119C)\sin(c+dx)}{30d\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```


```
[Out] -((9*A + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(2*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[c +
 d*x])^3*Sec[c + d*x]^(7/2)) - (2*C*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x
])^2*Sec[c + d*x]^(5/2)) - ((9*A + 119*C)*Sin[c + d*x])/(30*d*(a^3 + a^3*Co
s[c + d*x])*Sec[c + d*x]^(3/2)) + ((A + 11*C)*Sin[c + d*x])/(2*a^3*d*Sqrt[S
ec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+}{}}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} - \frac{2C\sin(c+dx)}{3ad(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} +$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} - \frac{2C\sin(c+dx)}{3ad(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} -$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} - \frac{2C\sin(c+dx)}{3ad(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} -$$

$$= -\frac{(9A+119C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(A+C)\sin}{5d(a+a\cos(c+d}$$

$$= -\frac{(9A+119C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+11C)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 4.15037, size = 573, normalized size = 2.3

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left((156A+1961C)\cos\left(\frac{1}{2}(c-dx)\right)+(114A+1609C)\cos\left(\frac{1}{2}(3c+dx)\right)+90A\cos\left(\frac{1}{2}(c+3dx)\right)+45A\cos\left(\frac{1}{2}(5c+3dx)\right)\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*((18*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (238*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((
2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (((156*A + 1961*C)*Cos[(c - d
*x)/2] + (114*A + 1609*C)*Cos[(3*c + d*x)/2] + 90*A*Cos[(c + 3*d*x)/2] + 11
65*C*Cos[(c + 3*d*x)/2] + 45*A*Cos[(5*c + 3*d*x)/2] + 620*C*Cos[(5*c + 3*d*
x)/2] + 27*A*Cos[(3*c + 5*d*x)/2] + 292*C*Cos[(3*c + 5*d*x)/2] + 65*C*Cos[(
7*c + 5*d*x)/2] + 5*C*Cos[(5*c + 7*d*x)/2] - 5*C*Cos[(9*c + 7*d*x)/2])*Csc[
c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*Sqrt[Sec[c + d*x]]) + 60*A*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 660*C*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))/(15*a^3*d*(1 + Cos[c +
d*x])^3)
```

Maple [A] time = 1.295, size = 465, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*C*cos(1/
2*d*x+1/2*c)^10+108*A*cos(1/2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*c
os(1/2*d*x+1/2*c)^5+54*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468
*C*cos(1/2*d*x+1/2*c)^8+330*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+714*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/
2*c)^6-1058*C*cos(1/2*d*x+1/2*c)^6+114*A*cos(1/2*d*x+1/2*c)^4+474*C*cos(1/2
*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c)^2-47*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/
a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2+A}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2
 + 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)
), x)
```

3.1203 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=290

$$\frac{7(7A+33C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(13A+63C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(13A+63C)\sin(c+dx)}{10d\sec^{\frac{5}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A+63C)\sqrt{\cos(c+d}{}$$

```
[Out] (7*(7*A + 33*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((13*A + 63*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3*Sec[c + d*x]^(9/2)) - (2*(A + 6*C)*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) - ((13*A + 63*C)*Sin[c + d*x])/(10*d*(
a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(5/2)) + (7*(7*A + 33*C)*Sin[c + d*x])
/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((13*A + 63*C)*Sin[c + d*x])/(6*a^3*d*Sqrt
[Sec[c + d*x]])
```

Rubi [A] time = 0.656942, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3042, 2977, 2748, 2635, 2641, 2639}

$$\frac{7(7A+33C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(13A+63C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(13A+63C)\sin(c+dx)}{10d\sec^{\frac{5}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A+63C)\sqrt{\cos(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```


```
[Out] (7*(7*A + 33*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((13*A + 63*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3*Sec[c + d*x]^(9/2)) - (2*(A + 6*C)*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) - ((13*A + 63*C)*Sin[c + d*x])/(10*d*(
a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(5/2)) + (7*(7*A + 33*C)*Sin[c + d*x])
/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((13*A + 63*C)*Sin[c + d*x])/(6*a^3*d*Sqrt
[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$=-\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+d}{}}{5a^2}$$

$$=-\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}-\frac{2(A+6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}$$

$$=-\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}-\frac{2(A+6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}$$

$$=-\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}-\frac{2(A+6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}$$

$$=-\frac{(A+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}-\frac{2(A+6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}$$

$$=\frac{7(7A+33C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A+63C)\sqrt{\cos(}}{}$$

Mathematica [C] time = 5.39056, size = 623, normalized size = 2.15

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(2(806A+3795C)\cos\left(\frac{1}{2}(c-dx)\right)+2(664A+3135C)\cos\left(\frac{1}{2}(3c+dx)\right)+940A\cos\left(\frac{1}{2}(c+3dx)\right)+530A\cos\left(\frac{1}{2}(5c\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)
),x]
```

```
[Out] -(Cos[(c + d*x)/2]^6*((98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))]))/E^(I*d*x) + (462*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + ((2*(806*A + 3795*C)*Cos[(c
- d*x)/2] + 2*(664*A + 3135*C)*Cos[(3*c + d*x)/2] + 940*A*Cos[(c + 3*d*x)/2
] + 4500*C*Cos[(c + 3*d*x)/2] + 530*A*Cos[(5*c + 3*d*x)/2] + 2430*C*Cos[(5*
c + 3*d*x)/2] + 234*A*Cos[(3*c + 5*d*x)/2] + 1110*C*Cos[(3*c + 5*d*x)/2] +
60*A*Cos[(7*c + 5*d*x)/2] + 276*C*Cos[(7*c + 5*d*x)/2] + 15*C*Cos[(5*c + 7*
d*x)/2] - 15*C*Cos[(9*c + 7*d*x)/2] - 3*C*Cos[(7*c + 9*d*x)/2] + 3*C*Cos[(1
1*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(16*Sqrt[Sec[c + d*x
]]) + 260*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
 + 1260*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))
/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 1.222, size = 479, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-192*C*cos(1/
2*d*x+1/2*c)^12+864*C*cos(1/2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+228*C*cos(1/2*d*x+1/2*c)^8+630*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*cos(1/2*d*x+1/2*c)^5+1386*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6-1590*C*cos(1/2*d*x+1/2*c)^6+264*A*cos
(1/2*d*x+1/2*c)^4+744*C*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2-57*C
*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
```

```
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^{2}+A}{\left(a^{3}\cos(dx+c)^{3}+3\,a^{3}\cos(dx+c)^{2}+3\,a^{3}\cos(dx+c)+a^{3}\right)\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2
 + 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(7/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)
), x)
```

3.1204 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=213

$$\frac{2a(16A+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a(16A+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)\sqrt{\sec(c}}{315d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (16*a*(16*A + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[
c + d*x]]) + (8*a*(16*A + 21*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqr
t[a + a*Cos[c + d*x]]) + (2*a*(16*A + 21*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.58155, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3044, 2980, 2772, 2771}

$$\frac{2a(16A+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a(16A+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)\sqrt{\sec(c}}{315d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (16*a*(16*A + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[
c + d*x]]) + (8*a*(16*A + 21*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqr
t[a + a*Cos[c + d*x]]) + (2*a*(16*A + 21*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\right.}{\cos^{\frac{11}{2}}(c+d\right.}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{\left(2\sqrt{\cos\right.}{}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\sec}{9d}$$

$$= \frac{2a(16A+21C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{7}{2}}(c+}{63d\sqrt{a+a}}$$

$$= \frac{8a(16A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a(16A+21C}{105d\sqrt{}}$$

$$= \frac{16a(16A+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{8a(16A+21}{315d}$$

Mathematica [A] time = 0.708371, size = 124, normalized size = 0.58

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(2(88A+63C)\cos(c+dx)+11(16A+21C)\cos(2(c+dx))+32A\cos(3}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(214*A + 189*C + 2*(88*A + 63*C)*Cos[c + d*x] +
 11*(16*A + 21*C)*Cos[2*(c + d*x)] + 32*A*Cos[3*(c + d*x)] + 42*C*Cos[3*(c
+ d*x)] + 32*A*Cos[4*(c + d*x)] + 42*C*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)
*Tan[(c + d*x)/2])/(315*d)
```

Maple [A] time = 0.22, size = 129, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(128A(\cos(dx+c))^4+168C(\cos(dx+c))^4+64A(\cos(dx+c))^3+84C(\cos(dx+c))^3+48A\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+168*C*cos(d*x+c)^4+64*A*cos(d*
x+c)^3+84*C*cos(d*x+c)^3+48*A*cos(d*x+c)^2+63*C*cos(d*x+c)^2+40*A*cos(d*x+c
)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c
)
```

Maxima [B] time = 1.77329, size = 890, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, al
gorithm="maxima")
```

```
[Out] 2/315*(A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 735*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10
7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(co
s(d*x + c) + 1)^10 + 1)) + 21*C*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x +
 c) + 1) - 55*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 82*sqrt
(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 66*sqrt(2)*sqrt(a)*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 + 31*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d
*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.49153, size = 311, normalized size = 1.46

$$\frac{2\left(8\,(16\,A+21\,C)\cos\left(dx+c\right)^{4}+4\,(16\,A+21\,C)\cos\left(dx+c\right)^{3}+3\,(16\,A+21\,C)\cos\left(dx+c\right)^{2}+40\,A\cos\left(dx+c\right)+35\,A\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, al
gorithm="fricas")
```

```
[Out] 2/315*(8*(16*A + 21*C)*cos(d*x + c)^4 + 4*(16*A + 21*C)*cos(d*x + c)^3 + 3*
(16*A + 21*C)*cos(d*x + c)^2 + 40*A*cos(d*x + c) + 35*A)*sqrt(a*cos(d*x + c
) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c
)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(11/
2), x)
```

3.1205 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=168

$$\frac{2a(24A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\,c}}{7d}$$

```
[Out] (4*a*(24*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a*(24*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(7*d)
```

Rubi [A] time = 0.505598, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3044, 2980, 2772, 2771}

$$\frac{2a(24A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\,c}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*a*(24*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a*(24*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+\right.}{\cos^{\frac{9}{2}}(c+d}$$

$$=\frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{\left(2\sqrt{co\right.}{}$$

$$=\frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}+\frac{2A\sqrt{a+a\cos(c+dx)}\,se}{7a}$$

$$=\frac{2a(24A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sec^{\frac{5}{2}}(c+}{35d\sqrt{a+}}$$

$$=\frac{4a(24A+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(24A+35}{105d}$$

Mathematica [A] time = 0.523482, size = 101, normalized size = 0.6

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(3(36A+35C)\cos(c+dx)+(24A+35C)\cos(2(c+dx))+24A\cos(3}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(54*A + 35*C + 3*(36*A + 35*C)*Cos[c + d*x] + (
24*A + 35*C)*Cos[2*(c + d*x)] + 24*A*Cos[3*(c + d*x)] + 35*C*Cos[3*(c + d*x
)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Maple [A] time = 0.189, size = 107, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(48A(\cos(dx+c))^3+70C(\cos(dx+c))^3+24A(\cos(dx+c))^2+35C(\cos(dx+c))^2+18A\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+70*C*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+35*C*cos(d*x+c)^2+18*A*cos(d*x+c)+15*A)*cos(d*x+c)*(1/cos(d*x+c))^(9/2
)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.8305, size = 765, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 2/105*(3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 35*C*(3*sq
rt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 12*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 6*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + sqrt(2)
*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.46804, size = 263, normalized size = 1.57

$$\frac{2\left(2\,(24\,A + 35\,C)\cos\left(dx + c\right)^{3} + (24\,A + 35\,C)\cos\left(dx + c\right)^{2} + 18\,A\cos\left(dx + c\right) + 15\,A\right)\sqrt{a\cos\left(dx + c\right) + a}\sin\left(dx + c\right)}{105\left(d\cos\left(dx + c\right)^{4} + d\cos\left(dx + c\right)^{3}\right)\sqrt{\cos\left(dx + c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(2*(24*A + 35*C)*cos(d*x + c)^3 + (24*A + 35*C)*cos(d*x + c)^2 + 18*A
*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c
)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(9/2
), x)
```

3.1206 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{2a(8A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*(8*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
 d*x]]) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*
d)
```

Rubi [A] time = 0.436867, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4221, 3044, 2980, 2771}

$$\frac{2a(8A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*(8*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
 d*x]]) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*
d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+\right.}{\cos^{\frac{7}{2}}(c+d}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{\left(2\sqrt{\mathrm{co}\right.}{}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}{5d}$$

$$= \frac{2a(8A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{3}{2}}(c+}{15d\sqrt{a+a}}$$

Mathematica [A] time = 0.292773, size = 73, normalized size = 0.59

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((8A+15C)\cos(2(c+dx))+8A\cos(c+dx)+14A+15C)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(14*A + 15*C + 8*A*Cos[c + d*x] + (8*A + 15*C)*
Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.175, size = 85, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(8A(\cos(dx+c))^2+15C(\cos(dx+c))^2+4A\cos(dx+c)+3A\right)\cos(dx+c)}{15d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+15*C*cos(d*x+c)^2+4*A*cos(d*x+c)+
3*A)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.71848, size = 640, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 2/15*(A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x +
```

```
c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 15*C*(sqrt(2)*sqrt(a)*sin(d*x + c)/
(cos(d*x + c) + 1) - 3*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 3*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(2)*sqrt(a)*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.38248, size = 213, normalized size = 1.73

$$\frac{2\left((8A+15C)\cos(dx+c)^2+4A\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*((8*A + 15*C)*cos(d*x + c)^2 + 4*A*cos(d*x + c) + 3*A)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x
+ c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos(dx+c)^2+A\right)\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2
), x)
```

3.1207 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=136

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin$$

```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.415969, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3044, 2980, 2774, 216}

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a + a\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+}{\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{\left(2\sqrt{\mathrm{co}\right.}{}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}{3a}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}{3a}$$

$$= \frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [A] time = 0.223471, size = 90, normalized size = 0.66

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2A\sin\left(\frac{3}{2}(c+dx)\right)+3\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[2]*
C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*A*Sin[(3*(c + d*x
))/2]))/(3*d)
```

Maple [B] time = 0.203, size = 271, normalized size = 2.

$$-\frac{2\cos(dx+c)(\sin(dx+c))^2}{3d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(3C\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+6*C*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*
cos(d*x+c)+3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c
))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(5/2)*sin(d*x+c)^2/(-
1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 2.32251, size = 1836, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/6*(3*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*sqrt(a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c)
- (cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + ((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))) + 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
```

```
2*d*x + 2*c)))) - 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 4*A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(
cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.60076, size = 333, normalized size = 2.45

$$-\frac{2\left(3\left(C\cos\left(dx+c\right)^{2}+C\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(2\,A\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*(C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c) + A
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2
), x)
```

3.1208 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=137

$$-\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{d}+\frac{\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a*(2*A - C)*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.43131, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3044, 2981, 2774, 216}

$$-\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{d}+\frac{\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a*(2*A - C)*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a + a\cos(c + dx)} \left(A + C\cos^2(c + dx)\right) \sec^{\frac{3}{2}}(c + dx)\, dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{\sqrt{a + a\cos(c + dx)}\left(A + \right.}{\cos^{\frac{3}{2}}(c + d}$$

$$= \frac{2A\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}\sin(c + dx)}{d} + \frac{\left(2\sqrt{\mathrm{co}\right.}{}$$

$$= -\frac{a(2A - C)\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}} + \frac{2A\sqrt{a + a\cos(c +}{}$$

$$= -\frac{a(2A - C)\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}} + \frac{2A\sqrt{a + a\cos(c +}{}$$

$$= \frac{\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}}{d} - $$

Mathematica [A] time = 0.267348, size = 100, normalized size = 0.73

$$\frac{\sec\left(\frac{1}{2}(c + dx)\right)\sqrt{\sec(c + dx)}\sqrt{a(\cos(c + dx) + 1)}\left(2\sin\left(\frac{1}{2}(c + dx)\right)(2A + C\cos(c + dx)) + \sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c + d\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*C*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + C*Cos[c + d*
x])*Sin[(c + d*x)/2]))/(2*d)
```

Maple [A] time = 0.193, size = 185, normalized size = 1.4

$$\frac{\cos(dx + c)}{d(1 + \cos(dx + c))}\left(C\sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}}\arctan\left(\frac{\sin(dx + c)}{\cos(dx + c)}\sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}}\right)\cos(dx + c) + C\sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}}\, a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+C*cos(d*x
+c)*sin(d*x+c)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*
x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [B] time = 2.2243, size = 1202, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C + 8*A*(sqrt(2)*sqrt
(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)
```

```
/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Fricas [A] time = 1.57315, size = 285, normalized size = 2.08

$$-\frac{(C\cos(dx+c)+C)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(C\cos(dx+c)+2A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -((C*cos(d*x + c) + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))) - (C*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

3.1209 $\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,d\dots$

Optimal. Leaf size=144

$$\frac{\sqrt{a}(8A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{2d\sqrt{\sec(c+dx)}}+\frac{aC\sin(c+\dots}{4d\sqrt{\sec(c+dx)}\sqrt{a\,c\dots}}$$

```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*C*Sin[c + d*x])/(4*d*Sq
rt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.427713, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3046, 2981, 2774, 216}

$$\frac{\sqrt{a}(8A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{2d\sqrt{\sec(c+dx)}}+\frac{aC\sin(c+\dots}{4d\sqrt{\sec(c+dx)}\sqrt{a\,c\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```



```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*C*Sin[c + d*x])/(4*d*Sq
rt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\right.}{\sqrt{\cos(c+dx}}$$

$$= \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{aC\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{C\sqrt{a+a\cos(c+dx}}{2d\sqrt{\sec(c+}}$$

$$= \frac{aC\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{C\sqrt{a+a\cos(c+dx}}{2d\sqrt{\sec(c+}}$$

$$= \frac{\sqrt{a}(8A+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{4d}$$

Mathematica [A] time = 0.33961, size = 118, normalized size = 0.82

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+3C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\left(2\sin\left(\frac{1}{2}(\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(8*A + 3*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*C*Sqrt[Co
s[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)
```

Maple [A] time = 0.218, size = 204, normalized size = 1.4

$$-\frac{(\cos(dx+c))^2-1}{4d(\sin(dx+c))^2}\left(2C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3C\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+8A\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.43338, size = 1629, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*(16*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
```

```
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1)))*C)/d
```

Fricas [A] time = 1.85704, size = 338, normalized size = 2.35

$$-\frac{((8A+3C)\cos(dx+c)+8A+3C)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2C\cos(dx+c)^2+3C\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(((8*A + 3*C)*cos(d*x + c) + 8*A + 3*C)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*C*cos(d*x + c)^2
+ 3*C*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)
), x)
```

3.1210 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+5C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{a\,\text{c}}}{3d\sec^{\frac{3}{2}}(c}$$

```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*C*Sin[c + d*x])/(12*d*S
qrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (C*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a*(8*A + 5*C)*Sin[c + d*x])/(8*d*S
qrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.505332, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3046, 2981, 2770, 2774, 216}

$$\frac{\sqrt{a}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+5C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{a\,\text{c}}}{3d\sec^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*C*Sin[c + d*x])/(12*d*S
qrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (C*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a*(8*A + 5*C)*Sin[c + d*x])/(8*d*S
qrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\left(A+C\right.$$

$$= \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\sqrt{\mathrm{co}}}{}$$

$$= \frac{aC\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\sqrt{a}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{}{12d\sqrt{}}$$

Mathematica [A] time = 0.509628, size = 134, normalized size = 0.71

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(8A+5C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+\right.\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2]*
(8*A + 5*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (24*A + 1
9*C + 10*C*Cos[c + d*x] + 4*C*Cos[2*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3
*(c + d*x))/2])))/(48*d)
```

Maple [A] time = 0.228, size = 274, normalized size = 1.5

$$\frac{(-1+\cos(dx+c))^2\cos(dx+c)}{24\,d\,(\sin(dx+c))^4}\left(8\,C\sin(dx+c)(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+10\,C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\mathrm{c}}{1+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+10*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2
4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+24*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))+15*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 2.93982, size = 3663, normalized size = 19.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/96*(24*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
```

```
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (4*(cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))
*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*cos(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arctan2(-(cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))
 + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
, (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3
```

```
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin
(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + arctan
2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) - 1)))*C)/d
```

Fricas [A] time = 1.89207, size = 385, normalized size = 2.04

$$-\frac{3\left((8A+5C)\cos(dx+c)+8A+5C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8C\cos(dx+c)^3+10C\cos(dx+c)^2+3(8A+5C)\cos(d\right.}{\sqrt{\cos(dx+c)}}}{24\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(3*((8*A + 5*C)*cos(d*x + c) + 8*A + 5*C)*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*C*cos(d*x + c)
^3 + 10*C*cos(d*x + c)^2 + 3*(8*A + 5*C)*cos(d*x + c))*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos(c+dx)+1\right)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + C*cos(c + d*x)**2)/sqrt(sec(c + d*
x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)
), x)
```

3.1211 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{a(48A+35C)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48A+}{64d\sqrt{\sec(c+}$$

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*C*Sin[c + d*x])/(24*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (C*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a*(48*A + 35*C)*Sin[c + d*x])/(
96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(48*A + 35*C)*Sin[c
+ d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.593696, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3046, 2981, 2770, 2774, 216}

$$\frac{a(48A+35C)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48A+}{64d\sqrt{\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*C*Sin[c + d*x])/(24*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (C*Sqrt[a + a*Cos[c + d*x]
]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a*(48*A + 35*C)*Sin[c + d*x])/(
96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(48*A + 35*C)*Sin[c
+ d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\left(A+\right.$$

$$= \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \mathrm{co}}{}$$

$$= \frac{aC\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+d}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+d}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+d}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+d}{4d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\sqrt{a}(48A+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{}{24}$$

Mathematica [A] time = 0.539651, size = 151, normalized size = 0.65

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(48A+35C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}\right.\right.\right.}{384}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2]*
(48*A + 35*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (144*A
+ 133*C + 2*(48*A + 53*C)*Cos[c + d*x] + 28*C*Cos[2*(c + d*x)] + 12*C*Cos[3
*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384*d)
```

Maple [A] time = 0.193, size = 344, normalized size = 1.5

$$-\frac{(-1+\cos(dx+c))^3\cos(dx+c)}{192\,d\,(\sin(dx+c))^6}\left(48\,C\sin(dx+c)(\cos(dx+c))^3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+56\,C\sin(dx+c)(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^3*(48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+56*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+96*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+70*C*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+144*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+105*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+144*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+10
5*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+
c))^(3/2)/sin(d*x+c)^6
```

Maxima [B] time = 3.81134, size = 10395, normalized size = 44.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/768*(48*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
```

2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((60*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 15*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 15*sin(4*d*x + 4*c)^3 + 60*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 15*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 60*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 31*cos(4*d*x + 4*c) + 15)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 31*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 15*cos(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 60*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (15*cos(4*d*x + 4*c)^3 + 4*(15*cos(4*d*x + 4*c)^3 + (15*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 38*cos(4*d*x + 4*c)^2 + 31*cos(4*d*x + 4*c) - 8)

```
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (15*cos(4*d*x + 4
*c) - 8)*sin(4*d*x + 4*c)^2 + 4*(15*cos(4*d*x + 4*c)^3 + (15*cos(4*d*x + 4*
c) - 8)*sin(4*d*x + 4*c)^2 + 22*cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c) - 8)*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 8*cos(4*d*x + 4*c)
^2 + (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x +
 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 31*cos(4*d*x + 4*c) + 15)*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 31*sin(4*
d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 15*cos(4
*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(15*c
os(4*d*x + 4*c)^3 + (15*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 23*cos(4
*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 15*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) - 4*(4*(15*cos(4*d*x + 4*c) - 8)*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (15*cos(4*d*x + 4*c) - 8)
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*si
n(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 6*(cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)^(1/4)*((32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(
4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 -
4*(3*sin(4*d*x + 4*c)^3 + 3*(cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*s
in(4*d*x + 4*c) - 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x
 + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 3*cos(4*d*x + 4*c)^2*sin(4
*d*x + 4*c) - 3*sin(4*d*x + 4*c)^3 - 4*(3*sin(4*d*x + 4*c)^3 + (3*cos(4*d*x
 + 4*c)^2 + 6*cos(4*d*x + 4*c) + 11)*sin(4*d*x + 4*c) + 32*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 32*(cos(4*d*x + 4*c
)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 - 2*(6*sin(4*d*x + 4*c)^3 + 6*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4
*c))*sin(4*d*x + 4*c) + 3*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))*sin(4*d*x + 4*c) - (64*cos(4*d*x + 4*c)^2 + 64*sin(4*d*x + 4*c)^2 - 61*
cos(4*d*x + 4*c) - 3)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*cos(1/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(16*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*cos(4*d*x + 4*c)^2 + 8*(2*cos(4*d*x +
4*c)^2 + 5*sin(4*d*x + 4*c)^2 - 32*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d
```

*x + 4*c), cos(4*d*x + 4*c))) - 2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 3*(cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 10*sin(4*d*x + 4*c)^2 - 61*sin(4*d*x +
 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (32*cos(4*d*x + 4*c)^2 + 32*sin(4*d
*x + 4*c)^2 + 3*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (32*(c
os(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 3*cos(4*d*x + 4*c)^3 - 4*(3*
cos(4*d*x + 4*c)^3 + (3*cos(4*d*x + 4*c) + 16)*sin(4*d*x + 4*c)^2 + 10*cos(
4*d*x + 4*c)^2 - 16*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x
+ 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 37*cos(4
*d*x + 4*c) + 24)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 -
3*(cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - 4*(3*cos(4*d*x + 4*c)^3 + 3*(
cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 + 30*cos(4*d*x + 4*c)^2 - 8*(cos(4
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 16*(cos(4*d*x + 4*c)^2 + sin(4*d*x
 + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 51*cos(4*d*x + 4*c) + 24)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 - 24*cos(4*d*x + 4*c)^2 - 2*(6*cos(4*d*x + 4*c)^3 + 2
*(3*cos(4*d*x + 4*c) + 22)*sin(4*d*x + 4*c)^2 + 38*cos(4*d*x + 4*c)^2 - (32
*cos(4*d*x + 4*c)^2 + 32*sin(4*d*x + 4*c)^2 - 29*cos(4*d*x + 4*c) - 3)*cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*sin(4*d*x + 4*c)*sin(1
/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*cos(4*d*x + 4*c))*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (16*cos(4*d*x + 4*c)^2 +
 16*sin(4*d*x + 4*c)^2 + 3*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2*sin(4*d*x + 4*c) - 8*((3*cos(4*d*x + 4*c) + 22)*sin(4*d*x + 4*c)
 - 16*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 6*(cos(4*d*x + 4*c
) + 8)*sin(4*d*x + 4*c) + 29*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))*sin(4*d*x + 4*c) + 3*(cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 3*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 10
5*((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^
2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 +
 sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x

```
 + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(
1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1))) + 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^
2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4
*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(
-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1
/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (4*(cos(4*d*x + 4*
c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^
2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
 cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + s
in(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
```

```
*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/
2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (4*(cos(4*d*x + 4*c)
^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
+ 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - c
os(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin
(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*C/(4*(cos(4*d*
x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x +
 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*
c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.29949, size = 440, normalized size = 1.88

$$-\frac{3\left((48\,A+35\,C)\cos\left(dx+c\right)+48\,A+35\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,C\cos(dx+c)^4+56\,C\cos(dx+c)^3+2\,(48\,A+3}{}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*((48*A + 35*C)*cos(d*x + c) + 48*A + 35*C)*sqrt(a)*arctan(sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*C*cos(d*
x + c)^4 + 56*C*cos(d*x + c)^3 + 2*(48*A + 35*C)*cos(d*x + c)^2 + 3*(48*A +
 35*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

3.1212 $\int (a+a\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=266

$$\frac{2a^2(28A+33C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{231d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(112A+143C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{385d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(112A+143C)\sin(c+d}{1155d\sqrt{a\cos(c+d}}$$

```
[Out] (16*a^2*(112*A + 143*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1155*d*Sqrt[a + a
*Cos[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(112*A + 143*C)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(385*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(28*A + 33*C)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(231*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqr
t[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(33*d) + (2*A*(a + a
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.836181, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2980, 2772, 2771}

$$\frac{2a^2(28A+33C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{231d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(112A+143C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{385d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(112A+143C)\sin(c+d}{1155d\sqrt{a\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2),x
]
```


```
[Out] (16*a^2*(112*A + 143*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1155*d*Sqrt[a + a
*Cos[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(112*A + 143*C)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(385*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(28*A + 33*C)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(231*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqr
t[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(33*d) + (2*A*(a + a
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c}$$

$$=\frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d}+$$

$$=\frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{33d}+\frac{2A}{}$$

$$=\frac{2a^2(28A+33C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{231d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+}}{}$$

$$=\frac{2a^2(112A+143C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{385d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(28}{}$$

$$=\frac{8a^2(112A+143C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(112}{}$$

$$=\frac{16a^2(112A+143C)\sqrt{\sec(c+dx)}\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}}+\frac{8a^2(11}{}$$

Mathematica [A] time = 0.839492, size = 146, normalized size = 0.55

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((4228A+4147C)\cos(c+dx)+2(728A+737C)\cos(2(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(1652*A + 1188*C + (4228*A + 4147*C)*Cos[c +
d*x] + 2*(728*A + 737*C)*Cos[2*(c + d*x)] + 1456*A*Cos[3*(c + d*x)] + 1859*
C*Cos[3*(c + d*x)] + 224*A*Cos[4*(c + d*x)] + 286*C*Cos[4*(c + d*x)] + 224*
A*Cos[5*(c + d*x)] + 286*C*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(c + d
*x)/2])/(2310*d)
```

Maple [A] time = 0.192, size = 152, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(896A(\cos(dx+c))^5+1144C(\cos(dx+c))^5+448A(\cos(dx+c))^4+572C(\cos(dx+c))^4+\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/1155/d*a*(-1+cos(d*x+c))*(896*A*cos(d*x+c)^5+1144*C*cos(d*x+c)^5+448*A*c
os(d*x+c)^4+572*C*cos(d*x+c)^4+336*A*cos(d*x+c)^3+429*C*cos(d*x+c)^3+280*A*
cos(d*x+c)^2+165*C*cos(d*x+c)^2+245*A*cos(d*x+c)+105*A)*cos(d*x+c)*(a*(1+co
s(d*x+c)))^(1/2)*(1/cos(d*x+c))^(13/2)/sin(d*x+c)
```

Maxima [B] time = 1.83423, size = 961, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] 4/1155*(7*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqrt(2
)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2
99*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)*a^(3/
2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(
```

```
cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 1)) + 11*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 455
*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 868*sqrt(2)*a^(3/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 962*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 + 653*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 247*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 38*sqrt(2)*
a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*C*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.49977, size = 383, normalized size = 1.44

$$\frac{2\left(8\,(112\,A+143\,C)a\cos\left(dx+c\right)^{5}+4\,(112\,A+143\,C)a\cos\left(dx+c\right)^{4}+3\,(112\,A+143\,C)a\cos\left(dx+c\right)^{3}+5\,(56\,A+33\right.}{1155\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)\sqrt{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] 2/1155*(8*(112*A + 143*C)*a*cos(d*x + c)^5 + 4*(112*A + 143*C)*a*cos(d*x +
c)^4 + 3*(112*A + 143*C)*a*cos(d*x + c)^3 + 5*(56*A + 33*C)*a*cos(d*x + c)^
2 + 245*A*a*cos(d*x + c) + 105*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(
(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(1
3/2), x)
```

3.1213 $\int (a+a\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=219

$$\frac{2a^2(52A+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(136A+189C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+189C)\sin(c+d}{315d\sqrt{a\cos(c+d}}$$

```
[Out] (4*a^2*(136*A + 189*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(52*A + 63*C)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x
]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*A*(a + a*Cos[c + d*x])^(3/2
)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.748089, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2980, 2772, 2771}

$$\frac{2a^2(52A+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(136A+189C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+189C)\sin(c+d}{315d\sqrt{a\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x
]
```


```
[Out] (4*a^2*(136*A + 189*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(52*A + 63*C)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x
]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*A*(a + a*Cos[c + d*x])^(3/2
)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}\left(\right.}{\cos^{\frac{11}{2}}(c}$$

$$=\frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}+-$$

$$=\frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{2A}{}$$

$$=\frac{2a^2(52A+63C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+}}{}$$

$$=\frac{2a^2(136A+189C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(52A}{}$$

$$=\frac{4a^2(136A+189C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(136}{}$$

Mathematica [A] time = 0.743927, size = 123, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((748A+567C)\cos(c+dx)+(748A+882C)\cos(2(c+dx))+136A}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
1/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(752*A + 693*C + (748*A + 567*C)*Cos[c + d*x]
 + (748*A + 882*C)*Cos[2*(c + d*x)] + 136*A*Cos[3*(c + d*x)] + 189*C*Cos[3*
(c + d*x)] + 136*A*Cos[4*(c + d*x)] + 189*C*Cos[4*(c + d*x)])*Sec[c + d*x]^
(9/2)*Tan[(c + d*x)/2])/(630*d)
```

Maple [A] time = 0.173, size = 130, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(272A(\cos(dx+c))^4+378C(\cos(dx+c))^4+136A(\cos(dx+c))^3+189C(\cos(dx+c))^3+1\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+378*C*cos(d*x+c)^4+136*A*cos
(d*x+c)^3+189*C*cos(d*x+c)^3+102*A*cos(d*x+c)^2+63*C*cos(d*x+c)^2+85*A*cos(
d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(
d*x+c)
```

Maxima [B] time = 1.85849, size = 836, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 4/315*((315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 94*s
qrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 63*(5*sqrt(2)*a^(3
/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 32*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
- 26*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 11*sqrt(2)*a^(3/
2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11)*C*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(co
```

```
s(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.48867, size = 327, normalized size = 1.49

$$\frac{2\left(2\,(136\,A+189\,C)a\cos\left(dx+c\right)^{4}+(136\,A+189\,C)a\cos\left(dx+c\right)^{3}+3\,(34\,A+21\,C)a\cos\left(dx+c\right)^{2}+85\,Aa\cos\left(dx+\right.\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/315*(2*(136*A + 189*C)*a*cos(d*x + c)^4 + (136*A + 189*C)*a*cos(d*x + c)^
3 + 3*(34*A + 21*C)*a*cos(d*x + c)^2 + 85*A*a*cos(d*x + c) + 35*A*a)*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt
(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(1
1/2), x)
```

3.1214 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=172

$$\frac{2a^2(4A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(104A+175C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)(a}{7d}$$

```
[Out] (2*a^2*(104*A + 175*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(4*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*S
qrt[a + a*Cos[c + d*x]]) + (6*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.654635, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3044, 2975, 2980, 2771}

$$\frac{2a^2(4A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(104A+175C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)(a}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(104*A + 175*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(4*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*S
qrt[a + a*Cos[c + d*x]]) + (6*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{(a + a\cos(c+dx))^{3/2} \left(\right.}{\cos^{\frac{9}{2}}(c-}$$

$$= \frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{(2}{}$$

$$= \frac{6aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A(}{}$$

$$= \frac{2a^2(4A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{6aA\sqrt{a+a\,c}}{}$$

$$= \frac{2a^2(104A+175C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(4A}{}$$

Mathematica [A] time = 0.55075, size = 102, normalized size = 0.59

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((468A+525C)\cos(c+dx)+2(52A+35C)\cos(2(c+dx))+104A}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(164*A + 70*C + (468*A + 525*C)*Cos[c + d*x]
+ 2*(52*A + 35*C)*Cos[2*(c + d*x)] + 104*A*Cos[3*(c + d*x)] + 175*C*Cos[3*(
c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.168, size = 108, normalized size = 0.6

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(104\,A\,(\cos(dx+c))^3+175\,C\,(\cos(dx+c))^3+52\,A\,(\cos(dx+c))^2+35\,C\,(\cos(dx+c))^2+39\right.}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+175*C*cos(d*x+c)^3+52*A*cos(
d*x+c)^2+35*C*cos(d*x+c)^2+39*A*cos(d*x+c)+15*A)*cos(d*x+c)*(1/cos(d*x+c))^
(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

<u>Maxima</u> [B] time = 1.76623, size = 711, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 4/105*((105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)) + 35*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c
) + 1) - 11*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sqrt(2
)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*a^(3/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*C*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

<u>Fricas</u> [A] time = 1.44365, size = 274, normalized size = 1.59

$$\frac{2\left((104\,A+175\,C)a\cos\left(dx+c\right)^{3}+(52\,A+35\,C)a\cos\left(dx+c\right)^{2}+39\,Aa\cos\left(dx+c\right)+15\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin(d}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*((104*A + 175*C)*a*cos(d*x + c)^3 + (52*A + 35*C)*a*cos(d*x + c)^2 +
39*A*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos
(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9
/2), x)
```

3.1215 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=183

$$\frac{2a^2(4A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c}{}$$

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 5*C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*(a + a*Cos[c + d*x])^(3/2
)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.619145, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2980, 2774, 216}

$$\frac{2a^2(4A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 5*C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*(a + a*Cos[c + d*x])^(3/2
)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^{3/2}\left(A\right.}{\cos^{\frac{7}{2}}(c+}$$

$$=\frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{\left(2\sqrt{\right.}}{}$$

$$=\frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2A(a}{}$$

$$=\frac{2a^2(4A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\,\mathrm{co}}}{}$$

$$=\frac{2a^2(4A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\,\mathrm{co}}}{}$$

$$=\frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.715512, size = 121, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((6A+5C)\cos(2(c+dx))+6A\cos(c+dx)+8A+5\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(5*Sqrt[2
]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + (8*A + 5*C + 6*A*
Cos[c + d*x] + (6*A + 5*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(5*d)
```

Maple [B] time = 0.204, size = 371, normalized size = 2.

$$\frac{2a\cos(dx+c)(\sin(dx+c))^4}{5d(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(5C\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 2/5/d*a*(5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3+15*C*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*cos(d*x+c)^2+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+5*C*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))+6*A*sin(d*x+c)*cos(d*x+c)^2+5*C*cos(d*x+c)^2*sin(d*x+c)+3*A*sin(d*x+c
)*cos(d*x+c)+A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3
```

Maxima [B] time = 2.42052, size = 2295, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/30*(5*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*a^(3/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 3
*((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*
arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)) + 1) - (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*
c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
```

```
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) - 1) - (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*
d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a*cos(2
*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) - 2*((6*(a*sin(4*d*x + 4*c) +
 2*a*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 3*a*sin(4*d*x + 4*c) - 7*a*sin(2*d*x + 2*c) - 6*(a*cos(4*d*x + 4*c) + 2*
a*cos(2*d*x + 2*c) + a)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (3*a*cos(4*d*
x + 4*c) + 7*a*cos(2*d*x + 2*c) + 6*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2
*c) + a)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*(a*sin(4*
d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 4*a)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 9*(a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) +
 a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*C/(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4) + 24
*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.75, size = 392, normalized size = 2.14

$$-\frac{2\left(5\left(Ca\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left((6A+5C)a\cos(dx+c)^2+3Aa\cos(dx+c)+Aa\right)\sqrt{a\cos}}{\sqrt{\cos(dx+c)}}\right.}{5\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] -2/5*(5*(C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((6*A + 5*C)*a*
cos(d*x + c)^2 + 3*A*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7
/2), x)
```

3.1216 $\int (a+a\cos(c+dx))^{3/2} \left(A + C\cos^2(c + dx)\right) \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$-\frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+\ldots}{3\ldots}$$

```
[Out] (3*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(8*A - 3*C)*Sin[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.639784, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2981, 2774, 216}

$$-\frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (3*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(8*A - 3*C)*Sin[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}\left(A\right.}{\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2A(a}{}$$

$$= -\frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c}}{}$$

$$= -\frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c}}{}$$

$$= \frac{3a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} -$$

Mathematica [A] time = 0.52912, size = 116, normalized size = 0.64

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(20A\cos(c+dx)+4A+3C\cos(2(c+dx))+3C)+9\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(9*Sqrt[2
]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*C + 20*A
*Cos[c + d*x] + 3*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Maple [A] time = 0.203, size = 290, normalized size = 1.6

$$-\frac{a\cos(dx+c)(\sin(dx+c))^2}{3d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(9C\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*a*(9*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+18*C*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))*cos(d*x+c)+9*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*C*cos(d*x+c)^2*sin(d*x+c)+10*A*s
in(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 2.30544, size = 1881, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(3*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 2*(3*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x
+ 2*c) + (a*cos(2*d*x + 2*c)^2*sin(d*x + c) + a*sin(2*d*x + 2*c)^2*sin(d*x
+ c) + 2*a*cos(2*d*x + 2*c)*sin(d*x + c) + a*sin(d*x + c))*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*(a*cos(2*d*x + 2*c) + a)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((a*cos(d*x + c) - a)
*cos(2*d*x + 2*c)^2 + (a*cos(d*x + c) - a)*sin(2*d*x + 2*c)^2 + 2*(a*cos(d*
x + c) - a)*cos(2*d*x + 2*c) + a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 3*((a*cos(2*d*x + 2*c)^2 + a*sin(
2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2(-(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - (a*cos(2*d*x + 2*c)^2 + a*sin(2
*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2(-(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2
```

```
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - (a*cos(2*d*x + 2*c)^2 + a*sin(2*
d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d
*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C
/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 16*(3
*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [A] time = 1.60115, size = 379, normalized size = 2.09

$$-\frac{9\left(Ca\cos\left(dx+c\right)^{2}+Ca\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(3\,Ca\cos(dx+c)^{2}+10\,Aa\cos(dx+c)+2\,Aa\right)\sqrt{a\cos(dx+c)+}}{\sqrt{\cos(dx+c)}}}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*(9*(C*a*cos(d*x + c)^2 + C*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (3*C*a*cos(d*x +
c)^2 + 10*A*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5
/2), x)
```

3.1217 $\int (a+a\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{a^{3/2}(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} - \frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}} - \frac{a(4A-C)\sin(c+}{2d\sqrt{s}}$$

```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^2*(8*A - 5*C)*Sin[c + d
*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*(4*A - C)*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Co
s[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.652538, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} - \frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}} - \frac{a(4A-C)\sin(c+}{2d\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^2*(8*A - 5*C)*Sin[c + d
*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*(4*A - C)*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Co
s[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}\left(A\right.}{\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2A(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{(2\sqrt{}}{}$$

$$= -\frac{a(4A-C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\,\mathrm{c}}{}$$

$$= -\frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{a(4A-C)\sqrt{a+a}}{2d\sqrt{}}$$

$$= -\frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{a(4A-C)\sqrt{a+a}}{2d\sqrt{}}$$

$$= \frac{a^{3/2}(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{4d}$$

Mathematica [A] time = 0.625655, size = 119, normalized size = 0.61

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+7C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
(8*A + 7*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A +
C + 7*C*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.206, size = 327, normalized size = 1.7

$$\frac{a\cos(dx+c)}{4d(1+\cos(dx+c))}\left(8A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)+2C(\cos(dx+c))^2\,\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*a*(8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+2*C*cos(d*x+c)^2*sin(d*x+c)+
7*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+7*C*cos(d*
x+c)*sin(d*x+c)+7*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*A*sin(d*x+c))*cos(d*x+c)*(1/co
s(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.96083, size = 362, normalized size = 1.86

$$-\frac{((8A+7C)a\cos(dx+c)+(8A+7C)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(2Ca\cos(dx+c)^2+7Ca\cos(dx+c)+8Aa)\sqrt{a\cos(d}}{\sqrt{\cos(dx+c)}}}{4(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(((8*A + 7*C)*a*cos(d*x + c) + (8*A + 7*C)*a)*sqrt(a)*arctan(sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*C*a*cos(d*x
 + c)^2 + 7*C*a*cos(d*x + c) + 8*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2), x)
```

3.1218 $\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}$

Optimal. Leaf size=191

$$\frac{a^{3/2}(24A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+19C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aC\sin(c+d}{4d\sqrt{}}$$

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 19*C)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*C*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (C*(a + a*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.643296, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3046, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(24A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+19C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aC\sin(c+d}{4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 19*C)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*C*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (C*(a + a*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}\left(\right.}{\sqrt{\cos(c}}$$

$$= \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\right.}}{}$$

$$= \frac{aC\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{C(a+a\cos(c+d}{3d\sqrt{\sec(}}$$

$$= \frac{a^2(24A+19C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{aC\sqrt{a+a\cos(}}{4d\sqrt{\mathrm{se}}}$$

$$= \frac{a^2(24A+19C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{aC\sqrt{a+a\cos(}}{4d\sqrt{\mathrm{se}}}$$

$$= \frac{a^{3/2}(24A+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}}{8d}$$

Mathematica [A] time = 0.667045, size = 133, normalized size = 0.7

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(24A+11C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(3*Sqrt[2]*(24*A + 11*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqr
t[Cos[c + d*x]]*(24*A + 37*C + 22*C*Cos[c + d*x] + 4*C*Cos[2*(c + d*x)])*Si
n[(c + d*x)/2]))/(48*d)
```

Maple [A] time = 0.203, size = 269, normalized size = 1.4

$$-\frac{a\left((\cos(dx+c))^2-1\right)}{24\,d\,(\sin(dx+c))^2}\left(8\,C\sin(dx+c)\,(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+22\,C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*a*(8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22
*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+72*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))+33*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1
/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.07427, size = 3707, normalized size = 19.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/96*(24*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
```

```
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
 (4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*
c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*
sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 3
3*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a*arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
```

```
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arctan2((co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4
)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 1.9246, size = 408, normalized size = 2.14

$$-\frac{3\left((24\,A+11\,C)a\cos\left(dx+c\right)+(24\,A+11\,C)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,Ca\cos(dx+c)^{3}+22\,Ca\cos(dx+c)^{2}+3\,(8\,A\cdots}{\sqrt{\cos\cdots}}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(3*((24*A + 11*C)*a*cos(d*x + c) + (24*A + 11*C)*a)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*C*a*
cos(d*x + c)^3 + 22*C*a*cos(d*x + c)^2 + 3*(8*A + 11*C)*a*cos(d*x + c))*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c)), x)
```

3.1219 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=238

$$\frac{a^2(16A+13C)\sin(c+dx)}{32d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(112A}{64d\sqrt{\sec(c+}}$$

```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(16*A + 13*C)*Sin
[c + d*x])/(32*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*C*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sec[c + d*x]^(3/2)) + (C*(a + a*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + (a^2*(112*A + 75*C)
*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.744797, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(16A+13C)\sin(c+dx)}{32d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(112A}{64d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```


```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(16*A + 13*C)*Sin
[c + d*x])/(32*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*C*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sec[c + d*x]^(3/2)) + (C*(a + a*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + (a^2*(112*A + 75*C)
*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
```

```
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,($$

$$= \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{aC\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+}{4d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(16A+13C)\sin(c+dx)}{32d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aC\sqrt{a+a\cos(c+dx)}\sin(c+}{8d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(16A+13C)\sin(c+dx)}{32d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aC\sqrt{a+a\cos(c+dx)}\sin(c+}{8d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(16A+13C)\sin(c+dx)}{32d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aC\sqrt{a+a\cos(c+dx)}\sin(c+}{8d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^{3/2}(112A+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} +$$

Mathematica [A] time = 0.647346, size = 150, normalized size = 0.63

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(112A+75C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(\right.\right.\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + 
d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
(112*A + 75*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (112*A
 + 95*C + (32*A + 62*C)*Cos[c + d*x] + 20*C*Cos[2*(c + d*x)] + 4*C*Cos[3*(c
 + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(128*d)
```

Maple [A] time = 0.178, size = 345, normalized size = 1.5

$$\frac{a\left(-1+\cos\left(dx+c\right)\right)^{2}\cos\left(dx+c\right)}{64\,d\left(\sin\left(dx+c\right)\right)^{4}}\left(16\,C\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{3}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+40\,C\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/64/d*a*(-1+cos(d*x+c))^2*(16*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+40*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+32*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+50*C*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+112*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+75*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+112*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*
C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+
c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c)
)^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 4.03674, size = 10799, normalized size = 45.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/256*(16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
```

```
*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) -
 a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (2*(cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/
4)*((a*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c)^3 + 4*(a*si
n(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*
d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*s
in(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(4
*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (2*a*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*s
in(4*d*x + 4*c) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 4*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*s
in(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*
a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*
a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c
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)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 17*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 17*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (a*cos(4*d*x + 4*c)^3 - 8*a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^3 - 10*a*cos(4*d*x + 4*c)^2 + (a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 17*a*cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^3 - 6*a*cos(4*d*x + 4*c)^2 + (a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 - 15*a*cos(4*d*x + 4*c) - 8*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 17*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 17*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(a*cos(4*d*x + 4*c)^3 - 9*a*cos(4*d*x + 4*c)^2 + (a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (2*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(a*cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((a*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c)^3 + 80*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c) + 76*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si n(4*d*x + 4*c) + 4*(a*sin(4*d*x + 4*c)^3 - 80*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a*cos(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) - 19*a)*sin(4*d*x + 4*c) + 76*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(2*a*sin(4*d*x + 4*c)^3 + a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + (152*a*cos(4*d*x + 4*c)^2 + 152*a*sin(4*d*x + 4*c)^2 - 153*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(10*a*cos(4*d*x + 4*c)^2 + 40*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 - 153*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*(5*a*cos(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)^2 - 76*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (76*a*cos(4*d*x + 4*c)^2 + 76*a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (a*cos(4*d*x + 4*c)^3 + 80*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 56*a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^3 - 38*a*cos(4*d*x + 4*c)^2 + (a*cos(4*d*x + 4*c) - 36*a)*sin(4*d*x + 4*c)^2 + 93*a*cos(4*d*x + 4*c) + 36*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 56*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (a*cos(4*d*x + 4*c) - 56*a)*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^3 - 54*a*cos(4*d*x + 4*c)^2 + (a*cos(4*d*x + 4*c) - 56*a)*sin(4*d*x + 4*c)^2 - 111*a*cos(4*d*x + 4*c) + 20*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 56*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(2*a*cos(4*d*x + 4*c)^3 - 104*a*cos(4*d*x + 4*c)^2 + 2*(a*cos(4*d*x + 4*c) - 51*a)*sin(4*d*x + 4*c)^2 - a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 112*a*cos(4*d*x + 4*c) + (72*a*cos(4*d*x + 4*c)^2 + 72*a*sin(4*d*x + 4*c)^2 - 73*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (36*a*cos(4*d*x + 4*c)^2 + 36*a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(160*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 73*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(36*a*cos(1/4

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*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a*cos(4*d
*x + 4*c) - 51*a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 2*(a*cos(4*d*x + 4*c) - 56*a)*sin(4*d*x + 4*c) - (a*cos(4*d
*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 75*((a*cos(4*d*x + 4*c)^2 + 4*(a*cos(
4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a
*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^
2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*c
os(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1))) + 1) - (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin
(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 +
a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)
^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
+ 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c)
```

, cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(c
os(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a
*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*
cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 +
 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c
))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x +
 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)) + 1) + (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*s
in(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2
+ a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*
c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*
x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2
(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*C/(4*(cos(4*d*x + 4*c
)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + si

```
n(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))))/d
```

Fricas [A] time = 2.50071, size = 456, normalized size = 1.92

$$-\frac{((112\,A+75\,C)a\cos(dx+c)+(112\,A+75\,C)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(16\,Ca\cos(dx+c)^4+40\,Ca\cos(dx+c)^3+2}{}}{64\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/64*(((112*A + 75*C)*a*cos(d*x + c) + (112*A + 75*C)*a)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (16*C*a
*cos(d*x + c)^4 + 40*C*a*cos(d*x + c)^3 + 2*(16*A + 25*C)*a*cos(d*x + c)^2
+ (112*A + 75*C)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt
(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x +
c)), x)
```

3.1220 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{a^2(176A+133C)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+67C)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+133C)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{128d}$$

```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A + 67*C)*S
in[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (3*a*C*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c + d*x]^(5/2)) + (C*(a + a
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(5/2)) + (a^2*(176*A +
 133*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) +
 (a^2*(176*A + 133*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])
```

Rubi [A] time = 0.823842, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(176A+133C)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+67C)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+133C)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```


```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A + 67*C)*S
in[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (3*a*C*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c + d*x]^(5/2)) + (C*(a + a
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(5/2)) + (a^2*(176*A +
 133*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) +
 (a^2*(176*A + 133*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\left(\right.$$

$$= \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{3aC\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{40d\sec^{\frac{5}{2}}(c+dx)} + \frac{C(a+a\cos(c+dx))^{3/2}\sin(c}{5d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{3aC\sqrt{a+a\cos(c+dx)}\sin}{40d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{3aC\sqrt{a+a\cos(c+dx)}\sin}{40d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{3aC\sqrt{a+a\cos(c+dx)}\sin}{40d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{3aC\sqrt{a+a\cos(c+dx)}\sin}{40d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^{3/2}(176A+133C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{128d}$$

Mathematica [A] time = 0.996422, size = 169, normalized size = 0.59

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(176A+133C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^
(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqrt[
2]*(176*A + 133*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (2
960*A + 2671*C + 2*(880*A + 1007*C)*Cos[c + d*x] + 4*(80*A + 181*C)*Cos[2*(
c + d*x)] + 228*C*Cos[3*(c + d*x)] + 48*C*Cos[4*(c + d*x)])*(-Sin[(c + d*x)
/2] + Sin[(3*(c + d*x))/2])))/(3840*d)
```

Maple [A] time = 0.197, size = 417, normalized size = 1.5

$$-\frac{a\left(-1+\cos\left(dx+c\right)\right)^{3}\cos\left(dx+c\right)}{1920\,d\left(\sin\left(dx+c\right)\right)^{6}}\left(384\,C\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+912\,C\sin\left(dx+c\right)\left(\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/1920/d*a*(-1+cos(d*x+c))^3*(384*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+912*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+640*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10
64*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1760*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1330*C*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2640*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+1995*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2640*A*
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+1995*C*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*
(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2
)/sin(d*x+c)^6
```

Maxima [B] time = 3.71757, size = 6035, normalized size = 21.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/7680*(80*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*s
qrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
```

cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*a
rctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqr
t(a))*A + (50*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin
(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((9*a*sin(4/5*arctan2(sin(5*d*
x + 5*c), cos(5*d*x + 5*c))) + 8*a*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c))) + 9*a*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*co
s(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - (9*a*cos(4/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 8*a*cos(3/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))) - 9*a*cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))) - 8*a)*sin(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)))*sqrt(a)
+ 6*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5
*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(8*(a*cos(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c)))^2*sin(5*d*x + 5*c) + a*sin(5*d*x + 5*c)*sin(2/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*a*cos(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c)))*sin(5*d*x + 5*c) + a*sin(5*d*x + 5*c))*cos(5/2*arcta
n2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 5*(9*a*sin(4/5*arctan2(sin(5*d*x
 + 5*c), cos(5*d*x + 5*c))) + 9*a*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c))) - 28*a*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 1

24*a*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*cos(1/2*arctan2(
sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 8*((a*cos(5*d*x + 5*c) - a)*cos(2/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + (a*cos(5*d*x + 5*c) - a)*
sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + a*cos(5*d*x + 5*c)
 + 2*(a*cos(5*d*x + 5*c) - a)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
 5*c))) - a)*sin(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 5*(9*a
*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 9*a*cos(3/5*arctan2
(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 28*a*cos(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))) - 68*a*cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
 5*c))) + 96*a)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)))*sqrt
(a) + 1995*(a*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
)^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*ar
ctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x
 + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*
arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5*d*x
+ 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1
/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arctan2(s
in(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*
x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))) + 1
) - a*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + si
n(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(
sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5
*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos
(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arctan2(sin(2/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)
)), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))) - 1) - a*ar
ctan2((cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x +

```
5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*
d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))) + 1)), (cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +
sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))) + 1)) + 1) + a*arctan2((cos(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*
sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2
/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)), (cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
+ 1)^(1/4)*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 1))*sqrt
(a))*C)/d
```

Fricas [A] time = 2.45599, size = 527, normalized size = 1.85

$$-\frac{15\left((176A+133C)a\cos(dx+c)+(176A+133C)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(384\,Ca\cos(dx+c)^{5}+912\,Ca\cos(dx-\cdots\right.}{\cdots}}{1920\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/1920*(15*((176*A + 133*C)*a*cos(d*x + c) + (176*A + 133*C)*a)*sqrt(a)*ar
ctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) -
(384*C*a*cos(d*x + c)^5 + 912*C*a*cos(d*x + c)^4 + 8*(80*A + 133*C)*a*cos(d
*x + c)^3 + 10*(176*A + 133*C)*a*cos(d*x + c)^2 + 15*(176*A + 133*C)*a*cos(
d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3
/2), x)
```

3.1221 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{15}{2}}(c+dx)\,dx$

Optimal. Leaf size=313

$$\frac{2a^2(136A+143C)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2717C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(83\cdots}{\cdots}$$

```
[Out] (16*a^3*(8368*A + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a
 + a*Cos[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(222
4*A + 2717*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*
Sin[c + d*x])/(1287*d) + (10*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(1
1/2)*Sin[c + d*x])/(143*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(
13/2)*Sin[c + d*x])/(13*d)
```

Rubi [A] time = 1.05602, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2980, 2772, 2771}

$$\frac{2a^2(136A+143C)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2717C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(83\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(15/2),x
]
```


```
[Out] (16*a^3*(8368*A + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a
 + a*Cos[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(222
4*A + 2717*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*
Sin[c + d*x])/(1287*d) + (10*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(1
1/2)*Sin[c + d*x])/(143*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(
13/2)*Sin[c + d*x])/(13*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{15}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{15}{2}}(c+}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{13}{2}}(c+dx)\sin(c+dx)}{13d}+\frac{(2}{}$$

$$=\frac{10aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{143d}+$$

$$=\frac{2a^2(136A+143C)\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(}{1287d}$$

$$=\frac{2a^3(2224A+2717C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(13}{}$$

$$=\frac{2a^3(8368A+10439C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15015d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(2}{}$$

$$=\frac{8a^3(8368A+10439C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}}+\frac{2a^3(8}{}$$

$$=\frac{16a^3(8368A+10439C)\sqrt{\sec(c+dx)}\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}}+\frac{8a^3(}{}$$

Mathematica [A] time = 0.927277, size = 171, normalized size = 0.55

$$\frac{a^2 \tan\left(\frac{1}{2}(c+dx)\right) \sec^{\frac{13}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(1120(347A+286C)\cos(c+dx)+14(30334A+32747C)\cos(2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(343612*A + 322751*C + 1120*(347*A + 286*C)
*Cos[c + d*x] + 14*(30334*A + 32747*C)*Cos[2*(c + d*x)] + 125520*A*Cos[3*(c
 + d*x)] + 141570*C*Cos[3*(c + d*x)] + 125520*A*Cos[4*(c + d*x)] + 156585*C
*Cos[4*(c + d*x)] + 16736*A*Cos[5*(c + d*x)] + 20878*C*Cos[5*(c + d*x)] + 1
6736*A*Cos[6*(c + d*x)] + 20878*C*Cos[6*(c + d*x)])*Sec[c + d*x]^(13/2)*Tan
[(c + d*x)/2])/(180180*d)
```

Maple [A] time = 0.207, size = 176, normalized size = 0.6

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(66944\,A\,(\cos(dx+c))^6+83512\,C\,(\cos(dx+c))^6+33472\,A\,(\cos(dx+c))^5+41756\,C\,(\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(15/2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+83512*C*cos(d*x+c)^6+3
3472*A*cos(d*x+c)^5+41756*C*cos(d*x+c)^5+25104*A*cos(d*x+c)^4+31317*C*cos(d
*x+c)^4+20920*A*cos(d*x+c)^3+18590*C*cos(d*x+c)^3+18305*A*cos(d*x+c)^2+5005
*C*cos(d*x+c)^2+11970*A*cos(d*x+c)+3465*A)*cos(d*x+c)*(1/cos(d*x+c))^(15/2)
*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.83182, size = 1030, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(15/2),x, al
gorithm="maxima")
```

```
[Out] 8/45045*((45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*sq
rt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 88
980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqrt(2)*a
^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c
) + 1)^10 + 1)) + 143*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1)
- 1575*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3654*sqrt(2)*a
^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5130*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 4595*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x +
 c) + 1)^9 - 2535*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 7
80*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 104*sqrt(2)*a^(5
/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*C*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + 1)))/d
```

Fricas [A] time = 1.54784, size = 478, normalized size = 1.53

$$\frac{2\left(8\,(8368\,A+10439\,C)a^2\cos(dx+c)^6+4\,(8368\,A+10439\,C)a^2\cos(dx+c)^5+3\,(8368\,A+10439\,C)a^2\cos(dx+c)^4 \cdots\right.}{45045\left(d\cos(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(15/2),x, al
gorithm="fricas")
```

```
[Out] 2/45045*(8*(8368*A + 10439*C)*a^2*cos(d*x + c)^6 + 4*(8368*A + 10439*C)*a^2
*cos(d*x + c)^5 + 3*(8368*A + 10439*C)*a^2*cos(d*x + c)^4 + 10*(2092*A + 18
59*C)*a^2*cos(d*x + c)^3 + 35*(523*A + 143*C)*a^2*cos(d*x + c)^2 + 11970*A*
a^2*cos(d*x + c) + 3465*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*co
```

```
s(d*x + c)^7 + d*cos(d*x + c)^6)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(15/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

3.1222 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=266

$$\frac{2a^2(32A+33C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{231d}+\frac{2a^3(232A+297C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(568A$$

```
[Out] (4*a^3*(568*A + 759*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(693*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(69
3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(232*A + 297*C)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(32*A + 33*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (10*a*A*(a +
a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + a
*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.957846, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2980, 2772, 2771}

$$\frac{2a^2(32A+33C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{231d}+\frac{2a^3(232A+297C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(568A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2),x
]
```


```
[Out] (4*a^3*(568*A + 759*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(693*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(69
3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(232*A + 297*C)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(32*A + 33*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (10*a*A*(a +
a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + a
*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{13}{2}}(c+\right.}$$

$$= \frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{(2}{}$$

$$= \frac{10aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2}{}$$

$$= \frac{2a^2(32A+33C)\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+}{231d}$$

$$= \frac{2a^3(232A+297C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(32A}{}$$

$$= \frac{2a^3(568A+759C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(232A}{}$$

$$= \frac{4a^3(568A+759C)\sqrt{\sec(c+dx)}\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(568A}{}$$

Mathematica [A] time = 1.1033, size = 149, normalized size = 0.56

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(2(5014A+4983C)\cos(c+dx)+52(71A+66C)\cos(2(c+dx))+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3628*A + 2673*C + 2*(5014*A + 4983*C)*Cos[
c + d*x] + 52*(71*A + 66*C)*Cos[2*(c + d*x)] + 3692*A*Cos[3*(c + d*x)] + 45
87*C*Cos[3*(c + d*x)] + 568*A*Cos[4*(c + d*x)] + 759*C*Cos[4*(c + d*x)] + 5
68*A*Cos[5*(c + d*x)] + 759*C*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(c
+ d*x)/2])/(2772*d)
```

Maple [A] time = 0.184, size = 154, normalized size = 0.6

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(1136\,A\left(\cos\left(dx+c\right)\right)^{5}+1518\,C\left(\cos\left(dx+c\right)\right)^{5}+568\,A\left(\cos\left(dx+c\right)\right)^{4}+759\,C\left(\cos\left(dx+c\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/693/d*a^2*(-1+cos(d*x+c))*(1136*A*cos(d*x+c)^5+1518*C*cos(d*x+c)^5+568*A
*cos(d*x+c)^4+759*C*cos(d*x+c)^4+426*A*cos(d*x+c)^3+396*C*cos(d*x+c)^3+355*
A*cos(d*x+c)^2+99*C*cos(d*x+c)^2+224*A*cos(d*x+c)+63*A)*cos(d*x+c)*(1/cos(d
*x+c))^(13/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.76597, size = 906, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] 8/693*((693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 13
00*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a^(5
/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/
```

```
(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
+ sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 33*(21*sqrt(2)*a^(5/2)*sin(d*
x + c)/(cos(d*x + c) + 1) - 98*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 196*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 218*sqr
t(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 143*sqrt(2)*a^(5/2)*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 - 52*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*
C*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
)/d
```

Fricas [A] time = 1.53941, size = 390, normalized size = 1.47

$$\frac{2\left(2\,(568\,A+759\,C)a^2\cos{(dx+c)}^5+(568\,A+759\,C)a^2\cos{(dx+c)}^4+6\,(71\,A+66\,C)a^2\cos{(dx+c)}^3+(355\,A+99\,C\cdots\right.}{693\left(d\cos{(dx+c)}^6+d\cos{(dx+c)}^5\right)\sqrt{\cos{(\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] 2/693*(2*(568*A + 759*C)*a^2*cos(d*x + c)^5 + (568*A + 759*C)*a^2*cos(d*x +
 c)^4 + 6*(71*A + 66*C)*a^2*cos(d*x + c)^3 + (355*A + 99*C)*a^2*cos(d*x + c
)^2 + 224*A*a^2*cos(d*x + c) + 63*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

3.1223 $\int (a+a\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=219

$$\frac{2a^2(64A+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a^3(8A+11C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(584A+90\ldots}{315a\ldots}$$

```
[Out] (2*a^3*(584*A + 903*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(8*A + 11*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 63*C)*Sqrt[a + a*Cos[c + d*x]]*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (10*a*A*(a + a*Cos[c + d*x])^(3/2
)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)
*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.862422, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3044, 2975, 2980, 2771}

$$\frac{2a^2(64A+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a^3(8A+11C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(584A+90\ldots}{315a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x
]
```


```
[Out] (2*a^3*(584*A + 903*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(8*A + 11*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 63*C)*Sqrt[a + a*Cos[c + d*x]]*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (10*a*A*(a + a*Cos[c + d*x])^(3/2
)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)
*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2} \left(A + C\cos^2(c + dx)\right) \sec^{\frac{11}{2}}(c + dx)\, dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{(a + a\cos(c + dx))^{5/2} \left(A\right.}{\cos^{\frac{11}{2}}(c +}$$

$$= \frac{2A(a + a\cos(c + dx))^{5/2} \sec^{\frac{9}{2}}(c + dx)\sin(c + dx)}{9d} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{10aA(a + a\cos(c + dx))^{3/2} \sec^{\frac{7}{2}}(c + dx)\sin(c + dx)}{63d} + \frac{2}{}$$

$$= \frac{2a^2(64A + 63C)\sqrt{a + a\cos(c + dx)}\sec^{\frac{5}{2}}(c + dx)\sin(c +}{315d}$$

$$= \frac{2a^3(8A + 11C)\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{15d\sqrt{a + a\cos(c + dx)}} + \frac{2a^2(64A + 6}{}$$

$$= \frac{2a^3(584A + 903C)\sqrt{\sec(c + dx)}\sin(c + dx)}{315d\sqrt{a + a\cos(c + dx)}} + \frac{2a^3(8A}{}$$

Mathematica [A] time = 0.968252, size = 127, normalized size = 0.58

$$\frac{a^2 \tan\left(\frac{1}{2}(c + dx)\right) \sec^{\frac{9}{2}}(c + dx)\sqrt{a(\cos(c + dx) + 1)}(4(698A + 441C)\cos(c + dx) + 4(803A + 966C)\cos(2(c + dx)) + 58}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
1/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(2908*A + 2961*C + 4*(698*A + 441*C)*Cos[c
+ d*x] + 4*(803*A + 966*C)*Cos[2*(c + d*x)] + 584*A*Cos[3*(c + d*x)] + 588*
C*Cos[3*(c + d*x)] + 584*A*Cos[4*(c + d*x)] + 903*C*Cos[4*(c + d*x)])*Sec[c
 + d*x]^(9/2)*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.181, size = 132, normalized size = 0.6

$$-\frac{2a^2(-1 + \cos(dx + c))\left(584A(\cos(dx + c))^4 + 903C(\cos(dx + c))^4 + 292A(\cos(dx + c))^3 + 294C(\cos(dx + c))^3 +\right.}{315d\sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+903*C*cos(d*x+c)^4+292*A*c
os(d*x+c)^3+294*C*cos(d*x+c)^3+219*A*cos(d*x+c)^2+63*C*cos(d*x+c)^2+130*A*c
os(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)
```

Maxima [B] time = 1.78378, size = 782, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 8/315*((315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 1)) + 21*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
65*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 113*sqrt(2)*a^(5/2
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*C*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.67505, size = 342, normalized size = 1.56

$$\frac{2\left((584\,A+903\,C)a^2\cos\left(dx+c\right)^4+2\,(146\,A+147\,C)a^2\cos\left(dx+c\right)^3+3\,(73\,A+21\,C)a^2\cos\left(dx+c\right)^2+130\,Aa^2\cos\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/315*((584*A + 903*C)*a^2*cos(d*x + c)^4 + 2*(146*A + 147*C)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 21*C)*a^2*cos(d*x + c)^2 + 130*A*a^2*cos(d*x + c) + 35*A*
a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x +
 c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

3.1224 $\int (a+a\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=230

$$\frac{2a^2(8A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^3(32A+49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}C\sqrt{\cos(c}}{}$$

```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 49*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*A*(a + a*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*A*(a + a*Co
s[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.799904, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2980, 2774, 216}

$$\frac{2a^2(8A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^3(32A+49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}C\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 49*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*A*(a + a*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*A*(a + a*Co
s[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}\left(\right.}{\cos^{\frac{9}{2}}(c}$$

$$= \frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{(2}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{}$$

$$= \frac{2a^2(8A+7C)\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+}{21d}$$

$$= \frac{2a^3(32A+49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+}{}$$

$$= \frac{2a^3(32A+49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+}{}$$

$$= \frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.42063, size = 151, normalized size = 0.66

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((93A+84C)\cos(c+dx)+(23A+7C)\cos(2(c\right.}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(84*Sqr
t[2]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 4*(29*A + 7*C
+ (93*A + 84*C)*Cos[c + d*x] + (23*A + 7*C)*Cos[2*(c + d*x)] + 23*A*Cos[3*(
```

```
c + d*x)] + 28*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(84*d)
```

Maple [B] time = 0.228, size = 473, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*a^2*(21*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+84*C*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))+126*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+84*C*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))+21*C*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+46*A*sin(d*x+c)*cos(d
*x+c)^3+56*C*sin(d*x+c)*cos(d*x+c)^3+23*A*sin(d*x+c)*cos(d*x+c)^2+7*C*cos(d
*x+c)^2*sin(d*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+3*A*sin(d*x+c))*cos(d*x+c)*(a
*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)*sin(d*x+c)^6/(-1+cos(d*x+c))^3/
(1+cos(d*x+c))^4
```

Maxima [B] time = 2.57932, size = 3163, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 1/210*(7*(6*(a^2*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 25*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*
c) + a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a) +
 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((15*a^2*sin(6*d*x + 6*c) + 50*a^2*sin(4*d*x + 4*c) + 58*a^2*sin(2*d*x + 2
*c) - 20*(3*a^2*sin(6*d*x + 6*c) + 10*a^2*sin(4*d*x + 4*c) + 11*a^2*sin(2*d
```

*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(3*a^2
*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 11*a^2*cos(2*d*x + 2*c) + 4*a
^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (15*a^2*cos(6*d*x + 6*c) + 50*a^2
*cos(4*d*x + 4*c) + 58*a^2*cos(2*d*x + 2*c) + 23*a^2 + 20*(3*a^2*cos(6*d*x
+ 6*c) + 10*a^2*cos(4*d*x + 4*c) + 11*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(7/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(3*a^2*sin(6*d*x + 6*c)
+ 10*a^2*sin(4*d*x + 4*c) + 11*a^2*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 25*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2
*cos(2*d*x + 2*c) + a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + 15*((a^2*cos(2*d*x + 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*
a^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c)
+ 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c
)^2 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
c) + 1)^(1/4)(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a
^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) +
 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)
^2 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a^
2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) +
2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)^
2 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x
+ 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*
d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*c
os(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)^2 + a^2)*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c

```
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1))*sqrt(a))*C/(cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4
 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 80*(21
*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/2)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*s
qrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.84983, size = 463, normalized size = 2.01

$$-\frac{2\left(21\left(Ca^2\cos(dx+c)^4+Ca^2\cos(dx+c)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,(23\,A+28\,C)a^2\cos(dx+c)^3+(23\,A+7\,C)a^2\cos\cdots\right.}{}\right.}{21\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] -2/21*(21*(C*a^2*cos(d*x + c)^4 + C*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*(23*A
+ 28*C)*a^2*cos(d*x + c)^3 + (23*A + 7*C)*a^2*cos(d*x + c)^2 + 12*A*a^2*cos
(d*x + c) + 3*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1225 $\int (a+a\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=230

$$-\frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{5d}+\frac{5a^{5/2}C\sqrt{\cos(c+dx}}{}$$

```
[Out] (5*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(64*A + 15*C)*Sin[c + d*x])/(15*d
*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(8*A + 5*C)*Sqrt[a +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*A*(a + a*Cos
[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c
 + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.855387, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2975, 2981, 2774, 216}

$$-\frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{5d}+\frac{5a^{5/2}C\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (5*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(64*A + 15*C)*Sin[c + d*x])/(15*d
*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(8*A + 5*C)*Sqrt[a +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*A*(a + a*Cos
[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c
 + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{7}{2}}(c+}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+$$

$$=\frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+$$

$$=\frac{2a^2(8A+5C)\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+d}{5d}$$

$$=-\frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=-\frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=\frac{5a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.00706, size = 141, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((112A+45C)\cos(c+dx)+4(43A+15C)\cos(2(\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(300*Sq
rt[2]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(196*A + 60
*C + (112*A + 45*C)*Cos[c + d*x] + 4*(43*A + 15*C)*Cos[2*(c + d*x)] + 15*C*
```

```
Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(120*d)
```

Maple [A] time = 0.211, size = 391, normalized size = 1.7

$$\frac{a^2\cos(dx+c)(\sin(dx+c))^4}{15d(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(75C\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 1/15/d*a^2*(75*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3+225*C*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*cos(d*x+c)^2+225*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+75*C*(cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))+15*C*sin(d*x+c)*cos(d*x+c)^3+86*A*sin(d*x+c)*cos(d*x+c)^2+30*C
*cos(d*x+c)^2*sin(d*x+c)+28*A*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c))*cos(d*x
+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4/(-1+cos(d*x+
c))^2/(1+cos(d*x+c))^3
```

Maxima [B] time = 2.42511, size = 2259, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/60*(5*(2*(5*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 3*(a^2*cos(2*d*x + 2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x +
 c) + 2*a^2*cos(2*d*x + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*((a^2*cos(d*x + c) - a^2)*c
os(2*d*x + 2*c)^2 + a^2*cos(d*x + c) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x +
 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a) + 15*((a^2*cos(2*d*x + 2*c)
```

```
^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x +
c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x
 + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(
-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos
(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x
 + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - (a^2*co
s(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 +
 a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*((12*a^2*sin(5*d*x + 5*c) + 15*a^2
*sin(4*d*x + 4*c) + 24*a^2*sin(3*d*x + 3*c) + 35*a^2*sin(2*d*x + 2*c) + 12*
a^2*sin(d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- (12*a^2*cos(5*d*x + 5*c) + 15*a^2*cos(4*d*x + 4*c) + 24*a^2*cos(3*d*x + 3
*c) + 35*a^2*cos(2*d*x + 2*c) + 12*a^2*cos(d*x + c) + 20*a^2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 27*(a^2*cos(2*d*x + 2*c)^2 +
a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4) + 32*(15*sqrt(2)*a^(5/2)*sin
(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sqr
t(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)))/d
```

Fricas [A] time = 1.72152, size = 452, normalized size = 1.97

$$-\frac{75\left(Ca^{2}\cos\left(dx+c\right)^{3}+Ca^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(15\,Ca^{2}\cos(dx+c)^{3}+2\,(43\,A+15\,C)a^{2}\cos(dx+c)^{2}+2\cdots\right.}{\sqrt{\cos(\cdots}}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] -1/15*(75*(C*a^2*cos(d*x + c)^3 + C*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (15*C*a^2
*cos(d*x + c)^3 + 2*(43*A + 15*C)*a^2*cos(d*x + c)^2 + 28*A*a^2*cos(d*x + c
) + 6*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1226 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=238

$$\frac{a^{5/2}(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-C)\sin(}{2d}$$

```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(56*A - 27*C)*Sin[c
+ d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(8*A - C)
*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (10*a*A*
(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a
 + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.852188, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3044, 2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-C)\sin(}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(56*A - 27*C)*Sin[c
+ d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(8*A - C)
*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (10*a*A*
(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a
 + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{5}{2}}(c+\ }$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{\left(2\sqrt{\right.}}{}$$

$$=\frac{10aA(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A}{}$$

$$=-\frac{a^2(8A-C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}+\frac{10aA(a+}{}$$

$$=-\frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{a^2(8A-C)\sqrt{a+}}{2d}$$

$$=-\frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{a^2(8A-C)\sqrt{a+}}{2d}$$

$$=\frac{a^{5/2}(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{4d}$$

Mathematica [A] time = 0.85915, size = 141, normalized size = 0.59

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(6\sqrt{2}(8A+19C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\sin\left(\frac{1}{2}\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(6*Sqrt
[2]*(8*A + 19*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(1
6*A + 33*C + (128*A + 9*C)*Cos[c + d*x] + 33*C*Cos[2*(c + d*x)] + 3*C*Cos[3
*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.214, size = 494, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/12/d*a^2*(24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+57*C*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*cos(d*x+c)^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+114*C*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))*cos(d*x+c)+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+57*C*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))+6*C*sin(d*x+c)*cos(d*x+c)^3+33*C*cos(d*x+c)^2*sin(d*x+c)+64*A*sin(d*x+
c)*cos(d*x+c)+8*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.15052, size = 459, normalized size = 1.93

$$-\frac{3\left((8A+19C)a^2\cos(dx+c)^2+(8A+19C)a^2\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(6Ca^2\cos(dx+c)^3+33Ca^2\right.}{}}{12\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/12*(3*((8*A + 19*C)*a^2*cos(d*x + c)^2 + (8*A + 19*C)*a^2*cos(d*x + c))*
sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
 + c))) - (6*C*a^2*cos(d*x + c)^3 + 33*C*a^2*cos(d*x + c)^2 + 64*A*a^2*cos(
d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1227 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=242

$$\frac{5a^{5/2}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-3C)\sin}{4a}$$

```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) - (a^3*(24*A - 49*C)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(8*A - 3
*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) - (a*(6
*A - C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) +
 (2*A*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.859843, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2976, 2981, 2774, 216}

$$\frac{5a^{5/2}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A-3C)\sin}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) - (a^3*(24*A - 49*C)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(8*A - 3
*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) - (a*(6
*A - C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) +
 (2*A*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{3}{2}}(c+}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{\left(2\sqrt{\right.}}{}$$

$$=-\frac{a(6A-C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2A(a+a}{}$$

$$=-\frac{a^2(8A-3C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}-\frac{a(6A-C}{}$$

$$=-\frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{a^2(8A-3C)\sqrt{a}}{4d}$$

$$=-\frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{a^2(8A-3C)\sqrt{a}}{4d}$$

$$=\frac{5a^{5/2}(8A+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{8d}$$

Mathematica [A] time = 1.00323, size = 142, normalized size = 0.59

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(8A+5C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(8*A + 5*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(4
8*A + 17*C + 3*(8*A + 27*C)*Cos[c + d*x] + 17*C*Cos[2*(c + d*x)] + 2*C*Cos[
3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [A] time = 0.168, size = 361, normalized size = 1.5

$$\frac{a^2\cos(dx+c)}{24\,d\,(1+\cos(dx+c))}\left(8\,C\sin(dx+c)(\cos(dx+c))^3+120\,A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/24/d*a^2*(8*C*sin(d*x+c)*cos(d*x+c)^3+120*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)+75*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+34*C*cos(d*x+c)^2*sin(d*x+c)+12
0*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))+24*A*sin(d*x+c)*cos(d*x+c)+75*C*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+75*C*cos(d*x+c)*sin(d*x+c)+48*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c)
)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.23903, size = 432, normalized size = 1.79

$$-\frac{15\left((8\,A+5\,C)a^2\cos(dx+c)+(8\,A+5\,C)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,Ca^2\cos(dx+c)^3+34\,Ca^2\cos(dx+c)^2+3\,(8\right.}{}}{24\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(15*((8*A + 5*C)*a^2*cos(d*x + c) + (8*A + 5*C)*a^2)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*C*a
^2*cos(d*x + c)^3 + 34*C*a^2*cos(d*x + c)^2 + 3*(8*A + 25*C)*a^2*cos(d*x +
c) + 48*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1228 $\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,$

Optimal. Leaf size=238

$$\frac{a^{5/2}(304A+163C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+299C)\sin(c+dx)}{192d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(16A+$$

```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A + 299*C)*
Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*(1
6*A + 17*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]]
) + (5*a*C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[c + d*x]
]) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.85195, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3046, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(304A+163C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+299C)\sin(c+dx)}{192d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(16A+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A + 299*C)*
Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*(1
6*A + 17*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]]
) + (5*a*C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[c + d*x]
]) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}\left(\right.}{\sqrt{\cos(c}}$$

$$=\frac{C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\right.}}{}$$

$$=\frac{5aC(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{24d\sqrt{\sec(c+dx)}}+\frac{C(a+a\cos(c}{4d\sqrt{}}$$

$$=\frac{a^2(16A+17C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{32d\sqrt{\sec(c+dx)}}+\frac{5aC(}{}$$

$$=\frac{a^3(432A+299C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{a^2(16A+17C}{}$$

$$=\frac{a^3(432A+299C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{a^2(16A+17C}{}$$

$$=\frac{a^{5/2}(304A+163C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{64d}$$

Mathematica [A] time = 0.763665, size = 153, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(304A+163C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\mathrm{si}\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt
[2]*(304*A + 163*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (
528*A + 581*C + (96*A + 362*C)*Cos[c + d*x] + 92*C*Cos[2*(c + d*x)] + 12*C*
Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384*d)
```

Maple [A] time = 0.181, size = 341, normalized size = 1.4

$$-\frac{a^2\left((\cos(dx+c))^2-1\right)}{192\,d\,(\sin(dx+c))^2}\left(48\,C\sin(dx+c)(\cos(dx+c))^3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+184\,C\sin(dx+c)(\cos(dx+c))^2\sqrt{\frac{\cos}{1+\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/192/d*a^2*(48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+184*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+96*A*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+326*C*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+528*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+489*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+912*A*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+489*C*arctan(s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1/
2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*
(cos(d*x+c)^2-1)
```

Maxima [B] time = 4.32445, size = 11549, normalized size = 48.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/768*(48*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2
*d*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(
2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a
^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
```

^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A - (
2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1)^(3/4)*((9*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c
) + 9*a^2*sin(4*d*x + 4*c)^3 + 36*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 36*(a^2*sin(4*d*x + 4*c)^3 + (a^2*
cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*a^2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*
c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(a
^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin
(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (40*a
^2*cos(4*d*x + 4*c)^2 + 40*a^2*sin(4*d*x + 4*c)^2 + 9*a^2*cos(4*d*x + 4*c)
+ 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x +
4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 160*(a
^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a
^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(80*a^2*cos(
4*d*x + 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 - 71*a^2*cos(4*d*x + 4*c) - 9*a^
2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(320*a^2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 71*a^2*si
n(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3
/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 36*(4*a^2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arcta
n2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(si

n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*a^2*cos(4*d*x + 4*c)^3 + 40*a
^2*cos(4*d*x + 4*c)^2 + 4*(9*a^2*cos(4*d*x + 4*c)^3 + 22*a^2*cos(4*d*x + 4*
c)^2 - 71*a^2*cos(4*d*x + 4*c) + (9*a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*
x + 4*c)^2 + 40*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + (9*a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 4*(9*a^2*cos(4*d*
x + 4*c)^3 + 58*a^2*cos(4*d*x + 4*c)^2 + 89*a^2*cos(4*d*x + 4*c) + (9*a^2*c
os(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 40*a^2)*sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - (40*a^2*cos(4*d*x + 4*c)^2 + 40*a^2*si
n(4*d*x + 4*c)^2 + 9*a^2*cos(4*d*x + 4*c) + 160*(a^2*cos(4*d*x + 4*c)^2 + a
^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4
*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 2*(80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(4*d*x
 + 4*c)^2 - 71*a^2*cos(4*d*x + 4*c) - 9*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) - 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) + 71*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 4*(9*a^2*cos(4*d*x + 4*c)^3 + 31*a^2*cos(4*d*x + 4*c)^2 -
40*a^2*cos(4*d*x + 4*c) + (9*a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c
)^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*a^2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4
*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) - 4*(4*(9*a^2*cos(4*d*x + 4*c) + 40*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (9*a^2*cos(4*d*x + 4*c) + 40*a^2)*s
in(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(
3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1)^(1/4)*((5*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 5*a^2*sin(
4*d*x + 4*c)^3 + 5*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + 192*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 +
2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^3 + 4*(5*a^2*sin(4*d*x + 4*c)^3 + 5*(a^2*cos(4*d*x + 4*c)^2 - 2*a^2
*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c) + 168*(a^2*cos(4*d*x + 4*c)^2 + a
^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 4*(5*a^2*sin(4*d*x + 4*c)^3 - 192*a^2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (5*a^2*cos(4*d*x + 4*c)^2
 + 10*a^2*cos(4*d*x + 4*c) - 43*a^2)*sin(4*d*x + 4*c) + 168*(a^2*cos(4*d*x
+ 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + 2*(10*a^2*sin(4*d*x + 4*c)^3 + 5*a^2*cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 10*(a^2*cos(4*

d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + (336*a^2*cos(4*d*x + 4*c)^2 + 336*a^2*sin(4*d*x + 4*c)^2 - 341*a^2*cos(4*d*x + 4*c) + 5*a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(24*a^2*cos(4*d*x + 4*c)^2 + 14*a^2*sin(4*d*x + 4*c)^2 - 341*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 96*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*(12*a^2*cos(4*d*x + 4*c)^2 + 7*a^2*sin(4*d*x + 4*c)^2 - 168*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*(a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (168*a^2*cos(4*d*x + 4*c)^2 + 168*a^2*sin(4*d*x + 4*c)^2 - 5*a^2*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (5*a^2*cos(4*d*x + 4*c)^3 - 120*a^2*cos(4*d*x + 4*c)^2 + 192*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 5*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(5*a^2*cos(4*d*x + 4*c)^3 - 82*a^2*cos(4*d*x + 4*c)^2 + 197*a^2*cos(4*d*x + 4*c) + (5*a^2*cos(4*d*x + 4*c) - 72*a^2)*sin(4*d*x + 4*c)^2 - 120*a^2 + 72*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 5*(a^2*cos(4*d*x + 4*c) - 24*a^2)*sin(4*d*x + 4*c)^2 + 4*(5*a^2*cos(4*d*x + 4*c)^3 - 110*a^2*cos(4*d*x + 4*c)^2 - 235*a^2*cos(4*d*x + 4*c) + 5*(a^2*cos(4*d*x + 4*c) - 24*a^2)*sin(4*d*x + 4*c)^2 - 120*a^2 + 48*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 72*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(10*a^2*cos(4*d*x + 4*c)^3 - 226*a^2*cos(4*d*x + 4*c)^2 - 5*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 240*a^2*cos(4*d*x + 4*c) + 2*(5*a^2*cos(4*d*x + 4*c) - 108*a^2)*sin(4*d*x + 4*c)^2 + (144*a^2*cos(4*d*x + 4*c)^2 + 144*a^2*sin(4*d*x + 4*c)^2 - 149*a^2*cos(4*d*x + 4*c) + 5*a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (72*a^2*cos(4*d*x + 4*c)^2 + 72*a^2*sin(4*d*x + 4*c)^2 - 5*a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(384*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 149*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(72*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (5*a^2*cos(4*d*x + 4*c) - 108*a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 10*(a^2*cos(4*d*x + 4*c) - 24*a^2)*sin(4*d*x + 4*c) - 5*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*

c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2
(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 489*((a^2*cos(4*d*x + 4*c
)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2
*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d
*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*ar
ctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - co
s(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d
*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*
c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2
*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4
*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*
c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos
(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*a

rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2
*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 +
a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*
d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)
^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*
d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 +
 a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^
2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*
cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x +
4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(
4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2
((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*C/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x +
 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x +
4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*
x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2
- 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c
) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))

```
)/d
```

Fricas [A] time = 2.46772, size = 486, normalized size = 2.04

$$-\frac{3\left((304\,A+163\,C)a^2\cos\left(dx+c\right)+(304\,A+163\,C)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,Ca^2\cos(dx+c)^4+184\,Ca^2\cos(\right.}{}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*((304*A + 163*C)*a^2*cos(d*x + c) + (304*A + 163*C)*a^2)*sqrt(a)*
arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
- (48*C*a^2*cos(d*x + c)^4 + 184*C*a^2*cos(d*x + c)^3 + 2*(48*A + 163*C)*a^
2*cos(d*x + c)^2 + 3*(176*A + 163*C)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

[Out] Timed out

3.1229 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=285

$$\frac{a^3(1040A+787C)\sin(c+dx)}{960d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(400A+283C)\sqrt{\cos(c+dx}}{}$$

```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*A + 787*C
)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*
(80*A + 79*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(240*d*Sec[c + d*x]^(3
/2)) + (a*C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(8*d*Sec[c + d*x]^(3/2
)) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (a^3*(400*A + 283*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])
```

Rubi [A] time = 0.948829, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(1040A+787C)\sin(c+dx)}{960d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(400A+283C)\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*A + 787*C
)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*
(80*A + 79*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(240*d*Sec[c + d*x]^(3
/2)) + (a*C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(8*d*Sec[c + d*x]^(3/2
)) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (a^3*(400*A + 283*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\left(A\right.$$

$$= \frac{C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{aC(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{C(a+a\cos(c+dx))^{5/2}\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(80A+79C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{240d\sec^{\frac{3}{2}}(c+dx)} + \frac{aC(a+a\cos(c+dx}{8d\sec^{\frac{3}{2}}(c}$$

$$= \frac{a^3(1040A+787C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(80A+79C)\sqrt{a+a\cos(c}}{240d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^3(1040A+787C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(80A+79C)\sqrt{a+a\cos(c}}{240d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^3(1040A+787C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(80A+79C)\sqrt{a+a\cos(c}}{240d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^{5/2}(400A+283C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{128d}$$

Mathematica [A] time = 1.30049, size = 170, normalized size = 0.6

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(400A+283C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c +
d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(400*A + 283*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] +
(6320*A + 5521*C + (2720*A + 3874*C)*Cos[c + d*x] + 4*(80*A + 331*C)*Cos[2*
(c + d*x)] + 348*C*Cos[3*(c + d*x)] + 48*C*Cos[4*(c + d*x)])*(-Sin[(c + d*x
)/2] + Sin[(3*(c + d*x))/2])))/(3840*d)
```

Maple [A] time = 0.18, size = 419, normalized size = 1.5

$$\frac{a^2(-1+\cos(dx+c))^2\cos(dx+c)}{1920\,d(\sin(dx+c))^4}\left(384\,C\sin(dx+c)(\cos(dx+c))^4\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+1392\,C\sin(dx+c)(\cos(dx\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/1920/d*a^2*(-1+cos(d*x+c))^2*(384*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+1392*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+640*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
2264*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2720*A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2830*C*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6000*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+4245*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6000*
A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4245*C*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(
a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1
/2)/sin(d*x+c)^4
```

Maxima [B] time = 4.01732, size = 6151, normalized size = 21.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/7680*(80*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos
(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
```

```
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)),
 (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)) - 1))*sqrt(a))*A + (10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*
d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2
*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((15*a^2*s
in(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*sin(3/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 15*a^2*sin(1/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
+ 1)) - (15*a^2*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a
^2*cos(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 15*a^2*cos(1/5*ar
ctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 88*a^2)*sin(3/2*arctan2(sin(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))) + 1)))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x + 5
*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)
*(8*(a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d*x +
 5*c) + a^2*sin(5*d*x + 5*c)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c)))^2 + 2*a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(5
```

```
*d*x + 5*c) + a^2*sin(5*d*x + 5*c))*cos(5/2*arctan2(sin(2/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))) + 1)) + 5*(5*a^2*sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
))) + 5*a^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 80*a^2*s
in(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 288*a^2*sin(1/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*cos(1/2*arctan2(sin(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5
*d*x + 5*c))) + 1)) - 8*(a^2*cos(5*d*x + 5*c) + (a^2*cos(5*d*x + 5*c) - a^2
)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + (a^2*cos(5*d*x +
 5*c) - a^2)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 - a^2 +
 2*(a^2*cos(5*d*x + 5*c) - a^2)*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))))*sin(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - 5*(5*a^2
*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 5*a^2*cos(3/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 80*a^2*cos(2/5*arctan2(sin(5*d*x
+ 5*c), cos(5*d*x + 5*c))) + 128*a^2*cos(1/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c))) - 208*a^2)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
 1)))*sqrt(a) + 4245*(a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5
*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +
2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*
arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arcta
n2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5
*c), cos(5*d*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), c
os(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))) + 1)^(1/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1
/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*ar
ctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*
x + 5*c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
) + 1))) + 1) - a^2*arctan2(-(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(
2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))*sin(1/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c))) - cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*si
n(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5
*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1))), (cos(2/5*arctan2(sin(
5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
 1)^(1/4)*(cos(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*cos(1/2*arc
tan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(
```

```
sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + sin(1/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c)))*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)
)) - 1) - a^2*arctan2((cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/
5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))) + 1)), (cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*
cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(
sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) + 1) + a^2*arctan2((cos(2/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)), (c
os(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
) + 1)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 2.67293, size = 547, normalized size = 1.92

$$-\frac{15\left(\left(400\,A+283\,C\right)a^{2}\cos\left(dx+c\right)+\left(400\,A+283\,C\right)a^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(384\,Ca^{2}\cos(dx+c)^{5}+1392\,Ca\cdots\right.}{\cdots}}{1920\left(d\cos\left(dx+c\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/1920*(15*((400*A + 283*C)*a^2*cos(d*x + c) + (400*A + 283*C)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
) - (384*C*a^2*cos(d*x + c)^5 + 1392*C*a^2*cos(d*x + c)^4 + 8*(80*A + 283*C
)*a^2*cos(d*x + c)^3 + 10*(272*A + 283*C)*a^2*cos(d*x + c)^2 + 15*(400*A +
283*C)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x +
c)), x)
```

3.1230 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=332

$$\frac{a^3(1304A+1015C)\sin(c+dx)}{768d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(136A+109C)\sin(c+dx)}{192d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(24A+23C)\sin(c+dx)\sqrt{a\cos(c+}{96d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(136*A + 109*
C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2
*(24*A + 23*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(96*d*Sec[c + d*x]^(5
/2)) + (a*C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(12*d*Sec[c + d*x]^(5/
2)) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Sec[c + d*x]^(5/2))
+ (a^3*(1304*A + 1015*C)*Sin[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]*Sec[
c + d*x]^(3/2)) + (a^3*(1304*A + 1015*C)*Sin[c + d*x])/(512*d*Sqrt[a + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.04587, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3046, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(1304A+1015C)\sin(c+dx)}{768d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(136A+109C)\sin(c+dx)}{192d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(24A+23C)\sin(c+dx)\sqrt{a\cos(c+}{96d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```


```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(136*A + 109*
C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2
*(24*A + 23*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(96*d*Sec[c + d*x]^(5
/2)) + (a*C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(12*d*Sec[c + d*x]^(5/
2)) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Sec[c + d*x]^(5/2))
+ (a^3*(1304*A + 1015*C)*Sin[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]*Sec[
c + d*x]^(3/2)) + (a^3*(1304*A + 1015*C)*Sin[c + d*x])/(512*d*Sqrt[a + a*Co
```

```
s[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\left(A\right.$$

$$= \frac{C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{6d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{aC(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{12d\sec^{\frac{5}{2}}(c+dx)} + \frac{C(a+a\cos(c+dx))^{5/2}\sin(c-}{6d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(24A+23C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{96d\sec^{\frac{5}{2}}(c+dx)} + \frac{aC(a+a\cos(c+dx}{12d\sec^{\frac{5}{2}}(c}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(24A+23C)\sqrt{a+a\cos(c}}{96d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(24A+23C)\sqrt{a+a\cos(c}}{96d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(24A+23C)\sqrt{a+a\cos(c}}{96d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(24A+23C)\sqrt{a+a\cos(c}}{96d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{a^{5/2}(1304A+1015C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{512d}$$

Mathematica [A] time = 1.25709, size = 192, normalized size = 0.58

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(1304A+1015C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\mathrm{s}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^
(3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt
[2]*(1304*A + 1015*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] +
 (4648*A + 4193*C + (2896*A + 3234*C)*Cos[c + d*x] + 4*(184*A + 315*C)*Cos[
2*(c + d*x)] + 96*A*Cos[3*(c + d*x)] + 428*C*Cos[3*(c + d*x)] + 112*C*Cos[4
*(c + d*x)] + 16*C*Cos[5*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))
/2])))/(3072*d)
```

Maple [A] time = 0.203, size = 491, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/1536/d*a^2*(-1+cos(d*x+c))^3*(256*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+896*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+384*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*x+c)+
1392*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1472*A*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1624*C*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2608*A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2030*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+3912*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+30
45*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3912*A*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3045*C*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1
/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.62512, size = 603, normalized size = 1.82

$$-\frac{3\left((1304\,A+1015\,C)a^2\cos\left(dx+c\right)+(1304\,A+1015\,C)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(256\,Ca^2\cos(dx+c)^6+896\,C\right.}{}}{1536\,(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/1536*(3*((1304*A + 1015*C)*a^2*cos(d*x + c) + (1304*A + 1015*C)*a^2)*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))) - (256*C*a^2*cos(d*x + c)^6 + 896*C*a^2*cos(d*x + c)^5 + 48*(8*A + 29*
C)*a^2*cos(d*x + c)^4 + 8*(184*A + 203*C)*a^2*cos(d*x + c)^3 + 2*(1304*A +
1015*C)*a^2*cos(d*x + c)^2 + 3*(1304*A + 1015*C)*a^2*cos(d*x + c))*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3
/2), x)
```

3.1231 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=289

$$\frac{2(19A+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(257A+273C)\sin(c+dx)\sqrt{\sec(c-}}{315d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(257*A + 273*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) - (2*(29*A + 21*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A + 21*C)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(9/2)*Sin[c + d*x])/
(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.02304, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2984, 12, 2782, 205}

$$\frac{2(19A+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(257A+273C)\sin(c+dx)\sqrt{\sec(c-}}{315d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x]],x
]
```


```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(257*A + 273*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) - (2*(29*A + 21*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A + 21*C)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(9/2)*Sin[c + d*x])/
(9*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(8A+9C}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+}}}{9a}$$

$$= -\frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+\right.}}{}$$

$$= \frac{2(19A+21C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A}{}$$

$$= -\frac{2(29A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(19A+21C)\sec^{\frac{5}{2}}(c+dx)\sin(c}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(257A+273C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(257A+273C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(257A+273C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A+21C)\sec^{\frac{3}{2}}(c+dx)\sin(}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + $$

Mathematica [C] time = 9.08628, size = 271, normalized size = 0.94

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(-315i(A+C)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}\sin\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d
*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((-315*I)*(A + C)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-1279*A - 1071*C + 2*(107*A + 63*C
)*Cos[c + d*x] - 8*(157*A + 168*C)*Cos[2*(c + d*x)] + 58*A*Cos[3*(c + d*x)]
 + 42*C*Cos[3*(c + d*x)] - 257*A*Cos[4*(c + d*x)] - 273*C*Cos[4*(c + d*x)])
*Sec[c + d*x]^(9/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2
])/4))/(315*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.194, size = 775, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/315/d*2^(1/2)/a*(315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*cos(d*x+c)^5+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^5+1575*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+1575*C*(cos(d*x+
c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+31
50*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*c
os(d*x+c)^3+3150*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*cos(d*x+c)^3+3150*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+3150*C*(cos(d*x+c)/(1+cos(d*x+c)))^
(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+1575*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+1575*C*(c
os(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)+315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))+257*A*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+273*C*2^(1/2)*cos(d*x+c)^4*sin(d*x
+c)-29*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-21*C*2^(1/2)*cos(d*x+c)^3*sin(d*x+
c)+57*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+63*C*2^(1/2)*cos(d*x+c)^2*sin(d*x+c
)-5*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)+35*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*(1/
cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^8/(-1+cos(d*x+c))^4/
(1+cos(d*x+c))^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83659, size = 529, normalized size = 1.83

$$\frac{\frac{315\sqrt{2}\left((A+C)a\cos(dx+c)^5+(A+C)a\cos(dx+c)^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((257A+273C)\cos(dx+c)^4-(29A+21C)\cos(dx+c)^3+3(19A+21C\cdots\right.}{\sqrt{\cos(dx\cdots}}}{315\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/315*(315*sqrt(2)*((A + C)*a*cos(d*x + c)^5 + (A + C)*a*cos(d*x + c)^4)*ar
ctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c)))/sqrt(a) + 2*((257*A + 273*C)*cos(d*x + c)^4 - (29*A + 21*C)*cos(d*x +
 c)^3 + 3*(19*A + 21*C)*cos(d*x + c)^2 - 5*A*cos(d*x + c) + 35*A)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*
cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{11}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(11/2)/sqrt(a*cos(d*x + c) +
a), x)
```

3.1232 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{2(31A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(43*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*(31*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqr
t[a + a*Cos[c + d*x]]) - (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a
 + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a
*Cos[c + d*x]])
```

Rubi [A] time = 0.829455, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2984, 12, 2782, 205}

$$\frac{2(31A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]],x]
```



```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(43*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*(31*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqr
t[a + a*Cos[c + d*x]]) - (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a
 + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a
*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(6A+7C}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a}}}{7a}$$

$$= -\frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+d}\right.}{}$$

$$= \frac{2(31A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2A}{7}$$

$$= -\frac{2(43A+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{2(43A+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{2(43A+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - 2($$

Mathematica [C] time = 10.2709, size = 2480, normalized size = 10.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*(-(C*Sin[c/2 + (d*x)/2])/(3*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(7/2)) + ((A + C)*Csc[c/2 + (d*x)/2]^9*(363825*Sin[c/2 + (d*x)/2]^2 - 472
9725*Sin[c/2 + (d*x)/2]^4 + 26785605*Sin[c/2 + (d*x)/2]^6 - 86790165*Sin[c/
```

2 + (d*x)/2]^8 + 177677808*Sin[c/2 + (d*x)/2]^10 - 239283044*Sin[c/2 + (d*x
)/2]^12 + 52080*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 560*HypergeometricPFQ[{2,
 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 213120160*Sin[c/2 + (d*x)/2]^14 - 168280
*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2
}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin
[c/2 + (d*x)/2]^14 - 121497024*Sin[c/2 + (d*x)/2]^16 + 212520*Hypergeometri
c2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin
[c/2 + (d*x)/2]^16 + 3360*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 1
3/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2
]^16 + 40125184*Sin[c/2 + (d*x)/2]^18 - 124320*Hypergeometric2F1[2, 11/2, 1
3/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]
^18 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 5840384*
Sin[c/2 + (d*x)/2]^20 + 28000*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 560*Hyperge
ometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 363825*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)] - 5336100*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 34636140*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 131060160*ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 320535600*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 530671680*ArcTanh[Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 604296000*ArcTan
h[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)
/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 468948480
*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2
+ (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 23
7726720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*S
in[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] - 70963200*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]]*Sin[c/2 + (d*x)/2]^18*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 9461760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]]*Sin[c/2 + (d*x)/2]^20*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)] - 1120*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 11/2},
{1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 +
 (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 280*Cos[(c + d*x)/2]^4*Hyperge

```
ometricPFQ[{2, 2, 11/2}, {1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(103 - 164*Sin[c/2 + (d*x)/2]^2 + 70*Sin
[c/2 + (d*x)/2]^4)))/(40425*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(9/2)*(-1 + 2*Sin[
c/2 + (d*x)/2]^2)) + (C*((5*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(7/2) + 2*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2) + 4*
(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)
/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))))/105))/(d*Sqrt[a*(1 + Cos[c + d*x]
)])
```

Maple [B] time = 0.229, size = 639, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/105/d*2^(1/2)/a*(105*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+105*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+420*A*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+420*C*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+630*
A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))+630*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))+420*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))+420*C*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+105*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+105*C*(cos(d*x+c)/(1+cos(d*x+c)
))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+43*A*2^(1/2)*cos(d*x+c)^3*sin(d
*x+c)+35*C*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-31*A*2^(1/2)*cos(d*x+c)^2*sin(d*
x+c)-35*C*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)
-15*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^6*(1/cos(d*x+c))^(9/2)*(a*(
1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^3/(1+cos(d*x+c))^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70353, size = 483, normalized size = 1.98

$$-\frac{\frac{105\sqrt{2}\left((A+C)a\cos(dx+c)^4+(A+C)a\cos(dx+c)^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((43A+35C)\cos(dx+c)^3-(31A+35C)\cos(dx+c)^2+3A\cos(dx\right.}{\sqrt{\cos(dx+c)}}}{105\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/105*(105*sqrt(2)*((A + C)*a*cos(d*x + c)^4 + (A + C)*a*cos(d*x + c)^3)*a
rctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
+ c)))/sqrt(a) + 2*((43*A + 35*C)*cos(d*x + c)^3 - (31*A + 35*C)*cos(d*x +
c)^2 + 3*A*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(
cos(d*x + c)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^{\frac{9}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(9/2)/sqrt(a*cos(d*x + c) + a
), x)
```

3.1233 $\int \frac{(A+C\cos^2(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=201

$$\frac{2(13A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(13*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*
Cos[c + d*x]]) - (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c +
 d*x]])
```

Rubi [A] time = 0.640555, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2984, 12, 2782, 205}

$$\frac{2(13A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(13*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*
Cos[c + d*x]]) - (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c +
 d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(4A+5}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}}{5a}$$

$$= -\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+\right.}{}$$

$$= \frac{2(13A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A}{}$$

$$= \frac{2(13A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A}{}$$

$$= \frac{2(13A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A}{}$$

$$= -\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} +$$

Mathematica [C] time = 7.98825, size = 1757, normalized size = 8.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*(-(C*Sin[c/2 + (d*x)/2])/(2*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(5/2)) - ((A + C)*Csc[c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825
*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x
)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10
- 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 51
8760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
```

```
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*S
in[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2
+ (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d
*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2
]^2)))/(675*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2
)) + (C*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2) + 4*(Sin
[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])
/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])))/30))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.203, size = 503, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*2^(1/2)/a*(15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+45*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+45*C*(cos(d*x+c)/(1+c
```

```
os(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+45*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)
+45*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
cos(d*x+c)+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))+13*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*C*2^(1/2)*cos(d*x+c)^2*sin
(d*x+c)-A*2^(1/2)*cos(d*x+c)*sin(d*x+c)+3*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*
(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^
2/(1+cos(d*x+c))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90142, size = 433, normalized size = 2.15

$$\frac{\frac{15\sqrt{2}\left((A+C)a\cos(dx+c)^3+(A+C)a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((13A+15C)\cos(dx+c)^2-A\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sin(\cdots}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((A + C)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*arct
an(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c
)))/sqrt(a) + 2*((13*A + 15*C)*cos(d*x + c)^2 - A*cos(d*x + c) + 3*A)*sqrt(
a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 +
a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a
), x)
```

3.1234 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{2A\sin(c+dx}{3d\sqrt{a\cos(c}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.46271, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3044, 2984, 12, 2782, 205}

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{2A\sin(c+dx}{3d\sqrt{a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(2A+3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}}{3a}$$

$$= -\frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+\right.}{}$$

$$= -\frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \left((A+C)\sqrt{c\right.$$

$$= -\frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2a(A+C)\right.}{}$$

$$= \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2}{}$$

Mathematica [C] time = 6.79832, size = 576, normalized size = 3.69

$$\frac{2\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{(A+C)\csc^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((-4*C*Sin[c/2 + (d*x)/2]^3)/(3*(1 - 2*Sin[c/2 + (d*x)/
2]^2)^(3/2)) + ((A + C)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(c + d*x)/2]^4*Hyperg
eometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 +
(d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Si
n[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 -
 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x
)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 -
```

```
20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2
]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh
[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2
 + (d*x)/2]^2))))/(63*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))))/(d*Sqrt[a*(1 +
Cos[c + d*x])])
```

Maple [B] time = 0.187, size = 366, normalized size = 2.4

$$\frac{\sqrt{2}\cos(dx+c)(\sin(dx+c))^2}{3\,ad(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(3\,A(\cos(dx+c))^2\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}+3\,C(\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*2^(1/2)/a*(3*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)+3*C*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6*A*cos(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6*C*cos(d*x+c)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*C*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+A*2^(1/2)*cos(d*x+c)*s
in(d*x+c)-A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2
)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95146, size = 382, normalized size = 2.45

$$-\frac{\frac{3\sqrt{2}\left((A+C)a\cos(dx+c)^2+(A+C)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2(A\cos(dx+c)-A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos(d*x + c))*arctan(
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
/sqrt(a) + 2*(A*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqr
t(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a
), x)
```

3.1235 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sqrt{\cos(c+ ...}}{}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.504376, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3044, 2982, 2782, 205, 2774, 216}

$$-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sqrt{\cos(c+ ...}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}aC\cos}{\sqrt{\cos(c+dx)}\sqrt{a+a}}}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.}{ad}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{}$$

Mathematica [C] time = 3.52424, size = 251, normalized size = 1.43

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(A+C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5\cos^2(c+dx)(\cos(c+dx)+2)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\right.\right.}{10\,\mathrm{co}}\right.}{d\sqrt{a(}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*C*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] - (2*C*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]] + ((
A + C)*Csc[(c + d*x)/2]^3*(5*Cos[c + d*x]^2*(2 + Cos[c + d*x])*(1 - Cos[c +
 d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt
[2 - 2*Sec[c + d*x]]) - Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(
c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]^2))/(10*Cos[c + d*x]^(5/2))
))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.181, size = 353, normalized size = 2.

$$\frac{\sqrt{2}\cos\left(dx+c\right)}{ad\left(1+\cos\left(dx+c\right)\right)}\left(C\sqrt{2}\cos\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)+A\arcsin\left(\frac{-1+c}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/a*(C*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+C*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)/cos(d*x+c))+C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*2^(1/2)*sin(d*x+c)+A*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+C*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2
)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.835, size = 454, normalized size = 2.59

$$-\frac{2\left(C\cos\left(dx+c\right)+C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}-\frac{2}{}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -(2*(C*cos(d*x + c) + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C
)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c)))/sqrt(a) - 2*sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a
), x)
```

3.1236 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] -((C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Si
n[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.505597, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3046, 2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Si
n[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$=\frac{C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(2A+}{\sqrt{\cos(c+}}}{a}$$

$$=\frac{C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,c}}{\sqrt{\cos}}}{2a}$$

$$=\frac{C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}}{ad}$$

$$=-\frac{C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.322944, size = 124, normalized size = 0.72

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2(A+C)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)-\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\sin\left(\frac{1}{2}\right.\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*
x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-(Sqrt[2]*C*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]]) + 2*(A + C)*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]] + 2*C*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Cos[c +
 d*x])])
```

Maple [A] time = 0.191, size = 185, normalized size = 1.1

$$\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{2\,ad\,(\sin(dx+c))^2}\left(-C\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+C\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+2\,A\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)/a*(-C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+C*
2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*C*arcsin((-1+cos(d*x+c))/sin(d*x+c)))
*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.8363, size = 447, normalized size = 2.58

$$\frac{\sqrt{a\cos(dx+c)+a}C\sqrt{\cos(dx+c)}\sin(dx+c)+(C\cos(dx+c)+C)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}((A+C)a\cos(}{}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*C*sqrt(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x +
c) + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) - sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*arctan(sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt
(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\sqrt{\sec\left(c + dx\right)}}{\sqrt{a\left(\cos\left(c + dx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) +
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sqrt{\sec\left(dx + c\right)}}{\sqrt{a\cos\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a
), x)
```

3.1237 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=223

$$\frac{(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{\sqrt{a}d}$$

```
[Out] ((8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(2*d
*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) - (C*Sin[c + d*x])/(4*d*Sqrt[
a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.687249, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3046, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((8*A + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(2*d
*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) - (C*Sin[c + d*x])/(4*d*Sqrt[
a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(}{}}{2a}$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} - \frac{C\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{(\sqrt{c}}{}$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} - \frac{C\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - ((A$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} - \frac{C\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{(2a}{}$$

$$= \frac{(8A+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tan^{-1}}{}$$

Mathematica [C] time = 1.21001, size = 496, normalized size = 2.22

$$-\frac{ie^{-3i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left((8A+7C)e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-8\sqrt{2}Ae^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] ((-I/16)*(1 + E^(I*(c + d*x)))*(-C + 2*C*E^(I*(c + d*x)) - 3*C*E^((2*I)*(c
+ d*x)) + 3*C*E^((3*I)*(c + d*x)) - 2*C*E^((4*I)*(c + d*x)) + C*E^((5*I)*(c
 + d*x)) + (8*A + 7*C)*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Ar
cSinh[E^(I*(c + d*x))] + Sqrt[2]*C*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x
))])] - 8*Sqrt[2]*A*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 7*Sqrt
[2]*C*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(-1 + E^(I*
(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 8*A*E^((2*I)*(c + d*
x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] -
7*C*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x))*Sqrt[a*(1 + C
os[c + d*x])])
```

Maple [A] time = 0.192, size = 270, normalized size = 1.2

$$\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{2}\cos\left(dx+c\right)}{8\,ad\left(\sin\left(dx+c\right)\right)^{4}}\left(2\,C\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\sin\left(dx+c\right)\cos\left(dx+c\right)-C\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\sin\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/8/d*2^(1/2)/a*(-1+cos(d*x+c))^2*(2*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)*cos(d*x+c)-C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)+8*A*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos
(d*x+c))+7*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+8*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))+8*C*arcsin((-1+cos(d*x+c))
/sin(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.4398, size = 529, normalized size = 2.37

$$-\frac{\left(8\,A+7\,C\right)\cos\left(dx+c\right)+8\,A+7\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{4\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)}}{\sqrt{a}\sin(}\right.}{\sqrt{a}}}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(((8*A + 7*C)*cos(d*x + c) + 8*A + 7*C)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2)*((A + C)*a
*cos(d*x + c) + (A + C)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) - (2*C*cos(d*x + c)^2 - C*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(
d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+C\cos^2\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d
*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{a\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c
))), x)
```

3.1238 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$-\frac{(8A+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A+7C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c-}{}$$

```
[Out] -((8*A + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTan[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(3*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) - (C*Sin[c + d*x])/(12*d*Sqr
t[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((8*A + 7*C)*Sin[c + d*x])/(8*d
*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.879979, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3046, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(8A+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A+7C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((8*A + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTan[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(3*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) - (C*Sin[c + d*x])/(12*d*Sqr
t[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((8*A + 7*C)*Sin[c + d*x])/(8*d
*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(}{}}{3a}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} - \frac{C\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(\sqrt{}}{}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} - \frac{C\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} - \frac{C\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} - \frac{C\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d}$$

$$= -\frac{(8A+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tan^{}}{}$$

Mathematica [C] time = 1.54667, size = 439, normalized size = 1.65

$$\frac{ie^{-4i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(3(8A+9C)e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+48\sqrt{2}(A+C)e^{3i(c+dx)}\sqrt{1+e^{2i(c}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((I/96)*(1 + E^(I*(c + d*x)))*(2*C - 3*C*E^(I*(c + d*x)) + 24*A*E^((2*I)*(c
 + d*x)) + 28*C*E^((2*I)*(c + d*x)) - 24*A*E^((3*I)*(c + d*x)) - 29*C*E^((3
*I)*(c + d*x)) + 24*A*E^((4*I)*(c + d*x)) + 29*C*E^((4*I)*(c + d*x)) - 24*A
*E^((5*I)*(c + d*x)) - 28*C*E^((5*I)*(c + d*x)) + 3*C*E^((6*I)*(c + d*x)) -
 2*C*E^((7*I)*(c + d*x)) + 3*(8*A + 9*C)*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + 48*Sqrt[2]*(A + C)*E^((3*I)*(c +
 d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])] - 24*A*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 27*C*E^((3*I)*(c +
d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
*Sqrt[Sec[c + d*x]])/(d*E^((4*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.166, size = 340, normalized size = 1.3

$$-\frac{\sqrt{2}\left(-1+\cos(dx+c)\right)^{3}\cos(dx+c)}{48\,ad\left(\sin(dx+c)\right)^{6}}\left(8\,C\left(\cos(dx+c)\right)^{2}\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-2\,C\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d*2^(1/2)/a*(-1+cos(d*x+c))^3*(8*C*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-2*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)*cos(d*x+c)+24*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)+21*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-24*A*2^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-27*C*2^
(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-48*A*
arcsin((-1+cos(d*x+c))/sin(d*x+c))-48*C*arcsin((-1+cos(d*x+c))/sin(d*x+c)))
*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/c
os(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 26.9935, size = 578, normalized size = 2.17

$$\frac{3\left((8A+9C)\cos\left(dx+c\right)+8A+9C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{24\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+}}{\sqrt{a}\sin}\right.}{\sqrt{a}}}{24\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(3*((8*A + 9*C)*cos(d*x + c) + 8*A + 9*C)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 24*sqrt(2)*((A + C
)*a*cos(d*x + c) + (A + C)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + (8*C*cos(d*x + c)^3 - 2*C*c
os(d*x + c)^2 + 3*(8*A + 7*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/
2)), x)
```

3.1239 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=315

$$\frac{(19A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A+7C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+}{2}$$

```
[Out] ((19*A + 11*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((1201*A + 665*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(210*a*d*Sq
rt[a + a*Cos[c + d*x]]) + ((397*A + 245*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A + 35*C)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(7/2)*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((11*A + 7*C)*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.08455, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3042, 2984, 12, 2782, 205}

$$\frac{(19A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A+7C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3/2),
x]
```


```
[Out] ((19*A + 11*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((1201*A + 665*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(210*a*d*Sq
rt[a + a*Cos[c + d*x]]) + ((397*A + 245*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A + 35*C)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(7/2)*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((11*A + 7*C)*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(11A+7\ldots}{\cos^{\frac{9}{2}}(c\ldots}}{2a^2}$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(11A+7C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{14ad\sqrt{a+a\cos(c+dx)}} + \ldots$$

$$= -\frac{(67A+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \ldots$$

$$= \frac{(397A+245C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} - \frac{(67A+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+\ldots}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(1201A+665C)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A+245C)\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{210ad\sqrt{a+a\cos(c+d\ldots}}$$

$$= -\frac{(1201A+665C)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A+245C)\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{210ad\sqrt{a+a\cos(c+d\ldots}}$$

$$= -\frac{(1201A+665C)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A+245C)\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{210ad\sqrt{a+a\cos(c+d\ldots}}$$

$$= \frac{(19A+11C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \ldots$$

Mathematica [C] time = 10.5468, size = 3121, normalized size = 9.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^
(3/2),x]
```

[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*((4*C*Sin[c/2 + (d*x)/2])/(7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) - ((A + C)*(1 - 2*Sin[c/2 + (d*x)/2]))/(28*(1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) + ((A + C)*(1 + 2*Sin[c/2 + (d*x)/2]))/(28*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) - ((A + C)*(315*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (5 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (11 + 17*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 + 71*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/70 + ((A + C)*(315*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (5 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (11 - 17*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 - 71*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/70 - ((-A + 7*C)*Csc[c/2 + (d*x)/2]^9*(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2]^4 + 26785605*Sin[c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d*x)/2]^8 + 177677808*Sin[c/2 + (d*x)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^12 + 52080*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 213120160*Sin[c/2 + (d*x)/2]^14 - 168280*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 121497024*Sin[c/2 + (d*x)/2]^16 + 212520*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 3360*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 40125184*Sin[c/2 + (d*x)/2]^18 - 124320*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 5840384*Sin[c/2 + (d*x)/2]^20 + 28000*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 363825*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 5336100*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 34636140*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 131060160*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 +

```
(d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 32053
5600*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[
c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] -
 530671680*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] + 604296000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] - 468948480*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin
[c/2 + (d*x)/2]^2)] + 237726720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)] - 70963200*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^18*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 9461760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^20*Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1120*Cos[(c + d*x)/2]^6*Hypergeomet
ricPFQ[{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 280*Co
s[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2}, {1, 13/2}, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(103 - 164*Sin[c
/2 + (d*x)/2]^2 + 70*Sin[c/2 + (d*x)/2]^4)))/(80850*(1 - 2*Sin[c/2 + (d*x)/
2]^2)^(9/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (8*C*((3*Sin[c/2 + (d*x)/2])/(
1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 +
(d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]
)))/35))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.244, size = 719, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/420/d*2^(1/2)/a^2*(-1995*A*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-1155*C*cos(d*x+c)^4*sin(d*x
+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-79
80*A*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)-4620*C*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-11970*A*cos(d*x+c)^2*sin(d*
x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-6
930*C*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c
```

```
)/(1+cos(d*x+c)))^(7/2)-7980*A*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-4620*C*cos(d*x+c)*sin(d*x+c)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-1995*
A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(7/2)-1155*C*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+
cos(d*x+c)))^(7/2)+1201*A*cos(d*x+c)^5*2^(1/2)+665*C*cos(d*x+c)^5*2^(1/2)-3
97*A*2^(1/2)*cos(d*x+c)^4-245*C*2^(1/2)*cos(d*x+c)^4-1000*A*cos(d*x+c)^3*2^
(1/2)-560*C*cos(d*x+c)^3*2^(1/2)+232*A*cos(d*x+c)^2*2^(1/2)+140*C*cos(d*x+c
)^2*2^(1/2)-96*A*cos(d*x+c)*2^(1/2)+60*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))
^(9/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^5/(-1+cos(d*x+c))^3/(1+cos(d*x+c
))^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.23885, size = 630, normalized size = 2.

$$-\frac{105\sqrt{2}\Big((19A+11C)\cos(dx+c)^5+2(19A+11C)\cos(dx+c)^4+(19A+11C)\cos(dx+c)^3\Big)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(\cdots}}{\sqrt{\cdots}}\right.}{420\left(a^2d\cos(dx+c)^5+2a^2d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/420*(105*sqrt(2)*((19*A + 11*C)*cos(d*x + c)^5 + 2*(19*A + 11*C)*cos(d*x
 + c)^4 + (19*A + 11*C)*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1201*A + 665*
C)*cos(d*x + c)^4 + 12*(67*A + 35*C)*cos(d*x + c)^3 - 28*(7*A + 5*C)*cos(d*
x + c)^2 + 36*A*cos(d*x + c) - 60*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*
```

```
cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.1240 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{(15A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{10ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+\cdots}{\cdots}$$

```
[Out] -((15*A + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) + ((49*A + 25*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a
 + a*Cos[c + d*x]]) - ((13*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*
d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A + 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.881977, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3042, 2984, 12, 2782, 205}

$$-\frac{(15A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{10ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3/2),
x]
```


```
[Out] -((15*A + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) + ((49*A + 25*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a
 + a*Cos[c + d*x]]) - ((13*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*
d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A + 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9A+}{\cos^{\frac{7}{2}}(}}{2a^2}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A+5C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(13A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{(49A+25C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(49A+25C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(49A+25C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{(13A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(15A+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} +$$

Mathematica [C] time = 8.05772, size = 2280, normalized size = 8.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*((4*C*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(5/2)) - ((A + C)*(1 - 2*Sin[c/2 + (d*x)/2]))/(20*(1 + Sin[c/2 + (d*x)
/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + ((A + C)*(1 + 2*Sin[c/2 + (d*x)/
```

2]))/(20*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (16 *C*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d *x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/15 - ((A + C)*(-105*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (4 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - (19 + 29*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/30 + ((A + C)*(-105*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c /2 + (d*x)/2]^2]] + (4 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - (19 - 29*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/30 + ((-A + 7*C)*Csc[c/2 + (d*x)/ 2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c /2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^1 0 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/ 2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hypergeometr icPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hy pergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/ 2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeo metric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] *Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F 1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 567 00*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/ 2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 2 91060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin [c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]* Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/ 2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/ 2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/ 2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si n[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2 *Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/ 2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(1350*(1 - 2*Sin[c/2 + (d*x)/ 2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*(a*(1 + Cos[c + d*x]))^(3/2

```
))
```

Maple [B] time = 0.205, size = 583, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/20/d*2^(1/2)/a^2*(75*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3*sin
(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+35*C*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+225*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))+105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x
+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+225*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+105*C*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))+75*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))+35*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))-49*A*2^(1/2)*cos(d*x+c)^4-25*C*2^(1/2)*cos(d*x+
c)^4+13*A*cos(d*x+c)^3*2^(1/2)+5*C*cos(d*x+c)^3*2^(1/2)+40*A*cos(d*x+c)^2*2
^(1/2)+20*C*cos(d*x+c)^2*2^(1/2)-8*A*cos(d*x+c)*2^(1/2)+4*A*2^(1/2))*cos(d*
x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^3/(-1+cos(d*x
+c))^2/(1+cos(d*x+c))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.36114, size = 567, normalized size = 2.12

$$\frac{5\sqrt{2}\left((15A+7C)\cos(dx+c)^4+2(15A+7C)\cos(dx+c)^3+(15A+7C)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{c}}{\sqrt{a}\sin(dx+c)}\right.}{20\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3+a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/20*(5*sqrt(2)*((15*A + 7*C)*cos(d*x + c)^4 + 2*(15*A + 7*C)*cos(d*x + c)^
3 + (15*A + 7*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((49*A + 25*C)*cos(d*x
 + c)^3 + 4*(9*A + 5*C)*cos(d*x + c)^2 - 4*A*cos(d*x + c) + 4*A)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a
^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.1241 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(11A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)}{2d(}$$

```
[Out] ((11*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((19*A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a +
a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Co
s[c + d*x])^(3/2)) + ((7*A + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*S
qrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.719054, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3042, 2984, 12, 2782, 205}

$$\frac{(11A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)}{2d(}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((11*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((19*A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a +
a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Co
s[c + d*x])^(3/2)) + ((7*A + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*S
qrt[a + a*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(7A+3C}{\cos^{\frac{5}{2}}(c+dx}}{2a^2}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} + $$

$$= -\frac{(19A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(19A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{(19A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} +$$

$$= \frac{(11A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(19}{}$$

Mathematica [C] time = 6.80915, size = 1055, normalized size = 4.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*((4*C*Sin[c/2 + (d*x)/2])/(3*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(3/2)) - ((A + C)*(1 - 2*Sin[c/2 + (d*x)/2]))/(12*(1 + Sin[c/2 + (d*x)
/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A + C)*(1 + 2*Sin[c/2 + (d*x)/
2]))/(12*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (8*
C*Sin[c/2 + (d*x)/2])/(3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - ((A + C)*(5*Ar
cTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 + Si
n[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2
]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/2 + ((
```

```
A + C)*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2
]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2
+ (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/
2])))/2 + ((A - 7*C)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(c + d*x)/2]^4*Hypergeom
etricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*
x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c
/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*
Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2
]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*
Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2
)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sq
rt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 +
(d*x)/2]^2))))/(126*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))))/(d*(a*(1 + Cos[c
+ d*x]))^(3/2))
```

Maple [B] time = 0.188, size = 445, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*2^(1/2)/a^2*(33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d
*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))+9*C*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))+66*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+18*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(
d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+9*C*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-19*A*cos(d*x+c)^3*2^(1/2
)-3*C*cos(d*x+c)^3*2^(1/2)+7*A*cos(d*x+c)^2*2^(1/2)+3*C*cos(d*x+c)^2*2^(1/2
)+16*A*cos(d*x+c)*2^(1/2)-4*A*2^(1/2))*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))
^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.40892, size = 521, normalized size = 2.36

$$-\frac{3\sqrt{2}\left((11A+3C)\cos(dx+c)^3+2(11A+3C)\cos(dx+c)^2+(11A+3C)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{c}}{\sqrt{a}\sin(dx+c)}\right.}{12\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*((11*A + 3*C)*cos(d*x + c)^3 + 2*(11*A + 3*C)*cos(d*x + c)
^2 + (11*A + 3*C)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((19*A + 3*C)*cos(d*x +
 c)^2 + 12*A*cos(d*x + c) - 4*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(
d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.1242 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{(7A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\mathrm{si}}{2d(a}$$

```
[Out] -((7*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(
3/2)*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x
])^(3/2)) + ((5*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*C
os[c + d*x]])
```

Rubi [A] time = 0.534772, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3042, 2984, 12, 2782, 205}

$$-\frac{(7A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A+C)\mathrm{si}}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3/2),
x]
```


```
[Out] -((7*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(
3/2)*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x
])^(3/2)) + ((5*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*C
os[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(5A+C)}{\cos^{\frac{3}{2}}(c+}}{2a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(7A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(A}{}$$

Mathematica [C] time = 4.66993, size = 460, normalized size = 2.67

$$\frac{2\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(A-7C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*(A + C)*Arc
Tan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]])/2 - (3*(A + C)*ArcTan[(1
+ 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]])/2 - ((A + C)*Sqrt[Cos[c + d*x]])
/(-1 + Sin[(c + d*x)/2]) + (4*C*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]] - ((A
+ C)*Sqrt[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]) + ((A + C)*(-1 + 2*Sin[(c +
 d*x)/2]))/(4*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2) -
 ((A + C)*(1 + 2*Sin[(c + d*x)/2]))/(4*Sqrt[Cos[c + d*x]]*(-1 + Sin[(c + d*
x)/2])) + ((A - 7*C)*Csc[(c + d*x)/2]^3*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c +
 d*x)])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)
]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*Hypergeometric2F1[2, 5/2, 7/2
```

```
, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]*Tan[c
 + d*x]))/(40*Cos[c + d*x]^(3/2))))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.185, size = 311, normalized size = 1.8

$$\frac{\sqrt{2}\cos(dx+c)}{4\,da^2\sin(dx+c)\,(1+\cos(dx+c))}\left(7\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-C\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*(7*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-C*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*A*cos(d
*x+c)^2*2^(1/2)-C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)-C*cos(d*x+c)^2*2^(1/2)+A*cos(d*x+c)*2^(1/2)+C*2^(1/2
)*cos(d*x+c)+4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c))
)^(1/2)/sin(d*x+c)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.06366, size = 427, normalized size = 2.48

$$\frac{\sqrt{2}\left((7\,A-C)\cos(dx+c)^2+2\,(7\,A-C)\cos(dx+c)+7\,A-C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\,((5\,A+C)\cos(dx}{}}{4\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((7*A - C)*cos(d*x + c)^2 + 2*(7*A - C)*cos(d*x + c) + 7*A - C
)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))) + 2*((5*A + C)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.1243 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{(3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a\cos(c+d}}\right.}{a^{3/2}d}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A - 5*C)*ArcTan[(Sqrt[a]*Sin[c +
 d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2
*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.556765, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3042, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a\cos(c+d}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3/2),
x]
```


```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A - 5*C)*ArcTan[(Sqrt[a]*Sin[c +
 d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2
*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\cdots}{\sqrt{\cdots}}}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c-\cdots}{4\cdots}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\cdots}{\cdots}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(3A-5C)\tan^{-1}\left(\frac{\cdots}{\sqrt{2\cdots}}\right.\cdots}{\cdots}$$

Mathematica [C] time = 1.65088, size = 245, normalized size = 1.32

$$-\frac{i\cos^3\left(\frac{1}{2}(c+dx)\right)\left(i(A+C)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+\cdots}}}\cdots}{2d(a(\cos(c+dx)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^
(3/2),x]
```

```
[Out] ((-I/2)*Cos[(c + d*x)/2]^3*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*C*ArcSinh[E^(I*(c + d*x))] - Sqr
t[2]*(3*A - 5*C)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))])] - 4*C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d
*x)) + I*(A + C)*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] -
Sin[(3*(c + d*x))/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.175, size = 283, normalized size = 1.5

$$\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(-A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-4C\sqrt{2}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-A*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*C*2^(1/2)*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-C*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)+A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*C*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)+C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos(dx+c)^2+A\right)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [A] time = 42.2666, size = 579, normalized size = 3.13

$$-\frac{\sqrt{2}\left((3A-5C)\cos(dx+c)^2+2(3A-5C)\cos(dx+c)+3A-5C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos}}{4\left(a^2d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((3*A - 5*C)*cos(d*x + c)^2 + 2*(3*A - 5*C)*cos(d*x + c) + 3*
A - 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(sqrt(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A + C)*sqrt(cos(d*x
+ c))*sin(d*x + c) + 8*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*ar
ctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(
a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3
/2), x)
```

3.1244 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=228

$$\frac{(A+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+}}\right.}{a^{3/2}d}$$

```
[Out] (-3*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A + 9*C)*ArcTan[(Sqrt[a]*Sin[c +
d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((A + 3*C)*Sin[c + d*x])
/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.722192, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3042, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(A+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] (-3*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A + 9*C)*ArcTan[(Sqrt[a]*Sin[c +
d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((A + 3*C)*Sin[c + d*x])
/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos}}{}}{2a}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A+3C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A+3C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A+3C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= -\frac{3C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A+9C)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{}$$

Mathematica [C] time = 1.68362, size = 251, normalized size = 1.1

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\left(\sin\left(\frac{3}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)(A + 2C\cos(c+dx) + 3C) + i\sqrt{2}e^{-\frac{1}{2}}\right.}{2d(a(\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d
*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(6*C*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*
(A + 9*C)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x
))])] - 6*C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x)) +
(A + 3*C + 2*C*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c
 + d*x)/2] + Sin[(3*(c + d*x))/2])))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.18, size = 321, normalized size = 1.4

$$-\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^2}{4\,da^2(\sin(dx+c))^5}\sqrt{a(1+\cos(dx+c))}\left(2\,C(\cos(dx+c))^2\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+A\sqrt{2}\sqrt{\frac{\cos(d}{1+\cos(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2
*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+6*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+C*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*C*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+9*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos
(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
 c))), x)
```

Fricas [A] time = 42.1055, size = 624, normalized size = 2.74

$$-\frac{\sqrt{2}\left((A+9C)\cos(dx+c)^2+2(A+9C)\cos(dx+c)+A+9C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-12\left(C\cos(dx\right.}{4\left(a^2d\cos(dx+c)^2+2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A + 9*C)*cos(d*x + c)^2 + 2*(A + 9*C)*cos(d*x + c) + A + 9*
C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 12*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*a
rctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) -
 2*(2*C*cos(d*x + c)^2 + (A + 3*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
 c))), x)
```

3.1245 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{\cos(c+dx)}}\right.}{2\sqrt{2}a^{3/2}d}$$

```
[Out] ((8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A + 13*C)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[
c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)) + ((A + 2*C)*
Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) - ((2*A +
 7*C)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.93092, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3042, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{\cos(c+dx)}}\right.}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),
x]
```


```
[Out] ((8*A + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A + 13*C)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[
c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)) + ((A + 2*C)*
Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) - ((2*A +
 7*C)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}}{}}{2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{(8A+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{3/2}d} - \frac{(5A+13C)\,\mathrm{ta}}{}$$

Mathematica [C] time = 6.19786, size = 385, normalized size = 1.35

$$-\frac{ie^{\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{2}e^{-2i(c+dx)}\left(e^{i(c+dx)}-e^{2i(c+dx)}+e^{3i(c+dx)}-1\right)\left(C\left(-3e^{i(c+dx)}-12e^{2i(c+dx)}-3e^{3i(c+dx)}+e^{4i(c+dx)}+1\right)-4Ae^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+\sqrt{2}(8A+\right.}{16d\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(
3/2)),x]
```

```
[Out] ((-I/16)*E^((I/2)*(c + d*x))*((Sqrt[2]*(-1 + E^(I*(c + d*x)) - E^((2*I)*(c
+ d*x)) + E^((3*I)*(c + d*x)))*(-4*A*E^((2*I)*(c + d*x)) + C*(1 - 3*E^(I*(c
 + d*x)) - 12*E^((2*I)*(c + d*x)) - 3*E^((3*I)*(c + d*x)) + E^((4*I)*(c + d
*x)))))/(E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]) + Sqrt[2]*(8*A
+ 19*C)*(1 + E^(I*(c + d*x)))^2*ArcSinh[E^(I*(c + d*x))] + 4*(5*A + 13*C)*(
1 + E^(I*(c + d*x)))^2*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((
2*I)*(c + d*x))])] - Sqrt[2]*(8*A + 19*C)*(1 + E^(I*(c + d*x)))^2*ArcTanh[S
qrt[1 + E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2])/(d*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(3/2)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(a*(1 + Cos[c +
d*x]))^(3/2))
```

Maple [A] time = 0.199, size = 404, normalized size = 1.4

$$-\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^3}{8da^2(\sin(dx+c))^7}\sqrt{a(1+\cos(dx+c))}\left(-2C(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+5C(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/8/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-
2*C*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*C*cos(d*x+c)^2
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8*A*2^(1/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+2*A*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+19*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+4*C*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)+10*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)-2*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+26*C*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*sin(d*x+c)-7*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

```
)/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + A}{(a\cos{(dx+c)} + a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(
3/2)), x)
```

Fricas [A] time = 75.4157, size = 701, normalized size = 2.46

$$\frac{\sqrt{2}\left((5\,A + 13\,C)\cos{(dx+c)}^2 + 2\,(5\,A + 13\,C)\cos{(dx+c)} + 5\,A + 13\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \left((8\,A + \right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A + 13*C)*cos(d*x + c)^2 + 2*(5*A + 13*C)*cos(d*x + c) + 5
*A + 13*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))/(sqrt(a)*sin(d*x + c))) - ((8*A + 19*C)*cos(d*x + c)^2 + 2*(8*A + 19*C)*
cos(d*x + c) + 8*A + 19*C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + (2*C*cos(d*x + c)^3 - 3*C*cos(d*x + c)
^2 - (2*A + 7*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(c
os(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(
3/2)), x)
```

3.1246 $\int \frac{(A+C\cos^2(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=315

$$\frac{(157A+45C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(787A+195C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(2671A+735C)\sin(c+dx)\sqrt{\sec(c}}{240a^2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -((283*A + 75*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) + ((2671*A + 735*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a^2
*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A + 195*C)*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*A + 5*C)*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((157*A + 45*C)*Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.08603, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3042, 2978, 2984, 12, 2782, 205}

$$\frac{(157A+45C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(787A+195C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(2671A+735C)\sin(c+dx)\sqrt{\sec(c}}{240a^2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5/2),
x]
```


```
[Out] -((283*A + 75*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) + ((2671*A + 735*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a^2
*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A + 195*C)*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*A + 5*C)*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((157*A + 45*C)*Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(13A}{\cos^{\frac{7}{2}}(c+}}{4a^2}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A+5C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A+5C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(787A+195C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{4d(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(2671A+735C)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A+195C)\sec^{\frac{3}{2}}(c+dx)\sin}{240a^2d\sqrt{a+a\cos(c+d}}$$

$$= \frac{(2671A+735C)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A+195C)\sec^{\frac{3}{2}}(c+dx)\sin}{240a^2d\sqrt{a+a\cos(c+d}}$$

$$= \frac{(2671A+735C)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A+195C)\sec^{\frac{3}{2}}(c+dx)\sin}{240a^2d\sqrt{a+a\cos(c+d}}$$

$$= -\frac{(283A+75C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 8.06, size = 261, normalized size = 0.83

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(50(521A+153C)\cos(c+dx)+108(157A+45C)\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-240*I)*(283*A + 75*C)*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (15053*
A + 4125*C + 50*(521*A + 153*C)*Cos[c + d*x] + 108*(157*A + 45*C)*Cos[2*(c
+ d*x)] + 9110*A*Cos[3*(c + d*x)] + 2550*C*Cos[3*(c + d*x)] + 2671*A*Cos[4*
(c + d*x)] + 735*C*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^3*Sec[c + d*x]^(5/2)*
Tan[(c + d*x)/2]))/(960*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.223, size = 717, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/480/d*2^(1/2)/a^3*(-4245*A*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-1125*C*cos(d*x+c)^4*sin(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-169
80*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))-4500*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^
3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-25470*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-6
750*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))-16980*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c
)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4500*C*(cos(d*x+c)/(1+cos(d
*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4245
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))-1125*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))+2671*A*cos(d*x+c)^5*2^(1/2)+735*C*cos(d*x+c)^5*2^(1/2)+
1884*A*2^(1/2)*cos(d*x+c)^4+540*C*2^(1/2)*cos(d*x+c)^4-2987*A*cos(d*x+c)^3*
2^(1/2)-795*C*cos(d*x+c)^3*2^(1/2)-1728*A*cos(d*x+c)^2*2^(1/2)-480*C*cos(d*
x+c)^2*2^(1/2)+256*A*cos(d*x+c)*2^(1/2)-96*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x
+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))/(1+cos(d*x+c
))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.05281, size = 717, normalized size = 2.28

$$\frac{15\sqrt{2}\left((283\,A+75\,C)\cos\left(dx+c\right)^{5}+3\,(283\,A+75\,C)\cos\left(dx+c\right)^{4}+3\,(283\,A+75\,C)\cos\left(dx+c\right)^{3}+(283\,A+75\,C)\right.}{480\left(a^{3}d\cos\left(dx+c\right)^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/480*(15*sqrt(2)*((283*A + 75*C)*cos(d*x + c)^5 + 3*(283*A + 75*C)*cos(d*x
 + c)^4 + 3*(283*A + 75*C)*cos(d*x + c)^3 + (283*A + 75*C)*cos(d*x + c)^2)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) + 2*((2671*A + 735*C)*cos(d*x + c)^4 + 5*(911*A + 255*C)*co
s(d*x + c)^3 + 32*(49*A + 15*C)*cos(d*x + c)^2 - 160*A*cos(d*x + c) + 96*A)
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
 c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c
)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5
/2), x)
```

3.1247 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=266

$$\frac{5(19A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{16\sqrt{}}$$

```
[Out] ((163*A + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((299*A + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*S
qrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(
a + a*Cos[c + d*x])^(5/2)) - ((17*A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (5*(19*A + 3*C)*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.917349, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3042, 2978, 2984, 12, 2782, 205}

$$\frac{5(19A+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5/2),
x]
```


```
[Out] ((163*A + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((299*A + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*S
qrt[a + a*Cos[c + d*x]]) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(
a + a*Cos[c + d*x])^(5/2)) - ((17*A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (5*(19*A + 3*C)*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(11A\cdots}{\cos^{\frac{5}{2}}(c\cdots}}{4a^2}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(299A+27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{4d(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(299A+27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{4d(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(299A+27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx\cdots}{4d(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(163A+19C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \cdots$$

Mathematica [C] time = 3.43765, size = 243, normalized size = 0.91

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3(163A+19C)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{8}i\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^
(5/2),x]
```

```
[Out] ((I/12)*Cos[(c + d*x)/2]^5*((3*(163*A + 19*C)*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (I/8)*(
878*A + 78*C + (1537*A + 81*C)*Cos[c + d*x] + 2*(503*A + 39*C)*Cos[2*(c + d
*x)] + 299*A*Cos[3*(c + d*x)] + 27*C*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*S
ec[c + d*x]^(3/2)*Tan[(c + d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.206, size = 573, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96/d*2^(1/2)/a^3*(-489*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos
(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))-57*C*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*sin(d*x+c)*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1467*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*
x+c))/sin(d*x+c))-171*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*
x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1467*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-171*C*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+299*A*2^(1/2)*cos(d*x+c)^4-489*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+27*C*2^(1/2)*cos(d*x+c)^4
-57*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+204*A*cos(d*x+c)^3*2^(1/2)+12*C*cos(d*x+c)^3*2^(1/2)-343*A*cos(d
*x+c)^2*2^(1/2)-39*C*cos(d*x+c)^2*2^(1/2)-192*A*cos(d*x+c)*2^(1/2)+32*A*2^(
1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(
1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.85552, size = 657, normalized size = 2.47

$$-\frac{3\sqrt{2}((163A+19C)\cos(dx+c)^4+3(163A+19C)\cos(dx+c)^3+3(163A+19C)\cos(dx+c)^2+(163A+19C)\,c}{96(a^3d\cos(dx+c)^4+3a^3d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/96*(3*sqrt(2)*((163*A + 19*C)*cos(d*x + c)^4 + 3*(163*A + 19*C)*cos(d*x
+ c)^3 + 3*(163*A + 19*C)*cos(d*x + c)^2 + (163*A + 19*C)*cos(d*x + c))*sqr
t(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) + 2*((299*A + 27*C)*cos(d*x + c)^3 + (503*A + 39*C)*cos(d*x +
c)^2 + 160*A*cos(d*x + c) - 32*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqr
t(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5
/2), x)
```

3.1248 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{(49A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{5(15A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-}{1}$$

```
[Out] (-5*(15*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c +
 d*x])^(5/2)) - ((13*A - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a +
 a*Cos[c + d*x])^(3/2)) + ((49*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*
a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.710174, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3042, 2978, 2984, 12, 2782, 205}

$$\frac{(49A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{5(15A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5/2),
x]
```


```
[Out] (-5*(15*A - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c +
 d*x])^(5/2)) - ((13*A - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a +
 a*Cos[c + d*x])^(3/2)) + ((49*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*
a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9A+}{\cos^{\frac{3}{2}}(c}}{4a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{5(15A-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} -$$

Mathematica [C] time = 2.16866, size = 213, normalized size = 0.97

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(10(17A+C)\cos(c+dx)+(49A+C)\cos(2(c+dx))\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-5*I)*(15*A - C)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((113*A + C +
 10*(17*A + C)*Cos[c + d*x] + (49*A + C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]
^3*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/4))/(4*d*(a*(1 + Cos[c + d*x]))^(5/
2))
```

Maple [B] time = 0.187, size = 457, normalized size = 2.1

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))\cos(dx+c)}{32\,da^3(\sin(dx+c))^3(1+\cos(dx+c))}\left(75\,A\sin(dx+c)(\cos(dx+c))^2\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(-1+cos(d*x+c))*(75*A*sin(d*x+c)*cos(d*x+c)^2*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*C*sin(d*x+c)*
cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+150*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)-49*A*cos(d*x+c)^3*2^(1/2)-10*C*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
-C*cos(d*x+c)^3*2^(1/2)+75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-36*A*cos(d*x+c)^2*2^(1/2)-5*C*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-4*C*co
s(d*x+c)^2*2^(1/2)+53*A*cos(d*x+c)*2^(1/2)+5*C*2^(1/2)*cos(d*x+c)+32*A*2^(1
/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/
(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.92116, size = 552, normalized size = 2.52

$$\frac{5\sqrt{2}\Big((15A-C)\cos(dx+c)^3+3(15A-C)\cos(dx+c)^2+3(15A-C)\cos(dx+c)+15A-C\Big)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(\cdots}}{\sqrt{\cdots}}\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/32*(5*sqrt(2)*((15*A - C)*cos(d*x + c)^3 + 3*(15*A - C)*cos(d*x + c)^2 +
3*(15*A - C)*cos(d*x + c) + 15*A - C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((49*A + C)*cos(d
*x + c)^2 + 5*(17*A + C)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5
/2), x)
```

3.1249 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{(19A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d}$$

```
[Out] ((19*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Se
c[c + d*x]]) - ((9*A - 7*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2
)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.523008, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3042, 2978, 12, 2782, 205}

$$\frac{(19A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5/2),
x]
```



```
[Out] ((19*A + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Se
c[c + d*x]]) - ((9*A - 7*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2
)*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}}{\sqrt{}}}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-7C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c}}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-7C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c}}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-7C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c}}$$

$$= \frac{(19A+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} -$$

Mathematica [C] time = 1.73779, size = 216, normalized size = 1.24

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left((19A+3C)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}i\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c\right.\right.\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^
(5/2),x]
```

```
[Out] ((I/4)*Cos[(c + d*x)/2]^5*(((19*A + 3*C)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) - (I/4)*(13*A
- 3*C + (9*A - 7*C)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(Si
n[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.177, size = 376, normalized size = 2.2

$$-\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(-9\,A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)*(-1+cos(d*x+c))^2*(-9*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^2+7*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*A*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)*cos(d*x+c)-4*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)+19*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+13*A*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)+3*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-
3*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.5243, size = 554, normalized size = 3.18

$$-\frac{\sqrt{2}\left((19\,A+3\,C)\cos\left(dx+c\right)^{3}+3\,(19\,A+3\,C)\cos\left(dx+c\right)^{2}+3\,(19\,A+3\,C)\cos\left(dx+c\right)+19\,A+3\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)\cdots}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((19*A + 3*C)*cos(d*x + c)^3 + 3*(19*A + 3*C)*cos(d*x + c)^2
 + 3*(19*A + 3*C)*cos(d*x + c) + 19*A + 3*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
```

```
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((9*A - 7*
C)*cos(d*x + c)^2 + (13*A - 3*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5
/2), x)
```

3.1250 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=232

$$\frac{(5A-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A - 43*C)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A - 11*C)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.719257, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3042, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),
x]
```



```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A - 43*C)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A - 11*C)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos}}{}}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(5A-11C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(5A-11C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(5A-11C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(5A-43C)\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [C] time = 2.36042, size = 262, normalized size = 1.13

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}((A-15C)\cos(c+dx)+5A-11C}{8d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d
*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(32*C*ArcSinh[E^(I*(c + d*x))] - Sqrt
[2]*(5*A - 43*C)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))])] - 32*C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c +
d*x)) + ((5*A - 11*C + (A - 15*C)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec
[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(8*d*(a*(1 + Cos
[c + d*x]))^(5/2))
```

Maple [B] time = 0.178, size = 475, normalized size = 2.1

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^3}{32\,da^3(\sin(dx+c))^7}\sqrt{a(1+\cos(dx+c))}\left(A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2-32\,C\sqrt{2}\arctan\left(\frac{\sin}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(A
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-32*C*2^(1/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*sin(d*
x+c)-15*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+5*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*sin(d*x+c)*cos(d*x+c)-32*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+4*C*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)-43*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x
+c)*cos(d*x+c)-5*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*A*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+11*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)-43*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^
(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
 c))), x)
```

Fricas [A] time = 77.7359, size = 756, normalized size = 3.26

$$-\frac{\sqrt{2}\left((5A-43C)\cos(dx+c)^3+3(5A-43C)\cos(dx+c)^2+3(5A-43C)\cos(dx+c)+5A-43C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}}{}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((5*A - 43*C)*cos(d*x + c)^3 + 3*(5*A - 43*C)*cos(d*x + c)^2
 + 3*(5*A - 43*C)*cos(d*x + c) + 5*A - 43*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 64*(C*cos(d*
x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((A - 15*
C)*cos(d*x + c)^2 + (5*A - 11*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
 c))), x)
```

3.1251 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{(3A+35C)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(3A+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sqrt{}}{}$$

```
[Out] (-5*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A + 115*C)*ArcTan[(Sqrt[a]*Sin[
c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x]
)/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((A - 15*C)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((3*A + 35*
C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.934547, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3042, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(3A+35C)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(3A+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),
x]
```


```
[Out] (-5*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A + 115*C)*ArcTan[(Sqrt[a]*Sin[
c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x]
)/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((A - 15*C)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((3*A + 35*
C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos}{\ }}{4}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-15C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-15C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-15C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-15C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{5C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A+115C)\tan^{-1}}{\ }$$

Mathematica [C] time = 2.74513, size = 274, normalized size = 0.99

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sec^4\left(\frac{1}{2}(c+dx)\right)((7A+55C)\cos(c+dx)+3A+}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(
3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(80*C*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]
*(3*A + 115*C)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
```

```
+ d*x))])] - 80*C*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*
x)) + ((3*A + 43*C + (7*A + 55*C)*Cos[c + d*x] + 8*C*Cos[2*(c + d*x)])*Sec[
(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]
))/2))/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [B] time = 0.188, size = 509, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*(
16*C*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*A*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+80*C*2^(1/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)+39*C*co
s(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-4*A*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*s
in(d*x+c)*cos(d*x+c)+80*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-20*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*cos(d*x+c)+115*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*
x+c)-3*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*A*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)-35*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+11
5*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(3/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(
3/2)), x)
```

Fricas [A] time = 78.3238, size = 795, normalized size = 2.87

$$-\frac{\sqrt{2}\Big((3\,A+115\,C)\cos{(dx+c)}^{3}+3\,(3\,A+115\,C)\cos{(dx+c)}^{2}+3\,(3\,A+115\,C)\cos{(dx+c)}+3\,A+115\,C\Big)\sqrt{a}\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A + 115*C)*cos(d*x + c)^3 + 3*(3*A + 115*C)*cos(d*x + c)
^2 + 3*(3*A + 115*C)*cos(d*x + c) + 3*A + 115*C)*sqrt(a)*arctan(sqrt(2)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 160*(C*c
os(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(16*
C*cos(d*x + c)^3 + (7*A + 55*C)*cos(d*x + c)^2 + (3*A + 35*C)*cos(d*x + c))
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^{2}+A}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(
3/2)), x)
```

3.1252 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=334

$$\frac{(7A+31C)\sin(c+dx)}{16a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(8A+39C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(11A+6}{16a^2d\sqrt{\sec(c+}}$$

```
[Out] ((8*A + 39*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(5/2)*d) - ((43*A + 219*C)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*S
in[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)) - ((3*A +
19*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2))
+ ((7*A + 31*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*
x]^(3/2)) - ((11*A + 63*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.15423, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3042, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(7A+31C)\sin(c+dx)}{16a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(8A+39C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(11A+6}{16a^2d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),
x]
```


```
[Out] ((8*A + 39*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(5/2)*d) - ((43*A + 219*C)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*S
in[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)) - ((3*A +
19*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2))
+ ((7*A + 31*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*
x]^(3/2)) - ((11*A + 63*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
```

```
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}}{}}{4a}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A+19C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A+19C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A+19C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A+19C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A+19C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+}$$

$$= \frac{(8A+39C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{5/2}d} - \frac{(43A+219C)}{}$$

Mathematica [C] time = 7.20035, size = 968, normalized size = 2.9

$$\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(-A\sin\left(\frac{dx}{2}\right)-C\sin\left(\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} - \frac{(A+C)\tan\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} + \frac{\sec\left(\frac{c}{2}\right)\left(19A\sin\left(\frac{dx}{2}\right)+35C\sin\left(\frac{dx}{2}\right)\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} + \frac{(19A}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(
5/2)),x]
```

```
[Out] (((-11*I)/4)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
d*x]))^(5/2)) - (((63*I)/4)*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(
a*(1 + Cos[c + d*x]))^(5/2)) + ((4*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x)
)] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(
d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2)) + ((39*I)*C*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSin
h[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2
 + (d*x)/2]^5)/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
 + (Cos[c/2 + (d*x)/2]^5*Sqrt[Sec[c + d*x]]*((-3*(5*A + 3*C)*Cos[(d*x)/2]*S
in[c/2])/(2*d) - (10*C*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d + (C*Cos[(5*d*x)/2]*S
in[(5*c)/2])/d - (3*(5*A + 3*C)*Cos[c/2]*Sin[(d*x)/2])/(2*d) + (Sec[c/2]*Se
c[c/2 + (d*x)/2]^4*(-(A*Sin[(d*x)/2]) - C*Sin[(d*x)/2]))/(2*d) + (Sec[c/2]*
Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] + 35*C*Sin[(d*x)/2]))/(4*d) - (10*C
*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d + (C*Cos[(5*c)/2]*Sin[(5*d*x)/2])/d + ((19*
A + 35*C)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) - ((A + C)*Sec[c/2 + (d*x)/2]^
3*Tan[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

Maple [B] time = 0.211, size = 642, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^5*(
-8*C*cos(d*x+c)^4*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+28*C*cos(d*x+c)
^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*A*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^2+32*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*C*cos(d*x+c)^2*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+156*C*2^(1/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*A*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+32*A*2^(1/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+43*A*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-32*C*2^(1/2)*(cos(d*x+c)/(1+c
```

```
os(d*x+c)))^(1/2)*cos(d*x+c)+156*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+219*C*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*sin(d*x+c)*cos(d*x+c)-11*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+43*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-63*C*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+219*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(
5/2)), x)
```

Fricas [A] time = 132.385, size = 886, normalized size = 2.65

$$\frac{\sqrt{2}\left((43\,A+219\,C)\cos(dx+c)^3+3\,(43\,A+219\,C)\cos(dx+c)^2+3\,(43\,A+219\,C)\cos(dx+c)+43\,A+219\,C\right)\sqrt{a}\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A + 219*C)*cos(d*x + c)^3 + 3*(43*A + 219*C)*cos(d*x + c
)^2 + 3*(43*A + 219*C)*cos(d*x + c) + 43*A + 219*C)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*((8
*A + 39*C)*cos(d*x + c)^3 + 3*(8*A + 39*C)*cos(d*x + c)^2 + 3*(8*A + 39*C)*
cos(d*x + c) + 8*A + 39*C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(8*C*cos(d*x + c)^4 - 20*C*cos(d*x +
 c)^3 - 5*(3*A + 19*C)*cos(d*x + c)^2 - (11*A + 63*C)*cos(d*x + c))*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 +
```

```
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(
5/2)), x)
```

3.1253 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=151

$$\frac{2B\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6B\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+$$

```
[Out] (-6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*C*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.137616, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3010, 2748, 2636, 2639, 2641}

$$\frac{2B\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6B\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*C*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,d$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B+C\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + \left(C\sqrt{\cos(c+\right.$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(\right.$$

$$= \frac{2C\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6B\sqrt{\sec(c+dx}}{5d}$$

$$= -\frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2C\sqrt{\cos(c+d}}{}$$

Mathematica [A] time = 0.318024, size = 97, normalized size = 0.64

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(21B\sin(c+dx)+9B\sin(3(c+dx))-36B\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10C\sin(2(c+dx))+20C\cos^{\frac{5}{2}}(c\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-36*B*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 2
0*C*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*B*Sin[c + d*x] + 10*C
*Sin[2*(c + d*x)] + 9*B*Sin[3*(c + d*x)]))/(30*d)
```

Maple [B] time = 2.656, size = 502, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x
+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(9/2), x)
```

3.1254 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2C\sqrt{\cos(}}{}$$

```
[Out] (-2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*B*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rubi [A] time = 0.122203, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3010, 2748, 2636, 2641, 2639}

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2C\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*B*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B+C\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \left(C\sqrt{\cos(c+d\right. \\
&= \frac{2C\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(B\right. \\
&= -\frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\cos(c+dx}}{}
\end{aligned}$$

Mathematica [A] time = 0.22534, size = 85, normalized size = 0.69

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2\sin(c+dx)(B+3C\cos(c+dx))+2B\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6C\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*C*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*C*Cos[c + d*x])*S
in[c + d*x]))/(3*d)
```

Maple [B] time = 2.335, size = 397, normalized size = 3.2

$$\frac{2}{3d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,B\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*B*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-12*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+6*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

3.1255 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d$$

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.105066, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3010, 2748, 2636, 2639, 2641}

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B+C\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(C\sqrt{\cos(c+d\right. \\
&= \frac{2C\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\sec(c+dx)}\,s}{d} \\
&= -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2C\sqrt{\cos(c+dx}}{}
\end{aligned}$$

Mathematica [A] time = 0.107871, size = 71, normalized size = 0.73

$$\frac{2\sqrt{\sec(c+dx)}\left(B\sin(c+dx)-B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-(B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x]))/d
```

Maple [A] time = 1.177, size = 148, normalized size = 1.5

$$-2\,\frac{B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-2\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2)}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2*(B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

3.1256 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=75

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.0932098, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4221, 3010, 2748, 2641, 2639}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B+C\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+\left(C\sqrt{\cos(c+dx)}\right.\\ &= \frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{\cos(c+dx)}}{}\end{aligned}$$

Mathematica [A] time = 0.0719839, size = 52, normalized size = 0.69

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(BF\left(\frac{1}{2}(c+dx)\middle|2\right)+CE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(C*EllipticE[(c + d*x)/2, 2] + B*EllipticF[(c + d*x)/
2, 2])*Sqrt[Sec[c + d*x]])/d
```

Maple [A] time = 0.797, size = 152, normalized size = 2.

$$-2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\left(B\mathrm{EllipticF}\left(\right.\right.}{\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

3.1257 $\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.105758, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3010, 2748, 2639, 2635, 2641}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(B+C\cos(c+dx))\,d\\ &= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx+\left(C\sqrt{\cos(c+d\right.\\ &= \frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\\ &= \frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2C\sqrt{\cos(c+dx)}}{}\end{aligned}$$

Mathematica [A] time = 0.131662, size = 76, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\left(\sin(2(c+dx))+2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*(
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)
```

Maple [A] time = 0.983, size = 229, normalized size = 2.3

$$\frac{2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-4\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+3\,B\sqrt{\left(\sin\left(1/2\,dx+c/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\cos(c + dx))\cos(c + dx)\sqrt{\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx + c)^2 + B\cos(dx + c)\right)\sqrt{\sec(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

3.1258 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=127

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{5d}$$

```
[Out] (6*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.119419, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3010, 2748, 2635, 2641, 2639}

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (6*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
```

```
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right) d\\
&= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(B+C\cos(c+dx))\,dx\\
&= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\,dx+\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.\\
&= \frac{2C\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}\\
&= \frac{6C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}
\end{aligned}$$

Mathematica [A] time = 0.311485, size = 88, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5B+3C\cos(c+dx))+10B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+18C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(18*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10
*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*B + 3*C*Cos[c + d*x])*
Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 1.984, size = 383, normalized size = 3.

$$-\frac{2B}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*B*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2/5*C*((2*cos(1/2*d
*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

3.1259 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10C\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10C\sqrt{\cos(c+}}{}$$

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*C*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.130235, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3010, 2748, 2635, 2639, 2641}

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10C\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10C\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sec[c + d*x]^(3/2),x]
```


```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*C*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3010

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
```

```
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{5}{2}}(c+dx)(B+C\cos(c+dx))\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{5}{2}}(c+dx)\,dx + \left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(3B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\mathrm{co}}$$

$$= \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10C\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{21d}$$

Mathematica [A] time = 0.533668, size = 99, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42B\cos(c+dx)+15C\cos(2(c+dx))+65C)+252B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100C\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
00*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*C + 42*B*Cos[c + d*
x] + 15*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] time = 1.079, size = 403, normalized size = 2.7

$$-\frac{2B}{5d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+8\,(\sin(1/2\,dx+c/2))^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/5*B*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d-2/21*C*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(48*cos(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2
*d*x+1/2*c)^5-72*cos(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/
2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/sec(d*x + c)^(3/2), x)
```

3.1260 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=163

$$\frac{2(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c}{5d}$$

```
[Out] (-2*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (2*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2
*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d)
```

Rubi [A] time = 0.153171, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4221, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (2*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2
*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + B\cos(c+dx) + C\cos^2(}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)\right.$$

$$= \frac{2(3A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(}{3d}$$

$$= -\frac{2(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + $$

Mathematica [A] time = 1.19919, size = 112, normalized size = 0.69

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(2\sin(c+dx)(3(3A+5C)\cos(2(c+dx))+15(A+C)+10B\cos(c+dx)) - 12(3A+5C)\cos^{\frac{5}{2}}(c+dx)E\left(\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*A + 5*C)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)
/2, 2] + 20*B*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(A + C)
+ 10*B*Cos[c + d*x] + 3*(3*A + 5*C)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 3.406, size = 799, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
```

```
/2*c)^3*(36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

3.1261 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=127

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B}{}$$

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.134662, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4221, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sqrt{}}{}$$

$$= -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(A+3C)}{}$$

Mathematica [A] time = 0.30723, size = 89, normalized size = 0.7

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2\sin(c+dx)(A+3B\cos(c+dx)) + 2(A+3C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6B\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
(A + 3*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(A + 3*B*Cos[c +
 d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 2.295, size = 500, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(2*sin(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

3.1262 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{2(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{d}$$

```
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/d + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.120765, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4221, 3021, 2748, 2641, 2639}

$$-\frac{2(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/d + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$=\frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$=-\frac{2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{}}{}$$

Mathematica [A] time = 0.177428, size = 75, normalized size = 0.74

$$\frac{2\sqrt{\sec(c+dx)}\left(-(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(c+dx)+B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/d
```

Maple [A] time = 1.03, size = 194, normalized size = 1.9

$$-2\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-2\,A\cos(1/2\,dx+c/2)\,(\sin(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

3.1263 $\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*d) + (2*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.119621, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4221, 3023, 2748, 2641, 2639}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*d) + (2*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + B\cos(c+dx) + C\cos^2(c}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(3A}{}$$

$$= \frac{2C\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c}$$

$$= \frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(3A+C)}{}$$

Mathematica [A] time = 0.17789, size = 80, normalized size = 0.76

$$\frac{\sqrt{\sec(c+dx)}\left(2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) + 6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + C\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(
3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])
)/(3*d)
```

Maple [A] time = 1.222, size = 274, normalized size = 2.6

$$-\frac{2}{3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,C\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+3\,A\sqrt{\left(\sin\left(1/2\,dx+c/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + B\cos{\left(c + dx \right)} + C\cos^{2}{\left(c + dx \right)}\right)\sqrt{\sec{\left(c + dx \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A\right)\sqrt{\sec\left(dx + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

3.1264 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=133

$$\frac{2(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

```
[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*B*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.135555, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4221, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*B*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(\right.$$

$$= \frac{2C\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}(5A\right.$$

$$= \frac{2C\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\,dx + \frac{1}{5}\left(\right.$$

$$= \frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2C\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sqrt{\cos(c+dx)}}{}$$

Mathematica [A] time = 0.287865, size = 94, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(6(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))(5B+3C\cos(c+dx))+10B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*B + 3*C*Cos[c
 + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] time = 1.104, size = 308, normalized size = 2.3

$$\frac{2}{15\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,C\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+(-20\,B-24\,C)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

3.1265 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{}}{}$$

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (2*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*B*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.156782, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4221, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (2*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*B*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Sqrt[S
ec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.$$
$$= \frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(7A+\right.$$
$$= \frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{5}{2}}(c+dx)\,dx+\frac{1}{7}\left((7\right.$$
$$= \frac{2C\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{1}{5}\left(3B\sqrt{\cos\right.$$
$$= \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}F}{2}$$

Mathematica [A] time = 0.651872, size = 108, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(70A+42B\cos(c+dx)+15C\cos(2(c+dx))+65C)+20(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2
0*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (70*A + 65*C +
 42*B*Cos[c + d*x] + 15*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 1.376, size = 342, normalized size = 2.1

$$-\frac{2}{105\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,C\,(\sin(1/2\,dx+c/2))^8\cos(1/2\,dx+c/2)+(-168\,B-360\,C)\right(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos{(c + dx)} + C\cos^{2}{(c + dx)}}{\sec^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A}{\sec\left(dx + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

3.1266 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=217

$$\frac{2a(5A+7(B+C))\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(3A+3B+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5A+7(B+C))\sqrt{\cos(}}{}$$

```
[Out] (-2*a*(3*A + 3*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(5*A + 7*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(3*A + 3*B + 5*C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*A + 7*(B + C))*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(21*d) + (2*a*(A + B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
+ (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.351826, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(5A+7(B+C))\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(3A+3B+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5A+7(B+C))\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(9/2),x]
```


```
[Out] (-2*a*(3*A + 3*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(5*A + 7*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(3*A + 3*B + 5*C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*A + 7*(B + C))*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(21*d) + (2*a*(A + B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
+ (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos}{}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} - \frac{1}{7}\left(2\sqrt{\cos}\right.$$

$$= \frac{2a(A+B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a}{}$$

$$= \frac{2a(A+B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a}{}$$

$$= \frac{2a(3A+3B+5C)\sqrt{\sec(c+dx)}\sin(c+d}{5d}$$

$$= -\frac{2a(3A+3B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 2.21569, size = 172, normalized size = 0.79

$$\frac{a\sec^{\frac{7}{2}}(c+dx)\left(40(5A+7(B+C))\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-168(3A+3B+5C)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(9/2),x]
```

```
[Out] (a*Sec[c + d*x]^(7/2)*(-168*(3*A + 3*B + 5*C)*Cos[c + d*x]^(7/2)*EllipticE[
(c + d*x)/2, 2] + 40*(5*A + 7*(B + C))*Cos[c + d*x]^(7/2)*EllipticF[(c + d*
x)/2, 2] + 2*(110*A + 70*B + 70*C + 21*(13*A + 13*B + 15*C)*Cos[c + d*x] +
10*(5*A + 7*(B + C))*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)] + 63*B*Cos[3*
(c + d*x)] + 105*C*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] time = 4.046, size = 849, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(-1/5
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2
*C+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*C*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/5*(1/2*A+1/2*B)/
(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(9/2), x)
```

3.1267 $\int (a+a\cos(c+dx)) \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \text{s}$ $dx)\,dx$

Optimal. Leaf size=179

$$\frac{2a(3A+5(B+C))\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5(B}{}$$

```
[Out] (-2*a*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*(B + C))*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3*d) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.315862, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+5(B+C))\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5(B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(7/2),x]
```


```
[Out] (-2*a*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*(B + C))*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3*d) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(\ldots}{\ldots}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\left(2\sqrt{\cos(\ldots}\right.$$

$$= \frac{2a(A+B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA\ldots}{\ldots}$$

$$= \frac{2a(A+B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA\ldots}{\ldots}$$

$$= \frac{2a(A+B+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\ldots\right.}{3d}$$

$$= -\frac{2a(3A+5(B+C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\ldots\right.}{5d}$$

Mathematica [A] time = 0.980072, size = 147, normalized size = 0.82

$$\frac{a\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(5(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3(3A+5(B+C))E\left(\frac{1}{2}(c\ldots\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*
x]]*(-3*(3*A + 5*(B + C))*EllipticE[(c + d*x)/2, 2] + 5*(A + B + 3*C)*Ellip
ticF[(c + d*x)/2, 2] + ((15*(A + B + C) + 10*(A + B)*Cos[c + d*x] + 3*(3*A
+ 5*(B + C))*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d
)
```

Maple [B] time = 3.262, size = 739, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*C+1/2*B)*(-(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/10*A/
(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+(B+C)a\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(7/2), x)
```

3.1268 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=140

$$\frac{2a(A+3(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

```
[Out] (-2*a*(A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d + (2*a*(A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.312637, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4221, 3031, 3021, 2748, 2641, 2639}

$$\frac{2a(A+3(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(5/2),x]
```

```
[Out] (-2*a*(A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d + (2*a*(A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos}{}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\left(2\sqrt{\cos}\right.$$

$$= \frac{2a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a}{}$$

$$= \frac{2a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a}{}$$

$$= -\frac{2a(A+B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d}$$

Mathematica [A] time = 0.690126, size = 99, normalized size = 0.71

$$\frac{a\sec^{\frac{3}{2}}(c+dx)\left(2(A+3(B+C))\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6(A+B-C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(5/2),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(-6*(A + B - C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x
)/2, 2] + 2*(A + 3*(B + C))*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
2*(A + 3*(A + B)*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 2.748, size = 515, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
```

```
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*A*(-1/6*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x
+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca\cos\left(dx+c\right)^3 + (B+C)a\cos\left(dx+c\right)^2 + (A+B)a\cos\left(dx+c\right) + Aa\right)\sec\left(dx+c\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(5/2), x)
```

3.1269 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{2a(3A+3B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*a*(A - B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d + (2*a*(3*A + 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]
) + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.262105, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4221, 3031, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+3B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(3/2),x]
```


```
[Out] (-2*a*(A - B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d + (2*a*(3*A + 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]
) + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(}{}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(2\sqrt{\cos(c\right.$$

$$= \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$= \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$= -\frac{2a(A-B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{d}$$

Mathematica [A] time = 0.459731, size = 101, normalized size = 0.72

$$\frac{a\sqrt{\sec(c+dx)}\left(2(3A+3B+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-6(A-B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(-6*(A - B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2] + 2*(3*A + 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
2*(3*A + C*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 1.242, size = 380, normalized size = 2.7

$$-\frac{2a}{3d}\left(4C(\sin(1/2dx+c/2))^4\cos(1/2dx+c/2)+3A\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*a*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}+\left(B+C\right)a\cos\left(dx+c\right)^{2}+\left(A+B\right)a\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

3.1270 $\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=147

$$\frac{2a(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{5d}$$

```
[Out] (2*a*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)) + (2*a*(B + C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.269949, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4221, 3033, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]],x]
```

```
[Out] (2*a*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)) + (2*a*(B + C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
```

```
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{co}}{}$$

$$= \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.$$

$$= \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a(5A+5B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 0.625854, size = 105, normalized size = 0.71

$$\frac{a\sqrt{\sec(c+dx)}\left(10(3A+B+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(5A+5B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2] + 10*(3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
] + (5*(B + C) + 3*C*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 1.217, size = 447, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(20*B+44*C)*sin(1/2*d*x+1/2*c)^4*cos(1/
```

```
2*d*x+1/2*c)+(-10*B-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca\cos(dx+c)^3 + (B+C)a\cos(dx+c)^2 + (A+B)a\cos(dx+c) + Aa\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

3.1271 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=184

$$\frac{2a(7A+7B+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+7B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3(B+C))\sqrt{\cos}}{}$$

```
[Out] (2*a*(5*A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(7*A + 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c
+ d*x]^(5/2)) + (2*a*(B + C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*
(7*A + 7*B + 5*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.297967, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(7A+7B+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+7B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+3(B+C))\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Sec
[c + d*x]],x]
```



```
[Out] (2*a*(5*A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(7*A + 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c
+ d*x]^(5/2)) + (2*a*(B + C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*
(7*A + 7*B + 5*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a + a\cos(c + dx))\left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)}{\sqrt{\sec(c + dx)}}\,dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \sqrt{\cos(c + dx)}(a + a\cos$$

$$= \frac{2aC\sin(c + dx)}{7d\sec^{\frac{5}{2}}(c + dx)} + \frac{1}{7}\left(2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int$$

$$= \frac{2aC\sin(c + dx)}{7d\sec^{\frac{5}{2}}(c + dx)} + \frac{2a(B + C)\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{1}{35}\left(4\sqrt{\cos}\right.$$

$$= \frac{2aC\sin(c + dx)}{7d\sec^{\frac{5}{2}}(c + dx)} + \frac{2a(B + C)\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{1}{7}\left(a(7A + \right.$$

$$= \frac{2a(5A + 3(B + C))\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

$$= \frac{2a(5A + 3(B + C))\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

Mathematica [A] time = 0.963751, size = 125, normalized size = 0.68

$$\frac{a\sqrt{\sec(c + dx)}\left(\sin(2(c + dx))(70A + 42(B + C)\cos(c + dx) + 70B + 15C\cos(2(c + dx)) + 65C) + 20(7A + 7B + 5C)\sqrt{c}\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(84*(5*A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 20*(7*A + 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2] + (70*A + 70*B + 65*C + 42*(B + C)*Cos[c + d*x] + 15*C*Cos[2*(c + d*
x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] time = 0.941, size = 481, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*C*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-528*C)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(140*A+308*B+448*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-70*A-112*B-122*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-105*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+(B+C)a\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{A\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(1/
2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)/sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integral(C*cos(c + d*x)**2/sqrt(sec(
c + d*x)), x) + Integral(C*cos(c + d*x)**3/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c) + a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

3.1272 $\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=217

$$\frac{2a(9A+9B+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5(B+C))\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{21d}$$

```
[Out] (2*a*(9*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*a*(7*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(9*d*Sec[c
 + d*x]^(7/2)) + (2*a*(B + C)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a
*(9*A + 9*B + 7*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*
(B + C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.345041, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(9A+9B+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5(B+C))\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[c +
d*x]^(3/2),x]
```

```
[Out] (2*a*(9*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*a*(7*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*C*Sin[c + d*x])/(9*d*Sec[c
 + d*x]^(7/2)) + (2*a*(B + C)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a
*(9*A + 9*B + 7*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*
(B + C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\,\text{co}$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{63}\left(4\sqrt{\text{co}}\right.$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{9}\left(a(9A\right.$$

$$= \frac{2aC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(9A+}{45}$$

$$= \frac{2a(9A+9B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{sec}}}{15d}$$

Mathematica [A] time = 0.860774, size = 149, normalized size = 0.69

$$\frac{a\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))(7(36A+36B+43C)\cos(c+dx)+5(84A+18(B+C)\cos(2(c+dx))+78B+7C\cos\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(336*(9*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 240*(7*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + 2*(7*(36*A + 36*B + 43*C)*Cos[c + d*x] + 5*(84*A + 78*B + 78*C +
 18*(B + C)*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(2
520*d)
```

Maple [B] time = 1.378, size = 512, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*C*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(720*B+2960*C)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-504*A-1584*B-3152*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(924*A+1344*B+1792*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-336
*A-366*B-408*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18
9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\cos\left(dx+c\right)^{3}+(B+C)a\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x
 + c) + A*a)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

3.1273 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=291

$$\frac{2a^2(19A+27B+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(5A+6B+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{4a^2(8A+9B+12C)\,\mathrm{s}}{1}$$

```
[Out] (-4*a^2*(8*A + 9*B + 12*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (4*a^2*(5*A + 6*B + 7*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(8*A + 9*B + 12*C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^2*(5*A + 6*B + 7*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(19*A + 27*B + 21*C)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(105*d) + (2*(4*A + 9*B)*(a^2 + a^2*Cos[c + d*x])*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d
*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.651897, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3043, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a^2(19A+27B+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(5A+6B+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{4a^2(8A+9B+12C)\,\mathrm{s}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(11/2),x]
```



```
[Out] (-4*a^2*(8*A + 9*B + 12*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (4*a^2*(5*A + 6*B + 7*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(8*A + 9*B + 12*C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^2*(5*A + 6*B + 7*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(19*A + 27*B + 21*C)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(105*d) + (2*(4*A + 9*B)*(a^2 + a^2*Cos[c + d*x])*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d
*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos}{}$$

$$= \frac{2A(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\sin}{9d}$$

$$= \frac{2(4A+9B)\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{7}{2}}(}{63d}$$

$$= \frac{2(4A+9B)\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{7}{2}}(}{63d}$$

$$= \frac{2a^2(19A+27B+21C)\sec^{\frac{5}{2}}(c+dx)\sin}{105d}$$

$$= \frac{2a^2(19A+27B+21C)\sec^{\frac{5}{2}}(c+dx)\sin}{105d}$$

$$= \frac{4a^2(8A+9B+12C)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$= -\frac{4a^2(8A+9B+12C)\sqrt{\cos(c+dx)}E\left(\right.}{15d}$$

Mathematica [A] time = 2.29949, size = 209, normalized size = 0.72

$$\frac{a^2\sec^{\frac{9}{2}}(c+dx)\left(240(5A+6B+7C)\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-336(8A+9B+12C)\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sec[c + d*x]^(9/2)*(-336*(8*A + 9*B + 12*C)*Cos[c + d*x]^(9/2)*Ellipti
cE[(c + d*x)/2, 2] + 240*(5*A + 6*B + 7*C)*Cos[c + d*x]^(9/2)*EllipticF[(c
+ d*x)/2, 2] + 2*(868*A + 819*B + 882*C + 90*(9*A + 8*B + 7*C)*Cos[c + d*x]
 + 14*(64*A + 72*B + 81*C)*Cos[2*(c + d*x)] + 150*A*Cos[3*(c + d*x)] + 180*
```

```
B*Cos[3*(c + d*x)] + 210*C*Cos[3*(c + d*x)] + 168*A*Cos[4*(c + d*x)] + 189*
B*Cos[4*(c + d*x)] + 252*C*Cos[4*(c + d*x)])*Sin[c + d*x]))/(1260*d)
```

Maple [B] time = 5.187, size = 1181, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*((1/4*B+1/
2*A)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))+1/4*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)
^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))))+(1/2*C+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1
/4*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)-1/5*(1/2*B+1/4*C+1/4*A)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4
+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
```

```
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^{2}\cos\left(dx+c\right)^{4}+(B+2\,C)a^{2}\cos\left(dx+c\right)^{3}+(A+2\,B+C)a^{2}\cos\left(dx+c\right)^{2}+(2\,A+B)a^{2}\cos\left(dx+c\right)+A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(11/2), x)
```

3.1274 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=255

$$\frac{2a^2(33A+49B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(3A+4B+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(6A+7B+14C)}{}$$

```
[Out] (-4*a^2*(3*A + 4*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^2*(6*A + 7*B + 14*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(3*A + 4*B + 5*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^2*(33*A + 49*B + 35*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*(4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.630896, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a^2(33A+49B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(3A+4B+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(6A+7B+14C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(9/2),x]
```


```
[Out] (-4*a^2*(3*A + 4*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^2*(6*A + 7*B + 14*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(3*A + 4*B + 5*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^2*(33*A + 49*B + 35*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*(4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos$$

$$= \frac{2A(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$= \frac{2(4A+7B)\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{5}{2}}(c+}{35d}$$

$$= \frac{2(4A+7B)\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{5}{2}}(c+}{35d}$$

$$= \frac{2a^2(33A+49B+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c}{105d}$$

$$= \frac{2a^2(33A+49B+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c}{105d}$$

$$= \frac{4a^2(6A+7B+14C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c-\right.}{21d}$$

$$= -\frac{4a^2(3A+4B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 2.56921, size = 177, normalized size = 0.69

$$\frac{a^2\sec^{\frac{7}{2}}(c+dx)\left(40(6A+7(B+2C))\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-168(3A+4B+5C)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sec[c + d*x]^(7/2)*(-168*(3*A + 4*B + 5*C)*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2] + 40*(6*A + 7*(B + 2*C))*Cos[c + d*x]^(7/2)*EllipticF[(c
+ d*x)/2, 2] + 2*(90*A + 70*B + 35*C + 21*(13*A + 14*B + 15*C)*Cos[c + d*x]
 + 5*(12*A + 14*B + 7*C)*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)] + 84*B*Co
```

```
s[3*(c + d*x)] + 105*C*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

Maple [B] time = 5.027, size = 932, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/4*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/
21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+(1/2*B+1/4*C+1/4*A)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*
C+1/4*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*
c)^2-1)-1/5*(1/4*B+1/2*A)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^2 \sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4+(B+2\,C)a^2\cos\left(dx+c\right)^3+(A+2\,B+C)a^2\cos\left(dx+c\right)^2+(2\,A+B)a^2\cos\left(dx+c\right)+Aa\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(9
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(9/2), x)
```

3.1275 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=214

$$\frac{2a^2(17A+25B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^2(A+2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2(4A}{}$$

```
[Out] (-4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(A + 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(17*A + 25*B + 15*C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(4*A + 5*B)*(a^2 + a^2*Cos[c + d*x])*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.608519, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{2a^2(17A+25B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^2(A+2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2(4A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(7/2),x]
```


```
[Out] (-4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(A + 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(17*A + 25*B + 15*C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(4*A + 5*B)*(a^2 + a^2*Cos[c + d*x])*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos}{}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$=\frac{2(4A+5B)\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{3}{2}}(c+}{15d}$$

$$=\frac{2(4A+5B)\left(a^2+a^2\cos(c+dx)\right)\sec^{\frac{3}{2}}(c+}{15d}$$

$$=\frac{2a^2(17A+25B+15C)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$=\frac{2a^2(17A+25B+15C)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$=-\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{5d}$$

Mathematica [A] time = 1.32794, size = 135, normalized size = 0.63

$$\frac{a^2\sec^{\frac{5}{2}}(c+dx)\left(40(A+2B+3C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)(3(8A+5(2B+C))\cos(2(c+dx))+10(2}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sec[c + d*x]^(5/2)*(-24*(4*A + 5*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 40*(A + 2*B + 3*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2
] + 2*(15*(2*A + 2*B + C) + 10*(2*A + B)*Cos[c + d*x] + 3*(8*A + 5*(2*B + C
))*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 3.902, size = 906, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(1/4*B+1/2*A)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*B+1/4*C+1/4*A)*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/20*A/(8*sin(
1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*
d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-
8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^{2}\cos\left(dx+c\right)^{4}+(B+2\,C)a^{2}\cos\left(dx+c\right)^{3}+(A+2\,B+C)a^{2}\cos\left(dx+c\right)^{2}+(2\,A+B)a^{2}\cos\left(dx+c\right)+Aa\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(7
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(7/2), x)
```

3.1276 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=212

$$-\frac{2a^2(5A+3B-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+3B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(4A+3B)\sin(c+d}{}$$

```
[Out] (-4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a^2*(2*A + 3*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(5*A + 3*B - C)*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]]) + (2*(4*A + 3*B)*(a^2 + a^2*Cos[c + d*x])*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.5886, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$-\frac{2a^2(5A+3B-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+3B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(4A+3B)\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(5/2),x]
```


```
[Out] (-4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a^2*(2*A + 3*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(5*A + 3*B - C)*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]]) + (2*(4*A + 3*B)*(a^2 + a^2*Cos[c + d*x])*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos}{}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$=\frac{2(4A+3B)\left(a^2+a^2\cos(c+dx)\right)\sqrt{\sec(c}}{3d}$$

$$=\frac{2(4A+3B)\left(a^2+a^2\cos(c+dx)\right)\sqrt{\sec(c}}{3d}$$

$$=-\frac{2a^2(5A+3B-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(4A+}{}$$

$$=-\frac{2a^2(5A+3B-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(4A+}{}$$

$$=-\frac{4a^2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A] time = 1.4375, size = 118, normalized size = 0.56

$$\frac{a^2\sec^{\frac{3}{2}}(c+dx)\left(8(2A+3B+2C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)(6(2A+B)\cos(c+dx)+2A+C\cos(2(c}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(5/2),x]
```

```
[Out] (a^2*Sec[c + d*x]^(3/2)*(-24*(A - C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)
/2, 2] + 8*(2*A + 3*B + 2*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
 2*(2*A + C + 6*(2*A + B)*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sin[c + d*x]))
/(6*d)
```

Maple [B] time = 3.35, size = 800, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+4*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*C*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
sin(1/2*d*x+1/2*c)^2-6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-4*C*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+7*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+3*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^4 + (B+2C)a^2\cos(dx+c)^3 + (A+2B+C)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(5
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(5/2), x)
```

3.1277 $\int (a+a\cos(c+dx))^2 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ ... $dx)\,dx$

Optimal. Leaf size=212

$$-\frac{2a^2(15A-5B-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{4a^2(3A+2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2(5A-C)\sin(c+d}{5d\sqrt{se}}$$

```
[Out] (4*a^2*(5*B + 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(3*A + 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(15*A - 5*B - 7*C)*Sin[c + d*x
])/(15*d*Sqrt[Sec[c + d*x]]) - (2*(5*A - C)*(a^2 + a^2*Cos[c + d*x])*Sin[c
+ d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.58893, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$-\frac{2a^2(15A-5B-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{4a^2(3A+2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2(5A-C)\sin(c+d}{5d\sqrt{se}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(3/2),x]
```

```
[Out] (4*a^2*(5*B + 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(3*A + 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(15*A - 5*B - 7*C)*Sin[c + d*x
])/(15*d*Sqrt[Sec[c + d*x]]) - (2*(5*A - C)*(a^2 + a^2*Cos[c + d*x])*Sin[c
+ d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos}{}$$

$$=\frac{2A(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$=-\frac{2(5A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{5d\sqrt{\sec(c+dx)}}$$

$$=-\frac{2(5A-C)\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{5d\sqrt{\sec(c+dx)}}$$

$$=-\frac{2a^2(15A-5B-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(5A}{}$$

$$=-\frac{2a^2(15A-5B-7C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2(5A}{}$$

$$=\frac{4a^2(5B+4C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{5d}$$

Mathematica [A] time = 0.700849, size = 121, normalized size = 0.57

$$\frac{a^2\sqrt{\sec(c+dx)}\left(2\sin(c+dx)(3(10A+C\cos(2(c+dx))+C)+10(B+2C)\cos(c+dx))+40(3A+2B+C)\sqrt{\cos(c+dx}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(24*(5*B + 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 40*(3*A + 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + 2*(10*(B + 2*C)*Cos[c + d*x] + 3*(10*A + C + C*Cos[2*(c + d*x)]))*Sin[c
+ d*x]))/(30*d)
```

Maple [B] time = 1.412, size = 595, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^2*(-12*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*B+16*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A+5*B+13*C)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+10*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^2 \sec\left(dx+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4+(B+2\,C)a^2\cos\left(dx+c\right)^3+(A+2\,B+C)a^2\cos\left(dx+c\right)^2+(2\,A+B)a^2\cos\left(dx+c\right)+Aa\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*se
c(d*x + c)^(3/2), x)
```

3.1278 $\int (a+a\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=219

$$\frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^2(14A+7B+6C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+4B+}{}$$

```
[Out] (4*a^2*(5*A + 4*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^2*(14*A + 7*B + 6*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(35*A + 49*B + 33*C)
*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin
[c + d*x])/(7*d*Sqrt[Sec[c + d*x]]) + (2*(7*B + 4*C)*(a^2 + a^2*Cos[c + d*x
])*Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.578959, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^2(14A+7B+6C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+4B+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec
[c + d*x]],x]
```


```
[Out] (4*a^2*(5*A + 4*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^2*(14*A + 7*B + 6*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(35*A + 49*B + 33*C)
*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin
[c + d*x])/(7*d*Sqrt[Sec[c + d*x]]) + (2*(7*B + 4*C)*(a^2 + a^2*Cos[c + d*x
])*Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

<u>Rule 2976</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

<u>Rule 2968</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 3023</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

<u>Rule 2748</u>

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos}{}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{(2\sqrt{}}{}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{2(7}{}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{2(7}{}$$

$$=\frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2C}{}$$

$$=\frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2C}{}$$

$$=\frac{4a^2(5A+4B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 0.991157, size = 133, normalized size = 0.61

$$\frac{a^2\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5(14A+28B+3C\cos(2(c+dx))+27C)+42(B+2C)\cos(c+dx))+40(14A+7B+6C\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(168*(5*A + 4*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 40*(14*A + 7*B + 6*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2] + (42*(B + 2*C)*Cos[c + d*x] + 5*(14*A + 28*B + 27*C + 3*C*Cos[2
*(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 1.319, size = 483, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-84*B-348*C)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(70*A+224*B+378*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-35*A-91*B-117*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-84*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{2}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^2\cos(dx+c)^4 + (B+2C)a^2\cos(dx+c)^3 + (A+2B+C)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)*sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sq
rt(sec(d*x + c)), x)
```

3.1279 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=255

$$\frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{21d}$$

```
[Out] (4*a^2*(12*A + 9*B + 8*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(15*d) + (4*a^2*(7*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(21*A + 27*B + 19*C
)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(9*d*Sec[c + d*x]^(3/2)) + (2*(9*B + 4*C)*(a^2 + a^2*Cos[c + d*
x])*Sin[c + d*x])/(63*d*Sec[c + d*x]^(3/2)) + (4*a^2*(7*A + 6*B + 5*C)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.611305, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (4*a^2*(12*A + 9*B + 8*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(15*d) + (4*a^2*(7*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(21*A + 27*B + 19*C
)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(9*d*Sec[c + d*x]^(3/2)) + (2*(9*B + 4*C)*(a^2 + a^2*Cos[c + d*
x])*Sin[c + d*x])/(63*d*Sec[c + d*x]^(3/2)) + (4*a^2*(7*A + 6*B + 5*C)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\,\mathrm{co}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(9B+4C)\left(a^2\right.}{63}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(9B+4C)\left(a^2\right.}{63}$$

$$= \frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{9d\sec}$$

$$= \frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{9d\sec}$$

$$= \frac{4a^2(12A+9B+8C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{15d}$$

$$= \frac{4a^2(12A+9B+8C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{15d}$$

Mathematica [A] time = 0.936758, size = 151, normalized size = 0.59

$$\frac{a^2\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))(7(36A+72B+79C)\cos(c+dx)+840A+90(B+2C)\cos(2(c+dx))+810B+35C\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(672*(12*A + 9*B + 8*C)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2] + 480*(7*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2] + 2*(840*A + 810*B + 780*C + 7*(36*A + 72*B + 79*C)*Cos[c + d*x
] + 90*(B + 2*C)*Cos[2*(c + d*x)] + 35*C*Cos[3*(c + d*x)])*Sin[2*(c + d*x)]
```

```
))/(2520*d)
```

Maple [A] time = 1.345, size = 514, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(360*B+1840*C)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-252*A-1044*B-2368*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(672*A+1134*B+1568*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-27
3*A-351*B-387*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-252*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
89*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+75*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos(dx+c)^4+(B+2C)a^2\cos(dx+c)^3+(A+2B+C)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sq
rt(sec(d*x + c)), x)
```

3.1280 $\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=291

$$\frac{4a^2(9A+8B+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(99A+121B+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^2(66A+55B+50C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{4a^2(66A$$

```
[Out] (4*a^2*(9*A + 8*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(15*d) + (4*a^2*(66*A + 55*B + 50*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(99*A + 121*B +
89*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])^
2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(5/2)) + (2*(11*B + 4*C)*(a^2 + a^2*Cos[
c + d*x])*Sin[c + d*x])/(99*d*Sec[c + d*x]^(5/2)) + (4*a^2*(9*A + 8*B + 7*C
)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (4*a^2*(66*A + 55*B + 50*C)*Sin
[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.65037, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3045, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(9A+8B+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(99A+121B+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^2(66A+55B+50C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{4a^2(66A$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[c
 + d*x]^(3/2),x]
```


```
[Out] (4*a^2*(9*A + 8*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(15*d) + (4*a^2*(66*A + 55*B + 50*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(99*A + 121*B +
89*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])^
2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(5/2)) + (2*(11*B + 4*C)*(a^2 + a^2*Cos[
c + d*x])*Sin[c + d*x])/(99*d*Sec[c + d*x]^(5/2)) + (4*a^2*(9*A + 8*B + 7*C
)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (4*a^2*(66*A + 55*B + 50*C)*Sin
[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(11B+4C)\left(a^2\right.}{99}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(11B+4C)\left(a^2\right.}{99}$$

$$= \frac{2a^2(99A+121B+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{11d\,\mathrm{se}}$$

$$= \frac{2a^2(99A+121B+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{11d\,\mathrm{se}}$$

$$= \frac{2a^2(99A+121B+89C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{11d\,\mathrm{se}}$$

$$= \frac{4a^2(9A+8B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{sec}}}{15d}$$

Mathematica [A] time = 1.39152, size = 174, normalized size = 0.6

$$\frac{a^2\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))(154(72A+79B+86C)\cos(c+dx)+5(36(11A+22B+27C)\cos(2(c+dx))+3564A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(14784*(9*A + 8*B + 7*C)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2] + 960*(66*A + 55*B + 50*C)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2] + 2*(154*(72*A + 79*B + 86*C)*Cos[c + d*x] + 5*(3564*A + 34
32*B + 3309*C + 36*(11*A + 22*B + 27*C)*Cos[2*(c + d*x)] + 154*(B + 2*C)*Co
```

```
s[3*(c + d*x)] + 63*C*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(55440*d)
```

Maple [A] time = 1.187, size = 545, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*B-37520*C)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(3960*A+20240*B+57040*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-11484*A-26048*B-46192*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(12474*A+17248*B+22022*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-3861*A-4257*B-4563*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2079*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+990*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1848*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+825*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+750
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\cos(dx+c)^4 + (B+2C)a^2\cos(dx+c)^3 + (A+2B+C)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\sec(dx+c)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^3 + (A + 2*B +
C)*a^2*cos(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c) + A*a^2)/sec(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/se
c(d*x + c)^(3/2), x)
```

3.1281 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=343

$$\frac{4a^3(210A+253B+264C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d}+\frac{4a^3(105A+121B+143C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{231d}+\frac{4a^3(15A-}{}$$

```
[Out] (-4*a^3*(15*A + 17*B + 21*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B + 143*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(15*A + 1
7*B + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(105*A + 121*B
 + 143*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(231*d) + (4*a^3*(210*A + 253*B
+ 264*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(1155*d) + (2*(105*A + 143*B + 99
*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d) + (2*
(6*A + 11*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9
9*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*
d)
```

Rubi [A] time = 0.843238, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3043, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^3(210A+253B+264C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d}+\frac{4a^3(105A+121B+143C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{231d}+\frac{4a^3(15A-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(13/2),x]
```


```
[Out] (-4*a^3*(15*A + 17*B + 21*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B + 143*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(15*A + 1
7*B + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(105*A + 121*B
 + 143*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(231*d) + (4*a^3*(210*A + 253*B
+ 264*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(1155*d) + (2*(105*A + 143*B + 99
*C)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d) + (2*
(6*A + 11*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9
9*a*d) + (2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*
```

```
d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\text{co}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{11}{2}}(c+dx)\sin(}{11d}$$

$$= \frac{2(6A+11B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{9}{2}}(}{99ad}$$

$$= \frac{2(105A+143B+99C)\left(a^3+a^3\cos(c+}{693d}$$

$$= \frac{2(105A+143B+99C)\left(a^3+a^3\cos(c+}{693d}$$

$$= \frac{4a^3(210A+253B+264C)\sec^{\frac{5}{2}}(c+dx)\,\text{s}}{1155d}$$

$$= \frac{4a^3(210A+253B+264C)\sec^{\frac{5}{2}}(c+dx)\,\text{s}}{1155d}$$

$$= \frac{4a^3(15A+17B+21C)\sqrt{\sec(c+dx)}\sin(}{15d}$$

$$= -\frac{4a^3(15A+17B+21C)\sqrt{\cos(c+dx)}E\Big(}{15d}$$

Mathematica [A] time = 3.14525, size = 242, normalized size = 0.71

$$\frac{a^3\sec^{\frac{11}{2}}(c+dx)\left(480(105A+121B+143C)\cos^{\frac{11}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-7392(15A+17B+21C)\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(13/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(11/2)*(-7392*(15*A + 17*B + 21*C)*Cos[c + d*x]^(11/2)*El
lipticE[(c + d*x)/2, 2] + 480*(105*A + 121*B + 143*C)*Cos[c + d*x]^(11/2)*E
llipticF[(c + d*x)/2, 2] + 2*(19530*A + 16830*B + 14850*C + 154*(375*A + 37
7*B + 396*C)*Cos[c + d*x] + 60*(336*A + 341*B + 319*C)*Cos[2*(c + d*x)] + 2
1945*A*Cos[3*(c + d*x)] + 24871*B*Cos[3*(c + d*x)] + 28413*C*Cos[3*(c + d*x
)] + 3150*A*Cos[4*(c + d*x)] + 3630*B*Cos[4*(c + d*x)] + 4290*C*Cos[4*(c +
d*x)] + 3465*A*Cos[5*(c + d*x)] + 3927*B*Cos[5*(c + d*x)] + 4851*C*Cos[5*(c
 + d*x)])*Sin[c + d*x]))/(27720*d)
```

Maple [B] time = 5.989, size = 1424, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*((3/8*A+3
/8*B+1/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+(1/8*B+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A
+1/8*B)*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3
-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))+1/8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1
/2*d*x+1/2*c)^2-1)-1/5*(1/8*A+3/8*B+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1
/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Elliptic
```

```
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*
A*(-1/352*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/15
4*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{5}+\left(B+3\,C\right)a^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,B+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+3\,B+C\right)a^{3}\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
```

```
3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(13/2), x)
```

3.1282 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx) \, dx$

Optimal. Leaf size=307

$$\frac{4a^3(32A+41B+42C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^3(17A+21B+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(73A+99B+63}{}$$

```
[Out] (-4*a^3*(17*A + 21*B + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B + 21*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(17*A + 21*B
+ 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(32*A + 41*B + 42*
C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*(73*A + 99*B + 63*C)*(a^3
+ a^3*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(2*A + 3*
B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*a*d) + (
2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.815777, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(32A+41B+42C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^3(17A+21B+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(73A+99B+63}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(11/2),x]
```


```
[Out] (-4*a^3*(17*A + 21*B + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B + 21*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(17*A + 21*B
+ 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(32*A + 41*B + 42*
C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*(73*A + 99*B + 63*C)*(a^3
+ a^3*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(2*A + 3*
B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*a*d) + (
2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\sin}{9d}$$

$$= \frac{2(2A+3B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{7}{2}}}{21ad}$$

$$= \frac{2(73A+99B+63C)\left(a^3+a^3\cos(c+d\right.}{315d}$$

$$= \frac{2(73A+99B+63C)\left(a^3+a^3\cos(c+d\right.}{315d}$$

$$= \frac{4a^3(32A+41B+42C)\sec^{\frac{3}{2}}(c+dx)\sin}{105d}$$

$$= \frac{4a^3(32A+41B+42C)\sec^{\frac{3}{2}}(c+dx)\sin}{105d}$$

$$= \frac{4a^3(11A+13B+21C)\sqrt{\cos(c+dx)}F\Big(}{21d}$$

$$= -\frac{4a^3(17A+21B+27C)\sqrt{\cos(c+dx)}E}{15d}$$

Mathematica [A] time = 2.34444, size = 209, normalized size = 0.68

$$\frac{a^3\sec^{\frac{9}{2}}(c+dx)\left(240(11A+13B+21C)\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-336(17A+21B+27C)\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(11/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(9/2)*(-336*(17*A + 21*B + 27*C)*Cos[c + d*x]^(9/2)*Ellip
ticE[(c + d*x)/2, 2] + 240*(11*A + 13*B + 21*C)*Cos[c + d*x]^(9/2)*Elliptic
```

```
F[(c + d*x)/2, 2] + 2*(1687*A + 1701*B + 1827*C + 45*(34*A + 30*B + 21*C)*C
os[c + d*x] + 14*(136*A + 153*B + 171*C)*Cos[2*(c + d*x)] + 330*A*Cos[3*(c
+ d*x)] + 390*B*Cos[3*(c + d*x)] + 315*C*Cos[3*(c + d*x)] + 357*A*Cos[4*(c
+ d*x)] + 441*B*Cos[4*(c + d*x)] + 567*C*Cos[4*(c + d*x)])*Sin[c + d*x]))/(
1260*d)
```

Maple [B] time = 6.28, size = 1262, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+(3/8*A+1/8*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+1/8*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*
d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2
*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))))+(1/8*A+3/8*B+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+(1/8*B+3/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/5*(3/8*A+3/8*B+1/8*C)/(8*sin(1/2*d*x+1/2*c)^
6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
```

```
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^3 \sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^3\cos\left(dx+c\right)^5 + (B+3\,C)a^3\cos\left(dx+c\right)^4 + (A+3\,B+3\,C)a^3\cos\left(dx+c\right)^3 + (3\,A+3\,B+C)a^3\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(11/2), x)
```

3.1283 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=271

$$\frac{4a^3(106A+147B+140C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7A+9B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d}$$

```
[Out] (-4*a^3*(7*A + 9*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(13*A + 21*B + 35*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(106*A + 147*B +
140*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 9*B + 5*C)*(a^3
 + a^3*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(6*A + 7*
B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (
2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.775516, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(106A+147B+140C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7A+9B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(9/2),x]
```


```
[Out] (-4*a^3*(7*A + 9*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(13*A + 21*B + 35*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(106*A + 147*B +
140*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 9*B + 5*C)*(a^3
 + a^3*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(6*A + 7*
B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (
2*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$= \frac{2(6A+7B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{5}{2}}(c}{35ad}$$

$$= \frac{2(7A+9B+5C)\left(a^3+a^3\cos(c+dx)\right)\mathrm{se}}{15d}$$

$$= \frac{2(7A+9B+5C)\left(a^3+a^3\cos(c+dx)\right)\mathrm{se}}{15d}$$

$$= \frac{4a^3(106A+147B+140C)\sqrt{\sec(c+dx)}\,\mathrm{si}}{105d}$$

$$= \frac{4a^3(106A+147B+140C)\sqrt{\sec(c+dx)}\,\mathrm{si}}{105d}$$

$$= -\frac{4a^3(7A+9B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 3.42645, size = 176, normalized size = 0.65

$$\frac{a^3\sec^{\frac{7}{2}}(c+dx)\left(80(13A+21B+35C)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-336(7A+9B+5C)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(9/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(7/2)*(-336*(7*A + 9*B + 5*C)*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2] + 80*(13*A + 21*B + 35*C)*Cos[c + d*x]^(7/2)*EllipticF[(c
 + d*x)/2, 2] + 2*(320*A + 210*B + 70*C + 21*(54*A + 58*B + 45*C)*Cos[c + d
*x] + 10*(26*A + 21*B + 7*C)*Cos[2*(c + d*x)] + 294*A*Cos[3*(c + d*x)] + 37
```

```
8*B*Cos[3*(c + d*x)] + 315*C*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] time = 4.606, size = 1097, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/8*A*(-1/56*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A
+3/8*B+1/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*A+3/8*B+3/8*C)
*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-
1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{5}+(B+3\,C)a^{3}\cos\left(dx+c\right)^{4}+(A+3\,B+3\,C)a^{3}\cos\left(dx+c\right)^{3}+(3\,A+3\,B+C)a^{3}\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(9/2), x)
```

3.1284 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=270

$$-\frac{4a^3(21A+20B+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(33A+35B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A+5(}{}$$

```
[Out] (-4*a^3*(9*A + 5*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(21*A + 20*B + 5*C)*S
in[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(33*A + 35*B + 15*C)*(a^3 + a^3
*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(6*A + 5*B)*(a^
2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*(a
 + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.790965, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$-\frac{4a^3(21A+20B+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(33A+35B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A+5(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(7/2),x]
```


```
[Out] (-4*a^3*(9*A + 5*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(21*A + 20*B + 5*C)*S
in[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(33*A + 35*B + 15*C)*(a^3 + a^3
*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(6*A + 5*B)*(a^
2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*(a
 + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\cos}{\quad}$$

$$=\frac{2A(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$=\frac{2(6A+5B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^{\frac{3}{2}}(c}{15ad}$$

$$=\frac{2(33A+35B+15C)\left(a^3+a^3\cos(c+dx)\right.}{15d}$$

$$=\frac{2(33A+35B+15C)\left(a^3+a^3\cos(c+dx)\right.}{15d}$$

$$=-\frac{4a^3(21A+20B+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(3}{\quad}$$

$$=-\frac{4a^3(21A+20B+5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(3}{\quad}$$

$$=-\frac{4a^3(9A+5B-5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 2.10876, size = 157, normalized size = 0.58

$$a^3 \sec^{\frac{5}{2}}(c+dx)\left(80(3A+5(B+C))\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-48(9A+5B-5C)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(7/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(5/2)*(-48*(9*A + 5*B - 5*C)*Cos[c + d*x]^(5/2)*EllipticE
[(c + d*x)/2, 2] + 80*(3*A + 5*(B + C))*Cos[c + d*x]^(5/2)*EllipticF[(c + d
*x)/2, 2] + 2*(5*(12*A + 4*B + 3*C)*Cos[c + d*x] + 6*(18*A + 5*(3*B + C))*C
os[2*(c + d*x)] + 5*(6*(4*A + 3*B + C) + C*Cos[3*(c + d*x)]))*Sin[c + d*x])
)/(60*d)
```

Maple [B] time = 4.743, size = 1328, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-120*C*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+90*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-20*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-60*C*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^4+60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+10
```

```
8*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+100*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*sin(1/2*d*x+1/2*c)^4-100*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2-60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*A*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2-100*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+60*B*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-60*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2+190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+246*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+4
0*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-180*B*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^6+100*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{5}+\left(B+3\,C\right)a^{3}\cos\left(dx+c\right)^{4}+\left(A+3\,B+3\,C\right)a^{3}\cos\left(dx+c\right)^{3}+\left(3\,A+3\,B+C\right)a^{3}\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(7/2), x)
```

3.1285 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=267

$$-\frac{4a^3(20A+5B-6C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2(35A+15B-3C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}} + \frac{4a^3(5A+5B+3C)\sqrt{\cos(c}{}$$

```
[Out] (-4*a^3*(5*A - 5*B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(20*A + 5*B - 6*C)*Si
n[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) - (2*(35*A + 15*B - 3*C)*(a^3 + a^3*C
os[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(2*A + B)*(a^2 +
a^2*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (2*A*(a + a*Co
s[c + d*x])^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.762245, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3043, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$-\frac{4a^3(20A+5B-6C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2(35A+15B-3C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}} + \frac{4a^3(5A+5B+3C)\sqrt{\cos(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(5/2),x]
```



```
[Out] (-4*a^3*(5*A - 5*B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(20*A + 5*B - 6*C)*Si
n[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) - (2*(35*A + 15*B - 3*C)*(a^3 + a^3*C
os[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(2*A + B)*(a^2 +
a^2*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (2*A*(a + a*Co
s[c + d*x])^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{co}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\sin(}{3d}$$

$$= \frac{2(2A+B)\left(a^2+a^2\cos(c+dx)\right)^2\sqrt{\sec(c}}{ad}$$

$$= -\frac{2(35A+15B-3C)\left(a^3+a^3\cos(c+dx\right.}{15d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2(35A+15B-3C)\left(a^3+a^3\cos(c+dx\right.}{15d\sqrt{\sec(c+dx)}}$$

$$= -\frac{4a^3(20A+5B-6C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2(3}{}$$

$$= -\frac{4a^3(20A+5B-6C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2(3}{}$$

$$= -\frac{4a^3(5A-5B-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 1.0416, size = 149, normalized size = 0.56

$$\frac{a^3\sec^{\frac{3}{2}}(c+dx)\left(80(5A+5B+3C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-48(5A-5B-9C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(5/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(-48*(5*A - 5*B - 9*C)*Cos[c + d*x]^(3/2)*EllipticE
[(c + d*x)/2, 2] + 80*(5*A + 5*B + 3*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d
*x)/2, 2] + 2*(20*A + 10*B + 30*C + 3*(60*A + 20*B + 3*C)*Cos[c + d*x] + 10
*(B + 3*C)*Cos[2*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sin[c + d*x]))/(60*d)
```

Maple [B] time = 4.498, size = 950, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-24*C*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+96*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*A*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2+50*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-90*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-30*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^2+50*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-5
0*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-54*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*s
in(1/2*d*x+1/2*c)^2+30*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-78*C
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-25*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+50*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+20*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+27*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+18*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^5 + (B+3C)a^3\cos(dx+c)^4 + (A+3B+3C)a^3\cos(dx+c)^3 + (3A+3B+C)a^3\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(5/2), x)
```

3.1286 $\int (a+a\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^{3/2}(c+dx)\,dx$

Optimal. Leaf size=269

$$-\frac{4a^3(35A-42B-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2(35A-7B-11C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{35d\sqrt{\sec(c+dx)}}+\frac{4a^3(35A+21B+13C)\sqrt{}}{}$$

```
[Out] (4*a^3*(5*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(35*A + 21*B + 13*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(35*A - 42*B - 41*
C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) - (2*(7*A - C)*(a^2 + a^2*Cos[c
 + d*x])^2*Sin[c + d*x])/(7*a*d*Sqrt[Sec[c + d*x]]) - (2*(35*A - 7*B - 11*C
)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]) + (2*A*(
a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.761866, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$-\frac{4a^3(35A-42B-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2(35A-7B-11C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{35d\sqrt{\sec(c+dx)}}+\frac{4a^3(35A+21B+13C)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(3/2),x]
```



```
[Out] (4*a^3*(5*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(35*A + 21*B + 13*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(35*A - 42*B - 41*
C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) - (2*(7*A - C)*(a^2 + a^2*Cos[c
 + d*x])^2*Sin[c + d*x])/(7*a*d*Sqrt[Sec[c + d*x]]) - (2*(35*A - 7*B - 11*C
)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]) + (2*A*(
a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\,\mathrm{co}}{}$$

$$=\frac{2A(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,\mathrm{sin}(}{d}$$

$$=-\frac{2(7A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{7ad\sqrt{\sec(c+dx)}}$$

$$=-\frac{2(7A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{7ad\sqrt{\sec(c+dx)}}$$

$$=-\frac{2(7A-C)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c}{7ad\sqrt{\sec(c+dx)}}$$

$$=-\frac{4a^3(35A-42B-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2}{}$$

$$=-\frac{4a^3(35A-42B-41C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-\frac{2}{}$$

$$=\frac{4a^3(5A+9B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 1.00429, size = 149, normalized size = 0.55

$$\frac{a^3\sqrt{\sec(c+dx)}\left(2\sin(c+dx)(5(28A+84B+113C)\cos(c+dx)+420A+42(B+3C)\cos(2(c+dx))+42B+15C\cos(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(3/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(336*(5*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 80*(35*A + 21*B + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] + 2*(420*A + 42*B + 126*C + 5*(28*A + 84*B + 113*C)*Cos[c + d*
x] + 42*(B + 3*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Sin[c + d*x]))/
(420*d)
```

Maple [B] time = 1.378, size = 727, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/105*a^3*(120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(7*B+36*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+21*B+43*C)*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(70*A+63*B+104*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+175*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)-105*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+105*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-189*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+65*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
```

```
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^{3}\cos\left(dx+c\right)^{5}+(B+3\,C)a^{3}\cos\left(dx+c\right)^{4}+(A+3\,B+3\,C)a^{3}\cos\left(dx+c\right)^{3}+(3\,A+3\,B+C)a^{3}\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*se
c(d*x + c)^(3/2), x)
```

3.1287 $\int (a+a\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=271

$$\frac{4a^3(42A+41B+32C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2(63A+99B+73C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{315d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B+11C)}{}$$

```
[Out] (4*a^3*(27*A + 21*B + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(21*A + 13*B + 11*C)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(42*A + 41*B +
 32*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])
^3*Sin[c + d*x])/(9*d*Sqrt[Sec[c + d*x]]) + (2*(3*B + 2*C)*(a^2 + a^2*Cos[c
 + d*x])^2*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]]) + (2*(63*A + 99*B + 73
*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.784267, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(42A+41B+32C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2(63A+99B+73C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{315d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B+11C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec
[c + d*x]],x]
```


```
[Out] (4*a^3*(27*A + 21*B + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(21*A + 13*B + 11*C)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(42*A + 41*B +
 32*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])
^3*Sin[c + d*x])/(9*d*Sqrt[Sec[c + d*x]]) + (2*(3*B + 2*C)*(a^2 + a^2*Cos[c
 + d*x])^2*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]]) + (2*(63*A + 99*B + 73
*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\,\mathrm{c}}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{(2}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{2(}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{2(}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{2(}{}$$

$$=\frac{4a^3(42A+41B+32C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2}{}$$

$$=\frac{4a^3(42A+41B+32C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2}{}$$

$$=\frac{4a^3(27A+21B+17C)\sqrt{\cos(c+dx)}E\Big(}{15d}$$

Mathematica [A] time = 1.54117, size = 153, normalized size = 0.56

$$\frac{a^3\sqrt{\sec(c+dx)}\Big(\sin(2(c+dx))(7(36A+108B+151C)\cos(c+dx)+5(252A+18(B+3C)\cos(2(c+dx))+330B+7C\cos\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(336*(27*A + 21*B + 17*C)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2] + 240*(21*A + 13*B + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2] + (7*(36*A + 108*B + 151*C)*Cos[c + d*x] + 5*(252*A + 330*
B + 318*C + 18*(B + 3*C)*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c
 + d*x)]))/(1260*d)
```

Maple [A] time = 1.49, size = 514, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*C*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(360*B+2200*C)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-252*A-1296*B-3412*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(882*A+1806*B+2702*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-37
8*A-624*B-738*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-567*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+195*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
441*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+165*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^5 + (B+3C)a^3\cos(dx+c)^4 + (A+3B+3C)a^3\cos(dx+c)^3 + (3A+3B+C)a^3\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sq
rt(sec(d*x + c)), x)
```

3.1288 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=307

$$\frac{4a^3(264A+253B+210C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(143A+121B+105C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(99A+143B+105C)\sin(c+dx)\left(\right.}{693d\sec^{\frac{3}{2}}(c+dx}$$

```
[Out] (4*a^3*(21*A + 17*B + 15*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(143*A + 121*B + 105*C)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(264*A + 2
53*B + 210*C)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c
 + d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*x]^(3/2)) + (2*(11*B + 6*C)*(a^2 +
 a^2*Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d*Sec[c + d*x]^(3/2)) + (2*(99*A +
 143*B + 105*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(693*d*Sec[c + d*x]^
(3/2)) + (4*a^3*(143*A + 121*B + 105*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d
*x]])
```

Rubi [A] time = 0.826813, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(264A+253B+210C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(143A+121B+105C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(99A+143B+105C)\sin(c+dx)\left(\right.}{693d\sec^{\frac{3}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (4*a^3*(21*A + 17*B + 15*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(143*A + 121*B + 105*C)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(264*A + 2
53*B + 210*C)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c
 + d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*x]^(3/2)) + (2*(11*B + 6*C)*(a^2 +
 a^2*Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d*Sec[c + d*x]^(3/2)) + (2*(99*A +
 143*B + 105*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(693*d*Sec[c + d*x]^
(3/2)) + (4*a^3*(143*A + 121*B + 105*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d
*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\,\mathrm{co}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(11B+6C)\left(a^2\right.}{99}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(11B+6C)\left(a^2\right.}{99}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(11B+6C)\left(a^2\right.}{99}$$

$$= \frac{4a^3(264A+253B+210C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\,\mathrm{co}}{11d}$$

$$= \frac{4a^3(264A+253B+210C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\,\mathrm{co}}{11d}$$

$$= \frac{4a^3(21A+17B+15C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

$$= \frac{4a^3(21A+17B+15C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

Mathematica [A] time = 1.56686, size = 174, normalized size = 0.57

$$\frac{a^3\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))(154(108A+151B+165C)\cos(c+dx)+5(36(11A+33B+49C)\cos(2(c+dx))+726\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(14784*(21*A + 17*B + 15*C)*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2] + 960*(143*A + 121*B + 105*C)*Sqrt[Cos[c + d*x]]*Elli
```

```
pticF[(c + d*x)/2, 2] + 2*(154*(108*A + 151*B + 165*C)*Cos[c + d*x] + 5*(72
60*A + 6996*B + 6741*C + 36*(11*A + 33*B + 49*C)*Cos[2*(c + d*x)] + 154*(B
+ 3*C)*Cos[3*(c + d*x)] + 63*C*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(55440
*d)
```

Maple [A] time = 1.523, size = 545, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*B-43680*C)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(3960*A+24200*B+77280*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-14256*A-37532*B-72240*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(19866*A+29722*B+39270*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-6864*A-8118*B-8820*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2145*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-4851*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1815*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3927*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1575*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
465*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^3}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5 + (B+3C)a^3\cos(dx+c)^4 + (A+3B+3C)a^3\cos(dx+c)^3 + (3A+3B+C)a^3\cos(dx+c}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sq
rt(sec(d*x + c)), x)
```

3.1289 $\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=343

$$\frac{4a^3(221A+195B+175C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{20a^3(286A+273B+236C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(121A+105B+95C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}$$

```
[Out] (4*a^3*(221*A + 195*B + 175*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(195*d) + (4*a^3*(121*A + 105*B + 95*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(286*A
 + 273*B + 236*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*C
os[c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c + d*x]^(5/2)) + (2*(13*B + 6*C)*(a
^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(143*a*d*Sec[c + d*x]^(5/2)) + (2*(1
43*A + 195*B + 145*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(1287*d*Sec[c
+ d*x]^(5/2)) + (4*a^3*(221*A + 195*B + 175*C)*Sin[c + d*x])/(585*d*Sec[c +
 d*x]^(3/2)) + (4*a^3*(121*A + 105*B + 95*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[
c + d*x]])
```

Rubi [A] time = 0.845113, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4221, 3045, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(221A+195B+175C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{20a^3(286A+273B+236C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(121A+105B+95C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (4*a^3*(221*A + 195*B + 175*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(195*d) + (4*a^3*(121*A + 105*B + 95*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(286*A
 + 273*B + 236*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (2*C*(a + a*C
os[c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c + d*x]^(5/2)) + (2*(13*B + 6*C)*(a
^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(143*a*d*Sec[c + d*x]^(5/2)) + (2*(1
43*A + 195*B + 145*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(1287*d*Sec[c
+ d*x]^(5/2)) + (4*a^3*(221*A + 195*B + 175*C)*Sin[c + d*x])/(585*d*Sec[c +
```

```
 d*x]^(3/2)) + (4*a^3*(121*A + 105*B + 95*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[
c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{c}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx}\right.}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(13B+6C)\left(a\right.}{1}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(13B+6C)\left(a\right.}{1}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(13B+6C)\left(a\right.}{1}$$

$$= \frac{20a^3(286A+273B+236C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a}{1}$$

$$= \frac{20a^3(286A+273B+236C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a}{1}$$

$$= \frac{20a^3(286A+273B+236C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a}{1}$$

$$= \frac{4a^3(221A+195B+175C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{195d}$$

<u>Mathematica</u> [A] time = 2.14915, size = 197, normalized size = 0.57

$$\frac{a^3\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))(154(3926A+4290B+4525C)\cos(c+dx)+5(936(33A+49B+59C)\cos(2(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(29568*(221*A + 195*B + 175*C)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 24960*(121*A + 105*B + 95*C)*Sqrt[Cos[c + d*x]]*
```

```
EllipticF[(c + d*x)/2, 2] + 2*(154*(3926*A + 4290*B + 4525*C)*Cos[c + d*x]
+ 5*(936*(33*A + 49*B + 59*C)*Cos[2*(c + d*x)] + 77*(52*A + 156*B + 245*C)*
Cos[3*(c + d*x)] + 3*(60632*A + 58422*B + 56290*C + 546*(B + 3*C)*Cos[4*(c
+ d*x)] + 231*C*Cos[5*(c + d*x)])))*Sin[2*(c + d*x)]))/(1441440*d)
```

Maple [A] time = 1.352, size = 576, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(131040*B+1058400*C)*sin(1/2*
d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-80080*A-567840*B-2122400*C)*sin(1/2*d*x+
1/2*c)^10*cos(1/2*d*x+1/2*c)+(314600*A+1004640*B+2331040*C)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-487916*A-939120*B-1535860*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(386386*A+510510*B+633710*C)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-105534*A-114660*B-121230*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)+23595*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-51051*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+20475*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-45045*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+18525*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-40425*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(a\cos\left(dx+c\right) + a\right)^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\cos(dx+c)^5+(B+3C)a^3\cos(dx+c)^4+(A+3B+3C)a^3\cos(dx+c)^3+(3A+3B+C)a^3\cos(dx+}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^4 + (A + 3*B +
3*C)*a^3*cos(d*x + c)^3 + (3*A + 3*B + C)*a^3*cos(d*x + c)^2 + (3*A + B)*a^
3*cos(d*x + c) + A*a^3)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/se
c(d*x + c)^(3/2), x)
```

3.1290 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=250

$$\frac{(7A-5B+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}-\frac{(5A-5B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(7A-5B+5C)\sin(c+dx)\sqrt{s}}{5ad}$$

```
[Out] (-3*(7*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) - ((5*A - 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (3*(7*A - 5*B + 5*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*a*d) - ((5*A - 5*B + 3*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d) + ((7*A - 5*B + 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d
) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.376678, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2748, 2636, 2639, 2641}

$$\frac{(7A-5B+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}-\frac{(5A-5B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(7A-5B+5C)\sin(c+dx)\sqrt{s}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos
[c + d*x]),x]
```


```
[Out] (-3*(7*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) - ((5*A - 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (3*(7*A - 5*B + 5*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*a*d) - ((5*A - 5*B + 3*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d) + ((7*A - 5*B + 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d
) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\left((5A-5B+3C)\right.}{}$$

$$= -\frac{(5A-5B+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{(7A-5B+5}{}$$

$$= -\frac{(5A-5B+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3ad}$$

$$= -\frac{3(7A-5B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+a}}{5ad}$$

Mathematica [A] time = 3.91208, size = 200, normalized size = 0.8

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(20(5A-5B+3C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+36(7A-5B+5C)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*(36*(7*A - 5*B + 5*C)*Cos[c + d*x]^
(5/2)*EllipticE[(c + d*x)/2, 2] + 20*(5*A - 5*B + 3*C)*Cos[c + d*x]^(5/2)*E
llipticF[(c + d*x)/2, 2] - (100*A - 40*B + 60*C + (173*A - 95*B + 135*C)*Co
s[c + d*x] + (76*A - 40*B + 60*C)*Cos[2*(c + d*x)] + 63*A*Cos[3*(c + d*x)]
- 45*B*Cos[3*(c + d*x)] + 45*C*Cos[3*(c + d*x)])*Tan[(c + d*x)/2]))/(30*a*d
*(1 + Cos[c + d*x]))
```

Maple [B] time = 4.773, size = 812, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-2*A+2*B)*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(2*A-2*B+2*C)*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*A/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+(-A+B-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos
(d*x + c) + a), x)
```

3.1291 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{(5A-3B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{(3A-3B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+a}{d(a\cos(c+dx)+a)}$$

```
[Out] ((3*A - 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d) + ((5*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((3*A - 3*B + C)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(a*d) + ((5*A - 3*B + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d
) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.348067, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2748, 2636, 2641, 2639}

$$\frac{(5A-3B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{(3A-3B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+a}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos
[c + d*x]),x]
```

```
[Out] ((3*A - 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d) + ((5*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((3*A - 3*B + C)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(a*d) + ((5*A - 3*B + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d
) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\left((3A-3B+C)\sqrt{\cos}\right.}{}$$

$$= -\frac{(3A-3B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(5A-3B+3C)}{}$$

$$= \frac{(3A-3B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} +$$

Mathematica [A] time = 2.50083, size = 162, normalized size = 0.79

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(2(5A-3B+3C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(3A-3B+C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx\right.\right.}{3ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*(6*(3*A - 3*B + C)*Cos[c + d*x]^(3/2
)*EllipticE[(c + d*x)/2, 2] + 2*(5*A - 3*B + 3*C)*Cos[c + d*x]^(3/2)*Ellipt
icF[(c + d*x)/2, 2] - (5*A - 9*B + 3*C + 4*(2*A - 3*B)*Cos[c + d*x] + 3*(3*
A - 3*B + C)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2]))/(3*a*d*(1 + Cos[c + d*x])
)
```

Maple [B] time = 3.162, size = 494, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*A*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-2*A+2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(A-B+C)*(cos(1/2*d*x+1/2*c)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos
(d*x + c) + a), x)
```

3.1292 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{(3A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\cos(c+dx)+a)}-\frac{(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{ad}$$

```
[Out] -(((3*A - B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d)) - ((A - B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*d) + ((3*A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x
]))
```

Rubi [A] time = 0.324195, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2748, 2636, 2639, 2641}

$$\frac{(3A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\cos(c+dx)+a)}-\frac{(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos
[c + d*x]),x]
```


```
[Out] -(((3*A - B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d)) - ((A - B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*d) + ((3*A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x
]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s}\right.}{}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\left((A-B-C)\sqrt{co}\right.}{}$$

$$= -\frac{(A-B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} +$$

$$= -\frac{(3A-B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}$$

Mathematica [A] time = 1.18292, size = 132, normalized size = 0.8

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\tan\left(\frac{1}{2}(c+dx)\right)((3A-B+C)\cos(c+dx)+2A)+(A-B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*((3*A - B + C)*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2] + (A - B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2] - (2*A + (3*A - B + C)*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*(1
+ Cos[c + d*x]))
```

Maple [A] time = 2.537, size = 353, normalized size = 2.1

$$-\frac{1}{ad}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B+C)*sin(1/2*d*x+1/2*c)^4
+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B+C)*sin(1/2*d*x
+1/2*c)^2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos
(d*x + c) + a), x)
```

3.1293 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)}+\frac{(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B+3C)\sqrt{\cos(c+d}{}$$

```
[Out] ((A - B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*d) + ((A + B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a*d) - ((A - B + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*
Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.294314, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4221, 3041, 2748, 2641, 2639}

$$-\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)}+\frac{(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B+3C)\sqrt{\cos(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos
[c + d*x]),x]
```



```
[Out] ((A - B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*d) + ((A + B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a*d) - ((A - B + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])*
Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^{2}}{\sqrt{\cos(c+dx)}(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(}\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left((A+B-C)\sqrt{\cos(c}\right.}{}$$

$$= \frac{(A-B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} +$$

Mathematica [A] time = 0.743101, size = 126, normalized size = 0.97

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-(A-B+C)\left(\sin(c+dx)-\tan\left(\frac{1}{2}(c+dx)\right)\right)+(A+B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*((A - B + 3*C)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2] + (A + B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2] - (A - B + C)*(Sin[c + d*x] - Tan[(c + d*x)/2])))/(a*d*(1 + Cos[c
 + d*x]))
```

Maple [A] time = 0.986, size = 281, normalized size = 2.2

$$\frac{1}{ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\left(A\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))+(2*A-2*B+2*C)*sin(1/2*d*x+1/2*c)^4+(-A+B-C)*sin(1/2*d*x+1/2*c)^2)/a
/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(
d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx+\int \frac{C\cos^{2}\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(B*cos(c +
d*x)*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(C*cos(c + d*x)**2
*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a), x)
```

3.1294 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{(3A-3B+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A-3B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3ad}$$

```
[Out] -(((A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d)) + ((3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((A - B + C)*Sin[c + d*x])/(d*(a + a*C
os[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A - 3*B + 5*C)*Sin[c + d*x])/(3*a*d*
Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.318428, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2748, 2639, 2635, 2641}

$$\frac{(3A-3B+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A-3B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sqrt[Sec[
c + d*x]]),x]
```

```
[Out] -(((A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d)) + ((3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((A - B + C)*Sin[c + d*x])/(d*(a + a*C
os[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A - 3*B + 5*C)*Sin[c + d*x])/(3*a*d*
Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2\right.}{a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} - \frac{\left((A-3B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{2a}$$

$$= -\frac{(A-3B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-B}{d(a+a\cos(}$$

$$= -\frac{(A-3B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A-3B+}{}$$

Mathematica [A] time = 0.781232, size = 163, normalized size = 0.94

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-(3A-3B+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(A-3B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.\right.}{3ad(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sqr
t[Sec[c + d*x]]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(3*(A - 3*B + 3*C)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2] - (3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + ((3*A - 3*B + 5*C + 2*C*Cos[c + d*x])*Sec[(c + d*x)/
2]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2))/(3*a*d*(1 + Cos[c + d*x])
)
```

Maple [A] time = 1.113, size = 300, normalized size = 1.7

$$-\frac{1}{3\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-8*C*sin(1/2*d*x+1/2*c)^6+(6*A-6*B+18*C)*sin(1/2*d*x+1
/2*c)^4+(-3*A+3*B-7*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt
(sec(d*x + c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\cos(c+dx)}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx+\int \frac{C\cos^2(c+dx)}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/sec(d*x+c)**(1/
2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x) + In
tegral(B*cos(c + d*x)/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))
), x) + Integral(C*cos(c + d*x)**2/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(
sec(c + d*x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

3.1295 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{(5A-5B+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{(3A-5B+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A-5B+5C)\sqrt{\cos(c}}{}$$

```
[Out] (3*(5*A - 5*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*a*d) - ((3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((A - B + C)*Sin[c + d*x])/(d*(a +
a*Cos[c + d*x])*Sec[c + d*x]^(5/2)) + ((5*A - 5*B + 7*C)*Sin[c + d*x])/(5*a
*d*Sec[c + d*x]^(3/2)) - ((3*A - 5*B + 5*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c
 + d*x]])
```

Rubi [A] time = 0.339559, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2748, 2635, 2641, 2639}

$$\frac{(5A-5B+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{(3A-5B+5C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A-5B+5C)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec[c + d
*x]^(3/2)),x]
```


```
[Out] (3*(5*A - 5*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*a*d) - ((3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((A - B + C)*Sin[c + d*x])/(d*(a +
a*Cos[c + d*x])*Sec[c + d*x]^(5/2)) + ((5*A - 5*B + 7*C)*Sin[c + d*x])/(5*a
*d*Sec[c + d*x]^(3/2)) - ((3*A - 5*B + 5*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c
 + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} - \frac{\left((3A-5B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{2a}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-5B+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(3A-5}{3a}$$

$$= \frac{3(5A-5B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(3A-5B+5}{}$$

Mathematica [A] time = 1.12279, size = 178, normalized size = 0.83

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-10(3A-5B+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+18(5A-5B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.\right.}{15ad(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec
[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(18*(5*A - 5*B + 7*C)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2] - 10*(3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2] + (15*A - 25*B + 22*C + (-10*B + 4*C)*Cos[c + d*x]
- 3*C*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*
x))/2])))/(15*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 1.235, size = 319, normalized size = 1.5

$$\frac{1}{15\,da}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right(15\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))-48*C*sin(1/2*d*x+1/2*c)^8+(40*B+56*C)*sin(1/2*d
*x+1/2*c)^6+(30*A-90*B+30*C)*sin(1/2*d*x+1/2*c)^4+(-15*A+35*B-23*C)*sin(1/2
*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(
d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/sec(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

3.1296 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{(5A-7B+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(7A-7B+9C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(7A-7B+9C)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+}{d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+}$$

```
[Out] (-3*(5*A - 7*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) + (5*(7*A - 7*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*a*d) - ((A - B + C)*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)) + ((7*A - 7*B + 9*C)*Sin[c + d*x])/
(7*a*d*Sec[c + d*x]^(5/2)) - ((5*A - 7*B + 7*C)*Sin[c + d*x])/(5*a*d*Sec[c
+ d*x]^(3/2)) + (5*(7*A - 7*B + 9*C)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x
]])
```

Rubi [A] time = 0.365452, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2748, 2635, 2639, 2641}

$$-\frac{(5A-7B+7C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(7A-7B+9C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(7A-7B+9C)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+}{d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec[c + d
*x]^(5/2)),x]
```


```
[Out] (-3*(5*A - 7*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) + (5*(7*A - 7*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*a*d) - ((A - B + C)*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)) + ((7*A - 7*B + 9*C)*Sin[c + d*x])/
(7*a*d*Sec[c + d*x]^(5/2)) - ((5*A - 7*B + 7*C)*Sin[c + d*x])/(5*a*d*Sec[c
+ d*x]^(3/2)) + (5*(7*A - 7*B + 9*C)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x
]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{5}{2}}(}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)} - \frac{\left((5A-7B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{2a}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)} + \frac{(7A-7B+9C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{(5A-}{5}$$

$$= -\frac{3(5A-7B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(A-}{d(a+a\,\mathrm{co}}$$

$$= -\frac{3(5A-7B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(7A-7}{}$$

Mathematica [A] time = 1.80131, size = 198, normalized size = 0.79

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-100(7A-7B+9C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+252(5A-7B+7C)\sqrt{\cos(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])*Sec
[c + d*x]^(5/2)),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(252*(5*A - 7*B + 7*C)*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2] - 100*(7*A - 7*B + 9*C)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + (350*A - 308*B + 438*C + (140*A - 56*B + 201*C
)*Cos[c + d*x] + 6*(7*B - 2*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Se
c[(c + d*x)/2]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(210*a*d*(1 + Co
s[c + d*x]))
```

Maple [A] time = 1.234, size = 341, normalized size = 1.4

$$-\frac{1}{105\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\,(17\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(175*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-175*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+225*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*C*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-480*C*sin(1/2*d*x+1/2*c)^10+(336*B+864*
C)*sin(1/2*d*x+1/2*c)^8+(-280*A-392*B-888*C)*sin(1/2*d*x+1/2*c)^6+(630*A-21
0*B+930*C)*sin(1/2*d*x+1/2*c)^4+(-245*A+161*B-321*C)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec
(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(
d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))/sec(d*x+c)**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec
(d*x + c)^(5/2)), x)
```

3.1297 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=251

$$\frac{(10A-5B+2C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{(7A-4B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(7A-4B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c$$

```
[Out] ((7*A - 4*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*d) + ((10*A - 5*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((7*A - 4*B + C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(a^2*d) + ((10*A - 5*B + 2*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(3*a^2*d) - ((7*A - 4*B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1
 + Cos[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a +
 a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.514424, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2978, 2748, 2636, 2641, 2639}

$$\frac{(10A-5B+2C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{(7A-4B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(7A-4B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos
[c + d*x])^2,x]
```



```
[Out] ((7*A - 4*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*d) + ((10*A - 5*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((7*A - 4*B + C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(a^2*d) + ((10*A - 5*B + 2*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(3*a^2*d) - ((7*A - 4*B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1
 + Cos[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a +
 a*Cos[c + d*x])^2)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}\right.}{}$$

$$= -\frac{(7A-4B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sec^{\frac{3}{2}}}{3d(a+a}$$

$$= -\frac{(7A-4B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sec^{\frac{3}{2}}}{3d(a+a}$$

$$= -\frac{(7A-4B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{(10A-5B+2C}{}$$

$$= \frac{(7A-4B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} +$$

Mathematica [A] time = 4.24249, size = 212, normalized size = 0.84

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(2(10A-5B+2C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(7A-4B+C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(3/2)*(6*(7*A - 4*B + C)*Cos[c + d*x]^(3
/2)*EllipticE[(c + d*x)/2, 2] + 2*(10*A - 5*B + 2*C)*Cos[c + d*x]^(3/2)*Ell
ipticF[(c + d*x)/2, 2] - ((56*A - 38*B + 8*C + (95*A - 60*B + 9*C)*Cos[c +
d*x] + (64*A - 38*B + 8*C)*Cos[2*(c + d*x)] + 21*A*Cos[3*(c + d*x)] - 12*B*
```

```
Cos[3*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])/4))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 4.203, size = 751, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(4*A*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(A-B+C)*(2*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4
-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+(-8*A+4*B)*(-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(4*A-2*B)*(cos(1/2
*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a^2*co
s(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^2, x)
```

3.1298 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=215

$$-\frac{(5A-2B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(5A-2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)\sin(c+}{a}$$

```
[Out] -(((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) - ((5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((4*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) - ((5*A - 2*B - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
 Cos[c + d*x])) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a
*Cos[c + d*x])^2)
```

Rubi [A] time = 0.481578, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2978, 2748, 2636, 2639, 2641}

$$-\frac{(5A-2B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(5A-2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)\sin(c+}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos
[c + d*x])^2,x]
```



```
[Out] -(((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) - ((5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((4*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) - ((5*A - 2*B - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
 Cos[c + d*x])) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a
*Cos[c + d*x])^2)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+}$$

$$=-\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$=-\frac{(5A-2B-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B+C)\sqrt{\sec}}{3d(a+a\,c}$$

$$=-\frac{(5A-2B-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B+C)\sqrt{\sec}}{3d(a+a\,c}$$

$$=-\frac{(5A-2B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}+$$

$$=-\frac{(4A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A}{}$$

Mathematica [A] time = 3.19114, size = 172, normalized size = 0.8

$$-\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2(5A-2B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(2(1}{3a^2d(\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(6*(4*A - B)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 2*(5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2] - ((24*A - 3*B + 2*(19*A - 4*B + C)*Cos[c + d*x] + 3*(4*A -
B)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/2))/(3*a^2*d*(1 +
 Cos[c + d*x])^2)
```

Maple [B] time = 3.256, size = 563, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B+C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B+C)*sin(1/2*d*x+1/2*c)^2)/sin(
1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x
+1/2*c)^2-1)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a^2*co
s(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^2, x)
```

3.1299 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=173

$$\frac{(2A+B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{(A-C)\sqrt{\cos(c+dx)}}{}$$

```
[Out] ((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + ((2*A + B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a^2*d) - ((A - C)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x]
)^2*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.436409, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2978, 2748, 2641, 2639}

$$\frac{(2A+B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{(A-C)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos
[c + d*x])^2,x]
```

```
[Out] ((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + ((2*A + B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a^2*d) - ((A - C)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x]
)^2*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^{\cdots}}{\sqrt{\cos(c+dx)}(a+a\cos(c\cdots}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\cdots\right.}{\cdots}$$

$$= -\frac{(A-C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{(A-B+C)\,\mathrm{si}\cdots}{3d(a+a\cos(c+dx)\cdots}$$

$$= -\frac{(A-C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{(A-B+C)\,\mathrm{si}\cdots}{3d(a+a\cos(c+dx)\cdots}$$

$$= \frac{(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A\cdots}{\cdots}$$

Mathematica [A] time = 2.00937, size = 164, normalized size = 0.95

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2(2A+B+2C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{2}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\mathrm{s}\cdots\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(6*(A - C)*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2] + 2*(2*A + B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2] + ((4*A - B - 2*C + 3*(A - C)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*
(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2))/(3*a^2*d*(1 + Cos[c + d*x])^
2)
```

Maple [B] time = 1.456, size = 507, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-12*C*cos(1/2*d*x+1/2*c)^6-4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(
1/2*d*x+1/2*c)^3-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*A*co
s(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+20*C*cos(1/2*d*x+1/2*c)^4+3*A*c
os(1/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2-9*C*cos(1/2*d*x+1/2*c)^2+A-B+C
)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*co
s(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a)^2, x)
```

3.1300 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=179

$$\frac{(A+2B-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{(A+2B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(B-4C)\sqrt{\cos(c+dx)}}{}$$

```
[Out] -(((B - 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + ((A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos
[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((A + 2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*
(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.444447, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2977, 2748, 2641, 2639}

$$\frac{(A+2B-5C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sqrt{\sec(c+dx)}}+\frac{(A+2B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(B-4C)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sqrt[Se
c[c + d*x]]),x]
```

```
[Out] -(((B - 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + ((A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - B + C)*Sin[c + d*x])/(3*d*(a + a*Cos
[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((A + 2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*
(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos}}{}}{3}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{(A+2B-5C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{(A+2B-5C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sqrt{\sec(c+dx)}}$$

$$= -\frac{(B-4C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B-5C)\sqrt{\cos}}{}$$

Mathematica [A] time = 2.10836, size = 162, normalized size = 0.91

$$-\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-2(A+2B-5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{2}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\right.}{3a^2d(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*S
qrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(6*(B - 4*C)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] - 2*(A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2] + ((A + 2*B - 5*C + 3*(B - 2*C)*Cos[c + d*x])*Sec[(c + d*x)/
2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2))/(3*a^2*d*(1 + Cos[c + d
*x])^2)
```

Maple [B] time = 1.577, size = 507, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-24*C*cos(1/2*d*x+1/2*c)^6-10*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^3-24*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*
cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4+38*C*cos(1/2*d*x+1/2*c)^4-3*
A*cos(1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2-15*C*cos(1/2*d*x+1/2*c)^2+A
-B+C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A}{\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*s
qrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A}{\left(a^2\cos\left(dx+c\right)^2 + 2\,a^2\cos\left(dx+c\right) + a^2\right)\sqrt{\sec\left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a
^2*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*s
qrt(sec(d*x + c))), x)
```

3.1301 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{(2A-5B+10C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A-4B+7C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sec^{\frac{3}{2}}(c+dx)}+\frac{(2A-5B+10C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(\right.}{3a^2d}$$

```
[Out] -(((A - 4*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*d)) + ((2*A - 5*B + 10*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - B + C)*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)) - ((A - 4*B + 7*C)*Sin[c + d*x])/
(3*a^2*d*(1 + Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((2*A - 5*B + 10*C)*Sin[c
 + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.496793, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(2A-5B+10C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A-4B+7C)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)\sec^{\frac{3}{2}}(c+dx)}+\frac{(2A-5B+10C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(\right.}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sec[c +
 d*x]^(3/2)),x]
```


```
[Out] -(((A - 4*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*d)) + ((2*A - 5*B + 10*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - B + C)*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)) - ((A - 4*B + 7*C)*Sin[c + d*x])/
(3*a^2*d*(1 + Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((2*A - 5*B + 10*C)*Sin[c
 + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} - \frac{(A-4B+7C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} - \frac{(A-4B+7C)\sin(c+dx)}{3a^2d(1+\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(A-4B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(A-}{3d(a+a\cos}$$

$$= -\frac{(A-4B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-5B+}{}$$

Mathematica [A] time = 2.74424, size = 183, normalized size = 0.83

$$-\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-2(2A-5B+10C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(A-4B+7C)\sqrt{\cos(c+dx)}E\left(\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*S
ec[c + d*x]^(3/2)),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(6*(A - 4*B + 7*C)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2] - 2*(2*A - 5*B + 10*C)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2] + ((2*A - 5*B + 11*C + (3*A - 6*B + 13*C)*Cos[c + d
*x] + C*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c
```

```
+ d*x))/2]))/2))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 1.332, size = 472, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*
C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+21*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*C*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+21*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)+16*C*sin(1/2*d*x+1/2*c)^8+(-12*A+24*B-76*C)*sin(1/2*d*x+1/2*c
)^6+(16*A-34*B+84*C)*sin(1/2*d*x+1/2*c)^4+(-5*A+11*B-25*C)*sin(1/2*d*x+1/2*
c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A}{(a\cos{(dx+c)} + a)^2 \sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*s
ec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a
^2*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*s
ec(d*x + c)^(3/2)), x)
```

3.1302 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$\frac{(20A-35B+56C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A-2B+3C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A-2B+3C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sec^{\frac{5}{2}}(c+dx)}-\frac{5(A-2B+3C)\sqrt{c}}{}$$

```
[Out] ((20*A - 35*B + 56*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a^2*d) - (5*(A - 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - B + C)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) - ((A - 2*B + 3*C)*Sin[c + d*x
])/(a^2*d*(1 + Cos[c + d*x])*Sec[c + d*x]^(5/2)) + ((20*A - 35*B + 56*C)*Si
n[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*(A - 2*B + 3*C)*Sin[c + d*x]
)/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.52053, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2977, 2748, 2635, 2641, 2639}

$$\frac{(20A-35B+56C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A-2B+3C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(A-2B+3C)\sin(c+dx)}{a^2d(\cos(c+dx)+1)\sec^{\frac{5}{2}}(c+dx)}-\frac{5(A-2B+3C)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*Sec[c +
 d*x]^(5/2)),x]
```

```
[Out] ((20*A - 35*B + 56*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a^2*d) - (5*(A - 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - B + C)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) - ((A - 2*B + 3*C)*Sin[c + d*x
])/(a^2*d*(1 + Cos[c + d*x])*Sec[c + d*x]^(5/2)) + ((20*A - 35*B + 56*C)*Si
n[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*(A - 2*B + 3*C)*Sin[c + d*x]
)/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{(A-2B+3C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} +$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{(A-2B+3C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} -$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{(A-2B+3C)\sin(c+dx)}{a^2d(1+\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{(20A-35B+56C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5(A-2B}{}$$

Mathematica [A] time = 3.64196, size = 200, normalized size = 0.79

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-50(A-2B+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(20A-35B+56C)\sqrt{\cos(c+dx)}E\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^2*S
ec[c + d*x]^(5/2)),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(6*(20*A - 35*B + 56*C)*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2] - 50*(A - 2*B + 3*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2] + ((50*A - 110*B + 158*C + (60*A - 130*B + 179*C)
```

```
*Cos[c + d*x] + (-10*B + 8*C)*Cos[2*(c + d*x)] - 3*C*Cos[3*(c + d*x)])*Sec[
(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/4))/(15*a^2*d*(1
+ Cos[c + d*x])^2)
```

Maple [A] time = 1.254, size = 491, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+75*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*C*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*cos(1/2*d*x+1/2*c)+96*C*sin(1/2*d*x+1/2*c)^10+(-80*B-128*C)*sin(1/2*d*
x+1/2*c)^8+(-120*A+380*B-328*C)*sin(1/2*d*x+1/2*c)^6+(170*A-420*B+526*C)*si
n(1/2*d*x+1/2*c)^4+(-55*A+125*B-171*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*
x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a
^2*cos(d*x + c) + a^2)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**2/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*s
ec(d*x + c)^(5/2)), x)
```

3.1303 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=310

$$\frac{(33A-13B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6a^3d}-\frac{(119A-49B+9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(119A-49B+9C)\sin(c}{30d\left(a^3\cos(c+\right.}$$

```
[Out] ((119*A - 49*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(10*a^3*d) + ((33*A - 13*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((119*A - 49*B + 9*C)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + ((33*A - 13*B + 3*C)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d*(a + a*Cos[c + d*x])^3) - ((2*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((119*A - 49*B + 9*C)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.710594, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2978, 2748, 2636, 2641, 2639}

$$\frac{(33A-13B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6a^3d}-\frac{(119A-49B+9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(119A-49B+9C)\sin(c}{30d\left(a^3\cos(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos
[c + d*x])^3,x]
```


```
[Out] ((119*A - 49*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(10*a^3*d) + ((33*A - 13*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((119*A - 49*B + 9*C)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + ((33*A - 13*B + 3*C)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d*(a + a*Cos[c + d*x])^3) - ((2*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((119*A - 49*B + 9*C)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c-}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\sec^{\frac{3}{2}}(c+}{3ad(a+a\,\mathrm{co}}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\sec^{\frac{3}{2}}(c+}{3ad(a+a\,\mathrm{co}}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\sec^{\frac{3}{2}}(c+}{3ad(a+a\,\mathrm{co}}$$

$$= -\frac{(119A-49B+9C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(33A-13}{}$$

$$= \frac{(119A-49B+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{10a^3d}$$

Mathematica [A] time = 5.84773, size = 249, normalized size = 0.8

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(10(33A-13B+3C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(119A-49B+9C)\cos^{\frac{3}{2}}(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sec[c + d*x]^(3/2)*(6*(119*A - 49*B + 9*C)*Cos[c + d*
x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(33*A - 13*B + 3*C)*Cos[c + d*x]^(3
/2)*EllipticF[(c + d*x)/2, 2] - ((3691*A - 1621*B + 261*C + 12*(533*A - 228
*B + 33*C)*Cos[c + d*x] + 8*(526*A - 221*B + 36*C)*Cos[2*(c + d*x)] + 1812*
A*Cos[3*(c + d*x)] - 752*B*Cos[3*(c + d*x)] + 132*C*Cos[3*(c + d*x)] + 357*
A*Cos[4*(c + d*x)] - 147*B*Cos[4*(c + d*x)] + 27*C*Cos[4*(c + d*x)])*Sec[(c
 + d*x)/2]^4*Tan[(c + d*x)/2])/16))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 5.271, size = 1040, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(8*A*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(4*A-2*B)*(2*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)
^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+(-24*A+8*B)*(-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(12*A-4*B)*(cos
(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(A-B+C)*(1/5*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+4/5*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+18/5*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-8/5*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18
/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a^3*co
s(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^3, x)
```

3.1304 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=277

$$\frac{(49A-9B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}}{6a^3d}$$

```
[Out] -((49*A - 9*B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(10*a^3*d) - ((13*A - 3*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - 9*B - C)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(10*a^3*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B - 2*C)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B - C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.68803, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2978, 2748, 2636, 2639, 2641}

$$\frac{(49A-9B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos
[c + d*x])^3,x]
```


```
[Out] -((49*A - 9*B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(10*a^3*d) - ((13*A - 3*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - 9*B - C)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(10*a^3*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B - 2*C)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B - C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s\right.}{}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{}}{15ad(a+}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{}}{15ad(a+}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{}}{15ad(a+}$$

$$= -\frac{(13A-3B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{6a^3d}$$

$$= -\frac{(49A-9B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{10a^3d}$$

Mathematica [A] time = 1.94575, size = 215, normalized size = 0.78

$$-\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(10(13A-3B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(49A-9B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(6*(49*A - 9*B - C)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2] + 10*(13*A - 3*B - C)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2] - ((992*A - 132*B - 8*C + (1621*A - 261*B + 11*C)*C
os[c + d*x] + 4*(188*A - 33*B - 2*C)*Cos[2*(c + d*x)] + 147*A*Cos[3*(c + d*
x)] - 27*B*Cos[3*(c + d*x)] - 3*C*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^4*Tan[
(c + d*x)/2])/8))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 1.602, size = 793, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*
B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(49*A-9*B-C)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(817*A-147*B-13*C)*sin(1/2*d*x+1/2*c)^6+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(248*A-43*B-2*C)*sin(1/2*
d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69
*B-C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a^3*co
s(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^3, x)
```

3.1305 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=233

$$-\frac{(9A+B-C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B-C)\sqrt{c}}{}$$

```
[Out] ((9*A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3*Sqrt[Sec[c + d*x]]) - ((6*A - B - 4*C)*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - ((9*A + B - C)*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.658936, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2978, 2748, 2641, 2639}

$$-\frac{(9A+B-C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B-C)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos
[c + d*x])^3,x]
```


```
[Out] ((9*A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3*Sqrt[Sec[c + d*x]]) - ((6*A - B - 4*C)*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - ((9*A + B - C)*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt{\cos(c+dx)}(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\text{se}\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} - \frac{(6A-B-4C}{15ad(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} - \frac{(6A-B-4C}{15ad(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} - \frac{(6A-B-4C}{15ad(a+a\cos(c+}$$

$$= \frac{(9A+B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} +$$

Mathematica [A] time = 1.47224, size = 188, normalized size = 0.81

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(10(3A+B+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(9A+B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(6*(9*A + B - C)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2] + 10*(3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2] + ((117*A - 7*B - 13*C + 4*(33*A + 2*B - 7*C)*Cos[c + d*x]
 + 3*(9*A + B - C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*(Sin[(c + d*x)/2] -
 Sin[(3*(c + d*x))/2]))/8))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 1.252, size = 624, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*c
os(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+1/2*c
)^6+24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2*d*x+1/2
*c)^4+3*A*cos(1/2*d*x+1/2*c)^2+7*B*cos(1/2*d*x+1/2*c)^2-17*C*cos(1/2*d*x+1/
2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*co
s(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a)^3, x)
```

3.1306 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=231

$$-\frac{(A-B-9C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B-9C)\sqrt{\cos}}{}$$

```
[Out] ((A - B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3*Sec[c + d*x]^(3/2)) + ((4*A + B - 6*C)*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - ((A - B - 9*C)*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.673277, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2977, 2978, 2748, 2641, 2639}

$$-\frac{(A-B-9C)\sin(c+dx)}{10d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B-9C)\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sqrt[Se
c[c + d*x]]),x]
```


```
[Out] ((A - B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3*Sec[c + d*x]^(3/2)) + ((4*A + B - 6*C)*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - ((A - B - 9*C)*Sin[c + d*x])/
(10*d*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+a\cos(c+dx))^3}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos}}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A+B-6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+d}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A+B-6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+d}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A+B-6C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sqrt{\sec(c+d}}$$

$$= \frac{(A-B-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+B+3C)\sqrt{c}}{}$$

Mathematica [A] time = 1.50811, size = 188, normalized size = 0.81

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(10(A+B+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(A-B-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*S
qrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(6*(A - B - 9*C)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2] + 10*(A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2] + ((-7*A - 13*B - 57*C + 4*(2*A - 7*B - 18*C)*Cos[c + d*x]
 + 3*(A - B - 9*C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*(Sin[(c + d*x)/2] -
 Sin[(3*(c + d*x))/2]))/8))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 1.228, size = 624, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8-30*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*cos(1/2*d*x+1/2*c)^5-54*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+198*C*cos(1/2*d*x+1/2*c
)^6+6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4-114*C*cos(1/2*d*x+1/
2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*B*cos(1/2*d*x+1/2*c)^2+27*C*cos(1/2*d*x+
1/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*s
qrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a
^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*s
qrt(sec(d*x + c))), x)
```

3.1307 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=235

$$\frac{(A+3B-13C)\sin(c+dx)}{6d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+3B-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B-49C)\sqrt{c}}{}$$

```
[Out] -((A + 9*B - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(10*a^3*d) + ((A + 3*B - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)) + ((2*A + 3*B - 8*C)*Sin[c + d*x]
)/(15*a*d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((A + 3*B - 13*C)*Si
n[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.650419, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3041, 2977, 2748, 2641, 2639}

$$\frac{(A+3B-13C)\sin(c+dx)}{6d\sqrt{\sec(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+3B-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B-49C)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c +
 d*x]^(3/2)),x]
```


```
[Out] -((A + 9*B - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(10*a^3*d) + ((A + 3*B - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)) + ((2*A + 3*B - 8*C)*Sin[c + d*x]
)/(15*a*d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((A + 3*B - 13*C)*Si
n[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+a\cos(c+dx))^3}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+d}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+d}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+d}$$

$$= -\frac{(A+9B-49C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+3B-1}{}$$

<u>Mathematica</u> [A] time = 2.16764, size = 190, normalized size = 0.81

$$-\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-10(A+3B-13C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(A+9B-49C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*S
ec[c + d*x]^(3/2)),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(6*(A + 9*B - 49*C)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2] - 10*(A + 3*B - 13*C)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2] + ((13*A + 57*B - 217*C + 4*(7*A + 18*B - 73*C)*Cos
[c + d*x] + 3*(A + 9*B - 29*C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*(Sin[(c
 + d*x)/2] - Sin[(3*(c + d*x))/2]))/8))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 1.413, size = 624, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*c
os(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*cos(1/2*d*x+1/2*c)^5-294*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6+578*C*cos(1/2*d*x
+1/2*c)^6-24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4-264*C*cos(1/
2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-27*B*cos(1/2*d*x+1/2*c)^2+37*C*cos
(1/2*d*x+1/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a
^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*s
ec(d*x + c)^(3/2)), x)
```

3.1308 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=272

$$\frac{(3A-13B+33C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(9A-49B+119C)\sin(c+dx)}{30d\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A-13B+33C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{6a^3d}$$

```
[Out] -((9*A - 49*B + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) + ((3*A - 13*B + 33*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])
/(5*d*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)) + ((B - 2*C)*Sin[c + d*x])
/(3*a*d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)) - ((9*A - 49*B + 119*C)*
Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A -
13*B + 33*C)*Sin[c + d*x])/(6*a^3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.685166, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(3A-13B+33C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(9A-49B+119C)\sin(c+dx)}{30d\sec^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A-13B+33C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c +
 d*x]^(5/2)),x]
```


```
[Out] -((9*A - 49*B + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) + ((3*A - 13*B + 33*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x])
/(5*d*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)) + ((B - 2*C)*Sin[c + d*x])
/(3*a*d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)) - ((9*A - 49*B + 119*C)*
Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A -
13*B + 33*C)*Sin[c + d*x])/(6*a^3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+a\cos(c+dx))^3}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} + \frac{(B-2C)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+a}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} + \frac{(B-2C)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+a}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)} + \frac{(B-2C)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+a}$$

$$= -\frac{(9A-49B+119C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(A}{5d(a+}$$

$$= -\frac{(9A-49B+119C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A-1}{}$$

Mathematica [A] time = 2.90274, size = 206, normalized size = 0.76

$$-\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-10(3A-13B+33C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(9A-49B+119C)\sqrt{\cos(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*S
ec[c + d*x]^(5/2)),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(6*(9*A - 49*B + 119*C)*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2] - 10*(3*A - 13*B + 33*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + ((57*A - 217*B + 567*C + (72*A - 292*B + 7
82*C)*Cos[c + d*x] + 3*(9*A - 29*B + 79*C)*Cos[2*(c + d*x)] + 10*C*Cos[3*(c
 + d*x)])*Sec[(c + d*x)/2]^5*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/8))
/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 1.493, size = 638, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*C*cos(1/
2*d*x+1/2*c)^10+108*A*cos(1/2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*c
os(1/2*d*x+1/2*c)^5+54*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348
*B*cos(1/2*d*x+1/2*c)^8-130*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*C*cos(1/2*d*x+1/
2*c)^8+330*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+714*C*cos(1/2*d
*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(
1/2*d*x+1/2*c)^6-1058*C*cos(1/2*d*x+1/2*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264
*B*cos(1/2*d*x+1/2*c)^4+474*C*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c)^
2+37*B*cos(1/2*d*x+1/2*c)^2-47*C*cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\left(a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3\right)\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a
^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c) + a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*s
ec(d*x + c)^(5/2)), x)
```

3.1309 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=313

$$\frac{7(7A-17B+33C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(13A-33B+63C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(13A-33B+63C)\sin(c+dx)}{10d\sec^{\frac{5}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-3\cdots}{}$$

```
[Out] (7*(7*A - 17*B + 33*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) - ((13*A - 33*B + 63*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x]
)/(5*d*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)) - ((2*A - 7*B + 12*C)*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) - ((13*A - 33
*B + 63*C)*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(5/2))
 + (7*(7*A - 17*B + 33*C)*Sin[c + d*x])/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((1
3*A - 33*B + 63*C)*Sin[c + d*x])/(6*a^3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.721488, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3041, 2977, 2748, 2635, 2641, 2639}

$$\frac{7(7A-17B+33C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(13A-33B+63C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(13A-33B+63C)\sin(c+dx)}{10d\sec^{\frac{5}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-3\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*Sec[c +
 d*x]^(7/2)),x]
```


```
[Out] (7*(7*A - 17*B + 33*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) - ((13*A - 33*B + 63*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sin[c + d*x]
)/(5*d*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)) - ((2*A - 7*B + 12*C)*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) - ((13*A - 33
*B + 63*C)*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^(5/2))
 + (7*(7*A - 17*B + 33*C)*Sin[c + d*x])/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((1
3*A - 33*B + 63*C)*Sin[c + d*x])/(6*a^3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+a\cos(c+dx))^3}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)} - \frac{(2A-7B+12C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)} - \frac{(2A-7B+12C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)} - \frac{(2A-7B+12C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)} - \frac{(2A-7B+12C)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+}$$

$$= \frac{7(7A-17B+33C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-3}{}$$

Mathematica [A] time = 3.12281, size = 229, normalized size = 0.73

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-10(13A-33B+63C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+42(7A-17B+33C)\sqrt{\cos(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^3*S
ec[c + d*x]^(7/2)),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(42*(7*A - 17*B + 33*C)*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2] - 10*(13*A - 33*B + 63*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + ((217*A - 567*B + 1062*C + 2*(146*A - 391*
B + 732*C)*Cos[c + d*x] + 3*(29*A - 79*B + 143*C)*Cos[2*(c + d*x)] - 10*B*C
os[3*(c + d*x)] + 12*C*Cos[3*(c + d*x)] - 3*C*Cos[4*(c + d*x)])*Sec[(c + d*
x)/2]^5*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/8))/(15*a^3*d*(1 + Cos[c
 + d*x])^3)
```

Maple [A] time = 1.556, size = 666, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-192*C*cos(1/
2*d*x+1/2*c)^12-160*B*cos(1/2*d*x+1/2*c)^10+864*C*cos(1/2*d*x+1/2*c)^10+348
*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-468*B*cos(1/2*d*x+1/
2*c)^8-330*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-714*B*cos(1/2*d
*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+228*C*cos(1/2*d*x+1/2*c)^8+630*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+1386*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^
6-1590*C*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1/2*d*x+
1/2*c)^4+744*C*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*
d*x+1/2*c)^2-57*C*cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^
5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a
^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(7/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**3/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*s
ec(d*x + c)^(7/2)), x)
```

3.1310 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=226

$$\frac{2a(16A+18B+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a(16A+18B+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+18B+2}{315d\sqrt{a}}$$

```
[Out] (16*a*(16*A + 18*B + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(16*A + 18*B + 21*C)*Sec[c + d*
x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(A + 9*B)*Se
c[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[
a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.646145, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3043, 2980, 2772, 2771}

$$\frac{2a(16A+18B+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a(16A+18B+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a(16A+18B+2}{315d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(11/2),x]
```



```
[Out] (16*a*(16*A + 18*B + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(16*A + 18*B + 21*C)*Sec[c + d*
x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(A + 9*B)*Se
c[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[
a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(}{9d}$$

$$= \frac{2a(A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2a(16A+18B+21C)\sec^{\frac{5}{2}}(c+dx)\sin(}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{8a(16A+18B+21C)\sec^{\frac{3}{2}}(c+dx)\sin(}{315d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{16a(16A+18B+21C)\sqrt{\sec(c+dx)}\sin}{315d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.954821, size = 155, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(2(88A+99B+63C)\cos(c+dx)+11(16A+18B+21C)\cos(2(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(214*A + 162*B + 189*C + 2*(88*A + 99*B + 63*C)
*Cos[c + d*x] + 11*(16*A + 18*B + 21*C)*Cos[2*(c + d*x)] + 32*A*Cos[3*(c +
d*x)] + 36*B*Cos[3*(c + d*x)] + 42*C*Cos[3*(c + d*x)] + 32*A*Cos[4*(c + d*x
)] + 36*B*Cos[4*(c + d*x)] + 42*C*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*Tan[
(c + d*x)/2])/(315*d)
```

Maple [A] time = 0.22, size = 171, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(128A(\cos(dx+c))^4+144B(\cos(dx+c))^4+168C(\cos(dx+c))^4+64A(\cos(dx+c))^3+72}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2
),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+168*C*cos(d
*x+c)^4+64*A*cos(d*x+c)^3+72*B*cos(d*x+c)^3+84*C*cos(d*x+c)^3+48*A*cos(d*x+
c)^2+54*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+40*A*cos(d*x+c)+45*B*cos(d*x+c)+35
*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.93485, size = 1331, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c)
)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 735*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10
7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(co
s(d*x + c) + 1)^10 + 1)) + 9*B*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x +
c) + 1) - 105*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sqr
t(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 142*sqrt(2)*sqrt(a)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 67*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 - 9*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)) + 21*C*(15*sqrt(2)*sqrt(a)*sin(d*x
 + c)/(cos(d*x + c) + 1) - 55*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 82*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 66*sqrt(2
)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 31*sqrt(2)*sqrt(a)*sin(d*x
+ c)^9/(cos(d*x + c) + 1)^9 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x +
```

```
c) + 1)^11)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.54835, size = 351, normalized size = 1.55

$$\frac{2\left(8\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{4}+4\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{3}+3\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{2}+5\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c)
)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*(16*A + 18*B + 21*C)*cos(d*x + c)^4 + 4*(16*A + 18*B + 21*C)*cos(d
*x + c)^3 + 3*(16*A + 18*B + 21*C)*cos(d*x + c)^2 + 5*(8*A + 9*B)*cos(d*x +
 c) + 35*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*co
s(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2)*(a+a*cos(d*x+
c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c)
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
sec(d*x + c)^(11/2), x)
```

3.1311 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{2a(24A+28B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+28B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+7B)\sin(c+\ldots}{35d\sqrt{a\cos(\ldots}}$$

```
[Out] (4*a*(24*A + 28*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*a*(24*A + 28*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(A + 7*B)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.592377, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3043, 2980, 2772, 2771}

$$\frac{2a(24A+28B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a(24A+28B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+7B)\sin(c+\ldots}{35d\sqrt{a\cos(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(9/2),x]
```


```
[Out] (4*a*(24*A + 28*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*a*(24*A + 28*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(A + 7*B)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{c}}}{}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$= \frac{2a(A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{2a(24A+28B+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c}{105d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{4a(24A+28B+35C)\sqrt{\sec(c+dx)}\sin(c}{105d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.710306, size = 121, normalized size = 0.68

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}(3(36A+42B+35C)\cos(c+dx)+(24A+28B+35C)\cos(2(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(54*A + 28*B + 35*C + 3*(36*A + 42*B + 35*C)*Co
s[c + d*x] + (24*A + 28*B + 35*C)*Cos[2*(c + d*x)] + 24*A*Cos[3*(c + d*x)]
+ 28*B*Cos[3*(c + d*x)] + 35*C*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c
+ d*x)/2])/(105*d)
```

Maple [A] time = 0.193, size = 138, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(48\,A(\cos(dx+c))^3+56\,B(\cos(dx+c))^3+70\,C(\cos(dx+c))^3+24\,A(\cos(dx+c))^2+28\,B(\right.}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+70*C*cos(d*x+
c)^3+24*A*cos(d*x+c)^2+28*B*cos(d*x+c)^2+35*C*cos(d*x+c)^2+18*A*cos(d*x+c)+
21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1
/2)/sin(d*x+c)
```

Maxima [B] time = 1.90256, size = 1145, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 7*B*(15*sq
rt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sqrt(2)*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 42*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 24*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sqrt
(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 35*C*(3*sqrt(2)*sqrt(a)*sin
(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 12*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*sqr
t(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + sqrt(2)*sqrt(a)*sin(d*x
+ c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((
sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.59926, size = 294, normalized size = 1.65

$$\frac{2\left(2\left(24A+28B+35C\right)\cos\left(dx+c\right)^{3}+\left(24A+28B+35C\right)\cos\left(dx+c\right)^{2}+3\left(6A+7B\right)\cos\left(dx+c\right)+15A\right)\sqrt{a\cos}}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(2*(24*A + 28*B + 35*C)*cos(d*x + c)^3 + (24*A + 28*B + 35*C)*cos(d*x
 + c)^2 + 3*(6*A + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
sec(d*x + c)^(9/2), x)
```

3.1312 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ dx) dx

Optimal. Leaf size=130

$$\frac{2a(8A+10B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\,c}}{5d}$$

```
[Out] (2*a*(8*A + 10*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*
Cos[c + d*x]]) + (2*a*(A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d)
```

Rubi [A] time = 0.500312, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4221, 3043, 2980, 2771}

$$\frac{2a(8A+10B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a(A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\,c}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(7/2),x]
```


```
[Out] (2*a*(8*A + 10*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*
Cos[c + d*x]]) + (2*a*(A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{\sqrt{a+a\,\mathrm{c}}}{}$$

$$=\frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$=\frac{2a(A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2}{}$$

$$=\frac{2a(8A+10B+15C)\sqrt{\sec(c+dx)}\sin(c+}{15d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.353633, size = 85, normalized size = 0.65

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((8A+10B+15C)\cos(2(c+dx))+2(4A+5B)\cos(c+dx)+14A+10}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(14*A + 10*B + 15*C + 2*(4*A + 5*B)*Cos[c + d*x
] + (8*A + 10*B + 15*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/
2])/(15*d)
```

Maple [A] time = 0.191, size = 105, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(8A(\cos(dx+c))^2+10B(\cos(dx+c))^2+15C(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+}{15d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+15*C*cos(d*x+c)
^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1
+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.84352, size = 957, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)
```

```
^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)
/(cos(d*x + c) + 1) - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(2)*sqrt(a)*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 15*C
*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sqrt(2)*sqrt(a)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.53206, size = 234, normalized size = 1.8

$$\frac{2\left((8A+10B+15C)\cos(dx+c)^2+(4A+5B)\cos(dx+c)+3A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*((8*A + 10*B + 15*C)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2
)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
sec(d*x + c)^(7/2), x)
```

3.1313 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=140

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.47332, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3043, 2980, 2774, 216}

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[a]*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,c}}{}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= \frac{2a(A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{2a(A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{2\sqrt{a}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx}}{d}$$

Mathematica [A] time = 0.387188, size = 105, normalized size = 0.75

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((2A+3B)\cos(c+dx)+A)+3\sqrt{2}C\sin^{-1}\left(\sqrt{2}\sin\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[2]*
C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (2*A + 3*B)*
Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] time = 0.202, size = 286, normalized size = 2.

$$-\frac{2\cos(dx+c)(\sin(dx+c))^2}{3d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(3C\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d*(3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+6*C*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*
cos(d*x+c)+3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*sin(d*x+c)*cos(d*x+c)+3*B*cos(d*x
+c)*sin(d*x+c)+A*sin(d*x+c))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x
+c))^(5/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 2.38839, size = 2088, normalized size = 14.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*sqrt(a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c)
- (cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + ((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))) + 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
```

```
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))) - 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 4*A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(
cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 12*B*(sqrt(2)*sqrt(a)*sin(d
*x + c)/(cos(d*x + c) + 1) - 2*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.65325, size = 344, normalized size = 2.46

$$-\frac{2\left(3\left(C\cos\left(dx+c\right)^{2}+C\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{((2A+3B)\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*(C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((2*A + 3*B)*cos(d*x
+ c) + A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
sec(d*x + c)^(5/2), x)
```

3.1314 $\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{3/2}(c+dx)\,dx$

Optimal. Leaf size=141

$$-\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{d}+\frac{\sqrt{a}(2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\cdots}{d}$$

```
[Out] (Sqrt[a]*(2*B + C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a*(2*A - C)*Sin[c + d*x])/(d*Sq
rt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.486167, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3043, 2981, 2774, 216}

$$-\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{d}+\frac{\sqrt{a}(2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\cdots}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (Sqrt[a]*(2*B + C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a*(2*A - C)*Sin[c + d*x])/(d*Sq
rt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + a*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{c}}}{}$$

$$= \frac{2A\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$= -\frac{a(2A-C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2A}{}$$

$$= -\frac{a(2A-C)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2A}{}$$

$$= \frac{\sqrt{a}(2B+C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c}}{d}$$

Mathematica [A] time = 0.303519, size = 104, normalized size = 0.74

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2A+C\cos(c+dx))+\sqrt{2}(2B+C)\sin^{-1}\left(\sqrt{2}\sin\left(\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2
*B + C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + C*Co
s[c + d*x])*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.202, size = 305, normalized size = 2.2

$$\frac{\cos(dx+c)}{d(1+\cos(dx+c))}\left(2B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)+C\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/d*(2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+C*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x
+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))+C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+C*cos(d*x+c)*sin(d*
x+c)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/
2)/(1+cos(d*x+c))
```

Maxima [B] time = 2.61003, size = 2288, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*B*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
```

```
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)) + (2*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x +
 c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a)
+ sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C + 8*A*(sqrt(2)*sqrt(a)*sin(d*x +
c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(3/2)))/d
```

Fricas [A] time = 2.08314, size = 304, normalized size = 2.16

$$-\frac{((2B+C)\cos(dx+c)+2B+C)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(C\cos(dx+c)+2A)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -(((2*B + C)*cos(d*x + c) + 2*B + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (C*cos(d*x + c) + 2*A)*sqrt(
```

```
a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
sec(d*x + c)^(3/2), x)
```

3.1315 $\int \sqrt{a + a\cos(c + dx)} \left(A + B\cos(c + dx) + C\cos^2(c + dx\right.$

Optimal. Leaf size=151

$$\frac{\sqrt{a}(8A + 4B + 3C)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} + \frac{a(4B + C)\sin(c + dx)}{4d\sqrt{\sec(c + dx)}\sqrt{a\cos(c + dx) + a}} + \frac{C\sin(c + d}{2d}$$

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*(4*B + C)*Sin[c +
 d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.486398, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3045, 2981, 2774, 216}

$$\frac{\sqrt{a}(8A + 4B + 3C)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} + \frac{a(4B + C)\sin(c + dx)}{4d\sqrt{\sec(c + dx)}\sqrt{a\cos(c + dx) + a}} + \frac{C\sin(c + d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*(4*B + C)*Sin[c +
 d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\cdots}}{\cdots}$$

$$= \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\mathrm{co}\cdots}\right.}{\cdots}$$

$$= \frac{a(4B+C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{C\cdots}{\cdots}$$

$$= \frac{a(4B+C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{C\cdots}{\cdots}$$

$$= \frac{\sqrt{a}(8A+4B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\cdots}{4d}$$

Mathematica [A] time = 0.427398, size = 123, normalized size = 0.81

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+4B+3C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\cdots\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(8*A + 4*B + 3*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqr
t[Cos[c + d*x]]*(4*B + 3*C + 2*C*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.209, size = 270, normalized size = 1.8

$$-\frac{(\cos(dx+c))^2-1}{4\,d\,(\sin(dx+c))^2}\left(2\,C\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4\,B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,C\sin(dx\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/4/d*(2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+8*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos
(d*x+c))+4*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)+3*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/c
os(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.77951, size = 2695, normalized size = 17.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + cos(d*x + c)) + 4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
```

```
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B +
 (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) -
 (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
1)))*C)/d
```

Fricas [A] time = 3.25351, size = 365, normalized size = 2.42

$$-\frac{((8A+4B+3C)\cos(dx+c)+8A+4B+3C)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2C\cos(dx+c)^2+(4B+3C)\cos(dx+c)\right)\cdot}{\sqrt{\cos(dx+c)}}}{4\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(((8*A + 4*B + 3*C)*cos(d*x + c) + 8*A + 4*B + 3*C)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*C*cos
(d*x + c)^2 + (4*B + 3*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)*(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*
sqrt(sec(d*x + c)), x)
```

3.1316 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=199

$$\frac{\sqrt{a}(8A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+6B+5C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a(6B}{12d\sec^{\frac{3}{2}}(c}$$

```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*(6*B + C)*Sin[c +
 d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (C*Sqrt[a + a*C
os[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a*(8*A + 6*B + 5*C)*
Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.562661, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3045, 2981, 2770, 2774, 216}

$$\frac{\sqrt{a}(8A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(8A+6B+5C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a(6B}{12d\sec^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
 d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*(6*B + C)*Sin[c +
 d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (C*Sqrt[a + a*C
os[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a*(8*A + 6*B + 5*C)*
Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+a\,\cdots}$$

$$= \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s\cdots}\right.}{\cdots}$$

$$= \frac{a(6B+C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(\cdots}}{3d\sec\cdots}$$

$$= \frac{a(6B+C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(\cdots}}{3d\sec\cdots}$$

$$= \frac{a(6B+C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(\cdots}}{3d\sec\cdots}$$

$$= \frac{\sqrt{a}(8A+6B+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx\cdots}}{8d}$$

Mathematica [A] time = 0.794893, size = 144, normalized size = 0.72

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(8A+6B+5C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\cdots\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*(8*A + 6*B + 5*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*S
qrt[Cos[c + d*x]]*(24*A + 18*B + 19*C + 2*(6*B + 5*C)*Cos[c + d*x] + 4*C*Co
s[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.2, size = 374, normalized size = 1.9

$$\frac{(-1+\cos(dx+c))^2\cos(dx+c)}{24\,d\,(\sin(dx+c))^4}\left(8\,C\sin(dx+c)\,(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12\,B\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cdots}{1\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1
0*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+18*B*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*C*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 3.4648, size = 5090, normalized size = 25.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
```

```
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + 6*(2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x
+ 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(
2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + si
n(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2
*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*
sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B + (4*
(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)
```

*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arctan2(-(co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))
 + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1

```
) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*C)/d
```

Fricas [A] time = 1.64047, size = 458, normalized size = 2.3

$$-\frac{3\left((8A+6B+5C)\cos(dx+c)+8A+6B+5C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)-1)}{2\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}\right)-\frac{2\left(8C\cos(dx+c)^3+2(6B+5C)\cos\cdots\right.}{\cdots}}{48(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(3*((8*A + 6*B + 5*C)*cos(d*x + c) + 8*A + 6*B + 5*C)*sqrt(a)*arctan(
1/2*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) - 1)/(sqrt(a)*sqrt(cos(d*x + c
))*sin(d*x + c))) - 2*(8*C*cos(d*x + c)^3 + 2*(6*B + 5*C)*cos(d*x + c)^2 +
3*(8*A + 6*B + 5*C)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqr
t(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a(\cos(c+dx)+1)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2
)/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
sqrt(sec(d*x + c)), x)
```

3.1317 $\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a(48A+40B+35C)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48\ldots}{64d\sqrt{s\ldots}}$$

```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*(8*B + C)*Sin
[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (C*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a*(48*A + 40*B +
 35*C)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (
a*(48*A + 40*B + 35*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])
```

Rubi [A] time = 0.653442, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3045, 2981, 2770, 2774, 216}

$$\frac{a(48A+40B+35C)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a(48\ldots}{64d\sqrt{s\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[
c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*(8*B + C)*Sin
[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (C*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a*(48*A + 40*B +
 35*C)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (
a*(48*A + 40*B + 35*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{c}}$$

$$= \frac{C\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}\right.}{}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(}}{4d\sec}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(}}{4d\sec}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(}}{4d\sec}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{C\sqrt{a+a\cos(}}{4d\sec}$$

$$= \frac{\sqrt{a}(48A+40B+35C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c-}}{64d}$$

Mathematica [A] time = 1.00554, size = 164, normalized size = 0.66

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(48A+40B+35C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*(48*A + 40*B + 35*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] +
2*Sqrt[Cos[c + d*x]]*(144*A + 152*B + 133*C + 2*(48*A + 40*B + 53*C)*Cos[c
+ d*x] + 4*(8*B + 7*C)*Cos[2*(c + d*x)] + 12*C*Cos[3*(c + d*x)])*Sin[(c + d
*x)/2]))/(384*d)
```

Maple [B] time = 0.196, size = 480, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^3*(48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+64*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c
)^2+56*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+96*A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*B*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+70*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)+144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+120*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+105*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+144*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c))+120*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))+105*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.93754, size = 490, normalized size = 1.98

$$-\frac{3\left(\left(48\,A+40\,B+35\,C\right)\cos\left(dx+c\right)+48\,A+40\,B+35\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,C\cos(dx+c)^{4}+8\,(8\,B+7\,C\right.}{}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/192*(3*((48*A + 40*B + 35*C)*cos(d*x + c) + 48*A + 40*B + 35*C)*sqrt(a)*
arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
- (48*C*cos(d*x + c)^4 + 8*(8*B + 7*C)*cos(d*x + c)^3 + 2*(48*A + 40*B + 35
*C)*cos(d*x + c)^2 + 3*(48*A + 40*B + 35*C)*cos(d*x + c))*sqrt(a*cos(d*x +
c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+a*cos(d*x+c))**(1/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/
sec(d*x + c)^(3/2), x)
```

3.1318 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \dots dx) \, dx$

Optimal. Leaf size=284

$$\frac{2a^2(84A+110B+99C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2(336A+374B+429C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d\sqrt{a\cos(c+dx)+a}} + \frac{8a^2(336A+3\dots}{3\dots}$$

```
[Out] (16*a^2*(336*A + 374*B + 429*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sq
rt[a + a*Cos[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*C)*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(336*A + 374*B +
429*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a^2*(84*A + 110*B + 99*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a*(3*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*
x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.90749, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2980, 2772, 2771}

$$\frac{2a^2(84A+110B+99C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2(336A+374B+429C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d\sqrt{a\cos(c+dx)+a}} + \frac{8a^2(336A+3\dots}{3\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(13/2),x]
```


```
[Out] (16*a^2*(336*A + 374*B + 429*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sq
rt[a + a*Cos[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*C)*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(336*A + 374*B +
429*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) +
 (2*a^2*(84*A + 110*B + 99*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a*(3*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*
x]^(11/2)*Sin[c + d*x])/(11*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
```

```
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\,\mathrm{c}}{}$$

$$=\frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\,\mathrm{s}}{11d}$$

$$=\frac{2a(3A+11B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c}{99d}$$

$$=\frac{2a^2(84A+110B+99C)\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}}{693d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{2a^2(336A+374B+429C)\sec^{\frac{5}{2}}(c+dx}{1155d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{8a^2(336A+374B+429C)\sec^{\frac{3}{2}}(c+dx}{3465d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{16a^2(336A+374B+429C)\sqrt{\sec(c+d}}{3465d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 1.14666, size = 187, normalized size = 0.66

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((12684A+12386B+12441C)\cos(c+dx)+(4368A+4862B+442}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(13/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(4956*A + 4114*B + 3564*C + (12684*A + 12386*
B + 12441*C)*Cos[c + d*x] + (4368*A + 4862*B + 4422*C)*Cos[2*(c + d*x)] + 4
368*A*Cos[3*(c + d*x)] + 4862*B*Cos[3*(c + d*x)] + 5577*C*Cos[3*(c + d*x)]
+ 672*A*Cos[4*(c + d*x)] + 748*B*Cos[4*(c + d*x)] + 858*C*Cos[4*(c + d*x)]
+ 672*A*Cos[5*(c + d*x)] + 748*B*Cos[5*(c + d*x)] + 858*C*Cos[5*(c + d*x)])
*Sec[c + d*x]^(11/2)*Tan[(c + d*x)/2])/(6930*d)
```

Maple [A] time = 0.221, size = 205, normalized size = 0.7

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(2688\,A\,(\cos(dx+c))^{5}+2992\,B\,(\cos(dx+c))^{5}+3432\,C\,(\cos(dx+c))^{5}+1344\,A\,(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2
),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(2688*A*cos(d*x+c)^5+2992*B*cos(d*x+c)^5+3432*C
*cos(d*x+c)^5+1344*A*cos(d*x+c)^4+1496*B*cos(d*x+c)^4+1716*C*cos(d*x+c)^4+1
008*A*cos(d*x+c)^3+1122*B*cos(d*x+c)^3+1287*C*cos(d*x+c)^3+840*A*cos(d*x+c)
^2+935*B*cos(d*x+c)^2+495*C*cos(d*x+c)^2+735*A*cos(d*x+c)+385*B*cos(d*x+c)+
315*A)*cos(d*x+c)*(1/cos(d*x+c))^(13/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.94937, size = 1438, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(21*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqrt(
2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
```

```
299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)*a^(3
/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + 1)) + 11*(315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 11
55*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2184*sqrt(2)*a^(3/
2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2586*sqrt(2)*a^(3/2)*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 1759*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
+ 1)^9 - 611*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 94*sqr
t(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*B*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 + 1)) + 33*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c)
+ 1) - 455*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 868*sqrt(2
)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 962*sqrt(2)*a^(3/2)*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 653*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 - 247*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 3
8*sqrt(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*C*(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(
-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 1)))/d
```

Fricas [A] time = 1.55082, size = 443, normalized size = 1.56

$$\frac{2\left(8\,(336\,A+374\,B+429\,C)a\cos\left(dx+c\right)^{5}+4\,(336\,A+374\,B+429\,C)a\cos\left(dx+c\right)^{4}+3\,(336\,A+374\,B+429\,C)a\cos\right.}{3465\left(d\cos\left(dx+c\right)^{6}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(8*(336*A + 374*B + 429*C)*a*cos(d*x + c)^5 + 4*(336*A + 374*B + 429
*C)*a*cos(d*x + c)^4 + 3*(336*A + 374*B + 429*C)*a*cos(d*x + c)^3 + 5*(168*
A + 187*B + 99*C)*a*cos(d*x + c)^2 + 35*(21*A + 11*B)*a*cos(d*x + c) + 315*
A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x +
```

```
c)^5)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(13/2), x)
```

3.1319 $\int (a+a\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \ldots dx) \, dx$

Optimal. Leaf size=232

$$\frac{2a^2(52A+72B+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(136A+156B+189C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+15\ldots}{3\ldots}$$

```
[Out] (4*a^2*(136*A + 156*B + 189*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(52*A + 72*B + 63*C)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) +
 (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.809637, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2980, 2772, 2771}

$$\frac{2a^2(52A+72B+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(136A+156B+189C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(136A+15\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(11/2),x]
```


```
[Out] (4*a^2*(136*A + 156*B + 189*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(52*A + 72*B + 63*C)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) +
 (2*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$=\frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,\mathrm{si}}{9d}$$

$$=\frac{2a(A+3B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+}{21d}$$

$$=\frac{2a^2(52A+72B+63C)\sec^{\frac{5}{2}}(c+dx)\,\mathrm{sin}}{315d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{2a^2(136A+156B+189C)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{4a^2(136A+156B+189C)\sqrt{\sec(c+dx)}}{315d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 1.05458, size = 157, normalized size = 0.68

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((748A+81(8B+7C))\cos(c+dx)+(748A+858B+882C)\cos(2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(752*A + 702*B + 693*C + (748*A + 81*(8*B + 7
*C))*Cos[c + d*x] + (748*A + 858*B + 882*C)*Cos[2*(c + d*x)] + 136*A*Cos[3*
(c + d*x)] + 156*B*Cos[3*(c + d*x)] + 189*C*Cos[3*(c + d*x)] + 136*A*Cos[4*
```

```
(c + d*x)] + 156*B*Cos[4*(c + d*x)] + 189*C*Cos[4*(c + d*x)])*Sec[c + d*x]^
(9/2)*Tan[(c + d*x)/2])/(630*d)
```

Maple [A] time = 0.198, size = 172, normalized size = 0.7

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(272\,A\,(\cos(dx+c))^4+312\,B\,(\cos(dx+c))^4+378\,C\,(\cos(dx+c))^4+136\,A\,(\cos(dx+c))^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2
),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+378*C*cos
(d*x+c)^4+136*A*cos(d*x+c)^3+156*B*cos(d*x+c)^3+189*C*cos(d*x+c)^3+102*A*co
s(d*x+c)^2+117*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+85*A*cos(d*x+c)+45*B*cos(d*
x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*
x+c)
```

Maxima [B] time = 1.90508, size = 1250, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 4/315*((315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 94*s
qrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 3*(105*sqrt(2)*a^(
3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 350*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 518*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
```

```
^5 - 444*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 209*sqrt(2)*
a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 38*sqrt(2)*a^(3/2)*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/
((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 1)) + 63*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 32*sq
rt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 26*sqrt(2)*a^(3/2)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 11*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*C*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) +
1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/
d
```

Fricas [A] time = 1.63049, size = 371, normalized size = 1.6

$$\frac{2\left(2\,(136\,A+156\,B+189\,C)a\cos\left(dx+c\right)^{4}+(136\,A+156\,B+189\,C)a\cos\left(dx+c\right)^{3}+3\,(34\,A+39\,B+21\,C)a\cos\left(dx+\right.\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(2*(136*A + 156*B + 189*C)*a*cos(d*x + c)^4 + (136*A + 156*B + 189*C)
*a*cos(d*x + c)^3 + 3*(34*A + 39*B + 21*C)*a*cos(d*x + c)^2 + 5*(17*A + 9*B
)*a*cos(d*x + c) + 35*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*
x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(11/2), x)
```

3.1320 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right.$ $dx)\, dx$

Optimal. Leaf size=184

$$\frac{2a^2(4A + 6B + 5C)\sin(c + dx)\sec^{\frac{3}{2}}(c + dx)}{15d\sqrt{a\cos(c + dx) + a}} + \frac{2a^2(104A + 126B + 175C)\sin(c + dx)\sqrt{\sec(c + dx)}}{105d\sqrt{a\cos(c + dx) + a}} + \frac{2a(3A + 7B)\sin(c}{}$$

```
[Out] (2*a^2*(104*A + 126*B + 175*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 6*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(3*A + 7*B)*Sqrt[a + a*Cos[c
+ d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.71429, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3043, 2975, 2980, 2771}

$$\frac{2a^2(4A + 6B + 5C)\sin(c + dx)\sec^{\frac{3}{2}}(c + dx)}{15d\sqrt{a\cos(c + dx) + a}} + \frac{2a^2(104A + 126B + 175C)\sin(c + dx)\sqrt{\sec(c + dx)}}{105d\sqrt{a\cos(c + dx) + a}} + \frac{2a(3A + 7B)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(104*A + 126*B + 175*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 6*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(3*A + 7*B)*Sqrt[a + a*Cos[c
+ d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{co}}{}$$

$$= \frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}}{7d}$$

$$= \frac{2a(3A+7B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+}{35d}$$

$$= \frac{2a^2(4A+6B+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2(104A+126B+175C)\sqrt{\sec(c+dx)}}{105d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.785061, size = 122, normalized size = 0.66

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((468A+462B+525C)\cos(c+dx)+2(52A+63B+35C)\cos(2(c+}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(164*A + 126*B + 70*C + (468*A + 462*B + 525*
C)*Cos[c + d*x] + 2*(52*A + 63*B + 35*C)*Cos[2*(c + d*x)] + 104*A*Cos[3*(c
+ d*x)] + 126*B*Cos[3*(c + d*x)] + 175*C*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/
2)*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.182, size = 139, normalized size = 0.8

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(104\,A\,(\cos(dx+c))^3+126\,B\,(\cos(dx+c))^3+175\,C\,(\cos(dx+c))^3+52\,A\,(\cos(dx+c))^2+63\right.}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+175*C*cos
(d*x+c)^3+52*A*cos(d*x+c)^2+63*B*cos(d*x+c)^2+35*C*cos(d*x+c)^2+39*A*cos(d*
x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)
))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.86711, size = 1064, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 4/105*((105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)) + 21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c
) + 1) - 15*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2
)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*a^(3/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 35*(3*sqrt(2)*a^(3/2)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 11*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 15*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*
a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9)*C*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.60012, size = 308, normalized size = 1.67

$$\frac{2\left((104\,A+126\,B+175\,C)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B+35\,C)a\cos\left(dx+c\right)^{2}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)+15\,Aa\right)\sqrt{}}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*((104*A + 126*B + 175*C)*a*cos(d*x + c)^3 + (52*A + 63*B + 35*C)*a*co
s(d*x + c)^2 + 3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c))
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(9/2), x)
```

3.1321 $\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=192

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(3A+5$$

```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(12*A + 20*B + 15*C)*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(3*A + 5*B)*S
qrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a +
 a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.661338, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2980, 2774, 216}

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(3A+5$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(7/2),x]
```


```
[Out] (2*a^(3/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(12*A + 20*B + 15*C)*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(3*A + 5*B)*S
qrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a +
 a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\ldots}{}$$

$$= \frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}\ldots}{5d}$$

$$= \frac{2a(3A+5B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c\ldots}{15d}$$

$$= \frac{2a^2(12A+20B+15C)\sqrt{\sec(c+dx)}\,\mathrm{si}\ldots}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2(12A+20B+15C)\sqrt{\sec(c+dx)}\,\mathrm{si}\ldots}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^{3/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+\ldots}}{d}$$

Mathematica [A] time = 0.881699, size = 134, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((18A+25B+15C)\cos(2(c+dx))+2(9A+5B)\ldots\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(30*Sqrt[
2]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(24*A + 25*B +
 15*C + 2*(9*A + 5*B)*Cos[c + d*x] + (18*A + 25*B + 15*C)*Cos[2*(c + d*x)])
*Sin[(c + d*x)/2]))/(30*d)
```

Maple [B] time = 0.205, size = 404, normalized size = 2.1

$$\frac{2\,a\cos\left(dx+c\right)\left(\sin\left(dx+c\right)\right)^{4}}{15\,d\left(-1+\cos\left(dx+c\right)\right)^{2}\left(1+\cos\left(dx+c\right)\right)^{3}}\left(15\,C\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{5/2}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)(\cos\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x)
```

```
[Out] 2/15/d*a*(15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3+45*C*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))*cos(d*x+c)^2+45*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+15*C*(cos(d*x+c)/(1
+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos
(d*x+c))+18*A*sin(d*x+c)*cos(d*x+c)^2+25*B*cos(d*x+c)^2*sin(d*x+c)+15*C*cos
(d*x+c)^2*sin(d*x+c)+9*A*sin(d*x+c)*cos(d*x+c)+5*B*cos(d*x+c)*sin(d*x+c)+3*
A*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^4*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)
))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3
```

Maxima [B] time = 2.50964, size = 2585, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*(5*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*a^(3/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 3
*((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*
arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
```

)) + 1) - (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) - 2*((6*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*a*sin(4*d*x + 4*c) - 7*a*sin(2*d*x + 2*c) - 6*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (3*a*cos(4*d*x + 4*c) + 7*a*cos(2*d*x + 2*c) + 6*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*a)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 9*(a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4) + 24*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 40*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d

Fricas [A] time = 1.85241, size = 420, normalized size = 2.19

$$-\frac{2\left(15\left(Ca\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left((18\,A+25\,B+15\,C)a\cos(dx+c)^{2}+(9\,A+5\,B)a\cos(d\right.}{\sqrt{\cos(dx+c)}}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*(C*a*cos(d*x + c)^3 + C*a*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((18*A + 25*B
 + 15*C)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*
x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(7/2), x)
```

3.1322 $\int (a+a\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=191

$$-\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(A+B)\sin(c$$

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(8*A + 6*B - 3*C)*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*(A + B)*Sqr
t[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + a*Cos[
c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.717627, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2981, 2774, 216}

$$-\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a(A+B)\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(5/2),x]
```

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(8*A + 6*B - 3*C)*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*(A + B)*Sqr
t[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + a*Cos[
c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\,\mathrm{co}}{}$$

$$=\frac{2A(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin}{3d}$$

$$=\frac{2a(A+B)\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+d}}{d}$$

$$=-\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=-\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=\frac{a^{3/2}(2B+3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos}}{d}$$

Mathematica [A] time = 0.714943, size = 128, normalized size = 0.67

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(5A+3B)\cos(c+dx)+4A+3C\cos(2(c+dx))+\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[2
]*(2*B + 3*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A +
3*C + 4*(5*A + 3*B)*Cos[c + d*x] + 3*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))
/(6*d)
```

Maple [B] time = 0.223, size = 490, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x)
```

```
[Out] -1/3/d*a*(6*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+9*C*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*cos(d*x+c)^2+12*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+18*C*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*cos(d*x+c)+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+9*C*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*C
*cos(d*x+c)^2*sin(d*x+c)+10*A*sin(d*x+c)*cos(d*x+c)+6*B*cos(d*x+c)*sin(d*x+
c)+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 2.83495, size = 3680, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((2*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - a*sin(2*d*x + 2*c) - 2*(a*cos(2*d*x + 2*c) + a)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + (2*a*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + a*cos(2*d*x + 2*c) + 2*(a*cos(2*d*x + 2*c) + a)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a)*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + ((a*cos(2*d*x + 2*c)^2 + a*sin(2*d
*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
```

```
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a*cos(2*d*x + 2*c)^2 + a
*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a*cos(2*d*x + 2*
c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^
2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 1))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1) + 3*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 2*(3*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*
d*x + 2*c) + (a*cos(2*d*x + 2*c)^2*sin(d*x + c) + a*sin(2*d*x + 2*c)^2*sin(
d*x + c) + 2*a*cos(2*d*x + 2*c)*sin(d*x + c) + a*sin(d*x + c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*(a*cos(2*d*x + 2*c) + a)*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((a*cos(d*x + c)
- a)*cos(2*d*x + 2*c)^2 + (a*cos(d*x + c) - a)*sin(2*d*x + 2*c)^2 + 2*(a*co
s(d*x + c) - a)*cos(2*d*x + 2*c) + a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 3*((a*cos(2*d*x + 2*c)^2 + a*
sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2(-(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - (a*cos(2*d*x + 2*c)^2 + a*s
in(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2(-(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2
```

```
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - (a*cos(2*d*x + 2*c)^2 + a*si
n(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos
(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a
))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 1
6*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

Fricas [A] time = 2.15974, size = 419, normalized size = 2.19

$$-\frac{3\left((2B+3C)a\cos(dx+c)^2+(2B+3C)a\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(3Ca\cos(dx+c)^2+2(5A+3B)a}{}}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*((2*B + 3*C)*a*cos(d*x + c)^2 + (2*B + 3*C)*a*cos(d*x + c))*sqrt(a)
*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
 - (3*C*a*cos(d*x + c)^2 + 2*(5*A + 3*B)*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(5/2), x)
```

3.1323 $\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=201

$$\frac{a^{3/2}(8A+12B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a(4A-C}{}$$

```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^2*(8*A - 4*B - 5
*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*(4
*A - C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (
2*A*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.71737, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(8A+12B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a(4A-C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^2*(8*A - 4*B - 5
*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*(4
*A - C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (
2*A*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{3/2}\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\ldots}{\ldots}$$

$$= \frac{2A(a + a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,\mathrm{s}\ldots}{d}$$

$$= -\frac{a(4A-C)\sqrt{a + a\cos(c+dx)}\sin(c+\ldots}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{a + a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \ldots$$

$$= -\frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{a + a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \ldots$$

$$= \frac{a^{3/2}(8A+12B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx\ldots}{\sqrt{a+a\cos(c+\ldots}}\right.\ldots}{4d}$$

Mathematica [A] time = 0.570374, size = 127, normalized size = 0.63

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(8A+12B+7C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)} + 2\,\mathrm{si}\ldots\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
(8*A + 12*B + 7*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*
(8*A + C + (4*B + 7*C)*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2])
)/(8*d)
```

Maple [B] time = 0.173, size = 462, normalized size = 2.3

$$\frac{a\cos(dx+c)}{4d(1+\cos(dx+c))}\left(8A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)+12B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x)
```

```
[Out] 1/4/d*a*(8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+12*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*
cos(d*x+c)+2*C*cos(d*x+c)^2*sin(d*x+c)+7*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c
)+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/cos(d*x+c))+4*B*cos(d*x+c)*sin(d*x+c)+12*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))+7*C*cos(d*x+c)*sin(d*x+c)+7*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*A*sin(d*x+c)
)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.47379, size = 392, normalized size = 1.95

$$-\frac{((8A+12B+7C)a\cos(dx+c)+(8A+12B+7C)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(2Ca\cos(dx+c)^2+(4B+7C)a\cos}{\sqrt{}}}{4(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(((8*A + 12*B + 7*C)*a*cos(d*x + c) + (8*A + 12*B + 7*C)*a)*sqrt(a)*ar
ctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) -
(2*C*a*cos(d*x + c)^2 + (4*B + 7*C)*a*cos(d*x + c) + 8*A*a)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(3/2), x)
```

3.1324 $\int (a+a\cos(c+dx))^{3/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right.$

Optimal. Leaf size=201

$$\frac{a^{3/2}(24A+14B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a(2B+}{}$$

```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 30*B
 + 19*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) +
 (a*(2*B + C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]
]) + (C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.726932, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3045, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(24A+14B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a(2B+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]],x]
```


```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 30*B
 + 19*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) +
 (a*(2*B + C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]
]) + (C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$= \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{a(2B+C)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{4d\sqrt{\sec(c+dx)}}$$

$$= \frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + $$

$$= \frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + $$

$$= \frac{a^{3/2}(24A+14B+11C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a+a\cos(c}}\right.}{8d}$$

Mathematica [A] time = 0.890116, size = 145, normalized size = 0.72

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(24A+14B+11C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(3*Sqrt[2]*(24*A + 14*B + 11*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]
+ 2*Sqrt[Cos[c + d*x]]*(24*A + 42*B + 37*C + 2*(6*B + 11*C)*Cos[c + d*x] +
4*C*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.185, size = 369, normalized size = 1.8

$$-\frac{a\left((\cos(dx+c))^2-1\right)}{24\,d\,(\sin(dx+c))^2}\left(8\,C\sin(dx+c)(\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12\,B\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+}{1+\cos(dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/24/d*a*(8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+12
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+24*A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+42*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+72*A*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+42*B*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))+33*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.57778, size = 5162, normalized size = 25.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
```

```
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
 6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arct
an2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + (4*(a*cos(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4
)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
```

(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))),
(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(c
os(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1))) - 1) - a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*
cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2

```
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)) - 1))*sqrt(a))*C)/d
```

Fricas [A] time = 3.62782, size = 450, normalized size = 2.24

$$-\frac{3\left((24A+14B+11C)a\cos(dx+c)+(24A+14B+11C)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8Ca\cos(dx+c)^3+2(6B+\right.}{}}{24\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*((24*A + 14*B + 11*C)*a*cos(d*x + c) + (24*A + 14*B + 11*C)*a)*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))) - (8*C*a*cos(d*x + c)^3 + 2*(6*B + 11*C)*a*cos(d*x + c)^2 + 3*(8*A + 1
4*B + 11*C)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(
d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)*sqrt(sec(d*x + c)), x)
```

3.1325 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+88B+75C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(11}{64d\sqrt{s}}$$

```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(48*A + 56
*B + 39*C)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2))
 + (a*(8*B + 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sec[c + d*x]
^(3/2)) + (C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/
2)) + (a^2*(112*A + 88*B + 75*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.806643, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(112A+88B+75C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(11}{64d\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```


```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(48*A + 56
*B + 39*C)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2))
 + (a*(8*B + 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sec[c + d*x]
^(3/2)) + (C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/
2)) + (a^2*(112*A + 88*B + 75*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\,\mathrm{c}$$

$$= \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(8B+3C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{C(a+}{}$$

$$= \frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(8B+3C)\sqrt{}}{2}$$

$$= \frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(8B+3C)\sqrt{}}{2}$$

$$= \frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(8B+3C)\sqrt{}}{2}$$

$$= \frac{a^{3/2}(112A+88B+75C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(}}{64d}$$

Mathematica [A] time = 0.930839, size = 167, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(112A+88B+75C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(112*A + 88*B + 75*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]
 + (336*A + 296*B + 285*C + 2*(48*A + 88*B + 93*C)*Cos[c + d*x] + 4*(8*B +
15*C)*Cos[2*(c + d*x)] + 12*C*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3
*(c + d*x))/2])))/(384*d)
```

Maple [B] time = 0.204, size = 481, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] 1/192/d*a*(-1+cos(d*x+c))^2*(48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+64*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)^2+120*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+96*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+176*B*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+150*C*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+336*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)+264*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+225*C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+336*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))+264*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))+225*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.93638, size = 514, normalized size = 2.03

$$-\frac{3\left((112\,A+88\,B+75\,C)a\cos\left(dx+c\right)+(112\,A+88\,B+75\,C)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,Ca\cos(dx+c)^4+8}{}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/192*(3*((112*A + 88*B + 75*C)*a*cos(d*x + c) + (112*A + 88*B + 75*C)*a)*
sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
 + c))) - (48*C*a*cos(d*x + c)^4 + 8*(8*B + 15*C)*a*cos(d*x + c)^3 + 2*(48*
A + 88*B + 75*C)*a*cos(d*x + c)^2 + 3*(112*A + 88*B + 75*C)*a*cos(d*x + c))
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
+ d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/sqrt(sec(d*x + c)), x)
```

3.1326 $\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{a^2(176A+150B+133C)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+150B+133C)\sqrt{\cos(c+dx)}}{128d}$$

```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A +
 90*B + 67*C)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5
/2)) + (a*(10*B + 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c +
 d*x]^(5/2)) + (C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(5/2)) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos
[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])
/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.90757, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(176A+150B+133C)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(176A+150B+133C)\sqrt{\cos(c+dx)}}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A +
 90*B + 67*C)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5
/2)) + (a*(10*B + 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c +
 d*x]^(5/2)) + (C*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(5/2)) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos
[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])
/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{c}$$

$$= \frac{C(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(10B+3C)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{40d\sec^{\frac{5}{2}}(c+dx)} + \frac{C(a+}{}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(10B+3C}{}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(10B+3C}{}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(10B+3C}{}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(10B+3C}{}$$

$$= \frac{a^{3/2}(176A+150B+133C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\mathrm{co}}}{128d}$$

Mathematica [A] time = 1.95439, size = 190, normalized size = 0.63

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(176A+150B+133C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(15*Sqrt[2]*(176*A + 150*B + 133*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/
2]] + 2*Sqrt[Cos[c + d*x]]*(2960*A + 2850*B + 2671*C + 2*(880*A + 930*B + 1
007*C)*Cos[c + d*x] + 4*(80*A + 150*B + 181*C)*Cos[2*(c + d*x)] + 120*B*Cos
[3*(c + d*x)] + 228*C*Cos[3*(c + d*x)] + 48*C*Cos[4*(c + d*x)])*Sin[(c + d*
x)/2]))/(3840*d)
```

Maple [B] time = 0.222, size = 589, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/1920/d*a*(-1+cos(d*x+c))^3*(384*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+480*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+912*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+64
0*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1200*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+1064*C*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1760*A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+1500*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+1330*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)+2640*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2250*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1995*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)+2640*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))+2250*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))+1995*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))
*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/c
os(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.92131, size = 595, normalized size = 1.96

$$-\frac{15\left(\left(176\,A+150\,B+133\,C\right)a\cos\left(dx+c\right)+\left(176\,A+150\,B+133\,C\right)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(384\,Ca\cos(d}{}}{1920}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1920*(15*((176*A + 150*B + 133*C)*a*cos(d*x + c) + (176*A + 150*B + 133*
C)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))) - (384*C*a*cos(d*x + c)^5 + 48*(10*B + 19*C)*a*cos(d*x + c)^4
 + 8*(80*A + 150*B + 133*C)*a*cos(d*x + c)^3 + 10*(176*A + 150*B + 133*C)*a
*cos(d*x + c)^2 + 15*(176*A + 150*B + 133*C)*a*cos(d*x + c))*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2
)/sec(d*x + c)^(3/2), x)
```

3.1327 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=334

$$\frac{2a^2(136A+182B+143C)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2522B+2717C)\sin(c+dx)\sec^{\frac{7}{2}}}{9009d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (16*a^3*(8368*A + 9230*B + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045
*d*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(8368*A + 9230*B + 10439*C)*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(8368*A
 + 9230*B + 10439*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(2224*A + 2522*B + 2717*C)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(9009*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(1287*d) + (2*a*(
5*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(1
43*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(13/2)*Sin[c + d*x])/(
13*d)
```

Rubi [A] time = 1.17617, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2980, 2772, 2771}

$$\frac{2a^2(136A+182B+143C)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2522B+2717C)\sin(c+dx)\sec^{\frac{7}{2}}}{9009d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(15/2),x]
```

```
[Out] (16*a^3*(8368*A + 9230*B + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045
*d*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(8368*A + 9230*B + 10439*C)*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(8368*A
 + 9230*B + 10439*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(2224*A + 2522*B + 2717*C)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(9009*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(1287*d) + (2*a*(
5*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(1
43*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(13/2)*Sin[c + d*x])/(
```

13*d)

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^{\frac{15}{2}}(c + dx)\, dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{(a + a}{}$$

$$= \frac{2A(a + a\cos(c + dx))^{5/2} \sec^{\frac{13}{2}}(c + dx)}{13d}$$

$$= \frac{2a(5A + 13B)(a + a\cos(c + dx))^{3/2} \mathrm{se}}{143d}$$

$$= \frac{2a^2(136A + 182B + 143C)\sqrt{a + a\cos}}{1287d}$$

$$= \frac{2a^3(2224A + 2522B + 2717C) \sec^{\frac{7}{2}}(c}{9009d\sqrt{a + a\cos(c + d}}$$

$$= \frac{2a^3(8368A + 9230B + 10439C) \sec^{\frac{5}{2}}(}{15015d\sqrt{a + a\cos(c +}}$$

$$= \frac{8a^3(8368A + 9230B + 10439C) \sec^{\frac{3}{2}}(}{45045d\sqrt{a + a\cos(c +}}$$

$$= \frac{16a^3(8368A + 9230B + 10439C)\sqrt{\mathrm{sec}}}{45045d\sqrt{a + a\cos(c +}}$$

Mathematica [A] time = 1.24411, size = 224, normalized size = 0.67

$$\frac{a^2 \tan\left(\frac{1}{2}(c + dx)\right) \sec^{\frac{13}{2}}(c + dx)\sqrt{a(\cos(c + dx) + 1)}(70(5552A + 5083B + 4576C)\cos(c + dx) + 14(30334A + 31850B}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(15/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(343612*A + 325910*B + 322751*C + 70*(5552*
A + 5083*B + 4576*C)*Cos[c + d*x] + 14*(30334*A + 31850*B + 32747*C)*Cos[2*
(c + d*x)] + 125520*A*Cos[3*(c + d*x)] + 138450*B*Cos[3*(c + d*x)] + 141570
```

```
*C*Cos[3*(c + d*x)] + 125520*A*Cos[4*(c + d*x)] + 138450*B*Cos[4*(c + d*x)]
 + 156585*C*Cos[4*(c + d*x)] + 16736*A*Cos[5*(c + d*x)] + 18460*B*Cos[5*(c
+ d*x)] + 20878*C*Cos[5*(c + d*x)] + 16736*A*Cos[6*(c + d*x)] + 18460*B*Cos
[6*(c + d*x)] + 20878*C*Cos[6*(c + d*x)])*Sec[c + d*x]^(13/2)*Tan[(c + d*x)
/2])/(180180*d)
```

Maple [A] time = 0.238, size = 240, normalized size = 0.7

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(66944\,A\,(\cos(dx+c))^6+73840\,B\,(\cos(dx+c))^6+83512\,C\,(\cos(dx+c))^6+33472\,A\,(\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(15/2
),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+73840*B*cos(d*x+c)^6+8
3512*C*cos(d*x+c)^6+33472*A*cos(d*x+c)^5+36920*B*cos(d*x+c)^5+41756*C*cos(d
*x+c)^5+25104*A*cos(d*x+c)^4+27690*B*cos(d*x+c)^4+31317*C*cos(d*x+c)^4+2092
0*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18590*C*cos(d*x+c)^3+18305*A*cos(d*x+
c)^2+14560*B*cos(d*x+c)^2+5005*C*cos(d*x+c)^2+11970*A*cos(d*x+c)+4095*B*cos
(d*x+c)+3465*A)*cos(d*x+c)*(1/cos(d*x+c))^(15/2)*(a*(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)
```

Maxima [B] time = 1.97296, size = 1542, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(15/2),x, algorithm="maxima")
```

```
[Out] 8/45045*((45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*sq
rt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 88
980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqrt(2)*a
```

```
^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c
) + 1)^10 + 1)) + 65*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
 3003*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6930*sqrt(2)*a^
(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10098*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 9053*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x +
 c) + 1)^9 - 4875*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 1
500*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 200*sqrt(2)*a^(
5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*B*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 1)) + 143*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
1575*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3654*sqrt(2)*a^(
5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5130*sqrt(2)*a^(5/2)*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 4595*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 2535*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 780
*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 104*sqrt(2)*a^(5/2
)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*C*(sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
+ 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) + 1
)^10 + 1)))/d
```

Fricas [A] time = 1.71169, size = 549, normalized size = 1.64

$$\frac{2\left(8\,(8368\,A + 9230\,B + 10439\,C)a^2\cos{(dx+c)}^6 + 4\,(8368\,A + 9230\,B + 10439\,C)a^2\cos{(dx+c)}^5 + 3\,(8368\,A + 9230}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(15/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(8*(8368*A + 9230*B + 10439*C)*a^2*cos(d*x + c)^6 + 4*(8368*A + 923
0*B + 10439*C)*a^2*cos(d*x + c)^5 + 3*(8368*A + 9230*B + 10439*C)*a^2*cos(d
*x + c)^4 + 5*(4184*A + 4615*B + 3718*C)*a^2*cos(d*x + c)^3 + 35*(523*A + 4
```

```
16*B + 143*C)*a^2*cos(d*x + c)^2 + 315*(38*A + 13*B)*a^2*cos(d*x + c) + 346
5*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^7 + d*cos(d
*x + c)^6)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(15/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1328 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$
$dx)\,dx$

Optimal. Leaf size=284

$$\frac{2a^2(32A+44B+33C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{231d}+\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)\sec^{\frac{5}{2}}(c}{3465d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a^3*(2840*A + 3212*B + 3795*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(1160*A + 13
64*B + 1485*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^
(7/2)*Sin[c + d*x])/(231*d) + (2*a*(5*A + 11*B)*(a + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*S
ec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 1.05841, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2980, 2772, 2771}

$$\frac{2a^2(32A+44B+33C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{231d}+\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)\sec^{\frac{5}{2}}(c}{3465d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(13/2),x]
```


```
[Out] (4*a^3*(2840*A + 3212*B + 3795*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(1160*A + 13
64*B + 1485*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^
(7/2)*Sin[c + d*x])/(231*d) + (2*a*(5*A + 11*B)*(a + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*S
ec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
```

```
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a}{}$$

$$= \frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)}{11d}$$

$$= \frac{2a(5A+11B)(a+a\cos(c+dx))^{3/2}\text{se}}{99d}$$

$$= \frac{2a^2(32A+44B+33C)\sqrt{a+a\cos(c+}}{231d}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sec^{\frac{5}{2}}(c}{3465d\sqrt{a+a\cos(c+d}}$$

$$= \frac{2a^3(2840A+3212B+3795C)\sec^{\frac{3}{2}}(c}{3465d\sqrt{a+a\cos(c+a}}$$

$$= \frac{4a^3(2840A+3212B+3795C)\sqrt{\sec(c}}{3465d\sqrt{a+a\cos(c+a}}$$

Mathematica [A] time = 0.981738, size = 190, normalized size = 0.67

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}((50140A+49654B+49830C)\cos(c+dx)+4(4615A+4642B+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(13/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(18140*A + 15356*B + 13365*C + (50140*A + 4
9654*B + 49830*C)*Cos[c + d*x] + 4*(4615*A + 4642*B + 4290*C)*Cos[2*(c + d*
x)] + 18460*A*Cos[3*(c + d*x)] + 20878*B*Cos[3*(c + d*x)] + 22935*C*Cos[3*(
c + d*x)] + 2840*A*Cos[4*(c + d*x)] + 3212*B*Cos[4*(c + d*x)] + 3795*C*Cos[
4*(c + d*x)] + 2840*A*Cos[5*(c + d*x)] + 3212*B*Cos[5*(c + d*x)] + 3795*C*C
os[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(c + d*x)/2])/(13860*d)
```

Maple [A] time = 0.22, size = 207, normalized size = 0.7

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(5680\,A\,(\cos(dx+c))^5+6424\,B\,(\cos(dx+c))^5+7590\,C\,(\cos(dx+c))^5+2840\,A\,(\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2
),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+7590
*C*cos(d*x+c)^5+2840*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+3795*C*cos(d*x+c)^4
+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3+1980*C*cos(d*x+c)^3+1775*A*cos(d*x
+c)^2+1430*B*cos(d*x+c)^2+495*C*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*
x+c)+315*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(13/2)/sin(d
*x+c)
```

Maxima [B] time = 1.97893, size = 1357, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="maxima")
```

```
[Out] 8/3465*(5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d
```

```
*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a
^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 11*(315*sqrt(2)*a^(5/2)*si
n(d*x + c)/(cos(d*x + c) + 1) - 1260*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 2394*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2736*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1859*sqrt(2)*a^(
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 676*sqrt(2)*a^(5/2)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 + 104*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x +
c) + 1)^13)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*
(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 1)) + 165*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 98
*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 196*sqrt(2)*a^(5/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 218*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 + 143*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 52*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 8*sqrt(2)*a^
(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*C*(sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Fricas [A] time = 1.51133, size = 464, normalized size = 1.63

$$\frac{2\left(2\,(2840\,A+3212\,B+3795\,C)a^2\cos\left(dx+c\right)^5+(2840\,A+3212\,B+3795\,C)a^2\cos\left(dx+c\right)^4+3\,(710\,A+803\,B+66}{3465\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(2*(2840*A + 3212*B + 3795*C)*a^2*cos(d*x + c)^5 + (2840*A + 3212*B
+ 3795*C)*a^2*cos(d*x + c)^4 + 3*(710*A + 803*B + 660*C)*a^2*cos(d*x + c)^3
 + 5*(355*A + 286*B + 99*C)*a^2*cos(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d
*x + c) + 315*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)
```

```
^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1329 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=234

$$\frac{2a^2(64A+90B+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a^3(8A+10B+11C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a^3*(584*A + 690*B + 903*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a^3*(8*A + 10*B + 11*C)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 90*B + 63*C)*Sqrt
[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*a*(5*A +
 9*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) +
(2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.953384, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3043, 2975, 2980, 2771}

$$\frac{2a^2(64A+90B+63C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2a^3(8A+10B+11C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(11/2),x]
```


```
[Out] (2*a^3*(584*A + 690*B + 903*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a^3*(8*A + 10*B + 11*C)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(64*A + 90*B + 63*C)*Sqrt
[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*a*(5*A +
 9*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) +
(2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^{\frac{11}{2}}(c + dx)\, dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{(a + a}{}$$

$$= \frac{2A(a + a\cos(c + dx))^{5/2} \sec^{\frac{9}{2}}(c + dx)}{9d}$$

$$= \frac{2a(5A + 9B)(a + a\cos(c + dx))^{3/2} \sec}{63d}$$

$$= \frac{2a^2(64A + 90B + 63C)\sqrt{a + a\cos(c +}}{315d}$$

$$= \frac{2a^3(8A + 10B + 11C)\sec^{\frac{3}{2}}(c + dx)\sin}{15d\sqrt{a + a\cos(c + dx)}}$$

$$= \frac{2a^3(584A + 690B + 903C)\sqrt{\sec(c + d}}{315d\sqrt{a + a\cos(c + dx)}}$$

Mathematica [A] time = 1.35216, size = 158, normalized size = 0.68

$$\frac{a^2 \tan\left(\frac{1}{2}(c + dx)\right) \sec^{\frac{9}{2}}(c + dx)\sqrt{a(\cos(c + dx) + 1)}(2(1396A + 1215B + 882C)\cos(c + dx) + 4(803A + 870B + 966C)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(2908*A + 2790*B + 2961*C + 2*(1396*A + 121
5*B + 882*C)*Cos[c + d*x] + 4*(803*A + 870*B + 966*C)*Cos[2*(c + d*x)] + 58
4*A*Cos[3*(c + d*x)] + 690*B*Cos[3*(c + d*x)] + 588*C*Cos[3*(c + d*x)] + 58
4*A*Cos[4*(c + d*x)] + 690*B*Cos[4*(c + d*x)] + 903*C*Cos[4*(c + d*x)])*Sec
[c + d*x]^(9/2)*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.203, size = 174, normalized size = 0.7

$$-\frac{2a^2(-1 + \cos(dx + c))\left(584A(\cos(dx + c))^4 + 690B(\cos(dx + c))^4 + 903C(\cos(dx + c))^4 + 292A(\cos(dx + c))^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2
),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+903*C*c
os(d*x+c)^4+292*A*cos(d*x+c)^3+345*B*cos(d*x+c)^3+294*C*cos(d*x+c)^3+219*A*
cos(d*x+c)^2+180*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+130*A*cos(d*x+c)+45*B*cos
(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)/sin
(d*x+c)
```

Maxima [B] time = 2.01837, size = 1169, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 8/315*((315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 1)) + 15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
77*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 119*sqrt(2)*a^(5/2
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)) + 21*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) +
 1) - 65*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 113*sqrt(2)*
a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*C*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
```

```
)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)))/d
```

Fricas [A] time = 1.51109, size = 382, normalized size = 1.63

$$\frac{2\left((584\,A + 690\,B + 903\,C)a^2\cos\left(dx + c\right)^4 + (292\,A + 345\,B + 294\,C)a^2\cos\left(dx + c\right)^3 + 3\,(73\,A + 60\,B + 21\,C)a^2\cos\left(\right.\right.}{315\left(d\cos\left(dx + c\right)^5 + d\cos\left(dx + c\right)^4\right)\sqrt{\cos\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*((584*A + 690*B + 903*C)*a^2*cos(d*x + c)^4 + (292*A + 345*B + 294*C)
*a^2*cos(d*x + c)^3 + 3*(73*A + 60*B + 21*C)*a^2*cos(d*x + c)^2 + 5*(26*A +
 9*B)*a^2*cos(d*x + c) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((
d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1330 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=242

$$\frac{2a^2(40A+56B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a^3(160A+224B+245C)\sin(c+dx)\sqrt{\sec(c+d}}{105d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(160*A + 224*B + 245*C)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(40*A +
 56*B + 35*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d) + (2*a*(5*A + 7*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c
 + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(7*d)
```

Rubi [A] time = 0.878197, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2980, 2774, 216}

$$\frac{2a^2(40A+56B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a^3(160A+224B+245C)\sin(c+dx)\sqrt{\sec(c+d}}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(9/2),x]
```


```
[Out] (2*a^(5/2)*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(160*A + 224*B + 245*C)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(40*A +
 56*B + 35*C)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d) + (2*a*(5*A + 7*B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c
 + d*x])/(35*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a}{}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,\mathrm{s}}{7d}$$

$$=\frac{2a(5A+7B)(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}}{35d}$$

$$=\frac{2a^2(40A+56B+35C)\sqrt{a+a\cos(c+}}{105d}$$

$$=\frac{2a^3(160A+224B+245C)\sqrt{\sec(c+dx}}{105d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{2a^3(160A+224B+245C)\sqrt{\sec(c+dx}}{105d\sqrt{a+a\cos(c+dx)}}$$

$$=\frac{2a^{5/2}C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{d}$$

Mathematica [A] time = 1.66847, size = 172, normalized size = 0.71

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((930A+987B+840C)\cos(c+dx)+2(115A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(420*Sq
rt[2]*C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 2*(290*A + 19
6*B + 70*C + (930*A + 987*B + 840*C)*Cos[c + d*x] + 2*(115*A + 98*B + 35*C)
*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos[3*(c + d*x)] + 280*C
*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(420*d)
```

Maple [B] time = 0.181, size = 522, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x)
```

```
[Out] -2/105/d*a^2*(105*C*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+420*C*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c))+630*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)
*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+420*C*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))+105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+230*A*sin(d*x+c)
*cos(d*x+c)^3+301*B*cos(d*x+c)^3*sin(d*x+c)+280*C*sin(d*x+c)*cos(d*x+c)^3+1
15*A*sin(d*x+c)*cos(d*x+c)^2+98*B*cos(d*x+c)^2*sin(d*x+c)+35*C*cos(d*x+c)^2
*sin(d*x+c)+60*A*sin(d*x+c)*cos(d*x+c)+21*B*cos(d*x+c)*sin(d*x+c)+15*A*sin(
d*x+c))*cos(d*x+c)*sin(d*x+c)^6*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/
2)/(-1+cos(d*x+c))^3/(1+cos(d*x+c))^4
```

Maxima [B] time = 2.89356, size = 3488, normalized size = 14.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 1/210*(7*(6*(a^2*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 25*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*
```

c) + a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((15*a^2*sin(6*d*x + 6*c) + 50*a^2*sin(4*d*x + 4*c) + 58*a^2*sin(2*d*x + 2*c) - 20*(3*a^2*sin(6*d*x + 6*c) + 10*a^2*sin(4*d*x + 4*c) + 11*a^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(3*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 11*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (15*a^2*cos(6*d*x + 6*c) + 50*a^2*cos(4*d*x + 4*c) + 58*a^2*cos(2*d*x + 2*c) + 23*a^2 + 20*(3*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 11*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(3*a^2*sin(6*d*x + 6*c) + 10*a^2*sin(4*d*x + 4*c) + 11*a^2*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 25*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 15*((a^2*cos(2*d*x + 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)^2 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)^2 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)^2 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x

```
+ 2*c)^4 + a^2*sin(2*d*x + 2*c)^4 + 4*a^2*cos(2*d*x + 2*c)^3 + 6*a^2*cos(2*
d*x + 2*c)^2 + 4*a^2*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2 + 2*a^2*c
os(2*d*x + 2*c) + a^2)*sin(2*d*x + 2*c)^2 + a^2)*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1))*sqrt(a))*C/(cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4
 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 80*(21
*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/2)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*s
qrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 112*(15*sqrt(2)*a^(5/2)*sin
(d*x + c)/(cos(d*x + c) + 1) - 50*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sq
rt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)
^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 1)))/d
```

Fricas [A] time = 1.85487, size = 504, normalized size = 2.08

$$-\frac{2\left(105\left(Ca^2\cos(dx+c)^4+Ca^2\cos(dx+c)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left((230A+301B+280C)a^2\cos(dx+c)^3+(115A+\cdots\right.}{\cdots}\right)}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(105*(C*a^2*cos(d*x + c)^4 + C*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((230*A
 + 301*B + 280*C)*a^2*cos(d*x + c)^3 + (115*A + 98*B + 35*C)*a^2*cos(d*x +
c)^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1331 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \ldots dx) \, dx$

Optimal. Leaf size=243

$$-\frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+10B+5C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{5d}+\frac{a^{5/2}(2B+5C\ldots}{\ldots}$$

```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(64*A + 70*B + 15*C)*Sin[
c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(8*A
+ 10*B + 5*C)*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*
d) + (2*a*(A + B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(5*d)
```

Rubi [A] time = 0.952403, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2975, 2981, 2774, 216}

$$-\frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+10B+5C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}{5d}+\frac{a^{5/2}(2B+5C\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(7/2),x]
```


```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(64*A + 70*B + 15*C)*Sin[
c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(8*A
+ 10*B + 5*C)*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*
d) + (2*a*(A + B)*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\,\mathrm{co}}{}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,\mathrm{si}}{5d}$$

$$=\frac{2a(A+B)(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}{3d}$$

$$=\frac{2a^2(8A+10B+5C)\sqrt{a+a\cos(c+dx)}}{5d}$$

$$=-\frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=-\frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=\frac{a^{5/2}(2B+5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\mathrm{cos}}}{d}$$

Mathematica [A] time = 1.40901, size = 156, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((112A+40B+45C)\cos(c+dx)+4(43A+40B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(60*Sqr
t[2]*(2*B + 5*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(1
96*A + 160*B + 60*C + (112*A + 40*B + 45*C)*Cos[c + d*x] + 4*(43*A + 40*B +
 15*C)*Cos[2*(c + d*x)] + 15*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(120*d)
```

Maple [B] time = 0.183, size = 673, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x)
```

```
[Out] 1/15/d*a^2*(30*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*C*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))*cos(d*x+c)^3+90*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+225*C*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*cos(d*x+c)^2+90*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+225*C*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))*cos(d*x+c)+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*C*(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))+15*C*sin(d*x+c)*cos(d*x+c)^3+86*A*sin(d*x+c)*cos(d*x+c)^2+80*B*
cos(d*x+c)^2*sin(d*x+c)+30*C*cos(d*x+c)^2*sin(d*x+c)+28*A*sin(d*x+c)*cos(d*
x+c)+10*B*cos(d*x+c)*sin(d*x+c)+6*A*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^4*(1/
cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))
^3
```

Maxima [B] time = 3.10637, size = 4362, normalized size = 17.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

[Out] 1/60*(10*(10*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*a^(5/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*((3*a^2*sin(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*sin(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c) + 4*a^2 + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(3*a^2*sin(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 15*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4) + 5*(2*(5*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 3*(a^2*cos(2*d*x + 2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x

+ c) + 2*a^2*cos(2*d*x + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*((a^2*cos(d*x + c) - a^2)
*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x
+ 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a) + 15*((a^2*cos(2*d*x + 2*
c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x
+ c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d
*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan
2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - c
os(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d
*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x
+ c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - (a^2*
cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2
+ a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*((12*a^2*sin(5*d*x + 5*c) + 15*a
^2*sin(4*d*x + 4*c) + 24*a^2*sin(3*d*x + 3*c) + 35*a^2*sin(2*d*x + 2*c) + 1
2*a^2*sin(d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - (12*a^2*cos(5*d*x + 5*c) + 15*a^2*cos(4*d*x + 4*c) + 24*a^2*cos(3*d*x +
3*c) + 35*a^2*cos(2*d*x + 2*c) + 12*a^2*cos(d*x + c) + 20*a^2)*sin(5/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 27*(a^2*cos(2*d*x + 2*c)^2
+ a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*C/(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4) + 32*(15*sqrt(2)*a^(5/2)*s
in(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*s
qrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A/((sin(d*x + c)/(cos(d
x + c) + 1) + 1)^(7/2)(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)))/d

Fricas [A] time = 2.30913, size = 502, normalized size = 2.07

$$-\frac{15\left(\left(2\,B+5\,C\right)a^{2}\cos\left(dx+c\right)^{3}+\left(2\,B+5\,C\right)a^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(15\,Ca^{2}\cos(dx+c)^{3}+2\,(43\,A\cdots\right.}{\cdots}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*((2*B + 5*C)*a^2*cos(d*x + c)^3 + (2*B + 5*C)*a^2*cos(d*x + c)^2)
*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c))) - (15*C*a^2*cos(d*x + c)^3 + 2*(43*A + 40*B + 15*C)*a^2*cos(d*x +
c)^2 + 2*(14*A + 5*B)*a^2*cos(d*x + c) + 6*A*a^2)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1332 $\int (a+a\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ dx) dx

Optimal. Leaf size=253

$$\frac{a^{5/2}(8A+20B+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} - \frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}} - \frac{a^2(8A}{}$$

```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(56*A + 12*B
- 27*C)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(a^2*(8*A + 4*B - C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c
 + d*x]]) + (2*a*(5*A + 3*B)*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(3*d)
```

Rubi [A] time = 0.948697, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3043, 2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(8A+20B+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} - \frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}} - \frac{a^2(8A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(5/2),x]
```


```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(56*A + 12*B
- 27*C)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(a^2*(8*A + 4*B - C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c
 + d*x]]) + (2*a*(5*A + 3*B)*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
```

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cdots}{\cdots}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}\cdots}{3d}$$

$$=\frac{2a(5A+3B)(a+a\cos(c+dx))^{3/2}\sqrt{\sec\cdots}}{3d}$$

$$=-\frac{a^2(8A+4B-C)\sqrt{a+a\cos(c+dx)}\,\mathrm{s}\cdots}{2d\sqrt{\sec(c+dx)}}$$

$$=-\frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\cdots$$

$$=-\frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\cdots$$

$$=\frac{a^{5/2}(8A+20B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+\cdots}{\sqrt{a+a\cos(c\cdots}}\right.}{4d}$$

Mathematica [A] time = 1.13666, size = 156, normalized size = 0.62

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(6\sqrt{2}(8A+20B+19C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\,\mathrm{s}\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(6*Sqrt
[2]*(8*A + 20*B + 19*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2)
 + 2*(16*A + 12*B + 33*C + (128*A + 48*B + 9*C)*Cos[c + d*x] + 3*(4*B + 11*
C)*Cos[2*(c + d*x)] + 3*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.184, size = 711, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x)
```

```
[Out] -1/12/d*a^2*(24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+60*B*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))+57*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+48*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))*cos(d*x+c)+120*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+114*C*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c))*cos(d*x+c)+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+60*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+6*C*sin(d*x+c)*cos(d*x+c)^3+57*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+12*B*cos
(d*x+c)^2*sin(d*x+c)+33*C*cos(d*x+c)^2*sin(d*x+c)+64*A*sin(d*x+c)*cos(d*x+c
)+24*B*cos(d*x+c)*sin(d*x+c)+8*A*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^2*(1/cos
(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.92421, size = 504, normalized size = 1.99

$$-\frac{3\left((8A+20B+19C)a^2\cos(dx+c)^2+(8A+20B+19C)a^2\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(6Ca}{}}{12\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*((8*A + 20*B + 19*C)*a^2*cos(d*x + c)^2 + (8*A + 20*B + 19*C)*a^2*
cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(s
qrt(a)*sin(d*x + c))) - (6*C*a^2*cos(d*x + c)^3 + 3*(4*B + 11*C)*a^2*cos(d*
x + c)^2 + 8*(8*A + 3*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1333 $\int (a+a\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ dx) dx

Optimal. Leaf size=251

$$\frac{a^{5/2}(40A+38B+25C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A}{}$$

```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) - (a^3*(24*A - 54*B
 - 49*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (a^2*(8*A - 2*B - 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Se
c[c + d*x]]) - (a*(6*A - C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/d
```

Rubi [A] time = 0.954069, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(40A+38B+25C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}-\frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{a^2(8A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) - (a^3*(24*A - 54*B
 - 49*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (a^2*(8*A - 2*B - 3*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Se
c[c + d*x]]) - (a*(6*A - C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]]) + (2*A*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cdots}{}$$

$$=\frac{2A(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,\mathrm{s}\cdots}{d}$$

$$=-\frac{a(6A-C)(a+a\cos(c+dx))^{3/2}\sin(c\cdots}{3d\sqrt{\sec(c+dx)}}$$

$$=-\frac{a^2(8A-2B-3C)\sqrt{a+a\cos(c+dx)}\cdots}{4d\sqrt{\sec(c+dx)}}$$

$$=-\frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\cdots$$

$$=-\frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\cdots$$

$$=\frac{a^{5/2}(40A+38B+25C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c\cdots}{\sqrt{a+a\cos(\cdots}}\right.}{8d}$$

Mathematica [A] time = 0.995426, size = 156, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(40A+38B+25C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt
[2]*(40*A + 38*B + 25*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]
] + 2*(48*A + 6*B + 17*C + 3*(8*A + 22*B + 27*C)*Cos[c + d*x] + (6*B + 17*C
)*Cos[2*(c + d*x)] + 2*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.185, size = 513, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x)
```

```
[Out] 1/24/d*a^2*(8*C*sin(d*x+c)*cos(d*x+c)^3+120*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)+114*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+12*B*cos(d*x+c)^2*sin(d*x+c)+7
5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+34*C*cos(d*x+c)^2*sin(d*x+c)+120*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))+24*A*sin(d*x+c)*cos(d*x+c)+114*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)+66*B*cos(d*x+c)*sin(d*x+c)+75*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*C*cos(d*x+c)*si
n(d*x+c)+48*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c))
)^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.26127, size = 478, normalized size = 1.9

$$-\frac{3\left((40\,A+38\,B+25\,C)a^2\cos\left(dx+c\right)+(40\,A+38\,B+25\,C)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,Ca^2\cos(dx+c)^3\cdots\right.}{\cdots}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*((40*A + 38*B + 25*C)*a^2*cos(d*x + c) + (40*A + 38*B + 25*C)*a^2)
*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c))) - (8*C*a^2*cos(d*x + c)^3 + 2*(6*B + 17*C)*a^2*cos(d*x + c)^2 + 3*
(8*A + 22*B + 25*C)*a^2*cos(d*x + c) + 48*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1334 $\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right.$

Optimal. Leaf size=253

$$\frac{a^{5/2}(304A+200B+163C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2}{}$$

```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A +
 392*B + 299*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a^2*(16*A + 24*B + 17*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(
32*d*Sqrt[Sec[c + d*x]]) + (a*(8*B + 5*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(24*d*Sqrt[Sec[c + d*x]]) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d
*x])/(4*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.940752, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3045, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(304A+200B+163C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]],x]
```


```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A +
 392*B + 299*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a^2*(16*A + 24*B + 17*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(
32*d*Sqrt[Sec[c + d*x]]) + (a*(8*B + 5*C)*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(24*d*Sqrt[Sec[c + d*x]]) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d
*x])/(4*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{c}}{}$$

$$=\frac{C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+$$

$$=\frac{a(8B+5C)(a+a\cos(c+dx))^{3/2}\sin(c+}{24d\sqrt{\sec(c+dx)}}$$

$$=\frac{a^2(16A+24B+17C)\sqrt{a+a\cos(c+dx}}{32d\sqrt{\sec(c+dx)}}$$

$$=\frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=\frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+$$

$$=\frac{a^{5/2}(304A+200B+163C)\sin^{-1}\left(\frac{\sqrt{a}\,\mathrm{si}}{\sqrt{a+a\,\mathrm{c}}}\right.}{64d}$$

Mathematica [A] time = 1.46623, size = 166, normalized size = 0.66

$$\frac{a^2\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(304A+200B+163C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]]*(3*Sqrt[2]*(304*A + 200*B + 163*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)
/2]] + 2*Sqrt[Cos[c + d*x]]*(528*A + 632*B + 581*C + (96*A + 272*B + 362*C)
*Cos[c + d*x] + 4*(8*B + 23*C)*Cos[2*(c + d*x)] + 12*C*Cos[3*(c + d*x)])*Si
```

```
n[(c + d*x)/2]))/(384*d)
```

Maple [B] time = 0.203, size = 477, normalized size = 1.9

$$-\frac{a^2\left((\cos(dx+c))^2-1\right)}{192\,d\,(\sin(dx+c))^2}\left(48\,C\sin(dx+c)\,(\cos(dx+c))^3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+64\,B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/192/d*a^2*(48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+64*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+184*C*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+96*A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+272*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+326*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+528*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+600*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+912*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))+600*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+489*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.95067, size = 541, normalized size = 2.14

$$-\frac{3\left((304\,A+200\,B+163\,C)a^2\cos\left(dx+c\right)+(304\,A+200\,B+163\,C)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(48\,Ca^2\cos}{}}{192\left(d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/192*(3*((304*A + 200*B + 163*C)*a^2*cos(d*x + c) + (304*A + 200*B + 163*
C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) - (48*C*a^2*cos(d*x + c)^4 + 8*(8*B + 23*C)*a^2*cos(d*x + c
)^3 + 2*(48*A + 136*B + 163*C)*a^2*cos(d*x + c)^2 + 3*(176*A + 200*B + 163*
C)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1335 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=301

$$\frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+110B+79C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(400A+326B+$$

```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*A
 + 950*B + 787*C)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x
]^(3/2)) + (a^2*(80*A + 110*B + 79*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(240*d*Sec[c + d*x]^(3/2)) + (a*(2*B + C)*(a + a*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(8*d*Sec[c + d*x]^(3/2)) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + (a^3*(400*A + 326*B + 283*C)*Sin[c + d*x])
/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.03823, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(80A+110B+79C)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(400A+326B+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```


```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*C
os[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*A
 + 950*B + 787*C)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x
]^(3/2)) + (a^2*(80*A + 110*B + 79*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(240*d*Sec[c + d*x]^(3/2)) + (a*(2*B + C)*(a + a*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(8*d*Sec[c + d*x]^(3/2)) + (C*(a + a*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + (a^3*(400*A + 326*B + 283*C)*Sin[c + d*x])
/(128*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a$$

$$= \frac{C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx\right.}}{}$$

$$= \frac{a(2B+C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{C(a}{}$$

$$= \frac{a^2(80A+110B+79C)\sqrt{a+a\cos(c+dx)}\sin(c+}{240d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(80A+}{}$$

$$= \frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(80A+}{}$$

$$= \frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(80A+}{}$$

$$= \frac{a^{5/2}(400A+326B+283C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{}}{128d}$$

Mathematica [A] time = 1.32975, size = 193, normalized size = 0.64

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(400A+326B+283C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(400*A + 326*B + 283*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c +
d*x]] + (6320*A + 5810*B + 5521*C + (2720*A + 3620*B + 3874*C)*Cos[c + d*x]
 + 4*(80*A + 230*B + 331*C)*Cos[2*(c + d*x)] + 120*B*Cos[3*(c + d*x)] + 348
*C*Cos[3*(c + d*x)] + 48*C*Cos[4*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c
 + d*x))/2])))/(3840*d)
```

Maple [B] time = 0.209, size = 591, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] 1/1920/d*a^2*(-1+cos(d*x+c))^2*(384*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+480*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+1392*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
640*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1840*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+2264*C*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2720*A*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+3260*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+2830*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+6000*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4890*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4245*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+6000*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+4890*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))+4245*C*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1
/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 5.19134, size = 614, normalized size = 2.04

$$-\frac{15\left(\left(400\,A+326\,B+283\,C\right)a^{2}\cos\left(dx+c\right)+\left(400\,A+326\,B+283\,C\right)a^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(384\,Ca}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1920*(15*((400*A + 326*B + 283*C)*a^2*cos(d*x + c) + (400*A + 326*B + 28
3*C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))) - (384*C*a^2*cos(d*x + c)^5 + 48*(10*B + 29*C)*a^2*cos(d*
x + c)^4 + 8*(80*A + 230*B + 283*C)*a^2*cos(d*x + c)^3 + 10*(272*A + 326*B
+ 283*C)*a^2*cos(d*x + c)^2 + 15*(400*A + 326*B + 283*C)*a^2*cos(d*x + c))*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) +
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/sqrt(sec(d*x + c)), x)
```

3.1336 $\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{a^3(1304A+1132B+1015C)\sin(c+dx)}{768d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(120A+156B+115C)\sin(c+d}{480d\sec^{\frac{5}{2}}(c+d}$$

```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(680
*A + 628*B + 545*C)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(5/2)) + (a^2*(120*A + 156*B + 115*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(480*d*Sec[c + d*x]^(5/2)) + (a*(12*B + 5*C)*(a + a*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(60*d*Sec[c + d*x]^(5/2)) + (C*(a + a*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(6*d*Sec[c + d*x]^(5/2)) + (a^3*(1304*A + 1132*B + 1015*C)*Si
n[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^3*(130
4*A + 1132*B + 1015*C)*Sin[c + d*x])/(512*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 1.15885, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3045, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(1304A+1132B+1015C)\sin(c+dx)}{768d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(120A+156B+115C)\sin(c+d}{480d\sec^{\frac{5}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(680
*A + 628*B + 545*C)*Sin[c + d*x])/(960*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(5/2)) + (a^2*(120*A + 156*B + 115*C)*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(480*d*Sec[c + d*x]^(5/2)) + (a*(12*B + 5*C)*(a + a*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(60*d*Sec[c + d*x]^(5/2)) + (C*(a + a*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(6*d*Sec[c + d*x]^(5/2)) + (a^3*(1304*A + 1132*B + 1015*C)*Si
n[c + d*x])/(768*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^3*(130
```

```
4*A + 1132*B + 1015*C)*Sin[c + d*x])/(512*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}$$

$$= \frac{C(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{6d\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(12B+5C)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{60d\sec^{\frac{5}{2}}(c+dx)} + \frac{C(a}{}$$

$$= \frac{a^2(120A+156B+115C)\sqrt{a+a\cos(c+dx)}\sin(c+}{480d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(120A+1}{}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(120A+1}{}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(120A+1}{}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(120A+1}{}$$

$$= \frac{a^{5/2}(1304A+1132B+1015C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{512d}$$

Mathematica [A] time = 1.77456, size = 227, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(1304A+1132B+1015C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(1304*A + 1132*B + 1015*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c
 + d*x]] + (23240*A + 22084*B + 20965*C + 2*(7240*A + 7748*B + 8085*C)*Cos[
c + d*x] + 4*(920*A + 1324*B + 1575*C)*Cos[2*(c + d*x)] + 480*A*Cos[3*(c +
d*x)] + 1392*B*Cos[3*(c + d*x)] + 2140*C*Cos[3*(c + d*x)] + 192*B*Cos[4*(c
+ d*x)] + 560*C*Cos[4*(c + d*x)] + 80*C*Cos[5*(c + d*x)])*(-Sin[(c + d*x)/2
] + Sin[(3*(c + d*x))/2])))/(15360*d)
```

Maple [B] time = 0.22, size = 699, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/7680/d*a^2*(-1+cos(d*x+c))^3*(1280*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+1536*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+4480*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+1920*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*x+c)+5568*B*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6960*C*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7360*A*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9056*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*cos(d*x+c)^2+8120*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+13040*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+11320*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1015
0*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19560*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+16980*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+15225*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19560*
A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+16980*B*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15225*C*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*
(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2
)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 5.37253, size = 701, normalized size = 1.99

$$-\frac{15\left((1304\,A+1132\,B+1015\,C)a^2\cos\left(dx+c\right)+(1304\,A+1132\,B+1015\,C)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/7680*(15*((1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c) + (1304*A + 1132*B
 + 1015*C)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) - (1280*C*a^2*cos(d*x + c)^6 + 128*(12*B + 35*C)*a^
2*cos(d*x + c)^5 + 48*(40*A + 116*B + 145*C)*a^2*cos(d*x + c)^4 + 8*(920*A
+ 1132*B + 1015*C)*a^2*cos(d*x + c)^3 + 10*(1304*A + 1132*B + 1015*C)*a^2*c
os(d*x + c)^2 + 15*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c))*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2
)/sec(d*x + c)^(3/2), x)
```

3.1337 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=305

$$\frac{2(19A-3B+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(257A-129B+273C)}{315d\sqrt{a\cos}}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sq
rt[a]*d)) + (2*(257*A - 129*B + 273*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(29*A - 93*B + 21*C)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A - 3*B + 21*C)*Sec
[c + d*x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 9*
B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*
Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.13197, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2984, 12, 2782, 205}

$$\frac{2(19A-3B+21C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B+21C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(257A-129B+273C)}{315d\sqrt{a\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2))/Sqrt[a +
a*Cos[c + d*x]],x]
```



```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sq
rt[a]*d)) + (2*(257*A - 129*B + 273*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(29*A - 93*B + 21*C)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A - 3*B + 21*C)*Sec
[c + d*x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 9*
B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*
Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+a\cos(c}$$

$$= \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(}{9d\sqrt{a+a\cos(c+}}$$

$$= \frac{2(19A-3B+21C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-9B)\,\mathrm{se}}{63d\sqrt{a}}$$

$$= -\frac{2(29A-93B+21C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(19A-3}{}$$

$$= \frac{2(257A-129B+273C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-}{}$$

$$= \frac{2(257A-129B+273C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-}{}$$

$$= \frac{2(257A-129B+273C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-}{}$$

$$= -\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{\sqrt{a}d}$$

Mathematica [C] time = 34.5876, size = 7123, normalized size = 23.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2))/Sqr
t[a + a*Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.21, size = 1131, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2
),x)
```

```
[Out] 1/315/d*2^(1/2)/a*(315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))+45*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)-129*B*2^(1/2)*cos(d*x
+c)^4*sin(d*x+c)-1575*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))-1575*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c))
)^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3150*B*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3150*B*cos(d*x+c)^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-315*
B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))+93*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-9*B*2^(1/2)*cos(d*x+c)^2*sin(
d*x+c)+315*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*cos(d*x+c)^5+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*cos(d*x+c)^5+1575*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+1575*C*(cos(d*x+c)/(1+cos(d*
x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+3150*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+
3150*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)^3+3150*A*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)^2+3150*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+1575*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+1575*C*(cos(d*x+c)/(1
+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+257*A*2^(
1/2)*cos(d*x+c)^4*sin(d*x+c)+273*C*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)-29*A*2^(
1/2)*cos(d*x+c)^3*sin(d*x+c)-21*C*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+57*A*2^(1
/2)*cos(d*x+c)^2*sin(d*x+c)+63*C*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-5*A*2^(1/2
)*cos(d*x+c)*sin(d*x+c)+315*C*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))+35*A*2^(1/2)*sin(d*x+c)-315*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(1/cos(d*x+c))^
(11/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^8/(-1+cos(d*x+c))^4/(1+cos(d*x+c
))^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c)
)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90098, size = 579, normalized size = 1.9

$$\frac{\frac{315\sqrt{2}\left((A-B+C)a\cos(dx+c)^5+(A-B+C)a\cos(dx+c)^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((257A-129B+273C)\cos(dx+c)^4-(29A-93B+21C)\cos(dx\right.}{}}{315\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c)
)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(315*sqrt(2)*((A - B + C)*a*cos(d*x + c)^5 + (A - B + C)*a*cos(d*x +
c)^4)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c)))/sqrt(a) + 2*((257*A - 129*B + 273*C)*cos(d*x + c)^4 - (29*A -
 93*B + 21*C)*cos(d*x + c)^3 + 3*(19*A - 3*B + 21*C)*cos(d*x + c)^2 - 5*(A
- 9*B)*cos(d*x + c) + 35*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2)/(a+a*cos(d*x+
c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sec(dx+c)^{\frac{11}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c)
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(11/2)/sqrt(
a*cos(d*x + c) + a), x)
```

3.1338 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=257

$$\frac{2(31A-7B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+}}{}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt
[a]*d) - (2*(43*A - 91*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sq
rt[a + a*Cos[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.909744, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2984, 12, 2782, 205}

$$\frac{2(31A-7B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt[a + a
*Cos[c + d*x]],x]
```



```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt
[a]*d) - (2*(43*A - 91*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sq
rt[a + a*Cos[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c-}$$

$$=\frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$=-\frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}+\frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(}{7d\sqrt{a+a\cos(c+}}$$

$$=\frac{2(31A-7B+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}-\frac{2(A-7B)\sec}{35d\sqrt{a}}$$

$$=-\frac{2(43A-91B+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2(31A-7}{}$$

$$=-\frac{2(43A-91B+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2(31A-7}{}$$

$$=-\frac{2(43A-91B+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2(31A-7}{}$$

$$=\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx}}{\sqrt{a}d}$$

Mathematica [C] time = 10.0211, size = 2646, normalized size = 10.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(7*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(7/2)) - (C*Sin[c/2 + (d*x)/2])/(3*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) +
 ((A - B + C)*Csc[c/2 + (d*x)/2]^9*(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*S
```

```
in[c/2 + (d*x)/2]^4 + 26785605*Sin[c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d
*x)/2]^8 + 177677808*Sin[c/2 + (d*x)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^1
2 + 52080*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin
[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 560*HypergeometricPFQ[{2, 2, 2,
 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]*Sin[c/2 + (d*x)/2]^12 + 213120160*Sin[c/2 + (d*x)/2]^14 - 168280*Hyper
geometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]*Sin[c/2 + (d*x)/2]^14 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1,
 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 +
 (d*x)/2]^14 - 121497024*Sin[c/2 + (d*x)/2]^16 + 212520*Hypergeometric2F1[2
, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 +
 (d*x)/2]^16 + 3360*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 +
 40125184*Sin[c/2 + (d*x)/2]^18 - 124320*Hypergeometric2F1[2, 11/2, 13/2, S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 -
2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 5840384*Sin[c/
2 + (d*x)/2]^20 + 28000*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 560*Hypergeometri
cPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 363825*ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)] - 5336100*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)] + 34636140*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 131060160*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 320535600*ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 530671680*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 604296000*ArcTanh[Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12
*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 468948480*ArcTa
nh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x
)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 23772672
0*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2
 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 7
0963200*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*S
in[c/2 + (d*x)/2]^18*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] + 9461760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)]]*Sin[c/2 + (d*x)/2]^20*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)] - 1120*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 11/2}, {1, 1,
 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)
```

```
/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 280*Cos[(c + d*x)/2]^4*Hypergeometri
cPFQ[{2, 2, 11/2}, {1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]*Sin[c/2 + (d*x)/2]^12*(103 - 164*Sin[c/2 + (d*x)/2]^2 + 70*Sin[c/2 +
 (d*x)/2]^4)))/(40425*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(9/2)*(-1 + 2*Sin[c/2 +
(d*x)/2]^2)) + (4*B*((3*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5
/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2
 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])))/35 + (C*((5*Sin[c/2 + (d*x
)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2) + 2*((3*Sin[c/2 + (d*x)/2])/(1 - 2
*Sin[c/2 + (d*x)/2]^2)^(5/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)
/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))))/
105))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.192, size = 927, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/105/d*2^(1/2)/a*(105*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))-105*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+105*C*cos(d*x+c)^4*(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+420*A*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-420*
B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))+420*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))+630*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))-630*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+630*C*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+420*A*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-420
*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))+420*C*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))+105*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-105*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))+105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))+43*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-91*B*2^(1/2)*cos(d*
x+c)^3*sin(d*x+c)+35*C*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-31*A*2^(1/2)*cos(d*x
+c)^2*sin(d*x+c)+7*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-35*C*2^(1/2)*cos(d*x+c
)^2*sin(d*x+c)+3*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)-21*B*2^(1/2)*cos(d*x+c)*si
```

```
n(d*x+c)-15*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)^6/(-1+cos(d*x+c))^3/(1+cos(d*x+c))^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97734, size = 522, normalized size = 2.03

$$-\frac{\frac{105\sqrt{2}\left((A-B+C)a\cos(dx+c)^4+(A-B+C)a\cos(dx+c)^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((43\,A-91\,B+35\,C)\cos(dx+c)^3-(31\,A-7\,B+35\,C)\cos(dx-}{\sqrt{\cos}}}{105\left(ad\cos\left(dx+c\right)^4+ad\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(105*sqrt(2)*((A - B + C)*a*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x +
 c)^3)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c)))/sqrt(a) + 2*((43*A - 91*B + 35*C)*cos(d*x + c)^3 - (31*A - 7
*B + 35*C)*cos(d*x + c)^2 + 3*(A - 7*B)*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^4 + a*d*cos(d
*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(a
*cos(d*x + c) + a), x)
```

3.1339 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=211

$$\frac{2(13A-5B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sq
rt[a]*d)) + (2*(13*A - 5*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*S
qrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(1
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.71706, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2984, 12, 2782, 205}

$$\frac{2(13A-5B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt[a + a
*Cos[c + d*x]],x]
```



```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sq
rt[a]*d)) + (2*(13*A - 5*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*S
qrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(1
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*
Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c}}$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin}{5d\sqrt{a+a\cos(c}}$$

$$= \frac{2(13A-5B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\,\mathrm{s}}{15d\sqrt{}}$$

$$= \frac{2(13A-5B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\,\mathrm{s}}{15d\sqrt{}}$$

$$= \frac{2(13A-5B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\,\mathrm{s}}{15d\sqrt{}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{\sqrt{a}d}$$

Mathematica [C] time = 8.01709, size = 1882, normalized size = 8.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(5/2)) - (C*Sin[c/2 + (d*x)/2])/(2*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) +
 (8*B*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 +
 (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/15 - ((A - B + C)*Csc[c/2 + (
d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*
Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)
/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyperge
```

```
ometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 17
70*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hyp
ergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeomet
ric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Si
n[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
- 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*S
in[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin
[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*(1 - 2*Sin[c/2 + (d
*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (C*((3*Sin[c/2 + (d*x)/2])
/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2) + 4*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2
+ (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^
2])))/30))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.227, size = 723, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/15/d*2^(1/2)/a*(15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)^3-15*B*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+45*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-45*B*cos(
d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)+45*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)^2+45*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*cos(d*x+c)-45*B*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+45*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+15*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-15*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*C*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+13*A*2^(1/2)*cos(d*x+c)^2*sin(d*x
+c)-5*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*C*2^(1/2)*cos(d*x+c)^2*sin(d*x+c
)-A*2^(1/2)*cos(d*x+c)*sin(d*x+c)+5*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)+3*A*2^(
1/2)*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)^4/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8877, size = 463, normalized size = 2.19

$$\frac{\frac{15\sqrt{2}\left((A-B+C)a\cos(dx+c)^3+(A-B+C)a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((13A-5B+15C)\cos(dx+c)^2-(A-5B)\cos(dx+c)+3A\right)\sqrt{a\cos}}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*x + c)
^2)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin
(d*x + c)))/sqrt(a) + 2*((13*A - 5*B + 15*C)*cos(d*x + c)^2 - (A - 5*B)*cos
(d*x + c) + 3*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(a
*cos(d*x + c) + a), x)
```

3.1340 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2A$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt
[a]*d) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[
c + d*x]]) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rubi [A] time = 0.506908, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3043, 2984, 12, 2782, 205}

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2A$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + a
*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt
[a]*d) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[
c + d*x]]) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}}$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin}{3d\sqrt{a+a\cos(c}}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin}{3d\sqrt{a+a\cos(c}}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin}{3d\sqrt{a+a\cos(c}}$$

$$= \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{\sqrt{a}d}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.219, size = 518, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/3/d*2^(1/2)/a*(3*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)-3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2+3*C*cos(d*x+c)^2*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6*A*cos(d*x+c)*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-6*B*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+6*C*cos
(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-
3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*
C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+A*2^
(1/2)*cos(d*x+c)*sin(d*x+c)-3*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)-A*2^(1/2)*sin
(d*x+c))*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1
/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94051, size = 404, normalized size = 2.48

$$-\frac{\frac{3\sqrt{2}\left((A-B+C)a\cos(dx+c)^2+(A-B+C)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*((A - B + C)*a*cos(d*x + c)^2 + (A - B + C)*a*cos(d*x + c))
*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c)))/sqrt(a) + 2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*
```

```
sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a
*cos(d*x + c) + a), x)
```

3.1341 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sqrt{\cos(c}{}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.55747, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3043, 2982, 2782, 205, 2774, 216}

$$-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2C\sqrt{\cos(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt[a + a
*Cos[c + d*x]],x]
```



```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c-}}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A}{}$$

Mathematica [C] time = 3.86226, size = 277, normalized size = 1.56

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(A-B+C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5\cos^2(c+dx)(\cos(c+dx)+2)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\tanh\right.\right.}{10}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*C*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + (2*B*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]] - (2
*C*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]] + ((A - B + C)*Csc[(c + d*x)/2]^3*(
5*Cos[c + d*x]^2*(2 + Cos[c + d*x])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[
c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - Hyp
ergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*
x)/2]^4*Sin[c + d*x]^2))/(10*Cos[c + d*x]^(5/2))))/(d*Sqrt[a*(1 + Cos[c + d
*x])])
```

Maple [B] time = 0.191, size = 439, normalized size = 2.5

$$\frac{\sqrt{2}\cos(dx+c)}{ad(1+\cos(dx+c))}\left(C\sqrt{2}\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+A\arcsin\left(\frac{-1+c}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/d*2^(1/2)/a*(C*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+C*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)+C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*2^(1/2)*sin(d*x+c)-B*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+C*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(
3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 68.4973, size = 464, normalized size = 2.61

$$-\frac{2(C\cos(dx+c)+C)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}((A-B+C)a\cos(dx+c)+(A-B+C)a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*(C*cos(d*x + c) + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A
 - B + C)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c)))/sqrt(a) - 2*sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c)/sq
rt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a
*cos(d*x + c) + a), x)
```

3.1342 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}}{\sqrt{a}d}$$

```
[Out] ((2*B - C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d*S
qrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.557908, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3045, 2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt[a + a
*Cos[c + d*x]],x]
```

```
[Out] ((2*B - C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d*S
qrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(}}$$

$$= \frac{C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left((2B-C)\sqrt{\cos(c+dx\right.}{}$$

$$= \frac{C\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left((2B-C)\sqrt{\cos(c+dx\right.}{}$$

$$= \frac{(2B-C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} +$$

Mathematica [A] time = 0.389264, size = 132, normalized size = 0.73

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2(A-B+C)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}(2B-C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt
[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2*B - C)*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(A - B + C)*ArcTan[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]] + 2*C*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1
+ Cos[c + d*x])])
```

Maple [A] time = 0.201, size = 247, normalized size = 1.4

$$\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{2\,ad\,(\sin(dx+c))^2}\left(-C\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-2\,B\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+C\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/2/d*2^(1/2)/a*(-C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*
B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+C
*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*
A*arcsin((-1+cos(d*x+c))/sin(d*x+c))-2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))
+2*C*arcsin((-1+cos(d*x+c))/sin(d*x+c)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 85.9044, size = 477, normalized size = 2.64

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(2\,B-C\right)\cos\left(dx+c\right)+2\,B-C\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*C*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*B - C)*co
s(d*x + c) + 2*B - C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A -
B + C)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
```

```
a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)\right)\sqrt{\sec\left(c + dx\right)}}{\sqrt{a\left(\cos\left(c + dx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/sqrt(a
*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A\right)\sqrt{\sec\left(dx + c\right)}}{\sqrt{a\cos\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a
*cos(d*x + c) + a), x)
```

3.1343 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=235

$$\frac{(8A-4B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\ldots}{\sqrt{2}\sqrt{\ldots}}\right.}{\sqrt{a}d}$$

```
[Out] ((8*A - 4*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)
*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c +
d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*B - C)*Sin[c
+ d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.777642, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3045, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A-4B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\ldots}{\sqrt{2}\sqrt{\ldots}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]),x]
```


```
[Out] ((8*A - 4*B + 7*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)
*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c +
d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*B - C)*Sin[c
+ d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+}}{}}{2a}$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4B-C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} +$$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4B-C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - ($$

$$= \frac{C\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4B-C)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} +$$

$$= \frac{(8A-4B+7C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} - \frac{\sqrt{2}(A-}{}$$

Mathematica [C] time = 27.462, size = 16885, normalized size = 71.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.218, size = 363, normalized size = 1.5

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))^2\cos(dx+c)}{8\,ad\,(\sin(dx+c))^4}\left(2\,C\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\cos(dx+c)+4\,B\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/8/d*2^(1/2)/a*(-1+cos(d*x+c))^2*(2*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)*cos(d*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*A*2^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-4*B*2^(1/
2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+7*C*2^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*A*arc
sin((-1+cos(d*x+c))/sin(d*x+c))-8*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))+8*C*
arcsin((-1+cos(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos{(c + dx)} + C\cos^2{(c + dx)}}{\sqrt{a(\cos{(c + dx)} + 1)}\sqrt{\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(a*(cos(c + d*x) + 1
))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx + c)}^2 + B\cos{(dx + c)} + A}{\sqrt{a\cos{(dx + c)} + a}\sqrt{\sec{(dx + c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*sqrt(sec(d*x + c))), x)
```

3.1344 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{(8A-14B+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A-2B+7C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+$$

```
[Out] -((8*A - 14*B + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B +
C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c
+ d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + ((6*B - C)*Sin[
c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((8*A - 2*B
+ 7*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.04055, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3045, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(8A-14B+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{(8A-2B+7C)\sin(c+dx)}{8d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)),x]
```


```
[Out] -((8*A - 14*B + 9*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B +
C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c
+ d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + ((6*B - C)*Sin[
c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((8*A - 2*B
+ 7*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(}{}}{3a}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{(6B-C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{(6B-C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{(6B-C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{C\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{(6B-C)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(8A-14B+9C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8\sqrt{a}d} + \frac{\sqrt{2}(}{}$$

Mathematica [C] time = 27.5513, size = 16934, normalized size = 60.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + a*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.187, size = 470, normalized size = 1.7

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{3}\cos\left(dx+c\right)}{48\,ad\left(\sin\left(dx+c\right)\right)^{6}}\left(8\,C\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+12\,B\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/48/d*2^(1/2)/a*(-1+cos(d*x+c))^3*(8*C*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+12*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*cos(d*x+c)*sin(d*x+c)-2*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)*cos(d*x+c)+24*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)-6*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+21*C*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-24*A*2^(1/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+42*B*2^(1/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-27*C*2^(1/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-48*A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))+48*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))-48*C*arcsin((-1+cos(d*
x+c))/sin(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d
*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)
*sec(d*x + c)^(3/2)), x)
```

3.1345 $\int \frac{(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=192

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(a-b)(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}}{\sqrt{a}d}$$

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(
A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (b*B*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.699061, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4221, 3045, 2982, 2782, 205, 2774, 216}

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(a-b)(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x
]])/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(
A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (b*B*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(Ab+aB)}{\sqrt{\cos(c+}}$$

$$=\frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+\right.}}{}$$

$$=\frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\left((a-b)(A\right.$$

$$=\frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{\left(2a(a-b)(\right.}{}$$

$$=\frac{(2Ab+2aB-bB)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c}}{\sqrt{a}d}$$

Mathematica [A] time = 0.440284, size = 143, normalized size = 0.74

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\sqrt{2}(2aB+2Ab-bB)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(a-b)(A-B)\tan^{-1}\left(\right.\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2*A*b + 2
*a*B - b*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(a - b)*(A - B)*ArcTan[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*b*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.219, size = 317, normalized size = 1.7

$$-\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{2ad(\sin(dx+c))^2}\left(B\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}b\sin(dx+c)+2A\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)b+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d
*x+c))^(1/2),x)
```

```
[Out] -1/2/d*2^(1/2)/a*(B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b*sin(d*x+c)+
2*A*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))
*b+2*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))*a-B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))*b-2*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a+2*A*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*b+2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a-2*B*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*b)*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(
a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos(dx + c)^2 + Aa + (Ba + Ab)\cos(dx + c)\right)\sqrt{\sec(dx + c)}}{\sqrt{a\cos(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*
x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.1346 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=333

$$\frac{(19A-15B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A-7B+7C)\sin(c+dx)\sec^{\frac{7}{2}}(c+}{14ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((19*A - 15*B + 11*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*S
qrt[2]*a^(3/2)*d) - ((1201*A - 1029*B + 665*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((397*A - 273*B + 245*C)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A - 63*B
 + 35*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]])
 - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^
(3/2)) + ((11*A - 7*B + 7*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[
a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.2265, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3041, 2984, 12, 2782, 205}

$$\frac{(19A-15B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A-7B+7C)\sin(c+dx)\sec^{\frac{7}{2}}(c+}{14ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/(a + a*Cos
[c + d*x])^(3/2),x]
```


```
[Out] ((19*A - 15*B + 11*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*S
qrt[2]*a^(3/2)*d) - ((1201*A - 1029*B + 665*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) + ((397*A - 273*B + 245*C)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A - 63*B
 + 35*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]])
 - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^
(3/2)) + ((11*A - 7*B + 7*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[
a + a*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{9}{2}}(c+dx)(a+a\cos(c+a}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(11A-7B+7C)\,\mathrm{se}}{14ad\sqrt{a+}}$$

$$= -\frac{(67A-63B+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C}{2d(a}$$

$$= \frac{(397A-273B+245C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} - \frac{(67A-6}{}$$

$$= -\frac{(1201A-1029B+665C)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A}{}$$

$$= -\frac{(1201A-1029B+665C)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A}{}$$

$$= -\frac{(1201A-1029B+665C)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A}{}$$

$$= \frac{(19A-15B+11C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 10.1321, size = 3136, normalized size = 9.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*((4*C*Sin[c/2 + (d*x)/2])/(7*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(7/2)) - ((A - B + C)*(1 - 2*Sin[c/2 + (d*x)/2]))/(28*(1 + Sin[c/2 + (
d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) + ((A - B + C)*(1 + 2*Sin[c/2
+ (d*x)/2]))/(28*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2
)) - ((A - B + C)*(315*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2
 + (d*x)/2]^2]] + (5 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 -
 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (11 + 17*Sin[c/2 + (d*x)/2])/((1 - Sin[c/
2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 + 71*Sin[c/2 + (d*x
)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sq
rt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/70 + ((A - B + C
)*(315*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]]
+ (5 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*
x)/2]^2)^(5/2)) - (11 - 17*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1
 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 - 71*Sin[c/2 + (d*x)/2])/((1 + Sin[
c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt[1 - 2*Sin[c/2
 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/70 - ((-A - 3*B + 7*C)*Csc[c/2 +
 (d*x)/2]^9*(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2]^4 + 2
6785605*Sin[c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d*x)/2]^8 + 177677808*Si
n[c/2 + (d*x)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^12 + 52080*Hypergeometri
c2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin
[c/2 + (d*x)/2]^12 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13
/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]
^12 + 213120160*Sin[c/2 + (d*x)/2]^14 - 168280*Hypergeometric2F1[2, 11/2, 1
3/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]
^14 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 12149702
4*Sin[c/2 + (d*x)/2]^16 + 212520*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 3360*Hyp
ergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 40125184*Sin[c/2 + (d*
x)/2]^18 - 124320*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 2240*HypergeometricPFQ[
{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 5840384*Sin[c/2 + (d*x)/2]^20 + 28000
*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^20 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}
, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[
c/2 + (d*x)/2]^20 + 363825*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
- 5336100*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]
```

```
*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)] + 34636140*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] - 131060160*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)] + 320535600*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)] - 530671680*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)] + 604296000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 468948480*ArcTanh[Sqrt[Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 237726720*ArcTanh[Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 70963200*ArcTanh[Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^18*Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 9461760*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
20*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1120*Cos[(c +
 d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2
+ (d*x)/2]^2) + 280*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2}, {1,
13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/
2]^12*(103 - 164*Sin[c/2 + (d*x)/2]^2 + 70*Sin[c/2 + (d*x)/2]^4)))/(80850*(
1 - 2*Sin[c/2 + (d*x)/2]^2)^(9/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (8*C*((3
*Sin[c/2 + (d*x)/2])/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2) + 4*(Sin[c/2 + (d*x
)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2])))/35))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.2, size = 1047, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 1/420/d*2^(1/2)/a^2*(-1201*A*cos(d*x+c)^5*2^(1/2)-1575*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+1155*C*(cos(d
*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+1
```

```
995*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
sin(d*x+c)-60*A*2^(1/2)-665*C*cos(d*x+c)^5*2^(1/2)+4620*C*(cos(d*x+c)/(1+co
s(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)
+11970*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*cos(d*x+c)^2*sin(d*x+c)-9450*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+6930*C*(cos(d*x+c)/(1+c
os(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c
)+7980*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*cos(d*x+c)*sin(d*x+c)-6300*B*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+4620*C*(cos(d*x+c)/(1+cos(d
*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-840*
B*2^(1/2)*cos(d*x+c)^3+168*B*2^(1/2)*cos(d*x+c)^2-84*B*2^(1/2)*cos(d*x+c)+1
029*B*cos(d*x+c)^5*2^(1/2)-273*B*cos(d*x+c)^4*2^(1/2)+397*A*2^(1/2)*cos(d*x
+c)^4+245*C*2^(1/2)*cos(d*x+c)^4+1000*A*cos(d*x+c)^3*2^(1/2)+560*C*cos(d*x+
c)^3*2^(1/2)-232*A*cos(d*x+c)^2*2^(1/2)-140*C*cos(d*x+c)^2*2^(1/2)+96*A*cos
(d*x+c)*2^(1/2)+1995*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)^4*sin(d*x+c)-1575*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*sin(d*x+c)+1155*C*(c
os(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)^4*sin(d*x+c)+7980*A*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-6300*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c))*cos(d*x+
c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^5/(-1+cos(d*x+c
))^3/(1+cos(d*x+c))^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.74924, size = 702, normalized size = 2.11

$$-\frac{105\sqrt{2}\Big((19\,A-15\,B+11\,C)\cos(dx+c)^5+2\,(19\,A-15\,B+11\,C)\cos(dx+c)^4+(19\,A-15\,B+11\,C)\cos(dx+c)^3\Big)}{420\,\big(a^2d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/420*(105*sqrt(2)*((19*A - 15*B + 11*C)*cos(d*x + c)^5 + 2*(19*A - 15*B +
 11*C)*cos(d*x + c)^4 + (19*A - 15*B + 11*C)*cos(d*x + c)^3)*sqrt(a)*arctan
(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
) + 2*((1201*A - 1029*B + 665*C)*cos(d*x + c)^4 + 12*(67*A - 63*B + 35*C)*c
os(d*x + c)^3 - 28*(7*A - 3*B + 5*C)*cos(d*x + c)^2 + 12*(3*A - 7*B)*cos(d*
x + c) - 60*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*cos
(d*x + c) + a)^(3/2), x)
```

3.1347 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{(15A-11B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+}{10ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -((15*A - 11*B + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*S
qrt[2]*a^(3/2)*d) + ((147*A - 95*B + 75*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(30*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((39*A - 35*B + 15*C)*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B + 5*
C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.980823, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3041, 2984, 12, 2782, 205}

$$-\frac{(15A-11B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B+5C)\sin(c+dx)\sec^{\frac{5}{2}}(c+}{10ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos
[c + d*x])^(3/2),x]
```


```
[Out] -((15*A - 11*B + 7*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*S
qrt[2]*a^(3/2)*d) + ((147*A - 95*B + 75*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(30*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((39*A - 35*B + 15*C)*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B + 5*
C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+}$$

$$=-\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}\right.}{}$$

$$=-\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(9A-5B+5C)\,\mathrm{s}}{10ad\sqrt{a}}$$

$$=-\frac{(39A-35B+15C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(A-B+}{2d(}$$

$$=\frac{(147A-95B+75C)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(39A-35}{3}$$

$$=\frac{(147A-95B+75C)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(39A-35}{3}$$

$$=\frac{(147A-95B+75C)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}}-\frac{(39A-35}{3}$$

$$=-\frac{(15A-11B+7C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 8.11092, size = 2295, normalized size = 8.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*((4*C*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(5/2)) - ((A - B + C)*(1 - 2*Sin[c/2 + (d*x)/2]))/(20*(1 + Sin[c/2 + (
d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + ((A - B + C)*(1 + 2*Sin[c/2
```

+ (d*x)/2]))/(20*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2
)) + (16*C*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[
c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/15 - ((A - B + C)*(-105*
ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (4 +
3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(3/2)) - (19 + 29*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 -
2*Sin[c/2 + (d*x)/2]^2]) - (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c
/2 + (d*x)/2])))/30 + ((A - B + C)*(-105*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/
Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (4 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/
2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - (19 - 29*Sin[c/2 + (d*x
)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - (67*Sqr
t[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/30 + ((-A - 3*B +
 7*C)*Csc[c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*
x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 65581
2*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c +
d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 +
 (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)
/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16
+ 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (
d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*Hyper
geometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(1350*(
1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*(a*(1

```
+ Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.235, size = 843, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -1/60/d*2^(1/2)/a^2*(-225*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3*
sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+165*B*cos(d*x+c)^3*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-105*C
*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-675*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin
(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+495*B*cos(d*x+c)^2*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-315*C*(c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))-675*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+495*B*cos(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-315*C*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))-225*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))+165*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)-105*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*
x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+147*A*2^(1/2)*cos(d*x+c)^4-95*B*cos
(d*x+c)^4*2^(1/2)+75*C*2^(1/2)*cos(d*x+c)^4-39*A*cos(d*x+c)^3*2^(1/2)+35*B*
2^(1/2)*cos(d*x+c)^3-15*C*cos(d*x+c)^3*2^(1/2)-120*A*cos(d*x+c)^2*2^(1/2)+8
0*B*2^(1/2)*cos(d*x+c)^2-60*C*cos(d*x+c)^2*2^(1/2)+24*A*cos(d*x+c)*2^(1/2)-
20*B*2^(1/2)*cos(d*x+c)-12*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)^3/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.52213, size = 632, normalized size = 2.23

$$\frac{15\sqrt{2}\left((15A-11B+7C)\cos(dx+c)^4+2(15A-11B+7C)\cos(dx+c)^3+(15A-11B+7C)\cos(dx+c)^2\right)\sqrt{a}\operatorname{arct}}{60\left(a^2d\cos(dx+c)^4+2a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] 1/60*(15*sqrt(2)*((15*A - 11*B + 7*C)*cos(d*x + c)^4 + 2*(15*A - 11*B + 7*C
)*cos(d*x + c)^3 + (15*A - 11*B + 7*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*
((147*A - 95*B + 75*C)*cos(d*x + c)^3 + 12*(9*A - 5*B + 5*C)*cos(d*x + c)^2
 - 4*(3*A - 5*B)*cos(d*x + c) + 12*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d
*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos
(d*x + c) + a)^(3/2), x)
```

3.1348 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{(11A-7B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((11*A - 7*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqr
t[2]*a^(3/2)*d) - ((19*A - 15*B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*
a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B + 3*C)*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.773033, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3041, 2984, 12, 2782, 205}

$$\frac{(11A-7B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B+3C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos
[c + d*x])^(3/2),x]
```


```
[Out] ((11*A - 7*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqr
t[2]*a^(3/2)*d) - ((19*A - 15*B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*
a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B + 3*C)*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+a}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B+3C)\sec}{6ad\sqrt{a+a}}$$

$$= -\frac{(19A-15B+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\,\mathrm{s}}{2d(a+}$$

$$= -\frac{(19A-15B+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\,\mathrm{s}}{2d(a+}$$

$$= -\frac{(19A-15B+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B+C)\,\mathrm{s}}{2d(a+}$$

$$= \frac{(11A-7B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 6.81555, size = 1070, normalized size = 4.59

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*((4*C*Sin[c/2 + (d*x)/2])/(3*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(3/2)) - ((A - B + C)*(1 - 2*Sin[c/2 + (d*x)/2]))/(12*(1 + Sin[c/2 + (
d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A - B + C)*(1 + 2*Sin[c/2
+ (d*x)/2]))/(12*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2
)) + (8*C*Sin[c/2 + (d*x)/2])/(3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - ((A -
B + C)*(5*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2
]] + (1 + Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2
+ (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/
```

```
2])))/2 + ((A - B + C)*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[
c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqr
t[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 +
Sin[c/2 + (d*x)/2])))/2 + ((A + 3*B - 7*C)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(c
 + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^
2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F
1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/
2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sq
rt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (
d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 -
7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/
2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2
))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(126*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2
))))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.209, size = 637, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -1/12/d*2^(1/2)/a^2*(-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos
(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))+21*B*cos(d*x+c)^2*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-9*C*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c
))/sin(d*x+c))-66*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))+42*B*cos(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-18*C*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))+21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)-9*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+19*A*cos(d*x+c)^3*2^(1/2)-15*B*2^(1/2)*cos(
d*x+c)^3+3*C*cos(d*x+c)^3*2^(1/2)-7*A*cos(d*x+c)^2*2^(1/2)+3*B*2^(1/2)*cos(
d*x+c)^2-3*C*cos(d*x+c)^2*2^(1/2)-16*A*cos(d*x+c)*2^(1/2)+12*B*2^(1/2)*cos(
d*x+c)+4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.43943, size = 563, normalized size = 2.42

$$-\frac{3\sqrt{2}\Big((11\,A-7\,B+3\,C)\cos(dx+c)^3+2\,(11\,A-7\,B+3\,C)\cos(dx+c)^2+(11\,A-7\,B+3\,C)\cos(dx+c)\Big)\sqrt{a}\arctan\Big(}{12\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*((11*A - 7*B + 3*C)*cos(d*x + c)^3 + 2*(11*A - 7*B + 3*C)*
cos(d*x + c)^2 + (11*A - 7*B + 3*C)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((19*
A - 15*B + 3*C)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) - 4*A)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^
2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^(3/2), x)
```

3.1349 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{(7A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-$$

```
[Out] -((7*A - 3*B - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[
2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*C
os[c + d*x])^(3/2)) + ((5*A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.568572, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3041, 2984, 12, 2782, 205}

$$-\frac{(7A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos
[c + d*x])^(3/2),x]
```



```
[Out] -((7*A - 3*B - C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[
2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*C
os[c + d*x])^(3/2)) + ((5*A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+a}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B+C)\sqrt{\sec}}{2ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B+C)\sqrt{\sec}}{2ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B+C)\sqrt{\sec}}{2ad\sqrt{a+a}}$$

$$= -\frac{(7A-3B-C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+d}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 5.6873, size = 481, normalized size = 2.66

$$\frac{2\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(A+3B-7C)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*(A - B + C)
*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]])/2 - (3*(A - B + C)*Ar
cTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]])/2 - ((A - B + C)*Sqrt[Co
s[c + d*x]])/(-1 + Sin[(c + d*x)/2]) + (4*C*Sin[(c + d*x)/2])/Sqrt[Cos[c +
d*x]] - ((A - B + C)*Sqrt[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]) + ((A - B +
 C)*(-1 + 2*Sin[(c + d*x)/2]))/(4*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/4] + Si
n[(c + d*x)/4])^2) - ((A - B + C)*(1 + 2*Sin[(c + d*x)/2]))/(4*Sqrt[Cos[c +
 d*x]]*(-1 + Sin[(c + d*x)/2])) + ((A + 3*B - 7*C)*Csc[(c + d*x)/2]^3*(5*(1
 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Se
c[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2
```

```
*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c
+ d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))/(40*Cos[c + d*x]^(3/2))))/(d*(a*(1
+ Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.204, size = 434, normalized size = 2.4

$$\frac{\sqrt{2}\cos(dx+c)}{4\,da^2\sin(dx+c)\,(1+\cos(dx+c))}\left(7\,A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-3\,B\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*(7*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-C*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+7*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)-5*A*cos(d*x+c)^2*2^(1/2)-3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+B*2^(1/2)*cos(d*x+c)^2-C*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-
C*cos(d*x+c)^2*2^(1/2)+A*cos(d*x+c)*2^(1/2)-B*2^(1/2)*cos(d*x+c)+C*2^(1/2)*
cos(d*x+c)+4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.1159, size = 456, normalized size = 2.52

$$\frac{\sqrt{2}\left((7\,A-3\,B-C)\cos\left(dx+c\right)^{2}+2\,(7\,A-3\,B-C)\cos\left(dx+c\right)+7\,A-3\,B-C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((7*A - 3*B - C)*cos(d*x + c)^2 + 2*(7*A - 3*B - C)*cos(d*x +
c) + 7*A - 3*B - C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((5*A - B + C)*cos(d*x + c) + 4*A)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^(3/2), x)

3.1350 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{(3A+B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos(}}\right.}{a^{3/2}d}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A + B - 5*C)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.598561, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3041, 2982, 2782, 205, 2774, 216}

$$\frac{(3A+B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos(}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos
[c + d*x])^(3/2),x]
```

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A + B - 5*C)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(}{\sqrt{\cos(c+dx)}(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((3A+B-5C)\sqrt{\mathrm{c}}\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((3A+B-5C)\sqrt{\mathrm{c}}\right.}{}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(3A+}{}$$

Mathematica [C] time = 27.788, size = 16028, normalized size = 84.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 a*Cos[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.183, size = 365, normalized size = 1.9

$$-\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(4\,C\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(4*C*2^(1/
2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+
c)+A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*A*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+C*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+5*C*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*sin(d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+B*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)-C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 166.405, size = 601, normalized size = 3.18

$$-\frac{\sqrt{2}\left((3\,A+B-5\,C)\cos\left(dx+c\right)^{2}+2\,(3\,A+B-5\,C)\cos\left(dx+c\right)+3\,A+B-5\,C\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+}}{\sqrt{a}\sin(dx+c)}\right.}{4\left(a^{2}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((3*A + B - 5*C)*cos(d*x + c)^2 + 2*(3*A + B - 5*C)*cos(d*x +
 c) + 3*A + B - 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A - B +
C)*sqrt(cos(d*x + c))*sin(d*x + c) + 8*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c)
 + C)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
```

```
in(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a)^(3/2), x)
```

3.1351 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=242

$$\frac{(A-5B+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-}}{a^{3/2}d}$$

```
[Out] ((2*B - 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B + 9*C)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((A - B
 + 3*C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.806847, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3041, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(A-5B+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-}}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/2)*Sqr
t[Sec[c + d*x]]),x]
```


```
[Out] ((2*B - 3*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B + 9*C)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((A - B
 + 3*C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2\right.}{(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-B+3C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-B+3C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-B+3C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+}}$$

$$= \frac{(2B-3C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A-5B+9C}{}$$

Mathematica [C] time = 27.8497, size = 16853, normalized size = 69.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/
2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.204, size = 450, normalized size = 1.9

$$-\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^2}{4\,da^2(\sin(dx+c))^5}\sqrt{a(1+\cos(dx+c))}\left(2\,C(\cos(dx+c))^2\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+A\sqrt{2}\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2
*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)+6*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*C*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+9*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x
+c))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*sqrt(sec(d*x + c))), x)
```

3.1352 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=300

$$\frac{(8A-12B+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A-9B+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\right.}{2\sqrt{2}a^{3/2}d}$$

```
[Out] ((8*A - 12*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*C
)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) -
((A - B + C)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/
2)) + ((A - B + 2*C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)) - ((2*A - 6*B + 7*C)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.05111, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3041, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(8A-12B+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(5A-9B+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\right.}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/2)*Sec
[c + d*x]^(3/2)),x]
```


```
[Out] ((8*A - 12*B + 19*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*C
)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) -
((A - B + C)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/
2)) + ((A - B + 2*C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)) - ((2*A - 6*B + 7*C)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-B+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-B+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-B+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A-B+2C)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+}$$

$$= \frac{(8A-12B+19C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{3/2}d} - \frac{(5A}{}$$

Mathematica [C] time = 28.1243, size = 17684, normalized size = 58.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(3/
2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.173, size = 567, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/8/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-
2*C*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-4*B*cos(d*x+c)^2
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*C*cos(d*x+c)^2*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+8*A*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+2*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)-12*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-2*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)+19*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))*sin(d*x+c)+4*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)-2*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*A*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)+6*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)-18*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-7*C*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+26*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c
))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(3/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/
2)*sec(d*x + c)^(3/2)), x)
```

3.1353 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=333

$$\frac{(157A-85B+45C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(787A-475B+195C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(2671A-1495B+7\ldots}{240a^2d\sqrt{\ldots}}$$

```
[Out] -((283*A - 163*B + 75*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(
16*Sqrt[2]*a^(5/2)*d) + ((2671*A - 1495*B + 735*C)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A - 475*B + 195*C)*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A -
 B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) -
 ((21*A - 13*B + 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c
 + d*x])^(3/2)) + ((157*A - 85*B + 45*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 1.24132, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3041, 2978, 2984, 12, 2782, 205}

$$\frac{(157A-85B+45C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(787A-475B+195C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(2671A-1495B+7\ldots}{240a^2d\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + a*Cos
[c + d*x])^(5/2),x]
```


```
[Out] -((283*A - 163*B + 75*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(
16*Sqrt[2]*a^(5/2)*d) + ((2671*A - 1495*B + 735*C)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A - 475*B + 195*C)*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A -
 B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) -
 ((21*A - 13*B + 5*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c
 + d*x])^(3/2)) + ((157*A - 85*B + 45*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B+5C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B+5C)}{16ad(a+}$$

$$= -\frac{(787A-475B+195C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B}{}$$

$$= \frac{(2671A-1495B+735C)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A}{}$$

$$= \frac{(2671A-1495B+735C)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A}{}$$

$$= \frac{(2671A-1495B+735C)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A}{}$$

$$= -\frac{(283A-163B+75C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 27.8309, size = 7162, normalized size = 21.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a +
 a*Cos[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.195, size = 1045, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] 1/480/d*2^(1/2)/a^3*(2671*A*cos(d*x+c)^5*2^(1/2)-96*A*2^(1/2)-4245*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1
125*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+735*C*cos(d*x+c)^5*2^(1/2)-4245*A*cos(d*x+c)^4*sin(d*x+c)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-1125*C*cos(d
*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)+1715*B*2^(1/2)*cos(d*x+c)^3+960*B*2^(1/2)*cos(d*x+c)^2-160*B*2
^(1/2)*cos(d*x+c)-1495*B*cos(d*x+c)^5*2^(1/2)-1020*B*cos(d*x+c)^4*2^(1/2)+2
445*B*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
cos(d*x+c)^4*sin(d*x+c)+2445*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-16980*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4500*C*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))-25470*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-6750*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-16980*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))-4500*C*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+c)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))+1884*A*2^(1/2)*cos(d*x+c)^4+540*C*2^(1/2
)*cos(d*x+c)^4-2987*A*cos(d*x+c)^3*2^(1/2)-795*C*cos(d*x+c)^3*2^(1/2)-1728*
A*cos(d*x+c)^2*2^(1/2)-480*C*cos(d*x+c)^2*2^(1/2)+256*A*cos(d*x+c)*2^(1/2)+
9780*B*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)+14670*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+9780*B*cos(d*x+c)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2))*
cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos
(d*x+c))/(1+cos(d*x+c))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.32035, size = 801, normalized size = 2.41

$$\frac{15\sqrt{2}\left((283\,A-163\,B+75\,C)\cos\left(dx+c\right)^{5}+3\,(283\,A-163\,B+75\,C)\cos\left(dx+c\right)^{4}+3\,(283\,A-163\,B+75\,C)\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(15*sqrt(2)*((283*A - 163*B + 75*C)*cos(d*x + c)^5 + 3*(283*A - 163*B
 + 75*C)*cos(d*x + c)^4 + 3*(283*A - 163*B + 75*C)*cos(d*x + c)^3 + (283*A
- 163*B + 75*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((2671*A - 1495*B + 735
*C)*cos(d*x + c)^4 + 5*(911*A - 503*B + 255*C)*cos(d*x + c)^3 + 32*(49*A -
25*B + 15*C)*cos(d*x + c)^2 - 160*(A - B)*cos(d*x + c) + 96*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3
*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos
(d*x + c) + a)^(5/2), x)
```

3.1354 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$\frac{(95A-39B+15C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B+19C)\sqrt{}}{}$$

```
[Out] ((163*A - 75*B + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16
*Sqrt[2]*a^(5/2)*d) - ((299*A - 147*B + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B + C)*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39*B
 + 15*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rubi [A] time = 1.01806, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3041, 2978, 2984, 12, 2782, 205}

$$\frac{(95A-39B+15C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B+19C)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + a*Cos
[c + d*x])^(5/2),x]
```


```
[Out] ((163*A - 75*B + 19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16
*Sqrt[2]*a^(5/2)*d) - ((299*A - 147*B + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B + C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B + C)*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39*B
 + 15*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]
])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+}$$

$$=-\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$=-\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(17A-9B+C)\,\mathrm{se}}{16ad(a+a}$$

$$=-\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{(17A-9B+C)\,\mathrm{se}}{16ad(a+a}$$

$$=-\frac{(299A-147B+27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}-\frac{(A-B}{4}$$

$$=-\frac{(299A-147B+27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}-\frac{(A-B}{4}$$

$$=-\frac{(299A-147B+27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}-\frac{(A-B}{4}$$

$$=\frac{(163A-75B+19C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 25.4205, size = 7114, normalized size = 25.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 a*Cos[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.247, size = 833, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] 1/96/d*2^(1/2)/a^3*(-489*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos
(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))+225*B*sin(d*x+c)*cos(d*x+c)^3*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-57*C*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)^3*arcsin((-1+cos(d*x
+c))/sin(d*x+c))-1467*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*
x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))+675*B*cos(d*x+c)^2*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-171*C*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c
))/sin(d*x+c))-1467*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+
c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+675*B*cos(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-171*C*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))+299*A*2^(1/2)*cos(d*x+c)^4-489*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*s
in(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-147*B*cos(d*x+c)^4*2^(1/2)+225
*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+27*C*2^(1/2)*cos(d*x+c)^4-57*C*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*si
n(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+204*A*cos(d*x+c)^3*2^(1/2)-108*
B*2^(1/2)*cos(d*x+c)^3+12*C*cos(d*x+c)^3*2^(1/2)-343*A*cos(d*x+c)^2*2^(1/2)
+159*B*2^(1/2)*cos(d*x+c)^2-39*C*cos(d*x+c)^2*2^(1/2)-192*A*cos(d*x+c)*2^(1
/2)+96*B*2^(1/2)*cos(d*x+c)+32*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(
a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.84258, size = 729, normalized size = 2.59

$$-\frac{3\sqrt{2}\Big((163A-75B+19C)\cos(dx+c)^4+3(163A-75B+19C)\cos(dx+c)^3+3(163A-75B+19C)\cos(dx+}{96\left(a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] -1/96*(3*sqrt(2)*((163*A - 75*B + 19*C)*cos(d*x + c)^4 + 3*(163*A - 75*B +
19*C)*cos(d*x + c)^3 + 3*(163*A - 75*B + 19*C)*cos(d*x + c)^2 + (163*A - 75
*B + 19*C)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((299*A - 147*B + 27*C)*cos(d*
x + c)^3 + (503*A - 255*B + 39*C)*cos(d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x +
 c) - 32*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*
cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos
(d*x + c) + a)^(5/2), x)
```

3.1355 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=231

$$\frac{(49A-9B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] -((75*A - 19*B - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*
Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a
+ a*Cos[c + d*x])^(5/2)) - ((13*A - 5*B - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((49*A - 9*B + C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.804608, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3041, 2978, 2984, 12, 2782, 205}

$$\frac{(49A-9B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + a*Cos
[c + d*x])^(5/2),x]
```

```
[Out] -((75*A - 19*B - 5*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*
Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a
+ a*Cos[c + d*x])^(5/2)) - ((13*A - 5*B - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((49*A - 9*B + C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s\right.}{}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B-3C)}{16ad(a+}$$

$$= -\frac{(75A-19B-5C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 25.0702, size = 7100, normalized size = 30.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 a*Cos[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.204, size = 649, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(-1+cos(d*x+c))*(75*A*sin(d*x+c)*cos(d*x+c)^2*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(
d*x+c))-5*C*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+150*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-49*A*cos(d*x+c)^3*2^(1/
2)-38*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)+9*B*2^(1/2)*cos(d*x+c)^3-10*C*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-C*cos
(d*x+c)^3*2^(1/2)+75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)-36*A*cos(d*x+c)^2*2^(1/2)-19*B*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*B*2^(1/2)
*cos(d*x+c)^2-5*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)-4*C*cos(d*x+c)^2*2^(1/2)+53*A*cos(d*x+c)*2^(1/2)-13*
B*2^(1/2)*cos(d*x+c)+5*C*2^(1/2)*cos(d*x+c)+32*A*2^(1/2))*cos(d*x+c)*(1/cos
(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69506, size = 616, normalized size = 2.67

$$\frac{\sqrt{2}\left((75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{2}+3\,(75\,A-19\,B-5\,C)\cos\left(dx+c\right)+75\,A-\right.}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((75*A - 19*B - 5*C)*cos(d*x + c)^3 + 3*(75*A - 19*B - 5*C)*c
os(d*x + c)^2 + 3*(75*A - 19*B - 5*C)*cos(d*x + c) + 75*A - 19*B - 5*C)*sqr
t(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) + 2*((49*A - 9*B + C)*cos(d*x + c)^2 + (85*A - 13*B + 5*C)*cos
(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^
3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos
(d*x + c) + a)^(5/2), x)
```

3.1356 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(19A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ((19*A + 5*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sq
rt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/
2)*Sqrt[Sec[c + d*x]]) - ((9*A - B - 7*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[
c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.573025, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3041, 2978, 12, 2782, 205}

$$\frac{(19A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + a*Cos
[c + d*x])^(5/2),x]
```


```
[Out] ((19*A + 5*B + 3*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sq
rt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/
2)*Sqrt[Sec[c + d*x]]) - ((9*A - B - 7*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[
c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt{\cos(c+dx)}(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s}\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B-}{16ad(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B-}{16ad(a+a\cos(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B-}{16ad(a+a\cos(c}$$

$$= \frac{(19A+5B+3C)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 24.8054, size = 7093, normalized size = 38.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 a*Cos[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.202, size = 524, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2)
,x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)*(-1+cos(d*x+c))^2*(-9*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^2+B*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*C*cos(d*x+
c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*A*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)+5*B*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*C*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*C*2^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)+19*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c)+13*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*B*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)-5*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*C*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*C*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.78746, size = 599, normalized size = 3.27

$$-\frac{\sqrt{2}\left((19\,A+5\,B+3\,C)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B+3\,C)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B+3\,C)\cos\left(dx+c\right)+19\,A+5\right.}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((19*A + 5*B + 3*C)*cos(d*x + c)^3 + 3*(19*A + 5*B + 3*C)*co
s(d*x + c)^2 + 3*(19*A + 5*B + 3*C)*cos(d*x + c) + 19*A + 5*B + 3*C)*sqrt(a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c))) + 2*((9*A - B - 7*C)*cos(d*x + c)^2 + (13*A - 5*B - 3*C)*cos(d*x
```

```
+ c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos
(d*x + c) + a)^(5/2), x)
```

3.1357 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=241

$$\frac{(5A+3B-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{a\,c\ldots}}\right)}{a^{5/2}d}$$

```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A + 3*B - 43*C)*ArcTan[(Sqrt[a]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A + 3*B
 - 11*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]
])
```

Rubi [A] time = 0.779074, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3041, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A+3B-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{a\,c\ldots}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sqr
t[Sec[c + d*x]]),x]
```


```
[Out] (2*C*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A + 3*B - 43*C)*ArcTan[(Sqrt[a]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A + 3*B
 - 11*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]
])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\mathrm{c}}}{}}{4}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(5A+3B-11C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c-}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(5A+3B-11C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c-}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(5A+3B-11C)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c-}}$$

$$= \frac{2C\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(5A+3B-43C)\tan}{}$$

Mathematica [C] time = 28.002, size = 16100, normalized size = 66.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/
2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.198, size = 624, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(A
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*B*cos(d*x+c)^2*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-15*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)-32*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)+5*A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*cos(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))-4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-43*C*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+4*C*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)-32*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+5*A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*sin(d*x+c)-5*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)-43*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+11*C*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2} + B\cos\left(dx+c\right) + A}{\left(a\cos\left(dx+c\right) + a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*sqrt(sec(d*x + c))), x)
```

3.1358 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{(3A-11B+35C)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(3A-43B+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((2*B - 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B + 115*C)*ArcTan
[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B
 + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(A + 7*B - 15*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d
*x]^(3/2)) + ((3*A - 11*B + 35*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.03566, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3041, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-11B+35C)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(3A-43B+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sec
[c + d*x]^(3/2)),x]
```


```
[Out] ((2*B - 5*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B + 115*C)*ArcTan
[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B
 + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(A + 7*B - 15*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d
*x]^(3/2)) + ((3*A - 11*B + 35*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
```

```
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\text{co}}{-}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+7B-15C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+7B-15C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+7B-15C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(A+7B-15C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c}$$

$$= \frac{(2B-5C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A-43B+}{}$$

Mathematica [C] time = 28.1435, size = 16926, normalized size = 57.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/
2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.211, size = 758, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*(
16*C*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*A*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-32*B*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-15*B*co
s(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*C*2^(1/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c
)+39*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*A*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)-43*B*sin(d*x+c)*cos(d*x+c)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))-32*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c))*sin(d*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)+115*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+
80*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*sin(d*x+c)-20*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*A*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*A*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+11*B*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+115*C*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*sin(d*x+c)-35*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x
+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*sec(d*x + c)^(3/2)), x)
```

3.1359 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=352

$$\frac{(7A-15B+31C)\sin(c+dx)}{16a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(8A-20B+39C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(11A-}{16a^2d\sqrt{\mathrm{se}}}$$

```
[Out] ((8*A - 20*B + 39*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(5/2)*d) - ((43*A - 115*B + 2
19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - ((A - B + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x
]^(7/2)) - ((3*A - 11*B + 19*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^
(3/2)*Sec[c + d*x]^(5/2)) + ((7*A - 15*B + 31*C)*Sin[c + d*x])/(16*a^2*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) - ((11*A - 35*B + 63*C)*Sin[c +
d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.27368, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3041, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(7A-15B+31C)\sin(c+dx)}{16a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{(8A-20B+39C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(11A-}{16a^2d\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/2)*Sec
[c + d*x]^(5/2)),x]
```


```
[Out] ((8*A - 20*B + 39*C)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(5/2)*d) - ((43*A - 115*B + 2
19*C)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - ((A - B + C)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x
]^(7/2)) - ((3*A - 11*B + 19*C)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^
(3/2)*Sec[c + d*x]^(5/2)) + ((7*A - 15*B + 31*C)*Sin[c + d*x])/(16*a^2*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) - ((11*A - 35*B + 63*C)*Sin[c +
d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
```

```
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A-11B+19C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A-11B+19C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A-11B+19C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A-11B+19C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{(3A-11B+19C)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c}$$

$$= \frac{(8A-20B+39C)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{5/2}d} - \frac{(43A}{}$$

Mathematica [C] time = 28.4879, size = 17757, normalized size = 50.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + a*Cos[c + d*x])^(5/
2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.191, size = 924, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^5*(
-8*C*cos(d*x+c)^4*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-16*B*cos(d*x+c)
^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+28*C*cos(d*x+c)^3*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*A*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)+15*A*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-80*B*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-39*B*cos(d*x
+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+156*C*2^(1/2)*arctan(sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)+75
*C*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+43*A*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+32*A*2^(1/2)*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-4*A*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-115*B*sin(d*x+c)*cos(d*x+c)*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))-80*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+20*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)+219*C*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)
*cos(d*x+c)+156*C*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c))*sin(d*x+c)-32*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)+43*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-11*A*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-115*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)+35*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+219*C*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)-63*C*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(5/2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/
2)*sec(d*x + c)^(5/2)), x)
```

3.1360 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=205

$$\frac{2a(5A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}}{7d}$$

```
[Out] (-2*b*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(5*d) + (2*a*(5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (
2*A*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*A*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(7*d)
```

Rubi [A] time = 0.295611, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3032, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(5A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2aA\sin(c+dx)\sec^{\frac{7}{2}}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-2*b*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(5*d) + (2*a*(5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (
2*A*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*A*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))\left(A+C\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$=\frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$=\frac{2Ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$=\frac{2Ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$=\frac{2b(3A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a(5A+7C)\sec^{\frac{3}{2}}}{2}$$

$$=-\frac{2b(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A] time = 2.52812, size = 155, normalized size = 0.76

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(2\sin(c+dx)(10a(5A+7C)\cos(2(c+dx))+110aA+70aC+21b(13A+15C)\cos(c+dx)+63Ab\cos(3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*b*(3*A + 5*C)*Cos[c + d*x]^(7/2)*EllipticE[(c + d
*x)/2, 2] + 40*a*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] +
 2*(110*a*A + 70*a*C + 21*b*(13*A + 15*C)*Cos[c + d*x] + 10*a*(5*A + 7*C)*C
os[2*(c + d*x)] + 63*A*b*Cos[3*(c + d*x)] + 105*b*C*Cos[3*(c + d*x)])*Sin[c
 + d*x]))/(420*d)
```

Maple [B] time = 4.181, size = 841, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*C*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*A*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*A*
(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
+2*C*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3+Ca\cos(dx+c)^2+Ab\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x
)
```

3.1361 $\int (a+b\cos(c+dx)) \left(A + C\cos^2(c+dx)\right) \sec^{\frac{7}{2}}(c+dx)\, dx$

Optimal. Leaf size=172

$$\frac{2a(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d} - \frac{2a(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aA\sin(c+dx)\sec\cdots}{5d}$$

```
[Out] (-2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*A*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.26713, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3032, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d} - \frac{2a(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aA\sin(c+dx)\sec\cdots}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*A*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))\left(A+C\right.}{\cos^{\frac{7}{2}}(c+dx}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2Ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{2Ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{2b(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} +$$

$$= -\frac{2a(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A] time = 1.63348, size = 122, normalized size = 0.71

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(2\sin(c+dx)(3a((3A+5C)\cos(2(c+dx))+5(A+C))+10Ab\cos(c+dx))-12a(3A+5C)\cos^{\frac{5}{2}}(c+dx\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*a*(3*A + 5*C)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*
x)/2, 2] + 20*b*(A + 3*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*
(10*A*b*Cos[c + d*x] + 3*a*(5*(A + C) + (3*A + 5*C)*Cos[2*(c + d*x)]))*Sin[
c + d*x]))/(30*d)
```

Maple [B] time = 3.631, size = 732, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*b
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a*A/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)+2*a*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2
*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x
)
```

3.1362 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{2b(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{d}$$

```
[Out] (-2*b*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*A*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.242031, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4221, 3032, 3021, 2748, 2641, 2639}

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{2b(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-2*b*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*A*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
```

```
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))\left(A+C\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2Ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3d}$$

$$= \frac{2Ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3d}$$

$$= -\frac{2b(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \cdots$$

Mathematica [A] time = 0.443414, size = 96, normalized size = 0.71

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2A\sin(c+dx)(a+3b\cos(c+dx))+2a(A+3C)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6b(A-C)\cos^{\frac{3}{2}}(c+dx)E\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*b*(A - C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2,
 2] + 2*a*(A + 3*C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*A*(a +
 3*b*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 2.783, size = 614, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*EllipticF(co
```

```
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*a*sin(1/2*d*x+1/2*c)^2+6*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b*sin(1/2*
d*x+1/2*c)^2-12*A*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*a*sin(1/2*d*x+1/2*c)^2-6*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b*sin(1/2
*d*x+1/2*c)^2-a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+2
*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*A*b*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2-3*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa\right)\sec(dx+c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x
)
```

3.1363 $\int (a+b\cos(c+dx)) \left(A+C\cos^2(c+dx)\right) \sec^{\frac{3}{2}}(c+dx)\, dx$

Optimal. Leaf size=135

$$-\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b(3A+C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{3d}$$

```
[Out] (-2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*A
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.235075, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4221, 3032, 3023, 2748, 2641, 2639}

$$-\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b(3A+C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*A
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
```

```
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))\left(A+C\right.}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2bC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{3}\left(4\sqrt{\right.$$

$$= \frac{2bC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(a(A-\right.$$

$$= -\frac{2a(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [A] time = 0.401319, size = 98, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)(3aA+bC\cos(c+dx)) - 6a(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2b(3A+C)\sqrt{\cos(c+d}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2] + 2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(3*a*A
 + b*C*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [A] time = 1.164, size = 294, normalized size = 2.2

$$-\frac{2}{3d}\left(4Cb\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^4 + 3Ab\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*C*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-6*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+C*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*C*b
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^{2}+A\right)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x
)
```

3.1364 $\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{5d}$$

```
[Out] (2*b*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.231809, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4221, 3034, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*b*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (2*a*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
```

```
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))\left(A+C\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2bC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{5aA}{2}}{}$$

$$= \frac{2bC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{15}\left(4\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2bC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a(3A+C)\sqrt{\cos(\right.$$

$$= \frac{2b(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +$$

Mathematica [A] time = 0.481177, size = 101, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(10a(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))(5a+3b\cos(c+dx))+6b(5A+3C)\sqrt{\cos(c\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*b*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 10*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*(5
*a + 3*b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 1.2, size = 363, normalized size = 2.6

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(20\,aC+24\,Cb\right)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*C*a+24*C*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*C*a-6*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15
*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+5*a*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x
)
```

3.1365 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{2a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(7A+5C)}{}$$

```
[Out] (2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*(7*A + 5*C)*Sin[c
+ d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.256132, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3034, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(7A+5C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*a*C*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*(7*A + 5*C)*Sin[c
+ d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(A+\right.$$

$$= \frac{2bC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\Big($$

$$= \frac{2bC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{35}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{2bC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(a(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.$$

$$= \frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2bC\sin(c}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b(7A+5}{}$$

Mathematica [A] time = 0.833644, size = 120, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42aC\cos(c+dx)+70Ab+15bC\cos(2(c+dx))+65bC)+84a(5A+3C)\sqrt{\cos(c+dx)}E\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2] + 20*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (
70*A*b + 65*b*C + 42*a*C*Cos[c + d*x] + 15*b*C*Cos[2*(c + d*x)])*Sin[2*(c +
 d*x)]))/(210*d)
```

Maple [A] time = 1.123, size = 401, normalized size = 2.3

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Big(240\,Cb\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-168\,aC-360\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*C*a-360*C*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A*b+168*C*a+280*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-70*A*b-42*C*a-80*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+35*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+25*C*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\left(a + b\cos\left(c + dx\right)\right)}{\sqrt{\sec\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\left(b\cos\left(dx + c\right) + a\right)}{\sqrt{\sec\left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x
)
```

3.1366 $\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{2a(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b(9A+7C)\text{si}}{45d\sec^{\frac{3}{2}}(c}$$

```
[Out] (2*b*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(15*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
)) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*(9*A + 7*C)*Sin[c
 + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*C)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.294256, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4221, 3034, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7A+5C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b(9A+7C)\text{si}}{45d\sec^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*b*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(15*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
)) + (2*a*C*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*(9*A + 7*C)*Sin[c
 + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*C)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(A+C\right.$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{9a}{2}\right.$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{63}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec\right.$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(9A+7C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A}{2}$$

$$= \frac{2b(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2a(7A+5C}{}$$

Mathematica [A] time = 1.23425, size = 141, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5(84aA+18aC\cos(2(c+dx))+78aC+7bC\cos(3(c+dx)))+7b(36A+43C)\cos(c+dx)\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*b*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 120*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 (7*b*(36*A + 43*C)*Cos[c + d*x] + 5*(84*a*A + 78*a*C + 18*a*C*Cos[2*(c + d
*x)] + 7*b*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [A] time = 1.315, size = 443, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*C*a+2240*C*b)*sin(1/2*d*x+1/2*
c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b-1080*C*a-2072*C*b)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(420*A*a+504*A*b+840*C*a+952*C*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-210*A*a-126*A*b-240*C*a-168*C*b)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+105*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*b+75*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2(c + dx)\right)(a + b\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*(a + b*cos(c + d*x))/sec(c + d*x)**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx + c)^2 + A\right)(b\cos(dx + c) + a)}{\sec(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x
)
```

3.1367 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{2\left(a^2(7A+9C)+4Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{2\left(a^2(7A+9C)+3b^2(3A+5C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2\left(a^2(\right.}{}$$

```
[Out] (-2*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (4*a*b*(5*A + 7*C)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(3*b^2*(3*A +
 5*C) + a^2*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a*b*(
5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(4*A*b^2 + a^2*(7*A
 + 9*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (8*a*A*b*Sec[c + d*x]^(7
/2)*Sin[c + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*S
in[c + d*x])/(9*d)
```

Rubi [A] time = 0.629894, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2\left(a^2(7A+9C)+4Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{2\left(a^2(7A+9C)+3b^2(3A+5C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2\left(a^2(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (-2*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (4*a*b*(5*A + 7*C)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(3*b^2*(3*A +
 5*C) + a^2*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a*b*(
5*A + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(4*A*b^2 + a^2*(7*A
 + 9*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (8*a*A*b*Sec[c + d*x]^(7
/2)*Sin[c + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)*S
in[c + d*x])/(9*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^2\left(A\right.}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2A(a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\right.$$

$$= \frac{8aAb\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A(a+b\cos(c+dx))}{}$$

$$= \frac{2\left(4Ab^2+a^2(7A+9C)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{8a}{}$$

$$= \frac{2\left(4Ab^2+a^2(7A+9C)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{8a}{}$$

$$= \frac{2\left(3b^2(3A+5C)+a^2(7A+9C)\right)\sqrt{\sec(c+dx)}\sin(c+}{15d}$$

$$= -\frac{2\left(3b^2(3A+5C)+a^2(7A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [A] time = 6.41383, size = 286, normalized size = 0.98

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}\left(7a^2A+9a^2C+9Ab^2+15b^2C\right)\sin(c+dx)+\frac{2}{45}\sec^2(c+dx)\left(7a^2A\sin(c+dx)+9a^2C\sin(c+dx)+9\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] ((2*(-49*a^2*A - 63*A*b^2 - 63*a^2*C - 105*b^2*C)*EllipticE[(c + d*x)/2, 2]
)/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(50*a*A*b + 70*a*b*C)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(105*d) + (Sqrt[S
ec[c + d*x]]*((2*(7*a^2*A + 9*A*b^2 + 9*a^2*C + 15*b^2*C)*Sin[c + d*x])/15
+ (2*Sec[c + d*x]^2*(7*a^2*A*Sin[c + d*x] + 9*A*b^2*Sin[c + d*x] + 9*a^2*C*
Sin[c + d*x]))/45 + (4*Sec[c + d*x]*(5*a*A*b*Sin[c + d*x] + 7*a*b*C*Sin[c +
 d*x]))/21 + (4*a*A*b*Sec[c + d*x]^2*Tan[c + d*x])/7 + (2*a^2*A*Sec[c + d*x
]^3*Tan[c + d*x])/9))/d
```

Maple [B] time = 5.346, size = 1179, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*C*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^2*(-1/144*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7
/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-2/5*(A*b^2+C*a^2)/(8*sin(1
```

```
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+4*a*A*b*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+
1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sec\left(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(11/2)
, x)
```

3.1368 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=243

$$\frac{2\left(a^2(5A+7C)+4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(a^2(5A+7C)+7b^2(A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

```
[Out] (-4*a*b*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a*b*(3*A + 5*C)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(4*A*b^2 + a^2*(5*A + 7*C))*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a*A*b*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])
/(7*d)
```

Rubi [A] time = 0.568128, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(a^2(5A+7C)+4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(a^2(5A+7C)+7b^2(A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-4*a*b*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a*b*(3*A + 5*C)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(4*A*b^2 + a^2*(5*A + 7*C))*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a*A*b*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])
/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^2\left(A\right.}{\cos^{\frac{9}{2}}(c+\right.}$$

$$=\frac{2A(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{1}{7}\left(2\right.$$

$$=\frac{8aAb\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2A(a+b\cos(c+dx))^2}{}$$

$$=\frac{2\left(4Ab^2+a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{8aA}{}$$

$$=\frac{2\left(4Ab^2+a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{8aA}{}$$

$$=\frac{2\left(7b^2(A+3C)+a^2(5A+7C)\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{21d}$$

$$=-\frac{4ab(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [A] time = 1.34922, size = 218, normalized size = 0.9

$$\frac{2\sec^{\frac{7}{2}}(c+dx)\left(5\left(a^2(5A+7C)+7b^2(A+3C)\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+15a^2A\sin(c+dx)+25a^2A\sin(c+dx)\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (2*Sec[c + d*x]^(7/2)*(-42*a*b*(3*A + 5*C)*Cos[c + d*x]^(7/2)*EllipticE[(c
+ d*x)/2, 2] + 5*(7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Cos[c + d*x]^(7/2)*Ell
ipticF[(c + d*x)/2, 2] + 15*a^2*A*Sin[c + d*x] + 25*a^2*A*Cos[c + d*x]^2*Si
n[c + d*x] + 35*A*b^2*Cos[c + d*x]^2*Sin[c + d*x] + 35*a^2*C*Cos[c + d*x]^2
*Sin[c + d*x] + 126*a*A*b*Cos[c + d*x]^3*Sin[c + d*x] + 210*a*b*C*Cos[c + d
*x]^3*Sin[c + d*x] + 21*a*A*b*Sin[2*(c + d*x)]))/(105*d)
```

Maple [B] time = 4.542, size = 930, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(
A*b^2+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*a*A*b/(8*sin(1/2
*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x
+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^
2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*A*a^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
```

```
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))+4*a*b*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2),
 x)
```

3.1369 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{2\left(a^2(3A+5C)+4Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(a^2(3A+5C)+5b^2(A-C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

```
[Out] (-2*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(4*A*b^2 + a^2*(3*A +
 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a*A*b*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(15*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(5*d)
```

Rubi [A] time = 0.528022, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3048, 3031, 3021, 2748, 2641, 2639}

$$\frac{2\left(a^2(3A+5C)+4Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(a^2(3A+5C)+5b^2(A-C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(4*A*b^2 + a^2*(3*A +
 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a*A*b*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(15*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^2\left(A\right.}{\cos^{\frac{7}{2}}(c+}$$

$$=\frac{2A(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\left(2\right.$$

$$=\frac{8aAb\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2A(a+b\cos(c+dx))^2}{}$$

$$=\frac{2\left(4Ab^2+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{8a}{}$$

$$=\frac{2\left(4Ab^2+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{8a}{}$$

$$=-\frac{2\left(5b^2(A-C)+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{5d}$$

Mathematica [A] time = 2.14974, size = 147, normalized size = 0.7

$$\frac{2\sec^{\frac{5}{2}}(c+dx)\left(3\sin(c+dx)\left(\left(a^2(3A+5C)+5Ab^2\right)\cos^2(c+dx)+a^2A\right)-3\left(a^2(3A+5C)+5b^2(A-C)\right)\cos^{\frac{5}{2}}(c+dx\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sec[c + d*x]^(5/2)*(-3*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*x]^(5
/2)*EllipticE[(c + d*x)/2, 2] + 10*a*b*(A + 3*C)*Cos[c + d*x]^(5/2)*Ellipti
cF[(c + d*x)/2, 2] + 3*(a^2*A + (5*A*b^2 + a^2*(3*A + 5*C))*Cos[c + d*x]^2)
```

```
*Sin[c + d*x] + 5*a*A*b*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 3.69, size = 913, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+4*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a*A*b*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*A*a^2/(8*sin(1/2*d*x+1/2*c)^6-12*si
n(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*(A*b^2+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*
x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sec\left(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
 x)
```

3.1370 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=194

$$\frac{2\left(a^2(A+3C)+b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4ab(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}$$

```
[Out] (-4*a*b*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(b^2*(3*A + C) + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(A - C)*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]) + (8*a*A*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (
2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.520299, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3048, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\left(a^2(A+3C)+b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4ab(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-4*a*b*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(b^2*(3*A + C) + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(A - C)*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]) + (8*a*A*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (
2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^2\left(A\right.}{\cos^{\frac{5}{2}}(c+}$$

$$=\frac{2A(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(2\right.$$

$$=\frac{8aAb\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A(a+b\cos(c+dx))^2}{}$$

$$=-\frac{2b^2(A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8aAb\sqrt{\sec(c+dx)}\sin(c+d}{3d}$$

$$=-\frac{2b^2(A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8aAb\sqrt{\sec(c+dx)}\sin(c+d}{3d}$$

$$=-\frac{4ab(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.57352, size = 133, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\left(a^2(A+3C)+b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(2a^2A+12aAb\cos(c+dx)+b^2C\cos(2(c+dx))+b\right.}{\cos^{\frac{3}{2}}(c+dx)}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-12*a*b*(A - C)*EllipticE[(c + d*x)
/2, 2] + 2*(b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2] + ((2*
a^2*A + b^2*C + 12*a*A*b*Cos[c + d*x] + b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x
])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 1.561, size = 871, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-8*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*b*(3*A*a+C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*a^2+6*A*a*b+C*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*b^2+6*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+3*C*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)*sin(1/2*d*x+1/2*c)^2+A*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+6*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a*b+3*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-6*C*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1
/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + 2\,Cab\cos(dx+c)^3 + 2\,Aab\cos(dx+c) + Aa^2 + \left(Ca^2 + Ab^2\right)\cos(dx+c)^2\right)\sec(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^2 \sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
 x)
```

3.1371 $\int (a+b\cos(c+dx))^2 \left(A + C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=206

$$-\frac{2\left(5a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4ab(3A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab(3A+C)\cdots}{}$$

```
[Out] (-2*(5*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a*b*(3*A + C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(5*A - C)*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) - (4*a*b*(3*A - C)*Sin[c + d*x])/(3*d*Sqrt[
Sec[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d
```

Rubi [A] time = 0.528284, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3048, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2\left(5a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4ab(3A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab(3A+C)\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(5*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a*b*(3*A + C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(5*A - C)*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) - (4*a*b*(3*A - C)*Sin[c + d*x])/(3*d*Sqrt[
Sec[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^2\left(A\right.}{\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2A(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{c\right.$$

$$= -\frac{2b^2(5A-C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+b\cos(c+dx))^2\sqrt{\sec}}{d}$$

$$= -\frac{2b^2(5A-C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4ab(3A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} +$$

$$= -\frac{2b^2(5A-C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4ab(3A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} +$$

$$= -\frac{2\left(5a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{5d}$$

Mathematica [A] time = 1.10257, size = 139, normalized size = 0.67

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(12\left(b^2(5A+3C)-5a^2(A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2\sin(c+dx)\left(30a^2A+20abC\cos(c+dx)+3b^2C\cos(2(c+\right.}{\sqrt{\cos(c+dx)}}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*(-5*a^2*(A - C) + b^2*(5*A + 3*C
))*EllipticE[(c + d*x)/2, 2] + 40*a*b*(3*A + C)*EllipticF[(c + d*x)/2, 2] +
 (2*(30*a^2*A + 3*b^2*C + 20*a*b*C*Cos[c + d*x] + 3*b^2*C*Cos[2*(c + d*x)])
*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(30*d)
```

Maple [B] time = 1.385, size = 694, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*b*(5*a+3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^2+10*C*a*b+3*C*b^2
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-15*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b
^2+10*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+2\,Cab\cos(dx+c)^3+2\,Aab\cos(dx+c)+Aa^2+\left(Ca^2+Ab^2\right)\cos(dx+c)^2\right)\sec(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
 x)
```

3.1372 $\int (a+b\cos(c+dx))^2 \left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{2\left(4a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2(3A+C)+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab}{}$$

```
[Out] (4*a*b*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(7*a^2*(3*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*C*Sin[c + d*x
])/(35*d*Sec[c + d*x]^(3/2)) + (2*(4*a^2*C + b^2*(7*A + 5*C))*Sin[c + d*x])
/(21*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.513227, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3050, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(4a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2(3A+C)+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (4*a*b*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(7*a^2*(3*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*C*Sin[c + d*x
])/(35*d*Sec[c + d*x]^(3/2)) + (2*(4*a^2*C + b^2*(7*A + 5*C))*Sin[c + d*x])
/(21*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d
*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^2\left(A+\right.}{\sqrt{\cos(c+d}}$$

$$=\frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$=\frac{8abC\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+$$

$$=\frac{8abC\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(4a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

$$=\frac{8abC\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(4a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

$$=\frac{4ab(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A] time = 1.07578, size = 148, normalized size = 0.7

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(70a^2C+84abC\cos(c+dx)+70Ab^2+15b^2C\cos(2(c+dx))+65b^2C\right)+20\left(7a^2(3A+C)\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*a*b*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 20*(7*a^2*(3*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2] + (70*A*b^2 + 70*a^2*C + 65*b^2*C + 84*a*b*C*Cos[c +
d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] time = 1.145, size = 532, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-336*C*a*b-360*C*b^2)*sin(1/2*d*x+1
/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^2+140*C*a^2+336*C*a*b+280*C*b^2)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*b^2-70*C*a^2-84*C*a*b-80*C*b^2)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35
*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-210*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+35*a^2*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+25*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{\sec\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
 x)
```

3.1373 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=245

$$\frac{2\left(4a^2C+b^2(9A+7C)\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

```
[Out] (2*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (4*a*b*(7*A + 5*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*C*Sin[c +
d*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(4*a^2*C + b^2*(9*A + 7*C))*Sin[c + d*
x])/(45*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
9*d*Sec[c + d*x]^(3/2)) + (4*a*b*(7*A + 5*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c
 + d*x]])
```

Rubi [A] time = 0.553583, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3050, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(4a^2C+b^2(9A+7C)\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (4*a*b*(7*A + 5*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*C*Sin[c +
d*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(4*a^2*C + b^2*(9*A + 7*C))*Sin[c + d*
x])/(45*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
9*d*Sec[c + d*x]^(3/2)) + (4*a*b*(7*A + 5*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c
 + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\left(A\right.$$

$$= \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{8abC\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{63}\left(4\sqrt{\cos}\right.$$

$$= \frac{8abC\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(4a^2C+b^2(9A+7C)\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+b}{}$$

$$= \frac{8abC\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(4a^2C+b^2(9A+7C)\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+b}{}$$

$$= \frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$

$$= \frac{2\left(3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$

Mathematica [A] time = 1.57533, size = 170, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7\left(36a^2C+36Ab^2+43b^2C\right)\cos(c+dx)+5b(168aA+36aC\cos(2(c+dx))+156aC+7\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2] + 240*a*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2] + (7*(36*A*b^2 + 36*a^2*C + 43*b^2*C)*Cos[c + d*x]
 + 5*b*(168*a*A + 156*a*C + 36*a*C*Cos[2*(c + d*x)] + 7*b*C*Cos[3*(c + d*x)
]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] time = 1.229, size = 587, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1440*C*a*b+2240*C*b^2)*sin(1/2*d
*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^2-504*C*a^2-2160*C*a*b-2072*C*b^2)
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(840*A*a*b+504*A*b^2+504*C*a^2+168
0*C*a*b+952*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-420*A*a*b-126*
A*b^2-126*C*a^2-480*C*a*b-168*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+210*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*b^2+150*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos(dx+c)^4 + 2\,Cab\cos(dx+c)^3 + 2\,Aab\cos(dx+c) + Aa^2 + \left(Ca^2 + Ab^2\right)\cos(dx+c)^2}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
 x)
```

3.1374 $\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{2\left(11a^2(7A+5C)+5b^2(11A+9C)\right)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2\left(11a^2(7A+5C)+5b^2(11A\right.}{}$$

```
[Out] (4*a*b*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (2*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a*b*C*Sin
[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*(4*a^2*C + b^2*(11*A + 9*C))*Sin[
c + d*x])/(77*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(11*d*Sec[c + d*x]^(5/2)) + (4*a*b*(9*A + 7*C)*Sin[c + d*x])/(45*d*Sec
[c + d*x]^(3/2)) + (2*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sin[c + d*x
])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.633486, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3050, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{2\left(11a^2(7A+5C)+5b^2(11A+9C)\right)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2\left(11a^2(7A+5C)+5b^2(11A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```



```
[Out] (4*a*b*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (2*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a*b*C*Sin
[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*(4*a^2*C + b^2*(11*A + 9*C))*Sin[
c + d*x])/(77*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(11*d*Sec[c + d*x]^(5/2)) + (4*a*b*(9*A + 7*C)*Sin[c + d*x])/(45*d*Sec
[c + d*x]^(3/2)) + (2*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sin[c + d*x
])/(231*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\left(A\right.$$

$$= \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{11}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{8abC\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{99}\left(4\sqrt{\cos}\right.$$

$$= \frac{8abC\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+}{}$$

$$= \frac{8abC\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+}{}$$

$$= \frac{8abC\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(4a^2C+b^2(11A+9C)\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+}{}$$

$$= \frac{4ab(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2\left(11a^2(\right.}{}$$

Mathematica [A] time = 2.44279, size = 209, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(5\left(36\left(11a^2C+11Ab^2+16b^2C\right)\cos(2(c+dx))+132a^2(14A+13C)+308abC\cos(3(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(14784*a*b*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2] + 480*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 2*(308*a*b*(36*A + 43*C)*Cos[c + d*x] + 5*
(132*a^2*(14*A + 13*C) + 3*b^2*(572*A + 531*C) + 36*(11*A*b^2 + 11*a^2*C +
16*b^2*C)*Cos[2*(c + d*x)] + 308*a*b*C*Cos[3*(c + d*x)] + 63*b^2*C*Cos[4*(c
 + d*x)]))*Sin[2*(c + d*x)]))/(55440*d)
```

Maple [B] time = 1.162, size = 649, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-24640*C*a*b-50400*C*b^2)*sin(1
/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^2+7920*C*a^2+49280*C*a*b+5688
0*C*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-11088*A*a*b-11880*A*b^2-
11880*C*a^2-45584*C*a*b-34920*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)+(4620*A*a^2+11088*A*a*b+9240*A*b^2+9240*C*a^2+20944*C*a*b+13860*C*b^2)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310*A*a^2-2772*A*a*b-2640*A*b^2-26
40*C*a^2-3696*C*a*b-2790*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-415
8*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+1155*A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A*
b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-3234*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+825*a^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+675*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2),
 x)
```

3.1375 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=333

$$\frac{2a\left(7a^2(7A+9C)+24Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d}+\frac{2b\left(9a^2(5A+7C)+8Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{63d}+\frac{2a\left(a^2(7\right.}{}$$

```
[Out] (-2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*b*(7*b^2*(A + 3*C) + 3*a^2*(5
*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(21*d) + (2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*b*(8*A*b^2 + 9*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(63*d) + (2*a*(24*A*b^2 + 7*a^2*(7*A + 9*C))*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(315*d) + (4*A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*
Sin[c + d*x])/(21*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c
 + d*x])/(9*d)
```

Rubi [A] time = 0.952832, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3048, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a\left(7a^2(7A+9C)+24Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d}+\frac{2b\left(9a^2(5A+7C)+8Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{63d}+\frac{2a\left(a^2(7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (-2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*b*(7*b^2*(A + 3*C) + 3*a^2*(5
*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(21*d) + (2*a*(9*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*b*(8*A*b^2 + 9*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(63*d) + (2*a*(24*A*b^2 + 7*a^2*(7*A + 9*C))*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(315*d) + (4*A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*
Sin[c + d*x])/(21*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c
 + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^3\left(A+\right.}{\cos^{\frac{11}{2}}(c+d}$$

$$= \frac{2A(a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{}\right.$$

$$= \frac{4Ab(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{2A(a}{}$$

$$= \frac{2a\left(24Ab^2+7a^2(7A+9C)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d} + \frac{4}{}$$

$$= \frac{2b\left(8Ab^2+9a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2a}{}$$

$$= \frac{2b\left(8Ab^2+9a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2a}{}$$

$$= \frac{2b\left(7b^2(A+3C)+3a^2(5A+7C)\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

$$= -\frac{2a\left(9b^2(3A+5C)+a^2(7A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [A] time = 6.54624, size = 324, normalized size = 0.97

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}a\left(7a^2A+9a^2C+27Ab^2+45b^2C\right)\sin(c+dx)+\frac{2}{45}\sec^2(c+dx)\left(7a^3A\sin(c+dx)+9a^3C\sin(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] ((2*(-49*a^3*A - 189*a*A*b^2 - 63*a^3*C - 315*a*b^2*C)*EllipticE[(c + d*x)/
2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(75*a^2*A*b + 35*A*b^3 +
 105*a^2*b*C + 105*b^3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(105*d) + (Sqrt[Sec[c + d*x]]*((2*a*(7*a^2*A + 27*A*b^2 + 9
*a^2*C + 45*b^2*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^2*(7*a^3*A*Sin[c + d*
```

```
x] + 27*a*A*b^2*Sin[c + d*x] + 9*a^3*C*Sin[c + d*x]))/45 + (2*Sec[c + d*x]*
(15*a^2*A*b*Sin[c + d*x] + 7*A*b^3*Sin[c + d*x] + 21*a^2*b*C*Sin[c + d*x]))
/21 + (6*a^2*A*b*Sec[c + d*x]^2*Tan[c + d*x])/7 + (2*a^3*A*Sec[c + d*x]^3*T
an[c + d*x])/9))/d
```

Maple [B] time = 5.788, size = 1270, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b^2+3*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3*(-1/144*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-2/
5*a*(3*A*b^2+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*A*a^2*b*(-1/56*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*C*a*b^2*(-(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(11/2)
, x)
```

3.1376 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=283

$$\frac{2a\left(5a^2(5A+7C)+24Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{6b\left(7a^2(3A+5C)+8Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{35d}+\frac{2a\left(a^2(5A\right.}{}$$

```
[Out] (-2*b*(5*b^2*(A - C) + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a*(21*b^2*(A + 3*C) + a^2*(5*A +
 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21
*d) + (6*b*(8*A*b^2 + 7*a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
35*d) + (2*a*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (12*A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(35*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d
)
```

Rubi [A] time = 0.861843, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2a\left(5a^2(5A+7C)+24Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{6b\left(7a^2(3A+5C)+8Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{35d}+\frac{2a\left(a^2(5A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-2*b*(5*b^2*(A - C) + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a*(21*b^2*(A + 3*C) + a^2*(5*A +
 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21
*d) + (6*b*(8*A*b^2 + 7*a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
35*d) + (2*a*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (12*A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(35*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d
)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^3 \left(A\right.}{\cos^{\frac{9}{2}}(c+\right.}$$

$$= \frac{2A(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\right.$$

$$= \frac{12Ab(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A}{}$$

$$= \frac{2a\left(24Ab^2+5a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} +$$

$$= \frac{6b\left(8Ab^2+7a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{35d} + \frac{2}{}$$

$$= \frac{6b\left(8Ab^2+7a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{35d} + \frac{2}{}$$

$$= -\frac{2b\left(5b^2(A-C)+3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 2.24872, size = 261, normalized size = 0.92

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(10a\left(a^2(5A+7C)+21b^2(A+3C)\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-42b\left(3a^2(3A+5C)+5b^2(A-C)\right)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-42*b*(5*b^2*(A - C) + 3*a^2*(3*A + 5*C))*Cos[c + d*x]
^(7/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(21*b^2*(A + 3*C) + a^2*(5*A + 7*C)
)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 30*a^3*A*Sin[c + d*x] + 50
*a^3*A*Cos[c + d*x]^2*Sin[c + d*x] + 210*a*A*b^2*Cos[c + d*x]^2*Sin[c + d*x
] + 70*a^3*C*Cos[c + d*x]^2*Sin[c + d*x] + 378*a^2*A*b*Cos[c + d*x]^3*Sin[c
 + d*x] + 210*A*b^3*Cos[c + d*x]^3*Sin[c + d*x] + 630*a^2*b*C*Cos[c + d*x]^
3*Sin[c + d*x] + 63*a^2*A*b*Sin[2*(c + d*x)]))/(105*d)
```

Maple [B] time = 4.525, size = 1113, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*(3*A*b^2
+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*A*a^2*b/(8*sin(1/2*d*
x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/
2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2
4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*A*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b^2+3*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+3\,Cab^2\cos\left(dx+c\right)^4+3\,Aa^2b\cos\left(dx+c\right)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
 x)
```

3.1377 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=269

$$\frac{2a\left(a^2(3A+5C)+8Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(3a^2(A+3C)+b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

[Out] (-2*a*(15*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*b*(b^2*(3*A + C) + 3*a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^3*(9*A - 5*C)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b^2 + a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rubi [A] time = 0.816824, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2a\left(a^2(3A+5C)+8Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(3a^2(A+3C)+b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]

[Out] (-2*a*(15*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*b*(b^2*(3*A + C) + 3*a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^3*(9*A - 5*C)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b^2 + a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*A*b*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^3\left(A+\right.}{\cos^{\frac{7}{2}}(c+dx}$$

$$= \frac{2A(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{c}\right.$$

$$= \frac{4Ab(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2A(a}{}$$

$$= \frac{2a\left(8Ab^2+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4Ab}{}$$

$$= -\frac{2b^3(9A-5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a\left(8Ab^2+a^2(3A+5C)\right)\sqrt{}}{5d}$$

$$= -\frac{2b^3(9A-5C)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a\left(8Ab^2+a^2(3A+5C)\right)\sqrt{}}{5d}$$

$$= -\frac{2a\left(15b^2(A-C)+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 2.09394, size = 216, normalized size = 0.8

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(10b\left(3a^2(A+3C)+b^2(3A+C)\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6a\left(a^2(3A+5C)+15b^2(A-C)\right)\cos^{\frac{5}{2}}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-6*a*(15*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*x]^(
5/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(b^2*(3*A + C) + 3*a^2*(A + 3*C))*Cos
[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 6*a^3*A*Sin[c + d*x] + 18*a^3*A
*Cos[c + d*x]^2*Sin[c + d*x] + 90*a*A*b^2*Cos[c + d*x]^2*Sin[c + d*x] + 30*
a^3*C*Cos[c + d*x]^2*Sin[c + d*x] + 10*b^3*C*Cos[c + d*x]^3*Sin[c + d*x] +
15*a^2*A*b*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 4.251, size = 1333, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^3*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))-4*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+6*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a^2*b*(-1/6*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(3*A*b^2+C*a
^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
```

```
1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+3\,Cab^2\cos\left(dx+c\right)^4+3\,Aa^2b\cos\left(dx+c\right)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
 x)
```

3.1378 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=258

$$\frac{2a\left(a^2(A+3C)+3b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(15a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+}}{5d}$$

```
[Out] (-2*b*(15*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a*(3*b^2*(3*A + C) + a^2*(A + 3*C
))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) -
 (2*b^3*(35*A - 3*C)*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)) - (2*a*b^2*(5*
A - C)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) + (4*A*b*(a + b*Cos[c + d*x])^2
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.82744, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2a\left(a^2(A+3C)+3b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(15a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```



```
[Out] (-2*b*(15*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a*(3*b^2*(3*A + C) + a^2*(A + 3*C
))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) -
 (2*b^3*(35*A - 3*C)*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)) - (2*a*b^2*(5*
A - C)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) + (4*A*b*(a + b*Cos[c + d*x])^2
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^3\left(A\right.}{\cos^{\frac{5}{2}}(c+}$$

$$=\frac{2A(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(2\right.$$

$$=\frac{4Ab(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2A(}{}$$

$$=-\frac{2b^3(35A-3C)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}+\frac{4Ab(a+b\cos(c+dx))^2}{d}$$

$$=-\frac{2b^3(35A-3C)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}-\frac{2ab^2(5A-C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

$$=-\frac{2b^3(35A-3C)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}-\frac{2ab^2(5A-C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

$$=-\frac{2b\left(15a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 2.6118, size = 179, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(20a\left(a^2(A+3C)+3b^2(3A+C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+12b\left(b^2(5A+3C)-15a^2(A-C)\right)E\left(\frac{1}{2}\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*b*(-15*a^2*(A - C) + b^2*(5*A +
3*C))*EllipticE[(c + d*x)/2, 2] + 20*a*(3*b^2*(3*A + C) + a^2*(A + 3*C))*El
lipticF[(c + d*x)/2, 2] + ((20*a^3*A + 30*a*b^2*C + 9*b*(20*a^2*A + b^2*C)*
Cos[c + d*x] + 30*a*b^2*C*Cos[2*(c + d*x)] + 3*b^3*C*Cos[3*(c + d*x)])*Sin[
c + d*x])/Cos[c + d*x]^(3/2)))/(30*d)
```

Maple [B] time = 4.072, size = 1267, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(30*C*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-18*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*s
in(1/2*d*x+1/2*c)^2+10*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-
45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-45*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b-15*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*A*a
^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*C*b^3*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2+72*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*C*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-48*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8-30*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-180*A*a
^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-120*C*a*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+90*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+30
*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+120*C*a*b^2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-5*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-15*a^3*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+30*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-90*C*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+90*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+90*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*
d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+3\,Cab^2\cos\left(dx+c\right)^4+3\,Aa^2b\cos\left(dx+c\right)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
 x)
```

3.1379 $\int (a+b\cos(c+dx))^3 \left(A + C\cos^2(c+dx)\right) \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=284

$$-\frac{2b\left(6a^2(7A-3C)-b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(21a^2(3A+C)+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c-\right.}{21d}$$

```
[Out] (-2*a*(5*a^2*(A - C) - 3*b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*b*(21*a^2*(3*A + C) + b^2*(7*A +
 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21
*d) - (2*a*b^2*(35*A - 11*C)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) - (2*b
*(6*a^2*(7*A - 3*C) - b^2*(7*A + 5*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) - (2*b*(7*A - C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c +
 d*x]]) + (2*A*(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.895452, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3049, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b\left(6a^2(7A-3C)-b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(21a^2(3A+C)+b^2(7A+5C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c-\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(5*a^2*(A - C) - 3*b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*b*(21*a^2*(3*A + C) + b^2*(7*A +
 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21
*d) - (2*a*b^2*(35*A - 11*C)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) - (2*b
*(6*a^2*(7*A - 3*C) - b^2*(7*A + 5*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) - (2*b*(7*A - C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c +
 d*x]]) + (2*A*(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^3\left(A\right.}{\cos^{\frac{3}{2}}(c+}$$

$$=\frac{2A(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\left(2\sqrt{c}\right.$$

$$=-\frac{2b(7A-C)(a+b\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{2A(a+b}{}$$

$$=-\frac{2ab^2(35A-11C)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b(7A-C)(a+b\cos(c}{7d\sqrt{\sec(c}}$$

$$=-\frac{2ab^2(35A-11C)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\left(6a^2(7A-3C)-b^2\right.}{21d\sqrt{\sec}}$$

$$=-\frac{2ab^2(35A-11C)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\left(6a^2(7A-3C)-b^2\right.}{21d\sqrt{\sec}}$$

$$=-\frac{2a\left(5a^2(A-C)-3b^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 1.96015, size = 193, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)\left(5b\left(84a^2C+28Ab^2+29b^2C\right)\cos(c+dx)+3\left(140a^3A+42ab^2C\cos(2(c+dx))+42ab^2C+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-168*a*(5*a^2*(A - C) - 3*b^2*(5*A + 3*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2] + 40*b*(21*a^2*(3*A + C) + b^2*(7*A + 5*C
))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5*b*(28*A*b^2 + 84*a^2
*C + 29*b^2*C)*Cos[c + d*x] + 3*(140*a^3*A + 42*a*b^2*C + 42*a*b^2*C*Cos[2*
(c + d*x)] + 5*b^3*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/(420*d)
```

Maple [B] time = 1.411, size = 943, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*(240*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*b^2*(7*a+5*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+28
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(5*A*b^2+15*C*a^2+1
8*C*a*b+10*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^3+35*A*b^3+105*C*a^2*b+63*C*
a*b^2+40*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+35*A
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+105*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a^3-315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+105*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
```

```
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+25*C*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-105*C*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3
-189*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
 x)
```

3.1380 $\int (a+b\cos(c+dx))^3 \left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{2b\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\left(8a^2C+63Ab^2+45b^2C\right)\sin(c+dx)}{63d\sqrt{\sec(c+dx)}}+\frac{2a\left(7a^2(3A+C)+3b^2(7A+5C\right.}{}$$

```
[Out] (2*b*(9*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*a*(7*a^2*(3*A + C) + 3*b^2*(7*
A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(21*d) + (2*b*(24*a^2*C + 7*b^2*(9*A + 7*C))*Sin[c + d*x])/(315*d*Sec[c + d
*x]^(3/2)) + (2*a*(63*A*b^2 + 8*a^2*C + 45*b^2*C)*Sin[c + d*x])/(63*d*Sqrt[
Sec[c + d*x]]) + (4*a*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Sqrt[Sec
[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Sqrt[Sec[c + d
*x]])
```

Rubi [A] time = 0.842625, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3050, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2b\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\left(8a^2C+63Ab^2+45b^2C\right)\sin(c+dx)}{63d\sqrt{\sec(c+dx)}}+\frac{2a\left(7a^2(3A+C)+3b^2(7A+5C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*b*(9*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*a*(7*a^2*(3*A + C) + 3*b^2*(7*
A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(21*d) + (2*b*(24*a^2*C + 7*b^2*(9*A + 7*C))*Sin[c + d*x])/(315*d*Sec[c + d
*x]^(3/2)) + (2*a*(63*A*b^2 + 8*a^2*C + 45*b^2*C)*Sin[c + d*x])/(63*d*Sqrt[
Sec[c + d*x]]) + (4*a*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Sqrt[Sec
[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Sqrt[Sec[c + d
*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^3\left(A\right.}{\sqrt{\cos(c+}}$$

$$=\frac{2C(a+b\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{1}{9}\left(2\sqrt{\cos(c+dx)}\right.$$

$$=\frac{4aC(a+b\cos(c+dx))^2\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2C(a+b\cos(c+}{9d\sqrt{\sec}}$$

$$=\frac{2b\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{4aC(a+b\,\mathrm{co}}{21d}$$

$$=\frac{2b\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\left(63Ab^2+\right.}{6}$$

$$=\frac{2b\left(24a^2C+7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\left(63Ab^2+\right.}{6}$$

$$=\frac{2b\left(9a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [A] time = 1.67249, size = 203, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7b\left(108a^2C+36Ab^2+43b^2C\right)\cos(c+dx)+5\left(84a^3C+252aAb^2+54ab^2C\cos(2(c+dx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*b*(9*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*a*(7*a^2*(3*A + C) + 3*b^2*(7*A +
5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*b*(36*A*b^2 + 108*a
^2*C + 43*b^2*C)*Cos[c + d*x] + 5*(252*a*A*b^2 + 84*a^3*C + 234*a*b^2*C + 5
4*a*b^2*C*Cos[2*(c + d*x)] + 7*b^3*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/
(1260*d)
```

Maple [B] time = 1.281, size = 718, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2160*C*a*b^2+2240*C*b^3)*sin(1/2
*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^3-1512*C*a^2*b-3240*C*a*b^2-2072
*C*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1260*A*a*b^2+504*A*b^3+420
*C*a^3+1512*C*a^2*b+2520*C*a*b^2+952*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+(-630*A*a*b^2-126*A*b^3-210*C*a^3-378*C*a^2*b-720*C*a*b^2-168*C*b^
3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-945*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^2*b-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+315*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
315*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-1
47*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+105*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
```

```
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*C*
a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+3\,Cab^2\cos\left(dx+c\right)^4+3\,Aa^2b\cos\left(dx+c\right)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
 x)
```

3.1381 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=335

$$\frac{2a\left(8a^2C+99Ab^2+77b^2C\right)\sin(c+dx)}{165d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\left(33a^2(7A+5C)+5b^2(11A+\right.}{231d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a*(a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11
*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(231*d) + (2*b*(8*a^2*C + 3*b^2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sec[c +
 d*x]^(5/2)) + (2*a*(99*A*b^2 + 8*a^2*C + 77*b^2*C)*Sin[c + d*x])/(165*d*Se
c[c + d*x]^(3/2)) + (4*a*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(33*d*Sec[c
 + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*
x]^(3/2)) + (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sin[c + d*x])/(2
31*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.927343, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3050, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a\left(8a^2C+99Ab^2+77b^2C\right)\sin(c+dx)}{165d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\left(33a^2(7A+5C)+5b^2(11A+\right.}{231d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11
*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(231*d) + (2*b*(8*a^2*C + 3*b^2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sec[c +
 d*x]^(5/2)) + (2*a*(99*A*b^2 + 8*a^2*C + 77*b^2*C)*Sin[c + d*x])/(165*d*Se
c[c + d*x]^(3/2)) + (4*a*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(33*d*Sec[c
 + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*
x]^(3/2)) + (2*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sin[c + d*x])/(2
31*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\left(A+\right.$$

$$= \frac{2C(a+b\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{11}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{4aC(a+b\cos(c+dx))^2\sin(c+dx)}{33d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))^3\sin(c+}{11d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)} + \frac{4aC(a+b\cos(c+dx))^2\sin}{33d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\left(99Ab^2+8a^2C+77b^2C\right.}{165d\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2b\left(8a^2C+3b^2(11A+9C)\right)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\left(99Ab^2+8a^2C+77b^2C\right.}{165d\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2a\left(a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

$$= \frac{2a\left(a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

Mathematica [A] time = 2.40187, size = 236, normalized size = 0.7

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(154a\left(12a^2C+36Ab^2+43b^2C\right)\cos(c+dx)+5b\left(12\left(33a^2C+11Ab^2+16b^2C\right)\cos(2(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(3696*a*(a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2] + 80*b*(33*a^2*(7*A + 5*C) + 5*b^2*(11*A
+ 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (154*a*(36*A*b^2 + 1
2*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*b*(1848*a^2*A + 572*A*b^2 + 1716*a^2*C
 + 531*b^2*C + 12*(11*A*b^2 + 33*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 154*a
*b*C*Cos[3*(c + d*x)] + 21*b^2*C*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(924
```

```
0*d)
```

Maple [B] time = 1.281, size = 793, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/1155*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6720*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*C*a*b^2-16800*C*b^3)*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(2640*A*b^3+7920*C*a^2*b+24640*C*a*b^2
+18960*C*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-5544*A*a*b^2-3960*A
*b^3-1848*C*a^3-11880*C*a^2*b-22792*C*a*b^2-11640*C*b^3)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(4620*A*a^2*b+5544*A*a*b^2+3080*A*b^3+1848*C*a^3+9240
*C*a^2*b+10472*C*a*b^2+4620*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-2310*A*a^2*b-1386*A*a*b^2-880*A*b^3-462*C*a^3-2640*C*a^2*b-1848*C*a*b^2-9
30*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*A*a^2*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+275*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1155*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a^3-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+825*a^2*b*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+225*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-693*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a^3-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos(dx+c)^5+3\,Cab^2\cos(dx+c)^4+3\,Aa^2b\cos(dx+c)+Aa^3+\left(3\,Ca^2b+Ab^3\right)\cos(dx+c)^3+\left(Ca^3+\right.}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
 x)
```

3.1382 $\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=386

$$\frac{2b\left(39a^2(9A+7C)+7b^2(13A+11C)\right)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\left(8a^2C+143Ab^2+117b^2C\right)\sin(c+dx)}{1001d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\left(24a^2C+11b^2(13A\right.}{1287d\sec}$$

```
[Out] (2*b*(39*a^2*(9*A + 7*C) + 7*b^2*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(195*d) + (2*a*(11*a^2*(7*A + 5*C) +
15*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(231*d) + (2*b*(24*a^2*C + 11*b^2*(13*A + 11*C))*Sin[c + d*x])/(
1287*d*Sec[c + d*x]^(7/2)) + (6*a*(143*A*b^2 + 8*a^2*C + 117*b^2*C)*Sin[c +
 d*x])/(1001*d*Sec[c + d*x]^(5/2)) + (12*a*C*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(143*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x
])/(13*d*Sec[c + d*x]^(5/2)) + (2*b*(39*a^2*(9*A + 7*C) + 7*b^2*(13*A + 11*
C))*Sin[c + d*x])/(585*d*Sec[c + d*x]^(3/2)) + (2*a*(11*a^2*(7*A + 5*C) + 1
5*b^2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.01634, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3050, 3049, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2b\left(39a^2(9A+7C)+7b^2(13A+11C)\right)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\left(8a^2C+143Ab^2+117b^2C\right)\sin(c+dx)}{1001d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\left(24a^2C+11b^2(13A\right.}{1287d\sec}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*b*(39*a^2*(9*A + 7*C) + 7*b^2*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(195*d) + (2*a*(11*a^2*(7*A + 5*C) +
15*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(231*d) + (2*b*(24*a^2*C + 11*b^2*(13*A + 11*C))*Sin[c + d*x])/(
1287*d*Sec[c + d*x]^(7/2)) + (6*a*(143*A*b^2 + 8*a^2*C + 117*b^2*C)*Sin[c +
 d*x])/(1001*d*Sec[c + d*x]^(5/2)) + (12*a*C*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(143*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x
])/(13*d*Sec[c + d*x]^(5/2)) + (2*b*(39*a^2*(9*A + 7*C) + 7*b^2*(13*A + 11*
C))*Sin[c + d*x])/(585*d*Sec[c + d*x]^(3/2)) + (2*a*(11*a^2*(7*A + 5*C) + 1
```

```
5*b^2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\left(A\right.$$

$$= \frac{2C(a+b\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{13}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{12aC(a+b\cos(c+dx))^2\sin(c+dx)}{143d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))^3\sin(c}{13d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(24a^2C+11b^2(13A+11C)\right)\sin(c+dx)}{1287d\sec^{\frac{7}{2}}(c+dx)} + \frac{12aC(a+b\cos(c+dx}{143d\sec^{\frac{5}{2}}(c}$$

$$= \frac{2b\left(24a^2C+11b^2(13A+11C)\right)\sin(c+dx)}{1287d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a\left(143Ab^2+8a^2C+\right.}{1001d\sec^{\frac{5}{2}}}$$

$$= \frac{2b\left(24a^2C+11b^2(13A+11C)\right)\sin(c+dx)}{1287d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a\left(143Ab^2+8a^2C+\right.}{1001d\sec^{\frac{5}{2}}}$$

$$= \frac{2b\left(24a^2C+11b^2(13A+11C)\right)\sin(c+dx)}{1287d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a\left(143Ab^2+8a^2C+\right.}{1001d\sec^{\frac{5}{2}}}$$

$$= \frac{2b\left(39a^2(9A+7C)+7b^2(13A+11C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}$$

Mathematica [A] time = 2.62382, size = 276, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(154b\left(78a^2(36A+43C)+b^2(1118A+1171C)\right)\cos(c+dx)+5\left(936a\left(11a^2C+33Ab^2+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7392*b*(39*a^2*(9*A + 7*C) + 7*b^2*(13*A + 11*C))*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6240*a*(11*a^2*(7*A + 5*C) + 15*
b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (154*b*(78
*a^2*(36*A + 43*C) + b^2*(1118*A + 1171*C))*Cos[c + d*x] + 5*(3432*a^3*(14*
A + 13*C) + 234*a*b^2*(572*A + 531*C) + 936*a*(33*A*b^2 + 11*a^2*C + 48*b^2
*C)*Cos[2*(c + d*x)] + 77*(52*A*b^3 + 156*a^2*b*C + 89*b^3*C)*Cos[3*(c + d*
```

```
x)] + 4914*a*b^2*C*Cos[4*(c + d*x)] + 693*b^3*C*Cos[5*(c + d*x)]))*Sin[2*(c
 + d*x)]))/(720720*d)
```

Maple [B] time = 1.377, size = 873, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*C
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(786240*C*a*b^2+1330560*C*b^3
)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^3-480480*C*a^2*b-19
65600*C*a*b^2-1798720*C*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(3088
80*A*a*b^2+320320*A*b^3+102960*C*a^3+960960*C*a^2*b+2218320*C*a*b^2+1379840
*C*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-216216*A*a^2*b-463320*A*a
*b^2-296296*A*b^3-154440*C*a^3-888888*C*a^2*b-1361880*C*a*b^2-666512*C*b^3)
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(60060*A*a^3+216216*A*a^2*b+360360
*A*a*b^2+136136*A*b^3+120120*C*a^3+408408*C*a^2*b+540540*C*a*b^2+198352*C*b
^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-30030*A*a^3-54054*A*a^2*b-102
960*A*a*b^2-24024*A*b^3-34320*C*a^3-72072*C*a^2*b-108810*C*a*b^2-27258*C*b^
3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-81081*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2*b-21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+15015*A*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+32175*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63063*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2*b-17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+10725*a^3*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+26325*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+3\,Cab^{2}\cos\left(dx+c\right)^{4}+3\,Aa^{2}b\cos\left(dx+c\right)+Aa^{3}+\left(3\,Ca^{2}b+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}\right.}{\sec\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + 3*C*a*b^2*cos(d*x + c)^4 + 3*A*a^2*b*cos(d
*x + c) + A*a^3 + (3*C*a^2*b + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*A*a*b^2)*
cos(d*x + c)^2)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2),
 x)
```

3.1383 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=417

$$\frac{4ab\left(a^2(673A+891C)+96Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3465d}+\frac{2\left(9a^2b^2(101A+143C)+15a^4(9A+11C)+64Ab^4\right)\sin(c}{693d}$$

```
[Out] (-8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(77*b^4*(A + 3*C) + 66*a^2*
b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A
+ 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(64*A*b^4 + 15*a^4*(9*
A + 11*C) + 9*a^2*b^2*(101*A + 143*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(69
3*d) + (4*a*b*(96*A*b^2 + a^2*(673*A + 891*C))*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(3465*d) + (2*(16*A*b^2 + 3*a^2*(9*A + 11*C))*(a + b*Cos[c + d*x])^2*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (16*A*b*(a + b*Cos[c + d*x])^3*Se
c[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c +
 d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 1.37543, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3048, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{4ab\left(a^2(673A+891C)+96Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3465d}+\frac{2\left(9a^2b^2(101A+143C)+15a^4(9A+11C)+64Ab^4\right)\sin(c}{693d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2),x]
```


```
[Out] (-8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(77*b^4*(A + 3*C) + 66*a^2*
b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a*b*(3*b^2*(3*A + 5*C) + a^2*(7*A
+ 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(64*A*b^4 + 15*a^4*(9*
A + 11*C) + 9*a^2*b^2*(101*A + 143*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(69
3*d) + (4*a*b*(96*A*b^2 + a^2*(673*A + 891*C))*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(3465*d) + (2*(16*A*b^2 + 3*a^2*(9*A + 11*C))*(a + b*Cos[c + d*x])^2*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (16*A*b*(a + b*Cos[c + d*x])^3*Se
```

```
c[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c +
 d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
```

```
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4\left(A+\right.}{\cos^{\frac{13}{2}}(c+d}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{1}{11}\left(2\right.$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2A(}{}$$

$$= \frac{2\left(16Ab^2+3a^2(9A+11C)\right)(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+}{231d}$$

$$= \frac{4ab\left(96Ab^2+a^2(673A+891C)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3465d}$$

$$= \frac{2\left(64Ab^4+15a^4(9A+11C)+9a^2b^2(101A+143C)\right)\sec^{\frac{3}{2}}}{693d}$$

$$= \frac{2\left(64Ab^4+15a^4(9A+11C)+9a^2b^2(101A+143C)\right)\sec^{\frac{3}{2}}}{693d}$$

$$= \frac{2\left(77b^4(A+3C)+66a^2b^2(5A+7C)+5a^4(9A+11C)\right)\sqrt{}}{231d}$$

$$= -\frac{8ab\left(3b^2(3A+5C)+a^2(7A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [A] time = 6.85254, size = 425, normalized size = 1.02

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{8}{15}ab\left(7a^2A+9a^2C+9Ab^2+15b^2C\right)\sin(c+dx)+\frac{2}{77}\sec^3(c+dx)\left(66a^2Ab^2\sin(c+dx)+9a^4A\sin(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2)
,x]
```

```
[Out] ((2*(-2156*a^3*A*b - 2772*a*A*b^3 - 2772*a^3*b*C - 4620*a*b^3*C)*EllipticE[
(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(225*a^4*A + 1
```

```
650*a^2*A*b^2 + 385*A*b^4 + 275*a^4*C + 2310*a^2*b^2*C + 1155*b^4*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1155*d) + (Sqrt
[Sec[c + d*x]]*((8*a*b*(7*a^2*A + 9*A*b^2 + 9*a^2*C + 15*b^2*C)*Sin[c + d*x
])/15 + (2*Sec[c + d*x]^3*(9*a^4*A*Sin[c + d*x] + 66*a^2*A*b^2*Sin[c + d*x]
 + 11*a^4*C*Sin[c + d*x]))/77 + (8*Sec[c + d*x]^2*(7*a^3*A*b*Sin[c + d*x] +
 9*a*A*b^3*Sin[c + d*x] + 9*a^3*b*C*Sin[c + d*x]))/45 + (2*Sec[c + d*x]*(45
*a^4*A*Sin[c + d*x] + 330*a^2*A*b^2*Sin[c + d*x] + 77*A*b^4*Sin[c + d*x] +
55*a^4*C*Sin[c + d*x] + 462*a^2*b^2*C*Sin[c + d*x]))/231 + (8*a^3*A*b*Sec[c
 + d*x]^3*Tan[c + d*x])/9 + (2*a^4*A*Sec[c + d*x]^4*Tan[c + d*x])/11))/d
```

Maple [B] time = 6.893, size = 1521, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8/5
*a*b*(A*b^2+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/
2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^4*(-1/352*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/154*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+2*b^2*(A*b^2+6*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+2*a^2*(6*A*b^2+C*a^2)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2
```

```
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+8*A*a^3*b*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-
7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+8*C*a*b^3*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(13/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\Big(\Big(Cb^4\cos\left(dx+c\right)^6+4\,Cab^3\cos\left(dx+c\right)^5+4\,Aa^3b\cos\left(dx+c\right)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos\left(dx+c\right)^4+4\Big(Ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(13
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(13/2)
, x)
```

3.1384 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=365

$$\frac{4ab\left(a^2(101A+147C)+32Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d}+\frac{2\left(7a^2b^2(155A+261C)+21a^4(7A+9C)+192Ab^4\right)\sin(c+$$

```
[Out] (-2*(15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*b^
2*(A + 3*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(21*d) + (2*(192*A*b^4 + 21*a^4*(7*A + 9*C) + 7*a^2*b^
2*(155*A + 261*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (4*a*b*(32*A*
b^2 + a^2*(101*A + 147*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) + (2*(4
8*A*b^2 + 7*a^2*(7*A + 9*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(315*d) + (16*A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c
 + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(9/2)*Sin[c + d*
x])/(9*d)
```

Rubi [A] time = 1.27745, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{4ab\left(a^2(101A+147C)+32Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d}+\frac{2\left(7a^2b^2(155A+261C)+21a^4(7A+9C)+192Ab^4\right)\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (-2*(15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4*(7*A + 9*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*b^
2*(A + 3*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(21*d) + (2*(192*A*b^4 + 21*a^4*(7*A + 9*C) + 7*a^2*b^
2*(155*A + 261*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (4*a*b*(32*A*
b^2 + a^2*(101*A + 147*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) + (2*(4
8*A*b^2 + 7*a^2*(7*A + 9*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(315*d) + (16*A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c
 + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(9/2)*Sin[c + d*
```

```
x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4 \left(A\right.}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\right.$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A}{}$$

$$= \frac{2\left(48Ab^2+7a^2(7A+9C)\right)(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+}{315d}$$

$$= \frac{4ab\left(32Ab^2+a^2(101A+147C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{315d}$$

$$= \frac{2\left(192Ab^4+21a^4(7A+9C)+7a^2b^2(155A+261C)\right)\sqrt{}}{315d}$$

$$= \frac{2\left(192Ab^4+21a^4(7A+9C)+7a^2b^2(155A+261C)\right)\sqrt{}}{315d}$$

$$= -\frac{2\left(15b^4(A-C)+18a^2b^2(3A+5C)+a^4(7A+9C)\right)\sqrt{}}{15d}$$

Mathematica [A] time = 6.74148, size = 356, normalized size = 0.98

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}\left(54a^2Ab^2+7a^4A+90a^2b^2C+9a^4C+15Ab^4\right)\sin(c+dx)+\frac{2}{45}\sec^2(c+dx)\left(54a^2Ab^2\sin(c+dx)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] ((2*(-49*a^4*A - 378*a^2*A*b^2 - 105*A*b^4 - 63*a^4*C - 630*a^2*b^2*C + 105
*b^4*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
+ 2*(100*a^3*A*b + 140*a*A*b^3 + 140*a^3*b*C + 420*a*b^3*C)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(105*d) + (Sqrt[Sec[c + d
*x]]*((2*(7*a^4*A + 54*a^2*A*b^2 + 15*A*b^4 + 9*a^4*C + 90*a^2*b^2*C)*Sin[c
```

```
 + d*x])/15 + (2*Sec[c + d*x]^2*(7*a^4*A*Sin[c + d*x] + 54*a^2*A*b^2*Sin[c
+ d*x] + 9*a^4*C*Sin[c + d*x]))/45 + (8*Sec[c + d*x]*(5*a^3*A*b*Sin[c + d*x
] + 7*a*A*b^3*Sin[c + d*x] + 7*a^3*b*C*Sin[c + d*x]))/21 + (8*a^3*A*b*Sec[c
 + d*x]^2*Tan[c + d*x])/7 + (2*a^4*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d
```

Maple [B] time = 6.142, size = 1451, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^4*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a*b*(A*b^2
+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8*A*a^3*b*(-1/56*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^4*(-1/14
4*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-
2/5*a^2*(6*A*b^2+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*s
in(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
```

```
n(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b^2+6*C*a^2)*(
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(11/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\,\left(C\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(11
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(11/2)
, x)
```

3.1385 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right) \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=356

$$\frac{4ab\left(a^2(101A+175C)+96Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}-\frac{2b^2\left(5a^2(5A+7C)+b^2(87A-35C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2\left(5a^2\right.}{}$$

```
[Out] (-8*a*b*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(7*b^4*(3*A + C) + 42*a^2*b^2*(A
 + 3*C) + a^4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(21*d) - (2*b^2*(b^2*(87*A - 35*C) + 5*a^2*(5*A + 7*C))*Si
n[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (4*a*b*(96*A*b^2 + a^2*(101*A + 17
5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(48*A*b^2 + 5*a^2*(5*A
+ 7*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (
16*A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*
A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.3014, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{4ab\left(a^2(101A+175C)+96Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}-\frac{2b^2\left(5a^2(5A+7C)+b^2(87A-35C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2\left(5a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-8*a*b*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(7*b^4*(3*A + C) + 42*a^2*b^2*(A
 + 3*C) + a^4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(21*d) - (2*b^2*(b^2*(87*A - 35*C) + 5*a^2*(5*A + 7*C))*Si
n[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (4*a*b*(96*A*b^2 + a^2*(101*A + 17
5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(48*A*b^2 + 5*a^2*(5*A
+ 7*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (
16*A*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*
A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4 \left(A+\right.}{\cos^{\frac{9}{2}}(c+dx}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{c}\right.$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A(a}{}$$

$$= \frac{2\left(48Ab^2+5a^2(5A+7C)\right)(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+d}{105d}$$

$$= \frac{4ab\left(96Ab^2+a^2(101A+175C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d}$$

$$= -\frac{2b^2\left(b^2(87A-35C)+5a^2(5A+7C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{4ab}{}$$

$$= -\frac{2b^2\left(b^2(87A-35C)+5a^2(5A+7C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{4ab}{}$$

$$= -\frac{8ab\left(5b^2(A-C)+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 3.27073, size = 296, normalized size = 0.83

$$\frac{2\sec^{\frac{7}{2}}(c+dx)\left(5\left(42a^2b^2(A+3C)+a^4(5A+7C)+7b^4(3A+C)\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-84ab\left(a^2(3A+5C)+5\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (2*Sec[c + d*x]^(7/2)*(-84*a*b*(5*b^2*(A - C) + a^2*(3*A + 5*C))*Cos[c + d*
x]^(7/2)*EllipticE[(c + d*x)/2, 2] + 5*(7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3
*C) + a^4*(5*A + 7*C))*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 15*a^
4*A*Sin[c + d*x] + 25*a^4*A*Cos[c + d*x]^2*Sin[c + d*x] + 210*a^2*A*b^2*Cos
[c + d*x]^2*Sin[c + d*x] + 35*a^4*C*Cos[c + d*x]^2*Sin[c + d*x] + 252*a^3*A
*b*Cos[c + d*x]^3*Sin[c + d*x] + 420*a*A*b^3*Cos[c + d*x]^3*Sin[c + d*x] +
```

```
420*a^3*b*C*Cos[c + d*x]^3*Sin[c + d*x] + 35*b^4*C*Cos[c + d*x]^4*Sin[c + d
*x] + 42*a^3*A*b*Sin[2*(c + d*x)]))/(105*d)
```

Maple [B] time = 5.316, size = 1531, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^4*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))-4*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^4*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+12*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*a^4*(-1/56*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(6*A*b^2+C*a^
2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-8/5*A*a^3*b/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^
2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
```

```
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+8*a*b*(A*b^2+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{4}\cos\left(dx+c\right)^{6}+4\,Cab^{3}\cos\left(dx+c\right)^{5}+4\,Aa^{3}b\cos\left(dx+c\right)+Aa^{4}+\left(6\,Ca^{2}b^{2}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+4\left(Ca\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(9/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2),
 x)
```

3.1386 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=361

$$-\frac{2b^2\left(3a^2(3A+5C)+b^2(59A-3C)\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}-\frac{4ab\left(3a^2(3A+5C)+2b^2(33A-5C)\right)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2\left(a^2(3A+5C)\right.}{}$$

```
[Out] (-2*(30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(b^2*(3*A
 + C) + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) - (2*b^2*(b^2*(59*A - 3*C) + 3*a^2*(3*A + 5*C))*Sin[c +
d*x])/(15*d*Sec[c + d*x]^(3/2)) - (4*a*b*(2*b^2*(33*A - 5*C) + 3*a^2*(3*A +
 5*C))*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(16*A*b^2 + a^2*(3*A +
5*C))*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (16*A
*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a
 + b*Cos[c + d*x])^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.26726, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b^2\left(3a^2(3A+5C)+b^2(59A-3C)\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}-\frac{4ab\left(3a^2(3A+5C)+2b^2(33A-5C)\right)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2\left(a^2(3A+5C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(b^2*(3*A
 + C) + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) - (2*b^2*(b^2*(59*A - 3*C) + 3*a^2*(3*A + 5*C))*Sin[c +
d*x])/(15*d*Sec[c + d*x]^(3/2)) - (4*a*b*(2*b^2*(33*A - 5*C) + 3*a^2*(3*A +
 5*C))*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(16*A*b^2 + a^2*(3*A +
5*C))*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (16*A
*b*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a
 + b*Cos[c + d*x])^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4\left(A\right.}{\cos^{\frac{7}{2}}(c+}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\right.$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= \frac{2\left(16Ab^2+a^2(3A+5C)\right)(a+b\cos(c+dx))^2\sqrt{\sec(c+}}{5d}$$

$$= -\frac{2b^2\left(b^2(59A-3C)+3a^2(3A+5C)\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(1\right.}{}$$

$$= -\frac{2b^2\left(b^2(59A-3C)+3a^2(3A+5C)\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)} - \frac{4ab}{}$$

$$= -\frac{2b^2\left(b^2(59A-3C)+3a^2(3A+5C)\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)} - \frac{4ab}{}$$

$$= -\frac{2\left(30a^2b^2(A-C)-b^4(5A+3C)+a^4(3A+5C)\right)\sqrt{\cos}}{5d}$$

Mathematica [A] time = 3.17118, size = 233, normalized size = 0.65

$$\frac{\sqrt{\sec(c+dx)}\left(80ab\left(a^2(A+3C)+b^2(3A+C)\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-12\left(30a^2b^2(A-C)+a^4(3A+5C)-b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-12*(30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3*A +
 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 80*a*b*(b^2*(3*A + C)
 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 36*a^4*A*S
in[c + d*x] + 360*a^2*A*b^2*Sin[c + d*x] + 60*a^4*C*Sin[c + d*x] + 3*b^4*C*
Sin[c + d*x] + 40*a*b^3*C*Sin[2*(c + d*x)] + 3*b^4*C*Sin[3*(c + d*x)] + 80*
```

```
a^3*A*b*Tan[c + d*x] + 12*a^4*A*Sec[c + d*x]*Tan[c + d*x]))/(30*d)
```

Maple [B] time = 5.208, size = 1622, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C*b^4*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+1/3*(16*C*a*b^3-12*C*b^4)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(
2*A*b^4+12*C*a^2*b^2-16*C*a*b^3+6*C*b^4)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))+8*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-2*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*a^2*b^2*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*C*a*b^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+8*A*a^3*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*A
*a^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
```

```
-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(6*A*b^2+C*a^2)*(-(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos\left(dx+c\right)^6+4\,Cab^3\cos\left(dx+c\right)^5+4\,Aa^3b\cos\left(dx+c\right)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos\left(dx+c\right)^4+4\left(C\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(7/
```

```
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^4 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2),
 x)
```

3.1387 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=340

$$-\frac{2b^2\left(3a^2(49A-13C)-b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(42a^2b^2(3A+C)+7a^4(A+3C)+b^4(7A+5C)\right)\sqrt{\cos(c+dx)}}{21d}$$

```
[Out] (-8*a*b*(5*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(42*a^2*b^2*(3*A + C) + 7*a^4*(A
 + 3*C) + b^4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(21*d) - (4*a*b^3*(175*A - 27*C)*Sin[c + d*x])/(105*d*Sec[
c + d*x]^(3/2)) - (2*b^2*(3*a^2*(49*A - 13*C) - b^2*(7*A + 5*C))*Sin[c + d*
x])/(21*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(21*A - C)*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(7*d*Sqrt[Sec[c + d*x]]) + (16*A*b*(a + b*Cos[c + d*x])^3*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.26081, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3048, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b^2\left(3a^2(49A-13C)-b^2(7A+5C)\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(42a^2b^2(3A+C)+7a^4(A+3C)+b^4(7A+5C)\right)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-8*a*b*(5*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(42*a^2*b^2*(3*A + C) + 7*a^4*(A
 + 3*C) + b^4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(21*d) - (4*a*b^3*(175*A - 27*C)*Sin[c + d*x])/(105*d*Sec[
c + d*x]^(3/2)) - (2*b^2*(3*a^2*(49*A - 13*C) - b^2*(7*A + 5*C))*Sin[c + d*
x])/(21*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(21*A - C)*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(7*d*Sqrt[Sec[c + d*x]]) + (16*A*b*(a + b*Cos[c + d*x])^3*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4 \left(A+\right.}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{c}\right.$$

$$= \frac{16Ab(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a}{}$$

$$= -\frac{2b^2(21A-C)(a+b\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}} + \frac{16Ab(a}{}$$

$$= -\frac{4ab^3(175A-27C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} - \frac{2b^2(21A-C)(a+b\cos}{7d\sqrt{\sec(}}$$

$$= -\frac{4ab^3(175A-27C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} - \frac{2b^2\left(3a^2(49A-13C)-\right.}{21d\sqrt{\mathrm{se}}}$$

$$= -\frac{4ab^3(175A-27C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} - \frac{2b^2\left(3a^2(49A-13C)-\right.}{21d\sqrt{\mathrm{se}}}$$

$$= -\frac{8ab\left(5a^2(A-C)-b^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 1.99578, size = 243, normalized size = 0.71

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(40\left(42a^2b^2(3A+C)+7a^4(A+3C)+b^4(7A+5C)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-672ab\left(5a^2(A-C)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-672*a*b*(5*a^2*(A - C) - b^2*(5*A
+ 3*C))*EllipticE[(c + d*x)/2, 2] + 40*(42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3
*C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2] + ((280*a^4*A + 140*A*b^4
+ 840*a^2*b^2*C + 145*b^4*C + 168*a*b*(20*a^2*A + 3*b^2*C)*Cos[c + d*x] + 2
0*(7*A*b^4 + 42*a^2*b^2*C + 8*b^4*C)*Cos[2*(c + d*x)] + 168*a*b^3*C*Cos[3*(
```

```
c + d*x)] + 15*b^4*C*Cos[4*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(
420*d)
```

Maple [B] time = 4.536, size = 1715, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(420*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b^3-630*a^2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+420*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^3*b+252*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-210*a^2*b^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+480*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10
+280*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+920*C*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6-280*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
-440*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+70*A*a^4*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+70*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+8
0*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-960*C*b^4*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^8-105*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-840*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-504*C*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b
^3*sin(1/2*d*x+1/2*c)^2+420*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+
1/2*c)^2+840*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-840*A*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2-35*A*a^4*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-35*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1680*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2016*C*a*b^3*cos(1
```

```
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1680*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4-1680*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1008*C*
a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+840*A*a^3*b*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2+420*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2+168*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1344*C*a*b^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-420*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+210*C*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+50*C*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*b^4*sin(1/2*d*x+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x
+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+1260*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos\left(dx+c\right)^6+4\,Cab^3\cos\left(dx+c\right)^5+4\,Aa^3b\cos\left(dx+c\right)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos\left(dx+c\right)^4+4\left(Ca\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(5/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2),
 x)
```

3.1388 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=360

$$-\frac{2b^2\left(3a^2(105A-41C)-7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}-\frac{4ab\left(a^2(63A-31C)-6b^2(7A+5C)\right)\sin(c+dx)}{63d\sqrt{\sec(c+dx)}}+\frac{8ab\left(7a^2(3A\right.}{}$$

```
[Out] (-2*(15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) - b^4*(9*A + 7*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*a^
2*(3*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(21*d) - (2*b^2*(3*a^2*(105*A - 41*C) - 7*b^2*(9*A + 7
*C))*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) - (4*a*b*(a^2*(63*A - 31*C) -
 6*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*d*Sqrt[Sec[c + d*x]]) - (2*a*b*(21*A
- 5*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (2*
b*(9*A - C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Sqrt[Sec[c + d*x]]) +
 (2*A*(a + b*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.34391, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3049, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b^2\left(3a^2(105A-41C)-7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}-\frac{4ab\left(a^2(63A-31C)-6b^2(7A+5C)\right)\sin(c+dx)}{63d\sqrt{\sec(c+dx)}}+\frac{8ab\left(7a^2(3A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) - b^4*(9*A + 7*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*a^
2*(3*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(21*d) - (2*b^2*(3*a^2*(105*A - 41*C) - 7*b^2*(9*A + 7
*C))*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) - (4*a*b*(a^2*(63*A - 31*C) -
 6*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*d*Sqrt[Sec[c + d*x]]) - (2*a*b*(21*A
- 5*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (2*
b*(9*A - C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Sqrt[Sec[c + d*x]]) +
 (2*A*(a + b*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4 \left(A \cdots\right.}{\cos^{\frac{3}{2}}(c+ \cdots}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{c \cdots}\right.$$

$$= -\frac{2b(9A-C)(a+b\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}} + \frac{2A(a+b \cdots}{\cdots}$$

$$= -\frac{2ab(21A-5C)(a+b\cos(c+dx))^2\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} - \frac{2b(9A \cdots}{\cdots}$$

$$= -\frac{2b^2\left(3a^2(105A-41C)-7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)} - \cdots$$

$$= -\frac{2b^2\left(3a^2(105A-41C)-7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)} - \cdots$$

$$= -\frac{2b^2\left(3a^2(105A-41C)-7b^2(9A+7C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)} - \cdots$$

$$= -\frac{2\left(15a^4(A-C)-18a^2b^2(5A+3C)-b^4(9A+7C)\right)\sqrt{c \cdots}}{15d}$$

Mathematica [A] time = 1.80494, size = 252, normalized size = 0.7

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)\left(120ab\left(28a^2C+28Ab^2+29b^2C\right)\cos(c+dx)+84\left(18a^2b^2C+3Ab^4+4b^4C\right)\cos(2(c+dx \cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-336*(15*a^4*(A - C) - 18*a^2*b^2*(5*A + 3*C) - b^4*(9
*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 960*a*b*(7*a^2*(3
*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2
*(2520*a^4*A + 252*A*b^4 + 1512*a^2*b^2*C + 301*b^4*C + 120*a*b*(28*A*b^2 +
 28*a^2*C + 29*b^2*C)*Cos[c + d*x] + 84*(3*A*b^4 + 18*a^2*b^2*C + 4*b^4*C)*
Cos[2*(c + d*x)] + 360*a*b^3*C*Cos[3*(c + d*x)] + 35*b^4*C*Cos[4*(c + d*x)]
```

```
)*Sin[c + d*x]))/(2520*d)
```

Maple [B] time = 1.743, size = 1209, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*(-1120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*b^3*(9*a+7*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c
)-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(63*A*b^2+378*
C*a^2+540*C*a*b+259*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+56*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(30*A*a*b^2+9*A*b^3+30*C*
a^3+54*C*a^2*b+60*C*a*b^2+17*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^4+140*A*a*
b^3+21*A*b^4+140*C*a^3*b+126*C*a^2*b^2+160*C*a*b^3+28*C*b^4)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-1890*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-189*A*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1260*
A*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)+420*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)-315*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-1134*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-147*C*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+420*a
^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+300*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^6+4\,Cab^3\cos(dx+c)^5+4\,Aa^3b\cos(dx+c)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos(dx+c)^4+4\,(C\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sec(d*x + c)^(3/
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2),
 x)
```

3.1389 $\int (a+b\cos(c+dx))^4 \left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=369

$$\frac{4ab\left(96a^2C+891Ab^2+673b^2C\right)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(9a^2b^2(143A+101C)+64a^4C+15b^4(11A+9C)\right)\sin(c+dx)}{693d\sqrt{\sec(c+dx)}}+\frac{2(1$$

```
[Out] (8*a*b*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(77*a^4*(3*A + C) + 66*a^2*b
^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a*b*(891*A*b^2 + 96*a^2*C + 673*b^2
*C)*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)) + (2*(64*a^4*C + 15*b^4*(11*A
 + 9*C) + 9*a^2*b^2*(143*A + 101*C))*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]
]) + (2*(16*a^2*C + 3*b^2*(11*A + 9*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(231*d*Sqrt[Sec[c + d*x]]) + (16*a*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])
/(99*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(11*
d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.24613, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3050, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{4ab\left(96a^2C+891Ab^2+673b^2C\right)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(9a^2b^2(143A+101C)+64a^4C+15b^4(11A+9C)\right)\sin(c+dx)}{693d\sqrt{\sec(c+dx)}}+\frac{2(1$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (8*a*b*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(77*a^4*(3*A + C) + 66*a^2*b
^2*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a*b*(891*A*b^2 + 96*a^2*C + 673*b^2
*C)*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)) + (2*(64*a^4*C + 15*b^4*(11*A
 + 9*C) + 9*a^2*b^2*(143*A + 101*C))*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]
]) + (2*(16*a^2*C + 3*b^2*(11*A + 9*C))*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(231*d*Sqrt[Sec[c + d*x]]) + (16*a*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])
/(99*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(11*
d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^4\left(A+\right.}{\sqrt{\cos(c+d}}$$

$$= \frac{2C(a+b\cos(c+dx))^4\sin(c+dx)}{11d\sqrt{\sec(c+dx)}} + \frac{1}{11}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{16aC(a+b\cos(c+dx))^3\sin(c+dx)}{99d\sqrt{\sec(c+dx)}} + \frac{2C(a+b\cos(c+}{11d\sqrt{\sec}}$$

$$= \frac{2\left(16a^2C+3b^2(11A+9C)\right)(a+b\cos(c+dx))^2\sin(c+d}{231d\sqrt{\sec(c+dx)}}$$

$$= \frac{4ab\left(891Ab^2+96a^2C+673b^2C\right)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(16a^2C\right.}{}$$

$$= \frac{4ab\left(891Ab^2+96a^2C+673b^2C\right)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(64a^4C\right.}{}$$

$$= \frac{4ab\left(891Ab^2+96a^2C+673b^2C\right)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(64a^4C\right.}{}$$

$$= \frac{8ab\left(3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{15d}$$

Mathematica [A] time = 1.75, size = 265, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(616ab\left(36a^2C+36Ab^2+43b^2C\right)\cos(c+dx)+5\left(36\left(66a^2b^2C+11Ab^4+16b^4C\right)\cos(2(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(29568*a*b*(3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 480*(77*a^4*(3*A + C) + 66*a^2*b^2
*(7*A + 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2] + 2*(616*a*b*(36*A*b^2 + 36*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*(1848*a
^4*C + 792*a^2*b^2*(14*A + 13*C) + 3*b^4*(572*A + 531*C) + 36*(11*A*b^4 + 6
```

```
6*a^2*b^2*C + 16*b^4*C)*Cos[2*(c + d*x)] + 616*a*b^3*C*Cos[3*(c + d*x)] + 6
3*b^4*C*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(55440*d)
```

Maple [B] time = 1.287, size = 924, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-49280*C*a*b^3-50400*C*b^4)*sin
(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^4+47520*C*a^2*b^2+98560*C*a
*b^3+56880*C*b^4)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-22176*A*a*b^3-1
1880*A*b^4-22176*C*a^3*b-71280*C*a^2*b^2-91168*C*a*b^3-34920*C*b^4)*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(27720*A*a^2*b^2+22176*A*a*b^3+9240*A*b^4+
4620*C*a^4+22176*C*a^3*b+55440*C*a^2*b^2+41888*C*a*b^3+13860*C*b^4)*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-13860*A*a^2*b^2-5544*A*a*b^3-2640*A*b^4-
2310*C*a^4-5544*C*a^3*b-15840*C*a^2*b^2-7392*C*a*b^3-2790*C*b^4)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3465*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6930*a^2*
A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-13860*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+1155*a^
4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*C*
b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-8316*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-6468*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos\left(dx+c\right)^6+4\,Cab^3\cos\left(dx+c\right)^5+4\,Aa^3b\cos\left(dx+c\right)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos\left(dx+c\right)^4+4\left(Ca\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)*sqrt(sec(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos (dx + c)^2 + A\right)(b \cos (dx + c) + a)^4 \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)),
 x)
```

3.1390 $\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=422

$$\frac{2\left(11a^2b^2(637A+491C)+192a^4C+77b^4(13A+11C)\right)\sin(c+dx)}{6435d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\left(96a^2C+1573Ab^2+1259b^2C\right)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+$$

```
[Out] (2*(39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C) + 7*b^4*(13*A + 11*C))*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(195*d) + (8*a
*b*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a*b*(1573*A*b^2 + 96*a^2*C
+ 1259*b^2*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (2*(192*a^4*C + 7
7*b^4*(13*A + 11*C) + 11*a^2*b^2*(637*A + 491*C))*Sin[c + d*x])/(6435*d*Sec
[c + d*x]^(3/2)) + (2*(48*a^2*C + 11*b^2*(13*A + 11*C))*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(1287*d*Sec[c + d*x]^(3/2)) + (16*a*C*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(143*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^4*
Sin[c + d*x])/(13*d*Sec[c + d*x]^(3/2)) + (8*a*b*(11*a^2*(7*A + 5*C) + 5*b^
2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.32403, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3050, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(11a^2b^2(637A+491C)+192a^4C+77b^4(13A+11C)\right)\sin(c+dx)}{6435d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\left(96a^2C+1573Ab^2+1259b^2C\right)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C) + 7*b^4*(13*A + 11*C))*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(195*d) + (8*a
*b*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a*b*(1573*A*b^2 + 96*a^2*C
+ 1259*b^2*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (2*(192*a^4*C + 7
7*b^4*(13*A + 11*C) + 11*a^2*b^2*(637*A + 491*C))*Sin[c + d*x])/(6435*d*Sec
[c + d*x]^(3/2)) + (2*(48*a^2*C + 11*b^2*(13*A + 11*C))*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(1287*d*Sec[c + d*x]^(3/2)) + (16*a*C*(a + b*Cos[c + d*x]
)^3*Sin[c + d*x])/(143*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^4*
```

```
Sin[c + d*x])/(13*d*Sec[c + d*x]^(3/2)) + (8*a*b*(11*a^2*(7*A + 5*C) + 5*b^
2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^4\left(A\right.$$

$$= \frac{2C(a+b\cos(c+dx))^4\sin(c+dx)}{13d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{13}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{16aC(a+b\cos(c+dx))^3\sin(c+dx)}{143d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))^4\sin(c}{13d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(48a^2C+11b^2(13A+11C)\right)(a+b\cos(c+dx))^2\sin(c+dx)}{1287d\sec^{\frac{3}{2}}(c+dx)} + \frac{16a}{}$$

$$= \frac{4ab\left(1573Ab^2+96a^2C+1259b^2C\right)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(48a^2C+11b^2(1\right.}{}$$

$$= \frac{4ab\left(1573Ab^2+96a^2C+1259b^2C\right)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(192a^4C+77b^4\right.}{}$$

$$= \frac{4ab\left(1573Ab^2+96a^2C+1259b^2C\right)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(192a^4C+77b^4\right.}{}$$

$$= \frac{2\left(39a^4(5A+3C)+78a^2b^2(9A+7C)+7b^4(13A+11C)\right)\sqrt{\cos(c+}}{195d}$$

$$= \frac{2\left(39a^4(5A+3C)+78a^2b^2(9A+7C)+7b^4(13A+11C)\right)\sqrt{\cos(c+}}{195d}$$

Mathematica [A] time = 2.7401, size = 303, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(154\left(156a^2b^2(36A+43C)+936a^4C+b^4(1118A+1171C)\right)\cos(c+dx)+5b\left(3744a\left(11\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(14784*(39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C) + 7
*b^4*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 49920*a*
b*(11*a^2*(7*A + 5*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c
```

```
 + d*x)/2, 2] + 2*(154*(936*a^4*C + 156*a^2*b^2*(36*A + 43*C) + b^4*(1118*A
 + 1171*C))*Cos[c + d*x] + 5*b*(312*a*(44*a^2*(14*A + 13*C) + b^2*(572*A +
531*C)) + 3744*a*(11*A*b^2 + 11*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 77*(52
*A*b^3 + 312*a^2*b*C + 89*b^3*C)*Cos[3*(c + d*x)] + 6552*a*b^2*C*Cos[4*(c +
 d*x)] + 693*b^3*C*Cos[5*(c + d*x)]))*Sin[2*(c + d*x)]))/(1441440*d)
```

Maple [B] time = 1.27, size = 1017, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*C
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(1048320*C*a*b^3+1330560*C*b^
4)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^4-960960*C*a^2*b^2
-2620800*C*a*b^3-1798720*C*b^4)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(4
11840*A*a*b^3+320320*A*b^4+411840*C*a^3*b+1921920*C*a^2*b^2+2957760*C*a*b^3
+1379840*C*b^4)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-432432*A*a^2*b^2-
617760*A*a*b^3-296296*A*b^4-72072*C*a^4-617760*C*a^3*b-1777776*C*a^2*b^2-18
15840*C*a*b^3-666512*C*b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(240240
*A*a^3*b+432432*A*a^2*b^2+480480*A*a*b^3+136136*A*b^4+72072*C*a^4+480480*C*
a^3*b+816816*C*a^2*b^2+720720*C*a*b^3+198352*C*b^4)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-120120*A*a^3*b-108108*A*a^2*b^2-137280*A*a*b^3-24024*A*b
^4-18018*C*a^4-137280*C*a^3*b-144144*C*a^2*b^2-145080*C*a*b^3-27258*C*b^4)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+60060*A*a^3*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+42900*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45045*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^4-162162*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-21021*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*b^4+42900*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35100*C*a*b^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-27027*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-126126*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*b^2-17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^4}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos\left(dx+c\right)^6+4\,Cab^3\cos\left(dx+c\right)^5+4\,Aa^3b\cos\left(dx+c\right)+Aa^4+\left(6\,Ca^2b^2+Ab^4\right)\cos\left(dx+c\right)^4+4\left(C\right.}{\sqrt{\sec\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + 4*C*a*b^3*cos(d*x + c)^5 + 4*A*a^3*b*cos(d
*x + c) + A*a^4 + (6*C*a^2*b^2 + A*b^4)*cos(d*x + c)^4 + 4*(C*a^3*b + A*a*b
^3)*cos(d*x + c)^3 + (C*a^4 + 6*A*a^2*b^2)*cos(d*x + c)^2)/sqrt(sec(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)),
 x)
```

3.1391 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5a^3d}-\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}$$

```
[Out] (-2*(5*A*b^2 + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (2*b*(A*b^2 + a^2*C)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*
(a + b)*d) + (2*(5*A*b^2 + a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*a^3*d) - (2*A*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + (2*A*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 1.20163, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5a^3d}-\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + b*Cos[c + d*x]),x]
```


```
[Out] (-2*(5*A*b^2 + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (2*b*(A*b^2 + a^2*C)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*
(a + b)*d) + (2*(5*A*b^2 + a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*a^3*d) - (2*A*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + (2*A*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5Ab}{2}+\frac{1}{2}a(3A+5C}{\cos^{\frac{5}{2}}(c}}{5a}$$

$$= -\frac{2Ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(4\sqrt{\cos(c+\right.}{}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5a^3d} - \frac{2Ab\sec^{\frac{3}{2}}(c+dx)\sin(c}{3a^2d}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5a^3d} - \frac{2Ab\sec^{\frac{3}{2}}(c+dx)\sin(c}{3a^2d}$$

$$= -\frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^3d} + \frac{2\left(5\right.}{}$$

$$= -\frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^3d} - \frac{2Ab}{}$$

Mathematica [B] time = 6.87407, size = 648, normalized size = 2.44

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2\left(3a^2A+5a^2C+5Ab^2\right)\sin(c+dx)}{5a^3} - \frac{2Ab\tan(c+dx)}{3a^2} + \frac{2A\tan(c+dx)\sec(c+dx)}{5a}\right)}{d} - \frac{-\frac{2\left(18a^3A+30a^3C+40aAb^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1}}{b\left(1-\cos^2(c\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] -((-2*(18*a^3*A + 40*a*A*b^2 + 30*a^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]
^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(19*a^
2*A*b + 45*A*b^3 + 45*a^2*b*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + ((9*a^2*A*b + 15*A*b^3 + 15*a^2*b*C)*Cos[2*(c +
d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE
```

```
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticP
i[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)
*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(30*a^3*d) + (Sqrt[Sec[c + d*x]]
*((2*(3*a^2*A + 5*A*b^2 + 5*a^2*C)*Sin[c + d*x])/(5*a^3) - (2*A*b*Tan[c + d
*x])/(3*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a)))/d
```

Maple [B] time = 4.78, size = 786, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(A*b^2+C*a^2)
*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/5*A/a/(8*sin(1/2*d*x+1/2*c)^6-12*
sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*
d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
-2*A/a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2+C*a^2)/a^3*(-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(b*cos(d*x + c) + a), x
)
```

3.1392 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2Ab\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2Ab\sqrt{\cos(c+dx)}\sqrt{\sec}}{a}$$

```
[Out] (2*A*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^
2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*a*d) + (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*A*b*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.844294, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2Ab\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2Ab\sqrt{\cos(c+dx)}\sqrt{\sec}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*A*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^
2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*a*d) + (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*A*b*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3Ab}{2}+\frac{1}{2}a(A+3C)}{\cos^{\frac{3}{2}}(c+}}{3a}$$

$$= -\frac{2Ab\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(4\sqrt{\cos(c+}\right.}{}$$

$$= -\frac{2Ab\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\left(4\sqrt{\cos(c+}\right.}{}$$

$$= \frac{2Ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{2Ab\sqrt{\sec(c+dx)}\sin(c+}{a^2d}$$

$$= \frac{2Ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3ad}$$

Mathematica [A] time = 2.90834, size = 220, normalized size = 1.1

$$-\frac{\cot(c+dx)\left(-2\left(a^2(A+3C)+3aAb+3Ab^2\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-a^2A\sec^{\frac{5}{2}}(c+dx)+a^2 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] -(Cot[c + d*x]*(-(a^2*A*Sec[c + d*x]^(5/2)) + a^2*A*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) + 6*a*A*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 2*(3*a*A*b + 3*A*b^2 + a^2*(A + 3*C))*EllipticF[ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*A*b^2*EllipticPi[-(a/b), -ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a^2*C*EllipticPi[-(a/
b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(3*a^3*d)
```

Maple [A] time = 3.33, size = 463, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b^2+C*a^2
)/a^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)
^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))-2/a^2*b*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin
(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x
)
```

3.1393 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=172

$$-\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(}}{a}$$

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
b*d) - (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a + b)*d) + (2*A*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(a*d)
```

Rubi [A] time = 0.615503, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3056, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
b*d) - (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a + b)*d) + (2*A*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$
$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{Ab}{2}-\frac{1}{2}a(A-C)\,c\ldots}{\sqrt{\cos(c+\ldots}}}{a}$$
$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx\ldots}}{a}$$
$$= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{2A\sqrt{\sec(c+dx)}\sin(c+d\ldots}{ad}$$
$$= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{2C\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\ldots\right.}{bd}$$

Mathematica [A] time = 1.23574, size = 128, normalized size = 0.74

$$\frac{2\cos(2(c+dx))\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(\left(a^2C+Ab^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+Ab(a+b)F\ldots\right.}{a^2bd\left(\sec^2(c+dx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),
x]
```

```
[Out] (2*Cos[2*(c + d*x)]*Csc[c + d*x]*(-(a*A*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]) + A*b*(a + b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b^2
+ a^2*C)*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*
Sqrt[-Tan[c + d*x]^2])/(a^2*b*d*(-2 + Sec[c + d*x]^2))
```

Maple [A] time = 2.424, size = 407, normalized size = 2.4

$$-\frac{1}{d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,\frac{C\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{Elliptic}}{b\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*(-A
*b^2-C*a^2)/a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x
)
```

3.1394 $\int \frac{(A+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{2(a^2C+Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aC\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2C\cdots}{\cdots}$$

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 - (2*a*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(b^2*d) + (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rubi [A] time = 0.36917, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3060, 2639, 3002, 2641, 2805}

$$\frac{2(a^2C+Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aC\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2C\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 - (2*a*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(b^2*d) + (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
```

```
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx \\
&= -\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-Ab+aC\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b} + \frac{\left(C\sqrt{\cos(c+dx)}\right.}{} \\
&= \frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{\left(aC\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{b^2} \\
&= \frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2aC\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{b^2d}
\end{aligned}$$

Mathematica [A] time = 6.45722, size = 242, normalized size = 1.67

$$\frac{\cot(c+dx)\left(2a^2C\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2Ab^2\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),
x]
```

```
[Out] (Cot[c + d*x]*(-(a*b*C*Sec[c + d*x]^(3/2)) - a*b*C*Cos[2*(c + d*x)]*Sec[c +
 d*x]^(3/2) + a*b*C*Sec[c + d*x]^(7/2) + a*b*C*Cos[2*(c + d*x)]*Sec[c + d*x
]^(7/2) - 2*a*b*C*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d
*x]^2] + 2*b*(A*b + a*C)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Ta
n[c + d*x]^2] + 2*A*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[-Tan[c + d*x]^2] + 2*a^2*C*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a*b^2*d)
```

Maple [A] time = 1.416, size = 259, normalized size = 1.8

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{\left(a-b\right)b^2\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(A\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b^2-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a*b+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+C*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\cos^2\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}}{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x
)
```

3.1395 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=190

$$\frac{2\left(3a^2C+b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.}{b^3d(a+b)}$$

```
[Out] (-2*a*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^2*d) + (2*(3*a^2*C + b^2*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) - (2*a*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a +
 b)*d) + (2*C*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.632715, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3050, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2C+b^2(3A+C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.}{b^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*a*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^2*d) + (2*(3*a^2*C + b^2*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) - (2*a*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a +
 b)*d) + (2*C*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
```

```
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2C\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aC}{2}+\frac{1}{2}b(3A+C)\cos(c+dx)-\frac{3}{2}aC\cos}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3b}$$

$$= \frac{2C\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}abC-\frac{1}{2}\left(3a^2C+b^2(3A+C)\right)\cos(c-}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3b^2}$$

$$= -\frac{2aC\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2C\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{\left(a\left(Ab^2\right.\right.}{}$$

$$= -\frac{2aC\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\left(3a^2C+b^2(3A+C)\right)\sqrt{\mathrm{co}}}{}$$

Mathematica [B] time = 6.74197, size = 539, normalized size = 2.84

$$-\frac{3C\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+b)\left(4a^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-2b^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\mathrm{s}}\right.\right.\right.}{b^2\left(1-\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] ((-2*(6*A*b + 2*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*C*Cos[c + d*x]^2*(Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(
(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (3*C*Cos[2*(c + d*x)]*(b + a*S
ec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b
)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*S
in[c + d*x])/(b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*
```

```
x]]*(2 - Sec[c + d*x]^2)))/(6*b*d) + (C*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)]
)/(3*b*d)
```

Maple [B] time = 1.37, size = 686, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*C*a*b^2-4*
C*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b^2+2*C*b^3)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b^2+3*
a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*a*b^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))*a^3+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^2)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\cos^2(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c) + a)\sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

3.1396 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{2a\left(C\left(3a^2+b^2\right)+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}+\frac{2\left(5a^2C+b^2(5A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5b^3d}$$

```
[Out] (2*(5*a^2*C + b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(5*b^3*d) - (2*a*(3*A*b^2 + (3*a^2 + b^2)*C)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*d) + (2*a^2*
(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(b^4*(a + b)*d) + (2*C*Sin[c + d*x])/(5*b*d*Sec[c + d
*x]^(3/2)) - (2*a*C*Sin[c + d*x])/(3*b^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.902225, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3050, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2a\left(C\left(3a^2+b^2\right)+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}+\frac{2\left(5a^2C+b^2(5A+3C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```


```
[Out] (2*(5*a^2*C + b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(5*b^3*d) - (2*a*(3*A*b^2 + (3*a^2 + b^2)*C)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*d) + (2*a^2*
(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(b^4*(a + b)*d) + (2*C*Sin[c + d*x])/(5*b*d*Sec[c + d
*x]^(3/2)) - (2*a*C*Sin[c + d*x])/(3*b^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2C\sin(c+dx)}{5bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+\frac{1}{2}b(5A+3C)\cos\right.}{a+b\cos(c+}}{5b}$$

$$= \frac{2C\sin(c+dx)}{5bd\sec^{\frac{3}{2}}(c+dx)} - \frac{2aC\sin(c+dx)}{3b^2d\sqrt{\sec(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5a^2C}{4}}{}}{}$$

$$= \frac{2C\sin(c+dx)}{5bd\sec^{\frac{3}{2}}(c+dx)} - \frac{2aC\sin(c+dx)}{3b^2d\sqrt{\sec(c+dx)}} - \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{5}{4}a^2bC}{}}{15b^3}$$

$$= \frac{2\left(5a^2C+b^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^3d} + \frac{2C\sin}{5bd\sec}$$

$$= \frac{2\left(5a^2C+b^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^3d} - \frac{2a\left(3A\right.}{}$$

Mathematica [B] time = 6.82922, size = 603, normalized size = 2.5

$$\frac{\frac{2\left(5a^2C+15Ab^2+9b^2C\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c+dx)+b)\left(\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{a\left(1-\cos^2(c+dx)\right)(a+b\cos(c+dx))}+\frac{\left(15a^2C+15Ab^2+\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] ((-16*a*C*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1
]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((a + b*Cos[c
 + d*x])*(1 - Cos[c + d*x]^2)) + (2*(15*A*b^2 + 5*a^2*C + 9*b^2*C)*Cos[c +
d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -Arc
Sin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]
*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((15*A*b^2 +
 15*a^2*C + 9*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*
Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
 + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ellipt
icPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/
(30*b^2*d) + (Sqrt[Sec[c + d*x]]*((C*Sin[c + d*x])/(10*b) - (a*C*Sin[2*(c +
 d*x)])/(3*b^2) + (C*Sin[3*(c + d*x)])/(10*b)))/d
```

Maple [B] time = 1.318, size = 948, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((24*C*a*b^3-2
4*C*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*C*a^2*b^2-44*C*a*b^3+2
4*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*C*a^2*b^2+16*C*a*b^3-
6*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
```

```
c),2^(1/2))-15*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^2*b^2+15*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*a^3*b*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+5*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5*C*a*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2*b^2+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^
4-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^4)/b^4/(a-b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
 x)
```

3.1397 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{2\left(7a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21b^3d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2b^2(3A+C)+21a^4C+b^4(7A+5C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{21b^5d}$$

```
[Out] (-2*a*(5*A*b^2 + 5*a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(5*b^4*d) + (2*(21*a^4*C + 7*a^2*b^2*(3*A + C) +
b^4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*b^5*d) - (2*a^3*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^5*(a + b)*d) + (2*C*Sin
[c + d*x])/(7*b*d*Sec[c + d*x]^(5/2)) - (2*a*C*Sin[c + d*x])/(5*b^2*d*Sec[c
 + d*x]^(3/2)) + (2*(7*a^2*C + b^2*(7*A + 5*C))*Sin[c + d*x])/(21*b^3*d*Sqr
t[Sec[c + d*x]])
```

Rubi [A] time = 1.25908, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3050, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(7a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21b^3d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2b^2(3A+C)+21a^4C+b^4(7A+5C)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{21b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-2*a*(5*A*b^2 + 5*a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(5*b^4*d) + (2*(21*a^4*C + 7*a^2*b^2*(3*A + C) +
b^4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*b^5*d) - (2*a^3*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^5*(a + b)*d) + (2*C*Sin
[c + d*x])/(7*b*d*Sec[c + d*x]^(5/2)) - (2*a*C*Sin[c + d*x])/(5*b^2*d*Sec[c
 + d*x]^(3/2)) + (2*(7*a^2*C + b^2*(7*A + 5*C))*Sin[c + d*x])/(21*b^3*d*Sqr
t[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5aC}{2}+\frac{1}{2}b(7A+5C)\cos(\ldots\right.}{a+b\cos(c+d\ldots}}{7b}$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} - \frac{2aC\sin(c+dx)}{5b^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c\ldots}}{\ldots}}{\ldots}$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} - \frac{2aC\sin(c+dx)}{5b^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21b^3d\sqrt{\sec(c+dx)}} + \ldots$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} - \frac{2aC\sin(c+dx)}{5b^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7a^2C+b^2(7A+5C)\right)\sin(c+dx)}{21b^3d\sqrt{\sec(c+dx)}} - \ldots$$

$$= -\frac{2a\left(5Ab^2+5a^2C+3b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^4d} + \frac{2C\,\mathrm{s}\ldots}{7bd\,\mathrm{s}\ldots}$$

$$= -\frac{2a\left(5Ab^2+5a^2C+3b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^4d} + \frac{2\left(21\ldots\right.}{\ldots}$$

Mathematica [B] time = 6.96731, size = 663, normalized size = 2.22

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(14a^2C+14Ab^2+13b^2C\right)\sin(2(c+dx))}{42b^3} - \frac{aC\sin(c+dx)}{10b^2} - \frac{aC\sin(3(c+dx))}{10b^2} + \frac{C\sin(4(c+dx))}{28b}\right)}{d} - \frac{-\frac{2\left(56a^2bC-70Ab^3-50b^3C\right)\sin(c+dx)\,\mathrm{c}\ldots}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] -((-2*(-70*A*b^3 + 56*a^2*b*C - 50*b^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(35*a
*A*b^2 + 35*a^3*C + 13*a*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
 + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
```

```
a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)) + ((105*a*A*b^2 + 105*a^3*C + 63*a*b^2*C)*Cos[2*(
c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ellip
ticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*Ellip
ticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(210*b^3*d) + (Sqrt[Sec[c +
d*x]]*(-(a*C*Sin[c + d*x])/(10*b^2) + ((14*A*b^2 + 14*a^2*C + 13*b^2*C)*Sin
[2*(c + d*x)])/(42*b^3) - (a*C*Sin[3*(c + d*x)])/(10*b^2) + (C*Sin[4*(c + d
*x)])/(28*b)))/d
```

Maple [B] time = 1.519, size = 1244, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((240*C*a*b^
4-240*C*b^5)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(168*C*a^2*b^3-528*C*a
*b^4+360*C*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a*b^4-140*A*
b^5+140*C*a^3*b^2-308*C*a^2*b^3+448*C*a*b^4-280*C*b^5)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-70*A*a*b^4+70*A*b^5-70*C*a^3*b^2+112*C*a^2*b^3-122*C*
a*b^4+80*C*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^3*b^2-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3+35*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^4-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^5-105*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))*a^3*b^2+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3-105*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b^4+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^5-105*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a^4*b+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
```

```
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b^2-35*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^3+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^4-25*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^5-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^5+105*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^4*b-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b^2+63*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b^3-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^4)/b^5
/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
 x)
```

3.1398 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=396

$$-\frac{\left(5Ab^2-a^2(2A-3C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{b\left(5Ab^2-a^2(4A-C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2C+Ab^2\right)\sin(}{ad\left(a^2-b^2\right)(a}$$

```
[Out] -((b*(5*A*b^2 - a^2*(4*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) - ((5*A*b^2 - a^2*(2*A - 3*C))*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2
- b^2)*d) - ((5*A*b^4 - a^2*b^2*(7*A - C) - 3*a^4*C)*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a
+ b)^2*d) + (b*(5*A*b^2 - a^2*(4*A - C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
a^3*(a^2 - b^2)*d) - ((5*A*b^2 - a^2*(2*A - 3*C))*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.56272, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(5Ab^2-a^2(2A-3C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{b\left(5Ab^2-a^2(4A-C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2C+Ab^2\right)\sin(}{ad\left(a^2-b^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -((b*(5*A*b^2 - a^2*(4*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) - ((5*A*b^2 - a^2*(2*A - 3*C))*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2
- b^2)*d) - ((5*A*b^4 - a^2*b^2*(7*A - C) - 3*a^4*C)*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a
+ b)^2*d) + (b*(5*A*b^2 - a^2*(4*A - C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
a^3*(a^2 - b^2)*d) - ((5*A*b^2 - a^2*(2*A - 3*C))*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\left(Ab^2 + a^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(-}{}}{}$$

$$= -\frac{\left(5Ab^2 - a^2(2A-3C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2 + a^2C\right)\sec^{\frac{3}{2}}(c+a}{a\left(a^2-b^2\right)d(a+b\,c}$$

$$= \frac{b\left(5Ab^2 - a^2(4A-C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(5Ab^2 - a^2(2A-3C)\right)s}{3a^2\left(a^2\right.}$$

$$= \frac{b\left(5Ab^2 - a^2(4A-C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(5Ab^2 - a^2(2A-3C)\right)s}{3a^2\left(a^2\right.}$$

$$= -\frac{b\left(5Ab^2 - a^2(4A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} + \frac{b\left(5\right.}{}$$

$$= -\frac{b\left(5Ab^2 - a^2(4A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} - \frac{\left(5A\right.}{}$$

Mathematica [A] time = 7.08297, size = 724, normalized size = 1.83

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{b\left(4a^2A-a^2C-5Ab^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)} + \frac{-a^2bC\sin(c+dx)-Ab^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{2A\tan(c+dx)}{3a^2}\right)}{d} + \frac{-\frac{2\left(-28a^3Ab+12a^3bC+40aAb^3\right)\sin(c+dx)\,c}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^
2,x]
```

```
[Out] ((-2*(-28*a^3*A*b + 40*a*A*b^3 + 12*a^3*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/
b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c +
d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-
4*a^4*A - 44*a^2*A*b^2 + 45*A*b^4 - 12*a^4*C + 9*a^2*b^2*C)*Cos[c + d*x]^2*
(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqr
```

```
t[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c
+ d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^2*A*b^2 + 1
5*A*b^4 + 3*a^2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*
b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Elli
pticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))
)/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((b*(4*a^2*A - 5*A*b^
2 - a^2*C)*Sin[c + d*x])/(a^3*(a^2 - b^2))) + (-(A*b^3*Sin[c + d*x]) - a^2*
b*C*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x
])/(3*a^2)))/d
```

Maple [B] time = 5.693, size = 1019, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*A*b^3/a^3/(-
2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*(A*b^2+C*a^2)/a^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2
/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2
)))-4*A/a^3*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2,
 x)
```

3.1399 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=330

$$-\frac{\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\cdots}{abd\left(a\cdots\right.}$$

```
[Out] ((3*A*b^2 - a^2*(2*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^2 - b^2)*d) + ((3*A*b
^4 - a^4*C - a^2*b^2*(5*A + C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b
^2 - a^2*(2*A - C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) +
((A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*C
os[c + d*x]))
```

Rubi [A] time = 1.14817, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\cdots}{abd\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((3*A*b^2 - a^2*(2*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^2 - b^2)*d) + ((3*A*b
^4 - a^4*C - a^2*b^2*(5*A + C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b
^2 - a^2*(2*A - C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) +
((A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*C
os[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(-3}{}}{}$$

$$= -\frac{\left(3Ab^2-a^2(2A-C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d(a+b\cos}$$

$$= -\frac{\left(3Ab^2-a^2(2A-C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d(a+b\cos}$$

$$= \frac{\left(3Ab^2-a^2(2A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-\right.}{}$$

$$= \frac{\left(3Ab^2-a^2(2A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a\right.}{}$$

Mathematica [B] time = 7.00557, size = 682, normalized size = 2.07

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(2a^2A-a^2C-3Ab^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)} + \frac{a^2C\sin(c+dx)+Ab^2\sin(c+dx)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d} - \frac{-\frac{2\left(4a^3A-4a^3C-8aAb^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c}{b\left(1-\cos^2(c+dx)\right)(a+b\cos(c+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^
2,x]
```

```
[Out] -((-2*(4*a^3*A - 8*a*A*b^2 - 4*a^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]
*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(10*a^2*A
*b - 9*A*b^3 + a^2*b*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]
], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
+ d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 -
 Cos[c + d*x]^2)) + ((2*a^2*A*b - 3*A*b^3 - a^2*b*C)*Cos[2*(c + d*x)]*(b +
a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a
```

```
- b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
)*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]]*(2 - Sec[c + d*x]^2)))/(4*a^2*(a - b)*(a + b)*d) + (Sqrt[Sec[c + d
*x]]*(((2*a^2*A - 3*A*b^2 - a^2*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)) + (A*b^2
*Sin[c + d*x] + a^2*C*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/
d
```

Maple [B] time = 4.196, size = 899, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b^2+C*a^
2)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b^2-C*a^2)/a/b*(-1/a*b^2/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a
*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^
2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2,
 x)
```

3.1400 $\int \frac{(A+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=274

$$\frac{(a^2C+Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{(a^2(-C)+Ab^2+2b^2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a^2-b^2)}-\frac{(a^2($$

```
[Out] -(((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*b*(a^2 - b^2)*d)) - ((A*b^2 - a^2*C + 2*b^2*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - ((A*
b^4 + a^4*C - 3*a^2*b^2*(A + C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^
2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.805498, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3056, 3059, 2639, 3002, 2641, 2805}

$$\frac{(a^2C+Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{(a^2(-C)+Ab^2+2b^2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a^2-b^2)}-\frac{(a^2($$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*b*(a^2 - b^2)*d)) - ((A*b^2 - a^2*C + 2*b^2*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - ((A*
b^4 + a^4*C - 3*a^2*b^2*(A + C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^
2 + a^2*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c +
 d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{\ }$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{\ }$$

$$= -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ab\left(a^2-b^2\right)d} + \frac{\left(Ab^2\right.}{a\left(a^2-b^2\right)d(a}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ab\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a^2C+2\right.}{\ }$$

Mathematica [B] time = 6.88086, size = 657, normalized size = 2.4

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{ab\left(a^2-b^2\right)}+\frac{a^2C\sin(c+dx)+Ab^2\sin(c+dx)}{b\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{2\left(-4a^2A-a^2C+3Ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c+dx)+b)(\mathrm{I}}{a\left(1-\cos^2(c+dx)\right)(a+b\cos}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^
2,x]
```

```
[Out] ((-2*(4*a*A*b + 4*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4*a^2*A + 3*A*b^2 -
a^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Elliptic
```

```
Pi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + ((A*b^2 + a^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*S
ec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ellipti
cPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
c[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos
[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(
4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((A*b^2 + a^2*C)*Sin[c + d*x
])/(a*b*(a^2 - b^2)) + (A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x])/(b*(-a^2 +
 b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 2.994, size = 804, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/b
*a*C/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^2*(A*b^2+C*a^2)*(-1/a*b^2/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/
(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
```

```
c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^{2}\left(c + dx\right)\right)\sqrt{\sec\left(c + dx\right)}}{\left(a + b\cos\left(c + dx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2,
 x)
```

3.1401 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=277

$$-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{a\left(-3a^2C+Ab^2+4b^2C\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+$$

```
[Out] ((A*b^2 + 3*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + (a*(A*b^2 - 3*a^2*C + 4*b^2*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b
^2)*d) - ((A*b^4 - 3*a^4*C + a^2*b^2*(A + 5*C))*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^3*(a + b)^
2*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.787369, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3048, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{a\left(-3a^2C+Ab^2+4b^2C\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((A*b^2 + 3*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + (a*(A*b^2 - 3*a^2*C + 4*b^2*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b
^2)*d) - ((A*b^4 - 3*a^4*C + a^2*b^2*(A + 5*C))*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^3*(a + b)^
2*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} + \frac{a\left(Ab^2-3\right.}{}$$

Mathematica [B] time = 6.85765, size = 663, normalized size = 2.39

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{b^2\left(b^2-a^2\right)}+\frac{a^3(-C)\sin(c+dx)-aAb^2\sin(c+dx)}{b^2\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{2\left(a^2C-Ab^2-2b^2C\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c+dx)+b)(}{a\left(1-\cos^2(c+dx)\right)(a+b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] ((-2*(4*a*A*b + 4*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(A*b^2) + a^2*C - 2*
b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Elliptic
Pi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + ((A*b^2 + 3*a^2*C - 2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a
```

```
*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4
*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^
2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
 d*x]^2)))/(4*(a - b)*b*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((A*b^2 + a^2*C)*
Sin[c + d*x])/(b^2*(-a^2 + b^2)) + (-(a*A*b^2*Sin[c + d*x]) - a^3*C*Sin[c +
 d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 3.837, size = 834, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C/b^3/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-4/b^2*(A*b^2+3*C*a^2)/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-2*a*(A*b^2+C*a^2)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))
), x)
```

3.1402 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=352

$$\frac{\left(5a^2C+3Ab^2-2b^2C\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\left(a^2b^2(3A-16C)+15a^4C-2b^4(3A\right.}{3}$$

```
[Out] -((a*(A*b^2 + 5*a^2*C - 4*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) + ((a^2*b^2*(3*A - 16*C) + 15*a
^4*C - 2*b^4*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*b^4*(a^2 - b^2)*d) + (a*(3*A*b^4 - a^2*b^2*(A - 7*C) - 5*a
^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/((a - b)*b^4*(a + b)^2*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A*b^2 + 5*a^2*C
 - 2*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.11428, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(5a^2C+3Ab^2-2b^2C\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\left(a^2b^2(3A-16C)+15a^4C-2b^4(3A\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((a*(A*b^2 + 5*a^2*C - 4*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) + ((a^2*b^2*(3*A - 16*C) + 15*a
^4*C - 2*b^4*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*b^4*(a^2 - b^2)*d) + (a*(3*A*b^4 - a^2*b^2*(A - 7*C) - 5*a
^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/((a - b)*b^4*(a + b)^2*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A*b^2 + 5*a^2*C
 - 2*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^2+5a^2C-2b^2C\right)\sin(c}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+}}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^2+5a^2C-2b^2C\right)\sin(c}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+}}$$

$$= -\frac{a\left(Ab^2+5a^2C-4b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d} - \frac{}{b\left(a^2\right.}$$

$$= -\frac{a\left(Ab^2+5a^2C-4b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d} + \frac{\left(a^2b^2\right.}{}$$

Mathematica [A] time = 6.99681, size = 698, normalized size = 1.98

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a\left(a^2C+Ab^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)} - \frac{a^4(-C)\sin(c+dx)-a^2Ab^2\sin(c+dx)}{b^3\left(b^2-a^2\right)(a+b\cos(c+dx))} + \frac{C\sin(2(c+dx))}{3b^2}\right)}{d} + \frac{-\frac{2\left(8a^2bC+12Ab^3+4b^3C\right)\sin(c+dx)\cos^2(c+dx)\sqrt{}}{b\left(1-\cos^2\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] ((-2*(12*A*b^3 + 8*a^2*b*C + 4*b^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]
*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-3*a*A*b
^2 + 5*a^3*C - 8*a*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x
]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[
c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) + ((3*a*A*b^2 + 15*a^3*C - 12*a*b^2*C)*Cos[2*(c + d*x)]
*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcS
```

```
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2
*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a
/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(12*b^2*(-a + b)*(a + b)*d) + (Sqrt[S
ec[c + d*x]]*((a*(A*b^2 + a^2*C)*Sin[c + d*x])/(b^3*(a^2 - b^2)) - (-(a^2*A
*b^2*Sin[c + d*x]) - a^4*C*Sin[c + d*x])/(b^3*(-a^2 + b^2)*(a + b*Cos[c + d
*x])) + (C*Sin[2*(c + d*x)])/(3*b^2)))/d
```

Maple [B] time = 4.283, size = 1102, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^2*C*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4*C/b^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(A*b^2+3*C*a^2+2*C*a*b+C*b^2)/b^4*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a/b^3*(A*b^2+2
*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b^2+C*a^2)/b^4*(-1/a*b^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^
```

```
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^
2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + A}{(b\cos{(dx+c)} + a)^2 \sec{(dx+c)}^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)
), x)
```

3.1403 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=430

$$\frac{\left(7a^2C+5Ab^2-2b^2C\right)\sin(c+dx)}{5b^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}-\frac{a\left(7a^2C+3Ab^2-4b^2C\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}$$

```
[Out] ((3*a^2*b^2*(5*A - 8*C) + 35*a^4*C - 2*b^4*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^4*(a^2 - b^2)*d) - (a*(a
^2*b^2*(9*A - 20*C) + 21*a^4*C - 4*b^4*(3*A + C))*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b
^4 - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^5*(a + b)^2*d) - ((A*b
^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sec[c + d*x
]^(5/2)) + ((5*A*b^2 + 7*a^2*C - 2*b^2*C)*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*
d*Sec[c + d*x]^(3/2)) - (a*(3*A*b^2 + 7*a^2*C - 4*b^2*C)*Sin[c + d*x])/(3*b
^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.54768, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(7a^2C+5Ab^2-2b^2C\right)\sin(c+dx)}{5b^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}-\frac{a\left(7a^2C+3Ab^2-4b^2C\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*a^2*b^2*(5*A - 8*C) + 35*a^4*C - 2*b^4*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^4*(a^2 - b^2)*d) - (a*(a
^2*b^2*(9*A - 20*C) + 21*a^4*C - 4*b^4*(3*A + C))*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b
^4 - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^5*(a + b)^2*d) - ((A*b
^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sec[c + d*x
]^(5/2)) + ((5*A*b^2 + 7*a^2*C - 2*b^2*C)*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*
d*Sec[c + d*x]^(3/2)) - (a*(3*A*b^2 + 7*a^2*C - 4*b^2*C)*Sin[c + d*x])/(3*b
```

```
^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A + C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^2 + 7a^2C - 2b^2C\right)\sin(c}{5b^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^2 + 7a^2C - 2b^2C\right)\sin(c}{5b^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^2 + 7a^2C - 2b^2C\right)\sin(c}{5b^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{\left(3a^2b^2(5A-8C) + 35a^4C - 2b^4(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5b^4\left(a^2-b^2\right)d}$$

$$= \frac{\left(3a^2b^2(5A-8C) + 35a^4C - 2b^4(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5b^4\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.14896, size = 768, normalized size = 1.79

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{\left(10a^2Ab^2 - a^2b^2C + 10a^4C + b^4C\right)\sin(c+dx)}{10b^4\left(a^2-b^2\right)} - \frac{a^3Ab^2\sin(c+dx) + a^5C\sin(c+dx)}{b^4\left(b^2-a^2\right)(a+b\cos(c+dx))} - \frac{2aC\sin(2(c+dx))}{3b^3} + \frac{C\sin(3(c+dx))}{10b^2}\right)}{d} + \frac{-2\left(56a^3bC+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] ((-2*(60*a*A*b^3 + 56*a^3*b*C + 4*a*b^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b)
, -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*
x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(15*
a^2*A*b^2 - 30*A*b^4 + 35*a^4*C - 32*a^2*b^2*C - 18*b^4*C)*Cos[c + d*x]^2*(
```

```
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt
[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
 d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((45*a^2*A*b^2 - 30*
A*b^4 + 105*a^4*C - 72*a^2*b^2*C - 18*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c
+ d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*E
llipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c
+ d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]
*(2 - Sec[c + d*x]^2)))/(60*(a - b)*b^3*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-
((10*a^2*A*b^2 + 10*a^4*C - a^2*b^2*C + b^4*C)*Sin[c + d*x])/(10*b^4*(a^2 -
 b^2)) - (a^3*A*b^2*Sin[c + d*x] + a^5*C*Sin[c + d*x])/(b^4*(-a^2 + b^2)*(a
 + b*Cos[c + d*x])) - (2*a*C*Sin[2*(c + d*x)])/(3*b^3) + (C*Sin[3*(c + d*x)
])/(10*b^2)))/d
```

Maple [B] time = 4.954, size = 1337, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^2*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)-4/3/b^3*C*(2*a+3*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/b^4*(
A*b^2+3*C*a^2+4*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-
2*(2*A*a*b^2+A*b^3+4*C*a^3+3*C*a^2*b+2*C*a*b^2+C*b^3)/b^5*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*a^2/b^4*(3*
```

```
A*b^2+5*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2+C*a^2)/b^5*(-1/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+
1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)
), x)
```

3.1404 $\int \frac{(A+C\cos^2(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=554

$$\frac{\left(-a^2b^2(61A-3C)+a^4(8A-21C)+35Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{12a^3d\left(a^2-b^2\right)^2}-\frac{b\left(-a^2b^2(65A-3C)+3a^4(8A-3C)+35Ab^4\right)\text{s}}{4a^4d\left(a^2-b^2\right)^2}$$

```
[Out] (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*A - 3*C))*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((
35*A*b^4 + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A
*b^6 - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^4*(a - b)^2*(a + b)^3*d) - (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*
A - 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A
*b^4 + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(12*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((7*A*b^4 - 5*a^4*C -
a^2*b^2*(13*A + C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d
*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.23811, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-a^2b^2(61A-3C)+a^4(8A-21C)+35Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{12a^3d\left(a^2-b^2\right)^2}-\frac{b\left(-a^2b^2(65A-3C)+3a^4(8A-3C)+35Ab^4\right)\text{s}}{4a^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*A - 3*C))*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((
35*A*b^4 + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A
*b^6 - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
```

```
^4*(a - b)^2*(a + b)^3*d) - (b*(35*A*b^4 + 3*a^4*(8*A - 3*C) - a^2*b^2*(65*
A - 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A
*b^4 + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(12*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((7*A*b^4 - 5*a^4*C -
a^2*b^2*(13*A + C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d
*(a + b*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\left(Ab^2+a^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(-}{}}{}$$

$$= \frac{\left(Ab^2+a^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(7Ab^4-5a^4C-a^2b^2(13A+C)\right)\mathrm{se}}{4a^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{c}}$$

$$= \frac{\left(35Ab^4+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \frac{\left(Ab\right.}{2a}$$

$$= -\frac{b\left(35Ab^4+3a^4(8A-3C)-a^2b^2(65A-3C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} +$$

$$= -\frac{b\left(35Ab^4+3a^4(8A-3C)-a^2b^2(65A-3C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} +$$

$$= \frac{b\left(35Ab^4+3a^4(8A-3C)-a^2b^2(65A-3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{b\left(35Ab^4+3a^4(8A-3C)-a^2b^2(65A-3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.2569, size = 886, normalized size = 1.6

$$-\frac{2\left(160Aba^5-96bCa^5-512Ab^3a^3+24b^3Ca^3+280Ab^5a\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(16Aa^6+48\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^
3,x]
```

```
[Out] ((-2*(160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 96*a^5*b*C + 24*a^3*b^3*C
)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + (2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 3
15*A*b^6 + 48*a^6*C - 57*a^4*b^2*C + 27*a^2*b^4*C)*Cos[c + d*x]^2*(Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(
a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((72*a^4*A*b^2 - 195*a^2*A*b
^4 + 105*A*b^6 - 27*a^4*b^2*C + 9*a^2*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c
+ d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*E
llipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c
+ d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]
*(2 - Sec[c + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]
]*(-(b*(24*a^4*A - 65*a^2*A*b^2 + 35*A*b^4 - 9*a^4*C + 3*a^2*b^2*C)*Sin[c +
 d*x])/(4*a^4*(a^2 - b^2)^2) + (-(A*b^3*Sin[c + d*x]) - a^2*b*C*Sin[c + d*x
])/(2*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (-15*a^2*A*b^3*Sin[c + d*x]
 + 9*A*b^5*Sin[c + d*x] - 7*a^4*b*C*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x])/
(4*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 9.041, size = 2140, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-12*A*b^3/a^4/(
-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b^2+C*a^2)/a^2*(-1/2/a*b^2/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+
b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^3*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*A*b^2/a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+
2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2)))-6*A/a^4*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x
+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3,
 x)
```

3.1405 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=477

$$\frac{\left(-a^2b^2(29A+C)+a^4(8A-5C)+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^3d\left(a^2-b^2\right)^2}-\frac{\left(-a^2b^2(11A+3C)-3a^4C+5Ab^4\right)\sin(c+dx)\sqrt{\mathrm{s}\ldots}}{4a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}$$

```
[Out] -((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((5*A*b
^4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 + 3*a^6*C
 - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*b*(a
+ b)^3*d) + ((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*A*b^
4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2
*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.71832, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-a^2b^2(29A+C)+a^4(8A-5C)+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^3d\left(a^2-b^2\right)^2}-\frac{\left(-a^2b^2(11A+3C)-3a^4C+5Ab^4\right)\sin(c+dx)\sqrt{\mathrm{s}\ldots}}{4a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((5*A*b
^4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 + 3*a^6*C
 - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*b*(a
+ b)^3*d) + ((15*A*b^4 + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sqrt[Sec[c
```

```
+ d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*A*b^
4 - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2
*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{2}(\cdots}{\cdots}$$

$$= \frac{\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5Ab^4-3a^4C-a^2b^2(11A+3C)\right)\cdots}{4a^2\left(a^2-b^2\right)^2 d(a+b\cdots}$$

$$= \frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(Ab^2\cdots\right.}{2a\,(\cdots}$$

$$= \frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(Ab^2\cdots\right.}{2a\,(\cdots}$$

$$= -\frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\cdots}}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(15Ab^4+a^4(8A-5C)-a^2b^2(29A+C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\cdots}}{4a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.17404, size = 846, normalized size = 1.77

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(8Aa^4-5Ca^4-29Ab^2a^2-b^2Ca^2+15Ab^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2}+\frac{C\sin(c+dx)a^2+Ab^2\sin(c+dx)}{2a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{3C\sin(c+dx)a^4+11Ab^2\sin(c+dx)a^2+3b^2C\sin(c+\cdots}{4a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx)\cdots}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^
3,x]
```

```
[Out] -((-2*(16*a^5*A - 80*a^3*A*b^2 + 40*a*A*b^4 - 16*a^5*C - 8*a^3*b^2*C)*Cos[c
 + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c
+ d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 -
 Cos[c + d*x]^2)) + (2*(56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 + 9*a^4*b*C -
```

```
3*a^2*b^3*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + El
lipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqr
t[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)) + ((8*a^4*A*b - 29*a^2*A*b^3 + 15*A*b^5 - 5*a^4*b*C - a^2*b^3*C)*Cos
[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*E
llipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*E
llipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c +
 d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a^3*(a - b)^2*(a + b
)^2*d) + (Sqrt[Sec[c + d*x]]*(((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 - 5*a^4*C
 - a^2*b^2*C)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2) + (A*b^2*Sin[c + d*x] + a
^2*C*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (11*a^2*A*b^2
*Sin[c + d*x] - 5*A*b^4*Sin[c + d*x] + 3*a^4*C*Sin[c + d*x] + 3*a^2*b^2*C*S
in[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 6.506, size = 2023, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b^2-C*a^2)/a/b*(-1/2/a*b^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b
)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
```

```
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9
/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(-A*b^2+C*a^2
)/a^2/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2
/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^3*(-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorit
```

```
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3,
 x)
```

3.1406 $\int \frac{(A+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=405

$$-\frac{\left(-a^2b^2(9A+5C)+a^4(-C)+3Ab^4\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+\frac{\left(-7a^2b^2(A+C)\right.}{}$$

```
[Out] ((3*A*b^4 - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^2*d) + ((A*b^4 + a^4*C
- 7*a^2*b^2*(A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(4*a*b^2*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^2*b^4*(2*A - C) - a^
6*C + 5*a^4*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d) + ((A*
b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2*Sqrt[S
ec[c + d*x]]) - ((3*A*b^4 - a^4*C - a^2*b^2*(9*A + 5*C))*Sin[c + d*x])/(4*a
^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.25454, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3056, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(-a^2b^2(9A+5C)+a^4(-C)+3Ab^4\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+\frac{\left(-7a^2b^2(A+C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] ((3*A*b^4 - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^2*d) + ((A*b^4 + a^4*C
- 7*a^2*b^2*(A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(4*a*b^2*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^2*b^4*(2*A - C) - a^
6*C + 5*a^4*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d) + ((A*
b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2*Sqrt[S
ec[c + d*x]]) - ((3*A*b^4 - a^4*C - a^2*b^2*(9*A + 5*C))*Sin[c + d*x])/(4*a
^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx\right.}}{}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(3Ab^4-a^4C-a^2b^2(9A\right.}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(3Ab^4-a^4C-a^2b^2(9A\right.}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(3Ab^4-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2b\left(a^2-b^2\right)^2 d} +$$

$$= \frac{\left(3Ab^4-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2b\left(a^2-b^2\right)^2 d} +$$

Mathematica [B] time = 6.98887, size = 816, normalized size = 2.01

$$-\frac{\frac{2\left(-32Aba^3-24bCa^3+8Ab^3a\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(16Aa^4+5Ca^4-19Ab^2a^2+b^2Ca^2+9A\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^
3,x]
```

```
[Out] ((-2*(-32*a^3*A*b + 8*a*A*b^3 - 24*a^3*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b
), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d
*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16
*a^4*A - 19*a^2*A*b^2 + 9*A*b^4 + 5*a^4*C + a^2*b^2*C)*Cos[c + d*x]^2*(Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x
])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b^2 + 3*A*b^4
 - a^4*C - 5*a^2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a
*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
```

```
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ell
ipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b
*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)
))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((9*a^2*A*b^2 - 3*
A*b^4 + a^4*C + 5*a^2*b^2*C)*Sin[c + d*x])/(4*a^2*b*(a^2 - b^2)^2) + (A*b^2
*Sin[c + d*x] + a^2*C*Sin[c + d*x])/(2*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^
2) + (-7*a^2*A*b^2*Sin[c + d*x] + A*b^4*Sin[c + d*x] + a^4*C*Sin[c + d*x] -
 7*a^2*b^2*C*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 5.434, size = 1846, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b^2+C*a^2)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2
-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
```

```
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b
^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/
4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-4*a*C/b^2*(-1/a*b^2/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3,
 x)
```

3.1407 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=408

$$-\frac{\left(a^2b^2(5A+9C)-3a^4C+Ab^4\right)\sin(c+dx)}{4abd\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+\frac{\left(a^2b^2(3A-5C)+3\right.}{}$$

```
[Out] ((A*b^4 - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^2)^2*d) + ((a^2*b^2*(3*A -
 5*C) + 3*a^4*C + b^4*(3*A + 8*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) + ((A*b^6 - 3*a^4*b^2*(A -
 2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^3*(a + b)^3
*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2*Sqrt[Sec[c + d*x]]) - ((A*b^4 - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sin[c +
d*x])/(4*a*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.31802, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(a^2b^2(5A+9C)-3a^4C+Ab^4\right)\sin(c+dx)}{4abd\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+\frac{\left(a^2b^2(3A-5C)+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((A*b^4 - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^2)^2*d) + ((a^2*b^2*(3*A -
 5*C) + 3*a^4*C + b^4*(3*A + 8*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) + ((A*b^6 - 3*a^4*b^2*(A -
 2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^3*(a + b)^3
*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2*Sqrt[Sec[c + d*x]]) - ((A*b^4 - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sin[c +
d*x])/(4*a*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(Ab^4-3a^4C+a^2b^2(5A\right.}{4ab\left(a^2-b^2\right)^2 d(a+b\cos}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(Ab^4-3a^4C+a^2b^2(5A\right.}{4ab\left(a^2-b^2\right)^2 d(a+b\cos}$$

$$= \frac{\left(Ab^4-3a^4C+a^2b^2(5A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab^2\left(a^2-b^2\right)^2 d} -$$

$$= \frac{\left(Ab^4-3a^4C+a^2b^2(5A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab^2\left(a^2-b^2\right)^2 d} +$$

Mathematica [B] time = 7.00782, size = 821, normalized size = 2.01

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(3Ca^4-5Ab^2a^2-9b^2Ca^2-Ab^4\right)\sin(c+dx)}{4ab^2\left(a^2-b^2\right)^2} - \frac{C\sin(c+dx)a^3+Ab^2\sin(c+dx)a}{2b^2\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{-5C\sin(c+dx)a^4+3Ab^2\sin(c+dx)a^2+11b^2C\sin(c+dx)a^2+}{4b^2\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] -((-2*(-16*a^3*A*b - 8*a*A*b^3 - 8*a^3*b*C - 16*a*b^3*C)*Cos[c + d*x]^2*Ell
ipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[
1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + (2*(9*a^2*A*b^2 - 3*A*b^4 + a^4*C + 5*a^2*b^2*C)*Cos[c + d*x]^2*(Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Se
c[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*
x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-5*a^2*A*b^2 - A*b^4
+ 3*a^4*C - 9*a^2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*
a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
```

```
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*El
lipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a +
b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2
)))/(16*a*(a - b)^2*b*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((-5*a^2*A*b^2 -
A*b^4 + 3*a^4*C - 9*a^2*b^2*C)*Sin[c + d*x])/(4*a*b^2*(a^2 - b^2)^2) - (a*A
*b^2*Sin[c + d*x] + a^3*C*Sin[c + d*x])/(2*b^2*(-a^2 + b^2)*(a + b*Cos[c +
d*x])^2) + (3*a^2*A*b^2*Sin[c + d*x] + 3*A*b^4*Sin[c + d*x] - 5*a^4*C*Sin[c
 + d*x] + 11*a^2*b^2*C*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d
*x]))))/d
```

Maple [B] time = 6.118, size = 1934, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12/
b^2*C*a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b^2+C*a^2)/b^3*(-1/2/a*b^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
```

```
),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-
b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b
/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/b
^3*(A*b^2+3*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))
), x)
```

3.1408 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=405

$$-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{\left(a^2b^2(3A+11C)-5a^4C+3Ab^4\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{a\left(-a^2b^2(A+33C\right.}{}$$

```
[Out] -((b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - (a*(15*
a^4*C + b^4*(7*A + 24*C) - a^2*b^2*(A + 33*C))*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 +
15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b
^4*(a + b)^3*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((3*A*b^4 - 5*a^4*C + a^2*b^2*(3*A +
11*C))*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c
 + d*x]])
```

Rubi [A] time = 1.34049, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4221, 3048, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(a^2C+Ab^2\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{\left(a^2b^2(3A+11C)-5a^4C+3Ab^4\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{a\left(-a^2b^2(A+33C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```



```
[Out] -((b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - (a*(15*
a^4*C + b^4*(7*A + 24*C) - a^2*b^2*(A + 33*C))*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 +
15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b
^4*(a + b)^3*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((3*A*b^4 - 5*a^4*C + a^2*b^2*(3*A +
11*C))*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c
```

```
 + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A + C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^4 - 5a^4C + a^2b^2(3A\right.}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^4 - 5a^4C + a^2b^2(3A\right.}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= -\frac{\left(b^4(5A-8C) - 15a^4C + a^2b^2(A+29C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{4b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(b^4(5A-8C) - 15a^4C + a^2b^2(A+29C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{4b^3\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 7.06682, size = 824, normalized size = 2.03

$$\frac{-\frac{2\left(8bCa^3 - 24Ab^3a - 32b^3Ca\right)\Pi\left(-\frac{a}{b}; -\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(5Ca^4 + 5Ab^2a^2 - 7b^2Ca^2 + Ab^4 + 8b^4C\right)(}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] ((-2*(-24*a*A*b^3 + 8*a^3*b*C - 32*a*b^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b
), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d
*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(5*
a^2*A*b^2 + A*b^4 + 5*a^4*C - 7*a^2*b^2*C + 8*b^4*C)*Cos[c + d*x]^2*(Ellipt
icF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
 + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-(a^2*A*b^2) - 5*A*b^4 +
 15*a^4*C - 29*a^2*b^2*C + 8*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(
```

```
-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(
a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec
[c + d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-((-(a
^2*A*b^2) - 5*A*b^4 + 7*a^4*C - 13*a^2*b^2*C)*Sin[c + d*x])/(4*b^3*(a^2 - b
^2)^2) - (-(a^2*A*b^2*Sin[c + d*x]) - a^4*C*Sin[c + d*x])/(2*b^3*(-a^2 + b^
2)*(a + b*Cos[c + d*x])^2) + (a^3*A*b^2*Sin[c + d*x] - 7*a*A*b^4*Sin[c + d*
x] + 9*a^5*C*Sin[c + d*x] - 15*a^3*b^2*C*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^
2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 6.187, size = 1966, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-4/b^3*(A*b^2+6*C*a^2)/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*a^2*(A*b^2+C*a^2)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a
^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-
3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-4*a/b^4*(A*b^2+2*C*
a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)
), x)
```

3.1409 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=493

$$\frac{\left(a^2b^2(3A-61C)+35a^4C-b^4(21A-8C)\right)\sin(c+dx)}{12b^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{\left(a^2b^2(A+13C)-7a^4C+5Ab^4\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}-\frac{(a}{2bd\left(a^2-b^2\right.}$$

```
[Out] -(a*(a^2*b^2*(3*A - 65*C) - 3*b^4*(3*A - 8*C) + 35*a^4*C)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) + (
(a^4*b^2*(9*A - 223*C) - a^2*b^4*(15*A - 128*C) + 105*a^6*C + 8*b^6*(3*A +
C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^
5*(a^2 - b^2)^2*d) - (a*(15*A*b^6 + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A -
 21*C) + 35*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^5*(a + b)^3*d) - ((A*b^2 + a^2*C)*S
in[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
+ ((5*A*b^4 - 7*a^4*C + a^2*b^2*(A + 13*C))*Sin[c + d*x])/(4*b^2*(a^2 - b^2
)^2*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((a^2*b^2*(3*A - 61*C) - b
^4*(21*A - 8*C) + 35*a^4*C)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[
c + d*x]])
```

Rubi [A] time = 1.87154, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3048, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2b^2(3A-61C)+35a^4C-b^4(21A-8C)\right)\sin(c+dx)}{12b^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{\left(a^2b^2(A+13C)-7a^4C+5Ab^4\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}-\frac{(a}{2bd\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```


```
[Out] -(a*(a^2*b^2*(3*A - 65*C) - 3*b^4*(3*A - 8*C) + 35*a^4*C)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) + (
(a^4*b^2*(9*A - 223*C) - a^2*b^4*(15*A - 128*C) + 105*a^6*C + 8*b^6*(3*A +
C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^
5*(a^2 - b^2)^2*d) - (a*(15*A*b^6 + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A -
 21*C) + 35*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^5*(a + b)^3*d) - ((A*b^2 + a^2*C)*S
```

```
in[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
+ ((5*A*b^4 - 7*a^4*C + a^2*b^2*(A + 13*C))*Sin[c + d*x])/(4*b^2*(a^2 - b^2
)^2*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((a^2*b^2*(3*A - 61*C) - b
^4*(21*A - 8*C) + 35*a^4*C)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[
c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(a+b\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A + C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^4 - 7a^4C + a^2b^2(A\right.}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^4 - 7a^4C + a^2b^2(A\right.}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(}$$

$$= -\frac{\left(Ab^2 + a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^4 - 7a^4C + a^2b^2(A\right.}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(}$$

$$= -\frac{a\left(a^2b^2(3A-65C) - 3b^4(3A-8C) + 35a^4C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a\left(a^2b^2(3A-65C) - 3b^4(3A-8C) + 35a^4C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.22297, size = 863, normalized size = 1.75

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a\left(11Ca^4+3Ab^2a^2-17b^2Ca^2-9Ab^4\right)\sin(c+dx)}{4b^4\left(a^2-b^2\right)^2} - \frac{C\sin(c+dx)a^5+Ab^2\sin(c+dx)a^3}{2b^4\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{-13C\sin(c+dx)a^6-5Ab^2\sin(c+dx)a^4+19b^2C\sin(c+}{4b^4\left(b^2-a^2\right)^2(a+b\cos(c+dx}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] -((-2*(-24*a^2*A*b^3 - 48*A*b^5 + 56*a^4*b*C - 112*a^2*b^3*C - 16*b^5*C)*Co
s[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + (2*(3*a^3*A*b^2 + 15*a*A*b^4 + 35*a^5*C - 73*a^3*b^2
```

```
*C + 56*a*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]
+ EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])
*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)) + ((9*a^3*A*b^2 - 27*a*A*b^4 + 105*a^5*C - 195*a^3*b^2*C + 72*a*
b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2
 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*(a - b)^2*
b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((a*(3*a^2*A*b^2 - 9*A*b^4 + 11*a^4*
C - 17*a^2*b^2*C)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2) - (a^3*A*b^2*Sin[c +
d*x] + a^5*C*Sin[c + d*x])/(2*b^4*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (-
5*a^4*A*b^2*Sin[c + d*x] + 11*a^2*A*b^4*Sin[c + d*x] - 13*a^6*C*Sin[c + d*x
] + 19*a^4*b^2*C*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))
+ (C*Sin[2*(c + d*x)])/(3*b^3)))/d
```

Maple [B] time = 6.854, size = 2240, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^3*C*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2*C/b^4*(3*a+2*b)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))+2*(A*b^2+6*C*a^2+3*C*a*b+C*b^2)/b^5*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a/b^4*(3*A
*b^2+10*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2+C*a^2)/b^5*(-1/2
/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
```

2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2
-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/
a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*
b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2)))+2*a^2/b^5*(3*A*b^2+5*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*
b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2
-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)
), x)
```

3.1410 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=579

$$\frac{\left(a^2b^2(15A-101C)+63a^4C-b^4(45A-8C)\right)\sin(c+dx)}{20b^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}+\frac{a\left(-5a^2b^2(A-7C)-21a^4C+b^4(11A-8C)\right)\sin(c+dx)}{4b^4d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\cdots$$

```
[Out] -((a^2*b^4*(145*A - 192*C) - 3*a^4*b^2*(25*A - 187*C) - 315*a^6*C - 8*b^6*(
5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(20*b^5*(a^2 - b^2)^2*d) + (a*(a^2*b^4*(33*A - 64*C) - 3*a^4*b^2*(5*A - 4
3*C) - 63*a^6*C - 8*b^6*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*b^6*(a^2 - b^2)^2*d) + (a^2*(35*A*b^6 - a^2*b^4
*(38*A - 99*C) + 15*a^4*b^2*(A - 10*C) + 63*a^6*C)*Sqrt[Cos[c + d*x]]*Ellip
ticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^6*(
a + b)^3*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2*Sec[c + d*x]^(7/2)) + ((7*A*b^4 - a^2*b^2*(A - 15*C) - 9*a^4*C)
*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(5/
2)) + ((a^2*b^2*(15*A - 101*C) - b^4*(45*A - 8*C) + 63*a^4*C)*Sin[c + d*x])
/(20*b^3*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)) + (a*(b^4*(11*A - 8*C) - 5*a^2
*b^2*(A - 7*C) - 21*a^4*C)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[c
+ d*x]])
```

Rubi [A] time = 2.42712, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4221, 3048, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2b^2(15A-101C)+63a^4C-b^4(45A-8C)\right)\sin(c+dx)}{20b^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}+\frac{a\left(-5a^2b^2(A-7C)-21a^4C+b^4(11A-8C)\right)\sin(c+dx)}{4b^4d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] -((a^2*b^4*(145*A - 192*C) - 3*a^4*b^2*(25*A - 187*C) - 315*a^6*C - 8*b^6*(
5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(20*b^5*(a^2 - b^2)^2*d) + (a*(a^2*b^4*(33*A - 64*C) - 3*a^4*b^2*(5*A - 4
3*C) - 63*a^6*C - 8*b^6*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*b^6*(a^2 - b^2)^2*d) + (a^2*(35*A*b^6 - a^2*b^4
```

```
*(38*A - 99*C) + 15*a^4*b^2*(A - 10*C) + 63*a^6*C)*Sqrt[Cos[c + d*x]]*Ellip
ticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^6*(
a + b)^3*d) - ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2*Sec[c + d*x]^(7/2)) + ((7*A*b^4 - a^2*b^2*(A - 15*C) - 9*a^4*C)
*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(5/
2)) + ((a^2*b^2*(15*A - 101*C) - b^4*(45*A - 8*C) + 63*a^4*C)*Sin[c + d*x])
/(20*b^3*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)) + (a*(b^4*(11*A - 8*C) - 5*a^2
*b^2*(A - 7*C) - 21*a^4*C)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[c
+ d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4-a^2b^2(A-15C)\right.}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4-a^2b^2(A-15C)\right.}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4-a^2b^2(A-15C)\right.}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4-a^2b^2(A-15C)\right.}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(}$$

$$= -\frac{\left(a^2b^4(145A-192C)-3a^4b^2(25A-187C)-315a^6C-8b^6(5A+3C)\right)\sqrt{\cos}}{20b^5\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(a^2b^4(145A-192C)-3a^4b^2(25A-187C)-315a^6C-8b^6(5A+3C)\right)\sqrt{\cos}}{20b^5\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.42168, size = 931, normalized size = 1.61

$$-\frac{\frac{2\left(168bCa^5+40Ab^3a^3-256b^3Ca^3-160Ab^5a-32b^5Ca\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{2\left(105Ca^6+2\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)
),x]
```

```
[Out] ((-2*(40*a^3*A*b^3 - 160*a*A*b^5 + 168*a^5*b*C - 256*a^3*b^3*C - 32*a*b^5*C
)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + (2*(25*a^4*A*b^2 - 35*a^2*A*b^4 + 40*A*b^6 + 105
*a^6*C - 211*a^4*b^2*C + 112*a^2*b^4*C + 24*b^6*C)*Cos[c + d*x]^2*(Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(
a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((75*a^4*A*b^2 - 145*a^2*A*b
^4 + 40*A*b^6 + 315*a^6*C - 561*a^4*b^2*C + 192*a^2*b^4*C + 24*b^6*C)*Cos[2
*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ell
ipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*Ell
ipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d
*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(80*(a - b)^2*b^4*(a + b)^
2*d) + (Sqrt[Sec[c + d*x]]*(-((35*a^4*A*b^2 - 65*a^2*A*b^4 + 75*a^6*C - 107
*a^4*b^2*C + 4*a^2*b^4*C - 2*b^6*C)*Sin[c + d*x])/(20*b^5*(a^2 - b^2)^2) -
(-(a^4*A*b^2*Sin[c + d*x]) - a^6*C*Sin[c + d*x])/(2*b^5*(-a^2 + b^2)*(a + b
*Cos[c + d*x])^2) + (9*a^5*A*b^2*Sin[c + d*x] - 15*a^3*A*b^4*Sin[c + d*x] +
 17*a^7*C*Sin[c + d*x] - 23*a^5*b^2*C*Sin[c + d*x])/(4*b^5*(-a^2 + b^2)^2*(
a + b*Cos[c + d*x])) - (a*C*Sin[2*(c + d*x)])/b^4 + (C*Sin[3*(c + d*x)])/(1
0*b^3)))/d
```

Maple [B] time = 7.436, size = 2466, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^3*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
```

^(1/2)-4/b^4*C*(a+b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/b^5*(A*b^2+
6*C*a^2+6*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(3*A
*a*b^2+A*b^3+10*C*a^3+6*C*a^2*b+3*C*a*b^2+C*b^3)/b^6*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*a^2/b^5*(2*A*b^
2+5*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(A*b^2+C*a^2)/b^6*(-1/2/a*b
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2
c)^2)^(1/2)(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)
b^3(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)-4*a^3/b^6*(2*A*b^2+3*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-
b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/

```
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2
)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c) + a)^3 \sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)
), x)
```

3.1411 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=544

$$-\frac{2\left(6Ab^2-7a^2(7A+9C)\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d}+\frac{2b\left(a^2(13A+21C)+8Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^5*d*Sqrt[
Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(12*a*A*b^2 + 16*A*b^3 + 6*a^2*b*(6
*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(315*a^4*d*Sqrt[Sec[c + d*x]]) + (2*b*(8*A*b^2 + a^2*(13*A + 21*
C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^3*d) -
 (2*(6*A*b^2 - 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(315*a^2*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(63*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^
(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.95254, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3048, 3055, 2998, 2816, 2994}

$$-\frac{2\left(6Ab^2-7a^2(7A+9C)\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d}+\frac{2b\left(a^2(13A+21C)+8Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^5*d*Sqrt[
Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(12*a*A*b^2 + 16*A*b^3 + 6*a^2*b*(6
```

```
*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(315*a^4*d*Sqrt[Sec[c + d*x]]) + (2*b*(8*A*b^2 + a^2*(13*A + 21*
C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^3*d) -
 (2*(6*A*b^2 - 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(315*a^2*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(63*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^
(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+\right.}{\cos^{\frac{11}{2}}(c+}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{}\right.$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63ad} + \frac{2A\sqrt{}}{}$$

$$= -\frac{2\left(6Ab^2-7a^2(7A+9C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+}{315a^2d}$$

$$= \frac{2b\left(8Ab^2+a^2(13A+21C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{315a^3d}$$

$$= \frac{2b\left(8Ab^2+a^2(13A+21C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{315a^3d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(16Ab^4+6a^2b^2(4A+7C)-21a^4(7A+\right.}{}$$

Mathematica [B] time = 25.4292, size = 3619, normalized size = 6.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A - 24*a^2*A*b^2
- 16*A*b^4 + 189*a^4*C - 42*a^2*b^2*C)*Sin[c + d*x])/(315*a^4) + (2*Sec[c +
 d*x]^2*(49*a^2*A*Sin[c + d*x] - 6*A*b^2*Sin[c + d*x] + 63*a^2*C*Sin[c + d*
x]))/(315*a^2) + (2*Sec[c + d*x]*(13*a^2*A*b*Sin[c + d*x] + 8*A*b^3*Sin[c +
 d*x] + 21*a^2*b*C*Sin[c + d*x]))/(315*a^3) + (2*A*b*Sec[c + d*x]^2*Tan[c +
 d*x])/(63*a) + (2*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a*A)/(15*
Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*A*b^2)/(105*a*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^4)/(315*a^3*Sqrt[a + b*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]) - (3*a*C)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
```

+ d*x]]) + (2*b^2*C)/(15*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*A*b*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (4*A*b^3*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^4*Sqrt[a + b*Cos[c + d*x]]) - (2*b*C*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*C*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) - (7*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) + (8*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^4*Sqrt[a + b*Cos[c + d*x]]) - (3*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 6*a^2*b*(6*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^4*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 6*a^2*b*(6*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^4*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 6*a^2*b*(6*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^4*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 6*a^2*b*(6*A + 7*C) + 21*a^3

```
*(7*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x
])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])] - ((a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A + 7*C) + 21*a
^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2
)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(1
2*a*A*b^2 - 16*A*b^3 - 6*a^2*b*(6*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c +
d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))] - b*(16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 2
1*a^4*(7*A + 9*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*
x)/2] - (16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (16*A*b^4 + 6*
a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(12*a*A*b^2 - 16*A*b^3
- 6*a^2*b*(6*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x
)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2
)/(a + b)]) - ((a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)
/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*a^4*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-16*A*b^4 - 6*a^2*b^2*(4*A
 + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
 + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 6*a^2*b*
(6*A + 7*C) + 21*a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 6*a^2*b^2*(4*A + 7*C) - 21*a^
4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*
x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^4*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.503, size = 4134, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c))^(1/2),x)

[Out] 2/315/d/a^4*(35*A*a^5+105*C*cos(d*x+c)^5*a^4*b+84*C*cos(d*x+c)^3*a^4*b-189*C*cos(d*x+c)^6*a^4*b-21*C*cos(d*x+c)^6*a^3*b^2+42*C*cos(d*x+c)^6*a^2*b^3+42*C*cos(d*x+c)^5*a^3*b^2-42*C*cos(d*x+c)^5*a^2*b^3-21*C*cos(d*x+c)^4*a^3*b^2+22*A*cos(d*x+c)^3*a^4*b+2*A*cos(d*x+c)^3*a^2*b^3-A*cos(d*x+c)^2*a^3*b^2+40*A*cos(d*x+c)*a^4*b-147*A*cos(d*x+c)^6*a^4*b-13*A*cos(d*x+c)^6*a^3*b^2+24*A*cos(d*x+c)^6*a^2*b^3-8*A*cos(d*x+c)^6*a*b^4+85*A*cos(d*x+c)^5*a^4*b+24*A*cos(d*x+c)^5*a^3*b^2-26*A*cos(d*x+c)^5*a^2*b^3+16*A*cos(d*x+c)^5*a*b^4+16*A*cos(d*x+c)^6*b^5+189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-16*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-111*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-10*A*cos(d*x+c)^4*a^3*b^2-8*A*cos(d*x+c)^4*a*b^4-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-16*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-147*A*cos(d*x+c)^5*a^5-16*A*cos(d*x+c)^5*b^5+98*A*cos(d*x+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-189*C*cos(d*x+c)^5*a^5+126*C*cos(d*x+c)^4*a^5+63*C*cos(d*x+c)^2*a^5-16*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+42*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1

/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2+189*C*cos(d*x+c)^5*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
a^4*b-42*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2-42*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3-111*A*cos(d*x
+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4*b+24*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*A*cos(d*x+c)^4*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+16*A*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^4+147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-24*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-
24*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^3-16*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*C*cos(d*x+c)^4*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
a^4*b+42*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+189*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-42*C*cos(d*x+c
)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3*b^2-42*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+24*A*cos(d*x+c)^5*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*A*c
os(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^3+16*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos

```
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+147*A*cos(d*x+c)^5*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-
24*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b^2-24*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3)*cos(d*x+c)*(1/cos(
d*x+c))^(11/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c))^(1/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/
2), x)
```

3.1412 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=455

$$-\frac{2\left(4Ab^2-5a^2(5A+7C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\left(5a^2(5A+7C)+6aAb+8Ab^2\right.}{}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^2 + a^2*(19*A + 35*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a -
b)*Sqrt[a + b]*(6*a*A*b + 8*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*
b^2 - 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*a^2*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(35*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(7*d)
```

Rubi [A] time = 1.42003, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3048, 3055, 2998, 2816, 2994}

$$-\frac{2\left(4Ab^2-5a^2(5A+7C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\left(5a^2(5A+7C)+6aAb+8Ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^2 + a^2*(19*A + 35*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a -
b)*Sqrt[a + b]*(6*a*A*b + 8*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
```

```
[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*
b^2 - 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*a^2*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(35*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+\right.}{\cos^{\frac{9}{2}}(c+d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\right.$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35ad} + \frac{2A\sqrt{}}{}$$

$$= -\frac{2\left(4Ab^2-5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{105a^2d}$$

$$= -\frac{2\left(4Ab^2-5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{105a^2d}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(8Ab^2+a^2(19A+35C)\right)\sqrt{\cos(c+dx)}\,c}{10}$$

Mathematica [A] time = 19.2202, size = 478, normalized size = 1.05

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2b\left(19a^2A+35a^2C+8Ab^2\right)\sin(c+dx)}{105a^3}+\frac{2\sec(c+dx)\left(25a^2A\sin(c+dx)+35a^2C\sin(c+dx)-4Ab^2\sin(c+dx)\right)}{105a^2}+\frac{2A}{}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(a + b)*(8*A*b^2 + a^2*(19*A
 + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + 2*a*(a + b)*(-6*a*A*b + 8*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A*b^2 +
 a^2*(19*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(105*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^
2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(19*a^2*A + 8*A*b^
```

```
2 + 35*a^2*C)*Sin[c + d*x])/(105*a^3) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d
*x] - 4*A*b^2*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a^2) + (2*A*b*Sec
[c + d*x]*Tan[c + d*x])/(35*a) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.339, size = 2775, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/a^3*(15*A*a^4+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-2*
A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b+19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b-2*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
```

```
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)
^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^4-25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^4-25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b+35*C*cos(d*x+c)^4*a^2*b^2+70*C*cos(d*x+c)^3*a^3*b-35*C*cos(d*x+c)
^5*a^3*b-35*C*cos(d*x+c)^5*a^2*b^2-35*C*cos(d*x+c)^4*a^3*b+4*A*cos(d*x+c)^3
*a*b^3-A*cos(d*x+c)^2*a^2*b^2+18*A*cos(d*x+c)*a^3*b-25*A*cos(d*x+c)^5*a^3*b
-19*A*cos(d*x+c)^5*a^2*b^2+4*A*cos(d*x+c)^5*a*b^3-19*A*cos(d*x+c)^4*a^3*b+2
0*A*cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^4*a*b^3+26*A*cos(d*x+c)^3*a^3*b-25*
A*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+35*C*cos(d*x
+c)^2*a^4-8*A*cos(d*x+c)^5*b^4+8*A*cos(d*x+c)^4*b^4+19*A*cos(d*x+c)^4*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, alg
```

```
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2
), x)
```

3.1413 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=385

$$-\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)
*Sqrt[a + b]*(9*a*A + 2*A*b + 15*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*A*b*Sqrt[a + b*Cos[c +
d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*Sqrt[a + b*Cos[c + d
*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.04928, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3048, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)
*Sqrt[a + b]*(9*a*A + 2*A*b + 15*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*A*b*Sqrt[a + b*Cos[c +
d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*Sqrt[a + b*Cos[c + d
```

```
*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\mathrm{co}\right.$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2A\sqrt{a}}{}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2A\sqrt{a}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\,\mathrm{csc}}{15a}$$

Mathematica [A] time = 17.8056, size = 429, normalized size = 1.11

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(9a^2A+15a^2C-2Ab^2)\sin(c+dx)}{15a^2}+\frac{2Ab\tan(c+dx)}{15a}+\frac{2}{5}A\tan(c+dx)\sec(c+dx)\right)}{d}+\frac{2\sqrt{2}\sqrt{\frac{\cos}{(\cos}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((a + b)*((-2*A*b^2
 + 3*a^2*(3*A + 5*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + a*(2*A*b - 3*a*(3*A + 5*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]) + (2*A*b^2 - 3*a^2*(3*A + 5*C
))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/
(15*a^2*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c +
d*x)/2]^2)^(3/2)) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2
*A - 2*A*b^2 + 15*a^2*C)*Sin[c + d*x])/(15*a^2) + (2*A*b*Tan[c + d*x])/(15*
a) + (2*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.24, size = 2442, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a^2*(-3*A*a^3+A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos
(d*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b+A*cos(d*x+c)^4*a*b^2-2*A*cos(d*x+c)^
3*a*b^2-4*A*cos(d*x+c)*a^2*b-5*A*cos(d*x+c)^3*a^2*b-9*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b-2*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15*C*cos(d
*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3+2*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
```

d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15
*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*sin(d*x
+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2+15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*A*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a

```
+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)
/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2
), x)
```

3.1414 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=454

$$\frac{2Ab(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A\cdots}{\cdots}$$

```
[Out] (2*A*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A*b - a*(A + 3*C))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]
]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
 ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.937137, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3048, 3053, 2809, 2998, 2816, 2994}

$$\frac{2Ab(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*A*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A*b - a*(A + 3*C))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]
```

```
]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
 ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+C}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{co\right.$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{co\right.$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,co}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{d\sqrt{\sec(c+d}}$$

$$= \frac{2A(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{3a^2d\sqrt{\sec(c+}}$$

Mathematica [A] time = 12.3902, size = 400, normalized size = 0.88

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2Ab\sin(c+dx)}{3a}+\frac{2}{3}A\tan(c+dx)\right)}{d} - \frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-2a(a(A+3C)+b(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*A*b*(a + b)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(b*(A - 3*C) +
 a*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] + b*(12*a*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] + A*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])))/(3*a*d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*b*Sin[c + d*x])/(3*a) + (2*A*Tan[c
 + d*x])/3))/d
```

Maple [B] time = 0.25, size = 1489, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*b^2+6*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+3*C*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*C*co
s(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*C*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*a*b+A*cos(d*x+c)^3*a*b+A*cos(d*x+c)^3*b^2+A*cos(d*x+c)^2*a^2+A*cos(d*
x+c)^2*a*b-A*cos(d*x+c)^2*b^2-2*A*cos(d*x+c)*a*b-A*a^2)*cos(d*x+c)*(1/cos(d
*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2
), x)
```

3.1415 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=499

$$-\frac{(2A-C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}(2aA-aC-2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{ad\sqrt{\sec(c+dx}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a*A - 2*A*b - a*C)*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
 - (a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/d - ((2*A - C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.20425, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3048, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2A-C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}(2aA-aC-2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{ad\sqrt{\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a*A - 2*A*b - a*C)*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
```

```
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
 - (a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/d - ((2*A - C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+\right.}{\cos^{\frac{3}{2}}(c+d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{\text{co}\right.$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \frac{(2A-}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \frac{(2A-}{}$$

$$= -\frac{a\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{bd\sqrt{\sec(c+}}$$

$$= \frac{(a-b)\sqrt{a+b}(2A-C)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\right.}{ad\sqrt{\sec(c}}$$

Mathematica [A] time = 18.1954, size = 703, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2(a(A-C)+Ab)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[(1
 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a*A*Tan[(c + d*x)/2] - 2*A*b*Tan[(c + d*x)
/2] + a*C*Tan[(c + d*x)/2] + b*C*Tan[(c + d*x)/2] + 4*A*b*Tan[(c + d*x)/2]^
3 - 2*b*C*Tan[(c + d*x)/2]^3 + 2*a*A*Tan[(c + d*x)/2]^5 - 2*A*b*Tan[(c + d*
x)/2]^5 - a*C*Tan[(c + d*x)/2]^5 + b*C*Tan[(c + d*x)/2]^5 - 2*a*C*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
```

```
2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(2*A - C)*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)] + 2*(A*b + a*(A - C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(1 + Tan[(c + d*x)/
2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 +
 Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.202, size = 1596, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*C*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
```

```
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a-C*co
s(d*x+c)^3*b-2*A*cos(d*x+c)^2*b-C*cos(d*x+c)^2*a+C*cos(d*x+c)^2*b-2*A*cos(d
*x+c)*a+2*A*cos(d*x+c)*b+C*cos(d*x+c)*a+2*a*A)*cos(d*x+c)*(1/cos(d*x+c))^(3
/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

3.1416 $\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}$

Optimal. Leaf size=515

$$\frac{\sqrt{a+b}\left(a^2C-4b^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*A*b + (a + 2*b)*C)*Sqrt[Cos
[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) +
(Sqrt[a + b]*(a^2*C - 4*b^2*(2*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (a*C*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*d)
```

Rubi [A] time = 1.24476, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3050, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(a^2C-4b^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```



```
[Out] -((a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*A*b + (a + 2*b)*C)*Sqrt[Cos
[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) +
(Sqrt[a + b]*(a^2*C - 4*b^2*(2*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
```

```
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (a*C*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\right.}{\sqrt{\cos(c+dx}}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(\right.$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{aC\sqrt{a+b\cos(c+dx)}}{4}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{aC\sqrt{a+b\cos(c+dx)}}{4}$$

$$= \frac{\sqrt{a+b}\left(a^2C-4b^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\right.}{4b^2d\sqrt{}}$$

$$= -\frac{(a-b)\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\sqrt{}}\right.\right.}{4bd\sqrt{\sec(c+}}$$

Mathematica [C] time = 18.7787, size = 1391, normalized size = 2.7

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (-
(a^2*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]) - a*b*Sqrt[(a - b)/(a + b)]*
C*Tan[(c + d*x)/2] + 2*a*b*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^3 + a^2
*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*C*T
an[(c + d*x)/2]^5 + (16*I)*A*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt
[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] - (2*I)*a^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b
)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^2
*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x
)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
```

```
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (16*I)*A*b^2*EllipticPi[
(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a +
b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*C*Ellipti
cPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((
a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^2*C*Ell
ipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
 -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*(a - b)
*C*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(
a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*(4*
A*b + (a + 2*b)*C)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/
2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(4
*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c
 + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.198, size = 1818, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-2*C*cos(d*x+c)*a*b+3
*C*cos(d*x+c)^3*a*b-C*cos(d*x+c)^2*a*b-2*b^2*C*cos(d*x+c)^2+16*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-
8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-4*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
```

```
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*C*cos(d*x+c)*sin(
d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^
2-2*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+C*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+16*A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+C*co
s(d*x+c)^2*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)^4*b^2
-C*cos(d*x+c)*a^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
), x)
```

3.1417 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=613

$$-\frac{\left(3a^2C-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24b^2d}-\frac{\sqrt{a+b}\left(3a^2C-2abC-8b^2(3A+2C)\right)\sqrt{\cos(c+}}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*C - 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b
]*(3*a^2*C - 2*a*b*C - 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a + b]*(8*A*b^
2 + (a^2 + 4*b^2)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x]]) - (a*C*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]) - ((3*a^2*C - 8*b^2*(3*A + 2*C))*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b^2*d)
```

Rubi [A] time = 1.72548, antiderivative size = 613, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(3a^2C-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24b^2d}-\frac{\sqrt{a+b}\left(3a^2C-2abC-8b^2(3A+2C)\right)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*C - 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b
]*(3*a^2*C - 2*a*b*C - 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
```

```
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a + b]*(8*A*b^
2 + (a^2 + 4*b^2)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x]]) - (a*C*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]) - ((3*a^2*C - 8*b^2*(3*A + 2*C))*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(A+\right.$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+}{3bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+}{3bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+}{3bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{a\sqrt{a+b}\left(8Ab^2+\left(a^2+4b^2\right)C\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin\right.}{8b^3d\sqrt{\sec(c+}}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3a^2C-8b^2(3A+2C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{24ab^2d\sqrt{\sec(c+}}$$

Mathematica [B] time = 19.4969, size = 1317, normalized size = 2.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((C*Sin[c + d*x])/12 + (a*C*Si
n[2*(c + d*x)])/(24*b) + (C*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c +
d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(1 + Tan[(c + d*x)/2]^2)]*(-24*a*A*b^2*Tan[(c + d*x)/2] - 24*A*b^3*Tan[(c
+ d*x)/2] + 3*a^3*C*Tan[(c + d*x)/2] + 3*a^2*b*C*Tan[(c + d*x)/2] - 16*a*b^
2*C*Tan[(c + d*x)/2] - 16*b^3*C*Tan[(c + d*x)/2] + 48*A*b^3*Tan[(c + d*x)/2
]^3 - 6*a^2*b*C*Tan[(c + d*x)/2]^3 + 32*b^3*C*Tan[(c + d*x)/2]^3 + 24*a*A*b
^2*Tan[(c + d*x)/2]^5 - 24*A*b^3*Tan[(c + d*x)/2]^5 - 3*a^3*C*Tan[(c + d*x)
/2]^5 + 3*a^2*b*C*Tan[(c + d*x)/2]^5 + 16*a*b^2*C*Tan[(c + d*x)/2]^5 - 16*b
^3*C*Tan[(c + d*x)/2]^5 + 48*a*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*C*EllipticPi[-1, -ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b^2*C*E
llipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 48*a*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*C*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 24*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-24*A*b^2 + 3*a^2*C
 - 16*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*b*(-24*A*b + (a - 14*b)*C)*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)]))/(24*b^2*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Ta
n[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.262, size = 2528, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)

[Out] -1/24/d/b^2*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b^2+10*C*cos(d*x+c)^4*a*b^2-C*cos(d*x+c)^3*a^2*b+3*C*cos(d*x+c)^2*a^2*b+6*C*cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+8*C*cos(d*x+c)^5*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos

```
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-3*C*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*a^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*b^3+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*
b^3-3*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3+3*C*cos(d*x+c)*a^3+24*A*cos(
d*x+c)^3*b^3-24*A*cos(d*x+c)^2*b^3)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*co
s(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + C\cos^2(c + dx))\sqrt{a + b\cos(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx + c)^2 + A)\sqrt{b\cos(dx + c) + a}}{\sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)
), x)
```

3.1418 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=698

$$\frac{a\left(15a^2C+48Ab^2+28b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192b^3d}+\frac{\left(5a^2C+4b^2(4A+3C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b^3*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a
+ b]*(15*a^3*C - 10*a^2*b*C + 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 7*C))*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b^3*d*Sqrt[Sec[c +
 d*x]]) + (Sqrt[a + b]*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^4*
d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*b*d*
Sec[c + d*x]^(3/2)) + ((5*a^2*C + 4*b^2*(4*A + 3*C))*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(32*b^2*d*Sqrt[Sec[c + d*x]]) - (5*a*C*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (a*(48*A*b^2 + 15*a^2
*C + 28*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
92*b^3*d)
```

Rubi [A] time = 2.17408, antiderivative size = 698, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{a\left(15a^2C+48Ab^2+28b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192b^3d}+\frac{\left(5a^2C+4b^2(4A+3C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
```

```
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b^3*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a
+ b]*(15*a^3*C - 10*a^2*b*C + 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 7*C))*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b^3*d*Sqrt[Sec[c +
 d*x]]) + (Sqrt[a + b]*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^4*
d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*b*d*
Sec[c + d*x]^(3/2)) + ((5*a^2*C + 4*b^2*(4*A + 3*C))*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(32*b^2*d*Sqrt[Sec[c + d*x]]) - (5*a*C*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (a*(48*A*b^2 + 15*a^2
*C + 28*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
92*b^3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+C$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{}}{}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} - \frac{5aC(a+b\cos(c+dx))^{3/2}\sin(c+}{24b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{a+b\,c}}{32b^2d\sqrt{\sec(c+}}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{a+b\,c}}{32b^2d\sqrt{\sec(c+}}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{a+b\,c}}{32b^2d\sqrt{\sec(c+}}$$

$$= \frac{\sqrt{a+b}\left(5a^4C+8a^2b^2(2A+C)-16b^4(4A+3C)\right)\sqrt{\cos(c+dx)}\csc(c+}{64b^4d\sqrt{s}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(48Ab^2+15a^2C+28b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\big(}{192b^3d\sqrt{\sec(c+}}$$

Mathematica [B] time = 26.2278, size = 4845, normalized size = 6.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*C*Sin[c + d*x])/(96*b) + (
(48*A*b^2 - 5*a^2*C + 48*b^2*C)*Sin[2*(c + d*x)])/(192*b^2) + (a*C*Sin[3*(c
 + d*x)])/(96*b) + (C*Sin[4*(c + d*x)])/32))/d + (((A*b)/(2*Sqrt[a + b*Cos[
```

c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*C)/(96*b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (3*b*C)/(8*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (3*a*A*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]]) + (25*a*C*Sqrt[Sec[c + d*x]])/(96*Sqrt[a + b*Cos[c + d*x]]) + (5*a^3*C*Sqrt[Sec[c + d*x]])/(384*b^2*Sqrt[a + b*Cos[c + d*x]]) + (a*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]]) + (7*a*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(96*Sqrt[a + b*Cos[c + d*x]]) + (5*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(128*b^2*Sqrt[a + b*Cos[c + d*x]]))*(-2*a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 2*a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + 3*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(192*b^4*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)*(-(Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3*(-2*a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 2*a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + 3*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(96*b^4*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)^2) + (Sin[c + d*x]*(-2*a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 2*a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + 3*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +

d*x)/2]^4*Tan[(c + d*x)/2]))/(384*b^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x]*(-2*a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 2*a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + 3*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(384*b^4*Sqrt[a + b*Cos[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)) + (-(a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - (a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 2*a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 2*a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + a*b^2*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - 2*a*b*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 + (9*(5*a^4*C + 8*a^2*b^2*(2*A

```
 + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x])
 + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + (3*(5*a^4*C + 8*a
^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-(
(b*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)]) - (a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A
 + 3*C) + 4*a*b^2*(12*A + 11*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4)/(Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])
 - (a*b*(a + b)*(48*A*b^2 + 15*a^2*C + 28*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c
+ d*x)/2]^4*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(
c + d*x)/2]^2] + 3*(5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)]*Sec[c + d*x]*(((a - b)*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (b*Sec
[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])) + 3*(5*a^4*C + 8*a^2*b^2*(2*
A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x])/(192*b^4*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)))
```

Maple [B] time = 0.36, size = 3615, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d/b^3*(96*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-96*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+48*A*si
```

n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a^2*b^2+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-10*C*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*C*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b^2+72*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3*b+28*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*C*cos(d*x+c)^6*b^4+28*C*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^3+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+96*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-96*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*A
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b^2+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3-10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-48*C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
a^2*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3*b+28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d

```
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+28*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+56*C*cos(d*x+c)^5*a*b^3-2*
C*cos(d*x+c)^4*a^2*b^2+5*C*cos(d*x+c)^3*a^3*b-192*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4+384*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4-1
44*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^4-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^4+288*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+15*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+24*C*cos(d*x+c)^4*b^4-
96*A*cos(d*x+c)^2*b^4-72*C*cos(d*x+c)^2*b^4-15*C*cos(d*x+c)*a^4+144*A*cos(d
*x+c)^3*a*b^3+48*A*cos(d*x+c)^2*a^2*b^2+15*C*cos(d*x+c)^2*a^4+96*A*cos(d*x+
c)^4*b^4-28*C*cos(d*x+c)^2*a*b^3+10*C*cos(d*x+c)*a^3*b-28*C*cos(d*x+c)*a^2*
b^2-72*C*cos(d*x+c)*a*b^3+44*C*cos(d*x+c)^3*a*b^3-48*A*cos(d*x+c)^2*a*b^3-4
8*A*cos(d*x+c)*a^2*b^2-96*A*cos(d*x+c)*a*b^3-192*A*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+384*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*b^4-144*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*C*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*b^4-15*C*cos(d*x+c)^2*a^3*b+30*C*cos(d*x+c)^2*a^2*b^2)*cos
(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

3.1419 $\int (a+b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=542

$$\frac{2\left(7a^2(7A+9C)+3Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad} - \frac{4b\left(2Ab^2-a^2(44A+63C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d*Sqrt[
Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(6*a*A*b^2 + 8*A*b^3 - 21*a^3*(7*A
+ 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (4*b*(2*A*b^2 - a^2*(44*A + 63*
C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^2*d) +
 (2*(3*A*b^2 + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(315*a*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(21*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9
/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.95096, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3048, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(7a^2(7A+9C)+3Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad} - \frac{4b\left(2Ab^2-a^2(44A+63C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x
]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d*Sqrt[
```

```
Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(6*a*A*b^2 + 8*A*b^3 - 21*a^3*(7*A
+ 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (4*b*(2*A*b^2 - a^2*(44*A + 63*
C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^2*d) +
 (2*(3*A*b^2 + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(315*a*d) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(21*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9
/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}\,(}{\cos^{\frac{11}{2}}(c}$$

$$= \frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}$$

$$= \frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{2A}{}$$

$$= \frac{2\left(3Ab^2+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+}{315ad}$$

$$= -\frac{4b\left(2Ab^2-a^2(44A+63C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}}{315a^2d}$$

$$= -\frac{4b\left(2Ab^2-a^2(44A+63C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}}{315a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^4+21a^4(7A+9C)+3a^2b^2(11A\right.}{}$$

Mathematica [B] time = 25.7529, size = 3622, normalized size = 6.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
1/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 33*a^2*A*b^2
+ 8*A*b^4 + 189*a^4*C + 63*a^2*b^2*C)*Sin[c + d*x])/(315*a^3) + (2*Sec[c +
d*x]^2*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x
]))/(315*a) + (4*Sec[c + d*x]*(44*a^2*A*b*Sin[c + d*x] - 2*A*b^3*Sin[c + d*
x] + 63*a^2*b*C*Sin[c + d*x]))/(315*a^2) + (20*A*b*Sec[c + d*x]^2*Tan[c + d
*x])/63 + (2*a*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^2*A)/(15*Sq
rt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (11*A*b^2)/(105*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^4)/(315*a^2*Sqrt[a + b*Cos[c + d*x]
```

]*Sqrt[Sec[c + d*x]]) - (3*a^2*C)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*C)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*A*b*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Sqrt[Sec[c + d*x]])/(315*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) + (a*b*C*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (b^3*C*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (7*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (11*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) - (3*a*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x

```
])] + (a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 21*a^3*(7*A + 9*C) + a^2*(39*A*b +
 63*b*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(
1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])] - ((a + b)*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11
*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(-6*a*
A*b^2 + 8*A*b^3 + 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x
])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))] + b*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^
2*(11*A + 21*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)
/2] + (8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (8*A*b^4 + 21*a
^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 +
 21*a^3*(7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*
x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^
2)/(a + b)]) - ((a + b)*(8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 2
1*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x
)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*a^3*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(8*A*b^4 + 21*a^4*(7*A + 9
*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 21*a^3*(
7*A + 9*C) + a^2*(39*A*b + 63*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 21*a^4*(7*A + 9*C) + 3*a^2*b
^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
 d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sq
rt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.449, size = 4118, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)

[Out] -2/315/d/a^3*(-35*A*a^5-189*C*cos(d*x+c)^3*a^4*b+189*C*cos(d*x+c)^6*a^4*b+1
26*C*cos(d*x+c)^6*a^3*b^2+63*C*cos(d*x+c)^6*a^2*b^3+63*C*cos(d*x+c)^5*a^3*b
^2-63*C*cos(d*x+c)^5*a^2*b^3-189*C*cos(d*x+c)^4*a^3*b^2-52*A*cos(d*x+c)^3*a
^4*b+A*cos(d*x+c)^3*a^2*b^3-53*A*cos(d*x+c)^2*a^3*b^2-85*A*cos(d*x+c)*a^4*b
+147*A*cos(d*x+c)^6*a^4*b+88*A*cos(d*x+c)^6*a^3*b^2+33*A*cos(d*x+c)^6*a^2*b
^3-4*A*cos(d*x+c)^6*a*b^4-10*A*cos(d*x+c)^5*a^4*b+33*A*cos(d*x+c)^5*a^3*b^2
-34*A*cos(d*x+c)^5*a^2*b^3+8*A*cos(d*x+c)^5*a*b^4+8*A*cos(d*x+c)^6*b^5-189*
C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a^5+147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*
cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^5+189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5-189*C*cos(d*x+c)^4*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+186*A*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4*b-68*A*cos(d*x+c)^4*a^3*b^2-4*A*cos(d*x+c)^4*a*b^4+147*A
*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^5-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-8*A*cos(d*x+c)^5*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+189*C*c
os(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a^5+147*A*cos(d*x+c)^5*a^5-8*A*cos(d*x+c)^5*b^5-98*A*cos(d*x+
c)^4*a^5-14*A*cos(d*x+c)^2*a^5+189*C*cos(d*x+c)^5*a^5-126*C*cos(d*x+c)^4*a^
5-63*C*cos(d*x+c)^2*a^5-8*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+252*C*cos(d*x+c)^5*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b
+63*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(

1+cos(d*x+c)))^(1/2)*a^3*b^2-189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b-63*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2-63*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3+186*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+33*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+252*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-189*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-63*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+33*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/

```
(a+b))^(1/2))*a*b^4-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*A*cos(d*x+c)^5*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-
33*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c
))^(11/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(1
1/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(1
1/2), x)
```

3.1420 $\int (a+b\cos(c+dx))^{3/2} \left(A + C\cos^2(c + dx)\right) \sec^{\frac{9}{2}}(c+dx)\, dx$

Optimal. Leaf size=458

$$\frac{2\left(5a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\left(25a^2A+35a^2C-57aAb-105a\right.}{}$$

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(3*A*b^2 - a^2*(41*A + 70*C))*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(a -
 b)*Sqrt[a + b]*(25*a^2*A - 57*a*A*b - 6*A*b^2 + 35*a^2*C - 105*a*b*C)*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt[Sec[c + d
*x]]) + (2*(3*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (6*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.42318, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3048, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\left(25a^2A+35a^2C-57aAb-105a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(3*A*b^2 - a^2*(41*A + 70*C))*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(a -
 b)*Sqrt[a + b]*(25*a^2*A - 57*a*A*b - 6*A*b^2 + 35*a^2*C - 105*a*b*C)*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
```

```
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt[Sec[c + d
*x]]) + (2*(3*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (6*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}\left(\right.}{\cos^{\frac{9}{2}}(c-}$$

$$= \frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(\right.$$

$$= \frac{6Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A(}{}$$

$$= \frac{2\left(3Ab^2+5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{105ad}$$

$$= \frac{2\left(3Ab^2+5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{105ad}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\left(3Ab^2-a^2(41A+70C)\right)\sqrt{\cos(c+d}}{}$$

Mathematica [A] time = 19.595, size = 482, normalized size = 1.05

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{4b\left(-41a^2A-70a^2C+3Ab^2\right)\sin(c+dx)}{105a^2} + \frac{2\sec(c+dx)\left(25a^2A\sin(c+dx)+35a^2C\sin(c+dx)+3Ab^2\sin(c+dx)\right)}{105a} + \right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9
/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*(3*A*b^2 - a^2*(41*A
+ 70*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + a*(a + b)*(-6*A*b^2 + 5*a^2*(5*A + 7*C) + 3*a*b*(19*A + 35*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(
3*A*b^2 - a^2*(41*A + 70*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
 d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(-41*a^2
```

```
*A + 3*A*b^2 - 70*a^2*C)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d*x]*(25*a^2*
A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a) + (
16*A*b*Sec[c + d*x]*Tan[c + d*x])/35 + (2*a*A*Sec[c + d*x]^2*Tan[c + d*x])/
7))/d
```

Maple [B] time = 0.304, size = 2979, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] 2/105/d/a^2*(15*A*a^4-6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-82*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-51
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+140*C*cos(d*x+c)^4*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b+140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b^2-140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+82*A*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b+82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-82*A*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b-51*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos
```

```
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-6*A*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^4-25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-6*A*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4-25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+140*C*cos(d*x+c)
^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b+140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+82*A*cos
(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3*b+140*C*cos(d*x+c)^4*a^2*b^2+175*C*cos(d*x+c)^3*a^3*b-35*C*
cos(d*x+c)^5*a^3*b-140*C*cos(d*x+c)^5*a^2*b^2-140*C*cos(d*x+c)^4*a^3*b-105*
C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-105*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-3*A*cos(d*x+c)^3*a*b^
3+27*A*cos(d*x+c)^2*a^2*b^2+39*A*cos(d*x+c)*a^3*b-25*A*cos(d*x+c)^5*a^3*b-8
2*A*cos(d*x+c)^5*a^2*b^2-3*A*cos(d*x+c)^5*a*b^3-82*A*cos(d*x+c)^4*a^3*b+55*
A*cos(d*x+c)^4*a^2*b^2+6*A*cos(d*x+c)^4*a*b^3+68*A*cos(d*x+c)^3*a^3*b-25*A*
cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+35*C*cos(d*x+c
)^2*a^4+6*A*cos(d*x+c)^5*b^4-6*A*cos(d*x+c)^4*b^4+82*A*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3+Ca\cos(dx+c)^2+Ab\cos(dx+c)+Aa\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9
/2), x)
```

3.1421 $\int (a+b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=525

$$-\frac{2\sqrt{a+b}\left(a^2(3A+5C)-2ab(2A+5C)+Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{5ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b^2 + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(5*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(
A*b^2 - 2*a*b*(2*A + 5*C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(5*a*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*C*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[
c + d*x]]) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(5*d)
```

Rubi [A] time = 1.36384, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3048, 3047, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(3A+5C)-2ab(2A+5C)+Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{5ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b^2 + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(5*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(
A*b^2 - 2*a*b*(2*A + 5*C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x
```

```
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(5*a*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*C*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[
c + d*x]]) + (2*A*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^{3/2}\left(\right.}{\cos^{\frac{7}{2}}(c-}$$

$$=\frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}\Big($$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2A(}{}$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2A(}{}$$

$$=-\frac{2b\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{d\sqrt{\sec(c}}$$

$$=\frac{2(a-b)\sqrt{a+b}\left(Ab^2+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\csc(}{5}$$

Mathematica [B] time = 24.676, size = 6049, normalized size = 11.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7
/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.263, size = 2827, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)

[Out] -2/5/d/a*(-A*a^3-3*A*cos(d*x+c)^2*a*b^2-5*C*cos(d*x+c)^3*a^2*b+5*C*cos(d*x+c)^4*a^2*b+3*A*cos(d*x+c)^4*a^2*b+2*A*cos(d*x+c)^4*a*b^2+A*cos(d*x+c)^3*a*b^2-3*A*cos(d*x+c)*a^2*b-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-5*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+10*C*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-5*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+A*cos(d*x+c)^4*b^3+3*A*cos(d*x+c)^3*a^3-2*A*cos(d*x+c)^2*a^3+5*C*cos(d*x+c)^3*a^3-5*C*cos(d*x+c)^2*a^3-A*cos(d*x+c)^3*b^3+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b

```
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
2+10*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b+A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))
^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7
/2), x)
```

3.1422 $\int (a+b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=560

$$\frac{\sqrt{a+b}\left(2a^2(A+3C)-a(8Ab-3bC)+6Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*b*Sqrt[a + b]*(8*A - 3*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(6*A*b^2 + 2*a^2*(A
 + 3*C) - a*(8*A*b - 3*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*
A*b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - (b*(8*A -
 3*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*
A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.69599, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3048, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(2a^2(A+3C)-a(8Ab-3bC)+6Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*b*Sqrt[a + b]*(8*A - 3*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(6*A*b^2 + 2*a^2*(A
 + 3*C) - a*(8*A*b - 3*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcS
```

```
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*
A*b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - (b*(8*A -
 3*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*
A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}\left(A\right.}{\cos^{\frac{5}{2}}(c+}$$

$$=\frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(2\right.$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2A(a}{}$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}-\frac{b(8A}{}$$

$$=\frac{2Ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}-\frac{b(8A}{}$$

$$=-\frac{3a\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+}}\right.\right.}{d\sqrt{\sec(c+}}$$

$$=\frac{(a-b)b\sqrt{a+b}(8A-3C)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{3ad\sqrt{\sec}}$$

Mathematica [B] time = 23.6384, size = 3392, normalized size = 6.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((8*A*b*Sin[c + d*x])/3 + (2*a*A*Tan[c + d*x])/3))/d + (((-4*a*A*b)/(3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*b*C)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) - (A*b^2*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (a^2*C*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]] + (b^2*C*Sqrt[Sec[c + d*x]])/(2*Sqrt[a + b*Cos[c + d*x]]) - (4*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(a + b)*(8*A - 3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*(3*A*b^2 + a^2*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 36*a*b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A - 3*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*b*(a + b)*(8*A - 3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*(3*A*b^2 + a^2*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 36*a*b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A - 3*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(6*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*b*(a + b)*(8*A - 3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*(3*A*b^2 + a^2*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 36*a*b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A - 3*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(6*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-(b*(8*A - 3*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - (b*(a + b)*(8*A - 3*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + C

os[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])] + (2*(3*A*b^2 + a^2*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2
- Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] -
(18*a*b*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*A - 3*C)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + (2*(3*A*b^2 + a^2*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (18*a*b*C*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))] + b^2*(8*A - 3*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^
2*Sin[c + d*x]*Tan[(c + d*x)/2] + b*(8*A - 3*C)*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - b*(8*A - 3*C)*Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (2*(3*A*b^2 + a^2
*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])
+ (18*a*b*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/
2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a +
b)]) - (b*(a + b)*(8*A - 3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1
- ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*b*(a + b)*(8*A -
3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + 4*(3*A*b^2 + a^2*(A + 3*C) + a*(4*A*b - 6*b*C))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 36*a*b*C*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A
- 3*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2
*Sec[c + d*x]*Tan[c + d*x]))/(6*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))

Maple [B] time = 0.22, size = 2134, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*(-10*A*cos(d*x+c)*a*b+8*A*cos(d*x+c)^2*a*b+2*A*cos(d*x+c)^3*a*b+3*C*
cos(d*x+c)^3*a*b-3*C*cos(d*x+c)^2*a*b-2*A*a^2+8*A*cos(d*x+c)^3*b^2+8*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*a*b+2*A*cos(d*x+c)^2*a^2-8*A*cos(d*x+c)^2*b^2+3*C*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^2+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^2+6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*C*cos(d*x+c)^3*b^2-12*C*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a*b+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-12*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+3*C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b+18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*a*b+3*C*cos(d*x+c)^4*b^2+2*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2
-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^2*a*b+18*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+8*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-8*
```

```
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*a*b+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+3*C*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2-8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*b^2+6*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+2*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a
^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5
/2), x)
```

3.1423 $\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=569

$$-\frac{\sqrt{a+b}\left(3a^2C+8Ab^2+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\ldots\right)}{4bd\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*A - 5*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*a*A - 16*A*b - 5*a*C
 - 2*b*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[S
ec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 3*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]
) - (b*(4*A - C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d
*x]]) - (a*(8*A - 5*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(4*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/d
```

Rubi [A] time = 1.72882, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3048, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2C+8Ab^2+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\ldots\right)}{4bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*A - 5*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*a*A - 16*A*b - 5*a*C
```

```
 - 2*b*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[S
ec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 3*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]
) - (b*(4*A - C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d
*x]]) - (a*(8*A - 5*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(4*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^{3/2}\left(\right.}{\cos^{\frac{3}{2}}(c}$$

$$=\frac{2A(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\left(2\right.$$

$$=-\frac{b(4A-C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}+\frac{2A(a+b}{}$$

$$=-\frac{b(4A-C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}-\frac{a(8A-5}{}$$

$$=-\frac{b(4A-C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}-\frac{a(8A-5}{}$$

$$=-\frac{\sqrt{a+b}\left(8Ab^2+3a^2C+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+}{}$$

$$=\frac{(a-b)\sqrt{a+b}(8A-5C)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{4d\sqrt{\sec}}$$

Mathematica [B] time = 18.373, size = 1178, normalized size = 2.07

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(2*a*A*Sin[c + d*x] + (b*C*Sin
[2*(c + d*x)])/4))/d + (8*a^2*A*Tan[(c + d*x)/2] + 8*a*A*b*Tan[(c + d*x)/2]
 - 5*a^2*C*Tan[(c + d*x)/2] - 5*a*b*C*Tan[(c + d*x)/2] - 16*a*A*b*Tan[(c +
d*x)/2]^3 + 10*a*b*C*Tan[(c + d*x)/2]^3 - 8*a^2*A*Tan[(c + d*x)/2]^5 + 8*a*
A*b*Tan[(c + d*x)/2]^5 + 5*a^2*C*Tan[(c + d*x)/2]^5 - 5*a*b*C*Tan[(c + d*x)
/2]^5 + 16*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + 6*a^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*C*EllipticPi[-1, -ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 16*A*b^2*Ellipt
icPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] + 6*a^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*C*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] + a*(a + b)*(8*A - 5*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(4*a
^2*(A - C) - 2*b^2*(2*A + C) + a*b*(8*A + C))*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(4
*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(
c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.254, size = 2618, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

[Out] -1/4/d*(-8*A*cos(d*x+c)*a*b+8*A*cos(d*x+c)^2*a*b-2*C*cos(d*x+c)*a*b+7*C*cos
(d*x+c)^3*a*b-5*C*cos(d*x+c)^2*a*b-8*A*a^2-2*b^2*C*cos(d*x+c)^2+16*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
b^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^2+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*C
*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+5*C*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-8*C*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2+5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+5*C*co
s(d*x+c)^2*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*

```
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+5*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)^4*b
^2-5*C*cos(d*x+c)*a^2+8*A*cos(d*x+c)*a^2+16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-8*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)*a^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)*cos(d*x+c)/(a+b*cos(d*x+
c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)+Aa\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + C*a*cos(d*x + c)^2 + A*b*cos(d*x + c) + A*a)
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos \left(dx + c\right)^{2} + A\right)\left(b \cos \left(dx + c\right) + a\right)^{\frac{3}{2}} \sec \left(dx + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2), x)
```

3.1424 $\int (a+b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sqrt{\sec(c+dx)}\, d$

Optimal. Leaf size=613

$$\frac{\left(3a^2C+8b^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\left(3a^2C+48aAb+14abC+24Ab^2+16b^2C\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2*C + 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]
*(48*a*A*b + 24*A*b^2 + 3*a^2*C + 14*a*b*C + 16*b^2*C)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a +
 b]*(24*A*b^2 - a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (a*C*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((3*a^2*C + 8*b^2*(3*A + 2*
C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```

Rubi [A] time = 1.90633, antiderivative size = 613, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2C+8b^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\left(3a^2C+48aAb+14abC+24Ab^2+16b^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```



```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2*C + 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]
*(48*a*A*b + 24*A*b^2 + 3*a^2*C + 14*a*b*C + 16*b^2*C)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
```

```
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a +
 b]*(24*A*b^2 - a^2*C + 12*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (a*C*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((3*a^2*C + 8*b^2*(3*A + 2*
C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^{3/2}\,(}{\sqrt{\cos(c}}$$

$$=\frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$

$$=\frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{C(a+b\cos(c+d}{3d\sqrt{\sec(}}$$

$$=\frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{C(a+b\cos(c+d}{3d\sqrt{\sec(}}$$

$$=\frac{aC\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{C(a+b\cos(c+d}{3d\sqrt{\sec(}}$$

$$=-\frac{a\sqrt{a+b}\left(24Ab^2-a^2C+12b^2C\right)\sqrt{\cos(c+dx)}\csc(c}{}$$

$$=-\frac{(a-b)\sqrt{a+b}\left(3a^2C+8b^2(3A+2C)\right)\sqrt{\cos(c+dx)}\,c}{}$$

Mathematica [B] time = 18.576, size = 1285, normalized size = 2.1

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*C*Sin[c + d*x])/12 + (7*a*
C*Sin[2*(c + d*x)])/24 + (b*C*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c
+ d*x)/2]^2)^(-1)]*(24*a*A*b^2*Tan[(c + d*x)/2] + 24*A*b^3*Tan[(c + d*x)/2]
 + 3*a^3*C*Tan[(c + d*x)/2] + 3*a^2*b*C*Tan[(c + d*x)/2] + 16*a*b^2*C*Tan[(
c + d*x)/2] + 16*b^3*C*Tan[(c + d*x)/2] - 48*A*b^3*Tan[(c + d*x)/2]^3 - 6*a
^2*b*C*Tan[(c + d*x)/2]^3 - 32*b^3*C*Tan[(c + d*x)/2]^3 - 24*a*A*b^2*Tan[(c
 + d*x)/2]^5 + 24*A*b^3*Tan[(c + d*x)/2]^5 - 3*a^3*C*Tan[(c + d*x)/2]^5 + 3
*a^2*b*C*Tan[(c + d*x)/2]^5 - 16*a*b^2*C*Tan[(c + d*x)/2]^5 + 16*b^3*C*Tan[
(c + d*x)/2]^5 - 144*a*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 72*a*b^2*C*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
144*a*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*C*EllipticPi[-1, -ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
 72*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(24*A*b^2 + 3*a^2*C + 16*b^
2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b*(24*a*A - 48*A*b + 7*a*C - 26*b*C)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)]))/(24*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]
)
```

Maple [B] time = 0.257, size = 2718, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

[Out] -1/24/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b^2+22*C*cos(d*x+c)^4*a*b^2+17*C*cos(d*x+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-6*C*cos(d*x+c)^2*a*b^2-14*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*C*cos(d*x+c)^5*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+

```
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+
48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*c
os(d*x+c)^3*b^3+3*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a
^3+24*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+c)^2*b^3)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c) + a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c)), x)
```

3.1425 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=698

$$\frac{a\left(-3a^2C+80Ab^2+52b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{64b^2d}-\frac{\left(3a^2C-4b^2(4A+3C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32bd\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(80*A*b^2 - 3*a^2*C + 52*b^2*C)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a +
b]*(3*a^3*C - 2*a^2*b*C - 8*b^3*(4*A + 3*C) - 4*a*b^2*(20*A + 13*C))*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]
]) - (Sqrt[a + b]*(3*a^4*C + 24*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3*d*Sq
rt[Sec[c + d*x]]) - ((3*a^2*C - 4*b^2*(4*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d*x]]) - (a*C*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(8*b*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (a*(80*A*b^2 - 3*a^2*C + 52*b^2*
C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*b^2*d)
```

Rubi [A] time = 2.36423, antiderivative size = 698, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{a\left(-3a^2C+80Ab^2+52b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{64b^2d}-\frac{\left(3a^2C-4b^2(4A+3C)\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(80*A*b^2 - 3*a^2*C + 52*b^2*C)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
```

```
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a +
b]*(3*a^3*C - 2*a^2*b*C - 8*b^3*(4*A + 3*C) - 4*a*b^2*(20*A + 13*C))*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]
]) - (Sqrt[a + b]*(3*a^4*C + 24*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3*d*Sq
rt[Sec[c + d*x]]) - ((3*a^2*C - 4*b^2*(4*A + 3*C))*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d*x]]) - (a*C*(a + b*Cos[c + d*x])^(3/2
)*Sin[c + d*x])/(8*b*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (a*(80*A*b^2 - 3*a^2*C + 52*b^2*
C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\left(\right.$$

$$= \frac{C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{aC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\cos(c+dx))^{5/2}\sin(c}{4bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{\left(3a^2C-4b^2(4A+3C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} - \frac{aC(a+b}{}$$

$$= -\frac{\left(3a^2C-4b^2(4A+3C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} - \frac{aC(a+b}{}$$

$$= -\frac{\left(3a^2C-4b^2(4A+3C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} - \frac{aC(a+b}{}$$

$$= -\frac{\sqrt{a+b}\left(3a^4C+24a^2b^2(2A+C)+16b^4(4A+3C)\right)\sqrt{\cos(c+dx)}\,\mathrm{cs}}{64b^3}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(80Ab^2-3a^2C+52b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E}{64b^2d\sqrt{\sec(c}}$$

Mathematica [A] time = 11.1392, size = 536, normalized size = 0.77

$$\frac{\frac{2\tan(c+dx)(a+b\cos(c+dx))\left(a^2C+12abC\cos(c+dx)+16Ab^2+4b^2C\cos(2(c+dx))+16b^2C\right)}{b} - \frac{ab\left(3a^2C-80Ab^2-52b^2C\right)\tan\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos(c+dx)\sec^2\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c +
d*x]],x]
```

```
[Out] (-((a*b*(a + b)*(-80*A*b^2 + 3*a^2*C - 52*b^2*C)*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*S
```

```
ec[(c + d*x)/2]^2)/(a + b)] + a*(a + b)*(3*a^3*C - 6*a^2*b*C + 8*b^3*(4*A +
 3*C) + 4*a*b^2*(12*A + 7*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] - (3*a^4*C + 24*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*((a - b)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + a*b*(-80*A*b^2 + 3*a^2*C - 5
2*b^2*C)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c
 + d*x]*Tan[(c + d*x)/2])/(b^3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2))) +
(2*(a + b*Cos[c + d*x])*(16*A*b^2 + a^2*C + 16*b^2*C + 12*a*b*C*Cos[c + d*x
] + 4*b^2*C*Cos[2*(c + d*x)])*Tan[c + d*x])/b)/(64*d*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.569, size = 3803, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/64/d/b^2*(-128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+32*A*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+96*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^2*b^2+80*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+80*A*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+2*C*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3*b-76*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+24*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+48*C*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
```

/(a+b))^(1/2))*a^2*b^2-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+52*C*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+16*
C*cos(d*x+c)^6*b^4+52*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+32*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^
3+96*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*a^2*b^2+80*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+80*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+2*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-76*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3+48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+52*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+52*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+40
*C*cos(d*x+c)^5*a*b^3+26*C*cos(d*x+c)^4*a^2*b^2-C*cos(d*x+c)^3*a^3*b-64*A*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
x+c),(-(a-b)/(a+b))^(1/2))(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*b^4+128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*b^4-48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^4+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+96*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt

```
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-3*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*C*
cos(d*x+c)^4*b^4-32*A*cos(d*x+c)^2*b^4-24*C*cos(d*x+c)^2*b^4+3*C*cos(d*x+c)
*a^4+112*A*cos(d*x+c)^3*a*b^3+80*A*cos(d*x+c)^2*a^2*b^2-3*C*cos(d*x+c)^2*a^
4+32*A*cos(d*x+c)^4*b^4-52*C*cos(d*x+c)^2*a*b^3-2*C*cos(d*x+c)*a^3*b-52*C*c
os(d*x+c)*a^2*b^2-24*C*cos(d*x+c)*a*b^3+36*C*cos(d*x+c)^3*a*b^3-80*A*cos(d*
x+c)^2*a*b^3-80*A*cos(d*x+c)*a^2*b^2-32*A*cos(d*x+c)*a*b^3-64*A*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^4+128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+6*C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^4+96*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*b^4+3*C*cos(d*x+c)^2*a^3*b+26*C*cos(d*x+c)^2*a^2
*b^2-128*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b^2*sin(d*x+c))*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x +
c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x +
c)), x)
```

3.1426 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=627

$$\frac{2\left(3a^2(9A+11C)+5Ab^2\right)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{231d}+\frac{2b\left(a^2(229A+297C)+3Ab^2\right)\sin(c+dx)\sec}{693ad}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A +
957*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^4*d*Sq
rt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9
*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) - 6*a^3*b*(101*A + 132*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*d*Sqrt[Sec[c + d*x]]) -
 (2*(4*A*b^4 - 15*a^4*(9*A + 11*C) - a^2*b^2*(205*A + 297*C))*Sqrt[a + b*Co
s[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(693*a^2*d) + (2*b*(3*A*b^2 +
a^2*(229*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(693*a*d) + (2*(5*A*b^2 + 3*a^2*(9*A + 11*C))*Sqrt[a + b*Cos[c + d*x]]*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (10*A*b*(a + b*Cos[c + d*x])^(3/
2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + b*Cos[c + d*x])^(5/2
)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 2.61462, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3048, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(3a^2(9A+11C)+5Ab^2\right)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{231d}+\frac{2b\left(a^2(229A+297C)+3Ab^2\right)\sin(c+dx)\sec}{693ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(13/2),x
]
```


```
[Out] (2*(a - b)*b*Sqrt[a + b]*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A +
957*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
```

```
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^4*d*Sq
rt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9
*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) - 6*a^3*b*(101*A + 132*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*d*Sqrt[Sec[c + d*x]]) -
 (2*(4*A*b^4 - 15*a^4*(9*A + 11*C) - a^2*b^2*(205*A + 297*C))*Sqrt[a + b*Co
s[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(693*a^2*d) + (2*b*(3*A*b^2 +
a^2*(229*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(693*a*d) + (2*(5*A*b^2 + 3*a^2*(9*A + 11*C))*Sqrt[a + b*Cos[c + d*x]]*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (10*A*b*(a + b*Cos[c + d*x])^(3/
2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A*(a + b*Cos[c + d*x])^(5/2
)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\left(A \ldots\right.}{\cos^{\frac{13}{2}}(c+ \ldots}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d} + \frac{1}{11} \ldots$$

$$= \frac{10Ab(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2 \ldots}{\ldots}$$

$$= \frac{2\left(5Ab^2+3a^2(9A+11C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+ \ldots}{231d}$$

$$= \frac{2b\left(3Ab^2+a^2(229A+297C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(\ldots}{693ad}$$

$$= -\frac{2\left(4Ab^4-15a^4(9A+11C)-a^2b^2(205A+297C)\right)\sqrt{a \ldots}}{693a^2d}$$

$$= -\frac{2\left(4Ab^4-15a^4(9A+11C)-a^2b^2(205A+297C)\right)\sqrt{a \ldots}}{693a^2d}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(8Ab^4+3a^2b^2(17A+33C)+a^4(741A \ldots\right.}{\ldots}$$

Mathematica [B] time = 26.1459, size = 3885, normalized size = 6.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(741*a^4*A + 51*a^2*A*b^
2 + 8*A*b^4 + 957*a^4*C + 99*a^2*b^2*C)*Sin[c + d*x])/(693*a^3) + (2*Sec[c
```

+ d*x]^3*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x] + 99*a^2*C*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(229*a^2*A*b*Sin[c + d*x] + 3*A*b^3*Sin[c + d*x] + 297*a^2*b*C*Sin[c + d*x]))/(693*a) + (2*Sec[c + d*x]*(135*a^4*A*Sin [c + d*x] + 205*a^2*A*b^2*Sin[c + d*x] - 4*A*b^4*Sin[c + d*x] + 165*a^4*C*S in[c + d*x] + 297*a^2*b^2*C*Sin[c + d*x]))/(693*a^2) + (46*a*A*b*Sec[c + d* x]^3*Tan[c + d*x])/99 + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/11))/d + (2*((-247*a^2*A*b)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (17*A*b^ 3)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^5)/(693*a^2*S qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a^2*b*C)/(21*Sqrt[a + b*C os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^3*C)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt [Sec[c + d*x]]) + (15*a^3*A*Sqrt[Sec[c + d*x]])/(77*Sqrt[a + b*Cos[c + d*x]]) - (26*a*A*b^2*Sqrt[Sec[c + d*x]])/(231*Sqrt[a + b*Cos[c + d*x]]) - (7*A* b^4*Sqrt[Sec[c + d*x]])/(99*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Sqrt[Sec [c + d*x]])/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) + (5*a^3*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*b^2*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*C*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d *x]]) - (247*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[a + b*C os[c + d*x]]) - (17*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(693*a ^3*Sqrt[a + b*Cos[c + d*x]]) - (29*a*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d* x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d *x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]] *(-2*b*(a + b)*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Sq rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2* a*(a + b)*(-6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 3 3*C) + a^3*(606*A*b + 792*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4* (741*A + 957*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(693*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]* ((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*b*(a + b)*(8*A*b ^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) + a^3*(606*A*b + 792*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/ ((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/ (a + b)] - b*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(693*a^ 3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*b*(a + b)*(8*A*b^4 + 3*a^2*b^2*(1 7*A + 33*C) + a^4*(741*A + 957*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^3 + 8*A*b^4 + 15*a

^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) + a^3*(606*A*b + 792*b*C))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A
*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Cos[c + d*x]*(a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(693*a^3*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(-(b*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Cos[c +
d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - (b*(a + b)*(8*A*b^4 + 3*a
^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + C
os[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(-6*a*A*b
^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) + a^3*(606*A*b
+ 792*b*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x]
)/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4
*(741*A + 957*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])
^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*
(-6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) + a^3
*(606*A*b + 792*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b^2*(8*
A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Cos[c + d*x]*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + b*(8*A*b^4 + 3*a^2*b^2*(17*A +
33*C) + a^4*(741*A + 957*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c
+ d*x]*Tan[(c + d*x)/2] - b*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A
+ 957*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]^2 + (a*(a + b)*(-6*a*A*b^3 + 8*A*b^4 + 15*a^4*(9*A + 11*C) + 3*a^2*b^
2*(19*A + 33*C) + a^3*(606*A*b + 792*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)
/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)
/(a + b)]) - (b*(a + b)*(8*A*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 9
57*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*
x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(693*a^3*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*b*(a + b)*(8*A*b^4 + 3*a^2*b^2*(1
7*A + 33*C) + a^4*(741*A + 957*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^3 + 8*A*b^4 + 15*a
^4*(9*A + 11*C) + 3*a^2*b^2*(19*A + 33*C) + a^3*(606*A*b + 792*b*C))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos

```
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(8*A
*b^4 + 3*a^2*b^2*(17*A + 33*C) + a^4*(741*A + 957*C))*Cos[c + d*x]*(a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
 + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/
(693*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.62, size = 4702, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/693/d/a^3*(8*A*cos(d*x+c)^7*b^6-63*A*a^6-363*C*cos(d*x+c)^6*a^4*b^2+99*C
*cos(d*x+c)^6*a^3*b^3-99*C*cos(d*x+c)^6*a^2*b^4-566*A*cos(d*x+c)^5*a^5*b-14
0*A*cos(d*x+c)^5*a^3*b^3-4*A*cos(d*x+c)^5*a*b^5-726*C*cos(d*x+c)^5*a^5*b-8*
A*cos(d*x+c)^6*b^6+741*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b-396*C*cos(d*x+c)^5*a^3*b^3-16
0*A*cos(d*x+c)^4*a^4*b^2+A*cos(d*x+c)^4*a^2*b^4-594*C*cos(d*x+c)^4*a^4*b^2-
86*A*cos(d*x+c)^3*a^5*b+135*A*cos(d*x+c)^7*a^5*b+741*A*cos(d*x+c)^7*a^4*b^2
+205*A*cos(d*x+c)^7*a^3*b^3+51*A*cos(d*x+c)^7*a^2*b^4-4*A*cos(d*x+c)^7*a*b^
5+165*C*cos(d*x+c)^7*a^5*b+957*C*cos(d*x+c)^7*a^4*b^2+297*C*cos(d*x+c)^7*a^
3*b^3+99*C*cos(d*x+c)^7*a^2*b^4+741*A*cos(d*x+c)^6*a^5*b-307*A*cos(d*x+c)^6
*a^4*b^2+51*A*cos(d*x+c)^6*a^3*b^3-52*A*cos(d*x+c)^6*a^2*b^4+8*A*cos(d*x+c)
^6*a*b^5+957*C*cos(d*x+c)^6*a^5*b+135*A*cos(d*x+c)^6*a^6+165*C*cos(d*x+c)^6
*a^6-99*C*cos(d*x+c)^2*a^6-54*A*cos(d*x+c)^4*a^6-66*C*cos(d*x+c)^4*a^6-18*A
*cos(d*x+c)^2*a^6-116*A*cos(d*x+c)^3*a^3*b^3-274*A*cos(d*x+c)^2*a^4*b^2-224
*A*cos(d*x+c)*a^5*b-396*C*cos(d*x+c)^3*a^5*b+663*A*sin(d*x+c)*cos(d*x+c)^5*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2+5
1*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b^3+2*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+8*A*sin(d*x+c)*cos(d*
x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^5-741*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
```

c),(-(a-b)/(a+b))^(1/2))*a^5*b-741*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2-51*A*sin(d*x+c)
*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b^3-51*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4-8*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5+957*C*sin
(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^5*b+891*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2+99*C*sin(d*x+c)*cos(d*x+c)^5
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3-
957*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^5*b-957*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2-99*C*sin(d*x+c)*cos
(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b^3-99*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+741*A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b+663*A*sin(d
*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4*b^2+51*A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3+2*A*sin(d*x+c)*cos(d*x+c)^6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+8*
A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^5-741*A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b-741*A*sin(d*x+c)*cos(d*x+
c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*
b^2-51*A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c

```
),(-(a-b)/(a+b))^(1/2))*a^3*b^3-51*A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4-8*A*sin(d*x+c)*
cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^5+957*C*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b+891*C*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2+99*C*sin(
d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b^3-957*C*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b-957*C*sin(d*x+c)*cos(d*x+c)^6
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2-
99*C*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b^3-99*C*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+135*A*sin(d*x+c)*co
s(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^6-8*A*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^6+165*C*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6+135*A*sin(d*x+c)*cos(
d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^6-8*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^6+165*C*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6)*cos(d*x+c)*(1/cos(d*x+
c))^(13/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(1
3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + 2\,Cab\cos(dx+c)^3 + 2\,Aab\cos(dx+c) + Aa^2 + \left(Ca^2 + Ab^2\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, al
gorithm="giac")
```

[Out] Timed out

3.1427 $\int (a+b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=544

$$\frac{2\left(7a^2(7A+9C)+15Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2b\left(a^2(163A+231C)+5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}}{315ad}$$

[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 + 21*a^3*(7*A + 9*C) + 15*a*b^2*(11*A + 21*C) - 6*a^2*b*(19*A + 28*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b*(5*A*b^2 + a^2*(163*A + 231*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a*d) + (2*(15*A*b^2 + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (10*A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)

Rubi [A] time = 2.00723, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3048, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(7a^2(7A+9C)+15Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2b\left(a^2(163A+231C)+5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}}{315ad}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(11/2),x]

[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sq

```
rt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 + 21*a^3*(7*A + 9*C) +
 15*a*b^2*(11*A + 21*C) - 6*a^2*b*(19*A + 28*C))*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b*(5*A*b^2
+ a^2*(163*A + 231*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(315*a*d) + (2*(15*A*b^2 + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]
]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (10*A*b*(a + b*Cos[c + d*x])^(
3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^(5
/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\,(}{\cos^{\frac{11}{2}}(c}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}$$

$$= \frac{10Ab(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} +$$

$$= \frac{2\left(15Ab^2+7a^2(7A+9C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c}{315d}$$

$$= \frac{2b\left(5Ab^2+a^2(163A+231C)\right)\sqrt{a+b\cos(c+dx)}\sec}{315ad}$$

$$= \frac{2b\left(5Ab^2+a^2(163A+231C)\right)\sqrt{a+b\cos(c+dx)}\sec}{315ad}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(10Ab^4-21a^4(7A+9C)-3a^2b^2(93}{}$$

Mathematica [A] time = 21.7389, size = 621, normalized size = 1.14

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2\left(-279a^2Ab^2-147a^4A-483a^2b^2C-189a^4C+10Ab^4\right)\sin(c+dx)}{315a^2}+\frac{2}{315}\sec^2(c+dx)\left(49a^2A\sin(c+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(1
1/2),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((a + b)*((-10*A*b^
4 + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)] - a*(-10*A*b^3 + 21*a^3*(7*A + 9*C) + 15*a*b^
```

```
2*(11*A + 21*C) + 6*a^2*b*(19*A + 28*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]) - (-10*A*b^4 + 21*a
^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[(1 + Cos[c + d*x])^
(-1)]*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-147*a^4*A - 279*a^2*A*b^2 + 10*A*b^4
- 189*a^4*C - 483*a^2*b^2*C)*Sin[c + d*x])/(315*a^2) + (2*Sec[c + d*x]^2*(4
9*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x]))/315
+ (2*Sec[c + d*x]*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x] + 231*a^
2*b*C*Sin[c + d*x]))/(315*a) + (38*a*A*b*Sec[c + d*x]^2*Tan[c + d*x])/63 +
(2*a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.465, size = 4338, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d/a^2*(-35*A*a^5+105*C*cos(d*x+c)^5*a^4*b-294*C*cos(d*x+c)^3*a^4*b+1
89*C*cos(d*x+c)^6*a^4*b+231*C*cos(d*x+c)^6*a^3*b^2+483*C*cos(d*x+c)^6*a^2*b
^3+483*C*cos(d*x+c)^5*a^3*b^2-483*C*cos(d*x+c)^5*a^2*b^3-714*C*cos(d*x+c)^4
*a^3*b^2-82*A*cos(d*x+c)^3*a^4*b-80*A*cos(d*x+c)^3*a^2*b^3-170*A*cos(d*x+c)
^2*a^3*b^2-130*A*cos(d*x+c)*a^4*b+147*A*cos(d*x+c)^6*a^4*b+163*A*cos(d*x+c)
^6*a^3*b^2+279*A*cos(d*x+c)^6*a^2*b^3+5*A*cos(d*x+c)^6*a*b^4+65*A*cos(d*x+c
)^5*a^4*b+279*A*cos(d*x+c)^5*a^3*b^2-199*A*cos(d*x+c)^5*a^2*b^3-10*A*cos(d*
x+c)^5*a*b^4-10*A*cos(d*x+c)^6*b^5-189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+147*A*cos(d*x+c
)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^5-147*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+10*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+189*C*cos(d*x+c)
^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a^5-189*C*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
```

d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+261*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-272*A*cos(d*x+
c)^4*a^3*b^2+5*A*cos(d*x+c)^4*a*b^4+147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*A*cos(d*x+
c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^5+10*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+189*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5+147*A*cos(d*x+c
)^5*a^5+10*A*cos(d*x+c)^5*b^5-98*A*cos(d*x+c)^4*a^5-14*A*cos(d*x+c)^2*a^5+1
89*C*cos(d*x+c)^5*a^5-126*C*cos(d*x+c)^4*a^5-63*C*cos(d*x+c)^2*a^5+10*A*cos
(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^4+357*C*cos(d*x+c)^5*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+483*C*cos(d*x+c)^5*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2-1
89*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a^4*b-483*C*cos(d*x+c)^5*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2-483*C*cos(d*x+c)^5*s
in(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a^2*b^3+261*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*A*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^3-10*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*A*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b^2-279*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-

```
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*A*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^4+357*C*cos(d*x+c)^4*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a^4*b+483*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-189*C*cos(d*x
+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4*b-483*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-483*C*cos(d*x+c)^4*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+279
*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b^2+155*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*A*cos(d*x+c)^5*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^4-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*A*cos(d*x
+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b^3+315*C*sin(d*x+c)*cos(d*x+c)^5*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3+315*C*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3)*
cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
```

```
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(1
1/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+2\,Cab\cos(dx+c)^3+2\,Aab\cos(dx+c)+Aa^2+\left(Ca^2+Ab^2\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

3.1428 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=600

$$\frac{2\left(a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}-\frac{2\sqrt{a+b}\left(a^2b(29A+49C)+a^3(-(5A+7C))-9\right.}{}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(3*A*b^2 + a^2*(29*A + 49*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(21*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a
 + b]*(3*A*b^3 - 9*a*b^2*(3*A + 7*C) - a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C
))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(21*a*d*Sqrt[Sec[c
 + d*x]]) - (2*b^2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(3*A*b^2 + a^2*(5*A + 7
*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*
A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*
A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.82355, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3048, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\left(a^2(5A+7C)+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}-\frac{2\sqrt{a+b}\left(a^2b(29A+49C)+a^3(-(5A+7C))-9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(3*A*b^2 + a^2*(29*A + 49*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(21*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a
```

```
 + b]*(3*A*b^3 - 9*a*b^2*(3*A + 7*C) - a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C
))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(21*a*d*Sqrt[Sec[c
 + d*x]]) - (2*b^2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(3*A*b^2 + a^2*(5*A + 7
*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*
A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*
A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{9}{2}}(c+}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\Big(2$$

$$= \frac{2Ab(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2A}{}$$

$$= \frac{2\left(3Ab^2+a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}{21d}$$

$$= \frac{2\left(3Ab^2+a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}{21d}$$

$$= -\frac{2b^2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right.\right.}{d\sqrt{\sec(c+}}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(3Ab^2+a^2(29A+49C)\right)\sqrt{\cos(c+dx)}}{2}$$

Mathematica [B] time = 25.3569, size = 3979, normalized size = 6.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(29*a^2*A + 3*A*b^2 + 49
*a^2*C)*Sin[c + d*x])/(21*a) + (2*Sec[c + d*x]*(5*a^2*A*Sin[c + d*x] + 9*A*
b^2*Sin[c + d*x] + 7*a^2*C*Sin[c + d*x]))/21 + (6*a*A*b*Sec[c + d*x]*Tan[c
+ d*x])/7 + (2*a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-29*a^2*A*b)
```

```
/(21*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^3)/(7*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*a^2*b*C)/(3*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) + (b^3*C)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
 + (5*a^3*A*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*A*b^2*
Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (A*b^4*Sqrt[Sec[c + d*x
]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) + (a^3*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[a +
 b*Cos[c + d*x]]) + (2*a*b^2*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*
x]]) - (29*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[
c + d*x]]) - (A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Co
s[c + d*x]]) - (7*a*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[a +
b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(a + b)*(3*A*
b^2 + a^2*(29*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + 2*a*(3*b^3*(A - 7*C) + 9*a*b^2*(3*A + 7*C) + a^3*(
5*A + 7*C) + a^2*b*(29*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] - 84*a*b^3*C*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(3*A*b^2 + a^2*(29*A +
 49*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]))/(21*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*b*(a + b)*(3*A*b^2 + a^2*(29
*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + 2*a*(3*b^3*(A - 7*C) + 9*a*b^2*(3*A + 7*C) + a^3*(5*A + 7*C) +
a^2*b*(29*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] - 84*a*b^3*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)] - b*(3*A*b^2 + a^2*(29*A + 49*C))*Cos[c
 + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21*a*(a
 + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*b*(a + b)*(3*A*b^2 + a^2*(29*A + 49*C
))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + 2*a*(3*b^3*(A - 7*C) + 9*a*b^2*(3*A + 7*C) + a^3*(5*A + 7*C) + a^2*b*(29
*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] - 84*a*b^3*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] - b*(3*A*b^2 + a^2*(29*A + 49*C))*Cos[c + d*x]*(
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21*a*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(-(b*(3*A*b^2 + a^2*(29*A + 49*C))*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^4)/2 - (b*(a + b)*(3*A*b^2 + a^2*(29*A + 49*C))*Sqrt[(a
```

```
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^
2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
 + (a*(3*b^3*(A - 7*C) + 9*a*b^2*(3*A + 7*C) + a^3*(5*A + 7*C) + a^2*b*(29*
A + 49*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] - (42*a*b^3*C*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
 + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(3*A*b^2 + a^
2*(29*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^
2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(3*b^3*(A
 - 7*C) + 9*a*b^2*(3*A + 7*C) + a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (42*a*b^3*C*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))] + b^2*(3*A*b^2 + a^2*(29*A + 49*C))*Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + b*(3*A*b^2 + a^2*(29*
A + 49*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*
x)/2] - b*(3*A*b^2 + a^2*(29*A + 49*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(3*b^3*(A - 7*C) + 9*a*b^2*(3*A +
 7*C) + a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c +
d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2
]^2)/(a + b)]) + (42*a*b^3*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1
 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c +
 d*x)/2]^2)/(a + b)]) - (b*(a + b)*(3*A*b^2 + a^2*(29*A + 49*C))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]
)/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(21*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c
 + d*x)/2]^2]) + ((-2*b*(a + b)*(3*A*b^2 + a^2*(29*A + 49*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*b^3*(A
 - 7*C) + 9*a*b^2*(3*A + 7*C) + a^3*(5*A + 7*C) + a^2*b*(29*A + 49*C))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 84*a
*b^3*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
```

```
+ b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] - b*(3*A*b^2 + a^2*(29*A + 49*C))*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*S
in[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*a*Sqr
t[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]))
```

Maple [B] time = 0.325, size = 3381, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d/a*(-3*A*a^4-3*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+29*A*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+27*A
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2+3*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-49*C*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-
49*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2+49*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-29*A*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b-29*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+29*A*cos(d*x+c)^3
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3*b+27*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
```

(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+7*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*A*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^4+5*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4+7*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*A*cos(d*x+c)^3*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
4+5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^4+3*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-49*C*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-49*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2+49*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-29*A*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b-49*C*cos(d*x+c)^4*a^2*b^2-56*C*cos(d*x+c)^3*a^3*b+7*C*cos(d*x+c)^5*a^
3*b+49*C*cos(d*x+c)^5*a^2*b^2+49*C*cos(d*x+c)^4*a^3*b+63*C*sin(d*x+c)*cos(d
x+c)^3(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^2+63*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-12*A*cos(d*x+c)^3*a*b^3-18*A*cos(d*x+c)^
2*a^2*b^2-12*A*cos(d*x+c)*a^3*b+5*A*cos(d*x+c)^5*a^3*b+29*A*cos(d*x+c)^5*a^
2*b^2+9*A*cos(d*x+c)^5*a*b^3+29*A*cos(d*x+c)^4*a^3*b-11*A*cos(d*x+c)^4*a^2*
b^2+3*A*cos(d*x+c)^4*a*b^3-22*A*cos(d*x+c)^3*a^3*b+5*A*cos(d*x+c)^4*a^4+7*C
*cos(d*x+c)^4*a^4-2*A*cos(d*x+c)^2*a^4-7*C*cos(d*x+c)^2*a^4+3*A*cos(d*x+c)^
5*b^4-3*A*cos(d*x+c)^4*b^4-29*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*C*sin(d*x+c)*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3+42*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*

```
x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-21*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+42*C*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*a*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/si
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1429 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=666

$$\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{5d}-\frac{\left(6a^2(3A+5C)+b^2(46A-15C)\right)\sin(c+dx)\sqrt{\mathrm{s}}}{15d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt
[a + b]*(30*A*b^3 - a*b^2*(46*A - 15*C) - 6*a^3*(3*A + 5*C) + a^2*(34*A*b +
 90*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqr
t[Sec[c + d*x]]) - (5*a*b*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^2 + a^2*(
3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d)
 - ((b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 2.33868, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3048, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\left(a^2(3A+5C)+5Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{5d}-\frac{\left(6a^2(3A+5C)+b^2(46A-15C)\right)\sin(c+dx)\sqrt{\mathrm{s}}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
```

```
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt
[a + b]*(30*A*b^3 - a*b^2*(46*A - 15*C) - 6*a^3*(3*A + 5*C) + a^2*(34*A*b +
 90*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqr
t[Sec[c + d*x]]) - (5*a*b*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^2 + a^2*(
3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d)
 - ((b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*b*(a + b*Cos[c + d*x])^(3/2)*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\left(\ldots\right.}{\cos^{\frac{7}{2}}(c \ldots}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(\ldots\right.$$

$$= \frac{2Ab(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2\ldots}{\ldots}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+d\ldots}}{5d}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+d\ldots}}{5d}$$

$$= \frac{2\left(5Ab^2+a^2(3A+5C)\right)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+d\ldots}}{5d}$$

$$= -\frac{5ab\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\ldots\right.\right.}{d\sqrt{\sec(c\ldots}}$$

$$= \frac{(a-b)\sqrt{a+b}\left(b^2(46A-15C)+6a^2(3A+5C)\right)\sqrt{\cos(\ldots}}{\ldots}$$

Mathematica [B] time = 25.5476, size = 6720, normalized size = 10.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7
/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.325, size = 3497, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*(-6*A*a^3-68*A*cos(d*x+c)^2*a*b^2+15*C*cos(d*x+c)^4*a*b^2-30*C*cos(
d*x+c)^3*a^2*b+30*C*cos(d*x+c)^4*a^2*b+18*A*cos(d*x+c)^4*a^2*b+22*A*cos(d*x
+c)^4*a*b^2+46*A*cos(d*x+c)^3*a*b^2-28*A*cos(d*x+c)*a^2*b+10*A*cos(d*x+c)^3
*a^2*b-15*C*cos(d*x+c)^3*a*b^2+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3+30*A*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^3+15*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*b^3+15*C*cos(d*x+c)^5*b^3-18*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
+150*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a*b^2-90*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+150*C*cos(d*x+c)^
2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*a*b^2-90*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+15*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+46*A*cos(
d*x+c)^4*b^3+18*A*cos(d*x+c)^3*a^3-12*A*cos(d*x+c)^2*a^3+30*C*cos(d*x+c)^3*
a^3+30*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*b^3-15*C*cos(d*x+c)^4*b^3-30*C*cos(d*x+c)^2*a^3-46*
A*cos(d*x+c)^3*b^3+18*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-18*A*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-46*A*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^3+30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
```

```
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3-18*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-46*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*C*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3-30*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-46*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+90*C*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b-30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+34*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+46*A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2-18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-46*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+90*C*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b-30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+34*A*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+46*A*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2+15*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)/(a+b*cos(d*x+c))^
(1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1430 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=627

$$\frac{\sqrt{a+b}\left(8a^2(A+3C)-a(56Ab-27bC)+6b^2(12A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{12d\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*b*Sqrt[a + b]*(56*A - 27*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(12*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(6*b^2*(12*A + C)
+ 8*a^2*(A + 3*C) - a*(56*A*b - 27*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(12*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 15*
a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (b^2*(8*A - C)*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - (a*b*(56*A - 27*C)*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (10*A*b*(a + b*Cos[c +
d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 2.27993, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {4221, 3048, 3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(8a^2(A+3C)-a(56Ab-27bC)+6b^2(12A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{12d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2),x]
```



```
[Out] ((a - b)*b*Sqrt[a + b]*(56*A - 27*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
```

```
*x]))/(a - b)])/(12*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(6*b^2*(12*A + C)
+ 8*a^2*(A + 3*C) - a*(56*A*b - 27*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(12*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 15*
a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (b^2*(8*A - C)*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - (a*b*(56*A - 27*C)*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (10*A*b*(a + b*Cos[c +
d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\left(A\right.}{\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\right.$$

$$= \frac{10Ab(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= -\frac{b^2(8A-C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{10Ab(a+}{}$$

$$= -\frac{b^2(8A-C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - \frac{ab(56A-}{}$$

$$= -\frac{b^2(8A-C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - \frac{ab(56A-}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+15a^2C+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+d}{4}$$

$$= \frac{(a-b)b\sqrt{a+b}(56A-27C)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\mathrm{s}\right.}{12d\sqrt{\mathrm{se}}}$$

<u>Mathematica</u> [B] time = 25.5437, size = 4880, normalized size = 7.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((14*a*A*b*Sin[c + d*x])/3 + (
b^2*C*Sin[2*(c + d*x)])/4 + (2*a^2*A*Tan[c + d*x])/3))/d + (((-7*a^2*A*b)/(
3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*b^3)/(Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (3*a^2*b*C)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) + (b^3*C)/(2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a
^3*A*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*a*A*b^2*Sqrt[Sec
```

[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (a^3*C*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]] + (11*a*b^2*C*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]]) - (7*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (9*a*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]]))*(2*a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 4*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(12*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(56*A - 27*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(12*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-(Tan[(c + d*x)/2]*(2*a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 4*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(12*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(56*A - 27*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(12*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)^2) + (b*Sin[c + d*x]*(2*a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 4*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(12*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(56*A - 27*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(24*(a + b*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) - (Tan[(c + d*x)/2]*(2*a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 4*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(12*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(56*A - 27*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(8*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + ((a*b*(a + b)*(56*A - 27*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (2*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (2*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + 2*a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 4*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - (2*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A + C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a*b*(a + b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2] + b*((a*(56*A - 27*C)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 + (6*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (6*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c

```
+ d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))] + 12*(8*A*b^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2] - a*b*(56*A - 27*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^4*
Sin[c + d*x]*Tan[(c + d*x)/2] - a*(56*A - 27*C)*(a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + 2*a*(56*A - 27*C)*Cos[c + d*x]
*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 - (6*(8*A*b^2 +
 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4)/(Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]
^2)/(a + b)]))))/(12*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqr
t[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) - ((2*a*b*(a
+ b)*(56*A - 27*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 4*(4*a^2*b*(7*A - 9*C) - 6*b^3*(2*A +
 C) + 3*a*b^2*(12*A + C) + 4*a^3*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(12*(8*A*b
^2 + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(56*A - 27*C)*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))*(-(Cos[(c +
d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Ta
n[c + d*x]))/(24*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*(Cos[(
c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2))))
```

Maple [B] time = 0.299, size = 3203, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/12/d*(-8*A*a^3-56*A*cos(d*x+c)^2*a*b^2+33*C*cos(d*x+c)^4*a*b^2+27*C*cos(
d*x+c)^3*a^2*b-27*C*cos(d*x+c)^2*a^2*b-6*C*cos(d*x+c)^2*a*b^2+56*A*cos(d*x+
c)^3*a*b^2-64*A*cos(d*x+c)*a^2*b+8*A*cos(d*x+c)^3*a^2*b-27*C*cos(d*x+c)^3*a
*b^2+6*C*cos(d*x+c)^5*b^3-56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-72*C*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
```

lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+6
*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*a*b^2+27*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+27*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+72*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^2+6*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+56*A*cos(d*x+c)^2*a^2*b+56*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b+8*A*cos(d*x+c)^2*a^3-24*A*sin(d*x+c)*cos(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3-24*A*sin(d*x+c)
*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*b^3-6*C*cos(d*x+c)^3*b^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+24*C*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b+72*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-56*A*sin(d*x+c)*cos(d
x+c)^2(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b-56*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^2-72*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+27*C*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b+24*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-56*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c

```
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+
90*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^2*b+27*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*C*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3
+24*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*b^3+90*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+48*A*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*b^3-12*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+24*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+48*A*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*
x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)+Aa^{2}+\left(Ca^{2}+Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1431 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=669

$$-\frac{\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}-\frac{\sqrt{a+b}\left(a^2(48A-33C)-2ab(72A+\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt
[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C) - 2*a*b*(72*A + 13*C))*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*Sqrt[Sec[c + d*x]])
- (5*a*Sqrt[a + b]*(8*A*b^2 + (a^2 + 4*b^2)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) - (a*b*(8*
A - 3*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) -
(b*(6*A - C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]]) - ((a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 2.41889, antiderivative size = 669, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3048, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}-\frac{\sqrt{a+b}\left(a^2(48A-33C)-2ab(72A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
```

```
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt
[a + b]*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C) - 2*a*b*(72*A + 13*C))*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*Sqrt[Sec[c + d*x]])
- (5*a*Sqrt[a + b]*(8*A*b^2 + (a^2 + 4*b^2)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) - (a*b*(8*
A - 3*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) -
(b*(6*A - C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]]) - ((a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\left(\right.}{\cos^{\frac{3}{2}}(c-}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\right.$$

$$= -\frac{b(6A-C)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2A(a+}{}$$

$$= -\frac{ab(8A-3C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} - \frac{b(6A-}{}$$

$$= -\frac{ab(8A-3C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} - \frac{b(6A-}{}$$

$$= -\frac{ab(8A-3C)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} - \frac{b(6A-}{}$$

$$= -\frac{5a\sqrt{a+b}\left(8Ab^2+\left(a^2+4b^2\right)C\right)\sqrt{\cos(c+dx)}\csc(c+}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(a^2(48A-33C)-8b^2(3A+2C)\right)\sqrt{\cos(}}{}$$

Mathematica [B] time = 19.4282, size = 1405, normalized size = 2.1

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((24*a^2*A + b^2*C)*Sin[c + d
*x])/12 + (13*a*b*C*Sin[2*(c + d*x)])/24 + (b^2*C*Sin[3*(c + d*x)])/12))/d
+ (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-48*a^3*A*Tan[(c + d*x)/2] - 48*a^2
*A*b*Tan[(c + d*x)/2] + 24*a*A*b^2*Tan[(c + d*x)/2] + 24*A*b^3*Tan[(c + d*x
)/2] + 33*a^3*C*Tan[(c + d*x)/2] + 33*a^2*b*C*Tan[(c + d*x)/2] + 16*a*b^2*C
*Tan[(c + d*x)/2] + 16*b^3*C*Tan[(c + d*x)/2] + 96*a^2*A*b*Tan[(c + d*x)/2]
```

```
^3 - 48*A*b^3*Tan[(c + d*x)/2]^3 - 66*a^2*b*C*Tan[(c + d*x)/2]^3 - 32*b^3*C
*Tan[(c + d*x)/2]^3 + 48*a^3*A*Tan[(c + d*x)/2]^5 - 48*a^2*A*b*Tan[(c + d*x
)/2]^5 - 24*a*A*b^2*Tan[(c + d*x)/2]^5 + 24*A*b^3*Tan[(c + d*x)/2]^5 - 33*a
^3*C*Tan[(c + d*x)/2]^5 + 33*a^2*b*C*Tan[(c + d*x)/2]^5 - 16*a*b^2*C*Tan[(c
 + d*x)/2]^5 + 16*b^3*C*Tan[(c + d*x)/2]^5 - 240*a*A*b^2*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^3*C*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] - 120*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] - 240*a*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^
3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] - 120*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a
+ b)*(a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C))*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*
a*(24*a^2*(A - C) + a*b*(72*A + 13*C) - 2*b^2*(36*A + 19*C))*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)]))/(24*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.345, size = 3521, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*(-48*A*a^3+24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b^2+34*C*cos(d
*x+c)^4*a*b^2+59*C*cos(d*x+c)^3*a^2*b-33*C*cos(d*x+c)^2*a^2*b-18*C*cos(d*x+
c)^2*a*b^2-26*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2-48*A*cos(d*x+c)*a^2*
b+144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+8*C*cos(d*x+c)^5*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+120*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*a*b^2+26*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-76*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+33*C*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+240*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-144*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^2+48*A*cos(d*x+c)*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+240*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-
144*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*a*b^2+120*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+26*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+33*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+30*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+33*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)

```
)*cos(d*x+c)*a^3+48*A*cos(d*x+c)^2*a^2*b+144*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^
3+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*a^3+33*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3+33*C*cos(d*x+c)
^2*a^3-16*C*cos(d*x+c)^2*b^3-33*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A
*cos(d*x+c)^2*b^3-48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*C*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b-48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+2\,Cab\cos(dx+c)^3+2\,Aab\cos(dx+c)+Aa^2+\left(Ca^2+Ab^2\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1432 $\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,d$

Optimal. Leaf size=695

$$\frac{a\left(15a^2C+432Ab^2+284b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\left(5a^2C+4b^2(4A+3C)\right)\sin(c+dx)\sqrt{a+b}}{32d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a
+ b]*(15*a^3*C + 24*b^3*(4*A + 3*C) + 2*a^2*b*(192*A + 59*C) + 4*a*b^2*(108
*A + 71*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d*
Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a^4*C - 120*a^2*b^2*(2*A + C) - 16*b^
4*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + ((5*a^2*C + 4*b^2*(4*A + 3*C))*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]]) + (5*a*C*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos
[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (a*(432*A*b^2 + 1
5*a^2*C + 284*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(192*b*d)
```

Rubi [A] time = 2.32322, antiderivative size = 695, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{a\left(15a^2C+432Ab^2+284b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\left(5a^2C+4b^2(4A+3C)\right)\sin(c+dx)\sqrt{a+b}}{32d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
```

```
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a
+ b]*(15*a^3*C + 24*b^3*(4*A + 3*C) + 2*a^2*b*(192*A + 59*C) + 4*a*b^2*(108
*A + 71*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d*
Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a^4*C - 120*a^2*b^2*(2*A + C) - 16*b^
4*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + ((5*a^2*C + 4*b^2*(4*A + 3*C))*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]]) + (5*a*C*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos
[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (a*(432*A*b^2 + 1
5*a^2*C + 284*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(192*b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}\left(A\right.}{\sqrt{\cos(c+}}$$

$$= \frac{C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{}\right.$$

$$= \frac{5aC(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24d\sqrt{\sec(c+dx)}} + \frac{C(a+b\cos(c+}{4d\sqrt{\text{se}}}$$

$$= \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(5a^2C+4b^2(4A+3C)\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32d\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(5a^4C-120a^2b^2(2A+C)-16b^4(4A+3C)\right)\sqrt{\text{c}}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(432Ab^2+15a^2C+284b^2C\right)\sqrt{\cos(c+}}{}$$

<u>**Mathematica**</u> **[A]** time = 20.2947, size = 603, normalized size = 0.87

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{1}{192}\left(59a^2C+48Ab^2+48b^2C\right)\sin(2(c+dx))+\frac{17}{96}abC\sin(c+dx)+\frac{17}{96}abC\sin(3(c+dx\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((17*a*b*C*Sin[c + d*x])/96 +
((48*A*b^2 + 59*a^2*C + 48*b^2*C)*Sin[2*(c + d*x)])/192 + (17*a*b*C*Sin[3*(
```

```
c + d*x)])/96 + (b^2*C*Sin[4*(c + d*x)])/32))/d + (Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a*b*(a + b)*(432*A*b^2
 + 15*a^2*C + 284*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] + a*(a + b)*(15*a^3*C - 30*a^2*b*C - 24*b^3*(4*A + 3*C) - 4*a*b^2*(84*A
 + 53*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*
x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 3*(5*a^4*
C - 120*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*EllipticF[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)] + a*b*(432*A*b^2 + 15*a^2*C + 284*b^2*C)*(a + b
*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c
+ d*x)/2]))/(192*b^2*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.392, size = 3993, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/192/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-1152*A*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a^2*b^2+96*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+1440*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+432*A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a^2*b^2+432*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+118*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*C*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^2+72*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+720*C*sin(d*x+c)*cos(d*x+c)*(co
```

s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+
15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b+284*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*C*cos(d*x+c)^6*b^4+28
4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^3+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+96*A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+1440*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*
b^2+432*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^2+432*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+118*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3+720*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a^2*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+284*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+284*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+184*C*cos(
d*x+c)^5*a*b^3+254*C*cos(d*x+c)^4*a^2*b^2+133*C*cos(d*x+c)^3*a^3*b-192*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
b^4+384*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*b^4-144*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d

```
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+15*C*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+2
4*C*cos(d*x+c)^4*b^4-96*A*cos(d*x+c)^2*b^4-72*C*cos(d*x+c)^2*b^4-15*C*cos(d
*x+c)*a^4+384*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+528*A*cos(d*x+c)^3*a*b^3+432*A*cos(d*x+c
)^2*a^2*b^2+15*C*cos(d*x+c)^2*a^4+96*A*cos(d*x+c)^4*b^4-284*C*cos(d*x+c)^2*
a*b^3-118*C*cos(d*x+c)*a^3*b-284*C*cos(d*x+c)*a^2*b^2-72*C*cos(d*x+c)*a*b^3
+172*C*cos(d*x+c)^3*a*b^3-432*A*cos(d*x+c)^2*a*b^3-432*A*cos(d*x+c)*a^2*b^2
-96*A*cos(d*x+c)*a*b^3-192*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+384*A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-14
4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*C*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-1
5*C*cos(d*x+c)^2*a^3*b+30*C*cos(d*x+c)^2*a^2*b^2+384*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-1152*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*
x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + 2\,Cab\cos(dx+c)^3 + 2\,Aab\cos(dx+c) + Aa^2 + \left(Ca^2 + Ab^2\right)\cos(dx+c)^2\right)\sqrt{b\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1433 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=806

$$\frac{C\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}-\frac{3aC\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}-\frac{\left(15a^2C-16b^2(5A+4C)\right)\sin(c+dx)(a+}{240bd\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(45*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A +
141*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d
*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*C - 30*a^3*b*C - 256*b^4*(5*A +
 4*C) - 12*a^2*b^2*(220*A + 141*C) - 8*a*b^3*(260*A + 193*C))*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[Sec[c + d*x]]) -
(a*Sqrt[a + b]*(3*a^4*C + 40*a^2*b^2*(2*A + C) + 80*b^4*(4*A + 3*C))*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(128*b^3*d*Sqrt
[Sec[c + d*x]]) + (a*(240*A*b^2 - 15*a^2*C + 172*b^2*C)*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(320*b*d*Sqrt[Sec[c + d*x]]) - ((15*a^2*C - 16*b^2*(5*A
 + 4*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x
]]) - (3*a*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c +
d*x]]) + (C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*
x]]) - ((45*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C))*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1920*b^2*d)
```

Rubi [A] time = 3.09615, antiderivative size = 806, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3050, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{C\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}-\frac{3aC\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}-\frac{\left(15a^2C-16b^2(5A+4C)\right)\sin(c+dx)(a+}{240bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(45*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A +
141*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d
*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*C - 30*a^3*b*C - 256*b^4*(5*A +
 4*C) - 12*a^2*b^2*(220*A + 141*C) - 8*a*b^3*(260*A + 193*C))*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[Sec[c + d*x]]) -
(a*Sqrt[a + b]*(3*a^4*C + 40*a^2*b^2*(2*A + C) + 80*b^4*(4*A + 3*C))*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(128*b^3*d*Sqrt
[Sec[c + d*x]]) + (a*(240*A*b^2 - 15*a^2*C + 172*b^2*C)*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(320*b*d*Sqrt[Sec[c + d*x]]) - ((15*a^2*C - 16*b^2*(5*A
 + 4*C))*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x
]]) - (3*a*C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c +
d*x]]) + (C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*
x]]) - ((45*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C))*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1920*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,($$

$$= \frac{C(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{3aC(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\cos(c+dx))^{7/2}\text{si}}{5bd\sqrt{\sec(c+dx}}$$

$$= -\frac{\left(15a^2C-16b^2(5A+4C)\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{240bd\sqrt{\sec(c+dx)}} - \frac{3a}{}$$

$$= \frac{a\left(240Ab^2-15a^2C+172b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{320bd\sqrt{\sec(c+dx)}} - \frac{(1}{}$$

$$= \frac{a\left(240Ab^2-15a^2C+172b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{320bd\sqrt{\sec(c+dx)}} - \frac{(1}{}$$

$$= \frac{a\left(240Ab^2-15a^2C+172b^2C\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{320bd\sqrt{\sec(c+dx)}} - \frac{(1}{}$$

$$= -\frac{a\sqrt{a+b}\left(3a^4C+40a^2b^2(2A+C)+80b^4(4A+3C)\right)\sqrt{\cos(c+dx}}{128}$$

$$= \frac{(a-b)\sqrt{a+b}\left(45a^4C-256b^4(5A+4C)-12a^2b^2(220A+141C)\right)}{19}$$

Mathematica [B] time = 22.1689, size = 2064, normalized size = 2.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Sec[c +
d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((80*A*b^2 + 93*a^2*C + 88*b^
2*C)*Sin[c + d*x])/960 + (a*(1040*A*b^2 + 15*a^2*C + 1024*b^2*C)*Sin[2*(c +
```

d*x)])/(1920*b) + ((80*A*b^2 + 93*a^2*C + 100*b^2*C)*Sin[3*(c + d*x)])/960
 + (21*a*b*C*Sin[4*(c + d*x)])/320 + (b^2*C*Sin[5*(c + d*x)])/80))/d + (Sqr
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-2640*a^3*A*b^2*Tan[(c + d*x)/2
] - 2640*a^2*A*b^3*Tan[(c + d*x)/2] - 1280*a*A*b^4*Tan[(c + d*x)/2] - 1280*
A*b^5*Tan[(c + d*x)/2] + 45*a^5*C*Tan[(c + d*x)/2] + 45*a^4*b*C*Tan[(c + d*
x)/2] - 1692*a^3*b^2*C*Tan[(c + d*x)/2] - 1692*a^2*b^3*C*Tan[(c + d*x)/2] -
 1024*a*b^4*C*Tan[(c + d*x)/2] - 1024*b^5*C*Tan[(c + d*x)/2] + 5280*a^2*A*b
^3*Tan[(c + d*x)/2]^3 + 2560*A*b^5*Tan[(c + d*x)/2]^3 - 90*a^4*b*C*Tan[(c +
 d*x)/2]^3 + 3384*a^2*b^3*C*Tan[(c + d*x)/2]^3 + 2048*b^5*C*Tan[(c + d*x)/2
]^3 + 2640*a^3*A*b^2*Tan[(c + d*x)/2]^5 - 2640*a^2*A*b^3*Tan[(c + d*x)/2]^5
 + 1280*a*A*b^4*Tan[(c + d*x)/2]^5 - 1280*A*b^5*Tan[(c + d*x)/2]^5 - 45*a^5
*C*Tan[(c + d*x)/2]^5 + 45*a^4*b*C*Tan[(c + d*x)/2]^5 + 1692*a^3*b^2*C*Tan[
(c + d*x)/2]^5 - 1692*a^2*b^3*C*Tan[(c + d*x)/2]^5 + 1024*a*b^4*C*Tan[(c +
d*x)/2]^5 - 1024*b^5*C*Tan[(c + d*x)/2]^5 + 2400*a^3*A*b^2*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 9600*a*
A*b^4*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] + 90*a^5*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] + 1200*a^3*b^2*C*EllipticPi[-1, -ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 7200*a*b^4*C*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] + 2400*a^3*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 9600*a*A*b^4*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + 90*a^5*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 1200*a^3*b^2*C*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 7200*a*b^4*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(45*a^4*
C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C))*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
 d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] - 2*a*b*(15*a^3*C - 6*a^2*b*(320*A + 191*C) + 4*a*b^2*(260*A + 193*C)
- 8*b^3*(380*A + 289*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*T

```
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(1920*b^2*d*Sqrt[1 + T
an[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2
)))
```

Maple [B] time = 0.525, size = 4726, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/1920/d/b^2*(-15*C*cos(d*x+c)^3*a^4*b+1752*C*cos(d*x+c)^5*a^2*b^3+774*C*c
os(d*x+c)^4*a^3*b^2+4720*A*cos(d*x+c)^3*a^2*b^3+2640*A*cos(d*x+c)^2*a^3*b^2
-3840*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b^2+2720*A*cos(d*x+c)^4*a*b^4+45*C*cos(d*x+c)*a^5+
384*C*cos(d*x+c)^7*b^5+128*C*cos(d*x+c)^5*b^5+640*A*cos(d*x+c)^5*b^5-45*C*c
os(d*x+c)^2*a^5+30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^4*b-2292*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1544*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-4624*C*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^4-45*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4*b+1692*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1692*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+1024*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+1200*C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^3*b^2+7200*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a*b^4+2080*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
```

((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-6080*A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^4+2640*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2640*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+1280*A*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^4+2400*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2+9600*A*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b
^4+30*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^4*b-2292*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1544*C*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^3-4624*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-45*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+1692*C*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3*b^2+1692*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+1024*C*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+1200*C*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a^3*b^2+7200*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4+640*A*cos(d*x+c)^3*b^5-
1280*A*cos(d*x+c)^2*b^5+512*C*cos(d*x+c)^3*b^5-1024*C*cos(d*x+c)^2*b^5-45*C
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^5+1024*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^5+90*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a

```
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5+1280*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-45*C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^5+1024*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^5+90*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5-3840*A*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+20
80*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b^3-6080*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^4+2640*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2640*A*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+1280*A*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^4+2400*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a^3*b^2+9600*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4+1280*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-30*C*cos(d*x+c)*a^
4*b-1692*C*cos(d*x+c)*a^3*b^2-1544*C*cos(d*x+c)*a^2*b^3-1024*C*cos(d*x+c)*a
*b^4+45*C*cos(d*x+c)^2*a^4*b+918*C*cos(d*x+c)^2*a^3*b^2-1692*C*cos(d*x+c)^2
*a^2*b^3-1032*C*cos(d*x+c)^2*a*b^4+664*C*cos(d*x+c)^4*a*b^4+1484*C*cos(d*x+
c)^3*a^2*b^3+1392*C*cos(d*x+c)^6*a*b^4-2640*A*cos(d*x+c)^2*a^2*b^3-1440*A*c
os(d*x+c)^2*a*b^4-2640*A*cos(d*x+c)*a^3*b^2-2080*A*cos(d*x+c)*a^2*b^3-1280*
A*cos(d*x+c)*a*b^4)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x +
c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\cos\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)+Aa^2+\left(Ca^2+Ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+\right.}}{\sqrt{\sec\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^3 + 2*A*a*b*cos(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x +
c)), x)
```

3.1434 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=469

$$\frac{2\left(5a^2(5A+7C)+24Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^3d}-\frac{2\sqrt{a+b}\left(-a^2(44Ab+70bC)-5a^3(5A+7C)+\right.}{}$$

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(24*A*b^2 + a^2*(22*A + 35*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^5*d*Sqrt[Sec[c + d*x]]) - (2*Sqr
t[a + b]*(12*a*A*b^2 - 48*A*b^3 - 5*a^3*(5*A + 7*C) - a^2*(44*A*b + 70*b*C)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec
[c + d*x]]) + (2*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^3*d) - (12*A*b*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 1.43348, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2(5A+7C)+24Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^3d}-\frac{2\sqrt{a+b}\left(-a^2(44Ab+70bC)-5a^3(5A+7C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (-4*(a - b)*b*Sqrt[a + b]*(24*A*b^2 + a^2*(22*A + 35*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^5*d*Sqrt[Sec[c + d*x]]) - (2*Sqr
t[a + b]*(12*a*A*b^2 - 48*A*b^3 - 5*a^3*(5*A + 7*C) - a^2*(44*A*b + 70*b*C)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
```

```
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec
[c + d*x]]) + (2*(24*A*b^2 + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^3*d) - (12*A*b*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{12Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35a^2d} + \frac{2A\sqrt{a+b\cos(c+dx}}{}$$

$$= \frac{2\left(24Ab^2 + 5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105a^3d} - \frac{1}{}$$

$$= \frac{2\left(24Ab^2 + 5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105a^3d} - \frac{1}{}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\left(24Ab^2 + a^2(22A+35C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{105a^5d\sqrt{\sec(c+d}}$$

Mathematica [B] time = 23.2581, size = 3164, normalized size = 6.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(22*a^2*A + 24*A*b^2 +
35*a^2*C)*Sin[c + d*x])/(105*a^4) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d*x]
+ 24*A*b^2*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a^3) - (12*A*b*Sec[c
 + d*x]*Tan[c + d*x])/(35*a^2) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(7*a)))/
d + (4*((44*A*b)/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*
A*b^3)/(35*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*C)/(3*a*
Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*A*Sqrt[Sec[c + d*x]])/(21
*Sqrt[a + b*Cos[c + d*x]]) + (32*A*b^2*Sqrt[Sec[c + d*x]])/(105*a^2*Sqrt[a
+ b*Cos[c + d*x]]) + (16*A*b^4*Sqrt[Sec[c + d*x]])/(35*a^4*Sqrt[a + b*Cos[c
 + d*x]]) + (C*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*C*
Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) + (44*A*b^2*Cos[2*(c +
 d*x)]*Sqrt[Sec[c + d*x]])/(105*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^4*C
```

```
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^4*Sqrt[a + b*Cos[c + d*x]]) + (2*
b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]])
)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*(24*A*b^2 + a^2*(22*A
+ 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + a*(-12*a*A*b^2 - 48*A*b^3 + 5*a^3*(5*A + 7*C) - 2*a^2*b*(22*A + 35
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + b*(24*A*b^2 + a^2*(22*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^4*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]*((2*b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*
x]*(2*b*(a + b)*(24*A*b^2 + a^2*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(-12*a*A*b^2 - 48*A*b^3 +
5*a^3*(5*A + 7*C) - 2*a^2*b*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(24*A*b^2 + a^2*(22*A + 35*C))
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*a^4*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*b*(a + b)*(24*A*b^2 + a^2*(
22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] + a*(-12*a*A*b^2 - 48*A*b^3 + 5*a^3*(5*A + 7*C) - 2*a^2*b*(22*A
 + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + b*(24*A*b^2 + a^2*(22*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^4*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((b*(24
*A*b^2 + a^2*(22*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^4)/2 + (b*(a + b)*(24*A*b^2 + a^2*(22*A + 35*C))*Sqrt[(a + b*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*
x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(-12*a*A
*b^2 - 48*A*b^3 + 5*a^3*(5*A + 7*C) - 2*a^2*b*(22*A + 35*C))*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Si
n[c + d*x]/(1 + Cos[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) +
 (b*(a + b)*(24*A*b^2 + a^2*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((
a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))] + (a*(-12*a*A*b^2 - 48*A*b^3 + 5*a^3*(5*A + 7*C) - 2*a^2*b*(22*
A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(
```

```
2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]) - b^2*(24*A*b^2
+ a^2*(22*A + 35*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
d*x)/2] - b*(24*A*b^2 + a^2*(22*A + 35*C))*(a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + b*(24*A*b^2 + a^2*(22*A + 35*C))*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*
(-12*a*A*b^2 - 48*A*b^3 + 5*a^3*(5*A + 7*C) - 2*a^2*b*(22*A + 35*C))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - (
(-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (b*(a + b)*(24*A*b^2 + a^2*(22*A +
 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d
*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*a^4*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(2*b*(a + b)*(24*A*b^2 + a^2*(22*A
 + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + a*(-12*a*A*b^2 - 48*A*b^3 + 5*a^3*(5*A + 7*C) - 2*a^2*b*(22*A + 3
5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + b*(24*A*b^2 + a^2*(22*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c
+ d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^4*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]))
```

Maple [B] time = 0.307, size = 2775, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d/a^4*(-15*A*a^4+48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-12*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2-48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^4*si
```

n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-12*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^4*a^2*b^2+35*C*cos(d*x+c)^3*a^3*b+35*C*

```
cos(d*x+c)^5*a^3*b-70*C*cos(d*x+c)^5*a^2*b^2-70*C*cos(d*x+c)^4*a^3*b+24*A*c
os(d*x+c)^3*a*b^3-6*A*cos(d*x+c)^2*a^2*b^2+3*A*cos(d*x+c)*a^3*b+25*A*cos(d*
x+c)^5*a^3*b-44*A*cos(d*x+c)^5*a^2*b^2+24*A*cos(d*x+c)^5*a*b^3-44*A*cos(d*x
+c)^4*a^3*b+50*A*cos(d*x+c)^4*a^2*b^2-48*A*cos(d*x+c)^4*a*b^3+16*A*cos(d*x+
c)^3*a^3*b+25*A*cos(d*x+c)^4*a^4+35*C*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^
4-35*C*cos(d*x+c)^2*a^4-48*A*cos(d*x+c)^5*b^4+48*A*cos(d*x+c)^4*b^4+44*A*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)/(a+b*cos(d*x+c))^(1/2
)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(9/2)/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(9/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(9/2)/sqrt(b*cos(d*x + c) + a
), x)
```

3.1435 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=394

$$\frac{2\sqrt{a+b}\left(-3a^2(3A+5C)+2aAb-8Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a +
 b]*(2*a*A*b - 8*A*b^2 - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (8*A*b*Sqrt[a + b*Co
s[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d) + (2*A*Sqrt[a + b*C
os[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 1.0099, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 3055, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(-3a^2(3A+5C)+2aAb-8Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a +
 b]*(2*a*A*b - 8*A*b^2 - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (8*A*b*Sqrt[a + b*Co
s[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d) + (2*A*Sqrt[a + b*C
```

```
os[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{8Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2A\sqrt{a+b\cos(c+dx)}}{5}$$

$$= -\frac{8Ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2A\sqrt{a+b\cos(c+dx)}}{5}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^2+3a^2(3A+5C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\right.\right.}{15a^4d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 22.1439, size = 2920, normalized size = 7.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 8*A*b^2 + 15*a^
2*C)*Sin[c + d*x])/(15*a^3) - (8*A*b*Tan[c + d*x])/(15*a^2) + (2*A*Sec[c +
d*x]*Tan[c + d*x])/(5*a)))/d + (2*((-3*A)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (8*A*b^2)/(15*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - C/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*A*b*Sqrt[Sec[c
+ d*x]])/(15*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^3*Sqrt[Sec[c + d*x]])/(15
*a^3*Sqrt[a + b*Cos[c + d*x]]) - (b*C*Sqrt[Sec[c + d*x]])/(a*Sqrt[a + b*Cos
[c + d*x]]) - (3*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*C
os[c + d*x]]) - (8*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[
a + b*Cos[c + d*x]]) - (b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*Sqrt[a
+ b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(8*A*
b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + 2*a*(2*a*A*b + 8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b
^2 + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]))/(15*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/
2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8
*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(2*a*A*b + 8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*
A*b^2 + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(15*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*
x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a
+ b)*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(2*a*A*b + 8*A*b^2 + 3*a^2*(3*A + 5*
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
] - (8*A*b^2 + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c
 + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^2 + 3*a
```

```
^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - (
(a + b)*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((
Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c +
 d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(2*a*A*b + 8*A*b^2 + 3*
a^2*(3*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])] - ((a + b)*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c
+ d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(2*a*A*b + 8*A*b^2 + 3*a^2*(3*A +
5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) +
((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(8*A*b^2 + 3*a^2*(3*A
 + 5*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (8
*A*b^2 + 3*a^2*(3*A + 5*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c +
 d*x]*Tan[(c + d*x)/2] - (8*A*b^2 + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(2*a*A*b + 8*A*b^2
 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)
*(8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1
 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(
15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(8
*A*b^2 + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(2*a*A*b + 8*A*b^2 + 3*a^2*(3*A + 5*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*
A*b^2 + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
 Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(15*a^3*Sqrt[a + b*Cos[c +
d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.247, size = 2243, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)

[Out] -2/15/d/a^3*(-3*A*a^3-4*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos(d*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2+8*A*cos(d*x+c)^3*a*b^2+A*cos(d*x+c)*a^2*b-10*A*cos(d*x+c)^3*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15*C*cos(d*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3-8*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos

```
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d
*x+c)*(1/cos(d*x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a
), x)
```

3.1436 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=323

$$\frac{2\sqrt{a+b}(a(A+3C)+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\ldots 4A$$

```
[Out] (-4*A*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a*(A + 3*C))*
Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c +
 d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3
*a*d)
```

Rubi [A] time = 0.679344, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3056, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a(A+3C)+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\ldots 4A$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (-4*A*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a*(A + 3*C))*
Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c +
 d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3
*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\left(2Ab\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{4A(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{3a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.2887, size = 303, normalized size = 0.94

$$\frac{2\left(\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(a(a(A+3C)-2Ab)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a\sec(c+dx)+b}{(a+b)(\sec(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{b-a}{a+b}\right)+Ab\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx\right.\right.}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}\right.}{3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] (2*((2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*A*b*(a + b)*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(
b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + a*(-2*A*b + a*(A + 3*C)
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d
*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + A*b*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[
Sec[(c + d*x)/2]^2] + A*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*(-2*b*Sin[c
 + d*x] + a*Tan[c + d*x])))/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.243, size = 1102, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a^2*(2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*
a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^2*a*b+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+2*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+3*C*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2+A*cos(d*x+c)^3*a*b-2*A*cos(d*x+c)^3*b^2+A*cos(d*x+c)^2*a^2-2*A*cos(d*x+c
)^2*a*b+2*A*cos(d*x+c)^2*b^2+A*cos(d*x+c)*a*b-A*a^2)*cos(d*x+c)*(1/cos(d*x+
c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a
), x)
```

3.1437 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=403

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2A\sqrt{a+b}\sqrt{}}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*A*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*C*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.619656, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3054, 2809, 12, 2801, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2A\sqrt{a+b}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]],x]
```



```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*A*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*C*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
```

```
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[S
ec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx+\left(C\sqrt{c\right.$$

$$=-\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

$$=-\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

$$=\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 15.9639, size = 624, normalized size = 1.55

$$\frac{2\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(a(A-C)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\left(\tan\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (2*S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-(a*A*Tan[(c + d*x)/2]) - A*b*Tan[(c +
d*x)/2] + 2*A*b*Tan[(c + d*x)/2]^3 + a*A*Tan[(c + d*x)/2]^5 - A*b*Tan[(c +
d*x)/2]^5 - 2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)] - 2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - A*(a + b)*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + a*(A - C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*d*(1 + Tan
[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.222, size = 1000, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d/a*(A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
```

```
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-C*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+A*cos(d*x+c)^2*
b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/c
os(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a
), x)
```

3.1438 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=453

$$\frac{\sqrt{a+b}(aC+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}+aC\sqrt{a+b}$$

```
[Out] -(((a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A*b + a*C)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) + (a*Sqr
t[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b^2*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.914585, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3062, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(aC+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}+aC\sqrt{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A*b + a*C)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) + (a*Sqr
t[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
```

```
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b^2*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3062

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -
Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*f*Sqrt[a + b*Sin[e + f*x]
]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - A*b
*d)*Sin[e + f*x] - C*(b*c + a*d)*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^
(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{a\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 11.5761, size = 340, normalized size = 0.75

$$\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(4Ab\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+C\cos(c+dx)\tan(\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*
x]],x]
```

```
[Out] (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*C*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*A*b*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*C*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + C*Cos
[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b*d*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [A] time = 0.222, size = 818, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
```

```
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b+C*cos(d*x+c)^3*b+C*cos(d*x+c)^2*a-C*cos(d*x+c)^2*b-C*cos(d*x+c)*a
)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\cos^2\left(c + dx\right)\right)\sqrt{\sec\left(c + dx\right)}}{\sqrt{a + b\cos\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sqrt{\sec\left(dx + c\right)}}{\sqrt{b\cos\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a
), x)
```

3.1439 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=515

$$-\frac{\sqrt{a+b}\left(3a^2C+4b^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|-}{4b^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (3*(a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - ((3*a - 2*b)*Sqrt[a + b]*C*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - (Sq
rt[a + b]*(3*a^2*C + 4*b^2*(2*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c + d*x]]) - (3*a*C*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d)
```

Rubi [A] time = 1.23886, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3050, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2C+4b^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|-}{4b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (3*(a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - ((3*a - 2*b)*Sqrt[a + b]*C*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - (Sq
```

```
rt[a + b]*(3*a^2*C + 4*b^2*(2*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c + d*x]]) - (3*a*C*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aC}{2}+b(2A+C)\,\mathrm{co}}{\sqrt{\cos(c+d}}}{2b}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3aC\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+}{4b^2d}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3aC\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+}{4b^2d}$$

$$= -\frac{\sqrt{a+b}\left(3a^2C+4b^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{co}}}{\sqrt{a+b}\sqrt{\mathrm{co}}}\right.\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{3(a-b)\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{4b^2d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 14.8489, size = 1399, normalized size = 2.72

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) +
(3*a^2*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2] + 3*a*b*Sqrt[(a - b)/(a + b
)]*C*Tan[(c + d*x)/2] - 6*a*b*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^3 -
3*a^2*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^5 + 3*a*b*Sqrt[(a - b)/(a +
b)]*C*Tan[(c + d*x)/2]^5 + (16*I)*A*b^2*EllipticPi[(a + b)/(a - b), I*ArcSi
nh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + (6*I)*a^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)
/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8
*I)*b^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(
c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
```

```
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (16*I)*A*b^2*Elli
pticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
-((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^2*C*
EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^
2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (3*I
)*a*(a - b)*C*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
-((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*
(4*A*b^2 + 3*a^2*C - a*b*C + 2*b^2*C)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a +
 b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)])/(4*b^2*Sqrt[(a - b)/(a + b)]*d*Sqrt[(1 + Tan[(c + d*x)/2]^
2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [B] time = 0.199, size = 1636, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d/b^2*(8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^2-16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-2*C*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
a*b+4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*b^2+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*C*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-6*C*cos(
```

```
d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^2-8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-16*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-2*C*cos(
d*x+c)^4*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b+4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-6*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-8*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^2+C*cos(d*x+c)^3*a*b+3*C*cos(d*x+c)^2*a^2-3*C*cos(d*x+c)^2*a*b+2*b^2*C*
cos(d*x+c)^2-3*C*cos(d*x+c)*a^2+2*C*cos(d*x+c)*a*b)*(1/cos(d*x+c))^(1/2)/si
n(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+C\cos^{2}\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x
))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+A}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c
))), x)
```

3.1440 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=534

$$-\frac{2\left(6Ab^2-a^2(A-5C)\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2b\left(8A\right.}{}$$

```
[Out] (-2*(16*A*b^4 - 2*a^2*b^2*(4*A - 5*C) - a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^5*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]
) - (2*(12*a*A*b^2 + 16*A*b^3 + 2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^4*Sqrt[a + b]*d*Sqrt
[Sec[c + d*x]]) + (2*b*(8*A*b^2 - a^2*(3*A - 5*C))*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d) + (2*(A*b^2 + a^2*C
)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
]) - (2*(6*A*b^2 - a^2*(A - 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.69996, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 3055, 2998, 2816, 2994}

$$-\frac{2\left(6Ab^2-a^2(A-5C)\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2b\left(8A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + b*Cos[c + d*x])^(3/2),
x]
```


```
[Out] (-2*(16*A*b^4 - 2*a^2*b^2*(4*A - 5*C) - a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^5*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]
) - (2*(12*a*A*b^2 + 16*A*b^3 + 2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt
```

```
[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^4*Sqrt[a + b]*d*Sqrt
[Sec[c + d*x]]) + (2*b*(8*A*b^2 - a^2*(3*A - 5*C))*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d) + (2*(A*b^2 + a^2*C
)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
]) - (2*(6*A*b^2 - a^2*(A - 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2\left(Ab^2+a^2C\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots}{\cdots}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-a^2(A-5C)\right)\sqrt{a+b\,\mathrm{co}\cdots}}{5a^2\left(a^2\cdots\right.}$$

$$= \frac{2b\left(8Ab^2-a^2(3A-5C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5a^3\left(a^2-b^2\right)d} + \frac{2\left(\cdots\right.}{\cdots}$$

$$= \frac{2b\left(8Ab^2-a^2(3A-5C)\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5a^3\left(a^2-b^2\right)d} + \frac{2\left(\cdots\right.}{\cdots}$$

$$= -\frac{2\left(16Ab^4-2a^2b^2(4A-5C)-a^4(3A+5C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\mathrm{si}\cdots\right.}{5a^5\sqrt{a+b}d\sqrt{\sec(c+\cdots}}$$

Mathematica [B] time = 25.6126, size = 3767, normalized size = 7.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + b*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A + 8*a^2*A*b^2 - 1
6*A*b^4 + 5*a^4*C - 10*a^2*b^2*C)*Sin[c + d*x])/(5*a^4*(a^2 - b^2)) + (2*(A
*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c
+ d*x])) - (6*A*b*Tan[c + d*x])/(5*a^3) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(
5*a^2)))/d + (2*((-3*a*A)/(5*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (8*A*b^2)/(5*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) + (16*A*b^4)/(5*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (a*C)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) + (2*b^2*C)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
- (4*A*b*Sqrt[Sec[c + d*x]])/(5*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (12
```

```
*A*b^3*Sqrt[Sec[c + d*x]])/(5*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (
16*A*b^5*Sqrt[Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) -
 (2*b*C*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3
*C*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]
]) - (8*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a^2*(a^2 - b^2)*Sqrt[
a + b*Cos[c + d*x]]) + (16*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a^
4*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*C*Cos[2*(c + d*x)]
*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(4*A - 5*C)
+ a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 2*a^2*b*(2*A + 5*C)
 + a^3*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] + (16*A*b^4 + 2*a^2*b^2*(-4*A + 5*C) - a^4*(3*A + 5*C))*C
os[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*a
^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-16*A*b^4 + 2*a
^2*b^2*(4*A - 5*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*b^3 -
2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 2*a^2*b^2*(-4*A + 5*C) -
 a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(5*a^4*(a^2 - b^2)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c +
d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(
a + b)*(-16*A*b^4 + 2*a^2*b^2*(4*A - 5*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(1
2*a*A*b^2 - 16*A*b^3 - 2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 2
*a^2*b^2*(-4*A + 5*C) - a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*a^4*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(((16*A*b^4 + 2*a^2*b^2*(-4*A + 5*C) - a^4*(3*A + 5*C))*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-16*A*b^4 + 2*a^2*b^2*(4*
A - 5*C) + a^4*(3*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*
x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(12*a*A*b^2 - 16*A*b^3 -
2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
```

```
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(
(Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
+ d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-16*A*b^4 + 2*a
^2*b^2*(4*A - 5*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + (a*(a + b)*(12*a*A*b^2 - 16*A*b^3 - 2*a^2*b*(2*A + 5*C) + a^3*(3*A
 + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
 + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(16*A*b^4 + 2*a^2*
b^2*(-4*A + 5*C) - a^4*(3*A + 5*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c +
 d*x]*Tan[(c + d*x)/2] - (16*A*b^4 + 2*a^2*b^2*(-4*A + 5*C) - a^4*(3*A + 5*
C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] +
 (16*A*b^4 + 2*a^2*b^2*(-4*A + 5*C) - a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(12*a*A*b^
2 - 16*A*b^3 - 2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Se
c[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c +
 d*x)/2]^2)/(a + b)]) - ((a + b)*(-16*A*b^4 + 2*a^2*b^2*(4*A - 5*C) + a^4*(
3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c
 + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(5*a^4*(a^2 - b^2)*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-16*A*b^4
 + 2*a^2*b^2*(4*A - 5*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(12*a*A*b^2 - 16*A*
b^3 - 2*a^2*b*(2*A + 5*C) + a^3*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (16*A*b^4 + 2*a^2*b^2*(-4*A +
5*C) - a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Co
s[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(5*a^4*(a^2 - b^2)*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
])))
```

Maple [B] time = 0.317, size = 4077, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d/(a+b)/(a-b)/a^4*(-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+A*a^3*b^2-A*a^5-5*C*cos
(d*x+c)^3*a^4*b+5*C*cos(d*x+c)^4*a^3*b^2-5*A*cos(d*x+c)^3*a^4*b-6*A*cos(d*x
+c)^3*a^2*b^3-6*A*cos(d*x+c)^2*a^3*b^2+2*A*cos(d*x+c)*a^4*b-16*A*cos(d*x+c)
^4*b^5+3*A*cos(d*x+c)^3*a^5+5*C*cos(d*x+c)^3*a^5-3*A*cos(d*x+c)^4*a^3*b^2+8
*A*cos(d*x+c)^4*a*b^4-2*A*cos(d*x+c)^2*a^5-5*C*cos(d*x+c)^2*a^5-10*C*cos(d*
x+c)^4*a^2*b^3-10*C*cos(d*x+c)^3*a^3*b^2+5*C*cos(d*x+c)^4*a^4*b+3*A*cos(d*x
+c)^4*a^4*b+8*A*cos(d*x+c)^4*a^2*b^3+8*A*cos(d*x+c)^3*a^3*b^2-16*A*cos(d*x+
c)^3*a*b^4+16*A*cos(d*x+c)^3*b^5+5*C*cos(d*x+c)^2*a^3*b^2+10*C*cos(d*x+c)^3
*a^2*b^3+8*A*cos(d*x+c)^2*a*b^4-2*A*cos(d*x+c)*a^2*b^3-16*A*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^4-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4*b+10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+10*C*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^3-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-10*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-3*A*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^4*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+16*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-A*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4*b+8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-4*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-16*A*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^4-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
```

)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+10*C*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+1
0*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^3-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-10*C*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b^2-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^4*b-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-8*A*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^3+16*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+8*A*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*b^2-4*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-5*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+5*C*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^5-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^5+16*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+3*A*sin(d*x+c)*cos(d
x+c)^2(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^5-5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^5+5*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-3*A*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)

```
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+16*
A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^5+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5)*cos(d*x+c)*(1/cos(d*x+c))^(7/2
)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

3.1441 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=432

$$-\frac{2\left(4Ab^2-a^2(A-3C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2(A\right.}{}$$

```
[Out] (2*b*(8*A*b^2 - a^2*(5*A - 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(6*a*A*b + 8*A*b^
2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 + a^2*C)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2
- a^2*(A - 3*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.19235, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 3055, 2998, 2816, 2994}

$$-\frac{2\left(4Ab^2-a^2(A-3C)\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2(A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3/2),
x]
```



```
[Out] (2*b*(8*A*b^2 - a^2*(5*A - 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(6*a*A*b + 8*A*b^
2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
```

```
*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 + a^2*C)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2
- a^2*(A - 3*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(3*a^2*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2\left(Ab^2+a^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(4Ab^2-a^2(A-3C)\right)\sqrt{a+b\,\mathrm{co}}}{3a^2\left(a^2\right.}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(4Ab^2-a^2(A-3C)\right)\sqrt{a+b\,\mathrm{co}}}{3a^2\left(a^2\right.}$$

$$= \frac{2b\left(8Ab^2-a^2(5A-3C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3a^4\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 22.9347, size = 3204, normalized size = 7.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*(5*a^2*A - 8*A*b^2 - 3*
a^2*C)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) - (2*(A*b^3*Sin[c + d*x] + a^2*b*C
*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])
/(3*a^2)))/d - (2*((5*A*b)/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (8*A*b^3)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (b*C)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) + (a*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (
7*A*b^2*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (8
*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (
a*C*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b^2*C*Sqr
t[Sec[c + d*x]])/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^2*Cos[2*
(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) -
 (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a +
b*Cos[c + d*x]]) - (b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^
```

2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a + b)*(-6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*(-(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a + b)*(-6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a + b)*(-6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((b*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (a*(a + b)*(-6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (a*(a + b)*(-6*a*A*b + 8*A*b^2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b^2*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - b*(8*A*b^2 + a^2*(-5*A + 3*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c

```
+ d*x]*Tan[(c + d*x)/2] + b*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (a*(a + b)*(-6*a*A*
b + 8*A*b^2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (
b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^
2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2
]^2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]
) - ((2*b*(a + b)*(8*A*b^2 + a^2*(-5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a + b)*(-6*a*A*b + 8*A*
b^2 + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] + b*(8*A*b^2 + a^2*(-5*A + 3*C))*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c +
 d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3
*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.219, size = 2676, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/(a+b)/(a-b)/a^3*(a^2*A*b^2-A*a^4+3*C*cos(d*x+c)^3*a^2*b^2-5*A*cos(d*
x+c)^3*a^2*b^2-5*A*cos(d*x+c)^2*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+8*A*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a*b^3+5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+3*C*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
```

```
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*C*cos(d*x+c)^3*a^3*b+8*A*cos(d*x+c)^3*b^
4-8*A*cos(d*x+c)^2*b^4-5*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-4*A*cos(d*x+c)^3*a*b^3+4*A*co
s(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b+A*cos(d*x+c)^3*a^3*b+A*cos(d*x+c)^2
*a^4+8*A*cos(d*x+c)^2*a*b^3-4*A*cos(d*x+c)*a*b^3+3*C*cos(d*x+c)^2*a^3*b-3*C
*cos(d*x+c)^2*a^2*b^2+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8
*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*A*cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
+8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^3-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
```

```
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b)*cos(d*x+c)*(1/cos(d*x+c
))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

3.1442 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=348

$$\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2Ab^2-a^2(A-C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(2*A*b^2 - a^2*(A - C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A - C))*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[S
ec[c + d*x]]) + (2*(A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.816186, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4221, 3056, 2998, 2816, 2994}

$$\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2Ab^2-a^2(A-C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2),
x]
```


```
[Out] (-2*(2*A*b^2 - a^2*(A - C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A - C))*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[S
ec[c + d*x]]) + (2*(A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$=\frac{2\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a\left(a\right.}$$

$$=\frac{2\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{\left((a-b)(2Ab+a(A-C))\sqrt{\cos(c\right.}{}$$

$$=-\frac{2\left(2Ab^2-a^2(A-C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{a^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 18.0122, size = 456, normalized size = 1.31

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(a^2A-a^2C-2Ab^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)}+\frac{2\left(a^2C\sin(c+dx)+Ab^2\sin(c+dx)\right)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d}+\frac{2\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\cos(c+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2*A - 2*A*b^2 - a^2*C)*
Sin[c + d*x])/(a^2*(a^2 - b^2)) + (2*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*
x]))/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[2]*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(3/2)*(-((a + b)*((-2*A*b^2 + a^2*(A - C))*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(2*A*b + a*(-A + C))*Ellipti
```

```
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c
+ d*x]) + (2*A*b^2 + a^2*(-A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*Sqrt[(1 + Cos[c + d*x])^
(-1)]*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.248, size = 2287, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/a^2/(a+b)/(a-b)*(a*A*b^2-A*a^3+A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a*b
^2-C*cos(d*x+c)^2*a^2*b+C*cos(d*x+c)*a^2*b-A*cos(d*x+c)*a^2*b-A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A
*cos(d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a^2*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a*b^2+A*cos(d*x+c)*a^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+2*A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-C*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+A*cos(d*x+c)^2*a^2*b
-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
```

```
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*b^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+C*cos(d*x+c)^2*a^3-C*cos(d*x+c)*a^3
-2*A*cos(d*x+c)^2*b^3-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-
A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b)*cos(d*x+c)*(1/c
os(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3
/2), x)
```

3.1443 $\int \frac{(A+C\cos^2(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=481

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin\right.}{a^2bd\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a
^2*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b - a*C)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) -
 (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.997279, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4221, 3052, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin\right.}{a^2bd\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2),
x]
```


```
[Out] (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a
^2*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b - a*C)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) -
```

```
(2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3052

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Dist[C/
(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[1/b
, Int[(A*b - a*C*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{Ab-aC\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{b} + \frac{\left(C\sqrt{\cos(c+d\right.}{}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{a^2b\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 17.9376, size = 1027, normalized size = 2.14

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(Ca^2+Ab^2\right)\sin(c+dx)}{ab\left(a^2-b^2\right)}+\frac{2\left(C\sin(c+dx)a^2+Ab^2\sin(c+dx)\right)}{b\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d}-\frac{2\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(Ab^3\tan^5\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^
(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b^2 + a^2*C)*Sin[c + d*
x])/(a*b*(a^2 - b^2)) + (2*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(b*(-
a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (2*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1
)]*(a*A*b^2*Tan[(c + d*x)/2] + A*b^3*Tan[(c + d*x)/2] + a^3*C*Tan[(c + d*x)
/2] + a^2*b*C*Tan[(c + d*x)/2] - 2*A*b^3*Tan[(c + d*x)/2]^3 - 2*a^2*b*C*Tan
[(c + d*x)/2]^3 - a*A*b^2*Tan[(c + d*x)/2]^5 + A*b^3*Tan[(c + d*x)/2]^5 - a
^3*C*Tan[(c + d*x)/2]^5 + a^2*b*C*Tan[(c + d*x)/2]^5 + 2*a^3*C*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a
*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + 2*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*b^2*C*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] + (a + b)*(A*b^2 + a^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - a*b*(a +
 b)*(A + C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*(a^3 - a*b^2)*d*(1 + Tan[(c + d*
x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.234, size = 2049, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/(a+b)/(a-b)/a/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-A*cos(d*
x+c)^2*a*b^2+A*cos(d*x+c)*a*b^2+C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)*a^2*b+A*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b-A*cos(d*x+c)*b^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+C*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*a*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+C*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*b^3+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
```

```
a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^3+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*cos(d*x+c)^2*a^3+C*cos(d*x+c)*a^3+A*c
os(d*x+c)^2*b^3)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3
/2), x)
```

3.1444 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=563

$$\frac{\left(3a^2C+2Ab^2-b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{(3a^2\cdots}{}$$

```
[Out] -(((2*A*b^2 + 3*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((2*A*b^2 + a*(3*a +
b)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a +
 b]*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^
2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + ((2*A*b^2 + 3*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.49766, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3048, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2C+2Ab^2-b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{(3a^2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),
x]
```


```
[Out] -(((2*A*b^2 + 3*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((2*A*b^2 + a*(3*a +
b)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
```

```
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a +
 b]*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^
2 + a^2*C)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + ((2*A*b^2 + 3*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}}\,dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{\sqrt{\cos(c + dx)}\left(A + C\cos^2(c + dx)\right)}{(a + b\cos(c + dx))^{3/2}}\,dx$$

$$= -\frac{2\left(Ab^2 + a^2C\right)\sin(c + dx)}{b\left(a^2 - b^2\right)d\sqrt{a + b\cos(c + dx)}\sqrt{\sec(c + dx)}} - \frac{\left(2\sqrt{\cos(c + dx)}\sqrt{\sec(c + d\right.}{}$$

$$= -\frac{2\left(Ab^2 + a^2C\right)\sin(c + dx)}{b\left(a^2 - b^2\right)d\sqrt{a + b\cos(c + dx)}\sqrt{\sec(c + dx)}} + \frac{\left(2Ab^2 + 3a^2C - b^2C\right)\sqrt{a\ }}{}$$

$$= -\frac{2\left(Ab^2 + a^2C\right)\sin(c + dx)}{b\left(a^2 - b^2\right)d\sqrt{a + b\cos(c + dx)}\sqrt{\sec(c + dx)}} + \frac{\left(2Ab^2 + 3a^2C - b^2C\right)\sqrt{a\ }}{}$$

$$= \frac{3a\sqrt{a + b}C\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^3d\sqrt{\sec(c + dx)}}$$

$$= -\frac{\left(2Ab^2 + 3a^2C - b^2C\right)\sqrt{\cos(c + dx)}\csc(c + dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{ab^2\sqrt{a + b}d\sqrt{\sec(c + dx)}}$$

Mathematica [B] time = 19.2505, size = 1163, normalized size = 2.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d
*x]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b^2 + a^2*C)*Sin[c + d*
x])/(b^2*(-a^2 + b^2)) - (2*(a*A*b^2*Sin[c + d*x] + a^3*C*Sin[c + d*x]))/(b
^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^
(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(
c + d*x)/2]^2)]*(2*a*A*b^2*Tan[(c + d*x)/2] + 2*A*b^3*Tan[(c + d*x)/2] + 3*
a^3*C*Tan[(c + d*x)/2] + 3*a^2*b*C*Tan[(c + d*x)/2] - a*b^2*C*Tan[(c + d*x)
/2] - b^3*C*Tan[(c + d*x)/2] - 4*A*b^3*Tan[(c + d*x)/2]^3 - 6*a^2*b*C*Tan[(
c + d*x)/2]^3 + 2*b^3*C*Tan[(c + d*x)/2]^3 - 2*a*A*b^2*Tan[(c + d*x)/2]^5 +
 2*A*b^3*Tan[(c + d*x)/2]^5 - 3*a^3*C*Tan[(c + d*x)/2]^5 + 3*a^2*b*C*Tan[(c
 + d*x)/2]^5 + a*b^2*C*Tan[(c + d*x)/2]^5 - b^3*C*Tan[(c + d*x)/2]^5 + 6*a^
3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
```

```
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] - 6*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2
*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(2*A*b^2 + 3*a^2*C - b^2*C)*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*b*(a + b)*(A*b + a*C)*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2
)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b^
2*(-a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2)
- a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.2, size = 2502, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d/(a+b)/(a-b)/b^2*(2*A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a*b^2+C*cos(d*x
+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)^2*a*b^2+2*C*cos(d*x+c)*a^2*
b+C*cos(d*x+c)*a*b^2-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*b^3+2*A*cos(d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+6*
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
```

```
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*b^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*C*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*a^3-2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-C*
cos(d*x+c)^3*b^3+3*C*cos(d*x+c)^2*a^3+C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3
-2*A*cos(d*x+c)^2*b^3)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
 c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2\right)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
 c))), x)
```

3.1445 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=664

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{a\left(15a^2C+8Ab^2-7b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{4b^3d\left(a^2-b^2\right)}+$$

```
[Out] ((8*A*b^2 + 15*a^2*C - 7*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - ((8*A*b^2 + (15*a^2 +
 5*a*b - 2*b^2)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*b^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 15*a^2*C +
 4*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*b^4*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A*b^2 + 5*a^2*C -
 b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*Sqrt[Se
c[c + d*x]]) - (a*(8*A*b^2 + 15*a^2*C - 7*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 1.98382, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3048, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{a\left(15a^2C+8Ab^2-7b^2C\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{4b^3d\left(a^2-b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),
x]
```



```
[Out] ((8*A*b^2 + 15*a^2*C - 7*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
```

```
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - ((8*A*b^2 + (15*a^2 +
 5*a*b - 2*b^2)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*b^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 15*a^2*C +
 4*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*b^4*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 + a^2*C)*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A*b^2 + 5*a^2*C -
 b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*Sqrt[Se
c[c + d*x]]) - (a*(8*A*b^2 + 15*a^2*C - 7*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4Ab^2+5a^2C-b^2C\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d\sqrt{}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4Ab^2+5a^2C-b^2C\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d\sqrt{}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4Ab^2+5a^2C-b^2C\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d\sqrt{}}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+15a^2C+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.\right.}{4b^4d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(8Ab^2+15a^2C-7b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{4b^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 16.9706, size = 2415, normalized size = 3.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(
3/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(A*b^2 + a^2*C)*Sin[c +
d*x])/(b^3*(a^2 - b^2)) + (2*(a^2*A*b^2*Sin[c + d*x] + a^4*C*Sin[c + d*x]))
/(b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x])) + (C*Sin[2*(c + d*x)])/(4*b^2)))/d
 + (8*a^2*A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] + 8*a*A*b^3*Sqrt[(a
- b)/(a + b)]*Tan[(c + d*x)/2] + 15*a^4*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*
```

x)/2] + 15*a^3*b*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2] - 7*a^2*b^2*Sqrt[
(a - b)/(a + b)]*C*Tan[(c + d*x)/2] - 7*a*b^3*Sqrt[(a - b)/(a + b)]*C*Tan[(
c + d*x)/2] - 16*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a^3*
b*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^3 + 14*a*b^3*Sqrt[(a - b)/(a + b
)]*C*Tan[(c + d*x)/2]^3 - 8*a^2*A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]^5 + 8*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*a^4*Sqrt[(a -
 b)/(a + b)]*C*Tan[(c + d*x)/2]^5 + 15*a^3*b*Sqrt[(a - b)/(a + b)]*C*Tan[(c
 + d*x)/2]^5 + 7*a^2*b^2*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - 7*a*b
^3*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^5 + (16*I)*a^2*A*b^2*EllipticPi
[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a +
 b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (16*I)*A*b^4*EllipticPi[(a + b)/(a -
b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (30*I)*a^4*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[
Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] - (22*I)*a^2*b^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a -
b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
(8*I)*b^4*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (16*I)*a^2*A*b^
2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (16*I)
*A*b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (
30*I)*a^4*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] - (22*I)*a^2*b^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] - (8*I)*b^4*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)
/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)] + I*a*(a - b)*(8*A*b^2 + 15*a^2*C - 7*b^2*C)*EllipticE[I*Arc
Sinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(a - b)*(15*a^3*C + 10*a^2*b*
C + 2*b^3*(2*A + C) + a*b^2*(8*A + C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x

```
)/2]^2)/(a + b)])/(4*b^3*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)*d*Sqrt[(1 + Tan[
(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-1 + Tan[(c + d*x)/2]^
4))
```

Maple [B] time = 0.271, size = 3555, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d/(a+b)/(a-b)/b^3*(-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-16*A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^
2+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-10*C*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4
*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+22*C*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^
2+15*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b-7*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-7*C*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+1
5*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
```

ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+22*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-7*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-7*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-2*C*cos(d*x+c)^4*a^2*b^2+5*C*cos(d*x+c)^3*a^3*b-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+2*C*cos(d*x+c)^4*b^4-2*C*cos(d*x+c)^2*b^4-15*C*cos(d*x+c)*a^4+8*A*cos(d*x+c)^2*a^2*b^2+15*C*cos(d*x+c)^2*a^4+7*C*cos(d*x+c)^2*a*b^3+10*C*cos(d*x+c)*a^3*b+7*C*cos(d*x+c)*a^2*b^2-2*C*cos(d*x+c)*a*b^3-5*C*cos(d*x+c)^3*a*b^3-8*A*cos(d*x+c)^2*a*b^3-8*A*cos(d*x+c)*a^2*b^2+8*A*cos(d*x+c)*a*b^3-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+16*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x

```
+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-30*C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^4+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*b^4-15*C*cos(d*x+c)^2*a^3*b-5*C*cos(d*x+c)^2*a^2*
b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(
3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(
3/2)), x)
```

3.1446 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=589

$$\frac{2\left(-a^2b^2(13A-C)+a^4(A-5C)+8Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{4\left(5a^2Ab^2+2a^4C-3Ab^4\right)\sin(\ldots}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\ldots}}$$

```
[Out] (-4*b*(8*A*b^4 + a^4*(4*A - 3*C) - a^2*b^2*(14*A - C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec
[c + d*x]]) - (2*(12*a*A*b^3 + 16*A*b^4 - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*
C) - a^3*(9*A*b - 3*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 + a^2
*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)) + (4*(5*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C)*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*A*b^4 + a^4*
(A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.88337, antiderivative size = 589, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 3055, 2998, 2816, 2994}

$$\frac{2\left(-a^2b^2(13A-C)+a^4(A-5C)+8Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{4\left(5a^2Ab^2+2a^4C-3Ab^4\right)\sin(\ldots}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5/2),
x]
```


```
[Out] (-4*b*(8*A*b^4 + a^4*(4*A - 3*C) - a^2*b^2*(14*A - C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec
```

```
[c + d*x]]) - (2*(12*a*A*b^3 + 16*A*b^4 - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*
C) - a^3*(9*A*b - 3*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 + a^2
*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)) + (4*(5*a^2*A*b^2 - 3*A*b^4 + 2*a^4*C)*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*A*b^4 + a^4*
(A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2(Ab^2+a^2C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{2(Ab^2+a^2C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{4(5a^2Ab^2-3Ab^4+2a^4C)\sec^{\frac{3}{2}}(}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\,c}}$$

$$= \frac{2(Ab^2+a^2C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{4(5a^2Ab^2-3Ab^4+2a^4C)\sec^{\frac{3}{2}}(}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\,c}}$$

$$= \frac{2(Ab^2+a^2C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{4(5a^2Ab^2-3Ab^4+2a^4C)\sec^{\frac{3}{2}}(}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\,c}}$$

$$= -\frac{4b(8Ab^4+a^4(4A-3C)-a^2b^2(14A-C))\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec}}$$

Mathematica [B] time = 26.0284, size = 3973, normalized size = 6.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(4*a^4*A - 14*a^2*A*b^2
 + 8*A*b^4 - 3*a^4*C + a^2*b^2*C)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)^2) - (2*
(A*b^3*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(a + b*Cos[
c + d*x])^2) - (2*(11*a^2*A*b^3*Sin[c + d*x] - 7*A*b^5*Sin[c + d*x] + 5*a^4
*b*C*Sin[c + d*x] - a^2*b^3*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Co
s[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d + (4*((8*a*A*b)/(3*(a^2 - b^2
)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (28*A*b^3)/(3*a*(a^2 - b
^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^5)/(3*a^3*(a^2
 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*b*C)/((a^2 -
```

b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^3*C)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^2*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (32*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^6*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (a^2*C*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*b^2*C*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*C*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (8*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (28*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a + b)*(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*b*(-3*A + C) + a^4*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((2*b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*b*(a + b)*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a + b)*(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*b*(-3*A + C) + a^4*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*b*(a + b)*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a + b)*(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*b*(-3*A + C) + a^4*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d

```
*x)/2]^4)/2 + (b*(a + b)*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c
+ d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])] + (a*(a + b)*(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*
b*(-3*A + C) + a^4*(A + 3*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c +
d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))
/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (b*(a + b)*(8*A*b^4 + a^4*(4*A
 - 3*C) + a^2*b^2*(-14*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
+ (a*(a + b)*(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*b*(-3*A +
 C) + a^4*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x])^2)))/(2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]) - b^2*
(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c + d*x]*Sec[(c + d*x
)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b
^2*(-14*A + C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c
 + d*x)/2] + b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c + d*
x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*
(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*b*(-3*A + C) + a^4*(A
+ 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (b*(a + b)*(8*A*b^4
+ a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2
]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)
/2]^2]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2
]^2]) + (2*(2*b*(a + b)*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a
*(a + b)*(12*a*A*b^3 - 16*A*b^4 + 2*a^2*b^2*(8*A - C) + 3*a^3*b*(-3*A + C)
+ a^4*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] + b*(8*A*b^4 + a^4*(4*A - 3*C) + a^2*b^2*(-14*A + C))*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c
+ d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*
Tan[c + d*x]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.363, size = 7095, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5
/2), x)
```

3.1447 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=489

$$-\frac{4\left(-a^2b^2(4A+C)+a^4(-C)+2Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2b(9A\right.}{}$$

```
[Out] (2*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c +
d*x]]) + (2*(6*a*A*b^2 + 8*A*b^3 - 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[Co
s[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^
2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (4*(2*A*b^4 - a^4*C -
a^2*b^2*(4*A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*
Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.34695, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 3055, 2998, 2816, 2994}

$$-\frac{4\left(-a^2b^2(4A+C)+a^4(-C)+2Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2b(9A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5/2),
x]
```


```
[Out] (2*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c +
d*x]]) + (2*(6*a*A*b^2 + 8*A*b^3 - 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[Co
s[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
```

```
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^
2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 + a^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (4*(2*A*b^4 - a^4*C -
a^2*b^2*(4*A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*
Sqrt[a + b*Cos[c + d*x]])
```

<u>Rule 4221</u>

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

<u>Rule 3056</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

<u>Rule 3055</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A + C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4\left(2Ab^4-a^4C-a^2b^2(4A+C)\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4\left(2Ab^4-a^4C-a^2b^2(4A+C)\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+}}$$

$$= \frac{2\left(8Ab^4+3a^4(A-C)-a^2b^2(15A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\right(}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c}}$$

<u>Mathematica</u> [B] time = 25.8027, size = 3741, normalized size = 7.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A - 15*a^2*A*b^2 +
8*A*b^4 - 3*a^4*C - a^2*b^2*C)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) + (2*(A*
b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x
])^2) + (4*(4*a^2*A*b^2*Sin[c + d*x] - 2*A*b^4*Sin[c + d*x] + a^4*C*Sin[c +
 d*x] + a^2*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))
))/d + (2*(-((a^2*A)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]])) + (5*A*b^2)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (8*A*b^4)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a^2*C)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) + (b^2*C)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (3*a*A*b*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (1
7*A*b^3*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) -
(8*A*b^5*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]])
```

- (a*b*C*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*C*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (a*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(8*A*b^4

```
+ 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + (a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C) - a^2*b*(9*A + C)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a
+ b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(8*A*b^4 + 3*a^4*(A - C)
- a^2*b^2*(15*A + C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
 d*x)/2] + (8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (8*A*b^4 + 3*a^4*(A
- C) - a^2*b^2*(15*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C)
- a^2*b*(9*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(8
*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*
x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c +
d*x)/2]^2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]) + ((-2*(a + b)*(8*A*b^4 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a
*(a + b)*(-6*a*A*b^2 + 8*A*b^3 + 3*a^3*(A - C) - a^2*b*(9*A + C))*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^4
+ 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c +
 d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*a^3*(a^2 - b^
2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.436, size = 6184, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5
/2), x)
```

3.1448 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=456

$$\frac{4b\left(Ab^2-a^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2(-(3A+C))\right.}{}$$

```
[Out] (-4*b*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b^2
 + 3*a*b*(A + C) - a^2*(3*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b
^2 + a^2*C)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqr
t[Sec[c + d*x]]) + (4*b*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.24055, antiderivative size = 456, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4221, 3056, 2993, 2998, 2816, 2994}

$$\frac{4b\left(Ab^2-a^2(3A+2C)\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2(-(3A+C))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5/2),
x]
```


```
[Out] (-4*b*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b^2
 + 3*a*b*(A + C) - a^2*(3*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
```

```
))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b
^2 + a^2*C)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqr
t[Sec[c + d*x]]) + (4*b*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{4b\left(Ab^2-a^2(3A+2C)\right)}{3a\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= \frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{4b\left(Ab^2-a^2(3A+2C)\right)}{3a\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= -\frac{4b\left(Ab^2-a^2(3A+2C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 23.0079, size = 3279, normalized size = 7.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^
(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*b*(3*a^2*A - A*b^2 + 2*a^2
*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) + (2*(A*b^2*Sin[c + d*x] + a^2*C*Si
n[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(-5*a^2*A*b^2*S
in[c + d*x] + A*b^4*Sin[c + d*x] + a^4*C*Sin[c + d*x] - 5*a^2*b^2*C*Sin[c +
 d*x]))/(3*a*b*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (4*((-2*a*A*b)/((a
^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/(3*a*(
a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*b*C)/(3*(a
^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[
c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b^2*Sqrt[Sec[c +
 d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c +
d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (a^2*C*Sqrt[Sec[c +
 d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (b^2*C*Sqrt[Sec[c + d*
x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*A*b^2*Cos[2*(c + d*x)]
*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
d*x]]) - (4*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqr
t[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*
(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] + a*(a + b)*(-2*A*b^2 + 3*a*b*(A + C) + a^2*(3*A +
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
] + b*(A*b^2 - a^2*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[(c + d*x)/2]^2]*((2*b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c
+ d*x]*(2*b*(a + b)*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a + b)*(-2*A*b^2 + 3*a*b*(
A + C) + a^2*(3*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] + b*(A*b^2 - a^2*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*(a + b*
Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*b*(a + b)*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a
+ b)*(-2*A*b^2 + 3*a*b*(A + C) + a^2*(3*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(A*b^2 - a^2*(3*A + 2*C))
```

```
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3
*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((b*(A*b^2 - a^2*(3*A + 2*C))*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (b*(a + b)*(A*b^2 - a^2*(3
*A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] + (a*(a + b)*(-2*A*b^2 + 3*a*b*(A + C) + a^2*(3*A + C))
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c
 + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]) + (b*(a + b)*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c
+ d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))] + (a*(a + b)*(-2*A*b^2 + 3*a*b*(A + C) + a^2*(3*A + C
))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a
 + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(2*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]) - b^2*(A*b^2 - a^2*(3*A
+ 2*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - b*(
A*b^2 - a^2*(3*A + 2*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*
x]*Tan[(c + d*x)/2] + b*(A*b^2 - a^2*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-2*A*b^2 + 3*a
*b*(A + C) + a^2*(3*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) +
(b*(a + b)*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*
Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^
2]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
+ (2*(2*b*(a + b)*(A*b^2 - a^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a + b)*(-2*A*b^2 + 3*a*b*(A
+ C) + a^2*(3*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + b*(A*b^2 - a^2*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d
*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*(
a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.293, size = 4579, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^2/(a+b)^2/(a-b)^2*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(3/2)*(-6*
A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3*b^2-6*A*cos(d*x+c)^2*a^4*b+C*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)+3*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)+6*A*cos(d
*x+c)^3*a^2*b^3+12*A*cos(d*x+c)^2*a^3*b^2+6*A*cos(d*x+c)*a^4*b+7*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b^2+C*cos(d*x+c)^3*a^5-C*cos(d*x+c)*a^5+4*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+3*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-
4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^4*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b^2-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)+3*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+C*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^5-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^3-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-12*A*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-
4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
```

b)/(a+b))^(1/2))*a^2*b^3+4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+5*C*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+7*C*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3*b^2+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-4*C*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-8*C*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b^2-4*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-5*C*cos(d*x+c)^3*a^3*b^2-5*A*co
s(d*x+c)^3*a^3*b^2+A*cos(d*x+c)^3*a*b^4-2*A*cos(d*x+c)^3*b^5+2*A*cos(d*x+c)
^2*b^5+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*b^5+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-6*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*A*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^3+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^4+4*C*cos(d*x+c)*a^4*b-3*C*cos(d*x+c)*a^3*b^2-4*C*cos(d*x+
c)^2*a^4*b+8*C*cos(d*x+c)^2*a^3*b^2-4*C*cos(d*x+c)^2*a^2*b^3+4*C*cos(d*x+c)
^3*a^2*b^3-4*A*cos(d*x+c)^2*a^2*b^3-4*A*cos(d*x+c)^2*a*b^4-7*A*cos(d*x+c)*a
^3*b^2-2*A*cos(d*x+c)*a^2*b^3+3*A*cos(d*x+c)*a*b^4-6*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^3+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^4+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+6*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b^2+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(

```
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^3-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-4*C*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b^2-4*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+4*C*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b^2+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^5+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)+3*C*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+9*A*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^4*b-6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5
/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5
/2), x)
```

3.1449 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=618

$$\frac{2\left(a^2b^2(3A+7C)-3a^4C+Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2bC\right.}{}$$

```
[Out] (-2*(A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(3*a^2*b^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*
x]]) - (2*(A*b^3 + 3*a^3*C + a^2*b*C - 3*a*b^2*(A + 2*C))*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[
Sec[c + d*x]]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 + a^2*C)*Si
n[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]
]) + (2*(A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.78356, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4221, 3048, 3051, 2809, 2993, 2998, 2816, 2994}

$$\frac{2\left(a^2b^2(3A+7C)-3a^4C+Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2bC\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),
x]
```



```
[Out] (-2*(A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
```

```
c[c + d*x]))/(a - b)])/(3*a^2*b^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*
x]]) - (2*(A*b^3 + 3*a^3*C + a^2*b*C - 3*a*b^2*(A + 2*C))*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[
Sec[c + d*x]]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 + a^2*C)*Si
n[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]
]) + (2*(A*b^4 - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\left(Ab^4-3a^4C+a^2b^2(3A+7C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{3a^2(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+}}$$

Mathematica [B] time = 19.6691, size = 1588, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d
*x]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-3*a^2*A*b^2 - A*b^4 + 3*
a^4*C - 7*a^2*b^2*C)*Sin[c + d*x])/(3*a*b^2*(a^2 - b^2)^2) - (2*(a*A*b^2*Si
n[c + d*x] + a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^
2) + (4*(a^2*A*b^2*Sin[c + d*x] + A*b^4*Sin[c + d*x] - 2*a^4*C*Sin[c + d*x]
 + 4*a^2*b^2*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))
/d + (2*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(3*a^3*A*b^2*Tan[(c + d*x)/2] +
 3*a^2*A*b^3*Tan[(c + d*x)/2] + a*A*b^4*Tan[(c + d*x)/2] + A*b^5*Tan[(c + d
*x)/2] - 3*a^5*C*Tan[(c + d*x)/2] - 3*a^4*b*C*Tan[(c + d*x)/2] + 7*a^3*b^2*
C*Tan[(c + d*x)/2] + 7*a^2*b^3*C*Tan[(c + d*x)/2] - 6*a^2*A*b^3*Tan[(c + d*
x)/2]^3 - 2*A*b^5*Tan[(c + d*x)/2]^3 + 6*a^4*b*C*Tan[(c + d*x)/2]^3 - 14*a^
```

```
2*b^3*C*Tan[(c + d*x)/2]^3 - 3*a^3*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^2*A*b^3*T
an[(c + d*x)/2]^5 - a*A*b^4*Tan[(c + d*x)/2]^5 + A*b^5*Tan[(c + d*x)/2]^5 +
 3*a^5*C*Tan[(c + d*x)/2]^5 - 3*a^4*b*C*Tan[(c + d*x)/2]^5 - 7*a^3*b^2*C*Ta
n[(c + d*x)/2]^5 + 7*a^2*b^3*C*Tan[(c + d*x)/2]^5 - 6*a^5*C*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*a^3
*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] - 6*a*b^4*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^5*C*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*a
^3*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^4*C*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
(a + b)*(-(A*b^4) + 3*a^4*C - a^2*b^2*(3*A + 7*C))*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] + a*b*(a + b)*(2*a^2*C - 3*a*b*(A + C) - b^2*(A + 3*C))*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[
(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/
(a + b)]))/(3*a*b^2*(a^2 - b^2)^2*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)
])
```

Maple [B] time = 0.502, size = 6427, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
 c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3 + 3\,ab^2\cos(dx+c)^2 + 3\,a^2b\cos(dx+c) + a^3\right)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)
^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c)
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
 c))), x)
```

3.1450 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=710

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}-\frac{\left(8Ab^4-C\left(-26a^2b^2+15a^4+3b^4\right)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos}}{3b^3d\left(a^2-b^2\right)^2}$$

```
[Out] ((8*A*b^4 - (15*a^4 - 26*a^2*b^2 + 3*b^4)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]
) - ((6*A*b^4 - a*b^3*(2*A - 3*C) - 15*a^4*C - 5*a^3*b*C + 21*a^2*b^2*C)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b
)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (5*a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d*Sqrt[Sec[c + d*x]]) - (2*(
A*b^2 + a^2*C)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)) + (2*(3*A*b^4 - 5*a^4*C + a^2*b^2*(A + 9*C))*Sin[c + d*
x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (
(8*A*b^4 - (15*a^4 - 26*a^2*b^2 + 3*b^4)*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 2.4282, antiderivative size = 710, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4221, 3048, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\left(a^2C+Ab^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}-\frac{\left(8Ab^4-C\left(-26a^2b^2+15a^4+3b^4\right)\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos}}{3b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] ((8*A*b^4 - (15*a^4 - 26*a^2*b^2 + 3*b^4)*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]
) - ((6*A*b^4 - a*b^3*(2*A - 3*C) - 15*a^4*C - 5*a^3*b*C + 21*a^2*b^2*C)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b
)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (5*a*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d*Sqrt[Sec[c + d*x]]) - (2*(
A*b^2 + a^2*C)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)) + (2*(3*A*b^4 - 5*a^4*C + a^2*b^2*(A + 9*C))*Sin[c + d*
x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (
(8*A*b^4 - (15*a^4 - 26*a^2*b^2 + 3*b^4)*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
```

```
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(3Ab^4-5a^4C+a^2b^2\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(3Ab^4-5a^4C+a^2b^2\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(3Ab^4-5a^4C+a^2b^2\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= \frac{5a\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{b^4d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(8Ab^4-\left(15a^4-26a^2b^2+3b^4\right)C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\mathrm{co}}}\right.\right.}{3a(a-b)b^3(a+b)^{3/2}d\sqrt{\sec(c+dx}}$$

<u>Mathematica</u> [B] time = 20.706, size = 1609, normalized size = 2.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(
3/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*(-2*A*b^4 + 3*a^4*C - 5*a
^2*b^2*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2) + (2*(a^2*A*b^2*Sin[c + d*x]
+ a^4*C*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(a^
3*A*b^2*Sin[c + d*x] - 5*a*A*b^4*Sin[c + d*x] + 7*a^5*C*Sin[c + d*x] - 11*a
^3*b^2*C*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d + (
```

```
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(8*a*A*b^4*Tan[(c + d*x)/2] +
 8*A*b^5*Tan[(c + d*x)/2] - 15*a^5*C*Tan[(c + d*x)/2] - 15*a^4*b*C*Tan[(c +
 d*x)/2] + 26*a^3*b^2*C*Tan[(c + d*x)/2] + 26*a^2*b^3*C*Tan[(c + d*x)/2] -
3*a*b^4*C*Tan[(c + d*x)/2] - 3*b^5*C*Tan[(c + d*x)/2] - 16*A*b^5*Tan[(c + d
*x)/2]^3 + 30*a^4*b*C*Tan[(c + d*x)/2]^3 - 52*a^2*b^3*C*Tan[(c + d*x)/2]^3
+ 6*b^5*C*Tan[(c + d*x)/2]^3 - 8*a*A*b^4*Tan[(c + d*x)/2]^5 + 8*A*b^5*Tan[(
c + d*x)/2]^5 + 15*a^5*C*Tan[(c + d*x)/2]^5 - 15*a^4*b*C*Tan[(c + d*x)/2]^5
 - 26*a^3*b^2*C*Tan[(c + d*x)/2]^5 + 26*a^2*b^3*C*Tan[(c + d*x)/2]^5 + 3*a*
b^4*C*Tan[(c + d*x)/2]^5 - 3*b^5*C*Tan[(c + d*x)/2]^5 - 30*a^5*C*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6
0*a^3*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)] - 30*a*b^4*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^5*C*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 60*a^3*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d
*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*b^4*C*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] - (a + b)*(-8*A*b^4 + (15*a^4 - 26*a^2*b^2 + 3*b^4)*C)*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] - 2*b*(a + b)*(3*A*b^3 - 5*a^3*C + 3*a^2*b*C + a*b^2*(A + 6*C))
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[1 + Tan[(c + d*x)/
2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.312, size = 6471, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(
3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + A\right)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3 + 3\,ab^2\cos(dx+c)^2 + 3\,a^2b\cos(dx+c) + a^3\right)\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)
^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(
3/2)), x)
```

3.1451 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\text{s}$ $dx)\,dx$

Optimal. Leaf size=230

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(5aA+7aC+7bB)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(3aB+3Ab+5bC)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{}$$

```
[Out] (-2*(3*A*b + 3*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(5*a*A + 7*b*B + 7*a*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(3*A*b + 3*a*B + 5*b
*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(5*a*A + 7*b*B + 7*a*C)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(A*b + a*B)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.376289, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(5aA+7aC+7bB)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(3aB+3Ab+5bC)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(9/2),x]
```



```
[Out] (-2*(3*A*b + 3*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(5*a*A + 7*b*B + 7*a*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(3*A*b + 3*a*B + 5*b
*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(5*a*A + 7*b*B + 7*a*C)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(A*b + a*B)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(}{}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} - \frac{1}{7}\left(2\sqrt{\cos(}\right.$$

$$= \frac{2(Ab+aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a}{}$$

$$= \frac{2(Ab+aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a}{}$$

$$= \frac{2(3Ab+3aB+5bC)\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= -\frac{2(3Ab+3aB+5bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 3.17863, size = 191, normalized size = 0.83

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(40\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+7aC+7bB)-168\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*(3*A*b + 3*a*B + 5*b*C)*Cos[c + d*x]^(7/2)*Ellipt
icE[(c + d*x)/2, 2] + 40*(5*a*A + 7*b*B + 7*a*C)*Cos[c + d*x]^(7/2)*Ellipti
cF[(c + d*x)/2, 2] + 2*(110*a*A + 70*b*B + 70*a*C + 21*(13*A*b + 13*a*B + 1
5*b*C)*Cos[c + d*x] + 10*(5*a*A + 7*b*B + 7*a*C)*Cos[2*(c + d*x)] + 63*A*b*
Cos[3*(c + d*x)] + 63*a*B*Cos[3*(c + d*x)] + 105*b*C*Cos[3*(c + d*x)])*Sin[
c + d*x]))/(420*d)
```

Maple [B] time = 4.913, size = 851, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*A*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b+C
*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*b*(-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*(A*b+B*a)/(8*sin(1/2
*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x
+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^
2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2} + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sec(dx+c)^{\frac{9}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)\sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(9/2), x)
```

3.1452 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=192

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(3aA+5aC+5bB)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)}{3d}-\frac{2\sqrt{\mathrm{co}}}{}$$

```
[Out] (-2*(3*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(A*b + a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a*A + 5*b*B + 5*a*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(3*d) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.34606, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(3aA+5aC+5bB)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)}{3d}-\frac{2\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(7/2),x]
```

```
[Out] (-2*(3*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(A*b + a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a*A + 5*b*B + 5*a*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(3*d) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos \ldots}{\ldots}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\left(2\sqrt{\cos \ldots}\right.$$

$$= \frac{2(Ab+aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a \ldots}{\ldots}$$

$$= \frac{2(Ab+aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a \ldots}{\ldots}$$

$$= \frac{2(Ab+aB+3bC)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\ldots\right.}{3d}$$

$$= -\frac{2(3aA+5bB+5aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\ldots\right.}{5d}$$

Mathematica [A] time = 1.223, size = 149, normalized size = 0.78

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(5F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)-3E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+5aC+5bB)+\frac{\sin(c+dx)(3(\cos \ldots}{\ldots}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(3*a*A + 5*b*B + 5*a*C)*Ellipt
icE[(c + d*x)/2, 2] + 5*(A*b + a*B + 3*b*C)*EllipticF[(c + d*x)/2, 2] + ((1
0*(A*b + a*B)*Cos[c + d*x] + 3*(5*b*B + 5*a*(A + C) + (3*a*A + 5*b*B + 5*a*
C)*Cos[2*(c + d*x)]))*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d)
```

Maple [B] time = 4.082, size = 742, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(A*
b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a*A/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^
2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*(B*b+C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin
(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3+(Ca+Bb)\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(7/2), x)
```

3.1453 $\int (a+b\cos(c+dx)) \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \mathrm{s}$ $dx)\,dx$

Optimal. Leaf size=151

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)}{3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab}{d}$$

```
[Out] (-2*(A*b + a*B - b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (2*(3*b*B + a*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/d + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.320446, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4221, 3031, 3021, 2748, 2641, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)}{3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(5/2),x]
```


```
[Out] (-2*(A*b + a*B - b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (2*(3*b*B + a*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/d + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(\ldots}{\ldots}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\left(2\sqrt{\cos(\ldots}\right.$$

$$= \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a\ldots}{\ldots}$$

$$= \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a\ldots}{\ldots}$$

$$= -\frac{2(Ab+aB-bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+d\ldots\right.}{d}$$

Mathematica [A] time = 1.06144, size = 112, normalized size = 0.74

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)-3E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab-bC)+\frac{\sin(c+dx)(3(aB+Ab)\cos\ldots}{\cos^{\frac{3}{2}}(c+d\ldots}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B - b*C)*EllipticE[(c
 + d*x)/2, 2] + (3*b*B + a*(A + 3*C))*EllipticF[(c + d*x)/2, 2] + ((a*A + 3
*(A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 2.967, size = 666, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*A*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^3+\left(Ca+Bb\right)\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(5/2), x)
```

3.1454 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=147

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a(A}{d}$$

```
[Out] (2*(b*B - a*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*(3*A*b + 3*a*B + b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c
+ d*x]]) + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.301595, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4221, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a(A}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^(3/2),x]
```


```
[Out] (2*(b*B - a*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*(3*A*b + 3*a*B + b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(3*d*Sqrt[Sec[c
+ d*x]]) + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(2\sqrt{\cos(c\right.$$

$$= \frac{2bC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(}{d}$$

$$= \frac{2bC\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{\sec(c+dx)}\sin(}{d}$$

$$= \frac{2(bB-a(A-C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}$$

Mathematica [A] time = 0.713809, size = 109, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)+6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(a(C-A)+bB)+2\,\mathrm{s}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(b*B + a*(-A + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 2*(3*A*b + 3*a*B + b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] + 2*(3*a*A + b*C*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 1.436, size = 388, normalized size = 2.6

$$-\frac{2}{3d}\left(4Cb\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+3Ab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*C*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-6*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+3*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+C*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*C*b*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

3.1455 $\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\ldots}$

Optimal. Leaf size=156

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(3A+C)+bB)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab\ldots}{5d}$$

```
[Out] (2*(5*A*b + 5*a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(b*B + a*(3*A + C))*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(5*d*Sec
[c + d*x]^(3/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.295874, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4221, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(3A+C)+bB)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab\ldots}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]],x]
```


```
[Out] (2*(5*A*b + 5*a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(b*B + a*(3*A + C))*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*C*Sin[c + d*x])/(5*d*Sec
[c + d*x]^(3/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
```

```
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2bC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec}\right.$$

$$= \frac{2bC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2bC\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2(5Ab+5aB+3bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 0.908504, size = 116, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+aC+bB)+6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+3bC)+s\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*A*b + 5*a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 10*(3*a*A + b*B + a*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2] + (5*b*B + 5*a*C + 3*b*C*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d
)
```

Maple [B] time = 1.324, size = 465, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*C*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*B*b+20*C*a+24*C*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*B*b-10*C*a-6*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+15*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
+5*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5*a*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

3.1456 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=194

$$\frac{2\sin(c+dx)(7aB+7Ab+5bC)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aB+7Ab+5bC)}{21d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}$$

```
[Out] (2*(5*a*A + 3*b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(7*A*b + 7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(5/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2
)) + (2*(7*A*b + 7*a*B + 5*b*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.312874, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\sin(c+dx)(7aB+7Ab+5bC)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aB+7Ab+5bC)}{21d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Sec
[c + d*x]],x]
```



```
[Out] (2*(5*a*A + 3*b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(7*A*b + 7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(5/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2
)) + (2*(7*A*b + 7*a*B + 5*b*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\,\mathrm{co}$$

$$= \frac{2bC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2bC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{35}\left(4\sqrt{\mathrm{c}}\right.$$

$$= \frac{2bC\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left((5aA\right.$$

$$= \frac{2(5aA+3bB+3aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{5d}$$

$$= \frac{2(5aA+3bB+3aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{5d}$$

Mathematica [A] time = 1.11189, size = 139, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42(aC+bB)\cos(c+dx)+70aB+70Ab+15bC\cos(2(c+dx))+65bC)+20\sqrt{\cos(c+dx}\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(5*a*A + 3*b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2] + 20*(7*A*b + 7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2] + (70*A*b + 70*a*B + 65*b*C + 42*(b*B + a*C)*Cos[c + d*x]
+ 15*b*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] time = 1.309, size = 515, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b-168*C*a-360*C*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b+140*B*a+168*B*b+168*C*a+280*C*b)*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*b-70*B*a-42*B*b-42*C*a-80*C*b)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+35*
A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+35*B*a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+25*C*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\cos(c + dx))\left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(1/
2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/sqrt
(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c) + A\right)(b\cos(dx + c) + a)}{\sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

3.1457 $\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{2\sin(c+dx)(9aB+9Ab+7bC)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)(7aA+5aC+5bB)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA}{21d}$$

```
[Out] (2*(9*A*b + 9*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (2*(7*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(9
*d*Sec[c + d*x]^(7/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/
2)) + (2*(9*A*b + 9*a*B + 7*b*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) +
(2*(7*a*A + 5*b*B + 5*a*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.358699, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4221, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\sin(c+dx)(9aB+9Ab+7bC)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)(7aA+5aC+5bB)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[c +
d*x]^(3/2),x]
```


```
[Out] (2*(9*A*b + 9*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (2*(7*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*C*Sin[c + d*x])/(9
*d*Sec[c + d*x]^(7/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/
2)) + (2*(9*A*b + 9*a*B + 7*b*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) +
(2*(7*a*A + 5*b*B + 5*a*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos($$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int c$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{63}\left(4\sqrt{co}\right.$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left((7aA+\right.$$

$$= \frac{2bC\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(9Ab+9}{45}$$

$$= \frac{2(9Ab+9aB+7bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}}{15d}$$

Mathematica [A] time = 1.52082, size = 165, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(7\cos(c+dx)(36aB+36Ab+43bC)+5(84aA+18(aC+bB)\cos(2(c+dx))+78aC+78b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(9*A*b + 9*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2] + 120*(7*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2] + (7*(36*A*b + 36*a*B + 43*b*C)*Cos[c + d*x] + 5*(84*a*A
 + 78*b*B + 78*a*C + 18*(b*B + a*C)*Cos[2*(c + d*x)] + 7*b*C*Cos[3*(c + d*x
)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] time = 1.242, size = 565, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b+720*C*a+2240*C*b)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b-504*B*a-1080*B*b-1080*C*a-2072*C*
b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a+504*A*b+504*B*a+840*B*b
+840*C*a+952*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a-126*A*b
-126*B*a-240*B*b-240*C*a-168*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1
05*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+75*b*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+75*a*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

3.1458 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=342

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^2(7A+9C)+18abB+4Ab^2\right)}{45d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2B+10aAb+14abC+7b^2\right.}{21d}$$

```
[Out] (-2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(10*a*A*b + 5*a^2*B
+ 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B +
14*a*b*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(4*A*b^2 + 18*a*b*B
+ a^2*(7*A + 9*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*a*(4*A*b +
9*a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^
2*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.763421, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^2(7A+9C)+18abB+4Ab^2\right)}{45d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2B+10aAb+14abC+7b^2\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(11/2),x]
```


```
[Out] (-2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(10*a*A*b + 5*a^2*B
+ 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B +
14*a*b*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(4*A*b^2 + 18*a*b*B
+ a^2*(7*A + 9*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*a*(4*A*b +
9*a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a + b*Cos[c + d*x])^
2*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2A(a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\sin(c}{9d}$$

$$= \frac{2a(4Ab+9aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d}$$

$$= \frac{2\left(4Ab^2+18abB+a^2(7A+9C)\right)\sec^{\frac{5}{2}}(c}{45d}$$

$$= \frac{2\left(4Ab^2+18abB+a^2(7A+9C)\right)\sec^{\frac{5}{2}}(c}{45d}$$

$$= \frac{2\left(18abB+3b^2(3A+5C)+a^2(7A+9C)\right.}{15d}$$

$$= -\frac{2\left(18abB+3b^2(3A+5C)+a^2(7A+9C\right.}{}$$

Mathematica [A] time = 6.65236, size = 357, normalized size = 1.04

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}\sin(c+dx)\left(7a^2A+9a^2C+18abB+9Ab^2+15b^2C\right)+\frac{2}{45}\sec^2(c+dx)\left(7a^2A\sin(c+dx)+9a^2C\sin(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(11/2),x]
```

```
[Out] ((2*(-49*a^2*A - 63*A*b^2 - 126*a*b*B - 63*a^2*C - 105*b^2*C)*EllipticE[(c
+ d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(50*a*A*b + 25*a^
2*B + 35*b^2*B + 70*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(105*d) + (Sqrt[Sec[c + d*x]]*((2*(7*a^2*A + 9*A*b^2 + 18*
a*b*B + 9*a^2*C + 15*b^2*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^3*(2*a*A*b*S
in[c + d*x] + a^2*B*Sin[c + d*x]))/7 + (2*Sec[c + d*x]^2*(7*a^2*A*Sin[c + d
*x] + 9*A*b^2*Sin[c + d*x] + 18*a*b*B*Sin[c + d*x] + 9*a^2*C*Sin[c + d*x]))
/45 + (2*Sec[c + d*x]*(10*a*A*b*Sin[c + d*x] + 5*a^2*B*Sin[c + d*x] + 7*b^2
```

```
*B*Sin[c + d*x] + 14*a*b*C*Sin[c + d*x]))/21 + (2*a^2*A*Sec[c + d*x]^3*Tan[
c + d*x])/9))/d
```

Maple [B] time = 6.62, size = 1196, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(2*A*b+B*a)
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+2*b*(B*b+2*C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*(A*b^2+2*B*
a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^2*(-1/144*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*b^2*C*(-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*
```

```
x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(11/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(11/2), x)
```

3.1459 $\int (a+b\cos(c+dx))^2 \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right)$ $dx)\,dx$

Optimal. Leaf size=288

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(5A+7C)+14abB+4Ab^2\right)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2B+6aAb+10abC+5b^2B\right)}{5d}$$

```
[Out] (-2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*b^2*(A + 3*C) +
 a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(5*d) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*(4*A*b + 7*a*B)*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d)
```

Rubi [A] time = 0.698136, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(5A+7C)+14abB+4Ab^2\right)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2B+6aAb+10abC+5b^2B\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(9/2),x]
```

```
[Out] (-2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*b^2*(A + 3*C) +
 a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(5*d) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*(4*A*b + 7*a*B)*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2A(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$= \frac{2a(4Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} +$$

$$= \frac{2\left(4Ab^2+14abB+a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c}{21d}$$

$$= \frac{2\left(4Ab^2+14abB+a^2(5A+7C)\right)\sec^{\frac{3}{2}}(c}{21d}$$

$$= \frac{2\left(14abB+7b^2(A+3C)+a^2(5A+7C)\right)}{2}$$

$$= -\frac{2\left(6aAb+3a^2B+5b^2B+10abC\right)\sqrt{\cos(}}{5d}$$

Mathematica [A] time = 4.36254, size = 221, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(21\sin(c+dx)\left(3a^2B+2ab(3A+5C)+5b^2B\right)+5\tan(c+dx)\left(a^2(5A+7C)+14abB+7Ab^2\right)+5\sqrt{\cos\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(3*a^2*B + 5*b^2*B + 2*a*b*(3*A + 5*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*
(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(3*a^2*B + 5
*b^2*B + 2*a*b*(3*A + 5*C))*Sin[c + d*x] + 5*(7*A*b^2 + 14*a*b*B + a^2*(5*A
 + 7*C))*Tan[c + d*x] + 21*a*(2*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x] + 15*a
^2*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

Maple [B] time = 5.326, size = 947, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A
*a^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))+2*(A*b^2+2*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5
*a*(2*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x
+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d
```

```
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(B*b+2*C*a)*(-(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(9/2), x)
```

3.1460 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=240

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(3A+5C)+10abB+4Ab^2\right)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-\right.}{3d}$$

```
[Out] (-2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*
(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*(4*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(5*d) + (2*a*(4*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*
d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.651969, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(3A+5C)+10abB+4Ab^2\right)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(7/2),x]
```


```
[Out] (-2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*
(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*(4*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(5*d) + (2*a*(4*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*
d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos}{}$$

$$=\frac{2A(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$=\frac{2a(4Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+$$

$$=\frac{2\left(4Ab^2+10abB+a^2(3A+5C)\right)\sqrt{\sec(c}}{5d}$$

$$=\frac{2\left(4Ab^2+10abB+a^2(3A+5C)\right)\sqrt{\sec(c}}{5d}$$

$$=-\frac{2\left(10abB+5b^2(A-C)+a^2(3A+5C)\right)}{}$$

Mathematica [A] time = 2.27464, size = 193, normalized size = 0.8

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(5F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)-3E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)+10abB+\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(10*a*b*B + 5*b^2*(A - C) + a^
2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2] + 5*(a^2*B + 3*b^2*B + 2*a*b*(A +
```

```
3*C))*EllipticF[(c + d*x)/2, 2] + ((15*(A*b^2 + 2*a*b*B + a^2*(A + C)) + 10
*a*(2*A*b + a*B)*Cos[c + d*x] + 3*(5*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Co
s[2*(c + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d)
```

Maple [B] time = 4.415, size = 1000, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a*b*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2+2*B*a*b+C*
a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1)-2/5*A*a^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(7/2), x)
```

3.1461 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=220

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}$$

```
[Out] (-2*(a^2*B - b^2*B + 2*a*b*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2
*b^2*(A - C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*(4*A*b + 3*a*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.619788, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(5/2),x]
```


```
[Out] (-2*(a^2*B - b^2*B + 2*a*b*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2
*b^2*(A - C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*(4*A*b + 3*a*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^2*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos$$

$$=\frac{2A(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$=\frac{2a(4Ab+3aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+$$

$$=-\frac{2b^2(A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(4Ab+3aB}{}$$

$$=-\frac{2b^2(A-C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(4Ab+3aB}{}$$

$$=-\frac{2\left(a^2B-b^2B+2ab(A-C)\right)\sqrt{\cos(c+dx}}{d}$$

Mathematica [A] time = 1.18173, size = 158, normalized size = 0.72

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)-6E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-C)\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-6*(a^2*B - b^2*B + 2*a*b*(A - C))*
EllipticE[(c + d*x)/2, 2] + 2*(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*Ell
ipticF[(c + d*x)/2, 2] + ((2*a^2*A + b^2*C + 6*a*(2*A*b + a*B)*Cos[c + d*x]
 + b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 3.646, size = 1303, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-6*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-12*B*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+8*
b^2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*b^2*C*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^4+6*B*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*C*b^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-A*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+2*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2+2*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*
sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-
6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-24*A*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+12*A*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*a*b*B*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+12*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b*sin(1/2*d*x+1/2*c)^2+12*B*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b*sin(1/2*d*x+1/2*c)^2-12*C*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(5/2), x)
```

3.1462 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=229

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*(
3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) - (2*b^2*(5*A - C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) - (2*b*(6*a
*A - b*B - 2*a*C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Cos[
c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.649228, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(3/2),x]
```


```
[Out] (2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*(
3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) - (2*b^2*(5*A - C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) - (2*b*(6*a
*A - b*B - 2*a*C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Cos[
c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$=\frac{2A(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$=-\frac{2b^2(5A-C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\cos}{}$$

$$=-\frac{2b^2(5A-C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b(6aA-bB}{3d\sqrt{}}$$

$$=-\frac{2b^2(5A-C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b(6aA-bB}{3d\sqrt{}}$$

$$=\frac{2\left(10abB-5a^2(A-C)+b^2(5A+3C)\right)\sqrt{}}{5}$$

Mathematica [A] time = 1.27998, size = 165, normalized size = 0.72

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)+12E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^2(A-C)+10abB\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*(10*a*b*B - 5*a^2*(A - C) + b^2*
(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*B + b^2*B + 2*a*b*(3*A +
 C))*EllipticF[(c + d*x)/2, 2] + (2*(10*b*(b*B + 2*a*C)*Cos[c + d*x] + 3*(1
```

```
0*a^2*A + b^2*C + b^2*C*Cos[2*(c + d*x)]))*Sin[c + d*x])/Sqrt[Cos[c + d*x]]
))/(30*d)
```

Maple [B] time = 1.51, size = 932, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b*(5*B*b+10*C*a+6*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^2+5*B*b^2
+10*C*a*b+3*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*a*A*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*b^2+15*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+5*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-30*B*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*a*b*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^2-9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*se
c(d*x + c)^(3/2), x)
```

3.1463 $\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=243

$$\frac{2\sin(c+dx)\left(4a^2C+14abB+7Ab^2+5b^2C\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2(3A+C)+14abB+b^2\right.}{21d}$$

```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*a^2*(3*A + C) +
b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*b*(7*b*B + 4*a*C)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)
) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] time = 0.635175, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\sin(c+dx)\left(4a^2C+14abB+7Ab^2+5b^2C\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2(3A+C)+14abB+b^2\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec
[c + d*x]],x]
```



```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*a^2*(3*A + C) +
b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*b*(7*b*B + 4*a*C)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)
) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c +
d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}} + \frac{1}{7}\big(2$$

$$= \frac{2b(7bB+4aC)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a+b\cos}{7a}$$

$$= \frac{2b(7bB+4aC)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2+1\right.}{}$$

$$= \frac{2b(7bB+4aC)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2+1\right.}{}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B+6abC\right)\sqrt{\cos(c}}{5d}$$

Mathematica [A] time = 0.981574, size = 183, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(5\left(14a^2C+28abB+14Ab^2+3b^2C\cos(2(c+dx))+13b^2C\right)+42b(2aC+bB)\cos(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(5*a^2*B + 3*b^2*B + 2*a*b*(5*A + 3*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2] + 40*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(
7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(42*b*(b*B + 2
*a*C)*Cos[c + d*x] + 5*(14*A*b^2 + 28*a*b*B + 14*a^2*C + 13*b^2*C + 3*b^2*C
```

```
*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(420*d)
```

Maple [B] time = 1.256, size = 706, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B*b^2-336*C*a*b-360*C*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*b^2+280*B*a*b+168*B*b^2+140*C*a
^2+336*C*a*b+280*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*b^2-
140*B*a*b-42*B*b^2-70*C*a^2-84*C*a*b-80*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-210*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+105*A*a^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+70*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b+35*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sq
rt(sec(d*x + c)), x)
```

3.1464 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=291

$$\frac{2\sin(c+dx)\left(4a^2C+18abB+9Ab^2+7b^2C\right)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(7a^2B+14aAb+10abC+5b^2B\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}$$

```
[Out] (2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(14*a*A*b + 7*a^2*B +
 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*b*(9*b*B + 4*a*C)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(
5/2)) + (2*(9*A*b^2 + 18*a*b*B + 4*a^2*C + 7*b^2*C)*Sin[c + d*x])/(45*d*Sec
[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(3/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.686044, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\sin(c+dx)\left(4a^2C+18abB+9Ab^2+7b^2C\right)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(7a^2B+14aAb+10abC+5b^2B\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```

```
[Out] (2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(14*a*A*b + 7*a^2*B +
 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*b*(9*b*B + 4*a*C)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(
5/2)) + (2*(9*A*b^2 + 18*a*b*B + 4*a^2*C + 7*b^2*C)*Sin[c + d*x])/(45*d*Sec
[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(3/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\,c$$

$$= \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{9}\left(2\sqrt{\cos(c+d\right.$$

$$= \frac{2b(9bB+4aC)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))^2}{9d\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2b(9bB+4aC)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(9Ab^2+18abB+4a\right.}{45d\sec}$$

$$= \frac{2b(9bB+4aC)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(9Ab^2+18abB+4a\right.}{45d\sec}$$

$$= \frac{2\left(18abB+3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+}}{15d}$$

$$= \frac{2\left(18abB+3a^2(5A+3C)+b^2(9A+7C)\right)\sqrt{\cos(c+}}{15d}$$

Mathematica [A] time = 1.40069, size = 218, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(7\cos(c+dx)\left(36a^2C+72abB+36Ab^2+43b^2C\right)+5\left(84a^2B+168aAb+18b(2aC+b\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(336*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 240*(7*a^2*B + 5*b^2*B + 2*a*
b*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(7*(36*A*b^
2 + 72*a*b*B + 36*a^2*C + 43*b^2*C)*Cos[c + d*x] + 5*(168*a*A*b + 84*a^2*B
+ 78*b^2*B + 156*a*b*C + 18*b*(b*B + 2*a*C)*Cos[2*(c + d*x)] + 7*b^2*C*Cos[
3*(c + d*x)]))*Sin[2*(c + d*x)]))/(2520*d)
```

Maple [B] time = 1.284, size = 784, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^2+1440*C*a*b+2240*C*b^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^2-1008*B*a*b-1080*B*b^2-
504*C*a^2-2160*C*a*b-2072*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(8
40*A*a*b+504*A*b^2+420*B*a^2+1008*B*a*b+840*B*b^2+504*C*a^2+1680*C*a*b+952*
C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-420*A*a*b-126*A*b^2-210*B*
a^2-252*B*a*b-240*B*b^2-126*C*a^2-480*C*a*b-168*C*b^2)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2+210*a*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b+105*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*b^2*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+1
50*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)}{\sqrt{\sec\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sq
rt(sec(d*x + c)), x)
```

3.1465 $\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=345

$$\frac{2\sin(c+dx)\left(9a^2B+18aAb+14abC+7b^2B\right)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(4a^2C+22abB+11Ab^2+9b^2C\right)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(11a^2(\right.}{}$$

```
[Out] (2*(18*a*A*b + 9*a^2*B + 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(110*a*b*B + 11*a^2*(7*A + 5
*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(231*d) + (2*b*(11*b*B + 4*a*C)*Sin[c + d*x])/(99*d*Sec[c +
 d*x]^(7/2)) + (2*(11*A*b^2 + 22*a*b*B + 4*a^2*C + 9*b^2*C)*Sin[c + d*x])/(
77*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d*
Sec[c + d*x]^(5/2)) + (2*(18*a*A*b + 9*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c +
d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^
2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.749051, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3049, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\sin(c+dx)\left(9a^2B+18aAb+14abC+7b^2B\right)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(4a^2C+22abB+11Ab^2+9b^2C\right)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(11a^2(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (2*(18*a*A*b + 9*a^2*B + 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(110*a*b*B + 11*a^2*(7*A + 5
*C) + 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(231*d) + (2*b*(11*b*B + 4*a*C)*Sin[c + d*x])/(99*d*Sec[c +
 d*x]^(7/2)) + (2*(11*A*b^2 + 22*a*b*B + 4*a^2*C + 9*b^2*C)*Sin[c + d*x])/(
77*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d*
Sec[c + d*x]^(5/2)) + (2*(18*a*A*b + 9*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c +
d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*(110*a*b*B + 11*a^2*(7*A + 5*C) + 5*b^
2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{c}$$

$$= \frac{2C(a+b\cos(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{11}\left(2\sqrt{\cos(c+\right.$$

$$= \frac{2b(11bB+4aC)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2C(a+b\cos(c+dx))}{11d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2b(11bB+4aC)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(11Ab^2+22abB+4\right.}{77d\,\mathrm{se}}$$

$$= \frac{2b(11bB+4aC)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(11Ab^2+22abB+4\right.}{77d\,\mathrm{se}}$$

$$= \frac{2b(11bB+4aC)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(11Ab^2+22abB+4\right.}{77d\,\mathrm{se}}$$

$$= \frac{2\left(18aAb+9a^2B+7b^2B+14abC\right)\sqrt{\cos(c+dx)}E\left(\right.}{15d}$$

Mathematica [A] time = 2.07236, size = 259, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(154\cos(c+dx)\left(36a^2B+72aAb+86abC+43b^2B\right)+5\left(36\cos(2(c+dx))\left(11a^2C+22a\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7392*(9*a^2*B + 7*b^2*B + 2*a*b*(9*A + 7*C))*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2] + 480*(110*a*b*B + 11*a^2*(7*A + 5*C) +
 5*b^2*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(154*
(72*a*A*b + 36*a^2*B + 43*b^2*B + 86*a*b*C)*Cos[c + d*x] + 5*(3432*a*b*B +
132*a^2*(14*A + 13*C) + 3*b^2*(572*A + 531*C) + 36*(11*A*b^2 + 22*a*b*B + 1
1*a^2*C + 16*b^2*C)*Cos[2*(c + d*x)] + 154*b*(b*B + 2*a*C)*Cos[3*(c + d*x)]
 + 63*b^2*C*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(55440*d)
```

Maple [B] time = 1.498, size = 863, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^2-24640*C*a*b-50400*
C*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^2+15840*B*a*b+246
40*B*b^2+7920*C*a^2+49280*C*a*b+56880*C*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-11088*A*a*b-11880*A*b^2-5544*B*a^2-23760*B*a*b-22792*B*b^2-1188
0*C*a^2-45584*C*a*b-34920*C*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4
620*A*a^2+11088*A*a*b+9240*A*b^2+5544*B*a^2+18480*B*a*b+10472*B*b^2+9240*C*
a^2+20944*C*a*b+13860*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310
*A*a^2-2772*A*a*b-2640*A*b^2-1386*B*a^2-5280*B*a*b-1848*B*b^2-2640*C*a^2-36
96*C*a*b-2790*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*A*a^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+825*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4158*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b+1650*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
```

```
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+825*a^2*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+675*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3234*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{2}\cos\left(dx+c\right)^{4}+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{3}+Aa^{2}+\left(Ca^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/se
c(d*x + c)^(3/2), x)
```

3.1466 $\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \ldots dx) \, dx$

Optimal. Leaf size=397

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(7a^2(7A+9C)+99abB+24Ab^2\right)}{315d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(9a^2b(5A+7C)+15a^3B+\ldots\right.}{63d}$$

```
[Out] (-2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(5*a^
3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(27*a^2*b*B + 1
5*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(15*d) + (2*(8*A*b^3 + 15*a^3*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(63*d) + (2*a*(24*A*b^2 + 99*a*b*B + 7*a^2
*(7*A + 9*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(2*A*b + 3*a*B)
*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*A*(a +
 b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.0899, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(7a^2(7A+9C)+99abB+24Ab^2\right)}{315d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(9a^2b(5A+7C)+15a^3B+\ldots\right.}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(11/2),x]
```


```
[Out] (-2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(5*a^
3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(27*a^2*b*B + 1
5*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(15*d) + (2*(8*A*b^3 + 15*a^3*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(63*d) + (2*a*(24*A*b^2 + 99*a*b*B + 7*a^2
*(7*A + 9*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(2*A*b + 3*a*B)
*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*A*(a +
```

```
b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\text{co}}{}$$

$$= \frac{2A(a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\sin(}{9d}$$

$$= \frac{2(2Ab+3aB)(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c}{21d}$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\text{sec}}{315d}$$

$$= \frac{2\left(8Ab^3+15a^3B+54ab^2B+9a^2b(5A+\right.}{63d}$$

$$= \frac{2\left(8Ab^3+15a^3B+54ab^2B+9a^2b(5A+\right.}{63d}$$

$$= \frac{2\left(5a^3B+21ab^2B+7b^3(A+3C)+3a^2b\right.}{}$$

$$= -\frac{2\left(27a^2bB+15b^3B+9ab^2(3A+5C)+\right.}{}$$

Mathematica [A] time = 6.91125, size = 416, normalized size = 1.05

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}\sin(c+dx)\left(7a^3A+27a^2bB+9a^3C+27aAb^2+45ab^2C+15b^3B\right)+\frac{2}{45}\sec^2(c+dx)\left(7a^3A\sin(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(11/2),x]
```

```
[Out] ((2*(-49*a^3*A - 189*a*A*b^2 - 189*a^2*b*B - 105*b^3*B - 63*a^3*C - 315*a*b
^2*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) +
2*(75*a^2*A*b + 35*A*b^3 + 25*a^3*B + 105*a*b^2*B + 105*a^2*b*C + 105*b^3*C
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(105*d)
+ (Sqrt[Sec[c + d*x]]*((2*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B + 9
```

```
*a^3*C + 45*a*b^2*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^3*(3*a^2*A*b*Sin[c
+ d*x] + a^3*B*Sin[c + d*x]))/7 + (2*Sec[c + d*x]^2*(7*a^3*A*Sin[c + d*x] +
 27*a*A*b^2*Sin[c + d*x] + 27*a^2*b*B*Sin[c + d*x] + 9*a^3*C*Sin[c + d*x]))
/45 + (2*Sec[c + d*x]*(15*a^2*A*b*Sin[c + d*x] + 7*A*b^3*Sin[c + d*x] + 5*a
^3*B*Sin[c + d*x] + 21*a*b^2*B*Sin[c + d*x] + 21*a^2*b*C*Sin[c + d*x]))/21
+ (2*a^3*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d
```

Maple [B] time = 7.183, size = 1292, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
^2*(3*A*b+B*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)
^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+2*b*(A*b^2+3*B*a*b+3*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))-2/5*a*(3*A*b^2+3*B*a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^
3*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(
1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
```

```
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))))+2*b^2*(B*b+3*C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3+3\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sec(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(11/2), x)
```

3.1467 $\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=334

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+63abB+24Ab^2\right)}{105d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(21a^2b(3A+5C)+21a^3B+\right.}{35d}$$

```
[Out] (-2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^2*b
*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(24*A*b^3 + 21*a^3*
B + 98*a*b^2*B + 21*a^2*b*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35
*d) + (2*a*(24*A*b^2 + 63*a*b*B + 5*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(105*d) + (2*(6*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.986297, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+63abB+24Ab^2\right)}{105d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(21a^2b(3A+5C)+21a^3B+\right.}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(9/2),x]
```


```
[Out] (-2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^2*b
*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(24*A*b^3 + 21*a^3*
B + 98*a*b^2*B + 21*a^2*b*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35
*d) + (2*a*(24*A*b^2 + 63*a*b*B + 5*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(105*d) + (2*(6*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos}{}$$

$$=\frac{2A(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$=\frac{2(6Ab+7aB)(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c}{35d}$$

$$=\frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sec}{105d}$$

$$=\frac{2\left(24Ab^3+21a^3B+98ab^2B+21a^2b(3A\right.}{35d}$$

$$=\frac{2\left(24Ab^3+21a^3B+98ab^2B+21a^2b(3A\right.}{35d}$$

$$=-\frac{2\left(3a^3B+15ab^2B+5b^3(A-C)+3a^2b(\right.}{}$$

Mathematica [A] time = 4.2589, size = 255, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(21\sin(c+dx)\left(3a^2b(3A+5C)+3a^3B+15ab^2B+5Ab^3\right)+5a\tan(c+dx)\left(a^2(5A+7C)+21abB+21A\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*
(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(21*a^2*b*B +
 21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + 21*(5*A*b^3 + 3*a^3*B + 15*a*b^2*B + 3*a^2*b*(3*A + 5
*C))*Sin[c + d*x] + 5*a*(21*A*b^2 + 21*a*b*B + a^2*(5*A + 7*C))*Tan[c + d*x
] + 21*a^2*(3*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x] + 15*a^3*A*Sec[c + d*x]^
2*Tan[c + d*x]))/(105*d)
```

Maple [B] time = 5.757, size = 1205, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*b*(A*b^2+3*B*a*b+3*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1)+2*a*(3*A*b^2+3*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x
+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))+2*A*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
```

```
(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a^2*(3*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sec(d*x + c)^(9
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(9/2), x)
```

3.1468 $\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=313

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2(3A+5C)+35abB+24Ab^2\right)}{15d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(A+\right.}{3d}$$

```
[Out] (-2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^3*B + 9
*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(9*A*b + 5*a*B - 5*b*C)*
Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*(24*A*b^2 + 35*a*b*B + 3*a^2
*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(6*A*b + 5*a*B)*
(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a +
b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.962522, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2(3A+5C)+35abB+24Ab^2\right)}{15d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(A+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(7/2),x]
```


```
[Out] (-2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^3*B + 9
*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(9*A*b + 5*a*B - 5*b*C)*
Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*(24*A*b^2 + 35*a*b*B + 3*a^2
*(3*A + 5*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(6*A*b + 5*a*B)*
(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a +
b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2A(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{2(6Ab+5aB)(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c}{15d}$$

$$= \frac{2a\left(24Ab^2+35abB+3a^2(3A+5C)\right)\sqrt{\mathrm{se}}}{15d}$$

$$= -\frac{2b^2(9Ab+5aB-5bC)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a}{}$$

$$= -\frac{2b^2(9Ab+5aB-5bC)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a}{}$$

$$= -\frac{2\left(15a^2bB-5b^3B+15ab^2(A-C)+a^3(3\right.}{}$$

Mathematica [A] time = 2.22688, size = 276, normalized size = 0.88

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{5}a\sin(c+dx)\left(3a^2A+5a^2C+15abB+15Ab^2\right)+\frac{2}{3}\sec(c+dx)\left(3a^2Ab\sin(c+dx)+a^3B\sin(c+dx)\right)+\frac{2}{5}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(7/2),x]
```

```
[Out] ((2*(-9*a^3*A - 45*a*A*b^2 - 45*a^2*b*B + 15*b^3*B - 15*a^3*C + 45*a*b^2*C)
*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(15
*a^2*A*b + 15*A*b^3 + 5*a^3*B + 45*a*b^2*B + 45*a^2*b*C + 5*b^3*C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (Sqrt[Sec
[c + d*x]]*((2*a*(3*a^2*A + 15*A*b^2 + 15*a*b*B + 5*a^2*C)*Sin[c + d*x])/5
+ (2*Sec[c + d*x]*(3*a^2*A*b*Sin[c + d*x] + a^3*B*Sin[c + d*x]))/3 + (b^3*C
*Sin[2*(c + d*x)])/3 + (2*a^3*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 5.409, size = 1419, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^3*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*B*b^3+6*C*a*b^2-4*C*b^3)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a*b^2*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a^
2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-6*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+2*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+
```

```
1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(3*A*b^2+3*B
*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sec(d*x + c)^(7
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(7/2), x)
```

3.1469 $\int (a+b\cos(c+dx))^3 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx) \, dx$

Optimal. Leaf size=311

$$-\frac{2b\sin(c+dx)\left(6a^2B+3ab(5A-C)-b^2B\right)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2\right.}{3d}$$

```
[Out] (-2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*b*B
 + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(35*A*b + 15*a*B - 3*b*C
)*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)) - (2*b*(6*a^2*B - b^2*B + 3*a*b*(
5*A - C))*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*(2*A*b + a*B)*(a + b*
Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + b*Cos[c + d*
x])^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.0019, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b\sin(c+dx)\left(6a^2B+3ab(5A-C)-b^2B\right)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(5/2),x]
```



```
[Out] (-2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*b*B
 + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b^2*(35*A*b + 15*a*B - 3*b*C
)*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)) - (2*b*(6*a^2*B - b^2*B + 3*a*b*(
5*A - C))*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*(2*A*b + a*B)*(a + b*
Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*(a + b*Cos[c + d*
x])^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos$$

$$=\frac{2A(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$=\frac{2(2Ab+aB)(a+b\cos(c+dx))^2\sqrt{\sec(c+}}{d}$$

$$=-\frac{2b^2(35Ab+15aB-3bC)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}+$$

$$=-\frac{2b^2(35Ab+15aB-3bC)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}-$$

$$=-\frac{2b^2(35Ab+15aB-3bC)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)}-$$

$$=-\frac{2\left(5a^3B-15ab^2B+15a^2b(A-C)-b^3(5\right.}{}$$

Mathematica [A] time = 2.0632, size = 224, normalized size = 0.72

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)-12E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(15a^2b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-12*(5*a^3*B - 15*a*b^2*B + 15*a^2*
b*(A - C) - b^3*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*b*B + b^
3*B + 3*a*b^2*(3*A + C) + a^3*(A + 3*C))*EllipticF[(c + d*x)/2, 2] + ((20*a
^3*A + 10*b^3*B + 30*a*b^2*C + 3*(60*a^2*A*b + 20*a^3*B + 3*b^3*C)*Cos[c +
d*x] + 10*b^2*(b*B + 3*a*C)*Cos[2*(c + d*x)] + 3*b^3*C*Cos[3*(c + d*x)])*Si
n[c + d*x])/Cos[c + d*x]^(3/2)))/(30*d)
```

Maple [B] time = 4.546, size = 1837, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(30*C*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-18*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*s
in(1/2*d*x+1/2*c)^2+10*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-
45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-45*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b-15*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+40*B*b
^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*B*a^3*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^4-40*B*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+10*B*b^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+10*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2+6*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+72*C*b^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+30*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48*C*b^3*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-30*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*
d*x+1/2*c)^2-5*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
```

```
*a^3-180*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-120*C*a*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+90*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+30*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+120*C*a*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-5*A
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-15*a^3*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-45*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b^2-90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2
+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b*sin(1/2*d*x+1/2*c)^2+30*C*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-90*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b
*sin(1/2*d*x+1/2*c)^2+90*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c
)^2+90*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+30*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3*
sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sec(d*x + c)^(5
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(5/2), x)
```

3.1470 $\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=319

$$\frac{2b\sin(c+dx)\left(-6a^2(7A-3C)+21abB+b^2(7A+5C)\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)\right.}{21d}$$

```
[Out] (2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*B
+ 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (2*b^2*(35*a*A - 7*b*B
- 11*a*C)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*b*(21*a*b*B - 6*a^2*
(7*A - 3*C) + b^2*(7*A + 5*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (2
*b*(7*A - C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
+ (2*A*(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.994247, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2b\sin(c+dx)\left(-6a^2(7A-3C)+21abB+b^2(7A+5C)\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(3/2),x]
```



```
[Out] (2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*B
+ 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (2*b^2*(35*a*A - 7*b*B
- 11*a*C)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*b*(21*a*b*B - 6*a^2*
(7*A - 3*C) + b^2*(7*A + 5*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (2
*b*(7*A - C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
+ (2*A*(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2A(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\sin(}{d}$$

$$= -\frac{2b(7A-C)(a+b\cos(c+dx))^2\sin(c+}{7d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2(35aA-7bB-11aC)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} -$$

$$= -\frac{2b^2(35aA-7bB-11aC)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{2b^2(35aA-7bB-11aC)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2\left(15a^2bB+3b^3B-5a^3(A-C)+3ab^2(\right.}{}$$

Mathematica [A] time = 1.92404, size = 233, normalized size = 0.73

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(40F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)-168E\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-168*(-15*a^2*b*B - 3*b^3*B + 5*a^3
*(A - C) - 3*a*b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 40*(21*a^3*B +
21*a*b^2*B + 21*a^2*b*(3*A + C) + b^3*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2
] + (2*(420*a^3*A + 42*b^3*B + 126*a*b^2*C + 5*b*(28*A*b^2 + 84*a*b*B + 84*
a^2*C + 29*b^2*C)*Cos[c + d*x] + 42*b^2*(b*B + 3*a*C)*Cos[2*(c + d*x)] + 15
*b^3*C*Cos[3*(c + d*x)])*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(420*d)
```

Maple [B] time = 1.794, size = 1278, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*(240*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*b^2*(7*B*b+21*C*a+15*C*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+28*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(5*A*b^2+
15*B*a*b+6*B*b^2+15*C*a^2+18*C*a*b+10*C*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^3
+35*A*b^3+105*B*a*b^2+21*B*b^3+105*C*a^2*b+63*C*a*b^2+40*C*b^3)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+35*A*b^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+105*A*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-315*A*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+105*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-315*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*B*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+105*a^2*b*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)+25*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)-105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^3-189*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sec(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*se
c(d*x + c)^(3/2), x)
```

3.1471 $\int (a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=336

$$\frac{2b\sin(c+dx)\left(24a^2C+99abB+63Ab^2+49b^2C\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(54a^2bB+8a^3C+9ab^2(7A+5C)+15b^3B\right)}{63d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\mathrm{co}}}{}$$

```
[Out] (2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^
2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(63*A*b^2 + 99
*a*b*B + 24*a^2*C + 49*b^2*C)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2
*(54*a^2*b*B + 15*b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*
d*Sqrt[Sec[c + d*x]]) + (2*(3*b*B + 2*a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/
(9*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.00736, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2b\sin(c+dx)\left(24a^2C+99abB+63Ab^2+49b^2C\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(54a^2bB+8a^3C+9ab^2(7A+5C)+15b^3B\right)}{63d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec
[c + d*x]],x]
```


```
[Out] (2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^
2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(63*A*b^2 + 99
*a*b*B + 24*a^2*C + 49*b^2*C)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2
*(54*a^2*b*B + 15*b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*
d*Sqrt[Sec[c + d*x]]) + (2*(3*b*B + 2*a*C)*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/
(9*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\,c}{}$$

$$=\frac{2C(a+b\cos(c+dx))^3\sin(c+dx)}{9d\sqrt{\sec(c+dx)}}+\frac{1}{9}$$

$$=\frac{2(3bB+2aC)(a+b\cos(c+dx))^2\sin(c}{21d\sqrt{\sec(c+dx)}}$$

$$=\frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2\left(15a^3B+27ab^2B+9a^2b(5A+3C)+\right.}{}$$

Mathematica [A] time = 1.73522, size = 253, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(2(c+dx))\left(7b\cos(c+dx)\left(108a^2C+108abB+36Ab^2+43b^2C\right)+5\left(252a^2bB+84a^3C+18ab^2(14\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]],x]
```

[Out] (Sqrt[Sec[c + d*x]]*(336*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3
*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 240*(21*a^2*b*
B + 5*b^3*B + 7*a^3*(3*A + C) + 3*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2] + 2*(7*b*(36*A*b^2 + 108*a*b*B + 108*a^2*C + 43*b^2*
C)*Cos[c + d*x] + 5*(252*a^2*b*B + 78*b^3*B + 84*a^3*C + 18*a*b^2*(14*A + 1
3*C) + 18*b^2*(b*B + 3*a*C)*Cos[2*(c + d*x)] + 7*b^3*C*Cos[3*(c + d*x)]))*S
in[2*(c + d*x)]))/(2520*d)

Maple [B] time = 1.29, size = 975, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B*b^3+2160*C*a*b^2+2240*C*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*b^3-1512*B*a*b^2-1080*B*
b^3-1512*C*a^2*b-3240*C*a*b^2-2072*C*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(1260*A*a*b^2+504*A*b^3+1260*B*a^2*b+1512*B*a*b^2+840*B*b^3+420*C*a^
3+1512*C*a^2*b+2520*C*a*b^2+952*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-630*A*a*b^2-126*A*b^3-630*B*a^2*b-378*B*a*b^2-240*B*b^3-210*C*a^3-378
*C*a^2*b-720*C*a*b^2-168*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+315
*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+315*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-945*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+315*a^2*b
B(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+75*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-315*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^3-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+105*a^3*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+225*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2*b-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^3\cos\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sq
rt(sec(d*x + c)), x)
```

3.1472 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=401

$$\frac{2\sin(c+dx)\left(242a^2bB+24a^3C+33ab^2(9A+7C)+77b^3B\right)}{495d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\left(24a^2C+143abB+99Ab^2+81b^2C\right)}{693d\sec^{\frac{5}{2}}(c+dx)}+$$

```
[Out] (2*(27*a^2*b*B + 7*b^3*B + 3*a^3*(5*A + 3*C) + 3*a*b^2*(9*A + 7*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(77*a
^3*B + 165*a*b^2*B + 33*a^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*b*(99*A*
b^2 + 143*a*b*B + 24*a^2*C + 81*b^2*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5
/2)) + (2*(242*a^2*b*B + 77*b^3*B + 24*a^3*C + 33*a*b^2*(9*A + 7*C))*Sin[c
+ d*x])/(495*d*Sec[c + d*x]^(3/2)) + (2*(11*b*B + 6*a*C)*(a + b*Cos[c + d*x
])^2*Sin[c + d*x])/(99*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^3*
Sin[c + d*x])/(11*d*Sec[c + d*x]^(3/2)) + (2*(77*a^3*B + 165*a*b^2*B + 33*a
^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*
x]])
```

Rubi [A] time = 1.06061, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\sin(c+dx)\left(242a^2bB+24a^3C+33ab^2(9A+7C)+77b^3B\right)}{495d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\left(24a^2C+143abB+99Ab^2+81b^2C\right)}{693d\sec^{\frac{5}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (2*(27*a^2*b*B + 7*b^3*B + 3*a^3*(5*A + 3*C) + 3*a*b^2*(9*A + 7*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(77*a
^3*B + 165*a*b^2*B + 33*a^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*b*(99*A*
b^2 + 143*a*b*B + 24*a^2*C + 81*b^2*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5
/2)) + (2*(242*a^2*b*B + 77*b^3*B + 24*a^3*C + 33*a*b^2*(9*A + 7*C))*Sin[c
+ d*x])/(495*d*Sec[c + d*x]^(3/2)) + (2*(11*b*B + 6*a*C)*(a + b*Cos[c + d*x
])^2*Sin[c + d*x])/(99*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^3*
```

```
Sin[c + d*x])/(11*d*Sec[c + d*x]^(3/2)) + (2*(77*a^3*B + 165*a*b^2*B + 33*a
^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*
x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\,\mathrm{co}$$

$$= \frac{2C(a+b\cos(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{11}\left(2\sqrt{\cos(c+d\right.$$

$$= \frac{2(11bB+6aC)(a+b\cos(c+dx))^2\sin(c+dx)}{99d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C(a}{}$$

$$= \frac{2b\left(99Ab^2+143abB+24a^2C+81b^2C\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2b\left(99Ab^2+143abB+24a^2C+81b^2C\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2b\left(99Ab^2+143abB+24a^2C+81b^2C\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2\left(27a^2bB+7b^3B+3a^3(5A+3C)+3ab^2(9A+7C)\right)}{15d}$$

$$= \frac{2\left(27a^2bB+7b^3B+3a^3(5A+3C)+3ab^2(9A+7C)\right)}{15d}$$

Mathematica [A] time = 3.47653, size = 304, normalized size = 0.76

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(33a^2b(7A+5C)+77a^3B+165ab^2B+5b^3(11A+9C)\right)+3696E\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(3696*(27*a^2*b*B + 7*b^3*B + 3*a^3*
(5*A + 3*C) + 3*a*b^2*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2] + 240*(77*a^3*
B + 165*a*b^2*B + 33*a^2*b*(7*A + 5*C) + 5*b^3*(11*A + 9*C))*EllipticF[(c +
 d*x)/2, 2] + ((154*(108*a^2*b*B + 43*b^3*B + 36*a^3*C + 3*a*b^2*(36*A + 43
*C))*Cos[c + d*x] + 5*(1848*a^3*B + 5148*a*b^2*B + 396*a^2*b*(14*A + 13*C)
+ 3*b^3*(572*A + 531*C) + 36*b*(11*A*b^2 + 33*a*b*B + 33*a^2*C + 16*b^2*C)*
```

```
Cos[2*(c + d*x)] + 154*b^2*(b*B + 3*a*C)*Cos[3*(c + d*x)] + 63*b^3*C*Cos[4*
(c + d*x)]))*Sin[2*(c + d*x)])/Sqrt[Cos[c + d*x]]))/(27720*d)
```

Maple [B] time = 1.485, size = 1082, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^3-36960*C*a*b^2-5040
0*C*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^3+23760*B*a*b^2
+24640*B*b^3+23760*C*a^2*b+73920*C*a*b^2+56880*C*b^3)*sin(1/2*d*x+1/2*c)^8*
cos(1/2*d*x+1/2*c)+(-16632*A*a*b^2-11880*A*b^3-16632*B*a^2*b-35640*B*a*b^2-
22792*B*b^3-5544*C*a^3-35640*C*a^2*b-68376*C*a*b^2-34920*C*b^3)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A*a^2*b+16632*A*a*b^2+9240*A*b^3+4620*B
*a^3+16632*B*a^2*b+27720*B*a*b^2+10472*B*b^3+5544*C*a^3+27720*C*a^2*b+31416
*C*a*b^2+13860*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6930*A*a^2*
b-4158*A*a*b^2-2640*A*b^3-2310*B*a^3-4158*B*a^2*b-7920*B*a*b^2-1848*B*b^3-1
386*C*a^3-7920*C*a^2*b-5544*C*a*b^2-2790*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+3465*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-3465*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6237*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+1155*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2475*a*b^2*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-6237*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-1617*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3+2475*a^2*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*C*b^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-2079*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-4851*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\cos\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3+\left(Ca^3+3\,Ba\right.}{\sqrt{\sec\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sq
rt(sec(d*x + c)), x)
```

3.1473 $\int \frac{(a+b\cos(c+dx))^3\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=463

$$\frac{2\sin(c+dx)\left(39a^2b(9A+7C)+117a^3B+273ab^2B+7b^3(13A+11C)\right)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(338a^2bB+24a^3C+39ab^2(11A\right.}{1001d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*(117*a^3*B + 273*a*b^2*B + 39*a^2*b*(9*A + 7*C) + 7*b^3*(13*A + 11*C))*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(195*d) + (
2*(165*a^2*b*B + 45*b^3*B + 11*a^3*(7*A + 5*C) + 15*a*b^2*(11*A + 9*C))*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*
b*(143*A*b^2 + 195*a*b*B + 24*a^2*C + 121*b^2*C)*Sin[c + d*x])/(1287*d*Sec[
c + d*x]^(7/2)) + (2*(338*a^2*b*B + 117*b^3*B + 24*a^3*C + 39*a*b^2*(11*A +
 9*C))*Sin[c + d*x])/(1001*d*Sec[c + d*x]^(5/2)) + (2*(13*b*B + 6*a*C)*(a +
 b*Cos[c + d*x])^2*Sin[c + d*x])/(143*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*C
os[c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c + d*x]^(5/2)) + (2*(117*a^3*B + 27
3*a*b^2*B + 39*a^2*b*(9*A + 7*C) + 7*b^3*(13*A + 11*C))*Sin[c + d*x])/(585*
d*Sec[c + d*x]^(3/2)) + (2*(165*a^2*b*B + 45*b^3*B + 11*a^3*(7*A + 5*C) + 1
5*a*b^2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.14125, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3049, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\sin(c+dx)\left(39a^2b(9A+7C)+117a^3B+273ab^2B+7b^3(13A+11C)\right)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(338a^2bB+24a^3C+39ab^2(11A\right.}{1001d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```

```
[Out] (2*(117*a^3*B + 273*a*b^2*B + 39*a^2*b*(9*A + 7*C) + 7*b^3*(13*A + 11*C))*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(195*d) + (
2*(165*a^2*b*B + 45*b^3*B + 11*a^3*(7*A + 5*C) + 15*a*b^2*(11*A + 9*C))*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*
b*(143*A*b^2 + 195*a*b*B + 24*a^2*C + 121*b^2*C)*Sin[c + d*x])/(1287*d*Sec[
c + d*x]^(7/2)) + (2*(338*a^2*b*B + 117*b^3*B + 24*a^3*C + 39*a*b^2*(11*A +
```

```
 9*C))*Sin[c + d*x])/(1001*d*Sec[c + d*x]^(5/2)) + (2*(13*b*B + 6*a*C)*(a +
 b*Cos[c + d*x])^2*Sin[c + d*x])/(143*d*Sec[c + d*x]^(5/2)) + (2*C*(a + b*C
os[c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c + d*x]^(5/2)) + (2*(117*a^3*B + 27
3*a*b^2*B + 39*a^2*b*(9*A + 7*C) + 7*b^3*(13*A + 11*C))*Sin[c + d*x])/(585*
d*Sec[c + d*x]^(3/2)) + (2*(165*a^2*b*B + 45*b^3*B + 11*a^3*(7*A + 5*C) + 1
5*a*b^2*(11*A + 9*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\,c$$

$$= \frac{2C(a+b\cos(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{13}\left(2\sqrt{\cos(c+\right.$$

$$= \frac{2(13bB+6aC)(a+b\cos(c+dx))^2\sin(c+dx)}{143d\sec^{\frac{5}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{2b\left(143Ab^2+195abB+24a^2C+121b^2C\right)\sin(c+dx}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b\left(143Ab^2+195abB+24a^2C+121b^2C\right)\sin(c+dx}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b\left(143Ab^2+195abB+24a^2C+121b^2C\right)\sin(c+dx}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b\left(143Ab^2+195abB+24a^2C+121b^2C\right)\sin(c+dx}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2\left(117a^3B+273ab^2B+39a^2b(9A+7C)+7b^3(13A\right.}{195}$$

Mathematica [A] time = 3.87686, size = 355, normalized size = 0.77

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(154\cos(c+dx)\left(78a^2b(36A+43C)+936a^3B+3354ab^2B+b^3(1118A+1171C)\right)+5\left(77\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7392*(117*a^3*B + 273*a*b^2*B + 39*a^2*b*(9*A + 7*C) +
 7*b^3*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6240*(
165*a^2*b*B + 45*b^3*B + 11*a^3*(7*A + 5*C) + 15*a*b^2*(11*A + 9*C))*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (154*(936*a^3*B + 3354*a*b^2*B + 7
8*a^2*b*(36*A + 43*C) + b^3*(1118*A + 1171*C))*Cos[c + d*x] + 5*(78*(1716*a
^2*b*B + 531*b^3*B + 44*a^3*(14*A + 13*C) + 3*a*b^2*(572*A + 531*C)) + 936*
```

```
(33*a^2*b*B + 16*b^3*B + 11*a^3*C + 3*a*b^2*(11*A + 16*C))*Cos[2*(c + d*x)]
 + 77*b*(52*A*b^2 + 156*a*b*B + 156*a^2*C + 89*b^2*C)*Cos[3*(c + d*x)] + 16
38*b^2*(b*B + 3*a*C)*Cos[4*(c + d*x)] + 693*b^3*C*Cos[5*(c + d*x)]))*Sin[2*
(c + d*x)]))/(720720*d)
```

Maple [B] time = 1.567, size = 1188, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*C
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(262080*B*b^3+786240*C*a*b^2+
1330560*C*b^3)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^3-4804
80*B*a*b^2-655200*B*b^3-480480*C*a^2*b-1965600*C*a*b^2-1798720*C*b^3)*sin(1
/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(308880*A*a*b^2+320320*A*b^3+308880*B*a
^2*b+960960*B*a*b^2+739440*B*b^3+102960*C*a^3+960960*C*a^2*b+2218320*C*a*b^
2+1379840*C*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-216216*A*a^2*b-4
63320*A*a*b^2-296296*A*b^3-72072*B*a^3-463320*B*a^2*b-888888*B*a*b^2-453960
*B*b^3-154440*C*a^3-888888*C*a^2*b-1361880*C*a*b^2-666512*C*b^3)*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(60060*A*a^3+216216*A*a^2*b+360360*A*a*b^2+13
6136*A*b^3+72072*B*a^3+360360*B*a^2*b+408408*B*a*b^2+180180*B*b^3+120120*C*
a^3+408408*C*a^2*b+540540*C*a*b^2+198352*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-30030*A*a^3-54054*A*a^2*b-102960*A*a*b^2-24024*A*b^3-18018*B
*a^3-102960*B*a^2*b-72072*B*a*b^2-36270*B*b^3-34320*C*a^3-72072*C*a^2*b-108
810*C*a*b^2-27258*C*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15015*A*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+32175*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-81081*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2*b-21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+32175*a^
2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+8775*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27027*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^3-63063*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+10725*a
^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+26325*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
```

```
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63063*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b-17787*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^{3}\cos\left(dx+c\right)^{5}+\left(3\,Cab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{4}+Aa^{3}+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(Ca^{3}+3\,\right.}{\sec\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))/sec(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/se
c(d*x + c)^(3/2), x)
```

3.1474 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right.$ $dx)\,dx$

Optimal. Leaf size=515

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(2a^2b(673A+891C)+539a^3B+1353ab^2B+192Ab^3\right)}{3465d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(9a^2b^2\right.}{}$$

```
[Out] (-2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*
A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (2*(220*a^3*b*B + 308*a*b^3*B + 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A
 + 7*C) + 5*a^4*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(231*d) + (2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*
b^3*(3*A + 5*C) + 4*a^3*b*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*d) + (2*(64*A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a^4*(9*A + 11*C) + 9*a^
2*b^2*(101*A + 143*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(693*d) + (2*a*(192
*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2*a^2*b*(673*A + 891*C))*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(3465*d) + (2*(16*A*b^2 + 55*a*b*B + 3*a^2*(9*A + 11*C))
*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (2*(8*A*
b + 11*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d)
+ (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 1.59025, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(2a^2b(673A+891C)+539a^3B+1353ab^2B+192Ab^3\right)}{3465d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(9a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(13/2),x]
```


```
[Out] (-2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*
A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (2*(220*a^3*b*B + 308*a*b^3*B + 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A
 + 7*C) + 5*a^4*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(231*d) + (2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*
b^3*(3*A + 5*C) + 4*a^3*b*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
```

```
*d) + (2*(64*A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a^4*(9*A + 11*C) + 9*a^
2*b^2*(101*A + 143*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(693*d) + (2*a*(192
*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2*a^2*b*(673*A + 891*C))*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(3465*d) + (2*(16*A*b^2 + 55*a*b*B + 3*a^2*(9*A + 11*C))
*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (2*(8*A*
b + 11*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d)
+ (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{11}{2}}(c+dx)\sin(}{11d}$$

$$= \frac{2(8Ab+11aB)(a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(}{99d}$$

$$= \frac{2\left(16Ab^2+55abB+3a^2(9A+11C)\right)(a+}{23}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b\right.}{34}$$

$$= \frac{2\left(64Ab^4+660a^3bB+682ab^3B+15a^4(9\right.}{}$$

$$= \frac{2\left(64Ab^4+660a^3bB+682ab^3B+15a^4(9\right.}{}$$

$$= \frac{2\left(220a^3bB+308ab^3B+77b^4(A+3C)+\right.}{}$$

$$= -\frac{2\left(7a^4B+54a^2b^2B+15b^4B+12ab^3(3A\right.}{}$$

Mathematica [A] time = 7.37236, size = 563, normalized size = 1.09

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}\sin(c+dx)\left(28a^3Ab+54a^2b^2B+36a^3bC+7a^4B+36aAb^3+60ab^3C+15b^4B\right)+\frac{2}{77}\sec^3(c+dx)\left(66a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(13/2),x]
```

```
[Out] ((2*(-2156*a^3*A*b - 2772*a*A*b^3 - 539*a^4*B - 4158*a^2*b^2*B - 1155*b^4*B
 - 2772*a^3*b*C - 4620*a*b^3*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*
```

```
x]]*Sqrt[Sec[c + d*x]]) + 2*(225*a^4*A + 1650*a^2*A*b^2 + 385*A*b^4 + 1100*
a^3*b*B + 1540*a*b^3*B + 275*a^4*C + 2310*a^2*b^2*C + 1155*b^4*C)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1155*d) + (Sqrt[Se
c[c + d*x]]*((2*(28*a^3*A*b + 36*a*A*b^3 + 7*a^4*B + 54*a^2*b^2*B + 15*b^4*
B + 36*a^3*b*C + 60*a*b^3*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^4*(4*a^3*A*
b*Sin[c + d*x] + a^4*B*Sin[c + d*x]))/9 + (2*Sec[c + d*x]^3*(9*a^4*A*Sin[c
+ d*x] + 66*a^2*A*b^2*Sin[c + d*x] + 44*a^3*b*B*Sin[c + d*x] + 11*a^4*C*Sin
[c + d*x]))/77 + (2*Sec[c + d*x]^2*(28*a^3*A*b*Sin[c + d*x] + 36*a*A*b^3*Si
n[c + d*x] + 7*a^4*B*Sin[c + d*x] + 54*a^2*b^2*B*Sin[c + d*x] + 36*a^3*b*C*
Sin[c + d*x]))/45 + (2*Sec[c + d*x]*(45*a^4*A*Sin[c + d*x] + 330*a^2*A*b^2*
Sin[c + d*x] + 77*A*b^4*Sin[c + d*x] + 220*a^3*b*B*Sin[c + d*x] + 308*a*b^3
*B*Sin[c + d*x] + 55*a^4*C*Sin[c + d*x] + 462*a^2*b^2*C*Sin[c + d*x]))/231
+ (2*a^4*A*Sec[c + d*x]^4*Tan[c + d*x])/11))/d
```

Maple [B] time = 9.52, size = 1550, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
^3*(B*b+4*C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*
x+1/2*c)^2-1)+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+2*a^3*(4*A*b+B*a)*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*c
os(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
```

```
ticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-1/56*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*a*b*(
2*A*b^2+3*B*a*b+2*C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*
sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*a^4*(-1/352*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
(1/2*d*x+1/2*c)^2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-15/154*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x
+1/2*c)^2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{13}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos(dx+c)^4 + 2\left(2\,Ca^3b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(13/2), x)
```

3.1475 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=441

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(202Ab+294bC)+75a^3B+261ab^2B+64Ab^3\right)}{315d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(7a^2b^2(155A\right.}{}$$

```
[Out] (-2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^
4*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2*B + 21*b^4*B + 28*a*b^3*(A + 3*C) +
4*a^3*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*(192*A*b^4 + 756*a^3*b*B + 1098*a*b^3*B + 21*a^4*(7*
A + 9*C) + 7*a^2*b^2*(155*A + 261*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315
*d) + (2*a*(64*A*b^3 + 75*a^3*B + 261*a*b^2*B + a^2*(202*A*b + 294*b*C))*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) + (2*(48*A*b^2 + 117*a*b*B + 7*a^2*(
7*A + 9*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d)
 + (2*(8*A*b + 9*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(63*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9
*d)
```

Rubi [A] time = 1.49228, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(202Ab+294bC)+75a^3B+261ab^2B+64Ab^3\right)}{315d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(7a^2b^2(155A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(11/2),x]
```


```
[Out] (-2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^
4*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2*B + 21*b^4*B + 28*a*b^3*(A + 3*C) +
4*a^3*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*(192*A*b^4 + 756*a^3*b*B + 1098*a*b^3*B + 21*a^4*(7*
A + 9*C) + 7*a^2*b^2*(155*A + 261*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315
*d) + (2*a*(64*A*b^3 + 75*a^3*B + 261*a*b^2*B + a^2*(202*A*b + 294*b*C))*Se
```

```
c[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) + (2*(48*A*b^2 + 117*a*b*B + 7*a^2*(
7*A + 9*C))*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d)
 + (2*(8*A*b + 9*a*B)*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(63*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9
*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{c}}{}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\sin}{9d}$$

$$= \frac{2(8Ab+9aB)(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(}{63d}$$

$$= \frac{2\left(48Ab^2+117abB+7a^2(7A+9C)\right)(a}{3}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(20\right.}{31}$$

$$= \frac{2\left(192Ab^4+756a^3bB+1098ab^3B+21\right.}{}$$

$$= \frac{2\left(192Ab^4+756a^3bB+1098ab^3B+21\right.}{}$$

$$= -\frac{2\left(36a^3bB+60ab^3B+15b^4(A-C)+\right.}{}$$

Mathematica [A] time = 7.26472, size = 459, normalized size = 1.04

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2}{15}\sin(c+dx)\left(54a^2Ab^2+7a^4A+90a^2b^2C+36a^3bB+9a^4C+60ab^3B+15Ab^4\right)+\frac{2}{45}\sec^2(c+dx)\left(54a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(11/2),x]
```

```
[Out] ((2*(-49*a^4*A - 378*a^2*A*b^2 - 105*A*b^4 - 252*a^3*b*B - 420*a*b^3*B - 63
*a^4*C - 630*a^2*b^2*C + 105*b^4*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + 2*(100*a^3*A*b + 140*a*A*b^3 + 25*a^4*B + 210
*a^2*b^2*B + 105*b^4*B + 140*a^3*b*C + 420*a*b^3*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(105*d) + (Sqrt[Sec[c + d*x]]*((2
```

```
*(7*a^4*A + 54*a^2*A*b^2 + 15*A*b^4 + 36*a^3*b*B + 60*a*b^3*B + 9*a^4*C + 9
0*a^2*b^2*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^3*(4*a^3*A*b*Sin[c + d*x] +
 a^4*B*Sin[c + d*x]))/7 + (2*Sec[c + d*x]^2*(7*a^4*A*Sin[c + d*x] + 54*a^2*
A*b^2*Sin[c + d*x] + 36*a^3*b*B*Sin[c + d*x] + 9*a^4*C*Sin[c + d*x]))/45 +
(2*Sec[c + d*x]*(20*a^3*A*b*Sin[c + d*x] + 28*a*A*b^3*Sin[c + d*x] + 5*a^4*
B*Sin[c + d*x] + 42*a^2*b^2*B*Sin[c + d*x] + 28*a^3*b*C*Sin[c + d*x]))/21 +
 (2*a^4*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d
```

Maple [B] time = 7.894, size = 1550, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*C*a*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*b^4*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*A*a^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3
-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
```

```
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^3*(4*A*b+B*a)*(-1/56*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a^2*(6*A*b
^2+4*B*a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{4}\cos\left(dx+c\right)^{6}+\left(4\,Cab^{3}+Bb^{4}\right)\cos\left(dx+c\right)^{5}+Aa^{4}+\left(6\,Ca^{2}b^{2}+4\,Bab^{3}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+2\left(2\,Ca^{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)+a\right)^4 \sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(11/2), x)
```

3.1476 $\int (a+b\cos(c+dx))^4 \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=423

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(202Ab+350bC)+63a^3B+413ab^2B+192Ab^3\right)}{105d} - \frac{2b^2\sin(c+dx)\left(5a^2(5A+7C)+98\right.}{105d\sqrt{\sec(c+a}}$$

```
[Out] (-2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5
*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a
^4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) - (2*b^2*(98*a*b*B + b^2*(87*A - 35*C) + 5*a^2*(5*A + 7*C))*Si
n[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*a*(192*A*b^3 + 63*a^3*B + 413*a
*b^2*B + a^2*(202*A*b + 350*b*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d)
+ (2*(48*A*b^2 + 77*a*b*B + 5*a^2*(5*A + 7*C))*(a + b*Cos[c + d*x])^2*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*(8*A*b + 7*a*B)*(a + b*Cos[c + d*x
])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^4*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.46859, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(202Ab+350bC)+63a^3B+413ab^2B+192Ab^3\right)}{105d} - \frac{2b^2\sin(c+dx)\left(5a^2(5A+7C)+98\right.}{105d\sqrt{\sec(c+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(9/2),x]
```


```
[Out] (-2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5
*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a
^4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) - (2*b^2*(98*a*b*B + b^2*(87*A - 35*C) + 5*a^2*(5*A + 7*C))*Si
n[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*a*(192*A*b^3 + 63*a^3*B + 413*a
*b^2*B + a^2*(202*A*b + 350*b*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d)
```

```
+ (2*(48*A*b^2 + 77*a*b*B + 5*a^2*(5*A + 7*C))*(a + b*Cos[c + d*x])^2*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*(8*A*b + 7*a*B)*(a + b*Cos[c + d*x
])^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^4*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\sin(c}{7d}$$

$$= \frac{2(8Ab+7aB)(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c}{35d}$$

$$= \frac{2\left(48Ab^2+77abB+5a^2(5A+7C)\right)(a+}{105}$$

$$= \frac{2a\left(192Ab^3+63a^3B+413ab^2B+a^2(202\right.}{105}$$

$$= -\frac{2b^2\left(98abB+b^2(87A-35C)+5a^2(5A\right.}{105d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2\left(98abB+b^2(87A-35C)+5a^2(5A\right.}{105d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\left(3a^4B+30a^2b^2B-5b^4B+20ab^3(A-\right.}{}$$

Mathematica [A] time = 4.43897, size = 294, normalized size = 0.7

$$\frac{\sqrt{\sec(c+dx)}\left(42a\sin(c+dx)\left(4a^2b(3A+5C)+3a^3B+30ab^2B+20Ab^3\right)+10a^2\tan(c+dx)\left(a^2(5A+7C)+28abB+4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-42*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A -
C) + 4*a^3*b*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10
*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5
*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 42*a*(20*A*b^3 +
3*a^3*B + 30*a*b^2*B + 4*a^2*b*(3*A + 5*C))*Sin[c + d*x] + 35*b^4*C*Sin[2*(
c + d*x)] + 10*a^2*(42*A*b^2 + 28*a*b*B + a^2*(5*A + 7*C))*Tan[c + d*x] + 4
```

```
2*a^3*(4*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x] + 30*a^4*A*Sec[c + d*x]^2*Tan
[c + d*x]))/(105*d)
```

Maple [B] time = 7.289, size = 1624, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*C*b^4*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*B*b^4+8*C*a*b^3-4*C*b^4)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a*b^3*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^4*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a
^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-8*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*(-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a
^3*(4*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x
+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
```

```
(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^4*(-1/56*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos(dx+c)^4 + 2\left(2\,Ca^3b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
```

```
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c) + a)^4 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(9/2), x)
```

3.1477 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx) \, dx$

Optimal. Leaf size=426

$$-\frac{2b^2\sin(c+dx)\left(3a^2(3A+5C)+50abB+b^2(59A-3C)\right)}{15d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\sin(c+dx)\left(6a^3(3A+5C)+105a^2bB+4ab^2(33A-5C\right.}{15d\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(
3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A
 + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) - (2*b^2*(50*a*b*B + b^2*(59*A - 3*C) + 3*a^2*(3*A + 5*C))*Sin[c + d*x
])/(15*d*Sec[c + d*x]^(3/2)) - (2*b*(105*a^2*b*B - 5*b^3*B + 4*a*b^2*(33*A
- 5*C) + 6*a^3*(3*A + 5*C))*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(1
6*A*b^2 + 15*a*b*B + a^2*(3*A + 5*C))*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(5*d) + (2*(8*A*b + 5*a*B)*(a + b*Cos[c + d*x])^3*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.47176, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b^2\sin(c+dx)\left(3a^2(3A+5C)+50abB+b^2(59A-3C)\right)}{15d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\sin(c+dx)\left(6a^3(3A+5C)+105a^2bB+4ab^2(33A-5C\right.}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(7/2),x]
```


```
[Out] (-2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(
3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A
 + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) - (2*b^2*(50*a*b*B + b^2*(59*A - 3*C) + 3*a^2*(3*A + 5*C))*Sin[c + d*x
])/(15*d*Sec[c + d*x]^(3/2)) - (2*b*(105*a^2*b*B - 5*b^3*B + 4*a*b^2*(33*A
```

```
- 5*C) + 6*a^3*(3*A + 5*C))*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*(1
6*A*b^2 + 15*a*b*B + a^2*(3*A + 5*C))*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(5*d) + (2*(8*A*b + 5*a*B)*(a + b*Cos[c + d*x])^3*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{co}}{}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\sin(}{5d}$$

$$= \frac{2(8Ab+5aB)(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(}{15d}$$

$$= \frac{2\left(16Ab^2+15abB+a^2(3A+5C)\right)(a+}{5}$$

$$= -\frac{2b^2\left(50abB+b^2(59A-3C)+3a^2(3A\right.}{15d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\left(50abB+b^2(59A-3C)+3a^2(3A\right.}{15d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\left(50abB+b^2(59A-3C)+3a^2(3A\right.}{15d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\left(20a^3bB-20ab^3B+30a^2b^2(A-C)\right.}{}$$

Mathematica [A] time = 4.99125, size = 307, normalized size = 0.72

$$\frac{\sqrt{\sec(c+dx)}\left(20\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(A+3C)+18a^2b^2B+a^4B+4ab^3(3A+C)+b^4B\right)-12\sqrt{\cos(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-12*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^
4*(5*A + 3*C) + a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + 20*(a^4*B + 18*a^2*b^2*B + b^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A + 3*
C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 36*a^4*A*Sin[c + d*x] +
360*a^2*A*b^2*Sin[c + d*x] + 240*a^3*b*B*Sin[c + d*x] + 60*a^4*C*Sin[c + d*
```

```
x] + 3*b^4*C*Sin[c + d*x] + 10*b^4*B*Sin[2*(c + d*x)] + 40*a*b^3*C*Sin[2*(c
 + d*x)] + 3*b^4*C*Sin[3*(c + d*x)] + 80*a^3*A*b*Tan[c + d*x] + 20*a^4*B*Ta
n[c + d*x] + 12*a^4*A*Sec[c + d*x]*Tan[c + d*x]))/(30*d)
```

Maple [B] time = 6.056, size = 1884, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C*b^4*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+1/3*(4*B*b^4+16*C*a*b^3-12*C*b^4)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+(2*A*b^4+8*B*a*b^3-4*B*b^4+12*C*a^2*b^2-16*C*a*b^3+6*C*b^4)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+8*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*b^4*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a^2*b^2*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-8*a*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+8*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*a^2*b^2*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*C*a*b^3*(sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*a^3*(4*A*b+B*a)*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))-2/5*A*a^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^4\cos\left(dx+c\right)^6+\left(4\,Cab^3+Bb^4\right)\cos\left(dx+c\right)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos\left(dx+c\right)^4+2\left(2\,Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(7/2), x)
```

3.1478 $\int(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=413

$$-\frac{2b^2\sin(c+dx)\left(105a^2B+350aAb-54abC-21b^2B\right)}{105d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\sin(c+dx)\left(3a^2b(49A-13C)+42a^3B-28ab^2B-b^3(7A\right.}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3
*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b
^4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) - (2*b^2*(350*a*A*b + 105*a^2*B - 21*b^2*B - 54*a*b*C)*Sin[c +
 d*x])/(105*d*Sec[c + d*x]^(3/2)) - (2*b*(42*a^3*B - 28*a*b^2*B + 3*a^2*b*(
49*A - 13*C) - b^3*(7*A + 5*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (
2*b*(21*A*b + 7*a*B - b*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[S
ec[c + d*x]]) + (2*(8*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d)
```

Rubi [A] time = 1.45101, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$-\frac{2b^2\sin(c+dx)\left(105a^2B+350aAb-54abC-21b^2B\right)}{105d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\sin(c+dx)\left(3a^2b(49A-13C)+42a^3B-28ab^2B-b^3(7A\right.}{21d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(5/2),x]
```


```
[Out] (-2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3
*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b
^4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) - (2*b^2*(350*a*A*b + 105*a^2*B - 21*b^2*B - 54*a*b*C)*Sin[c +
 d*x])/(105*d*Sec[c + d*x]^(3/2)) - (2*b*(42*a^3*B - 28*a*b^2*B + 3*a^2*b*(
```

```
49*A - 13*C) - b^3*(7*A + 5*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (
2*b*(21*A*b + 7*a*B - b*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Sqrt[S
ec[c + d*x]]) + (2*(8*A*b + 3*a*B)*(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^4*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
```

```
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= \frac{2(8Ab+3aB)(a+b\cos(c+dx))^3\sqrt{\sec(c}}{3d}$$

$$= -\frac{2b(21Ab+7aB-bC)(a+b\cos(c+dx))}{7d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2\left(350aAb+105a^2B-21b^2B-54abC\right.}{105d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\left(350aAb+105a^2B-21b^2B-54abC\right.}{105d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\left(350aAb+105a^2B-21b^2B-54abC\right.}{105d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\left(5a^4B-30a^2b^2B-3b^4B+20a^3b(A-\right.}{}$$

Mathematica [A] time = 3.10274, size = 316, normalized size = 0.77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(40F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5C)\right)-16\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-168*(5*a^4*B - 30*a^2*b^2*B - 3*b^
4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] + 4
0*(84*a^3*b*B + 28*a*b^3*B + 42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(
7*A + 5*C))*EllipticF[(c + d*x)/2, 2] + ((280*a^4*A + 140*A*b^4 + 560*a*b^3
*B + 840*a^2*b^2*C + 145*b^4*C + 42*(80*a^3*A*b + 20*a^4*B + 3*b^4*B + 12*a
```

```
*b^3*C)*Cos[c + d*x] + 20*b^2*(7*A*b^2 + 28*a*b*B + 42*a^2*C + 8*b^2*C)*Cos
[2*(c + d*x)] + 42*b^4*B*Cos[3*(c + d*x)] + 168*a*b^3*C*Cos[3*(c + d*x)] +
15*b^4*C*Cos[4*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(420*d)
```

Maple [B] time = 5.94, size = 2507, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(420*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b^3-630*a^2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+420*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^3*b+252*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-210*a^2*b^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+480*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10
-336*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+280*A*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6+504*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6
+920*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-280*A*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4-420*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
-252*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-440*C*b^4*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4+70*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
70*A*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+210*B*a^4*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+42*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+80
*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-960*C*b^4*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^8+630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-105*a^4*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-420*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-140*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^3+840*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)
^2+280*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2-840*C*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
```

+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-504*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2+420*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2+840*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-840*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2-35*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-25*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1120*B*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+1680*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2016*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1680*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1120*B*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1680*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1008*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+840*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+280*B*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+420*C*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+168*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1344*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-420*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+210*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+50*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+70*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+210*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2-126*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+1260*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2-1260*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos\left(dx+c\right)^6+\left(4\,Cab^3+Bb^4\right)\cos\left(dx+c\right)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos\left(dx+c\right)^4+2\left(2\,Ca^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(5/2), x)
```

3.1479 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ $dx)\,dx$

Optimal. Leaf size=419

$$\frac{2b^2\sin(c+dx)\left(a^2(-(315A-123C))+162abB+7b^2(9A+7C)\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\left(a^3(-(126A-62C))+117a^2bB+12\right.}{63d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^4
*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(15*d) + (2*(21*a^4*B + 42*a^2*b^2*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4
*a*b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(21*d) + (2*b^2*(162*a*b*B - a^2*(315*A - 123*C) + 7*b^2*(9*A + 7
*C))*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*b*(117*a^2*b*B + 15*b^3*
B - a^3*(126*A - 62*C) + 12*a*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*d*Sqrt[Sec
[c + d*x]]) - (2*b*(21*a*A - 3*b*B - 5*a*C)*(a + b*Cos[c + d*x])^2*Sin[c +
d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (2*b*(9*A - C)*(a + b*Cos[c + d*x])^3*Sin
[c + d*x])/(9*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^4*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.44445, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2b^2\sin(c+dx)\left(a^2(-(315A-123C))+162abB+7b^2(9A+7C)\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\left(a^3(-(126A-62C))+117a^2bB+12\right.}{63d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^(3/2),x]
```


```
[Out] (2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^4
*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(15*d) + (2*(21*a^4*B + 42*a^2*b^2*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4
*a*b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(21*d) + (2*b^2*(162*a*b*B - a^2*(315*A - 123*C) + 7*b^2*(9*A + 7
*C))*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*b*(117*a^2*b*B + 15*b^3*
```

```
B - a^3*(126*A - 62*C) + 12*a*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*d*Sqrt[Sec
[c + d*x]]) - (2*b*(21*a*A - 3*b*B - 5*a*C)*(a + b*Cos[c + d*x])^2*Sin[c +
d*x])/(21*d*Sqrt[Sec[c + d*x]]) - (2*b*(9*A - C)*(a + b*Cos[c + d*x])^3*Sin
[c + d*x])/(9*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Cos[c + d*x])^4*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
```

```
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2A(a+b\cos(c+dx))^4\sqrt{\sec(c+dx)}\sin(c}{d}$$

$$= -\frac{2b(9A-C)(a+b\cos(c+dx))^3\sin(c+}{9d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b(21aA-3bB-5aC)(a+b\cos(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

$$= \frac{2b^2\left(162abB-a^2(315A-123C)+7b^2(9A\right.}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2\left(162abB-a^2(315A-123C)+7b^2(9A\right.}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2\left(162abB-a^2(315A-123C)+7b^2(9A\right.}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(60a^3bB+36ab^3B-15a^4(A-C)+18a\right.}{}$$

Mathematica [A] time = 2.79265, size = 327, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)\left(30b\cos(c+dx)\left(168a^2bB+112a^3C+4ab^2(28A+29C)+29b^3B\right)+84b^2\cos(2(c+dx))\left(18\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-336*(-60*a^3*b*B - 36*a*b^3*B + 15*a^4*(A - C) - 18*a
^2*b^2*(5*A + 3*C) - b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2] + 240*(21*a^4*B + 42*a^2*b^2*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4*a
*b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(2520*a^
4*A + 252*A*b^4 + 1008*a*b^3*B + 1512*a^2*b^2*C + 301*b^4*C + 30*b*(168*a^2
*b*B + 29*b^3*B + 112*a^3*C + 4*a*b^2*(28*A + 29*C))*Cos[c + d*x] + 84*b^2*
```

```
(3*A*b^2 + 12*a*b*B + 18*a^2*C + 4*b^2*C)*Cos[2*(c + d*x)] + 90*b^4*B*Cos[3
*(c + d*x)] + 360*a*b^3*C*Cos[3*(c + d*x)] + 35*b^4*C*Cos[4*(c + d*x)])*Sin
[c + d*x]))/(2520*d)
```

Maple [B] time = 2.28, size = 1652, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*(-1120*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+80*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*b^3*(9*B*b+36*C*a+28*C*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*
d*x+1/2*c)-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(63*A
*b^2+252*B*a*b+135*B*b^2+378*C*a^2+540*C*a*b+259*C*b^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+56*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*b*(30*A*a*b^2+9*A*b^3+45*B*a^2*b+36*B*a*b^2+15*B*b^3+30*C*a^3+54*C*a^2*b+
60*C*a*b^2+17*C*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(105*A*a^4+140*A*a*b^3+21*A*b^4+21
0*B*a^2*b^2+84*B*a*b^3+40*B*b^4+140*C*a^3*b+126*C*a^2*b^2+160*C*a*b^3+28*C*
b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1260*A*a^3*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+420*a*A*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
+315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^4-1890*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-189*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+315*a^4*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+630*a^2*b^2*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+
75*b^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)-1260*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
```

```
2*d*x+1/2*c),2^(1/2))*a^3*b-756*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+420*a^3*b*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+300*C*a*b^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-31
5*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^4-1134*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^2-147*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^4\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^4\cos\left(dx+c\right)^6+\left(4\,Cab^3+Bb^4\right)\cos\left(dx+c\right)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\cos\left(dx+c\right)^4+2\left(2\,Ca^3b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
```

```
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)^4 \sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*se
c(d*x + c)^(3/2), x)
```

3.1480 $\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$

Optimal. Leaf size=444

$$\frac{2b\sin(c+dx)\left(1353a^2bB+192a^3C+2ab^2(891A+673C)+539b^3B\right)}{3465d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(9a^2b^2(143A+101C)+682a^3bB+\right.}{693d\sqrt{\sec(c+}}$$

```
[Out] (2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*A
 + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (2*(308*a^3*b*B + 220*a*b^3*B + 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A
+ 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(231*d) + (2*b*(1353*a^2*b*B + 539*b^3*B + 192*a^3*C + 2
*a*b^2*(891*A + 673*C))*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)) + (2*(682
*a^3*b*B + 660*a*b^3*B + 64*a^4*C + 15*b^4*(11*A + 9*C) + 9*a^2*b^2*(143*A
+ 101*C))*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]) + (2*(33*A*b^2 + 55*a*b*
B + 16*a^2*C + 27*b^2*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(231*d*Sqrt[S
ec[c + d*x]]) + (2*(11*b*B + 8*a*C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9
9*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(11*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 1.46011, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2b\sin(c+dx)\left(1353a^2bB+192a^3C+2ab^2(891A+673C)+539b^3B\right)}{3465d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(9a^2b^2(143A+101C)+682a^3bB+\right.}{693d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec
[c + d*x]],x]
```

```
[Out] (2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*A
 + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (2*(308*a^3*b*B + 220*a*b^3*B + 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A
+ 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(231*d) + (2*b*(1353*a^2*b*B + 539*b^3*B + 192*a^3*C + 2
*a*b^2*(891*A + 673*C))*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)) + (2*(682
*a^3*b*B + 660*a*b^3*B + 64*a^4*C + 15*b^4*(11*A + 9*C) + 9*a^2*b^2*(143*A
+ 101*C))*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]) + (2*(33*A*b^2 + 55*a*b*
```

```
B + 16*a^2*C + 27*b^2*C)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(231*d*Sqrt[S
ec[c + d*x]]) + (2*(11*b*B + 8*a*C)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(9
9*d*Sqrt[Sec[c + d*x]]) + (2*C*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(11*d*S
qrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4 \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos}{}$$

$$= \frac{2C(a+b\cos(c+dx))^4\sin(c+dx)}{11d\sqrt{\sec(c+dx)}} + \frac{1}{11}($$

$$= \frac{2(11bB+8aC)(a+b\cos(c+dx))^3\sin(c}{99d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(33Ab^2+55abB+16a^2C+27b^2C\right)(a}{231d\sqrt{\sec(c+}}$$

$$= \frac{2b\left(1353a^2bB+539b^3B+192a^3C+2ab^2\right.}{3465d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2b\left(1353a^2bB+539b^3B+192a^3C+2ab^2\right.}{3465d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2b\left(1353a^2bB+539b^3B+192a^3C+2ab^2\right.}{3465d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2\left(15a^4B+54a^2b^2B+7b^4B+12a^3b(5A\right.}{}$$

Mathematica [A] time = 3.8864, size = 338, normalized size = 0.76

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(7A+5C)+77a^4(3A+C)+308a^3bB+220ab^3B+5b^4(11A+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(3696*(15*a^4*B + 54*a^2*b^2*B + 7*b
^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2
] + 240*(308*a^3*b*B + 220*a*b^3*B + 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A + 5
*C) + 5*b^4*(11*A + 9*C))*EllipticF[(c + d*x)/2, 2] + ((154*b*(216*a^2*b*B
+ 43*b^3*B + 144*a^3*C + 4*a*b^2*(36*A + 43*C))*Cos[c + d*x] + 5*(7392*a^3*
b*B + 6864*a*b^3*B + 1848*a^4*C + 792*a^2*b^2*(14*A + 13*C) + 3*b^4*(572*A
+ 531*C) + 36*b^2*(11*A*b^2 + 44*a*b*B + 66*a^2*C + 16*b^2*C)*Cos[2*(c + d*
x)] + 154*b^3*(b*B + 4*a*C)*Cos[3*(c + d*x)] + 63*b^4*C*Cos[4*(c + d*x)]))*
Sin[2*(c + d*x)])/Sqrt[Cos[c + d*x]]))/(27720*d)
```

Maple [B] time = 1.618, size = 1273, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*C*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*B*b^4-49280*C*a*b^3-5040
0*C*b^4)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(7920*A*b^4+31680*B*a*b^3
+24640*B*b^4+47520*C*a^2*b^2+98560*C*a*b^3+56880*C*b^4)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-22176*A*a*b^3-11880*A*b^4-33264*B*a^2*b^2-47520*B*a*
b^3-22792*B*b^4-22176*C*a^3*b-71280*C*a^2*b^2-91168*C*a*b^3-34920*C*b^4)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(27720*A*a^2*b^2+22176*A*a*b^3+9240*A
*b^4+18480*B*a^3*b+33264*B*a^2*b^2+36960*B*a*b^3+10472*B*b^4+4620*C*a^4+221
76*C*a^3*b+55440*C*a^2*b^2+41888*C*a*b^3+13860*C*b^4)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-13860*A*a^2*b^2-5544*A*a*b^3-2640*A*b^4-9240*B*a^3*b-8
316*B*a^2*b^2-10560*B*a*b^3-1848*B*b^4-2310*C*a^4-5544*C*a^3*b-15840*C*a^2*
b^2-7392*C*a*b^3-2790*C*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3465*A
*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
```

```
F(cos(1/2*d*x+1/2*c),2^(1/2))+6930*a^2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+825*A
*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-13860*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b^3+4620*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+3300*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b^3-3465*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12474*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1155
*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675
*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-8316*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-6468*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*sq
rt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{4}\cos\left(dx+c\right)^{6}+\left(4\,Cab^{3}+Bb^{4}\right)\cos\left(dx+c\right)^{5}+Aa^{4}+\left(6\,Ca^{2}b^{2}+4\,Bab^{3}+Ab^{4}\right)\cos\left(dx+c\right)^{4}+2\left(2\,Ca^{3}b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4*sq
rt(sec(d*x + c)), x)
```

3.1481 $\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=517

$$\frac{2\sin(c+dx)\left(11a^2b^2(637A+491C)+3458a^3bB+192a^4C+4004ab^3B+77b^4(13A+11C)\right)}{6435d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\left(2171a^2bB\right.}{}$$

```
[Out] (2*(468*a^3*b*B + 364*a*b^3*B + 39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C)
 + 7*b^4*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(195*d) + (2*(77*a^4*B + 330*a^2*b^2*B + 45*b^4*B + 44*a^3*b*
(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*b*(2171*a^2*b*B + 1053*b^3*B + 192*
a^3*C + 2*a*b^2*(1573*A + 1259*C))*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)
) + (2*(3458*a^3*b*B + 4004*a*b^3*B + 192*a^4*C + 77*b^4*(13*A + 11*C) + 11
*a^2*b^2*(637*A + 491*C))*Sin[c + d*x])/(6435*d*Sec[c + d*x]^(3/2)) + (2*(1
43*A*b^2 + 221*a*b*B + 48*a^2*C + 121*b^2*C)*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(1287*d*Sec[c + d*x]^(3/2)) + (2*(13*b*B + 8*a*C)*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(143*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^4
*Sin[c + d*x])/(13*d*Sec[c + d*x]^(3/2)) + (2*(77*a^4*B + 330*a^2*b^2*B + 4
5*b^4*B + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sin[c + d*x])/(231*
d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.52742, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\sin(c+dx)\left(11a^2b^2(637A+491C)+3458a^3bB+192a^4C+4004ab^3B+77b^4(13A+11C)\right)}{6435d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\left(2171a^2bB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (2*(468*a^3*b*B + 364*a*b^3*B + 39*a^4*(5*A + 3*C) + 78*a^2*b^2*(9*A + 7*C)
 + 7*b^4*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(195*d) + (2*(77*a^4*B + 330*a^2*b^2*B + 45*b^4*B + 44*a^3*b*
(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*b*(2171*a^2*b*B + 1053*b^3*B + 192*
```

```
a^3*C + 2*a*b^2*(1573*A + 1259*C))*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)
) + (2*(3458*a^3*b*B + 4004*a*b^3*B + 192*a^4*C + 77*b^4*(13*A + 11*C) + 11
*a^2*b^2*(637*A + 491*C))*Sin[c + d*x])/(6435*d*Sec[c + d*x]^(3/2)) + (2*(1
43*A*b^2 + 221*a*b*B + 48*a^2*C + 121*b^2*C)*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/(1287*d*Sec[c + d*x]^(3/2)) + (2*(13*b*B + 8*a*C)*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(143*d*Sec[c + d*x]^(3/2)) + (2*C*(a + b*Cos[c + d*x])^4
*Sin[c + d*x])/(13*d*Sec[c + d*x]^(3/2)) + (2*(77*a^4*B + 330*a^2*b^2*B + 4
5*b^4*B + 44*a^3*b*(7*A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sin[c + d*x])/(231*
d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^4\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\,\text{c}$$

$$= \frac{2C(a+b\cos(c+dx))^4\sin(c+dx)}{13d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{13}\left(2\sqrt{\cos(c+\right.$$

$$= \frac{2(13bB+8aC)(a+b\cos(c+dx))^3\sin(c+dx)}{143d\sec^{\frac{3}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{2\left(143Ab^2+221abB+48a^2C+121b^2C\right)(a+b\cos(}{1287d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(2171a^2bB+1053b^3B+192a^3C+2ab^2(1573A+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(2171a^2bB+1053b^3B+192a^3C+2ab^2(1573A+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(2171a^2bB+1053b^3B+192a^3C+2ab^2(1573A+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(468a^3bB+364ab^3B+39a^4(5A+3C)+78a^2b^2(9\right.}{}$$

$$= \frac{2\left(468a^3bB+364ab^3B+39a^4(5A+3C)+78a^2b^2(9\right.}{}$$

Mathematica [A] time = 4.43192, size = 400, normalized size = 0.77

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(154\cos(c+dx)\left(156a^2b^2(36A+43C)+3744a^3bB+936a^4C+4472ab^3B+b^4(1118A+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7392*(468*a^3*b*B + 364*a*b^3*B + 39*a^4*(5*A + 3*C) +
 78*a^2*b^2*(9*A + 7*C) + 7*b^4*(13*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 6240*(77*a^4*B + 330*a^2*b^2*B + 45*b^4*B + 44*a^3*b*(7*
```

```
A + 5*C) + 20*a*b^3*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2] + (154*(3744*a^3*b*B + 4472*a*b^3*B + 936*a^4*C + 156*a^2*b^2*(36*A + 4
3*C) + b^4*(1118*A + 1171*C))*Cos[c + d*x] + 5*(78*(616*a^4*B + 3432*a^2*b^
2*B + 531*b^4*B + 176*a^3*b*(14*A + 13*C) + 4*a*b^3*(572*A + 531*C)) + 1872
*b*(33*a^2*b*B + 8*b^3*B + 22*a^3*C + 2*a*b^2*(11*A + 16*C))*Cos[2*(c + d*x
)] + 77*b^2*(52*A*b^2 + 208*a*b*B + 312*a^2*C + 89*b^2*C)*Cos[3*(c + d*x)]
+ 1638*b^3*(b*B + 4*a*C)*Cos[4*(c + d*x)] + 693*b^4*C*Cos[5*(c + d*x)]))*Si
n[2*(c + d*x)]))/(720720*d)
```

Maple [B] time = 1.67, size = 1407, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*C
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(262080*B*b^4+1048320*C*a*b^3
+1330560*C*b^4)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-160160*A*b^4-640
640*B*a*b^3-655200*B*b^4-960960*C*a^2*b^2-2620800*C*a*b^3-1798720*C*b^4)*si
n(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(411840*A*a*b^3+320320*A*b^4+617760*
B*a^2*b^2+1281280*B*a*b^3+739440*B*b^4+411840*C*a^3*b+1921920*C*a^2*b^2+295
7760*C*a*b^3+1379840*C*b^4)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-43243
2*A*a^2*b^2-617760*A*a*b^3-296296*A*b^4-288288*B*a^3*b-926640*B*a^2*b^2-118
5184*B*a*b^3-453960*B*b^4-72072*C*a^4-617760*C*a^3*b-1777776*C*a^2*b^2-1815
840*C*a*b^3-666512*C*b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(240240*A
*a^3*b+432432*A*a^2*b^2+480480*A*a*b^3+136136*A*b^4+60060*B*a^4+288288*B*a^
3*b+720720*B*a^2*b^2+544544*B*a*b^3+180180*B*b^4+72072*C*a^4+480480*C*a^3*b
+816816*C*a^2*b^2+720720*C*a*b^3+198352*C*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-120120*A*a^3*b-108108*A*a^2*b^2-137280*A*a*b^3-24024*A*b^4-30
030*B*a^4-72072*B*a^3*b-205920*B*a^2*b^2-96096*B*a*b^3-36270*B*b^4-18018*C*
a^4-137280*C*a^3*b-144144*C*a^2*b^2-145080*C*a*b^3-27258*C*b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-45045*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-162162*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-21021*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+60060
*A*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+42900*a*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10
8108*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
```

```
icE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-84084*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^
3+15015*a^4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+64350*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
*b^2+8775*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-27027*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4-126126*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-17787*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*b^4+42900*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35100*C*a*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\cos(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\cos(dx+c)^4 + 2\left(2\,Ca^3b\right.}{\sqrt{\sec(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
```

```
2*b^2 + 2*A*a*b^3)*cos(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^4/sq
rt(sec(d*x + c)), x)
```

3.1482 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(3A+5C)-5abB+5Ab^2\right)}{5a^3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)\right.}{5a^3d}$$

```
[Out] (-2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (2*b*(A*b^2 -
a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5
*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*d) - (2*(A*b - a*B)*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5
*a*d)
```

Rubi [A] time = 1.422, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(3A+5C)-5abB+5Ab^2\right)}{5a^3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)\right.}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + b*Cos
[c + d*x]),x]
```



```
[Out] (-2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (2*b*(A*b^2 -
a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5
*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*d) - (2*(A*b - a*B)*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5
*a*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{a+b\cos(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c}$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{2(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\,\mathrm{si}}{5ad}$$

$$= \frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5a^3d} -$$

$$= \frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5a^3d} -$$

$$= -\frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{5a^3d}$$

$$= -\frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{5a^3d}$$

Mathematica [B] time = 7.05932, size = 698, normalized size = 2.37

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\left(3a^2A+5a^2C-5abB+5Ab^2\right)}{5a^3}+\frac{2\sec(c+dx)(aB\sin(c+dx)-Ab\sin(c+dx))}{3a^2}+\frac{2A\tan(c+dx)\sec(c+dx)}{5a}\right)}{d} - \frac{-\frac{2\sin(c+dx)\cos^2(}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] -((-2*(18*a^3*A + 40*a*A*b^2 - 40*a^2*b*B + 30*a^3*C)*Cos[c + d*x]^2*Ellipt
icPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 -
 Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)
) + (2*(19*a^2*A*b + 45*A*b^3 - 10*a^3*B - 45*a*b^2*B + 45*a^2*b*C)*Cos[c +
 d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -Ar
cSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2
]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((9*a^2*A*b
 + 15*A*b^3 - 15*a*b^2*B + 15*a^2*b*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x]
)*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x]
)/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 -
Sec[c + d*x]^2)))/(30*a^3*d) + (Sqrt[Sec[c + d*x]]*((2*(3*a^2*A + 5*A*b^2 -
 5*a*b*B + 5*a^2*C)*Sin[c + d*x])/(5*a^3) + (2*Sec[c + d*x]*(-(A*b*Sin[c +
d*x]) + a*B*Sin[c + d*x]))/(3*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a))
)/d
```

Maple [B] time = 5.628, size = 802, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(A*b^2-B*a*b+
C*a^2)*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-1/6*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2-B*a*b+C*a^2)/a^3*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
```

```
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*A/a/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*cos
(d*x + c) + a), x)
```

3.1483 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=218

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-}{}$$

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*(
A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.980958, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos
[c + d*x]),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*(
A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c}$$

$$=\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{}$$

$$=-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin}{3ad}$$

$$=-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin}{3ad}$$

$$=\frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{2(}{}$$

$$=\frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2}{}$$

Mathematica [A] time = 4.08985, size = 273, normalized size = 1.25

$$-\frac{\cot(c+dx)\left(-2\sqrt{-\tan^2(c+dx)}\left(a^2(A-3B+3C)+3ab(A-B)+3Ab^2\right)F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-a^2A\sec^{\frac{5}{2}}(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]*(-(a^2*A*Sec[c + d*x]^(5/2)) + a^2*A*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) - 6*a*(-(A*b) + a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[-Tan[c + d*x]^2] - 2*(3*A*b^2 + 3*a*b*(A - B) + a^2*(A - 3*B + 3*C))*
```

```
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*A*b^2*E
llipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] +
6*a*b*B*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d
*x]^2] - 6*a^2*C*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2]))/(3*a^3*d)
```

Maple [A] time = 4.031, size = 474, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b^2-B*a*b
+C*a^2)/a^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a), x)
```

3.1484 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx}}{}$$

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
b*d) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a + b)*d) + (2*A*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.667404, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos
[c + d*x]),x]
```



```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
b*d) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a + b)*d) + (2*A*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(a*d)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + B\cos(c+dx) + C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{a}$$

$$= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{2A\sqrt{\sec(c}}{}$$

$$= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{2C\sqrt{\cos(c}}{}$$

Mathematica [A] time = 1.49676, size = 140, normalized size = 0.79

$$\frac{2\cos(2(c+dx))\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(\left(a(aC-bB)+Ab^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+b(a(}{a^2bd\left(\sec^2(c+dx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[2*(c + d*x)]*Csc[c + d*x]*(-(a*A*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]) + b*(A*b + a*(A - B))*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] +
 (A*b^2 + a*(-(b*B) + a*C))*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]],
 -1])*Sec[c + d*x]*Sqrt[-Tan[c + d*x]^2])/(a^2*b*d*(-2 + Sec[c + d*x]^2))
```

Maple [A] time = 2.814, size = 411, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,\frac{C\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{Elliptic}}{b\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*(-A
*b^2+B*a*b-C*a^2)/a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a), x)
```

3.1485 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{b^2d}$$

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(b^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rubi [A] time = 0.434514, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4221, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos
[c + d*x]),x]
```

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(b^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt{\cos(c+dx)}(a+b\cos(c}$$

$$= -\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-Ab-(bB-aC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b}$$

$$= \frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{\left((-bB+a\right.}{}$$

$$= \frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(bB-aC}{}$$

Mathematica [A] time = 2.05548, size = 282, normalized size = 1.8

$$\frac{\cot(c+dx)\left(2a^2C\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2Ab^2\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 b*Cos[c + d*x]),x]
```

```
[Out] (Cot[c + d*x]*(-(a*b*C*Sec[c + d*x]^(3/2)) - a*b*C*Cos[2*(c + d*x)]*Sec[c +
 d*x]^(3/2) + a*b*C*Sec[c + d*x]^(7/2) + a*b*C*Cos[2*(c + d*x)]*Sec[c + d*x
]^(7/2) - 2*a*b*C*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d
*x]^2] + 2*b*(A*b + a*C)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Ta
n[c + d*x]^2] + 2*A*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[-Tan[c + d*x]^2] - 2*a*b*B*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a^2*C*EllipticPi[-(a/b), -ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a*b^2*d)
```

Maple [A] time = 1.48, size = 323, normalized size = 2.1

$$-2\,\frac{\sqrt{\left(2\,\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\,\left(\cos(1/2\,dx+c/2)\right)^2+1}}{(a-b)\,b^2\sqrt{-2\,\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin(1/2\,dx+c/2)\right)^2}\sin(1/2\,dx+c/2)\sqrt{2\,\left(\cos(1/2\,dx+c/2)\right)^2-1}d}\left(A\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-B*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))*a*b-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a*b+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+C*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)\right)\sqrt{\sec\left(c + dx\right)}}{a + b\cos\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c
)),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/(a + b
*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A\right)\sqrt{\sec\left(dx + c\right)}}{b\cos\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a), x)
```

3.1486 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=207

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(b^2(3A+C)-3a(bB-aC)\right)}{3b^3d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-a\right.}{b^3d(a+b)}$$

```
[Out] (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b^2*d) + (2*(b^2*(3*A + C) - 3*a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) - (2*a*(A*b^2 - a*(b*B
- a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(b^3*(a + b)*d) + (2*C*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]
])
```

Rubi [A] time = 0.753949, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(b^2(3A+C)-3a(bB-aC)\right)}{3b^3d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-a\right.}{b^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sqrt[Sec[
c + d*x]]),x]
```



```
[Out] (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b^2*d) + (2*(b^2*(3*A + C) - 3*a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) - (2*a*(A*b^2 - a*(b*B
- a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(b^3*(a + b)*d) + (2*C*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]
])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{a+b\cos(c+dx)}$$

$$= \frac{2C\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aC}{2}+\frac{1}{2}b(3A+C)\cos(c+dx)+\frac{3}{2}(b}{\sqrt{\cos(c+dx)}(a+b\cos}}{3b}$$

$$= \frac{2C\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}abC-\frac{1}{2}\left(b^2(3A+C)-3a(bB-a\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c}}{3b^2}$$

$$= \frac{2(bB-aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2C\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} -$$

$$= \frac{2(bB-aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\left(b^2(3A+C)-3a\right.}{}$$

Mathematica [B] time = 6.78048, size = 560, normalized size = 2.71

$$\frac{(3bB-3aC)\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+b)\left(4a^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-2b^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin\right.}{ab^2\left(1-\cos^2(c+dx)\right)\sqrt{\sec(c+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sqr
t[Sec[c + d*x]]),x]
```

```
[Out] ((-2*(6*A*b + 2*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(3*b*B - a*C)*Cos[c + d*x]
^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[
Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((3*b*B - 3*a*C)
*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a
*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
```

```
c[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b
^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos
[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(6*b*d) + (C*Sqrt[Se
c[c + d*x]]*Sin[2*(c + d*x)])/(3*b*d)
```

Maple [B] time = 1.674, size = 945, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*C*a*b^2-4*
C*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*C*a*b^2+2*C*b^3)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*a*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b^2-3*
a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*a*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+3*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3+3*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2*b-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/b^3/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos{(dx+c)}^2 + B\cos{(dx+c)} + A}{(b\cos{(dx+c)} + a)\sqrt{\sec{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos{(c+dx)} + C\cos^2{(c+dx)}}{(a + b\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/sec(d*x+c)**(1/
2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/((a + b*cos(c + d*x))*sqr
t(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

3.1487 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=270

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-3a^3C-ab^2(3A+C)+b^3B\right)}{3b^4d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c-\right.}{5}$$

```
[Out] (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(3*a^2*b*B + b^3*B - 3*a^3*
C - a*b^2*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b^4*d) + (2*a^2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^4*(a + b)*d)
 + (2*C*Sin[c + d*x])/(5*b*d*Sec[c + d*x]^(3/2)) + (2*(b*B - a*C)*Sin[c + d
*x])/(3*b^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.05872, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-3a^3C-ab^2(3A+C)+b^3B\right)}{3b^4d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c-\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec[c + d
*x]^(3/2)),x]
```


```
[Out] (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(3*a^2*b*B + b^3*B - 3*a^3*
C - a*b^2*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b^4*d) + (2*a^2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^4*(a + b)*d)
 + (2*C*Sin[c + d*x])/(5*b*d*Sec[c + d*x]^(3/2)) + (2*(b*B - a*C)*Sin[c + d
*x])/(3*b^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A + B\cos(c+dx) + C\cos^2(c\right.}{a+b\cos(c+dx)}$$

$$= \frac{2C\sin(c+dx)}{5bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aC}{2}+\frac{1}{2}b(5A+3C)\right.}{a+b\,c}}{5b}$$

$$= \frac{2C\sin(c+dx)}{5bd\sec^{\frac{3}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{3b^2d\sqrt{\sec(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2C\sin(c+dx)}{5bd\sec^{\frac{3}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{3b^2d\sqrt{\sec(c+dx)}} - \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2\left(5Ab^2 - 5abB + 5a^2C + 3b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^3d}$$

$$= \frac{2\left(5Ab^2 - 5abB + 5a^2C + 3b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^3d}$$

Mathematica [B] time = 7.01389, size = 632, normalized size = 2.34

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{(bB-aC)\sin(2(c+dx))}{3b^2} + \frac{C\sin(c+dx)}{10b} + \frac{C\sin(3(c+dx))}{10b}\right)}{d} - \frac{2\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\left(-5a^2C+5abB-15Ab^2-9b^2C\right)(a\sec(c+}{a\left(1-\cos^2(c+dx)\right)(a+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec
[c + d*x]^(3/2)),x]
```

```
[Out] -((-2*(-10*b^2*B - 8*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[
Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d
```

```
*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-15*A*b^2 + 5*a*b*
B - 5*a^2*C - 9*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + ((-15*A*b^2 + 15*a*b*B - 15*a^2*C - 9*b^2*C)*Cos[2*(c +
d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticP
i[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)
*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(30*b^2*d) + (Sqrt[Sec[c + d*x]]
*((C*Sin[c + d*x])/(10*b) + ((b*B - a*C)*Sin[2*(c + d*x)])/(3*b^2) + (C*Sin
[3*(c + d*x)])/(10*b)))/d
```

Maple [B] time = 3.438, size = 803, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b*(-4*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+4/3/b^2*(B*b-C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/b^3
*(A*b^2-B*a*b-2*B*b^2+C*a^2+2*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*(A*a*b^2+A*b^3-B*a^2*b-B*a*b^2-B*b^3+C*a^3+C*a^2*b+C*a*b^2+
C*b^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-4*a^2*(A*b^2-B*a*b+C*a^2)/b^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/
```

```
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/sec(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c) + a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

3.1488 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=345

$$\frac{2\sin(c+dx)\left(7a^2C-7abB+7Ab^2+5b^2C\right)}{21b^3d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(3A+C)+21a^3bB-\right.}{21b^5d}$$

```
[Out] (2*(5*a^2*b*B + 3*b^3*B - 5*a^3*C - a*b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^4*d) - (2*(21*a^3*b*B + 7
*a*b^3*B - 21*a^4*C - 7*a^2*b^2*(3*A + C) - b^4*(7*A + 5*C))*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*b^5*d) - (2*a^3*(A*b
^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(b^5*(a + b)*d) + (2*C*Sin[c + d*x])/(7*b*d*Sec[c
 + d*x]^(5/2)) + (2*(b*B - a*C)*Sin[c + d*x])/(5*b^2*d*Sec[c + d*x]^(3/2))
+ (2*(7*A*b^2 - 7*a*b*B + 7*a^2*C + 5*b^2*C)*Sin[c + d*x])/(21*b^3*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 1.4781, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\sin(c+dx)\left(7a^2C-7abB+7Ab^2+5b^2C\right)}{21b^3d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(3A+C)+21a^3bB-\right.}{21b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec[c + d
*x]^(5/2)),x]
```


```
[Out] (2*(5*a^2*b*B + 3*b^3*B - 5*a^3*C - a*b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^4*d) - (2*(21*a^3*b*B + 7
*a*b^3*B - 21*a^4*C - 7*a^2*b^2*(3*A + C) - b^4*(7*A + 5*C))*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*b^5*d) - (2*a^3*(A*b
^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(b^5*(a + b)*d) + (2*C*Sin[c + d*x])/(7*b*d*Sec[c
 + d*x]^(5/2)) + (2*(b*B - a*C)*Sin[c + d*x])/(5*b^2*d*Sec[c + d*x]^(3/2))
+ (2*(7*A*b^2 - 7*a*b*B + 7*a^2*C + 5*b^2*C)*Sin[c + d*x])/(21*b^3*d*Sqrt[S
ec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{a+b\cos(c+dx)}$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5aC}{2}+\frac{1}{2}b(7A+5C)\right.}{a+b\,c}}{7b}$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{5b^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{5b^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+7a^2C+5b^2\right.}{21b^3d\sqrt{\sec(c+d}}$$

$$= \frac{2C\sin(c+dx)}{7bd\sec^{\frac{5}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{5b^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+7a^2C+5b^2\right.}{21b^3d\sqrt{\sec(c+d}}$$

$$= \frac{2\left(5a^2bB+3b^3B-5a^3C-ab^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{5b^4d}$$

$$= \frac{2\left(5a^2bB+3b^3B-5a^3C-ab^2(5A+3C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{5b^4d}$$

Mathematica [A] time = 6.59163, size = 538, normalized size = 1.56

$$\frac{4b^2\sin(c+dx)\left(70a^2C+42b(bB-aC)\cos(c+dx)-70abB+70Ab^2+15b^2C\cos(2(c+dx))+65b^2C\right)-\frac{2\cos(c+dx)\cot(c+}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])*Sec
[c + d*x]^(5/2)),x]
```

```
[Out] (4*b^2*(70*A*b^2 - 70*a*b*B + 70*a^2*C + 65*b^2*C + 42*b*(b*B - a*C)*Cos[c
+ d*x] + 15*b^2*C*Cos[2*(c + d*x)])*Sin[c + d*x] - (2*Cos[c + d*x]*Cot[c +
d*x]*(b + a*Sec[c + d*x])*(4*a*b^2*(35*A*b^2 + 28*a*b*B - 28*a^2*C + 25*b^2
*C)*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[-Tan[c + d*x]^2] - 2*b^2*(35*a^2*b*B + 63*b^3*B - 35*a^3*C - a*b^2*(35
*A + 13*C))*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]
 - 21*(-5*a^2*b*B - 3*b^3*B + 5*a^3*C + a*b^2*(5*A + 3*C))*(4*a*b - 4*a*b*S
ec[c + d*x]^2 + 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] - 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 4*a^2*EllipticPi[-(
a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x
]^2] + 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
 + d*x]]*Sqrt[-Tan[c + d*x]^2])))/(a*(a + b*Cos[c + d*x])))/(420*b^5*d*Sqrt
[Sec[c + d*x]])
```

Maple [B] time = 4.052, size = 1097, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/105*C/b*(60*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-168*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4/5/b^2*(B*b-C*a-4
*C*b)*(-4*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+4/3/b^3*(A*b^2-B*a*b-3*B*b^2+C*a^2+3*C*a*b+6*C*b^2)*(2*si
```

```
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/b^4*(A*a*b^2+2*A*b^3-B*a^2*b-2*B*a*b^
2-3*B*b^3+C*a^3+2*C*a^2*b+3*C*a*b^2+4*C*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*(A*a^2*b^2+A*a*b^3+A*b^4-B*a^3*b-B*a^2*b^2-B*a*b^3-B*b^4+C*
a^4+C*a^3*b+C*a^2*b^2+C*a*b^3+C*b^4)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*a^3*(A*b^2-B*a*b+C*a^2)/b^4/
(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec
(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))/sec(d*x+c)**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec
(d*x + c)^(5/2)), x)
```

3.1489 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=452

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(-(2A-3C))-3abB+5Ab^2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b(4A-C)+2a^3B-3ab^2B+\right.}{a^3d\left(a^2-b^2\right)}$$

```
[Out] -(((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) - ((5*A*b^2
 - 3*a*b*B - a^2*(2*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d) - ((5*A*b^4 + 5*a^3*b*B - 3*a*b^3
*B - a^2*b^2*(7*A - C) - 3*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + ((5*A*b
^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^3*(a^2 - b^2)*d) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.73624, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(-(2A-3C))-3abB+5Ab^2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b(4A-C)+2a^3B-3ab^2B+\right.}{a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos
[c + d*x])^2,x]
```



```
[Out] -(((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) - ((5*A*b^2
 - 3*a*b*B - a^2*(2*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d) - ((5*A*b^4 + 5*a^3*b*B - 3*a*b^3
*B - a^2*b^2*(7*A - C) - 3*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + ((5*A*b
^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^3*(a^2 - b^2)*d) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Sec[c + d*x]
```

```
^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+}$$

$$=\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$=-\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d}+\frac{\left(\right.}{}$$

$$=\frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)\sqrt{\sec(c+dx)}\sin(c+}{a^3\left(a^2-b^2\right)d}$$

$$=\frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)\sqrt{\sec(c+dx)}\sin(c+}{a^3\left(a^2-b^2\right)d}$$

$$=-\frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{a^3\left(a^2-b^2\right)d}$$

$$=-\frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{a^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.27722, size = 791, normalized size = 1.75

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\left(-4a^2Ab+a^2bC+2a^3B-3ab^2B+5Ab^3\right)}{a^3\left(a^2-b^2\right)}+\frac{-a^2bC\sin(c+dx)+ab^2B\sin(c+dx)-Ab^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2A\tan(c+dx)}{3a^2}\right)}{d}+\frac{-\frac{2\sin(c+dx)}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*(-28*a^3*A*b + 40*a*A*b^3 + 12*a^4*B - 24*a^2*b^2*B + 12*a^3*b*C)*Cos[
c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + (2*(-4*a^4*A - 44*a^2*A*b^2 + 45*A*b^4 + 30*a^3*b*B -
27*a*b^3*B - 12*a^4*C + 9*a^2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[
Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*
(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c
 + d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^2*A*b^2 + 15*A*b^4 + 6*a^3*b*B - 9
*a*b^3*B + 3*a^2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a
*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ell
ipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b
*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)
))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((-4*a^2*A*b + 5*A*b^
3 + 2*a^3*B - 3*a*b^2*B + a^2*b*C)*Sin[c + d*x])/(a^3*(a^2 - b^2)) + (-(A*b
^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x] - a^2*b*C*Sin[c + d*x])/(a^2*(a^2 -
 b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] time = 7.284, size = 1038, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(2*A*b-B
*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
```

```
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*(A*b^2-B*a*b+C*a^2)/a^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*
a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2)))+2*(-2*A*b+B*a)/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^2, x)
```

3.1490 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=366

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(-(2A-C))-abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\sqrt{\cos(c+dx}}{}$$

```
[Out] ((3*A*b^2 - a*b*B - a^2*(2*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^2 - b
^2)*d) + ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C - a^2*b^2*(5*A + C))*Sqrt[
Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b^2 - a*b*B - a^2*(2*A - C))*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.27374, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(-(2A-C))-abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos
[c + d*x])^2,x]
```


```
[Out] ((3*A*b^2 - a*b*B - a^2*(2*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^2 - b
^2)*d) + ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C - a^2*b^2*(5*A + C))*Sqrt[
Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*(a - b)*b*(a + b)^2*d) - ((3*A*b^2 - a*b*B - a^2*(2*A - C))*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(\sqrt{\cos(c+d}\right.}{}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{(A}{}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{(A}{}$$

$$= \frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.06388, size = 723, normalized size = 1.98

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\left(2a^2A-a^2C+abB-3Ab^2\right)}{a^2\left(a^2-b^2\right)}+\frac{a^2C\sin(c+dx)-abB\sin(c+dx)+Ab^2\sin(c+dx)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d} - \frac{-\frac{2\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\left(4a^3A+4\right.}{b\left(1-c\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 b*Cos[c + d*x])^2,x]
```

```
[Out] -((-2*(4*a^3*A - 8*a*A*b^2 + 4*a^2*b*B - 4*a^3*C)*Cos[c + d*x]^2*EllipticPi
[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec
[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
(2*(10*a^2*A*b - 9*A*b^3 - 4*a^3*B + 3*a*b^2*B + a^2*b*C)*Cos[c + d*x]^2*(E
llipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[
Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((2*a^2*A*b - 3*A*b^3
 + a*b^2*B - a^2*b*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b
*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ellip
ticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*C
os[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))
/(4*a^2*(a - b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((2*a^2*A - 3*A*b^2 + a*b
*B - a^2*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)) + (A*b^2*Sin[c + d*x] - a*b*B*S
in[c + d*x] + a^2*C*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 4.791, size = 903, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b^2+C*a^
2)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b^2+B*a*b-C*a^2)/a/b*(-1/a*b^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
```

```
/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(
a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*A/a^2*(-(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^2, x)
```

3.1491 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-C)-abB+Ab^2+2b^2C\right)}{b^2d\left(a^2-b^2\right)}$$

```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(a*b*(a^2 - b^2)*d)) - ((A*b^2 - a*b*B - a^2*C + 2*b^2*C)*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2
- b^2)*d) - ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.862256, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-C)-abB+Ab^2+2b^2C\right)}{b^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos
[c + d*x])^2,x]
```


```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(a*b*(a^2 - b^2)*d)) - ((A*b^2 - a*b*B - a^2*C + 2*b^2*C)*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2
- b^2)*d) - ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(}{\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$=\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$=\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}-\frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$=-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{ab\left(a^2-b^2\right)d}$$

$$=-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{ab\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.91404, size = 688, normalized size = 2.27

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\left(a^2C-abB+Ab^2\right)}{ab\left(a^2-b^2\right)}+\frac{a^2C\sin(c+dx)-abB\sin(c+dx)+Ab^2\sin(c+dx)}{b\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d}+\frac{2\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\left(-4a^2A-a^2C+abB\right.}{a(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*(4*a*A*b - 4*a^2*B + 4*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Si
```

```
n[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4*a^2*A +
3*A*b^2 + a*b*B - a^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + ((A*b^2 - a*b*B + a^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c
 + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c
 + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]
]*(2 - Sec[c + d*x]^2)))/(4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((
A*b^2 - a*b*B + a^2*C)*Sin[c + d*x])/(a*b*(a^2 - b^2)) + (A*b^2*Sin[c + d*x
] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c +
 d*x]))))/d
```

Maple [B] time = 3.699, size = 815, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(B*b-2*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^2*(A*b^2-B*a*b+C*a^2)*(-1/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+
```

```
1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)\right)\sqrt{\sec\left(c + dx\right)}}{\left(a + b\cos\left(c + dx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/(a + b
*cos(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a)^2, x)
```

3.1492 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=311

$$-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2bB-3a^3C+ab^2(A+4C)\right.}{b^3d\left(a^2-b^2\right)}$$

```
[Out] ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + ((a^2*b*B - 2*b^3*B - 3*a^
3*C + a*b^2*(A + 4*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C
+ a^2*b^2*(A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - ((A*b^2 - a*(b*B - a
*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
)
```

Rubi [A] time = 0.880677, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2bB-3a^3C+ab^2(A+4C)\right.}{b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sqrt[Se
c[c + d*x]]),x]
```


```
[Out] ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + ((a^2*b*B - 2*b^3*B - 3*a^
3*C + a*b^2*(A + 4*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C
+ a^2*b^2*(A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - ((A*b^2 - a*(b*B - a
*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
)
```


Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^2}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{}{b}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} + \frac{(}{}$$

Mathematica [B] time = 7.00413, size = 695, normalized size = 2.23

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\left(a^2C-abB+Ab^2\right)}{b^2\left(b^2-a^2\right)}+\frac{a^2bB\sin(c+dx)+a^3(-C)\sin(c+dx)-aAb^2\sin(c+dx)}{b^2\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{2\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\left(a^2C+abB-A\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*S
qrt[Sec[c + d*x]]),x]
```

```
[Out] ((-2*(4*a*A*b - 4*b^2*B + 4*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Si
```

```
n[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(A*b^2) +
a*b*B + a^2*C - 2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Cos[2*(c + d*x)]*
(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*
(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/
b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[
Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*b*(a + b)*d) + (Sqrt[Sec[c
+ d*x]]*(((A*b^2 - a*b*B + a^2*C)*Sin[c + d*x])/(b^2*(-a^2 + b^2)) + (-(a*A
*b^2*Sin[c + d*x]) + a^2*b*B*Sin[c + d*x] - a^3*C*Sin[c + d*x])/(b^2*(-a^2
+ b^2)*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 4.384, size = 862, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)-4/b^2*(A*b^2-2*B*a*b+3*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a
*(A*b^2-B*a*b+C*a^2)/b^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/
2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
```

```
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*s
qrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*s
qrt(sec(d*x + c))), x)
```

3.1493 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=403

$$\frac{\sin(c+dx)\left(5a^2C-3abB+3Ab^2-2b^2C\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{}$$

```
[Out] ((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((9*a^3*b*B
- 12*a*b^3*B - a^2*b^2*(3*A - 16*C) - 15*a^4*C + 2*b^4*(3*A + C))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*(a^2 - b^2)*
d) + (a*(3*A*b^4 + 3*a^3*b*B - 5*a*b^3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*Sqr
t[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/((a - b)*b^4*(a + b)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a
^2 - b^2)*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A*b^2 - 3*a*b*B
+ 5*a^2*C - 2*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.32545, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\sin(c+dx)\left(5a^2C-3abB+3Ab^2-2b^2C\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sec[c +
 d*x]^(3/2)),x]
```


```
[Out] ((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((9*a^3*b*B
- 12*a*b^3*B - a^2*b^2*(3*A - 16*C) - 15*a^4*C + 2*b^4*(3*A + C))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*(a^2 - b^2)*
d) + (a*(3*A*b^4 + 3*a^3*b*B - 5*a*b^3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*Sqr
t[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/((a - b)*b^4*(a + b)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a
^2 - b^2)*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) + ((3*A*b^2 - 3*a*b*B
```

```
+ 5*a^2*C - 2*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(\right.}{(a+b\cos(c+dx))^2}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^2-3abB+5a^2C-2\right.}{3b^2\left(a^2-b^2\right)d\sqrt{\mathrm{se}}}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^2-3abB+5a^2C-2\right.}{3b^2\left(a^2-b^2\right)d\sqrt{\mathrm{se}}}$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{sec}}}{b^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{sec}}}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.10937, size = 748, normalized size = 1.86

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\left(a^2C-abB+Ab^2\right)}{b^3\left(a^2-b^2\right)} - \frac{-a^2Ab^2\sin(c+dx)+a^3bB\sin(c+dx)+a^4(-C)\sin(c+dx)}{b^3\left(b^2-a^2\right)(a+b\cos(c+dx))} + \frac{C\sin(2(c+dx))}{3b^2}\right)}{d} + \frac{-\frac{2\sin(c+dx)\cos^2(c+dx)\sqrt{}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*S
ec[c + d*x]^(3/2)),x]
```

```
[Out] ((-2*(12*A*b^3 - 12*a*b^2*B + 8*a^2*b*C + 4*b^3*C)*Cos[c + d*x]^2*EllipticP
i[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Se
c[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
 (2*(-3*a*A*b^2 - 3*a^2*b*B + 6*b^3*B + 5*a^3*C - 8*a*b^2*C)*Cos[c + d*x]^2
*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sq
rt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c
 + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((3*a*A*b^2 - 9*a^
2*b*B + 6*b^3*B + 15*a^3*C - 12*a*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*
```

```
x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*Ellip
ticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*
x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
- Sec[c + d*x]^2)))/(12*b^2*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((a*(
A*b^2 - a*b*B + a^2*C)*Sin[c + d*x])/(b^3*(a^2 - b^2)) - (-(a^2*A*b^2*Sin[c
 + d*x]) + a^3*b*B*Sin[c + d*x] - a^4*C*Sin[c + d*x])/(b^3*(-a^2 + b^2)*(a
+ b*Cos[c + d*x])) + (C*Sin[2*(c + d*x)])/(3*b^2)))/d
```

Maple [B] time = 5.191, size = 1129, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^4*(4*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
6*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+9*a^2*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b-2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a/b^3*(2*A*b^2-3*B*a*b+4*C*a^2)/(
-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b^2-B*a*b+C*a^2)/b^4*(-1/a*b^2/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/
a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
```

```
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*s
ec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*s
ec(d*x + c)^(3/2)), x)
```

3.1494 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=505

$$\frac{\sin(c+dx)\left(7a^2C-5abB+5Ab^2-2b^2C\right)}{5b^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(5a^2bB-7a^3C-ab^2(3A-4C)-2b^3B\right)}{3b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)\left(\ldots\right.}{bd\left(a^2-b^2\right)\sec^{\frac{5}{2}}(c\ldots}$$

```
[Out] -((25*a^3*b*B - 20*a*b^3*B - 3*a^2*b^2*(5*A - 8*C) - 35*a^4*C + 2*b^4*(5*A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*b^4*(a^2 - b^2)*d) + ((15*a^4*b*B - 16*a^2*b^3*B - 2*b^5*B - a^3*b^2*(9*A
- 20*C) - 21*a^5*C + 4*a*b^4*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b^4 + 5*a^3
*b*B - 7*a*b^3*B - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*Sqrt[Cos[c + d*x]]*Ellipt
icPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^5*(a + b
)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])*Sec[c + d*x]^(5/2)) + ((5*A*b^2 - 5*a*b*B + 7*a^2*C - 2*b^2*C)*S
in[c + d*x])/(5*b^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) + ((5*a^2*b*B - 2*b^3
*B - a*b^2*(3*A - 4*C) - 7*a^3*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 1.80574, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\sin(c+dx)\left(7a^2C-5abB+5Ab^2-2b^2C\right)}{5b^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(5a^2bB-7a^3C-ab^2(3A-4C)-2b^3B\right)}{3b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)\left(\ldots\right.}{bd\left(a^2-b^2\right)\sec^{\frac{5}{2}}(c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*Sec[c +
 d*x]^(5/2)),x]
```



```
[Out] -((25*a^3*b*B - 20*a*b^3*B - 3*a^2*b^2*(5*A - 8*C) - 35*a^4*C + 2*b^4*(5*A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*b^4*(a^2 - b^2)*d) + ((15*a^4*b*B - 16*a^2*b^3*B - 2*b^5*B - a^3*b^2*(9*A
- 20*C) - 21*a^5*C + 4*a*b^4*(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^5*(a^2 - b^2)*d) - (a^2*(5*A*b^4 + 5*a^3
*b*B - 7*a*b^3*B - 3*a^2*b^2*(A - 3*C) - 7*a^4*C)*Sqrt[Cos[c + d*x]]*Ellipt
```

```
icPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^5*(a + b
)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])*Sec[c + d*x]^(5/2)) + ((5*A*b^2 - 5*a*b*B + 7*a^2*C - 2*b^2*C)*S
in[c + d*x])/(5*b^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) + ((5*a^2*b*B - 2*b^3
*B - a*b^2*(3*A - 4*C) - 7*a^3*C)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d*Sqrt[S
ec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^2}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^2-5abB+7a^2C-2b\right.}{5b^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^2-5abB+7a^2C-2b\right.}{5b^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^2-5abB+7a^2C-2b\right.}{5b^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}}$$

$$= -\frac{\left(25a^3bB-20ab^3B-3a^2b^2(5A-8C)-35a^4C+2b^4(5A+3C)\right)\sqrt{\cos(c+}}{5b^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(25a^3bB-20ab^3B-3a^2b^2(5A-8C)-35a^4C+2b^4(5A+3C)\right)\sqrt{\cos(c+}}{5b^4\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.35781, size = 837, normalized size = 1.66

$$-\frac{\frac{2\left(-20Bb^4+60aAb^3+4aCb^3-40a^2Bb^2+56a^3Cb\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{2\left(35Ca^4-25bBa^3+15\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^2*S
ec[c + d*x]^(5/2)),x]
```

```
[Out] ((-2*(60*a*A*b^3 - 40*a^2*b^2*B - 20*b^4*B + 56*a^3*b*C + 4*a*b^3*C)*Cos[c
+ d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + (2*(15*a^2*A*b^2 - 30*A*b^4 - 25*a^3*b*B + 40*a*b^3*B +
```

```
35*a^4*C - 32*a^2*b^2*C - 18*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[S
ec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(
b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c
+ d*x])*(1 - Cos[c + d*x]^2)) + ((45*a^2*A*b^2 - 30*A*b^4 - 75*a^3*b*B + 60
*a*b^3*B + 105*a^4*C - 72*a^2*b^2*C - 18*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec
[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*
b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin
[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]]*(2 - Sec[c + d*x]^2)))/(60*(a - b)*b^3*(a + b)*d) + (Sqrt[Sec[c + d*x]]
*(-((10*a^2*A*b^2 - 10*a^3*b*B + 10*a^4*C - a^2*b^2*C + b^4*C)*Sin[c + d*x]
)/(10*b^4*(a^2 - b^2)) - (a^3*A*b^2*Sin[c + d*x] - a^4*b*B*Sin[c + d*x] + a
^5*C*Sin[c + d*x])/(b^4*(-a^2 + b^2)*(a + b*Cos[c + d*x])) + ((b*B - 2*a*C)
*Sin[2*(c + d*x)])/(3*b^3) + (C*Sin[3*(c + d*x)])/(10*b^2)))/d
```

Maple [B] time = 5.506, size = 1382, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^2*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+4/3/b^3*(B*b-2*C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
/b^4*(A*b^2-2*B*a*b-2*B*b^2+3*C*a^2+4*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-2*(2*A*a*b^2+A*b^3-3*B*a^2*b-2*B*a*b^2-B*b^3+4*C*a^3+
3*C*a^2*b+2*C*a*b^2+C*b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
```

```
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4*a^2/b^4*(3*A*b^2-4*B*a*b+5*C*a^2)/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b^2-B*a*b+C*a^2)/b^5*(-1/a*b^2/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)
/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**2/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*s
ec(d*x + c)^(5/2)), x)
```

3.1495 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=669

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-a^2b^2(61A-3C)+a^4(8A-21C)+33a^3bB-15ab^3B+35Ab^4\right)}{12a^3d\left(a^2-b^2\right)^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-\right.}{}$$

```
[Out] ((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a^
2*b^3*(65*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B +
a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A*b^6 - 35*a
^5*b*B + 38*a^3*b^3*B - 15*a*b^5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A
 - 2*C) + 15*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) - ((35*A*b^5 - 8*a^
5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a^2*b^3*(65*A - 3*C
))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A*b^4 +
33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*C)
)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^3*B - 5*a^4*C - a^2*b^2*(13*A + C))*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.81586, antiderivative size = 669, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-a^2b^2(61A-3C)+a^4(8A-21C)+33a^3bB-15ab^3B+35Ab^4\right)}{12a^3d\left(a^2-b^2\right)^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos
[c + d*x])^3,x]
```



```
[Out] ((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a^
2*b^3*(65*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B +
```

```
a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^3*(a^2 - b^2)^2*d) + ((35*A*b^6 - 35*a
^5*b*B + 38*a^3*b^3*B - 15*a*b^5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A
 - 2*C) + 15*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) - ((35*A*b^5 - 8*a^
5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a^2*b^3*(65*A - 3*C
))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((35*A*b^4 +
33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*C)
)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^3*B - 5*a^4*C - a^2*b^2*(13*A + C))*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + B\cos(c+dx) + C\cos^2}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c-}$$

$$= \frac{\left(Ab^2 - a(bB - aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(\sqrt{\cos(c+}\right.}{}$$

$$= \frac{\left(Ab^2 - a(bB - aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(7Ab^4 + 9a\right.}{}$$

$$= \frac{\left(35Ab^4 + 33a^3bB - 15ab^3B + a^4(8A-21C) - a^2b^2(61A-\right.}{12a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(35Ab^5 - 8a^5B + 29a^3b^2B - 15ab^4B + 3a^4b(8A-3C) - a\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(35Ab^5 - 8a^5B + 29a^3b^2B - 15ab^4B + 3a^4b(8A-3C) - a\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(35Ab^5 - 8a^5B + 29a^3b^2B - 15ab^4B + 3a^4b(8A-3C) - a^2\right.}{4a^4\left(a^2-\right.}$$

$$= \frac{\left(35Ab^5 - 8a^5B + 29a^3b^2B - 15ab^4B + 3a^4b(8A-3C) - a^2\right.}{4a^4\left(a^2-\right.}$$

Mathematica [A] time = 7.66202, size = 1019, normalized size = 1.52

$$-\frac{\frac{2\left(-48Ba^6+160Aba^5-96bCa^5+240b^2Ba^4-512Ab^3a^3+24b^3Ca^3-120b^4Ba^2+280Ab^5a\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 b*Cos[c + d*x])^3,x]
```

[Out] ((-2*(160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 48*a^6*B + 240*a^4*b^2*B - 120*a^2*b^4*B - 96*a^5*b*C + 24*a^3*b^3*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 - 168*a^5*b*B + 285*a^3*b^3*B - 135*a*b^5*B + 48*a^6*C - 57*a^4*b^2*C + 27*a^2*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((72*a^4*A*b^2 - 195*a^2*A*b^4 + 105*A*b^6 - 24*a^5*b*B + 87*a^3*b^3*B - 45*a*b^5*B - 27*a^4*b^2*C + 9*a^2*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((-24*a^4*A*b + 65*a^2*A*b^3 - 35*A*b^5 + 8*a^5*B - 29*a^3*b^2*B + 15*a*b^4*B + 9*a^4*b*C - 3*a^2*b^3*C)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2) + (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x] - a^2*b*C*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (-15*a^2*A*b^3*Sin[c + d*x] + 9*A*b^5*Sin[c + d*x] + 11*a^3*b^2*B*Sin[c + d*x] - 5*a*b^4*B*Sin[c + d*x] - 7*a^4*b*C*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d

---

Maple [B] time = 12.504, size = 2165, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^2*(3*A*b-B*a)/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-3*A*b+B*a)/a^4*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1

/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(A*b^2-B*a*b+C*a
^2)/a^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-
b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)
^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2)))+2*A/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(2
*A*b-B*a)/a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)
b(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^

```
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^3, x)
```

3.1496 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=562

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b^2(29A+C)+a^4(8A-5C)+9a^3bB-3ab^3B+15Ab^4\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b^2(\right.}{4a^2d\left(a^2-\right.}$$

```
[Out] -((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a
^2 - b^2)^2*d) - ((5*A*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4
*a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 - 15*a^5*b*B + 6*a^3*b^3*B - 3*a*b^5*B
 + 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a -
b)^2*b*(a + b)^3*d) + ((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C)
- a^2*b^2*(29*A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2
*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*A*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C
 - a^2*b^2*(11*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2
)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 2.08982, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b^2(29A+C)+a^4(8A-5C)+9a^3bB-3ab^3B+15Ab^4\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b^2(\right.}{4a^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos
[c + d*x])^3,x]
```


```
[Out] -((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a
^2 - b^2)^2*d) - ((5*A*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4
*a^2*b*(a^2 - b^2)^2*d) - ((15*A*b^6 - 15*a^5*b*B + 6*a^3*b^3*B - 3*a*b^5*B
```

```
 + 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C))*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a -
b)^2*b*(a + b)^3*d) + ((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C)
- a^2*b^2*(29*A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2
*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*A*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C
 - a^2*b^2*(11*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2
)^2*d*(a + b*Cos[c + d*x]))
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(\sqrt{\cos(c+}\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5Ab^4+7a\right.}{}$$

$$= \frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C\right)}{4a^3\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C\right)}{4a^3\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 7.48883, size = 950, normalized size = 1.69

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(8Aa^4-5Ca^4+9bBa^3-29Ab^2a^2-b^2Ca^2-3b^3Ba+15Ab^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2} + \frac{C\sin(c+dx)a^2-bB\sin(c+dx)a+Ab^2\sin(c+dx)}{2a\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{3C\sin(c+dx)a^4-7b}{}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 b*Cos[c + d*x])^3,x]
```

```
[Out] -((-2*(16*a^5*A - 80*a^3*A*b^2 + 40*a*A*b^4 + 32*a^4*b*B - 8*a^2*b^3*B - 16
*a^5*C - 8*a^3*b^2*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(
b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(56*a^4*A*b - 95*a^2*A*b^
```

```
3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B - 9*a*b^4*B + 9*a^4*b*C - 3*a^2*b^3*
C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-
(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[
c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (
(8*a^4*A*b - 29*a^2*A*b^3 + 15*A*b^5 + 9*a^3*b^2*B - 3*a*b^4*B - 5*a^4*b*C
- a^2*b^3*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c +
d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a
/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a^3*
(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((8*a^4*A - 29*a^2*A*b^2 + 15
*A*b^4 + 9*a^3*b*B - 3*a*b^3*B - 5*a^4*C - a^2*b^2*C)*Sin[c + d*x])/(4*a^3*
(a^2 - b^2)^2) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d
*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (11*a^2*A*b^2*Sin[c + d*x]
- 5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c + d*x] + a*b^3*B*Sin[c + d*x] + 3*
a^4*C*Sin[c + d*x] + 3*a^2*b^2*C*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*
Cos[c + d*x]))))/d
```

Maple [B] time = 8.282, size = 2027, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^3/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(-A*b^2+B*a*b-C*a^2)/a/b*(-1/2/a*b^2/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
```

```
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2
-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*
(-A*b^2+C*a^2)/a^2/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a
+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b
+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^3, x)
```

3.1497 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=473

$$-\frac{\sin(c+dx)\left(-a^2b^2(9A+5C)+5a^3bB+a^4(-C)+ab^3B+3Ab^4\right)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+-\frac{\sqrt{}}{}$$

```
[Out] ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^2
*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B + a^4*C - 7*a^2*b^2*(A + C))*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^
2)^2*d) + ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2*b^4*(2*A -
 C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d
) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^2*Sqrt[Sec[c + d*x]]) - ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a
^2*b^2*(9*A + 5*C))*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.50817, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\left(-a^2b^2(9A+5C)+5a^3bB+a^4(-C)+ab^3B+3Ab^4\right)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+-\frac{\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos
[c + d*x])^3,x]
```



```
[Out] ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^2
*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B + a^4*C - 7*a^2*b^2*(A + C))*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^
2)^2*d) + ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2*b^4*(2*A -
 C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d
) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
```

```
+ d*x])^2*Sqrt[Sec[c + d*x]]) - ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a
^2*b^2*(9*A + 5*C))*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(}{\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(3Ab^4+5a\right.}{4a^2\left(\right.}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(3Ab^4+5a\right.}{4a^2\left(\right.}$$

$$= \frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos(c+dx}}{4a^2b\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos(c+dx}}{4a^2b\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.29419, size = 909, normalized size = 1.92

$$-\frac{2\left(16Ba^4-32Aba^3-24bCa^3+8b^2Ba^2+8Ab^3a\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{2\left(16Aa^4+5Ca^4-9bBa\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-32*a^3*A*b + 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B - 24*a^3*b*C)*Cos[c
+ d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + (2*(16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4 - 9*a^3*b*B + 3*a*
b^3*B + 5*a^4*C + a^2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c +
d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*S
ec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b^2 + 3*A*b^4 + 5*a^3*b*B + a*b^3*B - a
^4*C - 5*a^2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*S
ec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*Ellipti
cPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
c[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos
[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(
16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((9*a^2*A*b^2 - 3*A*b^
4 - 5*a^3*b*B - a*b^3*B + a^4*C + 5*a^2*b^2*C)*Sin[c + d*x])/(4*a^2*b*(a^2
- b^2)^2) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x])/
(2*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (-7*a^2*A*b^2*Sin[c + d*x] + A*
b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[c + d*x] + 3*a*b^3*B*Sin[c + d*x] + a^4*C*
Sin[c + d*x] - 7*a^2*b^2*C*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*(a + b*Cos[c
+ d*x]))))/d
```

Maple [B] time = 6.544, size = 1857, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*C/b/(-2*a*b+
2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+2*(A*b^2-B*a*b+C*a^2)/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b
)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9
/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(B*b-2*C*a)/b
^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
```

```
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a)^3, x)
```

3.1498 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=478

$$-\frac{\sin(c+dx)\left(a^2b^2(5A+9C)-a^3bB-3a^4C-5ab^3B+Ab^4\right)}{4abd\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+\frac{\sqrt{\cos}}{}$$

```
[Out] ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^2)^2
*d) + ((a^3*b*B - 7*a*b^3*B + a^2*b^2*(3*A - 5*C) + 3*a^4*C + b^4*(3*A + 8*
C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3
*(a^2 - b^2)^2*d) + ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^
2*(A - 2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^3*(a
+ b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a +
b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a
^4*C + a^2*b^2*(5*A + 9*C))*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*d*(a + b*Cos
[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.58171, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\left(a^2b^2(5A+9C)-a^3bB-3a^4C-5ab^3B+Ab^4\right)}{4abd\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}+\frac{\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sqrt[Se
c[c + d*x]]),x]
```


```
[Out] ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^2)^2
*d) + ((a^3*b*B - 7*a*b^3*B + a^2*b^2*(3*A - 5*C) + 3*a^4*C + b^4*(3*A + 8*
C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3
*(a^2 - b^2)^2*d) + ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^
2*(A - 2*C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^3*(a
+ b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a +
```

```
b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a
^4*C + a^2*b^2*(5*A + 9*C))*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*d*(a + b*Cos
[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^3}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(Ab^4-a^3bB-5ab^3B-3\right.}{4ab\left(a^2-b^2\right)^2d(a-}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} - \frac{\left(Ab^4-a^3bB-5ab^3B-3\right.}{4ab\left(a^2-b^2\right)^2d(a-}$$

$$= \frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{4ab^2\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{4ab^2\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.23928, size = 908, normalized size = 1.9

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(3Ca^4+bBa^3-5Ab^2a^2-9b^2Ca^2+5b^3Ba-Ab^4\right)\sin(c+dx)}{4ab^2\left(a^2-b^2\right)^2} - \frac{C\sin(c+dx)a^3-bB\sin(c+dx)a^2+Ab^2\sin(c+dx)a}{2b^2\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{-5C\sin(c+dx)a^4+bB\sin(c+d\right.}{}}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*S
qrt[Sec[c + d*x]]),x]
```

```
[Out] -((-2*(-16*a^3*A*b - 8*a*A*b^3 + 24*a^2*b^2*B - 8*a^3*b*C - 16*a*b^3*C)*Cos
[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[
c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) + (2*(9*a^2*A*b^2 - 3*A*b^4 - 5*a^3*b*B - a*b^3*B + a^4
*C + 5*a^2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]
 + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x]
)*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c
 + d*x]^2)) + ((-5*a^2*A*b^2 - A*b^4 + a^3*b*B + 5*a*b^3*B + 3*a^4*C - 9*a^
2*b^2*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]
```

```
^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*
(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a*(a - b
)^2*b*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((-5*a^2*A*b^2 - A*b^4 + a^3*b*B
+ 5*a*b^3*B + 3*a^4*C - 9*a^2*b^2*C)*Sin[c + d*x])/(4*a*b^2*(a^2 - b^2)^2)
- (a*A*b^2*Sin[c + d*x] - a^2*b*B*Sin[c + d*x] + a^3*C*Sin[c + d*x])/(2*b^2
*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (3*a^2*A*b^2*Sin[c + d*x] + 3*A*b^4
*Sin[c + d*x] + a^3*b*B*Sin[c + d*x] - 7*a*b^3*B*Sin[c + d*x] - 5*a^4*C*Sin
[c + d*x] + 11*a^2*b^2*C*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Cos[c +
 d*x]))))/d
```

Maple [B] time = 6.814, size = 1950, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
^2*(B*b-3*C*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^3*(A*b^2-2*B*a*b+3*C*a^2)
*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a*(A*b^2-B*a*b+C*
a^2)/b^3*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2
-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2
)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b
^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*s
qrt(sec(d*x + c))), x)
```

3.1499 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=483

$$-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{\sin(c+dx)\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+3Ab^4\right)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\sqrt{\cos}}{}$$

```
[Out] -((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a
^2 - b^2)^2*d) + ((3*a^4*b*B - 5*a^2*b^3*B + 8*b^5*B - 15*a^5*C - a*b^4*(7*
A + 24*C) + a^3*b^2*(A + 33*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^5*b*B + 6*a
^3*b^3*B - 15*a*b^5*B + 15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*
C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c +
d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((3*A
*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A + 11*C))*Sin[c + d*x])/
(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.68323, antiderivative size = 483, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4221, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{\sin(c+dx)\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+3Ab^4\right)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}+\frac{\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c +
 d*x]^(3/2)),x]
```


```
[Out] -((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a
^2 - b^2)^2*d) + ((3*a^4*b*B - 5*a^2*b^3*B + 8*b^5*B - 15*a^5*C - a*b^4*(7*
A + 24*C) + a^3*b^2*(A + 33*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) + ((3*A*b^6 - 3*a^5*b*B + 6*a
^3*b^3*B - 15*a*b^5*B + 15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*
C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
```

```
 + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c +
d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)) + ((3*A
*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A + 11*C))*Sin[c + d*x])/
(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^3}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^4+a^3bB-7ab^3B-\right.}{4b^2\left(a^2-b^2\right)^2d(a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3Ab^4+a^3bB-7ab^3B-\right.}{4b^2\left(a^2-b^2\right)^2d(a}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)\sqrt{\cos(c+dx)}E}{4b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)\sqrt{\cos(c+dx)}E}{4b^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.34853, size = 921, normalized size = 1.91

$$-\frac{2\left(16Bb^4-24aAb^3-32aCb^3+8a^2Bb^2+8a^3Cb\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{2\left(5Ca^4-bBa^3+5Ab^2a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*S
ec[c + d*x]^(3/2)),x]
```

```
[Out] ((-2*(-24*a*A*b^3 + 8*a^2*b^2*B + 16*b^4*B + 8*a^3*b*C - 32*a*b^3*C)*Cos[c
+ d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + (2*(5*a^2*A*b^2 + A*b^4 - a^3*b*B - 5*a*b^3*B + 5*a^4*C
- 7*a^2*b^2*C + 8*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + ((-(a^2*A*b^2) - 5*A*b^4 - 3*a^3*b*B + 9*a*b^3*B + 15*
a^4*C - 29*a^2*b^2*C + 8*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a
*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4
*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^
2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
 d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-((-(a^2*A
*b^2) - 5*A*b^4 - 3*a^3*b*B + 9*a*b^3*B + 7*a^4*C - 13*a^2*b^2*C)*Sin[c + d
*x])/(4*b^3*(a^2 - b^2)^2) - (-(a^2*A*b^2*Sin[c + d*x]) + a^3*b*B*Sin[c + d
*x] - a^4*C*Sin[c + d*x])/(2*b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^
3*A*b^2*Sin[c + d*x] - 7*a*A*b^4*Sin[c + d*x] - 5*a^4*b*B*Sin[c + d*x] + 11
*a^2*b^3*B*Sin[c + d*x] + 9*a^5*C*Sin[c + d*x] - 15*a^3*b^2*C*Sin[c + d*x])
/(4*b^3*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 7.813, size = 2000, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-4/b^3*(A*b^2-3*B*a*b+6*C*a^2)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a/b^4*(2*A*b^2-3*B*a*b+4*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*a^2*(A*b^2-B*a*b+C*a^2)/b^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^

```
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/
2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/
2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*s
ec(d*x + c)^(3/2)), x)
```

3.1500 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=596

$$-\frac{\sin(c+dx)\left(-a^2b^2(3A-61C)+15a^3bB-35a^4C-33ab^3B+b^4(21A-8C)\right)}{12b^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)\left(a^2b^2(A+13C)+3a^3bB-\right.}{4b^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)(a}$$

```
[Out] ((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*A
 - 8*C) - 35*a^5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*b^4*(a^2 - b^2)^2*d) - ((45*a^5*b*B - 99*a^3*b^3*B + 72*a*b^5*
B - a^4*b^2*(9*A - 223*C) + a^2*b^4*(15*A - 128*C) - 105*a^6*C - 8*b^6*(3*A
 + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12
*b^5*(a^2 - b^2)^2*d) - (a*(15*A*b^6 - 15*a^5*b*B + 38*a^3*b^3*B - 35*a*b^5
*B + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A - 21*C) + 35*a^6*C)*Sqrt[Cos[c +
 d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a
- b)^2*b^5*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)) + ((5*A*b^4 + 3*a^3*b*B
 - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A + 13*C))*Sin[c + d*x])/(4*b^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) - ((15*a^3*b*B - 33*a*b^3*B
 - a^2*b^2*(3*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sin[c + d*x])/(12*b^
3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 2.12979, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\left(-a^2b^2(3A-61C)+15a^3bB-35a^4C-33ab^3B+b^4(21A-8C)\right)}{12b^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)\left(a^2b^2(A+13C)+3a^3bB-\right.}{4b^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c +
 d*x]^(5/2)),x]
```


```
[Out] ((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*A
 - 8*C) - 35*a^5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*b^4*(a^2 - b^2)^2*d) - ((45*a^5*b*B - 99*a^3*b^3*B + 72*a*b^5*
B - a^4*b^2*(9*A - 223*C) + a^2*b^4*(15*A - 128*C) - 105*a^6*C - 8*b^6*(3*A
```

```
 + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12
*b^5*(a^2 - b^2)^2*d) - (a*(15*A*b^6 - 15*a^5*b*B + 38*a^3*b^3*B - 35*a*b^5
*B + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A - 21*C) + 35*a^6*C)*Sqrt[Cos[c +
 d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a
- b)^2*b^5*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)) + ((5*A*b^4 + 3*a^3*b*B
 - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A + 13*C))*Sin[c + d*x])/(4*b^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)) - ((15*a^3*b*B - 33*a*b^3*B
 - a^2*b^2*(3*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sin[c + d*x])/(12*b^
3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^3}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^4+3a^3bB-9ab^3B\right.}{4b^2\left(a^2-b^2\right)^2d(a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^4+3a^3bB-9ab^3B\right.}{4b^2\left(a^2-b^2\right)^2d(a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(5Ab^4+3a^3bB-9ab^3B\right.}{4b^2\left(a^2-b^2\right)^2d(a}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A-8C)-35a^5C\right)\sqrt{}}{4b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A-8C)-35a^5C\right)\sqrt{}}{4b^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.6471, size = 978, normalized size = 1.64

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a\left(11Ca^4-7bBa^3+3Ab^2a^2-17b^2Ca^2+13b^3Ba-9Ab^4\right)\sin(c+dx)}{4b^4\left(a^2-b^2\right)^2} - \frac{C\sin(c+dx)a^5-bB\sin(c+dx)a^4+Ab^2\sin(c+dx)a^3}{2b^4\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{-13C\sin(c+dx)a^6+9}{}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*S
ec[c + d*x]^(5/2)),x]
```

```
[Out] -((-2*(-24*a^2*A*b^3 - 48*A*b^5 - 24*a^3*b^2*B + 96*a*b^4*B + 56*a^4*b*C -
112*a^2*b^3*C - 16*b^5*C)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x
])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(3*a^3*A*b^2 + 15*a*A
```

```
*b^4 - 15*a^4*b*B + 21*a^2*b^3*B - 24*b^5*B + 35*a^5*C - 73*a^3*b^2*C + 56*
a*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellipt
icPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) + ((9*a^3*A*b^2 - 27*a*A*b^4 - 45*a^4*b*B + 87*a^2*b^3*B - 24*b^5*B + 10
5*a^5*C - 195*a^3*b^2*C + 72*a*b^4*C)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])
*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])
/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - S
ec[c + d*x]^2)))/(48*(a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((a*(
3*a^2*A*b^2 - 9*A*b^4 - 7*a^3*b*B + 13*a*b^3*B + 11*a^4*C - 17*a^2*b^2*C)*S
in[c + d*x])/(4*b^4*(a^2 - b^2)^2) - (a^3*A*b^2*Sin[c + d*x] - a^4*b*B*Sin[
c + d*x] + a^5*C*Sin[c + d*x])/(2*b^4*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2)
+ (-5*a^4*A*b^2*Sin[c + d*x] + 11*a^2*A*b^4*Sin[c + d*x] + 9*a^5*b*B*Sin[c
+ d*x] - 15*a^3*b^3*B*Sin[c + d*x] - 13*a^6*C*Sin[c + d*x] + 19*a^4*b^2*C*S
in[c + d*x])/(4*b^4*(-a^2 + b^2)^2*(a + b*Cos[c + d*x])) + (C*Sin[2*(c + d*
x)])/(3*b^3)))/d
```

Maple [B] time = 9.055, size = 2267, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^5*(4*b^2*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*a*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+18*a^2*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b-2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a/b^4*(3*A*b^2-6*B*a*b+10*C*a^2)
```

/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^5*(3*A*b^2-4*B*a*b+5*C*a^2)*(-1/a*b
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/
a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a^3*(A*b^2-B*a*b+C*a^2)/b
^5*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/
a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+
b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-
2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a

```
-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*s
ec(d*x + c)^(5/2)), x)
```

3.1501 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=714

$$-\frac{\sin(c+dx)\left(-a^2b^2(15A-101C)+35a^3bB-63a^4C-65ab^3B+b^4(45A-8C)\right)}{20b^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(-15a^3b^2(A-7C)-61a\right.}{12b^4d\,(}$$

```
[Out] -((175*a^5*b*B - 325*a^3*b^3*B + 120*a*b^5*B + a^2*b^4*(145*A - 192*C) - 3*
a^4*b^2*(25*A - 187*C) - 315*a^6*C - 8*b^6*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(20*b^5*(a^2 - b^2)^2*d) + ((
105*a^6*b*B - 223*a^4*b^3*B + 128*a^2*b^5*B + 8*b^7*B + 3*a^3*b^4*(33*A - 6
4*C) - 9*a^5*b^2*(5*A - 43*C) - 189*a^7*C - 24*a*b^6*(3*A + C))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^6*(a^2 - b^2)^2
*d) + (a^2*(35*A*b^6 - 35*a^5*b*B + 86*a^3*b^3*B - 63*a*b^5*B - a^2*b^4*(38
*A - 99*C) + 15*a^4*b^2*(A - 10*C) + 63*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^6*(a +
b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) + ((7*A*b^4 + 5*a^3*b*B - 11*a*b^3*B -
a^2*b^2*(A - 15*C) - 9*a^4*C)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x])*Sec[c + d*x]^(5/2)) - ((35*a^3*b*B - 65*a*b^3*B - a^2*b^2*(15*
A - 101*C) + b^4*(45*A - 8*C) - 63*a^4*C)*Sin[c + d*x])/(20*b^3*(a^2 - b^2)
^2*d*Sec[c + d*x]^(3/2)) + ((35*a^4*b*B - 61*a^2*b^3*B + 8*b^5*B + 3*a*b^4*
(11*A - 8*C) - 15*a^3*b^2*(A - 7*C) - 63*a^5*C)*Sin[c + d*x])/(12*b^4*(a^2
- b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 3.04849, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4221, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\sin(c+dx)\left(-a^2b^2(15A-101C)+35a^3bB-63a^4C-65ab^3B+b^4(45A-8C)\right)}{20b^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(-15a^3b^2(A-7C)-61a\right.}{12b^4d\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*Sec[c +
 d*x]^(7/2)),x]
```



```
[Out] -((175*a^5*b*B - 325*a^3*b^3*B + 120*a*b^5*B + a^2*b^4*(145*A - 192*C) - 3*
a^4*b^2*(25*A - 187*C) - 315*a^6*C - 8*b^6*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*
```

```
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(20*b^5*(a^2 - b^2)^2*d) + ((
105*a^6*b*B - 223*a^4*b^3*B + 128*a^2*b^5*B + 8*b^7*B + 3*a^3*b^4*(33*A - 6
4*C) - 9*a^5*b^2*(5*A - 43*C) - 189*a^7*C - 24*a*b^6*(3*A + C))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^6*(a^2 - b^2)^2
*d) + (a^2*(35*A*b^6 - 35*a^5*b*B + 86*a^3*b^3*B - 63*a*b^5*B - a^2*b^4*(38
*A - 99*C) + 15*a^4*b^2*(A - 10*C) + 63*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^6*(a +
b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2*Sec[c + d*x]^(7/2)) + ((7*A*b^4 + 5*a^3*b*B - 11*a*b^3*B -
a^2*b^2*(A - 15*C) - 9*a^4*C)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x])*Sec[c + d*x]^(5/2)) - ((35*a^3*b*B - 65*a*b^3*B - a^2*b^2*(15*
A - 101*C) + b^4*(45*A - 8*C) - 63*a^4*C)*Sin[c + d*x])/(20*b^3*(a^2 - b^2)
^2*d*Sec[c + d*x]^(3/2)) + ((35*a^4*b*B - 61*a^2*b^3*B + 8*b^5*B + 3*a*b^4*
(11*A - 8*C) - 15*a^3*b^2*(A - 7*C) - 63*a^5*C)*Sin[c + d*x])/(12*b^4*(a^2
- b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^3}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4+5a^3bB-11ab^3B\right.}{4b^2\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4+5a^3bB-11ab^3B\right.}{4b^2\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4+5a^3bB-11ab^3B\right.}{4b^2\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(7Ab^4+5a^3bB-11ab^3B\right.}{4b^2\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{\left(175a^5bB-325a^3b^3B+120ab^5B+a^2b^4(145A-192C)-3a^4b^2(25A-187\right.}{20b^5\left(a^2\right.}$$

$$= -\frac{\left(175a^5bB-325a^3b^3B+120ab^5B+a^2b^4(145A-192C)-3a^4b^2(25A-187\right.}{20b^5\left(a^2\right.}$$

Mathematica [A] time = 7.9263, size = 1065, normalized size = 1.49

$$-\frac{\frac{2\left(80Bb^6-480aAb^5-96aCb^5+560a^2Bb^4+120a^3Ab^3-768a^3Cb^3-280a^4Bb^2+504a^5Cb\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{b(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^3*S
ec[c + d*x]^(7/2)),x]
```

```
[Out] ((-2*(120*a^3*A*b^3 - 480*a*A*b^5 - 280*a^4*b^2*B + 560*a^2*b^4*B + 80*b^6*
B + 504*a^5*b*C - 768*a^3*b^3*C - 96*a*b^5*C)*Cos[c + d*x]^2*EllipticPi[-(a
/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c +
 d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(
75*a^4*A*b^2 - 105*a^2*A*b^4 + 120*A*b^6 - 175*a^5*b*B + 365*a^3*b^3*B - 28
0*a*b^5*B + 315*a^6*C - 633*a^4*b^2*C + 336*a^2*b^4*C + 72*b^6*C)*Cos[c + d
*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcS
in[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((225*a^4*A*b
^2 - 435*a^2*A*b^4 + 120*A*b^6 - 525*a^5*b*B + 975*a^3*b^3*B - 360*a*b^5*B
+ 945*a^6*C - 1683*a^4*b^2*C + 576*a^2*b^4*C + 72*b^6*C)*Cos[2*(c + d*x)]*(
b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(
2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b
), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x
]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[S
ec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(240*(a - b)^2*b^4*(a + b)^2*d) + (Sqrt
[Sec[c + d*x]]*(-((35*a^4*A*b^2 - 65*a^2*A*b^4 - 55*a^5*b*B + 85*a^3*b^3*B
+ 75*a^6*C - 107*a^4*b^2*C + 4*a^2*b^4*C - 2*b^6*C)*Sin[c + d*x])/(20*b^5*(
a^2 - b^2)^2) - (-(a^4*A*b^2*Sin[c + d*x]) + a^5*b*B*Sin[c + d*x] - a^6*C*S
in[c + d*x])/(2*b^5*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (9*a^5*A*b^2*Sin
[c + d*x] - 15*a^3*A*b^4*Sin[c + d*x] - 13*a^6*b*B*Sin[c + d*x] + 19*a^4*b^
3*B*Sin[c + d*x] + 17*a^7*C*Sin[c + d*x] - 23*a^5*b^2*C*Sin[c + d*x])/(4*b^
5*(-a^2 + b^2)^2*(a + b*Cos[c + d*x])) + ((b*B - 3*a*C)*Sin[2*(c + d*x)])/(
3*b^4) + (C*Sin[3*(c + d*x)])/(10*b^3)))/d
```

Maple [B] time = 9.429, size = 2520, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*C/b^3*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
```

^(1/2)+4/3/b^4*(B*b-3*C*a-3*C*b)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
/b^5*(A*b^2-3*B*a*b-2*B*b^2+6*C*a^2+6*C*a*b+3*C*b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-2*(3*A*a*b^2+A*b^3-6*B*a^2*b-3*B*a*b^2-B*b^3+10*C*a^3
+6*C*a^2*b+3*C*a*b^2+C*b^3)/b^6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b^5*a^2*(6*A*b^2-10*B*a*b+15*C*a^2)/(
-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(A*b^2-B*a*b+C*a^2)/b^6*(-1/2/a*b^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
b+3/8/(a+b)/(a^2-b^2)/a^2(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b
^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2/b^
6*a^3*(4*A*b^2-5*B*a*b+6*C*a^2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+

```
a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/a*b/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b
^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b
),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**3/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A}{\left(b\cos\left(dx+c\right)+a\right)^3 \sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*s
ec(d*x + c)^(7/2)), x)
```

3.1502 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\ldots dx$

Optimal. Leaf size=592

$$-\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(-7a^2(7A+9C)-9abB+6Ab^2\right)\sqrt{a+b\cos(c+dx)}}{315a^2d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2b(13A\ldots\right.}{\ldots}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*
A + 7*C) - 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(315*a^5*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(16*A*b^3
 + 12*a*b^2*(A - 2*B) + 6*a^2*b*(6*A - 3*B + 7*C) + 3*a^3*(49*A - 25*B + 63
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d*Sqrt[
Sec[c + d*x]]) + (2*(8*A*b^3 + 75*a^3*B - 12*a*b^2*B + a^2*b*(13*A + 21*C))
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^3*d) - (2
*(6*A*b^2 - 9*a*b*B - 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(315*a^2*d) + (2*(A*b + 9*a*B)*Sqrt[a + b*Cos[c + d
*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*a*d) + (2*A*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 2.24721, antiderivative size = 592, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(-7a^2(7A+9C)-9abB+6Ab^2\right)\sqrt{a+b\cos(c+dx)}}{315a^2d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2b(13A\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(11/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*
A + 7*C) - 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
```

```
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(315*a^5*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(16*A*b^3
 + 12*a*b^2*(A - 2*B) + 6*a^2*b*(6*A - 3*B + 7*C) + 3*a^3*(49*A - 25*B + 63
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d*Sqrt[
Sec[c + d*x]]) + (2*(8*A*b^3 + 75*a^3*B - 12*a*b^2*B + a^2*b*(13*A + 21*C))
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^3*d) - (2
*(6*A*b^2 - 9*a*b*B - 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(315*a^2*d) + (2*(A*b + 9*a*B)*Sqrt[a + b*Cos[c + d
*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*a*d) + (2*A*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c}{9d}$$

$$= \frac{2(Ab+9aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+}{63ad}$$

$$= -\frac{2\left(6Ab^2-9abB-7a^2(7A+9C)\right)\sqrt{a+}}{315a}$$

$$= \frac{2\left(8Ab^3+75a^3B-12ab^2B+a^2b(13A+\right.}{}$$

$$= \frac{2\left(8Ab^3+75a^3B-12ab^2B+a^2b(13A+\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(16Ab^4-57a^3bB-24ab\right.}{}$$

Mathematica [B] time = 27.7588, size = 4669, normalized size = 7.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A - 24*a^2*A*b^2
- 16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B + 189*a^4*C - 42*a^2*b^2*C)*Sin[c + d*
x])/(315*a^4) + (2*Sec[c + d*x]^3*(A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/
(63*a) + (2*Sec[c + d*x]^2*(49*a^2*A*Sin[c + d*x] - 6*A*b^2*Sin[c + d*x] +
9*a*b*B*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x]))/(315*a^2) + (2*Sec[c + d*x]*
(13*a^2*A*b*Sin[c + d*x] + 8*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] - 1
2*a*b^2*B*Sin[c + d*x] + 21*a^2*b*C*Sin[c + d*x]))/(315*a^3) + (2*A*Sec[c +
 d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a*A)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (8*A*b^2)/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d
```

*x]]) + (16*A*b^4)/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (19*b*B)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*b^3*B)/(105 *a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*a*C)/(5*Sqrt[a + b*C os[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*C)/(15*a*Sqrt[a + b*Cos[c + d*x]] *Sqrt[Sec[c + d*x]]) - (4*A*b*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d* x]]) + (4*A*b^3*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16 *A*b^5*Sqrt[Sec[c + d*x]])/(315*a^4*Sqrt[a + b*Cos[c + d*x]]) + (5*a*B*Sqrt [Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (17*b^2*B*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*b^4*B*Sqrt[Sec[c + d*x]])/(105*a^3 *Sqrt[a + b*Cos[c + d*x]]) - (2*b*C*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*C*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) - (7*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) + (8*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^4*Sqrt[a + b*Cos[c + d*x]]) - (19*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a* Sqrt[a + b*Cos[c + d*x]]) - (8*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt[a + b*Cos[c + d*x]]) - (3*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d* x]])/(5*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d *x]]*(-2*(a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7* C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b* Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/ 2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-16*A*b^3 + 12*a*b^2 *(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*Ellipt icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x) /2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a ^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^4*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2) *((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-16*A* b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C)) *Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S ec[(c + d*x)/2]^2 + a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6* A + 3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x) /2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (16*A*b^4 - 57* a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^4*(a + b*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (Sqrt[Cos[(c + d*x)/ 2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-16*A*b^4 + 57*a^3*b*B + 2 4*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]* EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3*a

^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(105*a^4*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + ((-2*(a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^4*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - ((a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 2*(a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - b*(16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + 2*(16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 + (3*a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + (a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3

```
*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((b*Sec[(c + d
*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)]) + (a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A +
3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*S
ec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)]*Sec[c + d*x])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[
(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B -
 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*
x)/2]^4*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c +
d*x)/2]^2] + a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B
 + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))/(315*a^4*
Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 0.65, size = 5979, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c))^(1/2
),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c)
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c)
)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(11/2)*(a+b*cos(d*x+
c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2)*(a+b*cos(d*x+c)
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(11/2), x)
```

3.1503 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=487

$$-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-5a^2(5A+7C)-7abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+\ldots}{\ldots}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*
C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[S
ec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + 2*a*b*(3*A - 7*B) + a^2*(
25*A - 63*B + 35*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(105*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A + 7*C))
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^2*d) + (2
*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3
5*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*
d)
```

Rubi [A] time = 1.58891, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-5a^2(5A+7C)-7abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(9/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*
C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[S
ec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + 2*a*b*(3*A - 7*B) + a^2*(
```

```
25*A - 63*B + 35*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(105*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A + 7*C))
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^2*d) + (2
*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3
5*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*
d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+}{7d}$$

$$= \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+}{35ad}$$

$$= -\frac{2\left(4Ab^2-7abB-5a^2(5A+7C)\right)\sqrt{a+b}}{105a^2}$$

$$= -\frac{2\left(4Ab^2-7abB-5a^2(5A+7C)\right)\sqrt{a+b}}{105a^2}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^3+63a^3B-14ab^2B+\right.}{}$$

Mathematica [B] time = 25.7949, size = 3574, normalized size = 7.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(19*a^2*A*b + 8*A*b^3 + 63
*a^3*B - 14*a*b^2*B + 35*a^2*b*C)*Sin[c + d*x])/(105*a^3) + (2*Sec[c + d*x]
^2*(A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]*(25*a^
2*A*Sin[c + d*x] - 4*A*b^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x] + 35*a^2*C*S
in[c + d*x]))/(105*a^2) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-1
9*A*b)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(105*a
^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*a*B)/(5*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*B)/(15*a*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (b*C)/(3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
+ (5*a*A*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (17*A*b^2*Sqrt
[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*
x]])/(105*a^3*Sqrt[a + b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/(15*Sq
rt[a + b*Cos[c + d*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*C
```

os[c + d*x]]) + (a*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) - (b^
2*C*Sqrt[Sec[c + d*x]])/(3*a*Sqrt[a + b*Cos[c + d*x]]) - (19*A*b^2*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt[a + b*Cos[c + d*x]]) - (3*
b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) + (2*
b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]
) - (b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*Sqrt[a + b*Cos[c + d*x
]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(8*A*b^3 + 63*a^3*B
 - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B
) + a^2*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
 + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*
A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]))/(105*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8*A*b^3 + 63*
a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A
 + 7*B) + a^2*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*
b*(19*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(105*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2])
 - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(8*A
*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 -
2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 63*a^3*B - 14*a*b^
2*B + a^2*b*(19*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x
)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^3 + 63*a^3*B
 - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^4)/2 - ((a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A
 + 35*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1
 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 6
3*B + 35*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x]
)/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] - ((a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(

```
19*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))
)/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A
*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c
+ d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))] + b*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A +
35*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (8*A
*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (8*A*b^3 + 63*a^3*B - 14*a
*b^2*B + a^2*b*(19*A + 35*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*
(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)
*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1
 - Tan[(c + d*x)/2]^2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]) + ((-2*(a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 3
5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B + 35*C))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Cos[c + d*x]*(a + b
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec
[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x])
)/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.418, size = 4345, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2)
,x)
```

[Out] 2/105/d/a^3*(15*A*a^4+8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-2*
A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+35*C*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b+19*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b-2*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+42*B*cos(d*x+c)^3*a^4+21*B*cos(d*x+c)*
a^4-63*B*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^4-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*cos(d*x+c)^3*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^4-63*B*cos(d*x+c)^5*a^3*b-7*B*cos(d*x+c)^5*a^2*b^2+14*B*cos
(d*x+c)^5*a*b^3+35*B*cos(d*x+c)^4*a^3*b+14*B*cos(d*x+c)^4*a^2*b^2-8*A*cos(d

*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^3+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-35*C*
cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3*b+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+35*C*cos(d*x+c)^4*a^2*b^2+70
*C*cos(d*x+c)^3*a^3*b-35*C*cos(d*x+c)^5*a^3*b-35*C*cos(d*x+c)^5*a^2*b^2-35*
C*cos(d*x+c)^4*a^3*b-7*B*cos(d*x+c)^3*a^2*b^2+28*B*cos(d*x+c)^2*a^3*b+4*A*c
os(d*x+c)^3*a*b^3-A*cos(d*x+c)^2*a^2*b^2+18*A*cos(d*x+c)*a^3*b-25*A*cos(d*x
+c)^5*a^3*b-19*A*cos(d*x+c)^5*a^2*b^2+4*A*cos(d*x+c)^5*a*b^3-19*A*cos(d*x+c
)^4*a^3*b+20*A*cos(d*x+c)^4*a^2*b^2-8*A*cos(d*x+c)^4*a*b^3+26*A*cos(d*x+c)^
3*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-25*A*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^4
*a^4+10*A*cos(d*x+c)^2*a^4+35*C*cos(d*x+c)^2*a^4-8*A*cos(d*x+c)^5*b^4+8*A*c
os(d*x+c)^4*b^4+19*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-14*B*cos(d*x+c)^4*a*b^3+63*B*si
n(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d
*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*a^4-14*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-49*B*sin(d*x+c)*cos(d*x+c)^4*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+14*B*sin(
d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a^2*b^2+63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^3*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-14

```
*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a*b^3-49*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+14*B*sin(d*x+c)*cos(d*x+c
)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b
^2+63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*a^3*b)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)/(a+b*cos(d*x+
c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)*(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(9/2), x)
```

3.1504 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{7/2}(c+dx)\,dx$

Optimal. Leaf size=400

$$-\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) -
(2*(a - b)*Sqrt[a + b]*(2*A*b + a*(9*A - 5*B + 15*C))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(A*b +
5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) +
 (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.10123, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(7/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) -
(2*(a - b)*Sqrt[a + b]*(2*A*b + a*(9*A - 5*B + 15*C))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(A*b +
```

```
5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) +
 (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,c}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(}{5d}$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+}{15ad}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(2Ab^2-5abB-3a^2(3A\right.}{}$$

Mathematica [A] time = 19.9845, size = 466, normalized size = 1.16

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sin(c+dx)\left(9a^2A+15a^2C+5abB-2Ab^2\right)}{15a^2}+\frac{2\sec(c+dx)(5aB\sin(c+dx)+Ab\sin(c+dx))}{15a}+\frac{2}{5}A\tan(c+dx)\mathrm{sec}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((a + b)*((-2*A*b^2
 + 5*a*b*B + 3*a^2*(3*A + 5*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] - a*(9*a*A - 2*A*b + 5*a*(B + 3*C))*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a
 + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]) - (-2*A*b^2 +
 5*a*b*B + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^4*Tan[(c + d*x)/2]))/(15*a^2*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a +
 b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (Sqrt[a + b*Cos[c + d*x]]*Sq
rt[Sec[c + d*x]]*((2*(9*a^2*A - 2*A*b^2 + 5*a*b*B + 15*a^2*C)*Sin[c + d*x])
/(15*a^2) + (2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a)
 + (2*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.299, size = 3343, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 2/15/d/a^2*(3*A*a^3-A*cos(d*x+c)^2*a*b^2+15*C*cos(d*x+c)^3*a^2*b-15*C*cos(d
*x+c)^4*a^2*b-9*A*cos(d*x+c)^4*a^2*b-A*cos(d*x+c)^4*a*b^2+2*A*cos(d*x+c)^3*
a*b^2+4*A*cos(d*x+c)*a^2*b+10*B*cos(d*x+c)^2*a^2*b+5*B*cos(d*x+c)^3*a*b^2+5
*A*cos(d*x+c)^3*a^2*b-5*B*cos(d*x+c)^3*a^2*b-5*B*cos(d*x+c)^4*a^2*b-5*B*cos
(d*x+c)^4*a*b^2+5*B*cos(d*x+c)*a^3-5*B*cos(d*x+c)^3*a^3+5*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
```

E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*
a*b^2+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+5*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*
a^2*b+5*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a^2*b+5*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-5*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*
a^2*b+2*A*cos(d*x+c)^4*b^3-9*A*cos(d*x+c)^3*a^3+6*A*cos(d*x+c)^2*a^3-15*C*c
os(d*x+c)^3*a^3+15*C*cos(d*x+c)^2*a^3-2*A*cos(d*x+c)^3*b^3-9*A*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3-2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-15*C*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(
d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-2*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3-15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+15*C*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-2*A*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+15*C*sin(d*x+c)*cos(d*x+c)^3*(c

```
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*A*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*s
in(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+15*C*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*A
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-5*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^2*a^3)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/s
in(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)*(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(7/2), x)
```

3.1505 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=467

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-2\sqrt{a}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(b*(A - 3*B)
- a*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
 b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.0064, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-2\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(5/2),x]
```



```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(b*(A - 3*B)
- a*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
```

```
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
 b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
```

```
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,c}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c}{3d}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\Big(}{}$$

$$= \frac{2(a-b)\sqrt{a+b}(Ab+3aB)\sqrt{\cos(c+dx)}}{}$$

Mathematica [B] time = 24.454, size = 5188, normalized size = 11.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.225, size = 2323, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 2/3/d/a*(2*A*cos(d*x+c)*a*b-A*cos(d*x+c)^2*a*b-3*B*cos(d*x+c)^3*a*b-A*cos(d
*x+c)^3*a*b+3*B*cos(d*x+c)^2*a*b+A*a^2-A*cos(d*x+c)^3*b^2+3*B*sin(d*x+c)*co
```

s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^2*a*b-A*cos(d*x+c)^2*a^2+A*cos(d*x+c)^2*b^2-3*C*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2+3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^2-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*cos(d*x+c)*a^2+3*C*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a*b+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-6*C*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
a*b+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)*a*b-3*B*cos(d*x+c)^2*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2-3*C*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)

```
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x
+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos \left(dx+c\right)^{2}+B\cos \left(dx+c\right)+A\right)\sqrt{b\cos \left(dx+c\right)+a}\sec \left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)*(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(5/2), x)
```

3.1506 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ dx) dx

Optimal. Leaf size=509

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)(2Ab-a(2A-2B-C))\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*A*b - a*(2*A - 2*B - C
))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c +
d*x]]) - (Sqrt[a + b]*(2*b*B + a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*A - C)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.3209, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)(2Ab-a(2A-2B-C))\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^(3/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(2*A - C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*A*b - a*(2*A - 2*B - C
))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
```

```
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c +
d*x]]) - (Sqrt[a + b]*(2*b*B + a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*A - C)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c-}{d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c-}{d}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c-}{d}$$

$$= -\frac{\sqrt{a+b}(2bB+aC)\sqrt{\cos(c+dx)}\csc(c+}{}$$

$$= \frac{(a-b)\sqrt{a+b}(2A-C)\sqrt{\cos(c+dx)}\csc(c}{}$$

Mathematica [A] time = 18.3235, size = 904, normalized size = 1.78

$$\frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2aA\tan^5\left(\frac{1}{2}(c+dx)\right)-2Ab\tan^5\left(\frac{1}{2}(c+dx)\right)-aC\,\mathrm{t}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[(1
 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a*A*Tan[(c + d*x)/2] - 2*A*b*Tan[(c + d*x)
/2] + a*C*Tan[(c + d*x)/2] + b*C*Tan[(c + d*x)/2] + 4*A*b*Tan[(c + d*x)/2]^
3 - 2*b*C*Tan[(c + d*x)/2]^3 + 2*a*A*Tan[(c + d*x)/2]^5 - 2*A*b*Tan[(c + d*
x)/2]^5 - a*C*Tan[(c + d*x)/2]^5 + b*C*Tan[(c + d*x)/2]^5 - 4*b*B*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
```

```
2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] - 4*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*C*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] - (a + b)*(2*A - C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(b*(A - B) + a*(A +
 B - C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2
]^2)])
```

Maple [B] time = 0.224, size = 2150, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] 1/d*(-C*cos(d*x+c)^3*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*a*A-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-4*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-4*B*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*b-C*cos(d*x+c)^2*a+2*A*cos(d*x+c)*b-2*A*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
```

```
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*A*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)^2*b+C*cos(d*x+c)^2*
b-2*A*cos(d*x+c)*a+C*cos(d*x+c)*a+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*cos(d*x+c)*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a)*cos(d*x+c)*(1
/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))
```

```
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)*(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(3/2), x)
```

3.1507 $\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$

Optimal. Leaf size=543

$$-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(4*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*A*b + a*C + 2
*b*(2*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sq
rt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec
[c + d*x]]) + ((4*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(4*b*d)
```

Rubi [A] time = 1.35335, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[S
ec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B + a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(4*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*A*b + a*C + 2
*b*(2*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
```

```
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sq
rt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec
[c + d*x]]) + ((4*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(4*b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
```

```
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\mathrm{c}}\right.$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4bB}{}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4bB}{}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+4abB-a^2C+4b^2C\right)\sqrt{\mathrm{c}}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}(4bB+aC)\sqrt{\cos(c+dx)}\,\mathrm{c}}{}$$

Mathematica [C] time = 19.4518, size = 1816, normalized size = 3.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (-
4*a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 4*b^2*Sqrt[(a - b)/(a + b)
]*B*Tan[(c + d*x)/2] - a^2*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2] - a*b*S
qrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2] + 8*b^2*Sqrt[(a - b)/(a + b)]*B*Tan
[(c + d*x)/2]^3 + 2*a*b*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^3 + 4*a*b*
Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 4*b^2*Sqrt[(a - b)/(a + b)]*B*
Tan[(c + d*x)/2]^5 + a^2*Sqrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - a*b*S
qrt[(a - b)/(a + b)]*C*Tan[(c + d*x)/2]^5 + (16*I)*A*b^2*EllipticPi[(a + b)
/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[(a + b)/(a - b), I*Arc
Sinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 -
```

```
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] - (2*I)*a^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a -
b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
(8*I)*b^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (16*I)*A*b^2*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*
B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*
a^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8
*I)*b^2*C*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(
c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
- I*(a - b)*(4*b*B + a*C)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c
+ d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] + (2*I)*(a - b)*(4*A*b + (a + 2*b)*C)*EllipticF[I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)])/(4*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[(1 - Tan[(c + d*x)/2
]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2
)])
```

Maple [B] time = 0.244, size = 2618, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-2*C*cos(d*x+c)*a*b+4
*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b+3*C*cos(d*x+c)^3*a*b-C*cos(d*x+c)^2*
a*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
```

```
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^2-2*b^2*C*cos(d*x+c)^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+16*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*C*cos(d*x+c)*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-2*
C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+C*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+16*A*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^3*b
^2-4*B*cos(d*x+c)^2*b^2+8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+C*cos(d*x+c)^2*a^2+2*C*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+C
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b+2*C*cos(d*x+c)^4*b^2-C*cos(d*x+c
```

```
)*a^2-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+4*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*
sqrt(sec(d*x + c)), x)
```

3.1508 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=646

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(2a^2bB+a^3(-C)-4ab^2(2A+C)-8b^3B\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\right.\right.}{8b^3d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(8*b^2*(3*A + 2*C) + 3*a*(2*b*B - a*C))*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) +
(Sqrt[a + b]*(24*A*b^2 + (a + 2*b)*(6*b*B - 3*a*C + 8*b*C))*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqr
t[a + b]*(2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x
]]) + ((2*b*B - a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Sec
[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c
 + d*x]]) + ((8*b^2*(3*A + 2*C) + 3*a*(2*b*B - a*C))*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b^2*d)
```

Rubi [A] time = 1.97946, antiderivative size = 646, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(2a^2bB+a^3(-C)-4ab^2(2A+C)-8b^3B\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\right.\right.}{8b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Sec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(8*b^2*(3*A + 2*C) + 3*a*(2*b*B - a*C))*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
```

```
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) +
(Sqrt[a + b]*(24*A*b^2 + (a + 2*b)*(6*b*B - 3*a*C + 8*b*C))*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqr
t[a + b]*(2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x
]]) + ((2*b*B - a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Sec
[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c
 + d*x]]) + ((8*b^2*(3*A + 2*C) + 3*a*(2*b*B - a*C))*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{co}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}\right.}{}$$

$$= \frac{(2bB-aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\,\mathrm{co}}{3}$$

$$= \frac{(2bB-aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\,\mathrm{co}}{3}$$

$$= \frac{(2bB-aC)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{C(a+b\,\mathrm{co}}{3}$$

$$= \frac{\sqrt{a+b}\left(2a^2bB-8b^3B-a^3C-4ab^2(2A+C)\right)\sqrt{\cos(c}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(8b^2(3A+2C)+3a(2bB-aC)\right)\sqrt{\cos(}}{}$$

Mathematica [A] time = 11.5974, size = 513, normalized size = 0.79

$$\frac{\frac{2\tan(c+dx)(a+b\cos(c+dx))(aC+6bB+4bC\cos(c+dx))}{b} - \frac{-b\tan\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-3a^2C+6abB+24Ab^2+16b^2C\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^{3/2}(a+b\cos(c+}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-((-(b*(a + b)*(24*A*b^2 + 6*a*b*B - 3*a^2*C + 16*b^2*C)*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + a*(a + b)*(24*A*b^2 + 3*a^2*C - 6*a
*b*(B + C) + 4*b^2*(3*B + 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)] + 3*(2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*((a - b)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(24*A*b^2 + 6*a*b*B - 3*a^
2*C + 16*b^2*C)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2
)*Sec[c + d*x]*Tan[(c + d*x)/2])/(b^3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/
2))) + (2*(a + b*Cos[c + d*x])*(6*b*B + a*C + 4*b*C*Cos[c + d*x])*Tan[c + d
*x])/b)/(24*d*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.346, size = 3767, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/24/d/b^2*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b^2+10*C*cos(d*x+c)^
4*a*b^2-C*cos(d*x+c)^3*a^2*b+3*C*cos(d*x+c)^2*a^2*b+6*C*cos(d*x+c)^2*a*b^2-
2*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2+6*B*cos(d*x+c)^2*a^2*b-6*B*cos(d
*x+c)^2*a*b^2-6*B*cos(d*x+c)*a^2*b-12*B*cos(d*x+c)*a*b^2+18*B*cos(d*x+c)^3*
a*b^2+12*B*cos(d*x+c)^4*b^3+8*C*cos(d*x+c)^5*b^3+6*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+2*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-28*
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*a*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+48*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+48*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^2-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+6*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-3
*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*a^3-12*B*cos(d*x+c)^2*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3-3*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^
3+3*C*cos(d*x+c)*a^3+24*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+c)^2*b^3+6*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
```

```
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*b^3-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*B*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^
2*b+12*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^2+48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)
/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos{(c+dx)}}\left(A+B\cos{(c+dx)}+C\cos^{2}{(c+dx)}\right)}{\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/
sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
sqrt(sec(d*x + c)), x)
```

3.1509 $\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=766

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2bB-15a^3C-4ab^2(12A+7C)-128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192b^3d}+\frac{\sin(c+dx)\left(5a^2C-8a\right.}{3}$$

```
[Out] ((a - b)*Sqrt[a + b]*(24*a^2*b*B - 128*b^3*B - 15*a^3*C - 4*a*b^2*(12*A + 7
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^3*d*Sqr
t[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3*C - 2*a^2*b*(12*B + 5*C) + 4*a*b^2*
(12*A + 4*B + 7*C) + 8*b^3*(12*A + 16*B + 9*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(192*b^3*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8
*a^3*b*B + 32*a*b^3*B - 5*a^4*C - 8*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^4*
d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*b*d*
Sec[c + d*x]^(3/2)) + ((16*A*b^2 - 8*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(32*b^2*d*Sqrt[Sec[c + d*x]]) + ((8*b*B - 5*a*
C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b^2*d*Sqrt[Sec[c + d*x]]) -
 ((24*a^2*b*B - 128*b^3*B - 15*a^3*C - 4*a*b^2*(12*A + 7*C))*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^3*d)
```

Rubi [A] time = 2.65667, antiderivative size = 766, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2bB-15a^3C-4ab^2(12A+7C)-128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192b^3d}+\frac{\sin(c+dx)\left(5a^2C-8a\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sec[
c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(24*a^2*b*B - 128*b^3*B - 15*a^3*C - 4*a*b^2*(12*A + 7
*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^3*d*Sqr
t[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3*C - 2*a^2*b*(12*B + 5*C) + 4*a*b^2*
(12*A + 4*B + 7*C) + 8*b^3*(12*A + 16*B + 9*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(192*b^3*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8
*a^3*b*B + 32*a*b^3*B - 5*a^4*C - 8*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^4*
d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*b*d*
Sec[c + d*x]^(3/2)) + ((16*A*b^2 - 8*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(32*b^2*d*Sqrt[Sec[c + d*x]]) + ((8*b*B - 5*a*
C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b^2*d*Sqrt[Sec[c + d*x]]) -
 ((24*a^2*b*B - 128*b^3*B - 15*a^3*C - 4*a*b^2*(12*A + 7*C))*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
```

```
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s}\right.}{}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{(8bB-5aC)(a+}{24b}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(16Ab^2-8abB+\right.}{}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(16Ab^2-8abB+\right.}{}$$

$$= \frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(16Ab^2-8abB+\right.}{}$$

$$= -\frac{\sqrt{a+b}\left(8a^3bB+32ab^3B-5a^4C-8a^2b^2(2A+C)+\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(24a^2bB-128b^3B-15a^3C-4ab^2(12A\right.}{}$$

Mathematica [A] time = 15.3834, size = 854, normalized size = 1.11

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(8bB+aC)\sin(c+dx)}{96b}+\frac{\left(-5Ca^2+8bBa+48Ab^2+48b^2C\right)\sin(2(c+dx))}{192b^2}+\frac{(8bB+aC)\sin(3(c+dx))}{96b}+\frac{1}{32}C\sin(4\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*b*B + a*C)*Sin[c + d*x])/
(96*b) + ((48*A*b^2 + 8*a*b*B - 5*a^2*C + 48*b^2*C)*Sin[2*(c + d*x)])/(192*
b^2) + ((8*b*B + a*C)*Sin[3*(c + d*x)])/(96*b) + (C*Sin[4*(c + d*x)])/32))/
d - (-((a - b)*b*(-24*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(12*A + 7*C)
)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)*(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)) - b*(-a + b)*(a + b)*(-24*a^2*b*B + 128*b^3*B + 15
*a^3*C + 4*a*b^2*(12*A + 7*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + a*(a - b)*(a + b
)*(15*a^3*C - 6*a^2*b*(4*B + 5*C) - 8*b^3*(12*A + 16*B + 9*C) + 4*a*b^2*(12
*A + 12*B + 11*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 3*(a - b)*(-8*a^3*b*B - 32*a
*b^3*B + 5*a^4*C + 8*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a - b)*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)])/(192*(a - b)*b^4*d*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.503, size = 5307, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(a+b*cos(d*x+c))**(1/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/
sec(d*x + c)^(3/2), x)
```

3.1510 $\int (a+b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx) \, dx$

Optimal. Leaf size=590

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(7a^2(7A+9C)+72abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{315ad}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-2a^2b(44A\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A +
9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(315*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3
 + 6*a*b^2*(A - 3*B) + 3*a^2*b*(13*A - 57*B + 21*C) - 3*a^3*(49*A - 25*B +
63*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqr
t[Sec[c + d*x]]) - (2*(4*A*b^3 - 75*a^3*B - 9*a*b^2*B - 2*a^2*b*(44*A + 63*
C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^2*d) +
 (2*(3*A*b^2 + 72*a*b*B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c
 + d*x]^(5/2)*Sin[c + d*x])/(315*a*d) + (2*(A*b + 3*a*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*A*(a + b*Cos[c + d*x])^
(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 2.21878, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(7a^2(7A+9C)+72abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{315ad}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-2a^2b(44A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A +
9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
```

```
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(315*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3
 + 6*a*b^2*(A - 3*B) + 3*a^2*b*(13*A - 57*B + 21*C) - 3*a^3*(49*A - 25*B +
63*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqr
t[Sec[c + d*x]]) - (2*(4*A*b^3 - 75*a^3*B - 9*a*b^2*B - 2*a^2*b*(44*A + 63*
C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a^2*d) +
 (2*(3*A*b^2 + 72*a*b*B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c
 + d*x]^(5/2)*Sin[c + d*x])/(315*a*d) + (2*(A*b + 3*a*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*A*(a + b*Cos[c + d*x])^
(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)}{9d}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(}{21d}$$

$$= \frac{2\left(3Ab^2+72abB+7a^2(7A+9C)\right)\sqrt{a}}{3}$$

$$= -\frac{2\left(4Ab^3-75a^3B-9ab^2B-2a^2b(44\right.}{}$$

$$= -\frac{2\left(4Ab^3-75a^3B-9ab^2B-2a^2b(44\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^4+246a^3bB-18\right.}{}$$

<u>Mathematica</u> [B] time = 27.8527, size = 4669, normalized size = 7.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 33*a^2*A*b^2
+ 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 189*a^4*C + 63*a^2*b^2*C)*Sin[c + d*
x])/(315*a^3) + (2*Sec[c + d*x]^3*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]
))/63 + (2*Sec[c + d*x]^2*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 7
2*a*b*B*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]*(8
8*a^2*A*b*Sin[c + d*x] - 4*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 9*a
*b^2*B*Sin[c + d*x] + 126*a^2*b*C*Sin[c + d*x]))/(315*a^2) + (2*a*A*Sec[c +
 d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^2*A)/(15*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (11*A*b^2)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
```

d*x]]) - (8*A*b^4)/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82*a*b*B)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^3*B)/(35*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*a^2*C)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*C)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*A*b*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Sqrt[Sec[c + d*x]])/(315*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) + (a*b*C*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (b^3*C*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (7*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (11*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) - (82*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) - (3*a*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^3*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^3*(a + b*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*

A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + ((-2*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - ((a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 2*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + b*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - 2*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 + (3*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B +

```
 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((b*
Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)]) + (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*
(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*Sec[(c + d*x)/2]^2*(Cos[
c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)]*Sec[c + d*x])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a
 + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(8*A*b^4 + 246*a^3*b*B - 18*
a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
Sec[(c + d*x)/2]^4*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1
- Tan[(c + d*x)/2]^2] + a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(1
3*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x])
)/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 0.645, size = 5965, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2
),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{11}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(11/2), x)
```

3.1511 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right.$ $dx)\,dx$

Optimal. Leaf size=490

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+42abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105ad} - \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B - 2*a^2*b*(41*A +
70*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqr
t[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B + 35*
C) + 3*a*b*(19*A - 7*B + 35*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(105*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(3*A*b^2 + 42*a*b*B + 5*a^2*
(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105
*a*d) + (2*(3*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c
 + d*x])/(7*d)
```

Rubi [A] time = 1.56764, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+42abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105ad} - \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(9/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B - 2*a^2*b*(41*A +
70*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqr
t[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B + 35*
```

```
C) + 3*a*b*(19*A - 7*B + 35*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(105*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(3*A*b^2 + 42*a*b*B + 5*a^2*
(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105
*a*d) + (2*(3*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c
 + d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,\mathrm{s}}{7d}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(}{35d}$$

$$= \frac{2\left(3Ab^2+42abB+5a^2(5A+7C)\right)\sqrt{a}}{10}$$

$$= \frac{2\left(3Ab^2+42abB+5a^2(5A+7C)\right)\sqrt{a}}{10}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(6Ab^3-63a^3B-21ab\right.}{}$$

Mathematica [B] time = 26.0037, size = 3611, normalized size = 7.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-82*a^2*A*b + 6*A*b^3 -
63*a^3*B - 21*a*b^2*B - 140*a^2*b*C)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d
*x]^2*(8*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a
^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x] + 35*a^2*C
*Sin[c + d*x]))/(105*a) + (2*a*A*Sec[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((
-82*a*A*b)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/(3
5*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*a^2*B)/(5*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (4*a*b*C)/(3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) + (5*a^2*A*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^2*
Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c +
d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) + (a*b*B*Sqrt[Sec[c + d*x]])/(5*Sq
rt[a + b*Cos[c + d*x]]) - (b^3*B*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c
```

+ d*x]]) + (a^2*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) - (b^2*C
*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) - (82*A*b^2*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Cos[2*(c
+ d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) - (3*a*b*B*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (b^3*B*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (4*b^2*
C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2
*B + 2*a^2*b*(41*A + 70*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C
) + 3*a*b*(19*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)] - (-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b
*(41*A + 70*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(105*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*(
(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-6*A*b^3
 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + a
^2*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-6*A*b^3 + 63*a^3*B
+ 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt
[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x
)/2]*(-2*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
 2*a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*
C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] - (-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Cos[c + d*x
]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*(-((-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-6*A*b
^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(
-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2
 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]

```
- ((a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos
[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(-6*A*b^2 + a^2*(25*A +
 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/(
(a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))] + b*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))
*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*A*b^3
+ 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-6*A*b^3 + 63*a^3*B + 21*a*b
^2*B + 2*a^2*b*(41*A + 70*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C
) + 3*a*b*(19*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])
- ((a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a
+ b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2
*a^2*b*(41*A + 70*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*
a*b*(19*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - (-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A
 + 70*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]
^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(
c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.416, size = 4534, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x)
```

```
[Out] 2/105/d/a^2*(15*A*a^4-6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-82*A*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-51
*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+140*C*cos(d*x+c)^4*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b+140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b^2-140*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+82*A*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b+82*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-82*A*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b-51*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+42*B*cos(d*x+c)^3*a^4+21*B*cos(d*
x+c)*a^4-63*B*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-6*A*cos(d*x+c)^4*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4-25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^4-35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-6*A*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-2
5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4-63*B*cos(d*x+c)^5*a^3*b-42*B*cos(d*x+c)^5*a^2*b^2-21
*B*cos(d*x+c)^5*a*b^3-21*B*cos(d*x+c)^4*a^2*b^2+6*A*cos(d*x+c)^3*sin(d*x+c)
```

*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-140*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+82*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+140*C*cos(d*x+c)^4*a^2*b^2+175*C*cos(d*x+c)^3*a^3*b-35*C*cos(d*x+c)^5*a^3*b-140*C*cos(d*x+c)^5*a^2*b^2-140*C*cos(d*x+c)^4*a^3*b-105*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-105*C*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+63*B*cos(d*x+c)^3*a^2*b^2+63*B*cos(d*x+c)^2*a^3*b-3*A*cos(d*x+c)^3*a*b^3+27*A*cos(d*x+c)^2*a^2*b^2+39*A*cos(d*x+c)*a^3*b-25*A*cos(d*x+c)^5*a^3*b-82*A*cos(d*x+c)^5*a^2*b^2-3*A*cos(d*x+c)^5*a*b^3-82*A*cos(d*x+c)^4*a^3*b+55*A*cos(d*x+c)^4*a^2*b^2+6*A*cos(d*x+c)^4*a*b^3+68*A*cos(d*x+c)^3*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-25*A*cos(d*x+c)^4*a^4-35*C*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+35*C*cos(d*x+c)^2*a^4+6*A*cos(d*x+c)^5*b^4-6*A*cos(d*x+c)^4*b^4+82*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+21*B*cos(d*x+c)^4*a*b^3+63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+21*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-84*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-21*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+63*B*

```
sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+21*B*sin(d*x+c)*cos(d*x+c)
^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3-
84*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a^3*b-21*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+63*B*sin(d*x+c)*cos(d
*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
^3*b)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(9/2), x)
```

3.1512 $\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{7/2}(c+dx)\,dx$

Optimal. Leaf size=550

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(9A-5B+15C)-2ab(6A-10B+15C)+3b^2(A-5B)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\ldots}{15ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) -
(2*Sqrt[a + b]*(3*b^2*(A - 5*B) - 2*a*b*(6*A - 10*B + 15*C) + a^2*(9*A - 5*
B + 15*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sq
rt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(3*A*b + 5*a*B)*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a +
 b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.43014, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3047, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(9A-5B+15C)-2ab(6A-10B+15C)+3b^2(A-5B)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\ldots}{15ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) -
```

```
(2*Sqrt[a + b]*(3*b^2*(A - 5*B) - 2*a*b*(6*A - 10*B + 15*C) + a^2*(9*A - 5*
B + 15*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sq
rt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(3*A*b + 5*a*B)*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*A*(a +
 b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{5d}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(}{15d}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(}{15d}$$

$$= -\frac{2b\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\mathrm{I}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3Ab^2+20abB+3a^2(3\right.}{}$$

Mathematica [B] time = 26.1924, size = 6852, normalized size = 12.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.331, size = 3930, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x)
```

[Out] -2/15/d/a*(-3*A*a^3-9*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*cos
(d*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+c)^4*a*b^2+3*A*cos(d*x+c
)^3*a*b^2-9*A*cos(d*x+c)*a^2*b-25*B*cos(d*x+c)^2*a^2*b-20*B*cos(d*x+c)^3*a*
b^2+20*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b+20*B*cos(d*x+c)^4*a*b^2-
5*B*cos(d*x+c)*a^3+5*B*cos(d*x+c)^3*a^3-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-9*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+30*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+30*
C*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+15*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*
b^2-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^3*a^2*b-20*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-20*B*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b^2+3*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^2*a^3+15
*C*cos(d*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3-3*A*cos(d*x+c)^3*b^3+9*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c

os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*
cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(
d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+1
2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-
3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^2+30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*C*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b+12*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3
+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(

```
d*x+c)*cos(d*x+c)^2*a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(
7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(7/2), x)
```

3.1513 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^{5/2}(c+dx)\, dx$

Optimal. Leaf size=588

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(2a^2(A-3B+3C)-ab(8A-3(4B+C))+6Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*A*b + 6*a*B - 3*b*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(6*A*b^2
+ 2*a^2*(A - 3*B + 3*C) - a*b*(8*A - 3*(4*B + C)))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(d*Sqrt[Sec[c + d*x]]) + (2*(A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d - ((8*A*b + 6*a*B - 3*b*C)*Sqrt[a + b*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^(3
/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.86722, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(2a^2(A-3B+3C)-ab(8A-3(4B+C))+6Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(5/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(8*A*b + 6*a*B - 3*b*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
```

```
ec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(6*A*b^2
+ 2*a^2*(A - 3*B + 3*C) - a*b*(8*A - 3*(4*B + C)))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(d*Sqrt[Sec[c + d*x]]) + (2*(A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d - ((8*A*b + 6*a*B - 3*b*C)*Sqrt[a + b*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^(3
/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{\quad}$$

$$= \frac{2A(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{3d}$$

$$= \frac{2(Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c}}{d}$$

$$= \frac{2(Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c}}{d}$$

$$= \frac{2(Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c}}{d}$$

$$= -\frac{\sqrt{a+b}(2bB+3aC)\sqrt{\cos(c+dx)}\csc(}{\quad}$$

$$= \frac{(a-b)\sqrt{a+b}(8Ab+6aB-3bC)\sqrt{\cos}}{\quad}$$

Mathematica [B] time = 25.8819, size = 7588, normalized size = 12.9

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.283, size = 3360, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x)
```

```
[Out] -1/3/d*(-10*A*cos(d*x+c)*a*b+8*A*cos(d*x+c)^2*a*b+6*B*cos(d*x+c)^3*a*b+2*A*
cos(d*x+c)^3*a*b-6*B*cos(d*x+c)^2*a*b+3*C*cos(d*x+c)^3*a*b-3*C*cos(d*x+c)^2
*a*b-2*A*a^2+8*A*cos(d*x+c)^3*b^2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a
*b+2*A*cos(d*x+c)^2*a^2-8*A*cos(d*x+c)^2*b^2+3*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^2+6*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+12*B*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*b^2-6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*b^2+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-6*B*cos(d*x+c)*a^2-3*C*cos(d
*x+c)^3*b^2-12*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-12*C*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a*b+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b+18*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
```

```
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+3*C*cos(d*x+c)^4*b^2+2
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^2*a^2+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+18*C*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+6*B*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+8*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+3*C*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*
b^2+6*B*cos(d*x+c)^2*a^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+6*C*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*b^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+12*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*a^2+12*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2)*cos(d*x+c)/(a
+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}},x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c)+a)^{\frac{3}{2}}\sec (dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(5/2), x)
```

3.1514 $\int (a+b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right.$ $dx)\,dx$

Optimal. Leaf size=595

$$-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(3a^2C+12abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{4bd\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(8*a*A - 4*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(a*(8*A -
 8*B - 5*C) - 2*b*(8*A + 2*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 12*a*b*B +
 3*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) - (b*(4*A - C)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((8*a*A - 4*b*B - 5*a*C)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (2*A*(a + b*Cos
[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.88902, antiderivative size = 595, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(3a^2C+12abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{4bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(3/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(8*a*A - 4*b*B - 5*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
```

```
ec[c + d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(a*(8*A -
 8*B - 5*C) - 2*b*(8*A + 2*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 12*a*b*B +
 3*a^2*C + 4*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) - (b*(4*A - C)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((8*a*A - 4*b*B - 5*a*C)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (2*A*(a + b*Cos
[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\cdots}{\cdots}$$

$$=\frac{2A(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,\mathrm{s}\cdots}{d}$$

$$=-\frac{b(4A-C)\sqrt{a+b\cos(c+dx)}\sin(c+\cdots}{2d\sqrt{\sec(c+dx)}}$$

$$=-\frac{b(4A-C)\sqrt{a+b\cos(c+dx)}\sin(c+\cdots}{2d\sqrt{\sec(c+dx)}}$$

$$=-\frac{b(4A-C)\sqrt{a+b\cos(c+dx)}\sin(c+\cdots}{2d\sqrt{\sec(c+dx)}}$$

$$=-\frac{\sqrt{a+b}\left(8Ab^2+12abB+3a^2C+4b^2\cdots\right.}{\cdots}$$

$$=\frac{(a-b)\sqrt{a+b}(8aA-4bB-5aC)\sqrt{\cos\cdots}}{\cdots}$$

Mathematica [B] time = 19.3452, size = 1469, normalized size = 2.47

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(2*a*A*Sin[c + d*x] + (b*C*Sin
[2*(c + d*x)])/4))/d + (8*a^2*A*Tan[(c + d*x)/2] + 8*a*A*b*Tan[(c + d*x)/2]
 - 4*a*b*B*Tan[(c + d*x)/2] - 4*b^2*B*Tan[(c + d*x)/2] - 5*a^2*C*Tan[(c + d
*x)/2] - 5*a*b*C*Tan[(c + d*x)/2] - 16*a*A*b*Tan[(c + d*x)/2]^3 + 8*b^2*B*T
an[(c + d*x)/2]^3 + 10*a*b*C*Tan[(c + d*x)/2]^3 - 8*a^2*A*Tan[(c + d*x)/2]^
5 + 8*a*A*b*Tan[(c + d*x)/2]^5 + 4*a*b*B*Tan[(c + d*x)/2]^5 - 4*b^2*B*Tan[(
c + d*x)/2]^5 + 5*a^2*C*Tan[(c + d*x)/2]^5 - 5*a*b*C*Tan[(c + d*x)/2]^5 + 1
6*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)] + 24*a*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*C*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*C*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 16*A*b
^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*C
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/
2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)] + 8*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(8
*a*A - 4*b*B - 5*a*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(4*a^2*(A + B - C) - 2*
b^2*(2*A + C) + a*b*(8*A - 8*B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(4*d*Sqrt[(
1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/
2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 +
 Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.309, size = 3606, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x)
```

```
[Out] 1/4/d*(8*A*cos(d*x+c)*a*b-8*A*cos(d*x+c)^2*a*b+2*C*cos(d*x+c)*a*b-4*B*cos(d
*x+c)^2*a*b+4*B*cos(d*x+c)*a*b-7*C*cos(d*x+c)^3*a*b+5*C*cos(d*x+c)^2*a*b-4*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^2+8*A*a^2+2*b^2*C*cos(d*x+c)^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-16*A*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*
A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^2-5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a^2-8*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*C*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*C*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*C*cos(d
*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*b^2-6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2-8*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-5*C*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2-16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
```

cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+8*C*cos(d*x+c)*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
-5*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)^3*b^2+4*B*cos(d*x+c)^2*b^2-16*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+8
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2-8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-5*C*cos(d*x+c)^2*a^2-2*C
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a*b-5*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b-2*C*cos(d*
x+c)^4*b^2+5*C*cos(d*x+c)*a^2+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*A*cos(d*x+c)*a^2-16*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-24*B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a*b+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x

```
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2-8*A*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+8*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2)*co
s(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sec(d*x + c)^(3/2), x)
```

3.1515 $\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$

Optimal. Leaf size=647

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C+30abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(3a^2\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[Cos[c
 + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) +
 (Sqrt[a + b]*(3*a^2*C + 4*b^2*(6*A + 3*B + 4*C) + 2*a*b*(24*A + 15*B + 7*C
))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c
 + d*x]]) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d
*Sqrt[Sec[c + d*x]]) + ((2*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]) + ((24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```

Rubi [A] time = 1.98092, antiderivative size = 647, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C+30abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[Cos[c
 + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) +
```

```
 (Sqrt[a + b]*(3*a^2*C + 4*b^2*(6*A + 3*B + 4*C) + 2*a*b*(24*A + 15*B + 7*C
))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c
 + d*x]]) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d
*Sqrt[Sec[c + d*x]]) + ((2*b*B + a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*d*Sqrt[Sec[c + d*x]]) + (C*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]) + ((24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{c}}{}$$

$$=\frac{C(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}$$

$$=\frac{(2bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{4d\sqrt{\sec(c+dx)}}$$

$$=\frac{(2bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{4d\sqrt{\sec(c+dx)}}$$

$$=\frac{(2bB+aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{4d\sqrt{\sec(c+dx)}}$$

$$=-\frac{\sqrt{a+b}\left(6a^2bB+8b^3B-a^3C+12ab^2(\right.}{}$$

$$=-\frac{(a-b)\sqrt{a+b}\left(24Ab^2+30abB+3a^2C\right.}{}$$

Mathematica [B] time = 24.2464, size = 4966, normalized size = 7.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*C*Sin[c + d*x])/12 + ((6*b
*B + 7*a*C)*Sin[2*(c + d*x)])/24 + (b*C*Sin[3*(c + d*x)])/12))/d + (Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*((2*a*A*b)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) + (a^2*B)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2
*B)/(2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*b*C)/(12*Sqrt[a
```

```
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c + d*x]])/Sqrt[a
+ b*Cos[c + d*x]] + (A*b^2*Sqrt[Sec[c + d*x]])/(2*Sqrt[a + b*Cos[c + d*x]])
 + (7*a*b*B*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]]) + (17*a^2*C*Sq
rt[Sec[c + d*x]])/(48*Sqrt[a + b*Cos[c + d*x]]) + (b^2*C*Sqrt[Sec[c + d*x]]
)/(3*Sqrt[a + b*Cos[c + d*x]]) + (A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]]
)/(2*Sqrt[a + b*Cos[c + d*x]]) + (5*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x
]])/(8*Sqrt[a + b*Cos[c + d*x]]) + (a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x
]])/(16*Sqrt[a + b*Cos[c + d*x]]) + (b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*
x]])/(3*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(b
*(a + b)*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2)/(a + b)] + a*(a + b)*(-24*A*b^2 + 3*a^2*C - 6*a*b*(3*
B + C) - 4*b^2*(3*B + 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + 3*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*((a - b)*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(24*A*b^2 + 30*a*b*B + 3*a^2*
C + 16*b^2*C)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*
Sec[c + d*x]*Tan[(c + d*x)/2]))/(24*b^2*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c
+ d*x)/2]^2)^(3/2)*((Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(b*(a + b)*(24*A*b^2 + 30*a*b*B + 3*a^2*
C + 16*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + a*(a
+ b)*(-24*A*b^2 + 3*a^2*C - 6*a*b*(3*B + C) - 4*b^2*(3*B + 4*C))*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 3*(6*a^2*b*B + 8*b^3*B - a^3*
C + 12*a*b^2*(2*A + C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] + b*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(48*b*
(a + b*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (Sqrt[Cos[c + d*x]
*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]
*(b*(a + b)*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + a*(a + b)*(-24*A*b^2 + 3*a^2*C - 6*a*b*
(3*B + C) - 4*b^2*(3*B + 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] + 3*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*((a - b)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(24*A*b^2 + 30*a*b*B + 3*a
^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/
2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(16*b^2*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c
```

+ d*x)/2]^2)^(3/2)) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(3/2)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(b*(a + b)*(24*A*b^2 + 30*a*b*B + 3
*a^2*C + 16*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Se
c[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
a*(a + b)*(-24*A*b^2 + 3*a^2*C - 6*a*b*(3*B + C) - 4*b^2*(3*B + 4*C))*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 3*(6*a^2*b*B + 8*b^3*B -
a^3*C + 12*a*b^2*(2*A + C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] + b*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x]
)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(
48*b^2*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (Sqrt[Cos[c +
d*x]*Sec[(c + d*x)/2]^2]*(b*(a + b)*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^
2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]
^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + a*(a + b)*(-24
*A*b^2 + 3*a^2*C - 6*a*b*(3*B + C) - 4*b^2*(3*B + 4*C))*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 3*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*
b^2*(2*A + C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(
c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(
24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])*(Cos[c + d*x
]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)
/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c
+ d*x]))/(48*b^2*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((b*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 1
6*b^2*C)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(3/2)*Sec[c + d*x])/2 + b*(a + b)*(24*A*b^2 + 30*a*b*B + 3*a^2*C +
16*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*
x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*
x)/2] + a*(a + b)*(-24*A*b^2 + 3*a^2*C - 6*a*b*(3*B + C) - 4*b^2*(3*B + 4*C
))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] +
3*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*((a - b)*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + (3*b*(24*A*b^2 + 30*a*
b*B + 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d
*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x])
+ Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + (b*(a + b)*(24*A*
b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(

```
a + b)) + ((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b
)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + (a*(a + b
)*(-24*A*b^2 + 3*a^2*C - 6*a*b*(3*B + C) - 4*b^2*(3*B + 4*C))*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sec[(c +
d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)]) + (3*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*((a -
 b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*(-((b*
Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)]) + (a*(a + b)*(-24*A*b^2 + 3*a^2*C - 6*a*b*(3*B + C)
- 4*b^2*(3*B + 4*C))*Sec[(c + d*x)/2]^4*Sqrt[((a + b*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Ta
n[(c + d*x)/2]^2)/(a + b)]) + (b*(a + b)*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 1
6*b^2*C)*Sec[(c + d*x)/2]^4*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - Tan[(
c + d*x)/2]^2]) + 3*(6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A + C))*Sec[
(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*(((a
 - b)*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b
)*Tan[(c + d*x)/2]^2)/(a + b)]) + (b*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c +
 d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)
/(a + b)])) - b^2*(24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*(Cos[c + d*x]*
Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]*Tan[c + d*x] + b*(24*A*b^2 + 30*
a*b*B + 3*a^2*C + 16*b^2*C)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]*Tan[c + d*x]))/(24*b^2*Sqrt[a +
 b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 0.344, size = 4147, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/24/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)^2*a*
b^2-24*A*cos(d*x+c)*a*b^2+22*C*cos(d*x+c)^4*a*b^2+17*C*cos(d*x+c)^3*a^2*b-3
*C*cos(d*x+c)^2*a^2*b-6*C*cos(d*x+c)^2*a*b^2-14*C*cos(d*x+c)*a^2*b-16*C*cos
(d*x+c)*a*b^2+30*B*cos(d*x+c)^2*a^2*b-30*B*cos(d*x+c)^2*a*b^2-30*B*cos(d*x+
c)*a^2*b-12*B*cos(d*x+c)*a*b^2+42*B*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^
```

3+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b+8*C*cos(d*x+c)^5*b^3+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-52*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2+144*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b^2-96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3+144*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-
96*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+14*C*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)

/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*a^3+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-12*B*cos(d*x+c)^2*b^3-48*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*a^2*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x+c)^3*b^3+3
*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3+24*A*cos(d*x+c
)^3*b^3-24*A*cos(d*x+c)^2*b^3+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin(d*x+c)*cos(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^2-48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+48*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*b^3+30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b+30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)

```
)^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)*sqrt(sec(d*x + c)), x)
```

3.1516 $\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=764

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2bB-9a^3C+12ab^2(20A+13C)+128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192b^2d}-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A +
13*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d*S
qrt[Sec[c + d*x]]) - (Sqrt[a + b]*(9*a^3*C - 6*a^2*b*(4*B + C) - 8*b^3*(12*
A + 16*B + 9*C) - 4*a*b^2*(60*A + 28*B + 39*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(192*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8
*a^3*b*B - 96*a*b^3*B - 3*a^4*C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3
*d*Sqrt[Sec[c + d*x]]) + ((4*b^2*(4*A + 3*C) + a*(8*b*B - 3*a*C))*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d*x]]) + ((8*b*B - 3*a*C
)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d*Sqrt[Sec[c + d*x]]) + (C
*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + ((24
*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d)
```

Rubi [A] time = 2.76907, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2bB-9a^3C+12ab^2(20A+13C)+128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192b^2d}-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A +
13*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d*S
qrt[Sec[c + d*x]]) - (Sqrt[a + b]*(9*a^3*C - 6*a^2*b*(4*B + C) - 8*b^3*(12*
A + 16*B + 9*C) - 4*a*b^2*(60*A + 28*B + 39*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(192*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8
*a^3*b*B - 96*a*b^3*B - 3*a^4*C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^3
*d*Sqrt[Sec[c + d*x]]) + ((4*b^2*(4*A + 3*C) + a*(8*b*B - 3*a*C))*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d*x]]) + ((8*b*B - 3*a*C
)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d*Sqrt[Sec[c + d*x]]) + (C
*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + ((24
*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
```

```
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b$$

$$= \frac{C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$= \frac{(8bB-3aC)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd\sqrt{\sec(c+dx)}} + \frac{C(}{}$$

$$= \frac{\left(4b^2(4A+3C)+a(8bB-3aC)\right)\sqrt{a+b\cos(c+dx}}{32bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(4b^2(4A+3C)+a(8bB-3aC)\right)\sqrt{a+b\cos(c+dx}}{32bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(4b^2(4A+3C)+a(8bB-3aC)\right)\sqrt{a+b\cos(c+dx}}{32bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(8a^3bB-96ab^3B-3a^4C-24a^2b^2(2A+C)\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24a^2bB+128b^3B-9a^3C+12ab^2(\right.}{}$$

Mathematica [A] time = 15.3309, size = 603, normalized size = 0.79

$$\frac{\frac{2\tan(c+dx)(a+b\cos(c+dx))\left(3a^2C+4b(9aC+8bB)\cos(c+dx)+56abB+48Ab^2+12b^2C\cos(2(c+dx))+48b^2C\right)}{b}-\frac{-b\tan\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24a^2bB-9a^3C+12ab^2\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-((-(b*(a + b)*(24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*S
qrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + a*(a + b)*(9*a^3*
C - 6*a^2*b*(4*B + 3*C) + 12*a*b^2*(12*A + 4*B + 7*C) + 8*b^3*(12*A + 16*B
+ 9*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 3*(8*a^3*b*
B - 96*a*b^3*B - 3*a^4*C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*((a -
 b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[(
(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(24*a^2*b*B + 128*b^3
*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*(a + b*Cos[c + d*x])*(Cos[c + d*x]*S
ec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])/(b^3*(Cos[c + d*x]*
Sec[(c + d*x)/2]^2)^(3/2))) + (2*(a + b*Cos[c + d*x])*(48*A*b^2 + 56*a*b*B
+ 3*a^2*C + 48*b^2*C + 4*b*(8*b*B + 9*a*C)*Cos[c + d*x] + 12*b^2*C*Cos[2*(c
 + d*x)])*Tan[c + d*x])/b)/(192*d*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/
2))
```

Maple [B] time = 0.508, size = 5495, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\cos\left(dx+c\right)^3+\left(Ca+Bb\right)\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2 + A*a + (B*a + A*
b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2
)/sqrt(sec(d*x + c)), x)
```

3.1517 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=705

$$\frac{2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\left(3a^2(9A+11C)+44abB+5Ab^2\right)\sqrt{a+b\cos(c+dx)}}{231d}+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(5a^2b(229\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*
B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a
 - b)*Sqrt[a + b]*(40*A*b^4 + 10*a*b^3*(3*A - 11*B) + 15*a^2*b^2*(19*A - 12
1*B + 33*C) + 3*a^4*(225*A - 539*B + 275*C) - 6*a^3*b*(505*A - 209*B + 660*
C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d*Sqrt[
Sec[c + d*x]]) - (2*(20*A*b^4 - 1793*a^3*b*B - 55*a*b^3*B - 75*a^4*(9*A + 1
1*C) - 5*a^2*b^2*(205*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3465*a^2*d) + (2*(15*A*b^3 + 539*a^3*B + 825*a*b^2*B + 5*
a^2*b*(229*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3465*a*d) + (2*(5*A*b^2 + 44*a*b*B + 3*a^2*(9*A + 11*C))*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (2*(5*A*b + 11*a*B
)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A
*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 3.37183, antiderivative size = 705, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\left(3a^2(9A+11C)+44abB+5Ab^2\right)\sqrt{a+b\cos(c+dx)}}{231d}+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(5a^2b(229\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(13/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*
B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a
 - b)*Sqrt[a + b]*(40*A*b^4 + 10*a*b^3*(3*A - 11*B) + 15*a^2*b^2*(19*A - 12
1*B + 33*C) + 3*a^4*(225*A - 539*B + 275*C) - 6*a^3*b*(505*A - 209*B + 660*
C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d*Sqrt[
Sec[c + d*x]]) - (2*(20*A*b^4 - 1793*a^3*b*B - 55*a*b^3*B - 75*a^4*(9*A + 1
1*C) - 5*a^2*b^2*(205*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3465*a^2*d) + (2*(15*A*b^3 + 539*a^3*B + 825*a*b^2*B + 5*
a^2*b*(229*A + 297*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3465*a*d) + (2*(5*A*b^2 + 44*a*b*B + 3*a^2*(9*A + 11*C))*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(231*d) + (2*(5*A*b + 11*a*B
)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*A
*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,c}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,s}{11d}$$

$$= \frac{2(5Ab+11aB)(a+b\cos(c+dx))^{3/2}\sec}{99d}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a}}{23}$$

$$= \frac{2\left(15Ab^3+539a^3B+825ab^2B+5a^2b(2\right.}{}$$

$$= -\frac{2\left(20Ab^4-1793a^3bB-55ab^3B-75a\right.}{}$$

$$= -\frac{2\left(20Ab^4-1793a^3bB-55ab^3B-75a\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(40Ab^5+1617a^5B+30\right.}{}$$

Mathematica [A] time = 21.6258, size = 959, normalized size = 1.36

$$\frac{2\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-(a+b)\left(1617Ba^5+15b(247A+319C)a^4+3069b^2Ba^3+15b^3(17A+33C)a^2-110b^4Ba+40Ab^5\right)E\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(13/2),x]
```

```
[Out] (2*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*((40*A*b^5 + 1617*a^5*B + 3069*a^3*b
^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*T
```

```
an[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)*(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2) - (a + b)*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 11
0*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*
(19*A + 121*B + 33*C) + 3*a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 20
9*B + 660*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3465*a^3*d*(1 + Tan[(c + d*x)/2]
^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + T
an[(c + d*x)/2]^2)]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(37
05*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a
*b^4*B + 4785*a^4*b*C + 495*a^2*b^3*C)*Sin[c + d*x])/(3465*a^3) + (2*Sec[c
+ d*x]^4*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x]))/99 + (2*Sec[c + d
*x]^3*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x] + 209*a*b*B*Sin[c + d
*x] + 99*a^2*C*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(1145*a^2*A*b*Sin[c +
 d*x] + 15*A*b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c
+ d*x] + 1485*a^2*b*C*Sin[c + d*x]))/(3465*a) + (2*Sec[c + d*x]*(675*a^4*A*
Sin[c + d*x] + 1025*a^2*A*b^2*Sin[c + d*x] - 20*A*b^4*Sin[c + d*x] + 1793*a
^3*b*B*Sin[c + d*x] + 55*a*b^3*B*Sin[c + d*x] + 825*a^4*C*Sin[c + d*x] + 14
85*a^2*b^2*C*Sin[c + d*x]))/(3465*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*
x])/11))/d
```

Maple [B] time = 0.911, size = 7237, normalized size = 10.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2
),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1518 $\int (a+b\cos(c+dx))^{5/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right.$ dx) dx

Optimal. Leaf size=592

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(7a^2(7A+9C)+90abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2b(163A\right.}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 435*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A
+ 9*C) - 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A
*b^3 + 15*a*b^2*(11*A - 3*B + 21*C) - 6*a^2*b*(19*A - 60*B + 28*C) + 3*a^3*
(49*A - 25*B + 63*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^3 + 75*a^3*B + 135*a*b^2*B + a^
2*b*(163*A + 231*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(315*a*d) + (2*(15*A*b^2 + 90*a*b*B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(5*A*b + 9*a*B)*(a
 + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a
+ b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 2.28393, antiderivative size = 592, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(7a^2(7A+9C)+90abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2b(163A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(11/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(10*A*b^4 - 435*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A
+ 9*C) - 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
```

```
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A
*b^3 + 15*a*b^2*(11*A - 3*B + 21*C) - 6*a^2*b*(19*A - 60*B + 28*C) + 3*a^3*
(49*A - 25*B + 63*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^3 + 75*a^3*B + 135*a*b^2*B + a^
2*b*(163*A + 231*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(315*a*d) + (2*(15*A*b^2 + 90*a*b*B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Cos
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(5*A*b + 9*a*B)*(a
 + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*A*(a
+ b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cdots}{\cdots}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\cdots}{9d}$$

$$= \frac{2(5Ab+9aB)(a+b\cos(c+dx))^{3/2}\,\mathrm{se}\cdots}{63d}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{\cdots}}{3\cdots}$$

$$= \frac{2\left(5Ab^3+75a^3B+135ab^2B+a^2b(16\cdots\right.}{\cdots}$$

$$= \frac{2\left(5Ab^3+75a^3B+135ab^2B+a^2b(16\cdots\right.}{\cdots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(10Ab^4-435a^3bB-\cdots\right.}{\cdots}$$

Mathematica [B] time = 27.9508, size = 4718, normalized size = 7.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-147*a^4*A - 279*a^2*A*b
^2 + 10*A*b^4 - 435*a^3*b*B - 45*a*b^3*B - 189*a^4*C - 483*a^2*b^2*C)*Sin[c
 + d*x])/(315*a^2) + (2*Sec[c + d*x]^3*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin
[c + d*x]))/63 + (2*Sec[c + d*x]^2*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c
+ d*x] + 135*a*b*B*Sin[c + d*x] + 63*a^2*C*Sin[c + d*x]))/315 + (2*Sec[c +
d*x]*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*
x] + 135*a*b^2*B*Sin[c + d*x] + 231*a^2*b*C*Sin[c + d*x]))/(315*a) + (2*a^2
*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^3*A)/(15*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a*A*b^2)/(35*Sqrt[a + b*Cos[c + d*x]]*Sq
```

rt[Sec[c + d*x]]) + (2*A*b^4)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a^2*b*B)/(21*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^3*B)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*a^3*C)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (23*a*b^2*C)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (38*a^2*A*b*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*A*b^3*Sqrt[Sec[c + d*x]])/(315*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) + (5*a^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*B*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) + (8*a^2*b*C*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (8*b^3*C*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) - (29*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) - (3*a^2*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (23*b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*a^2*(a + b*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-10*A*b^3 + 6*a^2*b

*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + ((-2*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - ((a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 2*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + b*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + (-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - 2*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 + (3*a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x

```
]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + (a*(a + b)*(-10
*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b
^2*(11*A + 3*(B + 7*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((b*Sec[(c + d*x
)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)]) + (a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(
49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*Sec[(c + d*x)/2]^2*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)]*Sec[c + d*x])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((
-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-10*A*b^4 + 435*a^3*b*B +
 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*Sec[(c + d*x)/2]^4*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sq
rt[1 - Tan[(c + d*x)/2]^2] + a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B +
28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/
2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c +
 d*x]*Tan[c + d*x]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)
^(3/2))))
```

Maple [B] time = 0.679, size = 6184, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2
),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1519 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \ldots dx$

Optimal. Leaf size=640

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+56abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2b\ldots\right.}{\ldots}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(15*A*b^3 + 63*a^3*B + 161*a*b^2*B + 5*a^2*b*(29*A +
 49*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sq
rt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(15*b^3*(A - 7*B) - a^3*(25*A - 63*B + 3
5*C) + a^2*b*(145*A - 119*B + 245*C) - a*b^2*(135*A - 161*B + 315*C))*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a*d*Sqrt[Sec[c + d*x]
]) - (2*b^2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(
5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*
d) + (2*(5*A*b + 7*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c
 + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(7*d)
```

Rubi [A] time = 1.99399, antiderivative size = 640, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+56abB+15Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2b\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(15*A*b^3 + 63*a^3*B + 161*a*b^2*B + 5*a^2*b*(29*A +
 49*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
```

```
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sq
rt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(15*b^3*(A - 7*B) - a^3*(25*A - 63*B + 3
5*C) + a^2*b*(145*A - 119*B + 245*C) - a*b^2*(135*A - 161*B + 315*C))*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a*d*Sqrt[Sec[c + d*x]
]) - (2*b^2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(
5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*
d) + (2*(5*A*b + 7*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c
 + d*x])/(35*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(7*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,\mathrm{s}}{7d}$$

$$= \frac{2(5Ab+7aB)(a+b\cos(c+dx))^{3/2}\,\mathrm{sec}}{35d}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{}}{1}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{}}{1}$$

$$= -\frac{2b^2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(15Ab^3+63a^3B+161a\right.}{}$$

Mathematica [B] time = 28.3667, size = 7891, normalized size = 12.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.444, size = 5151, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^(9/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1520 $\int (a+b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)$ dx) dx

Optimal. Leaf size=703

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(3A+5C)+10abB+5Ab^2\right)\sqrt{a+b\cos(c+dx)}}{5d} - \frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(6a^2(3A+5C)\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(70*a*b*B + b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x
]]) + (Sqrt[a + b]*(30*A*b^3 - 2*a^3*(9*A - 5*B + 15*C) + 2*a^2*b*(17*A - 3
5*B + 45*C) - a*b^2*(46*A - 15*(6*B + C)))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[a + b]*(2*b*B +
 5*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) - ((70*a*b*B + b^2*(
46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(15*d) + (2*(A*b + a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 2.55626, antiderivative size = 703, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(3A+5C)+10abB+5Ab^2\right)\sqrt{a+b\cos(c+dx)}}{5d} - \frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(6a^2(3A+5C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(7/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(70*a*b*B + b^2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x
]]) + (Sqrt[a + b]*(30*A*b^3 - 2*a^3*(9*A - 5*B + 15*C) + 2*a^2*b*(17*A - 3
5*B + 45*C) - a*b^2*(46*A - 15*(6*B + C)))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[a + b]*(2*b*B +
 5*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^2 + 10*a*b*B + a^2*(3*A + 5*C))*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) - ((70*a*b*B + b^2*(
46*A - 15*C) + 6*a^2*(3*A + 5*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(15*d) + (2*(A*b + a*B)*(a + b*Cos[c + d*x])^(3/2)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d
*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
```

```
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{5d}$$

$$= \frac{2(Ab+aB)(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(}{3d}$$

$$= \frac{2\left(5Ab^2+10abB+a^2(3A+5C)\right)\sqrt{a+}}{5}$$

$$= \frac{2\left(5Ab^2+10abB+a^2(3A+5C)\right)\sqrt{a+}}{5}$$

$$= \frac{2\left(5Ab^2+10abB+a^2(3A+5C)\right)\sqrt{a+}}{5}$$

$$= -\frac{b\sqrt{a+b}(2bB+5aC)\sqrt{\cos(c+dx)}\,\mathrm{cs}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(70abB+b^2(46A-15C\right.}{}$$

Mathematica [A] time = 20.4753, size = 1372, normalized size = 1.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (18*a^3*A*Tan[(c + d*x)/2] + 18*a^2*A*b*Tan[(c + d*x)/2] + 46*a*A*b^2*Tan[(
c + d*x)/2] + 46*A*b^3*Tan[(c + d*x)/2] + 70*a^2*b*B*Tan[(c + d*x)/2] + 70*
a*b^2*B*Tan[(c + d*x)/2] + 30*a^3*C*Tan[(c + d*x)/2] + 30*a^2*b*C*Tan[(c +
d*x)/2] - 15*a*b^2*C*Tan[(c + d*x)/2] - 15*b^3*C*Tan[(c + d*x)/2] - 36*a^2*
A*b*Tan[(c + d*x)/2]^3 - 92*A*b^3*Tan[(c + d*x)/2]^3 - 140*a*b^2*B*Tan[(c +
 d*x)/2]^3 - 60*a^2*b*C*Tan[(c + d*x)/2]^3 + 30*b^3*C*Tan[(c + d*x)/2]^3 -
18*a^3*A*Tan[(c + d*x)/2]^5 + 18*a^2*A*b*Tan[(c + d*x)/2]^5 - 46*a*A*b^2*Ta
n[(c + d*x)/2]^5 + 46*A*b^3*Tan[(c + d*x)/2]^5 - 70*a^2*b*B*Tan[(c + d*x)/2
]^5 + 70*a*b^2*B*Tan[(c + d*x)/2]^5 - 30*a^3*C*Tan[(c + d*x)/2]^5 + 30*a^2*
b*C*Tan[(c + d*x)/2]^5 + 15*a*b^2*C*Tan[(c + d*x)/2]^5 - 15*b^3*C*Tan[(c +
d*x)/2]^5 + 60*b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] + 150*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 60*b^3*B*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + 150*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(70*a*b*B + b^
2*(46*A - 15*C) + 6*a^2*(3*A + 5*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(15*b^3*
(A - B) + a*b^2*(23*A + 45*(B - C)) + a^2*b*(17*A + 35*B + 45*C) + a^3*(9*A
 + 5*(B + 3*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(15*d*Sqrt[(1 - Tan[(c + d*x)/2
]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2
)]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 23*A*b^2
+ 35*a*b*B + 15*a^2*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]*(11*a*A*b*Sin[c +
 d*x] + 5*a^2*B*Sin[c + d*x]))/15 + (2*a^2*A*Sec[c + d*x]*Tan[c + d*x])/5))
/d
```

Maple [B] time = 0.426, size = 4994, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)
,x)
```

```
[Out] 1/15/d*(6*A*a^3+68*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^4*a*b^2+30*C*cos(d*
x+c)^3*a^2*b-30*C*cos(d*x+c)^4*a^2*b-18*A*cos(d*x+c)^4*a^2*b-22*A*cos(d*x+c
)^4*a*b^2-46*A*cos(d*x+c)^3*a*b^2+28*A*cos(d*x+c)*a^2*b+80*B*cos(d*x+c)^2*a
^2*b+70*B*cos(d*x+c)^3*a*b^2-10*A*cos(d*x+c)^3*a^2*b-70*B*cos(d*x+c)^3*a^2*
b+15*C*cos(d*x+c)^3*a*b^2-10*B*cos(d*x+c)^4*a^2*b-70*B*cos(d*x+c)^4*a*b^2+1
0*B*cos(d*x+c)*a^3-10*B*cos(d*x+c)^3*a^3-60*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-15*C*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*b^3-30*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*cos(d*x+c)^3*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-60*B*cos(
d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*b^3-15*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-15*C*cos
(d*x+c)^5*b^3+70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+18*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-150*C*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b^2+90*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-150*C*cos(d*x+c)^2*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*
b^2-70*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^3*a^2*b-90*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+70*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3
```

*a^2*b+70*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+70*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-70*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*a^2*b-90*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+90*C*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*C*cos(d*x+
c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-46*A*cos(d*x+c)^4*b^3-18*A*cos(d*x+c)^3*a^3+12*A*cos(d*x+c)^2*
a^3-30*C*cos(d*x+c)^3*a^3-30*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+15*C*cos(d*x+c)^4*b^3+30*
C*cos(d*x+c)^2*a^3+46*A*cos(d*x+c)^3*b^3-18*A*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3+46*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+30*C*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-18*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+18*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+46*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3-30*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3+30*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+46*A*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2-90*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a

```
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-34*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b-46*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+46*A*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-90*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-34*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b-46*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-10*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*a^3-15*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)
*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
```

```
)*sec(d*x + c)^(7/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1521 $\int (a+b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=682

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2B+ab(56A-27C)-12b^2B\right)\sqrt{a+b\cos(c+dx)}}{12d}-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(-8\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12*a*d*Sqrt[Sec[c + d*x]]) - (Sq
rt[a + b]*(a*b*(56*A - 72*B - 27*C) - 6*b^2*(12*A + 2*B + C) - 8*a^2*(A - 3
*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12*d*Sqrt
[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 20*a*b*B + 15*a^2*C + 4*b^2*C)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[S
ec[c + d*x]]) - (b*(8*A*b + 4*a*B - b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*
Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (2*(5*A*
b + 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 2.42832, antiderivative size = 682, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2B+ab(56A-27C)-12b^2B\right)\sqrt{a+b\cos(c+dx)}}{12d}-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(-8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(5/2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
```

```
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12*a*d*Sqrt[Sec[c + d*x]]) - (Sq
rt[a + b]*(a*b*(56*A - 72*B - 27*C) - 6*b^2*(12*A + 2*B + C) - 8*a^2*(A - 3
*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(12*d*Sqrt
[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 + 20*a*b*B + 15*a^2*C + 4*b^2*C)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[S
ec[c + d*x]]) - (b*(8*A*b + 4*a*B - b*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*
Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (2*(5*A*
b + 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{3d}$$

$$= \frac{2(5Ab+3aB)(a+b\cos(c+dx))^{3/2}\sqrt{\mathrm{se}}}{3d}$$

$$= -\frac{b(8Ab+4aB-bC)\sqrt{a+b\cos(c+dx}}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(8Ab+4aB-bC)\sqrt{a+b\cos(c+dx}}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(8Ab+4aB-bC)\sqrt{a+b\cos(c+dx}}{2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2+20abB+15a^2C+4b\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(24a^2B-12b^2B+ab(56\right.}{}$$

Mathematica [B] time = 20.2095, size = 1640, normalized size = 2.4

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (56*a^2*A*b*Tan[(c + d*x)/2] + 56*a*A*b^2*Tan[(c + d*x)/2] + 24*a^3*B*Tan[(
c + d*x)/2] + 24*a^2*b*B*Tan[(c + d*x)/2] - 12*a*b^2*B*Tan[(c + d*x)/2] - 1
2*b^3*B*Tan[(c + d*x)/2] - 27*a^2*b*C*Tan[(c + d*x)/2] - 27*a*b^2*C*Tan[(c
+ d*x)/2] - 112*a*A*b^2*Tan[(c + d*x)/2]^3 - 48*a^2*b*B*Tan[(c + d*x)/2]^3
+ 24*b^3*B*Tan[(c + d*x)/2]^3 + 54*a*b^2*C*Tan[(c + d*x)/2]^3 - 56*a^2*A*b*
Tan[(c + d*x)/2]^5 + 56*a*A*b^2*Tan[(c + d*x)/2]^5 - 24*a^3*B*Tan[(c + d*x)
```

```
/2]^5 + 24*a^2*b*B*Tan[(c + d*x)/2]^5 + 12*a*b^2*B*Tan[(c + d*x)/2]^5 - 12*
b^3*B*Tan[(c + d*x)/2]^5 + 27*a^2*b*C*Tan[(c + d*x)/2]^5 - 27*a*b^2*C*Tan[(
c + d*x)/2]^5 + 48*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 90*a^2*b*C*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
24*b^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*B*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 90*a^2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*b^3*C*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)] + (a + b)*(24*a^2*B - 12*b^2*B + a*b*(56*A - 27*C))*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] - 2*(4*a^2*b*(7*A + 9*B - 9*C) - 6*b^3*(2*A + C) + 3*
a*b^2*(12*A - 12*B + C) + 4*a^3*(A + 3*(B + C)))*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)
/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])
/(12*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]*((2*a*(7*A*b + 3*a*B)*Sin[c + d*x])/3 + (b^2*C*Sin[2*(c + d*x)])/4
+ (2*a^2*A*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.398, size = 4897, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)
,x)
```

[Out] 1/12/d*(8*A*a^3+56*A*cos(d*x+c)^2*a*b^2-33*C*cos(d*x+c)^4*a*b^2-27*C*cos(d*x+c)^3*a^2*b+27*C*cos(d*x+c)^2*a^2*b+6*C*cos(d*x+c)^2*a*b^2-56*A*cos(d*x+c)^3*a*b^2+64*A*cos(d*x+c)*a^2*b+24*B*cos(d*x+c)^2*a^2*b+12*B*cos(d*x+c)^2*a*b^2-12*B*cos(d*x+c)^3*a*b^2-8*A*cos(d*x+c)^3*a^2*b-24*B*cos(d*x+c)^3*a^2*b+27*C*cos(d*x+c)^3*a*b^2+12*B*cos(d*x+c)^3*b^3+24*B*cos(d*x+c)*a^3-12*B*cos(d*x+c)^4*b^3-6*C*cos(d*x+c)^5*b^3-24*B*cos(d*x+c)^2*a^3+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+72*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-27*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-27*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-72*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-6*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-56*A*cos(d*x+c)^2*a^2*b+24*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b-12*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2-56*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*cos(d*x+c)^2*a^3-72*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+24*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+24*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+6*C*cos(d*x+c)^3*b^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*si

n(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3-24*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-56*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-72*A*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2+56*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+56*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*C*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b-27*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-24*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+24*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2
*a^3-12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*b^3-24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3+56*A*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3-90*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-27*C*cos(d*x+c)^2*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^2+12*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(

```
-(a-b)/(a+b))^(1/2))*b^3-24*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+24*B*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-12
*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*b^3-120*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-90*C*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
2*b-48*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*b^3+12*C*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-24*C*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*b^3-48*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-120*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+
c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1522 $\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal. Leaf size=707

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(-(48A-33C))+54abB+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}}{24d}-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*
x]]) - (Sqrt[a + b]*(a^2*(48*A - 48*B - 33*C) - 4*b^2*(6*A + 3*B + 4*C) - 2
*a*b*(72*A + 27*B + 13*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(24*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(30*a^2*b*B + 8*b^3*B + 5*a^
3*C + 20*a*b^2*(2*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) - (b*(8*a*A - 2*b*B - 3*a*C)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) - (b*(6*A - C
)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((54*
a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 2.5902, antiderivative size = 707, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(-(48A-33C))+54abB+8b^2(3A+2C)\right)\sqrt{a+b\cos(c+dx)}}{24d}-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*
x]]) - (Sqrt[a + b]*(a^2*(48*A - 48*B - 33*C) - 4*b^2*(6*A + 3*B + 4*C) - 2
*a*b*(72*A + 27*B + 13*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(24*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(30*a^2*b*B + 8*b^3*B + 5*a^
3*C + 20*a*b^2*(2*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) - (b*(8*a*A - 2*b*B - 3*a*C)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) - (b*(6*A - C
)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((54*
a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (2*A*(a + b*Cos[c + d*x])^(5/2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cdots}{}$$

$$= \frac{2A(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,\mathrm{s}\cdots}{d}$$

$$= -\frac{b(6A-C)(a+b\cos(c+dx))^{3/2}\sin(c\cdots}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(8aA-2bB-3aC)\sqrt{a+b\cos(c+d\cdots}}{4d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(8aA-2bB-3aC)\sqrt{a+b\cos(c+d\cdots}}{4d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b(8aA-2bB-3aC)\sqrt{a+b\cos(c+d\cdots}}{4d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(30a^2bB+8b^3B+5a^3C+20a\cdots\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(54abB-a^2(48A-33C\cdots\right.}{}$$

Mathematica [B] time = 20.6276, size = 1940, normalized size = 2.74

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((24*a^2*A + b^2*C)*Sin[c + d
*x])/12 + (b*(6*b*B + 13*a*C)*Sin[2*(c + d*x)])/24 + (b^2*C*Sin[3*(c + d*x)
])/12))/d + (48*a^3*A*Tan[(c + d*x)/2] + 48*a^2*A*b*Tan[(c + d*x)/2] - 24*a
*A*b^2*Tan[(c + d*x)/2] - 24*A*b^3*Tan[(c + d*x)/2] - 54*a^2*b*B*Tan[(c + d
*x)/2] - 54*a*b^2*B*Tan[(c + d*x)/2] - 33*a^3*C*Tan[(c + d*x)/2] - 33*a^2*b
*C*Tan[(c + d*x)/2] - 16*a*b^2*C*Tan[(c + d*x)/2] - 16*b^3*C*Tan[(c + d*x)/
```

2] - 96*a^2*A*b*Tan[(c + d*x)/2]^3 + 48*A*b^3*Tan[(c + d*x)/2]^3 + 108*a*b^
2*B*Tan[(c + d*x)/2]^3 + 66*a^2*b*C*Tan[(c + d*x)/2]^3 + 32*b^3*C*Tan[(c +
d*x)/2]^3 - 48*a^3*A*Tan[(c + d*x)/2]^5 + 48*a^2*A*b*Tan[(c + d*x)/2]^5 + 2
4*a*A*b^2*Tan[(c + d*x)/2]^5 - 24*A*b^3*Tan[(c + d*x)/2]^5 + 54*a^2*b*B*Tan
[(c + d*x)/2]^5 - 54*a*b^2*B*Tan[(c + d*x)/2]^5 + 33*a^3*C*Tan[(c + d*x)/2]
^5 - 33*a^2*b*C*Tan[(c + d*x)/2]^5 + 16*a*b^2*C*Tan[(c + d*x)/2]^5 - 16*b^3
*C*Tan[(c + d*x)/2]^5 + 240*a*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a^2*b*B*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*b^3*B*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 30*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 240*a*A*b^2*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + 180*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*b^3*B*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] + 30*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*C*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)] + (a + b)*(-54*a*b*B + a^2*(48*A - 33*C) - 8*b^2*(3*A + 2*C)
)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d
x)/2]^2](1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] - 2*(-12*b^3*B + 24*a^3*(A + B - C) + a^2*b*(72
*A - 72*B + 13*C) - 2*a*b^2*(36*A - 3*B + 19*C))*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)
/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])
/(24*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])

Maple [B] time = 0.434, size = 5138, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4 + \left(2\,Cab + Bb^2\right)\cos\left(dx+c\right)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos\left(dx+c\right)^2 + \left(Ba^2 + 2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(3/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(3/2),x, algorithm="giac")

[Out] Timed out
```

3.1523 $\int (a+b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$

Optimal. Leaf size=760

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(264a^2bB+15a^3C+4ab^2(108A+71C)+128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\sin(c+dx)\left(5a^2C+\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A
+ 71*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d*S
qrt[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3*C + 8*b^3*(12*A + 16*B + 9*C) + 2
*a^2*b*(192*A + 132*B + 59*C) + 4*a*b^2*(108*A + 52*B + 71*C))*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) - (S
qrt[a + b]*(40*a^3*b*B + 160*a*b^3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16
*b^4*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + ((16*A*b^2 + 24*a*b*B + 5*a^2*C +
12*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]])
+ ((8*b*B + 5*a*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[
c + d*x]]) + (C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Sqrt[Sec[c +
d*x]]) + ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A + 71*C))*Sqr
t[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rubi [A] time = 2.80298, antiderivative size = 760, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(264a^2bB+15a^3C+4ab^2(108A+71C)+128b^3B\right)\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\sin(c+dx)\left(5a^2C+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt
[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A
+ 71*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
```

```
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d*S
qrt[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3*C + 8*b^3*(12*A + 16*B + 9*C) + 2
*a^2*b*(192*A + 132*B + 59*C) + 4*a*b^2*(108*A + 52*B + 71*C))*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) - (S
qrt[a + b]*(40*a^3*b*B + 160*a*b^3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16
*b^4*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + ((16*A*b^2 + 24*a*b*B + 5*a^2*C +
12*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c + d*x]])
+ ((8*b*B + 5*a*C)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Sqrt[Sec[
c + d*x]]) + (C*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Sqrt[Sec[c +
d*x]]) + ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A + 71*C))*Sqr
t[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
```

```
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,c}{}$$

$$=\frac{C(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{1}{4}$$

$$=\frac{(8bB+5aC)(a+b\cos(c+dx))^{3/2}\sin(c}{24d\sqrt{\sec(c+dx)}}$$

$$=\frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+}}{32d\sqrt{\sec(c+dx}}$$

$$=\frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+}}{32d\sqrt{\sec(c+dx}}$$

$$=\frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+}}{32d\sqrt{\sec(c+dx}}$$

$$=-\frac{\sqrt{a+b}\left(40a^3bB+160ab^3B-5a^4C+1\right.}{}$$

$$=-\frac{(a-b)\sqrt{a+b}\left(264a^2bB+128b^3B+15\right.}{}$$

Mathematica [B] time = 25.6615, size = 5555, normalized size = 7.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.568, size = 5875, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4 + \left(2\,Cab + Bb^2\right)\cos\left(dx+c\right)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\cos\left(dx+c\right)^2 + \left(Ba^2 + 2\,Aab\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
```

```
*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1524 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=894

$$\frac{C\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}+\frac{(10bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}+\frac{\left(-15Ca^2+50bBa+80Ab^2+64\right.}{240bd}$$

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A
 + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(1920*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*C
 - 30*a^3*b*(5*B + C) - 16*b^4*(80*A + 45*B + 64*C) - 8*a*b^3*(260*A + 355*
B + 193*C) - 4*a^2*b^2*(660*A + 295*B + 423*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(
10*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5*C - 40*a^3*b^2*(2*A + C) - 80
*a*b^4*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(128*b^3*d*Sqrt[Sec[c + d*x]]) + ((50*a^2*b*B + 120*b^3*B - 15*a
^3*C + 4*a*b^2*(60*A + 43*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b
*d*Sqrt[Sec[c + d*x]]) + ((80*A*b^2 + 50*a*b*B - 15*a^2*C + 64*b^2*C)*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x]]) + ((10*b*B
 - 3*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c + d*x
]]) + (C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*x]]
) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a^2*
b^2*(220*A + 141*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(1920*b^2*d)
```

Rubi [A] time = 4.05813, antiderivative size = 894, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{C\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}+\frac{(10bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}+\frac{\left(-15Ca^2+50bBa+80Ab^2+64\right.}{240bd}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A
 + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(1920*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*C
 - 30*a^3*b*(5*B + C) - 16*b^4*(80*A + 45*B + 64*C) - 8*a*b^3*(260*A + 355*
B + 193*C) - 4*a^2*b^2*(660*A + 295*B + 423*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(
10*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5*C - 40*a^3*b^2*(2*A + C) - 80
*a*b^4*(4*A + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(128*b^3*d*Sqrt[Sec[c + d*x]]) + ((50*a^2*b*B + 120*b^3*B - 15*a
^3*C + 4*a*b^2*(60*A + 43*C))*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b
*d*Sqrt[Sec[c + d*x]]) + ((80*A*b^2 + 50*a*b*B - 15*a^2*C + 64*b^2*C)*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x]]) + ((10*b*B
 - 3*a*C)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c + d*x
]]) + (C*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*x]]
) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a^2*
b^2*(220*A + 141*C))*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(1920*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
```

```
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b$$

$$= \frac{C(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$= \frac{(10bB-3aC)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd\sqrt{\sec(c+dx)}} + \frac{C}{}$$

$$= \frac{\left(80Ab^2+50abB-15a^2C+64b^2C\right)(a+b\cos(c+}{240bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2bB+120b^3B-15a^3C+4ab^2(60A+43C)\right)\sqrt{}}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2bB+120b^3B-15a^3C+4ab^2(60A+43C)\right)\sqrt{}}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2bB+120b^3B-15a^3C+4ab^2(60A+43C)\right)\sqrt{}}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(10a^4bB-240a^2b^3B-96b^5B-3a^5C-40a\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(150a^3bB+2840ab^3B-45a^4C+2\right.}{}$$

Mathematica [C] time = 20.6159, size = 667, normalized size = 0.75

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2\sin(c+dx)\left(590a^2bB+15a^3C+16ab^2(65A+64C)+480b^3B\right)}{b}+2\tan(c+dx)\left(93a^2C+170abB+80Ab^2+88b^2C\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
```

```
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*(((-I)*((a - b)*(-150*a^3*b*B - 2840*a*b^3*B + 45
*a^4*C - 256*b^4*(5*A + 4*C) - 12*a^2*b^2*(220*A + 141*C))*EllipticE[I*ArcS
inh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - 2*(a - b
)*(-720*b^4*B - 30*a^3*b*(5*B - C) + 45*a^4*C - 4*a^2*b^2*(180*A + 185*B +
129*C) - 8*a*b^3*(220*A + 45*B + 161*C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + 30*(-10*a^4*b*B + 240*a^2*
b^3*B + 96*b^5*B + 3*a^5*C + 40*a^3*b^2*(2*A + C) + 80*a*b^4*(4*A + 3*C))*E
llipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]
], -((a + b)/(a - b))])*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b)])/(b^2*Sqrt[(a - b)/(a + b)]*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec
[(c + d*x)/2]^2]) + (2*(590*a^2*b*B + 480*b^3*B + 15*a^3*C + 16*a*b^2*(65*A
 + 64*C))*Sin[c + d*x])/b + 2*(80*A*b^2 + 170*a*b*B + 93*a^2*C + 100*b^2*C)
*Sec[c + d*x]*Sin[3*(c + d*x)] + 6*b*(10*b*B + 21*a*C)*Sec[c + d*x]*Sin[4*(
c + d*x)] + 24*b^2*C*Sec[c + d*x]*Sin[5*(c + d*x)] + ((150*a^3*b*B + 2840*a
*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Tan[(
c + d*x)/2])/b^2 + 2*(80*A*b^2 + 170*a*b*B + 93*a^2*C + 88*b^2*C)*Tan[c + d
*x]))/(1920*d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.753, size = 7064, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right) + a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^2\cos\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right.\right.}{\sqrt{\sec\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2
)/sqrt(sec(d*x + c)), x)
```

3.1525 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=506

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)-28abB+24Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^3d}+\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(2\right.}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 7
0*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^5*d*Sq
rt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(48*A*b^3 - 4*a*b^2*(3*A + 14*B) + a^3*(
25*A - 63*B + 35*C) + 2*a^2*b*(22*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(24*A*b^
2 - 28*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(105*a^3*d) - (2*(6*A*b - 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 1.6078, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)-28abB+24Ab^2\right)\sqrt{a+b\cos(c+dx)}}{105a^3d}+\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt[a + b
*Cos[c + d*x]],x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 7
0*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^5*d*Sq
rt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(48*A*b^3 - 4*a*b^2*(3*A + 14*B) + a^3*(
```

```
25*A - 63*B + 35*C) + 2*a^2*b*(22*A + 7*(B + 5*C)))*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(24*A*b^
2 - 28*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(105*a^3*d) - (2*(6*A*b - 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7ad} + \frac{\left(2\sqrt{\cos(c}\right.}{}$$

$$= -\frac{2(6Ab-7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35a^2d}$$

$$= \frac{2\left(24Ab^2-28abB+5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec}{105a^3d}$$

$$= \frac{2\left(24Ab^2-28abB+5a^2(5A+7C)\right)\sqrt{a+b\cos(c+dx)}\sec}{105a^3d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(48Ab^3-63a^3B-56ab^2B+a^2(44Ab+70\right.}{}$$

Mathematica [B] time = 25.9143, size = 3704, normalized size = 7.32

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(9/2))/Sqrt
[a + b*Cos[c + d*x]],x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-44*a^2*A*b - 48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 70*a^2*b*C)*Sin[c + d*x])/(105*a^4) + (2*Sec[c + d*x]^2*(-6*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/(35*a^2) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d*x] + 24*A*b^2*Sin[c + d*x] - 28*a*b*B*Sin[c + d*x] + 35*a^2*C*Sin[c + d*x]))/(105*a^3) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(7*a)))/d + (2*((44*A*b)/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^3)/(35*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*b^2*B)/(15*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*C)/(3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*A*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) + (32*A*b^2*Sqrt[Sec[c + d*x]])/(105*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^4*Sqrt[Sec[c + d*x]])/(35*a^4*Sqrt[a + b*Cos[c + d*x]]) - (7*b*B*Sqrt[Sec[c + d*x]])/(15*a*Sqrt[a + b*Cos[c + d*x]]) - (8*b^3*B*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (C*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) + (44*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^4*Sqrt[a + b*Cos[c + d*x]]) - (3*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (8*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^4*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^4*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 3

5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^4*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (48*A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*a^4*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-48*A*b^3 + 4*a*b^2*(-3*A + 14*B) - 2*a^2*b

```
*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (48*A*b^3 - 63*a^3*B - 56*
a*b^2*B + a^2*(44*A*b + 70*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)
/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^4*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
)))
```

Maple [B] time = 0.44, size = 4345, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/105/d/a^4*(-15*A*a^4+48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-12*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2-48*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b+70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-70*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)
^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b+44*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-44*A*cos(d
```

```
*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3*b-12*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-42*B*cos(d*x+c)^3*a^4-21*B*cos(
d*x+c)*a^4+63*B*cos(d*x+c)^4*a^4+35*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^4+25*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^4+35*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+48*A*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4+25*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^4+63*B*cos(d*x+c)^5*a^3*b-28*B*cos(d*x+c)^5*a^2*b^
2+56*B*cos(d*x+c)^5*a*b^3-70*B*cos(d*x+c)^4*a^3*b+56*B*cos(d*x+c)^4*a^2*b^2
-48*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^3+70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^3*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^2-70*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b+44*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+70*C*cos(d*x+c)^4
*a^2*b^2+35*C*cos(d*x+c)^3*a^3*b+35*C*cos(d*x+c)^5*a^3*b-70*C*cos(d*x+c)^5*
a^2*b^2-70*C*cos(d*x+c)^4*a^3*b-28*B*cos(d*x+c)^3*a^2*b^2+7*B*cos(d*x+c)^2*
a^3*b+24*A*cos(d*x+c)^3*a*b^3-6*A*cos(d*x+c)^2*a^2*b^2+3*A*cos(d*x+c)*a^3*b
+25*A*cos(d*x+c)^5*a^3*b-44*A*cos(d*x+c)^5*a^2*b^2+24*A*cos(d*x+c)^5*a*b^3-
44*A*cos(d*x+c)^4*a^3*b+50*A*cos(d*x+c)^4*a^2*b^2-48*A*cos(d*x+c)^4*a*b^3+1
6*A*cos(d*x+c)^3*a^3*b-56*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+25*A*cos(d*x+c)^4*a^4+35
*C*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4-35*C*cos(d*x+c)^2*a^4-48*A*cos(d*
x+c)^5*b^4+48*A*cos(d*x+c)^4*b^4+44*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-56*B*cos(d*x+c
```

```
)^4*a*b^3-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4-63*B*sin(d*x+c)*c
os(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a^4-56*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+14*B*sin(d*x+c)*co
s(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a^3*b+56*B*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-56*B*sin(d*x+
c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*a^2*b^2-56*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+14*B*sin(d*x+c)*cos(d*x+c)^3*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b+56*B*sin
(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b)*cos(d*x+c)*(1/cos(d*x+c))^(9
/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(b*
cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(9/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(b
*cos(d*x + c) + a), x)
```

3.1526 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=412

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(9A-5B+15C)-2ab(A+5B)+8Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) -
(2*Sqrt[a + b]*(8*A*b^2 - 2*a*b*(A + 5*B) + a^2*(9*A - 5*B + 15*C))*Sqrt[Co
s[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]
) - (2*(4*A*b - 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*a*d)
```

Rubi [A] time = 1.09383, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(9A-5B+15C)-2ab(A+5B)+8Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt[a + b
*Cos[c + d*x]],x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) -
(2*Sqrt[a + b]*(8*A*b^2 - 2*a*b*(A + 5*B) + a^2*(9*A - 5*B + 15*C))*Sqrt[Co
s[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
```

```
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]
) - (2*(4*A*b - 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c\right.}}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(8Ab^2-10abB+3a^2(3A+5C)\right)\sqrt{\cos(c+}}{1}$$

Mathematica [B] time = 24.8762, size = 3208, normalized size = 7.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/Sqrt
[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 8*A*b^2 - 10*a*
b*B + 15*a^2*C)*Sin[c + d*x])/(15*a^3) + (2*Sec[c + d*x]*(-4*A*b*Sin[c + d*
```

x] + 5*a*B*Sin[c + d*x]))/(15*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a))
)/d + (2*((-3*A)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^2
)/(15*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*B)/(3*a*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - C/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (7*A*b*Sqrt[Sec[c + d*x]])/(15*a*Sqrt[a + b*Cos[c + d*x]])
 - (8*A*b^3*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt
[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[Sec[c + d*x]])
/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) - (b*C*Sqrt[Sec[c + d*x]])/(a*Sqrt[a + b*
Cos[c + d*x]]) - (3*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a +
b*Cos[c + d*x]]) - (8*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sq
rt[a + b*Cos[c + d*x]]) + (2*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*
a^2*Sqrt[a + b*Cos[c + d*x]]) - (b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(
a*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a +
 b)*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B)
 + a^2*(9*A + 5*(B + 3*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] - (8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^3*d
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A
 + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*(B + 3*C)))*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^2
 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(
c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(
-2*(a + b)*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A
 - 5*B) + a^2*(9*A + 5*(B + 3*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
5*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(8*A*b^2 - 10*
a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x
]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(
9*A + 5*(B + 3*C)))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]

```
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c
 + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A +
5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) +
((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(8*A*b^2 + 2*a*b*(A
- 5*B) + a^2*(9*A + 5*(B + 3*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1
 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)] + b*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*Sec[(c + d*x)/
2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C
))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] -
(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2
*(9*A + 5*(B + 3*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)
*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x
)/2]^2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2
*(a + b)*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A -
 5*B) + a^2*(9*A + 5*(B + 3*C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^2 - 10*a*b*B + 3*a^2*(3*A + 5*C))*C
os[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Co
s[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c +
d*x]*Tan[c + d*x]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^
2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.304, size = 3142, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

[Out] -2/15/d/a^3*(-3*A*a^3-4*A*cos(d*x+c)^2*a*b^2-15*C*cos(d*x+c)^3*a^2*b+15*C*c
os(d*x+c)^4*a^2*b+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2+8*A*cos(d*x
+c)^3*a*b^2+A*cos(d*x+c)*a^2*b+5*B*cos(d*x+c)^2*a^2*b+10*B*cos(d*x+c)^3*a*b
^2-10*A*cos(d*x+c)^3*a^2*b-10*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b-1
0*B*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)*a^3+5*B*cos(d*x+c)^3*a^3+10*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^3*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
(1/(a+b)(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+10*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)^3*a^2*b+10*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b+10*B*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2-10*B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^2*a^2*b+8*A*cos(d*x+c)^4*b^3+9*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)
^2*a^3+15*C*cos(d*x+c)^3*a^3-15*C*cos(d*x+c)^2*a^3-8*A*cos(d*x+c)^3*b^3+9*A
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-15*C*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*

```
x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^3*a^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3)*cos(d*x+c)*(1/cos(d*
x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*
cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b
*cos(d*x + c) + a), x)
```

3.1527 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=333

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)(a(A-3B+3C)+2Ab)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a
*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.712295, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)(a(A-3B+3C)+2Ab)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + b
*Cos[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a
*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + B\cos(c+dx) + C\cos^2}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c}\right.}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\left((2Ab-3\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{3a^3d\sqrt{\sec}}$$

Mathematica [B] time = 21.9363, size = 2616, normalized size = 7.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt
[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-2*A*b + 3*a*B)*Sin[c + d
*x])/(3*a^2) + (2*A*Tan[c + d*x])/(3*a)))/d + (2*((2*A*b)/(3*a*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - B/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) + (A*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^2*Sq
rt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) - (b*B*Sqrt[Sec[c + d*x]
])/(a*Sqrt[a + b*Cos[c + d*x]]) + (C*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c +
 d*x]] + (2*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Co
s[c + d*x]]) - (b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*Sqrt[a + b*Cos[
c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-2*A*b + 3*a
*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
```

] + 2*a*(-2*A*b + a*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b + a*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b + a*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(-2*A*b + 3*a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(-2*A*b + a*(A + 3*(B + C)))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(-2*A*b + a*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(2*A*b - 3*a*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (2*A*b - 3*a*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(-2*A*b + a*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]

```
^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*a^2*Sqrt[a + b*Cos[c + d*x]
]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b + a
*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x
)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*a^2*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])
))
```

Maple [B] time = 0.22, size = 1740, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d/a^2*(A*cos(d*x+c)*a*b-2*A*cos(d*x+c)^2*a*b+3*B*cos(d*x+c)^3*a*b+A*co
s(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b-A*a^2-2*A*cos(d*x+c)^3*b^2-3*B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*cos(d*x+c)^2*a^2+2*A*cos(d*x+c)^2*b^2+3*C
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2-3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-3*B*cos(d*x+c)*
a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^2*a^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-3*B*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+3*B*
```

```
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+3*B*cos(d*x+c)
^2*a^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^2*b^2+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-3
*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(
a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*
cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b
*cos(d*x + c) + a), x)
```

3.1528 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=407

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A-B)}{}$$

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A - B)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b
]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*
Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.680827, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3053, 2809, 2998, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}(A-B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt[a + b
*Cos[c + d*x]],x]
```


```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A - B)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b
```

```
]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*
Sqrt[Sec[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c-}$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{bd\sqrt{\sec(c+dx)}}$$

$$= \frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.7911, size = 625, normalized size = 1.54

$$\frac{2\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(a(A+B-C)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/Sqrt
[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (2*S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-(a*A*Tan[(c + d*x)/2]) - A*b*Tan[(c +
d*x)/2] + 2*A*b*Tan[(c + d*x)/2]^3 + a*A*Tan[(c + d*x)/2]^5 - A*b*Tan[(c +
d*x)/2]^5 - 2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)] - 2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - A*(a + b)*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + a*(A + B - C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*d*(1 +
 Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.242, size = 1182, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -2/d/a*(A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a
-C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
```

```
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)*cos(d*x+c)*(1/c
os(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*
cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b
*cos(d*x + c) + a), x)
```

3.1529 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=461

$$\frac{\sqrt{a+b}(aC+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}(2bB}{}$$

```
[Out] -(((a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A*b + a*C)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[
a + b]*(2*b*B - a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.928468, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(aC+2Ab)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{abd\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}(2bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt[a + b
*Cos[c + d*x]],x]
```


```
[Out] -(((a - b)*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A*b + a*C)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[
```

```
a + b]*(2*b*B - a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= -\frac{\sqrt{a+b}(2bB-aC)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\right.}{b^2d\sqrt{\sec(c}}$$

$$= -\frac{(a-b)\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{abd\sqrt{\sec(c+a}}$$

Mathematica [A] time = 17.6595, size = 777, normalized size = 1.69

$$\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2b(A-B)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\left(\tan\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/Sqrt
[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*C*Tan[(c + d*x)/2] + b*C*Tan[(c + d
*x)/2] - 2*b*C*Tan[(c + d*x)/2]^3 - a*C*Tan[(c + d*x)/2]^5 + b*C*Tan[(c + d
*x)/2]^5 - 4*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + 2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*b*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
2*a*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
```

```
b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*C*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2
)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b
*(A - B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2]^2)])
```

Maple [B] time = 0.272, size = 1188, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+C*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a+C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+C*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
```

```
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+C*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+C*c
os(d*x+c)^3*b+C*cos(d*x+c)^2*a-C*cos(d*x+c)^2*b-C*cos(d*x+c)*a)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b
*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*
cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sqrt(sec(c + d*x))/sqrt(a
 + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b
*cos(d*x + c) + a), x)
```

3.1530 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(3*a*C - 2*b
*(2*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*
Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*
Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt
[Sec[c + d*x]]) + ((4*b*B - 3*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(4*b^2*d)
```

Rubi [A] time = 1.29753, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(4*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(3*a*C - 2*b
*(2*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
```

```
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*A*b^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*
Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*
Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt
[Sec[c + d*x]]) + ((4*b*B - 3*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(4*b^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aC}{2}+b(2A+C}{\sqrt{}}}{2b}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4bB-3aC)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{4b^2d}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4bB-3aC)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{4b^2d}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2-4abB+3a^2C+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}(4bB-3aC)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{4ab^2d\sqrt{\sec(c+dx)}}$$

<u>Mathematica</u> [B] time = 21.234, size = 1376, normalized size = 2.52

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (C*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-4*a*b*B*Tan[(c + d*x)/2] -
 4*b^2*B*Tan[(c + d*x)/2] + 3*a^2*C*Tan[(c + d*x)/2] + 3*a*b*C*Tan[(c + d*x
)/2] + 8*b^2*B*Tan[(c + d*x)/2]^3 - 6*a*b*C*Tan[(c + d*x)/2]^3 + 4*a*b*B*Ta
n[(c + d*x)/2]^5 - 4*b^2*B*Tan[(c + d*x)/2]^5 - 3*a^2*C*Tan[(c + d*x)/2]^5
+ 3*a*b*C*Tan[(c + d*x)/2]^5 + 16*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*b*B*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*C*E
llipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
```

```
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 8*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 16*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*b*B*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] + 6*a^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*C*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*(-4*b*B + 3*a*C)*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*
b*(4*A*b - a*C + 2*b*C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b^2*d*Sqrt[1 + Tan[(
c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.242, size = 2251, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d/b^2*(-2*C*cos(d*x+c)*a*b+4*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b-C*c
os(d*x+c)^3*a*b+3*C*cos(d*x+c)^2*a*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-2*b^2*C*cos(d*x+c)^2+4*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b+16*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
```

cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+6*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^
2-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^2-4*C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*b^2+6*C*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2+8*C*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
x+c),-1,(-(a-b)/(a+b))^(1/2))(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*b^2-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-8*A*cos(d*x+c)*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2
-3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)^3*b^2-4*B*cos(d*x+c)^2*b^2-3*C*cos(d*x+
c)^2*a^2+2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a*b-3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b
+2*C*cos(d*x+c)^4*b^2+3*C*cos(d*x+c)*a^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-8*B
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*a*b)*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(a + b*cos(c + d*x))
*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*sqrt(sec(d*x + c))), x)
```

3.1531 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=653

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(15a^2C-18abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24b^3d}+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(15a\right.}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^3*d*Sqrt[Sec[c + d*x]]
) + (Sqrt[a + b]*(24*A*b^2 - 18*a*b*B + 12*b^2*B + 15*a^2*C - 10*a*b*C + 16
*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^3*d*Sqr
t[Sec[c + d*x]]) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - 5*a^3*C - 4*a*b^2*(2
*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(8*b^4*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
*b*d*Sec[c + d*x]^(3/2)) + ((6*b*B - 5*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(12*b^2*d*Sqrt[Sec[c + d*x]]) + ((24*A*b^2 - 18*a*b*B + 15*a^2*C +
16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b^3
*d)
```

Rubi [A] time = 2.08621, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(15a^2C-18abB+24Ab^2+16b^2C\right)\sqrt{a+b\cos(c+dx)}}{24b^3d}+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\left(15a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + b*Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(24*A*b^2 - 18*a*b*B + 15*a^2*C + 16*b^2*C)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
```

```
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^3*d*Sqrt[Sec[c + d*x]]
) + (Sqrt[a + b]*(24*A*b^2 - 18*a*b*B + 12*b^2*B + 15*a^2*C - 10*a*b*C + 16
*b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^3*d*Sqr
t[Sec[c + d*x]]) - (Sqrt[a + b]*(6*a^2*b*B + 8*b^3*B - 5*a^3*C - 4*a*b^2*(2
*A + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(8*b^4*d*Sqrt[Sec[c + d*x]]) + (C*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
*b*d*Sec[c + d*x]^(3/2)) + ((6*b*B - 5*a*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(12*b^2*d*Sqrt[Sec[c + d*x]]) + ((24*A*b^2 - 18*a*b*B + 15*a^2*C +
16*b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b^3
*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A + B\cos(c+dx) + C\cos^2(c\right.}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{}}{}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd\sec^{\frac{3}{2}}(c+dx)} + \frac{(6bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{12b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd\sec^{\frac{3}{2}}(c+dx)} + \frac{(6bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{12b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{C\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd\sec^{\frac{3}{2}}(c+dx)} + \frac{(6bB-5aC)\sqrt{a+b\cos(c+dx)}\sin(c+d}{12b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\left(6a^2bB + 8b^3B - 5a^3C - 4ab^2(2A+C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Gamma}{8b^4d\sqrt{\sec(c+}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(24Ab^2 - 18abB + 15a^2C + 16b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx}{24ab^3d\sqrt{\sec(c+}}$$

Mathematica [B] time = 27.5488, size = 15695, normalized size = 24.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)),x]
```

[Out] Result too large to show

Maple [B] time = 0.329, size = 3583, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)
,x)
```

```
[Out] -1/24/d/b^3*(24*A*cos(d*x+c)^2*a*b^2-24*A*cos(d*x+c)*a*b^2-2*C*cos(d*x+c)^4
*a*b^2+5*C*cos(d*x+c)^3*a^2*b-15*C*cos(d*x+c)^2*a^2*b+18*C*cos(d*x+c)^2*a*b
^2+10*C*cos(d*x+c)*a^2*b-16*C*cos(d*x+c)*a*b^2-18*B*cos(d*x+c)^2*a^2*b+18*B
*cos(d*x+c)^2*a*b^2+18*B*cos(d*x+c)*a^2*b-12*B*cos(d*x+c)*a*b^2-6*B*cos(d*x
+c)^3*a*b^2+12*B*cos(d*x+c)^4*b^3+8*C*cos(d*x+c)^5*b^3-18*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*a*b^2-24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-10*C*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a
^2*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*a*b^2+15*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+16*C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2
-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a*b^2+16*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-48*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+24*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
a*b^2-24*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
```

```
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a*b^2-10*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-4*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+15*C*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*C*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2
4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*b^3-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+15*C*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-12*B*c
os(d*x+c)^2*b^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*b^3-30*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+15*C*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*C*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+8*C*cos(d*x
+c)^3*b^3+15*C*cos(d*x+c)^2*a^3-16*C*cos(d*x+c)^2*b^3-15*C*cos(d*x+c)*a^3+2
4*A*cos(d*x+c)^3*b^3-24*A*cos(d*x+c)^2*b^3-18*B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-24*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*b^3-18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b-18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
```

```
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*b^3-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(
1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*
sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)
*sec(d*x + c)^(3/2)), x)
```

3.1532 $\int \frac{\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=445

$$\frac{\sqrt{a+b}(2A+B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}(aB+$$

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A + B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b*d*Sqrt[Sec[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d
```

Rubi [A] time = 1.25442, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4221, 3029, 3003, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}(2A+B)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{a+b}(aB+$$

Antiderivative was successfully verified.

```
[In] Int[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x
]])/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A + B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
```

```
A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b*d*Sqrt[Sec[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3029

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^n*(b*B - a*C + b*C*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(Ab+aB)\,c}{\sqrt{\cos(c+}}$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(-abB+\right.}{\sqrt{\cos}}}{b^2}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \cdots$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \cdots$$

$$= -\frac{\sqrt{a+b}(2Ab+aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\cdots\right.}{\cdots}$$

$$= -\frac{(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\cdots\right.}{ad\sqrt{\cdots}}$$

Mathematica [A] time = 6.13656, size = 795, normalized size = 1.79

$$\frac{aB\tan^5\left(\frac{1}{2}(c+dx)\right)-bB\tan^5\left(\frac{1}{2}(c+dx)\right)+2bB\tan^3\left(\frac{1}{2}(c+dx)\right)+4Ab\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\sqrt{1-\tan\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-(a*B*Tan[(c + d*x)/2]) - b*B*Tan[(c + d*x)/2] + 2*b*B*Tan[(c + d*x)/2]^3
+ a*B*Tan[(c + d*x)/2]^5 - b*B*Tan[(c + d*x)/2]^5 + 4*A*b*EllipticPi[-1, -A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*B*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
```

```
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
(a + b)*B*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(A*b + a*(-A + B))*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)])/(d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [B] time = 0.254, size = 1369, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d
*x+c))^(1/2),x)
```

```
[Out] -1/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*c
os(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*(cos(d*x+c)/(1+cos
```

```
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c
)-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*sin(d*x+c)+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*
cos(d*x+c)*a)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*
x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}\sqrt{\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\cos(dx + c)^2 + Aa + (Ba + Ab)\cos(dx + c)\right)\sqrt{\sec(dx + c)}}{\sqrt{b\cos(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*
x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.1533 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=585

$$-\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^2(-(A-5C))-5abB+6Ab^2\right)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)}+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab^2-a(bB\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx}}$$

```
[Out] (-2*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*
A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^5*Sqr
t[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(48*A*b^3 + 4*a*b^2*(9*A - 10*B) + 6*a^
2*b*(2*A - 5*B + 5*C) + a^3*(9*A - 5*B + 15*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(15*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(
24*A*b^3 + 5*a^3*B - 20*a*b^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[a + b*Cos[c +
d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d) + (2*(A*b^2 -
 a*(b*B - a*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
b*Cos[c + d*x]]) - (2*(6*A*b^2 - 5*a*b*B - a^2*(A - 5*C))*Sqrt[a + b*Cos[c
+ d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.95549, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^2(-(A-5C))-5abB+6Ab^2\right)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)}+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab^2-a(bB\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a + b*Cos
[c + d*x])^(3/2),x]
```



```
[Out] (-2*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*
A + 5*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^5*Sqr
```

```
t[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(48*A*b^3 + 4*a*b^2*(9*A - 10*B) + 6*a^
2*b*(2*A - 5*B + 5*C) + a^3*(9*A - 5*B + 15*C))*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(15*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(
24*A*b^3 + 5*a^3*B - 20*a*b^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[a + b*Cos[c +
d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d) + (2*(A*b^2 -
 a*(b*B - a*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
b*Cos[c + d*x]]) - (2*(6*A*b^2 - 5*a*b*B - a^2*(A - 5*C))*Sqrt[a + b*Cos[c
+ d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+a}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(6Ab^2-5\right.}{}$$

$$= \frac{2\left(24Ab^3+5a^3B-20ab^2B-a^2(9Ab-15bC)\right)\sqrt{a+b\cos(c}}{15a^3\left(a^2-b^2\right)d}$$

$$= \frac{2\left(24Ab^3+5a^3B-20ab^2B-a^2(9Ab-15bC)\right)\sqrt{a+b\cos(c}}{15a^3\left(a^2-b^2\right)d}$$

$$= -\frac{2\left(48Ab^4+25a^3bB-40ab^3B-6a^2b^2(4A-5C)-3a^4(3A\right.}{}$$

Mathematica [B] time = 28.3326, size = 4900, normalized size = 8.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(7/2))/(a +
 b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^4*A + 24*a^2*A*b^2 -
48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 15*a^4*C - 30*a^2*b^2*C)*Sin[c + d*x])
/(15*a^4*(a^2 - b^2)) + (2*Sec[c + d*x]*(-9*A*b*Sin[c + d*x] + 5*a*B*Sin[c
+ d*x]))/(15*a^3) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2
*C*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x
]*Tan[c + d*x])/(5*a^2)))/d + (2*((-3*a*A)/(5*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^2)/(5*a*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^4)/(5*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) + (5*b*B)/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) - (8*b^3*B)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) - (a*C)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) + (2*b^2*C)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (4*A*b*Sqrt[Sec[c + d*x]])/(5*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]) - (12*A*b^3*Sqrt[Sec[c + d*x]])/(5*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c
 + d*x]]) + (16*A*b^5*Sqrt[Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*Sqrt[a + b*Cos
[c + d*x]]) + (a*B*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*
x]]) + (7*b^2*B*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x
]]) - (8*b^4*B*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*
x]]) - (2*b*C*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) +
(2*b^3*C*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (
3*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) - (8*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a^2*(a^2 - b^2)
*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])
/(5*a^4*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (5*b^2*B*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (8*b^4*B*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*
x]]) - (b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Co
s[c + d*x]]) + (2*b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^
2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a
+ b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(
3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2 + a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B)
- 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*S
```

qrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (48*A
*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1
5*a^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*((b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(-48*A*b^4
- 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
 + d*x)/2]^2 + a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A +
 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (48*A*b^4 + 25*a^3*b*
B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*Cos[c + d*x]*(a
 + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(15*a^4*(a^2 - b^2
)*(a + b*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-48*A*b^4 - 25*a^3*b*B
 + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
 + a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B + C))
+ a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (48*A*b^4 + 25*a^3*b*B - 40*a*b^3
*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c +
 d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(5*a^4*(a^2 - b^2)*Sqrt[a + b*
Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + ((-2*(a + b)*(-48*A*b^4 - 25*a^
3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2 + a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B +
 C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (48*A*b^4 + 25*a^3*b*B - 40*
a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
+ d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(
15*a^4*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]]*(((48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3
*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - ((a +
b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*
A + 5*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d
*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-48*A*b^4 - 25*a^3*b*B +

```
40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Se
c[(c + d*x)/2]^2*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a +
b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 2*(a + b)*(-48*A*b^4 - 25*a^3
*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2] - b*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(
4*A - 5*C) - 3*a^4*(3*A + 5*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x
]*Tan[(c + d*x)/2] - (48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A -
 5*C) - 3*a^4*(3*A + 5*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c +
d*x]*Tan[(c + d*x)/2] + 2*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(
4*A - 5*C) - 3*a^4*(3*A + 5*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^4*Tan[(c + d*x)/2]^2 + (3*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*
B) - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*
Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec
[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]))/2 + (a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5
*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((b*
Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)]) + (a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a
^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*Sec[(c + d*x)/2]^2*(Cos[c
 + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)]*Sec[c + d*x])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a
+ b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*
a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec
[(c + d*x)/2]^4*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - T
an[(c + d*x)/2]^2] + a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*
(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))/(1
5*a^4*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 0.451, size = 5893, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{7}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(7/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

3.1534 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=464

$$-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(-(A-3C))-3abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx}}$$

```
[Out] (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]
) + (2*(8*A*b^2 + 6*a*b*(A - B) + a^2*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])
+ (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt
[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.28252, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(-(A-3C))-3abB+4Ab^2\right)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos
[c + d*x])^(3/2),x]
```

```
[Out] (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]
) + (2*(8*A*b^2 + 6*a*b*(A - B) + a^2*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
```

```
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])
+ (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt
[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+a}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(4Ab^2-3\right.}{}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(4Ab^2-3\right.}{}$$

$$=\frac{2\left(8Ab^3+3a^3B-6ab^2B-a^2(5Ab-3bC)\right)\sqrt{\cos(c+dx)}\csc}{3a^4\sqrt{a+}}$$

Mathematica [B] time = 26.0806, size = 3736, normalized size = 8.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 b*Cos[c + d*x])^(3/2),x]
```

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B + 3*a^2*b*C)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) - (2*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^2)))/d + (2*((5*A*b)/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*B)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*B)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b*C)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (7*A*b^2*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (a*C*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b^2*C*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[

(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))

Maple [B] time = 0.292, size = 4201, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] -2/3/d/(a+b)/(a-b)/a^3*(a^2*A*b^2-A*a^4+3*C*cos(d*x+c)^3*a^2*b^2-5*A*cos(d*
x+c)^3*a^2*b^2-5*A*cos(d*x+c)^2*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2+8*A*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a*b^3+5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3+3*C*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*B*cos(d*x+c)*a^4-8
*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^4+A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^4+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*B*(cos(d*x+c)/(1+cos(
```

d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-3*C*cos(d*x+c)^3*a^3*b+3*B*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^2*a^3*b+6*B*cos(d*x+c)^2*a*b^3+3*B*cos(d*x+c)*a^2*b^2+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^4-5*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b-4*A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b+A*cos(d*x+c)^3*a^3*b+A*cos(d*x+c)^2*a^4+8*A*cos(d*x+c)^2*a*b^3-4*A*cos(d*x+c)*a*b^3+3*B*cos(d*x+c)^2*a^4+3*B*cos(d*x+c)^3*a^3*b-6*B*cos(d*x+c)^3*a*b^3-6*B*cos(d*x+c)^2*a^2*b^2+3*C*cos(d*x+c)^2*a^3*b-3*C*cos(d*x+c)^2*a^2*b^2+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(

```
a+b))^(1/2))*a^2*b^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*B*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*
B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^3-3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-6*B*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*B*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3*b+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)*cos(d*x
+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

3.1535 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=362

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx}}$$

```
[Out] (-2*(2*A*b^2 - a*b*B - a^2*(A - C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A -
B - C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a +
 b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.842377, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos
[c + d*x])^(3/2),x]
```


```
[Out] (-2*(2*A*b^2 - a*b*B - a^2*(A - C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A -
B - C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a +
 b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{\left(2\sqrt{\cos(c}\right.}{}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{\left(\left(2Ab^2-\right.\right.}{}$$

$$=-\frac{2\left(2Ab^2-abB-a^2(A-C)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\mathrm{s}\right.}{a^3\sqrt{a+b}d\sqrt{}}$$

Mathematica [A] time = 20.3682, size = 482, normalized size = 1.33

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sin(c+dx)\left(a^2A-a^2C+abB-2Ab^2\right)}{a^2\left(a^2-b^2\right)}+\frac{2\left(a^2C\sin(c+dx)-abB\sin(c+dx)+Ab^2\sin(c+dx)\right)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d}+\frac{2\sqrt{2}\sqrt{\frac{\cos(c+}{(\cos(c+d}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2*A - 2*A*b^2 + a*b*B -
 a^2*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)) + (2*(A*b^2*Sin[c + d*x] - a*b*B*Si
n[c + d*x] + a^2*C*Sin[c + d*x]))/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d
+ (2*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[
(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((a + b)*((-2*A*b
```

```
^2 + a*b*B + a^2*(A - C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + a*(2*A*b - a*(A + B - C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((a + b*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]) + (2*A*b^2 - a*b*B + a^2*(-
A + C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/
2]))/(a^2*(a^2 - b^2)*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*
x]]*(Sec[(c + d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.29, size = 3093, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2)
,x)
```

```
[Out] 2/d/a^2/(a+b)/(a-b)*(-a*A*b^2+A*a^3-A*cos(d*x+c)^2*a*b^2+2*A*cos(d*x+c)*a*b
^2+C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)*a^2*b+A*cos(d*x+c)*a^2*b+B*cos(d*x+c)^
2*a^2*b-B*cos(d*x+c)^2*a*b^2-B*cos(d*x+c)*a^2*b+B*cos(d*x+c)*a*b^2+A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)*b^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*a^2*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-A*cos(d*x+c)*a^3+2*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
```

)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-C*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*a^3-A*cos(d*x+c)^2*a^2*b+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-2*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
b^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3-C*cos(d*x+c)^2*a^3+C*cos(d*x+c)*a^3+2*A*cos(d*x+c)^2*b^3-B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+C*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d

```
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos
(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(3/2), x)
```

3.1536 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=496

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+\cdots}{a-\cdots}}\cdots}{a^2bd\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a^2*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b + b*B - a*C)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Se
c[c + d*x]]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*
C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rubi [A] time = 1.0693, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4221, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+\cdots}{a-\cdots}}\cdots}{a^2bd\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos
[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a^2*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b + b*B - a*C)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
```

```
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Se
c[c + d*x]]) - (2*Sqrt[a + b]*C*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*
C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(}{\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{Ab+(bB-aC)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{b} +$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{b^2d\sqrt{\sec(c+dx}}$$

$$= -\frac{2\sqrt{a+b}C\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{b^2d\sqrt{\sec(c+dx}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{a^2b\sqrt{a+b}d\sqrt{\sec(}}$$

Mathematica [B] time = 18.8523, size = 1141, normalized size = 2.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b^2 - a*b*B + a^2*C)*Si
n[c + d*x])/(a*b*(a^2 - b^2)) + (2*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x]
 + a^2*C*Sin[c + d*x]))/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (2*Sqrt
[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*b^2*Tan[(c + d*x)/2] + A*b^3*Tan[(c +
d*x)/2] - a^2*b*B*Tan[(c + d*x)/2] - a*b^2*B*Tan[(c + d*x)/2] + a^3*C*Tan[(
c + d*x)/2] + a^2*b*C*Tan[(c + d*x)/2] - 2*A*b^3*Tan[(c + d*x)/2]^3 + 2*a*b
^2*B*Tan[(c + d*x)/2]^3 - 2*a^2*b*C*Tan[(c + d*x)/2]^3 - a*A*b^2*Tan[(c + d
*x)/2]^5 + A*b^3*Tan[(c + d*x)/2]^5 + a^2*b*B*Tan[(c + d*x)/2]^5 - a*b^2*B*
Tan[(c + d*x)/2]^5 - a^3*C*Tan[(c + d*x)/2]^5 + a^2*b*C*Tan[(c + d*x)/2]^5
+ 2*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] - 2*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2
*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(A*b^2 + a*(-(b*B) + a*C))*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] - a*b*(a + b)*(A - B + C)*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)
/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])
)/(b*(a^3 - a*b^2)*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.293, size = 2859, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2)
,x)

[Out] -2/d/(a+b)/(a-b)/a/b*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-A*cos(d*
x+c)^2*a*b^2+A*cos(d*x+c)*a*b^2+C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)*a^2*b+B*c
os(d*x+c)^2*a^2*b-B*cos(d*x+c)^2*a*b^2-B*cos(d*x+c)*a^2*b+B*cos(d*x+c)*a*b^
2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*cos(d*x+c)*b^3-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*C*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*a*b^2+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-C*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*a^2*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*C*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+C*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b+C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-C*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
```

```
d*x+c)*a^3+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+2*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-
C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3-C*cos(d*x+c)^2*a^3+C*cos(d*x+c)*a^3+A*cos(d*x+c)^2*b^3+B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/sin(
d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a)^(3/2), x)
```

3.1537 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=595

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C-2abB+2Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+d}}$$

```
[Out] -(((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((2*A*b^2 -
 a*(b*(2*B - C) - 3*a*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*b*B - 3*a*C
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*
Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3
*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.68573, antiderivative size = 595, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4221, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C-2abB+2Ab^2-b^2C\right)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/2)*Sqr
t[Sec[c + d*x]]),x]
```


```
[Out] -(((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((2*A*b^2 -
```

```
 a*(b*(2*B - C) - 3*a*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*b*B - 3*a*C
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*
Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3
*a^2*C - b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
b^2*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^{3/2}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(2Ab^2-2abB+3a^2C-b^2\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(2Ab^2-2abB+3a^2C-b^2\right.}{}$$

$$= -\frac{\sqrt{a+b}(2bB-3aC)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\left(2Ab^2-2abB+3a^2C-b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{ab^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 21.0686, size = 1683, normalized size = 2.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/
2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b^2 - a*b*B + a^2*C)*Si
n[c + d*x])/(b^2*(-a^2 + b^2)) - (2*(a*A*b^2*Sin[c + d*x] - a^2*b*B*Sin[c +
 d*x] + a^3*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d -
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-2*a*A*b^2*Tan[(c + d*x)/2]
 - 2*A*b^3*Tan[(c + d*x)/2] + 2*a^2*b*B*Tan[(c + d*x)/2] + 2*a*b^2*B*Tan[(c
 + d*x)/2] - 3*a^3*C*Tan[(c + d*x)/2] - 3*a^2*b*C*Tan[(c + d*x)/2] + a*b^2*
```

```
C*Tan[(c + d*x)/2] + b^3*C*Tan[(c + d*x)/2] + 4*A*b^3*Tan[(c + d*x)/2]^3 -
4*a*b^2*B*Tan[(c + d*x)/2]^3 + 6*a^2*b*C*Tan[(c + d*x)/2]^3 - 2*b^3*C*Tan[(
c + d*x)/2]^3 + 2*a*A*b^2*Tan[(c + d*x)/2]^5 - 2*A*b^3*Tan[(c + d*x)/2]^5 -
 2*a^2*b*B*Tan[(c + d*x)/2]^5 + 2*a*b^2*B*Tan[(c + d*x)/2]^5 + 3*a^3*C*Tan[
(c + d*x)/2]^5 - 3*a^2*b*C*Tan[(c + d*x)/2]^5 - a*b^2*C*Tan[(c + d*x)/2]^5
+ b^3*C*Tan[(c + d*x)/2]^5 + 4*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*b^3*B*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*C*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 6*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + 4*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*b^3*B*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 6*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a*b^2*C*Elli
pticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] - (a + b)*(2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] + 2*b*(a + b)*(A*b - b*B + a*C)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)]))/(b^2*(-a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*
x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.246, size = 3698, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/d/(a+b)/(a-b)/b^2*(2*A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a*b^2+C*cos(d*x
+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)^2*a*b^2+2*C*cos(d*x+c)*a^2*
b+C*cos(d*x+c)*a*b^2-2*B*cos(d*x+c)^2*a^2*b+2*B*cos(d*x+c)^2*a*b^2+2*B*cos(
d*x+c)*a^2*b-2*B*cos(d*x+c)*a*b^2-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-2*B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*cos(
d*x+c)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2-2*C*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b-C*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^2
-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a*b^2-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+2*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+6*C*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*a*b^2-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b-2*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-C*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-
6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
```

(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3-2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-C*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-C*cos(d*x+c)^3*b^3+3*C*cos(d*x+c)^2*a^3+C*cos(d*x+c)^2*b^3-3*C*cos(d*x+c)*a^3-2*A*cos(d*x+c)^2*b^3-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2\right)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*sqrt(sec(d*x + c))), x)
```

3.1538 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=720

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(12a^2bB-15a^3C-ab^2(8A-7C)-4b^3B\right)\sqrt{a+}}{4b^3d\left(a^2-b^2\right)}$$

```
[Out] -((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a*b^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]
]) - ((8*A*b^2 - a*b*(12*B - 5*C) + 15*a^2*C - 2*b^2*(2*B + C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*Sqrt[a + b]*d*Sqrt[Sec[c
+ d*x]]) - (Sqrt[a + b]*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^4*d*Sqrt[Sec[
c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A*b^2 - 4*a*b*B + 5*a^2*C -
b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*Sqrt[Sec
[c + d*x]]) + ((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 2.34542, antiderivative size = 720, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4221, 3047, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(12a^2bB-15a^3C-ab^2(8A-7C)-4b^3B\right)\sqrt{a+}}{4b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/2)*Sec
[c + d*x]^(3/2)),x]
```

```
[Out] -((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a*b^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]
]) - ((8*A*b^2 - a*b*(12*B - 5*C) + 15*a^2*C - 2*b^2*(2*B + C))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*Sqrt[a + b]*d*Sqrt[Sec[c
+ d*x]]) - (Sqrt[a + b]*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^4*d*Sqrt[Sec[
c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A*b^2 - 4*a*b*B + 5*a^2*C -
b^2*C)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*Sqrt[Sec
[c + d*x]]) + ((12*a^2*b*B - 4*b^3*B - a*b^2*(8*A - 7*C) - 15*a^3*C)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c\right.}{(a+b\cos(c+dx))^{3/2}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4Ab^2-4abB+5a^2C-b^2\right.}{2b^2\left(a^2-b\right.}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4Ab^2-4abB+5a^2C-b^2\right.}{2b^2\left(a^2-b\right.}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(4Ab^2-4abB+5a^2C-b^2\right.}{2b^2\left(a^2-b\right.}$$

$$= -\frac{\sqrt{a+b}\left(8Ab^2-12abB+15a^2C+4b^2C\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\right.}{4b^4d\sqrt{\sec(c+dx}}$$

$$= -\frac{\left(12a^2bB-4b^3B-ab^2(8A-7C)-15a^3C\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin\right.}{4ab^3\sqrt{a+b}d\sqrt{\sec(c+}}$$

Mathematica [C] time = 25.7838, size = 3665, normalized size = 5.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/((a + b*Cos[c + d*x])^(3/
2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(A*b^2 - a*b*B + a^2*C)*
Sin[c + d*x])/(b^3*(a^2 - b^2)) + (2*(a^2*A*b^2*Sin[c + d*x] - a^3*b*B*Sin[
c + d*x] + a^4*C*Sin[c + d*x]))/(b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x])) + (
C*Sin[2*(c + d*x)])/(4*b^2)))/d + (Sqrt[a + b*Cos[c + d*x]]*((A*b)/((-a^2 +
 b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*B)/((-a^2 + b^2)*Sq
```

rt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*C)/(2*b*(-a^2 + b^2)*Sqrt
[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b*C)/(2*(-a^2 + b^2)*Sqrt[a + b
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*B*Sqrt[Sec[c + d*x]])/(2*b*(-a^2
+ b^2)*Sqrt[a + b*Cos[c + d*x]]) + (b*B*Sqrt[Sec[c + d*x]])/(2*(-a^2 + b^2)
*Sqrt[a + b*Cos[c + d*x]]) - (5*a*C*Sqrt[Sec[c + d*x]])/(8*(-a^2 + b^2)*Sqr
t[a + b*Cos[c + d*x]]) + (5*a^3*C*Sqrt[Sec[c + d*x]])/(8*b^2*(-a^2 + b^2)*S
qrt[a + b*Cos[c + d*x]]) + (a*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2
 + b^2)*Sqrt[a + b*Cos[c + d*x]]) - (3*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c +
d*x]])/(2*b*(-a^2 + b^2)*Sqrt[a + b*Cos[c + d*x]]) + (b*B*Cos[2*(c + d*x)]*
Sqrt[Sec[c + d*x]])/(2*(-a^2 + b^2)*Sqrt[a + b*Cos[c + d*x]]) - (7*a*C*Cos[
2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*(-a^2 + b^2)*Sqrt[a + b*Cos[c + d*x]])
 + (15*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*b^2*(-a^2 + b^2)*Sqrt[a
 + b*Cos[c + d*x]]))*(((-I)*Sqrt[(a - b)/(a + b)]*((-12*a^2*b*B + 4*b^3*B +
 a*b^2*(8*A - 7*C) + 15*a^3*C)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Ta
n[(c + d*x)/2]], -((a + b)/(a - b))] - 2*(-2*a^2*b*(6*B - 5*C) + 15*a^3*C +
 2*b^3*(2*A + C) + a*b^2*(8*A - 8*B + C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + 2*(a + b)*(8*A*b^2 - 12*a
*b*B + 15*a^2*C + 4*b^2*C)*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a -
b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))])*Sqrt[Cos[c + d*x]*Sec[(
c + d*x)/2]^2]*Sec[c + d*x])/Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] - (-12*a^2*b*B + 4*b^3*B + a*b^2*(8*A - 7*C) + 15*a^3*C)*Tan[(c +
 d*x)/2]))/(4*b^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]*(((-I)*S
qrt[(a - b)/(a + b)]*((-12*a^2*b*B + 4*b^3*B + a*b^2*(8*A - 7*C) + 15*a^3*C
)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
 - b))] - 2*(-2*a^2*b*(6*B - 5*C) + 15*a^3*C + 2*b^3*(2*A + C) + a*b^2*(8*A
 - 8*B + C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -
((a + b)/(a - b))] + 2*(a + b)*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sec[c + d*x])/
Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-12*a^2*b*B + 4*
b^3*B + a*b^2*(8*A - 7*C) + 15*a^3*C)*Tan[(c + d*x)/2]))/(8*b^2*(a^2 - b^2)
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*Sin[c + d*x]*(((-I)*Sqrt[(a - b)/(a + b)]*((-12*a^2*b*B +
 4*b^3*B + a*b^2*(8*A - 7*C) + 15*a^3*C)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - 2*(-2*a^2*b*(6*B - 5*C) +
15*a^3*C + 2*b^3*(2*A + C) + a*b^2*(8*A - 8*B + C))*EllipticF[I*ArcSinh[Sqr
t[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + 2*(a + b)*(8*A*
b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*EllipticPi[(a + b)/(a - b), I*ArcSinh[
Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))])*Sqrt[Cos[c +
d*x]*Sec[(c + d*x)/2]^2]*Sec[c + d*x])/Sqrt[((a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] - (-12*a^2*b*B + 4*b^3*B + a*b^2*(8*A - 7*C) + 15*a^3*C
)*Tan[(c + d*x)/2]))/(8*b^3*(a^2 - b^2)) + (Sqrt[a + b*Cos[c + d*x]]*(-((-1
2*a^2*b*B + 4*b^3*B + a*b^2*(8*A - 7*C) + 15*a^3*C)*Sec[(c + d*x)/2]^2)/2 -
 ((I/2)*Sqrt[(a - b)/(a + b)]*((-12*a^2*b*B + 4*b^3*B + a*b^2*(8*A - 7*C) +
 15*a^3*C)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((

```
a + b)/(a - b))] - 2*(-2*a^2*b*(6*B - 5*C) + 15*a^3*C + 2*b^3*(2*A + C) + a
*b^2*(8*A - 8*B + C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2]], -((a + b)/(a - b))] + 2*(a + b)*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*
b^2*C)*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]], -((a + b)/(a - b))])*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c +
d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(Sqrt[Cos[c + d*
x]*Sec[(c + d*x)/2]^2]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)]) + ((I/2)*Sqrt[(a - b)/(a + b)]*((-12*a^2*b*B + 4*b^3*B + a*b^2*(8*A -
7*C) + 15*a^3*C)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]
], -((a + b)/(a - b))] - 2*(-2*a^2*b*(6*B - 5*C) + 15*a^3*C + 2*b^3*(2*A +
C) + a*b^2*(8*A - 8*B + C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(
c + d*x)/2]], -((a + b)/(a - b))] + 2*(a + b)*(8*A*b^2 - 12*a*b*B + 15*a^2*
C + 4*b^2*C)*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Ta
n[(c + d*x)/2]], -((a + b)/(a - b))])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]
*Sec[c + d*x]*(-((b*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(((a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b))^(3/2) - (I*Sqrt[(a - b)/(a + b)]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*Sec[c + d*x]*(((-I)*Sqrt[(a - b)/(a + b)]*(-
2*a^2*b*(6*B - 5*C) + 15*a^3*C + 2*b^3*(2*A + C) + a*b^2*(8*A - 8*B + C))*S
ec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 + ((a - b)*Tan[(c +
 d*x)/2]^2)/(a + b)]) + ((I/2)*Sqrt[(a - b)/(a + b)]*(-12*a^2*b*B + 4*b^3*B
 + a*b^2*(8*A - 7*C) + 15*a^3*C)*Sec[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2])/Sqrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (I*Sqrt[(a - b)/(a
+ b)]*(a + b)*(8*A*b^2 - 12*a*b*B + 15*a^2*C + 4*b^2*C)*Sec[(c + d*x)/2]^2)
/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)])))/Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] - (I*Sqrt[(a - b)/(a + b)]*((-12*a^2*b*B + 4*b^3*B + a*b^2*(8*A
 - 7*C) + 15*a^3*C)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))] - 2*(-2*a^2*b*(6*B - 5*C) + 15*a^3*C + 2*b^3*(2*A
 + C) + a*b^2*(8*A - 8*B + C))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Ta
n[(c + d*x)/2]], -((a + b)/(a - b))] + 2*(a + b)*(8*A*b^2 - 12*a*b*B + 15*a
^2*C + 4*b^2*C)*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]], -((a + b)/(a - b))])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]
^2]*Sec[c + d*x]*Tan[c + d*x])/Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)]))/(4*b^3*(a^2 - b^2)*Sqrt[Sec[c + d*x]])))
```

Maple [B] time = 0.332, size = 5218, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)
```

```
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(3/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/
2)*sec(d*x + c)^(3/2)), x)
```

3.1539 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=660

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-a^2b^2(13A-C)+a^4(A-5C)+8a^3bB-4ab^3B+8Ab^4\right)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2\sin(c+dx}{}$$

```
[Out] (-2*(16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) +
 a^4*(8*A*b - 6*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a^5*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) - (2*(16*A*b^4 + 4*a*
b^3*(3*A - 2*B) - 3*a^3*b*(3*A - 3*B - C) - 2*a^2*b^2*(8*A + 3*B - C) - a^4
*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*
C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a^3*b*B + 3*a*b^3*B + 4*a^4*C)*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B + a^4*(A - 5*C) - a^2*b^2*(13*A
- C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2
 - b^2)^2*d)
```

Rubi [A] time = 2.73415, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-a^2b^2(13A-C)+a^4(A-5C)+8a^3bB-4ab^3B+8Ab^4\right)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a + b*Cos
[c + d*x])^(5/2),x]
```


```
[Out] (-2*(16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) +
 a^4*(8*A*b - 6*b*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
```

```
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a^5*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) - (2*(16*A*b^4 + 4*a*
b^3*(3*A - 2*B) - 3*a^3*b*(3*A - 3*B - C) - 2*a^2*b^2*(8*A + 3*B - C) - a^4
*(A - 3*B + 3*C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*
C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a^3*b*B + 3*a*b^3*B + 4*a^4*C)*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B + a^4*(A - 5*C) - a^2*b^2*(13*A
- C))*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2
 - b^2)^2*d)
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+a}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{\left(2\sqrt{\cos(c+}\right.}{}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2\left(10a^2Ab^2\right.}{}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2\left(10a^2Ab^2\right.}{}$$

$$=\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2\left(10a^2Ab^2\right.}{}$$

$$=-\frac{2\left(16Ab^5-3a^5B+15a^3b^2B-8ab^4B-2a^2b^3(14A-C)+a^{\cdot}\right.}{}$$

Mathematica [A] time = 22.097, size = 867, normalized size = 1.31

$$\frac{2\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-(a+b)\left(3Ba^5+(6bC-8Ab)a^4-15b^2Ba^3+2b^3(14A-C)a^2+8b^4Ba-16Ab^5\right)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/(a +
 b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*((-16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B
 + 8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^4*(-8*A*b + 6*b*C))*Tan[(c + d*x)/2
]*(-1 + Tan[(c + d*x)/2]^2)*(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2) - (a + b)*(-16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^
```

```
3*(14*A - C) + a^4*(-8*A*b + 6*b*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + a*(a + b)*
(-16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*a^3*b*(-3*
A - 3*B + C) + a^4*(A + 3*(B + C)))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a^4*(a^2
 - b^2)^2*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]*((2*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5
*B - 15*a^3*b^2*B + 8*a*b^4*B + 6*a^4*b*C - 2*a^2*b^3*C)*Sin[c + d*x])/(3*a
^4*(a^2 - b^2)^2) - (2*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C
*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(11*a^2*A*b
^3*Sin[c + d*x] - 7*A*b^5*Sin[c + d*x] - 8*a^3*b^2*B*Sin[c + d*x] + 4*a*b^4
*B*Sin[c + d*x] + 5*a^4*b*C*Sin[c + d*x] - a^2*b^3*C*Sin[c + d*x]))/(3*a^3*
(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 0.505, size = 10935, normalized size = 16.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*co
s(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos
(d*x + c) + a)^(5/2), x)
```

3.1540 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=535

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-2a^2b^2(4A+C)+5a^3bB-2a^4C-ab^3B+4Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB\right.}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

```
[Out] (2*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*S
qrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^
2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(8*A*b^3 + 2*a*b^2*(3*A - B) - 3*a^3*(A
 - B - C) - a^2*b*(9*A + 3*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*
b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^(3/2)) - (2*(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2
*a^2*b^2*(4*A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d
*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.64606, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4221, 3055, 2998, 2816, 2994}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-2a^2b^2(4A+C)+5a^3bB-2a^4C-ab^3B+4Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB\right.}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a + b*Cos
[c + d*x])^(5/2),x]
```


```
[Out] (2*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*S
qrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^
2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(8*A*b^3 + 2*a*b^2*(3*A - B) - 3*a^3*(A
```

```
 - B - C) - a^2*b*(9*A + 3*B + C))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*
b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^(3/2)) - (2*(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2
*a^2*b^2*(4*A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d
*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(c}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+a}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4Ab^4+\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4Ab^4+\right.}{}$$

$$= \frac{2\left(8Ab^4+6a^3bB-2ab^3B+3a^4(A-C)-a^2b^2(15A+C)\right)\sqrt{}}{3a^4(a}$$

Mathematica [A] time = 21.0117, size = 790, normalized size = 1.48

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sin(c+dx)\left(-15a^2Ab^2+3a^4A-a^2b^2C+6a^3bB-3a^4C-2ab^3B+8Ab^4\right)}{3a^3\left(a^2-b^2\right)^2}+\frac{2\left(a^2C\sin(c+dx)-abB\sin(c+dx)+Ab^2\sin(\right.}{3a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(3/2))/(a +
 b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A - 15*a^2*A*b^2 +
8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B - 3*a^4*C - a^2*b^2*C)*Sin[c + d*x])/(3*a^3
*(a^2 - b^2)^2) + (2*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c
 + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(8*a^2*A*b^2*Sin[c
+ d*x] - 4*A*b^4*Sin[c + d*x] - 5*a^3*b*B*Sin[c + d*x] + a*b^3*B*Sin[c + d*
x] + 2*a^4*C*Sin[c + d*x] + 2*a^2*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2
*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*((8*A*b
^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Tan[(c + d
*x)/2]*(-1 + Tan[(c + d*x)/2]^2)*(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2) - (a + b)*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^
2*b^2*(15*A + C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + a*(a + b)*(8*A*b^3 - 2*a*b^2*
(3*A + B) + 3*a^3*(A + B - C) - a^2*b*(9*A - 3*B + C))*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)]))/(3*a^3*(a^2 - b^2)^2*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.498, size = 8934, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3 + 3\,ab^2\cos(dx+c)^2 + 3\,a^2b\cos(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
ec(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*co
s(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos
(d*x + c) + a)^(5/2), x)
```

3.1541 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-2a^2b(3A+2C)+3a^3B+ab^2B+2Ab^3\right)}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

```
[Out] (-2*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[
c + d*x]]) - (2*(2*A*b^2 - a^2*(3*A + 3*B + C) + a*b*(3*A + B + 3*C))*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 -
b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*
(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(2*A*b^3
+ 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.37637, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4221, 3055, 2993, 2998, 2816, 2994}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-2a^2b(3A+2C)+3a^3B+ab^2B+2Ab^3\right)}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a + b*Cos
[c + d*x])^(5/2),x]
```


```
[Out] (-2*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[
c + d*x]]) - (2*(2*A*b^2 - a^2*(3*A + 3*B + C) + a*b*(3*A + B + 3*C))*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
```

```
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 -
b^2)*d*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*
(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(2*A*b^3
+ 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)+C\cos^2(}{\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(}\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(2Ab^3\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(2Ab^3\right.}{}$$

$$= -\frac{2\left(2Ab^3+3a^3B+ab^2B-2a^2b(3A+2C)\right)\sqrt{\cos(c+dx)}\,\mathrm{cs}}{3a^3(a-b)(}$$

Mathematica [B] time = 26.2244, size = 3853, normalized size = 7.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]])/(a +
 b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-6*a^2*A*b + 2*A*b^3 + 3
*a^3*B + a*b^2*B - 4*a^2*b*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) + (2*(A*b
^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*b*(-a^2 + b^
2)*(a + b*Cos[c + d*x])^2) + (2*(-5*a^2*A*b^2*Sin[c + d*x] + A*b^4*Sin[c +
d*x] + 2*a^3*b*B*Sin[c + d*x] + 2*a*b^3*B*Sin[c + d*x] + a^4*C*Sin[c + d*x]
 - 5*a^2*b^2*C*Sin[c + d*x]))/(3*a*b*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/
d + (2*((-2*a*A*b)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) + (2*A*b^3)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) + (a^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
+ (b^2*B)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (
4*a*b*C)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a
^2*A*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b^
2*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4
*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*
B*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*S
qrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (a^2*C*Sq
rt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (b^2*C*Sqrt[
Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*A*b^2*Cos[2*
(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (
2*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a +
b*Cos[c + d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)
^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (4*b^2*C*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*
(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) + a*b*(3*A - B + 3
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*d*
Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a
^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
```

d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) + a*b*(3*A - B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) + a*b*(3*A - B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) + a*b*(3*A - B + 3*C))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) + a*b*(3*A - B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) + a*b*(3*A - B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(

```
1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^
2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(2*A*b^3 + 3*a^3*B + a
*b^2*B - 2*a^2*b*(3*A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
 + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*B + C) +
 a*b*(3*A - B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*C
os[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Co
s[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c +
d*x]*Tan[c + d*x]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c
 + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.356, size = 7005, normalized size = 14.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*co
s(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos
(d*x + c) + a)^(5/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
    ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```